Entrepreneurial Innovation, Patent Protection, and Industry Dynamics *

Gerard Llobet
CEMFI

Javier Suarez
CEMFI, CEPR, and ECGI

May 2009

Abstract

We assess the effects of imitation and intellectual property (IP) protection in a model of industry dynamics in which the value of IP is eroded by further innovations and imitations. Innovations result from the development of ideas engendered by entrepreneurs. We find that innovation and welfare are decreasing in the protection of IP against further innovations, while their relationship with the protection against imitations typically has an inverted-U shape (partly because imitation reduces the resistance of incumbents to innovators). We also find that the welfare gains from increasing IP protection increase if entrepreneurs are financially constrained.

*We thank Sudipto Bhattacharya, Michele Boldrin, Catherine Casamatta, Corinne Langinier, Carlos Serrano, participants at the Society for Economic Dynamics Meetings 2008, the 2nd Ricafe2 Conference, the 9th Conference on Innovation and Intellectual Property in Financial Services, and the Simposio de Análisis Económico, and seminar audiences at Boston University, CEMFI, Universidad Carlos III, Universidad de Navarra, Universidad de Murcia, Universitat Autònoma de Barcelona, Universitat de València, and Universitat Pompeu Fabra, for their useful comments. Financial support from the European Commission (grant CIT5-CT-2006-028942) and the Spanish Ministry of Science and Innovation through the Consolider-Ingenio 2010 Project "Consolidating Economics" is gratefully acknowledged. Address for correspondence: CEMFI, Casado del Alisal 5, 28014 Madrid, Spain. Tel: +34-914290551. Fax: +34-914291056. Email: llobet@cemfi.es, suarez@cemfi.es.

1 Introduction

Intellectual property (IP) protection and the importance of the financial constraints faced by independent high-tech start-ups are important themes in academic and non-academic discussions on the trade-offs relevant for the design of a welfare-promoting innovation policy. Some of these trade-offs are genuinely dynamic and far from obvious. First, protecting an innovation engendered today may discourage the generation of further innovations tomorrow. Second, while it is clear that imitation hurts the owners of existing IP, it speeds up the process whereby consumers appropriate the surplus of prior of innovations. Third, as we emphasize in this paper, imitation facilitates the entry of new innovators through the weakening of the opposition coming from existing IP holders. Finally, financial constraints elevate the hurdle for independent innovators, reducing the innovation flow and, hence, the threat on existing IP, which alters the trade-offs relevant for the design of an optimal innovation policy. This paper presents a model of industry dynamics that allows us to compactly formalize and analyze these trade-offs.

We consider an industry made up of a continuum of business niches; each niche can be thought of as a market for a distinct product. The successful developers of new versions of each product ("new products") contribute to welfare and appropriate temporary monopoly profits like in a standard quality ladder model with limit pricing. These temporary monopolies are based on the protection granted by IP, and are threatened by the entry of the developers of even newer products, as well as imitators. The success of the developers of new products is compromised by the competition coming from other contemporaneous developers and by the opposition of incumbent monopolists, who use their IP to fight imitators and innovators alike.[1] We assume that IP provides incumbent monopolists with (random) protection against them, thereby affecting their survival as monopolists and the barriers faced by their potential challengers.[2]

[1]Formally, we model the generation of new products as an uncoordinated costly-entry process initiated by entrepreneurs and subject to congestion. From the perspective of a niche (and its occupants), innovation and imitation in our model are random (albeit endogenous)arrival processes.

[2]This modeling allows us to abstract from the traditional distinction between patent length and patent

An important feature of this model is that we assume that in non-monopolized niches the hurdle for innovative entry is lower than in monopolized niches, since in the former the incumbents have both less incentives and less capability to defend their IP. These differences can be due to at least three reasons. First, with competing incumbent producers, it tends to be easier and cheaper for the entrant to obtain a license for one of the substitute technologies. Second, previously successful imitation may signal that the patent of the previous monopolist was invalid or had expired, in which case his resistance would lack legal support. Finally, to the extent that court damages due to patent infringement tend to be related to foregone profits, the entrant may expect to reach a more favorable settlement with incumbents when the pre-entry profits in a niche were low.[3]

Adjusting IP protection in this setup involves several non-trivial dynamic trade-offs. The stronger protection of IP against future innovation implies a larger expected duration of the monopoly obtained by the developers of IP who enter successfully, but it also implies a stronger protection of the incumbents and, hence, a higher hurdle for successful entry. From the perspective of the incentives of a potential innovator, reducing the hurdle for successful entry and lengthening the duration of the monopoly obtained afterwards are substitute forms of rewarding the required investments. We find, however, that the former is a more effective means of increasing the steady-state rate of innovation and our measure of social welfare. This effect is essentially due to discounting.

Stronger protection of existing IP against imitation also lengthens the expected duration of the monopoly obtained by successful innovative entrants but has no direct effect on their entry hurdle. There is, however, an indirect effect on the entry hurdle associated with the fact that, at the industry level, increasing the protection against imitation increases the steady-state fraction of business niches monopolized by IP holders, which increases the effective opposition faced by later entrants across all possible niches. We find that this trade-off often leads to an interior level of protection against imitation that

breadth; see Scotchmer (2004) for a review of the treatment of these issues in the literature.

[3]The evidence in Cockburn and MacGarvie (2006) for the software industry is consistent with these views.

maximizes the steady-state values of the innovation rate and welfare. We show that this result holds even when imitation is *endogenized*, that is, when it is modeled as the result of a costly and risky entry process similar to the one that leads to innovative entry.

In a natural extension of the baseline model, we consider the case in which innovating entrepreneurs face financial constraints at the stage in which they need to develop their innovative ideas into new marketable products. A simple moral hazard problem affects the relationship between entrepreneurs and their external financiers, as in Holmstrom and Tirole (1997). However, instead of considering the development investments as part of a single non-transferable investment project, we assume that each entrepreneurial innovation has a continuum of possible development paths that are patented and can be separately developed by either the entrepreneur or a licensee (after incurring some cost associated with transferring the relevant knowledge). We find that, in spite of the inefficiency due to the knowledge transfer costs, there are circumstances under which it is optimal for entrepreneurs to relax their financial constraints through the partial out-licensing of their innovations. Licensing some innovation paths has the double effect of providing entrepreneurs with royalty income and reducing the size of their in-house development investments, which altogether means that entrepreneurs operate with higher internal financing ratios and, hence, less severe moral hazard problems vis-a-vis external financiers.[4]

This extension of the baseline model yields the third key result concerning IP protection. If financial constraints get tighter, IP protection should increase. The intuition for this result is the following. Financial constraints push innovating entrepreneurs into costly transfers of technology and, thus, make innovation less profitable. At an industry level, innovation falls, losing importance relative to imitation. As stressed in this paper,

[4]Other motivations for licensing considered in the literature include the strategic concerns that shape the patent licensing contract (see, for instance, Kamien (1992), Muto (1993), and Wang (1998)), the disclosure strategy of the inventor (Anton and Yao (2002), Bhattacharya and Guriev (2006)), and various dimensions of the competition and cooperation among patent holders with complementary innovations (such as cross-licensing agreements in Fershtman and Kamien (1992), patent pools in Lerner and Tirole (2004), and royalty staking in Lemley and Shapiro (2004)).

however, innovation and imitation are complementary, so this change renders imitation socially less desirable and, hence, shifts the relevant policy trade-offs towards an increased level of IP protection.

The increase in IP protection and the development of alternatives for the financing of high-tech start-ups (most notably, venture capital) are typically listed among the factors that facilitated the unprecedented prosperity of innovative entrepreneurial activities during the 1990s and early 2000s.[5] Although the impact of each of these factors has been the object of intense debates in separate strands of the literature, their interaction has been, as we explain next, largely unexplored.

Many papers focus on the effectiveness and adequacy of IP protection in the United States. It is commonly understood that the creation of a unique Court of Appeals of the Federal Circuit in 1982 strengthened the position of patent holders against potential infringers, and that other legislative changes, such as the 1984's Semiconductors Act or the extension of patent duration to 20 years, also reinforced the protection of IP. However, there is considerable empirical and theoretical controversy on whether these reforms actually promote innovation.[6] Some quantitative assessments based on US data conclude that higher protection would induce more innovation (Denicolò (2007)), while others suggest that the recent reforms might have actually reduced innovation (Levin et al. (1985), Hall and Ziedonis (2001)). Practitioners express their doubts regarding the role of IP protection by referring to issues such as the "tragedy of the anti-commons" that deems strategic patenting and patent stacking as obstacles to innovation (Heller and Eisenberg (1998)). At a theoretical level, although earlier literature, starting with Nordhaus (1969), emphasized the positive welfare implications of patent protection, recent papers such as Hunt (2004) or Bessen and Maskin (2006) suggest that in the context

[5] The list of contributing factors also includes the opportunities and technological changes associated with the information technology revolution, the reduction in setup costs and other barriers to the creation and development of new firms, and the emergence of a new "entrepreneurial culture" that has increased the social recognition and other rents associated with being and succeeding as an entrepreneur; see Greenwood and Jovanovic (1999), Kortum and Lerner (1998), and Kortum and Lerner (2000), among others.

[6] See Gallini (2002) for a review of the reforms and their effect on patenting activity.

of sequential innovation, patent protection is socially less desirable, especially in very innovative sectors. To the best of our knowledge, papers in this tradition do not explicitly deal with financial constraints and, hence, also ignore the possibility that changes in the importance of these constraints over time or across industries might qualify the expected effects of IP protection.

Regarding the financing of innovative start-ups, the importance of the access to informal sources of capital (such as private equity financing from friends and relatives, or from *business angels*) and venture capital is widely admitted, since these start-ups typically lack the collateral required for the access to more conventional financing sources (such as bank loans).[7] It is also admitted that the availability and degree of sophistication of these sources of capital (and, hence, the incidence of financial constraints) may vary notably across industries, countries, and time periods.[8] Most papers on entrepreneurial financing consider the traditional partial equilibrium setup of corporate finance and focus on understanding microeconomic issues such as the staging of finance (Gompers (1995) and Neher (1999)), the use of convertible securities (Casamatta (2003) and Schmidt (2003)), or optimal contracting when venture capitalists play an advising role (Repullo and Suarez (2004)). Some papers, including Holmstrom and Tirole (1997), Inderst and Muller (2004), and Michelacci and Suarez (2004), examine the equilibrium implications of financial constraints, but make no explicit reference to IP protection.

Finally, papers such as Aghion et al. (2001) or O'Donoghue and Zweimüller (2004) analyze the implications of IP protection from the perspective of the endogenous growth models.[9] The first studies R&D competition in a quality ladder model where each good is sold by two firms. These firms have a different innovative level that may improve by investing in R&D or in the case of a laggard, imitating the leader. The authors focus on

[7]See Gaston (1989), Gompers (1999), and Gompers and Lerner (2001).

[8]For instance, it has been argued that the lower development of European private equity markets may be the reason why Europe lags behind the US in terms of entrepreneurship and innovation (see Bottazzi and Da Rin (2002)).

[9]Papers in this tradition also include Grossman and Lai (2004), who study the effect of intellectual property on international trade, and Boldrin and Levine (2006), who measure the optimal degree of IP protection in a growth setup.

the protection against this imitation and show that, as in our paper, it has an inverse U-shaped effect on growth. The logic behind their effect is, however, very different. In Aghion et al. (2001), imitation helps backward firms to catch up with industry leaders who, in response, may increase their investment as a way to scape from the ensuing competition. With too much imitation, though, leadership is too short-lived, which discourages innovation. O'Donoghue and Zweimüller (2004) study the effect on growth of various aspects of patent policy (i.e. leading breadth and patentability requirements) in a general equilibrium context. Their paper emphasizes demand effects and effects associated with the reallocation of resources across sectors which are typically neglected in partial equilibrium models. Our work differs from the papers in this tradition in that we are interested in the effects at the industry level and do not attempt to provide a proper general equilibrium model with endogenous growth.

Our paper is mainly a contribution to the microeconomic literature on IP protection, although we take elements from the literature on start-up financing in order to model the financial constraints potentially faced by the innovating entrepreneurs. In relation to the IP literature, we contribute a richer analysis of the dynamic linkage between imitation and innovation, as well as a first explicit analysis of the effects of financial-constraints and their implications for the socially optimal level of IP protection. Our analysis highlights industry equilibrium effects that establish bidirectional relationships between the blanket of overlapping IP claims (that arise from the interaction between innovation, imitation, and financial constraints) and the innovation process (affected by disincentives created by the blanket of existing IP claims and the incentives that new IP protection might offer).

The rest of the paper proceeds as follows. Section 2 introduces our baseline industry dynamics setup. Section 3 analyzes its equilibrium and steady-state properties. Section 4 incorporates financial constraints in the analysis. Section 5 explores the welfare implications of IP protection and discusses optimal IP policies. Section 6 shows that the robustness of the results in the more general setup where imitation is endogenous. Section

7 concludes. All proofs are relegated to the Appendix.

2 The Model

Consider an infinite horizon, discrete time model of an industry. All agents are risk-neutral and the industry consists of a measure-one continuum of business niches. Each niche can be interpreted as the market for a different product.[10] At each date t there is a proportion $x_t \in [0, 1]$ of niches monopolized by producers protected by an active patent. Monopolists during their incumbency earn a per period profit flow of $a > 0$. The remaining niches are either empty or occupied by (symmetric) firms that compete a la Bertrand and make zero profits.

Active patents become worthless whenever their niche is successfully occupied by an imitator or the holder of a patent on a newer product.[11] At each date t, each monopolized niche is challenged by at least an imitator with a probability $\delta > 0$, which is exogenous and independent across niches.[12] When challenged by imitators, the patent allows the incumbent producer to successfully fight the imitation and preserve the monopoly on its niche with a probability λ_1. If this protection fails, the niche becomes Bertrand competitive.

The entry of the developers of new products occurs once the imitation process is completed. We assume that each niche (monopolized or not) is challenged by one new product with an endogenous probability q_t. In monopolized niches, patent protection allows the challenged incumbent to prevail against the innovative entrant with probability λ_2. Otherwise, the entrant becomes the new monopolist and its patent joins the stock of active patents.

Under these assumptions, the value of an active patent at date t, which is the present

[10]This simplification allows us to abstract from cross-product competition and to focus on competition related with concomitant and future entry into each niche.

[11]In Section 5 we interpret the introduction of newer products in terms of a standard quality ladder model with limit pricing in which a is the quality improvement brought about by each successful innovation in the corresponding niche.

[12] In Section 6 we endogenize the entry of imitators and show that the results are qualitatively unchanged.

value of the monopoly profits that it yields to its holder, can be recursively written as

$$v_t = a + \beta[1 - (1 - \lambda_1)\delta][1 - (1 - \lambda_2)q_{t+1}]v_{t+1}, \tag{1}$$

where the two terms in brackets represent the probability of surmounting the entry of imitators and innovators, respectively, at date $t + 1$. We will denote by x_t the stock of active patents and the mass of monopolized niches, which coincide at all dates. The law of motion of x_t can be written as

$$x_t = [1 - (1 - \lambda_1)\delta]x_{t-1} + \{1 - [1 - (1 - \lambda_1)\delta]x_{t-1}\}q_t. \tag{2}$$

The first term in the right hand side of this expression accounts for the niches that, being monopolized at $t-1$, remain monopolized after the entry of imitators at t; the second term accounts for those non-monopolized niches that become monopolized by the subsequent entry of a new patented product at t.[13]

We assume that at each date there is an infinite number of potential entrepreneurs who may attempt to engender and develop an innovation. This process is, however, costly and risky. For future use, we distinguish between a *non-pecuniary entry cost* $\Phi > 0$ and a *pecuniary development cost* normalized to one.[14] These costs are incurred one period before the potential new product is generated. The risks faced by the innovator include overcoming the competition of the developers of alternative new products that covet the same market niche and the opposition of the incumbent monopolist, if there is any at the relevant niche.

To capture the former, we model congestion drawing from the literature on markets with search frictions, where agents cannot coordinate their search efforts.[15] If $e_t \in [0, \infty)$ denotes the mass of innovations subject to development between dates $t - 1$ and t, we

[13]Notice that the entry of newer products in already monopolized niches implies the replacement of previously active patents with new ones that contributes to firm turnover but it does not change the size of the stock x_t.

[14]In Section 4 we consider an extension in which a moral hazard problem interferes with the external financing of the pecuniary development cost. Section 6 will also make use of the distinction between entry and development cost.

[15]Mortensen (1982) and Pissarides (1985) are classical references in the labor literature.

postulate that each of these innovations becomes the challenging product of a niche with an identical and independent probability $1/(1+e_t)$. With this formulation, the probability of success goes to one as the measure of simultaneously developed innovations goes to zero, and to zero as the measure of potential entrants goes to infinity.[16] Also, like in a reduced-form patent race among symmetric contestants, the probability of success of any given innovation declines with the number of competing innovations.[17]

Obviously, the probability q_t with which a business niche is challenged by a new developer at date t must equal the product of the number of innovations subject to development at that date, e_t, and the probability with which each of them gives rise to a challenger product, $1/(1+e_t)$. Thus, we must have $q_t = e_t/(1+e_t)$, which is increasing in e_t. We use q_t as an alternative measure of the *entry flow*.

To simplify the analysis, we assume that when the developer of a new product reaches an empty or competitive niche, it immediately becomes its monopolist. As mentioned above, in an already monopolized niche, the entrant must overcome the opposition from the incumbent (based on a legal dispute on patent rights) and only becomes the new monopolist with probability $1-\lambda_2$. Thus, the ex-ante probability of success in the development of an innovation between dates $t-1$ and t is defined as

$$p_t \equiv \{1-\lambda_2[1-(1-\lambda_1)\delta]x_{t-1}\}(1-q_t), \tag{3}$$

where we have used the equality $1/(1+e_t) = 1-q_t$ to rewrite the probability that the innovation becomes a challenging product. The term $[1-(1-\lambda_1)\delta]x_{t-1}$ reflects the fraction of niches that remain monopolized when developers reach them.

Finally, notice that for the entry flow of entrepreneurs to be finite, the net gain from entering and trying to develop an innovation must be zero or strictly negative at all dates,

$$-(1+\Phi)+\beta p_t v_t \leq 0, \tag{4}$$

[16] Of course, coordination and congestion problems could be modeled in many other ways. For example, the explicitly probabilistic *urn-ball process* postulated by the literature on random matching would imply a success probability of $[1-\exp(-e)]/e$ for each innovation. Our formulation is simply more tractable.

[17] Opposite to classical models in the patent-race literature (e.g., Loury (1979) and Lee and Wilde (1980)), we abstract from the timing of innovation.

but, if it is strictly negative at some date, then the entry flow q_t must be zero at that date. Using (3) to substitute for p_t, the previous inequality can be rewritten as

$$-(1+\Phi)+\beta\{1-\lambda_2[1-(1-\lambda_1)\delta]x_{t-1}\}(1-q_t)v_t \leq 0. \quad (5)$$

The condition for entry to be zero in case of strict inequality can be guaranteed by further imposing

$$q_t[-(1+\Phi)+\beta\{1-\lambda_2[1-(1-\lambda_1)\delta]x_{t-1}\}(1-q_t)v_t]=0. \quad (6)$$

We will refer to (5) and (6) as the *free-entry* inequality and the *complementary slackness* condition, respectively.

3 Equilibrium

In this section we define the dynamic equilibrium of the industry and analyze its steady-state properties. Equilibrium conditions determine the three key endogenous variables at each date t: the entry flow q_t, the stock of active patents x_t, and the value of a patent v_t.

Definition 1 *Given an initial stock of active patents x_0, an equilibrium is a sequence of non-negative triples (x_t, v_t, q_t), for $t = 1, ...\infty$, that satisfy the valuation equation (1), the law of motion (2), the free-entry inequality (5), and the complementary slackness condition (6).*

When entry is strictly positive along the equilibrium sequence, the set of equilibrium conditions described in Definition 1 can be reduced to a bi-dimensional first-order non-linear system of difference equations in x_t and v_t. Specifically, (2) can always be used to produce an expression for q_t in terms of x_{t-1} and x_t while, if entry is positive, (6) implies that (5) must hold with equality. The substitution of the expression for q_t in this equality and in (1), respectively, yields the two difference equations of the reduced system:

$$\beta(1-x_t)\frac{1-\lambda_2[1-(1-\lambda_1)\delta]x_{t-1}}{1-[1-(1-\lambda_1)\delta]x_{t-1}}v_t-(1+\Phi) = 0, \quad (7)$$

$$\beta[1-(1-\lambda_1)\delta]\frac{1-(1-\lambda_2)x_t-\lambda_2[1-(1-\lambda_1)\delta]x_{t-1}}{1-[1-(1-\lambda_1)\delta]x_{t-1}}v_t-v_{t-1}+a = 0. \quad (8)$$

This reduced system is sufficient to describe the dynamics of equilibrium in the neighborhood of any steady-state equilibrium (SS) with a strictly positive stock of active patents, since keeping this stock necessarily requires a positive entry flow.[18]

When (7) and (8) are evaluated in a steady-state equilibrium with $x_t = x_{t-1} = x_{ss}$ and $v_t = v_{t-1} = v_{ss}$ for all t, we obtain

$$\beta(1-x_{ss})\frac{1-\lambda_2[1-(1-\lambda_1)\delta]x_{ss}}{1-[1-(1-\lambda_1)\delta]x_{ss}}v_{ss}-(1+\Phi) \quad = \quad 0, \tag{9}$$

$$\left[1-\beta[1-(1-\lambda_1)\delta]\frac{1-[1-\lambda_2(1-\lambda_1)\delta]x_{ss}}{1-[1-(1-\lambda_1)\delta]x_{ss}}\right]v_{ss}-a \quad = \quad 0, \tag{10}$$

and the steady-state entry variable q_{ss} can be obtained as a function of x_{ss} using (2):

$$q_{ss} = \frac{(1-\lambda_1)\delta x_{ss}}{1-[1-(1-\lambda_1)\delta]x_{ss}}. \tag{11}$$

In the next lemma we provide a necessary and sufficient condition for the existence of a steady-state equilibrium with a strictly positive stock of active patents. We also show that, when it exists, this equilibrium is unique and locally stable.

Lemma 1 *There exists a unique steady-state equilibrium with $x_{ss} > 0$ if and only if*

$$\frac{\beta a}{1-\beta[1-(1-\lambda_1)\delta]} \geq 1+\Phi. \tag{12}$$

This equilibrium is locally stable and exhibits monotonic convergence in the state variable x_t and saddle-path convergence in the jump variable v_t.

The steady-state stock of active patents x_{ss} and the steady-state value of a patent v_{ss} can be described as the coordinates of the intersection between the two curves depicted in Figure 1: the *free-entry curve* defined by equation (9) and the *present-value curve* defined by (10). As shown in the proof of the previous lemma, equation (9) describes an increasing relationship between x_{ss} and v_{ss}. Quite intuitively, this free-entry curve

[18]Specifically, having $x_t = x_{t-1} = x_{ss} > 0$ for all t requires $q_{ss} > 0$, by (2). Intuitively, the additions to the stock of active patents due to entry must be sufficient to offset the subtractions due to imitation.

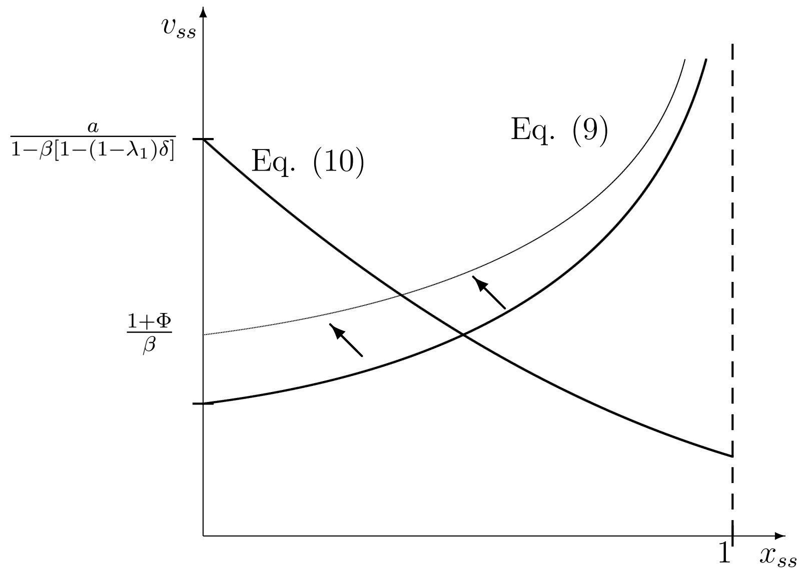

Figure 1: Characterization of the steady state. It is easy to study the comparative-statics implications for most parameters. Here, we illustrate the effect of an increase in Φ.

reflects that, when the stock of active patents is larger, the developers of new products are more likely to find opposition from incumbents and, thus, less likely to enter successfully, so a larger (contingent-on-success) value of patents is necessary to encourage them to innovate. Equation (10) expresses the value of a patent as a discounted sum of the one-period monopoly profits a, and establishes a negative relationship between x_{ss} and v_{ss}. The reason behind the negative slope of this present-value curve is that, as shown in (11), steady-state entry is positively related to x_{ss}, and entry increases the risk that a patent becomes worthless, and hence erodes v_{ss}. The existence condition provided in the lemma is equivalent to requiring that the intercept of the free-entry curve (9) is lower than the intercept of the present-value curve (10). Its economic interpretation is that if only one innovator were ever to arrive, it would obtain positive profits.

Figure 1 is also useful to perform comparative statics regarding the effects of most parameters on x_{ss} and v_{ss}, although, for some of them, graphical arguments are ambiguous and an analytical proof is required. The next proposition summarizes these effects.

Proposition 1 *In a steady-state equilibrium with* $x_{ss} > 0$*, the effects of infinitesimal changes in the parameters of the model on the steady-state variables* x_{ss}*,* v_{ss}*, and* q_{ss} *have the signs shown in Table 1.*

	a	β	Φ	δ	λ_1	λ_2
x_{ss}	+	+	−	−	+	−
v_{ss}	+	+	+	−	+	+
q_{ss}	+	+	−	?	?	−

Table 1: Comparative statics.

We shall start commenting the effects of each parameter on x_{ss} and v_{ss}, and later move to the effects on q_{ss}. The monopoly rents a and the discount factor β have the standard positive effects on the value of innovation and, thus, increase both the stock of active patents x_{ss} and their value v_{ss}. Increases in the entry cost Φ make the equilibrium flow of innovation less intense. This reduces the stock of active patents x_{ss} (which is continuously eroded by imitation), but it also increases the expected duration of the monopoly granted by each patent and, thus, its value v_{ss}. The rise in v_{ss} allows entering innovators to be compensated (on expectation) for the larger entry cost.

As one could expect, a smaller imitation risk (a reduction in δ) or a stronger protection of intellectual property rights against imitators (an increase in λ_1) or against further innovators (an increase in λ_2) result in an increase in the value of each active patent, as it reduces the probability that the patent becomes worthless. Interestingly, however, imitation risk and innovation risk have opposite implications for the steady-state stock of active patents. Lowering the effective imitation risk faced by each patent, $(1-\lambda_1)\delta$, merely expands the expected valuable life of each patent, resulting in an increase in x_{ss}, while more protection against innovative entry, λ_2, has the additional effect of weakening the incentives for would-be entrepreneurs to enter. This entry preemption effect dominates the life-expectancy effect on x_{ss} because, from the perspective of a potential entrant, the future protection granted by a larger λ_2 is discounted vis-a-vis the extra hurdle to current entry that it imposes.[19] This will have important implications for the social undesirability of increasing λ_2, that we establish below.

We now turn to the effects on steady-state entry. Due to the positive relationship between q_{ss} and x_{ss} described by (11), the effect of most parameters on both variables is of the same sign (steady state entry must be just sufficient to compensate the attrition in the stock of active patents due to imitation) and requires little comment. The exceptions are the parameters δ and λ_1, which determine the effective imitation risk faced by each patent. These parameters have a direct effect on (11), as well as some indirect effects channeled through x_{ss}. Mathematically, one can immediately see that the direct effect of $(1-\lambda_1)\delta$ is positive due to the above-mentioned compensate-for-attrition effect; in contrast, the indirect effect is negative since, as already explained, imitation risk reduces x_{ss}. Economically, the opposite sign of the effects is explained by the fact that, on the one hand, imitation increases the fraction of competitive niches, which facilitates entry, but, on the other hand, it also erodes the expected profits of the successful developer of a new product and, hence, reduces the incentives to innovate.

We have verified, using numerical simulations, that it is possible to find examples in which either the positive effect or the negative effect dominates. In many cases, as in the parametrization illustrated in Figure 2, entry is maximized at some interior value of the probability $(1-\lambda_1)\delta$. Hence, the trade-offs involved in the choice of an innovator's protection against imitators are not trivial—and they would be even less so if it were not possible to separately manage the protection against imitators, λ_1, and that against innovators, λ_2. We will return to these issues below, when discussing the implications of our analysis for optimal patent protection.

Before closing this section, it is worth to briefly comment on the significant case in which, in a part of the transition path towards steady state, equilibrium dynamics are not characterized by equations (7) and (8) because entry becomes zero at some dates.

[19] This effect can be appreciated in an unorthodox way by noting that the expression in equation (30) (in the proof of proposition 1) would be positive if β were greater than one.

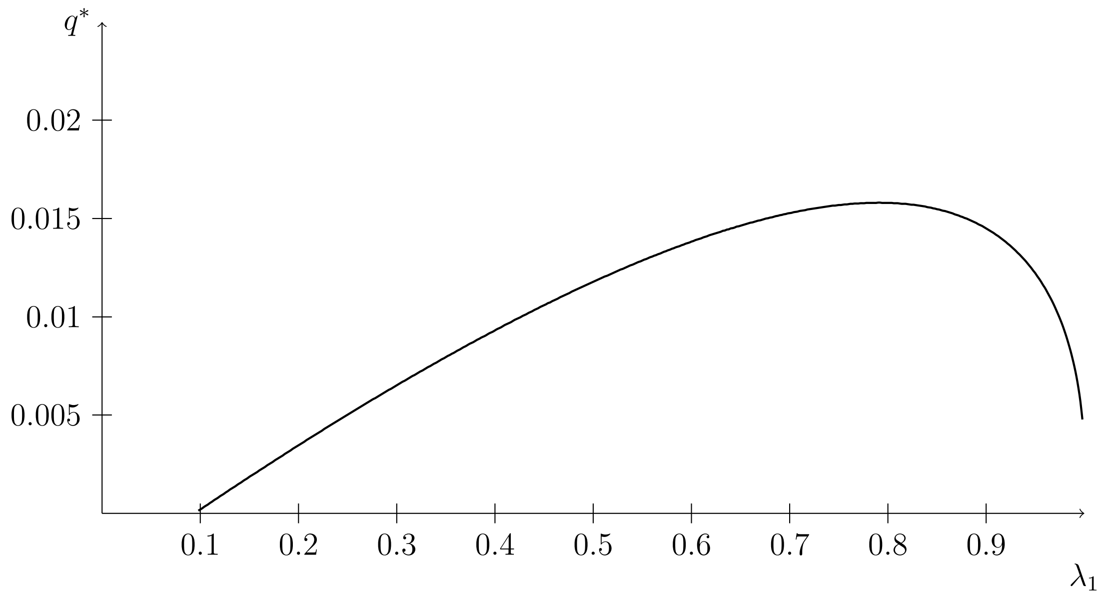

Figure 2: Steady-state entry and imitation risk. This graph depicts q_{ss} as a function λ_1. The underlying parameter values are $a = 0.1$, $\beta = 0.96$, $\Phi = 0.15$, $\delta = 0.05$, and $\lambda_2 = 0.5$.

Specifically, suppose that the initial stock of active patents x_0 is well above its steady-state value x_{ss}. How will the steady-state be reached? If the initial proportion of monopolized niches is too large, there will be a few periods in which entry is not profitable and, thus, $q_t = 0$. As time passes, however, imitation will erode the stock of active patents (and increase expected discounted profits) up to a point where entry is reestablished and the dynamics of the system is again described by (7) and (8).[20]

4 Adding External Financing Frictions

In this section we extend the model to consider frictions in the external financing of the pecuniary costs of developing an innovation. We assume that potential entrepreneurs are penniless and subject to limited liability. Each entrepreneur can incur the non-pecuniary cost Φ and engender an invention without any external support. This invention can be developed into a potentially marketable new product using a measure-one continuum of

[20] In this transition with zero entry for some periods, the reduction in the stock of active patents due to imitation will typically lead to a situation with $x_t > x_{ss} > [1 - (1 - \lambda_1)\delta]x_t$ just one period before reaching steady-state.

alternative development paths. For simplicity, we assume that only one of these paths can lead to a new marketable product at t, and ex-ante all paths are equally likely to lead to such a product. The new product will give to its developer the chance to occupy a niche in the industry as in the baseline version of the model. So a successful developer has a probability p_t of obtaining a monopoly position with value v_t.

For each path, the entrepreneur has a choice between in-house development and the licensing to an outside developer with deep pockets. The former requires the external financing of a pecuniary cost normalized to one, as in the baseline model. In the latter, the pecuniary development cost is assumed to be $1 + c$, where $c > 0$ reflects some friction in the transferring of the relevant technology to the licensee.[21]

In spite of the extra cost c, in-house development does not necessarily dominate licensing because external financing is afflicted by a moral hazard problem. Specifically, we assume that the quality of management is an unobservable decision of the developer that can take two values (*diligent* or *negligent*) and, as in Holmstrom and Tirole (1997), affects the probability of success in the development of each path (conditional on that path being the one that leads to the marketable product). This probability is one under diligent management and zero under negligent management, but under the latter the developer obtains a non-verifiable private benefit of $b > 0$.[22]

We will adopt specific assumptions below to guarantee that $\beta p_t v_t > b$ at all dates, so that diligent management is first-best optimal. However, if the entrepreneur pledges a sufficiently large fraction of her future profits to external financiers in exchange for their funding, the entrepreneur might be tempted to be negligent, rendering the financing deal unfeasible.

When licensing a path, the entrepreneur fully relinquishes its development to the

[21]The cost c may capture the cost of acquiring some relevant know-how that the entrepreneur already has as well as the legal cost associated with licensing the corresponding development path.

[22]The normalization of the probability of success under negligent management to 0 is only used to simplify the exposition. The same results would hold if, instead, success occurred with probability $1 - \Delta$ for $\Delta \in (0, 1]$. Formally, in such a setup, all final equations would be the same except for the fact that our parameter b would have to be replaced by b/Δ.

licensee who, if successful, appropriates the profits generated by the new product. We assume that there is a large pool of potential licensees with deep pockets (say, incumbent firms with previously accumulated internal funds) for whom the development of a path involves no moral hazard problem. By virtue of competition, these licensees pay royalties that leave the whole expected net present value of the external development of each path to the entrepreneur.

In order to guarantee that licensing can ameliorate the moral hazard problem of the entrepreneur, we assume $b > c$, and we will check below that the net present value of outside development is positive, $\beta p_t v_t > 1 + c$, whenever relevant.

4.1 Licensing as a result of financial frictions

In the setup just described, it is optimal for the entrepreneur to undertake as much development as financially feasible in-house. Since the development technology is linear, the division of licensed paths across (one or more) licensees is irrelevant and the licensing decision can be simply summarized by the total proportion of out-licensed paths, $\alpha_t \in [0, 1]$. Licensing helps the entrepreneur solve her financial problem in two ways: first, by reducing the scale of the in-house development problem and, thus, the implied financing needs to $1 - \alpha_t$, and, second, by allowing her to use the royalty proceeds $T_t = \alpha_t[\beta p_t v_t - (1 + c)]$ to cover internally some of those needs.[23]

If the entrepreneur pledges to her financiers a part $R_t \leq v_t$ of the discounted value of the conditional-on-success profits of any product arising from the paths left for in-house development, her incentive compatibility condition for diligent development can be written as

$$(1 - \alpha_t)\beta p_t(v_t - R_t) \geq (1 - \alpha_t)b, \tag{13}$$

while the financiers' individual rationality condition becomes

$$(1 - \alpha_t)\beta p_t R_t \geq (1 - \alpha_t) - T_t. \tag{14}$$

[23]Due to the extra revenue (specially valuable if used for internal financing purposes), the licensing of the paths that the entrepreneur does not develop in-house, if feasible, clearly dominates the alternative of leaving some paths undeveloped, which can be safely ignored in the rest of the discussion.

Obviously, it will always be optimal for the entrepreneur to choose R_t so as to make (14) hold with equality. But then, using the resulting equality together with the expression for T_t to substitute for R_t in (13), we can conclude that a licensing decision $\alpha_t \in [0,1]$ is feasible if and only if

$$\beta p_t v_t - 1 - c\alpha_t \geq (1-\alpha_t)b, \tag{15}$$

where the left hand side consists of the total net present value appropriated by the entrepreneur with the proposed arrangement if she behaves diligently, and the right hand side corresponds to what she could get under negligent management.

Clearly, the feasibility condition (15) is easier to satisfy with larger values of α_t insofar as $c < b$, as we have assumed. The results summarized in the following lemma are based on the fact that the optimal value of α_t is the lowest number in the range $[0,1]$ that satisfies (15), if it exists.

Lemma 2 *When $\beta p_t v_t - 1 \geq b$, entering entrepreneurs develop their inventions fully in-house, obtaining a net payoff $\beta p_t v_t - 1 - \Phi$. When $c \leq \beta p_t v_t - 1 < b$, they out-license a fraction*

$$\alpha_t^* = \frac{b - (\beta p_t v_t - 1)}{b - c} \tag{16}$$

of the development paths and develop the remaining fraction in-house, obtaining a net payoff $(1-\alpha_t^*)b - \Phi$. Finally, when $\beta p_t v_t - 1 < c$, the development is not feasible.

Notice that the parameters related to the moral hazard problem, b, and the technology transfer cost, c, play a crucial role in the partial equilibrium results shown in this lemma. When the net present value of diligent in-house development, $\beta p_t v_t - 1$, is larger than b, full in-house development is feasible and, hence, optimal. When it is smaller than b but larger than c, licensing becomes part of the second-best solution. If this present value is smaller than c, the development of the innovation becomes inviable.[24] Notice that in the

[24]So $\beta p_t v_t - 1 < c$ can only possibly occur in periods with no innovative entry. This situation cannot occur in a steady state with $x_{ss} > 0$ but it might occur in the transition to such a steady state if the industry starts with some pre-determined x_{t-1} sufficiently larger than x_{ss}. See footnote 20.

case with non-trivial licensing, the optimal licensed fraction α_t^* is increasing in b and c, and decreasing in $\beta p_t v_t - 1$.

4.2 Equilibrium with financial frictions

We now reconsider the dynamic equilibrium of the industry under external financing frictions. We will focus the discussion on the case in which the moral hazard problem is sufficiently severe ($b > \beta p_t v_t - 1$) in the steady-state equilibrium of the industry, since otherwise nothing changes relative to the baseline model (Lemma 2). The following proposition writes the relevant condition in terms of primitive parameters and shows that previous equilibrium conditions can be easily adapted to deal with this case.

Proposition 2 *If $b > \Phi$, in any date with positive entry, entering entrepreneurs out-license a fraction $\alpha^* = 1 - \frac{\Phi}{b}$ of the development paths of their inventions. Around steady-state, equilibrium conditions, as well as the existence condition (12), remain as in the baseline model except in that the entry cost parameter Φ has to be replaced by $\hat{\Phi} = (1 - \frac{c}{b})\Phi + c$.*

As in the baseline version of the model, entrepreneurs reap all the present value of the inventions, net of pecuniary and non-pecuniary investments. This is due to the competition among the incumbents who try to obtain the licensed paths, as well as the external financiers who provide the funds for the in-house investments. The net present value appropriated by the entrepreneur is, however, smaller than in the baseline case because of the technology-transfer costs $c\alpha_t^*$.

It turns out that the free-entry condition makes the licensing decision in equilibrium equal to the same constant $\alpha^* = 1 - \frac{\Phi}{b}$ in all periods with positive entry. Thus, the effective transfer costs become $c\left(1 - \frac{\Phi}{b}\right) = c - \frac{c}{b}\Phi$ and enter the problem in exactly the same way as the non-pecuniary entry cost Φ in the baseline model. So all results of the baseline model go through if Φ is replaced by $\hat{\Phi} = \Phi + c - \frac{c}{b}\Phi = (1 - \frac{c}{b})\Phi + c$, which is increasing in Φ, b, and c. Increasing the moral hazard parameter b or the technology cost parameter c

has, then, the effects that were already illustrated on Figure 1. Financial frictions reduce innovative entry, reduce the mass of active patents (or monopolized niches), and increase the profits from incumbency.

5 Welfare Analysis

So far we have not explicitly referred to the demand side of the industry. In this section we fill this gap in order to perform a meaningful analysis of the welfare and policy implications of the model. We interpret the innovation process in terms of a standard quality ladder model with limit pricing. The demand configuration and the proposed welfare measure are inspired in Hopenhayn et al. (2006), that applies them in a similar sequential innovation setup. Notice that entrepreneurs' and outside developers' direct net contribution to aggregate social surplus is zero in equilibrium since they operate under a free-entry or break-even conditions. So our welfare analysis only needs to focus on consumers' surplus.

Suppose that there is a unit mass of infinitely-lived homogeneous consumers willing to buy at most one unit of the product from each niche $j \in [0, 1]$ at each date t. Utility is additive across goods and dates, the intertemporal discount factor is $\beta < 1$, and the net utility flow from buying good j at price P_{jt} is $U_{jt} = A_{jt} - P_{jt}$, where A_{jt} is the quality of the good. Suppose that the successful entry of an innovation in a given niche improves the quality of the best good available in that niche by a units, while the successful entry of an imitator in the niche makes the production technology of the best quality good freely available to him (in addition to the previous monopolist). Finally, suppose, for simplicity, that production costs are zero.

How are goods priced in each niche? How does consumers' utility evolve over time? To answer these questions, notice that active monopolists are always able to charge a price $P_{jt} = a$ that captures the full quality advantage of their product vis-a-vis the best competing product. So the quality improvement associated with an innovation does not *directly and immediately* translate into an increase in consumers' net utility flow. In other words, if an innovation is attained in a non-monopolized niche, consumers will enjoy the greater quality of the new good but will also pay a higher price, so their net utility gain will be zero. The increase in consumers' net utility occurs when, later on, the monopolized niche experiences the entry of a competitor of either equal quality (an imitator) or greater quality (an innovator). Consumers will then enjoy an extra surplus of a per period for all periods ahead either because of the smaller price (zero) paid for the same old good (after imitation) or for enjoying (after innovation) a better quality good at the same price as before.

Clearly, in this setup, consumers' net utility in the steady state equilibrium grows linearly over time, so we can measure the social welfare associated with a steady state, W_{ss}, through the present value of consumers' *incremental* net utility flows due to the imitation and innovation processes completed in a typical date:

$$W_{ss} = \{(1-\lambda_1)\delta + [1-(1-\lambda_1)\delta](1-\lambda_2)q_{ss}\}x_{ss}\frac{a}{1-\beta}. \tag{17}$$

To explain (17), notice that utility additions only occur over monopolized business niches, whose measure is x_{ss}, and are associated with either imitation, which occurs at rate $(1-\lambda_1)\delta$ over those niches, or innovation, which occurs at rate $(1-\lambda_2)q_{ss}$ over the remaining proportion of monopolized niches $1-(1-\lambda_1)\delta$. Any of these entry processes imply a perpetual addition of a to consumers' net utility flow and $a/(1-\beta)$ is just the discounted value of such a perpetuity.

Expression (17) allows us to decompose the total effect of any model parameter θ on social welfare in up to a direct effect and two indirect effects channeled through the steady-state variables x_{ss} and q_{ss}:

$$\frac{dW_{ss}}{d\theta} = \frac{\partial W_{ss}}{\partial \theta} + \frac{\partial W_{ss}}{\partial x_{ss}}\frac{dx_{ss}}{d\theta} + \frac{\partial W_{ss}}{\partial q_{ss}}\frac{dq_{ss}}{d\theta}, \tag{18}$$

where $\partial W_{ss}/\partial x_{ss} = W_{ss}/x_{ss} > 0$ and $\partial W_{ss}/\partial q_{ss} = [1-(1-\lambda_1)\delta](1-\lambda_2)x_{ss}a/(1-\beta) > 0$. Direct inspection of (17) and the results in Propositions 1 and 2 allow us to construct the following table:

	a	β	$\widehat{\Phi}$	δ	λ_1	λ_2
$\frac{\partial W_{ss}}{\partial \theta}$	+	+	0	+	−	−
$\frac{\partial W_{ss}}{\partial x_{ss}} \frac{dx_{ss}}{d\theta}$	+	+	−	−	+	−
$\frac{\partial W_{ss}}{\partial q_{ss}} \frac{dq_{ss}}{d\theta}$	+	+	−	?	?	−
$\frac{dW_{ss}}{d\theta}$	+	+	−	?	?	−

Table 2: Decomposition of the model parameters' welfare effects. Recall that in the model with external financing frictions $\widehat{\Phi}$ is an increasing function of Φ, b, and c.

The positive effects of the quality-improvement parameter a and the discount factor β, as well as the negative effects of the parameters behind $\widehat{\Phi}$ (the entry cost Φ, the moral hazard parameter b, and the technology transfer cost c) are self-explanatory, once we recall their effects on the steady-state level of innovation and the stock of active patents, and notice that their direct effects on our welfare measure are either of the same sign as their indirect effects (in the case of a and β) or zero (in the case of $\widehat{\Phi}$).

The effects of the parameters related to imitation risk and IP protection are more intriguing. In Subsection 5.1 we discuss the welfare effects of changes in λ_1 (or δ) and λ_2, and the implications for the determination of the optimal overall level of IP protection when the degree of protection against imitation and innovation cannot be separately determined (say, because λ_1 and λ_2 are bound to be equal). In Subsection 5.2 we study the effects of external financing frictions and the optimal overall level of IP protection.

5.1 Optimal IP protection

Analytically, the total effect of the protection against imitation, λ_1, on welfare is ambiguous, although numerical examples show that the underlying relationship frequently has an inverted-U shape. As indicated in Table 5, the potential non-monotonicity arises from several sources. First, the direct effect of λ_1 on social welfare is *negative* because increasing λ_1 slows down the process whereby consumers attain the price reductions associated with the successful imitation of the products sold in monopolized niches. Second, the effect

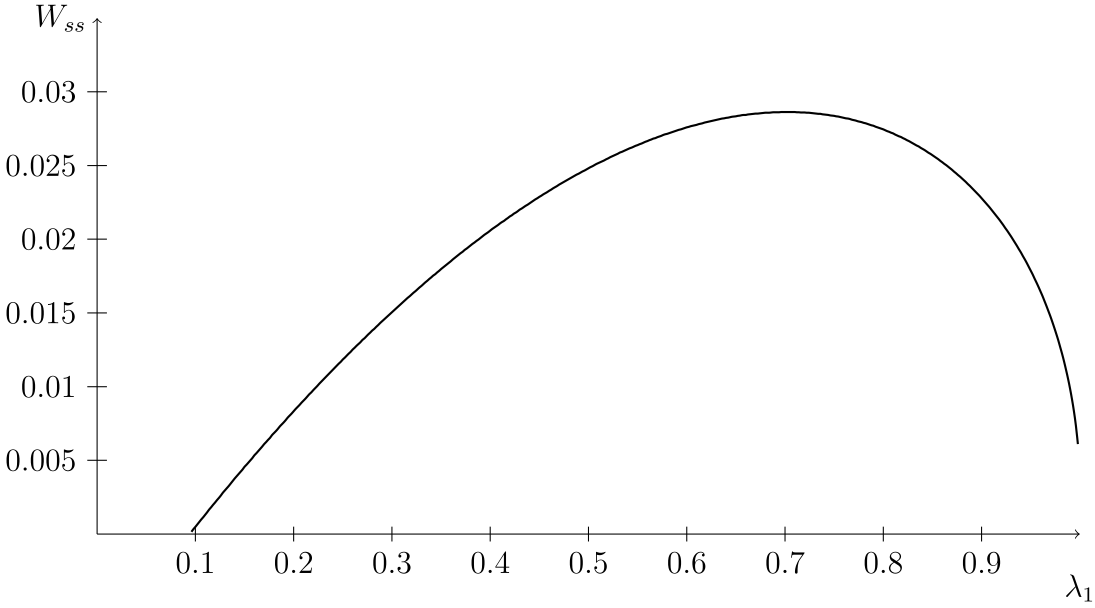

Figure 3: Steady-state welfare and imitation risk. This graph depicts W_{ss} as a function of λ_1. The underlying parameter values are $a = 0.1$, $\beta = 0.96$, $\hat{\Phi} = 0.15$, $\delta = 0.05$, and $\lambda_2 = 0.5$.

channeled through x_{ss} is *positive* because the protection against imitators contributes to maintain a larger stock of active patents. Finally, the effect channeled through q_{ss} is per se *ambiguous* since, as already discussed at the end of Section 3 and illustrated in Figure 2, the effect of imitation on steady-state entry is potentially non-monotonic: imitation frees up niches from protected incumbents, facilitating entry, but also erodes the value of incumbency and, hence, the incentives to enter.

The resolution of the ambiguity can point in any direction: depending on parameter values, W_{ss} can reach a maximum at $\lambda_1 = 0$, at $\lambda_1 = 1$, or at some interior level. Figure 3 shows that, under the same parametrization used in Figure 2, the socially optimal level of imitation risk is interior. When the solution is interior, the pro-competitive direct effect of imitation explains why we typically obtain that the optimal level of protection is lower than the one that maximizes the rate of technological progress (compare the locus of the maximum in figures 2 and 3).

The discussion of the welfare effects of increasing the protection of patent holders against subsequent innovations is much simpler, since in this case the direct effect and

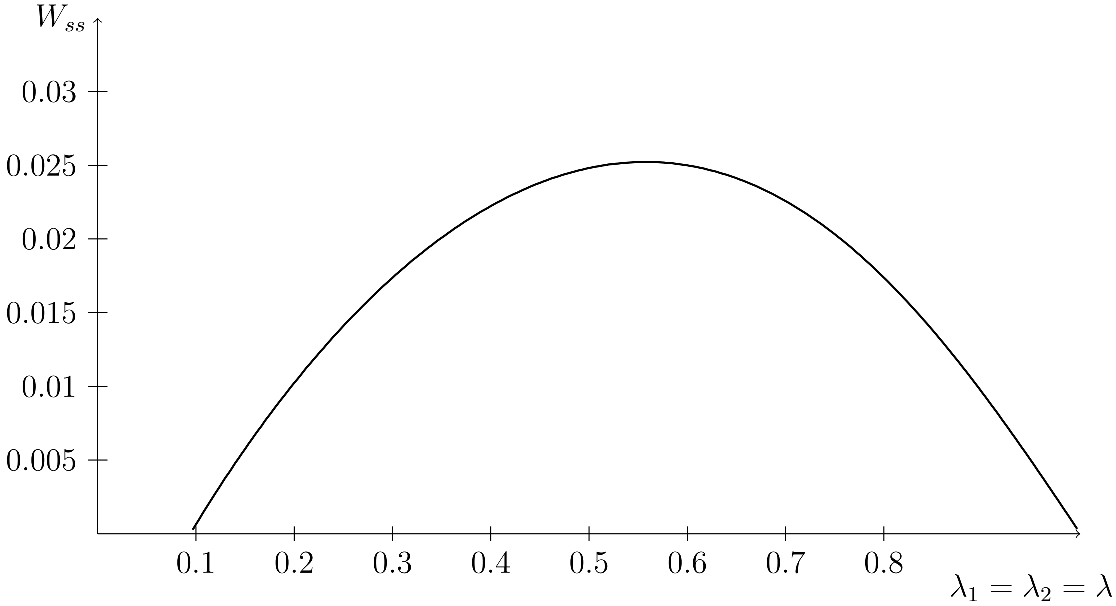

Figure 4: Steady-state welfare and IP protection for $\lambda_1 = \lambda_2 = \lambda$. Other parameters are as in Figure 3.

the two indirect effects have all the same negative sign. Reinforcing what we already said, explaining that λ_2 increases the hurdle for innovation and this effect more than compensates the incentive effects of increasing the value of incumbency, the direct effect of λ_2 on welfare reflects that, for a given entry rate q_{ss}, a higher λ_2 implies a lower rate of *successful* entry. Hence, either in welfare terms or in maximizing innovation terms, the conclusion is that the protection of IP against further innovation should be zero.

So far we have discussed the effects of λ_1 and λ_2 separately. But it may be argued that, in practical legal terms, it is difficult to make a clear distinction between imitation and innovation, and protect IP differently against each of them. As a result, the patent statute is likely to hinder both kinds of entry in a related manner. Figure 4 provides an example of the welfare implications of our model in one such case: when we impose $\lambda_1 = \lambda_2 = \lambda$ under the parametrization used in previous figures (and with $\delta = 0.05$). In this case, the overall degree of IP protection has an inverted-U shaped effect on social welfare. Intuitively, social welfare is maximized at a level of protection between zero (which would be the optimal value for an independently set level of protection against

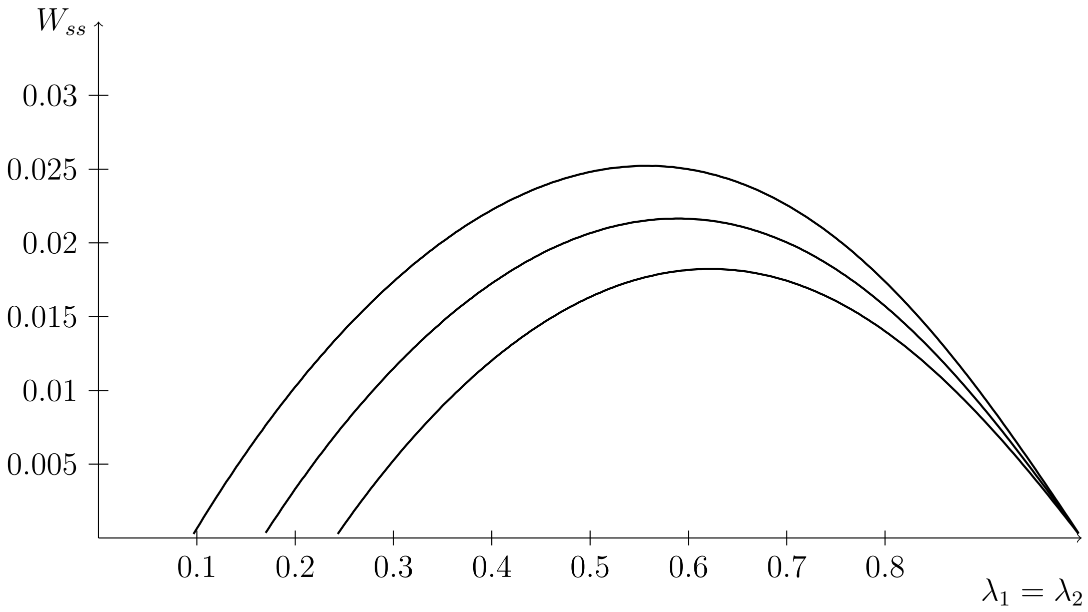

Figure 5: Changes in welfare and the optimal level of IP protection for different tightness of the financial constraints. The underlying parameters are as in Figure 4.

innovation λ_2) and the optimal level of protection against imitation λ_1 in Figure 3.

5.2 IP response to external financing frictions

To keep the discussion on how financial constraints interact with IP protection brief, we focus on the case with $\lambda_1 = \lambda_2 = \lambda$. Figure 5 displays, for the same parameter values used before, the relationship between λ and social welfare in three regimes that differ in the tightness of the financial constraints, as determined by the moral hazard parameter b. Holmstrom and Tirole (1997) model the effect of the monitoring of financial intermediaries on their borrowers as a reduction in b. Along the same lines, in an innovation financing context, one could interpret a reduction in b as the result of greater or more effective monitoring provided by an expert venture capitalist or a diligent bank.

The top curve in Figure 5 is the same as in Figure 4. The intermediate curve only differs in that b takes an intermediate value, that prevents the entrepreneur from developing the innovation fully in-house. The bottom curve exhibits the case in which the moral hazard problem is so severe that entrepreneurs are forced to license their innovations in full to outsiders. In addition to illustrating the detrimental effect of financial constraints, the figure clearly shows that these constraints also increase the optimal level of IP protection. Hence, a level of IP protection that is optimal under tight financial constraints may become excessive after the development of institutions such as venture capital financing or an improvement in monitoring technologies lessen the relevant financial constraints.

The reason why this effect occurs is different from what transpires of a typical static analysis. Remember that, with the dynamic effects captured in the model, increasing λ_2 has the net effect of discouraging innovation so one cannot immediately argue that increasing the overall protection λ seeks to compensate for the negative effect of financial constraints on innovation. In fact, the result arises from the complementarity between the imitation and innovation activities. The protection against imitation has two opposite effects on innovation. On the one hand, it increases the expected duration of an incumbent's monopoly position. On the other hand, it makes the entry of future innovators less likely. When financial constraints reduce the probability of future entry the second force becomes less important, pushing up the optimal value of λ_1 and, hence, λ.

6 Endogenizing Imitation

Throughout the paper we have taken imitation as exogenous and consisting of a constant flow of new imitators that challenged patent holder monopolies with a probability δ. We now proceed to endogenize the intensity of this threat which, in general, might not be constant over time and, hence, we denote by δ_t.

In parallel to our modeling of innovation, suppose there is an infinite supply of potential imitating entrepreneurs (or "imitators"). Their entry is subject to the same type of congestion postulated for the entry of innovative entrepreneurs. Similarly, these entrants cannot target their product to a particular market niche. Instead, they might enter a competitive niche in which they obtain zero profits or be assigned to a monopolized niche where, with probability $1-\lambda_1$, they replace the current monopolist.[25] We assume that,

[25] In principle, they could also enter an empty niche, if there were any, but such a situation never happens in and around steady state, so we ignore it.

only in this last case, an imitators' product provides a marginal quality improvement on the product of the incumbent monopolist, allowing the imitator to temporarily obtain per period profits $\varepsilon \in (0, a)$.[26] Opposite to innovators, this temporary monopoly position involves no protection against any form of future entry.

Finally, we assume that imitation entails the non-pecuniary entry cost Φ but does not require any development investment. As in the case of innovation, entry occurs one period after undertaking the decision on entry (and incurring its cost).

We denote as e_t^i the flow of imitators in period t, which leads to a proportion of niches being challenged

$$\delta_t = \frac{e_t^i}{1 + e_t^i}. \tag{19}$$

We denote as p_t^i the probability that an imitator reaches a position to (temporarily) obtain profits ε. This probability is the product of the probabilities of being assigned to a monopolized niche, succeeding in court against the established monopolist, and not being replaced by an innovator before the end of the period, that is,

$$p_t^i = (1 - \lambda_1) x_{t-1} \frac{1}{1 + e_t^i} (1 - q_t) = (1 - \lambda_1) x_{t-1} (1 - \delta_t)(1 - q_t). \tag{20}$$

The present value of profits of an imitator, v_t^i, can be written as

$$v_t^i = \varepsilon + \beta(1 - \delta_{t+1})(1 - q_{t+1}) v_{t+1}^i. \tag{21}$$

Notice that the discounting of future profits differs with respect to the case of an innovator (see (1)) in that the imitator enjoys no protection against further imitation or innovation (so λ_1 and λ_2 do not appear in the expression). Finally, the free-entry condition for an imitator becomes

$$\beta p_t^i v_t^i \leq \Phi,$$

and the complementary slackness condition imposes $\delta_t(\beta p_t^i v_t^i - \Phi) = 0$, so that imitative entry is zero if the net present value of imitative entry is negative.

[26] Our assumptions that profits are only obtained when an imitator replaces an previous innovator capture the idea that the fewer competitors the imitator faces, the larger the profits that she can obtain from her slight modification of an existing product.

Replacing v_t^i from the free-entry condition in the expression for the present value of profits, we obtain

$$\frac{1}{\beta x_{t-1}(1-\delta_t)(1-q_t)} = \frac{(1-\lambda_1)\varepsilon}{\Phi} + \frac{1}{x_t}.$$

We can now characterize the steady state as

$$\frac{1}{\beta x_{ss}(1-\delta_{ss})(1-q_{ss})} = \frac{(1-\lambda_1)\varepsilon}{\Phi} + \frac{1}{x_{ss}},$$

where we can replace q_{ss} from the law of motion of x_{ss}, which yields, as before,

$$q_{ss} = \frac{(1-\lambda_1)\delta_{ss}x_{ss}}{1-(1-(1-\lambda_1)\delta_{ss})x_{ss}}. \tag{22}$$

Further manipulation leads to the following steady-state condition for the flow of imitators

$$\delta_{ss} = \frac{(1-x_{ss})(\beta\varepsilon(1-\lambda_1)x_{ss}-(1-\beta)\Phi)}{\Phi(1-\lambda_1)x_{ss}+(\beta\varepsilon(1-\lambda_1)x_{ss}+\beta\Phi)(1-x_{ss})}. \tag{23}$$

This equation defines an *imitation curve* which can be added to the steady state conditions of the baseline model—the free-entry curve and the present-value curve in Figure 1—to fully characterize the steady state of this extended version of the model.

Numerical examples show that (23) describes an inverse U-shaped relationship between x_{ss} and δ_{ss}. When x_{ss} is low, the returns from imitation are small, since the probability that a firm occupies a previously monopolized niche is small. When x_{ss} is very large, however, the steady-state flow of new innovators is large, which reduces the expected duration of the span for which an imitator reaps profits of ε, so again the expected returns from imitation are small. Solving for the steady state essentially involves three simultaneous equations with three unknowns x_{ss}, v_{ss}, and δ_{ss}, since q_{ss} can be obtained recursively.

Social welfare W_{ss} can be computed in a way similar to what has been described for the baseline model. The arguments that led to equation (17) still apply since monopolized niches that are successfully challenged by either an innovator or an imitator generate welfare gains. However, now turnover at the niches occupied by imitators that previously replaced an innovator also contributes to welfare, adding a new term to the expression for W_{ss}. In particular, we obtain:

$$\begin{aligned} W_{ss} &= \{(1-\lambda_1)\delta_{ss} + [1-(1-\lambda_1)\delta_{ss}](1-\lambda_2)q_{ss}\}x_{ss}\frac{a}{1-\beta} + \\ &+ [\delta_{ss} + (1-\delta_{ss})q_{ss}](1-x_{ss})\frac{(1-\lambda_1)\delta_{ss}x_{ss}(1-q_{ss})}{q_{ss}+\delta_{ss}(1-q_{ss})}\frac{\varepsilon}{1-\beta}. \end{aligned} \quad (24)$$

The new term is proportional to the discounted value of the permanent quality improvement ε brought by each imitation. Such value is passed on to consumers whenever a monopolist imitator is subsequently challenged by either an innovator or another imitator. After properly accounting for these risks, as well as the steady state proportion of niches monopolized by unchallenged imitators we obtain the last term in the expression above.

The effects of changes in the intellectual property protection parameters λ_1 and λ_2 on the steady state variables are discussed with the help of Figures 6 and 7, respectively. The figures only describe the variables δ_{ss}, q_{ss}, and W_{ss} for brevity.

Increasing the protection against imitation, λ_1, leads to a monotonic increase in the proportion of monopolized niches. As shown in Figure 6, however, the effects on imitation δ_{ss} and innovation q_{ss} tend to be non-monotonic. The reason is that when λ_1 is low, the proportion of niches operating under monopoly is small, making the entry of imitators unprofitable. At the opposite end, a large λ_1 directly discourages imitation, increasing the proportion of monopolized niches and, in turn, the hurdle for future innovation (when $\lambda_2 > 0$). In the case depicted in the figure, the non-monotonic effect on innovation extends to welfare, for the reasons discussed in the baseline version of the model. So the optimal level of protection against imitation may well be, as in the baseline model, interior.

Increasing the protection against innovation, λ_2, also leads to a monotonic increase in the proportion of monopolized niches. Figure 7 shows that both imitation and innovation decrease as λ_2 increases, which explains why, as in the baseline model, $\lambda_2 = 0$ maximizes innovation and social welfare.

All in all, the simulations in Figures 6 and 7 show that making imitation endogenous

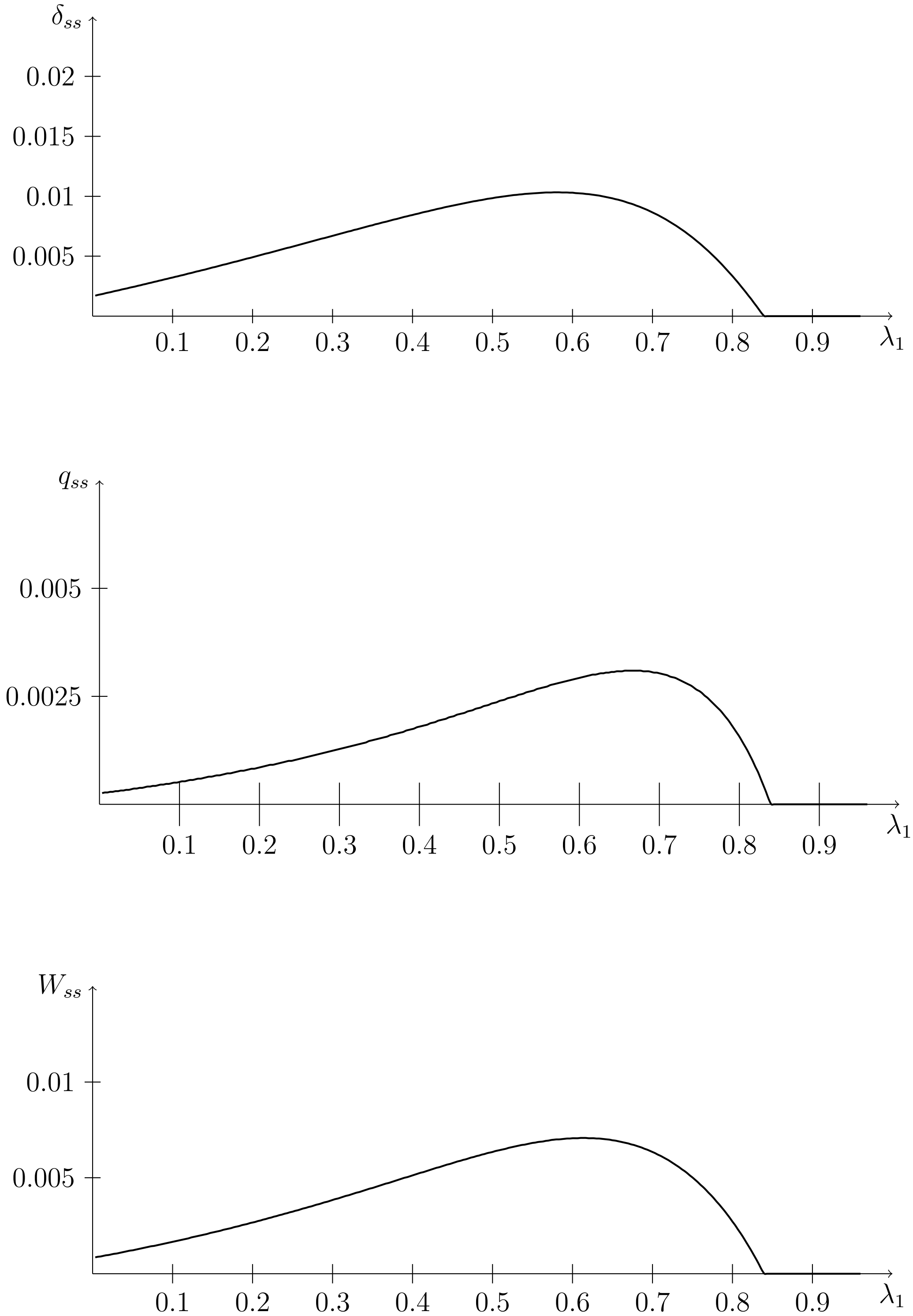

Figure 6: Imitation, innovation, and welfare for different values of λ_1. The underlying parameter values are $a = 0.1$, $\beta = 0.96$, $\Phi = 0.15$, $\lambda_2 = 0.5$, and $\varepsilon = 0.05$.

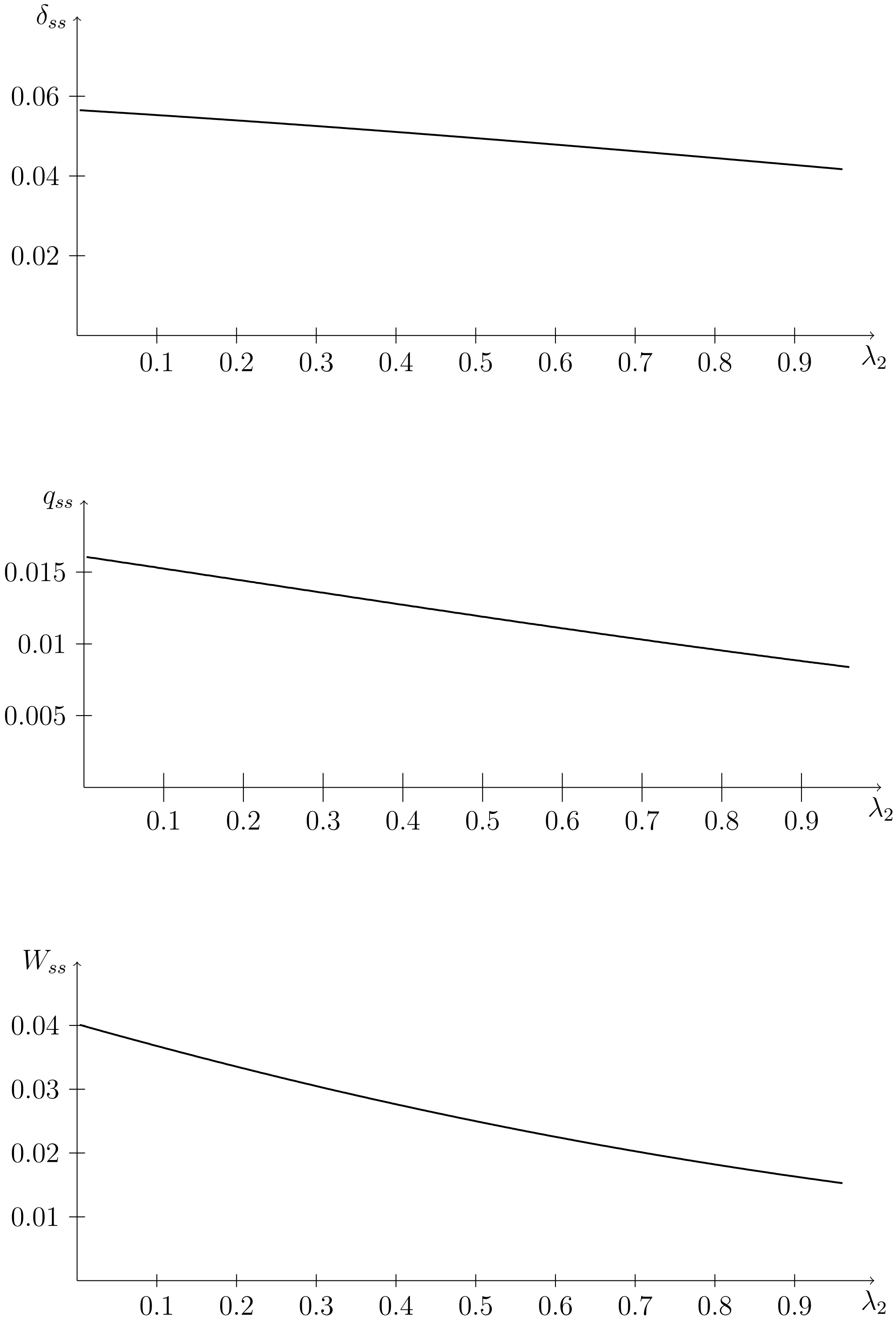

Figure 7: Imitation, innovation, and welfare for different values of λ_2. The underlying parameters are $a = 0.1$, $\beta = 0.96$, $\Phi = 0.15$, $\lambda_1 = 0.5$, and $\varepsilon = 0.05$.

does not change qualitatively the predictions obtained for the baseline model. That is, protection against innovation is always detrimental to welfare, while the optimal degree of protection against imitation is typically interior.

7 Concluding Remarks

Innovation is considered key to industry dynamics. Entry, exit, and innovation are complex interrelated phenomena in every industry, and especially so in the youngest and more technology-intensive industries. Many of these industries rely on intellectual property (IP) as the source of temporary monopoly power that allows the successful innovators to obtain a return for their previous research and development (R&D) investments. IP protection, however, is a double cutting edge knife for the dynamics of innovative industries, as the protection of incumbent innovators may be an obstacle to the success of novel innovators. This paper contributes to the growing literature that analyzes the role of IP protection by embedding it in an industry dynamics setting where innovation and imitation are different, interrelated processes modeled along similar lines. We also consider the implications of explicitly adding financial constraints to the innovation process, an aspect of obvious relevance for entrepreneurial innovators that has received almost no attention in the IP literature.

We explicitly distinguish between innovative entry and imitative entry, and the protection against each of them granted by IP to incumbents. This feature has allowed us to identify some novel (and somewhat surprising) trade-offs concerning the role of imitation. Specifically, we find that the pro-competitive effect of imitation may make imitation overall beneficial for innovation and welfare, since the hurdle for the entry of innovators is lower when there are less incumbent monopolists defending their business niches. For a wide range of parameter values, the relationship between imitation risk and welfare (as well as innovation) has an inverted-U shape, and such a shape extrapolates to the relationship between the overall degree of IP protection and welfare.

We have also shown that financial constraints provide a rationale for the use of the partial out-licensing of innovations as part of entrepreneurs' strategy for the financing of their R&D investments. Licensing involves a knowledge transfer cost that decreases the overall profitability of innovation. At an industry level, the resulting lower level of innovation alters the trade-offs underlying the choice of a socially optimal degree of IP protection. When innovators are less likely to come by, the positive role of imitation associated with facilitating entry through the erosion of existing monopoly power becomes less important, and the optimal protection of IP should increase. The reverse argument applies if financial constraints get relaxed: IP protection should diminish.

References

Aghion, P., Harris, C., Howitt, P., and Vickers, J., 2001, "Competition, Imitation and Growth with Step-by-Step Innovation," *Review of Economic Studies*, 68: 467–92.

Anton, J. J., and Yao, D. A., 2002, "The Sale of Ideas: Strategic Disclosure, Property Rights, and Contracting," *Review of Economic Studies*, 69: 513–31.

Bessen, J., and Maskin, E., 2006, "Sequential Innovation, Patents, and Imitation," *Rand Journal of Economics*, forthcoming.

Bhattacharya, S., and Guriev, S., 2006, "Patents vs. Trade Secrets: Knowledge Licensing and Spillover," *Journal of the European Economic Association*, 4: 1112–1147.

Boldrin, M., and Levine, D. K., 2006, "Growth and Intellectual Property," NBER Working Papers 12769, National Bureau of Economic Research, Inc.

Bottazzi, L., and Da Rin, M., 2002, "Venture Capital in Europe and the Financing of Innovative Companies," *Economic Policy*, 17: 229–270.

Casamatta, C., 2003, "Financing and Advising: Optimal Financial Contracts with Venture Capitalists," *Journal of Finance*, 58: 2059–2086.

Cockburn, I. M., and MacGarvie, M. J., 2006, "Entry, Exit and Patenting in the Software Industry," NBER Working Papers 12563, National Bureau of Economic Research, Inc.

Denicolò, V., 2007, "Do Patents Over-compensate Innovators?" *Economic Policy*, 22: 679–729.

Fershtman, C., and Kamien, M. I., 1992, "Cross licensing of complementary technologies," *International Journal of Industrial Organization*, 10: 329–348.

Gallini, N. T., 2002, "The Economics of Patents: Lessons from Recent U.S. Patent Reform," *Journal of Economic Perspectives*, 16: 131–154.

Gaston, R., 1989, "The Scale of Informal Capital Markets," *Small Business Economics*, 1: 223–230.

GOMPERS, P. A., 1995, "Optimal Investment, Monitoring, and the Staging of Venture Capital," *Journal of Finance*, 50: 1461–89.

——, 1999, *The Venture Capital Cycle*, MIT Press, Cambridge, MA.

GOMPERS, P. A., AND LERNER, J., 2001, "The Venture Capital Revolution," *Journal of Economic Perspectives*, 15: 145–168.

GREENWOOD, J., AND JOVANOVIC, B., 1999, "The Information-Technology Revolution and the Stock Market," *The American Economic Review, Papers and Proceedings*, 89: 116–122.

GROSSMAN, G. M., AND LAI, E. L. C., 2004, "International Protection of Intellectual Property," *The American Economic Review*, 94: 1635–1653.

HALL, B. H., AND ZIEDONIS, R. H., 2001, "The Patent Paradox Revisited: An Empirical Study of Patenting in the U.S. Semiconductor Industry, 1979-1995," *Rand Journal of Economics*, 32: 101–28.

HELLER, M. A., AND EISENBERG, R. S., 1998, "Can Patents Deter Innovation? The Anticommons in Biomedical Research," *Science*, 280: 698–701.

HOLMSTROM, B., AND TIROLE, J., 1997, "Financial Intermediation, Loanable Funds, and the Real Sector," *Quarterly Journal of Economics*, 112: 663–91.

HOPENHAYN, H., LLOBET, G., AND MITCHELL, M., 2006, "Rewarding Sequential Innovations: Prizes, Patents and Buyouts," *Journal of Political Economy*, 104: 1041–1068.

HUNT, R. M., 2004, "Patentability, Industry Structure, and Innovation," *Journal of Industrial Economics*, 52: 401–425.

INDERST, R., AND MULLER, H. M., 2004, "The Effect of Capital Market Characteristics on the Value of Start-up Firms," *Journal of Financial Economics*, 72: 319–356.

KAMIEN, M. I., 1992, "Patent Licensing," in R. AUMANN AND S. HART, eds., *Handbook of Game Theory with Economic Applications*, *Handbook of Game Theory with Economic Applications*, volume 1, chapter 11, Elsevier, 331–354.

KORTUM, S., AND LERNER, J., 1998, "Stronger protection or technological revolution: what is behind the recent surge in patenting?" *Carnegie-Rochester Conference Series on Public Policy*, 48: 247–304.

——, 2000, "Assessing the Contribution of Venture Capital to Innovation," *RAND Journal of Economics*, 31: 674–692.

LEE, T., AND WILDE, L. L., 1980, "Market Structure and Innovation: A Reformulation," *Quarterly Journal of Economics*, 94: 429–36.

LEMLEY, M. A., AND SHAPIRO, C., 2004, "Patent Holdup and Royalty Stacking," *Texas Law Review*, 85: 1991–2049.

LERNER, J., AND TIROLE, J., 2004, "Efficient Patent Pools," *American Economic Review*, 94: 691–711.

LEVIN, R. C., COHEN, W. M., AND MOWERY, D. C., 1985, "R&D Appropriability, Opportunity, and Market Structure: New Evidence on Some Schumpeterian Hypotheses," *American Economic Review*, 75: 20–24.

LOURY, G. C., 1979, "Market Structure and Innovation," *Quarterly Journal of Economics*, 93: 395–410.

MICHELACCI, C., AND SUAREZ, J., 2004, "Business Creation and the Stock Market," *Review of Economic Studies*, 71: 459–481.

MORTENSEN, D. T., 1982, "The Matching Process as a Non-Cooperative Bargaining Game," in J. MCCALL, ed., *The Economics of Information and Uncertainty*, NBER Conference Volume, New York.

MUTO, S., 1993, "On Licensing Policies in Bertrand Competition," *Games and Economic Behavior*, 5: 257–267.

NEHER, D. V., 1999, "Staged Financing: An Agency Perspective," *Review of Economic Studies*, 66: 255–74.

NORDHAUS, W., 1969, *Invention, Growth, and Welfare: A Theoretical Treatment of Technological Change*, The MIT Press.

O'DONOGHUE, T., AND ZWEIMÜLLER, J., 2004, "Patents in a Model of Endogenous Growth," *Journal of Economic Growth*, 9: 81–123.

PISSARIDES, C. A., 1985, "Short-run Equilibrium Dynamics of Unemployment Vacancies, and Real Wages," *American Economic Review*, 75: 676–90.

Repullo, R., and Suarez, J., 2004, "Venture Capital Finance: A Security Design Approach," *Review of Finance*, 8: 75–108.

Schmidt, K. M., 2003, "Convertible Securities and Venture Capital Finance," *Journal of Finance*, 58: 1139–1166.

Scotchmer, S., 2004, *Innovation and Incentives*, MIT Press, Cambridge, MA.

Wang, X. H., 1998, "Fee versus Royalty Licensing in a Cournot Duopoly Model," *Economics Letters*, 60: 55–62.

Appendix

Proof of Lemma 1: This proof has two parts. First we discuss the uniqueness and existence of a SS equilibrium. Then we discuss the local stability of the SS equilibrium.

Existence and uniqueness of the SS equilibrium: For brevity we eliminate the subscripts from x_{ss} and v_{ss} and rewrite (9) and (10) abstractly as:

$$f_1(x, v; \theta) = 0, \tag{25}$$

$$f_2(x, v; \theta) = 0, \tag{26}$$

where θ is the vector of parameters of the model. We will save on notation by referring to a single parameter $\psi \equiv (1 - \lambda_1)\delta$ rather than δ and λ_1 separately.

To establish the sign of the monotonic relationship between x_{ss} and v_{ss} in each of the equations, notice that

$$\frac{\partial f_1}{\partial v} = \beta(1 - x)\frac{1 - \lambda_2(1 - \psi)x}{1 - (1 - \psi)x} > 0,$$

and

$$\frac{\partial f_1}{\partial x} = \beta v \frac{\lambda_2(1 - \psi)[2x - (1 - \psi)x^2] - (1 - \lambda_2)\psi - \lambda_2}{[1 - (1 - \psi)x]^2}.$$

The numerator in the last expression is increasing in x and, hence, maximum at $x = 1$, but if we evaluate the numerator at $x = 1$ we obtain

$$-\lambda_2\psi^2 - (1 - \lambda_2)\psi < 0,$$

so $\frac{\partial f_1}{\partial x} < 0$ for all x. This implies that (9) defines an upward slopping curve in (x_{ss}, v_{ss}) space. Moreover, it is immediate to check that v_{ss} goes to infinity as x_{ss} approaches one.

As for (10), it can be verified that

$$\frac{\partial f_2}{\partial x} = \frac{\beta\psi(1 - \psi)(1 - \lambda_2)}{1 - (1 - \psi)x} v > 0$$

and

$$\frac{\partial f_2}{\partial v} = 1 - \beta(1 - \psi)\frac{1 - (1 - \lambda_2\psi)x}{1 - (1 - \psi)x} > 0,$$

so (10) describes a downward slopping curve. Obviously, the upward and downward sloping curves just described can intersect at most once and such an intersection, if it exists, defines the unique SS equilibrium. Since (9) has a vertical asymptote at $x = 1$, the necessary and sufficient condition for existence of the SS equilibrium is that the intercept of (9), $a/[1 - \beta(1 - \psi)]$, is lower than the intercept of (10), $(1 + \Phi)/\beta$, which explains condition (12).

Stability of the SS equilibrium: To analyze the local stability of the system around steady state, we proceed to log-linearize (7) and (8) around the SS point (v, x). Log-linearizing (7) yields

$$-\frac{1}{1 - x}dx_t + \frac{(1 - \psi)(1 - \lambda_2)}{[1 - (1 - \lambda_2\psi)x][1 - (1 - \psi)x]}dx_{t-1} + \frac{1}{v}dv_t = 0.$$

Log-linearizing (8) yields

$$-\frac{1 - \lambda_2}{1 - (1 - \lambda_2\psi)x}dx_t + \frac{1}{v}dv_t - \frac{1}{v - a}dv_{t-1} + \frac{(1 - \psi)(1 - \lambda_2)(1 - x)}{[1 - (1 - \lambda_2\psi)x][1 - (1 - \psi)x]}dx_{t-1} = 0$$

These expressions can be written as the following system of equations

$$
\begin{aligned}
dx_t - \frac{1-x}{v} dv_t &= \frac{(1-\psi)(1-\lambda_2)(1-x)}{[1-(1-\lambda_2\psi)x][1-(1-\psi)x]} dx_{t-1}, \\
-\frac{(1-\lambda_2)v}{(1-(1-\lambda_2\psi)x)} dx_t + dv_t &= -\frac{(1-\psi)(1-\lambda_2)(1-x)v}{[1-(1-\lambda_2\psi)x][1-(1-\psi)x]} dx_{t-1} + \frac{v}{v-a} dv_{t-1},
\end{aligned}
$$

or in matrix form as

$$
\begin{bmatrix} 1 & w_{12} \\ w_{21} & 1 \end{bmatrix} \begin{bmatrix} dx_t \\ dv_t \end{bmatrix} = \begin{bmatrix} z_{11} & 0 \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} dx_{t-1} \\ dv_{t-1} \end{bmatrix} \tag{27}
$$

where

$$
\begin{aligned}
w_{11} &= 1, & z_{11} &= \tfrac{(1-\psi)(1-\lambda_2)(1-x)}{[1-(1-\lambda_2\psi)x][1-(1-\psi)x]} > 0, \\
w_{12} &= -\tfrac{1-x}{v} < 0, & z_{12} &= 0, \\
w_{21} &= -\tfrac{(1-\lambda_2)v}{1-(1-\lambda_2\psi)x} < 0, & z_{21} &= -\tfrac{(1-\psi)(1-\lambda_2)(1-x)v}{[1-(1-\lambda_2\psi)x][1-(1-\psi)x]} < 0, \\
w_{22} &= 1, & z_{22} &= \tfrac{v}{v-a} > 1.
\end{aligned}
$$

Pre-multiplying both sides of (27) by the inverse of matrix W and pre-multiplying both sides of by it, the system becomes

$$
\begin{bmatrix} dv_t \\ dx_t \end{bmatrix} = Y \begin{bmatrix} dv_{t-1} \\ dx_{t-1} \end{bmatrix},
$$

with

$$
Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \equiv \frac{1}{1-w_{12}w_{21}} \begin{bmatrix} z_{11} - w_{12}z_{21} & -w_{12}z_{22} \\ -w_{21}z_{11} + z_{21} & z_{22} \end{bmatrix}.
$$

The two eigenvalues, μ_1 and μ_2, of matrix Y can be found as the solutions to the equation

$$
\det(Y - \mu I) = 0
$$

where I is the identity matrix of rank 2. Proving saddle-path convergence towards SS in the log-linearized system amounts to showing that, in absolute value, one of the eigenvalues of matrix Y is greater than 1 and the other is less than 1. We will further show that both eigenvalues are positive.

Since the function $D(\mu) \equiv \det(Y - \mu I)$ describes a parabola that tends to infinity when μ tends to both plus and minus infinity, then showing that $D(0) > 0 > D(1)$ would be enough for our proof. Consider first the sign of

$$
D(0) = \det(Y) = \frac{z_{11}z_{22}}{1 - w_{21}w_{12}}.
$$

Clearly, $z_{11}z_{22} > 0$, so proving that $D(0) > 0$ boils down to showing that

$$
1 - w_{12}w_{21} = 1 - \frac{1-x}{v}\frac{(1-\lambda_2)v}{1-(1-\lambda_2\psi)x} > 1 - \frac{(1-\lambda_2)(1-x)}{1-(1-\lambda_2\psi)x} = \frac{\lambda_2[1-(1-\psi)x]}{1-(1-\lambda_2\psi)x} \in (0,1). \tag{28}
$$

Now, as for

$$
D(1) = \det(Y - I) = \frac{(y_{11}-1)(y_{22}-1) - y_{12}y_{21}}{(1-w_{21}w_{12})^2},
$$

notice that we can ignore the denominator and prove the negativity of

$$
\begin{aligned}
(y_{11}-1)(y_{22}-1) - y_{12}y_{21} &= [z_{11} - w_{12}z_{21} - (1 - w_{12}w_{21})][z_{22} - (1 - w_{12}w_{21})] - w_{12}w_{21}z_{11}z_{22} + w_{12}z_{22}z_{21} \\
&= (1 - w_{12}w_{21})\left[w_{12}(z_{21} - w_{21}) - (z_{11} - 1)(1 - z_{22})\right].
\end{aligned}
$$

We already know, from (28), that $(1 - w_{12}w_{21}) > 0$. Moreover, from the expressions above, we clearly have $w_{12} < 0$, $1 - z_{22} < 0$, and

$$z_{21} - w_{21} = \frac{(1-\lambda_2)\psi v}{[1-(1-\lambda_2\psi)x][1-(1-\psi)x]} > 0.$$

It only remains to show that $z_{11} - 1 < 0$, where

$$z_{11} - 1 = \frac{-\psi - (1-\psi)\lambda_2 + 2\lambda_2(1-\psi)x - \lambda_2(1-\psi)^2x^2}{[1-(1-\lambda_2\psi)x][1-(1-\psi)x]}.$$

The denominator of this expression is clearly positive, while the numerator is maximized at $x = 1$. But at $x = 1$ the denominator becomes $-\psi[1-\lambda_2(1-\psi)] < 0$, so the denominator must be negative for all x.■

Proof of Proposition 1: For the sake of brevity, we will refer to the equilibrium equations using the same notation as in the proof of Lemma 1.

Effect of a: The parameter a only operates through equation (10). It is immediate that $\frac{\partial f_2}{\partial a} = -1 < 0$. As a result, increases in a shift upward the curve defined by (10) in Figure 1, resulting in an increase in the SS values of v and x.

Effect of β: The effect of β on x is immediate, since an increase in β produces an upward shift in the curve defined by (9) and a downward shift in the curve defined by (10) in Figure 1. Regarding the effect on v, let us implicitly define $x_2(v;\theta)$ from the equation $f_2(x_2(v;\theta), v;\theta) = 0$, recalling that f_2 is the left hand side of (10). Also, define

$$g(v;\theta) \equiv f_1(x_2(v;\theta), v;\theta), \tag{29}$$

so that v_{ss} solves $g(v;\theta) = 0$. Using the Implicit Function Theorem, it is enough for the result to show that g is increasing in v and decreasing in β. With respect to the first,

$$\frac{\partial g}{\partial v} = \frac{\partial f_1}{\partial v} + \frac{\partial f_1}{\partial x}x_2'(v;\theta) > 0$$

since $\partial x_2/\partial v = -(\partial f_2/\partial x)/(\partial f_2/\partial v) < 0$. Regarding the second, we obtain

$$\frac{\partial g}{\partial \lambda_2} = -\frac{\lambda_2 v\{[v-a-\beta(1-\psi)v]^2 + \beta^2\psi(1-\psi)(1-\lambda_2)v^2\}}{(1-\psi)[v-a-\beta(1-\psi)v]^2} < 0,$$

where $\psi \equiv (1-\lambda_1)\delta$, as already defined in the proof of Lemma 1.

Effect of Φ: The parameter Φ only operates through equation (9). It is immediate that $\frac{\partial f_1}{\partial \Phi} = -1 < 0$. As a result, increases in Φ shift upward the curve defined by (9) in Figure 1, resulting in an increase in v and a decrease in x.

Effect of δ and λ_1: Because of the way δ and λ_1 enter all expressions, the effects of these two parameters are colinear, but with the opposite sign. For brevity, we will refer to the effect of λ_1 only. Similarly to the case of β, the effect of increasing λ_1 on x is immediate from the upward shift of the curve defined by (9) and the downward shift of the curve defined by (10). Regarding the effect on v_{ss}, and using the function g defined in (29) above, it is enough to show that $\partial g/\partial\lambda_1 < 0$. In particular, this derivative can be written as

$$\frac{\partial g}{\partial \lambda_1} = \delta\frac{v[a-(1-\beta)v]\{-(v-a)[a-(1-\beta)v]+\beta v[\beta v\lambda_2^2(1-\psi)^2 - (v-a)(1+\lambda_2 - 2\varphi\lambda_2)]\}}{(1-\psi)^2 v[v-a+\beta(1-\varphi\lambda_2)v]^2}$$

Notice that $x \in [0,1]$ and $v \in [a/[1-\beta(1-\psi)], a/[1-\beta(1-\psi)\lambda_2]]$, so $a-(1-\beta)v > 0$. Moreover, the last term in the expression in curly brackets will be negative as long as

$$v \geq \frac{a}{1 - \dfrac{\beta\lambda_2^2(1-\psi)^2}{1+\lambda_2 - 2\varphi\lambda_2}},$$

which is true since

$$\frac{a}{1 - \dfrac{\beta\lambda_2^2(1-\psi)^2}{1+\lambda_2 - 2\varphi\lambda_2}} < \frac{a}{1-\beta\lambda_2(1-\psi)} < v.$$

Effect of λ_2*:* The effect on v is immediate, since an increase in λ_2 entails an upward shift of the two curves depicted in Figure 1. Regarding the effect on x, define $v_2(x;\theta)$ from the equation $f_2(x, v_2(x;\theta);\theta) = 0$, recalling that f_2 is the left hand side of (10). Also, define

$$h(x;\theta) \equiv f_1(x, v_2(x;\theta);\theta),$$

so that x_{ss} solves $h(x;\theta) = 0$. Using the Implicit Function Theorem, it is enough for the result to show that h is decreasing in both x and λ_2. With respect to the first,

$$\frac{\partial h}{\partial x} = \frac{\partial f_1}{\partial x} + \frac{\partial f_1}{\partial v}\frac{\partial v_2}{\partial x} < 0$$

since $\partial v_2/\partial x = -(\partial f_2/\partial v)/(\partial f_2/\partial x) < 0$. Regarding the second, we obtain

$$\frac{\partial h}{\partial \lambda_2} = -\beta(1-x)x(1-\psi)\frac{(1-\beta)[1-(1-\psi)x]a}{\{1-(1-\psi)x-\beta(1-\psi)[1-(1-\lambda_2\psi)x]\}^2} < 0.\blacksquare \quad (30)$$

Proof of Lemma 2: Notice that when $\beta p_t v_t - 1 \geq b$, the incentive compatibility constrained written in (13) holds for $\alpha_t = 0$ and, thus, the first-best allocation (full in-house development) is feasible and, therefore optimal, yielding net gains from innovative entry equal to $\beta p_t v_t - (1+\Phi)$. When $c \leq \beta p_t v_t - 1 < b$, there always exists a unique $\alpha_t^* \in (0,1)$ for which (13) holds with equality. Any other feasible α would be larger and, from the arguments given in the text, suboptimal. Profits under α_t^* can be computed as

$$\beta p_t v_t - 1 - c\alpha_t^* - \Phi = (1-\alpha_t^*)b - \Phi, \quad (31)$$

where the last equality arises, again, from (13).$\blacksquare$

Proof of Proposition 2: From (31), the net gains from entry in the case where licensing occurs in equilibrium can be rewritten as $\beta p_t v_t - (1+\Phi) - c\alpha_t^*$. Therefore, when the external financing frictions lead to licensing, the counterpart of the free-entry conditions (4) and (6) are

$$\begin{aligned} \beta p_t v_t - (1+\Phi) - c\alpha_t^* &\leq 0, \\ q_t[\beta p_t v_t - (1+\Phi) - c\alpha_t^*] &= 0. \end{aligned}$$

Moreover, when there is positive entry in a given period, the first equation holds with equality and together with (16), it pins down the value of the equilibrium licensing decision to a constant:

$$\alpha_t^* = \alpha^* = 1 - \frac{\Phi}{b} > 0.$$

Replacing this expression for α_t^* in (31), we can rewrite the net gain from entry as

$$\beta p_t v_t - (1 + \Phi) - c\left(1 - \frac{\Phi}{b}\right) = \beta p_t v_t - (1 + \hat{\Phi}),$$

where $\hat{\Phi} = (1 - \frac{c}{b})\Phi + c$, which is increasing in Φ, b, and c. Hence, all the results and conditions obtained for the baseline model are valid for the case with external financing frictions if the original parameter Φ is replaced by $\hat{\Phi}$.■