Long-Term Care Needs: Implications for Savings, Welfare, and Public Policy

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Abstract

Contrary to the predictions of standard life cycle models, individuals dissave slowly during retirement. I address this puzzle by investigating the role of long-term care (LTC) needs as a determinant of the savings decisions of the elderly and quantify their importance relative to alternative explanations such as medical expenses and bequests. For this purpose, I develop and estimate a model for retired single individuals who are heterogeneous in their access to informal care and make an optimal choice of care hours bought in the market. In order to take into account heterogeneity in both LTC needs and survival probabilities, I model LTC needs using a dynamic latent variable model that summarizes the rich information contained in health surveys into four parsimonious health groups. The main result is that LTC is a key driver of savings for the high-income elderly and significantly more important than bequest motives and medical expenses. In addition, the model highlights the inefficiencies of means-tested LTC programs and shows that 40% of the cross-country variation in dissaving rates can be explained by differences in the provision of LTC by the public sector.

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1 Introduction

Contrary to the predictions of a standard life-cycle model (Huggett, 1996), many elderly dissave slowly during retirement. In OECD countries, the risk of needing long-term care (LTC) is likely a very important driver of savings because it is insured to a lesser extent than medical expenses and it is expensive when paid out-of-pocket. Indeed, simple cross-country correlations suggest that dissaving of the old is more affected by public expenditures in LTC than in medical care. As shown in the left panel of Figure 1, in countries with higher levels of public spending in LTC, the old dissave faster. Higher levels of public spending in medical care, however, are not correlated with the dissaving pattern of the elderly across countries (right panel of Figure 1).

**Figure 1. Dissaving and public expenditure across countries: long-term care (left panel) and medical care (right panel)**

Sources: OECD health statistics, HRS, and SHARE. Long-term care expenditures: in thousands of 2011 current PPPs divided by population over age 65. Medical expenditures: expenditures in curative and rehabilitative care (inpatient and outpatient care) in thousands of 2011 current PPPs divided by total population. Dissaving: one minus predicted median assets at age 85 over predicted median assets at age 70 from median regression of assets against an age polynomial, gender and education dummies.\(^1\)

This paper has three goals: First, it analyzes to what extent LTC needs, defined as

\(^1\)Details on Figure 1 in section 4.2
assistance to perform basic tasks of everyday life, drives the savings decision of individuals late in life; Second, it compares the relative importance of LTC needs over medical expenses and bequest motives. And third, it evaluates the effect of different LTC policies on welfare, redistribution, and savings.

To address these goals, it is crucial to separate LTC needs from actual LTC expenditure choices taken by individuals. For this purpose, using the Health and Retirement Study (HRS) and the methodology in Amengual, Bueren, and Crego (2017), I document that LTC needs can be parsimoniously represented by four latent health states. The four health states summarize the information contained in 12 variables reporting difficulties with Activities of Daily Living (ADLs) and Instrumental Activities of Daily Living (IADLs). Healthy individuals do not need help with daily self-care activities. In contrast, physically and mentally frail individuals are in need of assistance with activities related to mobility and cognition, respectively. Finally, impaired individuals are in need of assistance with both physical and cognitive tasks. The estimated transition probabilities uncover sizeable differences in expected LTC needs across gender and income groups; in particular, females and the poor face significantly larger average duration of their LTC needs.

Based on the previous health classification, I present new empirical facts consistent with the idea that observed LTC expenses reflect choices and hence, cannot be used at face value to measure risks. First, spending on formal care (FC) hours provided by paid helpers increases as health deteriorates. For example, physically frail individuals consume 0.7 hours of FC per day or $5,365 per year, on average. When impaired, the average consumption of FC rises to 2.8 hours per day or $21,460 per year. Given that the median income of a single retired individual is around $16,840, LTC constitutes a significant financial risk faced by the elderly. Second, conditional on health, richer individuals spend significantly more on FC. For example, an individual in the top quartile of the income distribution consumes 1.2 hours more of care per day than an individual in the bottom quartile when impaired. And third, conditional on health, individuals who have greater access to informal care (IC) provided by relatives consume fewer hours of FC.

Motivated by these facts, I develop and estimate a model of single retired individuals
allowing for heterogeneity in both LTC needs and family types, as well as in gender, permanent income, medical shocks, and wealth. Family types differ in the hours of IC provided by children when the old is in need of LTC. Agents in the model derive utility from regular consumption and from care hours. Families provide IC for free and agents can also decide to buy extra care at a market price. The marginal utility of care hours is allowed to differ across agents’ LTC needs so that individuals are able to adjust their LTC expenses. Family type affects the agents’ savings decisions in two important ways. First, agents belonging to families that provide more IC do not need to carry too much wealth late in life for LTC expenses. Second, I allow the intensity of the bequest motive to vary across family types. Finally, agents have the option to access a government means-tested program that provides a consumption floor and some LTC services if necessary.

I use HRS data on savings, reported FC hours, and Medicaid decisions taken by single retirees to estimate the parameters in the model with the method of simulated moments. The estimated model is able to match the pattern of the targeted features of the data as well as other important non-targeted dimensions. The estimated preference parameters imply that as health deteriorates, agents optimally decide to increase the share of spending devoted to FC. For example, an individual in the top quartile of the permanent income (PI) distribution spends around 16%, 33%, and 53% of his consumption in FC hours if physically frail, mentally frail and impaired respectively. Furthermore, the estimated bequest parameters imply that individuals in close families show larger bequest intensity than individuals on their own, who have negligible bequest motives.

Counterfactual simulations show that LTC is a crucial driver of dissavings for the elderly rich. For instance, when I set to zero the utility derived from LTC expenses in all health states, median assets for individuals in the top quartile of the permanent income distribution decline much faster than in the baseline model. By age 90, median assets for this group would be one-third or $50,000 lower. Simulations also show that bequest motives induce individuals receiving IC from their relatives to accumulate assets beyond standard models’ predictions.

Different from previous studies, agents in the model self-insure little for medical expenses. This is because the estimated medical expenses’ process with a dynamic latent
variable model of health implies that, as in the data, people who suffer from large medical costs also face a large probability of dying (Pauly, 1990). In the data, I document that the two-year probability of dying for individuals aged 70 and over is 9.3%. Meanwhile, individuals in the top 90, 95, and 99 percentiles of the medical expenses distribution face a mortality rate of 15.9, 18.2, and 23.2, respectively. Using simulations from my estimated medical expenses process, mortality rates across the distribution of medical expenses are somewhat smaller than in the data but very much in line with them (12.3, 13.2, and 16.3, respectively). In contrast, when using two health groups based on self-reported health as in De Nardi, French, and Jones (2010) one finds very low mortality rates (10.3, 10.8, and 11.3, respectively) or in case it is considered to be health independent as in Kopecky and Koreshkova (2014) (9.3 for all quantiles). Thus, the positive correlation between medical expenses and mortality rates limits the underlying financial risk that agents face.

Finally, the estimated model highlights two important features related to public policy. First, I show that current means-tested programs are inefficient. Adding small public subsidies to the purchase of private LTC increases the welfare of all individuals and is self-financing because it encourages the savings of the poor who want to avoid to rely on Medicaid. More precisely, a 4.5% subsidy in the price of formal care is revenue neutral, reduces the progressivity of government transfers, and increases welfare for both the rich and the poor. Second, I evaluate the effect of a “Medicaid for all” reform (i.e. expand Medicaid LTC services to the whole population by eliminating eligibility criteria) on savings behavior. In particular, I vary the hours of care provided by Medicaid to match public expenditures in LTC in 10 European countries. The model implies that 40% of the cross-country variation can be explained by differences in the provision of LTC by the public sector.

This paper is related to the literature on the importance of health expenses (medical and LTC) on the savings behavior of the elderly. It is the first study to quantify the financial risk implied by LTC needs on the savings pattern of the elderly in a model that incorporates heterogeneous LTC needs and where LTC expenses are endogenous. Furthermore, I provide a new set of estimates on the importance of bequest and medical expenses as drivers of late-in-life dissaving.
Kotlikoff et al. (1989) underline the importance of health expenditures for understanding the lack of dissaving of the elderly. However, Hubbard, Skinner, and Zeldes (1994) and Palumbo (1999) find such expenses to have a small effect. Using more recent data, De Nardi et al. (2010) for the US and Dobrescu (2015) for Europe find health-related expenses to be crucial drivers of savings. My results are consistent with theirs in the sense that health expenditures (medical plus LTC) are an important driver of savings. In addition, my model allows identifying the independent contribution played by medical and LTC expenses. My estimates show that LTC needs are much more important than medical expenses as drivers of the elderly rich savings which is key for addressing effective policies aiming at increasing the welfare of the old.

Kopecky and Koreshkova (2014) estimate the independent contribution of medical and nursing home expenses on aggregate wealth. They find that savings for out-of-pocket health expenses account for 13.5 percent of the aggregate wealth, half of which is due to nursing home expenses. In their model nursing home is an exogenous state where individuals are forced to spend around $35,000 (94% of average earnings) for basic care services plus the level of consumption they choose. In their model, middle-income individuals save more for nursing home expenses than those at the top of the permanent earnings distribution because they face higher expenses relative to their income. Besides, Lockwood (2014), using a model where LTC choices are exogenous, finds bequest motives to be significantly more important than LTC as a driver of savings. Through the lens of his model, in order to match the dissaving pattern of the upper-middle class, a strong bequest motive is necessary. In contrast, I find that the rich spend significantly more on FC when in need of LTC than the poor (in line with the data), which increases late in life risk faced by the elderly upper-middle class and the rich. As a result, the elderly rich save more than the middle class for LTC. Thus, the proposed model estimates a weaker bequest motive than the previous literature.

In a recent work that uses high-quality data on survey answers to hypothetical scenarios, Ameriks, Briggs, Caplin, Shapiro, and Tonetti (2015) show that the marginal utility of consumption increases strongly when in need of LTC; they do not quantify, however, their effect on dissaving behavior. In contrast to their work, I allow for heterogeneity in LTC needs
and the role of the family as important determinant of care. Firstly, Ameriks et al. (2015) only consider one LTC status. The estimated preferences parameters in my model imply that as health deteriorates, individuals derive higher utility from consuming care. Secondly, I allow for heterogeneity in the help provided by the family and its link to bequest motives. Answers to hypothetical scenarios in their data reveal large heterogeneity in the bequest motives which is absent in their modeling strategy. The proposed model is able to predict the large fraction of individuals who are not willing to leave any bequest even at relatively large wealth levels.

Given all the potential LTC arrangements (continued independence, cohabitation, institutional care) and its interaction with care choices, one model cannot accommodate all possible care alternatives. I therefore disentangle regular consumption and LTC choices from LTC needs taking family care as given. In this model individuals in a nursing home will be those receiving relatively little IC from relatives and deriving high utility from care consumption as a result of deteriorated health status. As opposed, a recent strand of the macroeconomics literature analyzes family care arrangements as the outcome of a bargaining process between an elderly parent and her adult child fixing LTC expenses as exogenous across arrangements (Barczyk and Kredler 2017; Fahle 2015; Ko 2017; Mommaerts 2015). By abstracting from children’s problem, I am able to include heterogeneity in LTC transitions across income levels and gender, a persistent shock to medical expenses, and, more importantly, to allow agents to adjust LTC spending. By doing so, I am able to match wealth trajectories for different levels of permanent income.

Finally, this paper is complementary to empirical studies on bequest behavior in response to children’s attention and caregiving. M. Brown (2006) and Groneck (2016) find that end-of-life transfers favor both current and expected caregiver. In line with their results, I estimate that individuals receiving or expecting to receive more IC hold stronger bequest motives.

The rest of the paper is organized as follows. In Section 2, I explain the institutional framework of LTC in the US and I describe facts on LTC choices by level of need. Then, I propose a model that is able to accommodate these facts in Section 3. In Section 4, I present counterfactual experiments to quantify the forces affecting the saving behavior and
investigate the effects of different policies. Section 5 concludes.

2 Long-Term Care in the US

In this section, I explain how LTC expenses are financed in the US. Having described the institutional background, I describe how I identify LTC needs using the HRS data. Finally, given needs, I show how individuals adjust their LTC expenses based on the financial resources and their access to IC.

2.1 Institutional Background

In 2013, formal LTC costs in the US added up to $310 billion which corresponds to around 1.5 percent of GDP. These are financed through two main sources: public through Medicaid or private through out-of-pocket spending. In constrast, Medicare offers very limited coverage for LTC as it covers nursing home stays up to only 20 days following a hospital stay and copayment from day 21 to day 100 ($164.5 coinsurance per day). Besides, Medicare does not cover either health aide services.

Public expenditures on LTC are almost entirely paid by Medicaid. Medicaid is, however, means-tested and therefore only available to impoverished elderly. Medicaid is funded jointly by the federal and state governments. There are two main alternatives for qualifying for Medicaid. First, the “categorically needy” are individuals who are eligible for Supplemental Security Income (SSI). SSI pays a monthly benefit of around $650 for individuals satisfying income and asset test. Although there is some variation across states, individual’s income has to be below $1,400 per month and assets below $2,000. Second, the “medically needy” are individuals with income or assets above these thresholds but who do not have enough resources to cover their medical expenses.

Medicaid offers coverage of both home-based services as well as institutional care. Medicaid only covers basic care needs so that individuals can be reluctant to access it (Ameriks, Caplin, Laufer, & Van Nieuwerburgh, 2011). In the community, Medicaid offers homemaker
(including light housekeeping, grocery shopping or laundry), personal care (assistance with daily routines) and meals (provides meals at their homes or in senior centers) services. In nursing homes, Medicaid covers room, board, and care. By 2014, Medicaid spending on home services was $80.6 billion or 53% of total Medicaid spending on LTC versus 47% on nursing homes.

Since only the impoverished and those who suffer catastrophic medical expenses have access to Medicaid, most individuals pay LTC out-of-pocket. LTC in the US is expensive. The average annual rate for a semi-private nursing home is around $48,000 per year (Stewart, Grabowski, & Lakdawalla, 2009) and the hour of FC is estimated to be around $21 per hour. Given that 75% of the elderly will be in need of LTC and that 40% of the population will enter in a nursing at some point (J. R. Brown & Finkelstein, 2007), LTC expenses is likely one of the largest financial risks faced by the elderly in the US.

Although LTC expenses could thus deplete individuals’ wealth fast, few elderly decide to hold LTC insurance. The literature has identified two important reasons to explain this puzzle and that make LTC expenses to be fundamentally different from medical expenses. First, Ko (2017) and Mommaerts (2015) identify asymmetric information about the availability of IC as a key factor limiting the size of the market. Second, Braun, Kopecky, and Koreshkova (2017) underline high monitoring costs as an important reason to understand why the take-up rate is small even for the affluent.

### 2.2 LTC needs

In order to quantify the effect of LTC expenditures on the the savings behavior of the elderly, I need to separate LTC needs from the actual LTC choices taken by individuals. The HRS contains a wide array of variables about different aspects of elderly’s health. Amengual et al. (2017) propose a latent variable model which exploits both the cross-sectional and the time series dimension of the HRS and classifies individuals based on a discrete number of LTC need groups. Specifically, they exploit the information contained in 12 dummy variables.

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2LongTermCare.gov
that characterize individual’s reported difficulty with ADLs and IADLs. Each variable is equal to 1 if the individual reports difficulty and 0 otherwise. ADLs were proposed by Katz, Ford, Moskowitz, Jackson, and Jaffe (1963) as a measure of how independent a patient is with basic personal tasks of everyday life like being able to get in or out of bed. IADLs, in contrast, consist of activities more closely related to cognition. Examples of the latter include the ability to use a phone or controlling her medication.³

Amengual et al. (2017) assume that the main source of heterogeneity in the population is represented by a finite number of possible health groups that are not observed by the researcher. They consider that each individual belongs to one health group and that health groups differ in the probability of reporting a difficulty with each I-ADLs.⁴ Therefore, individuals within the same health group have the same probabilities of experiencing problems with any I-ADL but these probabilities might vary if individuals belong to a different group. Similarly, the same individual will face a different likelihood of experiencing each I-ADL when she changes groups during her life. Appendix A describes in detail the econometric model.

The estimation of the econometric model shows that variation in LTC needs can be parsimoniously represented by four clearly different health states. The healthy are those individuals whose probability of declaring problems with I-ADLs is close to 0 for every I-ADL. The physically frail have problems with ADLs but low probabilities with IADLs. The mentally frail have problems mainly with activities related with cognition. Finally, the impaired show problems with both cognitive and physical activities.

For each individual in the HRS, I can compute \( p(h_{it}|x_{iT}) \), a \( 4 \times 1 \) vector which contains the smoothed probability that each individual \( i \) belongs to each health group \( h \) at any point in time \( t \) where \( x_{iT} \) represents individual’s past, current, and future information on I-ADLs and potential death events. In Appendix B, I describe in detail how smoothed probabilities are computed based on estimated parameter values. \( p(h_{it}|x_{iT}) \) represents the health measure

³ADLs: Some difficulty with dressing (DRESS), using the toilet (TOILET), bathing (BATH), getting in or out of bed (BED), to walk across a room (WALK) and eating (EAT). IADLs: Some difficulty with preparing a hot meal (MEALS), shopping for groceries (SHOP), managing money (MONEY), taking medications (MEDS), using a phone (PHONE), and using a map (MAP).

⁴I use I-ADLs to refer to both ADLs and IADLs
that I use throughout the paper.

The proposed measure of LTC has two main advantages with respect to others used in the previous literature. First, the measure relies on LTC needs indicators and not on observed LTC choices. De Nardi, French, and Jones (2016), Kopecky and Koreshkova (2014), Lockwood (2014) or Barczyk and Kredler (2017) rely on endogenous LTC measures (for example, nursing home utilization or hours of care). These assumptions might imply large discrepancies between actual and modeled LTC risks. For example, if richer individuals consumed more care hours than the poor as a pure income effect, using FC hours as a measure for LTC needs would overestimate the LTC risk faced by the rich. The focus on reported difficulties allows to eliminate the bias. Second, the classification maximizes the representativeness of the observed difficulties with I-ADLs observed in the population. Ameriks et al. (2015) consider individuals as in need of LTC if they report one or more difficulties with ADLs. Ko (2017) classifies individuals as healthy if they report none or one difficulty with ADLs, in light need if has two or three and as impaired in case she reports difficulty with more than three ADLs. In contrast, our estimation results show that not all I-ADLs are as informative for predicting need of care and that there is large heterogeneity in the implied LTC needs for individuals reporting the same number of I-ADLs. For example, if a person has difficulties with getting in or out of bed, she belongs to the physically frail group with a probability higher than one-third and to the mentally frail with probability less than 5%. On the other hand, an individual incapable of eating is much more likely to belong to the impaired rather than to the physically frail group.

In order to exploit future and past individuals’ information, health states, transitions, and survival probabilities are jointly estimated. For this purpose, I assume that transitions are logistic functions of age, gender, PI quartile, and current health status (Markov). Table 1 summarizes expected LTC needs across groups by displaying average duration of each health status at age 70 by PI and gender. The first column is the sum of average duration across all possible status or individual’s life expectancy. The table shows large differences in life expectancy. A female in the top of the income distribution expects to live 7 more years than a man at the bottom of the income distribution. Furthermore, there are large differences in
healthy life expectancy across PI groups. Richer individuals, in spite of living longer, spend shorter periods of time in need of LTC. Females live on average 3.3 more years than men and of these, 2 years are spent in need of LTC.

### Table 1. Expected duration of each health state at age 70

<table>
<thead>
<tr>
<th>Permanent income quartile</th>
<th>Life Expectancy =</th>
<th>Physically frail +</th>
<th>Mentally frail +</th>
<th>Impaired</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Healthy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Males</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottom</td>
<td>10.0</td>
<td>6.4</td>
<td>1.9</td>
<td>0.9</td>
</tr>
<tr>
<td>Lower-middle</td>
<td>11.6</td>
<td>8.1</td>
<td>1.9</td>
<td>0.9</td>
</tr>
<tr>
<td>Upper-middle</td>
<td>12.4</td>
<td>9.2</td>
<td>1.8</td>
<td>0.8</td>
</tr>
<tr>
<td>Top</td>
<td>13.9</td>
<td>11.3</td>
<td>1.4</td>
<td>0.6</td>
</tr>
<tr>
<td>Gradient</td>
<td>3.9</td>
<td>4.9</td>
<td>-0.5</td>
<td>-0.3</td>
</tr>
<tr>
<td>Females</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottom</td>
<td>13.4</td>
<td>7.7</td>
<td>3.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Lower-middle</td>
<td>14.9</td>
<td>9.4</td>
<td>3.1</td>
<td>1.2</td>
</tr>
<tr>
<td>Upper-middle</td>
<td>15.9</td>
<td>10.7</td>
<td>2.9</td>
<td>1.1</td>
</tr>
<tr>
<td>Top</td>
<td>17.0</td>
<td>12.6</td>
<td>2.3</td>
<td>1.0</td>
</tr>
<tr>
<td>Gradient</td>
<td>3.6</td>
<td>4.9</td>
<td>-0.9</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

*Source*: HRS 1998-2014. All individuals in the sample.

*Notes*: Gradient=Top-Bottom. Permanent income quartile compute from distribution of non-asset income across waves.

Females live longer because conditional on health status, they face larger chances of surviving. Estimated transition parameters imply large differences in the two-year probability of survival across health status. Being impaired is an important predictor of mortality: the two year-mortality rate is around 2.5 times larger for the impaired than for the healthy group. Differences in life expectancy across PI groups are mainly driven by:

- Differences in health already present at age 70.
- Conditioning on surviving, poor individuals show higher probabilities of moving to a health group with higher mortality during first years after retirement.
- Richer individuals have larger probabilities of recovery from a LTC need status.
2.3 LTC Choices

In this section, I provide evidence that individuals adjust their LTC expenses depending on their needs, financial resources and help provided by relatives. For this purpose, I use the HRS helpers files which contain information about hours of care and the identity of the caregiver. I restrict the analysis to singles. I classify care as IC if the caregiver is a relative or a friend and as FC in case she is a paid helper or a professional. I compute statistics of care hours by health status based on the previous health classification. To do so, I construct 10,000 bootstrap samples using \( p(h_{it}|x_i^{T_t}) \), compute statistics for each sample and report the statistic’s average across samples. Thus, a given individual in a given wave might have different health status across simulated samples.

The last row of Table 2 shows that FC consumption increases as health deteriorates. The physically frail show an average consumption of FC of 0.7 hours per day. Then, the mentally frail seem to be in slightly higher need since they consume 1 hour per day. Finally, the impaired are the ones showing the largest consumption of FC with 2.8 hours. These correspond to around $5,365 per year (or 24% of the observed average non-asset income), $7,665 (or 35% of the observed average non-asset income) and $21,462 (or 100% of the observed average non-asset income) when physically frail, mentally frail and impaired, respectively.

Furthermore, the top panel in Table 2 shows that individuals adjust their level of LTC spending to their available financial resources as the rich consume more care than the poor. Besides, differences across income groups widen as health deteriorates. Physically frail individuals in the top PI consume 0.1 more hours of FC than those in the bottom. If impaired, individuals in the top consume 1.3 more hours of care.

Moreover, I present evidence that care provided by relatives alleviates significantly expenses when in need of LTC. Before doing so, I summarize the observed heterogeneity in IC hours provided by relatives in three types of families in the population. For this purpose, I split the distribution of IC hours into three equally likely groups by health status:

- Individuals belonging to close families are at the top tercile of the IC hours distribution.
- Individuals belonging to distant families are at the middle tercile of the IC hours distribution.
Table 2. Average formal care hours

<table>
<thead>
<tr>
<th></th>
<th>Physically frail</th>
<th>Mentally frail</th>
<th>Impaired</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>By permanent income quartile</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottom</td>
<td>0.8</td>
<td>1.0</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>(0.1)</td>
<td>(0.1)</td>
<td>(0.2)</td>
</tr>
<tr>
<td>Second</td>
<td>0.6</td>
<td>0.9</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td>(0.1)</td>
<td>(0.1)</td>
<td>(0.2)</td>
</tr>
<tr>
<td>Third</td>
<td>0.5</td>
<td>1.2</td>
<td>2.7</td>
</tr>
<tr>
<td></td>
<td>(0.1)</td>
<td>(0.1)</td>
<td>(0.2)</td>
</tr>
<tr>
<td>Top</td>
<td>0.9</td>
<td>1.3</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>(0.1)</td>
<td>(0.2)</td>
<td>(0.3)</td>
</tr>
<tr>
<td><strong>By family type</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>On your own</td>
<td>1.4</td>
<td>2.1</td>
<td>4.9</td>
</tr>
<tr>
<td></td>
<td>(0.1)</td>
<td>(0.2)</td>
<td>(0.3)</td>
</tr>
<tr>
<td>Distant</td>
<td>0.3</td>
<td>0.9</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>(0.0)</td>
<td>(0.1)</td>
<td>(0.3)</td>
</tr>
<tr>
<td>Close</td>
<td>0.3</td>
<td>0.5</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>(0.0)</td>
<td>(0.1)</td>
<td>(0.1)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>0.7</td>
<td>1.0</td>
<td>2.8</td>
</tr>
</tbody>
</table>


- Individuals on their own are at the bottom tercile of IC hours distribution.

Table 2 shows average FC hours by family type. Controlling by health status, individuals with closer relatives, significantly reduce their consumption of FC hours. The presence of close relatives therefore, reduces spending when in need of LTC. Individuals on their own purchase between 3 and 4 times more care than those who belong to close families.

3 The Model

Motivated by the previous section, I build a structural model that can reproduce the main features of the data: (i) as health deteriorates, individuals increase their consumption of
FC hours, (ii) richer individuals consume more care hours, and (iii) individuals with higher access to IC, consume less FC.

The model follows closely De Nardi et al. (2010) but at the same time incorporates new ingredients to match LTC choices observed in the data. First, agents in the model suffer LTC need shocks that affect individuals’ marginal utility of care consumption. Second, there are two types of care: FC provided by paid helpers and IC provided by families. Agents are heterogeneous over family types and decide FC consumption taking as given the decision of their relatives. Family types differ in hours of IC provided to the elderly in need of LTC. Thirdly, I allow the marginal utility of leaving bequests to depend on the family type.

Preferences.— Agents start their life at age $a = 70$ and live at most 100 years old. Every period lasts for two years of time: $a \in \{70, 72, ..., 100\}$. Individuals derive utility from regular consumption and care hours. Individual’s utility depend on health status $h$ which can take five values: healthy ($h = 1$), physically frail ($h = 2$), mentally frail ($h = 3$), impaired ($h = 4$) and dead ($h = 5$). The marginal utility of consuming care hours is allowed to vary depending on health. Furthermore, individuals when in need of LTC ($1 < h < 5$), receive IC hours ($l_{ic}$) depending on their family type ($F$) and their health status. There are three types of families in the model:

1. When the elderly are on their own ($F = 0$), children provide little/no care.
2. In distant families ($F = 1$), children provide moderate IC when their parents are in need of LTC.
3. In close families ($F = 2$), children provide intense IC when their parents are in need of LTC.

Each period their utility flow is given by,

$$u(c, l_{fc}; h, F) = \frac{c^{1-\sigma}}{1-\sigma} + \mu(h)^{\nu} \left[ l_{fc} + \omega \cdot l_{ic}(h, F) \right]^{1-\nu}$$

where, $c$ is regular consumption expressed in dollar values, $l_{fc}$ is FC hours. $\mu$ is the LTC needs shifter, which affects the marginal utility of consuming care hours when individuals
have difficulties with I-ADLs. \( \omega \) is an equivalence scale that maps IC into FC hours. \( \sigma \) and \( \nu \) are the risk aversion parameters of regular consumption and care hours, respectively.

When the person dies, individuals derive utility from leaving bequest following:

\[
\phi(k; F) = \lambda(F)^\sigma \frac{(k + \delta)^{1-\sigma}}{1 - \sigma},
\]

where \( \delta \) captures the extent to which bequests is a luxury good or a necessity and \( \lambda(F) \) captures the intensity of the bequest motive. Notice that I allow \( \lambda \) to be family specific so that different family types might differ over the preferences for leaving bequests.

**Health and medical expenses uncertainty.** — Health and survival probabilities vary across gender, permanent income quartile, age, and current health status (Markov). Moreover, individuals face uncertainty in out-of-pocket medical expenses (\( m \)). I follow French and Jones (2004) and model log health costs as the sum of a white noise process and a persistent AR(1). I allow the variance of the transitory component to be dependent on health status to capture the large dispersion in data when individuals are in need of LTC.

\[
\ln m_{i,t} = m(h, a, s, PI) + \psi_{i,t}(h) \quad (3)
\]

\[
\psi_{i,t}(h) = \xi_{i,t} + \zeta_{i,t}(h), \; \zeta_{i,t}(h) \sim N(0, \sigma^2_{\zeta}(h)) \quad (4)
\]

\[
\xi_{i,t} = \rho \xi_{i,t-1} + \epsilon_{i,t}, \; \epsilon_{i,t} \sim N(0, \sigma^2_{\epsilon}) \quad (5)
\]

Therefore in the model, health and medical expenses are considered as shocks. A different approach, based on Grossman (1972), is to consider medical expenses as investment in health (Ozkan 2017; Yogo 2016). However, many studies in the empirical literature have found such effects to be small: Brook et al. (1983), Fisher et al. (2003) or Finkelstein and McKnight (2008).

**Government insurance.** — Agents have the option of using a government program which is means-tested. The cost of using it is that consumer’s wealth is set to zero \(^5\). The government provides a consumption floor (\( c \)) and care hours (\( l(h) \)) if the agent is in need of LTC.

\(^5\)In reality, Medicaid has an asset disregard threshold which modal value across states is \$2,000 (see section 2). For simplicity, I set this threshold to zero.
\( h \in \{2, 3, 4\} \). \( G = 1 \) if the consumer chooses to use the program and \( G = 0 \), otherwise. Government transfers are given by:

\[
\max\{0, c + p_{fc}l(h) + m - b - (1 + r)k\},
\]

(6)

where \( p_{fc} \) denotes the market price of an hour of FC.

**Timing, budget constraints.**— I assume that \( b(PI, s) \) is a constant function of gender and PI. At the beginning of the period, the individual has wealth \((1 + r)k_t \geq 0\). \( r \) is the risk-free interest rate on savings. The individual receives non-asset income \( b(PI, s) \), and medical expenses \( m \) are realized. Children provide for free IC to their parents. Then, the individual decides how much to consume \( c \), how many hours of FC to purchase \( l_{fc} \) and how much to save or to access government insurance \( G \). Finally, the health shock hits and individuals who die leave all their assets as bequests. Therefore next period’s assets are given by:

\[
k' = (1 + r)k + b - m - p_{fc}l_{fc}.
\]

(7)

Individuals face a borrowing constraint such that \( a_{t+1} \geq 0 \).

**Solution method.**— To save on state variables, I redefine the problem in terms of cash in hand, \( x \):

\[
x = (1 + r)k + b(s, PI) - m.
\]

(8)

Given a set of parameter values, I can solve the model numerically by backward induction starting at age \( a = 100 \). We can write the model in recursive form in terms of cash in hand. \( \beta \) represents the discount factor. The value function is given by:

\[
V_a(x, h, \zeta, s, PI, F) = \max_{c, l_{fc}, G} \left\{ u(c, l_{fc}; h, F) + \beta \pi_{h'=5, h, a, s, PI} E_t[V_{a+2}(x', h', \zeta', s, PI, F)] + \beta \pi_{h'=5, h, a, s, PI} \phi(k'; F) \right\}
\]

(9)
subject to

\[ x' = (1 - G) \left[ (1 + r)(x - c - pfclfc) - m' \right] \]  

(10)

\[ G = 1 \iff \begin{cases} c = c_l \\ l_{fc} = l(h) \end{cases} \]  

(11)

3.1 Estimation

I estimate the model with a two step Method of Simulated Moments (MSM) estimator following Gourinchas and Parker (2002) and Cagetti (2003). In the first step, I estimate all the parameters that can be identified out of the model. In the second stage, the remaining parameters are estimated using the model and taking as given the parameters from the first step.

First stage parameters include non-asset income levels, health transitions, hours of care received in each type of family, hours of care provided by Medicaid LTC services and medical expenses. In the second stage, I estimate the set of parameters \( \theta = (\beta, \sigma, \nu, \delta, \omega, \mu, c) \) that minimize the distance between simulated wealth, Medicaid recipiency rates and care hours moments with their empirical counterparts.

3.1.1 First Stage Parameters

*Permanent income.* — I first compute individual non-asset income in each wave. Non-asset income is the sum of Social Security benefits, defined benefit pension benefits, and annuities. Since in the model agents have access to social insurance, I do not include means-tested government transfers such as Supplemental Security Income or food stamps. I, then, compute permanent income as the average non-asset income across waves for each individual in which she is observed. Finally, I split the distribution into four quartiles of PI. Each individual in the simulation is given the median non-asset income by gender of the PI quartile to which she belongs. Table 3 shows that median annual income ranges from around $8,000 per year in the bottom PI to $32,000 per year in the top PI. The representative female earns on
Table 3. First stage parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-asset income</td>
<td>b(s, PI) Permanent income quartile Females: $8,150/$13,000/$18,050/$28,800 (bottom/second/third/top) Males: $8,550/$13,300/$18,750/$32,250</td>
<td></td>
</tr>
<tr>
<td>Care hours by health status per day</td>
<td>l_c(h, F) Provided by families On your own: 0.0/0.1/0.1 (physically/mentally/impaired) Distant: 0.7/1.3/1.3 Close: 4.7/7.4/9.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>l(h) Provided by the Government 1.0/1.2/ 2.6 (physically/mentally/impaired)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Estimated from HRS sample 1998-2014. Only single individuals in the sample. All monetary values are expressed in 2015 dollars per year. Statistics conditional on latent health status or family type are averages across 10,000 bootstrap samples.

average between 5% and 10% less than the median male by PI quartile.

Medical expenses. — The health expenditure model is estimated using HRS data 1996-2014. A major difference with (De Nardi et al., 2010), is that health in my setup is latent. For this reason, I resort to a Gibbs sampler which includes a health sampling block. Details on the estimation procedure can be found in Appendix C. I drop from the sample individuals living in nursing homes or in Medicaid since LTC expenditures and government transfers are modeled explicitly in the model. I estimate jointly the mean and variance of log-medical expenditures. The mean is modeled as a age, age square, sex, PI ranking dummies, health dummies and PI interacted with health.

To better interpret the estimates, I simulate medical expenses histories based on artificial individuals. Figure 2 shows average medical spending as individuals age for different PI. Medical expenditures increase fast as health deteriorates from around $4,500 every two years at age 70 to around $7,500 at age 96. Relatively poorer individuals, in spite of being in worse health, tend to spend less on medical expenditures with differences widening as individuals age. Females spend similar amounts in their 70’s and around 30% more in their 90’s. Figure 3 shows average medical expenditures for healthy and impaired individuals. We observe that conditional on health status, age has a very small effect on medical spending. Therefore, the
secular increase in medical expenditures over age is driven by worsening health of survivors.

Table 4 shows estimates of the persistent component and the transitory component. The variance of the transitory component increases as individual’s health deteriorates. Furthermore, the persistent component has an autocorrelation coefficient of 0.93 so that innovations to the persistent component have long lived effects.

*Hours of care provided by the family and by Medicaid.* — IC hours by family type and FC hours by health status for individuals in Medicaid are set at their average in the HRS data. These are the ones I use in the estimation. Table 3 shows that as health deteriorates, Medicaid and relatives provide an increasing amount of care.

### 3.1.2 Second Stage Moments

*Empirical wealth moments.* — I select single retired individuals who were interviewed in 1998. Wealth moments track the evolution of wealth over time as members of the sample age. I split the sample into 4 6-year birth cohort by PI and by family type. The age ranges of these
Figure 3. Two Year Average Medical Expenses in Thousands by permanent income quartile. Females. Left Panel: Healthy; Right Panel: Impaired

Notes: Estimated from HRS sample 1998-2014 excluding nursing home residents and individuals in Medicaid.

Table 4. Medical expense process

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Autocorrelation, persistent part</td>
<td>0.93 (0.01)</td>
</tr>
<tr>
<td>$\sigma^2_\epsilon$</td>
<td>Innovation variance, persistent part</td>
<td>0.07 (0.01)</td>
</tr>
</tbody>
</table>

Innovation variance, transitory part:

- $\sigma^2_\zeta(h = 1)$ Healthy: 0.98 (0.01)
- $\sigma^2_\zeta(h = 2)$ Physically frail: 1.09 (0.03)
- $\sigma^2_\zeta(h = 3)$ Conginitively frail: 1.32 (0.05)
- $\sigma^2_\zeta(h = 4)$ Impaired: 1.52 (0.07)

Notes: The table provides the median of the posterior distribution of each parameter. Standard deviation of the posterior distribution in parentheses. Estimated from HRS sample 1998-2014 excluding nursing home residents and individuals in Medicaid. Couples and single individuals in the sample.
cohorts in 1998 are 70-75, 76-81, 82-87, and 88-92. For each cohort and PI/family type, I compute the median wealth in each wave from 1998 to 2014. Thus there are 128 wealth moments (4 PI $\times$ 8 waves $\times$ 4 cohorts) plus 96 wealth moments (3 family types $\times$ 8 waves $\times$ 4 cohorts)\(^6\). Each cohort’s wealth moments trace the evolution over time of the median wealth of its surviving members. Later waves contain fewer people due to death. Of the 2,958 individuals in the sample in 1998, 375 are still alive in 2014.

Medicaid and FC hours moments.— The empirical LTC moments are FC hours described in section 2.3 and Medicaid recipiency rates by health, PI, and family type. Thus, I have 21 moments for FC hours and 28 moment for Medicaid recipiency rates.

3.2 Simulation Procedure

I simulate a large number of artificial individuals. Each of these individuals is endowed with a value of the state vector \((a, s, k, b, h, \zeta, \xi, F)\). \((a, s, k, b)\) are drawn from the data distribution in 1998. \(\zeta\) and \(\xi\) are Monte Carlo draws from discretized versions of the estimated shock processes.

Sampling family types.— Following the empirical section, individuals in close families, distant families or on their own belong to the top, middle or bottom tercile of the IC distribution, respectively. Two complications arise when assigning families to individuals: first, individuals showing no difficulties with I-ADLs do not report IC and second individuals might report a different amount of IC such that they might change from one tercile to another one across waves. To deal with these two issues, I proceed as follows:

1. Individuals who report only one type of family are given their observed family type. For example, consider an individual that has no difficulties with I-ADLs from 1998 to 2004. In 2006, she is impaired and she reports belonging to a close family (top tercile of the IC distribution for impaired individuals). In 2008, dies. In such instances, I consider these individuals belonging to a close family with probability 1.

2. For individuals who report more than one family type (25% of individuals with observed

\(^6\)I drop cells with less than thirty observations
family type), I take a random draw based on the observed frequency. For example, consider an individual belonging to a distant family in one wave and to a close family in the following wave. I, thus, assign this individual to a close family with 50% probability and to a distant family with 50% probability.

3. For individuals for which IC hours is missing, the probability of belonging to each family type is estimated based on individuals specific covariates. Using individuals in cases 1 and 2, I run an ordered logit model of the family type against time invariant covariates that have been identified on the literature as determinants of IC.7

Sampling health status.— First, following De Nardi et al. (2010), the simulation uses each individual’s survival history in 2000-2014 to ensure that individuals contribute to the same wealth moments in the simulation as in the data. This protects against the mortality bias arising from the fact that richer individuals face larger chances of survival than in the parsimonious health model. Second, in order to ensure that the simulated health draws have the same persistence as the estimated health process, I use the Kim smoother proposed by Kim (1994) (Appendix B derives the smoother equations).

Procedure.— Given a guess for my parameter vector \( \theta \), I solve the model using value function iteration. This yields a set of policy function which allows me to simulate for each artificial individual her savings decision, consumption, FC hours consumed and whether she access Medicaid. The optimal choice of \( \hat{\theta} \) is the solution to the criterion function:

\[
\hat{\theta} = \arg \min_{\theta} (m_{\text{data}} - m_{\text{sim}}(\theta)) W (m_{\text{data}} - m_{\text{sim}}(\theta))'
\]  

(12)

The estimation of \( \theta \) is based on 273 moment conditions. The weighting matrix (W) used in the estimation is the inverse of the diagonal of the estimated variance-covariance matrix of the moment conditions (Altonji and Segal (1996) show potential biases when using the optimal weighting matrix). More precisely estimated moments receive greater weight in the estimation.

7 covariates: gender, permanent income quartile, number of children, race, religion, marital status (widow, divorced or ever single), education, children education, has daughter.
3.3 Parameter Identification

Before I present the estimation results, I explain how FC and Medicaid moments allows for the identification of parameters.

Firstly, the model can produce higher dis-saving rates for individuals both with increasing generosity of government insurance or lower bequest motives. By requiring the model to match Medicaid recipiency rates by health status, I am restricting the size of consumption floor and simultaneously identifying the intensity of the bequest motive. Furthermore, as individuals with worse health status are closer to death, differences on Medicaid recipiency rates allows further identification of the bequest motive.

Secondly, from intra-temporal first order conditions, I get the optimal relative share of FC consumption to regular consumption.

\[
\frac{p_{fc} \ell_{fc}}{c} = \mu(h) p_{fc}^{1-1/\nu} c^{\sigma/\nu-1}.
\]

Gradients in FC hours allows me to identify the relative size of both coefficients of risk aversion. If \( \sigma \) is larger than \( \nu \) relatively wealthier individuals (large \( c \)) will spend a larger share of their resources on care. Values of \( \sigma \) and \( \nu \) will be therefore identified by the observed differences on FC hours between the rich and the poor. At the same time, given that these coefficients have to be consistent with the observed wealth trajectories, the size of the risk aversion parameters are identified.

3.4 Estimated Parameter values

Table 5 shows the estimated preference parameters. The estimate of the discount factor \( \beta \) (0.97) is in line with the previous literature. The estimate for \( \sigma \), the coefficient of risk aversion for regular consumption is 4.2. This value is close to typically used in the savings literature. (De Nardi et al., 2010) estimated \( \sigma = 3.8 \) while Ameriks et al. (2015) estimated \( \sigma = 5.6 \). My estimate of \( \nu \), the coefficient of risk aversion for care hours, is 3.0. These estimates imply that relatively rich individuals spend a larger share of their resources on FC
Table 5. Estimated Preference Parameters

<table>
<thead>
<tr>
<th>Homogeneous preferences</th>
<th>Family preferences</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta ): discount factor</td>
<td>( \lambda(F = 0) ): On your own</td>
</tr>
<tr>
<td>0.97 (0.01)</td>
<td>0.06 (0.36)</td>
</tr>
<tr>
<td>( \sigma ): RRA, consumption</td>
<td>( \lambda(F = 1) ): Distant</td>
</tr>
<tr>
<td>4.24 (0.10)</td>
<td>0.62 (0.07)</td>
</tr>
<tr>
<td>( \nu ): RRA, care hours</td>
<td>( \lambda(F = 2) ): Close</td>
</tr>
<tr>
<td>3.03 (0.07)</td>
<td>0.83 (0.06)</td>
</tr>
<tr>
<td>( \delta ): bequest curvature ( \times 10^3 )</td>
<td>( \mu(h = 2) ): Physically frail</td>
</tr>
<tr>
<td>13.2 (1.75)</td>
<td>1.01 (0.11)</td>
</tr>
<tr>
<td>( \zeta ): consumption floor ( \times 10^3 )</td>
<td>( \mu(h = 3) ): Mentally frail</td>
</tr>
<tr>
<td>10.9 (0.10)</td>
<td>2.71 (0.27)</td>
</tr>
<tr>
<td>( \omega ): IC-FC equivalence</td>
<td>( \mu(h = 4) ): Impaired</td>
</tr>
<tr>
<td>0.87 (0.12)</td>
<td>6.55 (0.63)</td>
</tr>
</tbody>
</table>

Health preferences

<table>
<thead>
<tr>
<th>Health preferences</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu(h) ):</td>
</tr>
<tr>
<td>( h = 2 ): Physically frail</td>
</tr>
<tr>
<td>1.01 (0.11)</td>
</tr>
<tr>
<td>( h = 3 ): Mentally frail</td>
</tr>
<tr>
<td>2.71 (0.27)</td>
</tr>
<tr>
<td>( h = 4 ): Impaired</td>
</tr>
<tr>
<td>6.55 (0.63)</td>
</tr>
</tbody>
</table>

Notes: Parameters estimated with method of simulated moments. Weighting matrix is the diagonal of the variance-covariance matrix of the data. Standard errors in parentheses computed using sandwich formulas, taking as given first step estimates.

than relatively the poor. This result is in line with De Nardi et al. (2010) where the authors find that health related expenses (medical and nursing home expenses) to be luxury goods as they are much higher for individuals with higher PI.

The consumption floor is estimated at $10,916 every two years, similar to the value in De Nardi et al. (2016) ($6,670 per year) and relatively lower to the SSI benefit for singles ($7,800 per year).

The estimated care multiplier across different health status imply that, as expected, individuals marginal utility of consuming care increases with deteriorated health conditions.

Strength of the bequest motive. — Following Ameriks et al. (2015), I compare the intensity of the bequest motive implied by my estimated preference parameter to those estimated in lead papers. In Figure 4, I present the optimal bequest allocation that a healthy individual would choose one period before death when calibrated according to each paper’s baseline estimate of risk aversion and bequest parameters. The individual would be solving the following maximization problem:

\[
\max_{c,k} \frac{c^{1-\sigma}}{1 - \sigma} + \lambda \frac{(k + \delta)^{1-\sigma}}{1 - \sigma}
\]

s.t. \( W = c + k \)

(13)
The figure shows that the bequest motive in the model for individuals in close or distant families becomes active around the same level as in De Nardi et al. (2016) and Lockwood (2014). Individuals on their own, on the contrary, do not show any bequest motive. De Nardi et al. (2016) and Lockwood (2014) also use the HRS data for estimating their models. In the HRS, 70% of the sample owns less than $100,000 in assets the period before dying. It is therefore important to notice that all these studies point towards the idea that the intensity of bequest motives is moderate for asset poor individuals.

On the other hand, the estimated bequest motive is relatively closer to Ameriks et al. (2015) for relatively larger wealth levels. In Ameriks et al. (2015), authors estimate preference parameters in their model using strategic survey questions (SSQ) and wealth data for a richer population than the one in HRS. Therefore the estimated bequest intensity at higher wealth level lines up well with preferences parameters from a sample of relatively rich individuals.

Figure 5 show the optimal bequest allocation when individuals are in bad health across different models. Lockwood (2014) has no health dependent preferences so the bequest motive is unchanged. In De Nardi et al. (2010) differences across health status are very modest. In contrast, in Ameriks et al. (2015) and in the proposed model, the bequest motive
becomes active at much larger wealth levels (around $50,000). For relatively higher wealth levels, individuals in the model have lower bequest motives.

Finally, individuals’ answers to SSQ in Ameriks et al. (2015) show that a large fraction of the elderly are not willing to leave any bequest even at large levels of wealth ($100,000-$200,000). In line with this result, individuals on their own hold no bequest motives. Allowing for heterogeneity in the bequest motives, therefore, seems to be important for matching the data.

### 3.5 Model Fit

*Targeted moments.* — Figure 6 and 7 show median assets in the data (solid line) and in the model (dotted line) across PI groups and family types, respectively. The model matches well wealth trajectories in both cases and is able to reproduce the fact that the top PI group dissaves at a lower rate than poorer income groups.

Table 6 shows that the model reproduces the observed increase in FC hours as health deteriorates. In spite of matching difference across income groups, the model overestimates
Figure 6. Median Net Worth by Permanent Income Quartile

Notes: Figure shows data (solid) and simulated (dotted) median assets by permanent income quartile and cohort.

Table 7 shows that the model is also able to reproduce the patterns of Medicaid usage across health status and PI.

Table 6 shows how the model fits FC hours across family types. As in the data, individuals with more access to IC, consume less FC. In general, the model fits the data also quantitatively except individuals on their own where the model slightly under predicts FC hours consumed. Finally, Table 7 shows that the model is able to reproduce the fact that when healthy, the share of individuals in Medicaid is larger for individuals in close families than for individuals on their own. The opposite is true when individuals are impaired. The model tends to exaggerate the differences across family types when individuals are impaired.

Informal validation.— The utility function in the model assumes that regular consumption is not affected by individuals health status. Therefore if this assumption turned out not to be true, the model would be unable to fit the asset decumulation profile for individuals in different health status. For example, if the marginal utility derived from regular consumption decreased in bad health, the model would overestimate the dissaving pattern
Figure 7. Median Net Worth by Family Type

Notes: Figure shows data (solid) and simulated (dotted) median assets by family type and cohort.
### Table 6. Model fit: formal care hours

<table>
<thead>
<tr>
<th></th>
<th>Physically frail</th>
<th>Mentally frail</th>
<th>Impaired</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>By permanent income quartile</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottom</td>
<td>0.8</td>
<td>1.0</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>[0.6]</td>
<td>[1.0]</td>
<td>[2.5]</td>
</tr>
<tr>
<td>Second</td>
<td>0.6</td>
<td>0.9</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td>[0.3]</td>
<td>[0.8]</td>
<td>[2.3]</td>
</tr>
<tr>
<td>Third</td>
<td>0.5</td>
<td>1.2</td>
<td>2.7</td>
</tr>
<tr>
<td></td>
<td>[0.4]</td>
<td>[1.1]</td>
<td>[3.0]</td>
</tr>
<tr>
<td>Top</td>
<td>0.9</td>
<td>1.3</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>[0.9]</td>
<td>[2.1]</td>
<td>[4.5]</td>
</tr>
<tr>
<td><strong>By family type</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>On your own</td>
<td>1.4</td>
<td>2.1</td>
<td>4.9</td>
</tr>
<tr>
<td></td>
<td>[0.9]</td>
<td>[1.7]</td>
<td>[3.7]</td>
</tr>
<tr>
<td>Distant</td>
<td>0.3</td>
<td>0.9</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>[0.5]</td>
<td>[1.2]</td>
<td>[3.3]</td>
</tr>
<tr>
<td>Close</td>
<td>0.3</td>
<td>0.5</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>[0.2]</td>
<td>[0.5]</td>
<td>[1.4]</td>
</tr>
</tbody>
</table>

*Notes:* Table reports data and simulated formal care consumption. Simulated statistics are given in brackets.
Table 7. Model fit: Medicaid recipiency rates

<table>
<thead>
<tr>
<th></th>
<th>Physically Healthy</th>
<th>Mentally frail</th>
<th>Mentally frail</th>
<th>Impaired</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>By permanent income quartile</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottom</td>
<td>39</td>
<td>56</td>
<td>53</td>
<td>74</td>
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<td>[42]</td>
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<td>[79]</td>
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<td>[7]</td>
<td>[10]</td>
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<td>3</td>
<td>8</td>
<td>13</td>
<td>32</td>
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<td>3</td>
<td>5</td>
<td>12</td>
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<td></td>
<td>[5]</td>
<td>[8]</td>
<td>[7]</td>
<td>[12]</td>
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<tr>
<td><strong>By family type</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>On your own</td>
<td>9</td>
<td>23</td>
<td>28</td>
<td>46</td>
</tr>
<tr>
<td></td>
<td>[11]</td>
<td>[24]</td>
<td>[30]</td>
<td>[54]</td>
</tr>
<tr>
<td>Distant</td>
<td>9</td>
<td>19</td>
<td>23</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>[13]</td>
<td>[18]</td>
<td>[24]</td>
<td>[46]</td>
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<tr>
<td>Close</td>
<td>12</td>
<td>25</td>
<td>28</td>
<td>43</td>
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<td></td>
<td>[14]</td>
<td>[26]</td>
<td>[31]</td>
<td>[38]</td>
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</tbody>
</table>

*Notes: Table reports data and simulated medicaid recipiency rates in percentage. Simulated statistics are given in brackets.*
Figure 8. Median Net Worth by Health Status

Notes: Median net worth data (solid) and simulated (dotted) by cohort relative to first wave. Always healthy: individuals sampled as healthy in all periods observed. Switch to LTC need: individuals who are sampled at least once as physically frail, mentally frail or impaired.

of individuals in need of LTC since, in reality, these would be switching expenditures from regular consumption to formal care. To test this assumption, I split individuals in a given cohort between those who are always in good health and those which at any point in time become in need of LTC (physically, mentally or impaired). Figure 8 displays median asset for individuals showing different health trajectories relative to median wealth in the first wave by cohort. The model is able to match the data even if these moments are not included in the estimation procedure.

In my estimation, I target median wealth by PI and family type but no the whole asset distribution. Figure 8 shows that the model can match the reduced dispersion in assets as individuals age. As a cohort grows older, health deteriorates. Individuals with enough accumulated resources dissave strongly to meet their LTC needs and the asset dispersion decreases.
4 Results

In this section I analyze what are the main determinants of savings in retirement. Then, I use the model to conduct a series of policy experiments. More precisely, I compute a welfare cost analysis from changing different features of the current public provision of LTC in the US and their implications for the dissavings pattern.

4.1 Counterfactuals

In order to identify the key mechanism in the model, I run a set of counterfactual scenarios. For this purpose, I fix the estimated parameters at their benchmark and change one feature of the model at a time. I then compute the optimal savings decisions, simulate the model, and compare the resulting asset accumulation profile to the asset profile generated by the baseline model. I display the asset profiles for individuals who were aged 70-75 in 1998. Following (De Nardi et al., 2010) and to focus on the underlying changes in savings, I rule out attrition and simulate health transitions such that individuals live until age 100.
picture with mortality bias is very similar.

4.1.1 How much Do the Elderly Save for LTC needs?

To determine the importance of LTC needs, I simulate the model by setting the utility derived for LTC across health status equal to zero ($\mu(.) = 0$). This exercise would be identical to assuming that individuals remain healthy until death leaving survival and medical expenses uncertainty as in the benchmark model. The left panel of Figure 10 shows that expenses associated with LTC are a big determinant of the elderly savings for individuals at the top PI. The solid lines represent the simulated benchmark model while the dotted line represents the economy without LTC needs. While being healthy, individuals in the benchmark model reduce consumption from the first years after retirement until late in life (around age 90). Then, they dissave strongly by consuming care hours when in need of LTC. For a given level of initial wealth individuals in the top PI would hold 32% fewer assets by age 90 in a world without LTC needs. LTC has also a large impact on the dissaving pattern of individuals in the upper-middle income PI as the median net worth would be around 25% lower by age 90 given the observed asset levels at age 70.

The right panel of Figure 10 displays the same simulation across family types. The figure shows that LTC affects very differently the savings decision depending on how much IC an individual can access. Median net worth for individuals in close families is not affected by LTC needs. Family care constitutes, therefore, a good insurance against LTC needs. On the contrary, individuals on their own are forced to self-insure until very late in life. The model suggests that at the age of 90, around 50 percent of the assets held by these individuals are LTC related precautionary savings. Individuals in distant families need also to hold large amounts of precautionary savings against LTC as their level of wealth would be around 25% lower by age 90.
4.1.2 How much Do the Elderly Save for Medical Expenses?

I now ask whether out-of-pocket medical expenditures are important drivers of old-age savings. To answer this question, I set all out-of-pocket medical expenditures to zero for everyone and look at the corresponding profiles. This could be seen as an extreme form of insurance provided by the Medicare.

Figure 11 shows that medical expenditures play a minor role in explaining why rich individuals do not dissave until very late in life. Individuals in the top PI dissave slightly faster such that they would hold only 5% fewer assets by age 90 in a world without medical expenses.

As discussed in French and Jones (2004), the persistence of catastrophic medical shocks constitutes an important reason why individuals might hold a large amount of assets until late in life. Previous studies have underlined the importance of this persistence, however, little attention has been given to the relation between catastrophic medical expenses and mortality (Pauly, 1990). This relation is important because a positive correlation would bound the risk of persistently high medical expenses. The first row in Table 8 shows the
two-year probability of dying across medical expenses quantiles in the data for individuals aged 70 and over in the HRS. Given that the average two-year probability of dying in the data is 9.3%, the table shows that an individual in the top 5% of the distribution of medical expenses has around 2 times more chances of dying than the average while an individual in the top 1 has around 2.5. Therefore, catastrophic medical expenses are positively correlated with death.

I then compare the empirical correlation with correlation derived from simulating the latent health and medical expenses model. The second row in Table 8 shows that I am able to generate a strong positive correlation between medical expenses and death even if the increase is not as steep as it is in the data. Finally, I compare the empirical correlation with the correlation derived by using other health process used in previous literature. The third and fourth row in Table 8 shows the correlation implied when I use the health variable in De Nardi et al. (2010) (DFJ) and Kopecky and Koreshkova (2014) (KK), respectively. In De Nardi et al. (2010) the authors use a health classification based on two levels of self-reported health while in Kopecky and Koreshkova (2014) all individuals might suffer any
medical shock independent of their health status. The correlation is much weaker when using the self-reported health classification and zero when not using any health covariate. Therefore individuals in these models will overestimate the risk of persistently high medical expenses.

Table 8. Two year probability of dying across quantiles of the medical expenses distribution.

<table>
<thead>
<tr>
<th></th>
<th>90th</th>
<th>95th</th>
<th>99th</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&gt; $6,300</td>
<td>&gt; $9,600</td>
<td>&gt; $25,400</td>
</tr>
<tr>
<td>Data</td>
<td>15.9</td>
<td>18.2</td>
<td>23.2</td>
</tr>
<tr>
<td></td>
<td>(0.5)</td>
<td>(0.7)</td>
<td>(1.9)</td>
</tr>
<tr>
<td>Model</td>
<td>12.3</td>
<td>13.2</td>
<td>16.3</td>
</tr>
<tr>
<td></td>
<td>(0.4)</td>
<td>(0.6)</td>
<td>(1.5)</td>
</tr>
<tr>
<td>DFJ</td>
<td>10.6</td>
<td>10.8</td>
<td>11.3</td>
</tr>
<tr>
<td></td>
<td>(0.4)</td>
<td>(0.5)</td>
<td>(1.3)</td>
</tr>
<tr>
<td>KK</td>
<td>9.3</td>
<td>9.3</td>
<td>9.3</td>
</tr>
<tr>
<td></td>
<td>(0.1)</td>
<td>(0.1)</td>
<td>(0.1)</td>
</tr>
</tbody>
</table>

Notes: DFJ: De Nardi et al. (2010); KK: Kopecky and Koreshkova (2014). Bootstrapped standard errors in parentheses.

4.1.3 On the Strength of the Bequest Motive

I now ask whether the bequests motives that I estimate from the data are important drivers of old-age savings. To answer this question, I set the utility from leaving bequests to zero ($\lambda(.) = 0$) and look at the corresponding profiles.

The left panel of Figure 12 shows that across PI groups, bequest motives are relatively more important for poor individuals. In the economy without bequest motives, the top, upper-middle and lower-middle PI individuals dissave relatively faster such that they hold 9%, 20% and 45% lower wealth at age 90. This fact is surprising given that I estimated bequests motive to be luxury goods. In the absence of bequest motives, relatively poorer individuals have a higher incentives to over consume their financial resources in order to become eligible for Medicaid.

The right panel of figure 12 shows how strong bequests motives are across different
family types. As expected from preferences parameters, the figure shows that the closer the family, the stronger bequest motives are.

4.2 Policy Experiments

In this final section, I use the estimated model to evaluate the impact of two LTC reforms. First, I evaluate the effect of small public subsidies to the price of formal care on government spending, welfare, and redistribution. Second, I analyze changes in the saving behavior of the elderly in case Medicaid LTC services were not means-tested: “Medicaid for all”.

4.2.1 A Pareto Efficient Reform

Since Medicaid LTC services are mean-tested, individuals are encouraged to consume their wealth and income faster in order to become eligible for public aid (Hubbard et al., 1994). By subsidizing the price of FC, poor individuals are incentivized to avoid relying on Medicaid. The first column of Table 9 shows the present discounted value of government benefits at age
Table 9. The cost and benefits of subsidizing the price of formal care by 4.5%

<table>
<thead>
<tr>
<th>Permanent income</th>
<th>Government transfers</th>
<th>Δ Government transfers</th>
<th>Equivalent variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom</td>
<td>37.5</td>
<td>-1.3</td>
<td>3.0</td>
</tr>
<tr>
<td>L-M</td>
<td>15.0</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>U-M</td>
<td>13.0</td>
<td>0.4</td>
<td>1.4</td>
</tr>
<tr>
<td>Top</td>
<td>11.4</td>
<td>0.7</td>
<td>1.8</td>
</tr>
<tr>
<td>Mean</td>
<td>18.0</td>
<td>0.0</td>
<td>1.8</td>
</tr>
</tbody>
</table>

70. It shows that an individual in the bottom quartile of the PI distribution receives around 3.3 times larger benefits than an individual at the top. The second column shows the change in government transfers by PI when a subsidy of 4.5% in the price of FC is introduced. The policy decreases transfers to the poor and increases them for the rich making public LTC expenses more regressive. All in all, and as the second column of the last row shows, the effect is neutral to public expenditure. Subsidies below 4.5% decrease government expenditures below current levels while subsidies above 4.5% increase public expenditures.

To measure the welfare gains introduced by this policy, I compute the equivalent variation; that is the transfer under current system that would leave the retiree as well off as after the reform. More specifically, the equivalent variation ($TR$) at age 70 is computed as:

$$V_{70}(s, k + TR, b, h, \zeta, F; \text{current}) = V_{70}(s, k, b, h, \zeta, F; \text{experiment}).$$

The last column of table 9 shows average equivalent variations for individuals in different PI quartiles. Individuals value the reform at more than its cost for all PI groups as equivalent variation is larger than the increase in present discount value of government transfers. This result implies that current means-tested programs are inefficient as there exists deviations that are Pareto improving for all individuals and that do not increase public expenditures.
4.2.2 Dissaving and Cross-County Public Provision of Long-Term Care

Finally, I assess the model performance at explaining dissaving rates across countries based exclusively on differences in public provision of LTC. In the introduction I showed that in countries with higher levels of public spending in LTC, the elderly tend to dissave faster.

For constructing Figure 1, I rely on the aforementioned HRS plus the Survey of Health, Ageing and Retirement (SHARE). SHARE is a cross-national panel database harmonized with the HRS. I include in the analysis waves 1, 2, 4, and 5 (2004-2013) and restrict to countries already present in the first wave: Austria, Belgium, Denmark, France, Germany, Italy, the Netherlands, Spain, Sweden, and Switzerland. For each country, I have individual-level data on wealth (sum of all wealth components including housing), education\(^8\) and gender.

In a first stage and in order to analyze differences in the dissaving pattern across countries, I ran median wealth regression on a quartic in age and include gender and education dummies. In a second stage, I collected information from OECD health statistics on public long-term care expenditures and medical expenditure in 2011. In order to be able to compare expenditure levels across countries, I converted expenditure levels into 2011 current PPPs and divide long-term care and medical expenditures by the population aged over 65 and by the total population, respectively.

Given that public provision of LTC is universal in European countries, in the model I eliminate the eligibility criteria for LTC services provided in means-tested programs and make public LTC available for all the retired population. I then simulate the benchmark economy adjusting the number of hours provided by the government so that I match public LTC expenditures country by country.\(^9\)

Figure 13 shows the dissaving rate in the data and the one implied by the model. In

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\(^8\)HRS: 5 levels (drop-outs, GED, high-school, some college, and college), age and gender. SHARE: 7 levels based on ISCED classification

\(^9\)I match the relative increase in public long-term care. For example, in the Netherlands public expenditure in LTC per individuals aged 65 or over are 3.4 times larger than in the US. Therefore, I compute the increase in hours provided by a government under universal care such that government transfers for individuals in the simulation are 3.4 times larger than in the benchmark model.
the model and as expected, dissaving increases monotonically with public provision in LTC. Monotonicity is slightly broken by the means-tested nature of the public LTC program in the US where median dissaving is larger than for France and Austria, even though public expenditures are relatively lower. The relation implied between dissaving and LTC expenses is somewhat smaller in the model than in the data. In the data, the dissaving rate from age 70 to 85 increases 2.6 p.p per $1,000 spent in LTC while in the model it is about 1.5 p.p.

Figure 14 shows the relation between the dissaving rates implied by the model and the observed ones. The model is able to explain 40% of the observed variation in dissaving rates in the data. Thus, differences in the public provision of LTC across countries are a key factor for understanding differences in the savings behavior observed across countries.

**Figure 13. Dissaving and public provision of long-term care across countries: data (left panel) and model (right panel)**

*Sources: OECD health statistics, HRS, and SHARE. Long-term care expenditures: in thousands of 2011 current PPPs divided by population over age 65. Medical expenditures: expenditures in curative and rehabilitative care (inpatient and outpatient care) in thousands of 2011 current PPPs divided by total population. Dissaving: one minus predicted median assets at age 85 over predicted median assets at age 70 from median regression of assets against an age polynomial, gender and education dummies.*
Figure 14. Dissaving across countries: data and model

Notes: Dissaving in the data is one minus predicted median assets at age 85 over predicted median assets at age 70 from median regression of assets against an age polynomial, gender and education. Dissaving in the model is one minus median assets at age 85 over median assets at age 70 from simulated individuals.

5 Conclusions

In this paper, I estimate a model of savings for retired single individuals with heterogeneous LTC needs and where LTC expenses are endogenous. It is important to do so for several reasons. First, I show that allowing for different levels of health deterioration is important to measure the risk of being unhealthy for long periods of time. As a result, I find that medical expenses are significantly less crucial as a driver of savings than what previous literature has suggested.

Second, my estimated preferences parameters imply that LTC expenses are luxury goods; that is they are much higher for individuals with more financial resources. Thus, LTC is a more important driver of savings for richer individuals which explains the asymmetry in the dissaving pattern across the income distribution. As a result, I am able to match wealth profiles with weaker bequest motives than previous literature.

Third, I find that access to IC significantly alleviates the financial risk of LTC. However, given that individuals receiving more IC do not show high dissaving rates, the model
estimates a stronger bequest motive for these individuals.

All in all, I find LTC expenses to be the main important channel explaining the lack of dissaving among the affluent. Finally, the model identifies variation in the provision of public LTC as a key channel for identifying differences in the dissaving rates across countries.
References


Kim, C.-J. (1994). Dynamic linear models with markov-switching. *Journal of Econometrics,


Appendix A  Latent Health Model

In this appendix I describe the econometric model used for estimating latent health states and transition probabilities. The model is a slight modification from the original one in Amengual et al. (2017) where transition probabilities differ across permanent income groups.

The HRS is an unbalanced panel of individuals $i = 1, \ldots, N$ followed for $t_i = 1, \ldots, T_i$ periods which correspond from ages $a_{i1}^i$ to age $a_{iT_i}^i$. We consider that an individual $i$ at time $t$ belongs to a latent health group $h_{i,t}$ out of $H$ possible ones. If the individual belonged to group $g$, the probability of reporting difficulties with the $k$'th I-ADL, say $x_{i,k,t} = 1$, is $\iota_{k,g}$.

Under the assumption that I-ADLs are independently distributed conditional on the health status, the joint distribution of $x_{i,t} = (x_{1,i,t}, x_{2,i,t}, \ldots, x_{K,i,t})'$ is characterized by

$$p(x_{i,t}|\iota_g, h_{i,t} = g) = \prod_{k=1}^{K} \iota_{x_{k,i,t}}^{x_{k,i,t}} (1 - \iota_{k,g})^{1-x_{k,i,t}},$$

where $\iota_g = (\iota_{1,g}, \iota_{2,g}, \ldots, \iota_{K,g})'$. We take into account health dynamics by explicitly modeling the transition probabilities across groups. In particular, an individual $i$ at time $t$, with gender $s$ and in PI quantile $b$ who belongs to group $g$ transits to group $c$ with probability

$$\pi_{g,c}(a_{it}, s_i, b_i) = \frac{\exp[f_{g,c}(a_{it}, s_i, b_i)]}{1 + \sum_{c \in \mathcal{H}} \exp[f_{g,c}(a_{it}, s_i, b_i)]}$$

where $\mathcal{H}$ is the set that contains the $H$ health groups. The remaining possible event is that the individual dies, which is an observable state that occurs with probability

$$\pi_{g,D}(a_{it}, s_i, b_i) = \frac{1}{1 + \sum_{c \in \mathcal{H}} \exp[f_{g,c}(a_{it}, s_i, b_i)]}.$$

This specification allows health groups to own distinct dynamics as parameters differ according to the current health group. Moreover, to capture within-group heterogeneity, transition probabilities can depend on age, gender, PI ranking (I split PI distribution in

\footnote{Along the paper I use I-ADLs to denote the set of both ADLs and IADLs}
quartiles, $Q = 4$) and interaction terms through the function $f_{g,c}(a, s, b)$:

$$
 f_{g,c}(a, s, b) = \beta_{1,g,c} + \beta_{2,g,c}a + \beta_{3,g,c}s + \sum_{q=2}^{Q} \beta_{4,g,c,q}(1_{q=b} + \beta_{5,q,c}(a \times s)) + \sum_{q=2}^{Q} \beta_{6,g,c,q}(1_{q=b} \times a) \quad (3)
$$

In practice, I set the number of latent health groups $H = 4^{11}$. Estimation of the econometric model delivers two sets of parameters: $[\hat{\beta}, \hat{\iota}]$. $\hat{\iota}$ shows that individuals are classified as physically frail, mentally frail, impaired or healthy, represent individuals’ LTC needs suitably. Figure A1 shows the probability of reporting difficulty with I-ADLs in each LTC need group. The impaired have physical and cognitive limitations while the healthy have no or light difficulties with I-ADLs. In turn, the physically frail have limited mobility, while the mentally frail have difficulties with more cognitive tasks such as managing money.

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11For details on the estimation procedure and how we select the optimal number of health groups, the reader is referred to the original paper.
Appendix B  Smoothed Probabilities

In this appendix I explain the computation of smoothed probabilities. These are used for computing statistics by health status given our estimates of $\hat{\beta}$ and $\hat{\mu}$. The derivation is split in two parts: the filtered probabilities based on Hamilton (1989) and the smoothed probabilities based on Kim (1994).

Filtered probabilities.— For computing the filtered probabilities, I need first to obtain

$$p(x_{i,t+1}, h_{i,t+1}, h_{i,t} | x_i^t) = p(x_{i,t+1} | x_i^t, h_{i,t+1}, h_{i,t}) \cdot p(h_{i,t+1} | x_i^t, h_{i,t}) \cdot p(h_{i,t} | x_i^t)$$

$$= p(x_{i,t+1} | h_{i,t+1}) \cdot p(h_{i,t+1} | h_{i,t}) \cdot p(h_{i,t} | x_i^t)$$

where $p(x_{i,t+1} | h_{i,t+1})$ is given by equation 1, $p(h_{i,t+1} | h_{i,t})$ is given equation 2 and $p(h_{i,t} | x_i^t)$ is available by recursion. Then,

$$p(x_{i,t+1} | x_i^t) = \sum_{k,l} p(x_{i,t+1}, h_{i,t+1} = k, h_{i,t} = l | x_i^t)$$

I can thus compute the filtered probabilities as,

$$p(h_{i,t+1} | x_i^{t+1}) = \sum_l p(x_{i,t+1}, h_{i,t+1} = h_{i,t} = l | x_i^{t+1})$$

Smoothed probabilities.— I observe,

$$p(h_{i,t+1}, h_{i,t} | x_i^T) = p(h_{i,t+1} | x_i^T) \cdot p(h_{i,t} | h_{i,t+1}, x_i^T) = p(h_{i,t+1} | x_i^T) \cdot p(h_{i,t} | h_{i,t+1}, x_i, t)$$

$$= p(h_{i,t+1} | x_i^T) \cdot \frac{p(h_{i,t+1} | h_{i,t}) \cdot p(h_{i,t} | x_i^t)}{\sum_l p(h_{i,t+1} | h_{i,t} = l) \cdot p(h_{i,t} = l | x_i^t)}$$

Therefore, if we sum over all values of $h_{i,t+1}$, I get my target, $p(h_{i,t} | x_i^T)$.

Sample path for health states, given all the data.— I begin by drawing $h_{i,T}$ from the filtered $p(h_{i,T} | x_i^T)$, I then draw using:

$$p(h_{i,T-1} | h_{i,T}, x_i^T) = \frac{p(h_{i,T-1} | h_{i,T} = l) \cdot p(h_{i,T} | x_i^T) \cdot p(h_{i,T-1} | x_i^{T-1})}{\sum_l p(h_{i,T-1} | h_{i,T-1} = l) \cdot p(h_{i,T-1} = l | x_i^{T-1})}$$ (4)
Appendix C  Medical Expenditures: Estimation

Following French and Jones (2004), I estimate the following model:

\[ \ln m_{it} = X_{it}'\beta + \xi_{it} + \zeta(h)_{it}, \quad \zeta(h)_{it} \sim N(0, \sigma^2_{\zeta(h)_{it}}) \quad (5) \]
\[ \xi_{it} = \rho \xi_{it-1} + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, \sigma^2_{\epsilon}) \quad (6) \]

\( X_{it} \) consists of a polynomial in age, gender, permanent income dummies, health dummies and permanent income dummies interacted with health dummies. Thus the parameter vector to be estimated is  \( \theta = (\beta, \sigma^2_{\epsilon}, \sigma^2_{\zeta(h = 1)}, \sigma^2_{\zeta(h = 2)}, \sigma^2_{\zeta(h = 3)}, \sigma^2_{\zeta(h = 4)}, \rho) \). I aim at the posterior distribution \( p(\theta|X) \). I specify flat priors for all the parameters in \( \theta \). In order to sample from the joint posterior distribution, I enlarge the parameter space to \( (\theta, \xi) \) and apply the following Gibbs algorithm:

- Block 0: sample health status using Kim’s smoother.
- Block 1: \( p(\xi|\beta, \sigma^2_{\epsilon}, \sigma^2_{\zeta}, \rho) \)
- Block 2: \( p(\rho|\beta, \sigma^2_{\epsilon}, \sigma^2_{\zeta}, \xi) \)
- Block 3: \( p(\sigma^2_{\epsilon}|\beta, \sigma^2_{\zeta}, \rho, \xi) \)
- Block 4: \( p(\sigma^2_{\zeta}|\beta, \sigma^2_{\epsilon}, \rho, \xi) \)
- Block 5: \( p(\beta|\sigma^2_{\epsilon}, \sigma^2_{\zeta}, \rho, \xi) \)

Block 0: Using the filtered and transition probabilities from Amengual et al. (2017), sample health status using the Kim Smoother.

Block 1: Given linearity and normality, the Kalman smoother provides the distribution of each state at each time conditional on all available data. To sample the latent state \( \xi_{it} \), I have to simulate from the smoothed states. Given that the simulation smoother is backwards recursion which requires the Kalman filter output, it is convenient to rewrite the process in
the canonical state-space form as:

\[ \xi_{t+1} = F \xi_t + v_{t+1} \]  
\[ y_t = A' x_t + H' \xi_t + w_t \]

with

\[ E[v_t v_\tau] = \begin{cases} 
Q, & \text{for } t = \tau \\
0, & \text{otherwise}
\end{cases} \]  
\[ E[w_t w_\tau] = \begin{cases} 
R, & \text{for } t = \tau \\
0, & \text{otherwise}
\end{cases} \]

where, \( F = \rho, A = \beta, H = 1, Q = \sigma^2_h, R = \sigma^2_\xi(h_{it}). \) The initial condition for the forecast of \( \xi_{1|0} \) based on no observations of \( y \) or \( x \) is set to 0 and the associated mean square error to the unconditional variance: \( P_{1|0} = \sigma^2_h / (1 - \rho^2). \) For \( t > 0 \), we have the standard Kalman filter equations that we need to estimate for each individual at a time:

\[ K_t = FP_{t|t-1}H(H'P_{t|t-1}H + R)^{-1} \]  
\[ \xi_{t+1|t} = F \xi_{t|t-1} + (y_t - A' x_t - H' \xi_{t|t-1}) \]  
\[ P_{t|t} = P_{t|t-1} - P_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}H'P_{t|t-1} \]  
\[ P_{t+1|t} = FP_{t|t}F' + Q \]  
\[ \xi_{t|t} = \xi_{t|t-1} + P_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}(y_t - A' x_t - H' \xi_{t|t-1}) \]

To sample the states, I use Carter and Kohn (1994) algorithm that recursively updates, backwards through time the Kalman smoothed conditional densities of each state given the draw at time \( t + 1 \) following:

\[ \xi_{t|T} = \xi_{t|t} + P_{t|t}F'P_{t+1|t}^{-1}(\xi_{t+1} - F \xi_{t|t}) \]  
\[ P_{t|T} = P_{t|t} - P_{t|t}F'P_{t+1|t}^{-1}FP_{t|t} \]

then draw \( \xi_t \) from \( N(\xi_{t|T}, P_{t|T}). \)
Block 2 and 5 are identical to the problem of drawing from the posterior coefficients in a linear regression model. While, block 3 and 4 are identical to sample the posterior variance in a linear regression model.