Optimal Life Cycle Unemployment Insurance

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Abstract

We argue that US welfare would rise if unemployment insurance were to be increased for young workers and decreased for old. This is because young workers have little means to smooth consumption during unemployment, and want jobs to accumulate high-return human capital. So unemployment insurance is highly valuable to them while the induced moral hazard problem is mild. We consider a life cycle model with unemployment risk and endogenous search effort, that we calibrate to match US labor market institutions. We find that allowing unemployment replacement rates and other government transfers to decline with age yields sizeable welfare gains which amount to ninety percent of the gains attained under the unconstrained optimal scheme for unemployment insurance over the life cycle.
1 Introduction

The principle that government transfers and taxes should be conditioned on observable, immutable indicators of skills goes back at least to Akerlof (1978). More recently Kremer (2001), Erosa and Gervais (2002), Gervais (2004), Farhi and Werning (2013), Gorry and Oberfield (2012), Mirrlees et al. (2010), and Weinzierl (2011) have also stressed the importance of conditioning labor and capital income tax rates on age when designing an efficient tax system. In principle the same logic applies for the optimal design of unemployment insurance and other labor market institutions. Indeed several important economic variables (such as wages, wealth, consumption, and unemployment duration) vary over the life cycle which suggests that workers’ incentive to search for a job as well as their ability to cope with unemployment risk also vary over the life-cycle. Here we argue that, given current US labor market institutions, welfare would rise if unemployment insurance were to be increased for relatively young workers (in their mid twenties and early thirties) and decreased for old workers (in their forties and mid-fifties).

The idea is that unemployment insurance is highly valuable to young workers—because they typically have little means to smooth consumption during an unemployment spell—while the costs of the implied moral hazard problem are mild—because young workers want jobs to improve life-time career prospects, and to accumulate human capital whose marginal return is high when young. The intuition for this claim can be seen using a simple intuitive formula. Consider a government who uses one dollar to finance an increase in the level of unemployment benefits $b_n$ for a given age group $n$. Denote by $\mu_n$ the mass of unemployed workers in the age group, by $c_{un}$ their consumption level when unemployed and by $u'(c_{un})$ the associated marginal utility of consumption. If all currently unemployed workers receive a unit of money, welfare would increase by $\mu_n u'(c_{un})$. But standard moral hazard problems imply that more generous government transfers increase unemployment, and each unemployed worker receives benefits $b_n$. So a marginal increase in government transfers yields only $1/[\mu_n + b_n d\mu_n/db_n] = 1/[\mu_n(1 + \eta_n)]$ units of income to a currently unemployed worker, where $\eta_n$ is the elasticity of group $n$ unemployment to the corresponding unemployment benefits. By multiplying the two terms we find the following welfare gains from the marginal change in government transfers:

$$\varrho_n = \frac{u'(c_{un})}{1 + \eta_n}.$$ 

Intuitively the numerator measures the marginal value of the increase in Unemployment Insurance, the denominator the incentive costs of the induced moral hazard problem. Generally a revenue neutral change in unemployment insurance that increases benefits for a given age group $n$ while decreases them for another age group
$m$ is welfare improving whenever $\varrho_n > \varrho_m$, which can be used to identify possible gains from redistributing unemployment insurance over the life cycle.

To document how $\varrho_n$ varies across age groups, we first use data from the Panel Study of Income Dynamics (PSID) and show that consumption of unemployed workers is strictly increasing in age. Roughly speaking an unemployed worker in his thirties consumes 30 per cent less goods than a unemployed worker in his fifties. We also use data from the Current Population Survey (CPS) and from the Survey of Income and Program Participation (SIPP) to analyze how the unemployment level of different age groups responds to changes in unemployment benefits. As in Chetty (2008) we exploit changes in the level of benefits within US states over time. We find that, while the unemployment elasticity to unemployment benefits is small and statistically insignificant for workers in their mid twenties and early thirties, the elasticity is positive and significant for workers in their mid forties and fifties. Similar results are found by Meyer and Mok (2007). Gritz and MaCurdy (1992) also show that changes in benefits have insignificant effects on the unemployment level of young workers. This evidence indicates that providing additional insurance to young worker is highly valuable, while the incentive costs of the induced moral hazard problem are small, which implies that $\varrho_n$ is unambiguously larger for young than for old workers.

The data also provide more direct evidence that unemployment insurance is highly valuable to young workers and it has small moral hazard costs for them. We show that consumption losses upon unemployment are more pronounced for young than for old workers, and that the search behavior of young workers is strongly responsive to the provision of severance payments at the time of job loss. This indicates that young workers have little ability to smooth consumption during unemployment and require more liquidity and insurance. Chetty (2008) notices that the effects of benefits on the unemployment of wealthy workers—who arguably have great ability to smooth consumption during unemployment—measures the severity of the moral hazard problem. We find that the unemployment duration of old workers with high level of assets is highly affected by benefits, while the unemployment duration of young wealthy workers is little sensitive to benefits. This suggests that the moral hazard problem is severe for old workers while it is minor for young workers. This squares well with the idea that young workers want jobs not only to increase current income net of benefits but also to acquire labor market skills and to improve working life career prospects.

To study the magnitude of the potential welfare gains of age dependent unemployment insurance we consider a conventional life cycle model with decreasing returns to labor market experience and ongoing unemployment risk. Workers are born with no human capital and no assets and can save in a riskless bond. When
employed, they accumulate human capital, they receive wages and pay income taxes that are used to finance the unemployment insurance program and retirement pensions. Workers can lose their job and when unemployed they choose how intensively to search for a new job. During unemployment they receive unemployment benefits which are a constant fraction of past wages. The model is calibrated to match US labor market institutions and other key features of the life cycle of workers.

We optimally choose age-dependent replacement rates and/or income tax rates to maximize the worker’s initial expected utility. We find that under the optimal age dependent policy, replacement rates are increased from the current value of 50 per cent to around 80 percent for workers in their mid twenties and to 60 per cent for workers in their thirties. Workers in their forties and fifties, instead, obtain benefits equal to less than 10 percent of their past wage. When allowing for just age-dependent replacement rates, welfare gains are equivalent to just less than a 1 percent increase in life time consumption. When we combine age-dependent unemployment insurance with age-dependent taxes, gains go up to the equivalent of more than a 3 percent increase in life time consumption.

To analyze whether age dependent policies exhaust an important part of the existing unexploited gains present in the current US system, we consider the problem of an agency that optimally choose benefits, taxes, and pensions as function of the entire worker’s history. The agency can observe workers’ assets as well as search effort, so unemployment insurance induces no moral hazard problems. Although age dependent policies can only imperfectly reproduce the solution of the optimal program, we surprisingly find that the combination of age-dependent unemployment insurance with age-dependent taxes yields gains that amounts to ninety percent of the welfare gains obtained under the optimal programm.

Further relation to the literature Using different methodologies, several authors have argued that the level of unemployment benefit is close to optimal in the US, see for example Davidson and Woodbury (1997), Shimer and Werning (2007), Pavoni (2007), and Chetty (2008). Our results show that, although benefits are about optimal on average, there are still sizable welfare gains from redistributing unemployment insurance over the life cycle—increasing it for the young and decreasing it for the old.

This paper relates to the ongoing literature that starting with Hopenhayn and Nicolini (1997) has analyzed the optimal design of labor market institutions, see also Pavoni and Violante (2007), Shimer and Werning (2008), Pavoni (2009), Rendahl

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1 An alternative would be to have replacement rates and taxes being conditioned on the current level of assets rather than on age. Although this policy would distort saving incentives and it is in principle inferior to an age dependent policy, it could still yield important welfare gains. This is one of the point made by Conesa, Kitao, and Krueger (2009) and Rendahl (2012).
The literature typically focuses on the problem of an initially unemployed worker who becomes permanently employed after finding his first job. With the exception of Hopenhayn and Nicolini (2009) the issue of recurrent unemployment spells is typically neglected. The literature has also abstracted away from life cycle effects due to non-linear returns to labor market experience and asset accumulation, which are instead the main focus of this paper.

Baily (1978) and Chetty (2006) have proposed simple formulas to evaluate whether unemployment benefits are optimal on average. The formula \( \varrho \) is similar to theirs but it focuses on possible gains from redistributing unemployment insurance over the life-cycle or more generally across any groups of workers classified by observable, immutable skill characteristics including gender or race. The formula \( \varrho \) works exactly in the stylized model of Section 2. But the quantitative analysis also indicates that the key forces emphasized by \( \varrho \) dominate in the existing US labor market institutions. To be sure, the simple formula \( \varrho \) neglects the effects of age-specific changes in benefits on tax income or on the unemployment of age groups not directly targeted by the policy change. And we show that these considerations lead to an extended redistribution formula which works exactly in the quantitative model. But although the simple and extended formula could be different, we find that, in our laboratory economy, the two formulas exhibit a remarkably similar age profile.

Shimer and Werning (2007) and Chetty (2008) have criticized the Baily’s formula on the grounds that its use relies on specifying highly controversial preference parameters. Our formula, is less subject to their criticism in that its ability to identify redistributions gains just relies on signing the relative magnitude of \( \varrho \) across skill groups. This is often possible by just comparing unemployment elasticities and consumption levels when unemployed across skill groups, without having to specify any preference parameter.

Chéron, Hairault, and Langot (2012, 2011) have studied the role of age dependent labor market policies in a Mortensen and Pissarides (1994) search model with finitely lived workers. Our paper is obviously related to theirs but with some important differences. They emphasize the demand side of the labor market and the role of age-dependent policies in solving the conventional search inefficiencies in vacancy creation typically present in random search models; see Pissarides (2000) for an introduction to this class of models. Search inefficiencies naturally vanish in extended versions of the search model where firms post wage contracts, workers observe them and direct search accordingly, see for example Moen (1997), Acemoglu and Shimer (2001), Shimer (2005) and more recently Menzio and Shi (2011). Here we emphasize labor supply effects and that the trade-off between the gains from unemployment insurance and the incentive costs of the induced moral hazard problem varies over the life cycle.
Section 2 uses a stylized life cycle model to discuss the formula in (1) and its extension. Section 3 contains preliminary evidence. Section 4 presents our laboratory economy. Section 5 solves for the first best problem. Section 6 studies age dependent policies. Section 7 discusses robustness. Section 8 concludes. An Online Appendix provides exhaustive details on data and computation.

2 A stylized life cycle model

We present a simple stylized life-cycle model where our simple formula holds exactly. We then extend the model to incorporate additional effects into the analysis that lead to an extended formula. We later show that these formulas work well in a more conventional life-cycle model more suitable for quantitative analysis.

2.1 The worker’s problem

In this stylized model workers live for six periods $i = 1–6$. They are young, $n = y$, during the first three $i = 1–3$, and old, $n = o$, during the last three $i = 4–6$. Unemployment is the only source of risk in the model. Workers are employed with probability one in all periods except in period two and five when they have to search for a job. This characterizes the fact that unemployment risk is recurrent, it affects both young and old, and it has transitory effects. Unemployment is endogenous due to search intensity decisions. Search intensity reduces the probability of unemployment and the amount of leisure enjoyed by the worker. We assume that a worker who is unemployed with probability $\mu$ at the end of period two or five enjoys utility from leisure equal to $\psi(\mu)$, with $\psi'(\mu) > 0$ and $\psi''(\mu) < 0$. Workers initially have no wealth. They can not borrow and they can save in a riskless bond that pays a constant interest rate $r$ equal to the subjective discount rate of workers. So the workers’ subjective discount factor is equal to $\beta = 1/(1 + r)$. Following well established evidence from wage regressions, we assume that wages $w_i$, $i = 1–3$ increase over time when young, while they are flat and equal to $\bar{w}$ when old, with $w_1 < w_2 < w_3 < \bar{w}$. If unemployed at age $n = y,o$ (end of period two or five) workers obtain unemployment benefits $b_n$. Consumption utility in a period is $u(c)$.

We assume that consumption is equal to income for young workers: a young worker expects future increases in labor income and would like to borrow to smooth consumption, but he can not due to the borrowing constraint.\(^2\) This simplifying assumption implies that old workers’ decisions are unaffected by the employment

\(^2\)Even if wages are growing and the interest rate is equal to the worker’s subjective discount rate, young workers might want to accumulate some precautionary savings to insure the risk of unemployment in period two. Here we assume that the demand for consumption smoothing dominates the precautionary savings motive so that $u'(w_1) \geq \mu_y u'(b_y) + (1 - \mu_y) u'(w_2)$ where $\mu_y$ is the equilibrium unemployment probability in period two.
history when young, which in turn guarantees that changes in benefits when young (old) do not affect unemployment when old (young). As discussed in Section 2.3, this separability property is required for the formula to hold exactly. Separability implies that worker’s initial expected utility can be expressed as equal to

$$W(b_y, b_o) \equiv Y(b_y) + O(b_o)$$

(2)

where $Y(b_y) = \max_\mu \tilde{Y}(b_y, \mu)$ and $O(b_o) = \max_\mu \tilde{O}(b_o, \mu)$ are the sum of discounted utilities when young, $i = 1–3$, and when old, $i = 4–6$, respectively. In the expression

$$\tilde{Y}(b_y, \mu) \equiv u(w_1) + \beta [\psi(\mu) + \mu u(b_y) + (1 - \mu)u(w_2)] + \beta^2 u(w_3),$$

(3)

is the sum of utilities obtained by young workers for given unemployment probability $\mu$ in period two, while

$$\tilde{O}(b_o, \mu) \equiv \beta^3 \max_{a \geq 0} \left\{ u(\bar{w} - a) + \beta \psi(\mu) + \beta \mu \left[ u \left( b_o + \frac{a}{\beta} \right) + \beta u(\bar{w}) \right] + \beta(1 - \mu) (1 + \beta) u \left( \bar{w} + \frac{a}{1 + \beta} \right) \right\}$$

(4)

is the analogous sum for old worker when the unemployment probability $\mu$ in period five is taken as given. In (4), $a$ denotes the precautionary savings that the household accumulates in period four to finance consumption during unemployment in period five, which occurs with endogenously determined probability $\mu$. If the worker instead remains employed, $a$ is used to increase consumption equally in period five and six. This accounts for the last term in (4).

2.2 The government problem

As standard in the optimal unemployment insurance literature, see for example Hopenhayn and Nicolini (1997) and Shimer and Werning (2007, 2008), we assume that government interventions are actuarially fair so that the present value of UI transfers is equal to the present value of some exogenous government income $T$ used to finance the UI program. In the quantitative model below this income is obtained by taxing labor. The government chooses $b_n$, $n = y, o$, so as to maximize worker’s expected utility $W$ in (2) subject to the budget constraint

$$\beta_y \mu_y(b_y)b_y + \beta_o \mu_o(b_o)b_o = T$$

(5)

where $\beta_y = \beta$ and $\beta_o = \beta^4$ are the discount factors, while the functions $\mu_y(b_y)$ and $\mu_o(b_o)$ determine the age specific unemployment probabilities $\mu_y$ and $\mu_o$ given

In equilibrium $a$ will always be in the interval $(0, \bar{w} - b_o)$, so the constraint $a \geq 0$ will be slack, while the borrowing constraint will be binding in period five if the worker is unemployed.
the age specific benefits levels \( b_y \) and \( b_o \), respectively. Given (3) and (4) these functions are implicitly defined by the conditions \( \mu_y = \arg \max \tilde{Y}(b_y, \mu) \) and \( \mu_o = \arg \max \tilde{O}(b_o, \mu) \), respectively. The Lagrangian of the problem reads as

\[
L(b_y, b_o, \lambda) = Y(b_y) + O(b_o) + \lambda \left[ T - \beta_y \mu_y(b_y) b_y - \beta_o \mu_o(b_o) b_o \right]
\]

where \( \lambda \) is the Lagrange multiplier of the budget constraint in (5). By taking the first order condition with respect to \( b_n \), \( n = y, o \), and after using the envelope theorem, we immediately obtain that it is optimal to increase \( b_n \) if

\[
\beta_n \mu_n u'(c_{un}) > \lambda \beta_n \mu_n + \lambda \beta_n \frac{d\mu_n}{db_n} b_n
\]

(6)

where \( c_{un} \) denotes consumption when unemployed at age \( n \). By rearranging we obtain that the above condition is equivalent to

\[
\varrho_n \equiv \frac{u'(c_{un})}{1 + \eta_n} > \lambda
\]

(7)

where \( \eta_n \equiv \frac{d\ln \mu_n}{d\ln b_n} \) is the unemployment elasticity to benefits of age-group \( n \). The ratio in the left hand side is the net welfare gain of marginally increasing government transfers to unemployed workers of age \( n \); the numerator measures the value of the marginal increase in UI benefits; the denominator the cost of the induced increase in unemployment. Optimal life cycle unemployment insurance requires having \( \varrho_n = \lambda \) for any age group \( n \). Generally there are welfare gains from increasing transfers to young unemployed workers at the expense of the old whenever

\[
\varrho_y > \varrho_o
\]

(8)

Interestingly, the comparison does not involve evaluating consumption losses upon displacement. This is simply because the government compares the gains of increasing transfers to unemployed workers of different age, whose marginal value is measured by their state contingent marginal utility of consumption. The derivation that leads to (8) is little, if at all, affected when considering several extensions of the baseline model. In particular the formula remains valid when:

1. Differences in workers demand and/or supply

   The utility from leisure is age-specific, \( \psi_n(\mu) \), \( n = y, o \), with \( \psi'_n(\mu) > 0 \) and \( \psi''_n(\mu) < 0 \). This accounts for possible differences in the demand for workers of different age as well as in their labor supply, which can both affect job finding probabilities.\(^4\)

\(^4\)To see why an age dependent \( \Psi \) function subsumes age effects in both labor demand and labor supply assume that, as in standard search models (Pissarides, 2000), the unemployment probability of workers of age \( n \) is a decreasing function of both their search effort \( s \) and market tightness \( \theta_n \), so that \( \mu = \mu(s, \theta_n) \). Age specific differences in workers demand are reflected in \( \theta_n \). The disutility of search effort is \( \tilde{\Psi}_n(s) \), which is age specific to characterize age differences in labor supply. We can then invert the function \( \mu \) to express search effort as function of \( \mu \) and \( \theta_n \) so as to obtain the simple formulation in the text based on \( \Psi_n(\mu) \equiv \tilde{\Psi}_n(\mu^{-1}(\mu, \theta_n)) \).
2. Varying job loss probabilities. Workers search for a job in period two and five with age specific probability \( \delta_n, n = y, o \) (in the baseline model \( \delta_y = \delta_o = 1 \)). This takes into account that the risk of job loss varies over the life cycle.

3. Other income sources. Workers have access to other sources of income \( y_n \) (say due to the spouse income), whose relative importance varies over the life cycle.

4. Changing household size. The household is represented by a simple unitary model with consumption utility \( m_n u(C/m_n) \), where \( m_n \) denotes household size when household head has age \( n \), while \( C \) denotes household total consumption expenditures. This takes into account that household size changes over the life cycle due to marriage, children birth or old children leaving the household. Due again to the envelope theorem, the marginal value of a unitary increase in benefits is \( u'(C/m_n) \). This just implies that \( c_{un} \) in (7) has to be interpreted as per capita household consumption when age \( n \) household head is unemployed.

2.3 The extended redistribution formula \( \tilde{\varrho} \)

We now discuss how the simple redistribution formula \( \varrho \) in (7) gets modified when we extend the analysis along three dimensions. First we allow young workers in period one to save. Second we allow benefits to affect the present value of the government income available for the UI programm, which is a natural equilibrium outcome when, as in the quantitative analysis of Section 4, this income come from labor income taxes that depend on workers’ employment and workers’ human capital. As a result the government budget constraint becomes

\[
T(b_y, b_o) \geq \beta_y \mu_y b_y + \beta_o \mu_o b_o. \tag{9}
\]

Third the optimal choice of benefits is now subject to some feasibility constraints, that impose that benefits can not fall below a minimum level \( \bar{b}_n \) so that

\[
b_n \geq \bar{b}_n, \quad \forall n = y, o. \tag{10}
\]

In the quantitative analysis of Section 4 this minimum level is set to zero.

Since young workers can save, the employment state when young affects workers decisions when old. Generally choices for assets and unemployment probabilities at any time \( i \) are now contingent on the existing history at that time. Moreover, since assets choices are forward looking, the equilibrium unemployment probability at a given age is function of both \( b_y \) and \( b_o \) so that we now have \( \mu_y = \mu_y(b_y, b_o) \), and \( \mu_o = \mu_o(b_y, b_o) \). The complete analysis of the extended model is in Appendix B where we show that the value of marginally increasing benefit transfers to unemployed
workers of age \( n \)—i.e. the analogous of \( \varrho_n \) in (7)— is now given by

\[
\tilde{\varrho}_n = \frac{E[u'(c_{un})] + \kappa_n \mu_n}{1 + \eta_n - \frac{\partial T}{\partial b_n} \cdot \frac{\beta_n}{\mu_n}}.
\] (11)

In the above expression \( E[u'(c_{un})] \) is the expected marginal utility of consumption of unemployed workers of age \( n \), \( \kappa_n \geq 0 \), \( n = y, o \) is the current value Lagrange multiplier of the benefits feasibility constraint in (10), while

\[
\tilde{\eta}_n = \sum_{i=y, o} \frac{\partial \mu_i}{\partial b_n} \cdot \frac{\beta_i b_i}{\beta_n \mu_n}
\] (12)

is the modified unemployment elasticity to account for the fact that changing UI benefits for a given age group \( n \) could potentially affect the unemployment level of any other age group. Finally, \( \frac{\partial T}{\partial b_n} \) is the partial derivative of tax income with respect to change in benefits. Generally there are welfare gains from increasing transfers to young unemployed workers at the expense of the old whenever

\[
\tilde{\varrho}_y > \tilde{\varrho}_o.
\] (13)

There are four simple reasons that make \( \tilde{\varrho}_n \) different from \( \varrho_n \).

1. **Heterogeneity in assets** Since assets depend on employment histories, unemployed workers of the same age can now enjoy different consumption levels. This is why the expected marginal utility of consumption enters the numerator of (11).

2. **Unemployment cross derivatives** Since the unemployment probability at a given age is function of the entire age profile of benefits, increasing benefits for an age group \( n \) can affect the unemployment level of any age group. So, the present value of UI total expenditures generally increase by \( \beta_n \mu_n (1 + \tilde{\eta}_n) \).

3. **Reduction in tax income** Benefits reduce government income \( T \), whose cost is measured by the derivative \( -\frac{\partial T}{\partial b_n} \).

4. **Positive benefits** When \( \kappa_n \) is positive (the constraint in (10) is binding), the government would like to decrease benefits further for unemployed workers of age \( n \) since their consumption is inefficiently high. In the quantitative analysis of Section 4, this constraint will be binding for old workers.

Although \( \tilde{\varrho}_n \) and \( \varrho_n \) are generally different, we will see that, in the baseline calibration of the laboratory economy of Section 4, \( \tilde{\varrho}_n \) and \( \varrho_n \) exhibit a remarkably similar age profiles which indicate similar welfare gains from redistributing unemployment insurance over the life cycle. Differences start to be significant only after choosing the optimal values for age dependent benefits. A simple interpretation is that differences between \( \tilde{\varrho}_n \) and \( \varrho_n \) start to matter only when policies are close to optimal, while the key forces emphasized by the simple formula in (7) dominate under the existing US labor market institutions.
3 Some empirical evidence

We now show that in the US the unemployment elasticity to Unemployment Insurance (UI) benefits and the consumption while unemployed are both lower when young than when old. This indicates that the inequality (8) holds both because young workers’ incentives to search for a job are less affected by benefits—the denominator in (7) is smaller for young than for old—and because young workers value unemployment insurance more—the numerator is higher. We then provide more direct evidence i) that the moral hazard problem induced by unemployment insurance is mild for young workers, and ii) that young workers have little means to smooth consumption during unemployment and thereby value highly the insurance and liquidity provided by UI. We later use this evidence to evaluate the quantitative properties of the model of Section 4. We start by discussing very briefly the data sets used in the analysis; for exhaustive details on data construction and sample selection criteria see Appendix A.

3.1 Data

Our data come from the Survey of Income and Program Participation (SIPP), the Current Population Survey (CPS), the Panel Study of Income Dynamics (PSID) and surveys collected by Mathematica on behalf of the Department of Labor. The SIPP and Mathematica data are used to perform an unemployment duration analysis at the individual level; the CPS to estimate aggregate effects of benefits on unemployment; the PSID to provide evidence on consumption. In all cases the analysis focuses on working-age males. Sample periods vary but roughly cover the 80’s up to the early 00’s. Sample selection in the SIPP and the Mathematica data is exactly as in Chetty (2008). As far as possible we apply the same criteria to the construction of the CPS and PSID samples.

We use two measures of UI benefits. One is the imputation of individual benefits in the SIPP data by Chetty (2008). The other is a measure of the average benefits received by unemployed workers of different age groups in each US state and year. The construction of this latter measure closely mimics the procedure in Chetty (2008) but using CPS data: we first use the March CPS survey to impute pre-unemployment wages for each unemployed worker in CPS and then calculate individual benefits using the UI benefits calculator by Cullen and Gruber (2000). The resulting individual benefits are then averaged at the age-group state year level.

Consumption in PSID is measured using either food consumption at home, that is reported directly by PSID, or total consumption expenditures in non durables which is imputed using the methodology by Blundell, Pistaferri, and Preston (2008) as in Hryshko, Luengo-Prado, and Sorensen (2010). The imputation covers both the
core and the SEO sample in PSID which allows us to consider a more representa-
tive sample than the one originally considered by Blundell, Pistaferri, and Preston
(2008). Consumption corresponds to the average per capita weekly expenditures in
the household, which, following Blundell et al. (2008), we interpret as measuring
household consumption in an average week around the survey week.

3.2 Unemployment elasticity to benefits

To calculate the unemployment elasticity to benefits for workers of different age, we
start splitting the SIPP sample in two age groups depending on whether workers
have 20 to 40 or 41 to 60 years of age. The split by age is justified by the fact that,
after 40 years of age, the return to labor market experience substantially flattens
while assets increase significantly. We show later that this matters for determining
the insurance value and the moral hazard costs of unemployment insurance. For each
sample, we then estimate the following semi-parametric Cox proportional-hazards
regression for unemployment duration:

\[ \ln h_{it} = \beta \ln b_{it} + \theta X_{it} + \text{err.} \]  

where \( i \) denotes the worker, \( t \) the duration of the current unemployment spell, \( h_{it} \)
is the job finding probability at unemployment duration \( t \), \( b_{it} \) is the level of UI
benefits, and \( X_{it} \) are set of controls including worker’s age, years of education, a
marital status dummy, previous job tenure, a spline in past logged wages, dummies
for year, states, and unemployment duration and the interaction of benefits with
unemployment duration. The effects of benefits are identified using a difference-in-
differences identification strategy that exploits changes in the UI regulation of US
states through time. Table 1 reports the results for the two different measures of
benefits. Panel (a) deals with individual benefits, panel (b) with the age specific
average benefits measure.\(^5\) The first column of panel (a) deals with the full sample
estimates, that are analogous to those in Chetty (2008). Here the elasticity of the
job finding probability to benefits is very close to one third and strongly statisti-
cally significant. The results in the following two columns show that the full sample
estimates in Chetty (2008) hide some important heterogeneity across workers of
different age. When considering the sample of workers from 20 to 40 years of age
the effects of benefits on job finding are quantitatively small and not statistically
significant for either measure of benefits. In the sample of older workers the es-

timated elasticity is instead close to one and strongly statistically significant with

\(^5\)Much of the variation by age in UI replacement rates is due to the fact that wages are typically
replaced by a constant percent, usually 50%, but only up to a certain maximum that differs by
state. Since wages generally increase with age, this implies that effective replacement rates are
lower for older than for younger workers.
either benefits measure.\(^{6}\)

Table 1: Job finding elasticity to benefits, SIPP

<table>
<thead>
<tr>
<th>(a) Individual UI benefits</th>
<th>All</th>
<th>20-40 yrs</th>
<th>41-60 yrs</th>
<th>(b) Age-specific average UI benefits</th>
<th>All</th>
<th>20-40 yrs</th>
<th>41-60 yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln ben.</td>
<td>-.36(\ast\ast\ast)</td>
<td>- .23</td>
<td>- .86(\ast\ast\ast)</td>
<td>ln ben.</td>
<td>-.34*</td>
<td>- .19</td>
<td>-1.36(\ast\ast\ast)</td>
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<tr>
<td></td>
<td>(.11)</td>
<td>(.16)</td>
<td>(.19)</td>
<td></td>
<td>(.20)</td>
<td>(.25)</td>
<td>(.46)</td>
</tr>
<tr>
<td>N. of spells</td>
<td>4529</td>
<td>2858</td>
<td>1522</td>
<td>N. of spells</td>
<td>4380</td>
<td>2858</td>
<td>1522</td>
</tr>
</tbody>
</table>

Notes: Estimates of \(\beta\) in the Cox regression (14) using SIPP data. In panel (a) benefits are individually imputed, in panel (b) are age-specific state-year averages. First column deals with full sample, second and third with workers of age from 20 to 40 years, and from 41 to 60 years, respectively. Standard errors clustered by state in parenthesis. \(\ast\ast\ast\) indicates significance at 1%, \(\ast\ast\) at 5%, \(\ast\) at 10%.

We now split the data into finer age group of workers. To maintain sample size, we estimate the unemployment duration regression in (14) using nine partly overlapping samples of workers with age differences of ten years. To measure the unemployment elasticity to benefits, we use the relation

\[
\frac{d \ln u/d \ln b}{d \ln f/d \ln b} = -\left(1 - u\right)d \ln f/d \ln b,
\]

where \(d \ln f/d \ln b\) is the estimated job finding elasticity while \(u\) and \(f\) are the sample average of the unemployment rate and finding rate, respectively. The relation is exact if benefits affects unemployment only through the job finding rate. Panel (a) in Figure 1 reports the age profile of the resulting unemployment elasticity when using individual benefits. The results with the age-specific average measure of benefits are in Figure A1 in Appendix A. The dotted lines represent 90 percent confidence intervals. The unemployment elasticity is around twenty percent for workers in their twenties and early thirties while it is around one for workers in their mid forties and early fifties. For workers close to retirement it tends to fall, but confidence intervals are very large indicating little precision in the estimates.

So far we have focused on how UI benefits affects the job finding rates. But benefits can also affect unemployment through labor force participation or through the unemployment inflow rate and they can have aggregate equilibrium effects not properly measured by unemployment duration regressions. To address some of these concerns, we use US states aggregate unemployment data from CPS and the age-specific average measure of benefits to estimate the following regression:

\[
\ln u_{itj} = \sum_n \beta_n q^n_j \ln b_{itj} + \theta X_{itj} + \text{err.} \quad (15)
\]

where \(i\) stands for state, \(t\) for period (semester-year) and \(j\) for age group, \(u_{itj}\) is the unemployment over population ratio of age group \(j\) in state \(i\) in period \(t\), \(q^n_j\)

\(^{6}\)We checked that results are robust to including as controls the log of individual wealth or of net liquid assets at the time of the job loss, or to using a Weibull regression for unemployment duration. We have also split the sample in three educational groups (less than high school, high school graduates, some college or more) and found similar results in each of the three groups.
Figure 1: Unemployment elasticity to benefits by age group

Notes: Unemployment elasticity to benefits by workers age. Panel (a) estimates are based on (14) using SIPP data and individual benefits. Unemployment elasticities are calculated using the formula \( \frac{d\ln u}{d\ln b} = - (1 - u) \frac{d\ln f}{d\ln b} \), where \( u \) and \( f \) are the sample average of the unemployment rate and finding rate, respectively. Panel (b) are estimates of \( \beta_n \) in (15) using US states aggregate unemployment data from CPS. Dotted lines are 90 percent confidence intervals.

is a dummy variable which is equal to one if the observation corresponds to age group \( n \), \( b_{itj} \) is the imputed age-specific average benefit level deflated with the CPI index. The variables \( X_{itj} \) are a set of controls, including time, state, and age group dummies, the imputed logged average pre-unemployment wages (again deflated with the CPI index), the proportion in the group of white, of married workers, of workers with working spouse, and of unemployed workers with five different educational levels. Standard errors are clustered at the state level, since different US states are considered as (at least) partially segmented labor markets. Panel (b) of Figure 1 plots the estimated values of \( \beta_n \) in (15), which measure the unemployment elasticity to benefits for workers of age \( n \). Dotted lines are ninety percent confidence intervals. The estimated unemployment elasticities are again increasing by age. They are very close to zero for workers in their twenties and around 0.7 for workers in their fifties. Estimates are comparable to those from the unemployment duration analysis in panel (a), although they are now slightly smaller and there is no longer any evidence that the elasticity falls towards zero for workers close to retirement.⁷

⁷The CPS results are robust to controlling for the maximum duration of benefits in the state or to instrumenting benefits using their own lagged value in an attempt to account for endogeneity problems—say because average benefits change over the business cycle due to a changing composition in the pool of unemployed workers, see Mueller (2010). The IV estimates are larger and more in line with the estimates from the unemployment duration analysis, which might indicate that compositional changes make replacement rates increase in recessions.
3.3 Consumption while unemployed

To estimate how the consumption of unemployed workers varies with age, we run the following regression on PSID data:

\[
\ln c_{it} = \sum_n \beta_n^e e_{it}^n + \sum_n \beta_n^u u_{it}^n + \theta X_{it} + \text{err.} \tag{16}
\]

where \( i \) denotes the worker, \( t \) is the year, \( c_{it} \) is consumption per capita in the household, \( e_{it}^n \) and \( u_{it}^n \) are employment status dummies that are equal to one if, at the interview date, the household head of age \( n \) is employed or unemployed, respectively. Finally \( X_{it} \) are set of controls, including dummies for the educational level and the race of the household head, time dummies and the number of household members. To account for serial correlation in the errors, the regression is estimated using a GLS random-effects estimator. Figure 2 shows the estimated age profile of consumption of employed workers as a dashed line and of unemployed workers as a solid line. Panel (a) deals with food consumption, panel (b) with total consumption in nondurables. Consumption of employed workers increases with age reaching a peak at around 50 years of age. Consumption of unemployed workers also increases with age and it is generally lower than consumption of employed workers.\(^8\)

Figure 2: Food and total consumption by age, PSID

Notes: Life cycle profile of logged household per capita consumption. Estimates are obtained from estimating (16) on PSID data. Left column is for food consumption, right column for total consumption expenditures in nondurables. The log consumption of employed workers of 50-55 years of age is normalized to zero.

\(^8\)Results are robust to including temporary laid off workers in the pool of unemployed workers, to weighting observations, to using total expenditures in food either at home or out of home, and to dropping observations with consumption levels below the bottom or above the top percentile of the consumption distribution. We also found that consumption of unemployed workers increases with age not only on average but also in the first-order stochastic dominance sense.
3.4 Moral hazard and liquidity effects

The previous results indicate that unemployment insurance induces mild incentive costs and it is highly valuable to young workers. We now provide more direct evidence that i) the moral hazard problem induced by unemployment insurance is mild for young workers and ii) that young workers value highly unemployment insurance because they have little means to smooth consumption during unemployment.

Moral hazard effects by age As shown by Chetty (2008), UI benefits increase unemployment duration due to a conventional moral hazard effect—benefits reduce the net income gains from finding a job—and due to a liquidity effect—benefits allow to better equalize the marginal utility of consumption when employed and when unemployed. So the evidence that the unemployment elasticity to benefits increases with age does not necessarily indicate that the moral hazard problem is milder for young than for old workers. Chetty (2008) argues that the severity of the moral hazard problem is measured by the job finding response to benefits of workers with high asset levels: wealthy workers have great ability to smooth consumption during unemployment, so liquidity effects are absent and benefits increase unemployment just due to moral hazard. To pursue this logic, we use the SIPP data to estimate the following Cox regression for unemployment duration analogous to (14):

$$\ln h_{it} = \sum_n \beta_n q^n_{it} \ln b_{it} + \theta X_{itj} + \text{err.}$$

where $q^n_{it}$ is an indicator variable that is one if worker’s wealth is in quartile $n$ (with higher $n$ indicating greater wealth). Wealth quartiles are calculated for the entire sample. Results changes little when wealth quartiles are age-specific. Controls are as in the estimation of equation (14) with the additional inclusion of wealth dummies and their interaction with unemployment duration. Table 2 reports the estimated $\beta_n$ coefficients in the full sample, and in the samples of ‘young’ and ‘old’ workers. There is evidence that benefits reduces job finding rates of old workers with assets in the top third or fourth quartile of the wealth distribution. The effects are somewhat stronger when measuring benefits with state averages. Standard significance tests also indicate that for old workers we can not reject the null hypothesis that the effects of benefits is the same for rich as for poor workers. This is indirect evidence that benefits increase unemployment duration of old workers mainly because of a moral hazard problem, with liquidity effects being somewhat less important. For young wealthy workers UI benefits have no significant effects on unemployment. Overall this evidence is consistent with the claim that the moral hazard problem of unemployment insurance is more severe for old than for young workers.
Table 2: Job finding elasticity to benefits by assets, SIPP

(a) Individual UI benefits

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>20-40 yrs</th>
<th>41-60 yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q&lt;sub&gt;1&lt;/sub&gt; x ln ben.</td>
<td>-.64*** (.24)</td>
<td>-.55* (.30)</td>
<td>-1.32*** (.43)</td>
</tr>
<tr>
<td>Q&lt;sub&gt;2&lt;/sub&gt; x ln ben.</td>
<td>-.76*** (.22)</td>
<td>-.93*** (.24)</td>
<td>-.26 (.55)</td>
</tr>
<tr>
<td>Q&lt;sub&gt;3&lt;/sub&gt; x ln ben.</td>
<td>-.56*** (.16)</td>
<td>-.31 (.25)</td>
<td>-1.11*** (.35)</td>
</tr>
<tr>
<td>Q&lt;sub&gt;4&lt;/sub&gt; x ln ben.</td>
<td>.02 (.26)</td>
<td>.66 (.35)</td>
<td>-.79* (.47)</td>
</tr>
</tbody>
</table>

Q<sub>j</sub>, j = 1, 2, 3, 4 are the quartile of the wealth distribution in the entire sample. Further details are as in Table 1.

(b) Age-specific average benefits

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>20-40 yrs</th>
<th>41-60 yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q&lt;sub&gt;1&lt;/sub&gt; x ln ben.</td>
<td>.12 (.20)</td>
<td>-.49 (.52)</td>
<td>-1.40* (.74)</td>
</tr>
<tr>
<td>Q&lt;sub&gt;2&lt;/sub&gt; x ln ben.</td>
<td>.02 (.20)</td>
<td>-.49 (.47)</td>
<td>-1.62* (.96)</td>
</tr>
<tr>
<td>Q&lt;sub&gt;3&lt;/sub&gt; x ln ben.</td>
<td>.09 (.20)</td>
<td>.39 (.47)</td>
<td>-1.86*** (.96)</td>
</tr>
<tr>
<td>Q&lt;sub&gt;4&lt;/sub&gt; x ln ben.</td>
<td>.14 (.21)</td>
<td>.95 (.71)</td>
<td>-1.80*** (.50)</td>
</tr>
</tbody>
</table>

Q<sub>1</sub>=Q<sub>4</sub> p-val | .09 (.26) | .01 (.35) | .34 (.47) |
Q<sub>1</sub>+Q<sub>2</sub>=Q<sub>3</sub>+Q<sub>4</sub> p-val | .06 (.06) | .00 (.00) | .67 (.25) |
Q<sub>1</sub>=Q<sub>2</sub>=Q<sub>3</sub>=Q<sub>4</sub> p-val | .18 (.18) | .00 (.00) | .25 (.00) |

Notes: Estimates of $\beta_n$ in the Cox regression (17) using SIPP data. Q<sub>j</sub>, j = 1, 2, 3, 4 are the quartile of the wealth distribution in the entire sample. Further details are as in Table 1.

Liquidity effects by age  Table 2 provide some evidence that UI benefits increase the unemployment probability of young poor workers, especially when focusing on the individual measure of benefits. This is coherent with the idea that UI benefits provide some valuable liquidity to young workers which allows them to better smooth consumption during unemployment. We now provide two additional pieces of evidence consistent with this view. We first borrow from Chetty (2008) the idea that severance payments provide liquidity to unemployed workers with no moral hazard costs. By comparing the search behavior of unemployed workers who have received severance payments with the behavior of similar workers who have not, we can identify the importance of liquidity effects. As in Chetty (2008) we then exploit the fact that the Mathematica data contain information on whether displaced workers have received severance payments at the time of the job loss, which allows us to estimate the following Cox proportional hazards regression analogous to (14):

$$\ln h_{it} = \beta_{Sev_i} + \theta X_{it} + \text{err.}$$  \hspace{1cm} (18)

where Sev<sub>i</sub> is an indicator variable which is equal to one if the displaced worker has received some severance payments. The additional controls $X_{it}$ include worker’s age, four education dummies, a spline in past tenure, one in past wages, the log of unemployment benefits, fixed effects for state, occupation and industry, unemployment duration dummies and the interaction of the severance payment dummy with unemployment duration. Again the model is estimated for the full sample and for

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9Here we focus on the effects on search effort, but of course severance payments can affect workers’ incentive to accumulate precautionary savings and in this sense they also induce a moral hazard problem.
the two age groups of workers. The resulting estimate for \( \beta \) is reported in Table 3. The first column reproduces the full sample results in Chetty (2008), which indicate that unemployed workers with severance pay experiences a percentage reduction in job finding rates of around one quarter. When we split the sample by workers’ age we find that the reduction in finding rates for young workers is around one third, while for old workers it is close to zero and not statistically significant at conventional levels. This is coherent with the idea that young workers have little means to smooth consumption during unemployment.

Table 3: Job finding elasticity to severance pay, Mathematica data

<table>
<thead>
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<th></th>
<th>All</th>
<th>20-40 yrs</th>
<th>41-60 yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Severance pay</td>
<td>-.23***</td>
<td>-.35***</td>
<td>-.08</td>
</tr>
<tr>
<td></td>
<td>(.07)</td>
<td>(.09)</td>
<td>(.11)</td>
</tr>
<tr>
<td>Number of spells</td>
<td>2428</td>
<td>1514</td>
<td>790</td>
</tr>
</tbody>
</table>

Notes: Estimates of \( \beta \) in (18) using Mathematica data. Further details are as in Table 1.

The age pattern of consumption losses upon unemployment also indicates that young workers find difficult to smooth consumption during unemployment. To estimate consumption losses, we follow Gruber (1997) and estimate equation (16) but now including individual fixed effects and dummy variables characterizing changes in employment status. The resulting regression is estimated using a fixed-effects (within) regression estimator. The coefficient of the change in employment status from employment to unemployment characterizes the size of the average consumption loss upon unemployment. We allow this effect to vary by age. Figure 3 shows the age profile of consumption losses for food consumption (left panel) and total consumption in nondurables (right panel). Consumption losses are around 17% for workers in their twenties and thirties and fall to less than 5% for workers in their fifties and sixties.\(^\text{10}\) Consumption losses are slightly larger when considering total consumption expenditures in nondurables, but in either case they fall significantly with age. This is again coherent with the idea that young workers have little precautionary savings and limited liquidity to smooth consumption during unemployment.

4 Laboratory economy

We now consider a life cycle model with ongoing unemployment risk that we use as a laboratory economy to answer three questions. First we study the magnitude of

\(^{10}\)There is a large literature on measuring consumption losses upon unemployment, see Gruber (1997), Browning and Crossley (2001), Bloemen and Stancanelli (2005) and Sullivan (2008). All studies point out that average consumption losses result from aggregating vastly heterogenous individual responses. Our results indicates that part of this heterogeneity is due to the life cycle.
Figure 3: Consumption losses upon unemployment

(a) Food consumption losses
(b) Total consumption losses

Notes: Consumption losses upon unemployment by age, PSID data. Dotted lines are 90 percent confidence intervals.

the welfare gains of age dependent unemployment insurance. Second we compare these gains with those attained under the unconstrained optimal scheme for unemployment insurance over the life cycle. Third we analyze how accurately the simple formulas discussed in Section 2 identifies welfare gains of age dependent policies. We first characterize the economy. Then we turn to calibration and discuss key properties of the calibrated economy. The study of the first best policy is in Section 5 while the analysis of age dependent policies is in Section 6.

4.1 Assumptions

There is a mass one of workers who live for $\bar{n}_w + \bar{n}_r$ periods. They are active in the labor market in the first $\bar{n}_w$ periods, retired in the last $\bar{n}_r$ periods. Allowing for retirement is needed to have an empirically plausible age profile of assets. Workers have discount factor $\beta$ and receive utility from consumption $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, with $\sigma > 0$. They are born with no job, no human capital, $e = 0$, no assets, $a = 0$, and can save in a riskless bond who pays a constant interest rate $r$ that satisfies $\beta = \frac{1}{1+r}$. Workers have limited ability to borrow and their assets cannot be lower than the borrowing limit $l$. In each period of employment, workers accumulate one unit of human capital and they receive wages $w(e)$ that satisfies $w' \geq 0$ and $w'' \leq 0$. This formalizes the notion that there are positive but decreasing returns to labor market experience. Employed workers of age $n$ lose their job with probability $\delta_n$ and when unemployed they choose how intensively to search for a new job. We allow the separation rate to be age dependent to match the age profile of unemployment in the data. Search intensity reduces the probability of unemployment and the
amount of leisure enjoyed by the worker.\textsuperscript{11} We assume that a worker who receives job offers with probability $1 - \mu$ enjoys utility from leisure $\psi(\mu)$, with $\psi'(\mu) > 0$ and $\psi''(\mu) < 0$. Here $\mu$ denotes the within period unemployment probability of a worker searching for a job. We adopt the same timing convention as in Lentz and Tranaes (2005) and Chetty (2008), whereby successful search in a period leads to a job in the same period. If a worker of age $n$ is jobless at the end of the period, he receives unemployment benefits which are a fraction $\rho_n$ of his last wage in the job. At the end of each period of unemployment there is a probability $\gamma$ that the worker’s human capital gets depreciated to an amount $\kappa(n,e) \leq e$, which is dependent of worker’s age $n$ and worker’s human capital in his previous job $e$. If, at some point during the unemployment spell, worker’s human capital has depreciated, the worker is re-employed with human capital $\kappa(n,e)$. This induces wage losses upon displacement, which as documented in Davis and von Wachter (2011) and Johnson and Mommaerts (2011) substantially increase with age. Unemployment and the associated human capital losses are the only source of risk in the model. During the last $\bar{n}_r$ periods of their life, workers receive retirement pensions $\pi$ which, as in Conesa, Kitao, and Krueger (2009), are independent of workers’ earnings history. During employment, workers of age $n$ pay income taxes that are a fraction $\tau_n$ of their labor income. Taxes are used to finance the unemployment insurance program and retirement pensions. As in Hopenhayn and Nicolini (1997) and Shimor and Werning (2007, 2008), we assume that workers and government face the same interest rate and that government policies are actuarially fair. This implies that the expected present discounted value of all transfers received by the worker is equal to the present value of the tax income he expects to pay over his working life.\textsuperscript{12}

4.2 Worker’s maximization problem

Let $c_e(n,e,a,a') = (1 - \tau_n)w(e) + (1+r)a - a'$ denote the consumption of an employed worker of age $n \leq \bar{n}_w$ with human capital $e$ and assets $a$, who chooses asset level $a'$ for next period. Since $a'$ should be greater than the borrowing limit $l$, the value of

\textsuperscript{11}We model the moral hazard of UI by relying on search effort decisions. There is indeed evidence from time use survey that job search intensity is inversely related to the generosity of unemployment benefits, see Krueger and Mueller (2010). But the moral hazard problem induced by UI generally leads both to a decrease in search effort and to an increase in reservation wages. As Shimer and Werning (2007, 2008), we believe that the main implications of the paper are little affected by whether the moral hazard problem of UI is characterized in terms of search effort or reservation wages.

\textsuperscript{12}This government budget constraint can also be justified by assuming that in every period new cohorts of workers enter the labor market, that the size of these cohorts grow at rate $r$ over time and that the government budget is balanced, so that the total tax income net of transfers across cohorts is zero in each period.
being employed for this worker satisfies:

\[ V(n, e, a) = \max_{a' \geq l} u(c^*(n, e, a, a')) + \beta [1 - \delta_n] V(n + 1, e + 1, a') + \delta_n J(n + 1, e + 1, a') \]  

(19)

where the last term incorporates the fact that with probability \( \delta_n \) a worker of age \( n \) has to search for a new job which has value

\[ J(n, e, a) = \max_{\mu \in [0, 1]} \psi(\mu) + \mu U(n, e, a) + (1 - \mu)V(n, e, a) \]  

(20)

This uses the timing convention that search leads to a job in the period with probability \( 1 - \mu \) while otherwise the worker remains unemployed which has value

\[ U(n, e, a) = \max_{a' \geq l} u(c^u(n, e, a, a')) + \beta (1 - \gamma) J(n + 1, e, a') + \beta \gamma J^*(n + 1, e, a') \]  

(21)

where \( c^u(n, e, a, a') = \rho_n w(e) + (1 + r) a - a' \) denotes current period consumption when unemployed at age \( n \). With probability \( \gamma \) the worker experiences a loss in his human capital and the function \( J^* \) denotes the value of searching after such a loss. It satisfies the following Bellman equation

\[ J^*(n, e, a) = \max_{\mu \in [0, 1]} \psi(\mu) + \mu U^*(n, e, a) + (1 - \mu)V(n, \kappa(n, e), a), \]  

(22)

which incorporates the assumption that, after experiencing a loss in human capital, the worker is reemployed with human capital \( \kappa(n, e) \leq e \) where \( e \) refers to worker’s human capital in the previous job.\(^{13}\) In the expression above \( U^* \) denotes the value of being unemployed after a loss in human capital, which satisfies

\[ U^*(n, e, a) = \max_{a' \geq l} u(c^*(n, e, a, a')) + \beta J^*(n + 1, e, a') \]  

(23)

where \( c^*(n, e, a, a') = \rho_n w(e) + (1 + r) a - a' \) denotes per period consumption and \( e \) refers to worker’s human capital at the time of job displacement. In writing (19), (21) and (23) we adopted the convention that

\[ V(\bar{n}_w + 1, e, a) = U(\bar{n}_w + 1, e, a) = U^*(\bar{n}_w + 1, e, a) = \frac{1 - \beta^{\bar{n}_w}}{1 - \beta} u(c^*(a)) \]

where the last term is the value of retiring at \( n = \bar{n}_w + 1 \) with assets \( a \), which is equal to the discounted value of consuming in every remaining period \( c^*(a) = \pi + \frac{ra}{1 - \beta} \).

Government policies are actuarially fair in that the expected present value of the income taxes collected over the working life of a worker is equal to the present

\(^{13}\)Notice that the human capital loss \( e - \kappa(n, e) \) depends on worker age at reemployment and not at displacement. This is a simplifying assumption that allows us to economize on the number of state variables characterizing the problem of an unemployed worker.
value of the UI benefits and retirement pensions the worker expects to obtain over his entire life. This implies the condition
\[ \sum_{n=1}^{\bar{\bar{n}}_w} \beta^n \int_{R^+} \rho_n w(e) \chi^u(n, de) + \sum_{n=\bar{n}_w+1}^{\bar{n}_r} \beta^n \pi \chi^r(n) = \sum_{n=1}^{\bar{\bar{n}}_w} \beta^n \int_{R^+} \tau_n w(e) \chi^e(n, de) \] (24)
where integrals are conventionally defined Lebesgue integrals, see Stokey, Lucas, and Prescott (1989). Here \( \chi^e(n, e) \) denotes the measure of employed workers of age \( n \) and experience \( e \), \( \chi^u(n, e) \) denotes the mass of workers of age \( n \) who collect UI benefits and who were displaced with human capital \( e \) and finally \( \chi^r(n) = \int \chi^e(\bar{n}_w, de) + \int \chi^u(\bar{n}_w, de) = \chi^r \) denotes the measure of retired workers of age \( n \), which is constant and independent of age.\(^{14}\) Of course, since the mass of workers in the economy is one, these three measures taken together have the nature of a probability measure:
\[ \sum_{n=1}^{\bar{\bar{n}}_w} \left[ \int_{R^+} \chi^u(n, de) + \int_{R^+} \chi^e(n, de) \right] + \bar{n}_r \chi^r = 1. \]

4.3 Calibration

The model is calibrated at the quarterly frequency to data for male workers in the US. Model parameters are jointly determined to match the calibration targets reported in Table 4. This process can be seen as estimation by indirect inference, see for example Gourieroux, Monfort, and Renault (1993). The resulting parameters values are in Table 5. Appendix A and C contain details on the construction of calibration targets in the data and in the model, respectively. We now discuss how model parameters are identified starting from moment conditions.

Technology

We assume that workers are born at 20 years of age, they are active for 45 years in the labor market, \( \bar{n}_w = 180 \), and live twenty years after retirement, \( \bar{n}_r = 80 \). The wage function \( w(e) \) is restricted to be non decreasing and is characterized by a cubic spline at the ten skill knots reported in Table 5. The values at the knots are set to match the average wage levels for the eight age groups in Table 4, plus the normalization condition that \( w(0) = 1 \) and that wages are constant for workers in their sixties. The age profile of wages in the data is obtained from the CPS over the 1990-2010 period using a sample of working-age males: wages increase on average by around 90 per cent over the life cycle.

The separation rate function \( \delta_n \) is characterized by a five values cubic Hermite spline with age knots at \( n = 10, 40, 80, 120, 160 \). To help guaranteeing that \( \delta_n \) always lies in the interval \([0, 1]\) we impose the boundary constraints that for \( n \leq 10 \), \( \delta_n = \delta_{10} \) while for \( n \geq 160 \), \( \delta_n = \delta_{160} \). The five values of the spline are implicitly calibrated to

\(^{14}\)Notice that for expositional simplicity we do not make these measures explicitly dependent of some policy relevant state variables such as assets or depreciation of human capital.
Table 4: Calibration targets and model fit

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<th>Moment condition</th>
<th>Data</th>
<th>Model</th>
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<td></td>
<td></td>
</tr>
<tr>
<td>21 - 24 years</td>
<td>1.12</td>
<td>1.13</td>
<td>CPS</td>
</tr>
<tr>
<td>25 - 29 years</td>
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<tr>
<td>30 - 34 years</td>
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<td>CPS</td>
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<td>1.92</td>
<td>CPS</td>
</tr>
<tr>
<td>Unemployment rate:</td>
<td></td>
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<tr>
<td>21-24 years</td>
<td>.104</td>
<td>.104</td>
<td>CPS</td>
</tr>
<tr>
<td>25-34 years</td>
<td>.058</td>
<td>.058</td>
<td>CPS</td>
</tr>
<tr>
<td>35-44 years</td>
<td>.046</td>
<td>.046</td>
<td>CPS</td>
</tr>
<tr>
<td>45-54 years</td>
<td>.042</td>
<td>.042</td>
<td>CPS</td>
</tr>
<tr>
<td>55-64 years</td>
<td>.041</td>
<td>.041</td>
<td>CPS</td>
</tr>
<tr>
<td>Proportion of displaced workers with</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>benefits who experience a wage loss</td>
<td>.57</td>
<td>.57</td>
<td>SIPP</td>
</tr>
<tr>
<td>Median wage loss upon re-employment:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21-30 years</td>
<td>.00</td>
<td>.00</td>
<td>SIPP</td>
</tr>
<tr>
<td>31-50 years</td>
<td>-.07</td>
<td>-.07</td>
<td>SIPP</td>
</tr>
<tr>
<td>51-64 years</td>
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<td>SIPP</td>
</tr>
<tr>
<td>Unemployment duration (in weeks):</td>
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<td></td>
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<tr>
<td>21-30 years</td>
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<tr>
<td>35-45 years</td>
<td>20.2</td>
<td>20.6</td>
<td>CPS</td>
</tr>
<tr>
<td>50-60 years</td>
<td>25.8</td>
<td>25.7</td>
<td>CPS</td>
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<tr>
<td>Unemployment elasticity to benefits:</td>
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<tr>
<td>21-30 years</td>
<td>.24</td>
<td>.24</td>
<td>SIPP</td>
</tr>
<tr>
<td>35-45 years</td>
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<td>.60</td>
<td>SIPP</td>
</tr>
<tr>
<td>50-60 years</td>
<td>.80</td>
<td>.85</td>
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</tr>
<tr>
<td>UI benefits replacement rate</td>
<td>.50</td>
<td>.50</td>
<td>SIPP</td>
</tr>
<tr>
<td>Retir. pensions over mean wages</td>
<td>.39</td>
<td>.39</td>
<td>OECD</td>
</tr>
<tr>
<td>Minimum assets for workers ≤ 35 yrs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>over mean quarterly total income:</td>
<td>-.61</td>
<td>-.61</td>
<td>SCF</td>
</tr>
</tbody>
</table>

Notes: Unless otherwise specified all statistics are averages for either the entire working-age population or the corresponding age group. The age profiles of wages, unemployment rates, and unemployment duration are from CPS data on a sample of working-age males over the 1990-2010 period. The minimum asset level in the data comes from SCF in 2007 and it corresponds to the 5th percentile of the net worth of workers with less than 35 years of age over the mean quarterly total income in the working age population. Wage losses statistics are from SIPP over the 1996-2007 period when focusing on working-age white males displaced from their full time job as employee and who have cashed UI benefits at some point during their unemployment spell. Displaced workers are identified as in Johnson and Mommaerts (2011). Retirement pensions statistic is from OECD (2007). UI benefits replacement rate is as in Chetty (2008). See Appendix A and C for further details on calibration targets in data and model, respectively.

match the average unemployment rate of the five age groups in Table 4. Henceforth in the construction of age groups, we drop workers of 20 and 65 years of age because in the model they are mostly unemployed and just about to retire, respectively. The resulting $\delta_n$ function is plotted in Figure A4 in the Appendix. Mean separation rate
is 0.035 which is roughly consistent with data on average job tenure and with the mean separation rate from JOLTS over the period 2005-2007.

To calibrate the borrowing limit $l$, we focus on the distribution of net worth of young workers (less than 35 years of age) who are the most likely to be financially constrained in the model. In practice $l$ is set to be equal to minus sixty one percent of the mean quarterly total income (i.e. from both labor and capital) in the economy. In the Survey of Consumer Finances (SCF) in 2007 this value corresponds to the fifth percentile of the distribution of the net worth of workers with less than 35 years of age over average quarterly income (from labor and other sources) in the Survey.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{n}_w$</td>
<td>Working periods</td>
<td>180</td>
</tr>
<tr>
<td>$\bar{n}_r$</td>
<td>Periods in retirement</td>
<td>80</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>.99</td>
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<tr>
<td>$\rho$</td>
<td>UI benefits replacement rate</td>
<td>.50</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Retirement pensions level</td>
<td>.39</td>
</tr>
<tr>
<td>$l$</td>
<td>Borrowing constraint</td>
<td>$-1.12$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Risk aversion</td>
<td>2.0</td>
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<tr>
<td>$\tau$</td>
<td>Tax rate</td>
<td>.0707</td>
</tr>
<tr>
<td>$w(e)$</td>
<td>Wages at $e = 20j$, $j = 0, 1, ..., 9$</td>
<td>${1.0, 1.29, 1.56, 1.73, 1.84, 1.92, 1.95, 1.96, 1.97}$</td>
</tr>
<tr>
<td>$\delta_n$</td>
<td>Separation rate (in percentage) at $n = {10, 40, 80, 120, 160}$</td>
<td>${8.5, 3.49, 3.07, 2.44, 2.13}$</td>
</tr>
<tr>
<td>$\kappa_n$</td>
<td>Wage losses at $n = {1, 40, 80, 160, 180}$</td>
<td>${0, 0, .93, .899, .899}$</td>
</tr>
<tr>
<td>$\psi(\mu)$</td>
<td>Search effort function at $\mu = {0, .25, .47, .75, 1.0}$</td>
<td>${-7.25, 1.78, .50,.021, -.204}$</td>
</tr>
</tbody>
</table>

Notes: The functions $w(e)$, $\delta_n$, $\kappa_n$, and $\psi(\mu)$ are cubic splines through values in table.

Wage losses upon re-employment To calibrate the human capital loss function $\kappa(n, e)$ and the wage loss probability parameter $\gamma$, we use information on wage losses upon re-employment from SIPP over the 1996-2007 period. We focus on a sample of working-age white males displaced from their full time job as employee and who have cashed UI benefits at least at some point during their unemployment spell. Wage losses are measured as the log difference between the wage in the last job in the month before displacement and the wage in the new job in the first month after reemployment. The median wage loss in our data is zero for workers below thirty and it increases to around 10 percent for workers above fifty.

To characterize the human capital loss function $\kappa(n, e)$, we assume that if worker’s human capital has depreciated during the unemployment spell, the worker is re-
employed with a wage that is a fraction \( r_n \) of the worker’s wage in his previous job \( w(e) \). This implies that \( \kappa(n,e) = w^{-1}(r_n w(e)) \), where \( w^{-1} \) is the inverse function of \( w(e) \), which is well defined since \( w(e) \) is non decreasing in \( e \). The wage loss function \( r_n \) is characterized by a five values cubic Hermite spline with knots at the age levels \( n = 1, 40, 80, 160, 180 \). The five values at the age knots are chosen to match the median wage losses for the three age groups in Table 4 plus the boundary constraints that for \( n \leq 10, r_n = r_{40} \) while for \( n \geq 160, r_n = r_{160} \) which helps in guaranteeing that \( r_n \) always lie in the \([0,1]\) interval. The resulting \( r_n \) function is plotted in Figure A4 in the Appendix. The parameter \( \gamma \), which regulates the probability of a wage loss, is chosen so that a worker who collects UI benefits at some point during his job search spell has a 57 percent probability of experiencing a wage loss upon re-employment, which is in line with evidence from our SIPP sample.

**Search effort** To characterize the within-period unemployment probability function \( \psi(\mu) \) we start noticing that the second derivative of the function \( \psi \) plays a key role in determining the value of the unemployment elasticity to benefits. Because of this we decided to model explicitly its profile and to impose the constraint that it should always be non positive, \( \psi'' \leq 0 \), see Appendix C for further details. In practice the \( \psi \) function is approximated by a cubic spline evaluated at the five age knots reported in Table 5 where the intermediate knot corresponds to the endogenously determined value of \( \mu \) at which the second derivative of \( \psi \) reaches its maximum value (its minimum absolute value). The six moment conditions needed to pin down the function are the values for the average unemployment duration and the unemployment elasticity to benefits for the three age groups reported in Table 4.

In the model, the unemployment elasticity to benefits for workers of age \( n \), \( \eta_n \), are calculated considering changes in replacement rates at \( p \) consecutive quarters starting from age \( n \). To be sure, let \( \rho = \{\rho_1, ..., \rho_{\eta_n}\} \) denote the vector containing the age profile of UI replacement rates in the baseline economy. For every \( n \), the unemployment elasticity, \( \eta_n \), is calculated considering two economies one with lower and one with higher replacement rates at age \( n \) than in the baseline economy.\(^{17}\) The

\(^{16}\)For the range of values of \( \epsilon \) for which the \( w(\epsilon) \) function is constant, the inverse function \( w^{-1} \) is defined as selecting the minimum value of \( \epsilon \) over the corresponding range.

\(^{17}\)The lower and the higher replacement rates at age \( n \) are characterized by the vector \( \rho_n^i = \{\rho_1, ..., \rho_{n-1}, \theta_{n}^i, \theta_{n+1}^i, ..., \theta_{n+p-1}^i, \rho_n, ..., \rho_{\eta_n}\}, i = l, h \) where \( \theta_{n+j}^i = \rho_{n+j} - \frac{\epsilon}{2} \) and \( \theta_{n+j}^h = \rho_{n+j} + \frac{\epsilon}{2}, \forall j = 0,1, ..., p - 1 \), respectively. In the paper we work with \( \epsilon = 0.02 \) and \( p = 4 \), which corresponds to a change in benefits for an age group of one year. We checked that results are little affected when reducing \( \epsilon \) or \( p \). We consider changes in benefits for \( p \) consecutive quarters both to increase sample size and to reduce the likelihood that the policy change affects workers’ search effort decisions through effects on unemployment duration dependence in benefits, which is an issue somewhat unrelated to age dependent policies. To avoid this problem we could have indexed the level of replacement rates, rather than to current age, to the age at which the worker is displaced. But this alternative specification would require having an additional state variable in the worker problem, which would involve additional computational costs.
resulting $\psi(\mu)$ function is depicted in panel (a) of Figure 4.

*Remaining preferences* We set $\beta$ to .99, to match an annual interest rate of approximately 4%. The CRRA parameter $\sigma$ is chosen to be equal to two, which is as in Conesa, Kitao, and Krueger (2009) when using a specification with separable utility between consumption and leisure.

*Policy parameters* Benefit replacement rates $\rho_n$ are assumed to be equal to a constant value $\rho$, which following Chetty (2008) is calibrated to .5. The retirement pensions $\pi$ are set equal to 0.662, which yields a ratio of retirement pensions over mean quarterly labor income of 0.39 in line with aggregate statistics from OECD (2007). The tax rate $\tau = 7.07\%$ keeps the government budget constraint in (24) satisfied.

### 4.4 Further properties of the calibrated economy

Panels (b)-(h) of Figure 4 characterize the age profile of key variables in the model economy and in the data. In all panels, the blue solid line corresponds to the model, the dashed and dotted red lines to the data. To facilitate comparison between model and data we group workers in age groups of 21-25, 26-35, 36-45, 46-55, and 56-64 years of age. As before we exclude workers of 20 and 65 years of age because in the model they are mostly unemployed and just about to retire, respectively. We then calculate average for wages (panel b), unemployment rates (panel c), unemployment duration (panel e) and net assets over average quarterly total income in the economy (panel h). Data averages for unemployment elasticity to benefits (panel e), consumption when unemployed (panel f), and consumption differences between employed and unemployed (panel g) correspond to the analogous profiles in Figure 1 and 2. The model matches well the profile of wages, unemployment rates and unemployment duration, panel (b)-(d). All these were explicitly used as calibration targets. The model just tends to over-predict the unemployment duration of workers in their early sixties. This is because the $\psi$ function in panel (a) is strictly positive at a within-period unemployment probability equal to one, so unemployed workers close to retirement always tend to shirk. The unemployment risk faced by workers over their working life is sizeable: around twenty four per cent of workers have to search for a new job in at least one out of ten periods of their working life. The model also matches reasonably well the age profile of the unemployment elasticity to benefits in the data: the model counterpart tends to lie in between the estimated

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18In practice replacement rates in the US are not completely independent of age since wages are typically replaced by a constant percent but only up to a certain maximum. This implies that effective replacement rates are lower for groups with higher wages (such as older workers). Matching this feature of the UI system in the US would require making UI replacement rates function of both $n$ and $e$. In any case age differences in actual replacement rates are small (of the order of ten percent) relatively to the differences that arise under the optimal age dependent policies studied in Section 6.
Figure 4: Properties of laboratory economy

(a) Search effort function, $\psi(\mu)$

(b) Wages

(c) Unemployment rates, $u_n$

(d) Unemployment duration

(e) Unemp. elasticity to benefits, $\eta_n$

(f) Consumption of unemployed, $\ln c_{un}$

(g) Unemp. consumption loss, $\ln(c_{un}/c_{en})$

(h) Assets over aggregate income

Notes: With the exception of panel (a), blue solid lines correspond to model, dashed and dotted red lines to data. Dashed red lines in panel (b), (c), and (d) are from CPS. Dashed and dotted red lines correspond: in panel (c) to panel (a) and (b) of Figure 1, respectively; in panel (d) to red solid line in panel (a) and (b) of Figure 2, respectively; in panel (e) to difference between red solid line and blue dashed line in panel (a) and (b) of Figure 2, respectively. Red dashed line in panel (e) is the ratio between households’ net worth in the age group and average quarterly households’ total income in SCF. In panel (f) the log consumption of employed workers of 50-55 years of age is normalized to zero, which is as in Figure 2.
value based on the unemployment duration analysis in SIPP and the value obtained using aggregate state level data from CPS.

As regards consumption, the model approximates moderately well the age profile of consumption when unemployed in the data (panel f), although the profile in the model tends to reach a plateau a couple of years earlier than in the data. Also the profile of consumption losses upon unemployment—as measured by the log difference between the average consumption of employed and unemployed—in the model is reasonably in line with the data. Finally panel (h) focuses on the age profile of net assets. Asset levels are higher in the data than in the model, but overall the model reproduces well the average increase of assets over the life cycle. This is remarkable since in the calibration process we used no information on consumption and just limited information on assets.

4.5 Elasticities and redistribution formulas

Panel (a) of Figure 5 plots the age profiles of the simple redistribution formula \( \varrho_n \) in (7) as a blue solid line and of the extended redistribution formula \( \tilde{\varrho}_n \) in (11) as a dashed red line. The simple redistribution formula is calculated as

\[
\varrho_n = \frac{u'(c_{un})}{1+\eta_n} \frac{1}{\mu_n}
\]

where \( c_{un} \) denotes the expected consumption of unemployed workers of age \( n \). To calculate \( \tilde{\varrho}_n \) at each \( n \) we again exploit changes in replacement rates at \( p \) consecutive quarters starting from age \( n \). We use these policy changes to calculate the cross unemployment elasticity

\[
\tilde{\eta}_n = \frac{n}{\beta_n} \sum_{i=1}^{\beta_n} \frac{\partial \mu_i}{\partial \rho_n} \cdot \frac{\beta_i \rho_i}{\beta_n \mu_n},
\]

which is analogous to (12). Here \( \mu_n \equiv \int_{\mathbb{R}^+} \chi^n (n, de) \) denotes the mass of workers of age \( n \) who collect UI benefits. We also define the present value of total tax income in the economy as equal to

\[
T(\rho) = \sum_{n=1}^{\beta_n} \int_{\mathbb{R}^+} \tau_n w (e) \chi^n (n, de)
\]

and calculate the derivative of \( T \) with respect to the age dependent change in benefits. We then use (11) to calculate \( \tilde{\varrho}_n \), see Appendix C for further details.

The age profiles of \( \varrho_n \) and \( \tilde{\varrho}_n \) in Figure 5 are remarkably similar which indicates similar welfare gains from redistributing unemployment insurance over the life cycle. Either ratio is generally decreasing with age and it has a value close to 1.5 for workers in their twenties and close to one fourth for workers in their forties and early fifties. Overall this suggests that one unit of government money would yield six times larger welfare gains when assigned to young unemployed workers than to middle-aged unemployed workers. As implied by the discussion in Section 2.3, there
are three reasons why in the baseline calibration $\varrho_n$ differs from $\tilde{\varrho}_n$: (i) $\varrho_n$ focuses on the marginal utility of expected consumption rather than on the expected marginal utility of consumption; (ii) $\varrho_n$ misses the effects of age specific changes in benefits on the unemployment level of age groups different from those directly targeted by the change in benefits; and (iii) $\varrho_n$ neglects the effects of UI on tax income.\(^{19}\) Since the marginal utility of consumption is convex, the effect (i) tends to make $\varrho_n$ smaller than $\tilde{\varrho}_n$ while the effects (ii)-(iii) tend to make $\varrho_n$ greater than $\tilde{\varrho}_n$. To separately analyze the contribution of each factor in inducing the observed differences between $\varrho_n$ and $\tilde{\varrho}_n$, in panel (b) of Figure 5 we compute $\varrho_n$ adding one source of difference at a time: the solid blue line corresponds to the profile of $\varrho_n$ in panel (a); the dashed red line is analogous but where $\varrho_n$ is calculated using the expected marginal utility of consumption rather than the marginal utility of expected consumption; the green dash-dotted line corresponds to calculating $\varrho_n$ using the extended unemployment elasticity $\tilde{\eta}_n$ in (25) rather than the simple unemployment elasticity $\eta_n$; and finally the purple dotted line is obtained by calculating $\varrho_n$ after adding to the denominator the effect of taxes, as measured by $\frac{\partial T}{\partial b_n} \cdot \frac{1}{\mu_n}$. For workers below forty, consumption is low which makes the marginal utility of consumption highly convex. For these workers the positive effect on $\tilde{\varrho}_n$ of taking expectations cancels out almost exactly with the negative effects on $\tilde{\varrho}_n$ due to unemployment cross-derivatives and taxes. So the simple and extended formulas, $\tilde{\varrho}_n$ and $\varrho_n$, almost overlap in panel (a). But for workers above forty, consumption is high enough to make the marginal utility of consumption almost linear. For these workers the effects of cross derivatives and taxes necessarily dominate, so $\tilde{\varrho}_n$ falls below $\varrho_n$.

\(^{19}\)Notice that in the baseline calibration $\rho_n > 0$, $\forall n$, so the feasibility constraint is never binding and $z_n$ in (11) is always equal to zero.
5 Optimal life cycle unemployment insurance

At birth, workers have to search for a job, they have no experience and no assets so their welfare is given by \( W_s \equiv J(1, 0, 0) \). Before analyzing age dependent policies, we now study the first best problem faced by an agency that observes workers’ assets and workers search effort and maximizes initial worker’s utility \( W_s \) by choosing benefits \( \rho \), taxes \( \tau \), and pensions levels \( \pi \) as a function of the entire worker’s history. Government budget is balanced, so an expression analogous to (24) holds. Since assets are observable, we can think that the agency directly controls workers’ consumption. Moreover search effort is observable, so no moral hazard problem is present and the agency can achieve perfect consumption smoothing by guaranteeing the worker a constant consumption level \( c \) through his entire life. As a result consumption losses upon unemployment are zero. Let \( \Upsilon (n, e, c) \) denote the total net cost of providing a constant consumption flow \( c \) to a worker of age \( n \leq \bar{n}_w \) with human capital \( e \) who has just started searching for a job. This cost is equal to the difference between the present value of consumption expenditures and the expected present value of the income \( Y (n, e, c) \) produced by the worker:

\[
\Upsilon (n, e, c) = 1 - \beta^{\bar{n}_w + \bar{n}_r + 1 - n} c - Y (n, e, c) \tag{26}
\]

In each period the within-period unemployment probabilities are set to maximize the utility value of \( Y \) net of the disutility cost of searching, see Appendix C for details. The function \( \Upsilon (n, e, c) \) in (26) is decreasing in \( c \) because higher consumption implies greater expenditures as well as less future income \( Y \) since higher \( c \) reduces search effort due to a conventional income effect. The optimal value of \( c \), denoted by \( c^* \), is chosen to make \( \Upsilon (n, e, c) \) at worker’s birth equal to zero

\[
\Upsilon (1, 0, c^*) = 0.
\]

The solid line in panel (a) of Figure 6 characterizes the age profile of job finding rates under the optimal policy. The job finding rate of workers of age \( n \), \( f_n \), is simply equal to the ratio between the mass of workers of age \( n \) who find a job in a period and the pool of workers of the same age searching for a job. Job finding rates are slightly increasing with age until two years before retirement, when they start to fall rapidly to zero. Since the \( \psi \)-function is concave, the agency would like to smooth

\[20\]
search effort over time, but the opportunity cost of having an old, typically high skilled worker unemployed is high due to his high productivity. So job finding rates are generally (mildly) increasing in age. Just before retirement, investing in search is unprofitable since little time is left to capitalize any investment. So job finding rates drop to zero.

Figure 6: First best policy and optimal age-dependent policies

To analyze the profile of UI replacement rates under the optimal policy, we follow the optimal unemployment insurance literature (Hopenhayn and Nicolini 1997) and define $c^*_w(e)$ as the optimal replacement rate of a worker whose human capital at the time of displacement was equal to $e$. Similarly we can consider an employed worker with human capital $e$ and define the tax rate implied by the optimal policy as equal to $1 - \frac{c^*_w(e)}{w(e)}$. Figure 7 characterizes the age profile of average replacement rate (as a solid line) and average tax rates (as a dashed line) under the optimal policy. Since wages $w(e)$ tend to increase with age and the agency guarantee perfect consumption insurance to workers, we have that replacement rates are on average decreasing with age while tax rates are increasing. Table 6 compares welfare under the optimal policy and in the baseline economy.\(^{21}\) Gains relative to the status quo are sizable, roughly equivalent to a 3.4 per cent increase in per period consumption.

\(^{21}\)In the baseline economy average unemployment replacement rates might not be optimal. To better isolate the effects of policies, welfare gains are always measured relative to the economy with an optimal unemployment replacement rate. In practice, as many others (see Davidson and Woodbury 1997, Shimer and Werning 2007, Pavoni 2007, and Chetty 2008), we find that the optimal replacement rate is close to the actual US level—and equal to 0.51. Differences with the baseline economy of Section 4 are therefore minimal.
6 Age dependent policies

In the previous Section the government could condition transfers on workers’ entire labor market history as well as on their assets, age, experience, and employment status thereby guaranteeing perfect consumption insurance to the worker. We now study age dependent policies, where the government can condition UI replacement rates, $\rho_n$, and labor income tax rates, $\tau_n$, just on age $n$. Pensions levels are left unchanged, while tax levels are always adjusted to keep the government budget constraint (24) satisfied.

6.1 The problem

An optimal age-dependent replacement rate policy is a choice for the vector of replacement rates $\boldsymbol{\rho}$ that maximizes $W_s \equiv J(1, 0, 0)$ subject to the government budget constraint in (24), workers optimal choices as implied by (19)-(22) and a feasibility constraint that requires replacement rates to be non negative $\rho \geq 0$.

We model $\rho_n$ as the maximum between zero and a cubic spline at the ten age knots corresponding to $n = 1, 20, 40, 60, 80, 100, 120, 140, 160, 180$. We search for the value at the knots that maximize workers utility at birth $W_s$ and we check that results are little affected by increasing the number of knots. We then allow income tax rates also to vary with age. This problem is analogous to the previous one: the government chooses $\rho \geq 0$ and the vector of tax rates $\boldsymbol{\tau}$ to maximize $W_s$ subject to exactly the same constraints as before. To solve this problem we again assume that $\rho_n$ and $\tau_n$ are a cubic spline at the previously defined age knots where the former function is restricted to be non negative. For each policy, we study how replacement

\[22\] We impose this non negative constraint because the worker could always opt to drop out of the labor market and thereby cash zero benefits.
rate and tax rates vary by age and we analyze the properties of the $\varrho_n$ ratio in (1) as well of the modified redistribution formula $\tilde{\varrho}_n$ in (11). We then quantify the gains of age dependent policies and compare them with those attained under the optimal life cycle unemployment insurance problem of Section 5. When comparing welfare gains we also consider an economy where unemployment insurance replacement rates are maintained at the current US level, while the age profile of labor income tax rates $\tau$ is chosen to maximize $W_s$ subject to exactly the same constraints as before.

### 6.2 Optimal policies

The solid lines in the four panels of Figure 8 characterizes the economy with optimal age-dependent UI replacement rates and constant income tax rates. Dotted lines correspond to the baseline economy of Section 4. Panel (a) focuses on the optimal age profile of UI replacement rates, panel (b) on the profile of the marginal utility of the average consumption when unemployed, panel (c) on the unemployment elasticity to benefits, and panel (d) on the profile of $\varrho_n$ as previously defined. Under the optimal age dependent policy, replacement rates are increased from the current value of 50 per cent to around 80 percent for workers in their mid twenties and to 60 percent for workers in their thirties. Workers in their forties and in their fifties, instead, obtain almost no benefits. The age profile of the average marginal utility of consumption when unemployed is substantially flatter than in the baseline economy. The unemployment elasticity to benefits, $\eta_n$, is generally smaller than in the baseline economy and tends to decrease with age. Because of this, the age profile of the $\varrho_n$ ratio is now substantially flatter than in the baseline economy.

We now analyze why $\varrho_n$ does not get completely independent of age under the optimal age dependent UI benefits policy. In panel (a) of Figure 9 we plot the age profiles of $\varrho_n$ and $\tilde{\varrho}_n$ in the economy with optimal age dependent UI replacement rates. As expected the profile of $\tilde{\varrho}_n$ is approximately flat while $\varrho_n$ is greater than $\tilde{\varrho}_n$ for workers below forty while it is smaller for workers above forty. To study why the two profiles differ we perform a decomposition exercise identical to that in panel (b) of Figure 5 but where we now also take now into account that for workers above 40 years of age the feasibility constraint $\rho_n \geq 0$ is binding so that the Lagrange multiplier $\kappa_n$ in (11) is strictly positive. The contribution of the Lagrange multiplier corresponds to the blue bold dotted line in panel (b) which is obtained by calculating $\varrho_n$ after adding to the numerator in (1) the Lagrange multiplier $\frac{\kappa_n}{\mu_n}$ which is positive when the feasibility constraint $\rho_n \geq 0$ is binding. All other lines are as in panel (b) of Figure 5: the solid blue line corresponds to the profile of $\varrho_n$ in panel (a); the dashed red line is analogous but where $\varrho_n$ is calculated using the expected marginal utility of consumption rather than the marginal utility of
Figure 8: Age dependent replacement rates only

(a) Replacement rates, $\rho_n$

(b) Marginal utility of unemployed, $u'(c_{un})$

(c) Unemployment elasticity, $\eta_n$

(d) Redistribution formula, $\varrho_n$

Notes: Dotted lines correspond to the baseline economy, solid lines to the economy with optimal age-dependent UI replacement rates and constant income tax rates.

expected consumption; the green dash-dotted line corresponds to calculating $\varrho_n$ using the unemployment elasticity extended to include cross derivatives $\tilde{\eta}_n$, rather than the simple unemployment elasticity $\eta_n$; finally the purple dotted line is obtained by calculating $\varrho_n$ after adding to the denominator the effect of taxes, as measured by $\frac{\partial T}{\partial b_n} \cdot \frac{1}{\mu_n}$. For workers below forty, $\varrho_n$ is bigger than $\tilde{\varrho}_n$ mainly because $\tilde{\eta}_n$ is greater than $\eta_n$—i.e. because changes in benefits for an age group increase unemployment also for other age groups. For workers above forty, $\varrho_n$ falls below $\tilde{\varrho}_n$ just because the feasibility constraint $\rho_n \geq 0$ is binding, which makes the Lagrange multiplier $\kappa_n$ strictly positive.

Figure 10 is analogous to Figure 8 but where now we also optimally choose the age profile of labor income tax rates. Taxes are generally set to achieve a smooth age profile of consumption. Tax rates increase with age until the very late fifties when
they start to decrease quickly until retirement. Taxes before retirement are low to provide strong incentives to highly productive old workers, as well as to finance high consumption during retirement. The age profile of replacement rates is decreasing in age as in Figure 8, but now replacement rates are significantly smaller for workers in their thirties. Just before retirement benefits increase slightly, which follows from the analysis of $\tilde{\varrho}_n$ in (11): for this age group tax rates are negative, so $\frac{\partial T}{\partial b_n}$ is positive, which pushes up the value of $\tilde{\varrho}_n$ and thereby justifies increasing $\varrho_n$. The age profile of the marginal utility of consumption when unemployed and of the unemployment elasticity to benefits become substantially flatter than in the baseline economy. As a result the profile of $\varrho_n$ becomes almost invariant to age except for very young and very old workers, for whom $\varrho_n$ falls to around ten percentage points below its average. As expected, the age profile of $\tilde{\varrho}_n$ is completely flat, see dashed line in panel (d). A decomposition exercise analogous to the one performed in panel (b) of Figure 9 shows that almost all differences between $\varrho_n$ and $\tilde{\varrho}_n$ are due to the age profile of taxes: when taxes are negative, $\frac{\partial T}{\partial b_n}$ in (11) is positive, which makes $\tilde{\varrho}_n$ greater than $\varrho_n$; when taxes are positive, $\frac{\partial T}{\partial b_n}$ is negative and $\tilde{\varrho}_n$ falls below $\varrho_n$.

6.3 Welfare comparisons

Figure 6 characterizes the age profile of job finding rates (panel a), and consumption when unemployed (panel b) in the baseline economy (dotted line), in the economy

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**Notes:** Lines are as in figure 5 except for the blue bold dotted line in panel (b), which corresponds to the effects of the Lagrange multiplier on the feasibility constraint $\rho_n \geq 0$.  

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23 Since the retirement age is exogenous, workers in their sixties have little incentive to search for a job. Moreover we are not maximizing with respect to the level of retirement pensions $\pi$, which affects the choice for the age profile of taxes before retirement: when we double the value of $\pi$, taxes fall significantly less in the five years before retirement.
Figure 10: Age dependent replacement rates and tax rates

(a) Replacement rates $\rho_n$ and tax rates $\tau_n$

(b) Marginal utility of unemployed, $u'(c_{un})$

(c) Unemployment elasticity, $\eta_n$

(d) Redistribution formula, $\varrho_n$ and $\tilde{\varrho}_n$

Notes: In all panels dotted lines correspond to the baseline economy, solid lines to the economy with optimal age dependent UI replacement rates and labor income tax rates. In panel (d) red dashed line corresponds to the profile of $\tilde{\varrho}_n$ in the economy with optimal age dependent policies.

with optimal age dependent benefits (dashed line), in the economy with the combined age dependent policy for benefits and taxes (dash dotted line) and in the optimal problem studied in Section 5 (solid line). Age profiles do differ in the four economies. In the first best economy and in the age dependent policy economies job finding rates are mildly increasing with age. Both in the first best economy and in the economy with combined age dependent benefits and taxes, consumption is flat while consumption losses are small and relatively independent of age. In the baseline economy job finding rates are strongly decreasing in age, consumption is increasing and consumption losses are large for workers in their twenties and thirties.

Table 6 quantifies the welfare gains under the different allocations. The first best policy with observable search effort yields welfare gains equivalent to a 3.4% increase
in consumption. We normalize these gains to 100% and compare them with those attained under alternative age dependent policies. When allowing for age-dependent replacement rates, welfare gains are equivalent to just less than a 1 percent increase in life time consumption. When we combine age-dependent unemployment insurance with age-dependent taxes, gains go up to an equivalent of a 3.2 percent increase in life time consumption. Age-dependent policies reproduces more than 90% of the welfare gains attained under the optimal unemployment insurance program.\footnote{As in Shimer and Werning (2008), here there are small welfare gains from making UI benefits dependant on unemployment duration. As workers spend longer time into unemployment, their assets as well as their human capital fall which makes consumption decrease. This gives unemployed workers enough incentives to search for new jobs during the unemployment spell.} It is also useful to study the economy where unemployment insurance replacement rates are maintained at the current US level and labor income tax rates are allowed to vary with age. In this economy, tax rates are implicitly set to smooth the age profile of income. So consumption is relatively smooth over the life cycle but not across employment states. The economy with age dependent income tax rates yields welfare gains equivalent to about two thirds of the gains attained under the combined age dependent policy for replacement rates and taxes, with the remaining one third due to age dependent replacement rates. As further discussed below, an important part of the welfare gains come from relaxing financial constraints over the life cycle: when in the baseline economy of Section 4.3 we set the borrowing limit \( l \) at its natural level—so that no worker is financially constrained—, welfare increases by around three percent in consumption equivalent, which represents an important share of the gains from age dependent policies.\footnote{Notice that the natural borrowing limit is function of worker’s age \( n \) and workers’s human capital \( e \), \( l(n,e) \), see Appendix C for details.}

Table 6: Welfare comparisons

<table>
<thead>
<tr>
<th>Economy</th>
<th>Welfare gains (%)</th>
<th>Consum. equiv. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline economy with optimal replacement rate (51%)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Age dependent replacement rate</td>
<td>23.3</td>
<td>0.8</td>
</tr>
<tr>
<td>Age dependent tax rate</td>
<td>68.3</td>
<td>2.4</td>
</tr>
<tr>
<td>Age dependent replacement rate and tax rate</td>
<td>92.4</td>
<td>3.2</td>
</tr>
<tr>
<td>First best economy</td>
<td>100.0</td>
<td>3.4</td>
</tr>
</tbody>
</table>

Decomposing welfare gains. Welfare gains arise because of five different first order effects. There are gains due to better consumption smoothing over the life cycle, to better consumption smoothing across employment states, to a lower incidence of unemployment, to a changing allocation of search effort, and finally there

\[24\]
are production efficiency gains, since the unemployment rate falls among old highly productive workers, which increases output. Production efficiency gains are equal to the expected increase in the present value of output produced by a worker. To measure the contribution of the four other effects, we focus on the expected utility at birth of a fictional worker intended to be representative of a given economy, up to first order effects. Second order effects due to changes in the dispersion of consumption and search effort are measured as a residual. The representative worker is active in the labor market for $\bar{n}_w$ periods and retired in the remaining $\bar{n}_r$ periods of his life. At each age $n$ the worker has a probability $\nu_n$ of being unemployed, equal to the age specific unemployment rate in the economy. If employed, the worker has consumption level $c_n$ equal to the analogous average consumption level in the economy. If unemployed, his consumption level is $c_n (1 - \varphi_n)$, where $\varphi_n$ denotes the average consumption loss upon unemployment at age $n$ in the economy. The mass of people searching is $\delta_n = \frac{\mu_n}{1 - (1 - \delta_n)\bar{\mu}_n}$ and the within period unemployment probability is $\bar{\mu}_n = 1 - f_n$, equal to the average probability of remaining unemployed for a worker searching for a job at age $n$. The utility of the representative worker at birth is set equal to

$$ UR(\tilde{c}, \tilde{\varphi}, \tilde{\nu}, \tilde{\mu}) = \sum_{n=1}^{\bar{n}_w + \bar{n}_r} \beta^{n-1} \left[ (1 - \nu_n) u(c_n) + \nu_n u(c_n(1 - \varphi_n)) + \frac{\delta_n \psi(\bar{\mu}_n)}{1 - (1 - \delta_n)\bar{\mu}_n} \right] $$

which is function of the sequence of consumption $\tilde{c}$, of consumption losses upon unemployment $\tilde{\varphi}$, of the incidence of unemployment $\tilde{\nu}$, and of within period unemployment probabilities $\tilde{\mu}$. The last term in square brackets is set to zero for $n > \bar{n}_w$. We checked that $UR$ approximates reasonably well the utility at birth of the corresponding economy. This is because, after conditioning for age, cross sectional heterogeneity in consumption and search effort is relatively small. We calculate the value of $UR$ in the baseline economy and then measure how this value varies when replacing (one at a time) $\tilde{c}$, $\tilde{\varphi}$, $\tilde{\nu}$, and $\tilde{\mu}$ of the baseline economy with the analogous sequence for the economy with age dependent policies. This measures the gains from better consumption smoothing over the life cycle, from better consumption smoothing across employment states, from a lower incidence of unemployment, and from changing search effort, respectively. The sequence of consumption $\tilde{c}$ from the economy with age dependent policies is scaled down by the size of the production efficiency gains. Gains measures are converted into consumption equivalent units and correspond to percentage increases. The resulting gains are reported in Table 7 both for the economy with age dependent benefits only (in column two) and for the economy where both taxes and benefits optimally vary with age (in column three). In the economy with age dependent benefits only, most gains come from achieving better consumption smoothing across employment states. In the economy with
Table 7: Decomposing welfare gains of age dependent policies

<table>
<thead>
<tr>
<th>Source of gain</th>
<th>Age dependent benefits only</th>
<th>Age dependent benefits and taxes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production efficiency</td>
<td>0.05</td>
<td>0.58</td>
</tr>
<tr>
<td>Consumption smoothing over time</td>
<td>0.11</td>
<td>1.55</td>
</tr>
<tr>
<td>Consumption smoothing across states</td>
<td>0.46</td>
<td>1.07</td>
</tr>
<tr>
<td>Incidence of unemployment</td>
<td>0.12</td>
<td>0.27</td>
</tr>
<tr>
<td>Search effort over time</td>
<td>-0.06</td>
<td>-0.35</td>
</tr>
<tr>
<td>Sum</td>
<td>0.68</td>
<td>3.07</td>
</tr>
<tr>
<td>Residual (second order effects)</td>
<td>0.10</td>
<td>0.11</td>
</tr>
<tr>
<td>Total</td>
<td>0.78</td>
<td>3.18</td>
</tr>
</tbody>
</table>

Notes: Consumption equivalent percentage increases relative to the baseline economy.

age dependent benefits and taxes, there are also important gains from smoothing consumption over the life cycle, which represent almost a 2 percent increase in life time consumption. These gains are smaller but still present also in the economy with age dependent benefits only. This is because young workers use their high UI replacement rates to obtain a smoother consumption profile over the life cycle. As discussed in the next section the magnitude of these gains is affected by the financial constraint $l$. The contribution of the changing allocation of leisure to welfare is negative, since average search effort in the economy increases.

7 Further discussion

We next discuss the robustness of the result that UI replacement rates should generally decrease with age to alternative specifications of the baseline model. We first study the effects of relaxing the borrowing constraint $l$, and then analyze the effects of changing the return to skill. In analyzing the alternative specifications we always re-calibrate the economy to hit exactly the same targets as those discussed in Section 4.3. We also consider a version of the model where the government budget constraint in (24) is age specific so that no income can be redistributed across age groups. Finally we study the role of age dependent severance payments in insuring workers against unemployment risk over the life-cycle.

7.1 Relaxing the borrowing limit

To study the effects of relaxing the borrowing constraint $l$, we multiply its value by a factor of three—so we now have $l = -3.36$. The solid line in Figure 11 characterizes the new optimal profile of age dependent unemployment replacement rates in the economy with constant income tax rates. Replacement rates share the profile of the age dependent policy of the baseline economy, but they are on average smaller for young workers who are now less financially constrained. We have also studied the
welfare gains of optimally choosing age dependent replacement rate and tax rates. These gains are now smaller and are equivalent to an increase in consumption by 2.2%, relative to the economy with an optimally set constant over time replacement rate—which is now equal to 48%. This confirms that in our model an important part of the welfare gains from age dependent policies come from relaxing financial constraints over the life cycle.

Figure 11: Age dependent UI replacement rates with relaxed borrowing constraints

Notes: Profile of UI replacement rates in the economy with optimal age dependent UI replacement rates and constant income tax rates. Solid line corresponds to the economy with relaxed borrowing constraint, dotted line to baseline economy.

7.2 Changing the return to experience

The return to skill accumulated on the job varies substantially for different type of workers. For example, wage increases over the life cycle are substantially larger for workers with a college degree than for high school graduates: roughly speaking college (high school) graduates experience an increase in wage over the life cycle which is twenty percent higher (lower) than in our baseline economy. To analyze the sensitivity of our results to changes in the return to skill, we take the experience function \( w(e) \) with the normalization condition \( w(0) = 1 \), and then set the values of the spline at all age knots to \( 1 + \zeta [w(e) - 1] \). The constant \( \zeta - 1 \) represents a percentage change in the return to labor market experience. To study the effect of changing the return to experience by around twenty percent, we then study the optimal age profile of UI replacement rates in two economies one with \( \zeta = .9 \) and another with \( \zeta = 1.1 \). This provides some preliminary evidence for how the age profile of UI replacement rates should vary for different educational groups.\(^{26}\) We find that results vary little when changing the return to experience: there are always

\(^{26}\)Of course one should be careful in taking education as exogenous since the return to education and thereby the incentive to get educated is affected by labor market institutions. To be sure, here we are not advocating that UI replacement rates should be education specific.
welfare gains from allowing UI benefits replacement rates to decrease with age and the profile of UI replacements rate is also similar across groups, see Figure 12. But notice that a fall in the return to experience leads to a flatter age profile of UI replacement rates and to smaller welfare gains. When the return to experience falls, the government can insure less young workers because the moral hazard problem of UI is more severe. Moreover, with a lower return to experience, young workers are less financially constrained and thereby value less unemployment insurance.

Figure 12: Age dependent UI replacement rates and the return to experience

Notes: Profile of UI replacement rates in the economy with optimal age dependent UI replacement rates and constant income tax rates. Dotted lines correspond to the baseline economy, other lines correspond to economy with lower $\zeta = 0.9$ (as a solid line) and higher $\zeta = 1.1$ (as a dashed line) return to experience.

7.3 Age dependent government budget constraints

The budget constraint in (24) implies that part of the welfare gains of age dependent benefits arise because some tax income is redistributed from old wealthy workers to young poor workers. We now show that this is not the main reason why UI replacement rates should decrease with age. To show this, we study an economy where UI benefit expenditures for workers of a given age are financed through tax income levied just on workers of the same age. This prevents redistributing tax income across age groups.\(^{27}\) Let divide the population in $N$ mutually exclusive age groups with maximum age difference $k$ within the group, so that $Nk = \bar{n}_w$. The set of age levels for the $i$th age group, $i = 1, 2, ..., N$, is given by $\Gamma_i = \{(i - 1)k + 1, (i - 1)k + 2, ..., ik\}$. Income taxes are the sum of two rates, one used to financed UI benefits expenditures for the specific age group, the other to finance expenditures for retirement pensions, so that $\tau_n = \pi_n + \pi_0$. Here $\pi_0$ is the constant over-time income

\(^{27}\)We are thankful to Emmanuel Farhi, Juan Pablo Nicolini, and Robert Shimer for suggesting us this exercise.
The age dependent component of income tax rates $\tau_n$ is constant within its corresponding age group so that $\tau_n = \hat{\tau}_i \quad \forall n \in \Gamma_i$, where $\hat{\tau}_i$ satisfies the following group-$i$ specific budget constraint:

$$\sum_{n \in \Gamma_i} \beta^n \rho_n \chi^u(n, de) = \hat{\tau}_i \sum_{n \in \Gamma_i} \beta^n \int_{R^+} w(e) \chi^e(n, de), \quad \forall i = 1, ..., N \quad (28)$$

This constraint implies that an increase in benefits for a given age group $i$ can not be financed by increasing tax income for another age group. We then search for the age profile of UI replacement rate $\rho \geq 0$ which maximizes worker’s wealth at birth $W_s$ subject to the same constraints as before but where now the tax rates $\hat{\tau}_i$’s $i = 0, 1, ..., N$ satisfy the $N + 1$ government budget constraints in (27) and (28). In solving the problem we consider age groups of five years, $k = 20$.

The resulting optimal age dependent replacement rate under the age specific government budget constraints specified above corresponds to the solid line in Figure 13. For comparison, the optimal age dependent replacement rate of Figure 8 also appears as a dotted line. The UI replacement rate is again generally decreasing in age (at least for workers above twenty five), but, since no intergenerational redistribution is possible, the age profile is now marginally flatter.

Figure 13: Age dependent replacement rate with age specific budget constraint

Notes: The age specific budget constraint is satisfied for non overlapping age groups of five years. Solid line is the age profile of UI replacement rates, dashed line of income tax rates. Dotted line corresponds to the optimal profile of UI replacement rates in Figure 8.

7.4 Severance payments

To insure workers against the risk of wage losses upon displacement it might be useful to include severance payments in the optimal package for unemployment insurance.
We now show that age variation in severance payments helps little in improving welfare relative to the economy with optimal age dependent benefits and taxes. To show this, we extend our baseline economy to allow for age dependent severance payments: upon job displacement workers receive a government transfer equal to $\varsigma_n w(e)$ where worker’s age $n$ and worker’s human capital $e$ here refer to the last period before job displacement occurs. All the other model assumptions remain as in Section 4. The value of being employed in the economy with severance payments becomes equal to

$$V(n, e, a) = \max_{a' \geq a} u(c^e(n, e, a, a')) + \beta(1 - \delta_n)V(n + 1, e + 1, a') + \beta\delta_n J(n + 1, e + 1, a' + \varsigma_n w(e))$$  \hspace{1cm} (29)$$

where the last term incorporates the fact that upon displacement the worker receives a severance pay of $\varsigma_n w(e)$, which increases his wealth at the start of the current job search spell. Nothing else changes relative to (19) and all the other value functions remain as in Section 4, so equations (20)-(23) are left untouched. Of course, the government budget constraint in (24) has to be amended to include severance payments transfers. This yields the following constraint:

$$\sum_{n=1}^{\bar{n}_w} \beta^n \int_{R^+} \rho_n w(e) \chi^u(n, de) + \beta \sum_{n=1}^{\bar{n}_w} \beta^n \int_{R^+} \varsigma_n w(e) \delta_n \chi^e(n, de) = \sum_{n=1}^{\bar{n}_w} \beta^n \int_{R^+} \tau_n w(e) \chi^e(n, de) - \sum_{n=\bar{n}_w+1}^{\bar{n}_r} \beta^n \pi \chi^r(n).$$  \hspace{1cm} (30)$$

The second term in the first row takes into account that a fraction $\delta_n$ of the mass of employed workers of age $n$ and experience $e$, $\chi^e(n, e)$, will be displaced next period and will receive severance payments $\varsigma_n w(e)$.

Here we are interested in studying whether severance payments can improve welfare relative to the economy with optimal age dependent benefits and taxes. So we keep their age profile as given. To be sure, let $\rho_n^*$ and $\tau_n^*$ denote the optimal age profile of benefits and taxes as reported in Figure 10. Here we assume that $\rho_n = \rho_n^*$ and $\tau_n = \tau_n^* + \bar{\pi}$, where $\bar{\pi}$ is needed to keep the government budget constraint in (30) satisfied. We then search for the vectors of severance payments $\varsigma = \{\varsigma_1, \ldots, \varsigma_{\bar{n}_w}\}$ and the value of the tax rate $\bar{\pi}$ that maximize worker’s utility at birth $W_s \equiv J(1, 0, 0)$ subject to the new government budget constraint in (30), workers optimal choices as implied by (20)-(22) and (29), and the feasibility constraint $\rho \geq 0$.

Exactly as in Section 6 we assume that $\varsigma_n$ is a cubic spline at the previously defined ten age knots and search for the value at the knots that maximize $W_s$. When we restrict severance

\[\text{In the economy we also impose the restriction that } \varsigma_{\bar{n}_w} = 0, \text{ since this transfer would just have the nature of a retirement pension.}\]

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payments to be independent of age $\zeta_n = \zeta$, $\forall n$, we find that the optimal constant over age severance payment is $\zeta = 1.4$. This economy yields welfare gains equivalent to a 3.3 percent increase in life time consumption relative to the baseline economy. This is 0.1 percent higher than in the economy with optimal age dependent benefits and taxes. When we allow severance payment to vary with age, we virtually find no additional gains (up to the fourth order). We believe that age variation in severance payments yields small welfare gains because severance payments discourage workers from accumulating precautionary savings and thereby are more distortionary than a combination of UI benefits and subsidies to job creation. Since UI benefits together with labor income tax rates can mimic reasonably well the effects of a subsidy to job creation, age variation in severance payments can play little role in improving welfare in our economy.

8 Conclusions

Unemployed young workers have a high marginal utility from consumption, experience large consumption losses upon unemployment, and they respond little to changes in unemployment insurance benefits. This indicates that unemployment insurance is highly valuable to them while the induced moral hazard problem is mild. Using a life cycle model with unemployment risk and endogenous search effort, we find that under the optimal age dependent policy, replacement rates are increased from the current value of 50 per cent to around 80 percent for workers in their mid twenties and to 60 per cent for workers in their thirties. Workers in their forties and fifties, instead, obtain benefits equal to less than 10 percent of their past wage. Allowing unemployment replacement rates and other government transfers to decline with age yields sizeable welfare gains which amount to around ninety percent of the gains attained under the unconstrained optimal scheme for unemployment insurance over the life-cycle. The quantitative analysis also shows that the age variation in the ratio of the marginal utility of consumption when unemployed to one plus the unemployment elasticity to benefits identifies well the existence of welfare gains from redistributing unemployment insurance over the life-cycle. Results are robust to alternative specifications for the borrowing possibilities of workers, or to changes in the return from accumulating human capital on the job.

We purposely simplified the theoretical analysis in some dimensions. For example, we have assumed that job separation rates are exogenous, while in practice UI benefits affect the outside options of employed workers which can lead to higher separation and higher occupational mobility which we know from Kambourov and Manovskii (2008, 2009) to be higher for the young than for the old. Our modeling of wage losses upon displacement also relies on human capital depreciation during
unemployment. But in practice workers could have accumulated job specific human capital which is lost upon displacement independently of the duration of the future unemployment spell. Allowing for job specific human capital could challenge our conclusion that age dependent severance payments help little in reproducing the welfare gains obtained under the optimal program. Still we believe that our results on the optimal age profile of UI benefits are robust to alternative modeling choices for the process that leads to wage losses upon displacement.

Our analysis suggests that age dependent policies are Pareto-improving when applied just to new generations of workers entering the labor market, but, since policy reforms can not be typically applied just to specific cohorts, the introduction of age dependent labor market institutions might have to deal with important redistributions concerns. In studying age dependent labor market institutions, we have just focused on how unemployment insurance benefits should vary over the life cycle. But the analysis could be well extended to discuss additional features of UI systems (such as benefits duration, maximum benefits level, and eligibility) as well as several other labor market institutions, including policies for employment protection and poverty assistance. Along some of these dimensions it could well turn out that old workers require more protection than young workers do.

Future research should also evaluate the welfare gains of age dependent policies relative to unemployment insurance arrangements different from those currently in place in the US. In particular Feldstein and Altman (1998) and Feldstein (2005) have sponsored the introduction of individual saving accounts to reduce the moral hazard costs of unemployment insurance. The idea is that when employed the worker saves a fraction of his labor income in an individual saving account which the worker uses when unemployed to finance the UI benefits payments dictated by the current US system. At retirement, any residual positive balance is transferred back to the worker. The quantitative welfare gains of savings accounts systems have been studied by Ferrada (2010), Setty (2010), and Pallage and Zimmermann (2010). Our robustness exercise shows that replacement rates should decline with age also when workers face a loose borrowing constraint. Since savings accounts are essentially a means of providing greater liquidity to unemployed workers, this suggests that there should be welfare gains from having unemployment replacement rates decrease with age also in plausible implementations of the saving accounts proposal. This squares well with the conclusions by Setty (2010) who has already introduced elements favoring the young in his proposed implementation of the savings accounts system.
References


