Recursive Maturity Transformation*

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Abstract

We develop an infinite horizon model in which banks finance long term assets with non-tradable debt. Banks choose the amount and maturity of their debt taking into account investors' preference for short maturities (which stems from their exposure to idiosyncratic liquidity shocks) and the risk of systemic crises (during which banks' refinancing becomes especially expensive). Unregulated debt maturities are inefficiently short due to the interaction between pecuniary externalities in the market for funds during crises and banks' refinancing constraints. The impact of the externality on banks' continuation value amplifies the importance of the inefficiency. The role for debt maturity regulation prevails even in the presence of private or public liquidity insurance schemes.

JEL Classification: G01, G21, G32

Keywords: liquidity risk, liquidity regulation, liquidity insurance, pecuniary externalities, systemic crises

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1 Introduction

The recent financial crisis has extended the view among regulators that maturity mismatch in the financial system prior to the crisis was excessive and not properly addressed by the existing regulatory framework (see, for example, Tarullo, 2009). When the first losses on the subprime positions arrived in early 2007, investment banks, hedge funds and many commercial banks were heavily exposed to refinancing risk in wholesale debt markets. This risk was a key lever in generating, amplifying, and spreading the consequences of the collapse of money markets during the crisis (Brunnermeier, 2009; Gorton, 2009).

We develop a recursive infinite horizon model in which banks finance long-term assets by placing non-tradable debt among unsophisticated savers. Short maturities are attractive to these savers because they buy bank debt when they are patient but may suffer *shocks* that turn them impatient, in which case waiting until the debt matures to recover its principal is a source of disutility.

We assume, however, that banks are exposed to *systemic liquidity crises*: sudden episodes in which they are unable to place debt among the unsophisticated savers and they have to (temporarily) rely on the more expensive funding provided by some *crisis financiers*.[1] These financiers are sophisticated investors with outside investment opportunities. The heterogeneity in the value of these opportunities produces an upward slopping aggregate supply of funds during crises.

At the initial (non-crisis) period, banks decide the overall principal, interest rate and maturity of their debt (and, residually, their equity financing) by trading off the lower interest cost of short-maturity debt with the anticipated cost of refinancing during crises. Banks choose longer debt maturities (which imply smaller refinancing needs) if they anticipate crises to be more costly. The intersection between crisis financiers' supply of funds and banks' refinancing needs produces a unique equilibrium cost of crisis financing, and some unique equilibrium leverage and debt maturity decisions by banks associated with it.

Importantly, the debt maturity chosen by banks in the unregulated competitive equilibrium is inefficiently short. The reason for this is the combination of pecuniary externalities

[1] To abstract from the difficulties of endogenizing the emergence of these crises, we model them as an exogenous "sudden stop" of the type introduced by Calvo (1998) in the emerging markets literature. See Bianchi, Hatchondo, and Martinez (2013) for a recent application.

with the financial constraints faced by banks in their *maturity transformation* activity.[2] Specifically, we focus on parameterizations for which banks avoid going bankrupt (which we assume would imply their liquidation) during crises and, hence, keep sufficient equity value so as to be able to absorb the excess cost of funding in a crisis by diluting their equity. So, quite intuitively, guaranteeing the access to financing in a crisis, imposes a limit on ex ante leverage and debt maturity choices (that we call the *crisis financing constraint*).

In their uncoordinated, competitive capital structure decisions, banks neglect the impact of their refinancing needs on the equilibrium cost of crisis financing, which tightens the crisis financing constraint of all banks, reduces the total leverage that the banking industry can sustain, and damages the efficiency of the maturity transformation process. We show that a regulator can improve the overall surplus extracted from maturity transformation activities by inducing a lower use of their intensive margin (i.e. the choice of *longer* maturities) and a larger use of their extensive margin (i.e. the issuance of *more* debt) in a way that reduces aggregate refinancing needs during crises. We show the possibility of restoring efficiency through either the direct regulation of debt maturity or with a Pigovian tax on banks' refinancing needs.[3] We also show that the impact of the externality on banks' continuation value amplifies the importance of the inefficiency.

In the two main extensions of the model, we examine the impact of introducing private and public liquidity insurance schemes. The common result from these extensions is that, although smoothly spreading the cost of crisis financing across states of the world helps banks enhance their maturity transformation function, the need for maturity regulation does not vanish. A fairly-priced private liquidity insurance arrangement, if feasible, increases welfare but does not remove the basic pecuniary externality which still operates through the competitive cost of crisis insurance. Under a public liquidity insurance arrangement (e.g. a central bank acting as a lender of last resort during crises), the conclusion is similar. In this case, insurance premia and maturity regulation are complementary tools in attaining the objective of maximizing the surplus generated by banks' maturity transformation while covering the costs of the arrangement for the public insurer.

[2] We refer to maturity transformation to the extent that, in the model, banks issue debt with finite expected maturities backed with infinitely lived assets.

[3] We restrict attention to policy interventions involving no subsidization (no net positive use of government funds) and no greater informational requirements than the unregulated competitive equilibrium.

The paper is organized as follows. Section 2 places the contribution of the paper in the context of the existing literature. Section 3 presents the ingredients of the model. Section 4 defines equilibrium and covers the various steps necessary for its characterization. Section 5 examines the efficiency properties of the equilibrium and possible regulatory interventions. Section 6 extends the analysis to the introduction of private or public liquidity insurance schemes. Section 7 discusses robustness and several potential extensions of the analysis. Section 8 concludes. All the proofs are in the appendices.

2 Related literature

Our paper is in the interface of several literature strands. Our work is first related to the contributions in the infinite-horizon capital structure literature that incorporate debt refinancing risk. Leland and Toft (1996) explore the connection between credit risk and refinancing risk in a model a la Leland (1994) and show that short debt maturities increase the threshold of the firm's fundamental value below which costly bankruptcy occurs. He and Xiong (2012a) further explore the connections between credit risk and liquidity risk by considering shocks to market liquidity that increase the cost of debt refinancing. In a related model, He and Milbradt (2012) explore the feedback between credit risk and the liquidity of a secondary market for corporate debt subject to trading frictions (modeled as search frictions). With a scope closer to the banking literature, He and Xiong (2012b) show that "dynamic" debt runs may occur when lenders stop rolling over maturing debt in fear that future lenders would do the same before the debt now offered to them matures. Cheng and Milbradt (2012) show that this type of dynamic runs may have, up to some point, a beneficial effect on an asset substitution problem. Finally, Brunnermeier and Oehmke (2013) formalize a conflict of interest between long-term and short-term creditors during debt crises that pushes firms to choose debt maturities which are inefficiently short from an individual value maximization perspective.

Of course, many of the underlying themes have also been analyzed in models with simpler time structures. Flannery (1994) emphasizes the disciplinary role of short-term debt in a corporate finance context, and Calomiris and Kahn (1991), Diamond and Rajan (2001), and Huberman and Repullo (2010) in a banking context. In Flannery (1986) and Diamond (1991), short-term debt allows firms with private information to profit from future rating upgrades, while in Diamond and He (2012) short maturities have a non-trivial impact on a classical debt overhang problem. The emergence of roll-over risk as the result of a coordination problem between short-term creditors is also analyzed by Rochet and Vives (2004), Goldstein and Pauzner (2005), and Martin, Skeie, and von Thadden (2013), among others. Various papers, including Acharya and Viswanathan (2011) and Acharya, Gale, and Yorulmazer (2011), study the implications of roll-over risk for risk-shifting incentives, fire sales, and the collateral value of risky securities.

Our work is also connected to recent papers focused on the normative implications of externalities associated with banks' funding decisions. In Perotti and Suarez (2011), the externalities are modeled in reduced form, assuming that, when banks use short-term funding to expand their credit activity, they neglect their (non-pecuniary) contribution to the generation of systemic risk. In Farhi and Tirole (2012), the externalities operate through collective bail-out expectations: time-consistent liquidity support to distressed institutions during crises (e.g. via central bank lending) makes bank leverage decisions strategic complements, producing excessive short-term borrowing and social gains from the introduction of a cap on such borrowing. Finally, in Stein (2012), like in our paper, the inefficiency in banks' debt maturity choices comes from the combination of pecuniary externalities and financial constraints.[4] He develops a three-date model where the fraction of bank debt which is fully safe offers a money-like convenience yield to investors, providing banks with cheaper funding for their assets. Banks keep their short-term debt safe in bad times by incurring in asset sales whose effect on equilibrium prices is not internalized.[5] These pecuniary externalities interact with banks' financing constraints, causing inefficiency. In our recursive infinite horizon setup, the importance of the pecuniary externality is amplified via banks' endogenous continuation value, which is sensitive to the anticipated cost of all future crises and is a key determinant of banks' refinancing capacity.

Like in Bryant (1980), Diamond and Dybvig (1983), and many subsequent contributions,

[4] Pecuniary externalities are a common source of inefficiency in models with financial constraints (e.g. Lorenzoni, 2008) and more generally in economies with incomplete markets (Geanakoplos and Polemarchakis (1986), Greenwald and Stiglitz (1986)). The usual emphasis in the existing papers (including the recent contributions of Bianchi and Mendoza, 2011, Korinek, 2011, and Gersbach and Rochet, 2012) is on their potential to cause excessive fluctuations in credit and excessive credit.

[5] Asset sales as a means to accommodate refinancing needs have a long tradition in banking; see, for example, Allen and Gale (1998) and Acharya and Viswanathan (2011). Fire-sale prices in Stein (2012) play formally the same role in causing the pecuniary externality as the excess cost of crisis financing in our model.

investors in our model are subject to idiosyncratic liquidity shocks and bank debt is non-tradable. However, we do not assume bank debt to be demandable at will and, instead, we endogenize its maturity as the result of trading off investors' higher valuation of short maturities with banks' concerns about refinancing costs in a crisis. The importance of the lack of tradability assumption (as its connection with the banking literature) is further discussed in subsection 7.4.

3 The model

We consider an infinite horizon economy in which time is discrete $t = 0, 1, 2,...$ and a special class of agents called *experts* own and manage a continuum of measure one of *banks*—describable as an exogenous pool of long-term assets. The economy alternates between typically long *normal* phases ($s_t = N$) in which banks can place their debt among *savers* and short *crisis* episodes ($s_t = C$) in which they cannot. This episodes represent systemic liquidity crises in a reduced-form manner. For tractability, we assume $\Pr[s_{t+1} = C \mid s_t = N] = \varepsilon$ and $\Pr[s_{t+1} = C \mid s_t = C] = 0$, so that crises have a constant probability of following any normal period but last for just one period (so a period is the standard duration of a crisis). Finally, we assume that $s_0 = N$.

3.1 Agents

Both experts and savers are long-lived risk-neutral agents who are assumed to (potentially) enter the economy in a steady flow of sufficiently large measure per period and to exit it whenever their investment and consumption activities are completed.[6] Each entering agent is endowed with a unit of funds.

3.1.1 Experts

Experts are relatively impatient. They discount future consumption at rate ρ_H. When entering the economy, each expert has the opportunity to invest his endowment either in banks' claims or in an indivisible private investment project with a net present value of z, heterogeneously distributed over the entrants.[7] The distribution of z has support $[0, \overline{\phi}]$ and the

[6] This formulation guarantees that the entering agents are always sufficient for their endowments to cover banks' refinancing needs, while the measure of actually active (or non-exited) agents remains bounded.

[7] Experts who opt for their own projects exit the economy immediately.

measure of agents with $z \leq \phi$ is described by a differentiable and strictly increasing function $F(\phi)$, with $F(0) = 0$ and $F(\overline{\phi}) = \overline{F}$.

3.1.2 Savers

Entering savers are relatively *patient.* They start discounting next period utility from consumption at rate $\rho_L < \rho_H$. However, in every period they face an idiosyncratic probability γ of turning irreversibly *impatient* and start discounting the utility of any future consumption at rate ρ_H from that point onwards.

Savers are unsophisticated investors with no other investment opportunity that bank debt. So, in normal periods they decide between buying bank debt or consuming their endowment, while in crisis times they simply consume their endowments. Savers whose bank debt matures face the same possibilities as the contemporaneous entering experts with respect to the use of their recovered funds.

3.2 Banks

Each of the banks possesses a pool of long-term assets that, if not liquidated, yields a constant cash flow $\mu > 0$ per period. If liquidated, bank assets produce a terminal payoff L. Banks are owned and managed by experts who, while in this role, will be called *bankers.*

Bankers profit from the lower discount rates of the patient savers by issuing some non-tradable debt among them.[8] Debt is issued at par in the form of (infinitesimal) contracts with a principal normalized to one. At the initial period ($t = 0$), bankers choose a debt structure which can be described as a triple (D, r, δ), where D is the overall principal, r is the per-period interest rate, and δ is the constant probability with which each contract matures in each period. So debt maturity is random, which helps for tractability, and has the property that the expected time to maturity of any non-matured contract is equal to $1/\delta$. We assume that contract maturity arrives independently across contracts.[9] Failure to pay interest or repay the maturing debt in any period will lead the bank to be liquidated

[8] The lack of tradability might be structurally thought as the result of savers' geographical dispersion and the lack of access to centralized trading. The relevance of this assumption in the model (and in the context of the banking literature) is further discussed in Section 7.4.

[9] The case of perfectly correlated maturities within a bank (and independent across banks) is also tractable but makes banks more vulnerable to crises.

at value L, which is assumed to be low enough for bankers to choose initial debt structures under which liquidation is avoided at all times.[10]

In normal periods, the refinancing of maturing debt δD is is done by replacing the maturing contracts with identical contracts placed among patient savers. So the bank generates a free cash flow of $\mu - rD$ that is paid to bankers as a dividend.[11]

In crisis periods, refinancing the maturing debt requires banks to turn to experts. With the sole purpose of simplifying the algebra, we assume that bankers learn about their banks' refinancing problems in a crisis after having received and consumed the normal dividends.[12] Thus, they require δD units of *crisis financing*. In order to obtain these funds, existing bankers offer to some of the entering experts a fraction α of the continuation value of their bank (i.e. of its future free cash flows). Debt in hands of savers during a crisis reduces to $(1-\delta)D$ and, once the crisis is over, the bank restores its original debt structure (D, r, δ) by placing an extra amount of debt δD among savers. The proceeds from such placement are paid as a dividend and, hence, are part of the future value used to compensate to the funding experts.

3.3 The cost of crisis financing

By virtue of competition, the fraction α of a bank's continuation value offered to experts in compensation for their crisis financing will have to be enough to compensate the marginal entering expert for the opportunity cost of her funds, which we denote by ϕ. Given the heterogeneity in the value z of experts' private investment opportunities and the size δD of banks' aggregate refinancing needs, clearing the market for crisis financing requires $F(\phi) = \delta D$. So the market-clearing *excess cost* of crisis financing can be found as $\phi = F^{-1}(\delta D) \equiv \Phi(\delta D)$, where $\Phi(\cdot)$ is strictly increasing and differentiable, with $\Phi(0) = 0$ and $\Phi(\overline{F}) = \overline{\phi}$. We

[10] In Section 7.1 we explicitly discuss the condition under which bankers find it optimal to avoid liquidation in crises (and then obviously in normal periods too).

[11] We have considered an extension in which banks (or bankers) can invest the free cash flow in a perfectly liquid storage technology as a means to maintain a buffer of *liquidity* with which to partially cover their refinancing needs in a crisis. We have checked that if the probability of suffering a systemic crisis and/or the cost of liquidity in a crisis are not too large, then holding liquidity is strictly suboptimal. All the parameterizations explored in the figures below satisfy this property. With parameterizations not satisfying this property, analytical tractability is lost.

[12] Otherwise, they would find it optimal to cancel the dividends and reduce the bank's funding needs to $\delta D - (\mu - rD)$. The algebra in this case would be more tedious. However, under realistic parameterizations (such as those behind the fugures inserted below), the results would be barely affected because the dividends $\mu - rD$ are very small relative to the refinancing needs δD.

will refer to $\Phi(\cdot)$ as the *inverse supply* of crisis financing.

4 Equilibrium analysis

In this section we stick to the following definition of equilibrium:

Definition 1 *An* equilibrium with crisis financing *is a tuple* $(\phi^e, (D^e, r^e, \delta^e))$ *describing an excess cost of crisis financing* ϕ^e *and a debt structure for banks* (D^e, r^e, δ^e) *such that:*

1. *Patient savers accept the debt contracts involved in* (D^e, r^e, δ^e).
2. *Among the class of debt structures that allow banks to be refinanced during crises,* (D^e, r^e, δ^e) *maximizes the value of each bank to its initial owners.*
3. *The market for crisis financing clears in a way compatible with the refinancing of all banks, i.e.* $\phi^e = \Phi(\delta^e D^e)$.

In the next subsections we undertake the steps necessary to prove the existence and uniqueness of this equilibrium, and establish its properties.

4.1 Savers' required maturity premium

Let us first analyze the conditions upon which the debt contracts associated with some debt structure (D, r, δ) are acceptable to savers during normal times. Since the bank will fully pay back its maturing debt even in crisis periods, a saver's valuation of such contract does not depend on the aggregate state of the economy but only on whether the saver is patient ($i = L$) or impatient ($i = H$). The ex-coupon values of the contract in each of these individual states, U_L and U_H, must satisfy the following system of equations:

$$\begin{aligned} U_L &= \frac{1}{1+\rho_L}\{r + \delta + (1-\delta)[(1-\gamma)U_L + \gamma U_H]\}, \\ U_H &= \frac{1}{1+\rho_H}\left[r + \delta + (1-\delta)U_H\right]. \end{aligned} \tag{1}$$

The different discount factors multiply the payoffs and continuation values relevant in each state. The contract pays r with probability one in each next period. Additionally it matures with probability δ, in which case it pays its face value of one. With probability $1-\delta$, it does not mature and then its continuation value is U_L and U_H depending on the investor's individual state in the next period. The terms multiplying these variables in the right hand side of the equations reflect the probability of being in each individual state next period.

When banks place debt among savers, patient savers are abundant enough to acquire all the issue, so the acceptability of the terms (r,δ) requires

$$U_L(r,\delta) = \frac{r+\delta}{\rho_H+\delta}\frac{\rho_H+\delta+(1-\delta)\gamma}{\rho_L+\delta+(1-\delta)\gamma} \geq 1, \tag{2}$$

which uses the solution for U_L arising from (1). Obviously, for any given δ, bankers' value is maximized by issuing contracts with the minimal interest rate r that satisfies $U_L(r,\delta)=1$.

Proposition 1 *The minimal interest rate acceptable to patient savers for each maturity δ is given by the function*

$$r(\delta) = \frac{\rho_H\rho_L+\delta\rho_L+(1-\delta)\gamma\rho_H}{\rho_H+\delta+(1-\delta)\gamma}, \tag{3}$$

which is strictly decreasing and convex, with $r(0)=\rho_H\frac{\rho_L+\gamma}{\rho_H+\gamma} \in (\rho_L,\rho_H)$ and $r(1)=\rho_L$.

This result evidences the advantage of offering short debt maturities to the savers in our model. For any expected maturity $1/\delta$ longer than one, the saver bears the risk of turning impatient and having to postpone his consumption until his contract matures. Compensating the cost of waiting via a larger interest rate generates a maturity premium $r(\delta)-\rho_L>0$, which is increasing in $1/\delta$. Figure 1 illustrates the behavior of $r(\delta)$ in a specific numerical example.[13]

4.2 Banks' optimal debt structures

From now on, we will take savers' required maturity premium into account by assuming that the debt structures (D,r,δ) offered by banks always have $r=r(\delta)$. This allows us to refer banks' debt structures as simply (D,δ). To further save on notation, in most equations we will refer to $r(\delta)$ as simply r.

[13] All our figures rely on a baseline parameterization in which one period is one month, agents' annualized discount rates are $\rho_L=2\%$ and $\rho_H=6\%$, the annualized yield on bank assets is $\mu=4\%$, the expected time until the arrival of an idiosyncratic preference shock is one year ($\gamma=1/12$), and the expected time between systemic crises is ten years ($\varepsilon=1/120$). We also assume $\Phi(x)=ax^2$, with $a=1$ unless otherwise specified.

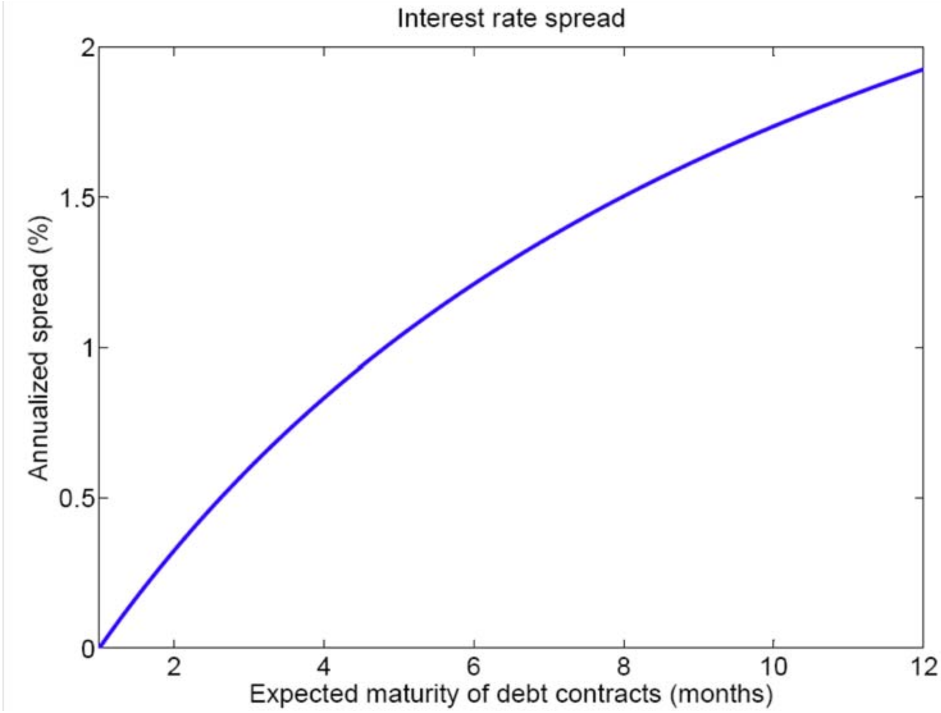

Figure 1: Interest rate spread vs. $1/\delta$

4.2.1 Value of bank equity in normal times

The continuation value of a bank to its owners in a normal period depends both on its debt structure (D, δ) and the fraction α of its continuation value in a crisis that has to be relinquished in order to obtain crisis financing. Let $E(D, \delta; \alpha)$ denote the ex dividend continuation value of the bank to its owners (i.e. the value of *bank equity*) in a normal period. This value satisfies the following recursive equation:

$$\begin{aligned} E(D,\delta;\alpha) &= \frac{1}{1+\rho_H}\Big\{(\mu - rD) + (1-\varepsilon)E(D,\delta;\alpha) + \\ &\qquad \varepsilon(1-\alpha)\frac{1}{1+\rho_H}[\mu-(1-\delta)rD+\delta D+E(D,\delta;\alpha)]\Big\}. \end{aligned} \tag{4}$$

To explain the equation, recall that bankers' discount rate is ρ_H and after each normal period they receive (and consume) a dividend of $\mu - rD$. With probability $1-\varepsilon$, the next period is a normal period so bankers' continuation value is $E(D, \delta; \alpha)$ once again. With probability ε, a systemic crisis arrives and refinancing the bank involves relinquishing a fraction α of its equity value to the crisis financiers.

The factor $\frac{1}{1+\rho_H}[\mu-(1-\delta)rD+\delta D+E(D,\delta;\alpha)]$ represents the total value of the bank's equity after it gets financed in the crisis period. Such value is is expressed in terms of payoffs received one period ahead: $\mu-(1-\delta)rD$ is the asset cash flow net of interest payments to savers (which reflects that the debt in hands of savers is temporarily reduced to $(1-\delta)D$), δD is the revenue from reissuing the debt that was financed by experts during the crisis period (paid as a special dividend to bank owners), and the last term reflects that, one period after the crisis, the original debt structure is fully restored so the bank's equity value is $E(D,\delta;\alpha)$ again.

Competition between crisis financiers implies that bankers will obtain the funds δD in exchange for the minimal α that satisfies

$$\alpha\frac{1}{1+\rho_H}[\mu-(1-\delta)rD+\delta D+E(D,\delta;\alpha)]\geq(1+\phi)\delta D. \tag{5}$$

Under such α, (5) holds with equality and can be used to substitute for α in (4) and to obtain the following Gordon-type formula for the value of bank equity value:

$$E(D,\delta;\phi)=\frac{1}{\rho_H}\left[\mu-r(\delta)D-\frac{\varepsilon}{1+\rho_H+\varepsilon}\{[(1+\rho_H)\phi+\rho_H]-r(\delta)\}\delta D\right]. \tag{6}$$

The interpretation of this expression is very intuitive. Equity is valued as a perpetuity with payoffs discounted at impatient rate ρ_H:

1. μ is the unlevered cash flow of the bank.

2. $r(\delta)$ is the interest rate paid on the debt placed among savers.

3. $\frac{\varepsilon}{1+\rho_H+\varepsilon}\{[(1+\rho_H)\phi+\rho_H]-r(\delta)\}$ reflects the (discounted) differential cost of refinancing the amount of maturing debt δD each time a crisis arrives.

These elements provide the basis of our trade-off theory of banks' debt structure decisions, as shown in the sections below.

Finally, taking into account that (5) holds with equality and α cannot be larger than one, the feasibility of crisis financing can be summarized by the condition:

$$\mu-(1-\delta)rD+\delta D+E(D,\delta;\phi)\geq(1+\rho_H)(1+\phi)\delta D, \tag{7}$$

which we will call the *crisis financing constraint* (CF) in the analysis that follows. Quite intuitively, the equity value of the bank after the crisis must be no lower than the amount needed to compensate, at rate ρ_H, the cost $1+\phi$ of each unit of crisis financing.

4.2.2 Optimal debt structure problem

Since the bankers appropriate D out of what savers pay for the bank's debt at $t = 0$, their goal when choosing the bank's initial debt structure is to maximize the total market value of the bank, $V(D, \delta; \phi) = D + E(D, \delta; \phi)$, which using (6) can be written as:

$$V(D, \delta; \phi) = \frac{\mu}{\rho_H} + \frac{\rho_H - r(\delta)}{\rho_H} D - \frac{1}{\rho_H}\frac{\varepsilon(\rho_H - r(\delta))}{1 + \rho_H + \varepsilon}\delta D - \frac{1}{\rho_H}\frac{\varepsilon(1 + \rho_H)\phi}{1 + \rho_H + \varepsilon}\delta D. \tag{8}$$

The first term in this expression is the value of the unlevered bank. The second term is the value obtained by financing the bank with debt claims held by savers' initially more patient than the bankers (notice that $r(\delta) < \rho_H$, by Proposition 1). The third term reflects the fact that refinancing during crises is made by impatient experts whose discount rate is ρ_H instead of by patient savers that require a yield $r()$. The last term accounts for the excess cost coming from having to compensate all crisis financing according to the excess opportunity cost of funds ϕ of the marginal crisis financier.

So bankers solve the following problem:

$$\begin{array}{lll} \max\limits_{D \geq 0,\ \delta \in [0,1]} & V(D, \delta; \phi) = D + E(D, \delta; \phi) & (9) \\ \text{s.t.} & E(D, \delta; \phi) \geq 0 & \text{(LL)} \\ & \mu - (1 - \delta)rD + \delta D + E(D, \delta; \phi) \geq (1 + \rho_H)(1 + \phi)\delta D & \text{(CF)} \end{array}$$

The first constraint imposes the non-negativity of the bank's equity value in normal periods, and we will refer to it as bankers' limited liability constraint (LL).[14] The second constraint is the crisis financing constraint (7), which can be interpreted as an expression of bankers' limited liability in crisis times. It can be shown that the two constraints boil down to the same constraint on D for $\delta = 0$, but (CF) is tighter than (LL) for $\delta > 0$.[15] Thus (LL) can be ignored.

Adopting the following technical assumptions help us prove the existence and uniqueness of the solution to the bank's optimization problem:[16]

A1. $\overline{\phi} < 2\frac{1+\rho_L}{1+\rho_H} - 1$.

[14] Satisfying (LL) implies in particular the non-negativity of bankers dividends, $\mu - r(\delta)D \geq 0$.

[15] See the proof of Proposition 2 in Appendix A.

[16] A1 and A2 are sufficient conditions that impose rather mild restrictions on the parameters. For instance, in the baseline parameterization behind our figures (see footnote 13), A1 and A2 impose $\overline{\phi} < 0.9936$ and $\gamma < 0.4976$. Moreover, we have checked numerically that the results in Proposition 2 hold well beyond the region of parameters delimited by these assumptions.

A2. $\gamma < \frac{1-\rho_H}{2}$.

Proposition 2 *The bank's maximization problem has a unique solution* (D^*, δ^*)*. In the solution:*

1. *The crisis financing constraint is binding, i.e. crisis financiers take 100% of the bank's equity.*
2. *Optimal debt maturity* $1/\delta^*$ *is increasing in* ϕ *and the optimal amount of maturing debt per period* $\delta^* D^*$ *is decreasing in* ϕ*; in fact, if* $\delta^* \in (0, 1)$*, both* δ^* *and* $\delta^* D^*$ *are strictly decreasing in* ϕ*.*

The intuition for these results is as follows. First, even if the bank does not get involved in maturity transformation ($\delta = 0$), its value is increasing in D, making it interested in choosing the maximum feasible leverage. If maturity transformation generates value, this tendency remains, so (CF) is binding at the optimum.[17] Second, as the excess cost of crisis financing ϕ increases, the value of maturity transformation diminishes which implies the choice of a longer expected maturity. The tightening of (CF) forces banks to reduce the amount of funding $\delta^* D^*$ demanded to crisis financiers.[18]

Interestingly, the debt structure decisions just described determine, as a residual, the bank's *capital ratio*, that is, the ratio of its equity value E to its total market value V. Figure 2 depicts E/V under banks' optimal choices as a function of the excess cost of crisis financing ϕ. The ratio is strictly increasing in ϕ, reflecting that, by (CF), each bank must keep enough equity value in normal times so as to be able to pay the excess cost of funding during a crisis.[19] Under the illustrated parameterization, the model yields capital ratios in a realistic 4% to 8% range for a wide range of values of ϕ.[20]

[17] The full dilution of bankers' equity in each crisis is an implication of the simplifying assumption that all crises have the same severity. With heterogeneity in this dimension (for example, due to random shifts in $\Phi(x)$), the corresponding crisis financing constraint might only be binding (or even not satisfied, inducing bankruptcy) in the most severe crises.

[18] In all our numerical examples, total debt D^* is also decreasing in ϕ.

[19] Even with $\phi = 0$ banks need strictly positive equity because crisis financiers demand a return ρ_H larger than r for their funds.

[20] By construction, the capital ratios depicted in Figure 2 are the minimal ones compatible with banks being able to avoid going bankrupt during a systemic liquidity crisis. These might be the relevant regulatory capital ratios imposed on banks in an extended version of the model in which bankers did not want to avoid bankruptcy during crises (perhaps because they do not internalize some social costs of bank failures). See Appendix C for a discussion of the case in which banks default in crises.

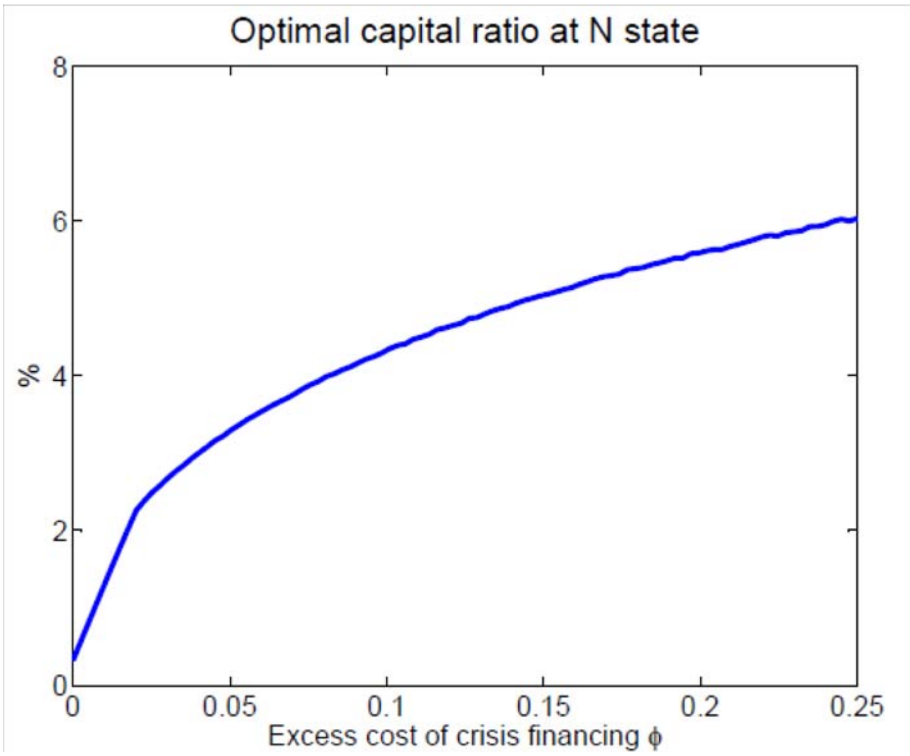

Figure 2: Banks' optimal capital ratio as a function of ϕ

4.3 The competitive equilibrium

Banks' optimization problem for any given excess cost of crisis financing ϕ embeds savers' participation constraint so the only condition for equilibrium that remains to be imposed is the clearing of the market for crisis financing. The continuity and monotonicity in ϕ of the function that describes excess demand in such market guarantees that there exists a unique excess cost of crisis financing ϕ^e for which the market clears:

Proposition 3 *The equilibrium of the economy* $(\phi^e, (D^e, r^e, \delta^e))$ *exists and is unique.*

The effects on equilibrium outcomes of shifts in the supply of crisis financing are summarized in the following proposition:

Proposition 4 *If the inverse supply of crisis financing* $\Phi(x)$ *shifts upwards, the equilibrium changes as follows: expected debt maturity* $1/\delta^e$ *increases, total refinancing needs* $\delta^e D^e$ *fall, bank debt yields* r^e *increase, and the excess cost of crisis financing* ϕ^e *increases. If initially* $\delta^e \in (0,1)$*, all these variations are strict.*

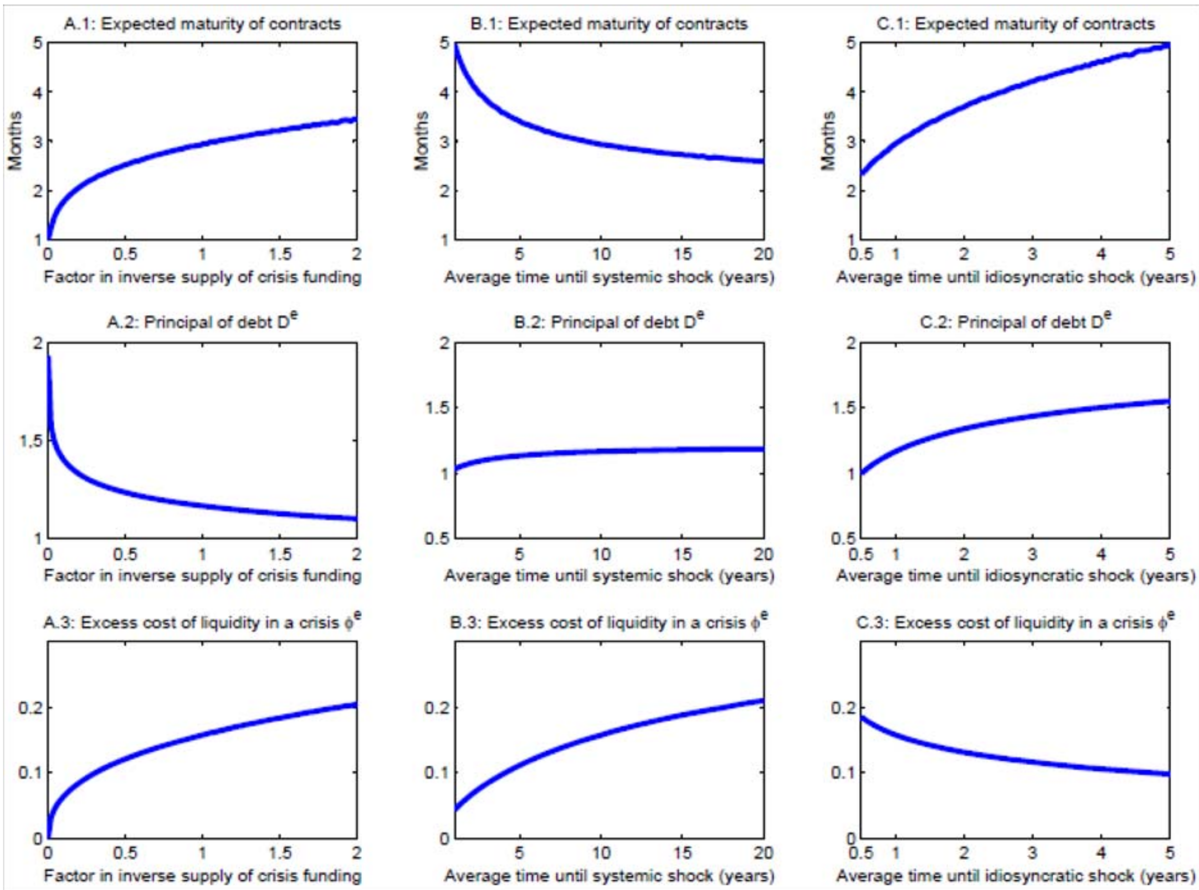

Figure 3: Effect of changes in the parameters on the competitive equilibrium

The results in Proposition 4 are illustrated, together with other comparative statics results, in Figure 3, where the inverse supply of crisis financing is parameterized as $\Phi_a(x) = ax^2$. The graphs in the first column plot various equilibrium variables (expected debt maturity, total debt, and excess cost of crisis financing) against the parameter a of this function. The second and third columns show the equilibrium effects of increasing the average time to the arrival of a systemic shock, $1/\varepsilon$, and an idiosyncratic shock, $1/\gamma$, respectively. These results are quite self-explanatory.

5 Efficiency and regulatory implications

In this section we solve the problem of a social planner who has the ability to control banks' funding decisions subject to the same constraints that banks face when solving their private value maximization problems. We find that bank debt maturity in the unregulated competitive equilibrium is inefficiently short because of the interaction of a pecuniary externality with banks' crisis financing constraints. We find that banks' endogenous continuation values amplify the significance of the inefficiency. Finally, we analyze two specific alternatives that would restore efficiency: directly regulating maturity decisions and the introduction of a suitable Pigovian tax on banks' refinancing needs.

5.1 Inefficiency of the unregulated equilibrium

Suppose that a social planner can regulate both the amount D and the maturity parameter δ of banks' debt. In our economy only existing bankers and future crisis financiers appropriate a surplus. So the natural objective function for the social planner is the sum of the present value of such surpluses. Crisis financiers appropriate the difference between the equilibrium excess cost of crisis financing, $\phi = \Phi(\delta D)$, and the net present value of their alternative investment opportunity z (which is positive for all but the marginal crisis financiers). Hence, crisis financiers' surplus is:

$$u(D,\delta) = \int_0^{\delta D} \left(\Phi(\delta D) - \Phi(x)\right) dx = \delta D \Phi(\delta D) - \int_0^{\delta D} \Phi(x) dx. \tag{10}$$

Evaluated at a normal period, the present value of the expected surpluses of potential crisis financiers along future crises can be written as:[21]

$$U(D,\delta) = \frac{1}{\rho_H} \frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon} u(D,\delta).$$

Hence, using (8), the objective function of the social planner can be expressed as:

$$\begin{aligned} W(D,\delta) &= V(D,\delta;\Phi(\delta D)) + U(D,\delta) \\ &= \frac{\mu}{\rho_H} + \frac{\rho_H - r(\delta)}{\rho_H} D - \frac{1}{\rho_H} \frac{\varepsilon(\rho_H - r(\delta))}{1+\rho_H+\varepsilon} \delta D - \frac{1}{\rho_H} \frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon} \int_0^{\delta D} \Phi(x) dx, \end{aligned} \tag{11}$$

[21] $U(D,\delta)$ satisfies the following recursive equation:

$$U(D,\delta) = \frac{1}{1+\rho_H} \left[(1-\varepsilon) U(D,\delta) + \varepsilon \left(u(D,\delta) + \frac{1}{1+\rho_H} U(D,\delta) \right) \right].$$

The first term in square brackets takes into account that, with probability $1-\varepsilon$, next period is a normal one and crisis financiers' continuation surplus remains equal to $U(D,\delta)$. The second term captures that with probability ε there is a crisis and the crisis financiers obtain $u(D,\delta)$ plus the continuation surplus that, one more period ahead, is again $U(D,\delta)$.

which contains four terms: the value of an unlevered bank, the value added by maturity transformation in the absence of systemic crises, the value loss due to financing the bank with impatient experts during liquidity crises, and the value loss due to the sacrifice of the NPV of the investment projects given up by those experts who act as banks' crisis financiers.

Thus, the social planner's problem can be written as:[22]

$$\begin{array}{lll} \max\limits_{D\geq 0,\ \delta\in[0,1]} & W(D,\delta) & (12) \\ \text{s.t.} & \mu-(1-\delta)rD+\delta D+E(D,\delta;\Phi(\delta D))\geq(1+\rho_H)(1+\Phi(\delta D))\delta D & \text{(CF')} \end{array}$$

This problem differs from banks' optimization problem (9) in two dimensions. First, the objective function includes the surplus of the crisis financiers. Second, the social planner internalizes the effect of banks' funding decisions on the market-clearing excess cost of crisis financing, so (CF') contains $\Phi(\delta D)$ in the place occupied by ϕ in (CF) constraint (see problem (7)).

Comparing the unregulated equilibrium with the solution of the social planner's problem, we obtain the following result:

Proposition 5 *If the competitive equilibrium features $\delta^e\in(0,1)$ then a social planner can increase social welfare by choosing a longer expected debt maturity than in the competitive equilibrium, i.e. some $1/\delta^s>1/\delta^e$.*

The root of the discrepancy between the competitive and the socially optimal allocations is at the way individual banks and the social planner perceive the frontier of the set of maturity transformation possibilities: banks choose their individually optimal (D,δ) along the (CF) constraint (where ϕ^e is taken as given) whereas the social planner does it along the (CF') constraint (where $\phi=\Phi(\delta D)$). Each of these constraints and the corresponding decisions are illustrated in Figure 4.

At the equilibrium allocation (D^e,δ^e) both the social planner's and the initial bankers' indifference curves are tangent to (CF). Moreover, (CF) and (CF') intersect at (D^e,δ^e) (since the competitive equilibrium obviously satisfies $\phi^e=\Phi(\delta^e D^e)$). However, the social planner's indifference curve is not tangent to (CF') at (D^e,δ^e), implying that this allocation does not maximize welfare. In the neighborhood of (D^e,δ^e), (CF') allows for a larger increase in D,

[22] Recall that the constraint called (LL) in (9) can be ignored because it is implied by the bridge financing constraint.

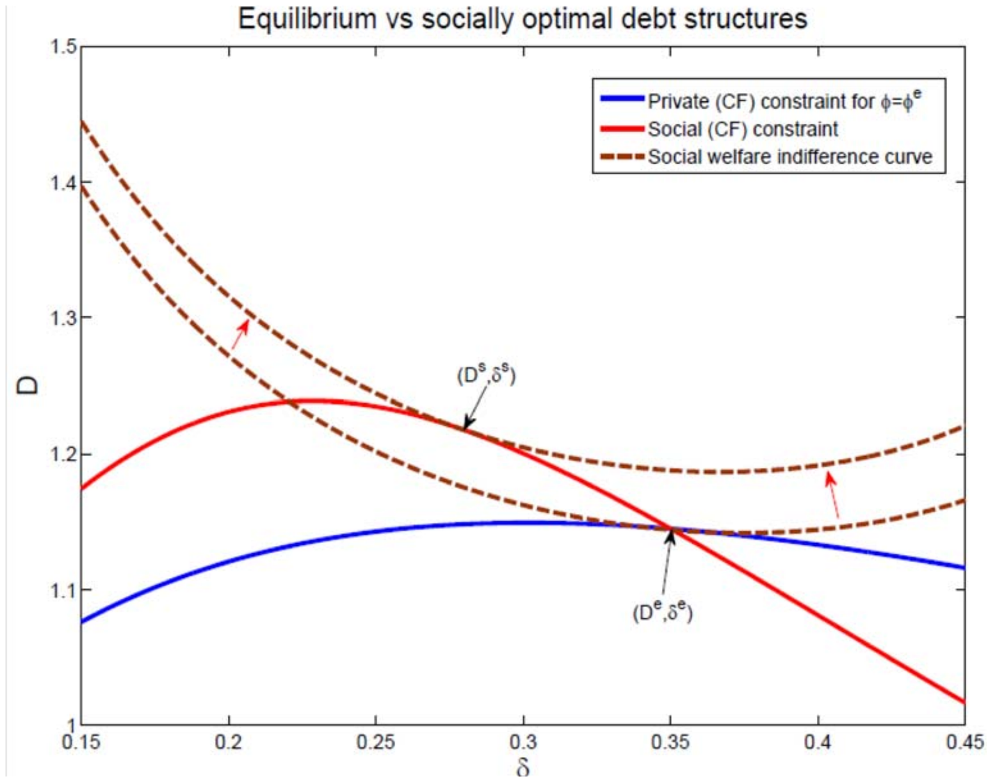

Figure 4: Maturity transformation possibilities from private and social perspectives

by reducing δ, than what seems implied by (CF) (where ϕ remains constant). It turns out that maturity transformation can produce a larger surplus with a larger use of its extensive margin (leverage) and a lower use of its intensive margin (short maturities), like at (D^s, δ^s) in the figure.

This finding offers a new perspective for the joint assessment of some of the regulatory proposals emerged in the aftermath of the recent crisis, which defend reducing *both* banks' leverage *and* their reliance on short-term funding. In the context of the current model, once debt maturity is regulated, limiting banks' leverage would be counterproductive. It also indicates that simply limiting banks' leverage (say, through higher capital requirements) does not correct, and would actually worsen, the impact of the externality associated with refinancing needs. In terms of Figure 4, forcing banks to choose debt lower than D^e would induce them to, in the new regulated equilibrium, move along (CF') in the direction that implies a larger δ, i.e. a shorter expected debt maturity, and lower aggregate welfare.

5.2 Amplification via continuation value

The source of inefficiency in our model is the impact on banks' financing constraints of pecuniary externalities generated in the market for crisis financing. As in Stein (2012), where the pecuniary externalities operate through asset sales, the unregulated equilibrium involves excessive refinancing needs. An important difference between both papers is that in our infinite horizon model crises are recurrent and the continuation value of a bank depends on the anticipated cost of funds in future crises. In our setup, reducing banks' refinancing needs reduces the excess cost of financing in each future crisis, ϕ, producing two effects that the social planner problem internalizes (but banks do not): (i) the direct effect of ϕ on the RHS of (CF'); and (ii) the indirect effect of ϕ on banks' continuation value in a crisis and, hence, on the LHS of (CF'). The second effect emanates from our recursive formulation and reinforces the first in relaxing (CF').

In order to quantify the importance of each effect we conduct the following exercise. We consider a social planner that only regulates debt structures up to the arrival of the first crisis. For the sake of the calculation, at the normal period following the first crisis, we allow banks to repay all their debt and reissue debt under the structure that is optimal in the unregulated equilibrium. The recursive equations that determine the market value of banks, the relevant crisis financing constraint, the rents obtained by crisis financiers and, henceforth, the problem of the *one-off* (rather than *recursive*) social planner are developed in Appendix B. Figure 5 compares expected debt maturity and total debt across the unregulated equilibrium and the one-off and recursive regulated equilibria. Clearly, regulated maturities and total debt are larger in the recursive welfare problem than in the one-off problem, and the size of the differences in total debt (which is the source of the initial dividend through which bankers appropriate the surplus from maturity transformation) suggests that the effects channeled through banks' continuation value are quantitatively very important.[23]

[23] In terms of Figure 4, the recursive planner's impact on bank continuation value implies that the frontier of maturity transformation possibilities over which he optimizes, (CF'), is steeper around (D^e, δ^e) than the frontier relevant to the one-off planner (which would, in turn, be steeper than banks' crisis financing constraint, (CF)).

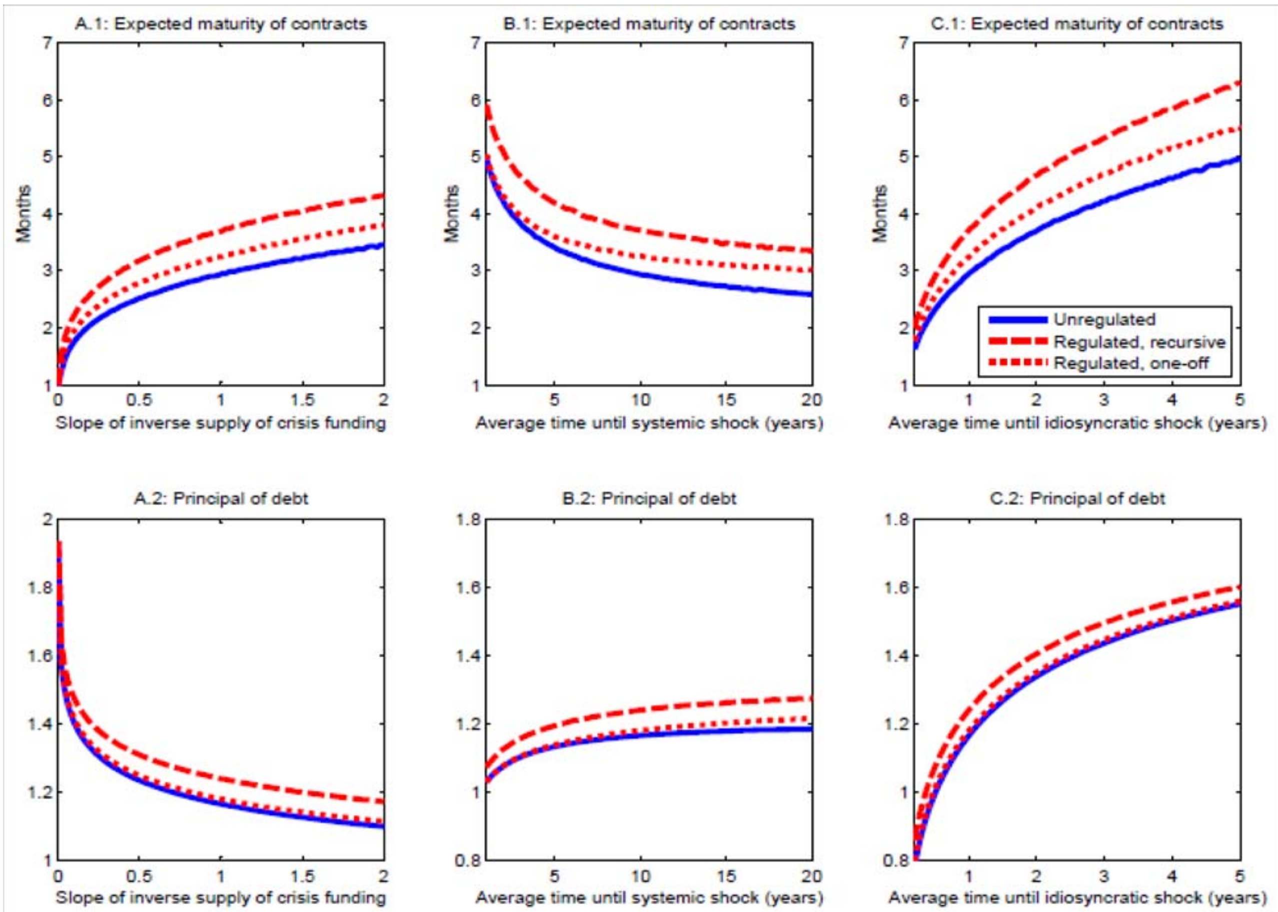

Figure 5: Competitive equilibrium vs. socially-efficient funding structures

5.3 Restoring efficiency

In order to achieve the socially efficient debt structure (D^s, δ^s) as a regulated competitive equilibrium, the most straightforward intervention in the context of the model would be to impose an upper limit δ^s to banks' maturity decision δ. Given that the inverse of δ is the expected maturity of a bank's debt (and our banks' assets have infinite maturity), such limit could be interpreted as equivalent to introducing a minimum net stable funding ratio like the one considered in Basel III. Anticipating a cost of crisis financing ϕ^s, banks in our model would find such a requirement binding and would choose to issue the maximum debt compatible with (CF) given ϕ^s and δ^s, which is D^s.

As shown by Perotti and Suarez (2011), adding unobservable heterogeneity across banks may undermine the efficiency of one-size-fits-all quantity-based liquidity regulation and alternatives such as Pigovian taxes may be superior.[24] With this motivation in perspective (but without explicitly adding heterogeneity), we next check whether a Pigovian tax on banks' refinancing needs might implement (D^s, δ^s) as a regulated competitive equilibrium. We consider the following class of *non-subsidized* Pigovian schemes:

1. Each bank pays a *proportional tax* of rate τ per period on its refinancing needs δD.

2. The social planner pays to each bank a *lump-sum transfer* $M \leq \tau\delta D$ per period.

Since $\tau\delta D$ is the revenue from the Pigovian tax, the constraint $M \leq \tau\delta D$ rules out the possibility of subsidizing banks via the lump-sum transfer M. We can prove analytically the following result:

Proposition 6 *If the unregulated competitive equilibrium features $\delta^e \in (0,1)$, there exists a Pigovian tax scheme (τ^P, M^P) that induces the socially optimal allocation (D^s, δ^s). This scheme satisfies $\tau^P > 0$ and $M^P = \tau^P\delta^s D^s$, and is unique if $\delta^s > 0$.*

The scheme uses some $\tau^P > 0$ to push banks towards funding decisions involving lower refinancing needs than in the unregulated competitive equilibrium. Interestingly, in order to reach the socially efficient allocation, all the revenue raised by the tax τ^P has to be rebated to the banks through M^P. Values of τ and M which induce δ^s but involve $M < \tau\delta^P D^P$ would reduce the value of bank equity relative to the situation in which δ^s is directly regulated, which would in turn tighten banks' crisis financing constraint pushing banks towards a leverage D^P strictly lower than D^s.

The need for rebating the revenue from the Pigovian tax is a novel insight relative to the non-pecuniary externality setup of Perotti and Suarez (2011). The general intuition is that, when pecuniary externalities cause inefficiency due to their interaction with financial constraints, regulators must be cautious not to address one of the manifestations of the inefficiency (excessively short maturities) in a way (non-rebated taxes) that, by tightening the relevant constraints (here, reducing banks' continuation values) may partly undo the potential gains from the intervention.

[24] Specifically, these authors show that if banks unobservably differ in their opportunities to extract value from maturity transformation, a flat rate Pigovian tax on refinancing needs can induce the marginal internalization of the relevant externalities while allowing the most efficient banks to operate with larger maturity mismatches than the less efficient ones.

6 Liquidity insurance

Some recent discussions on the final shape of bank liquidity risk regulation suggest the need to consider in parallel the possibility that banks' refinancing needs in a crisis are covered by explicit liquidity insurance schemes (Stein, 2013). In the next two subsections we consider the potential welfare contribution of private and government-based liquidity insurance arrangements. In both parts we conclude that liquidity insurance is socially beneficial but does not eliminate the desirability of debt maturity regulation, the intensity of which should generally depend, among other things, on the price elasticity of the opportunity cost of insurers' funds.

6.1 Private liquidity insurance

The fact that in both the competitive and the regulated allocations banks' crisis financing constraints are binding, while the normal times limited liability constraints are not, suggests that some form of insurance against systemic liquidity crises might increase welfare. To keep things tractable, we focus on simple one-period refinancing insurance arrangements subscribed by individual banks and entering experts at the beginning of each period, prior to the realization of uncertainty regarding the occurrence of a crisis.

Specifically, the arrangements we consider establish that:

1. Except in the period immediately after each crisis,[25] the bank pays a per-period premium p for each unit of insured refinancing $\lambda\delta D > 0$ to a measure $\lambda\delta D$ of entering experts, where $\lambda \in [0, 1]$ is the insured fraction of refinancing needs.

2. If there is a systemic crisis, the insuring experts supply the bank with funds $\lambda\delta D$ in the crisis and receive a gross repayment of $[1 + r(\delta) + p]\lambda\delta D$ just after the crisis.

Under this arrangement the refinancing of $\lambda\delta D$ is just as costly as if no crisis had occurred ($r(\delta)$ is the normal yield of savers' debt). The part $p\lambda\delta D$ of the repayment to the insuring experts after the crisis is included to offset the impact on the banks' net income of the fact that insurance is unneeded (and hence not paid for) immediately after a crisis.

[25] Insurance in the period immediately after a crisis is unneeded because, according to our assumptions, crisis periods are always followed by a normal period.

For the sale of insurance to be attractive to an entering expert with funds that can earn NPV of z in normal periods and $\max\{z,\phi\}$ in crisis periods, the insurance premium p must satisfy

$$p+\varepsilon\frac{1+r+p}{1+\rho_H}\geq\varepsilon\max\{1+z,1+\phi\},\tag{13}$$

where the second term in the LHS is the present value of the post-crisis repayments described above. Competition among entering experts will lead to a situation in which (13) is binding for the marginal provider of either insurance or crisis financing, who will have $z=\phi$.[26] Solving for p in such equality yields

$$p=\frac{\varepsilon}{1+\rho_H+\varepsilon}\{[(1+\rho_H)\phi+\rho_H]-r(\delta)\},\tag{14}$$

which is identical to the factor (within the large square brackets) that multiplies δD in (6).

On the other hand, the value of equity in a normal period of a bank that decides to insure a fraction λ of its refinancing needs can be written as

$$E(D,\delta,\lambda;\phi)=\frac{1}{\rho_H}\left\{\mu-r(\delta)D-p\lambda\delta D-\frac{\varepsilon}{1+\rho_H+\varepsilon}\{[(1+\rho_H)\phi+\rho_H]-r(\delta)\}(1-\lambda)\delta D\right\},$$

which is an extended version of (6). Now, using (14) to substitute for p, it becomes clear that:

$$E(D,\delta,\lambda;\phi)=E(D,\delta,0;\phi)=E(D,\delta;\phi),\tag{15}$$

which can be interpreted as a Modigliani-Miller type result: moving the fraction λ of fairly-priced insured funding for given value of D and δ, does not per se create or destroy value..

Insurance is, however, relevant for the bank's overall optimization problem because it alters the bank's crisis financing constraint. In the presence of insurance, the crisis financing constraint can be written as:

$$(\mu-\lambda p\delta D)-[1-(1-\lambda)\delta]rD+(1-\lambda)\delta D+E(D,\delta;\phi)\geq(1+\rho_H)(1+\phi)(1-\lambda)\delta D,\tag{CFI}$$

which differs from (7) in the presence of the insurance premia subtracting from the cash flow and adaptations that reflect that the bank's effective refinancing needs in the crisis are reduced to the uninsured fraction of its debt, $(1-\lambda)\delta D$.

[26] Clearing the market for crisis financing requires $\phi=\Phi(\delta D)$ irrespectively of the fraction of δD covered with insurance.

It is easy to check that $\lambda = 1$ implies the maximal relaxation of this constraint.[27] On the other hand, by (15), the bank's limited liability constraint remains the same as (LL) in (9). Therefore, the bank will solve the counterpart of the value maximization problem in (9) by getting fully insured against systemic crises ($\lambda = 1$). By doing so, its net cash flow becomes $\mu - rD - p\delta D$ and the constraints (CFI) and (LL) collapse into simply requiring that this cash flow is not negative.

The following proposition describes the positive welfare implications of adding insurance when funding decisions (i.e. δ) are optimally regulated. It also shows that, with liquidity insurance, banks in the unregulated economy would opt for inefficiently short debt maturities.

Proposition 7 *In a* regulated *economy, adding a private liquidity insurance scheme strictly increases welfare. With liquidity insurance, expected debt maturity in the* unregulated *equilibrium is too short.*

Intuitively, when liquidity insurance is introduced, $\mu - rD - p\delta D \geq 0$ becomes banks' only relevant constraint, which implies expanding the set of maturity transformation possibilities faced by both banks and the social planner. Hence, a social planner can definitely produce more social welfare with insurance than without insurance. However, the pecuniary externality regarding banks' debt maturity decisions remains present, now operating through the $\mu - rD - p\delta D \geq 0$ constraint. Intuitively, bank decisions affect the excess cost of crisis financing ϕ, which in turn affects the cost of insurance p, and ends up tightening this constraint.[28] So the main policy message from this subsection is that, if arranging for systemic liquidity insurance is at all feasible, it should be promoted but not as a substitute but as a complement to debt maturity regulation.

6.2 Public liquidity provision

We now turn to explore a simple reinterpretation of the model under which the marginal supplier of funds during a crisis is a government-sponsored lender (e.g. a central bank acting

[27] It suffices to realize that (14) implies $(1 + \rho_H)(1 + \phi) - 1 - r > p$.

[28] We are not able to prove that the introduction of insurance increases welfare in the unregulated economy, but this is actually the case in all the parameterizations that we have explored. The theoretical ambiguity comes from the fact that, with full insurance, unregulated banks will tend to choose funding structures that put upward pressure on ϕ^e and, in principle, a sufficiently large increase in ϕ^e might fully offset the gains due to the introduction of insurance.

as a lender of last resort) which is constrained to offer its funds on a non-subsidized basis. Specifically, suppose that the supply of funds from entering experts in a crisis is too small, say zero, but the government is able to obtain alternative funds x with a marginal (excess) opportunity cost that, to save on notation, we describe with the function $\Phi(x)$, which is analogous to our previous inverse supply of crisis financing.[29]

We are going to compare two public liquidity insurance regimes in which the government commits to cover, on a non-subsidized basis, banks' crisis financing needs:[30]

I. Public liquidity insurance only In each period, the government charges an insurance premium $p\delta D$ to each bank with refinancing needs δD and commits to cover these needs in each crisis period in exchange for a repayment of $[1+r(\delta)]\delta D$ in the period after the crisis.[31]

II. Public liquidity insurance cum maturity regulation In addition to the arrangements of the previous regime, the government regulates banks' maturity decision δ.

As with private insurance, banks' optimal debt structure decisions maximize the total market value of the bank (like in (9)), but only subject to the limited liability constraint $\mu - rD - p\delta D \geq 0$. The NPV of the revenues and costs that accrue to the government in its role as an insurer are:

$$G = \frac{1}{\rho_H}\left[p\delta D - \frac{\varepsilon\left(\rho_H - r(\delta)\right)}{1+\rho_H+\varepsilon}\delta D - \frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon}\int_0^{\delta D}\Phi(x)dx\right], \tag{16}$$

where the last term accounts for the (excess) opportunity cost of the funds lent in a crisis. Aggregate welfare in this setting can be defined as the sum of the total market value of the insured banks, V, and the net present value of the government's stake, G. The resulting expression for welfare is analogous to the expression for W in (11). We assume that the government maximizes W, subject to $G \geq 0$, i.e. we do not allow for positive NPV transfers

[29] We refer to $\Phi(x)$ as an *excess* cost because it comes on top of the normal opportunity cost of funds implied by assuming that the government has the same discount rate ρ_H as impatient agents. This excess cost may here reflect the NPV of the deadweight losses due to future distortionary taxes.

[30] The government might instead commit to supply liquidity during crises at some fixed excess cost $\widehat{\phi}$ à la Bagehot (1873). Numerical simulations show that this alternative policy is dominated by the arrangements that we analyze. The reason is that, as in the case of private liquidity insurance analyzed above, spreading the excess cost of crisis liquidity over time expands the set of maturity transformation possibilities.

[31] For simplicity, we assume that $p\delta D$ is paid to the government in *every* period (including periods after a crisis where the probability of suffering another crisis is zero).

from the government to the bank owners.[32]

The difference between the two public liquidity insurance regimes described above stems from the tools through which the government may influence banks' decisions. In Regime I, the government can only set p and will do so taking into account the impact of p on $(D, \delta) = (D(p), \delta(p))$. So, in this regime the premium p has to play the dual role of regulating banks' refinancing needs δD and guaranteeing $G \geq 0$. In Regime II, the government can set *both* p and δ, taking into account the impact of both parameters on $(D, \delta) = (D(p, \delta), \delta)$, and it turns out that the second tool is useful too. In fact, when the only tool is p, the welfare maximizing solution involves $G > 0$ in all our simulations while, when the two tools are available, it is possible to prove that the optimum involves $G = 0$.

Based on the same underlying parameterization as in previous figures, Figure 6 illustrates the outcomes under each of the two commented regimes. The horizontal axes represent different excess costs of government funds, measured by the factor a of a cost function specified as $\Phi(x) = ax$. Panel A represents welfare gains relative to a benchmark scenario without liquidity insurance in which banks fail if they have refinancing needs during a crisis.[33] These gains decrease as the excess cost of government funds increases and are significantly greater when maturity regulation is allowed. Quite intuitively, expected debt maturity (depicted in Panel B) is shorter when it can only be regulated using the premium p than when δ is controlled by the regulator. Finally, Panel C shows the positive government surplus $G > 0$ associated with Regime I and the zero surplus $G = 0$ associated with Regime II.[34]

As in the case of private liquidity insurance, the results in this subsection deliver the message that liquidity insurance is not a substitute but a complement to maturity regulation. With or without liquidity insurance, unregulated banks tend to choose equilibrium debt maturities that are excessively short, in that they imply refinancing costs during crises that tighten banks' financial constraints and impede them to collectively provide maturity

[32] Notice that this constraint is more flexible than the constraint associated with private liquidity insurance, where the *marginal* (rather than the *average*) provider of insurance must break even.

[33] The liquidation value L in case of default has been calibrated so that it equals 80% of the total market value of the bank. This choice only affects the scale of the vertical axis in Panel A. Details about the case in which banks fail in crises are provided in Section 7.1 and Appendix C.

[34] Consistent with the result in Proposition 6, a solution based on the combination of Pigovian taxes and lump-sum rebates might replace the direct regulation of δ. The resulting Pigovian insurance scheme would charge an excess insurance premium so as to induce the same maturity decision as the one directly regulated in Regime II and it would then rebate the government surplus to the banks in a periodic manner so as to compensate the negative effects of the excessive premia on equity values.

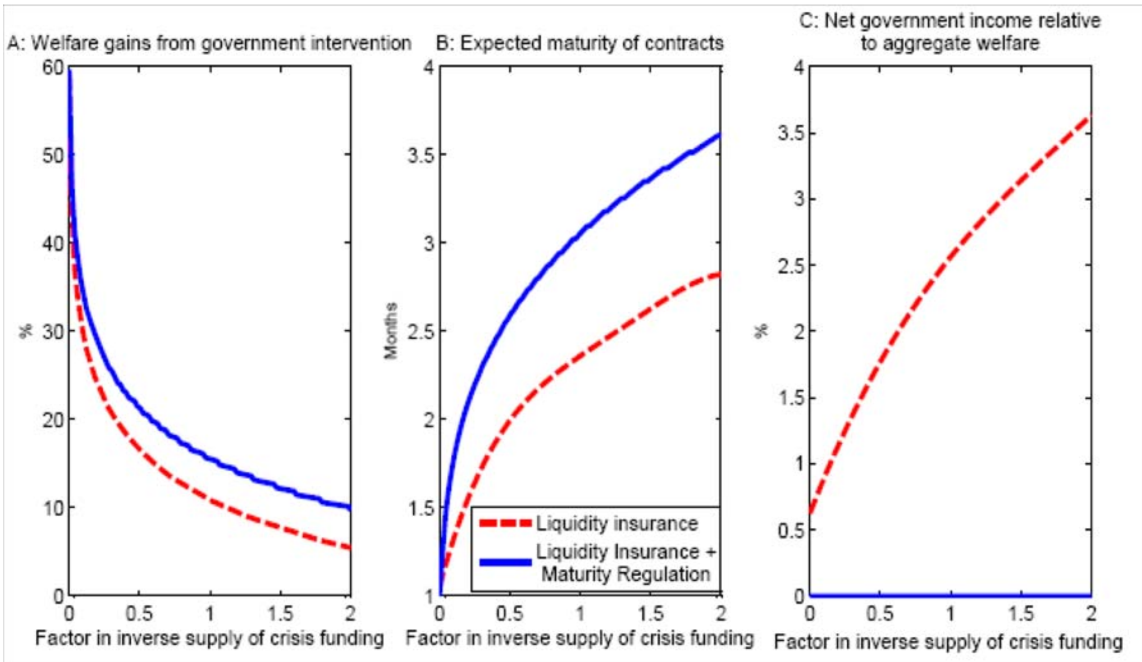

Figure 6: Optimal policy interventions vs. the government cost of funds

transformation in the socially most valuable manner.

7 Discussion and extensions

In this section we comment on the key assumptions and possible extensions of our model.

7.1 Optimality of not defaulting during crises

We have so far assumed that the liquidation value of banks in case of default, L, is small enough for banks to find it optimal to rely on funding structures that satisfy the (CF) constraint. How small L has to be (and what happens if it is not) is discussed next.

If a bank were not able to refinance its maturing debt, it would default, and we assume that this would precipitate its liquidation. For simplicity, we assume that, if the bank defaults, the liquidation value L is orderly distributed among all debtholders, which excludes the possibility of preemptive runs à la He and Xiong (2011a). If a bank were expected to default in a crisis, savers would require r to include a compensation for credit risk.

Based on the derivations provided in Appendix C, Figure 7 depicts for each possible equilibrium excess cost of crisis financing, ϕ^e, the maximum liquidation value $L^{\max}(\phi^e)$ for

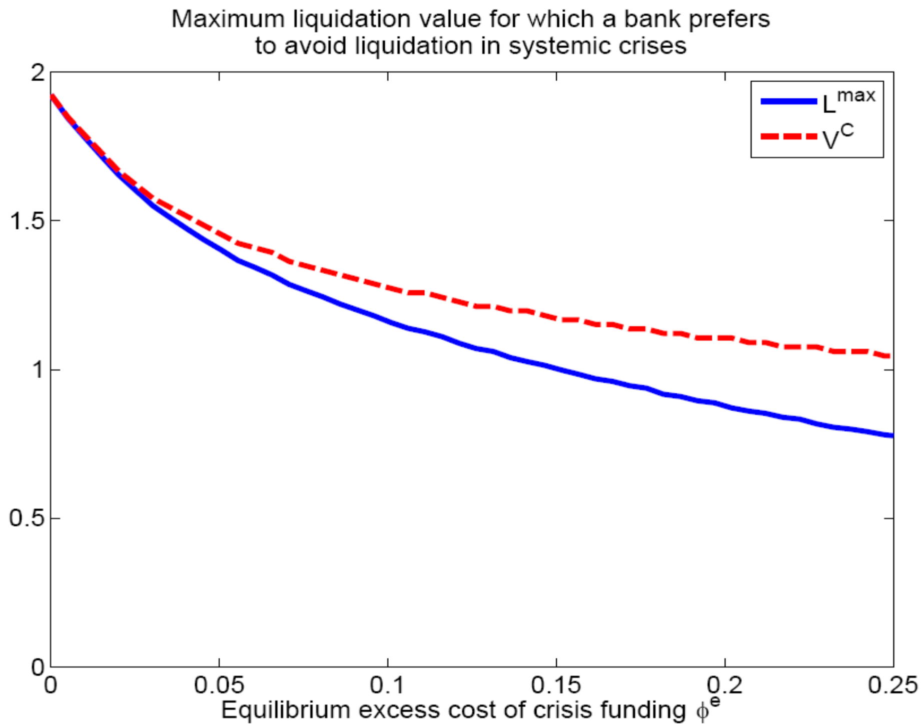

Figure 7: Conditions for the optimality of not defaulting during crises

which, when all other banks opt for crisis financing, an individual bank also prefers to rely on crisis financing. The variation of ϕ^e in this figure can be thought of as a general representation of shifts in the inverse supply of crisis financing $\Phi(\delta D)$ (which only affects the banks opting for crisis financing) so that both dimensions of the figure account for shifts in exogenous parameters. For configurations of parameters with $L \leq L^{\max}(\phi^e)$, the candidate equilibrium with crisis financing gets confirmed as an equilibrium.

$L^{\max}(\phi^e)$ is decreasing, so the higher the excess cost of crisis funding, the stronger the incentives for banks to opt for funding structures that imply defaulting in a crisis. To reinforce intuitions, Figure 7 also shows the total market value in a crisis of a bank that relies on crisis financing, $V^C(\phi^e)$.[35] The fact that $V^C(\phi^e) > L^{\max}(\phi^e)$ reflects that, for the values of L contained between the two curves, liquidation in case of a crisis is ex-post inefficient. Yet, opting for liquidation in a crisis, the bank can get rid of the (CF) constraint in (9) and expand its leverage up to the level allowed by (LL), so bankers may find it ex-ante optimal.

In situations with $L > L^{\max}(\phi^e)$ at least some banks will opt for being exposed to

[35] Since (CF) is binding, the value of a bank's pre-existing equity at a crisis is 0 and thus $V^C(\phi^e) = D^e(\phi^e)$.

liquidation during each systemic crisis. Given the absence of new bank formation in our model, one may wonder whether such a configuration of parameters would lead to the full collapse of the banking sector after sufficiently many crisis. However, for $L < L^{\max}(0)$, the answer is no, since there is a self-equilibrating mechanism: Bank exit will adjust ϕ down after each crisis, leading to a steady state in which all surviving banks eventually rely on crisis financing.[36] So for $L < L^{\max}(0)$, the type of equilibrium with crisis financing on which we have focused in the main sections of the paper is the long term equilibrium of the banking industry.

7.2 Deterministic vs random maturity

For tractability we have assumed that debt contracts have random maturity. It would be more realistic to assume that the bank chooses an integer T that describes the deterministic maturity of its debt contracts. In this setting it is possible to determine savers' required maturity premium $r^{\det}(T)$ as we did in Section 4.1. It can also be shown that for $T = 1/\delta$, we have $r^{\det}(T) < r(\delta)$ because discounting is a convex function of time and thus the random variation in maturity realizations produces disutility to impatient savers.

With deterministic maturities, the model would lose some of the Markovian properties that make it tractable. In the period after a crisis the initial funding structure would not be immediately reestablished since, in addition to the debt with principal $\frac{1}{T}D$ that matures and has to be refinanced, the bank would also have to issue the debt with face value $\frac{1}{T}D$ that was bridge financed during the crisis. Thus, in order for the bank to keep a constant fraction $1/T$ of debt maturing in each period, half of the debt issued by the bank in the after-crisis period should have maturity $T-1$, but this would introduce heterogeneity in interest rate payments across the various debts. The description would become further complicated if a new crisis arrives prior to the maturity of the debt with maturity $T-1$.

Therefore, assuming random maturities implies some loss of banks' value but is essential to the simplicity of our recursive valuation formulas. However, there is no reason to think that deterministic rather than random maturities would qualitatively change any of the

[36] In such an equilibrium the mass of banks would be $m < 1$ and the excess cost of crisis financing would equal the unique ϕ^m that satisfies $L^{\max}(\phi^m) = L$. Now, if (D^m, δ^m) denotes the funding decision under ϕ^m of banks subject to the (CF) constraint, then m can be found as the unique value that solves $\Phi(m\delta^m D^m) = \phi^m$. If the mass of banks were at any point larger than m, then a mass m of banks would use (D^m, δ^m), surviving each crisis, while the remaining ones would be exposed to liquidation in each crisis.

trade-offs behind the key results of the paper.

7.3 Resetting debt structures over time

For the sake of clarity, we have assumed that the debt structure (D, δ) decided at $t = 0$ is kept constant over normal periods and restored immediately after each crisis. What would happen if bankers could re-optimize in periods different from $t = 0$?

To narrow down the question, suppose, in particular, that outstanding debt were exogenously (and unexpectedly) maturing all at a time in a single normal period and bankers were allowed to decide a new debt structure from thereon. It is obvious from the Markovian structure of the model that their optimal decision would coincide with the initial one.[37]

The more general case in which at every date the bank could decide to roll-over part of its maturing debt at perhaps some new terms, while keeping constant the structure of its non-maturing debt, is hard to analyze because describing the debt structures that a bank might end up having requires a very complicated space of state variables. However, we find no obvious reasons to expect that those more general funding structures would increase the value to the bank. Intuition from simpler models suggests that altering the terms of new debt as maturing debt is rolled over might only create value to shareholders at the expense of non-maturing debt holders, but this (i) would have a negative repercussion on the value of such a debt when issued (and hence on initial shareholder value) and (ii) could be prevented by including proper covenants in the preexisting debt contracts.[38]

7.4 Tradability of debt

The non-tradability of banks' debt plays a key role in the model. Savers who turn impatient suffer disutility from delaying consumption until their debt matures because there is no secondary market where to sell the debt (or where to sell it at a sufficiently good price). If bank debt could be traded without frictions, impatient savers would sell their debts to

[37] The formal argument goes as follows: denote the bank's current debt structure by $(\overline{D}, \overline{\delta})$. In a N state that does not follow a C state, the market value of total outstanding debt is $\overline{D}$. Current shareholders would maximize $V(D, \delta; \phi) - \overline{D}$ subject to the same financing constraints as at $t = 0$ and the optimal solution would be the same as at $t = 0$, since the only difference between the initial optimization problems and the current one is the (constant) $\overline{D}$ now subtracted from the objective function. In a N state that follows a C state, the market value of outstanding debt would be $(1 - \overline{\delta})\overline{D}$ and current shareholders would maximize $V(D, \delta; \phi) - (1 - \overline{\delta})\overline{D}$ but again the solution would not change.

[38] See Brunnermeier and Oehmke (2013).

patient savers. Banks could issue perpetual debt ($\delta = 0$) at some initial period and get rid of refinancing concerns. In practice a lot of bank debt, starting with retail deposits, but including also certificates of deposit placed among the public, interbank deposits, debt involved in sales with repurchase agreements (repos), and commercial paper are commonly issued over the counter (OTC) and have no liquid secondary market.

Our model does not contain an explicit justification for the lack of tradability. Arguably, it might stem from administrative, legal compliance, and operational costs associated with the trading (specially using centralized trade) of heterogenous debt instruments issued in small amounts, with a short life or among a dispersed mass of unsophisticated investors. In fact, if other banks (or some other sophisticated traders) could possess better information about banks than ordinary savers, then costs associated with asymmetric information (e.g. exposure to a winners' curse problem in the acquisition of bank debt) might make the secondary market for bank debt unattractive to ordinary savers (Gorton and Pennacchi, 1990). This view is consistent with the common description of interbank markets as markets where peer monitoring is important (Rochet and Tirole, 1996).

Additionally, the literature in the Diamond and Dybvig (1983) tradition has demonstrated that having markets for the secondary trading of bank claims might damage the insurance role of bank deposits.[39] Yet, Diamond (1997) makes the case for the complementarity between banks and markets when, at least for some agents, the access to markets is not guaranteed.

Our model could be extended to describe situations in which debt is tradable but in a non-centralized secondary market characterized by search frictions (like in the models of OTC markets recently explored by Duffie, Gârleanu, and Pedersen, 2005, Vayanos and Weill, 2008, and Lagos and Rocheteau, 2009). In such setting, shortening the maturity of debt would have the effect of increasing the outside option of an impatient saver who is trying to find a buyer for his non-matured debt.[40] This could allow sellers to obtain better prices in the secondary market, making them willing to pay more for the debt in the first place and encouraging banks to issue short-term debt.[41]

[39] See von Thadden (1999) for an insightful review of the results obtained in this tradition.

[40] He and Milbradt (2012) and Bruche and Segura (2013) explicitly model the secondary market for corporate debt as a market with search frictions.

[41] In Bruche and Segura (2013), these trade-offs imply a privately optimal maturity for bank debt. The empirical evidence in Mahanti et al. (2008) and Bao, Pan, and Wang (2011), among others, shows that

8 Conclusion

We have developed an infinite horizon equilibrium model in which banks with long-lived assets decide the overall principal, interest rate payments, and maturity of their debt (and, as residual, their equity financing). Savers' preference for short maturities comes from their exposure to idiosyncratic preference shocks and the lack of tradability of bank debt. Banks' incentive not to set debt maturities as short as savers might ceteris paribus prefer, comes from the fact that there are episodes (systemic liquidity crises) in which their access to savers' funding fails and their refinancing becomes more expensive.

We identify a pecuniary externality in the market for crisis financing that, when combined with the financial constraints faced by banks, renders the unregulated competitive equilibrium socially inefficient. It turns out that, if a social planner coordinates the banks in the choice of longer debt maturities, then banks' total leverage and the social value of their overall maturity transformation activity increases.

We have explored alternatives for restoring efficiency, including forcing banks to issue debt of longer maturities or inducing them to do so with a Pigovian tax on their refinancing needs. We have assessed the amplifying impact of banks' endogenous continuation values on the size of the inefficiency and the impact of regulation. We have also considered the implications of adding private or public liquidity insurance schemes, finding that the case for regulating maturity decisions does not disappear, so that liquidity insurance and liquidity risk regulation can be considered complements rather than substitutes in dealing with the systemic liquidity risk.

short-term bonds are indeed more "liquid" (as measured by the narrowness of the bid-ask spread) than long-term bonds.

Appendix

A Proofs

This appendix contains the proofs of the propositions included in the body of the paper.

Proof of Proposition 1 Using (3) it is a matter of algebra to obtain that:

$$r'(\delta) = \frac{-\gamma(1+\rho_H)(\rho_H - \rho_L)}{\{\rho_H + \delta + (1-\delta)\gamma\}^2} < 0,$$

$$r''(\delta) = \frac{2\gamma\,(1-\gamma)\,(1+\rho_H)(\rho_H - \rho_L)}{\{\rho_H + \delta + (1-\delta)\gamma\}^3} > 0.$$

The other properties stated in the proposition are immediate.■

Proof of Proposition 2 The proof is organized in a sequence of steps.

1. If (CF) is satisfied then (LL) is strictly satisfied Using equation (6) we have that (LL) can be written as:

$$0 \leq E(D,\delta;\phi) = \frac{1}{\rho_H}(\mu - rD) - \frac{1}{\rho_H}\frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon}\left(1+\phi-\frac{1+r}{1+\rho_H}\right)\delta D,$$

while (CF) can be written, using (7), as

$$0 \leq \frac{1}{1+\rho_H}[\mu - r(1-\delta)D + \delta D + E(D,\delta;\phi)] - (1+\phi)\delta D =$$
$$= \frac{1}{\rho_H}(\mu - rD) - \left(1+\frac{1}{\rho_H}\frac{\varepsilon}{1+\rho_H+\varepsilon}\right)\left(1+\phi-\frac{1+r}{1+\rho_H}\right)\delta D.$$

Now, since $1+\frac{1}{\rho_H}\frac{\varepsilon}{1+\rho_H+\varepsilon} > \frac{(1+\rho_H)\varepsilon}{\rho_H(1+\rho_H+\varepsilon)}$ we conclude that whenever (CF) is satisfied, (LL) is strictly satisfied.

2. Notation and useful bounds Using equation (6) we can write:

$$V(D,\delta;\phi) = D + E(D,\delta;\phi) = \frac{1}{\rho_H}\mu + D\Pi(\delta;\phi),$$

where

$$\Pi(\delta,\phi) = 1 - \frac{1}{\rho_H}\left[\left(1-\frac{\varepsilon}{1+\rho_H+\varepsilon}\delta\right)r + \frac{(1+\rho_H)\varepsilon}{1+\rho_H+\varepsilon}\delta\left(\phi+\frac{\rho_H}{1+\rho_H}\right)\right]$$

can be interpreted as the value the bank generates to its shareholders per unit of debt. Using Proposition 1 we can see that the function $\Pi(\delta,\phi)$ is concave in δ.

(CF) in equation (7) can be rewritten as:

$$\mu + V(D, \delta; \phi) \geq [(1 + \rho_H)(1 + \phi)\delta + (1 + r)(1 - \delta)]D,$$

and if we define $C(\delta, \phi) = (1+\rho_H)(1+\phi)\delta + (1+r)(1-\delta)$, (CF) can be written in the more compact form that will be used from now onwards:

$$\frac{1 + \rho_H}{\rho_H}\mu + [\Pi(\delta, \phi) - C(\delta, \phi)]D \geq 0. \tag{17}$$

Using Proposition 1 we can see that the function $C(\delta, \phi)$ is convex in δ.

We have the following relationship:

$$\Pi(\delta, \phi) = 1 - \frac{1}{\rho_H}\frac{1 + \rho_H}{1 + \rho_H + \varepsilon}\left[r(\delta) + \frac{\varepsilon}{1 + \rho_H}(C(\delta, \phi) - 1)\right] \tag{18}$$

Assumption A1 implies $(1 + \rho_H)(1 + \phi) \leq 2(1 + \rho_L) \leq 2(1 + r(\delta))$ for all δ, and we can check that the following bounds (that are independent from ϕ) hold:

$$\begin{aligned} C(\delta, \phi) &\geq 1 + r(\delta). \\ \frac{\partial C(\delta, \phi)}{\partial \delta} &\leq 2(1 + r(\delta)) - (1 + r(\delta)) = 1 + r(\delta). \end{aligned} \tag{19}$$

Using assumption A2, it is a matter of algebra to check that, for all δ,

$$\frac{d^2 r}{d\delta^2} + \frac{dr}{d\delta} \geq 0.$$

And, from this inequality, $\frac{dr}{d\delta} < 0$, and $r < \rho_H$, it is possible to check that:

$$\frac{\partial^2 \Pi(\delta, \phi)}{\partial \delta^2} + \frac{\partial \Pi(\delta, \phi)}{\partial \delta} < -\frac{1}{\rho_H}\left(1 - \frac{\varepsilon}{1 + \rho_H + \varepsilon}\delta\right)\left(\frac{dr}{d\delta} + \frac{d^2 r}{d\delta^2}\right) \leq 0. \tag{20}$$

To save on notation, we will drop from now on the arguments of these functions when it does not lead to ambiguity.

3. $D^* = 0$ is not optimal It suffices to realize that $\frac{\partial V(D,0;\phi)}{\partial D} = \Pi(0, \phi) = 1 - \frac{r(0)}{\rho_H} > 0$.

4. The solution (D^*, δ^*) of the maximization problem in equation (9) exists, is unique, and satisfies (CF) with equality, i.e. $\frac{1+\rho_H}{\rho_H}\mu + (\Pi(\delta^*, \phi) - C(\delta^*, \phi))D^* = 0$

We are going to prove existence and uniqueness in the particular case that there exist $\delta_\Pi, \delta_C \in [0, 1]$ such that $\frac{\partial \Pi(\delta_\Pi,\phi)}{\partial \delta} = \frac{\partial C(\delta_C,\phi)}{\partial \delta} = 0$. This will ensure that the solution of the maximization problem is interior in δ. The other cases are treated in an analogous way but might give rise to corner solutions in δ.[42]

[42] More precisely, if for all $\delta \in [0, 1]$ $\frac{\partial C(\delta,\phi)}{\partial \delta} > 0$ we might have $\delta^* = 0$ and if for all $\delta \in [0, 1]$, $\frac{\partial \Pi(\delta,\phi)}{\partial \delta} > 0$ we might have $\delta^* = 1$.

First, since $\Pi(\delta, \phi)$ is concave in δ we have that $\frac{\partial \Pi(\delta,\phi)}{\partial \delta} \geq 0$ iff $\delta \leq \delta_\Pi$. Since $C(\delta, \phi)$ is convex in δ we have that $\frac{\partial C(\delta,\phi)}{\partial \delta} \geq 0$ iff $\delta \geq \delta_C$. It is easy to prove from equation (18) that $\delta_C < \delta_\Pi$.

Now, let (D^*, δ^*) be a solution to the maximization problem. The first order conditions (FOC) that characterize an interior solution (D^*, δ^*) are:

$$\begin{aligned} (1+\theta)\Pi - \theta C &= 0, \\ (1+\theta)\frac{\partial \Pi}{\partial \delta} - \theta \frac{\partial C}{\partial \delta} &= 0, \\ \theta \left[\frac{1+\rho_H}{\rho_H}\mu + (\Pi - C)D^* \right] &\geq 0, \\ \theta &\geq 0, \end{aligned} \tag{21}$$

where θ is the Lagrange multiplier associated with (CF) and we have used that $D^* > 0$ in order to eliminate it from the second equation.

If $\theta = 0$ then the second equation implies $\delta^* = \delta_\Pi$ and thus $\Pi(\delta^*, \phi) \geq \Pi(0, \phi) > 0$ and the first equation is not satisfied. Therefore we must have $\theta > 0$ so that (CF) is binding at the optimum. Now we can eliminate θ from the previous system of equations, which gets reduced to:

$$\frac{\partial \Pi(\delta^*, \phi)}{\partial \delta} C(\delta^*, \phi) = \frac{\partial C(\delta^*, \phi)}{\partial \delta} \Pi(\delta^*, \phi), \tag{22}$$

$$\frac{1+\rho_H}{\rho_H}\mu = \left[C(\delta^*, \phi) - \Pi(\delta^*, \phi) \right] D^*. \tag{23}$$

We are going to show that equation (22) has a unique solution in δ. For $\delta \leq \delta_C < \delta_\Pi$, we have $\frac{\partial C}{\partial \delta} \leq 0 < \frac{\partial \Pi}{\partial \delta}$ and thus the left hand side (LHS) of (22) is strictly bigger than the RHS. For $\delta \geq \delta_\Pi > \delta_C$, we have $\frac{\partial \Pi}{\partial \delta} \leq 0 < \frac{\partial C}{\partial \delta}$ and thus RHS of (22) is strictly bigger.

Now, the function $\frac{\partial C(\delta,\phi)}{\partial \delta}\Pi(\delta, \phi)$ is strictly increasing in the interval (δ_C, δ_Π) since both terms are positive and increasing. Thus, it suffices to prove that for $\delta \in (\delta_C, \delta_\Pi)$ the function $\frac{\partial \Pi(\delta,\phi)}{\partial \delta} C(\delta, \phi)$ is decreasing.[43] Using the the bounds in (19), inequality (20) and $\frac{\partial^2 \Pi}{\partial \delta^2} < 0$, $\frac{\partial \Pi}{\partial \delta} > 0$ for $\delta \in (\delta_C, \delta_\Pi)$, we have:

$$\frac{\partial}{\partial \delta}\left(\frac{\partial \Pi}{\partial \delta} C \right) = \frac{\partial^2 \Pi}{\partial \delta^2} C + \frac{\partial \Pi}{\partial \delta}\frac{\partial C}{\partial \delta} \leq (1+r)\left(\frac{\partial^2 \Pi}{\partial \delta^2} + \frac{\partial \Pi}{\partial \delta} \right) \leq 0.$$

This concludes the proof on the existence and uniqueness of a δ^* that satisfies the necessary FOC in (22).

Now, for given δ^*, the other necessary FOC (23) determines D^* uniquely.[44]

[43] This is not trivial since $C(\delta, \phi)$ is increasing.

[44] Let us observe that for all δ, $C(\delta, \phi) \geq 1 > \Pi(\delta, \phi)$.

5. δ^* is independent from μ and D^* is strictly increasing in μ Equation (22) determines δ^* and is independent from μ. Then equation (23) shows that D^* is increasing in μ.

6. δ^* is decreasing in ϕ and, if $\delta^* \in (0,1)$, it is strictly decreasing Let $\delta(\phi)$ be the solution of the maximization problem of the bank for given ϕ. Let us assume that $\delta(\phi)$ satisfies the FOC (22). The case of corner solutions is analyzed in an analogous way.

We have proved in Step 3 above that the function $\frac{\partial \Pi}{\partial \delta} C - \frac{\partial C}{\partial \delta} \Pi$ is decreasing in δ around $\delta(\phi)$. In order to show that $\delta(\phi)$ is decreasing, it suffices to show that the derivative of this function w.r.t. ϕ is negative. Using the definitions of $C(\delta, \phi), \Pi(\delta, \phi)$ after some (tedious) algebra we obtain:

$$\frac{\partial}{\partial \phi}\left[\frac{\partial \Pi}{\partial \delta} C - \frac{\partial C}{\partial \delta}\Pi\right] = -(1+\rho_H) - \frac{1}{\rho_H}\frac{1+\rho_H}{1+\rho_H+\varepsilon}\left[(1+\rho_H)\left(\frac{dr}{d\delta}\delta - r\right) + \varepsilon\right].$$

Now we have $\frac{d}{d\delta}\left(\frac{dr}{d\delta}\delta - r\right) = \frac{d^2 r}{d\delta^2}\delta \geq 0$ and thus $\frac{dr}{d\delta}\delta - r \geq \frac{dr}{d\delta}\delta - r\big|_{\delta=0} = -r(0)$, and finally:

$$\begin{aligned}\frac{\partial}{\partial \phi}\left[\frac{\partial \Pi}{\partial \delta} C - \frac{\partial C}{\partial \delta}\Pi\right] &\leq -(1+\rho_H) - \frac{1}{\rho_H}\frac{1+\rho_H}{1+\rho_H+\varepsilon}\left[-(1+\rho_H)r(0) + \varepsilon\right] \\ &< -(1+\rho_H) + \frac{1}{\rho_H}(1+\rho_H)r(0) = -(1+\rho_H)\left(1 - \frac{r(0)}{\rho_H}\right) < 0.\end{aligned}$$

This conclude the proof that $\frac{d\delta}{d\phi} < 0$.[45]

7. $\delta^* D^*$ is decreasing with ϕ. If $\delta^* > 0$ it is strictly decreasing

Let $\delta(\phi), D(\phi)$ be the solution of the maximization problem of the bank for given ϕ. We have:

$$\frac{1+\rho_H}{\rho_H}\mu = \left[C(\delta(\phi), \phi) - \Pi(\delta(\phi), \phi)\right] D(\phi).$$

Let $\phi_1 < \phi_2$. In Step 5 we showed that $\delta(\phi_1) \geq \delta(\phi_2)$. If $\delta(\phi_2) = 0$ then trivially $\delta(\phi_1)D(\phi_1) \geq \delta(\phi_2)D(\phi_2) = 0$. Let us suppose that $\delta(\phi_2) > 0$. Since trivially $\Pi(\delta(\phi_1), \phi_1)D(\phi_1) \geq \Pi(\delta(\phi_2); \phi_2)D(\phi_2)$, we must have $C(\delta(\phi_1), \phi_1)D(\phi_1) \geq C(\delta(\phi_2), \phi_2)D(\phi_2)$. Now, suppose that $\delta(\phi_1)D(\phi_1) \leq \delta(\phi_2)D(\phi_2)$, then we have the following two inequalities:

$$\begin{aligned}(1+\rho_H)(1+\phi_1)\delta(\phi_1)D(\phi_1) &< (1+\rho_H)(1+\phi_2)\delta(\phi_2)D(\phi_2), \\ (1+r(\delta(\phi_1)))(1-\delta(\phi_1)) &\leq (1+r(\delta(\phi_2)))(1-\delta(\phi_2)),\end{aligned}$$

that imply $C(\delta(\phi_1), \phi_1)D(\phi_1) < C(\delta(\phi_2), \phi_2)D(\phi_2)$, but this contradicts our assumption. Thus, $\delta(\phi_1)D(\phi_1) > \delta(\phi_2)D(\phi_2)$.■

[45] In the case of corner solution $\delta^*(\phi) = 1$, we might have $\frac{d\delta^*}{d\phi} = 0$ and obviously for $\delta^*(\phi) = 0$, $\frac{d\delta^*}{d\phi} = 0$.

Proof of Proposition 3 Let us denote $(D(\phi), \delta(\phi))$ the solution of the bank's optimization problem for every excess cost of crisis financing $\phi \geq 0$. Proposition 2 states that $\delta(\phi)D(\phi)$ is decreasing in ϕ. For $\phi \in [0, \overline{\phi}]$ let us define $\Sigma(\phi) = \Phi(\delta(\phi)D(\phi)) - \phi$. This function represents the difference between the excess cost of financing during a crisis by banks' decisions and banks' expectation on such variable. Since Φ is an increasing function on the aggregate demand of funds during a crisis the function $\Sigma(\phi)$ is strictly decreasing. Because of the uniqueness of the solution to the problem that defines $(D(\phi), \delta(\phi))$, the function is also continuous. Moreover, we trivially have $\Sigma(0) \geq 0$ and $\Sigma(\overline{\phi}) \leq 0$. Therefore there exists a unique $\phi^e \in \mathbb{R}^+$ such that $\Sigma(\phi^e) = 0$. By construction $D(\phi^e), \delta(\phi^e), \phi^e$ is the unique equilibrium of the economy.■

Proof of Proposition 4 We are going to follow the notation used in the proof of Proposition 3. Let Φ_1, Φ_2 be two curves describing the inverse supply of financing during a crisis and assume they satisfy $\Phi_1(x) > \Phi_2(x)$ for all $x > 0$. Let us denote $\Sigma_i(\phi) = \Phi_i(\delta(\phi)D(\phi)) - \phi$ for $i = 1, 2$. By construction we have $\Sigma_1(\phi_1^e) = 0$. Let us suppose that $\phi_1^e < \phi_2^e$. Then we would have:

$$\Sigma_2(\phi_2^e) = \Phi_2(\delta(\phi_2^e)D(\phi_2^e)) - \phi_2^e \leq \Phi_1(\delta(\phi_2^e)D(\phi_2^e)) - \phi_2^e < \Phi_1(\delta(\phi_1^e)D(\phi_1^e)) - \phi_1^e = \Sigma_1(\phi_1^e) = 0, \tag{24}$$

where in the first inequality we use the assumption $\Phi_2(x) \leq \Phi_1(x)$ for $x \geq 0$, and in the second inequality we use that if $\phi_1^e < \phi_2^e$ then $\delta(\phi_2^e)D(\phi_2^e) \leq \delta(\phi_1^e)D(\phi_1^e)$ (Proposition 2), and that $\Phi_1(\cdot)$ is increasing.

Notice that the sequence of inequalities in (24) implies $\Sigma_2(\phi_2^e) < 0$, which contradicts the definition of ϕ_2^e. We must therefore have $\phi_1^e \geq \phi_2^e$. Now Proposition 2 implies that $\delta_1^e \leq \delta_2^e, \delta_1^e D_1^e \leq \delta_2^e D_2^e, r_1^e \geq r_2^e$. Let us suppose that $\delta_2^e \in (0, 1)$ then the first inequality in (24) is strict, since $\delta_2^e D_2^e > 0$, and we can straightforwardly check that the previous argument implies $\phi_1^e > \phi_2^e$. Now, since $\delta_2^e \in (0, 1)$, Proposition 2 implies that $\delta_1^e < \delta_2^e$, $\delta_1^e D_1^e < \delta_2^e D_2^e$, and $r_1^e > r_2^e$.■

Proof of Proposition 5 We are going to follow the notation used in the proof of Proposition 2. The proof is organized in five steps:

1. Preliminaries We have seen in the proof of Proposition **??** that:

$$\frac{\partial W(D, \delta)}{\partial \delta} = \frac{\partial V(D, \delta; \Phi(\delta D))}{\partial \delta} = D \frac{\partial \Pi(\delta, \Phi(\delta D))}{\partial \delta}. \tag{25}$$

Similarly we have

$$\frac{\partial W(D, \delta)}{\partial D} = \frac{\partial V(D, \delta; \Phi(\delta D))}{\partial D} = \Pi(\delta, \Phi(\delta D)). \tag{26}$$

2. (CF) is binding at the socially optimal debt structure This is a statement that has been done in the main text just before Proposition 5. The proof is analogous to the one for the maximization problem of the bank that we did in Step 4 of the proof of Proposition 2. The only difference is that ϕ is not taken as given but as the function $\Phi(\delta D)$ in D and δ.

3. Definition of function $D^c(\delta)$ and its properties Let $(\phi^e, (D^e, \delta^e))$ be the competitive equilibrium. Let us assume that $\delta^e < 1$. By definition of equilibrium we have $\phi^e = \Phi(\delta^e D^e)$. For every δ let $D^c(\delta)$ be the unique principal of debt such that (CF) is binding, i.e.:

$$\frac{1+\rho_H}{\rho_H}\mu = \left[C(\delta, \phi^e) - \Pi(\delta, \phi^e)\right] D^c(\delta). \tag{27}$$

Differentiating w.r.t. δ:

$$\left[\frac{\partial C(\delta, \phi^e)}{\partial \delta} - \frac{\partial \Pi(\delta, \phi^e)}{\partial \delta}\right] D^c(\delta) + \left[C(\delta, \phi^e) - \Pi(\delta, \phi^e)\right] \frac{dD^c(\delta)}{d\delta} = 0. \tag{28}$$

Using the characterization of δ^e in equation (22), the inequalities $C(\delta, \phi^e) \geq 1 > \Pi(\delta, \phi^e)$ imply $\frac{\partial C(\delta^e,\phi^e)}{\partial \delta} - \frac{\partial \Pi(\delta^e,\phi^e)}{\partial \delta} > 0$ and, then, we can deduce from the equation above that $\frac{dD^c(\delta^e)}{d\delta} < 0$. Since (CF) is binding at the optimal debt structure we can think of the bank problem as maximizing the univariate function $V(D^c(\delta), \delta; \phi^e)$. Hence δ^e must satisfy the necessary FOC for an interior solution to the maximization of $V(D^c(\delta), \delta; \phi^e)$:

$$\frac{dV(D^c(\delta), \delta^e; \phi^e)}{d\delta} = 0 \Leftrightarrow D^c(\delta^e)\frac{\partial \Pi(\delta^e, \phi^e)}{\partial \delta} + \Pi(\delta^e, \phi^e)\frac{dD^c(\delta^e)}{d\delta} = 0, \tag{29}$$

which multiplying by δ^e can be written as

$$D^c(\delta^e)\frac{\partial \Pi(\delta^e, \phi^e)}{\partial \delta^e}\delta^e = \Pi(\delta^e, \phi^e)\left(-\frac{dD^c(\delta^e)}{d\delta}\delta^e\right).$$

Since $\frac{\partial\left(\Pi - \frac{\partial \Pi}{\partial \delta}\delta\right)}{\partial \delta} = -\frac{\partial^2 \Pi}{\partial \delta^2}\delta \geq 0$ and $\Pi(0,\phi) - \frac{\partial \Pi(0,\phi)}{\partial \delta} 0 > 0$, we have $\Pi(\delta, \phi) > \frac{\partial \Pi(\delta,\phi)}{\partial \delta}\delta$ for all $\delta \in [0, 1]$ and the previous equation implies

$$D^c(\delta^e) > -\frac{dD^c(\delta^e)}{d\delta}\delta^e \Leftrightarrow \left.\frac{d\left(\delta D^c(\delta)\right)}{d\delta}\right|_{\delta=\delta^e} > 0.$$

4. Evaluation of $\left.\frac{d(D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}$ and $\left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}$ For every δ, let $D^s(\delta)$ be the unique principal of debt such that (CF) is binding, i.e.

$$\frac{1+\rho_H}{\rho_H}\mu = \left[C(\delta, \Phi(\delta D^s(\delta))) - \Pi(\delta, \Phi(\delta D^s(\delta)))\right] D^s(\delta).$$

Differentiating w.r.t. δ, we obtain

$$\left[\frac{\partial C(\delta,\Phi)}{\partial\delta}-\frac{\partial\Pi(\delta,\Phi)}{\partial\delta}\right]D^s(\delta)+\left[C(\delta,\Phi(\delta D^s(\delta)))-\Pi(\delta,\Phi(\delta D^s(\delta)))\right]\frac{dD^s(\delta)}{d\delta}+$$
$$+\left[\frac{\partial C(\delta,\Phi)}{\partial\phi}-\frac{\partial\Pi(\delta,\Phi)}{\partial\phi}\right]\Phi'(\delta D^s(\delta)))\frac{d\left(\delta D^s(\delta)\right)}{d\delta}=0. \tag{30}$$

By construction, $D^s(\delta^e)=D^c(\delta^e)=D^e$. Now, subtracting equation (28) from equation (30) at the point $\delta=\delta^e$ we obtain

$$[C(\delta^e,\phi^e)-\Pi(\delta^e,\phi^e)]\left(\frac{dD^s(\delta^e)}{d\delta}-\frac{dD^c(\delta^e)}{d\delta}\right)+\left[\frac{\partial C(\delta^e,\phi^e)}{\partial\phi}\right.$$
$$\left.-\frac{\partial\Pi(\delta^e,\phi^e)}{\partial\phi}\right]\Phi'(\delta^e D^e)\left.\frac{d\left(\delta D^s(\delta)\right)}{d\delta}\right|_{\delta=\delta^e}=0. \tag{31}$$

Suppose that $\left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}\leq 0$, then we would have $\frac{dD^s(\delta^e)}{d\delta}\geq\frac{dD^c(\delta^e)}{d\delta}$, since trivially $\frac{\partial C(\delta^e,\phi^e)}{\partial\phi}-\frac{\partial\Pi(\delta,\phi^e)}{\partial\phi}>0$. But then

$$\left.\frac{d\left(\delta D^s(\delta)\right)}{d\delta}\right|_{\delta=\delta^e}=D^s(\delta^e)+\frac{dD^s(\delta^e)}{d\delta}\delta^e>D^c(\delta^e)+\frac{dD^c(\delta^e)}{d\delta}\delta^e=\left.\frac{d\left(\delta D^c(\delta)\right)}{d\delta}\right|_{\delta=\delta^e}>0,$$

which contradicts the hypothesis. We must thus have $\left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}>0$, in which case equation (31) implies $\frac{dD^s(\delta^e)}{d\delta}<\frac{dD^c(\delta^e)}{d\delta}<0$.

5. Evaluation of $\left.\frac{dW(D^s(\delta),\delta)}{d\delta}\right|_{\delta=\delta^e}$ Using equations (25) and (26), we have:

$$\begin{aligned}\frac{dW(D^s(\delta),\delta)}{d\delta}&=\frac{\partial W(D^s(\delta),\delta)}{\partial\delta}+\frac{\partial W(D^s(\delta),\delta)}{\partial D}\frac{dD^s(\delta)}{d\delta}\\&=D^s(\delta)\frac{\partial\Pi(\delta,\Phi(\delta D^s(\delta)))}{\partial\delta}+\Pi(\delta,\Phi(\delta D^s(\delta)))\frac{dD^s(\delta)}{d\delta}.\end{aligned}$$

And, using $\frac{dD^s(\delta^e)}{d\delta}<\frac{dD^c(\delta^e)}{d\delta}$ and (29), we obtain:

$$\left.\frac{dW(D^s(\delta),\delta)}{d\delta}\right|_{\delta=\delta^e}<D^s(\delta^e)\frac{\partial\Pi(\delta^e,\phi^e)}{\partial\delta}+\Pi(\delta^e,\phi^e)\frac{dD^c(\delta^e)}{d\delta}=0.$$

Summing up, having

$$\left.\frac{dW(D^s(\delta),\delta)}{d\delta}\right|_{\delta=\delta^e}<0,\quad\left.\frac{dD^s(\delta)}{d\delta}\right|_{\delta=\delta^e}<0,\text{ and }\left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}>0,$$

implies that a social planner can increase welfare by fixing some $\delta^s<\delta^e$, and suggests that doing so will produce higher leverage and lower refinancing needs than in the unregulated competitive equilibrium.■

Proof of Proposition 6 Let $(\phi^s, (D^s, \delta^s))$ be the socially efficient equilibrium. The sketch of the proof is as follows:

1. For any fixed ϕ banks' optimal choice of δ depends only on τ (and not on the lump-sum tax rebate M) and as τ increases δ decreases. From here we can show that if banks' expectation on the excess cost of crisis financing is $\phi^s < \phi^e$ there exists a Pigovian tax $\tau^P > 0$ that induces the socially efficient choice of maturity.

2. For $\phi = \phi^s$ and Pigovian tax τ^P defined above, once banks have taken their maturity decision δ^s they issue as much debt D as (CF) allows and at this point the amount of the lump-sum transfer M matters. The effect of the net per period transfer $\tau^P\delta^s D - M \geq 0$ from banks to the social planner is to reduce banks' equity value at the N state and thus to strictly tighten (CF) with respect to the situation in which the social planner directly regulates maturity to its social optimum $1/\delta^s$ unless there is full rebate of the Pigovian tax, i.e. $M = \tau^P\delta^s D$. More precisely, it can be shown that D is strictly increasing with M and that $D = D^s$ if and only if $M = \tau^P\delta^s D^s$.

3. Our candidate for optimal Pigovian tax scheme is (τ^P, M^P) with $M^P = \tau^P\delta^s D^s$. By construction, under this tax scheme if banks' expectation on the excess cost of crisis financing is ϕ^s, then banks' optimal funding structure coincides with the socially efficient structure (D^s, δ^s), which in turn satisfies $\phi^s = \Phi(\delta^s D^s)$, confirming ϕ^s as an expectation compatible with the equilibrium.

The most cumbersome details of the proof are analogous to those in the proof of Proposition 2 and are omitted for brevity. They are available from the authors upon demand.■

Proof of Proposition 7 Let us recall that the introduction of (fairly priced) insurance does not change the value of equity at the N state, i.e. $E(D, \delta, \lambda; \phi) = E(D, \delta; \phi)$ for all λ. In addition banks choose full insurance, $\lambda = 1$, and the only financial constraint is (LL) that can be written $E(D, \delta, 1; \phi) = E(D, \delta; \phi) > 0$. For the next steps, we follow the notation introduced in the proof of Proposition 2.

1. Insurance increases social welfare in the regulated economy Let (D^s, δ^s) be the socially optimal debt structure in the absence of insurance. In the proof of Proposition 5 we showed that (CF) is binding at (D^s, δ^s). In fact, we have $\frac{\partial W(D^s,\delta^s)}{\partial D} > 0$. Step 1 in the proof of Proposition 2 states that (LL) is satisfied with slack, i.e.

$$E(D^s, \delta^s; \Phi(\delta^s D^s)) > 0,$$

and thus by continuity there are values $D' > D^s$ such that $E(D', \delta^s; \Phi(\delta^s D')) > 0$ and $W(D', \delta^s) > W(D^s, \delta^s)$. Introducing insurance makes debt structures such as (D', δ^s) feasible and, hence, increases welfare relative to the regulated economy without insurance.

2. Under insurance the competitive expected maturity is shorter than the socially optimal one When insurance is introduced the relevant financial constraint faced both by banks in the unregulated equilibrium (for given ϕ) and by the social planner (for $\phi = \Phi(\delta D)$) is (LL) and is binding. From here, the proof is analogous to that of Proposition 5 and we omit it for brevity.■

B Amplification via continuation value

In this appendix we describe how the objective function of the one-off social planner in Section 5.2 is found. Let (D^e, δ^e) denote the competitive equilibrium of the economy and let us denote $V^e = V(D^e, \delta^e; \Phi(\delta^e D^e))$ and $U^e = U(D^e, \delta^e)$ the normal period competitive equilibrium market value of a bank and the expected surplus of potential crisis financiers, respectively.

Let us suppose that the one-off social planner chooses a debt structure (D, δ) up to the first crisis. At the normal period following that crisis, banks are allowed to repay their outstanding debt and choose their optimal debt structures, which in equilibrium will be (D^e, δ^e). The value $\widetilde{E}(D, \delta; \alpha)$ of the equity of banks at a normal period before the first crisis (and when the fraction relinquished to crisis financiers to obtain their funds at that crisis is α) satisfies:

$$\begin{aligned}\widetilde{E}(D, \delta; \alpha) &= \frac{1}{1+\rho_H}\Big\{(\mu - rD) + (1-\varepsilon)\widetilde{E}(D, \delta; \alpha) + \\ &\quad \varepsilon(1-\alpha)\frac{1}{1+\rho_H}[\mu - (1-\delta)rD - (1-\delta)D + V^e]\Big\}. \end{aligned} \tag{32}$$

The only difference with respect to the analogous recursive equation (4) is that the continuation value of the bank at the first crisis takes into account that at the following normal period the bank will buy its outstanding debt (hence the term $-(1-\delta)D$) and will issue debt D^e with maturity δ^e (hence the term $V^e = D^e + E(D^e, \delta^e; \Phi(\delta^e D^e))$). Due to competition between crisis financiers, α satisfies:

$$\alpha\frac{1}{1+\rho_H}[\mu - (1-\delta)rD - (1-\delta)D + V^e] = (1+\Phi(\delta D))\delta D, \tag{33}$$

where we are taking into account that the one-off social planner internalizes the effect of his decisions on the excess cost of financing in the first crisis. Using (33) in (32) we can find the following expression for the value of bank equity:

$$\widetilde{E}(D, \delta) = \frac{\mu - rD}{\rho_H + \varepsilon} + \frac{\varepsilon}{\rho_H + \varepsilon}[\frac{\mu - (1-\delta)rD - (1-\delta)D + V^e}{1+\rho_H} - (1+\Phi(\delta D))\delta D]. \tag{34}$$

The value $\widetilde{U}(D,\delta)$ of the surplus of potential crisis financiers in this set-up satisfies the recursive equation:

$$\widetilde{U}(D,\delta)=\frac{1}{1+\rho_H}\left[(1-\varepsilon)\widetilde{U}(D,\delta)+\varepsilon\left(u(D,\delta)+\frac{1}{1+\rho_H}U^e\right)\right],$$

where $u(D,\delta)$ has been defined in equation (10). Solving this equation we find:

$$\widetilde{U}(D,\delta)=\frac{\varepsilon}{\rho_H+\varepsilon}\left(u(D,\delta)+\frac{1}{1+\rho_H}U^e\right).$$

Finally, the objective function of the one-off social planner is $\widetilde{W}(D,\delta)=D+\widetilde{E}(D,\delta)+\widetilde{U}(D,\delta)$ and, when maximizing this function, he is subject to the crisis financing constraint that arises from imposing $\alpha\leq 1$ in (33):

$$\mu-(1-\delta)rD-(1-\delta)D+V^e\geq(1+\rho_H)\left(1+\Phi(\delta D)\right)\delta D. \tag{35}$$

C Debt structures that induce default during crises

In this appendix we examine the possibility that a bank decides to expose itself to the risk of defaulting on its debt obligations and being liquidated during systemic crises. First, we describe the sequence of events following a bank's default. Second, we show how the debt of the bank is valued by savers who correctly anticipate this course of events. Finally, we analyze the bank's decision problem when default during crises is an explicit alternative.

Default and liquidation Liquidation following the bank's inability to satisfy its refinancing needs yields a residual value $L\geq 0$. Suppose that partial liquidation is not allowed and L is distributed equally among all debtholders independently of their contract having just matured or not. This eliminates the type of preemptive runs studied by He and Xiong (2012b). It is easy to realize that if the bank does not want to rely on crisis financing (exposing itself to possible default in a crisis), then it will find it optimal to make its debt mature in a perfectly correlated manner since this minimizes the probability of default during crises. Hence we assume that the debt issued by the bank when getting rid of the (CF) constraint has perfectly correlated maturities.

Savers' required maturity premium when default is anticipated From a saver's perspective, there are four states relevant for the valuation of a given debt contract: personal patience in a normal period ($i=LN$), personal patience in a crisis period ($i=LC$),personal impatience in a normal period ($i=HN$), and personal impatience in a crisis period ($i=HC$).

Let $l = L/D < 1$ be the fraction of the principal of debt which is recovered in case of liquidation and let Q_i be the present value of expected losses due to default as evaluated from each of the states i just after the uncertainty regarding the corresponding period has realized and conditional on the debt not having matured in such period. Losses are measured relative to the benchmark case without default in which at maturity savers recover 100% of the principal. These values satisfy the following system of recursive relationships:

$$Q_{LN} = \frac{1}{1+\rho_L}\left[\delta\varepsilon(1-l) + (1-\delta)\{(1-\varepsilon)[(1-\gamma)Q_{LN} + \gamma Q_{HN}] + \varepsilon[(1-\gamma)Q_{LC} + \gamma Q_{HC}]\}\right],$$

$$Q_{HN} = \frac{1}{1+\rho_H}\left[\delta\varepsilon(1-l) + (1-\delta)\left[(1-\varepsilon)Q_{LN} + \varepsilon Q_{HC}\right]\right],$$

$$Q_{LC} = \frac{1}{1+\rho_L}(1-\delta)[(1-\gamma)Q_{LN} + \gamma Q_{HN}],$$

$$Q_{HC} = \frac{1}{1+\rho_H}(1-\delta)Q_{HN}.$$

These expressions essentially account for the principal $1-l>0$ which is lost whenever the saver's debt contract matures in a state of crisis. First equation reflects that default as well as any of four states i may follow state LN. The second equation reflects that impatience is an absorbing state. The third and fourth equations reflect that a crisis period can only be followed by a normal period. We will denote the solution of this linear system of equations $Q_{LN}(D, \delta; L, \varepsilon)$ in order to highlight its dependence on these variables.

The value of a debt contract $(1, r, \delta)$ to a patient saver in a normal period, when default is expected if the bank runs into refinancing needs during a crisis, can then be written as

$$U_L^d(r, \delta) = U_L(r, \delta) - Q_{LN}(D, \delta; L, \varepsilon),$$

where $U_L(r, \delta)$ is the value of the same contract in the scenario in which the principal is always recovered at maturity, whose expression is given in (2).

Now, let $r^d(\delta)$ be the interest rate yield that the bank offers in the default setting, which satisfies $U_L^d(r^d(\delta), \delta) = 1$. Since the non-default yield $r(\delta)$ satisfies $U_L(r(\delta), \delta) = 1$, the equation $U_L^d(r^d(\delta), \delta) = U_L(r(\delta), \delta)$ allows us to express $r^d(\delta)$ as the sum of $r(\delta)$ and a default-risk premium:

$$r^d(\delta) = r(\delta) + \frac{(\rho_H + \delta)\left(\rho_L + \delta + (1-\delta)\gamma\right)}{\rho_H + \delta + (1-\delta)\gamma} Q_{LN}(D, \delta; L, \varepsilon).$$

One can prove that the default-risk premium $r^d(\delta) - r(\delta)$ is increasing in δ, increasing in ε, decreasing in L, and increasing in D. Given that δ increases the probability of default, $r^d(\delta)$ is not necessarily decreasing in δ.

Banks' optimal funding structure inducing default If the bank does not satisfy the crisis financing constraint and thus defaults whenever it faces refinancing needs during a crisis, its equity value in normal times $E^d(D,\delta)$ will satisfy the following recursive equation:

$$E^d(D,\delta) = \frac{1}{1+\rho_H}\left[\mu - r^d D + (1-\varepsilon)E^d(D,\delta) + \varepsilon\{\delta \cdot 0 + (1-\delta)\frac{1}{1+\rho_H}[\mu - r^d D + E^d(D,\delta)]\}\right],$$

whose solution yields:

$$E^d(D,\delta) = \frac{1+\rho_H+\varepsilon(1-\delta)}{(1+\rho_H)^2 - (1+\rho_H)(1-\varepsilon) - \varepsilon(1-\delta)}(\mu - r^d D).$$

In this context, the problem determining the bank's optimal debt structure decision in the absence of the crisis financing constraint can be written as:

$$\begin{array}{cl} \max\limits_{D\geq 0,\ \delta\in[0,1]} & V^d(D,\delta) = D + E^d(D,\delta), \\ \text{s.t.} & E^d(D,\delta) \geq 0, \quad \text{(LL)} \end{array} \tag{36}$$

where (LL) is trivially equivalent to $\mu - r^d D \geq 0$.

Figure 7 in the main text has been generated by numerically solving this problem for each value of ϕ^e and L, and finding $L^{\max}(\phi^e)$ as the (maximum) value of L for which the total market value of the bank under the best debt structure compatible with (CF) equals the total market value that the bank can attain solving (36).

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