

# CLOSURE RULES, MARKET POWER AND RISK-TAKING IN A DYNAMIC MODEL OF BANK BEHAVIOR\*

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November 1994<sup>†</sup>

## ABSTRACT

The value of bank charters is an important component of bankers' bankruptcy costs and may constitute an incentive for banks to adopt prudent decisions. Charter values had been considered exogenous by previous researchers. Dynamic programming techniques allow us to obtain simultaneously the (endogenous) value of a bank and its optimal investment and financial policies. This model predicts a bang-bang risk-taking behavior by banks which might explain the sudden appearance of solvency problems in the banking sector. Soft prudential regulation, low market power, and a high risk-free interest rate may shift a bank from safe to risky. So, capital and asset regulations, and entry and closure rules are alternative ways to preserve solvency. Policy implications for the regulatory debate in the U.S. and Europe are derived.

**J.E.L. Codes:** D92, G21, G28

**Keywords:** banking theory, closure, risk-taking, dynamic programming

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\*This paper was a chapter of my Ph.D. thesis at University Carlos III, Madrid. I am very grateful to my supervisor Rafael Repullo. I also thank Victor Aguirregabiria, Andres Almazan, Xavier Freixas, Jorge Padilla, and Jesus Saurina for helpful comments. Financial support and facilities provided by Universidad Carlos III, Fundación BBV, and CEMFI are gratefully acknowledged. All existing errors are mine.

<sup>†</sup>This is a retyped version of the original manuscript that appeared as LSE Financial Markets Group Discussion Paper No. 196 in November 1994, when Javier Suarez was a Lecturer in the Department of Economics at the London School of Economics and a member of the Financial Markets Group. Due to minor typographical changes, page numbers do not coincide with the original.

# I INTRODUCTION

During the last decade the performance of the U.S. banking sector has been remarkably poor. The average number of failures jumped from less than 2 per year in the 1970s to roughly 130 per year between 1982 and 1991. The insolvencies and reorganizations of hundreds of savings and loan institutions (S&L) and a significant number of banking holding companies have inflicted enormous costs upon tax-payers. The public opinion is still astonished by the gravity of the crisis.

While in the political arena these facts provided a powerful impetus for the debate on regulatory reform, academics wondered about the causes of the debacle. The question is why the safety net built after the Great Depression began to fail in the early eighties.

Initial explanations focused on the lack of discipline associated to the over-extended deposit insurance system and the risk-insensitive nature of complementary regulations such as deposit insurance premiums and capital requirements. The microeconomic evidence analyzed in Cole *et al* (1992) and Boyd and Gertler (1993) led these authors to conclude that a main source of problems was increased by risk-taking by banks, providing support to an explanation based on moral-hazard.

However, the deposit insurance system existed since 1933 and generalized solvency problems appeared in the eighties. Indeed, when the deposit insurance system was created, prudential regulation included also a variety of branching and geographical restrictions, interest rate ceilings and tough constraints on the range of banking activities. Most of them were progressively eliminated during the long period of stability. It has been argued that some of those rules, originally designated to protect banks from competition, became barriers that prevented banks from adapting to new market conditions, stimulating disintermediation and the loss of the banks' best customers, and forcing at last a costly market-forced deregulatory process.

Qualified opinions relate the solvency problems of the 1980s with a continuous deterioration of bank rents. This decrease would explain the decline in the banks' goodwill and, consequently, in the value of bank charters. They argue that the erosion of monopoly rents and charter values stimulates risk-taking.<sup>1</sup>

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<sup>1</sup>See White (1991) for the main arguments, and Keeley (1990) and Cole *et al* (1992) for the empirical

The traditional theoretical approach to the moral hazard problem in banking has been based on static models of bank behavior (see Furlong and Keeley, 1989). Under risk-insensitive deposit insurance, higher mean-preserving portfolio risk and leverage are to the benefit of bank shareholders and to the detriment of the deposit insurance agency. Bankers' payoffs have a lower bound at zero since they are protected by limited liability.

In a dynamic setting, the banker who goes bankrupt is likely to suffer losses related to future payoffs. Apart from reputational losses which would appear in other industries, banking regulation contains special provisions for promoters and managers of banks which become insolvent: they are typically excluded from business. Actually, bankers need a charter to run a bank and, when supervisors intervene in bankruptcy procedures, the charter of the failed institution is either canceled after liquidation or transferred to a new holding company after a purchase and assumption transaction. Clearly, the threat of loss of the value of the charter when the bank fails may act as a disciplinary device against risk-taking. Of course, for this to be the case, the value of the charter has to be strictly positive. It is not rare, however, that locational, informational and reputational rents surge in the normal course of the banking business, where switching costs and regulatory barriers to entry are very plausible sources of market power.

In a dynamic setting, the stream of potentially positive future expected profits determines the cost of bankruptcy to the banker. The higher the present value of such stream, the lower the incentive to adopt risky short run decisions. Present and future profits depend on market power as well as on the regulatory constraints and macroeconomic factors affecting banks.

This paper examines the behavior of a bank in a dynamic setup taking into account the interactions between closure rules, market power and capital and asset regulations. The scope of the paper is mainly positive, since much has to be done in understanding the behavior of financial intermediaries before going into rigorous welfare analysis. Several regulatory trade-offs which derive from the analysis will be pointed out through the text. For instance, when market power encourages banks to be prudent, I will qualify to which extent capital and asset regulations can compensate for the deterioration of soundness in an increasingly competitive environment.

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evidence.

Some authors have previously analyzed the prudential implications of closure rules in banking, but from different, more partial or less formal perspectives. Davis and McManus (1991) examine the behavior of a risk-averse bank manager who faces an exogenous bankruptcy cost in a standard one-period model, focusing on the regulatory choice of the range of net worth values at which the bank is closed. In a similar setup, Mailath and Mester (1993) analyze discretionary closure within a game theoretic model, dealing with the determination of the closure decision by the authority. Banking charters are specifically mentioned in Marcus (1984) and Keeley (1990), who relate their value to market power: their theoretical frameworks consist of simple one-period models where the charter is a shareholders' claim which is contingent on solvency and whose conditional-on-solvency value is essentially taken as given.

Endogenizing the value of the charter within an infinite horizon model which allows for bankruptcy and closure constitutes the main purpose of this paper. The resulting model is not only more satisfactory than existing ones in formal terms, but also richer in empirical implications.

The optimization program presented in Section II extends in a natural way the typical decision problem faced by a bank in a static setting to a dynamic one. While in the static model the bank chooses an investment decision once and for all, in the dynamic model it chooses a state-contingent sequence of investment decisions. When the returns on the bank's portfolio are serially uncorrelated, the state can be summarized by an indicator variable which represents whether the bank remains open or has been closed by the authorities in the corresponding period. As in the standard bankruptcy procedures, the bank is closed if its net worth (at the end of the previous period) turns out to be negative.

Dynamic programming techniques allow us to define the value function associated to the bank's problem and to obtain an implicit definition for the conditional-on-surviving value of the bank,  $v$ . Not surprisingly,  $v$  is the sum of current one-period profits and the (endogenous) value of the banking charter (which, in turn, is the discounted value of  $v$  times the probability of the bank being closed at the end of the current period). Equivalently,  $v$  is the expected present value of the stream of current and future one-period profits.

The optimal policies of the bank depend on such value. Under perfect competition rents

are zero,  $v$  is zero, and the solution to the dynamic problem coincides with the one of the static problem: closure rules are ineffective in disciplining banks. With positive expected future rents, the dynamic problem is more interesting. For the sake of simplicity, I explore the case where the bank is a (local) monopolist in the market for deposits.

Section III derives comparative statistics results that show the effects on the value of the bank of the capital requirement, the regulatory bound to the risk of bank assets, the degree of market power and the risk-free rate of interest. All these parameters may have large (unambiguous) influence on  $v$ , which is the crucial determinant of bank risk-taking.

Section IV closes the analysis of the basic model examining the behavior of the bank as a function of these parameters. Optimal bank policies are characterized and several regulatory trade-offs are discussed. Interestingly the model predicts a kind of bang-bang behavior by banks which can explain the sudden appearance of widespread solvency problems in a banking sector as a result of small accumulated changes in the economic and regulatory environment. In such cases, the model could explain some striking facts of the recent U.S. banking experience.

In Section V, I extend the model in order to deal with an alternative closure rule which allows for voluntary recapitalization by shareholders in the event of failure. Section VI concludes with some policy implications for the current regulatory debate in the U.S. and Europe.

## **II THE DYNAMIC OPTIMIZATION PROGRAM**

### ***A Banking Charters, Solvency and the Dynamic Program***

In this paper, a bank is conceived as the investment project of a group of shareholders called bankers. Bankers are risk-neutral, enjoy limited liability and are initially granted a banking charter. A banking charter is an official permission to keep the bank open and under the control of their shareholders. The charter is renewed at the beginning of each period provided that the bank is solvent. If this is not the case, the bank is intervened and banking authorities assume control. From the viewpoint of the bankers, intervention is equivalent to closure, since it entails the loss of the charter. Therefore, in analyzing the

behavior of a bank I will indistinctly refer to intervention or closure.<sup>2</sup>

The sequence of events in any period  $t$  in which the bank remains open is as follows. At the beginning of period  $t$  the bank raises deposits  $D_t$  and capital  $K_t$  in order to invest in a portfolio of assets. The gross return of the portfolio of assets is a random variable  $R(\sigma_t)$ , independently distributed across time, with  $E[R(\sigma_t)] = 1 + r$  and dispersion measured by  $\sigma_t$ .  $r$  is the risk-free rate of interest. Shareholders' decision at each period entails choosing  $D_t$ ,  $K_t$  and  $\sigma_t$ .

At the end of period  $t$ , once asset returns are observed, the net worth of the bank,  $N_{t+1}$ , is computed as the difference between the end-of-period- $t$  value of assets and liabilities. The value of assets results from applying the stochastic gross return  $R(\sigma_t)$  to the initial investment  $D_t + K_t$ . Liabilities are made up of promised payments to depositors (principal plus interest), which are fully insured by a deposit insurance agency, so that their cost does not depend on the default risk of the bank. They are modeled as an increasing function of  $D_t$ ,  $C(D_t)$ . Particular assumptions about the degree of market power possessed by the bank when taking deposits will determine the shape of this function. In particular, perfect competition in the market for deposits (i.e. a rate-taking behavior by the bank) would cause  $C(D)$  to be linear. Clearly, the net worth of the bank is a function of the decision variables and the realization of  $R(\sigma_t)$  :

$$N_{t+1} = R(\sigma_t)(D_t + K_t) - C(D_t).$$

After computation of  $N_{t+1}$ , several possibilities arise. On the one hand, the bank may be liquidated by the bankers or intervened by the authorities; in this case the final payoff to shareholders is  $\max\{N_{t+1}, 0\}$ . Alternatively, the bank may remain open and under the control of the shareholders, then they take decisions for the following period. In particular, by choosing  $K_{t+1}$ , the shareholders implicitly decide whether the bank *pays a dividend* ( $K_{t+1} < N_{t+1}$ ) or *raises more capital* ( $K_{t+1} > N_{t+1}$ ).

The dynamic problem of the bank is different from a sequence of static problems because of the existence of a charter whose renewal takes place according to a closure rule. I will consider first the simple and realistic case in which banking authorities deny renewal and

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<sup>2</sup>In practice, supervisors face a variety of alternative ways to resolve insolvencies and the one applied in each case depends upon considerations such as cost minimization and the preservation of confidence in the banking system (see Benston *et al* 1986).

close the bank if its net worth at the end of a period is negative. Section V deals with the case in which shareholders possess an option to recapitalize whose exercise restores solvency and avoids closure.

Let the indicator variable  $I_t$  represent whether the bank is open or close at the beginning of period  $t$  :

$$I_t = \begin{cases} 0 & \text{if the bank is closed} \\ 1 & \text{if the bank remains open.} \end{cases}$$

Then the dynamics of closure under this rule can be formalized as follows:

$$I_t = I_{t-1} \cdot g(N_t),$$

where

$$g(x) = \begin{cases} 0 & x < 0 \\ 1 & \text{otherwise.} \end{cases}$$

With this specification,  $I_t$  takes value 0 both when the bank has been previously closed ( $I_{t-1} = 0$ ) and when it becomes insolvent at the end of period  $t - 1$  ( $N_t < 0$ ). Otherwise,  $I_t = 1$ .

In terms of the dynamic program, the state variable is  $I_t$  and the vector of control variables is  $y_t = (D_t, K_t, \sigma_t)$ . In each period, the bank is subject to static capital and asset regulations. On the one hand, a capital requirement obliges the bank to hold capital in excess of a certain fraction  $k$  of deposits,  $K_t \geq kD_t$ . On the other, regulatory limits to risk-taking in the asset side (together with the level of systematic risk in the economy) determine an upper bound to the level of risk of bank portfolios,  $\sigma_t \leq \bar{\sigma}$ . I assume that the informational and institutional context is such that banking regulation cannot or does not directly depend on  $\sigma_t$ , although indirect constraints on portfolio composition, off-balance-sheet operations, short-selling, sectorial and geographical concentration and the associated surveillance techniques allow the regulator to influence the upper bound to  $\sigma_t$ . For the sake of simplicity, I will consider time-invariant  $k$  and  $\bar{\sigma}$  so that the regulatory framework is stationary. Accordingly, the set of feasible controls of the dynamic program is specified as follows:

$$\Gamma \equiv \{y = (D, K, \sigma) \in R_+^3 \mid K \geq kD \text{ and } \sigma \leq \bar{\sigma}\}, \quad (1)$$

where  $k$  and  $\bar{\sigma}$  are determined by the regulator.

The dynamic optimization problem solved by the bankers is:

$$\begin{aligned} & \text{Maximize} && E \left[ \sum_{t=0}^{\infty} (1+r)^{-t} \psi(I_t, y_t) \right] \\ & \{y_t(I_t)\}_{t=0}^{\infty} \\ & \text{subject to} && y_t \in \Gamma \quad t = 0, 1, 2, \dots \\ & && I_t = I_{t-1} \cdot g(N_t), \quad t = 0, 1, 2, \dots \\ & && I_0 = 1, \end{aligned}$$

where the state-dependent one-period profit function is given by:

$$\psi(I_t, y_t) \equiv \begin{cases} (1+r)^{-1} \max\{N_{t+1}, 0\} - K_t & \text{if } I_t = 1 \\ 0 & \text{if } I_t = 0. \end{cases} \quad (2)$$

The bankers' problem can be interpreted as finding an optimal rule to determine the sequence of financial and investment decisions  $\{D_t, K_t, \sigma_t\}$  in an infinite discrete-time horizon. Risk-neutral bankers maximize the expected discounted value of the stream of cash-flows from their investment in the bank. If the bank is open, the investment generates an outflow of  $K_t$  at the beginning of period  $t$  and an inflow of  $\max\{N_{t+1}, 0\}$  at the end (discounting applies). If the bank is closed, cash-flows are zero.

When solving this program, the bank takes into account the impact of its present decisions on the probability of being closed, since closure hinders its shareholders from obtaining potentially positive future profits.<sup>3</sup> When such profits are high, a clear incentive to adopt prudent short-run policies arises.

## B The Value Function

Dynamic programming techniques allow us to define the value function associated to the bank's dynamic optimization problem, which is time-invariant:

$$V(I_t) = \sup_{y_t \in \Gamma} E \left[ \psi(I_t, y_t) + (1+r)^{-1} V(I_{t+1}) \right]. \quad (3)$$

This functional equation has a simple structure, since  $I_t$  takes only two values ("open" and "closed") and the value of a closed bank is zero. Then,

$$V(I_t) = \begin{cases} 0 & \text{if } I_t = 0 \\ v & \text{if } I_t = 1 \end{cases} \quad (4)$$

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<sup>3</sup>Without these dynamic concerns, the bank would simply maximize  $E[\psi(1, y_0)]$ , subject to  $y_0 \in \Gamma$ .

where  $v$  is the conditional-on-survival value of the bank, a constant which henceforth I will simply call the *value of the bank*:

$$v = \sup_{y_t \in \Gamma} E \left[ \psi(I_t, y_t) + (1+r)^{-1} V(I_{t+1}) \mid I_t = 1 \right]. \quad (5)$$

Now, using expression (4) evaluated in period  $t+1$  and the definition of  $I_{t+1}$ , we can write:

$$E[V(I_{t+1}) \mid I_t = 1] = \text{Prob}[I_{t+1} = 1 \mid I_t = 1] \cdot v = \text{Prob}[N_{t+1} \geq 0] \cdot v, \quad (6)$$

and plugging (6) into (5) and denoting  $\text{Prob}[N_{t+1} \geq 0]$  by  $\Phi(y_t)$ , we get:

$$v = \sup_{y_t \in \Gamma} \left\{ E[\psi(1, y_t)] + (1+r)^{-1} \Phi(y_t) v \right\},$$

This equation is fully time and state independent, so we can drop the time indices and leave:

$$v = \sup_{y \in \Gamma} \left\{ \Pi(y) + (1+r)^{-1} \Phi(y) v \right\}, \quad (7)$$

where  $\Pi(y)$  denotes the conditional-on-survival expected one-period profit of the bank,  $E[\psi(1, y)]$ .

The set of optimal policies for a given  $v$  can be defined as follows:

$$Y(v) = \arg \max_{y \in \Gamma} \left\{ \Pi(y) + (1+r)^{-1} \Phi(y) v \right\}. \quad (8)$$

When the bank decides an optimal control at the outset of period  $t$ ,  $v$  is taken as given, since  $y$  does not affect the expected future value of the bank, but only the probability of it to remain open,  $\Phi(y)$ . The higher the value of  $v$ , the lower the incentives to adopt short-run decisions that could increase the likelihood of being closed.

### **C One-Period Profits and the Probability of Being Closed**

Notice that, if  $C(D)$  were linear and shareholders were able to raise as much capital as they wanted at an opportunity cost  $r$ , the bank's one-period profit function,  $\Pi(y)$ , would be homogeneous of degree 1 in  $D$  and  $K$ , whilst the capital requirement inequality and the probability of survival  $\Phi(y)$  would be homogeneous of degree 0:

$$\begin{aligned} \Pi(y) &= (1+r)^{-1} E[\max\{R(\sigma)(D+K) - C(D), 0\}] - K, \\ \Phi(y) &= \text{Prob}[R(\sigma)(D+K) - C(D) \geq 0]. \end{aligned}$$

In such a situation, the value function would also be homogeneous of degree 1 in  $D$  and  $K$ . Degree-one homogeneity implies that either  $\Pi(y)$  is zero and, hence,  $v$  is zero for the optimal values of  $D$  and  $K$  (and, accordingly, the optimal scale of the bank is indeterminate) or the optimization problem lacks strictly positive and bounded solution for  $D$  and  $K$ .

Assume, for example, that  $C(D) = (1 + r_D)D$ , where  $r_D$  stands for the (constant) interest rate paid on deposits. It is well-known that the equilibrium outcome in a perfectly competitive industry facing constant returns to scale entails that prices are such that a zero-profit condition holds. Similarly, perfect competition between banks like the one described above would necessarily lead to an equilibrium deposit interest rate  $r_D$  such that  $\Pi(y)$  and  $v$  would be equal to zero. If  $v$  equals zero, however, the optimal control simply maximizes one-period expected profits (see equation [8]), thus ignoring the impact of these decisions on the probability of remaining open. With this equilibrium argument, we can state that under perfect competition closure rules do not affect bank decisions. Therefore, dynamic considerations are not relevant to solving the optimization problem of the bank and static models are essentially valid to describe its behavior.

Previous literature has shown that the behavior of insured banks in static perfectly competitive frameworks is characterized by maximum leverage and asset risk (Furlong and Keeley, 1989). Such behavior increases the risk of failure and may affect the equilibrium returns on bank assets (Genotte, 1990; Genotte and Pyle, 1991) and the efficiency of investment decisions by banks. Our result means that closure rules as specified above are ineffective in lessening these problems.

Nevertheless, perfect competition might not be the most reasonable hypothesis. Locational, informational and reputational rents may arise in the normal course of the banking business; switching costs and regulatory barriers to entry are possible sources of market power.<sup>4</sup> For many years, direct controls on interest rates and commissions and regulatory restrictions on branching and geographical expansion imposed clear limits to competition. In fact, the structure of the banking sector in many countries is far from the idealized atomistic market structure where price-taking arises as a natural assumption.

Standard models of oligopolistic competition have long been applied to analyzing the

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<sup>4</sup>Sharpe (1990) offers a model of bank-customer relationships where informational rents for the bank are generated.

banking industry (see Gilbert 1984). More recently, several authors have successfully developed specific models of banking and financial intermediation inspired in modern industrial organization analysis (Repullo 1991, Matutes and Vives 1992).

The aim of this paper, however, is not to examine complex strategic interactions between banks in an imperfectly competitive framework, but the effect of closure rules on bank behavior. Up to now we know that such effect is null if there is nothing to lose in the event of closure. The easiest way to obtain positive rents in this model is to assume that the bank exercises monopoly power in the market for deposits. So, I will analyze that case in the following sections.

#### **D *Modelling Market Power***

Suppose that the bank has a local monopoly in the supply of deposits and bank deposits provide transaction and liquidity services to depositors, who are consequently willing to demand these assets even if their rate of return  $r_D$  is smaller than the risk free rate  $r$ .<sup>5</sup> Clearly, when  $r_D$  equals  $r$ , depositors will hold all of their financial wealth as deposits. When  $r_D$  is smaller than  $r$ , the opportunity cost of holding deposits is positive, but the utility of liquidity services may compensate for it and the demand for deposits may still be positive. Define the intermediation margin  $\mu$  as the difference between  $r$  and  $r_D$ . If the marginal utility of liquidity services is decreasing in the amount of deposits, the demand for deposits will be a decreasing function of  $\mu$ .

For notational convenience, we can normalize the (exogenously given) financial wealth of potential depositors to unity. Then, according with the intuition sketched above and in terms of the inverse demand for deposits, the behavior of depositors can be parametrized as follows:

$$(1 + r_D) = (1 + r) D^\eta \quad \eta \geq 0, 0 \leq D \leq 1, \quad (9)$$

so that, taking logs,  $\mu \simeq -\eta \log(D)$ . Notice that  $\mu$  decreases as  $D$  increases, and is strictly positive for  $D < 1$  and zero for  $D = 1$ .  $\eta$  is the semi-elasticity of  $\mu$  to changes in  $D$ . The

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<sup>5</sup>I will refer to the “supply of deposits” instead of the more usual “demand for deposits” because deposits are conceived as assets issued by banks and demanded by depositors as an alternative to other financial assets.

greater  $\eta$ , the greater the degree of market power of the monopolist bank. Equation (9) yields a simple specification for  $C(D)$ ,  $C(D) = (1+r)D^{\eta+1}$ .

In order to obtain a convenient closed-form for  $\Pi(y) = E[\psi(1, y)]$ , the following parametrization of  $R(\sigma)$  is also assumed:

$$R(\sigma) = (1+r) \exp(\sigma z - \sigma^2/2) \quad (10)$$

where  $z$  is a Gaussian white noise process. According to (10),  $R(\sigma)$  is a log-normally distributed random variable with expected value equal to  $(1+r)$  and standard deviation increasing in  $\sigma$ .<sup>6</sup>  $F(z)$  will denote the cumulative distribution function of  $z$  and  $f(z)$  the corresponding density function.

Putting all of their components together, we have

$$\begin{aligned} \psi(1, y) &= \max \{ \exp(\sigma z - \sigma^2/2) (D+K) - D^{\eta+1}, 0 \} - K, \\ \Phi(y) &= \text{Prob} [ \exp(\sigma z - \sigma^2/2) (D+K) - D^{\eta+1} \geq 0 ], \end{aligned}$$

(notice that the discount factor cancels out with the  $(1+r)$  terms in  $R(\sigma)$  and  $C(D)$ ). Now, since  $\exp(\sigma z - \sigma^2/2) (D+K) - D^{\eta+1} \geq 0$  if and only if

$$z \geq (1/\sigma) [(\eta+1) \log(D) - \log(D+K) + \sigma^2/2] \equiv w,$$

we can write

$$\begin{aligned} E[\psi(1, y)] &= \int_{-\infty}^{+\infty} \max \{ \exp(\sigma z - \sigma^2/2) (D+K) - D^{\eta+1}, 0 \} f(z) dz - K \\ &= \int_w^{+\infty} [ \exp(\sigma z - \sigma^2/2) (D+K) - D^{\eta+1} ] f(z) dz - K. \end{aligned}$$

From the normality of  $z$ ,  $E[\psi(1, y)]$  can be written in terms of the cumulative distribution function of a normal random variable. So, integrating by parts and rearranging,

$$\Pi(y) = E[\psi(1, y)] = F(x) (D+K) - F(x-\sigma) D^{\eta+1} - K, \quad (11)$$

where

$$x \equiv \sigma - w = (1/\sigma) [ \log(D+K) - (\eta+1) \log(D) + \sigma^2/2 ]. \quad (12)$$

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<sup>6</sup>Although non-crucial to get the main results, this parametrization simplifies notably the algebra and puts our valuation formulas in connection with those of the option-theoretic approach to deposit insurance, initiated by Merton (1977).

On the other hand,

$$\Phi(y) = \text{Prob}[z \geq w] = F(x - \sigma). \quad (13)$$

The first two terms in equation (11) represent, respectively, the value of assets and liabilities to the bankers. The first one is the product of the probability of the bank being able to pay off depositors at the end of the period,  $F(x - \sigma)$ , times the conditional on solvency present expected value of assets,  $[F(x) / F(x - \sigma)](D + K)$ .<sup>7</sup> The second is the expected present value of payments to depositors,  $F(x - \sigma)D^{\eta+1}$ . Expression (11) is akin to the Black-Scholes formula for the valuation of call options and has, as the latter, the interesting property of its partial derivative with respect to  $x$  being equal to zero.<sup>8</sup>

### III THE VALUE OF THE BANK

Expression (11) and (13) can be used to show that the value of the bank is well-defined in the sense that there exists a unique  $v$  that solves equation (7). If we denote by  $H(v)$  the auxiliary function defined by the right hand side of equation (7):

$$H(v) = \sup_{y \in \Gamma} \left\{ \Pi(y) + (1+r)^{-1} \Phi(y)v \right\}, \quad (14)$$

the equilibrium value of the bank,  $v^*$ , is a fixed point in  $H(v)$ ,  $v^* = H(v^*)$ . Using the Theorem of the Maximum,  $H(v)$  can be shown to be a continuous, increasing and positive function of  $v$ , and the Contraction Mapping Theorem guarantees the existence of a unique  $v^*$ . Diagrammatically,  $v$  is defined by the intersection between the graph of  $H(v)$  and the 45-degree line (see Figure 1).<sup>9</sup>

Comparative statics results can be obtained by examining how changes in the regulatory and structural parameters move the graph of  $H(v)$  in the neighborhood of  $v^*$ . Upward shifts increase the steady state value of the bank, whereas downward shifts reduce it. Comparative statics on  $v^*$  are interesting for two reasons. First, they provide insights into the

<sup>7</sup>The conditional on solvency present value of assets is the mean of a truncated log-normal variable (only non-bankruptcy values of  $z$  are relevant) and, hence, results from integrating  $\exp(\sigma z - \sigma^2/2)$  over  $[w, \infty)$  with respect to  $[1 - F(w)]^{-1} f(z) dz = [F(x - \sigma)]^{-1} f(z) dz$ .

<sup>8</sup>In order to derive this result, notice that  $\partial F(x - \sigma) / \partial x = f(x - \sigma) = \exp(\sigma^2/2 - \sigma x) f(x)$ , write  $\partial \Pi(y) / \partial x = f(x) [(D + K) - \exp(\sigma^2/2 - \sigma x) D^{\eta+1}]$ , and use the definition of  $x$  in order to show that the term in square brackets is zero.

<sup>9</sup>Notice that the slope of  $H(v)$  is positive but never greater than  $(1+r)^{-1} < 1$ , whilst the ordinate at origin,  $\sup \Pi(y)$ , is strictly positive when the bank has market power.

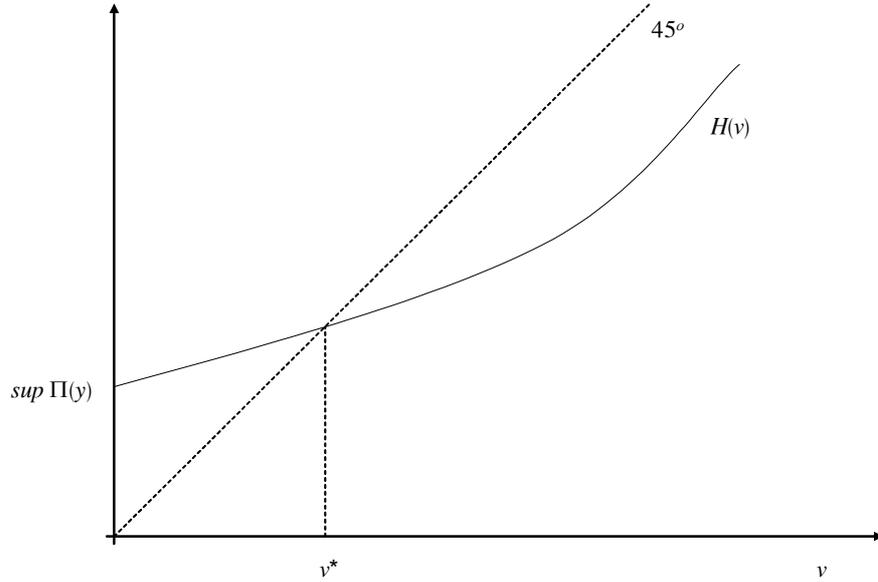


Figure 1: Existence and uniqueness of  $v^*$

fundamentals that determine the value of a chartered regulated bank. Some models of bank behavior have included exogenous charter values, but the value of a charter is intrinsically endogenous. Second, the optimal policy of the bank depends critically upon  $v^*$  (see equation [8]) and comparative statics will allow us to analyze in the next section the impact of structural and regulatory parameters on bank risk-taking.

In order to study the movements of  $H(v)$ , write  $H(v)$  as  $G(y(v), v)$ , where  $G(y, v) = \Pi(y) + (1+r)^{-1} \Phi(y)v$  and  $y(v) \in \{\arg \max G(y, v), y \in \Gamma\}$ . Comparative statics is notably simplified by the fact that  $y(v)$  represents an optimal choice of  $y$  given  $v$ . Table 1 contains the comparative statics for changes in the capital requirement  $k$ , the regulatory limit on the level of risk  $\bar{\sigma}$ , the risk free rate  $r$  and the degree of market power  $\eta$ .

TABLE 1  
REGULATORY AND STRUCTURAL DETERMINANTS  
OF THE VALUE OF A BANK

Parameter ( $w$ )	Sign of $dv^*/dw$	Implicit assumption
$k$	-	$k$ is binding <sup>†</sup>
$\bar{\sigma}$	+	$\bar{\sigma}$ is binding <sup>†</sup>
$r$	-	none
$\eta$	+	none

<sup>†</sup> Otherwise,  $dv^*/dw = 0$ .

Parameters  $k$  and  $\bar{\sigma}$  affect the definition of  $\Gamma$  but not that of  $G(y, v)$ . Besides, only when the constraints defined by  $k$  or  $\bar{\sigma}$  are binding at  $y(v)$ , a small change in them can change  $y(v)$  and  $H(v)$ . So, when the capital requirement is binding, higher (lower)  $k$  reduces (widens)  $\Gamma$  and causes  $H(v)$  and, then,  $v^*$  to be smaller (greater). When it is not, small variations in  $k$  are innocuous. Similarly, if the upper bound to portfolio risk is binding at  $y(v)$ , greater  $\bar{\sigma}$  can make  $H(v)$  and  $v^*$  to rise, whilst smaller  $\bar{\sigma}$  has the opposite effect.

Parameters  $r$  and  $\eta$  enter the definition of  $G(y, v)$  but not that of  $\Gamma$ . Thus, on the one hand, small changes in them cannot change the components of  $y(v)$  which represent a corner solution. On the other, the envelope theorem ensures that, if the interior components of  $y(v)$  vary, their variation will only have (negligible) second order effects on  $G(y, v)$ . In sum, only the *direct* effects of  $r$  and  $\eta$  on  $G(y, v)$  are relevant, and these effects can be computed as the corresponding partial derivatives of  $G(y, v)$  at  $y(v)$ . From equations (11) and (13):

$$\begin{aligned} \partial G(y, v) / \partial r &= -(1+r)^{-2} \Phi(y) v < 0, \\ \partial G(y, v) / \partial \eta &= - \left[ F(x - \sigma) D^{\eta+1} + (1/\sigma) (1+r)^{-1} f(x - \sigma) v \right] \log(D) > 0. \end{aligned}$$

(Notice that  $\log(D) < 0$ , since from our normalization  $D \in [0, 1]$ .) Intuitively, increases in  $r$  cause the present value of future expected profit to fall and, hence, reduce  $H(v)$  and the value of the bank. On the other hand, when the degree of market power increases, potentially wider intermediation margins and higher expected profits account for a greater value of  $H(v)$  and  $v^*$ .

These results support the intuitive idea that regulatory reforms that reduce the operative capacity of the bank cannot do bankers good and are consistent with the usual reluctance of the banking industry to accept harder regulatory constraints, except, perhaps, when they enhance directly or indirectly the degree of market power or the rents of the existing institutions. Note that such reluctance would not make sense in a perfectly competitive environment, because the equilibrium value of a bank would be zero anyway.

## IV THE EFFECTS OF REGULATION ON BANK RISK-TAKING

This section is devoted to analyze the impact of structural and regulatory parameters on bank risk-taking. First, I will characterize the behavior of the bank for all possible values of  $v$ . Because of non-convexities introduced by limited liability, the bank will follow one of two distinct types of policy: a safe policy (where  $\sigma = 0$  and the optimal capital structure is indeterminate) or a risky policy (where  $\sigma = \bar{\sigma}$  and the capital requirement is binding), the type that dominates depending on  $v$ . Later on, I will focus on the equilibrium policies -i.e. the policies which correspond to the equilibrium value of the bank  $v^*$ - in order to determine the impact of the structural and regulatory parameters of the model on the equilibrium type of policy and the solvency of the bank. Finally, I will discuss the empirical implications of the model and some trade-offs which may be relevant for the design of banking regulation.

### A *Characterizing Bank Behavior: Safe and Risk Policies*

The set of optimal policies for a given value of  $v$  is defined as follows

$$Y(v) = \arg \max_{y \in \Gamma} \left\{ \Pi(y) + (1+r)^{-1} \Phi(y)v \right\}.$$

The optimization problem underlying this definition has corner solutions for  $\sigma$ . This means that the solution found in static models under perfect competition also appear in this dynamic monopolistic framework. The difference, however, is that in this context the riskiest policy is not necessarily the optimal one. Briefly, according to the value of  $v$ , the bank decides to be safe or to be risky.

**Lemma 1** *There is no interior solution for  $\sigma$ . Depending on the parameters,  $\sigma(v)$  may*

be either 0 or  $\bar{\sigma}$ . Moreover, a policy involving  $\sigma = \bar{\sigma}$  can only be optimal if the capital requirement is binding,  $K = kD$ .

*Proof:* See the Appendix.

Lemma 1 implies that policies with  $0 < \sigma < \bar{\sigma}$  whatever  $k$ , or with  $\sigma = \bar{\sigma}$  with  $K > kD$  will never be optimal. Intuitively, the bank decides either to be safe in order to preserve its future rents or to exploit the deposit insurance system by means of the highest feasible volatility and leverage. Accordingly, we can condition the analysis of optimal policies upon the choice of  $\sigma$ , characterizing first the best policies for  $\sigma = 0$ ,  $Y_S(v)$  (the best safe policies), and then the best policies for and  $\sigma = \bar{\sigma}$  and  $K = kD$ ,  $Y_R(v)$  (the best risky policies). Clearly, the unconditional optimal policies are the best of  $Y_S(v)$  and  $Y_R(v)$ .

*The best safe policies.* Under the safe choice of  $\sigma$  the bank cannot fail, the capital structure is irrelevant and being or not closed becomes a deterministic outcome. The profit function takes a very simple expression:

$$\Pi(y) = \Pi(D, K, 0) = D - D^{\eta+1}. \quad (15)$$

The conditional optimal supply of deposits is defined by the first order condition derived from (15), which leads to  $D_S = [1/(1+\eta)]^{1/\eta}$ . This expression is always smaller than 1 when  $\eta$  is strictly higher than zero. As neither  $\Pi(D, K, 0)$  nor  $\Phi(D, K, 0) = 1$  depend on  $K$ , the bank will choose any  $K \geq kD_S$ . Notice that the best safe policies are independent of  $v$ . Finally, let

$$\Pi_S(y) = \max_{y \in \Gamma} \left\{ \Pi(D, K, 0) + (1+r)^{-1} \Phi(D, K, 0) v \right\} = \frac{\eta}{1+\eta} D_S + (1+r)^{-1} v.$$

*The best risky policies.* Under  $\sigma = \bar{\sigma}$  and  $K = kD$ , the probability of being closed at the end of a period is positive for all  $D > 0$ . The conditional optimal supply of deposits  $D_R(v)$  can be defined —after substituting  $kD$  for  $K$  in the expressions for  $\Pi(y)$  and  $\Phi(y)$ — as

$$D_R(v) = \arg \max_{0 \leq D \leq 1} \left\{ (1+k) F(x) D - F(x-\bar{\sigma}) D^{\eta+1} - kD + (1+r)^{-1} F(x-\bar{\sigma}) v \right\}, \quad (16)$$

where

$$x = (1/\bar{\sigma}) \left[ \log(1+k) - \eta \log D + \bar{\sigma}^2/2 \right]. \quad (17)$$

An unique interior solution to this problem does not necessarily exist. For one thing, the usual first order condition may yield a value of  $D$  which is greater than one. For another, even if it yields a single  $0 < D < 1$ , the second order condition does not necessarily hold, so that it can be a minimum, a maximum or a saddle point. Non-convexities might lead in principle to corner solutions such as  $D = 0$  and  $D = 1$ .

Computing  $D_R(v)$  will require, in general, numerical calculation, however the following lemma states that for values of  $v$  such that the best risky policies are better than the safe policies,  $D_R(v)$  is greater than  $D_S$ .

**Lemma 2** *For any  $v \geq 0$  such that  $\sigma(v) = \bar{\sigma}$ ,  $D_R(v) > D_S$ .*

*Proof: See the Appendix.*

Intuitively, when appropriating the subsidy to risk-taking which is associated to the deposit insurance system makes sense, the bank is willing to pay higher deposit rates than under the best safe policy, since the increase in the subsidy due to greater  $D$  pays for the fall in intermediation margins, the subsequent loss of monopoly rents and the increase in the probability of being closed.

Finally, notice that  $Y_R(v)$ , in contrast to  $Y_S(v)$ , may depend on  $v$ , and let

$$H_R(v) = \max_{0 \leq D \leq 1} \left\{ \Pi(D, kD, \bar{\sigma}) + (1+r)^{-1} \Phi(D, kD, \bar{\sigma}) v \right\}.$$

*The unconditional optimal policies.* Now, by comparing  $H_S(v)$  with  $H_R(v)$  we can obtain the unconditional optimal policy for each  $v$ . From previous results, the function  $H(v)$  defined in expression (14) is equal to  $\max\{H_S(v), H_R(v)\}$ . Proposition 3 shows that the optimal policy depends crucially but in a simple and intuitive way on  $v$ .

**Proposition 1** *There exists a unique  $\bar{v} \geq 0$  such that the optimal policy is the safe policy for all  $v \geq \bar{v}$  and the risky policy for all  $v \leq \bar{v}$ .*

*Proof: See the Appendix.*

Figure 2 depicts functions  $H_S(v)$  and  $H_R(v)$ . As shown in the picture, the ordinate at the origin is higher for  $H_S(v)$  than for  $H_R(v)$ , but the slope of  $H_S(v)$  is greater than that of  $H_R(v)$  given the positive probability of being closed (and losing the charter value) under the risk policy. As proved in the proposition, they intersect in a single point  $\bar{v}$ , that separates the values of  $v$  at which each policy dominates.

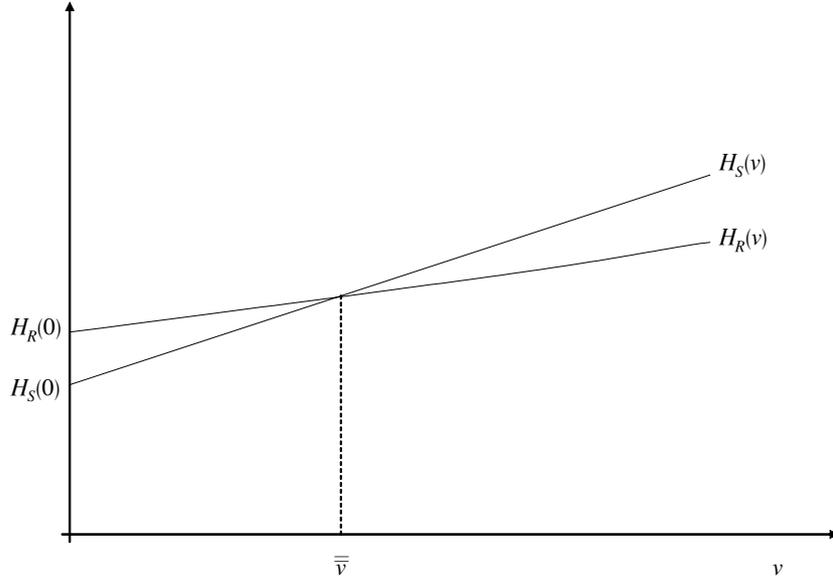


Figure 2: Choosing between the best safe and risky policies

## B Regulatory and Structural Determinants of Bank Risk-Taking

From Proposition 1 we can deduce that  $H(v)$  is a continuous function with a kink in  $\bar{v}$ . Whether the equilibrium policy of the bank is safe or risky depends upon the relative position  $\bar{v}$  with respect to the fixed point of  $H(v)$ ,  $v^*$ . Accordingly, in order to analyze the impact of the parameters on the bank's policy, I will consider three cases:  $v^* > \bar{v}$ ,  $v^* < \bar{v}$  and  $v^* = \bar{v}$ .

(i)  $v^* > \bar{v}$ . When the equilibrium value of the bank is relatively high, the bank chooses a safe policy,  $y(v^*) \in Y_S(v^*)$ . Capital and asset regulations are not binding and the probability of failing the solvency test is zero. As the bank is safe, small changes in the parameters are innocuous from a prudential point of view: the probability of failure is zero. The parameters  $k$  and  $\bar{\sigma}$  do not affect the equilibrium value of the bank. On the contrary,  $r$  and  $\eta$  have the effects shown in Table 1.

(ii)  $v^* < \bar{v}$ . When the equilibrium value of the bank is relatively low, the bank chooses a risky policy,  $y(v^*) \in Y_R(v^*)$ . Both capital and asset regulations are binding and the probability of failing the solvency test is strictly positive. The solvency of the bank, measured by  $F(x - \sigma)$ , depends directly on the parameters  $k$ ,  $\bar{\sigma}$  and  $\eta$  and on the optimal supply of

deposits. This, in turn, is affected by the parameters and by the equilibrium value of the bank, and is higher than with the safe policy. Finally, all the parameters  $k$ ,  $\bar{\sigma}$ ,  $r$  and  $\eta$  have non-null effects on the equilibrium value of the bank, as shown in Table 1 (see section III). So the comparative statics on the solvency of the bank involves three terms:

$$\frac{\partial F(x - \sigma)}{\partial w} + \frac{\partial F(x - \sigma)}{\partial D} \cdot \frac{\partial D_R}{\partial w} + \frac{\partial F(x - \sigma)}{\partial D} \cdot \frac{\partial D_R}{\partial v} \cdot \frac{\partial v^*}{\partial w}, \quad (18)$$

for  $w = k, \bar{\sigma}, r, \eta$ .<sup>10</sup> From equation (17), we can deduce that the direct effect (the first term in [18]) is positive for  $k$  and  $\eta$  (recall that  $D_R \leq 1$ ), negative for  $\bar{\sigma}$ , and null for  $r$ . If the supply of deposits does not vary, higher capitalization, a smaller upper bound on portfolio risk and increased market power enhance the solvency of the bank.

The effects coming from the shift in the supply of deposits depend on the sensitiveness of  $D_R(v^*)$  to  $v^*$  and the parameters  $w$ . If the equilibrium supply of deposits is unique and equals one,  $\partial D_R/\partial w$  and  $\partial D_R/\partial v$  equal zero and these effects disappear. When the equilibrium supply of deposits is smaller than one, the signs of  $\partial D_R/\partial w$  and  $\partial D_R/\partial v$  are ambiguous. Some numerical examples show that the supply-of-deposits effects are generally small compared to the direct effects.

(iii)  $v^* = \bar{v}$ . When the equilibrium value of the bank equals the critical value  $\bar{v}$ , the bank is indifferent between the risky policies in  $Y_R(v^*)$  and the safe policies in  $Y_S(v^*)$ . The situation under each of these policies is that described in (i) and (ii). In this case, however, changes in  $k$ ,  $\bar{\sigma}$ ,  $r$  and  $\eta$  may lead the bank to shift from safe to risky or vice versa.

For  $k$  and  $\bar{\sigma}$ , the result is immediate. Tighter regulation makes the bank to prefer a safe policy, since regulatory burdens reduce the value of the best risky policy, but not that of the best safe policy (remember Table 1). Diagrammatically, higher  $k$  and lower  $\bar{\sigma}$  move downward the curve  $H_R(v)$ , but do not alter  $H_S(v)$ . Thus,  $\bar{v}$  moves to the left, while  $v^*$  remains constant. Figure 3 illustrates this result.

Variations in  $r$  and  $\eta$  change the position of both  $H_R(v)$  and  $H_S(v)$  and diagrammatic analysis is not enough to clarify whether the risky or the safe policies dominate after the change. However, the results are unambiguous. Proposition 2 shows that an increase in the interest rate introduces an advantage for risky policies, since their probability rests

<sup>10</sup>Equation (18) holds for cases in which  $D_R(v^*)$  is unique; otherwise, differentiating  $D_R(v^*)$  makes no sense.

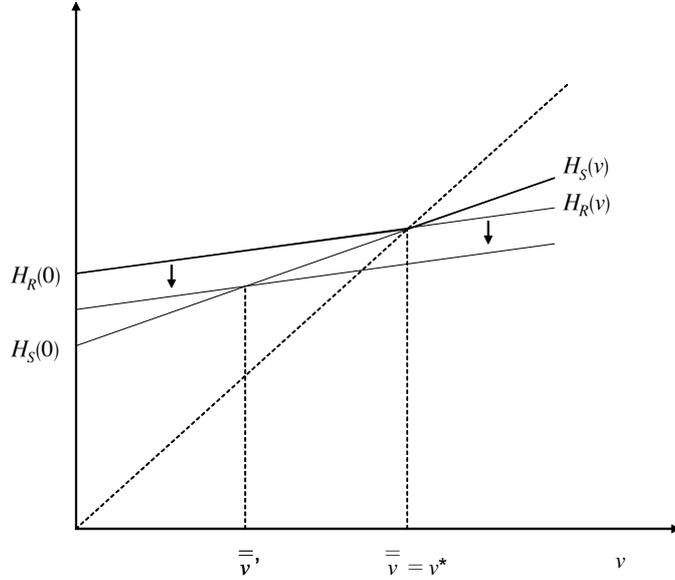


Figure 3: The effects of more stringent regulation

comparatively more on one-period profits and less on future discounted profits:

**Proposition 2** *When  $v^* = \bar{v}$ , small increases (decreases) in  $r$  will lead the bank to choose the risky (safe) policies.*

*Proof: See the Appendix.*

On the other hand, the equilibrium value of a bank is the expected discounted value of current and future one-period profits. Greater market power ( $\eta$ ) enhances the current and future profits of the bank under both the risky and the safe policies. Nevertheless, the benefit of greater  $\eta$  is higher the lower the supply of deposits (as an extreme case, when  $D$  equals one the benefit is zero). Accordingly, an increase in market power makes the safe policy better since, from Lemma 2,  $D_S$  is smaller than  $D_R(v^*)$  :

**Proposition 3** *When  $v^* = \bar{v}$ , small increases (decreases) in  $\eta$  will lead the bank to choose the safe (risky) policies.*

*Proof: See the Appendix.*

## C *Implications*

The results obtained in this section provide a clear understanding of how regulatory and structural parameters influence the safety of banks. Given the sort of bang-bang behavior implied by the model, a bank can suddenly switch from the safe to the risky policies not only as a result of sudden big changes in the economic environment, but also of small accumulated changes in the economic circumstances. For instance, small changes in the stringency of capital and asset regulations, in the degree of market power of incumbent banks or in the macroeconomic conditions as reflected in the interest rate and the level of systematic risk (which is likely to affect  $\bar{\sigma}$  for a given asset regulation) can increase the latent advantages of, say, risky policies versus safe policies. If changes in the same direction continue over time, a point can be reached where the bank jumps from the latter to the former, whilst in the meantime the potential deterioration of solvency remains hidden.<sup>11</sup>

Some authors have argued that the market value of equity could be used for monitoring the safety of banks and pricing deposit insurance guarantees.<sup>12</sup> With the previous result, however, situations can be identified in which the jump to risky policies may coincide with increases in the value of the bank, which may contribute to confuse the regulator.<sup>13</sup>

The results concerning the case  $v^* = \bar{v}$  can be used to analyze the effects of regulatory reforms and changes in  $r$  when regulation applies uniformly to a set of heterogeneous banks, say the banks in a national banking industry. Assume that banks only differ in their degree of (local) monopoly power and, then,  $\eta$  is distributed across banks in a certain fashion. For a given set of parameters,  $k$ ,  $\bar{\sigma}$  and  $r$ , the case  $v^* = \bar{v}$  will correspond to a particular  $\eta = \bar{\eta}$ . According to previous results, banks with  $\eta < \bar{\eta}$  will prefer a risky policy, whereas banks with  $\eta > \bar{\eta}$  will prefer a safe policy. Higher  $k$ , lower  $\bar{\sigma}$  and lower  $r$  imply a lower  $\bar{\eta}$ , reduce the set of banks which choose to be risky and, hence, the risk of the banking industry as a whole. The effects are the opposite if capital and asset regulations are lessened and the interest rate goes up.

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<sup>11</sup>These arguments are strictly valid within the stationary version of the model that has been presented in this paper if subsequent changes in the parameters are fully unanticipated and taken to be permanent.

<sup>12</sup>Marcus and Shaked (1984), and Ronn and Verma (1986), among others, have used option valuation formulas and equity values to infer the (underlying) value of bank assets.

<sup>13</sup>Assume, for example, that  $\bar{\sigma}$  increases so as to make the risky policy better. The rise in  $\bar{\sigma}$  does not change the value of the safe policy, but increases the value of the risky policy. Then, the shift to the risky policy will entail an increase in equity value, i.e. higher  $v^*$ !

The results under heterogeneity are very interesting from an aggregate point of view. Individual banks jump from safe to risky policies (or vice versa), but the industry as a whole may evolve more smoothly. Actually, changes in the aggregate will depend on the number and importance of the banks concentrated around  $\bar{\eta}$  in a particular situation.

From a prudential perspective, these results show to what extent capital and asset regulations can compensate of the deterioration of soundness associated to an increasingly competitive banking sector (i.e. a downward shift in the distribution of  $\eta$  across banks). On the other hand, although  $\eta$  has been taken so far as given, banking regulation by means of the chartering policy and the requirements for the creation of new banks or branches, among others, can create barriers to entry which protect and enhance the exercise of monopoly power by depository institutions. In this model, a higher  $\eta$  is an alternative to capital and asset regulations when the regulator tries to promote solvency. Since regulatory policies increasing  $\eta$  will obviously lessen depositors' welfare by decreasing the interest rate paid on deposits, some authors had previously referred to this trade-off as one between efficiency and solvency. Nevertheless, the costs of bank failures to the tax-payers and the welfare costs and the benefits of the alternative regulatory instruments should be taken into account before delivering normative conclusions. Further research could center on this issue from an optimal regulation perspective à la Laffont-Tirole.<sup>14</sup>

## V THE OPTION TO RECAPITALIZE AND THE EFFECTIVENESS OF CLOSURE

Throughout this paper, I have considered the simple (but realistic) closure rule under which banking authorities deny renewal of the charter and close the bank if its net worth at the end of a period is negative. The threat of being closed in such a context has been proved to be an effective way to induce the bank to be prudent when the present value of its future rents is sufficiently high. In this section, I show that this disciplinary effect of closure vanishes when an apparently minor change in the closure rule is introduced.

Assume that banking authorities, instead of directly closing the bank when its net worth at the end of a period is negative, allow shareholders to inject new funds into the bank (i.e.

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<sup>14</sup>See Laffont and Tirole (1993) and, for an application to banking, Bensaid *et al.* (1993).

to recapitalize) so as to afford promised payments to depositors and obtain the renewal of the charter. Authorities may find attractive the avoidance of closure and liquidation when recapitalization takes place.

Under this rule, shareholders have an option to retain the charter. The exercise price of such option is the additional capital (if any) that has to be raised in order to fully pay off depositors. In terms of the dynamic optimization program, the state variable,  $I_t$ , becomes also a control variable, whilst previous period net worth,  $N_t$ , becomes a state variable.  $I_t$  is chosen at the beginning of each period, once  $N_t$  is observed. Choosing  $I_t = 1$  requires  $I_{t-1} = 1$  (otherwise the bank would be closed) and entails keeping the charter and recapitalizing, i.e. injecting  $-N_t$  if  $N_t < 0$ , and 0 otherwise. On the contrary, choosing  $I_t = 0$  implies refusing the option.

The dynamic optimization problem of the bank is:

$$\begin{aligned} & \text{Maximize} && E \left[ \sum_{t=0}^{\infty} (1+r)^{-1} [\min \{N_t, 0\} \cdot I_t + \psi(I_t, y_t)] \right] \\ & \{I_t, y_t\}_{t=0}^{\infty} \\ & \text{subject to} && \left. \begin{array}{l} I_t \in \{0, 1\} \\ I_t \leq I_{t-1} \\ y_t \in \Gamma \\ N_0 = 0 \end{array} \right\} t = 0, 1, 2, \dots \end{aligned}$$

where the constraint  $I_t \leq I_{t-1}$  states that if shareholders refuse the option on the charter at any period, the charter is lost forever. The definitions of  $\Gamma$  and  $\psi(I_t, y_t)$  are those given by equations (1) and (2) in section II. Recall that  $\psi(I_t, y_t)$  depends on  $I_t$  but not on  $N_t$ .

Keeping the same notation as above, and with  $S_t = (I_{t-1}, N_t)$  as the vector of state variables, define the following value function:

$$V(S_t) = \sup_{I_t \leq I_{t-1}} \left\{ \min \{N_t, 0\} \cdot I_t + \sup_{y_t \in \Gamma} E \left[ \psi(I_t, y_t) + (1+r)^{-1} V(S_{t+1}) \right] \right\} \quad (19)$$

Let  $v$  be the conditional-on-continuation value of the bank:

$$v = \sup_{y_t \in \Gamma} E \left[ \psi(I_t, y_t) + (1+r)^{-1} V(S_{t+1}) \mid I_t = 1 \right], \quad (20)$$

which is time and state independent. If  $I_t = 1$ , shareholders pay  $-\min \{N_t, 0\}$  and get  $v$ , whilst, if  $I_t = 0$ , the bank is closed and the expression into big braces in (19) takes value zero.

Clearly, shareholders choose to maintain the charter whenever the costs of recapitalization are not higher than  $v$ . So, the choice of  $I_t$  can be characterized as follows:

$$I_t = I(S_t) = \begin{cases} 1, & \text{if } I_{t-1} = 1 \text{ and } N_t \geq -v \\ 0, & \text{otherwise.} \end{cases} \quad (21)$$

Taking into account the optimal choice of  $I_t$ , the expression for  $V(S_t)$  is quite simple

$$V(S_t) = \begin{cases} \min\{N_t, 0\} + v, & \text{if } I_{t-1} = 1 \text{ and } N_t \geq -v \\ 0, & \text{otherwise,} \end{cases}$$

and can be used to compute  $E[V(S_{t+1}) | I_t = 1]$ :

$$\begin{aligned} E[V(S_{t+1}) | I_t = 1] &= \text{Prob}[N_{t+1} \geq 0] \cdot v \\ &\quad + \text{Prob}[-v \leq N_{t+1} \leq 0] \cdot E[N_{t+1} + v | -v \leq N_{t+1} \leq 0]. \end{aligned}$$

On the other hand, from the definition of  $\psi(I_t, y_t)$ ,

$$E[\psi(I_t, y_t) | I_t = 1] = (1+r)^{-1} \text{Prob}[N_{t+1} \geq 0] \cdot E[N_{t+1} | N_{t+1} \geq 0] - K_t.$$

Thus, plugging the last two equations in (20), we have

$$v = \sup_{y_t \in \Gamma} \left\{ (1+r)^{-1} \text{Prob}[N_{t+1} + v \geq 0] \cdot E[N_{t+1} + v | N_{t+1} + v \geq 0] - K_t \right\}.$$

Now, we can drop the time indices and replace  $N$  by  $R(\sigma)(D+K) - C(D)$ :

$$v = \sup_{y \in \Gamma} \left\{ (1+r)^{-1} E[\max\{R(\sigma)(D+K) - C(D) + v, 0\}] - K \right\}. \quad (22)$$

According to this equation, the continuation value of the bank is that of investing  $K$  in a call option which is written not only on bank assets,  $R(\sigma)(D+K)$ , but on the sum of bank assets and the continuation value of the bank,  $v$ . The amount of promised payments to depositors,  $C(D)$ , is the strike price of such option.

Specifying  $R(\sigma)$  and  $C(D)$  as in section II.D, the value of this option can be computed following the same steps that led to  $\Pi(y)$ , yielding:

$$v = \sup_{y \in \Gamma} \Pi(y, v) + (1+r)^{-1} \Phi(y, v) v,$$

where

$$\begin{aligned} \Pi(y, v) &= F(x)(D+K) - F(x-\sigma)D^{\eta+1} - K, \\ \Phi(y, v) &= F(x-\sigma), \end{aligned}$$

and

$$x = (1/\sigma) \left\{ \log(D + K) - \log \left[ D^{\eta+1} - (1+r)^{-1} v \right] + \sigma^2/2 \right\}.$$

$\Pi(y, v)$  and  $\Phi(y, v)$  differ from  $\Pi(y)$  and  $\Phi(y)$  (in previous sections) because of the definition of  $x$ , that now depends on  $v$ . Such dependence reflects that the value of the charter affects shareholders' decision on continuation.

The next result states that, at least for  $r > \eta$ , the optimal policies for the bank are risky policies in the sense that capital and asset regulations are binding. Moreover, since this result does not depend on the values of  $k$  and  $\bar{\sigma}$ , there are situations where traditional regulations are ineffective as a means of inducing a safe policy.

**Proposition 4** *When recapitalization is allowed and  $r > \eta$ , the optimal policies for the bank are risky policies, whatever the values of  $k$  and  $\bar{\sigma}$ .*

*Proof: See the Appendix.*

This result has to do with the form of the payoffs associated to one-period decisions. Under the simple closure rule of previous sections, shareholders win  $N + v$  when  $N$  is positive, and 0 otherwise. On the contrary, when recapitalization is allowed, shareholders win  $N + v$  when  $N + v$  is positive and 0 otherwise.

Figure 4 represents, for both cases, the (discounted) payoffs to shareholders at the end of a period as a function of the (discounted) gross return on bank assets at that date. As can be seen, the difference between the two panels is in the range  $[D^{\eta+1} - (1+r)^{-1} v, D^{\eta+1}]$  of asset returns. With the first closure rule, the bank is closed when  $R(\sigma)(D + K)$  is the interior of that interval. With the second, shareholders recapitalize, paying  $D^{\eta+1} - R(\sigma)(D + K)$  to keep  $(1+r)^{-1} v$ . The non-convexity of the payoff (as a function of the asset returns) in the first case explains shareholders' aversion to risk when  $v$  is high enough. Conversely, the convexity of the payoff in the second case induces risk-loving, except when  $D^{\eta+1} - (1+r)^{-1} v$  becomes negative.

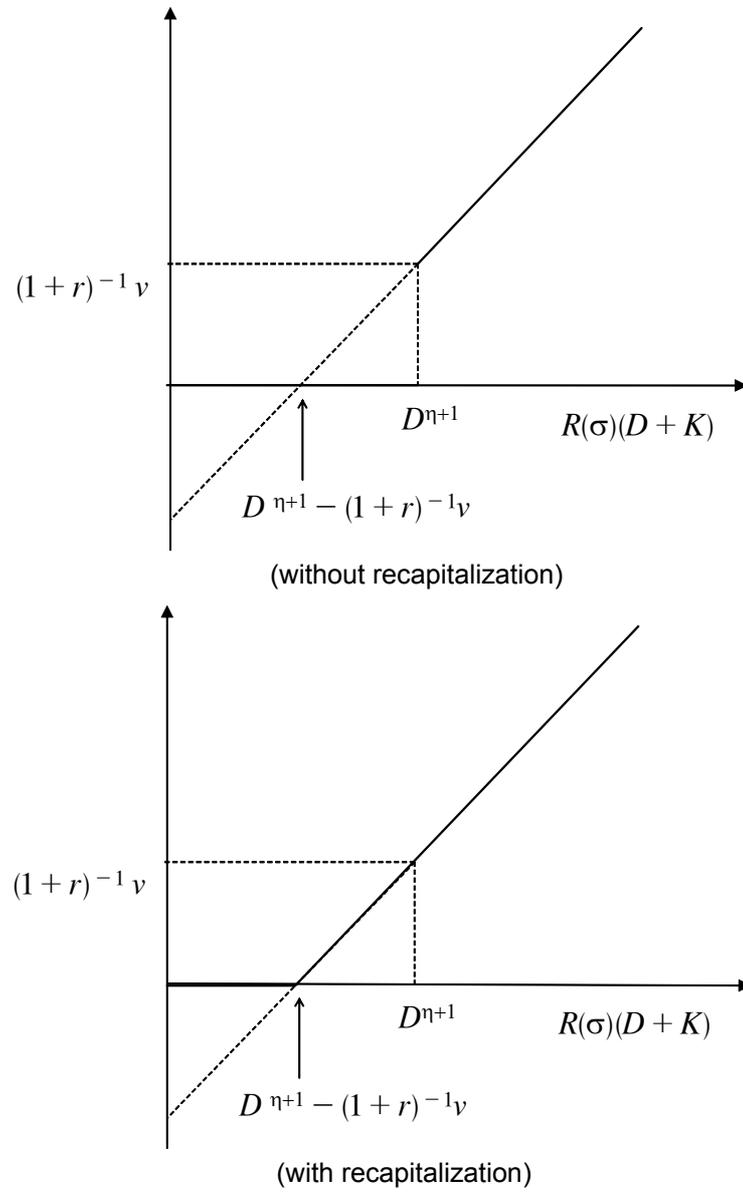


Figure 4: Shareholders' payoffs and the option to recapitalize

In practical terms, this section enters a caveat against a modification of the closure rule that might seem attractive for banking authorities. More concretely, once insolvency takes place, the supervisory agency may prefer the injection of capital by shareholders to its public involvement in the resolution of the crisis. These ex-post incentives to give to the shareholders an option to recapitalize may, however, go against the ex-ante need for

inducing discipline with a (credible) threat of closure.

## VI CONCLUSIONS AND POLICY IMPLICATIONS

In this paper I have analyzed the behavior of a bank in a dynamic model which allows for bankruptcy and closure. The effectiveness of chartering and closure policies and their relationship with market power, capital and asset regulations and risk-taking can be formally settled. Given closure policies and the provisions that exclude from business the promoters and managers of banks which become insolvent, the value of bank charters is an important endogenous component of bankruptcy costs to bankers and may constitute an incentive to adopt prudent decisions.

Dynamic programming techniques have allowed us to obtain the conditional-on-surviving value of a bank,  $v^*$ , together with the bank's optimal investment and financial policies. Comparative statics on  $v^*$  provide insights into the fundamentals that influence the value of a chartered bank. They show that tighter capital and asset regulations (when binding) are associated to a lower  $v^*$ , while smaller rates of interest and greater market power result in a higher  $v^*$ .

The equilibrium behavior of the bank depends crucially on  $v^*$ . Because of non-convexities derived from the limited liability of bankers, optimal policies may be of two distinct extreme types: safe or risky. The risky type of policy dominates when  $v^*$  is lower than a critical value  $\bar{v}$ , whereas the safe type of policy is optimal when  $v^*$  is higher than  $\bar{v}$ . Tough prudential regulation in general elicits the safe policy, since high capital requirements and strong limitations on portfolio risk cause the value of a risky policy to be low, whilst they do not affect the value of a safe policy. Similarly, I have proved that high market power and low interest rates favor safe policies, that rest comparatively more on long-run profits and less on short-run opportunistic exploitation of the deposit insurance system.

From the results, we can deduce that capital and asset regulation, on the one hand, and entry and closure policies, on the other, are alternative ways to preserve the solvency of banks. This trade-off should be taken into account in the design of banking regulation.

Even though some empirical implications of this model seem to be somewhat extreme, several facts give support to the kind of bang-bang behavior predicted here. In particular,

the sudden appearance of recent widespread solvency problems of the savings and loan institutions in the U.S., after a long period of well-functioning of the deposit insurance system, can be explained with this model.<sup>15</sup> Financial innovation and disintermediation accumulated along time had created an environment of tougher competition for depository institutions (smaller  $\eta$ ). Such environment together with the financial deregulation (greater  $\bar{\sigma}$ ) and the higher real interest rates of the late seventies and early eighties (greater  $r$ ) shaped a picture increasingly advantageous (according to our model) for the development of risky policies. Poor realizations of risky investments triggered off the debacle in the mid eighties.

Some regulatory mistakes contributed to aggravate the crisis. In its initial phases, the regulators shored up book value net worth with a variety of accounting changes and reduced the minimum regulatory capital requirements (thereby producing smaller  $k$ ). Their inadequate response included also excessive forbearance and delay in closing or intervening the thrifts in trouble (i.e. relaxing the closure rule and its disciplinary effects).

After a huge number of failures, the U.S. government and banking authorities tried to introduce reforms oriented to restore the prudent behavior which had been predominant during the post-war period. The safety net built after the Great Depression had been partially based upon branching and geographical restrictions and interest rate ceilings, which acted as legal guarantees of market power. But financial innovation during the seventies and eighties irrevocably damaged such conception of the banking business. Nowadays, disintermediation and largely unregulated non-bank financial intermediaries absorb a notable fraction of depositors' wealth, whereas modern information technologies give the banks' best debtors direct access to capital markets. As a consequence, regulatory reform faces the difficult task of restoring solvency in a necessarily more competitive context. This probably means that the future design of prudential regulation will have to rest more on asset and capital restrictions than in the past (the Basle Accord of 1988 and the recent Federal Deposit Insurance Corporation Improvement Act confirm this tendency), but it also calls for prudence in the management of the chartering and closure policies and the way to confront future deregulatory pressures.

In a different context, after the Second Banking Directive of the European Community

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<sup>15</sup>See U.S. Department of Treasury (1991) for a detailed description of the process.

(1989), European banks are able to provide their services throughout the Community with a single banking license from their home country. This rule raises potential threats to financial stability. For one thing, stronger competition and the erosion of charter values could be expected. For another, with banks potentially competing at a European level, domestic chartering policies become ineffective as a mean of controlling the degree of market power of banking institutions under the jurisdiction of home country authorities. Restoring their effectiveness would require coordination between national regulators.

A different but very related topic is the so-called too-big-to-fail problem, whose importance has been remarked by many practitioners. In fact, the evidence in Boyd and Gertler (1993) suggests that solvency problems detected in some large bank holding companies in the U.S. might reflect this problem. In terms of the model, the systematic reluctance of banking supervisors to close banks that are considered too big to fail means that the closure rule is not in force for such banks. All the disciplinary effect potentially related to the rents of bigger banks is lost. Paradoxically, the authorities confer a guarantee of survival upon big banks for fear of causing severe troubles to the financial system and losing the value of the banks as going concerns. This guarantee makes the optimal policy of big banks to be risky and increases the costs of the deposit insurance system.

Although preserving the value of the charter and avoiding the external costs of bankruptcy can make sense, rescue techniques should be designed so as to simultaneously discipline the bankers. Regulators should be allowed to take over banks which failed the established solvency tests, and the final payment to shareholders should only be the liquidation value of the net worth (intangible assets excluded), whatever the size, going-concern value and final destination of the insolvent bank. Accordingly, discipline would be preserved, while if rescued banks had positive going-concern values (so they might have a future under the control of new shareholders), the price paid for the institution by the successful bidders could partially or totally off-set the cost of the funds injected by the authorities to restore solvency.

## APPENDIX

**Proof of Lemma 1** The bank seeks to maximize:

$$G(y, v) = F(x)(D + K) - F(x - \sigma)D^{\eta+1} - K + (1 + r)^{-1}F(x - \sigma)v. \quad (\text{A1})$$

where

$$\frac{\partial G}{\partial \sigma} = f(x - \sigma) \left[ D^{\eta+1} - (1 + r)^{-1}(x/\sigma)v \right] \quad (\text{A2})$$

and

$$\begin{aligned} \frac{\partial^2 G}{\partial \sigma^2} &= -f'(x - \sigma) \left[ D^{\eta+1} - (1 + r)^{-1}(x/\sigma)v \right] x/\sigma^2 + \\ &\quad (1 + r)^{-1}f(x - \sigma) [\log(1 + k) - \eta \log(D)] v/\sigma^3. \end{aligned} \quad (\text{A3})$$

If an interior solution for  $\sigma$  existed,  $0 < \sigma(v) < \bar{\sigma}$ , (A2) would have to be zero, while (A3) would have to be negative. However, when (A2) equals zero, the first term in (A3) is zero, whilst the second is always positive. Thus, (A3) is positive, contradicting the necessary second order condition for a maximum. Consequently,  $\sigma(v)$  may be either 0 or  $\bar{\sigma}$ , but not  $0 < \sigma(v) < \bar{\sigma}$ . Now, let us examine the choice of  $K$  when  $\sigma(v) = \bar{\sigma}$ . From equation (A1), the first order condition associated to an interior solution for  $K$  is:

$$[F(x) - 1] + (1 + r)^{-1}f(x - \sigma)(x/\sigma) \frac{v}{\sigma(D + K)} = 0, \quad (\text{A4})$$

whereas, the necessary second order condition for a maximum is:

$$\frac{f(x)}{\sigma(D + K)} - (1 + r)^{-1}f(x - \sigma)(x/\sigma) \frac{v}{\sigma(D + K)^2} \leq 0$$

From (A4) we can substitute  $1 - F(x)$  for  $[\sigma(1 + r)(D + K)]^{-1}f(x - \sigma)v$  in the left hand side of this inequality. Reordering, we arrive at:

$$\frac{1 - F(x)}{\sigma(D + K)} \left[ \frac{f(x)}{1 - F(x)} - x \right]$$

which is positive, since  $f(x) / [1 - F(x)]$  is the hazard function of a standard normal random variable, that is greater than  $x$  for all  $x \geq 0$ . Then, the second order condition does not hold and no interior solution for  $K$  can exist. If an optimal policy entails  $\sigma = \bar{\sigma}$ , the optimal  $K$  is  $kD$ . The choice of  $\sigma = \bar{\sigma}$  and an infinite  $K$  makes no sense because the value of  $G(y, v)$

at  $(D, K, \bar{\sigma})$  when  $K$  tends to infinity, whatever  $D$ , can be attained with the same  $D$ , any finite  $K'$  and  $\sigma = 0$ .  $\square$

**Proof of Lemma 2** The definition of  $D_R(v)$  in equation (16) can be re-written in the following way:

$$D_R(v) = \arg \max_{0 \leq D \leq 1} \{J(D) + Q(D)\},$$

where

$$J(D) = D - D^{\eta+1} + (1+r)^{-1}v$$

and

$$Q(D) = [1 - F(x - \bar{\sigma})] [D^{\eta+1} - (1+r)^{-1}v] - (1+k) [1 - F(x)] D.$$

By definition,  $J(D)$  attains a unique global maximum at  $D_S$ , is increasing at  $D < D_S$  and decreasing at  $D > D_S$ . On the other hand, as  $\sigma(v) = \bar{\sigma}$ , Lemma 1 implies that the capital requirement is binding. Then,  $G(y, v)$ , defined in (A1), is decreasing in  $K$  at  $K = kD_R(v)$  :

$$\frac{\partial G}{\partial K} = [F(x) - 1] + (1+r)^{-1} f(x - \bar{\sigma}) [\bar{\sigma} (1+k) D_R(v)]^{-1} v < 0. \quad (\text{A5})$$

This inequality allow us to prove that  $Q(D)$  is increasing in  $D$  at  $D_R(v)$  :

$$\frac{\partial Q}{\partial D} = (\eta + 1) [1 - F(x - \bar{\sigma})] D^\eta + (1+k) F(x) - \eta (1+r)^{-1} f(x - \bar{\sigma}) (\bar{\sigma} D)^{-1} v.$$

But, from (A5),

$$(1+r)^{-1} f(x - \bar{\sigma}) [\bar{\sigma} D_R(v)]^{-1} v < (1+k) [1 - F(x)].$$

Then, at  $D_R(v)$ ,

$$\begin{aligned} \frac{\partial Q}{\partial D} &> (\eta + 1) [1 - F(x - \bar{\sigma})] D_R(v)^\eta + (1+k) F(x) - \eta (1+k) [1 - F(x)] \\ &= (\eta + 1) \{ [1 - F(x - \bar{\sigma})] D_R(v)^\eta - (1+k) [1 - F(x)] \} + (1+k) [1 - F(x)] > 0. \end{aligned}$$

(Notice that the term in braces is the value of deposit guarantees per unit of deposits  $(1+r)^{-1} E[\max\{-N, 0\}]/D$ , which is, by definition, positive.) Now we can prove the

final results. Suppose, on the contrary,  $D_R(v) < D_S$ . Then both  $J(D)$  and  $Q(D)$  are increasing at  $D_R(v)$  whereas values of  $D$  higher than  $D_R(v)$  are feasible. This contradicts the definition of  $D_R(v)$ .  $\square$

**Proof of Proposition 1** Using the Theorem of the Maximum,  $H_S(v)$  and  $H_R(v)$  can be shown to be continuous, increasing, positive and with a slope smaller than  $(1+r)^{-1}$ . Now, in order to prove the result, I will show that  $H_S(v)$  and  $H_R(v)$  have a unique intersection at a point  $\bar{v} > 0$ . Equation (A5) shows the partial derivative of  $G(y, v)$  with respect to  $\sigma$ . On the one hand,  $\partial G/\partial \sigma > 0$  at  $v = 0$ , so  $\sigma = 0$  cannot be optimal for  $v = 0$ , then  $H_S(0) < H_R(0)$ . On the other hand, the sign of  $\partial G/\partial \sigma$  is the sign of  $D^{\eta+1} - (1+r)^{-1}(x/\sigma)v$ , which attains to a maximum for  $D = 1$  and  $K = kD$ . Then, there exists a value  $\hat{v} = (1+r) [(1/2) + \log(1+k)/\bar{\sigma}^2]^{-1}$  such that  $\partial G/\partial \sigma < 0$  for all  $(D, K, \bar{\sigma}) \in \Gamma$  and  $v > \hat{v}$  [recall the definition of  $x$  in equation (12)]. Therefore,  $H_S(v) > H_R(v)$  at any  $v > \hat{v}$ . Thus, since  $H_S(v)$  and  $H_R(v)$  are continuous, they intersect at least at one point  $\bar{v}$ . Moreover, the intersection is unique because the slope of  $H_S(v)$  is greater than that of  $H_R(v)$  for all  $v : \Phi(y) = 1 > \Phi(y')$  for any  $y \in Y_S(v)$  and  $y' \in Y_R(v)$ . [Notice that either the Envelope Theorem (in interior solutions) or the fact that  $\Gamma$  does not depend on  $v$  (in corner solutions) ensure  $dH_S/dv = \partial H_S/\partial v = \Phi(y) = 1$  for any  $y \in Y_S(v)$  and  $dH_R/dv = \partial H_R/\partial v = \Phi(y) < 1$  for any  $y \in Y_R(v)$ .] Therefore,  $H_S(v) \geq H_R(v)$  for  $v \geq \bar{v}$  and  $H_S(v) \leq H_R(v)$  for  $v \leq \bar{v}$ , and the result follows.  $\square$

**Proofs of Propositions 2 and 3** Lemma 3 (below) provides necessary and sufficient conditions under which a type of policy dominates the other after the change in a parameter. Intuitively, for the risky type of policy to dominate, the upward (downward) movement of  $H_R(v)$  at  $v^*$  has to be great (small) as compared to the upward (downward) movement of  $H_S(v)$ ; otherwise, the safe type of policy dominates.

**Lemma 3** When  $v^* = \bar{v}$  and a small increase (decrease) in a parameter  $w = k, \bar{\sigma}, r, \eta$  takes place, the risky (safe) policies will dominate the safe (risky) policies if and only if the

following condition holds:

$$\partial H_R / \partial w \geq \left[ 1 + \frac{1 - \Phi(y)}{r} \right] \partial H_S / \partial w. \quad (\text{A6})$$

Otherwise, the safe (risky) policies will dominate the risky (safe) ones.

*Proof:* This proof is based on a geometrical argument which hinges upon a linearization of  $H_R(v)$  and  $H_S(v)$  around their intersection at  $\bar{v} = v^*$ . Figure A1 depicts in augmented scale a case in which the vertical movement of  $H_R(v)$  and  $H_S(v)$  (as a result of a change in a parameter  $w$ ) is such that indifference between risky policies in  $Y_R(v)$  and safe policies in  $Y_S(v)$  remains. Graphically, the situation is characterized by a sufficiently great vertical displacement of  $H_R(v)$ ,  $\overline{AC}$ , as compared with the displacement of  $H_S(v)$ ,  $\overline{AB}$ .

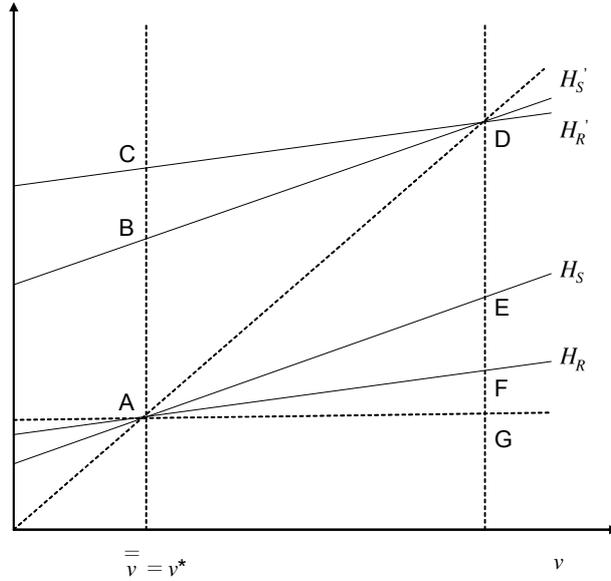


Figure A1

Clearly,  $\overline{AC} = \overline{DF} = \overline{DG} - \overline{FG} \simeq [1 - \partial H_R / \partial v] \overline{AG}$ . Similarly,  $\overline{AB} = \overline{DE} = \overline{DG} - \overline{EG} \simeq [1 - \partial H_S / \partial v] \overline{AG}$ . So, solving for  $\overline{AG}$  in the second equation and substituting back in the first, we get:

$$\overline{AC} = [1 - \partial H_S / \partial v]^{-1} [1 - \partial H_R / \partial v] \overline{AB}$$

Now, if  $\overline{AC}$  is approximated by  $(\partial H_R/\partial w) dw$  and  $\overline{AB}$  by  $(\partial H_S/\partial w) dw$ , the result is:

$$\partial H_R/\partial w = [1 - \partial H_S/\partial v]^{-1} [1 - \partial H_R/\partial v] \partial H_S/\partial w$$

If  $\partial H_R/\partial w$  were greater than in the case depicted in Figure A1, the new intersection of  $H_R(v)$  and  $H_S(v)$  would take place to the right of  $\hat{v}$  and the risky policies in  $Y_R(v)$  would dominate the safe ones in  $Y_S(v)$ . Expression (A6) arises when  $\partial H_S/\partial w$  and  $\partial H_R/\partial w$  are computed:

$$[1 - \partial H_S/\partial v]^{-1} [1 - \partial H_R/\partial v] = \left[ \frac{r}{1+r} \right]^{-1} \left[ \frac{r+1-\Phi(y)}{1+r} \right] = 1 + \frac{1-\Phi(y)}{r}. \square$$

**Proof of Proposition 2** From the Envelope Theorem, the impact of  $r$  on the opti-

mal risky and safe policies can be ignored and simple differentiation leads to  $\partial H_R/\partial r = -(1+r)^{-2} \Phi(y) v^*$  and  $\partial H_S/\partial v = -(1+r)^{-2} v^*$ , with  $y \in Y_R(v^*)$ . As  $\Phi(y) < 1$ , the term in square brackets of condition (A6) is greater than one and condition (A6) holds:

$$-(1+r)^{-2} \Phi(y) v^* \geq -(1+r)^{-2} \left[ 1 + \frac{1-\Phi(y)}{r} \right] v^* \Leftrightarrow \Phi(y) < 1 < 1 + \frac{1-\Phi(y)}{r}. \square$$

**Proof of Proposition 3** As in the proof of Proposition 2, the elements in condition (A6) have to be computed. If  $D_R(v^*)$  equals one (corner solution),  $\partial H_R/\partial \eta$  equals zero, whereas  $\partial H_S/\partial v$  is positive, then the results is clearly true. If  $D_R(v^*)$  is smaller than one (interior solution), we have:

$$\partial H_R/\partial \eta = - \left[ F(x - \bar{\sigma}) D_R(v^*)^{\eta+1} + (1+r)^{-1} f(x - \bar{\sigma}) v^*/\bar{\sigma} \right] \log D_R(v^*) > 0$$

and

$$\partial H_S/\partial v = - [D_S^{\eta+1}] \log D_S.$$

Now, from the first order conditions associated to the optimal choice of  $D_R(v^*)$  and  $D_S$  and the conditions  $H_R(v^*) = v^*$  and  $H_S(v^*) = v^*$ , the terms in brackets can be re-written, leading to:

$$\partial H_R/\partial \eta = -(1/\eta) \left[ \frac{r+1-\Phi(y)}{1+r} v^* \right] \log D_R(v^*)$$

and

$$\partial H_S / \partial v = - (1/\eta) \left[ \frac{r}{1+r} v^* \right] \log D_S > 0.$$

Then, recalling that  $D_R(v^*) > D_S$  and applying Lemma 3, the result follows.  $\square$

**Proof of Proposition 4** Let  $G(y, v)$  be  $\Pi(y, v) + (1+r)^{-1} \Phi(y, v) v$  and notice that the partial derivative of  $G(y, v)$  with respect to  $x$  is zero. Note also that, under  $D^{\eta+1} - (1+r)^{-1} v > 0$ ,  $\partial G(y, v) / \partial \sigma$  is positive and  $\partial G(y, v) / \partial K$  is negative so the bank would adopt risky policies. On the contrary, under  $D^{\eta+1} - (1+r)^{-1} v \leq 0$ , we have  $F(x) = F(x - \sigma) = \Phi(y, v) = 1$ , so the bank would be safe and would choose a safe policy as those described in section IV. In order to prove the result, I will show that when  $r > \eta$  the best safe policies,  $y \in Y_S$ , entail  $D_S^{\eta+1} - (1+r)^{-1} v_S > 0$  so that they cannot be optimal. For any  $y \in Y_S$ ,  $G(y, v) = [\eta / (1 + \eta)] D_S + (1+r)^{-1} v$ , where  $D_S = [1 / (1 + \eta)]^{1/\eta}$ . Then, the value  $v_S$  that solves  $v = G(y, v)$  for all  $y \in Y_S$  can be computed:  $v_S = (1+r) (\eta/r) (1 + \eta)^{-\eta/(1+\eta)}$ . We can easily check that condition  $D_S^{\eta+1} - (1+r)^{-1} v_S \leq 0$  holds if and only if  $r \leq \eta$ .  $\square$

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