Royalty Stacking and Validity Challenges: The Inverse Cournot Effect *

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Abstract

It has been argued that the licensing of complementary patents leads to excessively large royalties due to the well-known royalty-stacking problem. This paper shows that considering litigation when firms have patents of heterogeneous strength may mitigate or even eliminate this distortion due to a moderating force that we denote the Inverse Cournot effect. The lower the royalty rate that other patent holders charge, the lower the royalty rate that those more exposed to litigation by downstream producers will set. Interestingly, this effect is mitigated when all firms have weak patents, making royalty stacking a more relevant concern in that case. We also show that these forces are moderated when litigation is profitable for a firm with a weak patent. In that case, a firm with a strong patent (or a strong patent portfolio) might prefer to discourage litigation through an increase in the royalty rate.

JEL codes: L15, L24, O31, O34.


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1 Introduction

The fundamental nature of the patent system is under debate among claims on whether it fosters or hurts innovation. The main concerns focus on the impact of patent enforcement in the Information and Communications Technologies (ICT) industry. ICT products, such as laptops, tablets, or smartphones use a variety of technologies covered by complementary patents. The sum of all royalties that must be paid for multiple patented technologies in a single product is said to form a harmful “royalty stack” (Lemley and Shapiro, 2007). This distortion would harm the incentives for firms to invest and innovate in product markets due to the excessively high end-product prices it would entail.

The arguments supporting royalty stacking and the need for a profound reform of the patent system rely on theoretical models which reformulate the well-known problem of Cournot-complements in a licensing framework. In industries where each single product is covered by patents of multiple owners, a patent holder may not fully take into account that an increase in the royalty rate is likely to result in a cumulative rate that may be too high according to other licensors, the licensees, and their customers. Since this negative externality (or Cournot effect) is ignored by all patent holders, the royalty stack may prove inefficiently high.

In this paper we develop a model of the licensing of complementary innovations under the threat of litigation that explains the circumstances under which royalty stacking is likely to be a problem in practice. This model departs from the extant literature in only one natural dimension; we assume that manufacturers of products covered by multiple patented technologies may challenge in court those patents and, crucially, that the likelihood that a judge rules in favor of the patent holder is increasing in the strength of its patent portfolio. This assumption is reasonable. Downstream manufacturers commonly challenge the validity of the patents that cover their products when they litigate in court.
the licensing terms offered by patent holders. Those with large and high quality patent portfolios will not be constrained by the threat of litigation when setting their royalty rate. On the contrary, owners of weak portfolios will have to moderate their royalty claims in order to avoid litigation concerning patent validity.

Interestingly, our analysis uncovers a positive relationship between the royalty rate that patent holders optimally set when facing the threat of litigation from a downstream user of their technology. That is, the ability of a patent holder to charge a high royalty rate without triggering litigation increases in the aggregate royalty rate charged by all other patent holders. The intuition is as follows. A downstream producer will decide to go to court and try to invalidate the patents of an upstream innovator if the increase in profits associated to the corresponding reduction in the aggregate royalty rate compensates for the legal costs involved. Due to the convexity of the profit function with respect to its own costs, this difference is decreasing in the royalty rate set by the remaining patent holders. That is, when the total royalty rate set by all other patent holders is already high and, consequently, profits are low in any case, the marginal gains from invalidating the portfolio of an innovator are also low. This means that the innovator can also charge a higher royalty rate and avoid being challenged in court. This positive relationship is a novel and very general insight that we denote as the \textit{Inverse Cournot effect} and it represents a positive externality among owners of complementary inputs (in this case patents). We show that this effect arises regardless of whether royalty rates are per-unit or ad-valorem or if royalties are offered as part of a two-part tariff.

As a result of the Inverse Cournot effect, royalty rate reductions become more appealing compared to the case where the threat of litigation is ignored. A strong patent holder, by lowering the royalty rate, forces rivals with weaker portfolios to reduce theirs, boosting total production. When the threat of litigation faced by these rivals is significant, the increase in the production is large and it compensates the strong patent holder for the reduced margin from the lower royalty rate. As a result, the Inverse Cournot
effect becomes a moderating force, partially offsetting the royalty-stacking problem that arises from the Cournot effect. This is in contrast with the case of a single patent holder owning both perfectly complementarity portfolios, who would always be able to charge the monopoly rate as long as one of the patents is sufficiently strong.

This channel becomes less effective, however, among patent holders with weak patent portfolios. To illustrate that result, we consider the case in which a licensee decides to sue patent holders in an endogenous sequence. Because as the portfolio of a patentee is invalidated the aggregate royalty rate goes down, the incentives for the downstream producer to subsequently sue other patent holders become stronger. As a result of this litigation cascade, when a weak patentee considers whether to lower the royalty rate it ought to anticipate that, although it might benefit from a smaller royalty stack through an increase in sales, there is also a greater probability of itself being brought to court. The risk of a litigation cascade mitigates the Inverse Cournot effect and, therefore, allows patent holders to sustain a higher royalty rate, sometimes as high as what a single patent holder would choose. Correspondingly, the royalty-stacking problem might be milder when strong and weak patent holders coexist compared to the case where all patent holders are weak.

The model is extended in several dimensions. We discuss some features specific to Standard Setting Organization (SSOs), where Standard Essential Patents (SEPs) are licensed on Fair, Reasonable, and Non-Discriminatory (FRAND) terms. We also explore how the analysis can be extended to account for the effect of downstream competition, royalty renegotiation, and sequential royalty-rate setting. In all cases, a modified version of the Inverse Cournot effect emerges.

The analysis in most of the paper considers litigation as a threat but assumes away the possibility that patent holders might prefer to fight the validity of their patents in court. In a final section we explicitly account for this option. We show that in that case the Inverse Cournot effect still applies but a new force in the opposite direction emerges,
particularly when a weak patent holder might find it worthwhile to go to court. In that case, a strong patent holder might prefer to raise the royalty rate rather than decrease it. Doing so allows the weak patent holder to raise its own royalty rate without spurring litigation. This strategy might yield a higher revenue for the strong patent holder because the increase in the royalty rate, compared to the one that would emerge under litigation, compensates for the lower expected quantity sold as a result.

We present the model in section 2 and section 3 characterizes the single innovator benchmark. Section 4 analyzes the case with two innovators and discusses the circumstances under which the Inverse Cournot and the Litigation Cascade effect arise depending on the strength of the portfolio of each firm. Section 5 extends and discusses the robustness of the results to changing some of the assumptions. Section 6 considers equilibrium litigation and section 7 concludes.

1.1 Literature Review

The existing theoretical literature has observed that the licensing of complementary and essential patents by many developers could give rise to a royalty-stacking problem (Lemley and Shapiro, 2007) and works like Lerner and Tirole (2004) have pointed out that patent pools constitute a natural way to mitigate its effects. It is also known that this distortion is smaller when royalties are paid ad-valorem instead of per-unit (Llobet and Padilla, 2016). There is more debate on its practical relevance in markets like mobile telecommunications, with some works arguing its large effects (Lemley, 2002), while others (Geradin et al., 2007) emphasize that the conditions necessary for its emergence do not typically arise in practice.

Our paper is also related to a long literature on the decision of a patent holder to sue firms that might have infringed its probabilistic patents, including papers like Llobet (2003) and Farrell and Shapiro (2008). More recent works have aimed to capture the interaction of these conflicts in contexts like SSOs, analyzing the litigation between
producers and Non-Practicing Entities (NPEs). This is the case, for example, of Choi and Gerlach (2018) that studies the information externalities that arise when a NPE sequentially sues several producers.

The paper closest to ours is Choi and Gerlach (2015). They develop a model in which patent holders with weak patents facing the threat of litigation moderate their royalty claims so that the aggregate royalty payment falls below the one that would emerge from a patent pool. In their setup the positive relationship between the royalty rate of both firms arises from a mechanism that differs from the Inverse Cournot effect identified in our paper. If a downstream producer invalidates the patent portfolio of one of the firms, the rival can raise its own royalty rate. This means that the best response of a patent holder to the reduction in the royalty charged for the complementary portfolio of another patentee may be to reduce one’s own royalty in order to reduce the likelihood of a validity challenge against its own patent. This effect does not arise in our model, since the Inverse Cournot effect occurs even when one of the patentees has ironclad patents so that its decision to lower its royalty rate is not driven by the need to avoid going to court.

Bourreau et al. (2015) consider a setup similar to ours to study licensing and litigation in Standard Setting Organizations. The main difference with our paper is that in their setup litigation occurs after production has taken place. As a result, the total quantity produced does not depend on the outcome of this litigation but only on the aggregate royalty rate negotiated ex-ante. This assumption severs the link between the licensing decisions of patent holders and the litigation decisions of licensees, thus eliminating the Inverse Cournot effect that plays a crucial role in our setup.

Finally, this paper is related to the literature on patent holdout (Epstein and Noroozi, 2018). Small innovators might not find it worthwhile to enforce their patent portfolio due to the legal costs involved. In anticipation of that, it is optimal for users of the technology to infringe on those patents unless the royalty rate offered is sufficiently low. As Lichtman
(2006) points out, this implies that when complementary patents that cover the same technology are owned by a few firms, the commitment to litigate will be stronger and the royalty stacking problem will be more relevant than when ownership is disperse. Although patent holdout is present in our paper, our model emphasizes how the interaction with other patent holders might affect the incentives to litigate above and beyond this effect. We show that the Inverse Cournot Effect induces even strong patent holders that are unaffected by patent holdout to choose a lower royalty rate. Furthermore, we show that once we account for this effect, weaker portfolios might actually lead to a higher royalty stack.

2 The Model

Consider a market in which a downstream monopolist, firm $B$, faces a twice-continuously differentiable demand function $D(p)$, decreasing in the price $p$. The production of the good requires the usage of technologies patented by two pure upstream innovators. Innovator $i = 1, 2$ holds a patent with strength $x_i$ relevant for its own technology, with $x_1 \geq x_2$. Each innovator charges a per-unit royalty $r_i$ to license the patent that covers its technology.\footnote{As pointed out in Llobet and Padilla (2016) royalty-stacking problems are aggravated under per-unit royalties compared to the more frequent ad-valorem royalties, based on firm revenue. As discussed in section 5, the mechanisms discussed in this paper also operate when royalties are assumed to be ad-valorem but they lead to a more complicated exposition.} We denote the total royalty rate as $R \equiv r_1 + r_2$. We assume that there is no further cost of production so that the marginal cost of the final product is also equal to $R$.

Most of the results of the paper do not require that we explicitly model the pricing decision of the downstream producer in the final market. It is enough to make the following assumption on how the quantity sold depends on the aggregate royalty rate.

**Assumption 1.** Define as $\tilde{D}(R)$ the total quantity sold in the final market as a function of $R$. This function is a decreasing and log-concave function of $R$, with $\tilde{D}(0) > 0$ and $\tilde{D}(R) \to 0$ as $R \to \infty$. 
These are standard regularity conditions guaranteeing that the patent holders’ profit function is well-behaved. It is worth to discuss two extreme cases. When the downstream producer can extract all the surplus from consumers using perfect price discrimination, the previous assumptions imply that $D(p)$ is log-concave in $p$, as typically assumed in in the literature. At the other extreme, when the downstream producer chooses a unique monopoly price for the product, $\bar{p}(R)$, this assumption imposes conditions on $\bar{D}(R) \equiv D(\bar{p}(R))$. Double marginalization will arise in this last case.

We denote the profits of the downstream producer as $\Pi_B(R)$. Standard arguments allow us to show that $\Pi'_B(R) = -\bar{D}(R) < 0$ and the previous assumption implies that $\Pi''_B(R) = -\bar{D}'(R) > 0$, so that the profits of the downstream producer are convex in $R$.

The royalty rate for technology $i$ is set by innovator $i$ as a take-it-or-leave-it offer. The downstream producer, however, might challenge in court the patent that covers that technology. Litigation involves positive costs $L_B$ and $L_U$ for the downstream monopolist and any upstream patent holder, respectively. The success in court of innovator $i$ is based on the strength of its patent, $x_i$. In particular, the probability that a judge rules in favor of patent holder $i$, denoted as $g(x_i)$, is assumed to be increasing in $x_i$. To the extent that more substantial innovations translate into stronger patents, we can interpret the increasing function $g(x_i)$ as a reflection of this relationship. If innovator $i$ wins in court the downstream producer must pay the royalty rate $r_i$. Otherwise, the royalty rate of that innovator is reduced to 0. When indifferent we assume that the downstream producer prefers not to litigate.

We can summarize the payoffs of the downstream producer if it decides to only take innovator $i$ to court as

$$(1 - g(x_i))\Pi_B(r_j) + g(x_i)\Pi_B(r_i + r_j) - L_B,$$

where $r_j$ is the royalty rate charged by innovator $j \neq i$. If the patent of innovator $j$ had

\[\text{For simplicity we abstract from situations in which upstream patent holders own the rights for technologies that might be infringed by other upstream patent holders.}\]
already been litigated, profits in that case would become

$$(1 - g(x_i))\Pi_B(r_j) + g(x_i)\Pi_B(r_i + r_j) - 2L_B,$$

where the original royalty rate $r_j$ would be replaced by 0 if the patent of innovator $j$ had been invalidated, with probability $1 - g(x_j)$. Notice that this discussion assumes that litigation is sequential. Since royalty rates are established before any litigation takes place, the decision to simultaneously litigate both patents is equivalent to sequential litigation where the second trial occurs regardless of the initial outcome.

In the main sections of the paper we focus on the case in which $L_U$ is relatively high so that litigation is a significant threat but it never emerges in equilibrium. That is, it is always optimal for patent holders to choose a royalty rate that discourages litigation by the downstream producer, leading to profits of $r_i\tilde{D}(r_i + r_j)$ for innovator $i$ given $j \neq i$. This case can be understood as a situation in which innovators individually benefit relatively less from licensing than downstream producers from avoiding to pay the royalty rate. Doing so allows us to focus on the litigation incentives of the downstream producer and compare this situation with the standard royalty-stacking case where this threat does not exist. In section 6 we extend the analysis to consider lower values of $L_U$ and we explore the incentives for both patent holders to go to court. In that case, the expected profits of innovator $i$ would become

$$g(x_i)r_i\tilde{D}(r_i + r_j) - L_U,$$

where $r_j$ would be 0 if the patent of firm $j$ had been invalidated in a previous trial. We also discuss the conditions under which litigation arises in equilibrium and how it affects our conclusions.

The timing of the model is described in Figure 1. First, upstream innovators simultaneously choose their royalty rates. In the second stage the downstream producer chooses which patentees to take to court (if any) and the sequence. In the final stage, once
litigation has been resolved, and given the outstanding royalty rate $R$, the downstream producer sells in the final market.

In order to characterize the equilibrium of the game depending on the strength of the patent of each firm it is useful to start with the benchmark case of a single innovator that owns both patents.

### 3 A Single Innovator

Suppose that both patents are owned by the same innovator, who licenses them to the downstream producer. When litigation is not a relevant threat, the perfect complementarity between both technologies implies that any combination of royalty rates $r_1 + r_2 = R^M$, where $R^M = \arg\max_R R\tilde{D}(R)$, will maximize upstream profits.

The previous royalty rate arises in equilibrium as long as both patents are sufficiently strong — that is, if $g(x_i)$ is very close to 1 for $i = 1, 2$ — or, alternatively, when the legal costs of the downstream producer, $L_B$, are sufficiently high. In that case, it is not in the interest of the downstream producer to challenge in court any of the patents and it will pay the total royalty rate $R^M$.

Consider now the situation where one of the patents is weak. For simplicity, suppose that $g(x_1) = 1$ but $g(x_2) < 1$ so that the first patent will never be litigated by the downstream producer. This firm prefers not to challenge in court the second patent if and only if

$$(1 - g(x_2)) [\Pi_B(r_1) - \Pi_B(r_1 + r_2)] \leq L_B.$$ (1)
It is clear that this condition will hold if $r_1^* = R^M$ and $r_2^* = 0$. Hence, due to the perfect complementarity of the two patents, it is enough that one of them is sufficiently strong to guarantee that the monopolist innovator can attain the total royalty rate $R^M$ without triggering litigation by the downstream producer. Of course, the optimal apportionment of the total royalty rate may not be unique if $g(x_2) > 0$ and any combination $(r_1, r_2)$ such that $r_1 + r_2 = R^M$ that also satisfies equation (1) will yield the same profits.

Finally, suppose now that both patents are (equally) weak. To be more precise, assume that $g(x_1) = g(x_2) = g(x)$ is small and that

$$(1 - g(x)) \left[ \Pi_B(0) - \Pi_B(R^M) \right] > L_B. \quad (2)$$

This condition implies that if the total monopoly royalty rate is allocated to one of the patents the downstream producer finds it worthwhile to go to court and try to invalidate it. In this case, it is optimal for the patent holder to apportion the total royalty rate between the two patents. The highest royalty rate that the patent holder can demand for each patent is limited by the legal costs that the downstream producer must incur to challenge it in court.

Interestingly, litigation here might involve one or both patents. Sequential litigation allows the downstream producer to condition the decision to challenge a patent in court on the outcome of the previous trial. This strategy is optimal due to the convexity of $\Pi_B(R)$. This means that the litigation of a patent might be optimal when the other one has already been invalidated, but not when it has been proved to be valid. That is, it could be the case that

$$(1 - g(x)) \left[ \Pi_B(0) - \Pi_B(r_i) \right] > L_B > (1 - g(x)) \left[ \Pi_B(r_j) - \Pi_B(r_1 + r_2) \right],$$

for some $i = 1, 2$ and $j \neq i$. By waiting until the outcome of the first trial has been revealed, the downstream producer would benefit from the option value of going to court a second time only in those states of the world where it is worthwhile.
The next result summarizes the previous discussion and characterizes the optimal royalty rate when litigation is a relevant concern for both patents and the downstream producer chooses endogenously the sequence under which they are challenged.

**Proposition 1.** When \( g(x_1) \geq g(x_2) \) is sufficiently high so that condition (2) is not satisfied, unconstrained monopoly profits can be attained. If \( g(x_1) = g(x_2) = g(x) \) is sufficiently small so that condition (2) holds, then it is optimal to charge a positive royalty rate for both patents. Furthermore, when \( L_B \) is sufficiently low so that the total monopoly royalty rate, \( R^M \), cannot be attained without triggering litigation, the optimal combination of royalty rates is unique, with \( r_1^m = r_2^m = r^m \). The royalty rate is increasing in \( g(x) \) and \( L_B \), implicitly defined as

\[
g(x)\Pi_B(r^m) + (1 - g(x))\Pi_B(0) - \Pi_B(2r^m) = \frac{L_B}{1 - g(x)} + L_B. \tag{3}
\]

In order to interpret the previous result it is useful to focus first on the case in which both patents command the same royalty rate, \( r_1 = r_2 = r \). Because \( \Pi_B(R) \) is a convex function of \( R \), we have that \( \Pi_B(r) - \Pi_B(2r) < \Pi_B(0) - \Pi_B(r) \). This implies that if it is profitable for the downstream producer to challenge one of the patents, it will also be profitable to challenge the other one upon an initial success in court. It also means that the litigation of both patents will not be profitable if

\[
(1 - g(x)) [\Pi_B(r) - \Pi_B(2r)] + (1 - g(x)) \{(1 - g(x)) [\Pi_B(0) - \Pi_B(r)] - L_B\} \leq L_B. \tag{4}
\]

The first term in the left-hand side of the equation identifies the gains of the downstream producer to challenge in court one of the patents, as described in equation (1). The second term captures the *option value* that litigation may bring about. That is, if the downstream producer wins the first trial the profitability of challenging the other patent increases. We call this result a *litigation cascade.*

\[\footnote{In practice, litigation might take years and a second trial might start before the first one has concluded if the information uncovered by the downstream producer during the process indicates that the revised probability of success is sufficiently high. The implications of such a strategy are very similar to the fully sequential setup assumed here.}\]
Because the previous expression is increasing in $r$ and the innovator is constrained by the weak patents it owns, it is always optimal to choose the highest royalty rate compatible with not triggering the litigation of any of the patents, which determines equation (3) and, hence, $r^m$. Notice that the equality in equation (4) implies that challenging the first patent must yield an expected revenue lower than the cost of going to court, $L_B$. However, it also implies that the profits from the second trial, which occurs with probability $1 - g(x)$, compensate for the losses from going to court the first time. That is, when indifferent between going to court or not, the downstream producer is only motivated to litigate by the prospect of invalidating both patents. Of course, the previous condition also means that if the challenge to the first patent is unsuccessful, litigating the other one would be unprofitable, as the returns from that second trial would be identical to those faced for the first patent.

When both patents are equally weak it is optimal to offer them at the same royalty rate because, by doing so, the cost of the downstream producer to try to invalidate them is maximized as, in order for litigation to be profitable it would have to incur the legal costs twice (in the second case with probability $1 - g(x)$). In contrast, charging different royalty rates could foster the litigation of the patent with the highest rate and, possibly, imply a cascade. As this would reduce the costs of the downstream producer of going to court, the monopolist patent holder would need to charge a lower total royalty rate to fend off litigation.

The three cases characterized here are a useful reference for the situation discussed next where there are two innovators. Before we do that, however, we can informally discuss the optimal royalty rate when $1 > g(x_1) > g(x_2) > 0$ and no combination of royalty rates can attain the monopoly total rate $R_M$ without triggering litigation. By continuity, we know that when the difference between $g(x_1)$ and $g(x_2)$ is sufficiently large, patent 1 will command a higher royalty rate. The apportionment of the total rate will be such that litigating one of the patents will yield strictly negative profits and the prospect
of initiating a cascade will not compensate for this loss.

4 Two Innovators

We consider now the case where each patent is owned by a different innovator. Following the previous discussion, we analyze three different situations depending on whether the two patents are strong, only one is strong, or both patents are weak.

4.1 Two Strong Patents

Suppose that both upstream innovators have a sufficiently strong patent so that litigation by the downstream producer will never be a credible threat, $g(x_1) = g(x_2) = 1$. The profits of innovator $i$ can be defined as

$$\Pi_i(r_j) = \max_{r_i} r_i \tilde{D}(r_i + r_j),$$

where $j \neq i$. We denote the royalty rate that corresponds to the Nash Equilibrium of the game when firms are unconstrained by litigation as $r_i^u = r^u$ for all $i$. For completeness, we reproduce next the standard royalty-stacking result (see, for example, Lemley and Shapiro (2007)), which shows that this royalty rate is higher that the one we characterized in the previous section, where a unique firm maximized the profits from licensing both patents, $R_M$. Assumption 1 not only guarantees concavity of the patent holder’s problem but it also implies that royalty rates are strategic substitutes, delivering the following result.

**Proposition 2** (Royalty Stacking). When $g(x_1) = g(x_2) = 1$ the game has a unique equilibrium in which all innovators choose $r_i^u = r^u$. The total royalty rate is higher than the one that would emerge if both patents were owned by the same innovator, $2r^u > R_M$.

This result is a version of the Cournot-complements effect under which firms choosing quantities of complementary products induce final prices even higher than those of a monopolist. The intuition here is very similar. The decision of a patent holder to increase the royalty rate trades off the higher margin with the lower quantity sold but without
internalizing the fact that this decrease in the quantity has a negative effect on the royalty revenues of the other patent holder.$^4$

4.2 One Weak Patent: The Inverse Cournot Effect

Suppose now that $g(x_1) = 1$ but $g(x_2) < 1$ so that only patent holder 2 may face litigation by the downstream producer. Given the royalty rates chosen in the first stage, the downstream producer prefers not to challenge in court the patent of innovator 2 if and only if (1) holds. That is, litigation is unprofitable if the expected gains from avoiding to license the patent of innovator 2 are lower than the costs involved. The next lemma characterizes the values of $r_1$ for which litigation will emerge.

Lemma 3. If $L_B > (1 - g(x_2)) [\Pi_B(0) - \Pi_B(r_2)]$ innovator 2 will not be brought to court for any $r_1 > 0$. For lower values of $L_B$, the downstream producer will litigate the patent of innovator 2 if $r_1 < \tilde{r}_1(L_B, x_2, r_2)$. The threshold royalty rate $\tilde{r}_1$ is increasing in $r_2$ and decreasing in $L_B$ and $x_2$.

The previous lemma distinguishes two regions. When $L_B$ is high, litigation will not be a meaningful threat. For lower values of $L_B$, the decision of the downstream producer to sue innovator 2 depends on the royalty rate set by the other patent holder. In particular, a positive royalty rate $r_1 < \tilde{r}_1(L_B, x_2, r_2)$ chosen by innovator 1 will spur the litigation of the patent of innovator 2. The intuition is as follows. If $r_1$ is high, profits for the downstream producer are low, independently of whether the patent of innovator 2 is upheld in court or not. Thus, it is unlikely that the gains from litigation offset the costs involved. When $r_1$ is reduced, and due to the fact that the profit function $\Pi_B(R)$ is convex in $R$, the difference in profits when the patent of innovator 2 is upheld in court or invalidated increases, enticing downstream litigation.

An immediate consequence of this result is that if $L_B$ is sufficiently low royalty stacking is mitigated and the equilibrium characterized in Proposition 2 will fail to exist. More

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$^4$This result holds for a generic number of firms meaning that the royalty-stacking problem becomes more severe when the total number of patents is fragmented in the hands of more firms.
interesting is the fact that, as we will see next, the threat of litigation might operate even in the case in which the original equilibrium satisfies equation (1), i.e. when \( r^u \geq \bar{r}_1(L_B, x_2, r^u) \).

In the model without litigation, royalty stacking arises because royalty rates are strategic substitutes. Since both patent holders choose their royalty rate without anticipating that the reduction of the quantity sold downstream negatively affects the other patent holder, they engender a total royalty rate that becomes too high. The threat of litigation provides a moderating effect on the royalty rate that innovator 2 will offer to avoid being brought to court. Furthermore, innovator 1 anticipates that reducing \( r_1 \) induces a decrease of \( r_2 \). We denote this mechanism the Inverse Cournot effect and it operates in the opposite direction of the standard Cournot Effect.\(^5\) This new effect generates a positive relationship between \( r_1 \) and \( r_2 \), allowing innovator 1 to internalize the gains that a lower royalty rate would bring about due to the higher quantity sold in the final market.

In any equilibrium with royalty rates \( r_1^* \) and \( r_2^* \), innovator 2 will avoid being sued if (1) holds. However, this condition also implies that there will never be a Nash Equilibrium in which the downstream producer is indifferent between litigating innovator 2’s patent or not. The reason is that innovator 1 would always prefer to lower a bit the royalty rate and induce innovator 2 to be brought to court. At essentially no cost, with probability \( 1 - g(x_2) \) innovator 1 would become the only firm licensing the technology. This deviation would be profitable as it generates a discrete increase in the quantity sold downstream. If, instead, equation (1) held with inequality, innovator 2 would find optimal to raise its royalty rate unless it were already equal to \( r^u \). A consequence of this insight is that unless \( L_B \) is so high that the litigation threat is irrelevant and \( r_1^* = r_2^* = r^u \), there will be no pure-strategy equilibrium without litigation.

\textbf{Proposition 4.} An equilibrium in pure strategies and no litigation exists if and only if

\(^5\)Of course, this effect immediately generalizes to the case of \( N \) patent holders with a portfolio sufficiently strong so that it will never be litigated. In that case, the Inverse Cournot effect would indicate that the highest royalty that patentee 2 can charge is increasing in the sum of the royalty of all the other patent holders.
When \( L_B \) is small, given that demand is decreasing in the aggregate royalty rate, a Nash equilibrium in which litigation does not take place necessarily implies mixed strategies. Innovator \( i = 1, 2 \) randomizes according to a distribution \( F_i(r_i) \) in a support \([r_i^L, r_i^H]\). Innovator 2 when choosing a lower \( r_2 \) trades off a lower probability of being sued with a higher payoff when litigation occurs and it succeeds in court. This trade-off means that innovator 2 will choose a lower expected royalty rate than when litigation was not a threat. In the case of innovator 1 two effects go in opposite directions. On the one hand, due to the Inverse Cournot effect, the innovator has incentives to lower the royalty rate \( r_1 \) in order to enjoy monopoly profits with a higher probability. On the other hand, there is a positive probability that the other patent is invalidated for a given \( r_1 \) and, in that case, it becomes optimal to increase the royalty rate.

Figure 2 provides a numerical example that illustrates the previous trade-offs. Compared to the equilibrium royalty rate when both patents are strong, \( r^u = \frac{1}{3} \), innovator 2’s randomization uses a lower support and will always set a lower \( r_2 \). In contrast, innovator 1 sets a higher expected royalty rate. However, the Inverse Cournot effect is reflected in the fact that the support of \( r_1 \) also includes royalty rates below \( r^u \). As a result, the expected total royalty rate is lower in this case compared to when both patents are strong, as opposed to what occurs in the monopoly case. In section 6 we explore the case where litigation is a possible equilibrium outcome and we make further assumptions that allow us to discuss these different effects in more detail.

The positive effect of an increase in \( r_1 \) on the royalty rate of the weak patent holder that we uncover here is new in the literature. In particular, in Choi and Gerlach (2015) downstream profits are linearly decreasing in the total royalty rate (rather than convex as in our model) and, hence, the mechanism described in Lemma 3 does not operate. In other words, in their model reducing \( r_1 \) by itself does not make suing innovator 2 more profitable. Instead, in their setup it is assumed that when the patent of innovator 2 has

\[ r_1^* = r_2^* = r^u. \]
been successfully upheld in court the firm can raise its royalty rate. The room for an ex-post increase is lower when the royalty rate charged by innovator 1 is higher. As a result, the cost for the downstream producer of losing in court against innovator 2 decreases and it is more willing to litigate. This leads to a negative rather than a positive effect (i.e. litigation against innovator 2 becomes more attractive when $r_1$ goes up rather than down, as in our model).\footnote{A positive effect arises when both patent holders are weak. In that case, when the downstream producer is indifferent between suing either of them but not both due to the litigation costs involved, the decrease in the royalty rate of innovator 1 increases the probability that the competitor is brought to court, reducing, in turn, its royalty rate. The downstream producer’s upside is greater and its downside lower when suing the patent holder with the high royalty rate.}

Finally, it is useful to compare the results of our model to the case in which a monopolist patent holder sets both royalty rates at the same time. As discussed in section 3, due to the complementarity between both technologies, it is optimal for a monopolist to place most of the burden on the strong patent, for example, by choosing $r_2 = 0$ and $r_1 = R^M$. Hence, in the case of a single innovator, due to the perfect complementarity and absent any strategic effects, the total royalty rate would be unchanged as long as at least one of the patents is sufficiently strong so that litigation would never be profitable for the downstream producer. This is in contrast with the case discussed in this section, where the strategic complementarity induced by the Inverse Cournot effect provides incentives for innovators to set a total royalty rate lower than what would emerge under
royalty stacking. This occurs even when litigation does not take place in equilibrium and where, as discussed earlier, a royalty rate $r^n$ by both firms would not have made litigation worthwhile for the downstream producer.

4.3 Two Weak Patents: Strategic Effects of Litigation Cascades

We now turn to the case in which both patents are equally weak, $g(x_1) = g(x_2) = g(x) < 1$. As in the monopoly case discussed before, we assume that the downstream producer sues patent holders in an endogenous sequence that can be conditioned on the previous court outcome. Our first result characterizes the optimal order under which patents will be challenged in court.

**Lemma 5.** When both patents are equally weak, it is always optimal for the downstream producer to challenge first the patent associated to the highest royalty rate.

The higher the royalty rate of a patent holder the more likely it is that litigation pays off irrespective of the outcome of the lawsuit against the other patent holder. In contrast, the litigation of the patent with the low royalty rate is less profitable and whether it is optimal to go to court or not might hinge on the outcome of the other trial. Thus, it is optimal to postpone litigation of that patent until the resolution of the first court case. In the rest of the paper we assume that when both patent holders set the same royalty rate they are brought to court first with probability $1/2$.

The previous result is useful to anticipate the changes in the probability that innovators are brought to court as a result of variations in the royalty rate. We now explore the condition under which a symmetric equilibrium may exist. As discussed in section 3, if both innovators choose the same royalty rate, the downstream producer will not be willing to litigate if (4) holds. It is immediate that this condition is less likely to be satisfied than the one driving the decision to sue innovator 2 when only this firm is constrained, as illustrated in equation (1). This comparison would suggest that, before we account for the optimal response of the innovators to the increased litigation, the royalty-stacking
problem would become less severe when both patents are weak. As we will see next, the opposite may actually be true once we account for these strategic considerations.

Given \( r_1 \) and \( r_2 \) and the endogenous ordering implied by Lemma 5, we can compute the gains of the downstream producer from suing innovator 2 contingent on success in the first trial — to be compared to \( L_B + (1 - g(x))L_B \) — as

\[
\Phi(r_1, r_2) \equiv \begin{cases} 
\Pi_B(r_2) - \Pi_B(r_1 + r_2) + (1 - g(x))[\Pi_B(0) - \Pi_B(r_2)] & \text{if } r_1 > r_2, \\
\Pi_B(r) - \Pi_B(2r) + (1 - g(x))[\Pi_B(0) - \Pi_B(r)] & \text{if } r_1 = r_2 = r, \\
\Pi_B(r_1) - \Pi_B(r_1 + r_2) + (1 - g(x))[\Pi_B(0) - \Pi_B(r_1)] & \text{otherwise.}
\end{cases}
\]

These gross profits change with \( r_1 \) according to

\[
\frac{\partial \Phi}{\partial r_1} = \begin{cases} 
-\Pi_B'(r_1 + r_2) & \text{if } r_1 \geq r_2, \\
\Pi_B'(r_1) - \Pi_B'(r_1 + r_2) - (1 - g(x))\Pi_B'(r_1) & \text{otherwise.}
\end{cases}
\]

This expression implies that increases and decreases of \( r_1 \) around \( r_2 \) have an asymmetric effect on the willingness to litigate of the downstream producer. Consider an initial situation in which \( r_1 = r_2 \). As expected, an increase in \( r_1 \) raises the profitability of challenging the patent of innovator 1 as the downstream profits without litigation are smaller. In contrast, decreases in \( r_1 \) below \( r_2 \), lead to two opposite effects. On the one hand, the first two terms correspond to the Inverse Cournot effect and imply that innovator 2 is more likely to be brought to court and, in turn, trigger a litigation cascade. On the other hand, contingent on the patent of innovator 2 being invalidated, which occurs with probability \( 1 - g(x) \), a lower \( r_1 \) reduces the expected gains from trying to invalidate also the patent of innovator 1 by \((1 - g(x))\Pi_B'(r_1)\). Hence, the total effect of a decrease in \( r_1 \) in the chances that innovator 1 ends up in court is in general ambiguous. The following example illustrates this point.

**Example 1.** Under a linear demand function, \( D(p) = 1 - p \), a downstream monopoly price, and symmetric royalty rates \( r_1 = r_2 = r \), a decrease in the royalty rate of one of the innovators lowers the return from litigation of the downstream producer if and only if

\[
\frac{1 - g(x)}{2 - g(x)}.
\]

Notice that in the previous example, the unconstrained equilibrium royalty rate is
Thus, if \( g(x) < \frac{1}{4} \), the litigation cascade will dominate the Inverse Cournot effect, making a deviation of a patent holder from \( r^u \) unprofitable.

As opposed to the case of one weak patent, when both patents are weak the risk of a litigation cascade places a lower bound on the innovator’s decrease in the royalty rate. As the next proposition states, this limit may help sustain a symmetric Nash Equilibrium in pure strategies without litigation.

**Proposition 6.** Suppose that \( L_U \) is large so that there is no litigation in equilibrium. If patent holders are identical, the demand function is linear, and monopoly pricing is used, when \( g(x) \) and \( L_B \) are sufficiently small, a unique symmetric equilibrium in pure strategies exists, where \( r^*_1 = r^*_2 = r^* \) solves

\[
g(x)\Pi_B(r^*) + (1 - g(x))\Pi_B(0) - \Pi_B(2r^*) = \frac{L_B}{1 - g(x)} + L_B.
\]

and \( r^* < r^u \). The equilibrium royalty rate is increasing in \( g(x) \) and \( L_B \).

This result provides conditions under which a pure-strategy equilibrium without litigation that differs from the one in the case of strong patents might emerge. In order to interpret this outcome, consider a deviation in the royalty rate. An increase will surely foster litigation and will not pay off when the cost \( L_U \) is high. Lowering slightly the royalty rate below \( r^* \) implies that the patent of the other innovator is litigated first. However, given that \( g(x) \) is small, a litigation cascade might affect the deviating patent holder, undermining the profitability of this decision. Finally, a significant decrease in the royalty rate would discourage further litigation if the downstream producer were successful against innovator 2. The lower is \( L_B \) the lower this royalty rate must be and, again, the less profitable this deviation becomes.

The comparison with the case of a single innovator allows us to highlight the implications of the litigation cascades. By Proposition 1, when patents are sufficiently weak both situations deliver the same royalty rate, \( r^m = r^* \). The reason is that when patents are weak innovators are wary of a decrease in the royalty rate that might lead the other
firm to court and the additional litigation that this might bring about. Consequently, the strategic considerations captured by Inverse Cournot effect become muted and firms choose in equilibrium the highest royalty rate compatible with preventing litigation by the downstream producer.\footnote{Notice that the previous result does not mean that the royalty rate defined in (6) emerges under the same region of parameters in both cases. In particular, it could be that a single monopolist is not constrained by litigation and would choose $R^M$ whereas two innovators would be forced to choose a royalty rate $r^*$.}

The previous arguments allows us to identify the different strategic behavior when both patents are owned by different firms, compared to the case of a firm that owns both patents. In the case of a single innovator, an increase in the strength of a patent is associated with a (weakly) higher royalty rate. Interestingly, the Inverse Cournot effect suggests that this monotonicity does not need to hold when the patents are owned by two innovators and only one of them becomes stronger. In fact, when that patent becomes sufficiently strong, the innovator is no longer constrained by the risk of being brought to court and is willing to lower the royalty rate to induce the litigation of the other patent. In Appendix B we provide an example where this effect results in a lower equilibrium royalty rate in spite of the stronger overall intellectual property.

## 5 Robustness and Extensions

We now study the effect of changing some of the maintained assumptions throughout the paper.

### 5.1 Ad-Valorem Royalties

Although most of the literature on innovation has assumed that royalties are paid per unit sold in the downstream market, in many technological industries patents are licensed using ad-valorem royalties, understood as a percentage of the revenue of the licensee.\footnote{See, for example, Bousquet et al. (1998). As discussed in section 5.6, royalties are also combined with a fixed fee.} As Lloret and Padilla (2016) show, absent litigation, ad-valorem royalties mitigate the
royalty stacking problem.

In this section we show that the same moderating force introduced by the Inverse Cournot effect also arises under ad-valorem royalties. In particular, consider the case in which the downstream producer faces a demand $D(p)$ and it incurs in a marginal cost of production $c > 0$. When innovators 1 and 2 charge ad-valorem royalties $s_1$ and $s_2$ and the aggregate rate is $S \equiv s_1 + s_2$, the problem of the downstream producer that chooses a downstream price $p$ can be written as

$$\Pi_B(S) = \max_p [(1 - S)p - c] D(p).$$

The monopoly price, $p^M$, is increasing in $S$ under standard regularity conditions, such as the log-concavity of the demand function. This requirement is also enough to show that $\Pi_B(S)$ is decreasing and convex in $S$. As a result, if we consider the case in which $g(x_2) < g(x_1) = 1$, the downstream producer will challenge in court the patent of innovator 2 if $s_1$ is lower than a threshold level $\bar{s}_1$, defined as

$$(1 - g(x_2)) [\Pi_B(\bar{s}_1) - \Pi_B(\bar{s}_1 + s_2)] = L_B.$$  \hspace{1cm} (7)

It is immediate that a counterpart of Lemma 3 can be obtained in this case, with $\bar{s}_1$ increasing in $s_2$. As a result, patent holder 1 has incentives to lower $s_1$ in order to induce patentee 2 to lower $s_2$ and prevent being litigated. See Appendix B for an example.

### 5.2 Downstream Competition

In this section we show that as downstream competition increases, the Inverse Cournot Effect is moderated but it does not necessarily disappear. There are two reasons for this weaker effect. First, more competition leads not only to lower downstream profits but also to lower differential profits from invalidating a patent. Second, a free-riding problem arises. If courts invalidate the patent of one of the innovators the royalty rate that all downstream producers pay to that firm is also reduced to 0.

\textsuperscript{9}As it is well-known, the problem when $c = 0$ is trivial, since an ad-valorem royalty rate of 100% would always be optimal, as it would create no distortion in the final market.
Regarding the first effect, consider a downstream market with \( N \) identical competitors. Their only marginal cost of production is the total royalty rate \( R \). Denote profits as \( \Pi_B(R, N) \). Under standard conditions, \( \Pi_B(R, N) \) is decreasing in both arguments and convex in \( R \).

Consider the case where \( g(x_1) = 1 \). Suppose that if a total of \( n \leq N \) downstream firms challenge the patent of innovator 2 in court, it will be considered valid with probability \( g(x_2, n) \), weakly decreasing in \( n \). Each downstream producer incurs a litigation cost \( L_B \) by going to court.

Any downstream firm will be indifferent between challenging patent 2 or not, assuming that no other downstream firm goes to court too, if \( r_1 \leq \bar{r}_1 \), defined as

\[
(1 - g(x_2, 1)) [\Pi_B(\bar{r}_1, N) - \Pi_B(\bar{r}_1 + r_2, N)] = L_B.
\]

As in the baseline model, the Inverse Cournot effect arises due to the convexity of the profit function with respect to \( R \). Furthermore, if \( \frac{\partial \Pi_B}{\partial R \partial N} > 0 \) then \( \frac{dr_1}{dN} < 0 \). This condition holds under many of the typical demand specifications.

**Example 2** (Cournot Competition). Under a linear demand function \( P(Q) = a - Q \), where \( R < a \), \( \frac{\partial \Pi_B}{\partial R \partial N} = \frac{a - R}{2N} > 0 \). When demand is isoelastic, \( P(Q) = Q^{\frac{1}{\eta}} \), the cross derivative of the equilibrium profit function corresponds to

\[
\frac{\partial \Pi_B}{\partial R \partial N} = (\eta - 1)\eta^{-\eta}R^{-\eta}(\eta - N)^{\eta - 2} > 0.
\]

**Example 3** (Product Differentiation). Suppose that downstream producers sell differentiated products, with a degree of substitutability identified by the parameter \( \gamma \geq 0 \). Firm \( i \) faces a demand function

\[
q_i = \frac{1}{N} \left[ v - p_i(1 + \gamma) + \frac{\gamma}{N} \sum_{j=1}^{N} p_j \right].
\]

Using the expression for the profits in the symmetric equilibrium we have

\[
\frac{\partial \Pi_B}{\partial R \partial N} = \frac{2(v - R) [(N - 1)\gamma(3 + \gamma) + 2N]}{((N - 1)\gamma + 2N)^3} > 0.
\]
In the previous examples as the number of downstream firms increases the Inverse Cournot effect becomes weaker. That is, innovator 1 must decrease the royalty rate further in order to induce litigation against the other innovator. The second example indicates that as product differentiation increases, understood as a decrease in $\gamma$, the threshold value $r_1$ increases. The reason is that product differentiation operates as a decrease in competition.

Finally, in order to study the free-riding effect, let’s focus on the case with $N = 2$. The profits of a downstream producer when $n \leq 2$ firms litigate, gross of litigation costs, can be written as

$$V_B(n) = (1 - g(x_2, n))\Pi_B(r_1, 2) + g(x_2, n)\Pi_B(r_1 + r_2, 2).$$

Suppose that it is worthwhile for the two downstream firms to challenge in court patent 2. That is, $2V_B(2) - 2L_B > 2V_B(1) - L_B$. It is easy to see that if one of the firms litigates, the other firm will also litigate if and only if $V_B(2) - L_B > V_B(1)$. As a result, if $V_B(2) - V_B(1) < L_B < 2[V_B(2) - V_B(1)]$ litigation that would increase the overall value for downstream firms will not take place, due to the lack of coordination.

5.3 Royalty Renegotiation

The timing of the model assumes that once patent holder $i$ chooses the royalty rate $r_i$ the downstream producer will end up paying that amount unless it is brought to court and the patent invalidated. This means that when the patent is considered valid by the court its owner has no chance to increase the royalty rate. Papers like Choi and Gerlach (2015) allow for the possibility of renegotiation under these new circumstances. As we discuss next, royalty renegotiation weakens the Inverse Cournot effect but it does not qualitatively affect the results of the paper.

In the benchmark model, the maximum royalty rate that innovator 1 could set and induce litigation on innovator 2, $\bar{r}_1$, can be obtained by setting (1) with equality. Under

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10 An exception is the circular city, where the inelastic demand implies that the cross-derivative is 0 and, thus, the Inverse Cournot effect is independent of the number of firms.
royalty renegotiation, $\tilde{r}_1$ would now arise from

$$(1 - g(x_2)) [\Pi_B(\tilde{r}_1) - \Pi_B(\tilde{r}_1 + r_2)] + g(x_2) [\Pi_B(r^M) - \Pi_B(\tilde{r}_1 + r_2)] = L_B,$$

where, after the success of the downstream producer against innovator 2, the royalty rate increases to the monopoly one, $r^M$. In this case it is still true that the Inverse Cournot effect operates, since $\tilde{r}_1$ increases in $r_2$, but only when the patent of innovator 2 is sufficiently weak. This observation is in opposition to the results in Choi and Gerlach (2015), where the positive relationship between the royalty rate of both firms is generated precisely by the upside that royalty renegotiation provides.

### 5.4 FRAND Licensing

Most SSOs request participating firms to license the patents that are considered essential to the standard according to Fair, Reasonable, and Non-discriminatory (FRAND) terms. The ambiguity of this term and the different interpretation of patent holders and licensees has made FRAND a legally contentious issue. Courts have sometimes been asked to determine whether a royalty rate is FRAND or not and in some instances to set the FRAND rate.

The goal of this section is not to assert whether a royalty rate is FRAND or not but, rather, to study what is the effect of courts determining it on the previous results and, in particular, on the Inverse Cournot effect. In order to do so, we now extend the basic setup and assume that when a patent is valid, the downstream producer can ask the court to rule that it is essential to the standard and the royalty requested is not FRAND. We assume that the stronger is a patent the more likely it is that the technology it covers is considered essential to the standard. This probability is defined as $h(x_i)$, increasing in $x_i$. The arguments apply to the existence of $N$ patent holders, with $R_{-i}$ corresponding to the sum of the royalty rate of all patentees other than $i$.

If the patent is declared to be essential to the standard the court will determine the appropriate royalty rate, $\rho(x_i, r_i, R_{-i})$. We assume that this rate is an increasing function
of the strength of the patent, \( x_i \). As we discuss later, we also allow for the possibility that the court’s decision depends on the royalty announced by the patent holder or the total royalty established by the other patent holders.

Following the analysis in the benchmark model, the downstream monopolist will be interested in challenging patent \( i \) in court only if

\[
(1 - g(x_i)) \left[ \Pi_B(R_{-i} - R_{-i} + r_i) \right] \\
+ g(x_i) h(x_i) \left[ \Pi_B(R_{-i} + \rho(x_i, r_i, R_{-i})) - \Pi_B(R_{-i} + r_i) \right] > L_B.
\]

The previous expression has a straightforward interpretation. The downstream producer might benefit from litigation either because the patent is ruled invalid, which occurs with probability \( 1 - g(x_i) \), or because it is considered valid and essential to the standard, with probability \( g(x_i) h(x_i) \). In this latter case, the royalty rate drops from \( r_i \) to \( \rho(x_i, r_i, R_{-i}) \).

**Lemma 7.** Suppose \( \rho(x_i, r_i, R_{-i}) \) is independent of \( r_i \) and \( R_{-i} \), \( \rho(x_i) \). Then, there exists a unique critical value \( \bar{r}_i(x_i, R_{-i}, L_B) \) such that the producer prefers to challenge in court patent \( i \) if and only if \( r_i > \bar{r}_i \). Furthermore, this threshold is increasing in \( R_{-i} \) and \( L_B \).

This result indicates that the Inverse Cournot effect is qualitatively unaffected as long as the court determines the FRAND royalty only as a function of the strength of the patent. The main difference, however, is that the result does not guarantee that innovators with a stronger patent can indeed charge a higher royalty without triggering litigation by the downstream producer. Although a higher \( x_i \) reduces the probability that the court invalidates the patent, it also increases the probability that the patent is considered essential and, thus, that the royalty rate is diminished from \( r_i \) to \( \rho(x_i) \). This second effect prevails when increases in \( x_i \) have a large impact on \( h(x_i) \) but a small one on \( \rho(x_i) \).

The previous lemma establishes sufficient conditions and the result might still hold even if, as it is plausible, \( \rho(x_i, r_i, R_{-i}) \) increases in \( r_i \). An interesting case that it is worth to mention is the following: Suppose that a court would determine the FRAND royalty
rate as a function of $x_i$ but it would never choose $\rho(x_i, r_i, R_{-i})$ higher than $r_i$. It can be shown that the results are preserved in that case.

Finally, there have been instances in which courts have used existing licensing agreements in order to pin down the FRAND royalty rate for a patent (or patent portfolio). Interestingly, they have been used in two directions. In some cases, courts have adopted the so-called comparables approach and set the royalty rate according to the rate negotiated for comparable patents, even in the same standard.\textsuperscript{11} In those cases increases in $R_{-i}$ have a positive effect on $\rho(x_i, r_i, R_{-i})$ and strengthen the Inverse Cournot effect.

In other cases, and more specifically in the Microsoft v. Motorola case,\textsuperscript{12} it has been argued that the FRAND royalty rate of a patent holder should be lowered due to the already large royalty stack. This reasoning would make $\rho(x_i, r_i, R_{-i})$ non-increasing in $R_{-i}$. Interestingly, this result would undermine the Inverse Cournot effect and it might even reverse its sign, with self-defeating consequences. Large patent holders would anticipate that by choosing a higher royalty rate, weaker competitors facing litigation would be forced by the court to set a lower rate, exacerbating the royalty-stacking problem that courts were aiming to address in the first place.

### 5.5 Sequential Royalty Setting

In the benchmark model firms choose their royalty rates simultaneously. Let’s consider now the case in which the innovator with the strong patent, innovator 1, decides the royalty rate first. Innovator 2 is constrained by litigation due to its weak patent and chooses later. It is easy to see that the main forces at play in the simultaneous case will apply here. The decision of patent holder 1 in both cases would internalize the effect of $r_1$ on the incentives of patent holder 2 to lower $r_2$ and prevent litigation. Of course, in opposition to the simultaneous move case, here a pure strategy equilibrium will exist.

The similarity between the simultaneous and the sequential move game is useful to

\textsuperscript{11}See Leonard and Lopez (2014) for a discussion of this and other approaches used to determine FRAND royalty rates.

\textsuperscript{12}Microsoft Corp v. Morotola Inc, 854 F. Supp 2d 933 - Dist Court WD Washington 2012.
explain the behavior of large innovators that participate in SSOs. These firms devote substantial resources in developing technologies the profitability of which depends on the success in the final-good market of the products that embed them. The announcement of a low royalty rate early in the standardization process can, thus, be understood as a commitment that the royalty rate of complementary technologies developed by firms with a weaker patent portfolio would also be low, reducing the risk of royalty stacking. This interpretation is consistent with the adoption of some standards in recent years. For example, the main sponsors of the fourth-generation mobile telecommunications technology announced the licensing condition for their (essential) patents very early in the process.\textsuperscript{13}

\section*{5.6 Two-Part Contracts}

As widely assumed in the literature, innovators in the benchmark model can only use per-unit royalty rates. In this section we explore the implications of enlarging the kind of contracts that patent holders can use to accommodate two-part tariffs, combining royalty rates and fixed fees. These contracts have been documented, for example, in the biomedical industry (Hegde, 2014).

Consider the case of two patent holders \( i = 1, 2 \) that own one patent and offer a two-part tariff \((r_i, F_i)\), where \( r_i \) and \( F_i \) are the royalty rate and the fixed fee, respectively. Following the reduced-form specification proposed by Calzolari et al. (2020), we assume that a fixed fee \( F_i \geq 0 \) generates a distortion in the final market of \( \mu \geq 0 \). This parameter aims to capture some of the reasons that the literature has proposed to reconcile the fact that royalties are used in spite of the double-marginalization they generate. It has been argued that two-part contracts are optimal, for example, as part of a risk-sharing strategy (Bousquet et al., 1998) or in situations where there is downstream competition.

\textsuperscript{13} The mechanism used to spur the adoption of a new technology that this paper uncovers resembles contractual arrangements that we observe in other technological contexts. See Gambardella and Hall (2006) for a study of the public-good problem faced in software development when placed in the public domain. In the case of encryption technologies the risk that non-practicing entities might try and enforce their patents has encouraged agents more invested in the development of software to make it open source and royalty free. See “A rush to patent the blockchain is a sign of the technology’s promise” (2017, 14 January), The Economist (downloaded on 8 February 2017).
The next lemma summarizes the results when patents are ironclad, \( g(x_1) = g(x_2) = 1 \), and they are owned by different firms. They choose simultaneously the contract to be offered to the downstream producer, \((r_i, F_i)\) for \(i = 1, 2\). As in the benchmark model, for any \(\mu > 0\) the total royalty rate will be higher than in the single innovator case. As it is usually the case, we assume that fees must be non-negative.\(^{14}\) When \(\mu = 0\) it is immediate that \(r_1 = r_2 = 0\) is always optimal and no distortion arises.

**Lemma 8.** When both innovators own ironclad patents and offer two-part tariffs, their royalty rate in a symmetric equilibrium, \(r^u(\mu)\), leads to royalty stacking when \(\mu > 0\). That is, \(R^u(\mu) \equiv 2r^u(\mu) > R^M(\mu)\) for all \(\mu > 0\), where \(R^M(\mu)\) is the rate chosen by a single innovator. Royalty rates are increasing in \(\mu\). When \(\mu = 0\), \(r^u(0) = R^M(0) = 0\) and the royalty rate maximizes social welfare, regardless of the number of innovators.

Notice that, as expected, as fixed fees generate higher distortions, the royalty rate that innovators choose in equilibrium increases. In the limit, when \(\mu\) tends to infinity the equilibrium royalty rates coincides with those in Proposition 2. Royalty stacking occurs for all positive values of \(\mu\).

Consider now the case where one patent is strong and the other one is weak, \(g(x_2) < g(x_1) = 1\). As in the benchmark model, a single innovator will still be able to choose a total royalty that coincides with the case in which both patents are strong.

When the patents are owned by different firms, however, we now show that innovator 1 will have incentives to deviate and lower the royalty rate if the other patent is sufficiently weak, in line with the results in the rest of this paper. The condition that establishes that it is in the interest of the downstream producer to challenge patent 2 in court is

\[
(1 - g(x_2)) [\Pi_B(r_1) - \Pi_B(r_1 + r_2) + (1 + \mu)F_2] > L_B,
\]

where, \(\Pi_B(R)\) denotes the profits of the downstream firm gross of fixed fees. This condi-

\(^{14}\)Negative fees would attract potential licensees that have no intention to produce.
tion indicates that, similarly to what happens in the benchmark model with pure royalties, the lower is $r_1$ the lower should be $r_2$ and/or $F_2$ to guarantee that litigation is not profitable. Notice that the fixed fee $F_1$ is irrelevant for this decision, since it enters the profit function of the downstream producer linearly. Of course, $F_1$ affects the optimal level of $F_2$ as it must guarantee that the downstream producer is willing to participate. That is, in an equilibrium without litigation, it must be that $\Pi_B(r_1 + r_2) - (1 + \mu)(F_1 + F_2) \geq 0$.

The next result shows that if a pure strategy equilibrium exists it must lead to the royalty-stacking outcome described above.

**Proposition 9.** When $g(x_1) = 1 > g(x_2)$ an equilibrium in pure strategies without litigation and two-part tariffs exists if both firms set the royalty rate $r^u(\mu)$. This equilibrium only exists when $x_2$ is sufficiently high.

The previous result is the counterpart of Proposition 4 for the case of two-part tariffs. It implies that if condition (8) holds when both firms charge a royalty rate $r^u$ and $F_2 = 0$, a pure strategy equilibrium will fail to exist. This is likely to be the case when $x_2$ is low and it implies that the Inverse Cournot effect will re-emerge with similar implications to those highlighted in the baseline model. Innovator 1 would have incentives to lower the royalty rate below $r^u(\mu)$ in order to foster litigation against innovator 2. Notice that in that case, the innovator can benefit from the invalidation of the patent of the rival not only because of the increase in the quantity it can bring about but also due to the higher fixed fee that the downstream producer will be willing accept.

### 6 Equilibrium Litigation

A sustained assumption throughout the paper has been that it was always optimal for the innovators to avoid litigation. This assumption is consistent with high litigation costs of these upstream firms, $L_U$. In this section we analyze the implications of relaxing this assumption.
6.1 The Litigation Decision of Patent Holder 2

We concentrate in the situation where innovator 1 has a strong patent, \( g(x_1) = 1 \), whereas innovator 2’s patent is weak, \( g(x_2) < 1 \). Figure 3 illustrates the structure of the game and defines the relevant payoffs. Whereas in the benchmark model we assumed that the downstream firm always accepted in equilibrium the royalty rate offered, here we need to define the payoffs when it prefers to go to court. These profits are defined as

\[
\tilde{\Pi}_1(r_1, r_2) = g(x_2)\Pi_1(r_1, r_2) + (1 - g(x_2))\Pi_1(r_1, 0),
\]

\[
\tilde{\Pi}_2(r_1, r_2) = g(x_2)\Pi_2(r_1, r_2) - L_U,
\]

\[
\tilde{\Pi}_B(r_1, r_2) = g(x_2)\Pi_B(r_1 + r_2) + (1 - g(x_2))\Pi_B(r_1) - L_B,
\]

where \( \Pi_i(r_1, r_2) \) are the profits of innovator \( i \) when the downstream producer licenses both patents. The expression \( \tilde{\Pi}_B(r_1, r_2) \) has been implicitly used before, since its difference with respect to \( \Pi_B(r_1 + r_2) \) determines the results in Lemma 3.

From the previous expressions it is clear that, contingent on litigation, the optimal response of innovator 2 to a royalty \( r_1 \) coincides with the one that arises when litigation is not a threat. Denote this choice as \( r^*_{2}(r_1) \) which, due to Assumption 1, is decreasing in \( r_1 \).

For a given \( r_1 \) we obtain the royalty rate that guarantees that it is not worthwhile for the downstream producer to litigate, \( \tilde{r}_2(r_1) \), as the inverse of \( \tilde{r}_1(r_2) \) in (1). Using Lemma 3, when \( L_B \) is sufficiently low, this function is increasing in \( r_1, L_B, \) and \( g(x_2) \). Since \( r^*_{2}(r_1) \) converges to 0 as \( r_1 \) grows, it is immediate that there is a threshold value \( \hat{\rho}_1 \) so that for \( r_1 \geq \hat{\rho}_1, r^*_{2}(r_1) \leq \tilde{r}_2(r_1) \). In that case, litigation would not be a relevant threat.
Figure 6: Royalties and profits under litigation and accommodation with parameter values: $L_B = 0.015$, $g(x_2) = 0.4$, $L_U = 0.01$.

When $r_1 < \hat{\rho}_1$, innovator 2 trades off the increase in revenues originated by $\hat{r}_2^c(r_1)$ with the probability that the patent is invalidated, yielding a revenue of 0. The former will dominate if $r_1$ is sufficiently small, as the large decrease in the royalty rate necessary to fend off litigation is unlikely to be profitable.

In the rest of this section we rely on a specific demand structure to characterize the equilibrium. We consider the linear demand case, $D(p) = 1 - p$, where the downstream firm chooses a unique (monopoly) price. In that case $\tilde{D}(R) = \frac{1 - R}{2}$ and using the previous expressions we can show that

$$r_2^c(r_1) = \frac{1 - r_1}{2},$$

$$\hat{r}_2(r_1) = 1 - r_1 - \sqrt{(1 - r_1)^2 - \frac{4L_B}{1 - g(x_2)}}.$$ (9)

The next result characterizes the optimal royalty rate for innovator 2 in this case.

**Proposition 10.** Suppose that demand is $\tilde{D}(R) = \frac{1 - R}{2}$. When $L_U$ is sufficiently low, there exists a unique threshold value $\hat{\rho}_1(x_2, L_B, L_U) \in [0, \hat{\rho}_1)$ decreasing in $L_B$ and $L_U$ so that the optimal decision of innovator 2 becomes,

$$r_2^*(r_1) = \begin{cases} r_2^c(r_1) & \text{if } r_1 < \hat{\rho}_1, \\ \hat{r}_2(r_1) & \text{if } r_1 \geq \hat{\rho}_1. \end{cases}$$

This proposition shows that the optimal royalty rate of innovator 2 can be characterized by two regions. As explained before, when $r_1$ is low, it does not pay off for
innovator 2 to decrease the royalty rate to avoid litigation. At the other extreme, when \( r_1 \) is sufficiently high, a small decrease in \( r_2 \) is required and avoiding litigation increases profits. Under the linear demand function, there is a unique threshold, identified as \( \hat{\rho}_1 \), that determines the two regions. The discontinuity in profits of moving, for the same level of \( r_2 \), from a situation where litigation is averted to one in which it occurs explains why \( \hat{\rho}_1 < \hat{\rho}_1 \).

Figure 6 illustrates this result. The left panel characterizes the threshold \( \hat{\rho}_1 \) which determines when the royalty choice of innovator 2 is restricted by litigation. The right panel show that the profits of innovator 2 are decreasing in \( r_1 \) with and without litigation but for different reasons. When there is no litigation in equilibrium \( \Pi_2(r_1, \bar{r}_2(r_1)) \) is decreasing in \( r_1 \) due to the standard Cournot arguments. When litigation occurs in equilibrium, however, this effect is moderated by the fact that the decrease in the royalty rate required to avoid litigation is now smaller. This counteracting effect also explains why the threshold for which litigation is preferred is unique.

This figure also allows us to illustrate how the threshold \( \hat{\rho}_1 \) changes with the litigation costs.\(^{15}\) Increases in \( L_U \) lead to a downward shift in \( \bar{\Pi}_2(r_1, \bar{r}_2(r_1)) \) as litigation becomes less profitable for innovator 2. Similarly, in the case in which litigation does not occur, an increase in \( L_B \) allows innovator 2 to raise the royalty rate \( \bar{r}_2(r_1) \), increasing \( \Pi_2(r_1, \bar{r}_2(r_1)) \). In both cases, \( \hat{\rho}_1 \) moves to the left and litigation is less likely to arise in equilibrium for a given value of \( r_1 \). The effect of an increase in \( x_2 \) is ambiguous since it raises profits in both cases.

6.2 The Optimal Choice of \( r_1 \)

We now discuss how the optimal royalty rate of innovator 1 is affected by the possibility of litigation in equilibrium. To answer this question, we need to make an additional assumption. We depart from the structure of the baseline model and assume that innovator

\(^{15}\)It is immediate that \( \hat{\rho}_1 \) is decreasing in \( L_B \) and \( g(x_2) \).
moves first. As explained in section 5, this assumption has little impact in the case where one patent holder is strong and legal costs are high but it guarantees the existence of an equilibrium in pure strategies. We continue to focus on the linear demand case discussed in the last proposition, which guarantees that litigation only emerges for low values of \( r_1 \).

From the previous analysis, we need to distinguish two cases. If \( r_1 < \bar{\rho}_1 \), innovator 2 will choose \( r_2^c(r_1) \) and litigation will arise in equilibrium. If \( r_1 \geq \bar{\rho}_1 \), innovator 2 prefers to offer a lower royalty rate, \( \tilde{r}_2(r_1) \), and avoid being brought to court.

In the first case, innovator 1 maximizes

\[
\max_{r_1 < \bar{\rho}_1} r_1 \left[ g(x_2) \hat{D}(r_1 + r_2^c(r_1)) + (1 - g(x_2)) \hat{D}(r_1) \right],
\]

which implies an equilibrium royalty rate \( \tilde{r}_1^* = \min\{\bar{\rho}_1, \frac{1}{2}\} \). Notice that when \( \bar{\rho}_1 \) is sufficiently high, the royalty rate is a convex combination of the one that a monopolist and a Stackelberg leader would choose which, under a linear demand, coincide.

In the second case, a high \( r_1 \) is chosen and innovator 2 prefers to avoid litigation by setting a royalty rate \( \tilde{r}_2(r_1) \). Innovator 1 maximizes the profit function

\[
\max_{r_1 \geq \bar{\rho}_1} r_1 \frac{1 - r_1 - \tilde{r}_2(r_1)}{2},
\]

resulting in a candidate royalty rate \( \tilde{r}_1^* = \max\{\bar{\rho}_1, r_1^{ic}\} \), where \( r_1^{ic} = \frac{3}{4} - \frac{1}{4} \sqrt{\frac{32L^2}{1-g(x_2)} + 1} \). Notice that \( r_1^{ic} \leq \frac{1}{2} \) meaning that this royalty rate is lower than the unconstrained choice when litigation arises in equilibrium. That is, the unconstrained royalty rate of innovator 1 is lower in the case in which innovator 2 accommodates. This result is a version of the Inverse-Cournot effect: by reducing \( r_1 \) innovator 1 also fosters a reduction in \( r_2 \), mitigating the royalty-stacking distortion and increasing downstream sales and overall profits.

It can also be shown that if we, again, abstract from the constraints imposed by \( \bar{\rho}_1 \), innovator 1’s profits are always higher when \( r_1 = r_1^{ic} \) and litigation does not occur in equilibrium compared to when \( r_1 = \frac{1}{2} \) and innovator 2 is brought to court. The linear
The structure of the demand function implies that, for a given \( r_1 \), the option that maximizes profits is the one that leads to the lowest expected total royalty rate. This means that in some situations \( r_1 = \tilde{\rho}_1 \) will be optimal as a way to avoid litigation. The next result uses the previous insights to characterize the optimal royalty rate for innovator 1 as a function of the threshold \( \tilde{\rho}_1 \).

**Proposition 11.** The optimal royalty rate of innovator 1 can be characterized as

\[
\begin{align*}
r^*_1 &= \begin{cases} 
 r^{ic}_1 & \text{if } \tilde{\rho}_1 \leq r^{ic}_1, \\
 \tilde{\rho}_1 & \text{if } \tilde{\rho}_1 \in (r^{ic}_1, \rho^*], \\
 \frac{1}{2} & \text{otherwise},
\end{cases}
\end{align*}
\]

where \( \rho^* > \frac{1}{2} \).

Figure 7 illustrates the optimal royalty rate for different values of \( \tilde{\rho}_1 \). This figure identifies two regions. For values of \( \tilde{\rho}_1 \) below a threshold \( \rho^* \) it is optimal for innovator 1 to induce a low royalty rate \( r_2 \) that will be accepted by the downstream producer. When \( \tilde{\rho}_1 \) is higher than \( \rho^* \) inducing the litigation of patent 2 is optimal for innovator 1. The royalty rate that maximizes profits in that region is the unconstrained one.

In order to unpack the implications of the previous result, it is useful to illustrate the discussion by analyzing the effect of different values of \( L_U \). The comparative statics exercise in this case is simple since the litigation cost of innovator 2 has no direct impact.
on the profits of innovator 1 except through the changes in $\tilde{\rho}_1$. From Proposition 10 we know that increases in $L_U$ are associated with a decrease in $\tilde{\rho}_1$, as the higher the cost of innovator 2 to defend its patent in court the higher the royalty rate that innovator 1 can charge without triggering litigation. As it can be seen from the figure, when $L_U$ is low, and therefore $\tilde{\rho}_1$ is high, innovator 1 is likely to find optimal to choose $r_1 = \frac{1}{2}$. The reason is that discouraging litigation (i.e. innovator 2 chooses a low $r_2$) would require a very high royalty rate. As a result the total burden $r_1 + \bar{r}_2(r_1)$ would become very high and the quantity sold low. As $L_U$ increases, however, discouraging innovator 2 from litigating is easier and, eventually, when litigation costs are sufficiently high so that going to court is not a reasonable option, the Inverse-Cournot effect is the only relevant force. This effect pushes innovator 1 to choose a royalty rate lower than the one that would emerge when litigation was optimal. This case has been the focus of the main sections of the paper.

For intermediate values of $L_U$ we observe a region in which litigation does not take place (to the left of $\rho^*$) but the royalty rate of innovator 1 is higher not only than $r_1^{ic}$ but also than the one that would emerge under litigation. When $r_1^* = \tilde{\rho}_1$ the Inverse Cournot effect is relaxed, allowing innovator 2 to increase the royalty rate and making the option of avoiding litigation more profitable. Hence, for innovator 1, this higher royalty rate generates a trade-off. Choosing $\tilde{\rho}_1$, compared to $r_1 = \frac{1}{2}$, implies a higher individual royalty rate but a lower quantity due to the higher “expected” total royalty rate that emerges due to the lower probability that the patent of innovator 2 is invalidated.\footnote{For a given value of $r_1$ we can write the profits of patent holder 1 as $\Pi_1(r_1) = r_1 \frac{1}{2} - R$, where $R = r_1 + g(x_2)\bar{r}_2(r_1)$ and $\bar{R} = r_1 + \bar{r}_2(r_1)$ when litigation occurs in equilibrium and when patent holder 2 avoids it, respectively.} For values of $\tilde{\rho}_1$ below $\rho^*$, this trade-off is resolved in favor of the high royalty rate even if that implies an increase in $r_2$.

The effect of $L_B$ is similar in the sense that increases in this cost also shift the threshold value $\tilde{\rho}_1$ downwards. However, an increase in $L_B$ also raises $\bar{r}_2(r_1)$, reducing the profits from discouraging innovator 2 to defend its patent in court. Both effects go in the same
direction, suggesting that as $L_B$ increases the region under which promoting litigation is optimal for patent holder 1 expands.

The effect of $x_2$ is in general difficult to ascertain, as it affects the figure in several dimensions. First, we can observe that, both under litigation and under accommodation, the profits of innovator 1 decrease as $x_2$ increase, since the problem of royalty stacking becomes more relevant. However, an analytical comparison of the magnitude of the effect in both cases as well as the effect of $x_2$ on $\hat{\rho}_1$ is difficult to establish.

Finally, this example allows us to draw some implications for equilibrium royalty stacking. Trivially, when litigation emerges in equilibrium the expected royalty rate is lower than the one that arises when both patents are strong. The reason is that both innovators would choose the same royalty rate but the expected royalty rate decreases as the probability that the patent of innovator 2 is invalidated increases. At the other extreme, when $r^*_1 = r^{ic}_1$ the Inverse Cournot effect implies that the resulting royalty rate is lower than the one that would emerge under ironclad patents.

Interestingly, in the intermediate region, when $r^*_1 = \hat{\rho}_1$ and litigation is credible, the implications for royalty stacking are ambiguous. Innovator 1 can increase revenues by raising the royalty rate above $\frac{1}{2}$ even as this fosters a limited increase in $r_2$. Under some parameter configurations this may lead to a higher total royalty rate.

7 Concluding Remarks and Policy Implications

The existence of royalty stacking in the context of technology licensing has been argued in analogy with the classical case of Cournot complements. This paper shows, however, that these insights do not necessarily carry through when we explicitly consider patent litigation and, most specifically, the incentives that firms have to make strategic use of it.

The implications of reconsidering the idea of royalty stacking through the lens of a model of patent litigation are far-reaching. One of the main contexts in which these
changes apply is in the case of Standard Setting Organizations. Royalty stacking has been used to assess the desirability of patent consolidation or disaggregation. The concern about “privateers” — spin-offs of existing firms aimed at enforcing their intellectual property — and “patent assertion entities” is that they can be used to increase the royalty stack. In contrast, consolidation efforts through patent acquisitions or the creation of patent pools have been encouraged as they would contribute to lower the aggregate royalty rate.\footnote{See Lerner and Tirole (2004). Other papers, however, have pointed out that patent pools might reduce social welfare when they include non-essential patents (Quint, 2014) or when some licensors and producers are vertically integrated (Reisinger and Tarantino, 2019).}

Our model suggests that these rules should be implemented with caution and that the impact of the litigation threat should be factored in. Since litigation decisions depend on the strength of the patent portfolio, if patentees pool their patents they are likely to make enforcement more effective. This last effect implies that the formation of a patent pool or the merger of patent holders might make the royalty-stacking problem worse, particularly if not all patent holders are included and the portfolios become more similar in strength.\footnote{In our model, a patent pool including all firms will always eliminate the royalty stack and increase overall profits. Of course, to the extent that the Inverse Cournot effect reduces the size of this royalty stack, the incentives to form a pool are diminished.}

By the same token, to the extent that patent holders decide to disaggregate their patent portfolio into more asymmetric patent holdings, the outcome could be socially beneficial. To evaluate the impact of these decisions we should account for how the moderation force of large patent holders that the Inverse Cournot effect brings about is mitigated or strengthened.

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A Proofs

The main results of the paper are proved here.

**Proof of Proposition 1:** Following the discussion in the text, if \( g(x_1) \) is such that condition (2) is not satisfied, \( r_1^* = R^M \) and \( r_2^* = 0 \) allows the innovator to attain monopoly profits.

When \( g(x_1) = g(x_2) = g(x) \) is sufficiently small so that condition (2) is satisfied, it is immediate that a necessary condition to attain the monopoly total royalty rate \( R^M \) is that the innovator chooses \( r_1 > 0 \) and \( r_2 > 0 \).

The single innovator maximizes the following function

\[
\max_{r_1, r_2} (r_1 + r_2) \tilde{D}(r_1 + r_2)
\]

subject to

\[
(1 - g(x)) \{ \Pi_B(r_i) - \Pi_B(r_1 + r_2) \} + \max\{ (1 - g(x)) \Pi_B(0) - \Pi_B(r_i), 0 \} \leq L_B
\]

for \( i = 1, 2 \).

Suppose towards a contradiction that the two royalty rates that maximize profits are different and, without loss of generality, \( r_1^m > r_2^m \). It is immediate that in that case it cannot be that the initial litigation of both of the patents leads to a cascade or none of them does. For the former, suppose that both of them lead to a cascade. The two constraints can be written in this case as

\[
\Pi_B(r_1^m + r_2^m) \geq g(x)\Pi_B(r_1^m) + (1 - g(x))\Pi_B(0) - L_B - \frac{L_B}{1 - g(x)}.
\]

Since \( \Pi_B(r) \) is decreasing in \( r \) the constraint for litigating first patent 2 is binding. Hence, profits could be increased by diminishing \( r_1 \) and increasing \( r_2 \). A similar argument applies when there is no litigation cascade.

Hence, if different royalty rates are optimal, it has to be that a cascade only occurs in one of the cases. Furthermore, the cascade must occur only after success against patent 2 and only when

\[
\Pi_B(0) - \Pi_B(r_1^m) > \frac{L_B}{1 - g(x)} \geq \Pi_B(0) - \Pi_B(r_2^m).
\]

Hence, the optimal royalty rates must satisfy the constraints,

\[
\Pi_B(r_1^m + r_2^m) = g(x)\Pi_B(r_1^m) + (1 - g(x))\Pi_B(0) - L_B - \frac{L_B}{1 - g(x)}, \quad (12)
\]

\[
\Pi_B(r_1^m + r_2^m) = \Pi_B(r_2^m) - \frac{L_B}{1 - g(x)}. \quad (13)
\]
This cannot occur. In particular, combining (12) and (13) we have that
\[ g(x)\Pi_B(r_1^m) + (1 - g(x))\Pi_B(0) - L_B = \Pi_B(r_2^m) \geq \Pi_B(0) - \frac{L_B}{1 - g(x)}, \]
where the last inequality arises from (11). This implies that \( \Pi_B(0) - \Pi_B(r_1^m) \leq \frac{L_B}{1 - g(x)} \), which is a contradiction with the condition that litigating patent 1 after success in invalidating patent 2 is profitable. Hence \( r_1^m = r_2^m = r^m \).

Notice that \( \Pi_B(0) - \Pi_B(r_1^m) > \frac{L_B}{1 - g(x)} \), so that a second trial would take place after an initial success. A royalty rate that did not lead to a cascade would be lower and, therefore, yield lower profits.

Finally, since the left-hand side of equation (3) is increasing in \( r^m \) it is immediate, using the Implicit Function Theorem, that this royalty rate is increasing in \( L_B \) and \( g(x) \).

**Proof of Proposition 2:** The optimal royalty of patentee \( i \) resulting from (5) is determined using the first-order condition
\[ \frac{\partial}{\partial R} D(R) + r_i^u \frac{\partial}{\partial R} D'(R) = 0. \implies r_i^u = -\frac{\partial}{\partial R} D(R) \frac{\partial}{\partial R} D'(R). \]
Replacing \( r_i^u = r^* = \frac{R^u}{N} \) where \( N \leq 2 \) is the number of firms, we can use the Implicit Function Theorem to compute
\[ \frac{dR^u}{dN} = \frac{\frac{R^u}{N} \frac{\partial}{\partial R} D'(R^u)}{\frac{\partial}{\partial R} D'(R^u) + \frac{1}{N} \frac{\partial}{\partial R} D'(R^u)} > 0. \]

The last inequality arises from a negative numerator due to \( \frac{\partial}{\partial R} D'(R) < 0 \) and a negative denominator that it is also negative due to the quasiconcavity of \( D(R) \). In particular, this result implies that \( R^M = R^u(1) < R^u(2) = 2r^u \).

**Proof of Lemma 3:** Define \( \bar{r}_1 \) as the value of \( r_1 \) for which equation (1) is satisfied with equality. First, we establish that it is unique and well-defined for all positive values of \( L_B \). The left-hand side of that equation is always decreasing in \( r_1 \) for \( r_2 > 0 \). Furthermore, as \( D(R) \to 0 \) when \( R \to \infty \) we have that the left-hand side expression can be arbitrarily small as \( r_1 \) increases. When \( L_B < (1 - g(x_2))(\Pi_B(0) - \Pi_B(r_2)) \) the threshold value \( \bar{r}_1 \) is always positive.
Using the fact that $\Pi_B'(R) < 0$ and $\Pi_B''(R) > 0$, we can compute, for $r_2 > 0$,

\[
\frac{d\bar{r}_1}{dL_B} = \frac{1}{\Pi_B'(\bar{r}_1) - \Pi_B'(\bar{r}_1 + r_2)} < 0,
\]

\[
\frac{d\bar{r}_1}{dx_2} = \frac{g'(x_2)[\Pi_B'(\bar{r}_1) - \Pi_B'(\bar{r}_1 + r_2)]}{[\Pi_B'(\bar{r}_1) - \Pi_B'(\bar{r}_1 + r_2)]} < 0,
\]

\[
\frac{d\bar{r}_1}{dr_2} = \frac{\Pi_B'(\bar{r}_1 + r_2)}{\Pi_B'(\bar{r}_1) - \Pi_B'(\bar{r}_1 + r_2)} > 0.
\]

**Proof of Proposition 4:** Following the arguments in the text, suppose towards a contradiction that an equilibrium without litigation $(r_1^*, r_2^*)$ exists with $r_1^* > 0$ and $r_2^* > 0$, different from the unconstrained solution, $r^u$, as defined in Proposition 2. Since $r_2^* > 0$, this equilibrium must satisfy equation (1). Suppose that this condition is satisfied with strict equality. It is easy to see that in that case $r_1^*$ would not be optimal for patent holder 1, as it could be slightly diminished, leading to a discrete increase in final market sales from $\hat{D}(r_1^* + r_2^*)$ to almost $g(x_2)\hat{D}(r_1^* + r_2^*) + (1 - g(x_2))\hat{D}(r_1^*)$.

Hence, condition (1) must be satisfied with strict inequality in the equilibrium. This means that patent holder 1 chooses the royalty as the result of $r_1^* = \arg \max \ r_1 \hat{D}(r_1 + r_2^*)$. Since the equilibrium differs from the unconstrained one and condition (1) constitutes an upper bound for the royalty rate of patent holder 2, it has to be the case that $r_2^*$ is lower than the best response to $r_1^*$. But this is a contradiction, since patent holder 2 could always increase the royalty rate while the constraint still holds.

Alternatively, suppose that at least one of the royalty rates is 0 in equilibrium. This would be a contradiction, since both patent holders could guarantee positive profits and avoid litigation by choosing a positive royalty rate, unless $\hat{D}(r_1^* + r_2^*) = 0$. But in that case the patent holder with a positive royalty rate would obtain higher (and positive) profits by decreasing it.

**Proof of Lemma 5:** Suppose without loss of generality that $r_1 > r_2$. The optimal policy of the downstream producer can be described as arising from the following two stages. In the first stage, it decides whether to sue patent holder 1 or 2 or none at all. Upon observing the outcome of the first trial the downstream producer decides whether to sue the other patent holder or not.

Suppose that in the first stage patentee $i$ was brought to court. Then, if it is optimal for the downstream producer to sue patentee $j$ upon defeat it is also optimal to litigate...
upon success since, by convexity of $\Pi_B(R)$,

$$\Pi_B(r_i) - \Pi_B(r_i + r_j) \leq \Pi_B(0) - \Pi_B(r_j),$$

for $i = 1, 2$ and $j \neq i$. Furthermore, notice that

$$\Pi_B(r_1) - \Pi_B(r_1 + r_2) \leq \Pi_B(r_2) - \Pi_B(r_1 + r_2),$$

$$\Pi_B(0) - \Pi_B(r_2) \leq \Pi_B(0) - \Pi_B(r_1).$$

Hence, two possible orderings can arise depending on whether $\Pi_B(r_2) - \Pi_B(r_1 + r_2)$ is higher or lower than $\Pi_B(0) - \Pi_B(r_2)$. In order to determine the profits of the downstream producer in each case, we need to see how these profits compare with $\frac{L_B}{1 - g(x)}$.

(i) Suppose that when 1 is sued first it is always optimal to sue 2 afterwards. Obviously, if the opposite order yields the same order, both options are equivalent and profits are identical.

(ii) Suppose that when 1 is sued first it is only optimal to sue 2 after victory. This implies that $\Pi_B(r_1) - \Pi_B(r_1 + r_2) < \frac{L_B}{1 - g(x)} \leq \Pi_B(0) - \Pi_B(r_2)$. Profits become

$$g(x) [\Pi_B(r_1 + r_2) - L_B] + (1 - g(x)) [g(x)\Pi_B(r_2) + (1 - g(x))\Pi_B(0)] - L_B,$$

which, by definition, are higher than those that arise in the first case. If after litigating the portfolio of patent holder 2 it is then optimal to litigate the portfolio of the other patent holder always, this option would be, therefore, dominated by (i).

Alternatively, it could be that when patent holder 2 is sued first it is only optimal to sue patent holder 1 upon victory. Profits in that case would be

$$g(x) [\Pi_B(r_1 + r_2) - L_B] + (1 - g(x)) [g(x)\Pi_B(r_1) + (1 - g(x))\Pi_B(0)] - L_B,$$

which are lower than when patent holder 1 is sued first.

(iii) Suppose that when patent holder 1 is sued first it is never optimal to litigate the portfolio of patent holder 2 afterwards. Profits would be

$$g(x)\Pi_B(r_1 + r_2) + (1 - g(x))\Pi_B(r_2) - L_B.$$

If it is always optimal to sue patent holder 1 after patent holder 2 has been sued first, these profits are lower because, as in the previous case, they coincide with the profits
in the first option. If instead it was optimal to litigate only upon success, again, these profits are dominated by the second option. Finally, if it is never optimal to sue patent holder 1, profits become
\[ g(x)\Pi_B(r_1 + r_2) + (1 - g(x))\Pi_B(r_1) - L_B, \]
which are again lower.

(iv) Using the same argument, if \( \frac{L_B}{1 - g(x)} \) is sufficiently high so that it is never optimal to sue patent holder 1 only, bringing to court patent holder 2 only must also be dominated.

Proof of Proposition 6: Consider a symmetric equilibrium in which patent holder 1 and 2 are constrained. This implies that \( \Phi(r^*, r^*) = \frac{L_B}{1 - g(x)} + L_B \). Each firm obtains profits \( r^*\hat{D}(2r^*) \). It is immediate that \( r^* \) is increasing in \( L_B \) and \( g(x) \).

Three possible deviations of a patent holder, say patentee 1, should be considered:

(i) Patentee 1 might increase its royalty to \( r_1 > r^* \). This firm will be litigated first and profits, defined as \( \max_{r_1} g(x)r_1\hat{D}(r_1 + r^*) - L_U \), will be lower if \( L_U \) is sufficiently high.

(ii) Patentee 1 might deviate by lowering the royalty rate slightly. In this case, the sign of \( \frac{\partial \Phi}{\partial r_1} \) becomes relevant. In particular,
\[ \frac{\partial \Phi}{\partial r_1}(r_1, r_2) \geq 0 \iff g(x)\Pi'_B(r_1) - \Pi'_B(r_1 + r_2) = \hat{D}(r_1 + r_2) - g(x)\hat{D}(r_1) \geq 0, \]

If \( \frac{\partial \Phi}{\partial r_1} \geq 0 \), decreases in \( r_1 \) reduce the incentives for the downstream firm to litigate. Since royalties are strategic substitutes and \( r^* \) is below the unconstrained royalty this strategy can never be optimal.

Alternatively, if \( \frac{\partial \Phi}{\partial r_1} < 0 \), a deviation consisting in a slight decrease in \( r_1 \) induces litigation, first against patentee 2 and, upon success, against patentee 1. This implies that the profits of patentee 1 become
\[ g(x)r^*D(2r^*) + (1 - g(x)) [g(x)r^*D(r^*) - L_U], \]
This deviation is unprofitable if
\[ L_U > r^*D(p^M(2r^*)) - g(x)r^*D(p^M(r^*)), \]
which holds given that the right-hand side is negative when \( \frac{\partial \Phi}{\partial r_1}(r^*, r^*) < 0 \).
Finally, patent holder 1 could lower \( r_1 \) enough so that 
\[
(1 - g(x)) \left[ \Pi_B(0) - \Pi_B(r_1) \right] \leq L_B.
\]
In that case, patent holder 1 would not be brought to court. Again, two possibilities can arise here depending on whether the downstream producer is interested in suing patentee 2 or not. Notice that only if patentee 2 is sued this deviation might be profitable. Hence, the optimal deviation is \( \tilde{r}_1 = \min \{ r_1^A, r_1^B \} \), where the values \( r_1^A \) and \( r_1^B \) are defined as
\[
(1 - g(x)) \left[ \Pi_B(0) - \Pi_B(r_1^A) \right] = L_B,
\]
\[
(1 - g(x)) \left[ \Pi_B(r_1^B) - \Pi_B(r^* + r_1^B) \right] = L_B.
\]

When \( r^* \) is sufficiently high the first constraint will be binding. Profits in either case will be 
\[
g(x)r_1 \tilde{D}(r^* + \tilde{r}_1) + (1 - g(x))r_1 \tilde{D}(\tilde{r}_1).
\]

When \( g(x) \) is sufficiently small it is clear that the first deviation is always dominated since it would imply profits of \(-L_U\). The second deviation is also unprofitable since when \( g(x) = 0, \frac{\partial \Phi}{\partial r_1} \geq 0 \).

Regarding the last deviation, we know that \( \tilde{r}_1 \leq r_1^B \). Under a linear demand when \( g(x) = 0 \) and monopoly pricing, we have that 
\[
\Pi_B(0) - \Pi_B(2r^*) = 2 \left[ \Pi_B(r_1^B) - \Pi_B(r_1^B + r^*) \right]
\]
implies \( r_1^B = \frac{r^*}{2} \). Thus, for the deviation not to be profitable we only require
\[
r^* D(p^M(2r^*)) \geq \frac{r^*}{2} D\left(p^M\left(\frac{r^*}{2}\right)\right).
\]

When \( L_B = 0, r^* = 0 \) and the result holds trivially. The derivative of the profit functions evaluated at \( r^* = 0 \) are 
\[
D(p^M(0)) \quad \text{and} \quad \frac{1}{2} D(p^M(0)) \quad \text{for the left-hand side and the right-hand side expression, respectively.}
\]
Thus, the deviation is not profitable when \( L_B \) is sufficiently small.

We now show that there is no other symmetric pure-strategy equilibrium when the litigation constraint is relevant. First, notice that if \( r_1 = r_2 \) are lower than \( r^* \), each firm has incentives to increase its royalty since their problem is the same as they would face if they were unconstrained and royalties are strategic substitutes. If, instead, \( r_1 = r_2 = \tilde{r} \) are higher than \( r^* \) each firm obtains profits
\[
\frac{1}{2} \left[ g(x)\tilde{r} D(p^M(2\tilde{r})) - L_U \right] \quad \text{and} \quad \frac{1}{2} \left[ g(x)\tilde{r} D(p^M(2\tilde{r})) + (1 - g(x)) \left[ g(x)\tilde{r} D(p^M(2\tilde{r})) - L_U \right] \right]
\]
where each firm is brought to court first with probability \( \frac{1}{2} \) and the second firm is sued only if the downstream producer succeeds against the first. Notice that in this case it is
always optimal for one firm, say patentee 1, to undercut the other patentee. As a result
profits increase to
\[ g(x)\hat{D}(p^M(2\hat{r})) + (1 - g(x)) [g(x)\hat{D}(p^M(2\hat{r})) - L_U] . \]

\[ \square \]

**Proof of Lemma 7:** Define
\[ \Phi(r_i, x_i, L_B, R_{-i}) \equiv (1 - g(x_i)) [\Pi_B(R_{-i}) - \Pi_B(R_{-i} + r_i)] + \\
g(x_i)h(x_i) [\Pi_B(R_{-i} + \rho(x_i, r_i, R_{-i})) - \Pi_B(R_{-i} + r_i)] - L_B \]

Obviously, \( \frac{\partial \Phi}{\partial L_B} = -1 \). We can also compute
\[
\frac{\partial \Phi}{\partial r_i} = - (1 - g(x_i))\Pi'_B(R_{-i} + r_i) + g(x_i)h(x_i) \left[ \Pi'_B(R_{-i} + \rho(x_i, r_i, R_{-i})) \frac{\partial \rho}{\partial r_i} - \Pi'_B(R_{-i} + r_i) \right] \\
\frac{\partial \Phi}{\partial R_{-i}} = (1 - g(x_i)) [\Pi'_B(R_{-i}) - \Pi'_B(R_{-i} + r_i)] \\
+ g(x_i)h(x_i) \left[ \Pi'_B(R_{-i} + \rho(x_i, r_i, R_{-i})) \left( 1 + \frac{\partial \rho}{\partial R_i} \right) - \Pi'_B(R_{-i} + r_i) \right]
\]
Given that \( \Pi_B \) is convex, \( \rho(x_i, r_i, R_{-i}) \leq r_i \) and the assumption that \( \rho(x_i, r_i, R_{-i}) \) is independent of \( r_i \) and \( R_{-i} \) we can show that \( \frac{\partial \Phi}{\partial r_i} \geq 0 \) and \( \frac{\partial \Phi}{\partial R_{-i}} \leq 0 \). \( \square \)

**Proof of Lemma 8:** Consider the case of a single innovator. Given a total royalty rate \( R \) the firm set a fixed fee \( F \) that claws back all the surplus of the firm. That is \( (1 + \mu)F = \Pi_B(R) \). This means that the firm would choose a royalty rate to maximize
\[ \max_R R\hat{D}(R) + \frac{\Pi_B(R)}{1 + \mu} . \]
The first order condition results in an optimal royalty rate
\[ R^* \hat{D}'(R^*) + \hat{D}(R^*)\frac{\mu}{1 + \mu} = 0. \]

In the case of two innovators, firm \( i \) obtains a fixed fee \( F_i = \frac{\Pi_B(r_1 + r_2)}{1 + \mu} - F_j \) for \( j \neq i \). As a result, the maximization of firm \( i \) is
\[ \max_{r_i} r_i\hat{D}(r_1 + r_2) + \frac{\Pi_B(r_1 + r_2)}{1 + \mu} - F_j, \]
with a first order condition that in a symmetric equilibrium can be characterized as
\[ r^u\hat{D}'(2r^u) + \hat{D}(2r^u)\frac{\mu}{1 + \mu} = 0. \]
It is immediate that the log-concavity of \( \hat{D}(R) \) implies that \( R^M < 2r^a \). Otherwise,
\[
R^M = - \frac{\hat{D}(R^M)}{\hat{D}'(R^M)} \frac{\mu}{1 + \mu} = \frac{\hat{D}(2r^a)}{\hat{D}'(2r^a)} \frac{\mu}{1 + \mu} = r^u,
\]
which is a contradiction unless \( R^M = r^a = 0 \) which cannot occur if \( \mu > 0 \). Finally, notice that the first order condition in both cases is increasing in \( \mu \) implying that the higher the distortion from using fixed fees the higher the royalty rate.

**Proof of Proposition 9:** Using the same arguments as in the benchmark model, a pure-strategy equilibrium with royalty rates \( r^*_1 \) and \( r^*_2 \) can exist only if
\[
(1 - g(x_2)) \left[ \Pi_B(r^*_1) - \Pi_B(r^*_1 + r^*_2) + (1 + \mu)F_2 \right] < L_B.
\]
Otherwise, innovator 1 has incentives to lower the royalty rate and induce litigation on the patent of innovator 2. Furthermore, the optimal choice of innovator 2 has to be interior in the sense that it must be the solution to
\[
\max_{r_2} r_2 \hat{D}(r_1 + r_2) + F_2
\]
\[
s.t. \quad \Pi_B(r_1 + r_2) - (1 + \mu)(F_1 + F_2) \geq 0,
\]
\[
(1 - g(x_2)) \left[ \Pi_B(r_1) - \Pi_B(r_1 + r_2) + (1 + \mu)F_2 \right] < L_B.
\]
It is easy to see that this problem can be rewritten as
\[
\max_{r_2} r_2 \hat{D}(r_1 + r_2) + \frac{\Pi_B(r_1 + r_2)}{1 + \mu} - A,
\]
where \( A = \max\{\Pi_B(r_1) - \frac{L_B}{1 - g(x_2)}, (1 + \mu)F_1\} \). This means that the first order condition is identical to the one that arises in the case of ironclad patents,
\[
r_2 \hat{D}'(r_1 + r_2) + \hat{D}(r_1 + r_2) \frac{\mu}{1 + \mu} = 0,
\]
leading to a symmetric equilibrium (in royalty rates) \( r^*_1 = r^*_2 = r^a \).

**Proof of Proposition 10:** First notice that if \( \frac{L_B}{1 - g(x_2)} > \frac{3}{16} \) then \( r^*_2(r_1) < \tilde{r}_2(r_1) \) for all \( r_1 \). Otherwise, \( r^*_2(r_1) < \tilde{r}_2(r_1) \) if and only if \( r_1 < \hat{\rho}_1 \equiv 1 - \sqrt{\frac{16L_B}{3(1 - g(x_2))}} \).

Define \( \tilde{\rho}_1 \) as the value for which patent holder 2 will be indifferent between going to court or offering a royalty rate that the downstream producer will accept. That is,
\[
\tilde{\Pi}_2(\tilde{\rho}_1, \tilde{r}_2(\tilde{\rho}_1)) = g(x_2)\Pi_2(\tilde{\rho}_1, \tilde{r}_2(\tilde{\rho}_1)) - L_U = \Pi_2(\hat{\rho}_1, \tilde{r}_2(\hat{\rho}_1))
\]
where
\[
\tilde{\Pi}_2(r_1, \tilde{r}_2) = g(x_2)\left(1 - r_1\right)^2 8 - L_U,
\]
\[
\Pi_2(r_1, \tilde{r}_2) = \frac{4L_B}{1 - g(x_2)} - (1 - r_1)\tilde{r}_2(r_1).
\]
We now show that this threshold is unique and litigation is preferred by patent holder 2 when \( r_1 < \hat{\rho}_1 \). We can compute the effect of \( r_1 \) on both choices as

\[
\frac{d\tilde{\Pi}_2}{dr_1}(r_1, r_2^c(r_1)) = -g(x_2) \frac{1 - r_1}{4} < 0, \\
\frac{d\Pi_2}{dr_1}(r_1, \tilde{r}_2(r_1)) = -\frac{\tilde{r}_2(r_1)^2}{2\sqrt{(1 - r_1)^2 - \frac{4L_B}{1 - g(x_2)}}} < 0.
\]

Both derivatives are negative. However, notice that \( \frac{d\tilde{\Pi}_2}{dr_1}(r_1, r_2^c) \) is increasing in \( r_1 \) whereas \( \frac{d\Pi_2}{dr_1}(r_1, \tilde{r}_2(r_1)) \) is increasing in \( r_1 \). That is, \( \tilde{\Pi}_2(r_1, r_2^c) \) is convex in \( r_1 \) and \( \Pi_2(r_1, \tilde{r}_2) \) is concave in \( r_1 \). This implies that there might be 0, 1 or 2 points in which these functions cross. We can rule out the case in which the functions cross twice, because \( \tilde{\Pi}_2(\hat{\rho}_1, r_2^c(\hat{\rho}_1)) < \Pi_2(\rho_1, \tilde{r}_2(\hat{\rho}_1)) \) since in this case litigation does not imply a higher royalty rate. Hence, two possibilities remain: (i) the functions do not cross, which occurs if \( \tilde{\Pi}_2(r_1, r_2^c(r_1)) < \Pi_2(r_1, r_2^c(r_1)) \) for all values of \( r_1 \) or (ii) there is a single crossing point \( \hat{\rho}_1 \in (0, \tilde{\rho}_1) \), which occurs if \( \tilde{\Pi}_2(0, r_2^c(0)) > \Pi_2(0, r_2^c(0)) \). The second case arises when \( L_B \) is sufficiently low as stated in the lemma.

The effect of \( L_U \) and \( L_B \) can be characterized directly from the derivatives

\[
\frac{d\tilde{\Pi}_2}{dL_U}(r_1, r_2^c(r_1)) = -1, \\
\frac{d\tilde{\Pi}_2}{dL_B}(r_1, \tilde{r}_2(r_1)) = \frac{1}{2} - \frac{1 - r_1}{4\sqrt{(1 - r_1)^2 - \frac{4L_B}{1 - g(x_2)}}}.
\]

Obviously, the first expression is always negative. The second is positive if and only if \( r_1 \) is in the relevant range, \( r_1 \leq \hat{\rho}_1 \).

\textbf{Proof of Proposition 11:} To prove the result we only need to show that there are instances in which \( \hat{\rho}_1 > \frac{1}{2} \) so that \( r_1 = \frac{1}{2} \) would be feasible and it would induce litigation, but raising the royalty so that patent holder 2 would instead accommodate increases profits for patent holder 1.

First, notice that since \( \hat{\rho}_1 > \frac{1}{2} \) it has to be that \( \tilde{\Pi}_2(\frac{1}{2}, r_2^c(\frac{1}{2})) > \Pi_2(\frac{1}{2}, \tilde{r}_2(\frac{1}{2})) \). This condition implies that

\[
g(x_2) \frac{1}{32} - L_U > \tilde{r}_2(\frac{1}{2}) \frac{1}{2} - \tilde{r}_2(\frac{1}{2})
\]

or

\[
\tilde{r}_2(\frac{1}{2}) < \frac{1 - \sqrt{1 - g(x_2)} + 32L_U}{4}, \tag{14}
\]

given that \( \tilde{r}_2(\frac{1}{2}) < r_2^c(\frac{1}{2}) = \frac{1}{4} \).
We now show that when $\hat{\rho}_1 > \frac{1}{2}$, $\Pi_1(\frac{1}{2}, \bar{r}_2(\frac{1}{2})) > \tilde{\Pi}_1(\frac{1}{2}, \bar{r}_2'(\frac{1}{2}))$. This condition holds if the expected royalty rate is lower without litigation. That is, if

$$\frac{1}{2} + \bar{r}_2(\frac{1}{2}) < \frac{1}{2} + g(x_2)\frac{1}{4}.$$ 

This condition is satisfied given (14), since $\sqrt{1 - g(x_2) + 32L_U} > 1 - g(x_2)$.

By continuity, the previous conditions imply that if $\hat{\rho}_1$ is sufficiently close to $\frac{1}{2}$ then $\Pi_1(\hat{\rho}_1, \bar{r}_2(\hat{\rho}_1)) > \tilde{\Pi}_1(\frac{1}{2}, \bar{r}_2'(\frac{1}{2}))$. 

$\square$
B A Parametric Example

Consider the case where the downstream demand corresponds to a unique consumer with a valuation for one unit of the good. With probability $\alpha \in (0, 1)$ this valuation is 1. With probability $1 - \alpha$ the valuation is $v < 1$. Furthermore, we assume that the downstream firm chooses the price after the valuation has been realized. This timing implies that the downstream producer will always choose a price equal to the realized valuation of the consumer. That is, given $R$ the downstream producer captures all the surplus without generating the losses associated to double marginalization. As a result, expected downstream profits $\Pi_B(R)$ can be computed as

$$\Pi_B(R) = \begin{cases} 
\alpha + (1 - \alpha)v - R & \text{if } R \leq v, \\
\alpha(1 - R) & \text{if } R \in (v, 1], \\
0 & \text{otherwise.}
\end{cases} \quad (15)$$

These profits are decreasing and weakly convex in $R$.\(^{19}\) Notice that the demand is weakly log-concave in the price as expected from Assumption 1. However, the fact that profits are not linear everywhere is enough for our results to go through.

We start by characterizing the royalty rate that maximizes joint profits for the upstream patent holders when their portfolio is sufficiently strong so that $g(x_1) = g(x_2) = 1$. This royalty rate will be used as a benchmark for the case in which innovators decide independently.

**Proposition 12.** Under the two-point demand function, when $g(x_1) = g(x_2) = 1$ there is a continuum of undominated pure-strategy equilibria. The corresponding royalty rates $(r_1^u, r_2^u)$ can be characterized as follows:

1. If $v \geq \frac{2\alpha}{1 + \alpha}$, $R^u = r_1^u + r_2^u = v$ with $r_i^u \leq \frac{v - \alpha}{1 - \alpha}$ for $i = 1, 2$,

2. If $v \leq \frac{1 + \alpha}{2}$, $R^u = r_1^u + r_2^u = 1$ with $r_i^u \geq \frac{v - \alpha}{1 - \alpha}$ for $i = 1, 2$.

Both kinds of equilibria co-exist when $\frac{2\alpha}{1 + \alpha} \leq v \leq \frac{1 + \alpha}{2}$. All equilibria imply royalty stacking when $\alpha \leq v < \frac{2\alpha}{1 + \alpha}$.

**Proof.** Regarding the first case, contingent on selling with probability 1 the sum of royalties must be equal to $v$ or otherwise any patent holder would deviate and increase the

\(^{19}\)A dead-weight loss would arise if we assumed that the downstream producer chose the price before the demand is realized. In that case, the threshold value on $R$ in the profit function $\Pi_B(R)$ would change. That is, $p_M(R) = v$ if and only if $R \leq \tilde{R} \equiv \frac{v - \alpha}{1 - \alpha} < v$. Since double marginalization does not interact with the mechanisms explored in this paper (see Assumption 1), the main results would go through under this alternative assumption although at the cost of an increasing technical complexity.
royalty rate. Hence, take \( r_1^u \) and \( r_2^u = v - r_1^u \) and suppose without loss of generality that \( r_1^u \geq \frac{v}{2} \geq r_2^u \). The optimal deviation for patentee \( i \) is \( \hat{r}_i = 1 - r_j^u \) for \( j \neq i \) and it would be unprofitable if \( v - r_j^u \geq \alpha(1 - r_j^u) \) or \( r_j^u \leq \frac{v - \alpha}{1 - \alpha} \). Such a combination of royalties is only possible as long as \( \frac{v}{2} \leq r_1^u \leq \frac{v - \alpha}{1 - \alpha} \) or \( v \geq \frac{2\alpha}{1+\alpha} \).

For the second case, take \( r_1^u \) and \( r_2^u = 1 - r_1^u \) and suppose without loss of generality that \( r_1^u \geq \frac{1}{2} \geq r_2^u \). The optimal deviation for patentee \( i \) is \( \hat{r}_i = v - r_j^u \) for \( j \neq i \) if it leads to a positive royalty and it would be unprofitable if \( \alpha(1 - r_j^u) \geq v - r_j^u \) or \( r_j^u \geq \frac{v - \alpha}{1 - \alpha} \). Such a combination of royalties will be possible as long as \( \frac{v - \alpha}{1 - \alpha} \leq r_2^u \leq \frac{1}{2} \) or \( v \leq \frac{1+\alpha}{2} \).

Finally, notice that \( \frac{2\alpha}{1+\alpha} < \frac{1+\alpha}{2} \) for all \( \alpha \in [0, 1] \) so both equilibria can co-exist.

Intuitively, the equilibrium with a total royalty of 1 is likely to exist when \( v \) is small and \( \alpha \) is sufficiently close to 1. A deviation might exist if any patent holder prefers to decrease the royalty rate in order to cater the consumer regardless of her valuation. This deviation is illustrated in Figure 8. Given \( r_2^u \), innovator 1 can choose \( r_1^u = 1 - r_2^u \) or deviate and choose \( \hat{r}_1 = v - r_2^u \) so that the probability of selling increases from \( \alpha \) to 1. Such a deviation is unprofitable if \( r_2^u \) is sufficiently large and, thus, the low \( \hat{r}_1 \) does not allow the firm to benefit from the increase in sales. In the limit, when \( v = 0 \) or \( \alpha = 1 \) this equilibrium holds for any combination of royalties that sums up to 1.

Similarly, equilibria with a total royalty equal to \( R^u = v \) are likely to exist when \( v \) is sufficiently high and \( \alpha \) is sufficiently small. This time a deviation aims to capture the additional surplus when consumer valuation is 1, even if this surplus is materialized only with probability \( \alpha \). To prevent this deviation each innovator must set a modest royalty so that the other firm already obtains sufficiently high profits in equilibrium, thus reducing the appeal of raising the royalty rate and reducing the probability of sale. In the limit, when \( v = 1 \) or \( \alpha = 0 \) any combination of royalty rates that sums up to \( v \) would constitute an equilibrium. Such coordination would also maximize social welfare.

In contrast, a total royalty \( R = v \) would be chosen by a single innovator owning both patents if and only if \( v > \alpha \). Royalty stacking, which here takes the form of a total royalty rate equal to 1 when joint profit maximization requires \( R^M = v \), arises as a Nash equilibrium when \( \alpha \leq v < \frac{1+\alpha}{2} \) and it is unique when \( \alpha \leq v < \frac{2\alpha}{1+\alpha} \).

### B.1 One Constrained Patent Holder

Suppose now that \( g(x_1) = 1 \) and \( g(x_2) < 1 \) so that the downstream producer may only be interested in litigating the portfolio of innovator 2. We restrict our discussion to the
Figure 8: Equilibrium with $r_1^u + r_2^u = 1$. Profits for patent holder 1 correspond to the gray area. The striped area indicates the profits under the optimal deviation.

case where $v > \alpha$ so that, according to Proposition 12, in the previous benchmark a combination of royalties for which $r_1^* + r_2^* = 1$ constituted an equilibrium with royalty stacking. Once the threat of litigation is accounted for, such an equilibrium may fail to exist for two reasons. First, given $r_1^* + r_2^* = 1$, the downstream firm will obtain higher profits by going to court if

$$r_2^* > \bar{r}_2 = \left\{ \begin{array}{ll}
\frac{L_B}{\alpha(1-g(x_2))} & \text{if } \frac{L_B}{1-g(x_2)} < \alpha(1-v), \\
(1-\alpha)(1-v) & \text{otherwise.}
\end{array} \right. \quad (16)$$

Second, if $r_2^* \leq \bar{r}_2$, innovator 1 might benefit from deviating to a royalty below $r_1^*$ which induces litigation. Using a similar logic as in Lemma 3, we can show that when the litigation cost $L_B$ is sufficiently low there is a threshold $\bar{r}_1 > 0$ such that litigation will occur if $r_1 < \bar{r}_1$. The next lemma characterizes the region under which litigation may occur as a function of $r_1$.

**Lemma 13.** Under the two-point demand function, if $r_2 > \bar{r}_2$ there is no equilibrium with royalty stacking and no litigation. If

$$r_2 \leq \bar{r}_2 = \left\{ \begin{array}{ll}
\frac{L_B}{1-g(x_2)} & \text{if } \frac{L_B}{1-g(x_2)} \leq v, \\
\frac{L_B}{\alpha(1-g(x_2))} - \frac{L_B}{1-g(x_2)} & 1-\alpha v, \\
(1-\alpha)(1-v) & \text{otherwise,}
\end{array} \right. \quad (17)$$

innovator 2 will not be brought to court for any $r_1 \geq 0$. If $r_2 \in (\bar{r}_2, \bar{r}_2]$, litigation will occur if

$$r_1 < \tilde{r}_1(r_2) = v + \frac{\alpha}{1-\alpha} r_2 - \frac{L_B}{(1-\alpha)(1-g(x_2))} \leq v. \quad (18)$$

**Proof.** From the argument in the text it is immediate that for $r_2 > \bar{r}_2$ an equilibrium without litigation and with royalty stacking cannot arise, since for all $r_1 = 1-r_2$ litigation
will be profitable for the downstream firm. For the rest of the arguments, it is useful to distinguish two cases depending on the relationship between \( v \) and \( \frac{L_B}{1 - g(x_2)} \).

Suppose that \( \frac{L_B}{1 - g(x_2)} \leq v \). First notice that litigation will not occur for any value of \( r_1 \) if and only if \( r_2 \leq \bar{r}_2 = \frac{L_B}{1 - g(x_2)} \). From Lemma 3, the incentives to litigate are highest when \( r_1 = 0 \) and, in that case, the expected profits from going to court are \((1 - g(x_2))r_2 \leq L_B\). Consider now the case \( r_2 \in \left( \frac{L_B}{1 - g(x_2)}, \bar{r}_2 \right) \). By definition, when \( r_2 < \bar{r}_2 \) a royalty \( r_1 = 1 - r_2 \) induces litigation. Even if \( r_2 \) is sufficiently close to \( v \) a royalty \( r_1 = 0 \) litigation will always be profitable for the downstream producer since \((1 - g(x_2)) [\Pi_B(0) - \Pi_B(v)] = (1 - g(x_2))v > L_B\). The value of \( r_2 \) for which the downstream firm is indifferent between litigating or not is defined by (18).

Consider now the case in which \( v < \frac{L_B}{1 - g(x_2)} \). First suppose that \( r_2 \leq \bar{r}_2 = \frac{L_B}{\alpha(1 - g(x_2))} - \frac{1 - \alpha}{\alpha} v < \frac{L_B}{1 - g(x_2)} \). In that case, even \( r_1 \) will not induce litigation since the downstream profits from going to court will be \((1 - g(x_2))(1 - (1 - \alpha)v + \alpha r_2) \leq L_B\). Suppose now that \( r_2 \in \left( \frac{L_B}{\alpha(1 - g(x_2))} - \frac{1 - \alpha}{\alpha} v, \bar{r}_2 \right) \). By definition, when \( r_2 < \bar{r}_2 \) a royalty of \( r_1 = 1 - r_2 \) induces litigation. If, instead, \( r_2 \) is sufficiently close to \( \frac{L_B}{\alpha(1 - g(x_2))} - \frac{1 - \alpha}{\alpha} v < v \) a royalty \( r_1 = 0 \) will induce litigation since the downstream profits of going to court are \((1 - g(x_2))(1 - (1 - \alpha)v + \alpha r_2) > L_B\). The value of \( r_2 \) for which the downstream firm is indifferent between litigating or not is defined by (18).

Innovator 1 might benefit from lowering the royalty rate below \( \bar{r}_1 \) if, by causing litigation against patentee 2, the quantity sold expands from \( \alpha \) to 1, which would occur with probability \( 1 - g(x_2) \). Hence, a profitable deviation \( \hat{r}_1 \) must be lower than \( v \). Since \( \bar{r}_1(\bar{r}_2) \leq v \) it follows that the optimal deviation for innovator 1 when patentee 2 sets \( r_2^* \leq \bar{r}_2 \) is the highest royalty rate which guarantees that the patent of innovator 2 is litigated, \( \hat{r}_1 = \bar{r}_1(r_2^*) \).\(^{20}\) Innovator 1’s profits in that case would become

\[
\hat{\Pi}_1 = [\alpha + (1 - \alpha)(1 - g(x_2))] \hat{r}_1. \tag{19}
\]

That is, a deviation will lead to profits equal to \( \hat{r}_1 \) either because the valuation of the consumer is 1 or because the valuation is \( v \) but the patent of innovator 2 has been invalidated in court. This deviation will take place if profits, \( \hat{\Pi}_1 \), are higher than those in the candidate equilibrium, \( \Pi_1^* = \alpha r_1^* \). Notice that the lower are \( r_1^* \) or \( g(x_2) \) the more binding this condition becomes. The next proposition characterizes the circumstances under which \( \Pi_1^* \geq \hat{\Pi}_1 \) cannot hold while, as required by Proposition 12, \( r_2^* \geq \frac{\bar{r}_2 - \alpha}{1 - \alpha} \). In those situations, an equilibrium with royalty stacking and no litigation will fail to exist.

\(^{20}\)More precisely, given our assumptions, \( \hat{r}_1 \) should be slightly lower than \( \bar{r}_1(r_2^*) \).
Proposition 14. Consider the two-point demand function case and suppose \( v > \alpha \). If 
\[
\frac{L_B}{1-g(x_2)} < \frac{v-\alpha}{1-\alpha}
\]
and \( g(x_2) \) is sufficiently small, there is no equilibrium with royalty stacking and no litigation. If \( L_U \) is sufficiently large, only the efficient equilibrium exists, which involves 
\[
r'_2 \leq \frac{L_B}{1-g(x_2)} < v \quad \text{and} \quad r'_1 = v - r'_2.
\]

Proof. From Proposition 12, a necessary condition for a royalty-stacking equilibrium with no litigation to exist is that 
\[
r'_1 + r'_2 = 1 \quad \text{and} \quad r'_2 \geq \frac{v-\alpha}{1-\alpha} \quad \text{or, else, patent holder 1 would have incentives to lower its royalty rate.}
\]

Furthermore, a second deviation consisting in choosing a royalty slightly lower than \( \bar{r}_1(r'_2) \) might be profitable for patent holder 1 if it leads to profits 
\[
\hat{\Pi}_1 = [\alpha + (1 - \alpha)(1 - g(x_2))] \bar{r}_1(r'_2) > \alpha r'_1.
\]

This condition holds if 
\[
r'_2 < \rho(G) \equiv \frac{(1-\alpha)(\alpha - Gv) + G \frac{L_B}{1-g(x_2)}}{\alpha(G + (1 - \alpha))},
\]
where \( G \equiv \alpha + (1 - \alpha)(1 - g(x_2)) \in [\alpha, 1] \). Thus, in instances in which \( \rho(G) < \frac{v-\alpha}{1-\alpha} \) an equilibrium with royalty stacking will fail to exist. This inequality implies that 
\[
G < G^* \left( \frac{L_B}{1-g(x_2)} \right) = \frac{\alpha(1-\alpha)(1-v)}{(1-\alpha)\left(v - \frac{L_B}{1-g(x_2)}\right) - \alpha^2(1-v)}.
\]

This function is increasing in \( \frac{L_B}{1-g(x_2)} \) and \( G^* \left( \frac{L_B}{1-g(x_2)} \right) < G^* \left( \frac{v-\alpha}{1-\alpha} \right) = 1 \). Hence, there is always \( g(x_2) \) sufficiently small so that the deviation will be optimal.

We now consider conditions under which an equilibrium with \( R = v \) exists. Consider the case 
\[
r'_2 = \frac{L_B}{1-g(x_2)} < \frac{v-\alpha}{1-\alpha} \quad \text{and} \quad r'_1 = v - r'_2.
\]
From (17), \( r'_2 \) avoids litigation and by Proposition 12 patentee 1 has no incentive to deviate. Thus, the only deviation we need to consider from patentee 2 is such that \( R > v \). However, notice that 
\[
r'_1 = v - \frac{L_B}{1-g(x_2)} = v + \frac{\alpha}{1-\alpha}r'_2 - \frac{L_B}{(1-\alpha)(1-g(x_2))} = \bar{r}_1(r'_2),
\]
and so any higher \( r_2 \) will induce litigation. Hence, an equilibrium in pure strategies exists if and only if such a deviation is not profitable 
\[
\frac{L_B}{1-g(x_2)} \geq \alpha g(x_2) \left(1 - v + \frac{L_B}{1-g(x_2)}\right) - L_U.
\]
This condition is guaranteed if \( g(x_2) \) is sufficiently small or \( L_U \) sufficiently large. \( \square \)
This result indicates that when $L_B$ and/or $g(x_2)$ are sufficiently low, royalty stacking will not arise in an equilibrium without litigation. In order to interpret this result it is useful to start by considering the case under which such an equilibrium with royalty stacking may exist. From (17) we know that if $r_2^* \leq \frac{L_B}{1-g(x_2)}$ the Inverse Cournot effect has no bite since there is no positive value of $\hat{r}_1$ that triggers litigation. When $\frac{L_B}{1-g(x_2)} \geq \frac{v-\alpha}{1-\alpha}$ it is also possible to find $r_2^* \geq \frac{v-\alpha}{1-\alpha}$, satisfying the conditions of Proposition 12. Hence, it is optimal for innovator 1 to choose $r_1^* = 1 - r_2^*$ and an equilibrium with royalty stacking will arise in that case.

When $\frac{L_B}{1-g(x_2)} < \frac{v-\alpha}{1-\alpha}$, however, a deviation from the royalty-stacking equilibrium may exist. Starting from a combination of royalties $(r_1^*, r_2^*)$ with $r_i^* \geq \frac{v-\alpha}{1-\alpha}$ for $i = 1, 2$, patent holder 1 trades off a decrease in the royalty to $r_1^*(r_2^*)$ with an increase in the probability of sale from $\alpha$ to $\alpha + (1 - \alpha)(1 - g(x_2))$. The previous proposition shows that if $g(x_2)$ is sufficiently small this expansion effect dominates and the deviation is profitable. The reason for this result is, precisely, that when $v > \alpha$ eliminating royalty stacking raises total profits and the smaller is $g(x_2)$ the larger is the proportion of that increase that innovator 1 can appropriate.

The second part of the proposition also indicates that when the probability of success in court of innovator 2 is small two results concur. First, the royalty rate is commensurate with the strength of the patent portfolio and the cost of challenging those rights by the downstream producer, $r_2^* \leq \frac{L_B}{1-g(x_2)}$. This result arises from the fact that when $g(x_2)$ is small innovator 2 must choose a low royalty rate to discourage the downstream producer from engaging in litigation that will, most likely, result in a zero royalty. Second, and more interestingly, the joint profit maximizing equilibrium, consisting of $R^M = v$, may exist. The reason is that the low value of $r_2$ makes innovator 1 the residual claimant of the surplus generated. This can be seen using Figure 8, where the lower is $r_2$ the more innovator 1 internalizes the losses that a deviation towards a larger royalty rate may entail.

### B.2 Two Constrained Patent Holders

Suppose now that both firms have an identical patent holdings that does not confer full protection against litigation, $g(x_1) = g(x_2) = g(x) < 1$. As in the previous case we focus on the situation in which royalty stacking was an equilibrium when no litigation was feasible, $v > \alpha$. We study whether litigation affects the existence of an equilibrium.
with royalty stacking, so that \( r_1^* + r_2^* = 1 \). As in the general case, it is enough to focus on the symmetric case in which \( r_1^* = r_2^* = \frac{1}{2} \) as if this equilibrium did not exist no asymmetric equilibrium would exist either.\(^{21}\) We also explained that in the symmetric case it will never be optimal for the downstream producer to bring to court only one of the innovators. The next lemma characterizes the threshold values of \( \hat{r}_1 \) for which innovator 1 expects to be sued in case the other patentee loses in court.

**Lemma 15.** Under the two-point demand function with \( v > \alpha \), suppose that for \( r_1^* = r_2^* = \frac{1}{2} \) it is not profitable for the downstream producer to engage in litigation. If by deviating to \( \hat{r}_1 < r_1^* \) innovator 2 is sued and its patent invalidated, innovator 1 will also be sued if and only if \( \hat{r}_1 > \frac{L_B}{1-g(x)}. \)

**Proof.** First notice that if patent holder 2 loses in court patent holder 1 will be brought to court if and only if

\[
\Pi_B(0) - \Pi_B(\hat{r}_1) > \frac{L_B}{1-g(x)}
\]

or \( \hat{r}_1 > \frac{L_B}{1-g(x)} \). Also notice that, from the arguments in the text, if originally it was not optimal to engage in litigation it has to be that

\[
\Pi_B(1/2) - \Pi_B(1) \leq \frac{L_B}{1-g(x)}.
\]

Patent holder 1 would be sued after downstream producer loses against patent holder 2 if

\[
\Pi_B(1/2) - \Pi_B(1/2 + \hat{r}_1) > \frac{L_B}{1-g(x)}
\]

which is incompatible with the previous condition. \( \square \)

The deviations that this lemma characterizes determine two regions depending on whether \( \hat{r}_1 \) is higher or lower than \( \frac{L_B}{1-g(x)} \). Both deviations are less profitable than in the case in which \( g(x_1) = 1 \), albeit for slightly different reasons. In one of the regions, by choosing a low \( \hat{r}_1 \), innovator 1 eludes litigation but at the cost of a significant reduction in licensing revenues. In the second region, when \( \hat{r}_1 \) is higher, the lower profitability of the deviation arises from the probability that the innovator might not receive any licensing revenues from its patent if it is declared invalid, together with the corresponding litigation costs. In particular, in this last region, the profits from a deviation are

\[
\hat{\Pi}_1 = g(x)\alpha \hat{r}_1 + (1 - g(x)) [g(x)\hat{r}_1 - L_U].
\]

\(^{21}\)As discussed in previous sections, an equilibrium may fail to exist because one of the royalty rates is too low and, as a result, either the innovator decides to deviate and raise it even at the cost of being sued or the other patentee may benefit from lowering its own royalty rate and serve the whole market. By focusing on the symmetric royalty rate we are minimizing the profitability of these deviations.
That is, when the patent of the other innovator is upheld in court the expected quantity is $\alpha$. If, instead, the portfolio of innovator 2 is invalidated and the downstream producer also decides to sue innovator 1, the quantity sold is 1 but the royalty $\hat{r}_1$ is only paid if the corresponding patent is upheld in the second trial.

We now illustrate how the risk of a litigation cascade might foster the existence of an equilibrium with royalty stacking and no litigation. Take the case in which $L_U$ is very large so that the threat of litigation is particularly relevant for the upstream patent holders, and consider the situation in which $\nu \leq \frac{1}{2}$. Given $r^*_1 = r^*_2 = \frac{1}{2}$, two conditions must be satisfied for such an equilibrium to exist. First, using equation (4), the downstream producer should not be interested in going to court, which in this case it implies

$$\frac{L_B}{1 - g(x_2)} \geq \frac{1}{2 - g(x)} \left[ \frac{g(x) \alpha}{2} + (1 - g(x)(\alpha + (1 - \alpha)\nu)) \right].$$

Second, the cost of a litigation cascade implies that the optimal deviation of innovator $i$, for $i = 1, 2$, involves $\hat{r}_i = \min \{\nu, \frac{L_B}{1 - g(x)}\}$ and such a deviation is unprofitable if and only if $\Pi_1 \leq \Pi^*$ or

$$[\alpha + (1 - \alpha)(1 - g(x))] \hat{r}_i \leq \frac{\alpha}{2}. \tag{21}$$

Notice that because, as in the case of one constrained patent holder, $\hat{r}_i \leq v$ the expected demand expands if the patent of the other innovator is invalidated.

These two conditions provide a lower and upper bound, respectively, on $\frac{L_B}{1 - g(x)}$ for an equilibrium with royalty stacking and no litigation to exist. That is, the litigation costs of the downstream producer must be sufficiently large to discourage this firm from litigating but they must also be sufficiently small so that the decrease in the royalty rate necessary for a deviating firm to fend off litigation is large.

Although the previous conditions are highly non-linear in the main parameters of the model it is easy to find combinations that satisfy them. More interestingly, we can also find situations in which this equilibrium with a total royalty equal $R^* = 1$ is sustainable when both innovators have a very strong or a very weak patent but not in the case in which the patents have an asymmetric strength.

**Example 4.** Consider the parameter values $\alpha = 0.1$, $\nu = 0.3$, $g(x_2) = 0.68$, $L_B = 0.035$, and $L_U$ sufficiently large. If litigation were not possible, the parameter values would satisfy the conditions of Proposition 12 and an equilibrium with royalty stacking, $R^u = 1$, would exist.

Next, consider the case in which $g(x_1) = 1$ so that only the second patent holder is potentially constrained. By construction, $\frac{L_B}{1 - g(x_2)} < \frac{\nu - \alpha}{1 - \alpha}$, and it can be verified that...
innovator 1 has incentives to deviate from any candidate equilibrium \((r_1^*, r_2^*)\) and choose \(\tilde{r}_1(r_2^*)\), so that the royalty-stacking will not emerge in this case.

Finally, consider the case in which \(g(x_1) = g(x_2) = 0.68\). Equations (20) and (21) are satisfied and, thus, the royalty-stacking equilibrium exists when both innovators are similarly constrained.

The previous example illustrates that, as in the main sections of the paper, once we introduce litigation in the model the royalty rate is not necessarily monotonic in the strength of the patents. When patents are weaker but more evenly distributed the royalty-stacking problem might actually become more relevant.

### B.3 Ad-Valorem Royalties

In this section we show that, under ad-valorem royalties, royalty stacking can also be eliminated. We do so in the context of the parametric example of this section where we assume that the downstream producer faces a marginal cost \(c \in (0, v)\). Under ad-valorem royalties downstream profits can be written as

\[
\Pi_B(S) = \begin{cases} 
(1 - S)(\alpha + (1 - \alpha)v) - c & \text{if } S \leq 1 - \frac{c}{\alpha}, \\
\alpha(1 - S - c) & \text{if } S \in \left(1 - \frac{c}{\alpha}, 1 - c\right], \\
0 & \text{otherwise,}
\end{cases}
\]

We can now write a counterpart of Proposition 12.

**Proposition 16.** Under the two-point demand function, there exists a threshold \(\tilde{v}\) such that joint profit maximization implies a royalty \(S^M = 1 - c\) if \(v > \tilde{v}\) and \(S^M = 1 - \frac{c}{\alpha} v\) otherwise. Under competition there is a continuum of undominated pure-strategy equilibria. There exist values \(v_u\) and \(v_d\), such that \(\tilde{v} \leq v < v\) so that the equilibrium ad-valorem royalty rates \((s_{u1}^u, s_{u2}^u)\) can be characterized as follows:

1. If \(v \geq v_u\), \(S^n = s_1^u + s_2^u = 1 - \frac{c}{\alpha} v\) with \(s_i^u \leq 1 - \frac{(1 - 2\alpha)cv + ac}{(1 - \alpha)\alpha v}\) for \(i = 1, 2\).

2. If \(v \leq v_d\), \(S^n = s_1^u + s_2^u = 1 - c\) with \(s_i^u \geq 1 - \frac{(1 - 2\alpha)cv + ac}{(1 - \alpha)\alpha v}\) for \(i = 1, 2\).

As a result, royalty stacking emerges in equilibrium when \(\tilde{v} < v < v\).

**Proof.** As in the case of per-unit royalties only two ad-valorem rates can maximize joint profits, \(1 - \frac{c}{\alpha} v\) and \(1 - c\). The low rate dominates if

\[
\left(1 - \frac{c}{\alpha}\right) (\alpha + (1 - \alpha)v) \geq (1 - c)\alpha
\]
or

\[ v \leq \hat{v} \equiv \frac{(1 - 2\alpha) c + \sqrt{(4\alpha^2 - 4\alpha + 1) c^2 + (4\alpha - 4\alpha^2) c}}{2(1 - \alpha)}. \]

Regarding the Nash equilibria, suppose that patent holder \( j = 1, 2 \) chooses \( s_j \). Patent holder \( i \) will prefer \( s_i = 1 - \frac{c}{v} - s_j \) to \( s_i = 1 - c - s_j \) if

\[ \left(1 - \frac{c}{v} - s_j\right) (\alpha + (1 - \alpha)v) \geq (1 - c - s_j) \alpha \]

or \( s_j \leq \bar{s} \equiv \frac{(1-\alpha)\alpha - \alpha c}{(1-\alpha)^{\alpha}} \). Hence, for this equilibrium to exist we require that

\[ 2 \bar{s} \geq 1 - \frac{c}{v} \]

or\( s_j \geq \frac{c}{v} \equiv \frac{1 - \alpha \alpha - \alpha c}{(1-\alpha)^{\alpha}} \). Hence, for this equilibrium to exist we require that

\[ 2 \bar{s} \geq 1 - \frac{c}{v} \]

or\( s_j \leq \bar{s} \equiv \frac{(1-\alpha)\alpha - \alpha c}{(1-\alpha)^{\alpha}} \). Hence, for this equilibrium to exist we require that

\[ 2 \bar{s} \geq 1 - \frac{c}{v} \]

or\( s_j \geq \frac{c}{v} \equiv \frac{1 - \alpha \alpha - \alpha c}{(1-\alpha)^{\alpha}} \).

Comparison of the threshold expressions lead to \( \hat{v} \leq v \leq \bar{v} \) if \( \alpha < 0 \) and \( c \in (0,v) \).

As in the previous case, royalty stacking arises when \( v \) takes an intermediate value. Innovators individually charge a total royalty rate higher than what joint maximization would find optimal. The proof provides the specific expressions for the different thresholds.

We turn now to the case where innovator 1 has a portfolio of strength \( x_1 \) such that \( g(x_1) = 1 \) whereas \( x_2 \) is such that \( g(x_2) < 1 \). First notice that a necessary condition for an equilibrium with royalty stacking to exist, \( s_1^* + s_2^* = 1 - c \), is that the downstream producer does not have incentives to sue innovator 2. In particular, this implies that

\[ (1 - g(x_2)) [\Pi_B(1 - c - s_2^*) - \Pi_B(1 - c)] \leq L_B. \]

Two cases arise depending on whether the royalty of innovator 1, \( s_1^* = 1 - c - s_2^* \), is greater than \( 1 - \frac{c}{v} \) or not. As a result, litigation by the downstream producer will not be profitable if

\[
\begin{align*}
    s_2^* \leq \bar{s}_2 &= \left\{ \begin{array}{ll}
        \frac{L_B}{\alpha(1-g(x_2))} + \frac{L_B}{(1-\alpha)(1-v)} & \text{if } \frac{L_B}{1-g(x_2)} < \alpha \left( \frac{c}{v} - c \right), \\
        \frac{\alpha}{(1-\alpha)(1-v)} & \text{otherwise}.
    \end{array} \right.
\end{align*}
\]

For a given royalty rate \( s_2 \) set by innovator 2 we can define, using equation (7), the threshold royalty rate \( \bar{s}_1 \) as

\[
\bar{s}_1(s_2) = 1 - \frac{c}{v} - \frac{L_B}{(1 - g(x_2))(1 - \alpha)v} + \frac{\alpha}{(1-\alpha)v} s_2 \text{ if } s_2 \leq s_2 \leq \bar{s}_2.
\]
where for any \( s_2 < \bar{s}_2 \) we have \( \bar{s}_1 \leq 1 - \frac{\xi}{\alpha} \). As in the previous case, if \( s_1 < \bar{s}_1(s_2) \) the patent of innovator 2 will be litigated by the downstream producer. The lower threshold is defined as the highest value of \( s_2 \) for which it is not worthwhile to sue innovator 2 even when innovator 1 chooses \( s_1 = 0 \) and it can be written as

\[
s_2 < \bar{s}_2 = \begin{cases} 
\frac{L_B}{(1-g(x_2))(\alpha + (1-\alpha)v)} & \text{if } \frac{L_B}{1-g(x_2)} \leq (\alpha + (1-\alpha)v) \left( 1 - \frac{\xi}{v} \right), \\
\frac{1}{\alpha(1-g(x_2))} - \frac{1-\alpha}{\alpha}(v-c) & \text{otherwise.}
\end{cases}
\]  

(24)

Given \( s_2 \), innovator 1 will have incentives to deviate if, by choosing \( s_1 \leq \bar{s}_1(s_2) \), profits increase due to the increase in quantity when the patent of innovator 2 is invalidated. The next proposition shows that, as in the case in which royalties were paid per-unit, the royalty stacking equilibrium fails to exist when the portfolio of innovator 2 is sufficiently weak.

**Proposition 17.** Suppose that \( v > \hat{v} \). If \( \frac{L_B}{(1-g(x_2))(\alpha + (1-\alpha)v)} < 1 - \frac{(1-2\alpha)c\alpha+ac}{(1-\alpha)v^2} \) and \( g(x_2) \) is sufficiently small, there is no pure strategy equilibrium with royalty stacking.

*Proof.* From Proposition 16, a necessary condition for a royalty-stacking equilibrium to exist is that \( s_1^* + s_2^* = 1 - c \) and \( s_2^* \geq 1 - \frac{(1-2\alpha)c\alpha+ac}{(1-\alpha)v^2} \) or, else, patent holder 1 would have incentives to lower its royalty rate.

Furthermore, a second deviation consisting in choosing a royalty slightly lower than \( \bar{s}_1(s_2^*) \) might be profitable for patent holder 1 if it leads to profits

\[
\hat{\Pi}_1 = \left[ \alpha + (1-\alpha)(1-g(x_2))v \right] \bar{s}_1(s_2^*) > \alpha s_1^*.
\]

This condition holds if

\[
s_2^* < \sigma(G) \equiv \frac{(1-\alpha)v \left[ \alpha(1-c) - G \left( 1 - \frac{\xi}{\alpha} \right) \right] + G \frac{L_B}{1-g(x_2)}}{\alpha(G + (1-\alpha)v)},
\]

where \( G \equiv \alpha + (1-\alpha)(1-g(x_2))v \in \left[ \alpha, 1 \right] \). Thus, in instances in which \( \sigma(G) < 1 - \frac{(1-2\alpha)c\alpha+ac}{(1-\alpha)v^2} \) an equilibrium with royalty stacking will fail to exist. This inequality implies that

\[
G < G^* \left( \frac{L_B}{1-g(x_2)} \right) \equiv \frac{\alpha(1-\alpha)(1-v)c\alpha + (1-\alpha)v}{-\frac{L_B}{1-g(x_2)} \left( 1 - \alpha \right) v^2 + (1 - \alpha) v^2 ((1 - \alpha) (v-c) + \alpha) - (1-2\alpha)c\alpha-c}. 
\]

This function is increasing in \( \frac{L_B}{1-g(x_2)} \) and \( G^* \left( \frac{L_B}{1-g(x_2)} \right) < G^* \left( (\alpha + (1-\alpha)v) * \left( 1 - \frac{(1-2\alpha)c\alpha+ac}{(1-\alpha)v^2} \right) \right) = \alpha + (1-\alpha)v \). Hence, there is always \( g(x_2) \) sufficiently small so that the deviation will be optimal. \( \square \)