

Financial Markets and Wages

CLAUDIO MICHELACCI

CEMFI and CEPR

and

VINCENZO QUADRINI

University of Southern California, CEPR, and NBER

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We study a labour market equilibrium model in which firms sign optimal long-term contracts with workers. Firms that are financially constrained offer an increasing wage profile: they pay lower wages today in exchange for higher future wages once they become unconstrained. Because constrained firms grow faster, the model predicts a positive correlation between the growth of wages and the growth of the firm. Under some conditions, the model also generates a positive relation between firm size and wages. Using matched employer–employee data from Finland and the National Longitudinal Survey of Youth for the U.S., we show that the key dynamic properties of the model are supported by the data.

1. INTRODUCTION

This paper studies how financial market frictions affect the wage policy of the firm and, through this mechanism, how the dynamics of wages are related to the dynamics of the firm.

We develop a model in which firms sign optimal long-term (implicit) contracts with workers as in Harris and Holmstrom (1982), Wright (1988), and Burdett and Coles (2003).¹ Because of limited enforceability, some firms are financially constrained, forcing them to operate at a suboptimal scale. Under these conditions, the optimal employment contract offered to workers is characterized by increasing wage profiles until the firm becomes unconstrained. By paying lower wages today, the firm implicitly borrows from workers, and this allows the firm to grow faster. Therefore, a key prediction of the model is that the start-up wages of newly hired workers are negatively correlated to the future growth rate of the firm. Another prediction is that the within-job growth of wages is positively correlated to the growth rate of the firm. Using matched employer–employee data from Finland and the National Longitudinal Survey of Youth (NLSY) for the U.S., we find that these two dynamic predictions of the model are supported by the data.

In addition to generating predictions about the relation between the dynamics of the firm and the dynamics of wages, the model also generates a positive cross-sectional relation between firm size and wages, which is a well-known empirical finding. Brown and Medoff (1989) and Oi and Idson (1999) provide a review of the empirical studies. Along this dimension, our paper complements previous theoretical studies, such as Burdett and Mortensen (1998) and Zabojnik and Bernhardt (2001) that also propose explanations for the firm size–wage relation but abstract from financial market frictions. Indeed, despite a large set of empirical regularities about the link between the financial characteristics of firms and their size (see, for example, Fazzari, Hubbard and Petersen, 1988; Gilchrist and Himmelberg, 1996), the importance of financial factors for

1. The relevance of long-term wage contracts is supported by the empirical studies of Beaudry and DiNardo (1991) and McDonald and Worswick (1999).

the firm size–wage relation has not been previously studied in the theoretical literature.² We quantitatively evaluate the importance of financial factors by parameterizing the model to best fit some moments of the distribution of firms and worker turnover in the U.S. We estimate wage regressions on model-generated data similar to those considered in the empirical literature and we find that financial factors explain at least one third of the firm size effect found in empirical studies.

An important theoretical question is why firms are able to (implicitly) borrow from workers beyond what they can borrow from external investors. This is made possible by the fact that part of the accumulated capital is lost if a worker quits. The loss could derive from recruiting costs, training expenses, and/or worker's productivity enhanced through learning. The threat of quitting provides the worker with an (implicit) form of collateral that is not available to external investors. This guarantees that the firm does not renege the long-term wage contract.

The structure of the paper is as follows. In the next section we review the main findings in the empirical literature that are relevant to the questions addressed in the paper. Section 3 describes the basic theoretical framework and characterizes the firm's dynamics. Section 4 extends the model to allow for firm and worker turnover, derives the labour market equilibrium, and studies the importance of financial factors for the firm size effect. Section 5 tests some novel predictions about the relation between the dynamics of firms and the dynamics of wages. Section 6 concludes.

2. EXISTING EMPIRICAL FINDINGS

The paper has two main goals. The first is to characterize the dynamic properties of wages and firms induced by financial market frictions. The second is to show how these dynamic properties reproduce a set of regularities found in several empirical studies, with special attention to the firm size–wage relation. In this section, we describe the existing empirical regularities. Then in Section 5 we provide new empirical evidence about the relation between the dynamics of wages and the dynamics of firms predicted by our model.

1. *Larger firms pay higher wages.* The positive relation between firm size and wages is robust to the introduction of several controls for workers' and firms' characteristics. See Brown and Medoff (1989) and Oi and Idson (1999) for a review. It does not arise just because larger firms employ more skilled workers. Abowd and Kramarz (2000) report that, both in France and in the U.S., the variation in firms' characteristics explains about 70% of the firm size–wage differential.³
2. *Older firms do not pay higher wages.* Doms, Dunne and Troske (1997), Troske (1999), and Brown and Medoff (2003) find that the effect of firm age on wages is positive without controlling for worker's characteristics, but it becomes negative (albeit not significant) after controlling for worker's tenure on the job and firm size.
3. *Fast growing firms pay lower wages.* Bronars and Famulari (2001) and Hanka (1998) report that firm growth (in terms of employment and sales) has a negative effect on wages in a regression that controls for several workers' and firms' characteristics, including firm size.
4. *Firms under financial pressure have lower employment and pay lower wages.* Nickell and Wadhvani (1991) document a negative relation between debt and employment. Other

2. Brown and Medoff (1989) and Oi and Idson (1999) hint a potential link. They conjecture that financial market imperfections increase the cost of capital for small firms and induce them to choose lower capital intensity. In a model with wage bargaining, the lower capital intensity implies that these firms pay lower wages. However, in empirical studies, the firm size effect remains significant even if we control for the capital intensity and the productivity of the firm. As we will show in the next sections, the financial mechanism proposed in our paper does not rely on the capital intensity of the firm.

3. This study revises the previous estimate reported in Abowd, Kramarz and Margolis (1999) that was based on an approximation of the estimation problem.

studies provide some evidence that indicators of financial pressure (such as high debt or low net worth) are associated with lower wages. See Blanchflower, Oswald and Garrett (1990), Hanka (1998), Nickell and Nicolitsas (1999), and Bronars and Famulari (2001).

The first and second findings relate the level of wages to firm size and age. The third finding relates the level of wages to the dynamics of the firm. The fourth suggests that financial factors could be important for wages. In the following sections we show how financial factors affect the wage policy of the firm in a way that is consistent with the above empirical findings.

3. THE BASIC MODEL

We start describing a simple version of the model to illustrate the key dynamics of firms and wages. The analysis of the simple model will be convenient for understanding the properties of the general model studied in Section 4.

Consider a risk-neutral infinitely lived entrepreneur with initial wealth a_0 and lifetime utility $E_0 \sum_{t=0}^{\infty} \beta^t c_t$, where β is the inter-temporal discount factor and c_t is consumption. The entrepreneur has the managerial skills to run an investment project that generates revenues with the Leontief function $y_t = A \min\{K_t/\kappa, N_t\}$, where K_t is the input of capital, N_t the number of workers, and A is a constant. Given the structure of the production function, capital per worker is always chosen to be equal to κ , and without loss of generality we can write the revenue function as $y = AN$. The project is subject to the capacity constraint $N \leq \bar{N}$. In the general model studied in Section 4, the capacity constraint \bar{N} differs across entrepreneurs, and the productivity A is allowed to change stochastically.

The investment per worker κ has two components: fungible investment, κ_f , and worker-specific investment, κ_w .⁴ The first component, κ_f , has an external value and can be resold at no cost. The second component, κ_w , represents the cost incurred by the firm for hiring a new worker. This includes expenses associated with recruiting, training, and the purchase of equipment that cannot be easily used by other workers. This capital is lost if the worker quits or is fired. The total capital accumulated at the end of period t by a firm created at time 0 is $\kappa \sum_{\tau=0}^t n_{\tau}$, where n_{τ} is the number of workers hired at time τ (who start producing at time $\tau + 1$). The output produced by the firm at $t + 1$ is $y_{t+1} = A \sum_{\tau=0}^t n_{\tau}$.

To finance the capital, entrepreneurs sign optimal contracts with *investors* who are risk neutral and discount future payments at rate r . We assume that $\beta \leq 1/(1+r)$ so that internal financing does not dominate external financing. Financial contracts are not fully enforceable as firms can default. In case of default, firms retain their workers (and the worker-specific capital). Furthermore, as in Cooley, Marimon and Quadrini (2004), there is no market exclusion; that is, they can sign a new financial contract with other investors. Investors can only confiscate the fungible resources as specified below.

Workers are infinitely lived with lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t [U(c_t) + \ell_t], \quad U(c_t) = \frac{c^{1-\sigma} - 1}{1-\sigma},$$

where β is the discount factor, σ is the relative risk aversion, c_t is consumption, and $\ell_t \in \{0, \bar{\ell}\}$ denotes the utility of leisure, which is forgone when the worker provides effort. The assumption that there is some forgone utility is relevant only for the analysis of renegotiation studied in Appendix A. In equilibrium, workers provide effort and in the main analysis we impose $\ell_t = 0$.

4. The capital per worker κ can also be thought as a Leontief aggregation of κ_f and κ_w .

Workers do not save and cannot borrow by pledging their future labour income. Therefore, consumption is simply equal to their wages.⁵

When a worker is hired, the firm signs a long-term contract that specifies the whole sequence of wages. The labour market is *competitive*, so the initial lifetime utility provided by the firm to the worker is equal to the utility earned by re-entering the labour market. This utility, denoted by q_{res} , is for the moment exogenous. In the general model it will be derived as the clearing price for the labour market.⁶ Let w_{res} denote the constant wage that provides the reservation utility q_{res} to workers, that is, $\beta U(w_{\text{res}})/(1 - \beta) = q_{\text{res}}$. To guarantee that hiring workers is profitable to the entrepreneur, we assume that

$$\kappa \leq \frac{\beta(A - w_{\text{res}})}{1 - \beta}, \quad (1)$$

which is a natural outcome of a competitive equilibrium with positive workers demand.

We start assuming that firms and workers commit to the long-term contract. In Appendix A we formally discuss why the contract can be supported as a subgame perfect equilibrium of the repeated game played by the firm with each individual worker. The enforceability is made possible by the fact that the firm loses the worker-specific capital κ_w if the worker quits. The threat of quitting then guarantees that the firm never reneges the long-term wage contract.

3.1. The firm's problem

At time 0 the entrepreneur starts the firm by signing a financial contract with investors. The financial contract maximizes the value of the firm for the entrepreneur by choosing in every period the number of new workers, n_t , the dividend payments to the entrepreneur, d_t , the payments to external investors, m_t , and the wages $w_{\tau,t}$ paid to the different cohorts of workers hired at time $\tau \geq 0$. The total employment at time t is $N_t = \sum_{\tau=0}^{t-1} n_\tau$, and the total wage bill is $\sum_{\tau=0}^{t-1} n_\tau w_{\tau,t}$. Given the sequence of wages, we denote by $q_{\tau,t} = \sum_{j=1}^{\infty} \beta^j U(w_{\tau,t+j})$ the end-of-period t lifetime utility promised to workers hired at time τ .

The financial contract must be enforceable; that is, the entrepreneur cannot gain from defaulting. In case of default the firm re-enters the financial market retaining the labour force $\mathbf{n}_t = \{n_0, \dots, n_{t-1}\}$ to which it has promised the lifetime utilities $\mathbf{q}_t = \{q_{0,t}, \dots, q_{t-1,t}\}$. At this point, the net worth of the firm is only $\kappa_w \sum_{\tau=0}^{t-1} n_\tau$ because these are the only resources that cannot be confiscated. The value of defaulting is denoted by $V_t(\mathbf{n}_t, \mathbf{q}_t, \kappa_w \sum_{\tau=0}^{t-1} n_\tau)$, and the optimization problem can be written as follows:

$$V_0(a_0) = \max_{\{m_t, n_t, \{w_{\tau,t+j}\}_{j=1}^{\infty}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t d_t \quad (2)$$

subject to

$$d_t = A \sum_{\tau=0}^{t-1} n_\tau - \sum_{\tau=0}^{t-1} n_\tau w_{\tau,t} - \kappa n_t - m_t \geq 0, \quad (3)$$

5. The assumption that workers do not save is without loss of generality. Because we assume that the return from savings r is smaller than $1/\beta - 1$ and, as we will see, wages do not decrease over time, workers would not save even if they were allowed to. For the general model of Section 4, it is further required that β is sufficiently small.

6. In an alternative to a competitive labour market, we could assume the presence of search frictions. In this case q_{res} would be bargained after the match and it would be affected both by the outside opportunities of the worker and by the firm financial conditions. Although finding the bargaining solution for the initial q_{res} would be quite complex, the optimal long-term contract would have very similar features to those emphasized in the paper once q_{res} has been determined. See Rudanko (2008) for an analysis of a model with search frictions under optimal long-term wage contracts.

$$\sum_{j=0}^{\infty} \beta^j d_{t+j} \geq V_t \left(\mathbf{n}_t, \mathbf{q}_t, \kappa_w \sum_{\tau=0}^{t-1} n_{\tau} \right), \quad (4)$$

$$a_0 + \sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^j m_j \geq 0, \quad (5)$$

$$\sum_{j=1}^{\infty} \beta^j U(w_{t,t+j}) \geq q_{\text{res}}, \quad (6)$$

$$\sum_{\tau=0}^t n_{\tau} \leq \bar{N}, \quad (7)$$

which all have to hold for $t \geq 0$.

Constraint (3) defines the dividend payments to the entrepreneur through the budget constraint. This says that the firm cash flow (equal to the difference between firm revenue and wage payments) can be used to pay dividends, to hire new workers, and to pay investors. The constraint also imposes the non-negativity of dividend payments. Constraint (4) is the enforcement condition for the financial contract with investors. It requires that the discounted value of dividends under the optimal contract cannot be smaller than the value of defaulting. Constraint (5) is the participation constraint for investors. This requires that the discounted value of payments made to investors is not negative. Notice that we use the convention that at time 0 the entrepreneur transfers the whole wealth to investors. Therefore, the net payment received by investors at time 0 is $a_0 + m_0$.⁷ Condition (6) is the worker's participation constraint: the utility value of the sequence of wages paid to a new recruit cannot be smaller than the reservation utility q_{res} . Notice that new workers are hired at the end of the period and they receive the first payment starting in the next period. This constraint should be imposed not only when the worker is hired but also in all future periods. However, as we will see below, wages never decrease. Therefore, if the participation constraint is satisfied when the worker is hired, it will also be satisfied at any future date. The last condition imposes the capacity constraint.

3.2. Some properties

Let b_t denote the value of the financial contract for the investors at time $t \geq 0$. This is equal to

$$b_t = \sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^j m_{t+j}.$$

We will refer to b_t as *firm debt*. The following property, proved in Appendix B, provides a characterization of the enforcement constraint for the financial contract.

Property 1. *The enforcement constraint (4) is satisfied if and only if $b_t \leq \sum_{\tau=0}^{t-1} (\kappa_f + A - w_{\tau,t}) n_{\tau}$ for any $t \geq 0$.*

This property has a simple intuition. By defaulting the firm gains the cancellation of the debt, b_t , but it loses the fungible capital plus the cash flow, that is, $\sum_{\tau=0}^{t-1} (\kappa_f + A - w_{\tau,t}) n_{\tau}$. The

7. This is without loss of generality. Alternatively we could rewrite constraint (5) as $\sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^j m_j \geq 0$ and the budget constraint (3) at time 0 as $d_0 = a_0 - \kappa_n n_0 - m_0 \geq 0$.

property then says that the gain from defaulting cannot be bigger than the loss. Thanks to this property, we can replace constraint (4) with $b_t \leq \sum_{\tau=0}^{t-1} (\kappa_f + A - w_{\tau,t}) n_\tau$, so that there are no unknown functions in the constraints of problem (2).

Let γ_t be the Lagrange multiplier for constraint (3) and $\lambda_t n_t$ the multiplier for the participation constraint (6). Using Property 1, Appendix C shows that the first-order condition with respect to $w_{\tau,t}$ is

$$1 + \gamma_t = \beta(1+r)\lambda_\tau U_c(w_{\tau,t+1}), \quad (8)$$

where U_c denotes the marginal utility of consumption. The variable λ_τ is the marginal cost to the firm of providing one unit of utility to a worker hired at time τ . Thus, $\lambda_\tau U_c(w_{\tau,t+1})$ represents the marginal cost of reducing wages in the next period. The term $1 + \gamma_t$ is the value of one additional unit of internal funds. Therefore, equation (8) says that the optimal wage policy is such that the marginal cost of reducing wages is equal to the marginal value of internal funds. In other words, the firm “borrows” from a worker until the cost of borrowing is equal to the marginal value of internal funds.

The multiplier γ_t captures the tightness of the financial constraints. The following property, proved in Appendix D, characterizes the dynamic features of this variable.

Property 2. *The multiplier γ_t is monotonically decreasing over time.*

Essentially, as the firm retains earnings and reduces the tightness of the financial constraints, the value of internal funds, $1 + \gamma_t$, declines until it becomes 1. Then equation (8) implies

Property 3. *The wage received by each worker grows over time until the firm becomes unconstrained; that is, $\gamma_t = 0$.*

Equation (8) also implies that the ratio of marginal utilities between workers of different cohorts remains constant over time. Consider condition (8) for two cohorts, indexed by τ_1 and τ_2 , and divide side by side. We obtain

$$\frac{U_c(w_{\tau_1,t+1})}{U_c(w_{\tau_2,t+1})} = \frac{\lambda_{\tau_2}}{\lambda_{\tau_1}}.$$

Since the R.H.S. does not change over time, this condition implies

Property 4. *The relative marginal utilities of different cohorts of workers remain constant over time.*

In the next section we use this property to rewrite the problem recursively with a limited number of state variables. The recursive formulation will be crucial to study the general model of Section 4.

3.3. Recursive formulation of the firm’s problem

The lifetime utility promised at the end of period t to a worker hired at time $\tau \leq t$ follows the recursion

$$q_{\tau,t} = \beta[U(w_{\tau,t+1}) + q_{\tau,t+1}]. \quad (9)$$

With the utility $U(c) = (c^{1-\sigma} - 1)/(1 - \sigma)$, Property 4 implies that the ratios of wages paid to workers of different cohorts remain constant over time. This property also implies that the ratios of lifetime utilities promised to different cohorts of workers remain constant. Thus, if

we consider the last and first cohort of workers, their relative wages and lifetime utilities are linked by

$$\left(\frac{w_{t,t+1}}{w_{0,t+1}}\right)^{1-\sigma} = \frac{q_{t,t}}{q_{0,t}} = \frac{q_{\text{res}}}{q_{0,t}},$$

where the last equality uses the fact that the continuation utility of a newly hired worker is the reservation utility, that is, $q_{t,t} = q_{\text{res}}$. From this, the wage ratio between the cohort hired at time t and the cohort hired at time 0 can be expressed as

$$\frac{w_{t,t+1}}{w_{0,t+1}} = \left(\frac{q_{\text{res}}}{q_{0,t}}\right)^{1/(1-\sigma)} = \psi(q_{0,t}).$$

From now on we omit the zero subscript to identify the first cohort of workers (*i.e.* those hired at time 0). Therefore, w_{t+1} and q_{t+1} denote the wage and promised utility at time $t + 1$ of the first cohort of workers. Using the function ψ , the total wages paid by the firm at time $t + 1$ can be written as $H_t w_t$, where $H_t = \sum_{\tau=0}^{t-1} \psi(q_\tau) n_\tau$ evolves recursively according to

$$H_{t+1} = H_t + \psi(q_t) n_t. \tag{10}$$

The variable H_t summarizes the compensation structure of the workers inherited from previous periods.

To specify the problem recursively, we introduce the variable $a_t = (\kappa + A)N_t - H_t w_t - b_t$. This variable denotes the *net worth* of the firm after paying the wages at time t but before hiring new workers and before paying dividends to the entrepreneur. Using the fact that $m_t = b_t - b_{t+1}/(1+r)$, the budget constraint in (3) can be written as

$$d_t = a_t + \frac{b_{t+1}}{1+r} - \kappa N_{t+1}.$$

Furthermore, using Property 1, the enforcement constraint in (4) becomes

$$a_{t+1} \geq \kappa_w N_{t+1}.$$

We can then rewrite the optimization problem recursively as follows:

$$V(a, q, N, H) = \max_{w', q', b', N' \leq \bar{N}} \{d + \beta V(a', q', N', H')\} \tag{11}$$

subject to

$$d = a + \frac{b'}{1+r} - \kappa N' \geq 0, \tag{12}$$

$$a' \geq \kappa_w N', \tag{13}$$

$$q = \beta[U(w') + q'], \tag{14}$$

$$a' = (\kappa + A)N' - H' w' - b', \tag{15}$$

$$H' = H + \psi(q)(N' - N). \tag{16}$$

Constraint (12) imposes the non-negativity of dividends and (13) is the enforcement constraint for the financial contract. Equation (14) is the promise-keeping constraint for the first cohort of workers. Equations (15) and (16) are the laws of motion for the states a and H , respectively.

Let γ and λH be the Lagrange multipliers of constraints (12) and (14), respectively. The first-order conditions, derived in Appendix E, imply

$$1 + \gamma = \beta(1 + r)\lambda U_w', \quad (17)$$

$$\lambda = \lambda'. \quad (18)$$

The first condition is analogous to (8). The second condition simply says that λ is constant over time. Notice, however, that the Lagrange multiplier for the promise-keeping constraint is λH . As the firm grows, the compensation of workers, summarized in H , increases. Therefore, the cost of workers captured by the multiplier increases over time. These two conditions are equivalent to those derived in the previous section and confirms the equivalence between the original and the recursive formulation of the optimization problem.

3.4. A numerical example

Figure 1 shows some of the properties of the model with a numerical example. The parameter values are as follows: $r = 0.03$, $\beta = 0.934$, $\sigma = 1$, $q_{\text{res}} = U(0.6)/(1 - \beta)$, $\bar{N} = 1000$, $A = 1$, $\kappa_f = 0.45$, and $\kappa_w = 2.1$. The initial assets are $a_0 = 300$. The numerical example considered here is provided only for illustrative purposes. A formal quantitative exercise will be conducted in Section 4.3 after the specification of the general model.

Panel (a) in Figure 1 plots the employment dynamics. The firm starts with an initial employment of about 200 workers and then gradually grows until it reaches the optimal size $\bar{N} = 1000$. The transition takes place in seven periods. The panel (b) plots the wage paid to the first and to the last cohort of workers. The continuous line is the wage of the first cohort. This is increasing until the firm reaches the unconstrained status. The dashed line is the start-up wage of each cohort. As the firm gets closer to the optimal scale, it offers higher initial wages, and the wage profile of newer workers is less steep overall. Therefore, growing firms pay wages that are lower initially and grow over time. This is one of the properties that will be tested in the empirical analysis conducted in Section 5.

Panel (c) plots the average wage paid by the firm as a function of its age and panel (d) as a function of its size (measured by the number of employees). The average wage increases with the size and age of the firm. This is a direct consequence of the fact that, when the firm is young and constrained, it operates at a suboptimal scale and offers an increasing profile of wages.

The concavity of the utility function, σ , and the initial assets, a_0 , play an important role in shaping the dynamics of wages. As shown in panel (e), the size dependence of wages is stronger when the utility function is less concave. Clearly, with a smaller σ , workers are more willing to accept a non-flat consumption profile, and it becomes cheaper for the firm to borrow from workers. In the extreme case in which $\sigma = 0$ (linear utility), the financing premium required by the workers is 0. In this case it would be optimal for the firm to pay zero wages until it reaches the optimal scale.

The role of the initial assets is shown in panel (f). For a given \bar{N} , smaller values of a_0 imply tighter initial constraints, and the firm starts with a smaller scale. This also implies that the firm has a greater incentive to rely on the wage policy to finance its growth. As a result, it pays smaller wages initially. As we will see, the dependence of the wage dynamics on a_0 will be important for generating a firm size–wage relation in the general model.

4. GENERAL MODEL AND SIMULATED REGRESSIONS

In the simple model studied so far, the profile of wages is fully captured by the age of the firm. Therefore, once we control for age, firm size becomes irrelevant. However, in a cross-section

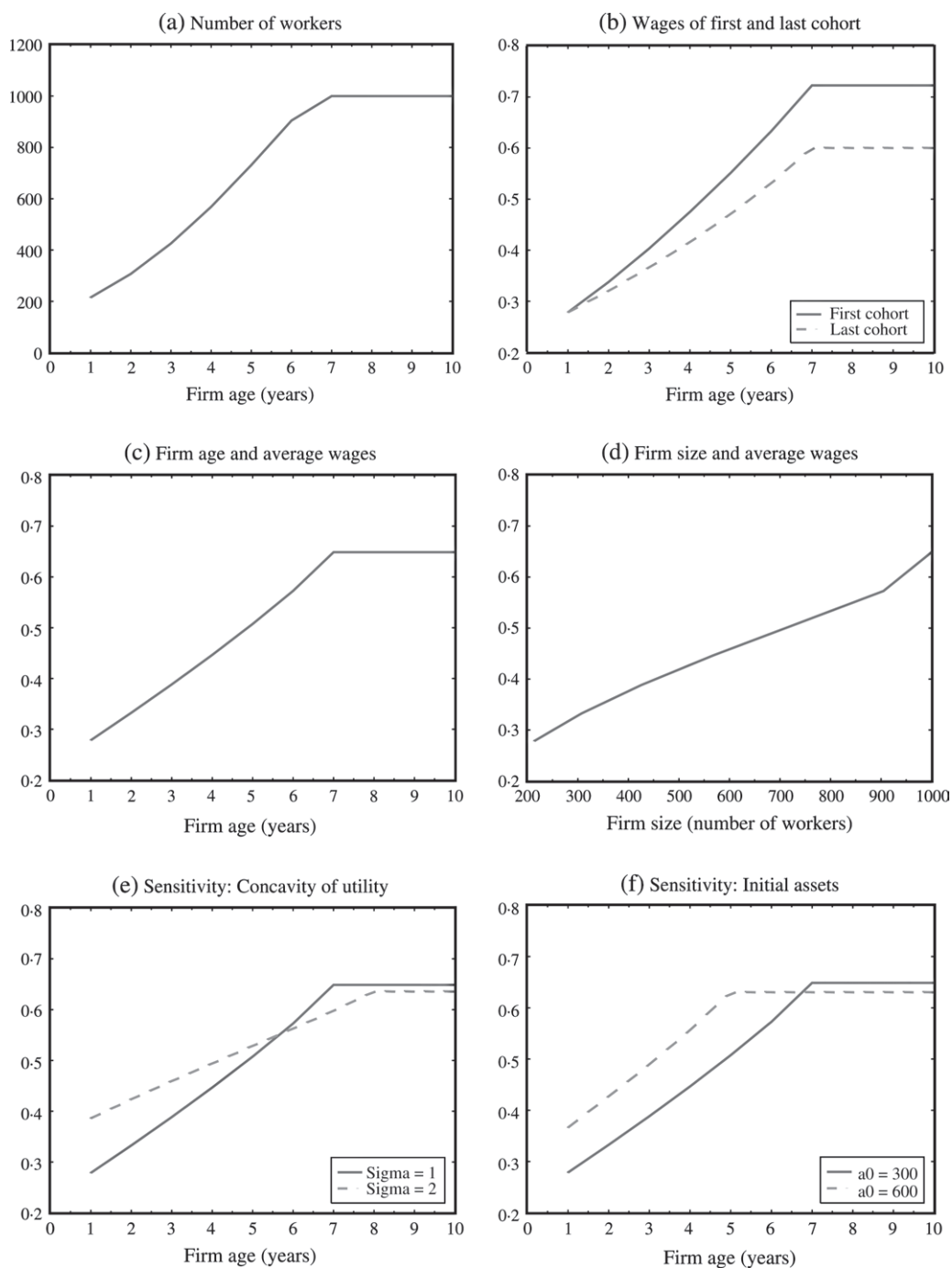


FIGURE 1

Employment dynamics and wage patterns over age and size

of firms, size could have an independent effect because of other sources of heterogeneity. In particular, firms could differ in capacity \bar{N} and in initial assets a_0 . To capture this additional heterogeneity, we need to extend the model and specify the whole industry structure, including entrance and exit.

We extend the model in four directions. We allow for (i) firm heterogeneity in technology \bar{N} and initial wealth a_0 ; (ii) firm entry and exit; (iii) turnover of workers; and (iv) shocks to productivity. The first extension allows us to generate a size distribution of firms similar to the data. The second guarantees that at each point in time there is a fraction of firms that are financially constrained. The third is introduced because workers' turnover affects the quantitative contribution of financial factors in generating the firm size–wage relation. The fourth extension is done for robustness.

4.1. Model description

At each point in time, workers die with probability $1 - \eta$, and firms become unproductive with probability $1 - p$. The workers of exiting firms re-enter the market but without worker-specific capital κ_w . The exit of firms and the subsequent entrance of new firms (entrepreneurs) guarantee that there are always some firms that are constrained.

New entrant firms are heterogeneous in the project capacity \bar{N} and in the initial wealth a_0 . The project capacity remains constant over time while the net worth changes according to the policy of the firm. The project capacity \bar{N} and the initial wealth a_0 are drawn from some joint distribution $\Gamma(\bar{N}, a_0)$.

We also allow for job-to-job mobility by making the following assumptions. In each period a firm matches with m employed workers. These workers can transfer the specific capital, and therefore, they do not require a new investment κ_w . As in Burdett and Vishwanath (1988), the matching technology is balanced in the sense that the number of matched workers is proportional to the size of the firm; that is, $m = \chi N$, where χ is constant. The worker-specific capital can be transferred only to firms with the same characteristics, that is, firms with the same project capacity \bar{N} and age.⁸

After matching with the worker, the firm makes a *take-it or leave-it* offer. Offers are private information and there is a cost to make an offer verifiable to the employer. This assumption implies that the worker is unable to let the current and new employers compete over his or her skills. Anticipating this, the contacting firm offers a contract that gives the worker the same utility q received in the incumbent firm. A formal characterization of this type of equilibrium is provided by Hashimoto (1981) and Anderlini and Felli (2001).⁹

To keep the model tractable we treat each firm as if it employs a continuum of workers. This implies that the death probability $1 - \eta$ is also the fraction of workers who die in an individual firm, and χ is the fraction of workers contacted by other firms.

The final assumption is that the productivity A is stochastic, and it can take a finite number of values. It is further assumed that A is independently and identically distributed across firms and times. The probability distribution is denoted by $\pi(A)$. We limit the analysis to the i.i.d. case because persistent productivity shocks make the problem highly untractable. In particular

8. Alternatively, we could assume that the worker-specific capital can be transferred with some probability to any firm, but this probability is higher for firms with the same characteristics. Assuming that the ability to contact workers is limited, firms will concentrate their search among employers with the same characteristics.

9. To make the offer verifiable to the employer, the worker needs to exercise some effort. The current employer would match the external offer if the worker demands to renegotiate the contract. However, because the renegotiation of the contract requires effort from the worker, the utility from renegotiating is smaller than the utility from accepting the external offer. This holds up the worker from renegotiating. Anticipating this, the expected utility offered by the poaching firm is only infinitesimally higher than the utility that the worker would earn by staying with the current employer.

Property 4 that we use to write the problem recursively may no longer hold when productivity shocks are persistent.¹⁰

4.2. Optimization problem for the general model

Let N be the number of workers employed by a surviving firm. Of those, $(1 - \eta)N$ are lost because of death and χN are lost because of being contacted by other firms. On the other hand, the firm matches with χN workers with transferable skills. Therefore, the net employment loss is $(1 - \eta)N$. By treating the firm as if it employs a continuum of workers, the utilities offered to the contacted workers are equal to the utilities of the workers who quit.

We limit the analysis to steady-state equilibria where the (utility) price for workers without transferable capital, q_{res} , is constant. Even though q_{res} is constant, the problem of the firm is not deterministic. In particular, there is a probability $1 - p$ that firms become unproductive, and the laid-off workers re-enter the market without transferable capital. This is in addition to the possibility that workers switch employer (with probability χ) or die (with probability $1 - \eta$). Furthermore, the productivity of the firm is now stochastic.

The stochastic nature of the firm's problem implies that the optimal sequences of wages and payments to investors are conditional on all the stochastic events affecting the firm. For some of these events, however, the wages delivered by the optimal contract can be easily determined. In particular, when a worker dies, the wages are obviously 0. The same is true when the worker switches to a new employer. Less obvious is the optimal compensation when the firm ceases to be productive and exits. In general, it would be optimal for the firm to insure workers against this event by promising severance payments. The problem is that the promise of these payments is not credible. As shown in Appendix A, the factor that prevents the firm from renegotiating the wage contract is the worker's threat of quitting, with consequent loss of worker-specific capital. When the firm becomes unproductive, however, the capital is already lost. Therefore, there is nothing that prevents the firm from renegotiating. Hence, without loss of generality, we can assume that workers do not receive any payment when the firm exits.¹¹

To write the promise-keeping constraint for the worker, we need first to introduce some notation. The optimal contract determines the whole sequence of decision variables as functions of the history of shocks, that is, $\mathbf{h}_t = \{A_1, \dots, A_t\}$. The history \mathbf{h}_t is conditional on the survival of the firm up to time t . Then the promise-keeping constraint at history \mathbf{h}_t of a worker hired at time τ and still surviving at time t is

$$q(\tau, \mathbf{h}_t) = \beta [U(w(\tau, \mathbf{h}_{t+1})) + \eta p \sum \pi(A_{t+1}) q(\tau, \mathbf{h}_{t+1}) + \eta(1 - p)q_{\text{res}}].$$

In words, the firm pays the wage $w(\tau, \mathbf{h}_{t+1})$ in the next period. After the payment of the wages, workers survive with probability η . In case of survival two events are possible. With probability p the firm survives and promises continuation utilities $q(\tau, \mathbf{h}_{t+1})$. With probability $1 - p$ the firm becomes unproductive, and the worker re-enters the labour market with continuation utility q_{res} . When the firm survives, some of the workers switch employer but they receive the same value $q(\tau, \mathbf{h}_{t+1})$. Therefore, we do not have to take into account the probability χ in the promise-keeping constraint. Notice that the viability of the project and the separation of the

10. The property is derived under the assumption that the participation constraint for workers is only binding when the worker is hired. Although this continues to hold when shocks are i.i.d. (as we show below) there is no guarantee that this still holds when shocks are persistent. Consequently, the history of wages promised to the different cohorts of workers can no longer be summarized with only two variables H and q .

11. In principle, severance payments could be promised by investors as part of the optimal contract. Moral hazard problems, however, limit the insurance that workers can receive against the risk of losing their job.

worker is observed after paying the current wage but before the new investment. Consequently, only the continuation utility is renegotiated, not the current wage.

For the analysis that follows it will be convenient to rescale the promised utility $q(\tau, \mathbf{h}_t)$ by the constant term $\eta(1-p)\beta q_{\text{res}}/(1-\eta p\beta)$, that is,

$$z(\tau, \mathbf{h}_t) = q(\tau, \mathbf{h}_t) - \frac{\eta(1-p)\beta q_{\text{res}}}{1-\eta p\beta}.$$

Using this transformation, the promise-keeping constraint can be written as

$$z(\tau, \mathbf{h}_t) = \beta \left[U(w(\tau, \mathbf{h}_{t+1})) + \eta p \sum \pi(A_{t+1}) z(\tau, \mathbf{h}_{t+1}) \right]. \quad (19)$$

Since the ratios of marginal utilities between different cohorts of workers is constant over time (*i.e.* Property 4 remains valid), the wage ratio between a newly hired worker who requires the investment κ_w and the first cohort of workers satisfies

$$\frac{w(t, \mathbf{h}_{t+1})}{w(0, \mathbf{h}_{t+1})} = \left(\frac{z(t, \mathbf{h}_t)}{z(0, \mathbf{h}_t)} \right)^{1/(1-\sigma)} = \left(\frac{q_{\text{res}}}{z(0, \mathbf{h}_t)} \right)^{1/(1-\sigma)} = \psi(z(\mathbf{h}_t)),$$

where $z(\mathbf{h}_t)$ is the (normalized) utility promised to the first cohort of workers.

As in the previous section, we use the variable H to summarize the compensation structure of the firm, which now evolves according to

$$H' = \eta H + \psi(z)(N' - \eta N), \quad (20)$$

where $N' - \eta N$ is the number of workers hired in the current period who require the investment κ_w . The problem solved by a surviving firm with capacity \bar{N} is

$$\begin{aligned} & V(a, z, N, H) \\ &= \max_{\substack{w(A'), z(A'), \\ b(A'), \tilde{b}(A'), N' \leq \bar{N}}} \left\{ d + \beta \sum \pi(A') [pV(a(A'), z(A'), N', H')(1-p)\tilde{a}(A')] \right\} \end{aligned} \quad (21)$$

subject to

$$d = a + \frac{p \sum \pi(A') b(A')}{1+r} + \frac{(1-p) \sum \pi(A') \tilde{b}(A')}{1+r} - \kappa N' \geq 0 \quad (22)$$

$$a(A') \geq \eta \kappa_w N' \quad (23)$$

$$\tilde{a}(A') \geq 0 \quad (24)$$

$$z = \beta \sum \pi(A') [U(w(A')) + \eta p z(A')] \quad (25)$$

$$a(A') = (\eta \kappa_w + \kappa_f + A') N' - H' w(A') - b(A') \quad (26)$$

$$\tilde{a}(A') = (\kappa_f + A') N' - H' w(A') - \tilde{b}(A') \quad (27)$$

$$H' = \eta H + \psi(z)(N' - \eta N). \quad (28)$$

The constraints are analogous to those in problem (11) except that now firm debt and wage payments are state contingent. If the firm survives the value of firm debt is $b(A')$ while the wage paid to the first cohort of workers is $w(A')$. The tilde denotes variables that are conditional on the exit of the firm. So $\tilde{b}(A')$ denotes the debt if the firm exits while $\tilde{a}(A')$ is the liquidation value received by the entrepreneur. See Appendix F for the derivation of the first order conditions.

The following property, proved in Appendix G, provides some characterization of the optimal contract.

Property 5. *Under the optimal contracts, wages, employment, and net worth do not depend on the productivity shock A' .*

There are two points of special interest in this property. The first is that the independence of wages from the realization of the shock implies that workers are insured against firm-specific shocks. The second point is that the independence of net worth from the realization of the shock implies that firms are insured by investors against random fluctuation in profits. This is obtained by making higher payments to investors after good realizations of A' and lower payments after low realizations of A' . Thanks to the insurance provided by investors, firms are also able to insure their workers. The property that firm-specific i.i.d. shocks do not affect wages finds empirical support in the study of Guiso, Pistaferri and Schivardi (2005).

It is important to point out that Property 5 holds only if productivity shocks are i.i.d. With persistent shocks it may still be optimal to make the payment of wages contingent on the realization of these shocks. The features of the optimal contracts when shocks are persistent cannot be characterized analytically, and the computation becomes highly complex.¹²

Using Property 5, a steady-state labour market equilibrium is defined as

Definition 1. A steady-state labour market equilibrium is defined by (i) policy rules $N(a, z, N, H)$, $b(a, z, N, H)$, $\tilde{b}(a, z, N, H)$, $w(a, z, N, H)$, $z(a, z, N, H)$, and value functions $V(a, z, N, H)$ for each firm type \bar{N} ; (ii) aggregate demand and supply of workers without transferable capital κ_w ; (iii) a market price for these workers q_{res} ; (iv) a distribution (measure) of firms $M(\bar{N}, a, z, N, H)$; and (v) a transition function for the distribution of firms such that (a) for each firm type \bar{N} , the policy rules solve the firm's problem (21) and $V(a, z, N, H)$ is the associated value function; (b) the labour market clears at the equilibrium price q_{res} ; (c) the transition function is consistent with the firms' policies and the distribution of project capacity and net worth of new entrant firms $\Gamma(\bar{N}, a_0)$; and (d) the next period distribution is equal to the current distribution.

4.3. Quantitative analysis

In this section, we study the properties of the model by estimating wage regressions on model-generated data similar to those estimated in the empirical literature. We will show that (i) the model generates a positive firm size–wage relation; (ii) the relation also holds after controlling for the age of the firm; and (iii) fast-growing firms pay on average lower wages. We first describe the parametrization of the model and then we report the results. The numerical procedure used to solve the model is described in Appendix H.

4.3.1. Parametrization. Because the majority of empirical studies looking at the firm size–wage relation use data from the U.S., the parametrization of the model is also based on U.S. data.

The interest rate on secured debt is set to $r = 0.03$ and the inter-temporal discount factor to $\beta = 0.934$. This implies a discount rate for entrepreneurs equal to $1/\beta - 1 \approx 0.07$, which is close to the post-war stock market return in the U.S. The risk-aversion parameter is set to $\sigma = 1$ (log-utility).

12. As previously discussed the recursive formulation of the firm's problem based on the variable H is no longer applicable when shocks are persistent. Moreover, even if we were able to provide conditions that allow us to write the problem recursively with a limited number of states, the dynamics of the firm is not deterministic. Therefore, we would have a dynamic stochastic problem with at least five state variables. With i.i.d. shocks, the problem of the firm still depends on four endogenous states. However, the problem is much simpler because the states evolve deterministically.

The survival probability of workers is set to $\eta = 0.9778$. This corresponds to a working life duration of about 45 years, consistent with the calibration of life-cycle models such as Auerbach and Kotlikoff (1987) and Rios-Rull (1996). The probability of firms' survival p and the matching probability χ determine the flow of workers who re-enter the labour market without transferable capital and the flow of workers with transferable capital who switch employers. We interpret the first group of workers as experiencing a transition from employment to unemployment and the second as a job-to-job transition. We set p to 0.95 and χ to 0.15. The values of p and χ , together with the value of η , imply that about 80% of workers have more than one year of tenure with their employer. This is the approximate number reported by Farber (1999) for the U.S. economy. The large value of χ relative to η and p comes from the fact that job-to-job transitions are more than twice the transitions from employment to unemployment. See Fallick and Fleischman (2001).

The per-worker investments, κ_f and κ_w , are chosen as follows. The fungible capital κ_f is key to determining the debt capacity of the firm. According to the Flow of Funds data for the most recent years 2001–2005, the credit market liabilities in the business sector (corporate and non-corporate) are about 85% of business GDP (see Jermann and Quadrini, 2006). Using Property 1, the total liabilities in the model are given by $b = (\kappa_f + \bar{A} - w)N$, that is, the fungible capital plus the capital income. After normalizing $\bar{A} = 1$, the ratio of total liabilities over output is equal to $\kappa_f + 1 - \bar{w}$, where \bar{w} is the average wage paid in the economy. With the normalization $\bar{A} = 1$ this is equal to the labour share. With a labour share of 0.6 (imposed below) and a debt-to-output ratio of 0.85, $\kappa_f = 0.45$.

To parameterize κ_w we use some findings in the human resource literature. According to Branham (2000), replacing an employee costs on average around 40–50% of the annual compensation per worker. This number is based on the cost of hiring and training a new employee, and it is in line with estimates of turnover costs typically found in the human resource literature.¹³ In our model, the cost of replacing a worker is 0 if the replacement does not require a new investment in worker-specific capital, and it is equal to κ_w if the firm needs to make the investment. The first arises with probability χ and the second with probability $1 - \eta$. Therefore, the average cost of replacing a worker is $(1 - \eta)\kappa_w / (\chi + 1 - \eta)$. According to the estimates in human resource studies, this must be equal to 0.45 \bar{w} , that is, 45% of the annual average wage. Since \bar{w} is the labour share of 0.6, and η and χ have been set above to 0.9778 and 0.15, the resulting value of κ_w is 2.09.

The joint distribution of a_0 and \bar{N} is specified as follows. We assume that the initial wealth is related to project capacity \bar{N} by the equation $a_0 = \alpha \bar{N}^\rho e^\varepsilon$, where α and ρ are parameters and ε is a stochastic variable normally distributed with zero mean and S.D. σ_ε . The project capacity is distributed according to $\Gamma(\bar{N})$. This is a simple way to formalize the idea that the initial wealth of the entrepreneur could be correlated with the project capacity \bar{N} . We assume that \bar{N} can take eight possible values. The values of \bar{N} and the corresponding probabilities $\Gamma(\bar{N})$ are determined jointly with the parameters α , ρ , and σ_ε .¹⁴ In the computation, the variable ε is discretized with three values, $(-\sigma_\varepsilon, 0, \sigma_\varepsilon)$.

We use a simulated method of moments to pin down these parameters. More specifically, we minimize the sum of square errors between specific moments generated by the model and those observed in the data. The moments are the size distribution of new and incumbent firms as

13. The 40–50% cost may be a lower bound for the actual cost of replacing a worker. The Saratoga-Institute (2007) Human Capital Effectiveness Report contains results from over 300 organizations. The total cost of replacing a worker averages 150% of the annual earnings of the worker, with the cost increasing with the skill of the worker.

14. We use this formulation for the joint distribution of \bar{N} and a_0 rather than a joint log-normal, because it does not impose any constraint on the degree of skewness and kurtosis in the initial distribution of firm capacities. We tried to impose a joint log-normal but the model could not fit the thick right tail of the firm size distribution reported in Table 1.

TABLE 1

Size distribution of firms in the U.S. economy and in the model

| Firm size (employees) | Firms (%) | | Employees (%) | | Employees Firms | |
|--------------------------|-----------|--------|---------------|--------|--------------------|----------|
| | Data | Model | Data | Model | Data | Model |
| <i>New firms</i> | | | | | | |
| 1-19 | 95.37 | 94.95 | 53.28 | 56.46 | 3.3 | 3.7 |
| 20-499 | 4.58 | 4.97 | 37.66 | 31.12 | 48.0 | 39.1 |
| 500+ | 0.05 | 0.08 | 9.06 | 12.42 | 1022.7 | 1044.7 |
| Total | 100.00 | 100.00 | 100.00 | 100.00 | 5.8 | 6.2 |
| <i>All firms</i> | | | | | | |
| 1-19 | 87.46 | 87.95 | 17.90 | 16.97 | 4.7 | 4.7 |
| 20-49 | 7.94 | 7.65 | 10.27 | 10.18 | 30.0 | 32.7 |
| 50-99 | 2.53 | 2.36 | 7.43 | 7.79 | 68.4 | 81.1 |
| 100-499 | 1.72 | 1.70 | 14.26 | 14.59 | 192.4 | 210.9 |
| 500-999 | 0.17 | 0.17 | 5.13 | 5.1 | 689.0 | 734.3 |
| 1000-1499 | 0.06 | 0.06 | 3.02 | 3.00 | 1217.4 | 1274.1 |
| 1500-2499 | 0.05 | 0.05 | 3.84 | 3.82 | 1915.8 | 1941.0 |
| 2500+ | 0.07 | 0.06 | 38.13 | 38.45 | 12,074.1 | 15,629.3 |
| Total | 100.00 | 100.00 | 100.00 | 100.00 | 23.2 | 24.6 |

Notes: The first column reports the percentage distribution of firms by size category in the data and in the model, both for new firms and for the all firms population; the second column deals with the percentage of workers in each size category; the third column reports the average number of workers per firm in each size category. Actual data are from Small Business Administration, Office of Advocacy, from U.S. Census Bureau, Statistics of U.S. Businesses, 2001, available at <http://www.sba.gov/advo/stats/data.html>.

reported in Table 1, plus a labour income share of 60%.¹⁵ The table also reports the moments generated by the model, which are very close to those in the data. The labour income share generated by the model is 0.6016 which is very close to the target of 0.6. The estimated parameters are reported in Table 2.

The estimated parameters imply that firms with larger projects face on average higher initial tightness because ρ is smaller than 1. This is a consequence of the fact that the distribution of new firms shown in Table 1 is much more concentrated towards small firms than the distribution of incumbent firms. The model replicates this by imposing tighter initial constraints on firms with higher capacity \bar{N} .

4.3.2. Regression from model-generated data. Using the steady-state distribution of firms, we estimate the following regression

$$\ln(\text{Wage}_{i,j}) = \bar{\alpha} + \alpha_T \text{WorkerTenure}_{i,j} + \alpha_{T^2} \text{WorkerTenure}_{i,j}^2 + \alpha_A \text{FirmAge}_j + \alpha_S \ln(\text{FirmSize}_j) + \alpha_G \cdot \text{FirmGrowth}_j. \quad (29)$$

The index i identifies the worker and j the firm where the worker is employed. This specification is similar to the one used in the empirical literature although we include a smaller set of controls consistent with the structure of the model. The goal of these regressions is to investigate whether the data generated by the model produce estimation results that are similar to those

15. The size distribution reported in Table 1 gives us 20 independent moments. With the addition of the labour income share we have 21 moments to match but only 18 parameters: eight values of \bar{N} , seven probabilities $\Gamma(\bar{N})$, plus α , ρ , and σ_ε .

TABLE 2
Estimated parameters

| \bar{N} | $\Gamma(\bar{N})$ | α | ρ | σ_ε |
|-----------|-------------------|----------|--------|----------------------|
| 4.8 | 0.86131 | 1.621 | 0.686 | 0.086 |
| 33.8 | 0.08760 | | | |
| 85.8 | 0.02498 | | | |
| 223.4 | 0.02142 | | | |
| 749.0 | 0.00253 | | | |
| 1293.4 | 0.00074 | | | |
| 1967.5 | 0.00072 | | | |
| 21,758.6 | 0.00069 | | | |

Notes: Parameters estimates using a simulated method of moments to match the statistics in Table 1 plus a labour share of 60%.

TABLE 3
Wage equation estimation from model-generated data

| Description | (1) | (2) | (3) | (4) | (5) | (6) |
|----------------------------------|---------|---------|---------|---------|---------|---------|
| Constant | -0.5747 | -0.6000 | -0.5405 | -0.5630 | -0.6839 | -0.6688 |
| Worker tenure | 0.0103 | 0.0104 | — | — | 0.0193 | 0.0208 |
| Worker tenure ² /1000 | -0.2741 | -0.3215 | — | — | -0.6456 | -0.6465 |
| Firm age | -0.0017 | — | -0.0014 | — | -0.0017 | — |
| Firm log-size | 0.0116 | 0.0099 | 0.0117 | 0.0101 | 0.0075 | 0.0092 |
| Firm growth | -0.8662 | -0.7541 | -0.9228 | -0.8331 | — | — |
| R^2 | 0.371 | 0.357 | 0.347 | 0.337 | 0.153 | 0.133 |

Notes: OLS estimates from running regression (29) with alternative controls on model-simulated data from steady-state distribution.

obtained in the empirical literature. The results are in Table 3.¹⁶ The first column reports the coefficient estimates when all variables are included in the regression. Of special interest are the coefficients of firm size and firm growth. The estimates for these two parameters are consistent with the findings of the empirical literature. In particular, while the size of the firm has a positive impact on wages, the effect of firm growth is negative. We discuss in detail each of the coefficient estimates.

The firm size effect. To understand the effect of firm size on wages, it is important to take into account that firms with different capacity \bar{N} face different financial constraints initially. This is determined by the parameter ρ estimated to be 0.687. A value of ρ smaller than 1 implies that the financial tightness of new firms increases with \bar{N} . As a result, we have that firms with higher \bar{N} pay a steeper profile of wages. This also implies that these firms pay higher wages than any other firm once they become unconstrained. Because these are also the largest firms, this mechanism generates a positive correlation between firm size and wages. If ρ was equal to 1—implying that all new firms face the same financial tightness—the differences in wages would be fully captured by the age of the firm.¹⁷

16. Since the equation is estimated using the actual-state distribution of firms, t -statistics do not have the usual small sample interpretation and we do not report them.

17. Indeed, if we constrain ρ to be 1 and we control for firm age, the estimated coefficient for the size of the firm becomes 0. On the other hand, the sign of the coefficient for size is not affected by α . The parameter α is important for the coefficient on firm growth.

The effect of firm size is important and comparable to those found in the empirical literature. Brown and Medoff (1989) survey the empirical studies and report estimates of the log-size coefficient that ranges from 0.01 to 0.03. Our estimated coefficient of 0.0116 implies that financial factors can account for at least one third of the firm size effect found in the data. The size differential between the largest size class of firms (those with more than 2500 employees) and the first size class of firms (those with less than 20 employees) accounts for a wage premium of about 9%.

The firm growth effect. The second important result is the negative effect of firm growth on wages. The intuition for this result arises naturally from the discussion above: firms that grow are those with binding financing constraints. Because of the constraints, these firms pay lower wages today in exchange for higher future wages when they operate at the optimal scale. Quantitatively, the estimates of this coefficient are not very different from those found in the empirical literature. Bronars and Famulari (2001) report a coefficient of firm growth that ranges from -0.4 to -0.35 .

Tenure and firm age. The other two variables included in the regression are the worker's tenure and the age of the firm. The positive effect of the worker's tenure derives from the fact that the wages paid by constrained firms increase over time, and, therefore, with the tenure of workers. The return to tenure is smaller than the one estimated by Topel (1991), but comparable to the estimates of Altonji and Shakotko (1987). The estimated coefficient for firm age is negative. However, the sign and magnitude of this coefficient depend on the variables we include in the regression. For instance, if we exclude worker's tenure, the coefficient of firm age decreases and it becomes positive if we also exclude firm size from the regression. In brief, the unconditional correlation between wage and firm age is positive while it becomes negative after controlling for some workers and firm's characteristics. This is consistent with the empirical findings discussed in Section 2.

Financial indicators. In the wage regression size matters for wages because it is an indicator of the firm's financial history. We have also estimated the wage regression with the addition of several financial indicators that are commonly used in the empirical literature. We found that financial leverage, b_t/a_t , debt per worker, b_t/N_t , and net worth per worker, a_t/N_t , have the expected sign. They increase somewhat the R^2 but they affect only marginally the coefficient for the size of the firm. This is because these indicators exhibit very little variability across firms and change little over each firm's financial history. This is in line with the empirical findings by Hanka (1998) and Bronars and Famulari (2001) as discussed in Section 2. There are also some financial variables that completely eliminate the effect of firm size on wages. The financial constraint of the firm in (23) implies that there is a linear relation between employment size and the value of net worth. So when we also control for firm net worth (or firm debt), the coefficient on firm size becomes 0.

These findings are not inconsistent with the data. We have experimented with the Finnish data (described later), which contain information on the financial structure of firms. We find that "leverage" does not reduce substantially the size effect. On the other hand, the total value of firm debt and firm equity do reduce significantly the size effect. We do not report these findings explicitly since controlling for the total debt or the total equity of the firm is like controlling for an alternative measure of firm size.

4.3.3. Sensitivity analysis. Table 4 reports the estimates for alternative coefficients of risk aversion σ . When $\sigma = 0.5$, the firm size–wage effect increases more than 40%. This derives from

TABLE 4
Sensitivity analysis

| Description | (1) $\sigma = 0.5$ | (2) $\sigma = 1.0$ | (3) $\sigma = 2.0$ |
|----------------------------------|-----------------------|-----------------------|-----------------------|
| Constant | -0.5693 | -0.5747 | -0.5499 |
| Worker tenure | 0.0130 | 0.0103 | 0.0063 |
| Worker tenure ² /1000 | -0.3369 | -0.2741 | -0.1677 |
| Firm age | -0.0028 | -0.0017 | -0.0008 |
| Firm log-size | 0.0169 | 0.0117 | 0.0061 |
| Firm growth | -1.6779 | -0.8662 | -0.3815 |
| R ² | 0.542 | 0.371 | 0.255 |

Notes: OLS estimates from running regression (29) on model-simulated data from steady-state distribution for alternative values of the curvature of the utility function σ .

the fact that the cost of offering an increasing wage profile is smaller when the inter-temporal elasticity of substitution is high. Consequently, firms offer a steeper wage profile and the effects of firm size and growth are stronger. The opposite is true when $\sigma = 2$. In the limit case in which $\sigma = \infty$, all firms would pay a constant wage.

5. TESTING FURTHER PREDICTIONS

We have seen that our model is consistent with several empirical findings. In particular, the fact that large firms pay higher wages (Brown and Medoff, 1989) and that, on average, fast growing firms pay lower wages (Hanka, 1998; Bronars and Famulari, 2001). In the model, the wage policy and the dynamics of the firm are both affected by the financial conditions. This interaction generates two specific predictions about the relation between the dynamics of wages and the dynamics of firms which lies at the core of the firm size–wage relation. The first prediction is that there is a positive correlation between the growth of the firm and the within-job wage growth. The second prediction is that start-up wages are lower in firms with higher future growth. We will discuss in detail each of these two predictions and how they will be tested. To our knowledge, they have not been tested before.

5.1. Firm growth and within-job wage growth

The first prediction of the model is based on the first-order condition (8). For the general model this condition is derived by combining conditions (50) and (52) in Appendix F. Assuming a constant elasticity of substitution utility function and taking logs, it can be written as

$$\ln \lambda_{\tau} - \sigma \ln w_{\tau,t} = \ln(1 + p\gamma_t), \quad \forall t > \tau.$$

Evaluating this condition at two points in time and then taking differences side by side, we obtain

$$\ln w_{\tau,t+1} - \ln w_{\tau,t} = \frac{1}{\sigma} \ln \left(\frac{1 + p\gamma_t}{1 + p\gamma_{t+1}} \right).$$

The L.H.S. is the within-job growth of wages and the R.H.S. is the ratio of multipliers at two points in time. This term is increasing in the growth of the firm. To see this remember that the

variable γ_t is the shadow value of internal funds and identifies the tightness of the financial constraints. As the firm retains earnings, the financial tightness is relaxed and the firm grows until γ_t converges to 0. A large decline in γ_t is obtained when the firm experiences a large size expansion and gets closer to the optimal scale. The R.H.S. is then bigger when the firm experiences faster growth. Thus, one key prediction of the model is that within-job wage growth is higher when the firm grows faster. This gives rise to the following testing relation:¹⁸

Test 1. *Individual wages grow faster in fast growing firms.*

To test this relation, we estimate the following equation:

$$\ln W_{ijt} - \ln W_{ijt-1} = \beta_x X_{ijt} + \alpha_{G1} G_{ijt} \quad (30)$$

where W_{ijt} is the real wage income earned by worker i in job j at time t ; X_{ijt} is a set of controls for the worker's and firm's characteristics as described below; G_{ijt} is the yearly growth rate of firm j . We are interested in the sign of the coefficient α_{G1} , which we expect to be positive.¹⁹

5.2. Firm growth and start-up wages

Firms with tight financial constraints pay lower start-up wages in the promise of higher future wages. To see this, consider the first-order condition for a new worker requiring the investment κ_w . This is the condition considered above when $\tau = t - 1$, that is,

$$\ln \lambda_{t-1} - \sigma \ln w_{t-1,t} = \ln(1 + p\gamma_t).$$

As observed above, the financial tightness of the firm is captured by the variable γ_t , which declines over time. As this variable decreases, the L.H.S. term must also decrease. This requires a higher wage. The variable λ_{t-1} also declines. However, it can be proved that the decline in λ_{t-1} is not sufficient to compensate for the decline in γ_t . This can also be seen in the numerical example presented in Figure 1 for the simplified model. Because firms with tighter constraints (higher γ_t) grow faster in the future, start-up wages are negatively correlated with the future growth rate of the firm.

In the general model, some of the new workers are hired with transferable capital. Therefore, their start-up wages depend on the promised utility achieved before switching to the new employer. However, independently of the promised utility previously achieved, the new wages will grow for a longer period of time if the workers are hired by firms with higher γ_t . In fact, a higher γ_t implies that a firm takes longer to reach \bar{N} . For a given lifetime utility, this implies that the start-up wage paid to a newly hired worker is lower. Thus, we have the following testing relation:

18. Although this property has been derived under the assumption that productivity shocks are i.i.d., we conjecture that it also holds when productivity shocks are persistent. Suppose that the productivity of the firm increases persistently. Given the higher productivity, there is an incentive to grow faster. In part this can be achieved by making the investor's payments smaller after the productivity increase. Because this increases the net worth and allows the firm to reach the optimal scale faster, the shadow value of internal funds γ converges faster to 0. This implies that wages grow faster since condition $1 + p\gamma = \beta(1+r)\lambda U_c(w(A'))$ also holds when shocks are persistent.

19. Equation (30) is different from the first difference of a standard wage equation, since the first difference is now taken only for workers within the same job. Indeed equation (30) is borrowed directly from Topel (1991) (see his equation (4)), who stresses that the use of within-job first differences eliminates any worker or job-specific fixed effect that could, in principle, bias the estimates. Notice that we are not interested in separately identifying whether firm growth increases the return to tenure or labour market experience, since, in the extended version of the model, both contribute to wage growth.

Test 2. *Start-up wages are lower in firms with higher future growth.*

To test this relation, we estimate the following equation:

$$\ln W_{ijt} = \beta_x X_{ijt} + \alpha_{G_2} G_{ijt+1}, \quad (31)$$

where W_{ijt} is the real wage income earned by the new hired worker i in firm j at time t ; X_{ijt} is a set of controls for the worker's characteristics as described below; G_{ijt+1} is the yearly growth rate of the employer j . We are interested in the sign of the coefficient α_{G_2} , which we expect to be negative.

5.3. Data sources

We use the Finnish Longitudinal Employer–Employee Data (FLEED) released by Statistics Finland. It contains information on *all* Finnish firms and *all* individuals in the age group 16–70 living in Finland between 1988 and 2002. Appendix I describes the variables and the sample selection.

To investigate whether similar results hold for the U.S., we also use the NLSY, started in 1979 (NLSY79). This is a nationally representative sample of 12,686 young men and women who were 14–22 years old when they were first interviewed in 1979. The detailed description is provided in Appendix I.

We have chosen these two data sets because of their longitudinal structure. This allows us to compute the growth rate in the size of firms and in individual wages. However, there are some drawbacks with the NLSY79 data. One problem is that only the size of the establishment is reported, not the size of the firm. Because we also have information on whether the employer has more than one establishment, we can conduct the analysis on single establishment firms. Another problem is that the size of the establishment is reported by the worker. Thus, the numbers are likely to be plagued by substantial measurement errors. A third problem is that we can calculate the growth rate of the firm only for those workers who remain in the same job for two consecutive years. These problems do not arise in the Finnish data because variables at the firm level are directly reported by the firm, not the employees. However, despite these problems, we obtain consistent results.

5.4. Empirical results

Table 5 reports some descriptive statistics for the variables used in the estimation of equations (30) and (31). Panel A is for the FLEED data and Panel B is for the NLSY79 sample.

We first investigate whether the set of relevant stylized facts documented in the literature for the U.S. also holds in Finland. We find that: (i) the magnitude of the firm size effect is of the same order of magnitude as those found using U.S. data; (ii) fast growing firms pay on average lower wages; and (iii) smaller firms grow faster.²⁰ For economy of space we do not report the detailed findings because they have been investigated in previous studies; see, for example, Nurmi (2004), Appelqvist (2007), Ilmakunnas and Maliranta (2007) and Kyyra (2007). Instead, we focus on the tests that have not been previously conducted in the literature.

20. In a standard OLS wage regression that includes all variables reported in Table 5, plus the square of tenure and age, a full set of year dummies, six industry dummies, and nine educational dummies, the coefficient for the current (logged) firm size is about 0.03. When we also add the current growth rate of the firm and its two lagged values, the sum of the three coefficients of growth is about –0.01.

TABLE 5
Sample statistics

| Variable | Mean | Std. Dev. |
|-----------------------|-----------|-----------|
| <i>A: FLEED Data</i> | | |
| Monthly wage income | 1.54 | 3.78 |
| Male | 0.63 | 0.48 |
| Age | 36.86 | 10.38 |
| Tenure | 6.54 | 8.09 |
| Firm size | 1656.7 | 3969.9 |
| Firm growth | 0.095 | 0.455 |
| Productivity growth | 0.0126 | 0.420 |
| No. of observations | 7,266,473 | |
| <i>B: NLSY79 Data</i> | | |
| Hourly wage | 12.58 | 15.68 |
| Male | 0.45 | 0.49 |
| Black | 0.06 | 0.24 |
| White | 0.88 | 0.32 |
| Experience | 13.64 | 4.95 |
| Tenure | 3.67 | 2.70 |
| Firm size | 52.16 | 260.91 |
| Firm growth | 0.013 | 0.18 |
| No. of observations | 1999 | |

Notes: Panel A (FLEED): Tenure and age in years. Monthly wage income is in local currency divided by the CPI index. The original measure of yearly income is divided by the number of months worked in the year. Firm growth rate is the yearly growth rate of the firm. Productivity growth is the yearly growth rate of the ratio between value added and total employment.

Panel B (NLSY79): Tenure and labour market experience are in years. Weekly tenure is converted into years dividing by 52. Hourly wages are in dollars. White refers to individuals that are neither black nor Hispanic. Firm growth rate is the yearly growth rate of the establishment for single establishment employers.

5.5. Firm growth and within-job wage growth

The top section of Table 6 reports the OLS estimations of equation (30) using the FLEED data.²¹ The basic estimation is reported in column 1. In column 2 we also add firm size. All regressions include the set of standard controls described at the bottom of the table plus the productivity growth measured as value added per worker.

The inclusion of productivity growth is important in two respects. First, we do not have information about hours worked. Therefore, we want to rule out the possibility that workers in fast growing firms experience faster wage growth simply because they work longer hours.²² If this is the case, then the growth in wages should be captured by the growth in the productivity of the firm, given that productivity is measured per worker. The second reason we want to control for productivity growth is because firms may become more productive as they grow. If there is some form of rent sharing, this could imply that growing firms pay growing wages. By controlling for productivity growth we rule out, at least partially, this channel.

21. In the regressions below, a Hausman specification test does not reject the null hypothesis that the fixed effects are uncorrelated with the independent variables at a 5% level. So the OLS estimates are consistent even in the presence of workers' heterogeneity.

22. For example, Belman and Groshen (1998) and Drolet and Morissette (1998) document a positive non-linear relation between firm size and hours per worker. In the NLSY79 we also find a positive relation between growth in hours per worker and firm growth. The coefficient is small (about 2%) and only marginally significant.

TABLE 6
Firm growth and within-job wage growth (Test 1)

| Description | (1) | (2) |
|-----------------------|---------------------|-----------------------|
| <i>A: FLEED Data</i> | | |
| Firm growth | 0.052** (0.0007) | 0.052** (0.001) |
| Productivity growth | 0.007** (0.0004) | 0.007** (0.0004) |
| Firm size | | -0.00005 (0.00007) |
| R^2 | 0.012 | 0.014 |
| No. of observations | 4,576,731 | 4,576,731 |
| <i>B: NLSY79 Data</i> | | |
| Firm growth | 0.087* (0.041) | 0.083* (0.041) |
| Firm size | | 0.004 (0.004) |
| R^2 | 0.016 | 0.017 |
| No. of observations | 1794 | 1794 |

Notes: OLS regression (30) on Finnish data (Panel A) and on U.S. data (Panel B).

Panel A: Finnish data The dependent variable is the within-job growth in the log-monthly real wage income. All regressions include age in level and squared; tenure in level and squared; fifteen year dummies; six industry dummies; a male dummy; and nine education dummies.

Panel B: U.S. data The dependent variable is the within-job growth in the log-hourly wage of workers with at least 35 working hours per week. All regressions include experience in level and squared; tenure in level and squared; 12-year dummies; a male dummy; four region dummies; a dummy for working in a metropolitan area; and 12 industry dummies.

*Significant at 5% level.

**Significant at 1% level.

The coefficient estimates of firm growth, α_{G1} , have the expected sign and are statistically significant at conventional levels. This remains true after controlling for productivity growth and firm size. Therefore, growing firms offer steeper wage profiles as predicted by the model.

The coefficient on firm size is not statistically different from 0. This shows that the impact of firm growth on within-job wage growth is more important than the impact of firm size. It also shows that the wage policy changes as the firm evolves over the life cycle, which is one of the key implications of the model. We find similar results when we restrict the sample to male workers, female workers or firms with less than 500 employees.

The second section of Table 6 reports the estimation results using the NLSY79 data. The basic estimation is reported in column 1. Column 2 adds the current size of the firm as a regressor. The full set of regressors is described at the bottom of the table. The coefficient estimates for the growth rate of the firm, α_{G1} , have the expected sign and are statistically significant at conventional levels. Firm size is not statistically significant as in the Finnish data. This is in line with Hu (2003) who finds that the return to tenure is not clearly associated with firm size.

5.6. Firm growth and start-up wages

To estimate equation (31) we need to observe both the initial wage of the worker in the job and the future dynamics of the firm. This information is readily available in the Finnish data because we have information about the firm for the whole sample period even if the worker quits. In the NLSY79, instead, this information is available only for workers who remain with the same

TABLE 7
Firm growth and start-up wages (test 2)

| FLEED Data | | |
|---|---------------------|---------------------|
| Description | (1) | (2) |
| Firm growth between t and $t + 1$ | -0.016** (0.001) | -0.005** (0.001) |
| Firm growth between $t + 1$ and $t + 2$ | -0.002* (0.001) | -0.006** (0.001) |
| Initial firm size | 0.030** (0.0002) | 0.024** (0.0003) |
| R^2 | 0.28 | 0.29 |
| No. of observations | 1,704,117 | 1,704,117 |

Notes: OLS estimates from running regression (31) on Finnish data. The dependent variable is the log-monthly real wage income of a worker in his first year of tenure. All regressions include age in level and squared; fifteen year dummies; six industry dummies; a male dummy; nine education dummies; and the (logged) firm productivity. In Column (2) we also control for the growth of firm productivity in the two years before and after hiring.

*Significant at 5% level.

**Significant at 1% level.

employer for at least two consecutive years. Because selecting only these workers would reduce the sample size considerably, this test is conducted only with FLEED data.

Table 7 reports the OLS estimation of equation (31), where initial wages are the earnings of workers with less than one year of tenure. The basic estimation is reported in column 1. In column 2 we also control for the dynamics of productivity in the years before and after the hiring date in order to rule out alternative mechanisms whereby the future growth rate of the firm could lead to lower initial wages. All regressions include the set of standard controls described at the bottom of the table.

The coefficient estimates of firm growth, α_{G_2} , have the expected sign and they are statistically significant at the conventional levels. We have also included the initial size of the firm which has a positive impact. This is also consistent with our model because firms that pay lower initial wages are those that are financially constrained and operate at a suboptimal scale (they are small). Therefore, small and fast growing firms pay lower initial wages to their workers. Given the results reported in Table 6, these workers will experience faster wage growth in subsequent periods.

6. CONCLUSION

This paper studies how financial constraints affect the compensation structure of workers. Firms that are financially constrained offer an increasing profile of wages to alleviate the financial restrictions. This allows firms to generate higher cash flows used to finance their growth. The key predictions of the model are tested using longitudinal matched employer-employees data from Finland and the NLSY for the U.S. The results support our theory. In particular, the within-job growth of wages is positively correlated with the growth of the firm and start-up wages are negatively correlated with the future growth rates of the firm.

The model can also generate a positive relation between firm size and wages, which is a well-known empirical finding. There are several theoretical contributions that try to explain why large firms pay higher wages. For example, large firms may employ workers with higher skills or human capital as in Zbojnik and Bernhardt (2001) or in Kremer and Maskin (1996). Others have suggested a theory based on efficiency wages à la Shapiro and Stiglitz (1984) where large

firms pay higher wages because detecting shirking is more difficult. Wage bargaining is another possibility if workers of larger firms have higher bargaining power. These theories capture only part of the relation between firm size and wages. In fact, even after controlling for variables that proxy for these explanations, firm size remains an important determinant of wages.²³ Burdett and Mortensen (1998) and the extension with optimal wage contracts of Burdett and Coles (2003) propose a theory based on wage posting and search frictions whose full implications have not been tested yet. Our paper provides an additional (and complementary) explanation that relies on financial market frictions.²⁴

The centrepiece of our theory is the result that financially constrained firms offer increasing wage profiles, implicitly borrowing from workers. This raises the question of why firms are able to borrow from workers beyond what they can borrow from financial markets. In our model this is possible because workers can use a punishment mechanism that is not available to external investors. An investor can punish the firm only by confiscating the tangible assets. A worker, instead, can punish the firm by quitting. This implies the loss of job-specific investment for the firm and gives the worker a credible punishment tool in the event of repudiation, that is not available to investors.

Indeed, there is both direct and indirect evidence that firms borrow from their employees. In some cases, the borrowing is explicit.²⁵ In others, the loan is implicit in the compensation structure of employees, as in our model. For example, the widespread use of stock options and/or stock grants to ordinary workers, such as middle-run managers, secretaries, and clerks—whose effort, when individually considered, is likely to have a negligible effect on the overall value of the firm—can hardly be justified as a way to provide incentives. This view is also expressed in Hall and Murphy (2003). Most likely, stock options are used to delay the cash compensation of employees and retain more funds in the firm. In accordance with this interpretation, Blasi, Kruse and Bernstein (2003) find that stock options were especially rewarding for workers hired before their companies went public—that is, companies that are more likely to be financially constrained. Also consistent with this interpretation is the finding of Core and Guay (2001) for which the use of stock options is more common in firms that are financially constrained.

In our set-up, stock options and especially stock grants could be substitutes for the promise of future wages. Stock options or grants are also attractive because, as shown in Clementi, Cooley and Wang (2006), they provide a commitment device against the firm's renegotiation of the promised compensation. In more general environments, however, there are limits to the optimality of stock options. With more general firm level uncertainty, stock grants may not be a perfect substitute for the promise of future wages because they impose some additional risk on workers. As long as workers are more risk averse than firms, wages may be preferred to stocks. Also, stocks may be an effective way to borrow from employees only if the firm goes public. In the absence of a liquid market for the firm's stocks, the liquidity risk further reduces the attractiveness of stocks for workers. More generally, while the firm remains private, stock options and wage promises are imperfect substitutes.

23. This is clearly stated in Troske (1999), who concludes: "After testing several possible explanations we are still left with the question: why do large firms pay higher wages?"

24. The Burdett and Mortensen mechanism may not be able to capture the negative correlation between wages and firm growth. In this model, firms face a trade-off between paying high wages to attract and retain a large number of workers or paying low wages but with fewer workers hired and retained. In equilibrium there are firms that pay low wages and remain small and firms that pay high wages and become large. Thus firms that grow faster should be the ones that pay higher wages. However, this is only a conjecture since the firm dynamics generated by this model have not been fully explored.

25. An example is Energy Services Group International, an energy-services engineering and construction company in Williamsburg, VA. The company got a new contract from an electric utility in Florida but it could not persuade banks to lend. Only employees came forward with investments that ranged from \$200 to \$74,000 in exchange for promissory notes. See Inc. Magazine, January 1992, <http://www.inc.com/magazine/19920101/3886.html>.

APPENDIX A. WAGE CONTRACTS IMPLEMENTATION

In the previous sections we have assumed that firms commit to the long-term wage contracts. The assumption of commitment is not an innocuous assumption because promised utilities increase over time until the firm operates at the unconstrained scale. More specifically, a new worker starts with $q_t = q_{\text{res}}$ and, as long as the firm survives, he or she receives $q_{t+j} \geq q_{\text{res}}$ for all $j > 0$. Because new workers can always be hired with initial utility q_{res} , the firm may have an incentive to renege promises that exceed q_{res} . The goal of this section is to discuss the conditions that prevent the firm from renegotiating the long-term contract.

Before continuing, it will be convenient to summarize the timing of the model. First, workers decide whether to provide effort—which has a cost $\bar{\ell}$ in forgone utility—and whether to quit the firm. Then production takes place and the firm observes whether the worker has provided effort. At this point the firm could renege the wage promises. Afterwards, it decides whether to renegotiate the financial contract. Renegotiation entitles the investors to seize the firm assets. After paying workers and investors, the survival of the firm is observed.

Suppose that the worker and the firm follow these strategies (which for simplicity are specified independently of the investors' past history):

- *Worker*: The worker provides effort as long as the firm pays the contracted wages. If one of the two parties has sometimes renege in the past (either the worker has shirked or the firm has paid a wage different from the one contracted), the worker withdraws effort and quits.
- *Firm*: The firm pays the contracted wages as long as the worker provides effort. If one of the two parties has sometimes renege in the past (either the worker has shirked or the firm has paid a wage different from the one contracted), it sets the wage to 0.

The equilibrium associated with these strategies is subgame perfect. To see this, let's consider first the worker. Providing low effort would trigger a wage cut which forces the worker to quit the firm and be left with the reservation value q_{res} starting from the next period. But the utility from doing so, $U(0) + \bar{\ell} + \eta q_{\text{res}}$, is not bigger than the utility obtained from providing effort; that is, $U(w_t) + p\eta q_t + (1-p)\eta q_{\text{res}}$. If the firm has sometimes paid a different wage from the one contracted, quitting is optimal since the firm will pay a zero wage today and in the future.

Consider now the firm. When the firm expects the worker to quit tomorrow, setting the wage to 0 today is always the firm's best response. Thus, given each worker's strategy, paying zero wages is optimal when the worker has sometimes shirked. Along the equilibrium path, the firm never finds it optimal to deviate from the promised long-term contract because, if the firm reneges its wage promises, the worker quits and the firm loses the sunk investment κ_w . Therefore, the assumption that part of the investment is worker-specific, is key to prevent renegotiation.

Of course, there is a limit to this. If the worker's utility becomes very big, the gains from reducing the wage obligations (by reneging the long-term contract and hiring a new worker) could be higher than the loss of sunk investment. This happens if the worker-specific investment, κ_w , and the initial assets, a_0 , are small. In this paper we have implicitly assumed that κ_w and a_0 are sufficiently large so that this never arises in equilibrium.

To show that the non-renegotiation condition is satisfied in the numerical exercises of Section 4.3, Table A1 reports the maximum gains that can be obtained by replacing an existing worker (and paying lower wages afterwards). The maximum gain can be achieved by firms with the largest capacity \bar{N} once they reach the unconstrained scale.²⁶ Denote by w_{max} the wage paid by these firms to the first cohort of workers (who receive the highest wages). These workers could be replaced by new workers with constant wage w_{res} ; that is, the wage that gives the reservation utility $q_{\text{res}} = \beta U(w_{\text{res}})/(1 - \beta\eta)$. By doing so, the firm saves $w_{\text{max}} - w_{\text{res}}$ in each period. The expected discounted value is

$$RG(\mathbf{P}) \equiv \frac{\beta(w_{\text{max}} - w_{\text{res}})}{1 - \beta p \eta (1 - \chi)},$$

where RG stands for Renegotiation Gains and \mathbf{P} are the model's parameters. Notice that the discount factor of future gains is $\beta p \eta (1 - \chi)$ because the firm survives with probability p and the worker with probability η . Moreover, conditional on firm's and worker's survival, the worker does not switch to a new employer with probability $1 - \chi$.

Table A1 reports the renegotiation gains for different curvatures of the utility function. As expected from the theoretical analysis, the renegotiation gains increase as we decrease σ . This is because with a lower σ it is cheaper to borrow from workers and the profile of wages is steeper. The renegotiation gains are compared to the loss of worker-specific capital κ_w , which in the parameterized model takes the value of 2.09. For the baseline parametrization with $\sigma = 1$, the non-renegotiation condition is satisfied. However, this is not satisfied for $\sigma = 0.5$.

26. It can be shown that the maximal promised utility for which the firm does not renegotiate is decreasing in the age of the firm. This together with the fact that the promised utility of workers increases with tenure (until the firm becomes unconstrained), proves that the incentive to renegotiate is the highest when the firm is unconstrained.

TABLE A1
Renegotiation gains for different curvatures of the utility function

| | $\sigma = 0.5$ | $\sigma = 1.0$ | $\sigma = 2.0$ |
|------------------|----------------|----------------|----------------|
| $RG(\mathbf{P})$ | 2.917 | 1.929 | 1.030 |

APPENDIX B. PROOF OF PROPERTY 1

At any point in time the firm can liquidate existing investors by paying the remaining firm debt, b_t , and signing a new financial contract. After the liquidation of previous investors, the net worth of the firm is $\sum_{\tau=0}^{t-1} (\kappa + A - w_{\tau,t})n_{\tau} - b_t$. The value of the new financial contract is equivalent to the value of defaulting with this net worth, that is, $V_t(\mathbf{n}_t, \mathbf{q}_t, \sum_{\tau=0}^{t-1} (\kappa + A - w_{\tau,t})n_{\tau} - b_t)$. The firm does not default if this value is weakly greater than the value of defaulting, that is, $V_t(\mathbf{n}_t, \mathbf{q}_t, \kappa_w \sum_{\tau=0}^{t-1} n_{\tau})$. Therefore, the enforcement condition imposes

$$V_t \left(\mathbf{n}_t, \mathbf{q}_t, \sum_{\tau=0}^{t-1} (\kappa + A - w_{\tau,t})n_{\tau} - b_t \right) \geq V_t \left(\mathbf{n}_t, \mathbf{q}_t, \kappa_w \sum_{\tau=0}^{t-1} n_{\tau} \right).$$

For given \mathbf{n}_t and \mathbf{q}_t the function V is obviously strictly increasing in the third argument (having more net worth must increase the value of the firm). Therefore, the above condition is satisfied if and only if $\sum_{\tau=0}^{t-1} (\kappa + A - w_{\tau,t})n_{\tau} - b_t \geq \kappa_w \sum_{\tau=0}^{t-1} n_{\tau}$. Remembering that $\kappa = \kappa_f + \kappa_w$ and rearranging we get $b_t \leq \sum_{\tau=0}^{t-1} (\kappa_f + A - w_{\tau,t})n_{\tau}$.

APPENDIX C. CHARACTERIZATION OF THE FIRM'S PROBLEM IN SECTION 3

Let γ_t , μ_t , and $\lambda_t n_t$ denote the Lagrange multipliers associated with the non-negativity of dividends, the enforcement constraint for the financial contract (Property 1) and the participation constraints for new workers. The Lagrangian is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \sum_{\tau=0}^{t-1} (A - w_{\tau,t})n_{\tau} - \kappa n_t - m_t + \gamma_t \left[\sum_{\tau=0}^{t-1} (A - w_{\tau,t})n_{\tau} - \kappa n_t - m_t \right] + \mu_t \left[\sum_{\tau=0}^{t-1} (\kappa_f + A - w_{\tau,t})n_{\tau} - b_t \right] + \lambda_t n_t \left[\sum_{j=1}^{\infty} \beta^j U(w_{t,t+j}) - q_{res} \right] \right\},$$

where $m_t = b_t - \frac{b_{t+1}}{1+r}$. For $t \geq 0$, the first-order conditions for $w_{\tau,t+1}$ and b_{t+1} are

$$\lambda_{\tau} U_c(w_{\tau,t+1}) = 1 + \gamma_{t+1} + \mu_{t+1} \tag{32}$$

and

$$1 + \gamma_t = \beta(1+r)(1 + \gamma_{t+1} + \mu_{t+1}), \tag{33}$$

respectively. Using (32) to substitute for $1 + \gamma_{t+1} + \mu_{t+1}$ in (33) yields (8).

APPENDIX D. PROOF OF PROPERTY 2

We prove the property by contradiction, by showing that, if γ_t increases for some t , the policy characterized by first-order conditions is not optimal. Suppose that for some t , $\gamma_{t-1} < \gamma_t$. From (8) this implies that wages fall between t and $t + 1$ for any cohort of workers, that is, $w_{\tau,t} > w_{\tau,t+1}$ for $\tau = 0, \dots, t - 1$. Suppose we reduce wages at time t and increase them at $t + 1$ so as to make the wage schedule flatter. Let's denote the changes by $\Delta w_{\tau,t}$ and $\Delta w_{\tau,t+1}$. Because of the concavity of the utility function, it is possible to find $\Delta w_{\tau,t} < 0$ and $\Delta w_{\tau,t+1} > 0$ such that

$$\Delta w_{\tau,t} + \beta \Delta w_{\tau,t+1} < 0, \tag{34}$$

and

$$U(w_{\tau,t}) + \beta U(w_{\tau,t+1}) = U(w_{\tau,t} + \Delta w_{\tau,t}) + \beta U(w_{\tau,t+1} + \Delta w_{\tau,t+1}), \tag{35}$$

for all $\tau = 0, \dots, t-1$. This means that, after the wage changes, workers remain equally well off, so their participation constraint in (6) remains satisfied. We will next show that these changes in wage policies can be used to improve the entrepreneur's utility. In particular, consider the following policy deviation

$$\begin{aligned}\Delta b_t &= - \sum_{\tau=0}^{t-1} \Delta w_{\tau,t} n_{\tau} \\ \Delta n_{t-1} &= \frac{\Delta b_t}{(1+r)\kappa} \\ \Delta b_{t+1} &= - \sum_{\tau=0}^{t-1} \Delta w_{\tau,t+1} n_{\tau} \\ \Delta n_t &= -\Delta n_{t-1}\end{aligned}$$

while all the other choice variables remain as under the original policy. Because $\Delta w_{\tau,t}$ is negative and $\Delta w_{\tau,t+1}$ is positive, we are increasing the borrowing at time $t-1$ and reducing it at time t , so as to make the enforcement constraint in Property 1 satisfied. With the extra borrowing at time $t-1$, we increase the number of new hires at time $t-1$ by Δn_{t-1} , and we reduce it in the next period by the same amount. In this way total employment increases only at time t . We further assume that the additional workers hired at $t-1$ are paid w_{res} at time t and $w_{t,t+s}$ at any $s > 0$. In other words, after period t , the additional hires receive the same wages as workers hired at time t under the original compensation plan. Note that the participation constraint of the additional hires is satisfied.

All we have to show is that this policy deviation satisfies all the constraints of the firm's problem while it allows the entrepreneur to increase dividends at time t , keeping dividends in all other periods unchanged. Consider first the enforcement constraints at t and $t+1$. With the policy deviation these constraints are

$$\begin{aligned}\sum_{\tau=0}^{t-1} (\kappa_f + A - w_{\tau,t} - \Delta w_{\tau,t}) n_{\tau} + (\kappa_f + A - w_{\text{res}}) \Delta n_{t-1} &\geq b_t + \Delta b_t, \\ \sum_{\tau=0}^t (\kappa_f + A - w_{\tau,t+1} - \Delta w_{\tau,t+1}) n_{\tau} &\geq b_{t+1} + \Delta b_{t+1},\end{aligned}$$

which, given the policy deviation above, continue to be satisfied. Given (3), dividends at time $t-1$, t , and $t+1$ become equal to

$$\begin{aligned}d_{t-1} &= \sum_{\tau=0}^{t-2} (A - w_{\tau,t-1}) n_{\tau} - \kappa(n_{t-1} + \Delta n_{t-1}) - b_{t-1} + \frac{b_t + \Delta b_t}{1+r} \\ d_t &= \sum_{\tau=0}^{t-1} (A - w_{\tau,t}) n_{\tau} - \sum_{\tau=0}^{t-1} (\Delta w_{\tau,t}) n_{\tau} + (A - w_{\text{res}}) \Delta n_{t-1} - \kappa(n_t + \Delta n_t) \\ &\quad - (b_t + \Delta b_t) + \frac{b_{t+1} + \Delta b_{t+1}}{1+r} \\ d_{t+1} &= \sum_{\tau=0}^t (A - w_{\tau,t+1}) n_{\tau} - \sum_{\tau=0}^{t-1} \Delta w_{\tau,t+1} n_{\tau} - \kappa n_{t+1} - (b_{t+1} + \Delta b_{t+1}) + \frac{b_{t+2}}{1+r}.\end{aligned}$$

With the policy deviation defined above, it can be verified that dividends remain unchanged at time $t-1$ and $t+1$, while at t they increase by

$$\begin{aligned}\Delta d_t &= - \left(\frac{1}{1+r} \right) \left[\left(\frac{A - w_{\text{res}}}{\kappa} + 1 \right) \sum_{\tau=0}^{t-1} \Delta w_{\tau,t} n_{\tau} + \sum_{\tau=0}^{t-1} \Delta w_{\tau,t+1} n_{\tau} \right] \\ &> - \left(\frac{1}{1+r} \right) \sum_{\tau=0}^{t-1} \Delta w_{\tau,t} n_{\tau} \left[\frac{A - w_{\text{res}}}{\kappa} + 1 - \frac{1}{\beta} \right] > 0,\end{aligned}$$

where the first inequality uses (34), the second (1) and $\Delta w_{\tau,t} < 0$.

APPENDIX E. FIRST-ORDER CONDITIONS FOR THE RECURSIVE
PROBLEM IN SECTION 3

The Lagrangian can be written as

$$\begin{aligned}\mathcal{L} = & a + \frac{b'}{1+r} - \kappa N' + \beta V(a', q', N', H') \\ & + \gamma \left(a + \frac{b'}{1+r} - \kappa N' \right) \\ & + \beta \mu' (a' - \kappa_w N') \\ & + \lambda H' [\beta (U(w') + q') - q],\end{aligned}$$

where γ , $\beta \mu'$, and $\lambda H'$ are Lagrange multipliers. The problem is also subject to the law of motion for the next period value of a and H , that is, (15) and (16). The first-order conditions are

$$w' : V_{a'} + \mu' = \lambda U_{w'} \quad (36)$$

$$q' : V_{q'} + \lambda H = 0 \quad (37)$$

$$b' : 1 + \gamma = \beta(1+r)[V_{a'} + \mu'] \quad (38)$$

$$N' : \beta [(V_{a'} + \mu')(\kappa + A) + V_{N'} + \psi(q)(V_{H'} - (V_{a'} + \mu')w') - \mu' \kappa_f] \geq (1 + \gamma)\kappa \quad (39)$$

where the last condition is satisfied with equality if $N' < \bar{N}$. The envelope conditions are

$$V_a = 1 + \gamma \quad (40)$$

$$V_q = \beta \psi_q [V_{H'} - (V_{a'} + \mu')w'] (N' - N) - \lambda H \quad (41)$$

$$V_N = -\beta \psi(q) [V_{H'} - (V_{a'} + \mu')w'] \quad (42)$$

$$V_H = \beta [V_{H'} - (V_{a'} + \mu')w']. \quad (43)$$

Equation (17) in the text comes from combining equations (36) and (38). To show that $\lambda = \lambda'$, let us first substitute (36) in (43) and we get

$$-V_H = \beta [\lambda w' U_{w'} - V_{H'}]. \quad (44)$$

Remembering the functional form for the utility function we have $\lambda w' U_{w'} = \lambda (w')^{1-\sigma} = \lambda(1-\sigma)U(w')$. Substituting in (44) yields

$$-V_H = \beta [\lambda(1-\sigma)U(w') - V_{H'}]. \quad (45)$$

Now consider the promise-keeping constraint $q = \beta[U(w') + q']$. Multiplying the L.H.S. and R.H.S. by $\lambda(1-\sigma)$ we get

$$\lambda(1-\sigma)q = \beta [\lambda(1-\sigma)U(w') + \lambda(1-\sigma)q']. \quad (46)$$

Equations (45) and (46) imply

$$-V_H = \lambda(1-\sigma)q \quad (47)$$

$$-V_{H'} = \lambda(1-\sigma)q'. \quad (48)$$

Updating the first term we also have that

$$-V_{H'} = \lambda'(1-\sigma)q'. \quad (49)$$

Condition (48) and (49) then imply that $\lambda = \lambda'$.

APPENDIX F. FIRST-ORDER CONDITIONS FOR THE GENERAL
MODEL IN SECTION 4

Let γ , $\beta p \pi(A') \mu(A')$, $\beta(1-p)\pi(A') \tilde{\mu}(A')$, and $\lambda H'$ be the Lagrange multipliers associated with constraints (22)–(25), respectively. Following similar steps to those in Appendix E, the first-order conditions of problem (21) can be

written as follows:

$$w(A') : 1 + \gamma(A') + \mu(A') = \lambda U_c(w(A')) \quad (50)$$

$$z(A') : V_{z(A')} + \eta \lambda H' = 0 \quad (51)$$

$$b(A') : 1 + \gamma = \beta(1+r)[1 + \gamma(A') + \mu(A')] \quad (52)$$

$$\tilde{b}(A') : 1 + \gamma = \beta(1+r)[1 + \tilde{\mu}(A')] \quad (53)$$

$$\begin{aligned} N' : & \beta \sum \pi(A') [(1 + \gamma(A') + \mu(A'))(\kappa_f + A' - \psi(z)w(A')) \\ & + \eta p(1 + \gamma(A'))\kappa_w + pV_{N'} + p\psi(z)V_{H'}] \geq (1 + \gamma)\kappa, \end{aligned} \quad (54)$$

where the last condition is satisfied with equality if $N' < \bar{N}$. Notice that the above conditions make use of the envelope condition $V_a = 1 + \gamma$. The remaining envelope conditions are

$$V_z = \psi_z(N' - \eta N)\beta \sum \pi(A') [pV_{H'} - (1 + \gamma(A') + \mu(A'))w(A')] - \lambda H' \quad (55)$$

$$V_N = -\psi(z)V_H \quad (56)$$

$$V_H = \eta\beta \sum \pi(A') [pV_{H'} - (1 + \gamma(A') + \mu(A'))w(A')]. \quad (57)$$

APPENDIX G. PROOF OF PROPERTY 5

The first-order conditions for problem (21) are derived in Appendix F. Combining the first-order conditions (50) and (52) we get $1 + \gamma = \beta(1+r)\lambda U_c(w(A'))$, where γ is the Lagrange multiplier for the non-negativity of the entrepreneur's payments. Because γ is the current multiplier, and therefore, it is independent of A' , the next period wage cannot depend on A' . To show that $a(A')$ does not depend on A' it is enough to prove that, if the enforcement constraint $a(A') \geq \kappa_w N'$ is binding for some A' then it must be binding for all A' . Suppose that the enforcement constraint is binding for $A' = A_1$ (so that $\mu(A_1) > 0$) while it is not binding for $A' = A_2$ (so that $\mu(A_2) = 0$). From the enforcement constraint this implies that $a(A_1) < a(A_2)$ and from condition (52) that $\gamma(A_1) < \gamma(A_2)$. In other words, after the realization of $A' = A_1$, the firm has lower net worth than after the realization of $A' = A_2$. However, the shadow value of internal funds, $\gamma(A_1)$, is smaller. This yields a contradiction because the shadow value of internal funds cannot increase with the value of internal funds. To prove that $z(A')$ is also independent of A' , one can use condition (51) and see that the marginal derivative of the value function with respect to the next period value of z is independent of A' (remember that H' does not depend on A' because it is fully determined in the current period). Because all the other state variables in the next period are independent of A' , the independence of $V_{z(A')}$ implies that $z(A')$ is also independent of A' . The final step is to show that future employment choices are not affected by past realizations of the shock. We have already shown that the next period states of the firm are not affected by A' . If the states are not affected, then future choices, including employment, will not be affected either. \parallel

APPENDIX H. COMPUTATION OF THE EQUILIBRIUM OF THE GENERAL MODEL IN SECTION 4

There are two main steps in solving for the steady-state equilibrium. The first step consists of solving the firm problem for a given equilibrium price q_{res} . The second step finds the equilibrium prices q_{res} .

H.1. Solving for the firm's problem

The key conditions that characterize the solution to the firm's problem are

$$d = a + \frac{b'}{1+r} - \kappa N' \quad (58)$$

$$a' \geq \eta \kappa_w N' \quad (59)$$

$$z = \beta [U(w') + \eta p z'] \quad (60)$$

$$a' = (\eta\kappa_w + \kappa_f + A)N' - H'w' - b' \quad (61)$$

$$H' = \eta H + \psi(z)(N' - \eta N) \quad (62)$$

$$1 + \gamma' + \mu' = \lambda U_c(w') \quad (63)$$

$$1 + \gamma = \beta(1+r)\lambda U_c(w') \quad (64)$$

$$\beta[\lambda U_c(w')(\kappa_f + A - \psi(z)w') + \eta p(1 + \gamma')\kappa_w + (1 - \sigma)\eta p\lambda(\psi(z') - \psi(z))z'] \geq (1 + \gamma)\kappa. \quad (65)$$

Conditions (58)–(62) are the budget constraint, the enforcement constraint, the promise keeping and the laws of motion for the next period states a and H , respectively. Conditions (63) and (64) are obtained by manipulating the first-order conditions (50), (52), and (54) after using the envelope conditions and the other first-order conditions.

Given q_{res} , and, therefore, z_{res} , the problem of a firm with capacity \bar{N} and initial states a_0 , $z_0 = z_{\text{res}}$, $N_0 = 0$, $H_0 = 0$, is solved as follows:

1. Guess λ and γ_0 .
2. At each point $t = 0, 1, 2, \dots$ solve for γ_{t+1} , μ_{t+1} , b_{t+1} , w_{t+1} , a_{t+1} , z_{t+1} , N_{t+1} and H_{t+1} using conditions (58)–(65) with the equality sign after imposing $d_t = 0$. This yields eight conditions in eight unknowns. Stop solving when $N_{t+1} \geq \bar{N}$.
3. Denote by T the period when the previous step was stopped. Therefore, $N_{T+1} \geq \bar{N}$. If at $t = T$, $\gamma_T = 0$, and $z_T = z_{T+1}$ the algorithm has converged. Otherwise, restart the procedure from step 1 until convergence is achieved.

H.2. Labour market equilibrium

To compute the labour market equilibrium we start by guessing the equilibrium value of z_{res} . Given this value we solve for the firm's problem as described above. After finding the invariant distribution of firms, we compute the aggregate demand of labour and check the clearing condition in the labour market. We update z_{res} until the labour market clears.

APPENDIX I. DATA APPENDIX

I.1. Finnish longitudinal employer–employee data

We use the sample of full-time employees in the age group 16–65. We eliminate those for which data on personal characteristics and/or wages are missing. The final sample includes 7,266,473 observations, corresponding to more than one million individuals. Following is a description of the variables we use in the analysis.

Gender Dummies. This is the variable SP in the survey.

Age. This is the variable IKA.

Firm Size. This is TPHENK from the financial statement of the firm.

Industry Dummies. This is the variable TAY. There are six industry groups: (1) Manufacturing; (2) Construction; (3) Trade, Hotels and Restaurants; (4) Transportation, Storage and Telecommunications; (5) Business Services and the Financial Sector; (6) Otherwise.

Firm Productivity. This is the ratio of the firm's value added (variable JAL) and the firm size. Value added is the sum of corrected operating profits, wages and salaries, and other personnel expenses.

Employer Tenure. Difference between the current year and the year when the employment relationship started (variable ALKU3).

New vs. Continuing Jobs. FLEED explicitly reports whether the worker has changed employer.

Firm Growth Rates. Log-change in firm size.

Monthly Wage Income. FLEED reports the total yearly wage income of the individual. When the worker is employed for the whole year, the monthly wage is the yearly wage income divided by 12. When the worker experiences unemployment, the monthly wage is determined by dividing the yearly wage income by the total number of months in employment.

Education Dummies. There are nine education groups: (1) pre-primary education; (2) primary education; (3) lower secondary education; (4) upper secondary education; (5) post-secondary and non-tertiary education; (6) lowest level tertiary education; (7) lower-degree level tertiary education; (8) higher-degree level tertiary education; and (9) doctorate or equivalent tertiary level.

I.2. National longitudinal survey of youth

We focus on a sample of 6111 individuals designed to be representative of the non-institutionalized civilian segment of the U.S. young population. We consider only the 13 more recent waves, from 1986 to 2002, because the size of the establishment is not always reported in the previous waves. NLSY79 contains information only on the size of the establishment, not the size of the firm. We also know, however, whether the employer has more than one establishment. To make sure that the size of the establishment is a good measure of the size of the firm, we select only workers employed in single establishment firms. The sample is restricted to full-time workers (working a minimum of 35 hours per week) with reliable data on wages and with positive labour market experience. We also restrict the sample to observations for which the annual growth rate of the firm is smaller than plus or minus 50%. This eliminates outliers that are likely the results of measurement errors. This leads to our final sample of 1991 observations for 771 individuals. Following is the description of the main variables.

Regional Dummies. There are four dummies constructed from the variable “Region of current residence”.

Schooling. This is the “Highest grade completed as of May 1 survey year”.

Experience. Age of worker at interview date, minus years of schooling, minus six.

Working Hours. Until 1993 the number of working hours per week is obtained from the variable “Hours per week usually worked at current/most recent job”. Starting in 1994, job 1 always coincides with the Current Population Survey (CPS) job and information about working hours is obtained from the variable “Hours per week worked at job 1”.

Metropolitan Area. This is obtained from the question “Is respondent current residence urban/rural?”.

Establishment size. Until 1993, this is the “Number of employees at location of current job”. Starting in 1994 we use “Number of employees at location of job 1”. We set to missing values observations with reported values of 99,995 or 99,996.

Multiple Establishments. Until 1993, the number of establishments is obtained from the question “Does employer at current job have greater-than-one location?”. Starting in 1994, we use the question “Does employer at job 1 have greater-than-one location?”

Industry Dummies. Until 1993 the industry dummies were constructed by using the variable “Type of business or industry of most recent job (Census 3 digit)”. Starting in 1994 we used the variable “Type of business or industry job 1 (Census 3 digit)”. From these variables we constructed 12 industry dummies.

Hourly wage. Until 1993 the hourly wage in dollars is obtained from the variable “Hourly rate of pay current job”. Starting in 1994 we used the variable “Hourly rate of pay of job 1”. To eliminate obvious data entry errors we drop observations whose hourly wage is greater than \$500 or less than half the minimum wage.

Employer tenure. This is obtained from the five variables “Total tenure in weeks with employer job 1 (2, 3, 4, 5)”. We then identify whether job 1, 2, 3, 4, or 5 corresponds to the CPS job by using the questions “Internal check: is job 1 (2, 3, 4, 5) the same as current job?”. After 1993 the CPS job corresponds to job 1.

New vs. Continuing Jobs. To identify whether the current CPS job is a new or a continuing job, we follow the procedure detailed in appendix 9 of the user’s guide.

Firm growth rates. Log-change in establishment size.

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