The Inverse Cournot Effect in Royalty Negotiations with Complementary Patents*

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Abstract

It has been argued that the licensing of complementary patents leads to excessively large royalties due to the well-known royalty-stacking problem. This paper shows that considering patent litigation and heterogeneity in portfolio size may mitigate or even eliminate this distortion due to a moderating force that we denote the Inverse Cournot effect. The lower the total royalty that a downstream producer pays, the lower the royalty that those patent holders exposed to the threat of litigation by downstream producers can charge. Interestingly, this effect is less relevant when all patent portfolios are weak, making royalty stacking more relevant. These forces are moderated when litigation is profitable for a weak patent holder. In that case, strong patent holders might prefer to discourage litigation through an increase in their royalty rate.

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1 Introduction

The fundamental nature of the patent system is under debate among claims on whether it fosters or hurts innovation. The main concerns focus on the impact of patent enforcement in the Information and Communications Technologies (ICT) industry. ICT products, such as laptops, tablets, or smartphones use a variety of technologies covered by complementary patents. The royalties that must be paid for multiple patented technologies in a single product added together are said to form a harmful "royalty stack" (Lemley and Shapiro, 2007). This in turn is claimed to result in excessively high end-product prices and a reduction in the incentives for firms to invest and innovate in product markets.

The arguments supporting royalty stacking and the need for a profound reform of the patent system rely on theoretical models which reformulate the well-known problem of Cournot-complements in a licensing framework. In industries where each single product is covered by multiple patents, a patent holder may not fully take into account that an increase in the royalty rate is likely to result in a cumulative rate that may be too high according to other licensors, the licensees, and their customers. Since this negative externality (or Cournot effect) is ignored by all patent holders, the royalty stack may prove inefficiently high.

In this paper we develop a model of licensing complementary innovations under the threat of litigation that explains the circumstances under which royalty stacking is likely to be a problem in practice. This model departs from the extant literature in only one natural dimension; we assume that manufacturers of products covered by multiple patented technologies may challenge in court those patents and, crucially, that the likelihood that a judge rules in favor of the patent holder is increasing in the strength of its patent portfolio. This assumption is reasonable. Downstream manufacturers commonly challenge the validity of the patents that cover their products when they litigate in court the licensing terms offered by patent holders. Those with large and high quality patent portfolios will not be constrained by the threat of litigation when setting their royalty rate. On the contrary, owners of weak portfolios will have to moderate their royalty claims in order to avoid litigation over patent validity.

More interestingly, our analysis shows that the ability of a patent owner to charge a high royalty rate without triggering litigation depends on the aggregate royalty rate charged by all other patent holders: the higher it is, the higher will be the royalty rate that any patent holder can charge. The intuition is that when the aggregate rate is higher the expected gains of a downstream licensee from invalidating the portfolio of a patent holder are less likely to compensate for the litigation costs incurred. This positive relationship is a novel and very general insight that we denote as the *Inverse Cournot effect* and it represents a positive externality among owners of complementary inputs (in this case patents). As a result, the Inverse Cournot effect makes royalty rate reductions more appealing. A strong patent holder by lowering the royalty rate forces rivals with weaker portfolios to reduce theirs, boosting total production. When the threat of litigation faced by these rivals is significant, the increase in the production is large and it compensates the reduced margin from the lower royalty rate. As a result, the Inverse Cournot effect becomes a moderating force, partially offsetting the royalty-stacking problem that arises from the Cournot effect.

This channel becomes less effective, however, among patent holders with weak patent portfolios. To illustrate that result, we consider the case in which a licensee decides to sue patent holders in an endogenous sequence. Because when the portfolio of a patentee is invalidated the aggregate royalty rate goes down, the incentives for the downstream producer to then sue other patentees become stronger. As a result of this *litigation cascade*, when a patent holder considers now whether to lower the royalty rate or not it ought to anticipate that, although it might benefit from a smaller royalty stack through an increase in sales, there is also a greater probability of itself being brought to court. Such a countervailing force implies that the Inverse Cournot effect is stronger when patent holdings are more skewed; the royalty-stacking problem might be milder when facing asymmetric but stronger patent holders compared to the case of weak but more similar ones.

The model is also extended in several dimensions. We discuss some features specific to Standard Essential Patents (SEPs), like the commitments to license patents according to Fair, Reasonable, and Non-Discriminatory (FRAND) terms. We also explore how the analysis can be extended to the case when firms license their patents using ad-valorem royalties or use two-part tariffs and we study the effect of downstream competition, royalty renegotiation, and sequential royalty-rate setting. In all cases, a modified version of the Inverse Cournot effect emerges.

Importantly, the analysis in most of the paper considers litigation as a threat but assumes away the possibility that patent holders might prefer to fight the validity of their patents in court. In a final section we explicitly account for this option. We show that in that case the Inverse Cournot effect still applies but a new force in the opposite direction emerges, particularly, when a weak patent holder might find it worthwhile to go to court. In that case, we show that a strong patent holder might prefer to raise the royalty rate, rather than decrease it. Doing so, allows the weak patent holder to raise its own royalty rate without spurring litigation. This strategy yields a higher revenue for the strong patent holder because the increase in the royalty rate, compared to the one that would emerge under litigation, compensates for the lower expected quantity that emerges as a result.

We start by presenting in section 2 a generic model where the Inverse Cournot and the Litigation cascade effect emerge depending on the strength of the portfolio of each firm. In section 3 we illustrate the results using a very stylized example which allows us to further characterize the equilibrium. In section 4 we extend and discuss the robustness of the results to changing some of the assumptions. Section 5 considers equilibrium litigation and section 6 concludes.

1.1 Literature Review

The existing literature has shown that the licensing of complementary and essential patents by many developers could give rise to a royalty-stacking problem (Lemley and Shapiro, 2007). This is not, however, a general result. Spulber (2016), for example, shows that when firms choose quantities but negotiate royalty rates the cooperative outcome will emerge.

The paper closest to ours is Choi and Gerlach (2015). They develop a model in which patent holders with *weak* patents facing the threat of litigation moderate their royalty claims so that the aggregate royalty payment falls below the one that would emerge from a patent pool. In their setup the positive relationship between the royalty rate of both firms arises from a mechanism that differs from the Inverse Cournot effect identified in our paper. If a downstream producer invalidates the patent portfolio of one of the firms, the rival can raise its own royalty rate. This means that the best response of a patent holder to the reduction in the royalty charged for the complementary portfolio of another patentee may be to reduce one's own royalty in order to make litigation relatively less attractive. This effect does not exist in our model, since the Inverse Cournot effect occurs even when one of the patentees has *ironclad* patents so that the decision to lower the royalty rate is not used to avoid going to court.

Bourreau et al. (2015) consider a setup similar to ours to study licensing and litigation in Standard Setting Organizations, as well as the decisions of firms to sell their IP to other innovators. The main difference with our paper, however, is that in their setup litigation occurs after production has taken place. As a result, the total quantity produced does not depend on the outcome of this litigation but only on the aggregate royalty rate negotiated ex-ante. This assumption severs the link between the licensing decision of different patent holders and the litigation decisions of licensees, thus eliminating the Inverse Cournot effect that plays a crucial role in our model.

2 The Model

Consider a market in which a downstream monopolist, firm D, faces a twice continuously differentiable demand function $D(p)$, decreasing in the price p. The production of the good requires the usage of technologies patented by two pure upstream firms. Upstream firm $i = 1, 2$ holds a portfolio of patents with strength x_i relevant for its own technology, with $x_1 \geq x_2$. Each patent holder charges a per-unit royalty r_i to license the necessary patents to make use of that technology.[1] We denote the total royalty rate as $R \equiv r_1 + r_2$. We assume that there is no further cost of production so that the marginal cost of the final product is also equal to R.

The royalty rate for technology i is set by patent holder i as a take-it-or-leave-it offer. The downstream producer, however, might challenge in court the patents that cover the technology. Litigation involves positive costs L_D and L_U for the downstream monopolist and any upstream patent holder, respectively. As we discuss later, the downstream producer can also choose to sue more than one patent holder, in an endogenous sequence. When indifferent we assume that the downstream producer prefers not to litigate.

The success in court is based on the strength of the portfolio of the patent holder. In particular, the probability that a judge rules in favor of patent holder i, denoted as $g(x_i)$, is assumed to be increasing in x_i. This assumption can be justified on several grounds. First, one of the most common ways for a downstream producer to dispute in court the licensing

[1] As pointed out in Llobet and Padilla (2016) royalty-stacking problems are aggravated under per-unit royalties compared to the more frequent ad-valorem royalties, based on firm revenue. As discussed in section 4, the mechanisms discussed in this paper also operate when royalties are assumed to be ad-valorem but they lead to a more complicated exposition.

terms offered is to challenge the validity of the patents that cover the technology. This strategy is less likely to succeed if the patent holder is stronger, in the sense of owning a larger portfolio and/or more valuable patents. Second, patent holders do not typically defend their technology with all their patent portfolio but, rather, they choose the patents that are most likely to be upheld in court or that are more relevant for the disputed application. It is more likely to find a suitable patent for litigation if choosing from a larger patent portfolio. Finally, the model is isomorphic to one in which each upstream patent holder i holds a unique patent of quality (or a number of patents of weighted quality) x_i. To the extent that more substantial innovations translate into stronger patents, we can interpret the increasing function $g(x_i)$ as a reflection of this relationship.[2]

In the main sections of the paper we will focus on the case in which L_U is relatively high so that litigation is a significant threat but it never emerges in equilibrium. That is, it is always optimal for patent holders to choose a royalty rate that does not spur litigation by the downstream producer. This case can be understood as a situation in which the technology developers individually benefit relatively less from licensing than downstream producers from avoiding to pay the royalty rate. Doing so allows us to highlight the mechanism that derives the main results of the paper and that stems from the litigation incentives of the downstream producer. In section 5 we extend the analysis by considering lower values of L_U and introducing the incentives for the upstream patent holders to go to court. In that case, we illustrate the conditions under which litigation arises in equilibrium and how this affects our results.

Most of the results of the paper do not require that we explicitly model the pricing decision of the downstream producer in the final market. It is enough to make the following assumptions on how the quantity sold depends on the aggregate royalty.

[2] For simplicity we abstract from situations in which upstream patent holders own the rights for technologies that might be infringed by other upstream patent holders.

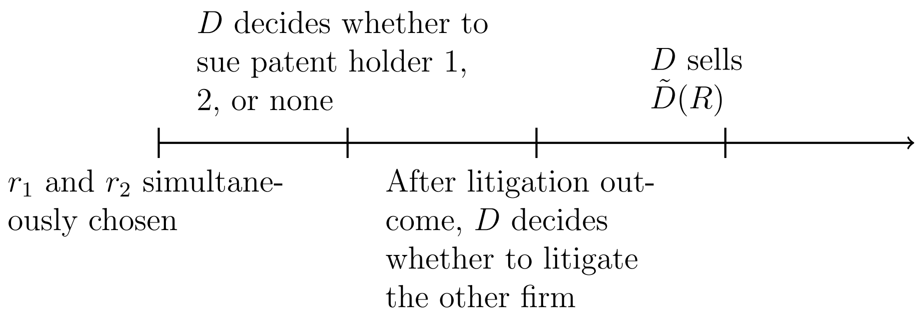

Figure 1: Timing of the model

Assumption 1. *The total quantity sold in the final market, $\tilde{D}(R)$, is a decreasing and log-concave function of R, with $\tilde{D}(0) > 0$ and $\tilde{D}(R) \to 0$ as $R \to \infty$.*

The profits of the downstream producer are denoted as $\Pi_D(R)$. Standard arguments allow us to show that $\Pi'_D(R) = -\tilde{D}(R) < 0$ and the previous assumption implies that $\Pi''_D(R) = -\tilde{D}(R) - R\tilde{D}(R) > 0$ so that the profits of the downstream producer are convex in R. It is also the standard regularity condition that guarantees that the profit function of the patent holders is well-behaved. When perfect price discrimination is possible and the downstream producer can extract all the surplus from consumers the previous assumptions imply that $D(p)$ is log-concave in p as typically assumed in in the literature. At the other extreme, when the downstream producer chooses a unique monopoly price for the product $p^M(R)$ it implies that $\tilde{D}(R) = D(p^M(R))$ is log-concave in R. Double marginalization will arise in this last case.

The timing of the model is described in Figure 1. First, upstream patent holders simultaneously choose their royalty rates. In the second stage the downstream producer chooses which patentees to take to court (if any) and the sequence. In the final stage, once litigation has been resolved, the downstream producer sells the quantity in the final market.

We now characterize the equilibrium of the game depending on the strength of the patent portfolio of each firm. We start with the case in which the parameters imply that litigation never plays a role in the model. This assumption will give rise to the standard *royalty-*

stacking result in the literature that we reproduce next.

2.1 Strong Patent Portfolios

Suppose that both patent holders have a sufficiently strong portfolio so that $g(x_1) = g(x_2) = 1$. In this case, litigation by the downstream producer will never be a credible threat.[3] The profits of patent holder i can be defined as

$$\Pi_i(r_j) = \max_{r_i} \; r_i \tilde{D}(r_i + r_j), \tag{1}$$

where $j \neq i$. We denote the royalty rate that corresponds to the Nash Equilibrium of the game when firms are unconstrained by litigation as $r_i^u = r^u$ for all i. For completeness, we reproduce next the standard royalty-stacking result (Lemley and Shapiro, 2007), which shows that this royalty rate would be higher than the one that would emerge if firms chose it cooperatively, r^M. It is important to notice that Assumption 1 not only guarantees concavity of the patent holder's problem but it also implies that royalty rates are strategic substitutes, delivering the result.

Proposition 1 (Royalty Stacking)**.** *When $g(x_1) = g(x_2) = 1$ the game has a unique equilibrium in which all patent holders choose $r_i^u = r^u$, independently of the size of their portfolio. This royalty is higher than the one that would emerge if firms maximized joint profits, $r^u > r^M$.*

We now discuss the effects of the litigation threat. We analyze two prototypical situations. First, we consider the case in which only one patentee is constrained by this threat. Later we study the situation in which both patentees are equally constrained.

[3] The same results would arise if, instead, we assumed that L_D is sufficiently high.

2.2 The Inverse Cournot Effect

Suppose now that $g(x_1) = 1$ but $g(x_2) < 1$ so that only patent holder 2 may face litigation by the downstream producer. Given the royalty rates chosen in the first stage, the downstream producer prefers not to challenge in court the portfolio of patentee 2 if and only if

$$(1 - g(x_2)) [\Pi_D(r_1) - \Pi_D(r_1 + r_2)] \leq L_D. \quad (2)$$

That is, litigation is unprofitable if the expected gains from avoiding to license the patent portfolio of patentee 2 are lower than the costs involved. The next lemma characterizes the values of r_1 for which litigation will emerge.

Lemma 2. *If $L_D > (1-g(x_2)) [\Pi_D(0) - \Pi_D(r_2)]$ patent holder 2 will not be brought to court for any $r_1 > 0$. For lower values of L_D, the downstream producer will litigate the portfolio of patent holder 2 if $r_1 < \bar{r}_1(L_D, x_2, r_2)$. This threshold royalty $\bar{r}_1$ is increasing in r_2 and decreasing in L_D and x_2.*

The previous lemma distinguishes two regions. When L_D is high litigation will never be a meaningful threat. For lower values of L_D, the decision of the downstream producer to sue patent holder 2 depends on the royalty rate set by the other patent holder. In particular, a positive royalty rate $r_1 < \bar{r}_1(L_D, x_2, r_2)$ chosen by patent holder 1 will spur the litigation of the portfolio of patent holder 2. The intuition of this effect is as follows. If r_1 is high, profits for the downstream producer are low, independently of whether the patent portfolio of firm 2 is upheld in court or not. Thus, it is unlikely that the gains from litigation offset the costs involved. As r_1 becomes lower, however, and due to the fact that the profit function $\Pi_D(R)$ is convex in R, the difference in profits when the portfolio of patent holder 2 is upheld in court or invalidated increases, enticing downstream litigation.

An immediate consequence of this result is that if L_D is sufficiently low royalty stacking would be mitigated and the equilibrium characterized in Proposition 1 would fail to exist.

More interesting, however, is the fact that the threat of litigation might operate even in the case in which the original equilibrium satisfies equation (2), i.e. when $r^u > \bar{r}_1(L_D, x_2, r^u)$.

In the model without litigation, royalty stacking arises because royalty rates are strategic substitutes. Because both patent holders choose their royalty rate without anticipating that the reduction of the quantity sold downstream negatively affects the other patent holder they generate a total royalty rate that becomes too high. The threat of litigation generates a moderating effect on the royalty rate that patent holder 2 will demand to avoid being brought to court. Furthermore, patent holder 1 anticipates that, in this case, reducing r_1 induces a decrease of r_2. We denote this mechanism the *Inverse Cournot effect* and it operates in the opposite direction of the standard Cournot Effect.[4] This new effect generates a positive relationship between r_1 and r_2 allowing patent holder 1 to internalize the gains that a lower royalty rate would bring about due to the higher quantity sold in the final market.

In any equilibrium with royalty rates r_1^* and r_2^* patentee 2 will avoid being sued if (2) holds. However, this condition also implies that there will never be a Nash Equilibrium in which the downstream producer is indifferent between litigating the portfolio of patentee 2 or not. The reason is that in that case patentee 1 would always have incentives to lower slightly the royalty rate, so that (2) does not hold and induce litigation against patentee 2. At essentially no cost, it becomes with probability $1 - g(x_2)$ the only firm licensing the technology. This deviation is profitable as it generates a discrete increase in the quantity sold downstream. If, instead, equation (2) held with inequality, patent holder 2 would find optimal to raise its royalty unless it were already equal to r^u. A consequence of this insight is that unless L_D is so high that the litigation threat is irrelevant and $r_1^* = r_2^* = r^u$, there will be no pure-strategy equilibrium without litigation.

[4] Of course, this effect immediately generalizes to the case of N patent holders with a portfolio sufficiently strong so that it will never be litigated. In that case, the Inverse Cournot effect would indicate that the highest royalty that patentee 2 can charge is increasing in the sum of the royalty of all the other patent holders.

Proposition 3. *An equilibrium in pure strategies and no litigation exists if and only if* $r_1^* = r_2^* = r^u$.

When L_D is small, given that demand is decreasing in the aggregate royalty rate, a Nash equilibrium in which litigation does not take place necessarily implies mixed strategies. Patent holders randomize in a support $[r_i^L, r_i^H]$ and according to a distribution $F_i(r_i)$ (with density $f_i(r_i)$) for $i = 1, 2$. Patentee 2 when choosing a lower r_2 trades off a lower probability of being sued with a higher payoff when litigation occurs but the firm succeeds in court. This trade-off means that patentee 2 will choose a lower expected royalty rate than when litigation was not a threat. In the case of patentee 1 two effects go in opposite directions. On the one hand, due to the Inverse Cournot effect the patent holder has incentives to lower the royalty rate r_1 in order to enjoy monopoly profits with a higher probability. On the other hand, there is a positive probability that the portfolio of the other patent holder is invalidated for a given r_1 and, hence, it becomes optimal to raise r_1. In section 5 we explore the case where litigation is a possible equilibrium outcome and we make further assumptions that allow us to discuss the different effects in more detail.

The positive effect of an increase in r_1 on the royalty rate of the weak patent holder (and vice versa) that we uncover here is new in the literature. In particular, in Choi and Gerlach (2015) downstream profits are linearly decreasing in the total royalty rate (rather than convex as in our model) and, hence, the mechanism described in Lemma 2 does not arise. In other words, in their model reducing r_1 by itself does not make litigation against patent holder 2 more profitable. Instead, in their setup when the patent portfolio of upstream patent holder 2 has been successfully upheld in court the firm can raise its royalty rate. The size of this ex-post increase is lower when the royalty rate charged by patent holder 1 is high. As a result, the cost for the downstream producer of losing in court against patent holder 2 decreases and it is more willing to litigate. This leads to a negative rather than a positive effect (i.e. litigation against patent holder 2 becomes more attractive when r_1 goes up rather than down, as in our model).[5]

2.3 Litigation Cascades and its Strategic Effects

We now turn to the case in which both patent holders have a similarly weak portfolio, $g(x_1) = g(x_2) = g(x) < 1$. As opposed to what happened in the previous case, litigation here might involve one or both upstream patent holders. We assume that in this case the downstream producer sues patent holders in an endogenous sequence that can potentially depend on the previous court outcome. Our first result characterizes the optimal order under which patent holders will be brought to court.

Lemma 4. *When both portfolios have the same strength, the downstream producer always prefers to sue first the patent holder that sets the highest royalty rate.*

The higher the royalty rate of a patent holder the more likely it is that litigation pays off irrespective of the outcome of the litigation against the other patent holder. In contrast, the profitability of litigating against the firm with the low royalty rate is lower and whether it is optimal or not to go to court might hinge on the outcome of the other trial. Thus, it is optimal to postpone litigation until the resolution of that other trial. In the rest of the paper we assume that when both patent holders set the same royalty rate the downstream producer brings to court each of them first with probability $1/2$.

The previous result is useful in order to anticipate the changes in the probability that patent holders are brought to court as a result of variations in the royalty rate. In particular, we now explore conditions under which a symmetric equilibrium $r_1^* = r_2^* = r^*$ exists. Because

[5] A positive effect arises when both patent holders are weak. In that case, when the downstream producer is indifferent between suing either of them but not both due to the litigation costs involved, the decrease in the royalty rate of patentee 1 increases the probability that the competitor is brought to court, reducing, in turn, its royalty rate. The downstream producer's upside is greater and its downside lower when suing the patent holder with the high royalty rate.

$\Pi_D(R)$ is a convex function of the total royalty rate, we have that $\Pi_D(r^*) - \Pi_D(2r^*) < \Pi_D(0) - \Pi_D(r^*)$. This implies that if it is profitable to litigate the portfolio of one of the patent holders it will also be profitable to litigate the other one upon an initial success in court. It also means that in a symmetric equilibrium, r^*, litigation against both patent holders will not be profitable if

$$(1 - g(x)) [\Pi_D(r^*) - \Pi_D(2r^*)] + (1 - g(x)) \{(1 - g(x)) [\Pi_D(0) - \Pi_D(r^*)] - L_D\} \leq L_D. \quad (3)$$

The first term is identical to the one that governs the decision of the downstream producer to go to court in the case of one constrained patent holder, as described in equation (2). The second term captures the *option value* that litigation may now bring about. That is, if the downstream producer wins the first trial the profitability of going to court against the other patent holder increases. We call this result a *litigation cascade*.[6]

In order to interpret this constraint it is useful to consider the situation in which the condition is satisfied with equality and the downstream producer is indifferent between engaging in litigation or not. Suppose that the order of litigation is such that patent holder 2 is brought to court first. In this scenario, equation (3) with equality implies that suing patentee 2 only must result in an increase in expected market revenues lower than the cost L_D. Since litigation against patentee 2 only is unprofitable and the problem the downstream producer faces with respect to patentee 1 is the same when it has not succeeded in court before, it will only litigate a second time upon an initial success. In other words, indifference between going to court or not implies that the profits from this second trial, which occurs with probability $1 - g(x)$, must compensate the losses from the first one. When indifferent between going to court or not, the downstream producer is only motivated to litigate due to

[6]In practice, litigation might take years and a second trial might start before the first one has concluded if the information uncovered by the downstream producer during the process indicates that the revised probability of success is sufficiently high. The implications of such a strategy are very similar to the fully sequential setup assumed here.

the prospect of invalidating the portfolio of both patent holders.

From the previous arguments it is immediate that equation (3) is less likely to be satisfied than the one driving the decision to sue patent holder 2 when only this firm is constrained, as illustrated in equation (2). This comparison would suggest that before we introduce strategic considerations in the patent holders' decisions – that is, before we account for the optimal response of the patent holders to the increased litigation risk associated with that option –, the royalty-stacking problem would become less relevant when they both have a weak portfolio. As we will see next, the opposite may hold once we introduce these strategic considerations.

We now characterize the incentives to go to court when patent holder 1 deviates from the symmetric candidate equilibrium. Given r_1 and r_2 and the endogenous ordering implied by Lemma 4, we can define the gains for the downstream producer arising from suing a second patent holder contingent on success in the first trial as

$$\Phi(r_1, r_2) \equiv \begin{cases} \Pi_D(r_2) - \Pi_D(r_1 + r_2) + (1 - g(x))\left[\Pi_D(0) - \Pi_D(r_2)\right] & \text{if } r_1 > r_2, \\ \Pi_D(r) - \Pi_D(2r) + (1 - g(x))\left[\Pi_D(0) - \Pi_D(r)\right] & \text{if } r_1 = r_2 = r, \\ \Pi_D(r_1) - \Pi_D(r_1 + r_2) + (1 - g(x))\left[\Pi_D(0) - \Pi_D(r_1)\right] & \text{otherwise.} \end{cases}$$

These gross profits change with r_1 according to

$$\frac{\partial \Phi}{\partial r_1} = \begin{cases} -\Pi_D'(r_1 + r_2) & \text{if } r_1 \geq r_2, \\ \Pi_D'(r_1) - \Pi_D'(r_1 + r_2) - (1 - g(x))\Pi_D'(r_1) & \text{otherwise.} \end{cases}$$

This expression implies that increases and decreases of r_1 around r_2 have an asymmetric effect on the willingness of the downstream producer to litigate. Consider an initial situation in which $r_1 = r_2$. As expected, an increase in r_1 raises the profitability of challenging the portfolio of patentee 1 as the downstream profits without litigation are smaller. Decreases in r_1 below r_2, however, lead to two opposing effects. On the one hand, the first two terms correspond to the Inverse Cournot effect and imply that patent holder 2 is more likely to be brought to court and, in turn, trigger a litigation cascade. On the other hand, contingent on the portfolio of patent holder 2 being invalidated, a lower r_1 reduces the expected gains from trying to invalidate the portfolio of patent holder 1 by $(1-g(x))\Pi_D'(r_1)$. Hence, the total effect of a decrease in r_1 in the chances that patentee 1 ends up in court is in general ambiguous. The following example illustrates this point.

Example 1. *Under a linear demand function, $D(p) = 1-p$, a downstream monopoly price, and symmetric royalty rates $r_1 = r_2 = r$, a decrease in the royalty rate lowers the return from litigation of the downstream producer if and only if $r > \frac{1-g(x)}{2-g(x)}$.*

Notice that in the previous example, the unconstrained equilibrium royalty rate is $r_1^u = r_2^u = \frac{1}{3}$. Thus, the litigation cascade will dominate the Inverse Cournot effect making a deviation from this equilibrium unprofitable if $g(x)$ is sufficiently small.

As opposed to the case of one weak patent holder, the risk of a litigation cascade places a lower bound on the decrease in the royalty rate that firms find optimal. As the next proposition shows, this limit may help sustain a symmetric Nash Equilibrium in pure strategies without litigation.

Proposition 5. *Suppose that L_U is large so that there is no litigation in equilibrium. If patent holders are identical and the demand function is linear, in a symmetric equilibrium in pure strategies, $r_1^* = r_2^* = r^*$, either $r^* = r^u$ or $r^* < r^u$, defined as*

$$g(x)\Pi_D(r^*) + (1-g(x))\Pi_D(0) - \Pi_D(2r^*) = \frac{L_D}{1-g(x)} + L_D.$$

This last equilibrium arises when $g(x)$ and L_D are sufficiently small. The equilibrium royalty rate is increasing in $g(x)$ and L_D.

The previous result provides conditions under which an equilibrium without litigation that differs from the one in the case of strong patents might emerge. In order to interpret this outcome, consider a deviation in the royalty rate. First, increases in the royalty rate might compensate entail high litigation cost L_U and the probability that the patent portfolio is invalidated. When $g(x)$ is small, and the probability that the portfolio is invalidated is large, the costs are likely to outweigh the benefits. Second, lowering the royalty rate below r^* implies that the patent portfolio of the other firm is litigated first. However, given that $g(x)$ is small, a litigation cascade might affect the deviating patent holder, making the decision less profitable. Finally, a significant decrease in the royalty rate is necessary to discourage further litigation if the downstream producer is successful against patent holder 2. The lower is L_D the lower this royalty rate must be and, again, the less profitable this deviation becomes.

In the next section we provide a simpler version of the model that allow us to illustrate the interplay between the Inverse Cournot effect and litigation cascades.

3 A Parametric Example

Let's consider the case where the downstream demand corresponds to a unique consumer with a valuation for one unit of the good. With probability $\alpha \in (0,1)$ this valuation is 1. With probability $1-\alpha$ the valuation is $v < 1$. Furthermore, we assume that the downstream firm chooses the price after the valuation has been realized. This timing implies that the downstream producer will always choose a price equal to the realized valuation of the consumer. That is, given R the downstream producer captures all the surplus without generating the losses associated to double marginalization. As a result, expected downstream profits $\Pi_D(R)$ can be computed as

$$\Pi_D(R) = \begin{cases} \alpha + (1-\alpha)v - R & \text{if } R \leq v, \\ \alpha(1-R) & \text{if } R \in (v,1], \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

These profits are decreasing and weakly convex in R.[7] Notice that the demand is weakly log-concave in the price as expected from Assumption 1. However, the fact that profits are not linear everywhere is enough for our results to go through.

We start by characterizing the royalty rate that maximizes joint profits for the upstream patent holders when their portfolio is sufficiently strong so that $g(x_1) = g(x_2) = 1$. This royalty rate will be used as a benchmark for the case in which patent holders decide independently.

Proposition 6. *Under the two-point demand function, when $g(x_1) = g(x_2) = 1$ there is a continuum of undominated pure-strategy equilibria. The corresponding royalty rates (r_1^u, r_2^u) can be characterized as follows:*

1. *If $v \geq \frac{2\alpha}{1+\alpha}$, $R^u = r_1^u + r_2^u = v$ with $r_i^u \leq \frac{v-\alpha}{1-\alpha}$ for $i = 1, 2$,*
2. *If $v \leq \frac{1+\alpha}{2}$, $R^u = r_1^u + r_2^u = 1$ with $r_i^u \geq \frac{v-\alpha}{1-\alpha}$ for $i = 1, 2$.*

Both kinds of equilibria co-exist when $\frac{2\alpha}{1+\alpha} \leq v \leq \frac{1+\alpha}{2}$. All equilibria imply royalty stacking when $\alpha \leq v < \frac{2\alpha}{1+\alpha}$.

Intuitively, the equilibrium with a total royalty of 1 is likely to exist when v is small and α is sufficiently close to 1. A deviation might exist if any patent holder prefers to decrease the royalty rate in order to cater the consumer regardless of her valuation. This deviation is illustrated in Figure 2. Given r_2^u, patent holder 1 can choose $r_1^u = 1 - r_2^u$ or deviate and choose $\hat{r}_1 = v - r_2^u$ so that the probability of selling increases from α to 1. Such a deviation is unprofitable if r_2^u is sufficiently large and, thus, the low $\hat{r}_1$ does not allow the firm to benefit

[7] A dead-weight loss would arise if we assumed that the downstream producer chose the price before the demand is realized. In that case, the threshold value on R in the profit function $\Pi_D(R)$ would change. That is, $p^M(R) = v$ if and only if $R \leq \tilde{R} \equiv \frac{v-\alpha}{1-\alpha} < v$. Since, as explained in previous sections, double marginalization does not interact with the mechanisms explored in this paper, the main results would go through under this alternative assumption although at the cost of an increasing technical complexity.

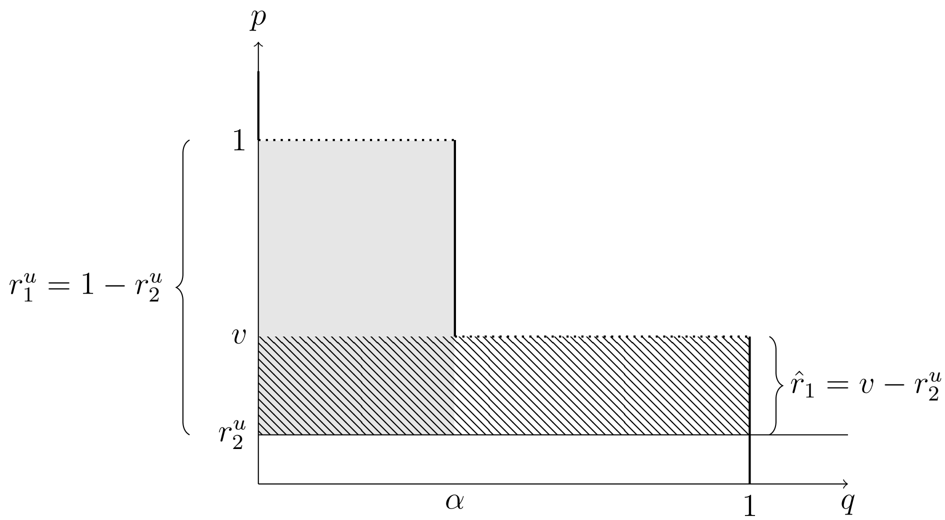

Figure 2: Equilibrium with $r_1^u + r_2^u = 1$. Profits for patent holder 1 correspond to the gray area. The striped area indicates the profits under the optimal deviation.

from the increase in sales. In the limit, when $v = 0$ or $\alpha = 1$ this equilibrium holds for any combination of royalties that sums up to 1.

Similarly, equilibria with a total royalty equal to $R^u = v$ are likely to exist when v is sufficiently high and α is sufficiently small. This time a deviation aims to capture the additional surplus when consumer valuation is 1, even if this surplus is materialized only with probability α. To prevent these deviations each patent holder must charge a modest royalty so that the other firm already obtains sufficiently high profits in equilibrium, thus reducing the appeal of raising the royalty rate and reducing the probability of sale. In the limit, when $v = 1$ or $\alpha = 0$ any combination of royalty rates that sums up to v would constitute an equilibrium. Such coordination would also maximize social welfare.

In contrast, a total royalty $R = v$ maximizes joint profits if and only if $v > \alpha$. Royalty stacking, which here takes the form of a total royalty rate equal to 1 when joint profit maximization requires $R^M = v$, arises as a Nash equilibrium when $\alpha \leq v < \frac{1+\alpha}{2}$ and it is unique when $\alpha \leq v < \frac{2\alpha}{1+\alpha}$.

3.1 One Constrained Patent Holder

Suppose now that $g(x_1) = 1$ and $g(x_2) < 1$ so that the downstream producer may only be interested in litigating the portfolio of patent holder 2. We restrict our discussion to the case where $v > \alpha$ so that, according to Proposition 6, in the previous benchmark a combination of royalties for which $r_1^* + r_2^* = 1$ constituted an equilibrium with royalty stacking. Once the threat of litigation is accounted for, such an equilibrium may fail to exist for two reasons. First, given $r_1^* + r_2^* = 1$, the downstream firm will obtain higher profits by going to court if

$$r_2^* > \bar{r}_2 = \begin{cases} \frac{L_D}{\alpha(1-g(x_2))} & \text{if } \frac{L_D}{1-g(x_2)} < \alpha(1-v), \\ (1-\alpha)(1-v) + \frac{L_D}{1-g(x_2)} & \text{otherwise.} \end{cases} \tag{5}$$

Second, if $r_2^* \leq \bar{r}_2$, patent holder 1 might benefit from deviating to a royalty below r_1^* which induces litigation. Using a similar logic as in Lemma 2, we can show that when the litigation cost L_D is sufficiently low there is a threshold $\bar{r}_1 > 0$ such that litigation will occur if $r_1 < \bar{r}_1$. The next lemma characterizes the region under which litigation may occur as a function of r_1.

Lemma 7. *Under the two-point demand function, if $r_2 > \bar{r}_2$ there is no equilibrium with royalty stacking and no litigation. If*

$$r_2 \leq \underline{r}_2 = \begin{cases} \frac{L_D}{1-g(x_2)} & \textit{if } \frac{L_D}{1-g(x_2)} \leq v, \\ \frac{L_D}{\alpha(1-g(x_2))} - \frac{1-\alpha}{\alpha} v & \textit{otherwise,} \end{cases} \tag{6}$$

patent holder 2 will not be brought to court for any $r_1 \geq 0$. If $r_2 \in (\underline{r}_2, \bar{r}_2]$, litigation will occur if

$$r_1 < \bar{r}_1(r_2) = v + \frac{\alpha}{1-\alpha} r_2 - \frac{L_D}{(1-\alpha)(1-g(x_2))} \leq v. \tag{7}$$

Patent holder 1 might benefit from lowering the royalty rate below $\bar{r}_1$ if, by causing litigation against patentee 2, the quantity sold expands from α to 1, which would occur with probability $1 - g(x_2)$. Hence, a profitable deviation $\hat{r}_1$ must be lower than v. Since $\bar{r}_1(\bar{r}_2) \leq v$ it follows that the optimal deviation for patent holder 1 when patentee 2 sets

$r_2^* \leq \bar{r}_2$ is the highest royalty rate which guarantees that the portfolio of patentee 2 is litigated, $\hat{r}_1 = \bar{r}_1(r_2^*)$.[8] Patent holder 1's profits in that case would become

$$\hat{\Pi}_1 = [\alpha + (1-\alpha)(1-g(x_2))]\,\hat{r}_1. \tag{8}$$

That is, a deviation will lead to profits equal to $\hat{r}_1$ either because the valuation of the consumer is 1 or because the valuation is v but the portfolio of patent holder 2 has been successfully litigated by the downstream producer. This deviation will take place if profits, $\hat{\Pi}_1$, are higher than those in the candidate equilibrium, $\Pi_1^* = \alpha r_1^*$. Notice that the lower are r_1^* or $g(x_2)$ the more binding this condition becomes. The next proposition characterizes the circumstances under which $\Pi_1^* \geq \hat{\Pi}_1$ cannot hold while, as required by Proposition 6, $r_2^* \geq \frac{v-\alpha}{1-\alpha}$. In those situations, an equilibrium with royalty stacking and no litigation will fail to exist.

Proposition 8. *Consider the two-point demand function case and suppose $v > \alpha$. If $\frac{L_D}{1-g(x_2)} < \frac{v-\alpha}{1-\alpha}$ and $g(x_2)$ is sufficiently small, there is no equilibrium with royalty stacking and no litigation. If L_U is sufficiently large, only the efficient equilibrium exists, which involves $r_2^* \leq \frac{L_D}{1-g(x_2)} < v$ and $r_1^* = v - r_2^*$.*

This result indicates that when L_D and/or $g(x_2)$ are sufficiently low, royalty stacking will not arise in an equilibrium without litigation. In order to interpret this result it is useful to start by considering the case under which such an equilibrium with royalty stacking may exist. From (6) we know that if $r_2^* \leq \frac{L_D}{1-g(x_2)}$ the Inverse Cournot effect has no bite since there is no positive value of $\hat{r}_1$ that triggers litigation. When $\frac{L_D}{1-g(x_2)} \geq \frac{v-\alpha}{1-\alpha}$ it is also possible to find $r_2^* \geq \frac{v-\alpha}{1-\alpha}$, satisfying the conditions of Proposition 6. Hence, it is optimal for patent holder 1 to choose $r_1^* = 1 - r_2^*$ and an equilibrium with royalty stacking will arise in that case.

[8] More precisely, given our assumptions, $\hat{r}_1$ should be slightly lower than $\bar{r}_1(r_2^*)$.

When $\frac{L_D}{1-g(x_2)} < \frac{v-\alpha}{1-\alpha}$, however, a deviation from the royalty-stacking equilibrium may exist. Starting from a combination of royalties (r_1^*, r_2^*) with $r_i^* \geq \frac{v-\alpha}{1-\alpha}$ for $i = 1, 2$, patent holder 1 trades off a decrease in the royalty to $\overline{r}_1(r_2^*)$ with an increase in the probability of sale from α to $\alpha + (1-\alpha)(1-g(x_2))$. The previous proposition shows that if $g(x_2)$ is sufficiently small this expansion effect dominates and the deviation is profitable. The reason for this result is, precisely, that when $v > \alpha$ eliminating royalty stacking raises total profits and the smaller is $g(x_2)$ the larger is the proportion of that increase that patentee 1 can appropriate.

The second part of the proposition also indicates that when the probability of success in court of patent holder 2 is small two results concur. First, the royalty rate is commensurate with the strength of the patent portfolio and the cost of challenging those rights by the downstream producer, $r_2^* \leq \frac{L_D}{1-g(x_2)}$. This result arises from the fact that when $g(x_2)$ is small patent holder 2 must choose a low royalty rate to discourage the downstream producer from engaging in litigation that will, most likely, result in a zero royalty. Second, and more interestingly, the joint profit maximizing equilibrium, consisting of $R^M = v$, may exist. The reason is that the low value of r_2 makes patent holder 1 the residual claimant of the surplus generated. This can be seen using Figure 2, where the lower is r_2 the more patent holder 1 internalizes the losses that a deviation towards a larger royalty rate may entail.

3.2 Two Constrained Patent Holders

Suppose now that both firms have identical patent holdings which do not confer full protection against litigation, $g(x_1) = g(x_2) = g(x) < 1$. As in the previous case we focus on the situation in which royalty stacking was an equilibrium when no litigation was feasible, $v > \alpha$. We study whether litigation affects the existence of an equilibrium with royalty stacking, so that $r_1^* + r_2^* = 1$. As in the general case, it is enough to focus on the symmetric case in which $r_1^* = r_2^* = 1/2$ as if this equilibrium did not exist no asymmetric equilibrium would exist either.[9] We also explained that in the symmetric case it will never be optimal for the downstream producer to bring to court only one of the patent holders. The next lemma characterizes the threshold values of $\hat{r}_1$ for which patentee 1 expects to be sued in case patentee 2 loses in court.

Lemma 9. *Under the two-point demand function with $v > \alpha$, suppose that for $r_1^* = r_2^* = 1/2$ it is not profitable for the downstream producer to engage in litigation. If by deviating to $\hat{r}_1 < r_1^*$ patent holder 2 is sued, patent holder 1 will also be sued if and only if patent holder 2 loses in court and $\hat{r}_1 > \frac{L_D}{1-g(x)}$.*

The deviations that this lemma characterizes determine two regions depending on whether $\hat{r}_1$ is higher or lower than $\frac{L_D}{1-g(x)}$. Both deviations are less profitable than in the case in which $g(x_1) = 1$, albeit for different reasons. In one of the regions, by choosing a low $\hat{r}_1$, patentee 1 eludes litigation but at the cost of a significant reduction in licensing revenues. In the second region, when $\hat{r}_1$ is higher, the lower profitability of the deviation arises from the probability that the patent holder might not receive any licensing revenues from its portfolio if the court declares it invalid, together with the corresponding litigation costs. In particular, in this last region, the profits from a deviation are

$$\hat{\Pi}_1 = g(x)\alpha\hat{r}_1 + (1 - g(x))\left[g(x)\hat{r}_1 - L_U\right].$$

That is, when the portfolio of the other patent holder is upheld in court the expected quantity is α. If, instead, the portfolio of patentee 2 is invalidated and the downstream producer also decides to sue patent holder 1, the quantity sold is 1 but the royalty $\hat{r}_1$ is only paid if the portfolio is upheld in the second trial.

[9] As discussed in previous sections, an equilibrium may fail to exist because one of the royalty rates is too low and, as a result, either the patent holder decides to deviate and raise it even at the cost of being sued or the other patent holder may benefit from lowering its own royalty rate and serve the whole market. By focusing on the symmetric royalty rate we are minimizing the profitability of these deviations.

We now illustrate how the risk of a litigation cascade might foster the existence of an equilibrium with royalty stacking and no litigation. Take the case in which L_U is very large so that the threat of litigation is particularly relevant for the upstream patent holders, and consider the situation in which $v \leq \frac{1}{2}$. Given $r_1^* = r_2^* = 1/2$, two conditions must be satisfied for such an equilibrium to exist. First, using equation (3), the downstream producer should not be interested in going to court, which in this case it implies

$$\frac{L_D}{1-g(x_2)} \geq \frac{1}{2-g(x)} \left[g(x)\frac{\alpha}{2} + (1-g(x)(\alpha+(1-\alpha)v) \right]. \tag{9}$$

Second, the cost of a litigation cascade implies that the optimal deviation of patent holder i, for $i = 1, 2$, involves $\hat{r}_i = \min\left\{v, \frac{L_D}{1-g(x)}\right\}$ and such a deviation is unprofitable if and only if $\hat{\Pi}_1 \leq \Pi^*$ or

$$\left[\alpha + (1-\alpha)(1-g(x))\right]\hat{r}_i \leq \frac{\alpha}{2}. \tag{10}$$

Notice that because, as in the case of one constrained patent holder, $\hat{r}_i \leq v$ the expected demand expands if the portfolio of the other patent holder is invalidated.

These two conditions provide a lower and upper bound, respectively, on $\frac{L_D}{1-g(x)}$ for an equilibrium with royalty stacking and no litigation to exist. That is, the litigation costs of the downstream producer must be sufficiently large to discourage this firm from litigating but they must also be sufficiently small so that the decrease in the royalty rate necessary for a deviating firm to fend off litigation is large.

Although the previous conditions are highly non-linear in the main parameters of the model it is easy to find combinations that satisfy them. More interestingly, we can also find situations in which this equilibrium with a total royalty equal $R^* = 1$ is sustainable when both patent holders have a very strong or a very weak portfolio but not in the case in which firms' patent portfolios are asymmetric.

Example 2. *Consider the parameter values $\alpha = 0.1$, $v = 0.3$, $g(x_2) = 0.68$, $L_D = 0.035$,* *and L_U sufficiently large. If litigation were not possible, the parameter values would satisfy the conditions of Proposition 6 and an equilibrium with royalty stacking, $R^u = 1$, would exist.*

Next, consider the case in which $g(x_1) = 1$ so that only the second patent holder is potentially constrained. By construction, $\frac{L_D}{1-g(x_2)} < \frac{v-\alpha}{1-\alpha}$, and it can be verified that patent holder 1 has incentives to deviate from any candidate equilibrium (r_1^*, r_2^*) and choose $\bar{r}_1(r_2^*)$, so that the royalty-stacking will not emerge in this case.

Finally, consider the case in which $g(x_1) = g(x_2) = 0.68$. Equations (9) and (10) are satisfied and, thus, the royalty-stacking equilibrium exists when both patent holders are similarly constrained.

The previous example illustrates that, as suggested in the previous section, once we introduce litigation in the model the royalty rate is not necessarily monotonic in the strength of the patent portfolios. When portfolios are weaker but more evenly distributed the royalty-stacking problem might actually become more relevant.

4 Robustness and Extensions

We now study the effect of changing some of the maintained assumptions throughout the paper. Some of the results are based on the parametric model discussed in the previous section.

4.1 Ad-Valorem Royalties

Although most of the literature in innovation has assumed that royalties are paid per unit sold in the downstream market, in many technological industries patents are typically licensed using ad-valorem royalties, understood as a percentage of the revenue of the licensee.[10] As

[10]See, for example, Bousquet et al. (1998). Interestingly, pure lump-sum payments are not common. Of course, if firms relied only on them the royalty-stacking problem would not be a relevant concern as only the distribution of the surplus would be affected. As discussed in section 4.6, two-part tariffs are often used, though.

Llobet and Padilla (2016) show, absent litigation, ad-valorem royalties mitigate the royalty stacking problem.

In this section we show that the same moderating force introduced by the Inverse Cournot effect also exists under ad-valorem royalties. In particular, consider the generic case in which the downstream producer faces a demand $D(p)$ and it incurs in a marginal cost of production $c > 0$.[11] When patent holders 1 and 2 charge ad-valorem royalties s_1 and s_2 and the aggregate rate is $S \equiv s_1 + s_2$, the problem of the downstream producer can be written as

$$\Pi_D(S) = \max_p \left[(1 - S)p - c\right] D(p).$$

The monopoly price, p^M, is increasing in S under standard regularity conditions, such as the log-concavity of the demand function. This requirement is also enough to show that $\Pi_D(S)$ is decreasing and convex in S. As a result, if we consider the case in which $g(x_2) < g(x_1) = 1$, the downstream producer will litigate the portfolio of patent holder 2 if s_1 is lower than a threshold level $\bar{s}_1$, defined as

$$(1 - g(x_2)) \left[\Pi_D(\bar{s}_1) - \Pi_D(\bar{s}_1 + s_2)\right] = L_D. \tag{11}$$

A counterpart of Lemma 2 can be obtained in this case, with $\bar{s}_1$ increasing in s_2. As a result, patent holder 1 has incentives to lower s_1 in order to induce patentee 2 to lower s_2 and prevent being litigated.

In order to illustrate that this effect might eliminate royalty stacking, we consider again the example discussed in section 3, with $v > c$. Under ad-valorem royalties downstream profits can be written as

$$\Pi_D(S) = \begin{cases} (1 - S)(\alpha + (1 - \alpha)v) - c & \text{if } S \le 1 - \frac{c}{v}, \\ \alpha(1 - S - c) & \text{if } S \in \left(1 - \frac{c}{v}, 1 - c\right], \\ 0 & \text{otherwise,} \end{cases} \tag{12}$$

We can now write a counterpart of Proposition 6.

[11] As it is well-known, the problem when $c = 0$ is trivial, since a royalty rate of 100% would always be optimal and it would create no distortion in the final market.

Proposition 10. *Under the two-point demand function, there exists a threshold $\tilde{v}$ such that joint profit maximization implies a royalty $S^M = 1 - c$ if $v > \tilde{v}$ and $S^M = 1 - \frac{c}{v}$ otherwise. Under competition there is a continuum of undominated pure-strategy equilibria. There exist values $\underline{v}$ and $\overline{v}$, such that $\tilde{v} \leq \underline{v} < \overline{v}$ so that the equilibrium ad-valorem royalty rates (s_1^u, s_2^u) can be characterized as follows:*

1. *If $v \geq \underline{v}$, $S^u = s_1^u + s_2^u = 1 - \frac{c}{v}$ with $s_i^u \leq 1 - \frac{(1-2\alpha)cv+\alpha c}{(1-\alpha)\, v^2}$ for $i = 1, 2$.*
2. *If $v \leq \overline{v}$, $S^u = s_1^u + s_2^u = 1 - c$ with $s_i^u \geq 1 - \frac{(1-2\alpha)cv+\alpha c}{(1-\alpha)v^2}$ for $i = 1, 2$.*

As a result, royalty stacking emerges in equilibrium when $\tilde{v} < v < \underline{v}$.

As in the previous case, royalty stacking arises when v takes an intermediate value. Patent holders individually charge a total royalty rate higher than what joint maximization would find optimal. The proof in the appendix provides the specific expressions for the different thresholds.

We turn now to the case where patent holder 1 has a portfolio of strength x_1 such that $g(x_1) = 1$ whereas x_2 is such that $g(x_2) < 1$. First notice that a necessary condition for an equilibrium with royalty stacking to exist, $s_1^* + s_2^* = 1 - c$, is that the downstream producer does not have incentives to sue patent holder 2. In particular, this implies that

$$(1 - g(x_2))\,[\Pi_D(1 - c - s_2^*) - \Pi_D(1 - c)] \leq L_D.$$

Two cases arise depending on whether the royalty of patent holder 1, $s_1^* = 1 - c - s_2^*$ is greater than $1 - \frac{c}{v}$ or not. As a result, litigation by the downstream producer will not be profitable if

$$s_2^* \leq \bar{s}_2 = \begin{cases} \frac{L_D}{\alpha(1-g(x_2))} & \text{if } \frac{L_D}{1-g(x_2)} < \alpha\left(\frac{c}{v} - c\right), \\ c\frac{(1-\alpha)(1-v)}{\alpha+(1-\alpha)v} + \frac{L_D}{(\alpha+(1-\alpha)v)(1-g(x_2))} & \text{otherwise.} \end{cases} \tag{13}$$

For a given royalty rate s_2 set by patent holder 2 we can define, using equation (11), the threshold royalty rate $\bar{s}_1$ as

$$\bar{s}_1(s_2) = 1 - \frac{c}{v} - \frac{L_D}{(1-g(x_2))(1-\alpha)v} + \frac{\alpha}{(1-\alpha)v} s_2 \text{ if } \underline{s}_2 \leq s_2 \leq \bar{s}_2.$$

where for any $s_2 < \bar{s}_2$ we have $\bar{s}_1 \leq 1 - \frac{c}{v}$. As in the previous case, if $s_1 < \bar{s}_1(s_2)$ the portfolio of patent holder 2 will be litigated by the downstream producer. The lower threshold is defined as the highest value of s_2 for which it is not worthwhile to sue patent holder 2 even when patent holder 1 chooses $s_1 = 0$ and it can be written as

$$s_2 < \underline{s}_2 = \begin{cases} \frac{L_D}{(1-g(x_2))(\alpha+(1-\alpha)v)} & \text{if } \frac{L_D}{1-g(x_2)} \leq (\alpha + (1-\alpha)v)\left(1 - \frac{c}{v}\right), \\ \frac{L_D}{\alpha(1-g(x_2))} - \frac{1-\alpha}{\alpha}(v-c) & \text{otherwise.} \end{cases} \tag{14}$$

Given s_2, patent holder 1 will have incentives to deviate if, by choosing $s_1 \leq \bar{s}_1(s_2)$, profits increase due to the increase in quantity when the patent portfolio of firm 2 is invalidated. The next proposition shows that, as in the case in which royalties were paid per-unit, the royalty stacking equilibrium fails to exist when the portfolio of patent holder 2 is sufficiently weak.

Proposition 11. *Suppose that $v > \tilde{v}$. If $\frac{L_D}{(1-g(x_2))(\alpha+(1-\alpha)v)} < 1 - \frac{(1-2\alpha)cv+\alpha c}{(1-\alpha)v^2}$ and $g(x_2)$ is sufficiently small, there is no pure strategy equilibrium with royalty stacking.*

4.2 Downstream Competition

In this section we show that as downstream competition increases, the Inverse Cournot Effect is moderated but it does not necessarily disappear. There are two reasons for this weaker effect. First, more competition leads not only to lower downstream profits but also to lower differential profits from invalidating the portfolio of a patent holder. Second, a free-riding problem arises. If courts invalidate the portfolio of one of the patent holders the royalty rate that all downstream producers pay to that firm is also reduced to 0.

Regarding the first effect, consider a downstream market with N identical competitors. Their only marginal cost of production is the total royalty R. Denote profits as $\Pi_D(R, N)$. Under standard conditions, $\Pi_D(R, N)$ is decreasing in both arguments and convex in R.

Consider a situation in which all downstream firms decide simultaneously whether to litigate one of the upstream patent holders (firm 2) or not and, in particular, $g(x_1) = 1$. Suppose that if a total of n downstream firms litigate the patent holder, the probability that the portfolio is considered valid is $g(x_2, n)$, weakly decreasing in n. Each downstream producer incurs a litigation cost L_D if it goes to court.

Notice that if no other downstream firm litigates upstream patent holder 2, any firm would be indifferent between litigating or not if $r_1 \leq \bar{r}_1$ defined as

$$(1 - g(x_2, 1)) \left[\Pi_D(\bar{r}_1, N) - \Pi_D(\bar{r}_1 + r_2, N)\right] = L_D.$$

As in the baseline model, the Inverse Cournot effect arises due to the convexity of the profit function with respect to R. Furthermore, if $\frac{\partial \Pi_D}{\partial R \partial N} > 0$, then $\frac{d\bar{r}_1}{dN} < 0$. This condition holds under many of the typical demand specifications.

Example 3 (Cournot Competition)**.** *Under a linear demand function $P(Q) = a - Q$, where $R < a$, $\frac{\partial \Pi_D}{\partial R \partial N} = \frac{a-R}{2N} > 0$. When demand is isoelastic, $P(Q) = Q^{-\frac{1}{\eta}}$, the cross derivative of the equilibrium profit function corresponds to*

$$\frac{\partial \Pi_D}{\partial R \partial N} = (\eta - 1)\eta^{-\eta} R^{-\eta} (\eta - N)^{\eta - 2} > 0.$$

Example 4 (Product Differentiation)**.** *Suppose that downstream producers sell differentiated products, with a degree of substitubility identified by the parameter $\gamma \geq 0$. Firm i faces a demand function*

$$q_i = \frac{1}{N}\left[v - p_i(1+\gamma) + \frac{\gamma}{N}\sum_{j=1}^{N} p_j\right].$$

Using the expression for the profits in the symmetric equilibrium we have

$$\frac{\partial \Pi_D}{\partial R \partial N} = \frac{2\left(v-R\right)\left[(N-1)\gamma(3+\gamma)+2N\right]}{\left((N-1)\gamma+2N\right)^3} > 0.$$

In all the previous examples, as the number of downstream firms increases the Inverse Cournot effect becomes weaker.[12] That is, patent holder 1 must decrease the royalty further in order to induce litigation against another patent holder.

From the previous example we can also show that as product differentiation increases, understood as a decrease in γ, the threshold value $\overline{r}_1$ increases. The reason is that product differentiation operates as a decrease in competition.

Finally, in order to study the free-riding effect, let's focus on the case with $N = 2$. The profits for each firm when $n \leq N$ downstream producers litigate, gross of litigation costs, can be written as

$$V_D(n) = (1 - g(x_2, n))\Pi_D(r_1, 2) + g(x_2, n)\Pi_D(r_1 + r_2, 2).$$

Suppose that it is worthwhile for the two firms to litigate the portfolio of patent holder 2. That is, $2V_D(2) - 2L_D > 2V_D(1) - L_D$. It is easy to see that if one of the firms litigates, the other firm will also litigate if and only if $V_D(2) - L_D > V_D(1)$. As a result, if $V_D(2) - V_D(1) < L_D < 2\left[V_D(2) - V_D(1)\right]$ litigation that would increase value for downstream firms will not take place.

4.3 Royalty Renegotiation

The timing of the model assumes that once patent holder i chooses the royalty rate r_i the downstream producer will end up paying that amount unless it is brought to court and the portfolio invalidated. In that case it would pay 0. This means that when the portfolio is considered valid by the court the patent holder has no chance to increase the royalty rate.

[12] An exception is the circular city, where the inelastic demand implies that the cross-derivative is 0 and, thus, the Inverse Cournot effect is independent of the number of firms.

Papers like Choi and Gerlach (2015) allow for the possibility of renegotiation under these new circumstances. As we discuss next, royalty renegotiation weakens the Inverse Cournot effect but it does not qualitatively affect the results of the paper.

In the benchmark model, the maximum royalty rate that patent holder 1 could charge and induce litigation on patent holder 2, $\bar{r}_1$, arises from (2). In the two-point demand case, allowing patent holder 2 to revise the royalty rate after the portfolio has been considered valid implies that this royalty would become $\tilde{r}_2 = 1 - \bar{r}_1$. Equation (2) would then be replaced by

$$(1 - g(x_2))\left[\Pi_D(\bar{r}_1) - \Pi_D(\bar{r}_1 + r_2)\right] + g(x_2)\left[\Pi_D(1) - \Pi_D(\bar{r}_1 + r_2)\right] = L_D.$$

It is still true that $\bar{r}_1$ increases in r_2 but only when the size of the portfolio of patent holder 2, x_2, is sufficiently small. So, the Inverse Cournot effect will continue to operate when the distribution of patent ownership is sufficiently skewed. This observation is in opposition to the results in Choi and Gerlach (2015), where the positive relationship between the royalty rate of both firms is generated precisely by the upside that royalty renegotiation provides.

4.4 FRAND Licensing

Most SSOs request participating firms to license the patents that are considered essential to the standard according to Fair, Reasonable, and Non-discriminatory (FRAND) terms. The ambiguity of this term and the different interpretation of patent holders and licensees has made FRAND a legally contentious issue. Courts have sometimes been asked to decide whether a royalty rate is FRAND or not and in some instances to determine the FRAND rate.

The goal of this section is not to assert whether a royalty is FRAND or not but, rather, to study what is the effect of courts determining it on the previous results and, in particular, on the Inverse Cournot effect. In order to do so, we now extend the basic model and assume that the downstream producer can sue a patent holder arguing, as before, that the portfolio is invalid and, in case it is not, to ask the court to rule that the patents are essential to the standard and the royalty requested is not FRAND. We assume that the larger is a patent portfolio the more likely it is that the technology it covers is considered essential to the standard. This probability is defined as $h(x_i)$, increasing in x_i. The arguments apply to the existence of N patent holders, with R_{-i} corresponding to the sum of the royalty rate of all patentees other than i.

If the portfolio is declared to include patents that are essential to the standard the court will determine the appropriate royalty. We assume that this royalty, $\rho(x_i, r_i, R_{-i})$, is an increasing function of the quality of the patent portfolio, x_i. As we discuss later, we also allow for the possibility that the court's decision depends on the royalty announced by the patent holder or the total royalty established by the other patent holders.

Following the analysis in the benchmark model, the downstream monopolist will be interested in litigating patentee i only if

$$\begin{aligned}&(1-g(x_i))\left[\Pi_D(R_{-i})-\Pi_D(R_{-i}+r_i)\right]\\&+g(x_i)h(x_i)\left[\Pi_D(R_{-i}+\rho(x_i,r_i,R_{-i}))-\Pi_D(R_{-i}+r_i)\right]>L_D.\end{aligned}$$

The previous expression has a straightforward interpretation. The producer might benefit from litigation either because the patent portfolio is invalidated, which occurs with probability $1-g(x_i)$, or because it is considered valid and essential to the standard, with probability $g(x_i)h(x_i)$. In this latter case, the royalty rate changes from r_i to $\rho(x_i, r_i, R_{-i})$.

Lemma 12. *Suppose that $\rho(x_i, r_i, R_{-i})$ is independent of r_i and R_{-i}. Then, there exists a unique critical value $\bar{r}_i(x_i, R_{-i}, L_D)$ such that the producer prefers to litigate patentee i if and only if $r_i > \bar{r}_i$. Furthermore, this threshold is increasing in R_{-i} and L_D.*

This result indicates that the Inverse Cournot effect is qualitatively unaffected as long as the court determines the FRAND royalty as only a function of the quality of the portfolio. The main difference, however, is that the result does not guarantee that patent holders with a stronger portfolio can indeed charge a higher royalty without enticing the downstream producer to litigate. The reason is that although a higher x_i reduces the probability that the court invalidates the patent portfolio, it also increases the probability that it considers the patents essential and, thus, that the royalty rate would be decreased from r_i to $\rho(x_i)$. This second effect prevails when increases in x_i have a large impact on $h(x_i)$ but a small one on $\rho(x_i)$.[13]

It is plausible, however, that $\rho(x_i, r_i, R_{-i})$ is increasing in r_i. Our results establish sufficient conditions and they might still hold even if ρ increases in r_i. An interesting case that it is worth to mention is the following: Suppose that a court would determine the FRAND royalty rate as a function of x_i but it would never choose $\rho(x_i, r_i, R_{-i})$ higher than r_i. It can be shown that the results are preserved in that case.

Finally, there have been instances in which courts have used existing licensing agreements in order to pin down the FRAND royalty rate for a patent portfolio. Interestingly, they have been used in two directions. In some cases, courts have adopted the so-called *comparables approach* and set the royalty rate according to the rate negotiated for comparable patent portfolios, even in the same standard.[14] In those cases increases in R_{-i} would have a positive effect on $\rho(x_i, r_i, R_{-i})$ and strengthen the Inverse Cournot effect.

In other cases, and more specifically in the Microsoft v. Motorola case,[15] it has been argued that the FRAND royalty rate of a patent holder should be lowered due to the already large royalty stack. This reasoning would make $\rho(x_i, r_i, R_{-i})$ non-increasing in R_{-i}. Interestingly, this result would undermine the Inverse Cournot effect and it might even reverse

[13] It stands to reason that if the latter effect dominated, large patent holders would anticipate it and decide to license some of their patents at a rate of 0 in order to prevent their portfolio being deemed as essential.

[14] See **?** for a discussion of this and other approaches used to determine FRAND royalty rates.

[15] Microsoft Corp v. Morotola Inc, 854 F. Supp 2d 933 - Dist Court WD Washington 2012.

its sign, with self-defeating consequences. Large patent holders would anticipate that by choosing a larger royalty, weaker competitors facing litigation would be forced by the court to set a lower rate, making worse the royalty-stacking problem that courts were aiming to mitigate in the first place.

4.5 Sequential Royalty Setting

In the benchmark model firms choose their royalty rates simultaneously. Let's consider now the case in which patent holder 1, with the strong portfolio, decides the royalty rate first, whereas the patent holder 2, constrained by litigation, chooses later. It is easy to see that the main forces at play in the simultaneous case will apply here. The decision of patent holder 1 in both cases would internalize the effect of r_1 on r_2 through $\bar{r}_2$. In the case where a pure strategy Nash equilibrium exists like in our parametric example, the equilibrium aggregate royalty rate would coincide. In those cases where only a mixed strategy equilibrium existed in the simultaneous move case, a pure strategy equilibrium will now exist.

This similarity between the simultaneous and the sequential move game is useful to explain the behavior of large innovators that participate in SSOs. These firms devote substantial resources in developing technologies that depend on the success in the final-good market of the products that embed them. The announcement of a low royalty rate early in the standardization process can, thus, be understood as a commitment that the royalty rate of complementary technologies developed by firms with a weaker patent portfolio would also be low, reducing the risk of royalty stacking. This interpretation is consistent with the adoption of some standards in recent years. For example, in the case of the fourth-generation mobile telecommunications technology (also denoted as LTE) its main sponsors announced the licensing condition for their (essential) patents very early in the process.

The mechanism used to spur the adoption of a new technology that this paper uncovers resembles contractual arrangements that we observe in other technological contexts. **?** study the public-good problem faced in software development when placed in the public domain. Developers create improvements to the software but use them to launch commercial applications instead of making them available to the rest of the users. In this context they analyze the option that the leader of the project has to attach a General Public License (GPL) to the software, which forces all improvements to be contributed back to the project. They show that a GPL has two opposite effects. On the one hand, it discourages applications to be created since their developers lose the competitive advantage that their improvements generate as they become available to everybody. On the other hand, the quality of the software improves, since some developers that would otherwise create commercial applications now decide to contribute to the project and improve it. More recently, in the case of encryption technologies the risk that non-practicing entities might try to enforce their patents has encouraged agents more invested in the development of software to make it open source and, therefore, royalty free.[16]

In our model, a large patent holder can use the litigation threat of the downstream producer, by means of the Inverse Cournot effect, to limit the incentives of other patent holders to charge a high royalty rate, fostering the adoption of the technology. In these other examples, the choice of a GPL or a royalty-free arrangement allows the leaders of a technology to internalize part of the potential distortions. Imposing restrictions on the behavior of other developers reduces the free-riding problem, promotes the contribution to a technology and helps in its take-up. Leaders might even find optimal to forgo royalty revenues altogether if they obtain profits from other sources related to the success of the technology, for example, through complementary services.

[16]See "A rush to patent the blockchain is a sign of the technology's promise" (2017, 14 January), The Economist (downloaded on 8 February 2017).

4.6 Two-Part Tariffs

As widely assumed in the literature, patent holders in our model can only use royalty rates. In this section we explore implications of enlarging the kind of contracts that patent holders can use to accommodate two-part tariffs, combining royalty rates and fixed fees. These contracts are common, for example, in the biomedical industry (**?**).

It is important to point out that, absent any frictions, the optimal contract that patent holders will offer typically include only fixed fees, as they avoid double-marginalization. The literature has found the usage of two-part tariffs optimal as part of a risk-sharing strategy (Bousquet et al., 1998) or in situations where there is downstream competition (Hernández-Murillo and Llobet (2006), Reisinger and Tarantino (2019)). In our setup we have abstracted from these motivations. However, it is still the case that a patent holder with a large patent portfolio has incentives to deviate from the usage of pure fixed fees, in the direction highlighted in the rest of this paper.

Suppose that patent holder $i = 1, 2$ offers a two-part tariff (r_i, F_i) for $i = 1, 2$. Absent litigation, to the extent that both patent holders offer their contract simultaneously, the symmetric equilibrium two-part tariff in this case would amount to $r_1^* = r_2^* = 0$.

Consider the case discussed in the main sections of the paper where only patent holder 1 has a strong portfolio. The condition that determines whether it is in the interest of the downstream producer to litigate the portfolio of patent holder 2 boils down to

$$(1 - g(x_2)) \left[(p^M(r_1) - r_1) D(p^M(r_1)) - (p^M(r_1 + r_2) - (r_1 + r_2) D(p^M(r_1 + r_2)) + F_2 \right] \geq L_D. \tag{15}$$

This condition indicates that, similarly to what happens in the benchmark model with pure royalties, decreases in r_1 should lead to a decrease in r_2 and/or F_2 to prevent litigation. The fixed fee F_1 is irrelevant for this decision, since it enters the profit function of the downstream producer linearly.

As discussed in the previous subsection, however, the existence of litigation turns patent holder 1 into a Stackelberg leader when establishing the licensing contract. This means that we can solve the game by backwards induction. Given (r_1, F_1) if patent holder 2 aims to avoid litigation, it will always choose $r_2^* = 0$ and the highest value of F_2 that makes equation (15) fail to hold. Patent holder 1 can only induce litigation by reducing r_1 below what it would be optimal otherwise. In this sense, the model operates exactly as in our benchmark case. Of course, negative royalties are unlikely to arise in practice. However, one can think of the previous result as a force that would lead to a lower royalty royalty than the one that would emerge when the motivations discussed earlier applied. Endogenizing these motivations is beyond the scope of this paper.

The mechanism described here is very similar to the one uncovered in Marx and Shaffer (1999) in the standard context of vertical relationships. In their case the downstream firm negotiates in sequence with the two upstream input suppliers. Under two-part tariffs, the authors show that in the negotiation with the first firm the per-unit price of the input is set below marginal cost in order to improve the bargaining position in the negotiation with the second firm.

5 Equilibrium Litigation

A sustained assumption throughout the paper has been that it was always optimal for the patent holders to avoid litigation. This assumption is consistent with high values of the litigation costs of these upstream firms, L_U. In this section we relax this assumption by considering lower values of L_U.

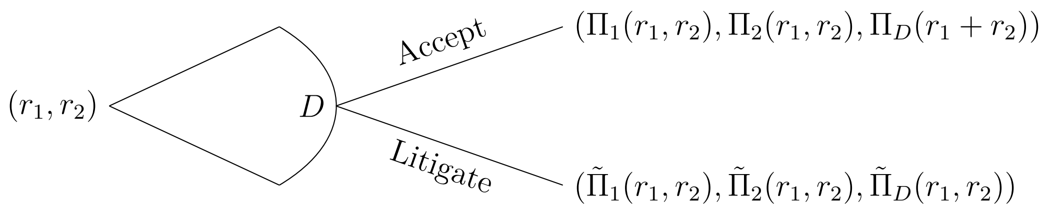

Figure 3: Timing of the game when litigation may occur in equilibrium.

5.1 The Litigation Decision of Patent Holder 2

To analyze this case, we concentrate in the situation where patent holder 1 has a strong portfolio, $g(x_1) = 1$, whereas patent holder 2 has a weak one, $g(x_2) < 1$. Figure 3 illustrates the structure of the game and defines the relevant payoffs. In particular, we define the profits from going to court as follows:

$$\tilde{\Pi}_1(r_1, r_2) = g(x_2)\Pi_1(r_1, r_2) + (1 - g(x_2))\Pi_1(r_1, 0),$$

$$\tilde{\Pi}_2(r_1, r_2) = g(x_2)\Pi_2(r_1, r_2) - L_U,$$

$$\tilde{\Pi}_D(r_1, r_2) = g(x_2)\Pi_D(r_1 + r_2) + (1 - g(x_2))\Pi_D(r_1) - L_D,$$

whereas $\Pi_i(r_1, r_2)$ are the profits of patent holder i when both r_1 and r_2 are paid by the downstream producer. The expression $\tilde{\Pi}_D(r_1, r_2)$ has been implicitly used before, since its difference with respect to $\Pi_D(r_1 + r_2)$ determines the results in Lemma 2.

From the previous expressions it is clear that, contingent on litigation, the optimal response of patent holder 2 to a royalty r_1 coincides with the one that arises when litigation is not a threat, the Cournot best response. Denote this choice as $r_2^c(r_1)$ which, due to Assumption 1, is decreasing in r_1.

For a given r_1 we can determine the royalty rate that guarantees that it is not worthwhile for the downstream producer to litigate, $\bar{r}_2(r_1)$, as the inverse of $\bar{r}_1(r_2)$ from equation (2). Using Lemma 2, when L_D is sufficiently low, this function is increasing in r_1, L_D, and $g(x_2)$. Since $r_2^c(r_1)$ converges to 0 as r_1 grows, it is immediate that $r_2^c(r_1) > \bar{r}_2(r_1)$ if r_1 is lower than a threshold value $\hat{\rho}_1$, decreasing in L_D and $g(x_2)$.

If r_1 is higher than $\hat{\rho}_1$ litigation is not a relevant threat, since $r_2^c(r_1)$ is lower than $\bar{r}_2(r_1)$. For lower values of r_1, patent holder 2 must trade off the increase in the revenues originated by $r_2^c(r_1)$ with the probability that the patent portfolio is invalidated and revenues become 0. The former will dominate if r_1 is sufficiently small, as the large decrease in the royalty rate necessary to fend off litigation is unlikely to be profitable. Additional conditions are necessary to guarantee that that this threshold, denoted as $\tilde{\rho}_1$, is unique, although it is satisfied by standard demand assumptions. For this reason, in the rest of this section we focus on the case in which demand is linear, $D(p) = 1 - p$, and the downstream firm chooses a unique (monopoly) price. In that case $\tilde{D}(R) = \frac{1-R}{2}$ and using the previous expressions we can show that

$$r_2^c(r_1) = \frac{1 - r_1}{2}, \tag{16}$$

$$\bar{r}_2(r_1) = 1 - r_1 - \sqrt{(1 - r_1)^2 - \frac{4L_D}{1 - g(x_2)}}. \tag{17}$$

The next result characterizes the optimal royalty rate for patent holder 2 in that case.

Proposition 13. *Suppose that demand is $\tilde{D}(R) = \frac{1-R}{2}$. When L_U is sufficiently low,*

1. *If $\frac{L_D}{1-g(x_2)} > \frac{3}{16}$ then $r_2^c(r_1) < \bar{r}_2(r_1)$ for all r_1. Otherwise, $r_2^c(r_1) < \bar{r}_2(r_1)$ if and only if $r_1 < \hat{\rho}_1 \equiv 1 - \sqrt{\frac{16L_D}{3(1-g(x_2))}}$.*

2. *There exists a unique threshold value $\tilde{\rho}_1(x_2, L_D, L_U)$ decreasing in L_D and L_U so that the optimal decision of patent holder 2 becomes,*

$$r_2^*(r_1) = \begin{cases} r_2^c(r_1) & \text{if } \; r_1 < \tilde{\rho}_1, \\ \bar{r}_2(r_1) & \text{if } \; r_1 \geq \tilde{\rho}_1, \end{cases}$$

where $\tilde{\rho}_1 < \hat{\rho}_1$. Furthermore, $\tilde{\rho}_1 > 0$ if $L_U \leq \frac{g(x_2)}{8} + \frac{1 - \frac{4L_D}{1-g(x_2)} - \sqrt{1 - \frac{4L_D}{1-g(x_2)}}}{2}$.

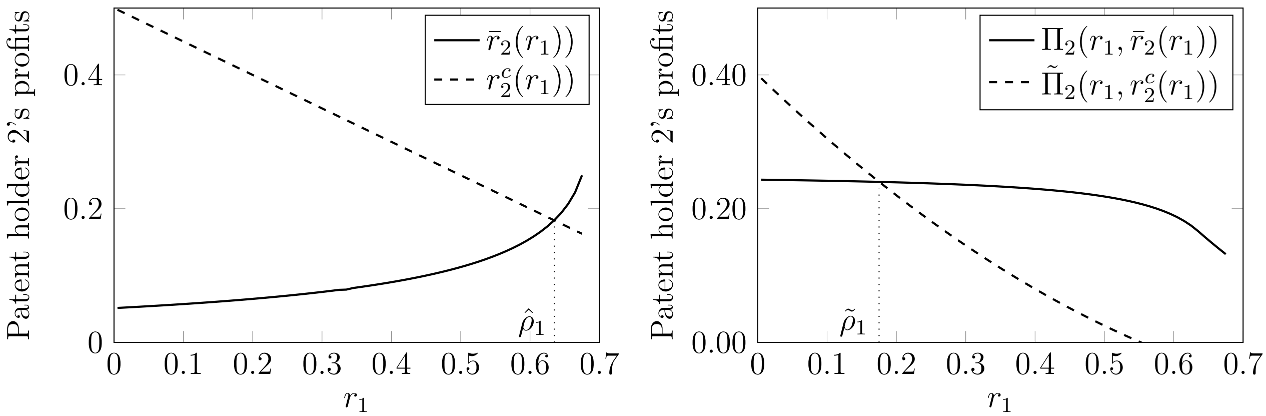

Figure 4: Profits and royalties under litigation and accommodation with parameter values: $L_D = 0.015$, $g(x_2) = 0.4$, $L_U = 0.01$.

As expected, when r_1 is low, the decrease in the royalty rate of patent holder 2 necessary to avoid litigation does not pay off. When r_1 is high, however, this decrease is not significant and avoiding litigation increases profits. The discontinuity in profits of moving, for the same level of r_2, from a situation where litigation is averted to one in which it occurs explains why $\tilde{\rho}_1 < \hat{\rho}_1$.

The right panel in Figure 4 illustrates this result. The profits of patent holder 2 are decreasing in r_1 with and without litigation but for different reasons. When there is no litigation in equilibrium $\Pi_2(r_1, \bar{r}_2(r_1))$ is decreasing due to the standard Cournot arguments. When litigation occurs in equilibrium, however, this effect is moderated by the fact that the decrease in the royalty required to avoid litigation is now smaller. This counteracting effect also explains why the threshold for which litigation is preferred is unique.

This figure also allows us to illustrate how the threshold on r_1 changes with the litigation costs. Increases in L_U lead to a downward shift in $\tilde{\Pi}_2(r_1, \bar{r}_2^c(r_1))$ as litigation becomes less profitable for patent holder 2. Similarly, in the case in which litigation does not occur, an increase in L_D allows patent holder 2 to raise the royalty rate $\bar{r}_2(r_1)$, increasing $\Pi_2(r_1, \bar{r}_2(r_1))$. In both cases, $\hat{\rho}_1$ moves to the left and litigation is less likely to arise in equilibrium for a given value of r_1. The effect of an increase in x_2 is ambiguous since it raises profits in both cases.

5.2 The Optimal Choice of r_1

We now discuss how the optimal royalty rate of patent holder 1 is affected by the possibility of litigation in equilibrium. In order to do that, we need to make two assumptions. First, we will continue to focus on the linear demand case discussed in the last proposition, which guarantees that litigation only emerges for low values of r_1. Second, we will depart from the structure of the rest of the paper and assume that patent holder 1 moves first. As explained in section 4, this assumption has little effect in the case when litigation is avoided but it guarantees the existence of an equilibrium in pure strategies. As a result, it helps us understand how lower litigation costs L_U would change the effects described in the main sections of the paper.

In particular, patent holder 1 chooses the royalty rate in the first stage knowing that if $r_1 < \tilde{\rho}_1$ patent holder 2 will choose $r_2^c(r_1)$ and litigation will ensue. Otherwise, if $r_1 \geq \tilde{\rho}_1$, patent holder 2 will prefer to offer to the downstream producer a lower royalty rate $\bar{r}_2(r_1)$ to avoid being brought to court. We start by analyzing both cases separately.

Suppose that litigation occurs in equilibrium. That is, the optimal royalty rate of patent holder r_1^* is lower than $\tilde{\rho}$. In that case, patent holder 1 maximizes

$$\max_{r_1 < \tilde{\rho}_1} r_1 \left[g(x_2)\tilde{D}(r_1 + r_2^c(r_1)) + (1 - g(x_2))\tilde{D}(r_1) \right],$$

which implies an equilibrium royalty rate $\tilde{r}_1^* = \min\left\{\tilde{\rho}_1, \frac{1}{2}\right\}$. Notice that when $\tilde{\rho}_1$ is sufficiently high, the royalty rate is a convex combination of the one that a monopolist and a Stackelberg leader would choose which, under a linear demand, coincide.

If, instead, r_1 is chosen so that there is no litigation in equilibrium – that is, the royalty rate of patent holder 2 is sufficiently small so that D prefers to accept it –, patent holder 1 maximizes the profit function

$$\max_{r_1 \geq \tilde{\rho}_1} r_1 \frac{1 - r_1 - \bar{r}_2(r_1)}{2},$$

resulting in a candidate royalty rate $\bar{r}_1^* = \max\{\tilde{\rho}_1, r_1^{ic}\}$, where $r_1^{ic} \equiv \frac{3}{4} - \frac{1}{4}\sqrt{\frac{32 L_D}{1-g(x_2)} + 1}$. It is important to notice that $r_1^{ic} \leq \frac{1}{2}$. That is, if we ignore the constraints that $\tilde{\rho}_1$ pose, patent holder 1 would choose a lower royalty rate when patent holder 2 will later accommodate. Again, the reason is the *Inverse-Cournot effect*: by reducing r_1 patent holder 1 also fosters a reduction in r_2, mitigating the royalty-stacking distortion and increasing downstream sales and overall profits.

It can also be shown that if we, again, abstract from the constraints imposed by $\tilde{\rho}_1$, profits are always higher when litigation does not occur in equilibrium. That is, the profits of patent holder 1 are higher under $r_1 = r_1^{ic}$ when there is no litigation than when $r_1 = \frac{1}{2}$ and litigation occurs in equilibrium. In general, due to the linear structure of the demand function it is easy to see that given the same r_1, the option that maximizes profits is the one that leads to the lowest expected total royalty rate. This means that in some situations $r_1 = \tilde{\rho}_1$ will be optimal as a way to avoid litigation. The next result uses the previous insights to characterize the optimal royalty rate for patent holder 1 as a function of the threshold $\tilde{\rho}_1$.

Proposition 14. *The optimal royalty rate of patent holder* 1 *can be characterized as*

$$r_1^* = \begin{cases} r_1^{ic} & \textit{if } \tilde{\rho}_1 \leq r_1^{ic}, \\ \tilde{\rho}_1 & \textit{if } \tilde{\rho}_1 \in (r_1^{ic}, \rho^*], \\ \frac{1}{2} & \textit{otherwise,} \end{cases}$$

where $\rho^* > \frac{1}{2}$.

Figure 5 illustrates the optimal royalty rate for different values of $\tilde{\rho}_1$. The figure determines two regions. For values of $\tilde{\rho}_1$ below a threshold ρ^* it is optimal for patent holder 1 to foster a low royalty rate r_2 that will be accepted by the downstream producer. When $\tilde{\rho}_1$ is higher than ρ^* litigation is optimal. The royalty that maximizes profits in that region is the unconstrained one.

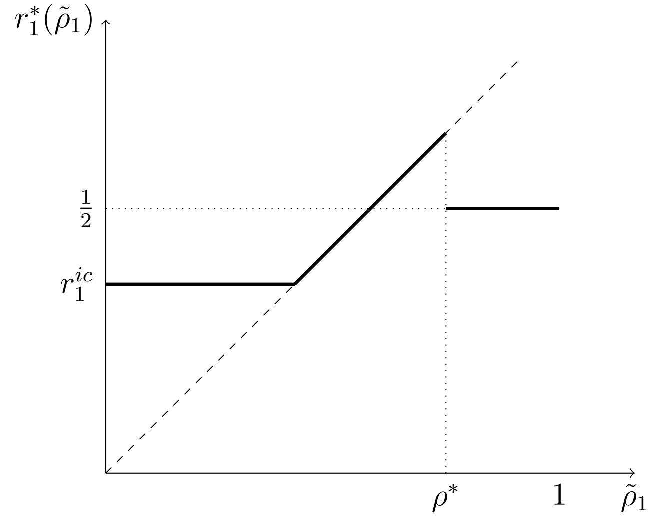

Figure 5: Optimal royalty rate of patent holder 1 as a function of $\tilde{\rho}_1$.

In order to unpack the implications of the previous figure, it is useful to start with a discussion on the effect of one the parameters, L_U. The comparative statics exercise in this case is simple since the litigation cost of patent holder 2 has no direct impact on the profits of patent holder 1 except through the changes in $\tilde{\rho}_1$. From Proposition 13 we know that increases in L_U are associated with a decrease in $\tilde{\rho}_1$, as the higher the cost of patent holder 2 to defend its patent portfolio in court the higher the royalty rate that patent holder 1 can charge and avoid litigation. As it can be seen from the figure, when L_U is low, and therefore $\tilde{\rho}_1$ is high, patent holder 1 is likely to find optimal to choose $r_1 = \frac{1}{2}$. The reason is that discouraging litigation by patent holder 2 would imply a very high royalty rate. As a result the total burden $r_1 + \bar{r}_2(r_1)$ would become very high and the quantity sold would be low. As L_U increases, however, discouraging patent holder 2 from litigating is easier and, eventually, when they are sufficiently high so that going to court is not a reasonable option, the Inverse-Cournot effect is the only relevant force. This effect pushes patent holder 1 to choose a royalty rate lower than the one that would emerge when litigation was optimal. This case has been the focus in the main sections of the paper.

For intermediate values of L_U we observe a region in which litigation does not take place (to the left of ρ^*) but the royalty rate is higher not only than r_1^{ic} but also than the one that would emerge under litigation. The higher royalty rate $r_1^* = \tilde{\rho}_1$ relaxes the Inverse Cournot effect and allows patent holder 2 to increase the royalty rate, making the option of avoiding litigation more profitable. Hence, for patent holder 1, this higher royalty rate implies a trade-off. Choosing $\tilde{\rho}_1$, compared to $r_1 = \frac{1}{2}$, implies a higher individual royalty rate but a lower total quantity due to the higher "expected" total royalty rate that emerges due to the lower probability that the portfolio of patent holder 2 is invalidated.[17] For values of $\tilde{\rho}_1$ below ρ^*, this trade-off is solved in favor of the high royalty rate even if that implies an increase in r_2.

The effect of L_D is similar in the sense that increases in this cost also shifts the threshold value $\tilde{\rho}_1$ downwards. However, an increase in L_D also raises $\bar{r}_2(r_1)$ reducing the profits from discouraging patent holder 2 from defending its portfolio in court. Both effects go in the same direction, suggesting that as L_D increases the region under which promoting litigation is optimal for patent holder 1 expands.

The effect of x_2 is in general difficult to determine, as it affects several dimensions of the picture. First, we can observe that, both under litigation and under accommodation, the profits of patent holder 1 decrease as x_2 increase, since the problem of royalty stacking becomes more relevant. However, an analytical comparison of the magnitude of the effect in both cases as well as the effect of x_2 on $\tilde{\rho}_1$ is difficult to establish.

Finally, this example allows us to draw some implications for equilibrium royalty stacking. Trivially, when litigation emerges in equilibrium the expected royalty rate is lower than the one that emerges when both patent portfolios are strong. The reason is that both patent holders would choose the same royalty rate but the expected royalty rate decreases as the

[17]For a given value of r_1 we can write the profits of patent holder 1 as $\Pi_1(r_1) = r_1 \frac{1-R}{2}$, where $R = r_1 + g(x_2) r_2^c(r_1)$ and $R = r_1 + \bar{r}_2(r_1)$ when litigation occurs in equilibrium and when patent holder 2 avoids it, respectively.

probability that the portfolio of patent holder 2 is invalidated increases. At the other extreme, when $r_1^* = r_1^{ic}$ the Inverse Cournot effect implies that the resulting royalty rate is lower than the one that would emerge under ironclad patents.

Interestingly, in the intermediate region, when $r_1^* = \tilde{\rho}_1$ and litigation is credible, the implications for royalty stacking are ambiguous. Patent holder 1 can increase revenues by raising the royalty rate above $\frac{1}{2}$ even as this fosters a limited increase in r_2. Under some parameter configurations this may lead to a higher total royalty rate.

6 Concluding Remarks and Policy Implications

The existence of royalty stacking has been argued by translating the insights that arise from the idea of Cournot complements to the context of technology licensing. This paper shows, however, that these insights do not carry through when we explicitly consider patent litigation and, most specifically, the incentives that firms have to make strategic use of it.

The implications of reconsidering the idea of royalty stacking through the lens of a model of patent litigation are far-reaching. One of the main contexts in which these changes apply is in the case of Standard Setting Organizations. Royalty stacking has been used to assess the desirability of patent consolidation or disaggregation. The concern about privateers, spin-offs of existing firms aimed at enforcing their intellectual property, and patent assertion entities has been seen as a way to increase the royalty stack. In contrast, consolidation efforts through patent acquisitions or the creation of patent pools have been encouraged as they would contribute to lower the aggregate royalty rate.[18]

Our model suggests that these rules should be implemented with caution and the impact of the litigation threat should be factored in. Since litigation decisions depend on the strength

[18]See Lerner and Tirole (2004). Other papers, however, have pointed out that patent pools might reduce social welfare when they include non-essential patents (Quint, 2014) or when some licensors and producers are vertically integrated (Reisinger and Tarantino, 2019).

of the patent portfolio, if firms pool their patents they are likely to make enforcement more effective. This last effect implies that the formation of a patent pool or the merger of patent holders might make the royalty-stacking problem worse, particularly, if not all firms are included and the portfolios become more similar in strength.[19] By the same token, to the extent that firms decide to disaggregate their patent portfolio into more asymmetric patent holdings, the outcome could be socially beneficial. To evaluate the impact of these decisions we should account for how the moderation force of large patent holders that the Inverse Cournot effect brings about is mitigated or strengthened.

References

Bourreau, Marc, Yann Meniere and Tim Pohlman, "The Market for Standard Essential Patents," 2015, working Paper.

Bousquet, Alain, Helmuth Cremer, Marc Ivaldi and Michel Wolkowicz, "Risk sharing in licensing," *International Journal of Industrial Organization*, September 1998, *16*(5), pp. 535–554.

Choi, Jay Pil and Heiko Gerlach, "Patent Pools, Litigation and Innovation," *RAND Journal of Economics*, 2015, *46*(3).

Hernández-Murillo, Ruben and Gerard Llobet, "Patent licensing revisited: Heterogeneous firms and product differentiation," *International Journal of Industrial Organization*, January 2006, *24*(1), pp. 149–175.

Lemley, Mark A. and Carl Shapiro, "Patent Holdup and Royalty Stacking," *Texas Law Review*, 2007, *85*, pp. 1991–2049.

[19]In our model, a patent pool including all firms will always eliminate the royalty stack and increase overall profits. Of course, to the extent that the Inverse Cournot effect reduces the size of this royalty stack, the incentives to form a pool are diminished.

Lerner, Josh and Jean Tirole, "Efficient Patent Pools," *American Economic Review*, 2004, *94* (3), pp. 691–711.

Llobet, Gerard and Jorge Padilla, "The Optimal Scope of the Royalty Base in Patent Licensing," *Journal of Law and Economics*, 2016, *59* (1), pp. 45–73.

Marx, Leslie M. and Greg Shaffer, "Predatory Accommodation: Below-Cost Pricing without Exclusion in Intermediate Goods Markets," *RAND Journal of Economics*, Spring 1999, *30* (1), pp. 22–43.

Quint, Daniel, "Pooling with Essential and Nonessential Patents," *American Economic Journal: Microeconomics*, 2014, *6* (1), pp. 23–57.

Reisinger, Markus and Emanuele Tarantino, "Patent Pools, Vertical Integration, and Downstream Competition," *RAND Journal of Economics*, Spring 2019, *50* (1), pp. 168–200.

Spulber, Daniel F., "Complementary Monopolies and Bargaining," 2016, Northwestern Law & Econ Research Paper No. 16-10.

7 Proofs

The main results of the paper are proved here.

Proof of Proposition 1: The optimal royalty of patentee i resulting from (1) is determined using the first-order condition

$$\tilde{D}(R) + r_i^u \tilde{D}'(R) = 0. \Longrightarrow r_i^u = -\frac{\tilde{D}(R)}{\tilde{D}'(R)}.$$

Replacing $r_i^u = r^* = \frac{R^u}{N}$ where N is the number of firms, we can use the Implicit Function Theorem to compute

$$\frac{dR^u}{dN} = \frac{\frac{R^u}{N}\tilde{D}'(R^u)}{\tilde{D}'(R^u) + \frac{R^*}{N}\tilde{D}'(R^u) + \frac{1}{N}\tilde{D}'(R^u)} > 0.$$

The last inequality arises from a negative numerator due to $\tilde{D}'(R) < 0$ and a negative denominator that it is also negative due to the quasiconcavity of $\tilde{D}(R)$. In particular, this result implies that $2r^M = R^U(1) < R^u(2) = 2r^u$. $\square$

Proof of Lemma 2: Define $\bar{r}_1$ as the value of r_1 for which equation (2) is satisfied with equality. First, we establish that it is unique and well-defined for all positive values of L_D. The left-hand side of that equation is always decreasing in r_1 for $r_2 > 0$. Furthermore, as $\tilde{D}(R) \to 0$ when $R \to \infty$ we have that the left-hand side expression can be arbitrarily small as r_1 increases. When $L_D < (1 - g(x_2))(\Pi_D(0) - \Pi_D(r_2))$ the threshold value $\bar{r}_1$ is always positive.

Using the fact that $\Pi'_D(R) < 0$ and $\Pi''_D(R) > 0$, we can compute, for $r_2 > 0$,

$$\begin{aligned}
\frac{d\bar{r}_1}{dL_D} &= \frac{1}{\Pi'_D(\bar{r}_1) - \Pi'_D(\bar{r}_1 + r_2)} < 0, \\
\frac{d\bar{r}_1}{dx_2} &= \frac{g'(x_2)\left[\Pi_D(\bar{r}_1) - \Pi_D(\bar{r}_1 + r_2)\right]}{\left[\Pi'_D(\bar{r}_1) - \Pi'_D(\bar{r}_1 + r_2)\right]} < 0, \\
\frac{d\bar{r}_1}{dr_2} &= \frac{\Pi'_D(\bar{r}_1 + r_2)}{\Pi'_D(\bar{r}_1) - \Pi'_D(\bar{r}_1 + r_2)} > 0.
\end{aligned}$$

$\square$

Proof of Proposition 3: Following the arguments in the text, suppose towards a contradiction that an equilibrium without litigation (r_1^*, r_2^*) exists with $r_1^* > 0$ and $r_2^* > 0$, different from the unconstrained solution, r^u, as defined in Proposition 1. Since $r_2^* > 0$, this equilibrium must satisfy equation (2). Suppose that this condition is satisfied with strict equality. It is easy to see that in that case r_1^* would not be optimal for patent holder 1, as it could be slightly diminished, leading to a discrete increase in final market sales from $\tilde{D}(r_1^* + r_2^*)$ to almost $g(x_2)\tilde{D}(r_1^* + r_2^*) + (1 - g(x_2))\tilde{D}(r_1^*)$.

Hence, condition (2) must be satisfied with strict inequality in the equilibrium. This means that patent holder 1 chooses the royalty as the result of $r_1^* = \arg\max r_1\tilde{D}(r_1 + r_2^*)$. Since the equilibrium differs from the unconstrained one and condition (2) constitutes an upper bound for the royalty rate of patent holder 2, it has to be the case that r_2^* is lower than the best response to r_1^*. But this is a contradiction, since patent holder 2 could always increase the royalty rate while the constraint still holds.

Alternatively, suppose that at least one of the royalty rates is 0 in equilibrium. This would be a contradiction, since both patent holders could guarantee positive profits and avoid litigation by choosing a positive royalty rate, unless $\tilde{D}(r_1^* + r_2^*) = 0$. But in that case the patent holder with a positive royalty rate would obtain higher (and positive) profits by decreasing it. $\square$

Proof of Lemma 4: Suppose without loss of generality that $r_1 > r_2$. The optimal policy of the downstream producer can be described as arising from the following two stages. In the first stage, it decides whether to sue patent holder 1 or 2 or none at all. Upon observing the outcome of the first trial the downstream producer decides whether to sue the other patent holder or not.

Suppose that in the first stage patentee i was brought to court. Then, if it is optimal for the downstream producer to sue patentee j upon defeat it is also optimal to litigate upon success since, by convexity of $\Pi_D(R)$,

$$\Pi_D(r_i) - \Pi_D(r_i + r_j) \leq \Pi_D(0) - \Pi_D(r_j),$$

for $i = 1, 2$ and $j \neq i$. Furthermore, notice that

$$\Pi_D(r_1) - \Pi_D(r_1 + r_2) \leq \Pi_D(r_2) - \Pi_D(r_1 + r_2),$$
$$\Pi_D(0) - \Pi_D(r_2) \leq \Pi_D(0) - \Pi_D(r_1).$$

Hence, two possible orderings can arise depending on whether $\Pi_D(r_2) - \Pi_D(r_1 + r_2)$ is higher or lower than $\Pi_D(0) - \Pi_D(r_2)$. In order to determine the profits of the downstream producer in each case, we need to see how these profits compare with $\frac{L_D}{1-g(x)}$.

(i) Suppose that when 1 is sued first it is always optimal to sue 2 afterwards. Obviously, if the opposite order yields the same order, both options are equivalent and profits are identical.

(ii) Suppose that when 1 is sued first it is only optimal to sue 2 after victory. This implies that $\Pi_D(r_1) - \Pi_D(r_1 + r_2) < \frac{L_D}{1-g(x)} \leq \Pi_D(0) - \Pi_D(r_2)$. Profits become

$$g(x)\left[\Pi_D(r_1 + r_2) - L_D\right] + (1 - g(x))\left[g(x)\Pi_D(r_2) + (1 - g(x))\Pi_D(0)\right] - L_D,$$

which, by definition, are higher than those that arise in the first case. If after litigating the portfolio of patent holder 2 it is then optimal to litigate the portfolio of the other patent holder always, this option would be, therefore, dominated by (i).

Alternatively, it could be that when patent holder 2 is sued first it is only optimal to sue patent holder 1 upon victory. Profits in that case would be

$$g(x)\left[\Pi_D(r_1+r_2)-L_D\right]+(1-g(x))\left[g(x)\Pi_D(r_1)+(1-g(x))\Pi_D(0)\right]-L_D,$$

which are lower than when patent holder 1 is sued first.

(iii) Suppose that when patent holder 1 is sued first it is never optimal to litigate the portfolio of patent holder 2 afterwards. Profits would be

$$g(x)\Pi_D(r_1+r_2)+(1-g(x))\Pi_D(r_2)-L_D.$$

If it is always optimal to sue patent holder 1 after patent holder 2 has been sued first, these profits are lower because, as in the previous case, they coincide with the profits in the first option. If instead it was optimal to litigate only upon success, again, these profits are dominated by the second option. Finally, if it is never optimal to sue patent holder 1, profits become

$$g(x)\Pi_D(r_1+r_2)+(1-g(x))\Pi_D(r_1)-L_D,$$

which are again lower.

(iv) Using the same argument, if $\frac{L_D}{1-g(x)}$ is sufficiently high so that it is never optimal to sue patent holder 1 only, bringing to court patent holder 2 only must also be dominated.

□

Proof of Proposition 5: Consider a symmetric equilibrium in which patent holder 1 and 2 are constrained. This implies that $\Phi(r^*,r^*)=\frac{L_D}{1-g(x)}+L_D$. Each firm obtains profits $r^*\tilde{D}(2r^*)$. It is immediate that r^* is increasing in L_D and $g(x)$.

Three possible deviations of a patent holder, say patentee 1, should be considered:

(i) Patentee 1 might increase its royalty to $r_1>r^*$. This firm will be litigated first and profits, defined as $\max_{r_1}\ g(x)r_1\tilde{D}(r_1+r^*)-L_U$, will be lower if L_U is sufficiently high.

(ii) Patentee 1 might deviate by lowering the royalty rate slightly. In this case, the sign of $\frac{\partial\Phi}{\partial r_1}$ becomes relevant. In particular,

$$\frac{\partial\Phi}{\partial r_1}(r_1,r_2)\geq 0 \Longleftrightarrow g(x)\Pi_D'(r_1)-\Pi_D'(r_1+r_2)=\tilde{D}(r_1+r_2)-g(x)\tilde{D}(r_1)\geq 0,$$

If $\frac{\partial \Phi}{\partial r_1} \geq 0$, decreases in r_1 reduce the incentives for the downstream firm to litigate. Since royalties are strategic substitutes and r^* is below the unconstrained royalty this strategy can never be optimal.

Alternatively, if $\frac{\partial \Phi}{\partial r_1} < 0$, a deviation consisting in a slight decrease in r_1 induces litigation, first against patentee 2 and, upon success, against patentee 1. This implies that the profits of patentee 1 become

$$g(x)r^*D(2r^*) + (1-g(x))\left[g(x)r^*D(r^*) - L_U\right],$$

This deviation is unprofitable if

$$L_U > r^*D(p^M(2r^*)) - g(x)r^*D(p^M(r^*)),$$

which holds given that the right-hand side is negative when $\frac{\partial \Phi}{\partial r_1}(r^*, r^*) < 0$.

(iii) Finally, patent holder 1 could lower r_1 enough so that $(1-g(x))\left[\Pi_D(0) - \Pi_D(r_1)\right] \leq L_D$. In that case, patent holder 1 would not be brought to court. Again, two possibilities can arise here depending on whether the downstream producer is interested in suing patentee 2 or not. Notice that only if patentee 2 is sued this deviation might be profitable. Hence, the optimal deviation is $\tilde{r}_1 = \min\{r_1^A, r_1^B\}$, where the values r_1^A and r_1^B are defined as

$$(1-g(x))\left[\Pi_D(0) - \Pi_D(r_1^A)\right] = L_D,$$
$$(1-g(x))\left[\Pi_D(r_1^B) - \Pi_D(r^* + r_1^B)\right] = L_D.$$

When r^* is sufficiently high the first constraint will be binding. Profits in either case will be $g(x)r_1\tilde{D}(r^* + \tilde{r}_1) + (1-g(x))r_1\tilde{D}(\tilde{r}_1)$.

When $g(x)$ is sufficiently small it is clear that the first deviation is always dominated since it would imply profits of $-L_U$. The second deviation is also unprofitable since when $g(x) = 0$, $\frac{\partial \Phi}{\partial r_1} \geq 0$.

Regarding the last deviation, we know that $\tilde{r}_1 \leq r_1^B$. Under a linear demand when $g(x) = 0$ and monopoly pricing, we have that $\Pi_D(0) - \Pi_D(2r^*) = 2\left[\Pi_D(r_1^B) - \Pi_D(r_1^B + r^*)\right]$ implies $r_1^B = \frac{r^*}{2}$. Thus, for the deviation not to be profitable we only require

$$r^*D(p^M(2r^*)) \geq \frac{r^*}{2}D\left(p^M\left(\frac{r^*}{2}\right)\right).$$

When L_D is 0, $r^* = 0$ and the result holds trivially. The derivative of the profit functions evaluated at $r^* = 0$ are $D(p^M(0))$ and $\frac{1}{2}D(p^M(0))$ for the left-hand side and the right-hand side expression, respectively. Thus, the deviation is not profitable when L_D is sufficiently small.

We now show that there is no other symmetric pure-strategy equilibrium when the litigation constraint is relevant. First, notice that if $r_1 = r_2$ are lower than r^*, each firm has incentives to increase its royalty since their problem is the same as they would face if they were unconstrained and royalties are strategic substitutes. If, instead, $r_1 = r_2 = \tilde{r}$ are higher than r^* each firm obtains profits

$$\frac{1}{2}\left[g(x)\tilde{r}D(p^M(2\tilde{r})) - L_U\right] + \frac{1}{2}\left[g(x)\tilde{r}D(p^M(2\tilde{r})) + (1 - g(x))\left[g(x)\tilde{r}D(p^M(2\tilde{r})) - L_U\right]\right]$$

where each firm is brought to court first with probability $\frac{1}{2}$ and the second firm is sued only if the downstream producer succeeds against the first. Notice that in this case it is always optimal for one firm, say patentee 1, to undercut the other patentee. As a result profits increase to

$$g(x)\tilde{r}D(p^M(2\tilde{r})) + (1 - g(x))\left[g(x)\tilde{r}D(p^M(2\tilde{r})) - L_U\right].$$

□

Proof of Proposition 6: Regarding the first case, contingent on selling with probability 1 the sum of royalties must be equal to v or otherwise any patent holder would deviate and increase the royalty rate. Hence, take r_1^u and $r_2^u = v - r_1^u$ and suppose without loss of generality that $r_1^u \geq \frac{v}{2} \geq r_2^u$. The optimal deviation for patentee i is $\hat{r}_i = 1 - r_j^u$ for $j \neq i$ and it would be unprofitable if $v - r_j^u \geq \alpha(1 - r_j^u)$ or $r_j^u \leq \frac{v-\alpha}{1-\alpha}$. Such a combination of royalties is only possible as long as $\frac{v}{2} \leq r_1^u \leq \frac{v-\alpha}{1-\alpha}$ or $v \geq \frac{2\alpha}{1+\alpha}$.

For the second case, take r_1^u and $r_2^u = 1 - r_1^u$ and suppose without loss of generality that $r_1^u \geq \frac{1}{2} \geq r_2^u$. The optimal deviation for patentee i is $\hat{r}_i = v - r_j^u$ for $j \neq i$ if it leads to a positive royalty and it would be unprofitable if $\alpha(1 - r_j^u) \geq v - r_j^u$ or $r_j^u \geq \frac{v-\alpha}{1-\alpha}$. Such a combination of royalties will be possible as long as $\frac{v-\alpha}{1-\alpha} \leq r_2^u \leq \frac{1}{2}$ or $v \leq \frac{1+\alpha}{2}$.

Finally, notice that $\frac{2\alpha}{1+\alpha} < \frac{1+\alpha}{2}$ for all $\alpha \in [0, 1]$ so both equilibria can co-exist. □

Proof of Lemma 7: From the argument in the text it is immediate that for $r_2 > \bar{r}_2$ an equilibrium without litigation and with royalty stacking cannot arise, since for all $r_1 = 1 - r_2$

litigation will be profitable for the downstream firm. For the rest of the arguments, it is useful to distinguish two cases depending on the relationship between v and $\frac{L_D}{1-g(x_2)}$.

Suppose that $\frac{L_D}{1-g(x_2)} \leq v$. First notice that litigation will not occur for any value of r_1 if and only if $r_2 \leq \underline{r}_2 = \frac{L_D}{1-g(x_2)}$. From Lemma 2, the incentives to litigate are highest when $r_1 = 0$ and, in that case, the expected profits from going to court are $(1 - g(x_2))r_2 \leq L_D$. Consider now the case $r_2 \in \left(\frac{L_D}{1-g(x_2)}, \bar{r}_2\right)$. By definition, when $r_2 < \bar{r}_2$ a royalty $r_1 = 1 - r_2$ induces litigation. Even if r_2 is sufficiently close to v a royalty $r_1 = 0$ litigation will always be profitable for the downstream producer since $(1 - g(x_2))\left[\Pi_D(0) - \Pi_D(v)\right] = (1 - g(x_2))v > L_D$. The value of r_2 for which the downstream firm is indifferent between litigating or not is defined by (7).

Consider now the case in which $v < \frac{L_D}{1-g(x_2)}$. First suppose that $r_2 \leq \underline{r}_2 = \frac{L_D}{\alpha(1-g(x_2))} - \frac{1-\alpha}{\alpha}v < \frac{L_D}{1-g(x_2)}$. In that case, even r_1 will not induce litigation since the downstream profits from going to court will be $(1 - g(x_2))\left[(1 - \alpha)v + \alpha r_2\right] \leq L_D$. Suppose now that $r_2 \in \left(\frac{L_D}{\alpha(1-g(x_2))} - \frac{1-\alpha}{\alpha}v, \bar{r}_2\right)$. By definition, when $r_2 < \bar{r}_2$ a royalty of $r_1 = 1 - r_2$ induces litigation. If, instead, r_2 is sufficiently close to $\frac{L_D}{\alpha(1-g(x_2))} - \frac{1-\alpha}{\alpha}v < v$ a royalty $r_1 = 0$ will induce litigation since the downstream profits of going to court are $(1 - g(x_2))\left[(1 - \alpha)v + \alpha r_2\right] > L_D$. The value of r_2 for which the downstream firm is indifferent between litigating or not is defined by (7). $\square$

Proof of Proposition 8: From Proposition 6, a necessary condition for a royalty-stacking equilibrium with not litigation to exist is that $r_1^* + r_2^* = 1$ and $r_2^* \geq \frac{v-\alpha}{1-\alpha}$ or, else, patent holder 1 would have incentives to lower its royalty rate.

Furthermore, a second deviation consisting in choosing a royalty slightly lower than $\bar{r}_1(r_2^*)$ might be profitable for patent holder 1 if it leads to profits

$$\hat{\Pi}_1 = \left[\alpha + (1 - \alpha)(1 - g(x_2))\right]\bar{r}_1(r_2^*) > \alpha r_1^*.$$

This condition holds if

$$r_2^* < \rho(G) \equiv \frac{(1 - \alpha)(\alpha - Gv) + G\frac{L_D}{1-g(x_2)}}{\alpha(G + (1 - \alpha))},$$

where $G \equiv \alpha + (1 - \alpha)(1 - g(x_2)) \in [\alpha, 1]$. Thus, in instances in which $\rho(G) < \frac{v-\alpha}{1-\alpha}$ an equilibrium with royalty stacking will fail to exist. This inequality implies that

$$G < G^*\left(\frac{L_D}{1 - g(x_2)}\right) = \frac{\alpha(1 - \alpha)(1 - v)}{(1 - \alpha)\left(v - \frac{L_D}{1-g(x_2)}\right) - \alpha^2(1 - v)}.$$

This function is increasing in $\frac{L_D}{1-g(x_2)}$ and $G^*\left(\frac{L_D}{1-g(x_2)}\right) < G^*\left(\frac{v-\alpha}{1-\alpha}\right) = 1$. Hence, there is always $g(x_2)$ sufficiently small so that the deviation will be optimal.

We now consider conditions under which an equilibrium with $R = v$ exists. Consider the case $r_2^* = \frac{L_D}{1-g(x_2)} < \frac{v-\alpha}{1-\alpha}$ and $r_1^* = v - r_2^*$. From (6), r_2^* avoids litigation and by Proposition 6 patentee 1 has no incentive to deviate. Thus, the only deviation we need to consider from patentee 2 is such that $R > v$. However, notice that

$$r_1^* = v - \frac{L_D}{1-g(x_2)} = v + \frac{\alpha}{1-\alpha} r_2^* - \frac{L_D}{(1-\alpha)(1-g(x_2))} = \bar{r}_1(r_2^*),$$

and so any higher r_2 will induce litigation. Hence, an equilibrium in pure strategies exists if and only if such a deviation is not profitable

$$\frac{L_D}{1-g(x_2)} \geq \alpha g(x_2)\left(1 - v + \frac{L_D}{1-g(x_2)}\right) - L_U.$$

This condition is guaranteed if $g(x_2)$ is sufficiently small or L_U sufficiently large. $\square$

Proof of Lemma 9: First notice that if patent holder 2 loses in court patent holder 1 will be brought to court if and only if

$$\Pi_D(0) - \Pi_D(\hat{r}_1) > \frac{L_D}{1-g(x)}$$

or $\hat{r}_1 > \frac{L_D}{1-g(x)}$. Also notice that, from the arguments in the text, if originally it was not optimal to engage in litigation it has to be that

$$\Pi_D(1/2) - \Pi_D(1) \leq \frac{L_D}{1-g(x)}.$$

Patent holder 1 would be sued after downstream producer loses against patent holder 2 if

$$\Pi_D(1/2) - \Pi_D(1/2 + \hat{r}_1) > \frac{L_D}{1-g(x)}$$

which is incompatible with the previous condition. $\square$

Proof of Proposition 10: As in the case of per-unit royalties only two ad-valorem rates can maximize joint profits, $1 - \frac{c}{v}$ and $1 - c$. The low rate dominates if

$$\left(1 - \frac{c}{v}\right)(\alpha + (1-\alpha)v) \geq (1-c)\alpha$$

or

$$v \leq \tilde{v} \equiv \frac{(1-2\alpha)\,c + \sqrt{(4\alpha^2 - 4\alpha + 1)\,c^2 + (4\alpha - 4\alpha^2)\,c}}{2(1-\alpha)}.$$

Regarding the Nash equilibria, suppose that patent holder $j = 1, 2$ chooses s_j. Patent holder i will prefer $s_i = 1 - \frac{c}{v} - s_j$ to $s_i = 1 - c - s_j$ if

$$\left(1 - \frac{c}{v} - s_j\right)(\alpha + (1-\alpha)v) \geq (1 - c - s_j)\alpha$$

or $s_j \leq \bar{s} \equiv \frac{(1-\alpha)\,v^2 - (1-2\,\alpha)\,c\,v - \alpha\,c}{(1-\alpha)\,v^2}$. Hence, for this equilibrium to exist we require that $2\bar{s} \geq 1 - \frac{c}{v}$ or

$$v \geq \underline{v} \equiv \frac{(1-3\alpha)\,c + \sqrt{(9\alpha^2 - 6\,\alpha + 1)\,c^2 + (8\alpha - 8\alpha^2)\,c}}{2(1-\alpha)}.$$

Similarly, an equilibrium with $S^u = 1 - c$ would exist if $s_j \geq \bar{s}$ and $2\bar{s} \leq 1 - c$ which can occur if

$$v \leq \overline{v} \equiv \frac{(1-2\,\alpha)\;c + \sqrt{(2\,\alpha^2 - 2\,\alpha + 1)\;c^2 + (2\,\alpha - 2\,\alpha^2)\,c}}{(1-\alpha)\,(1+c)}.$$

Comparison of the threshold expressions lead to $\tilde{v} \leq \underline{v} < \overline{v}$ if $\alpha < 0$ and $c \in (0, v)$. □

Proof of Proposition 11: From Proposition 10, a necessary condition for a royalty-stacking equilibrium to exist is that $s_1^* + s_2^* = 1 - c$ and $s_2^* \geq 1 - \frac{(1-2\alpha)cv + \alpha c}{(1-\alpha)v^2}$ or, else, patent holder 1 would have incentives to lower its royalty rate.

Furthermore, a second deviation consisting in choosing a royalty slightly lower than $\bar{s}_1(s_2^*)$ might be profitable for patent holder 1 if it leads to profits

$$\hat{\Pi}_1 = [\alpha + (1-\alpha)(1 - g(x_2))v]\,\bar{s}_1(s_2^*) > \alpha s_1^*.$$

This condition holds if

$$s_2^* < \sigma(G) \equiv \frac{(1-\alpha)v\left[\alpha(1-c) - G\left(1 - \frac{c}{v}\right)\right] + G\frac{L_D}{1-g(x_2)}}{\alpha(G + (1-\alpha)v)},$$

where $G \equiv \alpha + (1-\alpha)(1-g(x_2))v \in [\alpha, 1]$. Thus, in instances in which $\sigma(G) < 1 - \frac{(1-2\alpha)cv + \alpha c}{(1-\alpha)v^2}$ an equilibrium with royalty stacking will fail to exist. This inequality implies that

$$G < G^*\left(\frac{L_D}{1-g(x_2)}\right) \equiv \frac{\alpha(1-\alpha)(1-v)cv(\alpha + (1-\alpha)v)}{-\frac{L_D}{1-g(x_2)}\,(1-\alpha)\,v^2 + (1-\alpha)\,v^2\left((1-\alpha)\,(v-c) + \alpha\right) - (1-2\alpha)\,\alpha cv - \alpha^2 c}.$$

This function is increasing in $\frac{L_D}{1-g(x_2)}$ and $G^*\left(\frac{L_D}{1-g(x_2)}\right) < G^*\left((\alpha + (1-\alpha)v) * \left(1 - \frac{(1-2\alpha)cv + \alpha c}{(1-\alpha)v^2}\right)\right) = \alpha + (1-\alpha)v$. Hence, there is always $g(x_2)$ sufficiently small so that the deviation will be optimal. □

Proof of Lemma 12: Define

$$\Phi(r_i, x_i, L_D, R_{-i}) \equiv (1-g(x_i))\left[\Pi_D(R_{-i}) - \Pi_D(R_{-i}+r_i)\right] + \\ g(x_i)h(x_i)\left[\Pi_D(R_{-i}+\rho(x_i,r_i,R_{-i})) - \Pi_D(R_{-i}+r_i)\right] - L_D$$

Obviously, $\frac{\partial \Phi}{\partial L_D} = -1$. We can also compute

$$\frac{\partial \Phi}{\partial r_i} = -(1-g(x_i))\Pi_D'(R_{-i}+r_i) + g(x_i)h(x_i)\left[\Pi_D'(R_{-i}+\rho(x_i,r_i,R_{-i}))\frac{\partial \rho}{\partial r_i} - \Pi_D'(R_{-i}+r_i)\right]$$

$$\frac{\partial \Phi}{\partial R_{-i}} = (1-g(x_i))\left[\Pi_D'(R_{-i}) - \Pi_D'(R_{-i}+r_i)\right] \\ + g(x_i)h(x_i)\left[\Pi_D'(R_{-i}+\rho(x_i,r_i,R_{-i}))\left(1+\frac{\partial \rho}{\partial R_i}\right) - \Pi_D'(R_{-i}+r_i)\right]$$

Given that Π_D is convex, $\rho(x_i,r_i,R_{-i}) \leq r_i$ and the assumption that $\rho(x_i,r_i,R_{-i})$ is independent of r_i and R_{-i}) we can show that $\frac{\partial \Phi}{\partial r_i} \geq 0$ and $\frac{\partial \Phi}{\partial R_{-i}} \leq 0$. □

Proof of Proposition 13: The first part is immediate from the inspection of $r_2^c(r_1)$ and $\bar{r}_2(r_1)$.

Regarding the second part, define $\tilde{\rho}_1$ as the value for which patent holder 2 will be indifferent between going to court or offering a royalty rate that the downstream producer will accept. That is,

$$\tilde{\Pi}_2\left(\tilde{\rho}_1, r_2^c(\tilde{\rho}_1)\right) = g(x_2)\Pi_2\left(\tilde{\rho}_1, r_2^c(\tilde{\rho}_1)\right) - L_U = \Pi_2\left(\tilde{\rho}_1, \bar{r}_2(\tilde{\rho}_1)\right),$$

where

$$\tilde{\Pi}_2(r_1, r_2^c) = g(x_2)\frac{(1-r_1)^2}{8} - L_U,$$
$$\Pi_2(r_1, \bar{r}_2) = \frac{4L_D}{1-g(x_2)} - (1-r_1)\bar{r}_2(r_1).$$

We now show that this threshold is unique and litigation is preferred by patent holder 2 when $r_1 < \tilde{\rho}_1$. We can compute the effect of r_1 on both choices as

$$\frac{d\tilde{\Pi}_2}{dr_1}(r_1, r_2^c(r_1)) = -g(x_2)\frac{1-r_1}{4} < 0,$$
$$\frac{d\Pi_2}{dr_1}(r_1, \bar{r}_2(r_1)) = -\frac{\bar{r}_2(r_1)^2}{2\sqrt{(1-r_1)^2 - \frac{4L_D}{1-g(x_2)}}} < 0.$$

Both derivatives are negative. However, notice that $\frac{d\tilde{\Pi}_2}{dr_1}(r_1, r_2^c)$ is increasing in r_1 whereas $\frac{d\Pi_2}{dr_1}(r_1, \bar{r}_2(r_1))$ is increasing in r_1. That is, $\tilde{\Pi}_2(r_1, r_2^c)$ is convex in r_1 and $\Pi_2(r_1, \bar{r}_2)$ is concave in r_1. This implies that there might be 0, 1 or 2 points in which these functions cross. We can rule out the case in which the functions cross twice, because $\tilde{\Pi}_2(\hat{\rho}_1, r_2^c(\hat{\rho}_1)) < \Pi_2(\hat{\rho}_1, r_2^c(\hat{\rho}_1))$ since in this case litigation does not imply a higher royalty rate. Hence, two possibilities remain: (i) the functions do not cross, which occurs if $\tilde{\Pi}_2(r_1, r_2^c(r_1)) < \Pi_2(r_1, r_2^c(r_1))$ for all values of r_1 or (ii) there is a single crossing point $\hat{\rho}_1 \in (0, \tilde{\rho}_1)$, which occurs if $\tilde{\Pi}_2(0, r_2^c(0)) > \Pi_2(0, r_2^c(0))$. The second case arises when L_U is sufficiently low as stated in the lemma.

The effect of L_U and L_D can be characterized directly from the derivatives

$$\frac{d\tilde{\Pi}_2}{dL_U}(r_1, r_2^c(r_1)) = -1,$$
$$\frac{d\Pi_2}{dL_D}(r_1, \bar{r}_2(r_1)) = \frac{1}{2} - \frac{1 - r_1}{4\sqrt{(1-r_1)^2 - \frac{4L_D}{1-g(x_2)}}}.$$

Obviously, the first expression is always negative. The second is positive if and only if r_1 is in the relevant range, $r_1 \leq \hat{\rho}_1$. □

Proof of Proposition 14: To prove the result we only need to show that there are instances in which $\tilde{\rho}_1 > \frac{1}{2}$ so that $r_1 = 1/2$ would be feasible and it would induce litigation but raising the royalty so that patent holder 2 would instead induce accommodation increases profits for patent holder 1.

First, notice that for $\tilde{\rho}_1 > \frac{1}{2}$ it has to be that $\tilde{\Pi}_2\left(1/2, r_2^c(1/2)\right) > \Pi_2\left(1/2, \bar{r}_2(1/2)\right)$. This condition implies that

$$g(x_2)\frac{1}{32} - L_U > \frac{1}{2}\left(\frac{1}{2} - \sqrt{\frac{1}{4} - \frac{4L_D}{1-g(x_2)}}\right)\sqrt{\frac{1}{4} - \frac{4L_D}{1-g(x_2)}}.$$

The previous condition is satisfied if

$$\sqrt{\frac{1}{4} - \frac{4L_D}{1-g(x_2)}} \notin \left[\frac{1 - \sqrt{1 - g(x_2) + 32L_U}}{4}, \frac{1 + \sqrt{1 - g(x_2) + 32L_U}}{4}\right].$$

Since $\hat{\rho}_1 > \frac{1}{2}$ and this implies that $\frac{L_D}{1-g(x_2} > \frac{3}{64}$ the first expression is always higher than $\frac{1}{4}$ and hence, we have that only the values above $\frac{1+\sqrt{1-g(x_2)+32L_U}}{4}$ are relevant.

We now show that when $\tilde{\rho}_1 > \frac{1}{2}$, it has to be that $\Pi_1\left(1/2, r_2^c(1/2)\right) > \tilde{\Pi}_1\left(1/2, \bar{r}_2(1/2)\right)$. This condition holds if the expected royalty rate is lower without litigation. That is, if

$$\frac{1}{2} + g(x_2)\frac{1}{4} > \frac{1}{2} + \bar{r}_2(1/2).$$

The previous expression is equivalent to $\sqrt{1-g(x_2)+32L_U} > 1-g(x_2)$ which is always true.

By continuity, the previous conditions imply that if $\tilde{\rho}_1$ is sufficiently close to $\frac{1}{2}$ then $\Pi_1\left(\tilde{\rho}_1, r_2^c(\tilde{\rho}_1)\right) > \tilde{\Pi}_1\left({}^1/_2, \bar{r}_2({}^1/_2)\right)$.

$\square$