Auctions with Privately Known Capacities*

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Abstract

We study a multi-unit auction model in which bidders are privately informed about the maximum number of units they are willing to trade. No matter how big or small, private information in this dimension (which we refer to as 'capacity') changes the nature of the equilibrium as compared to when private information is on costs (or valuations). Privately known capacities also break the revenue equivalence between uniform-price and discriminatory auctions: the former lead to higher payments to firms even when capacities are independently drawn. Although our setup applies to a wide variety of contexts (from Central Banks' liquidity auctions to emissions trading), our analysis is motivated by the performance of electricity markets. The shift from conventional to renewable energies makes it necessary to model capacities as being privately known, with meaningful implications for market performance.

Keywords: multi-unit auctions, private information, electricity markets, renewables.

JEL Codes: L13, L94.

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1 Introduction

In many multi-unit auction settings, bidders are privately informed about the maximum number of units they are willing to buy or sell. This is the case of Treasury Bill auctions (Hortaçsu and McAdams, 2010; Kastl, 2011), in which banks are privately informed about their hedging needs and, consequently, on the volume of bonds they aim to acquire. Other examples include emission permit auctions (Cantillon and Slechten, 2018), spectrum auctions (Milgrom, 2004), Central Banks' liquidity auctions (Klemperer, 2010), electricity capacity markets (Fabra, 2018; Llobet and Padilla, 2018), or auctions for renewable investments (Cantillon, 2014; Fabra and Montero, 2020), to name a few.[1] In this paper we develop a model of strategic bidding behavior that captures this source of private information (which we generically refer to as 'privately-known capacities').[2] We highlight the differences and similarities between our equilibrium predictions and those of previous multi-unit auction analyses that typically assume private information on costs (or valuations) rather than on capacities.[3]

Because of their policy relevance, we motivate our analysis in the context of electricity markets, which have recently witnessed a rapid deployment of renewable energies. Whereas competition among conventional fossil-fuel generators is by now well understood (e.g., Borenstein, 2002; von der Fehr and Harbord, 1993; Green and Newbery, 1992) much less is known about competition among wind and solar producers (which we broadly refer to as *renewables*). Competition-wise, there are two key differences between conventional and renewable technologies. First, the marginal cost of conventional power plants de-

[1] Likewise, firms have private information on capacities in a wide range of markets that can be analyzed through the lens of auction theory (Klemperer, 2003), e.g., the markets for hotel bookings or ride-hailing services, in which firms are privately informed about the number of empty rooms or available cars. Needless to say, in these examples, firms or bidders might also be privately informed about other dimensions.

[2] Similar examples can be found in markets which are not organized as formal auctions: how much oil an oil producer is willing to sell depends on the remaining oil in the well; how many available cars a ride-hailing company has depends on how many drivers are on service, net of those who are already occupied; how many rooms a hotel is willing to offer online depends on how many rooms have been booked through other channels; how much cloud computing space a firm is willing to offer depends on how much excess capacity it has above its own data needs; or how much olive oil a firm is willing to sell depends on whether its harvest was good or bad.

[3] While incomplete information on capacities can be captured through incomplete information on costs or valuations, this approach typically requires marginal costs (or valuations) to be bounded and continuous. Some papers allow for discrete valuations with a maximum number of units demanded (e.g., Burkett and Woodward, 2020; Hortaçsu and McAdams, 2010), or focus on the empirical identification of the optimal bids from the First Order Condition of profit maximization rather than on a full equilibrium characterization.

pends on their efficiency rate as well as on the price at which they buy the fossil fuel. In contrast, the marginal cost of renewable generation is essentially zero, as plants produce electricity out of freely available natural resources (e.g., wind or sun). Second, the capacity of conventional power plants is well known, as they tend to be available at all times (absent rare outages). In contrast, the availability of renewable plants is uncertain and depends on weather conditions that are forecasted with error (Gowrisankaran et al., 2016).[4] Hence, the move from fossil-fuel generation towards renewable sources will imply a change in the competitive paradigm. Whereas the previous literature has analyzed environments in which production capacities are publicly known and marginal costs are either publicly or privately known (Fabra et al., 2006; Holmberg and Wolak, 2018), the relevant setting will soon be one in which marginal costs are known (and essentially zero) but firms' available capacities are private information. Our analysis aims to highlight the implications of this shift on the performance of future electricity markets.

We build a model in which production capacities are firms' private information. Producers compete to serve demand by submitting price-quantity pairs, which indicate the minimum price at which they are willing to produce up to the committed quantity. Firms produce in increasing price order until total demand is satisfied. The price they receive for their output depends on the auction format in place: either a uniform-price auction, which pays the winning producers at the market-clearing price, or a discriminatory auction, which pays each producer at their own bid. We characterize and compare equilibrium bidding behaviour and market outcomes under these two auction formats.

Under the uniform-price auction, firms exercise market power by offering all their capacity at a price above marginal cost or by withholding capacity. When a firm's realized capacity is below total demand, the firm adds a mark-up over its marginal cost that is decreasing in its realized capacity. This reflects the standard trade-off faced by competing firms, as decreasing the price leads to an output gain (*quantity effect*) but it also depresses the market price if the rival bids below (*price effect*). Since firms gain more from the *quantity effect* when their realized capacity is large, they are more eager to sacrifice part of their mark-up in exchange for selling at capacity. When a firm's realized capacity is above total demand, it exercises market power by withholding output in order to sell at capacity at the higher market price set by its rival.

[4]Our analysis applies mainly to wind and solar power, which are the most widely deployed renewable technologies. However, not all renewable technologies share these characteristics.

Bidding behaviour under the discriminatory auction is similar, with two main differences. First, given that firms are always paid according to their own bid, they tend to offer higher prices relative to the uniform-price auction. For the same reason, firms do not gain by withholding output as this does not affect the price they receive.

These equilibrium properties imply that, under both auction formats, market prices are lower (higher) at times of high (low) capacity availability relative to demand. Thus, price volatility is inherently linked to market power, not to capacity uncertainty *per se*, as in the absence of market power prices would remain unchanged at marginal cost regardless of capacity realizations. An increase in capacity investment shifts the whole distribution to the right, which depresses expected market prices until they converge towards marginal costs.[5]

Even though our model assumes that capacity realizations are independent across firms, it does not lead to the standard revenue-equivalence result that is found in the auction literature with privately known — and independently distributed — costs (Holmberg and Wolak, 2018). In particular, we show that the discriminatory auction leads to lower firms' profits and higher consumer surplus than the uniform-price format. The mechanism is akin to allowing for negatively correlated types in models with private information on costs, which reverses the result of a standard auction model with affiliated values (Milgrom and Weber, 1982).[6]

In particular, under both auction formats, a firm that has a higher capacity realization is willing to offer a lower price (*quantity effect*). However, having a higher capacity also implies that, conditionally on having a larger capacity and hence a lower bid than the rival, the rival's expected capacity goes up while its expected price offer goes down. This reduces the price that the low bidder expects to receive under the uniform-price auction and weakens the firm's incentives to bid aggressively. This effect is not present under the discriminatory format given that each firm is paid according to its own bid.

The above result also uncovers a fundamental difference between models with privately known — and independently distributed —- costs or capacities. When private information

[5]Bushnell and Novan (2018) reach a similar conclusion in a counterfactual exercise that uses data from the Californian electricity market.

[6]Vives (2011) also analyzes equilibrium bidding in a uniform-price auction with private information on costs. He assumes that firms receive an imprecise signal about their costs, which are correlated across firms. Thus, a key question in his set-up is whether the market price aggregates information and how this affects the shape of firms' bidding functions. This question does not arise in our set-up, nor in Holmberg and Wolak (2018)'s, given that we assume that firms know their realized capacities or costs.

is on cost, the output allocated to the low and higher bidders is the same regardless of their private information. This implies that the *quantity* and *price effects* depend on the firms' costs only through their optimal bids. This is in contrast to when private information is on capacity, in which case the *quantity* and *price effects* further depend on the firms' types through the output allocation conditionally on having the low or the high bid. This difference impacts the shape of the optimal bid functions across models. When private information is on cost, the optimal price offer is increasing in the cost realization and it is everywhere concave. In contrast, when private information is on capacity, the optimal price offer is decreasing in the realized capacity, and it tends to turn from being concave for low capacity realizations to being convex for high capacity realizations. Ultimately, these different shapes of the equilibrium bid functions affect how changes in private information affect market prices, firms' payments and price volatility. For instance, whereas a large capacity realization pushes firms' bids to rapidly converge towards marginal cost, a low cost realization leads to a more nuanced price convergence.

In order to understand how private information changes the nature of the equilibrium under uniform-price auctions, we also characterize two benchmarks: competition when all the information is either publicly known or unknown. In this regard, we show that the impact of private information is similar across models with privately known costs or capacities, despite the differences in equilibrium bidding behavior highlighted above.

Overall, we find that more information (be it on costs or capacities) strengthens firms' market power. Since private information introduces asymmetries, firms compete less fiercely as compared to when they do not observe their own costs or capacities. In contrast, when they observe each others' private information, they can condition on it to ease rivalry while sharing ex-ante expected profits symmetrically. As a consequence, the highest (lowest) profits are obtained when information is publicly known (unknown), with equilibrium profits under private information laying in between.

Furthermore, as shown by Lagerlöf (2016) for the case of privately known costs, we show that an increase in information precision regarding the rival's capacity leads to less competitive outcomes. In this respect, our results provide a theoretical explanation to the experimental findings in Hefti et al. (2019), which are reminiscent of the literature on Treasury auctions (LiCalzi and Pavan, 2005) showing that noise in the demand function rules out the seemingly collusive equilibria that would arise otherwise (Back and Zender, 2001).

The above results suggest that firms might be better off exchanging private information (be it on costs or capacities) in order to sustain higher equilibrium profits, at the consumers' expense. However, whereas information exchange regarding costs leads to improved productive efficiency, information exchange regarding capacities cannot be justified on these grounds.

Finally, we apply the model with privately known capacities to provide insight on the performance of future electricity markets. Using proprietary data of renewable investors in Spain, we provide evidence suggesting that plant owners indeed have private information about their own available capacities. System operators provide national-wide forecasts on renewables' availability, but firms' private information about their sites' idiosyncratic conditions allows them to perform more accurate forecasts. The main insight from our model is that renewables mitigate market power relative to conventional energies, but market power nevertheless remains. Market prices smoothly go down as renewable investments go up, but they do not converge towards marginal cost unless there exists sufficient excess capacity.

Within our framework, we can analyze the importance of further developments in future electricity markets, such as the increase in demand elasticity brought about by dynamic pricing policies, or the deployment of storage facilities. In particular, our main equilibrium characterization with inelastic demand extends very naturally to environments with a downward-sloping demand function. Indeed, while all equilibrium properties remain the same, we show that an increase in demand elasticity reduces the maximum price that firms are willing to offer, thereby making their bidding functions flatter. Ultimately, demand elasticity reduces prices, increases the pace at which prices converge towards marginal cost, and it is likely to reduce price volatility across time.

Previous papers have also analyzed competition among renewable power sources under capacity uncertainty (Acemoglu et al., 2017; Kakhbod et al., 2021). These papers, unlike ours, assume Cournot competition, i.e., they constrain firms to exercise market power only by withholding output.[7] Acemoglu et al. (2017) focus on the effects of joint ownership of

[7] In a context without uncertain renewables, Genc and Reynolds (2019) and Bahn et al. (2019) also assume Cournot competition to analyze the effects of the ownership structure of renewable plants on market outcomes. The trade-offs that arise when relying on a simple and tractable setup, like the Cournot model, versus one that more closely mimics the institutional details of electricity markets, like an auction model, have been extensively discussed in the previous literature. See among others, von der Fehr and

conventional and renewable plants. They show that joint ownership weakens the price-depressing effect of renewables as it pushes the strategic firms to withhold more output from their conventional power plants. This possibility to exercise market power in this way will not be available in 100% renewables markets, which is the case addressed in our paper. Kakhbod et al. (2021) focus on the heterogeneous availability of renewable sources across locations and show that firms withhold more output when their plants are more closely located, i.e., when their output is highly and positively correlated.

The remainder of the paper is structured as follows. Section 2 describes and solves the model with privately known capacities, both under the uniform-price auction as well as under the discriminatory auction. Section 3 studies the impact of private information and information precision on equilibrium outcomes. Section 4 revisits the model with privately known costs under both auction formats. Section 5 compares the equilibria and the market outcomes in models in which private information is on capacity or on cost. Section 6 uses the model to shed some light on the future performance of electricity markets. Last, Section 7 concludes. All proofs are relegated to the appendix.

2 Auctions with privately known capacities

Consider a model in which two ex-ante symmetric firms $i = 1, 2$ compete in a market to serve a perfectly price-inelastic demand $\theta > 0$. Firms can produce at a constant marginal cost $c \geq 0$ up to their available capacities, which are assumed to be random. In particular, the available capacity of firm i, denoted as k_i, is distributed according to $G(k_i)$, with positive density $g(k_i)$ in the whole interval $\left[\underline{k}, \overline{k}\right]$. We assume $2\underline{k} \geq \theta$ to make sure that there is always enough available capacity to cover total demand. Firm i can observe its capacity realization but not that of its rival, which is independent from its own.

Firms compete on the basis of the bids submitted to an auctioneer. Each firm simultaneously and independently submits a price-quantity pair (b_i, q_i), where b_i is the minimum price at which it is willing to supply the corresponding quantity q_i. We assume $b_i \in [0, P]$, where P denotes the "market reserve price." We also assume that firms cannot offer to produce above their available capacity or below their minimum capacity, $q_i \in [\underline{k}, k_i]$, for

Harbord (1993) and Wolfram (1998).

$i = 1, 2$.[8]

The auctioneer ranks firms according to their price offers, and calls them to produce in increasing rank order. In particular, if firms submit different prices, the low-bidding firm is ranked first. If firms submit equal prices, firm i is ranked first with probability $\alpha(q_i, q_j)$ and it is ranked second with probability $1 - \alpha(q_i, q_j)$. We assume a symmetric function $\alpha(q_i, q_j) = \alpha(q_j, q_i) \in (0, 1)$.[9] If firm i is ranked first it produces $\min\{\theta, q_i\}$, while if it is ranked second it produces $\max\{0, \min\{\theta - q_j, q_i\}\}$, where $j \neq i$.

Firms receive a uniform price per unit of output, which is set equal to the market-clearing price. For $b_i \leq b_j$, this market-clearing price is defined as

$$p = \begin{cases} b_i & \text{if } q_i > \theta, \\ b_j & \text{if } q_i \leq \theta \text{ and } q_i + q_j > \theta, \\ P & \text{otherwise.} \end{cases}$$

In words, the market-clearing price is set by the highest accepted bid, unless the quantity offered by the winning bid(s) is exactly equal to total demand. In this case, the market price is set equal to the lowest non-accepted bid, or to P if no such bid exists because all the quantity offered has been accepted.[10]

Firms, which are assumed to be risk neutral, bid so as to maximize their individual expected profits, given their realized capacities.

2.1 Uniform-price auction

In this section we characterize the symmetric Bayesian Nash Equilibria (BNE) of the uniform-price auction when capacities are private information. When $\underline{k} \geq \theta$ the characterization of the equilibrium is trivial. Since either firm can cover total demand regardless of their realized capacities, Bertrand forces drive equilibrium prices down to marginal cost. For this reason, in the remainder of the paper we turn attention to the remaining cases. It is useful to start by assuming $\overline{k} \leq \theta$ (small installed capacities). In this case, a firm's capacity can never exceed total demand, implying that the low bid

[8] The implicit assumption is that withholding below $\underline{k}$ would make it clear that the firm has strategically reduced output in order to raise prices, which could trigger regulatory intervention.

[9] Hence, when firms' quantity offers are equal, $\alpha(q, q) = 1/2$. We do not need to specify $\alpha(q_i, q_j)$ outside of the diagonal as this is inconsequential for equilibrium bidding.

[10] Assuming that the market price is set at the lowest non-accepted bid when the quantity offered by the winning bid(s) equals total demand is made for analytical convenience, with no impact on equilibrium outcomes. It avoids situations where firms want to offer a quantity slightly below total demand in order to push the market price up to the higher bid offered by the rival.

is always payoff irrelevant. We later analyze the case in which $\overline{k} > \theta$ (large installed capacities).

Small Installed Capacities

We first consider the case in which each firm's capacity never exceeds total demand, i.e., $\overline{k} \leq \theta$. Our first lemma identifies three key properties that any equilibrium must satisfy in this case.

Lemma 1. *If $\overline{k} \leq \theta$,*

(i) Capacity withholding is never optimal, $q_i^*(k_i) = k_i$.

(ii) All Bayesian Nash Equilibria must be in pure strategies.

(iii) The optimal price offer of firm i, $b_i^*(k_i)$, must be (strictly) decreasing in k_i.

The first part of the lemma rules out capacity withholding in equilibrium.[11] This follows from four observations: first, conditionally on having the low bid, the firm maximizes its output by offering to sell at capacity; second, conditionally on having the high bid, its profits do not depend on its quantity offer as the firm always serves the residual demand; third, the probability of having the low bid does not depend on q_i; and last, the market price remains unchanged with or without capacity withholding. It follows that expected profits are strictly increasing in q_i, and are thus maximized at $q_i = k_i$.

The second part of the lemma rules out non-degenerate mixed-strategy equilibria. The underlying reason is simple: a firm's profits at a mixed-strategy equilibrium depend on its realized capacity, which is non-observable by the rival. If the competitor randomizes in a way that makes the firm indifferent between two bids for a given capacity realization, the same randomization cannot make the firm indifferent for other capacity realizations as well. It follows that all equilibria must involve pure strategies.

The last part of the above lemma rules out bids that are non-decreasing in the firm's capacity. When a firm considers whether to reduce its bid marginally, two effects are at play (for a given bid of the rival): a profit gain due to the output increase (*quantity effect*), and a profit loss due to the reduction in the market price (*price effect*). On the one hand, the *quantity effect* is increasing in the firm's capacity, as if it bids below the

[11] In case of indifference between withholding or not, we assume without loss of generality that the firm chooses not to withhold.

rival, it sells at capacity rather than just the residual demand. On the other hand, the *price effect* is independent of the firm's capacity as, contingent on bidding higher than the rival, the firm always sells the residual demand. Combining these two effects, the incentives to bid low are (weakly) increasing in the firm's capacity, giving rise to optimal bids that are non-increasing in k_i. Finally, standard Bertrand arguments imply that the optimal price offer must be *strictly* decreasing in k_i: equilibrium bidding functions cannot contain flat regions, as firms would otherwise have incentives to slightly undercut those prices in order to expand their expected quantity without affecting the price.

Part (iii) of the lemma allows us to write the expected profits of firm i using the inverse of the bid function of firm j, $b_j(k_j)$, as follows

$$\pi_i(b_i, b_j(k_j)) = \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j) - c)k_i g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\overline{k}} (b_i - c)(\theta - k_j)g(k_j)dk_j.$$

When $k_j < b_j^{-1}(b_i)$, firm i has the low bid and sells up to capacity at the price set by firm j. Otherwise, firm i serves the residual demand and sets the market price at b_i.

Maximizing profits with respect to b_i and applying symmetry, we can characterize the optimal bid at a symmetric equilibrium.[12]

Proposition 1. *If $\overline{k} \leq \theta$, at the unique symmetric Bayesian Nash equilibrium when capacities are privately known, each firm $i = 1, 2$ offers all its capacity, $q^*(k_i) = k_i$, at a price given by*

$$b^*(k_i) = c + (P - c) \exp\left(-\omega(k_i)\right), \tag{1}$$

where

$$\omega(k_i) = \int_{\underline{k}}^{k_i} \frac{(2k - \theta)g(k)}{\int_{k}^{\overline{k}} (\theta - k_j)g(k_j)dk_j} dk.$$

Equation (1) characterizes the optimal price offer for all capacity realizations. As anticipated, the optimal price offer adds a markup above marginal cost that is strictly decreasing in k_i. In order to provide some intuition, it is useful to implicitly re-write the price offer as follows

$$-\frac{b'^*(k_i)}{b^*(k_i) - c_i} = \omega'(k_i) = \frac{2k_i - \theta}{\theta - E(k_j|k_j \leq k_i)} \frac{g(k_i)}{1 - G(k_i)}. \tag{2}$$

[12] The model also displays asymmetric equilibria where one firm always sets a price P, while the rival chooses a price sufficiently close to c. This equilibrium is usually ignored in the literature, as it may lead to coordination issues. It also makes this case difficult to compare with the discriminatory auction, where such an equilibrium does not arise.

This equation describes the incentives to marginally reduce the bid. The ratio on the right-hand side captures the trade-off between the *quantity effect* and the *price effect*.

On the numerator, the output gain from marginally reducing the firm's bid, or *quantity effect*, is relevant only when the two firms tie in prices, i.e., when the rival also has capacity k_i, an event that occurs with probability $g(k_i)$. Reducing the bid implies that the firm sells all its capacity rather than just the residual demand, i.e., its output expands in the amount $k_i - (\theta - k_i) = 2k_i - \theta$. On the denominator, the price loss from marginally reducing the bid, or *price effect*, is only relevant when the firm is setting the market price, i.e., when the rival firm's capacity is above k_i. In this case, reducing the bid implies that the firm sells the expected residual demand, $\theta - E(k_j|k_j \leq k_i)$, at a lower market price.

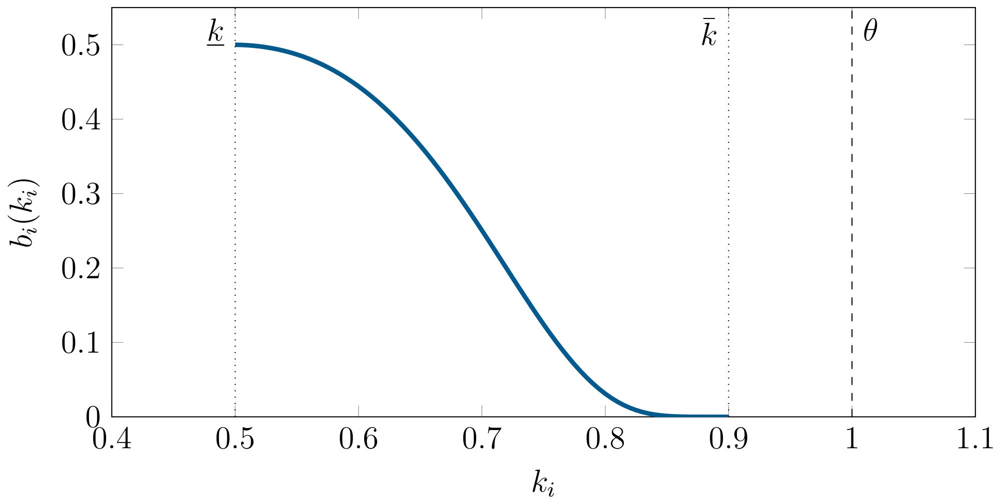
The ratio of these two effects gives shape to the bidding function. A bigger quantity effect increases a firm's incentives to undercut the rival. This means that in order to sustain this symmetric equilibrium the bidding function must become steeper — to require a larger bid reduction for a given quantity gain — and the mark-up must become smaller — to make undercutting less profitable. A smaller price effect, to the extent that it makes price increases less profitable, has a similar effect.

Figure 1 illustrates the equilibrium price offer as a function of k_i. The optimal bid ranges from P for the lowest possible capacity realization, to c for the largest one. When $k_i = \underline{k}$, firm j is guaranteed to have a higher capacity and thus a lower price. Since firm i serves the residual demand with probability one, it maximizes its profits by bidding at P. When $k_i = \overline{k}$, firm j is guaranteed to have a lower capacity and hence a higher price. In this case, only the *price effect* matters. Firm i then finds it optimal to bid at c in order to maximize its chances of selling at capacity at the rival's price. In this case, only the *quantity effect* matters.

Since an increase in k_i pushes the *quantity* and the *price effects* in opposite directions,[13] the optimal bid function is first concave and eventually becomes convex as k_i approaches $\overline{k}$. In the latter case, bidding incentives approach those under Bertrand competition as, for large k_i realizations, the *price effect* wanes. With only the *quantity effect* at play, the rival's bid must become increasingly flat at marginal cost in order to offset the firm's strong undercutting incentives.

Finally, given equilibrium bidding, each firm's expected profits are equal to the min-

[13]This is always the case whenever the distribution function is log-concave, which holds true for a large family of distribution functions.



Note: This figure depicts the equilibrium price offer as a function of k_i when $k_i \sim U[0.5, 0.9]$, with $\theta = 1$, $c = 0$, and $P = 0.5$. One can see that it starts at P for $k_i = \underline{k}$, and that it decreases in k_i until it takes the value $c = 0$ at $k_i = \overline{k} = 0.9$.

Figure 1: Equilibrium price offer

imax when $k_i = \underline{k}$, and they are strictly higher otherwise. The reason is simple: a firm can always pretend to be smaller by withholding output and replicating the smaller firm's bid. The fact that firms prefer to offer all their capacity means that larger firms make higher equilibrium profits than the smallest one, whose profits exactly coincide with the minimax.

Changes in parameter values affect the shape of the bidding functions, thereby impacting market outcomes. For instance, equilibrium price offers shift up as demand θ increases. This is driven by a weaker *quantity effect*, i.e., the quantity gain when undercutting the rival is smaller since the residual demand is larger, and a stronger *price effect*, i.e., the gain from increasing the price conditionally on being the high bidder goes up because the residual demand is larger. Consequently, equilibrium prices increase. Similar effects are at play when the capacity distribution shifts to the right in a first-order stochastic sense. On the one hand, the price effect becomes stronger as for a given k_i realization the rival's capacity is expected to be larger, making firm i more likely to set the market price. This pushes the equilibrium price offer up. On the other hand, firms' capacities are now larger on average, which shifts the equilibrium price offer to the right. Overall, the latter effect dominates, leading to lower equilibrium market prices. The total effect is illustrated in Figure 2.

Large Installed Capacities

We now turn to the case in which a single firm's capacity might exceed total demand, $\overline{k} > \theta$. In contrast to the case of small installed capacities, withholding is now optimal for firms whose capacity exceeds total demand, $k_i > \theta$, as shown in the following proposition.

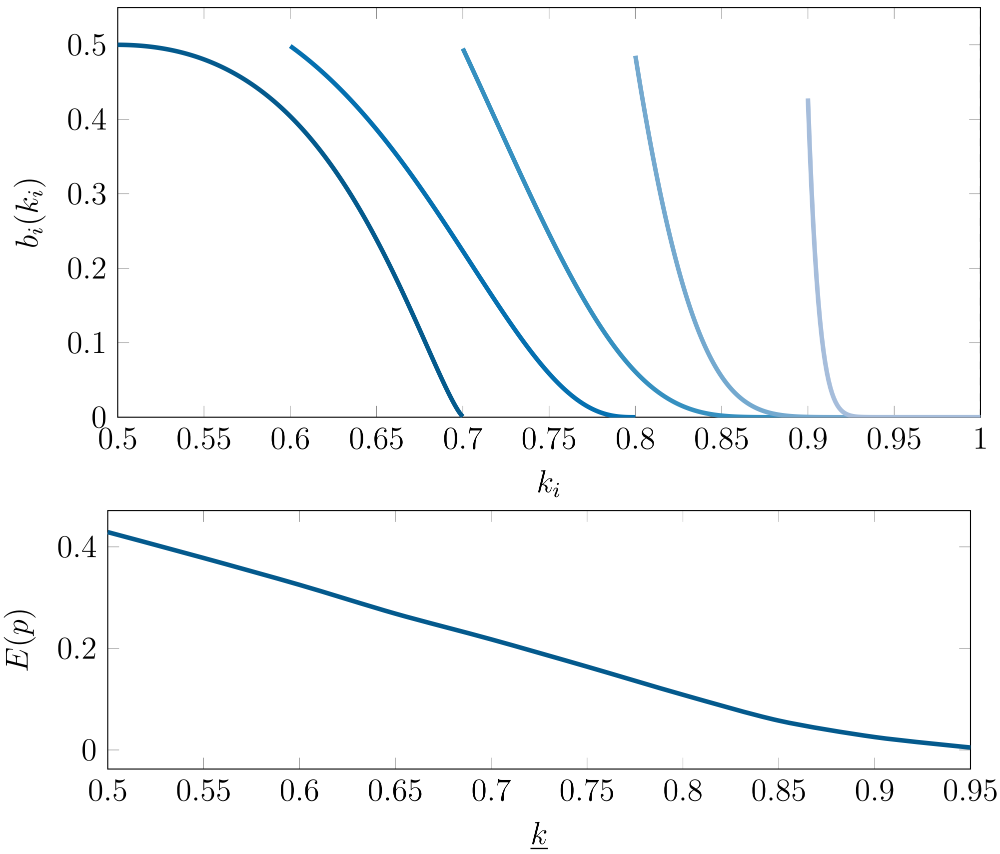
Proposition 2. *If $\overline{k} > \theta$, in equilibrium, $b^*(k_i) = c$ and $q_i^*(k_i) = \theta$ for all $k_i > \theta$, $i = 1, 2$. For $k_i \leq \theta$, $q_i^*(k_i) = k_i$ and $b^*(k_i)$ is defined as in Proposition 1, with k_i and k_j replaced by $q_i^*(k_i)$ and $q^*(k_j)$, $i = 1, 2$.*

For capacity realizations $k_i \leq \theta$, equilibrium bidding is just as in the case with small installed capacities. However, for $k_i > \theta$, offering to supply k_i is weakly dominated by offering to supply θ: in any event, the firm will never produce more than θ and, conditioning on having the low price, offering θ instead of k_i increases the chances that the rival's higher price offer will set the market price.[14] For these reasons, the equilibrium characterization is identical to the one in the previous proposition, the only difference being that the relevant distribution now has a mass point at θ. Note that withholding increases the market price but it implies no distortion in the quantity sold given that the withheld capacity would not have been used in any event.

Similarly to the case of small installed capacities, an increase in demand leads to a lower price. This effect is now enhanced as the increase in θ makes it less likely that both firms bid at c. Indeed, as θ goes up, the expected market price goes up. In contrast, if the whole capacity distribution $\left[\underline{k}, \overline{k}\right]$ shifts to the right, as also shown in Figure 2, expected prices smoothly converge towards marginal cost.

An interesting insight from our model is that capacity realizations determine whether firms find it optimal to compete either (i) by offering all their capacity at prices above marginal cost, or (ii) by withholding capacity in order to stop prices from falling when they bid at marginal cost. As we will see next, this pattern does not arise when each bidder receives its own price offer (discriminatory pricing), rather than the market-clearing price. Intuitively, firms would always bid above marginal cost in order to obtain positive profits and would then have no need to withhold. This difference has implications for the profit ranking of both auctions, as we show next.

[14] If instead of setting the market price at the lowest non-accepted bid, we set it equal to the highest accepted bid, firm i would optimally offer to produce a quantity slightly below total demand, θ, giving rise to the same market price and (almost) the same quantity allocation.



Note: The upper panel shows that the equilibrium price offers shift outwards as κ, and consequently, $\underline{k}$ increases. The lower panel shows that the expected market price smoothly goes down as a function of $\underline{k}$, which together with $\overline{k}$, shift out as κ increases. The figures assume $\theta = 1$, $c = 0$, and $P = 0.5$, and $k_i \sim U[\underline{k}, \underline{k} + 0.2]$, for $\underline{k} \in [0.5, 0.95]$.

Figure 2: Equilibrium price offers and expected market price as installed capacity increases.

2.2 Discriminatory Auction

We now characterize equilibrium bidding under the discriminatory auction.

Proposition 3. *In the discriminatory auction, the unique Bayesian Nash equilibrium when capacities are privately known is symmetric.*

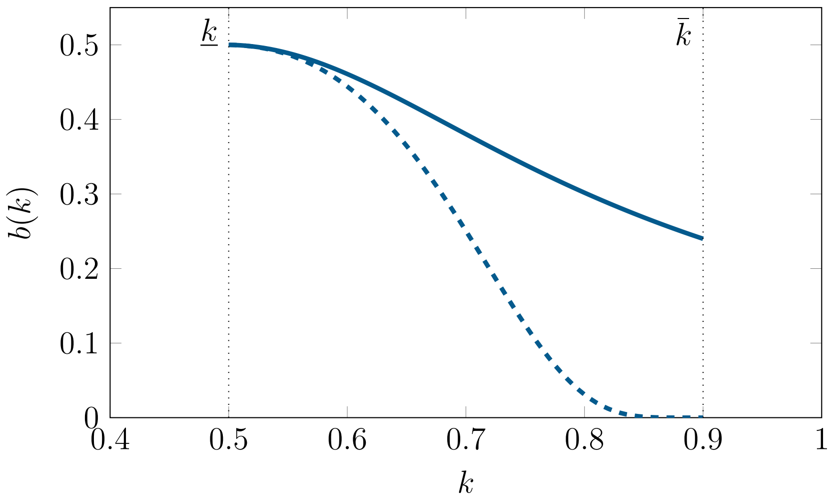
(i) If $\overline{k} \leq \theta$, each firm $i = 1, 2$ offers all its capacity, $q^*(k_i) = k_i$, at a price given by

$$b^*(k_i) = c + (P - c) \exp\left(-\omega\left(k_i\right)\right), \tag{3}$$

where

$$\omega\left(k_i\right) = \int_{\underline{k}}^{k_i} \frac{(2k - \theta) g(k)}{kG(k) + \int_{k}^{\overline{k}} (\theta - k_j) g(k_j) dk_j} dk.$$

(ii) If $\overline{k} > \theta$, in equilibrium $q_i^*(k_i) = k_i$, and $b_i^*(k_i)$ is given by (3), with k_i replaced by $\min\{\theta, k_i\}$ *for $i = 1, 2$.*



Note: The figure depicts the equilibrium price offers under the discriminatory auction (solid) and the uniform-price auction (dashed). One can see that firms always offer, for a given realized capacity, higher prices under the discriminatory auction. Parameter values: $k_i \sim U[0.5, 0.9]$, $c = 0$, $P = 0.5$ and $\theta = 1$.

Figure 3: Comparison between the optimal price offers across auctions.

Unlike the uniform-price auction, a higher price offer under the discriminatory auction always allows the firm to obtain a higher price for its output, even when the rival firm bids higher. Hence, firms now face stronger incentives to increase their price offers. In particular, under the discriminatory auction, the optimal bid is always strictly above marginal cost, even when $k_i = \overline{k}$. Because the price that firms receive does not depend on the quantity sold, they do not need to withhold output when their capacities exceed total demand. Their profits are the same if they withhold capacity or if they don't.

Since firms submit higher bids under the discriminatory auction, it follows that the market-clearing price is higher than under the uniform-price auction. However, this is not enough to rank payments under the two formats given that under the discriminatory auction firms do not receive the market-clearing price, but their own bid. Indeed, as we show in our next result, the uniform-price auction yields higher payments to firms than the discriminatory format. Since costs are the same across formats, this also implies that firms make higher profits.

Proposition 4. *Expected payments to firms are higher under the uniform-price auction relative to the discriminatory auction.*

In order to interpret this result, it is useful to highlight the different bidding incentives triggered by the two formats. Under both formats, having a large capacity is good news as the firm is likely to have the low bid and thus sell at capacity. However, under the uniform-price auction, this is also bad news as it means that the rival's capacity is likely to be large as well. This means that, conditionally on having the low bid, the firm will receive a lower price for its capacity, thus weakening firms' incentives to bid more aggressively. Note that this is the case despite capacity realizations being independent. The reason is that, conditionally on firm i having the low bid, the distribution of the rival's capacity is truncated at k_i. Hence, a higher k_i also implies a higher expected k_j. In contrast, this second effect is not present under the discriminatory format given that firms are paid according to their own bid, regardless of their rivals' capacity.

At the lowest capacity realization, $\underline{k}$, firms make minimax profits under the two auction formats. However, the previous argument indicates that, for higher capacity realizations, firms' profits increase faster under the uniform-price auction. The result in Proposition 4 immediately follows.

The previous arguments do not rely on capacity withholding and, indeed, they completely characterize the result in the small capacity case. Interestingly, allowing for withholding strengthens the result when capacities are large. The reason is that under the discriminatory auction firms have no incentives to withhold capacity because they receive their own bid. The equilibrium outcome is thus independent of whether this possibility is considered. However, as shown in Proposition 2, in the uniform-price auction firms find it optimal to withhold output to $q_i = \theta$ when their capacity exceeds that level. This decision is akin to a leftward shift in the distribution of capacities, which leads to higher equilibrium prices and profits.

3 The impact of private information

In this section we aim to understand the effect of private information on bidding behavior and market outcomes under a uniform-price auction. We perform two types of analyses. First, we compare the equilibrium outcomes when capacities are privately known with the ones that arise either when capacities are publicly known or when they are unknown to both firms prior to bidding. Second, we analyze the effects of information precision on equilibrium outcomes.

3.1 Known versus unknown capacities

First, suppose that firms observe realized capacities prior to submitting their price offers. The following lemma characterizes the level of profits that can be sustained in symmetric pure-strategy equilibria.

Lemma 2. *Suppose that realized capacities are publicly known prior to bidding. There exist symmetric pure-strategy Nash equilibria, resulting in expected joint profits* $(P-c)\theta$. *These profits can be sustained by the following bidding profiles: for* $i, j = 1, 2$ *and* $j \neq i$, *(i) if* $k_i < k_j$, $b_i^*(k_i) = P$ *and* $q_i^*(k_i) = k_i$, *while* $b_j^*(k_j) \in [c, \underline{b}_i]$ *and* $q_j^*(k_j) = \min\{\theta, k_j\}$, *with* $\underline{b}_i$ *low enough so as to make undercutting by firm* i *unprofitable. (ii) If* $k_i = k_j = k$, *firms play mixed-strategies, with expected joint profits* $2(P-c)(\theta-k)$ *if* $k < \theta$ *and* 0 *otherwise.*

The game with known capacities allows firms to sustain equilibria in which all their output is sold at P. These equilibria are characterized by asymmetric bidding once the capacities are realized, even though the equilibrium is ex-ante symmetric. Indeed, the large capacity firm bids low enough so as to make undercutting by the small firm unprofitable.[15] This firm maximizes profits over the residual demand by bidding at the highest possible price, P. Since both firms are equally likely to be the small or the large capacity firm, they share profits symmetrically. Observing realized capacities allows firms to overcome the coordination problem as to which firm bids low or high and this, in turn, allows them to attain maximum profits.[16]

Consider now the case in which firms do not observe realized capacities prior to bidding. They first choose prices before capacities are realized, and then choose their quantity offers once they have observed them.[17] The following lemma shows that the unique symmetric equilibrium involves mixed-strategy pricing.

Lemma 3. *If realized capacities* (k_i, k_j) *are known after firms have made their price offers, the unique symmetric Bayesian Nash equilibrium involves mixed-strategies, with*

[15] This holds true even if $k_i > \theta$ as in this case firms can escape Bertrand pricing by withholding output and choosing $q_i(k_i) = \theta$. This is in contrast to Fabra et al. (2006), who predict Bertrand competition when $k_i > \theta$. The difference is that they do not allow firms to choose both prices and quantities.

[16] The only exception is when firms realized capacities are equal. In this case, since observing realized capacities does not allow them to overcome the coordination problem, the unique symmetric equilibrium involves mixed-strategy pricing, with firms making lower expected profits. However, this case arises with a zero probability.

[17] The same results would arise if, instead, firms commit to sell all their capacity at the chosen price once capacities are realized.

firms randomizing their prices in the interval (c, P). *Expected equilibrium joint profits are* $2(P-c)\left[\theta - E(k|k \leq \theta)\right] G(\theta)$.

Since price offers cannot be conditioned on capacities, in a symmetric equilibrium both firms would either charge equal prices or use the same mixed-strategy to randomize their prices. The former is ruled out by standard Bertrand arguments, implying that the only symmetric equilibrium involves mixed-strategies. Since at P the rival firm is bidding below with probability one, and since all the prices in the equilibrium support yield equal expected profits, it follows that at the unique symmetric equilibrium each firm makes expected profits equal to $(P-c)(\theta - E[k|k \leq \theta])G(\theta)$. Note that the high bidder only makes positive profits when the rival's capacity turns out to be below θ, i.e., with probability $G(\theta)$.

We are now ready to rank expected prices arising at the symmetric equilibria across all three information treatments.

Proposition 5. *If firms play symmetric Bayesian Nash equilibria, expected prices are the highest with publicly known capacities, and the lowest with unknown capacities. Expected equilibrium prices with privately known capacities lay in between.*

The proposition above shows that the more information firms have, the higher the expected prices they can obtain at a symmetric equilibrium. When capacities are private information, the fact that bidding incentives differ across firms allows them to avoid fierce competition, but not as much as if both capacities were known: large (small) firms find it in their own interest to bid low (high), but not as low (high) as if they knew with certainty that the rival firm was bidding higher (lower). When capacities are unknown to both firms, they face fully symmetric incentives and they end up competing fiercely. As a result, private information leads to higher prices than in the case with unknown capacities, but lower than when capacities are publicly known. This suggests that firms would be better off if they could exchange their private information regarding their available capacities.[18]

[18]In fact, in our model firms would have unilateral incentives to share the realization of their own capacity with the rival, as this allows them to better coordinate and sustain higher equilibrium profits. The debate on the incentives for information transmission between firms dates back to classical papers like Vives (1984) and Gal-or (1986).

3.2 Information precision

Given the equilibrium characterization in Proposition 1, one may be tempted to conclude that an improvement in information precision leads to more competitive bidding, a conclusion that would be at odds with our previous results. Indeed, based on Proposition 1, as the range $\left[\underline{k}, \overline{k}\right]$ shrinks, equilibrium profits converge to those with symmetric and known capacities, for which the symmetric equilibrium involves mixed-strategies, giving rise to very low profits.

However, this approach is misleading as making $\left[\underline{k}, \overline{k}\right]$ narrower not only improves the information precision, but also increases the likelihood of capacities being ex-post symmetric. Since increased symmetry leads to more competitive outcomes, this latter effect confounds the true impact of information precision on bidding behavior.

Lagerlöf (2016) highlights a similar issue when private information is on costs. In particular, he notes that an open auction yields the same result as if firms had complete information, regardless of whether the quantity is fixed or variable (i.e., price elastic). Using the revenue equivalence theorem between sealed-bid and open-auctions for a fixed quantity, and using Hansen (1988)'s result that sealed-bid auctions for variable quantity lead to lower prices than for a fixed quantity, he concludes that cost uncertainty intensifies competition. We cannot use a similar approach because in our multi-unit setting with private information on capacities, none of these properties holds. In particular, the discriminatory and uniform-price auctions, which are the analogues of the first-price and second-price auctions, are not revenue equivalent, as shown in Lemma 4. Furthermore, the uniform-price auction does not deliver the same result when capacities are known or privately known, as shown in Proposition 5. Hence, to disentangle the effects of information precision from those of increased symmetry, we need to extend our model to allow for ex-ante asymmetric capacities.

Asymmetric firms

Suppose for simplicity that firm i's capacity is uniformly distributed in $[\underline{k}_i, \overline{k}_i]$ and that firms' aggregate capacity is always enough to cover total demand, $\underline{k}_1 + \underline{k}_2 \geq \theta$. Our next proposition characterizes equilibrium bidding for the case in which the two firms' installed capacities are small, $\overline{k}_2 \leq \overline{k}_1 \leq \theta$.

Proposition 6. *Assume that k_i is uniformly distributed in $[\underline{k}_i, \overline{k}_i]$. If $\overline{k}_1 \leq \theta$, in equilibrium each firm offers all its capacity, $q_i^*(k_i) = k_i$ for $i = 1, 2$. Furthermore:*

(i) If $\overline{k}_2 \geq \underline{k}_1$, there exists an equilibrium in which price offers are characterized by

$$b_i^*(k_i) = \begin{cases} P & \text{if } \underline{k}_2 \leq k_i \leq \underline{k}_1, \\ b^*(k_i) & \text{if } \underline{k}_1 < k_i < \overline{k}_2, \\ c & \text{if } \overline{k}_2 \leq k_i \leq \overline{k}_1, \end{cases}$$

where

$$b^*(k_i) = c + (P - c) \exp\left(-\omega(k_i)\right), \tag{4}$$

and

$$\omega(k_i) = \int_{\underline{k}_1}^{k_i} \frac{(2k - \theta)}{\int_k^{\overline{k}_2} (\theta - k_j) dk_j} dk.$$

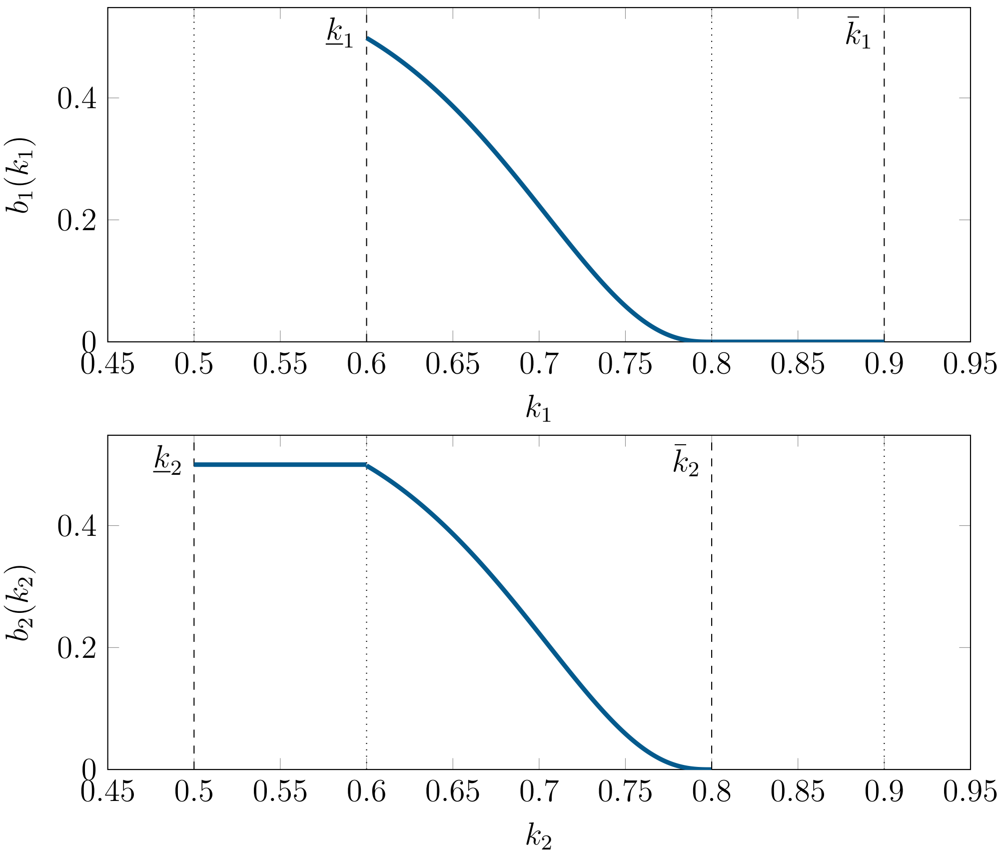
(ii) If $\overline{k}_2 < \underline{k}_1$ the only pure-strategy Bayesian Nash equilibrium is asymmetric, and it coincides with the one characterized in Lemma 2 (i).

Interestingly, this proposition shows that ex-ante capacity asymmetries move equilibrium bidding behavior from the symmetric equilibria provided in Proposition 1 to the asymmetric equilibria provided in Lemma 2. When the capacity intervals do not overlap, as in part (ii), one firm sets the market price at P and the other one chooses a sufficiently low bid that avoids undercutting. Equilibrium bidding departs from the one in Proposition 1, as it rests on firms being uncertain about the identity of the large firm and, therefore, about the identity of the low bidder.

In contrast, when the capacity intervals overlap, this uncertainty reemerges for capacities in the range $[\underline{k}_1, \overline{k}_2]$. Over this interval, the equilibrium price offers resemble those in Proposition 1, with firms pricing at P for $k_i = \underline{k}_1$ and at c for $k_i = \overline{k}_2$. For smaller capacity realizations, firm 2 bids at P. For higher capacity realizations, firm 1 bids at c. As a result, both price offers are continuous in the realized capacities. Figure 4 illustrates these bids.

These equilibria survive in the large installed capacities case when capacity withholding becomes optimal, as stated next.

Corollary 1. *If $\overline{k}_1 > \theta$, in equilibrium each firm offers $q_i^*(k_i) = \min\{\theta, k_i\}$ and prices according to Proposition 6, where the relevant threshold in part (ii) of the Proposition, $\overline{k}_2$, is replaced by $\min\{\theta, \overline{k}_2\}$.*



Note: This figure depicts the equilibrium price offers as a function of k_i when $k_1 \sim U[0.6, 0.9]$ and $k_2 \sim U[0.5, 0.8]$, $\theta = 1$, $c = 0$, and $P = 0.5$. One can see that the equilibrium is symmetric only in the area of capacity overlap, $[0.6, 0.8]$. For larger capacities $[0.8, 0.9]$, the large firm bids at c (upper panel), whereas for smaller capacities $[0.5, 0.6]$, the small firm bids at P (lower panel).

Figure 4: Equilibrium price offers when firms are ex-ante asymmetric

For the same reasons explained in the ex-ante symmetric capacities case, firms always find it optimal to withhold capacity whenever their realized capacity exceeds θ. As a result, firms behave in equilibrium as if their capacities were capped, with a mass point at θ.

This equilibrium characterization allows us to conclude that, keeping aggregate capacity as given, an increase in firms' ex-ante asymmetries results in higher expected prices. As firm 2 becomes smaller in expected terms, it bids at P with a higher probability, raising the expected equilibrium price. In the limit, when asymmetries are such that there is no capacity overlap, $\underline{k}_1 > \overline{k}_2$, the market price is P with probability 1.[19]

[19]It is important to notice, however, that the characterization of this equilibrium hinges on the density of each firm being identical in the range of capacity overlap, thanks to the assumption of uniformly and identically distributed idiosyncratic shocks. This guarantees that the two first order conditions that characterize optimal bidding are identical, allowing us to conclude that the equilibrium price offers are

What do these results tell us about the effects of information precision? To answer this question, suppose first that firms' capacities are so asymmetric that their intervals never overlap, $\underline{k}_1 > \overline{k}_2$. By Proposition 6, the equilibrium price in this case is P with probability one. Introducing a small amount of uncertainty around asymmetric capacities would have no impact on bidding behavior or market outcomes as long as the intervals do not overlap. Otherwise, adding more uncertainty would eventually imply $\overline{k}_2 > \underline{k}_1$, giving rise to equilibria with bids below P. As in Proposition 5, the expected market price would start falling below P, the more so the more noisy the forecasts about the rival's capacity become.

We can thus conclude that the less precise the signal about the rival's capacity, the weaker is market power, in line with our previous conclusions regarding the impact of private information.

4 Auctions with privately known costs

We now turn to the case where marginal costs are private information but capacities are known. This setup allows us to highlight how the different sources of private information affect equilibrium outcomes.

In particular, suppose that both firms have the same capacity, denoted by k, while their costs are the realization of two independent random variables. Firm $i = 1, 2$ has a cost c_i drawn from a distribution $F(c_i)$ in the interval $c_i \in [\underline{c}, \overline{c}]$, with a strictly positive density $f(c_i)$ in the whole support, and $\underline{c} \geq 0$. Firm i observes its own idiosyncratic cost but not that of its rival, i.e., costs are private information. We assume $2k \geq \theta$ to ensure that firms always have enough available capacity to cover total demand, so that competition is meaningful. As in the benchmark case, demand θ is assumed to be price-inelastic and bids cannot be raised above a price-cap P. This set-up is equivalent to Holmberg and Wolak (2018)'s, with minor variations.[20]

It is immediate, using arguments similar to those in the baseline model, that firms always find it optimal to withhold capacity if $k > \theta$. Hence, in equilibrium the quantity

symmetric. While we do not provide a characterization for generic distribution functions, we conjecture that the nature of the equilibrium would remain similar but explicit solutions for the optimal bids would be unlikely to come by.

[20]In their paper, they assume that $P = \overline{c}$. Also, they allow the capacity to be stochastic, but it is unrelated to the cost shocks. These differences have no impact on the results. They also analyze the more general case where costs are affiliated, which provides interesting insights for the comparison with our model as discussed later in the paper.

offered by firms is always equal to $q = \min\{\theta, k\}$. For this reason, in what follows we assume without loss of generality that $k \leq \theta$.

Our analysis follows the same structure as in previous sections. We first characterize equilibrium bidding in uniform-price and discriminatory auctions and then compare the equilibria across auction formats. We end by analyzing the effects of private information on equilibrium bidding and market outcomes.

4.1 Uniform-price auction

Our first lemma is analogous to Lemma 1 for the case in which costs are privately known.

Lemma 4. *When costs are private information,*

(i) All Bayesian Nash Equilibria must be in pure-strategies.

(ii) The optimal price offer of firm i, $b_i^*(c_i)$, must be (strictly) increasing in c_i.

We can rule out non-degenerate mixed-strategy equilibria for reasons similar as when capacities are privately known. Quite intuitively, the optimal price offer must be increasing in the firm's cost. A firm with a lower cost bids more aggressively since its profit margin is higher and, therefore, it benefits more from an increase in the quantity sold.

The previous result allows us to characterize firm i's profits as follows:

$$\pi_i\left(b_i, b_j|c_i\right) = \int_{\underline{c}}^{b_j^{-1}(b_i)} (b_i - c_i)(\theta - k) f(c_j) dc_j + \int_{b_j^{-1}(b_i)}^{\overline{c}} \left(b_j\left(c_j\right) - c_i\right) k f(c_j) dc_j. \quad (5)$$

When the rival bids below, firm i serves the residual demand $\theta - k$ at its own bid. Otherwise, it serves all its capacity at the price offered by the rival. Importantly, the firm's private information affects the probability of being the low or the high bidder, but it does not affect the quantity it produces conditionally on having the low or the high bid. This is in contrast with the model with privately known capacities, in which firms' private information affects both.

The next result characterizes the bidding function in the symmetric equilibrium of the game.

Proposition 7. *At the unique symmetric Bayesian Nash equilibrium when costs are privately known, each firm $i = 1, 2$ offers all its capacity, $q^*(k_i) = k_i$, at a price given by*

$$b_i(c_i) = c_i + (P - \overline{c}) \left(\frac{1}{F\left(c_i\right)}\right)^{-\frac{2k-\theta}{\theta-k}} + \int_{c_i}^{\overline{c}} \left(\frac{F\left(c\right)}{F\left(c_i\right)}\right)^{-\frac{2k-\theta}{\theta-k}} dc. \quad (6)$$

It is easy to verify that $b(\bar{c}) = P$ and $b(\underline{c}) = \underline{c}$. When the firm has the highest possible cost, it sells the residual demand and sets the equilibrium price with probability 1. As a result, the firm finds it optimal to choose the highest possible price. At the other extreme, when the firm has the lowest possible cost, it always sells at capacity and it never sets the equilibrium price. As a result, it is a dominant strategy for the firm to offer the lowest possible price. Note that the *quantity effect* and the *price effect* show up in the numerator and denominator of the exponent term, respectively. However, unlike in the model with privately known capacities, these effects are invariant to the firm's private information.

4.2 Discriminatory auction

We can carry out a similar exercise to characterize equilibrium bidding in the discriminatory auction. Firm i's profit function can be written as

$$\pi_i\left(b_i, b_j | c_i\right) = \left(b_i - c_i\right)\left[\int_{\underline{c}}^{b_j^{-1}(b_i)} (\theta - k) f(c_j) dc_j + \int_{b_j^{-1}(b_i)}^{\bar{c}} k f(c_j) dc_j\right]. \tag{7}$$

For a given bid profile, quantities are the same as under the uniform-price auction but prices are not, as firms are now always paid at their own bid.

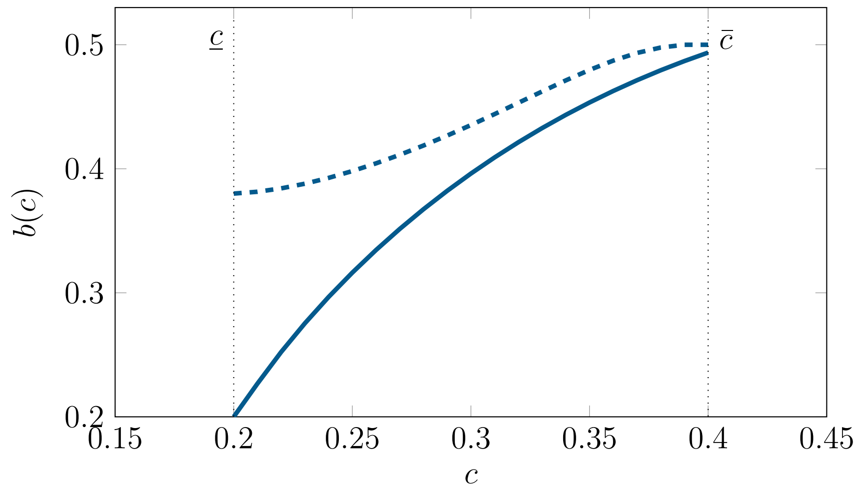
Proposition 8. *At the unique symmetric Bayesian Nash equilibrium of the discriminatory auction when costs are privately known, each firm* $i = 1, 2$ *offers all its capacity,* $q^*(k_i) = k_i$*, at a price given by*

$$b_i(c_i) = c_i + (P - \bar{c})\frac{H(\bar{c})}{H(c_i)} - \int_{c_i}^{\bar{c}} \frac{H(c)}{H(c_i)} dc \tag{8}$$

where

$$H(c) \equiv k(1 - F(c)) + (\theta - k)F(c).$$

Note that $H(c_i)$ represents firm i's expected output when its cost is c_i. It thus captures the *price effect.* Comparison of the equilibrium bids under the uniform-price and the discriminatory auctions shows that the former are always lower, as depicted in Figure 5. Intuitively, as we already discussed in the case with privately known capacities, a firm in a uniform-price auction offers lower prices knowing that, conditionally on being the low bidder, the price will be set by the rival. However, unlike that case, the two effects now exactly compensate each other, giving rise to *revenue equivalence* between the two auction formats.



Note: The figure depicts the equilibrium price offers under the discriminatory auction (solid) and the uniform-price auction (dashed) when costs are privately known. One can see that firms always offer, for a given realized cost, higher prices under the discriminatory auction. Parameter values: $c_i \sim U[0.2, 0.4]$, $k = 0.7$, $P = 0.5$ and $\theta = 1$.

Figure 5: Comparison between the optimal price offers across auctions when costs are privately known

Proposition 9. *When costs are independent, expected payments to firms are the same under the uniform-price and discriminatory auction formats.*

The reasoning goes as follows. At a symmetric equilibrium, small changes in costs affect prices but, due to the Envelope Theorem, this does not directly affect profits. Furthermore, contingent on having either the low or the high cost, the quantity produced by each firm is independent of its private information. Thus, since the probability that the two firms have the same cost is zero, the bid ranking and quantities allocated to the two firms are not affected by small cost shocks. Hence, revenue stays unchanged. The only effect of private information on profits is through changes in the cost of production, but this effect is the same across auction formats.

4.3 Known versus unknown costs

In this section we briefly show that a counterpart of the results provided in Section 3.1 regarding the effects of information on bidding behavior also goes through in this case.

We start by characterizing equilibrium profits when cost realizations are publicly known.

Lemma 5. *Suppose that realized costs are publicly known prior to bidding. There exist* *symmetric pure-strategy Nash equilibria, resulting in expected joint profits* $[P - E(c)]\,\theta$*. These profits are sustained by the following bid profiles: for* $i, j = 1, 2$ *and* $j \neq i$*, (i) if* $c_i > c_j$*,* $b_i^*(c_i) = P$ *and* $b_j^*(c_j) \in [c, \underline{b}_i]$*, with* $\underline{b}_i$ *low enough so as to make undercutting by firm* i *unprofitable. (ii) If* $c_i = c_j = c$*, firms play mixed-strategies, with expected joint profits* $2(P - c)(\theta - k)$*.*

The equilibrium when costs are unknown to both firms before they make their offers is characterized as follows.

Lemma 6. *If realized costs* (c_i, c_j) *are known only after firms have made their offers, the unique symmetric Bayesian Nash equilibrium involves mixed-strategies, with firms randomizing their prices in the interval* $(\underline{p}, P)$*, where* $\underline{p} > \underline{c}$*. Expected equilibrium joint profits are* $2\,[P - E(c)]\,(\theta - k)$*.*

Putting all results together, we can rank expected equilibrium prices across information treatments.

Proposition 10. *If firms play symmetric Bayesian Nash equilibria, expected prices are the highest with publicly known costs, and the lowest with unknown costs. Expected equilibrium prices with privately known costs lay in between.*

Several comments are in order. As in the case with heterogeneous capacities, more information allows firms to coordinate their behavior, giving rise to higher prices. However, it is important to notice that both when costs are private information as when they are publicly known, the equilibrium bid of the firm with the highest cost is always higher than that of the rival. As a result, the most efficient firm sells at capacity, leading to productive efficiency. When costs are unknown, however, since firms cannot condition neither on their own nor on the rival's cost, the identity of the firm that produces at capacity is independent of the cost realizations. This inefficiency reduces total welfare.

5 Private information on capacities *vs.* costs

In this section we use our previous results to discuss whether the source of private information –either capacities or costs– matters in shaping firms' bidding behaviour and market outcomes.

Let us start with the insights that are common to both cases; namely, the impact of private information and information precision on the intensity of competition (Propositions 5 and 10). As we have shown, in a uniform-price auction firms can attain more collusive outcomes when they can condition their bids on the realization of a random variable (be it on capacities or on costs) than when they cannot. Hence, moving from the case in which capacities or costs are privately known to one in which they are publicly known weakens competition in both cases. A similar impact arises when moving from a setting in which both capacities and costs are unknown to one in which they are privately known. Furthermore, as we show in Section 3.2 for the case of privately-known capacities — and Lagerlöf (2016) for the case of privately-known costs — the more precise the signal about the rival's capacity or cost the weaker is competition.

In contrast, the effects of private information on productive efficiency are substantially different across the two models. Whereas private information on capacities has no welfare implications, information asymmetries regarding costs harm productive efficiency. Therefore, whereas information provision or information exchange about costs improves productive efficiency, information provision or information exchange about capacities cannot be justified on those grounds.

The comparison of the bidding incentives across the two models also allows us to identify important differences regarding the shape of the bidding functions and their implications for market outcomes. For ease of exposition, we reproduce here the optimal markup for the case of privately known capacities

$$\frac{b'^*(k_i)}{b^*(k_i) - c_i} = -\frac{2k_i - \theta}{\theta - E(k_j|k_j \leq k_i)} \frac{g(k_i)}{1 - G(k_i)}. \tag{2}$$

When costs are heterogeneous, the optimal markup can instead be characterized as

$$\frac{b'^*(c_i)}{b^*(c_i) - c_i} = \frac{2k - \theta}{\theta - k} \frac{f(c_i)}{F(c_i)}. \tag{9}$$

As usual, these expressions reflect the ratio of the *quantity effect* over the *price effect*. Under private information on costs, this ratio only depends on the firm's private information through the *hazard rate* $f(c_i)/F(c_i)$, which is decreasing in c_i when $F(c_i)$ is log-concave, a property that is satisfied by most commonly used distributions. This means that the *quantity effect* becomes weaker relative to the *price effect* as c_i increases. To compensate for this reduced incentive, the bid must become less sensitive to c_i, leading to the concave shape shown in Figure 5 for the uniform-price format.

Private information on capacities also affects bidding incentives through the *failure rate* $g(k_i)/(1 - G(k_i))$, which is increasing in k_i when $G(k_i)$ is log-concave. However, equation (2) further depends on k_i as it affects the quantities produced by firm i conditionally on being the low or high bidder. This additional effect is not present when private information is on costs given that the low and high bidders produce the same regardless of their private information.

To see this in more detail, note first that k_i impacts the *quantity effect* in (2) through the output loss from being undercut, i.e., the $(2k_i - \theta)$ term. This means that, abstracting from the density term which is also present in (9), the *quantity effect* in (2) is stronger the higher k_i, while in (9) it equals $(2k - \theta)$ independently of c_i. Turning to the *price effect*, an increase in k_i reduces the firm's expected residual demand contingent on the rival bidding below, i.e., $(\theta - E(k_j | k_j \leq k_i))$ is decreasing in k_i. In contrast, the residual demand faced by the high bidder in the model with privately known costs remains constant at $(\theta - k)$ regardless of the firm's own cost.

Such differences in bidding incentives explain why the shape of the optimal bidding functions differ across models. When private information is on costs, the optimal bid function in Figure 5 is concave for all cost realizations. In contrast, when private information is on capacities, the optimal bid function in Figure 1 turns from being concave for low capacity realizations to being convex for high capacity realizations.

The above differences also underline an important dimension of the comparison between models; namely, the revenue ranking between the uniform-price and the discriminatory auction formats. The standard revenue equivalence result applies when costs are private information as long as they are independent across firms (Proposition 9). In contrast, when capacities are private information, the uniform-price auction yields higher payments to firms as compared to the discriminatory auction, despite capacity draws being independent (Proposition 4). The reason is that the rival's capacity is payoff relevant for each firm beyond the effect on its bids, giving rise to a similar effect as if there were negative affiliation across capacities. In the natural case when costs are positively affiliated, the opposite ranking holds, with the discriminatory auction resulting in higher payments to firms (Holmberg and Wolak, 2018). This is in line with Milgrom and Weber (1982)'s result for the single-unit case, under which the first-price auction delivers a higher price than the second-price auction when costs are positively affiliated.

6 Understanding competition in electricity markets

Our model with privately known capacities is particularly well suited to understand the performance of electricity markets. Needless to say, electricity markets are complex institutions, which differ across jurisdictions in several issues, including market rules or market structure. While our stylized model does not include all those ingredients, it nevertheless captures the main driving forces that will shape market performance once renewables become the dominant energy source in the near future. In this section, we discuss why this is the case, and use the model to derive some key lessons.

The main characteristics of electricity markets are very much in line with the model's assumptions. First, we have assumed that firms compete by submitting step-wise supply functions to serve a known and inelastic demand. This structure resembles competition in most electricity markets in practice, where firms submit a finite number of price-quantity pairs to an auctioneer who then allocates output and sets market prices according to such bids.[21] By the time firms submit their bids, they have very precise information about demand as system operators regularly publish highly accurate demand forecasts before the market opens. Furthermore, electricity demand is highly price-inelastic in the short-run because electricity retail prices rarely reflect movements in spot market prices. Even where they do, consumers typically do not have strong incentives or the necessary information to optimally respond to the high-frequency price changes.[22]

In current electricity markets, renewable energy sources coexist with conventional technologies. While our model does not explicitly capture the coexistence of various energy sources, the conventional technologies are implicitly present in the model through P, which can be interpreted as the marginal costs of coal or gas plants, as long as renewable producers do not own a significant proportion of the conventional-power capacity. In future stages of the energy transition, and consistently with most real electricity markets, P could be interpreted as an explicit price cap, or as an implicit one that triggers regulatory intervention. Our model assumes that P is known, but it could be extended

[21] Due to tractability reasons, our model restricts the number of steps that firms can use to just one. The same constraint applies to other papers in the electricity auctions literature (Holmberg and Wolak, 2018; Fabra et al., 2006). Analyzing the model with privately known capacities and multiple steps is beyond the scope of this paper.

[22] The empirical evidence shows that this is the case in the Spanish electricity market, the only country so far where Real Time Pricing has been implemented as the default option for all households (Fabra et al., 2021). However, this might change once automation devices become more broadly deployed.

to make it stochastic.[23]

One of the core assumptions of our model is that firms' capacities are subject to random shocks. In the context of electricity markets, the capacity of each renewable plant is subject to common and idiosyncratic availability shocks. In our model, these shocks could be captured by decomposing the available capacity of firm i in two additive components, $k_i \equiv \beta\kappa + \varepsilon_i$. The parameter $\beta \in [0, 1]$ is the common shock component that affects the availability of each firm's nameplate capacity κ. The idiosyncratic shock ε_i can be thought of as being distributed according to $\Phi(\varepsilon_i|\kappa)$ in an interval $[\underline{\varepsilon}, \overline{\varepsilon}]$, with $E(\varepsilon_i) = 0$. As a result, firm i's available capacity k_i is distributed according to $G(k_i) = \Phi(k_i - \beta\kappa|\kappa)$ in the interval $k_i \in \left[\underline{k}, \overline{k}\right]$, where $\underline{k} = \beta\kappa + \underline{\varepsilon}$ and $\overline{k} = \beta\kappa + \overline{\varepsilon}$, in line with the assumptions of the model. According to this interpretation, firms' available capacities are correlated through the common shock component, albeit imperfectly so due to the presence of idiosyncratic shocks.

While electricity system operators typically publish forecasts of the common weather shocks at the national or regional level, the idiosyncratic components remain each firm's private information. Indeed, through the monitoring stations installed at the renewable plants' sites, firms have access to local weather measurements that are not available to the competitors. Beyond weather conditions, the plants' availability might be subject to random outages and maintenance schedules that only their owners are aware of. Accordingly, in the presence of private information, each firm is better informed about its own available capacity than its competitors. Our model applies even in those settings in which the amount of private information is relatively small.[24]

To illustrate this claim empirically, we have collected data from the Spanish electricity market to perform and compare forecasts of a plant's production, with and without firms' private information. In particular, we have obtained proprietary data of six renewable plants corresponding to their hourly production and their own available forecasts at the time of bidding, for a two-year period. We have also gathered the forecasts computed

[23]If P were stochastic (either because it is interpreted as the marginal cost of the conventional producers or because it is the implicit price-cap that triggers regulatory intervention) the equilibrium market price would maximize the high bidder's profits, taking into account the distribution of P. Since the high bidder sells the expected residual demand, such a price would be independent of the firm's realized capacity.

[24]One caveat of our model however is that it assumes that all firms are symmetrically informed. In reality however, it is reasonable to expect that larger firms have more precise information about their rivals' capacities. Intuitively, having access to more accurate forecast could reinforce their market power, but this issue is out of the scope of the current paper.

Table 1: Forecast errors with public versus private information.

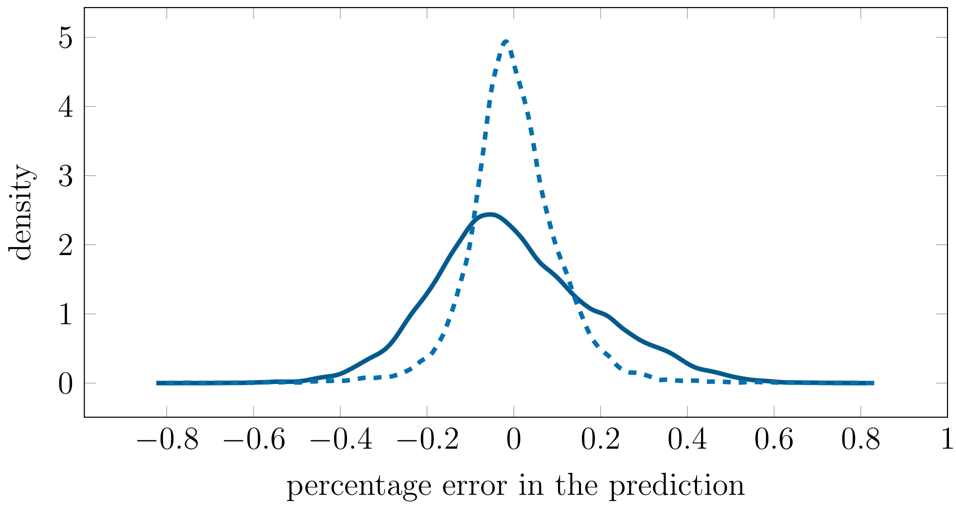
Variables	(1)	(2)
Public forecast	0.582***	0.070***
	(0.035)	(0.021)
Private forecast		0.657***
		(0.008)
Observations	36,671	36,671
R-squared	0.520	0.826
Standard deviation of the error	.18	.11

Note: The dependent variable is the plant's hourly production normalized by its nameplate capacity. Both regressions include weather data (temperature, wind speed and atmospheric pressure) as well as plant, hour and date fixed effects. The robust standard errors are in parentheses *** $p<0.01$, ** $p<0.05$, * $p<0.1$. One can see that using the plant's own forecast significantly reduces the forecast error, with the R^2 increasing from 0.520 to 0.826. When the private forecast is used, the public forecast is still statistically significant but it has a small impact on the prediction.

by the Spanish System Operator (Red Eléctrica de España) and the one-day ahead predictions of the Spanish weather agency (Agencia Estatal de Meteorología or AEMET) at provincial level, which is the most disaggregated data publicly available, close to the plant's location. We have used OLS to forecast each plants' hourly availability, with and without the firms' proprietary local data. Figure 6 plots the distribution of the forecast errors and Table 1 summarizes the mean and standard deviations of the corresponding forecast errors. The evidence is consistent with firms possessing private information that allows them to significantly improve the precision of the forecasts of their own plants' available capacities.[25] Interestingly, when the private forecast is used, the national forecast, while still statistically significant, has a small economic impact in the prediction.

As shown by these results, firms' forecast errors are centered around zero, but the standard deviations remain significant even when firms have private information about their own capacities. Nevertheless, the day-ahead market price and output allocation are computed using firms' committed quantities, even when these differ from their actual ones. Hence, for bidding purposes, what matters is that each firm knows exactly how much output it has offered in the day-ahead market, and not necessarily how much it

[25]We have also used more general specifications, such as a LASSO, with almost identical results.



Note: This figure depicts the densities of the forecast errors of the specifications in Table 1. Both distributions are centered around zero, but the standard deviation is larger when only publicly available data are used.

Figure 6: Kernel distribution of the forecasts errors using public (solid) or private information (dashed).

will be able to produce in real time. This is consistent with our model, since we have assumed that firms know their available capacity before submitting their bids.

6.1 Implications for future electricity markets

Since our model captures reasonably well the main features of electricity markets, we can use its results to shed light on the future performance of electricity markets in practice.

First, our model has shown that renewables will not in general make electricity markets immune to market power. Rather, firms will keep on exercising market power either by raising their bids or by withholding their output (Proposition 1 and 2). The fact that the price offers are decreasing in firms' capacities implies that mark-ups will be lower at times of more available capacity relative to demand, leading to price dispersion both within as well as across days, depending on weather conditions. This price volatility is not driven only by the intermittency of renewable plants but also by its combination with market power. Our assumption that demand can always be satisfied implies that, in the absence of market power, prices would remain stable around the renewable energies' marginal costs.

Second, our model suggests that renewable power tends to mitigate market power as compared to conventional technologies. This conclusion is derived by associating competition among renewable plants with the model of unknown capacities on the one hand, and competition among conventional power sources with the model of known costs on the other (Proposition 5). If we consider that the marginal costs of conventional technologies might also be privately known, this conclusion would also be confirmed by our model if we assume, as it seems natural, that the range of cost variation is more limited than the range of variation in renewable power availability. Taking into account our previous results on improved information precision leading to less competition (Proposition 6), this would suggest that competition is more vigorous when private information is on capacities rather than when it is on costs.

Third, the prevalence of positive mark-ups implies that the price-depressing effects of renewable power sources will not be as pronounced as those predicted under the assumption of perfect competition. Renewable power will have a stronger price-depressing effect in the long-run as installed capacity grows. The reduction in expected prices as a function of total investment will not be linear, but will rather be smoother at the late stages of the energy transition (Figure 2).

Fourth, our model also serves to predict that differences across renewable technologies (e.g., solar versus wind) will give rise to different market power impacts, depending on the correlation between demand and the expected availability of each technology. Competition among renewable power plants could also be affected by portfolio effects, as firms typically own a variety of technologies whose joint distribution will affect firms' optimal bidding strategies. In sum, future electricity markets would depict large price differences across the day and across the year, reflecting differences in weather conditions and the associated differences in firms' ability and incentives to charge positive mark-ups.

Elastic demand and storage

Notably, our stylized model does not capture two important ingredients of future electricity markets: the possibility that demand becomes more price-elastic as dynamic pricing and automation devices get more broadly deployed in the future; and the likely expansion of storage facilities.

Consider first the impact of allowing for price-elastic demand. As it is standard in oligopoly models, this will partially mitigate market power. However, beyond reducing expected prices, demand elasticity will also affect price volatility through its effect on the shape of the optimal bid function. In particular, the optimal bid function will tilt downwards, starting at the profit maximizing price of the residual monopolist (taking into account the expected capacity of the rival)[26] and ending at marginal cost. Our last Proposition characterizes optimal bidding behaviour in the uniform-price auction with elastic demand.

Proposition 11. *Suppose that market demand $D(p)$ is downward-sloping, continuously differentiable, log-concave, and such that $D(c) < 2\underline{k}$. Firms compete in a uniform-price auction.*

(i) If $\overline{k} < D(c)$, at the unique symmetric Bayesian Nash equilibrium when capacities are privately known, firm i offers all its capacity at a price that is strictly decreasing in k_i and which is decreasing in the price-elasticity of demand. The optimal bid function starts at $b(\underline{k})$, implicitly defined by

$$\frac{b(\underline{k}) - c}{b(\underline{k})} = \frac{1}{\epsilon\left(b(\underline{k})\right)}$$

where $\epsilon\left(b(\underline{k})\right)$ is the price-elasticity of the residual demand $D(p) - E[k]$ at a price $b(\underline{k})$, and it ends at $b(\overline{k}) = c$. For each k_i the optimal price offer $b(k_i)$ is below expression (2).

(ii) If $\overline{k} \geq D(c)$, the bidding function is as defined in part (i) for all $k_i < D(c)$. For $k_i \geq D(c)$, firm i bids at c and withholds capacity to $q_i = D(c)$.

The previous result shows that the main features of the model extend to environments with a downward-sloping demand.[27] Firms find it optimal to withhold capacity when they can individually cover the whole market at the competitive price. Instead, for lower capacity values, they offer all their capacity at prices that decrease in their own capacity all the way down to marginal cost, c. Interestingly, the highest price offer they ever submit, $b(\underline{k})$, is lower for higher $E[k]$, as this makes the residual demand more elastic. Therefore, when the distribution of capacities moves to the right — in the first

[26]We are implicitly assuming that the price cap P is so high that it is never binding. Otherwise, the optimal bid function would still start at P.

[27]Somogy and Vergote (2021) analyze the discriminatory auction with elastic demand, albeit in a simplified version of our setup. In their model, firms can only be either capacity-constrained or unconstrained, and capacity withholding is not allowed. In line with our result of decreasing bid functions, they also find that smaller (capacity-constrained) firms set higher prices than larger (unconstrained) firms.

order stochastic sense — the bidding function starts at a lower price, in contrast to the inelastic demand case in which the highest bid stays constant at P (Figure 2).

As the bidding function becomes flatter, price dispersion diminishes as compared to the inelastic demand case. However, demand elasticity also enlarges price differences across periods (with more or less abundant renewable energy available and higher or lower demand) as these shocks do not only shift the bidding functions outwards and inwards (as in Figure 2), but also change their slopes. Whether demand elasticity results in higher or lower price volatility will depend on the interplay between these two effects.

The deployment of storage facilities will allow for supply management. Firms owning storage capacity will engage in price arbitrage by moving production from periods when renewables' capacity is high (and/or demand is low) to periods when it is low (and/or demand is high). The extent of this shift will depend on the storage capacity and the market power of the firms that operate it. Our model can capture the effects of storage through a dampening in the variation of θ across time, which could be interpreted as demand net of storage.

Following the comparative statics we derived from Propositions 1 and 2, storage will reduce price differences across periods, both through the direct effects on net demand as well as through the indirect effects on the mark-ups. Whether average prices go up or down crucially depends on whether the price-depressing effect in the high priced periods is stronger than price-increasing effect in the low priced ones, as well as on the degree of market power that storage firms can exert. Andres-Cerezo and Fabra (2020) analyze this issue, albeit in a different setup. They conclude that storage depresses average prices since its effects are stronger when mark-ups are higher. This result would suggest that the combination of storage and uncertain renewable power sources would also depress average prices, as prices and mark-ups are higher when renewable power availability is low relative to demand.

7 Concluding Remarks

In this paper we have analyzed equilibrium bidding in multi-unit auctions when bidders' production capacities are private information. We have allowed changes in capacity availability to shape the bidding functions, both through changes in the prices and the quantities offered by firms. This is unlike other papers in the auction literature, which typically assume that the private information is on costs (or bidders' valuations) and which, with few exceptions, do not allow bidders to act on both the price and quantity dimensions.

From a broad economic perspective, we have shown that the nature of private information and the strategies available to firms have a key impact on equilibrium behavior. We have shown that, due to competition, when firms receive a positive capacity shock, they find it optimal to offer more output at prices that rapidly approach marginal cost. In contrast, when costs are private information, a low cost realization implies a more nuanced approach towards marginal cost bidding.

We have also shown that competition with privately known capacities does not give rise to revenue equivalence between the uniform-price and the discriminatory auctions, in contrast to models with privately known — and independently distributed — costs. In our model, we have shown that private information on independently distributed capacities is akin to having negative affiliation across cost shocks given that, for a given price, a firm's capacity has a negative effect on the rival's profits. As a result, allocation schemes that elicit more information, like the uniform-price auction, result in higher prices as compared to other schemes in which prices only depend on each firm's bid, like the discriminatory auction.

Although our setup applies to a variety of environments, it is particularly well suited to shed light on the future performance of electricity markets. We have provided suggestive evidence on the existence of private information regarding renewable plants' available capacity, whose marginal costs are broadly known to be close to zero. Understanding competition among renewables is of first order importance to guide policy making in this area. Our model predicts that electricity prices will go down as more renewables get deployed, although some market power will remain. Promoting demand elasticity as well as investments in energy storage — ingredients that can well be incorporated into our model — will mitigate market power and improve market performance along the way.

Finally, the model proposed in this paper can also be used for policy analysis. Fabra and Llobet (2019) extends the model to N firms and analyzes the effect of a merger that decreases the extent of the private information but increases market power. We also discuss the effect of prohibiting capacity withholding, as proposed in some jurisdictions.

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A Proofs

Proof of Lemma 1: For part (i) of the lemma, suppose that firm j chooses a bid according to a distribution $\Phi_j(b_j, q_j|k_j)$. Profits for firm i can be written as

$$\begin{aligned}\pi_i(b_i, q_i, \Phi_j|k_i) = \int_{(b,q\geq \underline{k})} & [(b-c)q_i \Pr(b_i \leq b) \\ & + (b_i - c)(\theta - q) \Pr(b_i > b)] \, d\Phi_j(b, q|k_j) g(k_j) dk_j.\end{aligned} \tag{10}$$

The above equation is increasing in q_i, indicating that the firm maximizes profits by choosing $q_i^*(k_i) = k_i$. In what follows we simplify the notation by eliminating q_i from the profit function π_i and by indicating that the randomization is only over prices, $\Phi_i(b_i|k_i)$.

For part (ii), we start by defining b_i^{min} as the lowest bid in the support of a firm with capacity k_i. We now show that a firm with capacity $k_i' > k_i$ maximizes profits by choosing a bid $b_i' \leq b_i^{min}$. Suppose that this is not the case and the firm with capacity k_i' chooses $b_i' > b_i^{min}$. By revealed preference,

$$\pi_i(b_i^{min}, \Phi_j|k_i) - \pi_i(b_i', \Phi_j|k_i) \geq 0 \geq \pi_i(b_i^{min}, \Phi_j|k_i') - \pi_i(b_i', \Phi_j|k_i').$$

Using (10), this is a contradiction since

$$\frac{\partial \left[\pi_i(b_i^{min}, \Phi_j|k_i) - \pi_i(b_i', \Phi_j|k_i)\right]}{\partial k_i} =$$
$$\int_{\underline{k}}^{\overline{k}} \int_b (b-c) \left[\Pr(b_i^{min} \leq b) - \Pr(b_i' \leq b)\right] d\Phi_j(b|k_j) g(k_j) dk_j > 0,$$

where the last inequality is due to the fact that, using Bertrand arguments, Φ_j cannot contain gaps in the support and, therefore, $\Pr(b_i^{min} \leq b) > \Pr(b_i' \leq b)$.

Notice that the previous result implies that each bid can be used by at most one capacity realization. That is, the bid support used for different capacity realizations do not overlap. Suppose now a firm with capacity k_i randomizes between two different bids b_i and $\hat{b}_i$ with $b_i < \hat{b}_i$. By Bertrand arguments, it has to be that case that all bids in between are also in the randomization support. However, since each capacity arises with probability 0, the firm will always prefer to choose the highest point in the support, $\hat{b}_i$, as the revenues increase but the probability of being outbid is essentially unchanged.

Part (iii) follows directly from the first part of the previous argument. Without randomization, a firm with capacity k_i chooses $b_i = b_i^{min}$ and for any $k_i' > k_i$ it has to be the case that $b_i' \leq b_i$. $\square$

Proof of Proposition 1: Expected profits can be written as

$$\pi_i\left(b_i, b_j | k_i\right) = \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j) - c) k_i g(k_j) dk_j$$
$$+ \int_{b_j^{-1}(b_i)}^{\overline{k}} (b_i - c)(\theta - k_j) g(k_j) dk_j, \quad (11)$$

and the first order condition that characterizes the optimal bid of firm i can be written as

$$\frac{\partial \pi_i}{\partial b_i} = b_j^{-1\prime}(b_i) g(b_j^{-1}(b_i))(b_i - c)(k_i + b_j^{-1}(b_i) - \theta) + \int_{b_j^{-1}(b_i)}^{\overline{k}} (\theta - k_j) g(k_j) dk_j = 0. \quad (12)$$

Under symmetry, $b_j(k) = b_i(k)$. Accordingly, we can rewrite the expression as

$$\frac{1}{b_i'(k_i)} g(k_i)(b_i(k_i) - c)(2k_i - \theta) + \int_{k_i}^{\overline{k}} (\theta - k_j) g(k_j) dk_j = 0. \quad (13)$$

The first term of the first order condition (13) is negative and the second term is positive, taking the form

$$b_i'(k_i) + a(k_i) b_i(k_i) = c a(k_i),$$

where

$$a(k) = \frac{(2k - \theta) g(k)}{\int_k^{\overline{k}} (\theta - k_j) g(k_j) dk_j}. \quad (14)$$

If we multiply both sides by $e^{\int_{\underline{k}}^{k} a(s) ds}$ and integrate from $\underline{k}$ to k_i we obtain

$$\int_{\underline{k}}^{k_i} \left(e^{\int_{\underline{k}}^{k} a(s) ds} b_i'(k) + a(k) e^{\int_{\underline{k}}^{k} a(s) ds} b_i(k) \right) dk_i = c \int_{\underline{k}}^{k_i} a(k_i) e^{\int_{\underline{k}}^{k} a(s) ds} dk_i.$$

We can now evaluate the integral as

$$e^{\int_{\underline{k}}^{k} a(k) dk} b_i(k) \Big]_{\underline{k}}^{k_i} = c e^{\int_{\underline{k}}^{k} a(s) ds} \Big]_{\underline{k}}^{k_i}.$$

This results in

$$e^{\int_{\underline{k}}^{k_i} a(k) dk} b_i(k_i) - b_i(\underline{k}) = c e^{\int_{\underline{k}}^{k_i} a(k) dk} - c.$$

Solving for $b_i(k_i)$ we obtain

$$b_i(k_i) = c + A e^{-\int_{\underline{k}}^{k_i} a(k) dk} = c + A e^{-\omega(k_i)},$$

where $A \equiv b_i(\underline{k}) - c$ and $\omega\left(k_i\right) \equiv \int_{\underline{k}}^{k_i} a(k) dk$.

A necessary condition for an equilibrium is that the resulting profits are at or above the minimax, which the firm can obtain by bidding at P. Hence, a necessary condition for equilibrium existence is that

$$\pi_i\left(b_i, b_j|k_i\right) \geq \int_{\underline{k}}^{\overline{k}} (P-c)(\theta-k_j)g(k_j)dk_j. \tag{15}$$

Hence, to rule out deviations to P, we now need to prove that minimax profits increase less in k_i as compared to equilibrium profits. The derivative of the minimax is

$$(P-c)\left(G\left(\theta-k_i\right)-g\left(\theta-k_i\right)k_i\right).$$

The derivative of profits is

$$\int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j)-c)k_i g(k_j)dk_j.$$

This derivative is greater than that of the minimax.

It follows that deviations to P are not profitable since equilibrium profits are always strictly greater than the minimax, except for $k_i = \underline{k}$ when equilibrium profits are exactly equal to the minimax.

Finally, we need to verify that the candidate equilibrium, indeed, maximizes profits for each of the firms. From the first order condition in (12) we can compute the second derivative of the profit function of firm i, when firm j uses a bidding function $b_j(k_j)$ as

$$\left(-\frac{b_j''\left(k_j\right)}{\left(b_j'\left(k_j\right)\right)^2}(b_i-c)(k_i+b_j^{-1}(b_i)-\theta)+\frac{1}{b_j'\left(k_j\right)}\frac{g'(b_j^{-1}(b_i))}{g(b_j^{-1}(b_i))}(b_i-c)(k_i+b_j^{-1}(b_i)-\theta)\right.$$
$$\left.+(k_i+b_j^{-1}(b_i)-\theta)+\frac{1}{b_j'\left(k_j\right)}(b_i-c)-(\theta-b_j^{-1}(b_i))\right)\frac{g(b_j^{-1}(b_i))}{b_j'\left(k_j\right)}.$$

Once we substitute the candidate equilibrium $b_i(k) = b_j(k)$ the previous expression becomes

$$\frac{\partial^2\pi_i}{\partial^2 b_i(k_i)} = \frac{g(k_i)}{b^{*\prime}\left(k_i\right)}\frac{1}{a\left(k_i\right)} < 0.$$

Because there is a unique solution to the first order condition, this implies that the profit function is quasiconcave, which guarantees the existence of the equilibrium. In particular, this rules out deviations where firms choose any lower bid, including c.

In order to show how the shape of the optimal bid function changes with k_i, we take derivatives on the right-hand side of $\omega\left(k_i\right)$, see equation (2). For the ease of exposition, we now write $\omega\left(k_i\right)$ as follows:

$$\omega\left(k_i\right) = \frac{(2k_i-\theta)}{d(k_i)}\frac{g(k)}{1-G(k)}, \tag{16}$$

where

$$d(k_i) \equiv \int_{k_i}^{\overline{k}} (\theta - k_j) \frac{g(k_j)}{1 - G(k_i)} dk_j.$$

The denominator $d(k_i)$ is decreasing in k_i since

$$d'(k_i) = \frac{g(k_i)}{1 - G(k_i)} \left[k_i - E(k_j | k_j > k_i) \right] \leq 0.$$

Hence, since the term $(2k_i - \theta)$ is increasing in k_i, it follows that the first ratio in (16) is increasing in k_i. It also follows that $\omega(k_i)$ is increasing if the second ratio, $\frac{g(k_i)}{1-G(k_i)}$, is increasing in k_i. A sufficient condition is that g is log-concave.

We can now assess how the slope of $b(k_i)$ changes with k_i. In particular, using equation (2),

$$b'(k_i) = -(b(k_i) - c)\omega(k_i).$$

Hence, taking the derivative with respect to k_i,

$$b''(k_i) = -b'(k_i)\omega(k_i) - (b(k_i) - c)\frac{d\omega(k_i)}{dk_i}.$$

As a result, the first term is positive (recall that $b'(k_i) < 0$) and the second term is negative, as we have just demonstrated above. In the limit, when capacity is $\overline{k}$, $\big(b(\overline{k}) - c\big) = 0$, so the total effect would be positive (and the bidding function convex). When the capacity is $\underline{k}$, note that $d'(\underline{k}) = 0$. $\square$

Proof of Proposition 2: We first show that, for $k_i \geq \theta$ and any price offer b_i, quantity $q_i > \theta$ is dominated by offering $q_i = \theta$. If the firm offers $q_i = \theta$, its expected profits are

$$\pi_i(b_i, b_j(k_j) | q_i = \theta) = \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j) - c)\theta g(k_j) dk_j + \int_{b_j^{-1}(b_i)}^{\overline{k}} (b_i - c)(\theta - k_j) g(k_j) dk_j. \quad (17)$$

Instead, if the firm offers $q_i > \theta$, its expected profits are

$$\pi_i(b_i, b_j(k_j) | q_i > \theta) = \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_i - c)\theta g(k_j) dk_j + \int_{b_j^{-1}(b_i)}^{\overline{k}} (b_i - c)(\theta - k_j) g(k_j) dk_j.$$

The inspection of the above equation in comparison with (17), shows that offering $q_i > \theta$ is dominated by $q_i = \theta$: the second term is the same as in equation (17), while the first term is now smaller since, over this range, $b_j(k_j) > b_i$. Given the optimality of $q_i = \theta$, the problem is the same as the one solved in Proposition 1, with $G(k_i)$ and $G(k_j)$ now adjusted to $G(q_i^*(k_i))$ and $G(q_j^*(k_j))$, $i, j = 1, 2$, $i \neq j$.

Proof of Proposition 3: Suppose $\overline{k} < \theta$. Expected profits under the discriminatory auction are given by:

$$\pi_i(b_i, b_j(k_j)|k_i) = (b_i - c)\left(\int_{\underline{k}}^{b_j^{-1}(b_i)} k_i g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\overline{k}} (\theta - k_j)g(k_j)dk_j\right). \quad (18)$$

Maximization with respect to b_i implies,

$$\left(\int_{\underline{k}}^{b_j^{-1}(b_i)} k_i g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\overline{k}} (\theta - k_j)g(k_j)dk_j\right) +$$
$$(b_i - c)b_j^{-1\prime}(b_i)\left(g(b_j^{-1}(b_i))(k_i + b_j^{-1}(b_i) - \theta)\right) = 0.$$

Under symmetry, $b_j(k) = b_i(k)$. Accordingly, we can rewrite the expression as

$$k_i G(k_i) + \int_{k_i}^{\overline{k}} (\theta - k_j)g(k_j)dk_j + (b_i - c)\frac{1}{b_i'(k_i)}g(k_i)(2k_i - \theta) = 0.$$

This expression is similar to equation (13) for the uniform-price auction, but it has an additional term, $k_i G(k_i)$, reflecting the fact that the firm is always paid according to its bid, also when it is the large firm and hence has the low bid. The rest of the proof follows the same steps as the proof of Proposition 1.

For the case with $\overline{k} \geq \theta$, similar arguments as those in the proof of Proposition 2 show that for $k_i \geq \theta$, offering a quantity $q_i > \theta$ is equivalent to offering $q_i = \theta$. Hence, the problem is the same as the one solved above, with $G(k_i)$ and $G(k_j)$ now adjusted to $G(q_i^*(k_i))$ and $G(q_j^*(k_j))$, $i, j = 1, 2$, $i \neq j$. □

Proof of Proposition 4: We show that expected profits under the uniform-price auction are weakly higher than under the discriminatory auction for all values of k_i, with a strict inequality for some capacities. Since the total cost and quantity are always the same, this implies that firms' payments are higher under that format. Denote the bid and profits under the uniform-price and discriminatory auctions with the subscripts u and d, respectively. Define $\Pi_s(k_i) = \pi_i(b^*, b^*|k_i)$ for $s = u, d$.

Start by assuming that there is no withholding and suppose that $\Pi_u(k_i) = \Pi_d(k_i)$ for some value of k_i. Since in equilibrium $b_d(k_i) > b_u(k_i)$, we know that

$$\int_{k_i}^{\bar{k}} (b_d(k_i) - c)(\theta - k_j)g(k_j)dk_j > \int_{k_i}^{\bar{k}} (b_u(k_i) - c)(\theta - k_j)g(k_j)dk_j.$$

Using (18) and (11), this implies that

$$\int_{\underline{k}}^{k_i} (b^d(k_i) - c)k_i g(k_j)dk_j < \int_{\underline{k}}^{k_i} (b^u(k_j) - c)k_i g(k_j)dk_j.$$

The previous condition implies that

$$\frac{d\Pi_u(k_i)}{dk_i} = \int_{\underline{k}}^{k_i} (b_u(k_i) - c)g(k_j)dk_j > \int_{\underline{k}}^{k_i} (b_d(k_j) - c)g(k_j)dk_j = \frac{d\Pi_d(k_i)}{dk_i}.$$

This means that for capacity values higher than k_i, the uniform-price auction yields higher profits. Since $\Pi_u(\underline{k}) = \Pi_d(\underline{k})$, it follows that there is no value of k_i for which $\Pi_u(k_i) < \Pi_d(k_i)$.

Consider now the possibility of withholding. Firms' profits under the discriminatory auction remain unchanged. In the uniform-price format, however, as withholding is equivalent to a leftward shift of $G(k)$ in the first-order stochastic sense. This implies even higher profits, reinforcing the previous result. □

Proof of Lemma 2: The proof follows similar steps as Fabra et al. (2006) for the case in which demand and capacities are known.

Consider first the second stage of the game when realized capacities are known to be (k_i, k_j). First, suppose that $k_i < k_j$ with $k_j \leq \theta$. Following Lemma 1, in all candidate equilibria we must have $q_j(k_i, k_j) = k_j > q_i(k_i, k_j) = k_i$. There cannot exist a pure-strategy equilibrium with $b_i(k_i, k_j) = b_j(k_i, k_j)$ given that either firm would be better off slightly undercutting the other in order to increase its production with no effect on the price. Consider equilibria with $b_i(k_i, k_j) > b_j(k_i, k_j)$. Since, conditionally on being the higher bidder, firm i's profits are strictly increasing in its bid, it follows that $b_i(k_i, k_j) = P$. In order to discourage firm i from undercutting firm j's bid, it must be the case that $(P - c)(\theta - k_j) \geq (b_j(k_i, k_j) - c) k_i$. Solving for b_j, it follows that $b_j(k_i, k_j) \leq \underline{b}_i \equiv c + (P - c)\frac{\theta - k_j}{k_i}$. Since the low bid is pay-off irrelevant, and firm j is selling all its capacity at P, it does not have incentives to deviate either. In equilibrium, firm i makes profits $(P - c)(\theta - k_j)$ and firm j makes profits $(P - c) k_j$.

Second, if $k_j > \theta$, one can apply the same argument as above by setting $q_j(k_i, k_j) = \min\{\theta, k_j\}$.

Last, if $k_i = k_j = k$, Bertrand arguments rule out any pure-strategy symmetric equilibrium. The only equilibrium is therefore in mixed-strategies. Since P must be part of the equilibrium support, it follows that expected equilibrium profits for each firm are $(P - c)(\theta - k)$ if $k < \theta$ and zero otherwise.

To conclude the proof, consider the first stage of the game. Since both firms are equally likely to be the small or the large firm, and since the event $k_i = k_j = k$ occurs with zero probability, it follows that in equilibrium firms make symmetric expected profits $(P-c)\,\theta/2$. $\square$

Proof of Lemma 3: The proof follows similar steps as in Fabra et al. (2006) for the case in which demand is unknown and capacities are symmetric and known.

Consider the second stage of the game in which capacities are known to be (k_i, k_j) and firms have to choose q_i. If $k_i < \theta$, firm i's profits are weakly increasing in k_i. Therefore, firm i does not find it optimal to withhold. If, instead, $k_i \geq \theta$, for the same reason as in the benchmark model, it finds it optimal to offer $q_i = \theta$. In this case, in the first stage when firms choose prices, they behave as if their capacity had a mass point at θ. See Proposition 2.

Consider now the first stage of the game in which firms have to choose their price offer without knowing their realized capacities. Since firms are symmetric in expected terms, a symmetric equilibrium would call them to offer the same price. However, this is ruled out by Bertrand arguments. The only equilibrium is therefore in mixed-strategies. Since P must be part of the equilibrium support, it follows that expected equilibrium profits for firm $i = 1, 2$ are $(P-c)\,[\theta - E(k|k \leq \theta)]\, G(\theta)$, given that a firm bidding at P only faces a positive residual demand in the event that its rival's capacity is below θ, which occurs with probability $G(\theta)$. $\square$

Proof of Proposition 5: It follows from combining the results of Propositions 1 and 2 and Lemmas 2 and 3. $\square$

Proof of Proposition 6: We show that there is no profitable deviation from the candidate equilibrium stated in the text of the proposition.

Regarding part (i), let's start by focusing on $k_i \in [\underline{k}_1, \overline{k}_2]$. It is easy to see that a counterpart of Lemma 1 applies in this case. As a result, the profit function of both firms can be written as

$$\pi_i(b_i, b_j(k_j)) = (P-c)k_i G_i(\underline{k}_1) + \int_{\underline{k}_1}^{b_j^{-1}(b_i)} (b_j(k_j) - c)k_i g_j(k_j)dk_j$$
$$+ \int_{b_j^{-1}(b_i)}^{\overline{k}_2} (b_i - c)(\theta - k_j)g_j(k_j)dk_j.$$

Under the assumption that $g_i(k_i)$ is uniformly distributed in an interval of the same length, we have $g_i(k_i) = g_j(k_j)$ for $k_i \in [\underline{k}_i, \overline{k}_i]$ and $i = 1, 2$. As a result, the profit function of the two firms is identical because the bid function in this range is the same. Hence, the first condition is also the same and it coincides with equation (13) in the proof of Proposition 1, leading to expression (4).

It remains to show that (1) $b_2(k_2) = P$ for $k_2 < \underline{k}_1$ and (2) $b_1(k_1) = c$ when $k_1 > \overline{k}_2$. Regarding (1), by definition of equilibrium we have that $\pi_2(\underline{k}_1, P) \geq \pi_2(\underline{k}_1, b)$ for $b < P$. Since firm 2 can always satisfy the residual demand, we also have that for $k_2 < \underline{k}_1$, the firm makes the same level of profits, $\pi_2(\underline{k}_1, P) = \pi_2(k_2, P)$. In turn, since profits are always increasing in capacity, we also have that for any $b < P$, $\pi_2(\underline{k}_1, b) > \pi_2(k_2, b)$. This shows that (1) is optimal.

With respect to (2), by definition of equilibrium we have that $\pi_1(\overline{k}_2, c) \geq \pi_1(\overline{k}_2, b)$ for any $b > c$. Furthermore, for all $k_1 \geq \overline{k}_2$, profits increase faster with capacity when the firm bids at c than when it bid at any $b > c$, $\frac{\partial \pi_1}{\partial k_1}(k_1, c) > \frac{\partial \pi_1}{\partial k_1}(k_1, b)$. This shows that (2) is optimal. □

Proof of Lemma 4: The structure is very similar to the one of Lemma 1. For part (i), suppose that the rival randomizes according to the function $\Phi_j(b|c_j)$. As a result, firm i's profits in the uniform-price auction can be written as

$$\pi_i\left(b_i, \Phi_j|c_i\right) = \int_{\underline{c}}^{\overline{c}} \int_b \left[(b_i - c_i)(\theta - k)\Pr(b_i > b) + (b - c_i)k\Pr(b_i \geq b)\right] d\Phi_j(b|c_j) f(c_j) dc_j. \tag{19}$$

Suppose that the highest bid of a firm with marginal cost c_i is b_i^{max}. We now show that a firm with marginal costs $c_i' > c_i$ maximizes profits by choosing $b_i' \geq b_i^{max}$. Suppose this is not the case and a firm with marginal cost c_i' chooses $b_i' < b_i^{max}$. By revealed preference,

$$\pi_i(b_i^{max}, \Phi_j|c_i) - \pi_i(b_i', \Phi_j|c_i) \geq 0 \geq \pi_i(b_i^{max}, \Phi_j|c_i') - \pi_i(b_i', \Phi_j|c_i').$$

Using (19), this is a contradiction, since

$$\frac{\partial\left[\pi_i(b_i', \Phi_j|c_i) - \pi_i(b_i, \Phi_j|c_i)\right]}{\partial c_i} = \int_{\underline{c}}^{\overline{c}} (2k - \theta)\left[\Pr(b_i \leq b)) - \Pr(b_i' \leq b)\right] d\Phi_j(b|c_j) f(c_j) dc_j < 0,$$

where the last inequality is due to the fact that, using Bertrand arguments, Φ_j cannot contain gaps in the support and, therefore, $\Pr(b_i^{max} \leq b) < \Pr(b_i' \leq b)$.

Notice that the previous result implies that each bid can be used by at most one cost realization. That is, bid support used under different cost realizations do not overlap. Suppose now a firm with cost c_i randomizes between two different bids b_i and $\hat{b}_i$ with $b_i < \hat{b}_i$. Using again Bertrand arguments, it has to be that case that all bids in between are also in the randomization support. However, since each capacity arises with probability 0, the firm will always prefer to choose the highest point in the support, $\hat{b}_i$, as the revenues increase but the probability of being outbid is essentially unchanged.

Part (ii) follows directly from the first part of the previous argument. Without randomization, a firm with cost c_i chooses $b_i = b_i^{max}$ and for any $c_i' > c_i$ it has to be the case that $b_i' \geq b_i$. $\square$

Proof of Proposition 7: Using the profit expression in (5) we obtain the following first order condition that characterizes the optimal bid of firm i

$$\frac{\partial \pi_i}{\partial b_i} = \int_{\underline{c}}^{b_j^{-1}(b_i)} (\theta - k) f(c_j) dc_j - b_j^{-1\prime}(b_i) f(b_j^{-1}(b_i))(2k - \theta)(b_i - c). \tag{20}$$

Under symmetry, $b_j(c) = b_i(c)$. Accordingly, we can rewrite the expression as

$$b_i'(c_i) + a(c_i) b_i(c_i) = a(c_i) c_i,$$

where

$$a(c_i) \equiv -\frac{2k - \theta}{\theta - k} \frac{f(c_i)}{F(c_i)}.$$

Note that a sufficient condition for $a(c_i)$ to be upward sloping is that f is log-concave. In this case, the optimal bid is concave.

Solving for $b_i(c_i)$ and using the fact that $b_i(\bar{c}) = P$ we obtain

$$b_i(c_i) = c_i + (P - \bar{c}) F(c_i)^{\frac{2k-\theta}{\theta-k}} + \int_{c_i}^{\bar{c}} \left(\frac{F(c)}{F(c_i)} \right)^{-\frac{2k-\theta}{\theta-k}} dc.$$

$\square$

Proof of Proposition 8: Expected profits in the discriminatory auction can be written as

$$\pi_i (b_i, b_j | c_i) = (b_i - c_i) \left[\int_{\underline{c}}^{b_j^{-1}(b_i)} (\theta - k) f(c_j) dc_j + \int_{b_j^{-1}(b_i)}^{\bar{c}} k f(c_j) dc_j \right], \tag{21}$$

leading to the following first order condition that characterizes the optimal bid of firm i

$$\frac{\partial \pi_i}{\partial b_i} = k \left[1 - F\left(b_j^{-1\prime}(b_i) \right) \right] + (\theta - k) F\left(b_j^{-1\prime}(b_i) \right) - (b_i - c) b_j^{-1\prime}(b_i) f(b_j^{-1}(b_i))(2k - \theta). \tag{22}$$

Under symmetry, $b_j(c) = b_i(c)$. Accordingly, we can rewrite the expression as

$$b_i'(c_i) + a(c_i) b_i(c_i) = a(c_i) c_i,$$

where

$$a(c_i) \equiv -\frac{(2k - \theta) f(c_i)}{k(1 - F(c_i)) + (\theta - k) F(c_i)}.$$

Solving for $b_i(c_i)$ and using the fact that $b_i(\bar{c}) = P$, we obtain

$$b_i(c_i) = c_i + (P - \bar{c})\frac{\theta - k}{k(1 - F(c_i)) + (\theta - k)F(c_i)} - \int_{c_i}^{\bar{c}} \frac{k(1 - F(c)) + (\theta - k)F(c)}{k(1 - F(c_i)) + (\theta - k)F(c_i)}dc.$$

□

Proof of Proposition 9: Denote with the subindex d and u the profits under the discriminatory and the uniform-price auction, respectively. Define $\Pi_s(c_i) = \pi_i(b^*, b^*|c_i)$ for $s = u, d$.

Notice that $\Pi_d(\bar{c}) = \Pi_u(\bar{c}) = (P - \bar{c})(\theta - k)$. Furthermore, for all c_i

$$\frac{d\Pi_u(c_i)}{dc_i} = kF(c_i) = \frac{d\Pi_d(c_i)}{dc_i}.$$

Hence, profits are the same for all values of c_i, meaning that the expected payment is also the same for both auction formats. □

Proof of Lemma 5: It follows similar steps as the proof of Lemma 2. See also Fabra et al. (2006). □

Proof of Lemma 6: It follows similar steps as the proof of Lemma 3. See also Fabra et al. (2006). □

Proof of Proposition 10: It follows from combining the result of Proposition 7 and Lemmas 5 and 6. □

Proof of Proposition 11: Applying the same arguments as in Lemma 1, the optimal price offer of a firm has to be decreasing in k_i. This implies that at $k_i = \underline{k}$, for a given b_i the firm makes expected profits $(b_i - c)\min\{D(b_i) - E[k], k_i\}$. Notice that $D(b_i) - E[k] < D(b_i) - \underline{k} < D(c) - \underline{k} < \underline{k}$, where the last inequality follows from the assumption $D(c) < 2\underline{k}$. This means that at $\underline{k}$ the firm can cover all the market and the optimal bid can be expressed as

$$b_i(\underline{k}) = \arg\max_{b_i} (b_i - c)(D(b_i) - E[k]).$$

By the log-concavity of $D(b)$, the resulting bid is uniquely defined using the standard inverse elasticity rule as

$$\frac{b_i(\underline{k}) - c}{b_i(\underline{k})} = \frac{1}{\epsilon(b_i(\underline{k}))},$$

where $\epsilon(b_i(\underline{k}))$ is the price-elasticity of the residual demand $D(b_i) - E[k]$ at a price $b_i(\underline{k})$. Note that the higher the demand elasticity, the lower the highest bid that firms offer in equilibrium.

Since the optimal bid is decreasing in capacity, we first note that in a symmetric equilibrium it has to be the case that $k_i > D(b_i) - b_j^{-1}(b_i)$. Towards a contradiction, suppose not. Then, $k_i < D(b_i) - b_j^{-1}(b_i) = D(b_i) - k_i$. Since firms never choose prices below marginal cost, it follows that $2\underline{k} \leq 2k_i < D(b_i) \leq D(c)$, which contradicts our initial assumption $D(c) < 2\underline{k}$. Hence, we only need to consider cases where $k_i > D(b_i) - b_j^{-1}(b_i)$, implying that a firm always has enough capacity to satisfy the residual demand if it turns out to be the high bidder.

Also notice that it is never optimal to chose a price-quantity pair (b_i, q_i) such that $D(b_i) < q_i = k_i$. In this case, the firm could increase its profits by withholding output to $q_i = D(b_i) < k_i$. This would not restrict its ability to serve the residual demand $D(b_i) - k_j$ if it turns out to be the higher bidder, but it would drive up the price from b_i to $b_j(k_j)$ if, instead, it is the low bidder. Hence, the expected profits can be expressed as

$$\pi_i(b_i, q_i | b_j, k_i) = \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j) - c) q_i g(k_j) dk_j + \int_{b_j^{-1}(b_i)}^{\overline{k}} (b_i - c)(D(b_i) - k_j) g(k_j) dk_j$$

where $q_i = \min\{D(b_i), k_i\}$.

If $q_i = D(b_i)$, profits do not depend on k_i. Hence, the optimal bid must be independent of k_i. Bertrand arguments rule out that such bid is greater than c as firms would have incentives to undercut it. It follows that the optimal bid is $b_i(k_i) = c$ for all $k_i > D(c)$.

Consider now $k_i < D(c)$. For the same arguments as in Lemma 1, the equilibrium does not exhibit withholding, $q_i = k_i$. It follows that the relevant first-order condition can be written as

$$\begin{aligned}\frac{\partial \pi_i}{\partial b_i} &= b_j^{-1\prime}(b_i) g(b_j^{-1}(b_i))(b_i - c)(k_i + b_j^{-1}(b_i) - D(b_i)) \\ &+ \int_{b_j^{-1}(b_i)}^{\overline{k}} (D(b_i) + D'(b_i) b_i - k_j) g(k_j) dk_j = 0.\end{aligned}$$

Applying symmetry, the optimal bid is the solution to

$$\frac{\partial \pi_i}{\partial b_i} = \frac{1}{b_i'(k_i)} g(k_i)(b_i(k_i) - c)(2k_i - D(b_i)) + \int_{k_i}^{\overline{k}} (D(b_i) + D'(b_i) b_i - k_j) g(k_j) dk_j = 0. \quad (23)$$

This expression is decreasing in the slope of the demand function, which enters into the integral. This implies that the optimal bid that solves (23) is lower than the optimal bid that solves the analogous first order condition for inelastic demand case, (13). The difference is greater the flatter demand. Since a flatter demand also implies, all else equal, a higher demand elasticity, lower equilibrium price offers are associated with more elastic demand functions.

Altogether, if $\overline{k} < D(c)$ there is never withholding, $q_i = k_i$, and the optimal price offer is equal to the solution to (23). The optimal price offer at $\overline{k}$ must equal marginal cost. Since the second term in (23) cancels out, the first term is also zero when $b_i(\overline{k}) = c$.

If $\overline{k} > D(c)$ there is no withholding for capacity realizations up to $D(c)$, with $q_i = k_i$ and the optimal price offer given by the solution to (23). Instead, for all capacity realizations $k_i > D(c)$, there is withholding to $q_i = D(c)$, and the optimal price offer is equal to marginal cost. $\square$