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Liquidity standards and the value of an informed lender of last resort*

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Abstract

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Keywords: Liquidity standards, lender of last resort, bank runs

JEL Classification: G01, G21, G28

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Abstract

We consider a dynamic model in which receiving support from the lender of last resort (LLR) may help banks to weather investor runs. We show the need for regulatory liquidity standards when the underlying social trade-offs make the uninformed LLR inclined to support troubled banks during a run. Liquidity standards increase the time available before the LLR must decide on supporting the bank. This facilitates the arrival of information on the bank's financial condition and improves the efficiency of the decision taken by the LLR, a role that can be modified but not replaced with the use of capital regulation.

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1 Introduction

Prior to the Great Recession, the focus of bank regulation was on bank capital. However, the liquidity problems that banks experienced since the onset of the financial crisis in 2007 brought to the forefront a debate about the potential value of regulating banks' liquidity.[1] Those problems also reignited the debate on the challenges that uncertainty about the financial condition of banks pose to the lender of last resort (LLR).[2] In this paper, we contribute to these debates by presenting a novel theory of banks' liquidity standards.

Our theory builds on what we believe is a distinct feature of an instrument such as the liquidity coverage ratio (LCR) of Basel III:[3] Once a crisis starts, liquidity buffers provide banks the capability to accommodate potential debt withdrawals for some time. Having time to resist without LLR support is valuable; it allows for the discovery of information on the bank's financial condition that is useful for the LLR's decision on whether to grant support. This generally improves the efficiency of the decisions regarding the continuation of the bank as a going concern or its liquidation and, on occasion, allows for a resolution of the crisis without an intervention by the LLR.

We consider a model in which a bank ex ante decides how to allocate its funds across liquid and illiquid assets. Illiquid assets are more profitable than liquid assets but their quality is vulnerable to the realization of an interim shock. If assets get damaged by the shock, the bank turns fundamentally insolvent and its early liquidation is efficient.[4] In contrast, if assets do not get damaged, the bank remains fundamentally solvent and its early liquidation is inefficient. A crucial problem is that discerning whether the assets are damaged takes time.

The bank is funded with equity and short-term debt, and faces rollover risk because each

[1]See Gorton (2009) and Shin (2010) for a discussion on the role of banks' liquidity problems during the Global Financial Crisis.

[2]Bagehot (1873) advocates that central banks should extend liquidity support to banks experiencing liquidity problems provided they are solvent. However Goodhart (1999) argues that the feasibility of establishing a clear-cut distinction between illiquidity and insolvency on the spot is a myth.

[3]For a description of the LCR, see Basel Committee on Banking Supervision (2010).

[4]Early liquidation may help reverse unprofitable investment strategies, stop "evergreening" strategies with respect to a portfolio of bad loans or any other form of gambling for resurrection.

period a portion of investors decide whether to rollover their short-term debt.[5] Under these circumstances, the shock to the bank's financial condition can trigger a run among investors, which if sustained for long enough, can lead the bank into failure, unless it borrows from the LLR. In making its lending decision, the LLR faces the classical problem that the bank seeking liquidity support might be fundamentally insolvent. While it is optimal to grant liquidity to solvent banks, early liquidation would be preferable in the case of an insolvent bank.

In general, assessing the financial condition of the bank in real time is quite difficult. Following this view, we assume that the LLR is initially uncertain about the financial condition of the bank (the quality of its illiquid assets) but may obtain the relevant information over time. Thus, liquidity standards, which lengthen the time a bank can sustain a liquidity shock without outside support, allow for more information on the bank's financial condition to come out prior to the LLR decision on whether to extend its emergency lending to the bank. Such information is valuable because it improves the efficiency of the implied continuation versus liquidation decision regarding the bank's illiquid assets. Our model, therefore, shows that postponing the time at which the bank needs liquidity support from the LLR may be conducive to a more efficient resolution of the crisis.

Our model also shows that, when the potential support received from the LLR involves a subsidy, the liquidity standards voluntarily adopted by bank owners may be lower than those that a regulator might like to set.[6] Specifically, if bank owners expect support to be granted when the LLR is uninformed about the quality of the assets once the bank exhausts its cash ("strong bank case"), they may prefer to opportunistically hold less liquidity than it would be socially optimal. By doing this, they shorten the spell over which the bank can resist the run without support and, thus, the chances of receiving support from the LLR. In this case, introducing a minimal regulatory liquidity standard can increase the overall

[5] We focus on short-term debt different from retail demand deposits that are typically protected with deposit insurance and, hence, more stable.

[6] In the extension in Section 7.1, we show that, for certain classes of banks, supporting the bank is overall efficient but cannot be made on an ex post break-even basis, so it must necessarily involve some degree of subsidization. In this sense, it might be argued that in the baseline case, our LLR consolidates two forms of government support: (unsubsidized) emergency lending and (subsidized) capital support. This does not undermine the interest of the analysis since most bank crises involve both types of support.

efficiency relative to the laissez faire benchmark.

Although some of the key trade-offs of our analysis could be illustrated in reduced form using a standard banking model with three dates, we formulate our model is in continuous time. This allows us to study explicitly the implications of liquidity buffers during the course of a run. During a run, the gradual withdrawal of funds by the holders of maturing debt reduces the bank's capacity to resist the run, while simultaneously the discovery of relevant information about the quality of its illiquid assets (modeled as a Poisson process) may arrive and either lead the run to self-resolve or render the LLR better informed when called to act. This modeling also allows us to formally examine the impact of liquidity on debtholders' incentive to rollover their debts or not after taking into account the subsequent unfolding of the run, as in the dynamic runs studied by He and Xiong (2012).

We investigate several extentions to our baseline model. In our main extension, we consider the interaction between liquidity standards and capital requirements. We endogenize the bank's capital structure as the result of its initial owners decision to contribute their wealth as equity financing, possibly subject to a regulatory capital requirement. To make this decision have an impact on the bank's fundamental solvency, we endogenize the probability that the bank's illiquid assets remain undamaged after the solvency shock by postulating that it gets affected by a costly hidden action of bank owners. This moral hazard problem adds a typical skin-in-the-game effect of bank capital on asset quality. Given the importance of the quality of the illiquid assets in the baseline model (including the determination of whether the bank would be supported or not by an uninformed LLR), the moral hazard problem renders a number of interesting interactions between the bank's capital and liquidity decisions, and regulations affecting each of them.

Our findings in this extension suggest that the informational role of liquidity standards highlighted in the baseline model can be modified but not replaced with the use of capital standards. In general capital requirements can at the margin either complement or partially substitute liquidity requirements, but, unless the capital requirement fully removes the risk of bank runs, it does not eliminate the role of liquidity holdings in allowing the LLR to "buy" the time needed to make better informed decisions in a run. This finding is critical because it shows that there is a unique role for liquidity regulation even in the presence of capital regulation.

In another extension to our baseline model, we show that requiring the LLR to lend on an expected break-even basis has important implications, but it does not eliminate the rationale for liquidity standards that we put forth. Yet in another extension, we show that allowing for temporary liquidity assistance does not necessarily improve upon the outcomes obtained when LLR support is treated as irreversible.

Until the Great Recession, there was no consensus among policy makers about the need for liquidity regulation. This was in contrast with an existing body of academic research that pointed to the existence of inefficiencies in worlds with a strictly private provision of liquidity, via either interbank markets (Bhattacharya and Gale, 1987) or credit line agreements (Holmström and Tirole, 1998). A common view was that liquidity regulation was costly for banks in spite of results pointing to its welfare enhancing effects, e.g. by reducing fire-sale effects in crises (Allen and Gale, 2004) or the risk of panics due to coordination failure (Rochet and Vives, 2004). Another view was that the effective action by the LLR rendered liquidity standards unnecessary.[7] There was also the view that, although the financial system was vulnerable to panics (Allen and Gale, 2000), the liquidation threat implied by short-term financing had positive incentive effects (Calomiris and Kahn, 1991; Chen and Hasan, 2006; Diamond and Rajan, 2005).

The severity of banks' liquidity problems during the recent crisis led to a consensus among policy makers about the need to introduce some form of liquidity regulation for banks.[8] Those problems also motivated new academic papers analyzing bank liquidity standards. Perotti and Suarez (2011), for example, rationalize liquidity regulation as a response to the existence of systemic externalities and analyze the relative advantages of price-based vs. quantity-based instruments. Calomiris, Heider, and Hoerova (2014) show that liquidity

[7] See Flannery (1996), Freixas, Parigi and Rochet (2000), and Rochet and Vives (2004).

[8] Banks' liquidity problems appear to have started in the summer of 2007 following the collapse of the asset-backed commercial paper (ABCP) market. These problems grew larger with the collapse or near collapse of several other markets, including the repo and the financial commercial paper markets, and even several segments of the interbank market, and with banks' shortages of collateral in part due to downward spirals in market and funding liquidity (Brunnermeier and Pedersen, 2009).

requirements may substitute for capital requirements in a moral hazard setup. Diamond and Kashyap (2016), in turn, show that liquidity holdings help deter runs that might otherwise occur as a probabilistic sunspot equilibrium. These studies, however, do not rationalize the time-dimension of Basel III LCR, and do not discuss the interaction between liquidity regulation and the provision of emergency liquidity by the LLR.

We contribute to close this gap in the literature with a theory that relies on a novel way of thinking about liquidity requirements – an instrument that, by making banks better able to withstand the initial phases of a crisis, allows the LLR to be better informed when it gets called into action.[9] Our paper is also related to the literature on the value of commitment to be tough in the context of lending of last resort or bank rescue policies (Mailath and Mester, 1994; Perotti and Suarez, 2002; Repullo, 2005; Acharya and Yorulmazer, 2007, 2008; Ratnovski, 2009; Farhi and Tirole, 2012; Chari and Kehoe, 2013), and to Acharya and Thakor (2016) who argue that the prospects of (unconditional) LLR support undermine investors' incentives to generate information about banks. We add to this literature by analyzing how liquidity requirements allow LLR support to be based on better information and, thus, less frequently unconditional, leaving a larger residual role for market discipline.

Prior studies on investors' incentives to run on banks include Diamond and Dybvig (1983), Jacklin and Bhattacharya (1988), Goldstein and Pauzner (2005), He and Xiong (2012), and He and Manela (2016). We share with He and Manela (2016) the modeling of a slow moving run that may be reversed by the arrival of good news, but instead of focusing on investors' incentives to acquire information along the run, we introduce a LLR and examine the impact of banks' cash holdings on the timing and efficiency of its support decisions.

Finally, there is a growing literature that, as in our main extention to the baseline model, jointly analyzes liquidity and capital regulation. This includes the contributions of Vives (2014), De Nicolo, Gamba and Luccheta (2014), Walther (2016), Goodhart et al. (2012), and Kashyap, Tsomocos and Vardulakis (2017). Vives (2014) analyzes how the regulation of

[9] Nosal and Ordoñez (2016) describe a setup in which a government delays intervention to learn more about the systemic dimension of a crisis. Their analysis focuses on the strategic interaction between banks, which can restrain from risk taking so as to avoid getting into trouble earlier than their peers, i.e. at a time in which the government is still not supporting banks in trouble.

capital and liquidity ratios affects the probabilities of insolvency and illiquidity in a model of bank runs and finds that, while both ratios affect bank fragility, they do not perfectly substitute each other, since the first is more effective in controlling the risk of insolvency and the second is more effective in controlling the risk of illiquidity.

De Nicolo, Gamba and Lucchetta (2014) develop a dynamic partial equilibrium model of a bank that undertakes maturity transformation under the protection of deposit insurance. While capital requirements help ameliorate excessive risk taking incentives due to leverage, liquidity requirements are found, in the absence of run risk, to be detrimental to welfare due to their negative impact on bank lending. Walther (2016) considers a setup in which liquidity regulation copes with a fire sale externality, while capital regulation copes with a social cost of bank default. Walther finds that, under some conditions, just one of the two regulations is sufficient to restore efficiency; otherwise, like in our extension, they are imperfect substitutes.

Goodhart et al. (2012) consider multiple regulations in a general equilibrium model with several imperfections, including fire sales induced by defaults on mortgages, while Kashyap, Tsomocos and Vardulakis (2017) analyze capital and liquidity regulations in a global game extension of Diamond and Dybvig (1983). Both contributions find that it is generally optimal to simultaneously rely on several regulatory tools, but neither of them consider the role of the LLR.

Our extension shares some features with these contributions. First, capital regulation primarily affects bank solvency, while liquidity regulation interferes with maturity transformation. Second, insolvency risk and maturity transformation may be excessive if a bank fails to internalize some of the negative effects of its fragility. In our strong bank case, those negative effects take the form of excessive public support during runs on banks with poor asset quality. Our contribution is unique in its emphasis on the informational value of liquidity requirements, which gets modified by capital requirements (via their impact on bank asset quality and the size of the refinancing needs during a run) but not fully replaced by them, unless the risk of a run is reduced to zero.

The rest of the paper is organized as follows. Section 2 introduces our dynamic model of runs. Section 3 analyzes several issues relevant for solving the model. Section 4 characterizes the *early run equilibrium*: the situation in which investors start canceling their debt immediately after the shock to the bank's financial condition. Section 5 considers social welfare and the rationale for liquidity standards. Section 6 presents the extension our baseline model to investigate the interaction between liquidity and capital standards. Section 7 discusses two other extensions. Section 8 concludes the paper. All proofs are in the Appendix. The Online Appendix analyzes the possibility of sustaining other types of equilibria in greater detail, expands the analysis on the determinants of optimal liquidity standards, and discusses some additional extensions to the baseline model.

2 The model

Consider a continuous time model of an individual bank in which time is indexed by $t \in \mathbb{R}$. There are three classes of agents: bank owners, investors, and a lender of last resort (LLR). All agents are risk neutral and discount future payoffs at a zero rate. Bank owners and investors care about the expected net present value of their own payoffs. The LLR is a benevolent maximizer of total expected net present value, with proper consideration of the cost of the subsidies embedded in its lending activity. The model puts particular focus on what happens to the bank after some shock arriving at $t = 0$ weakens its perceived solvency.

The bank exists from a foundation date, say $t = -1$. At that date, the bank's initial owners invest in assets of total size one, issue debt and equity among competitive investors, and, hence, appropriate as a surplus the difference between the value of the securities sold to the investors and the unit of funds needed to start up the bank.

2.1 *Assets and liabilities of the bank*

The assets of the bank consist of an amount C of a liquid asset (*cash*) and an amount $1 - C$ of *illiquid assets*. Illiquid assets pay some potentially risky per-unit final return equal to $\tilde{a}$ at termination and to $\tilde{q}$ in case of early liquidation. Early liquidation is feasible at any date prior to termination but cannot be partial.

The debt issued by the bank at $t = -1$ is uniformly distributed among a measure-one continuum of debtholders. Each debtholder is given the option to "put" her debt back to the bank in exchange for an early repayment of D at some exercise dates over the life of her contract. Alternatively, investors can obtain a late repayment $B = (1 + b)D$, with $b > 0$, at termination. Debt puttability is a convenient way to make investors face rollover decisions and banks face rollover risk similar to those that would emerge in a more complex environment with overlapping issues of short-term debt with fixed maturity.[10]

To facilitate tractability, we assume that both illiquid assets and the uncanceled debt of the bank mature at $T \to \infty$, which is a practical way to capture "the long run" in our model.[11] We also assume that debtholders' chances to put their debt arrive according to independent Poisson processes with intensity δ, so that $1/\delta$ can be interpreted as the average maturity of bank debt.[12] This parameter will determine the speed of the run and, therefore, the length of time the bank can survive the run with its available cash.

2.2 *Sequence of events after* $t = 0$

To focus the analysis of the model on the possibility of bank runs, and the way the bank and the LLR cope with them, we assume that the bank has a quiet life between dates $t = -1$ and $t = 0$. So the bank keeps the same debt and liquidity as initially chosen. At $t = 0$, there is a probability ε that the bank suffers a shock that impairs the quality of its illiquid assets, otherwise its life continues quiet forever.

The illiquid assets of the bank can be good (g) or bad (b). The final per-unit returns of good and bad assets are a_g and a_b, respectively, and their per-unit liquidation returns are $q_g < 1$ and $q_b < 1$, respectively. In the absence of the shock, assets are good with probability

[10] This debt is not intended to represent demand deposits, whose stability is guaranteed by deposit insurance, but the short-term wholesale debt at the root of liquidity problems during the 2007-2008 Global Financial Crisis.

[11] Equivalently, we can think of both illiquid assets and debt maturing randomly according to Poisson arrival processes with intensities going to zero (so that their expected life-spans go to infinity), which means that any other arrival process with positive intensity will arrive earlier on with probability one.

[12] One can think of the puttability of bank debt as a feature that under "normal circumstances" allows investors to cease their investment in the bank for idiosyncratic reasons (that the model abstracts from). In those circumstances, the bank would have no problem in simply replacing the exiting debtholders with new debtholders who would buy debt identical to the one canceled. See Segura and Suarez (2017) for a model with this type of recursive refinancing structure.

one. But when the shock hits, assets are good with probability μ and bad with probability $1-\mu$. We assume:

$$a_b < q_b \leq q_g < D < B < a_g, \tag{1}$$

so that the efficient continuation decision depends on the quality of the assets and gets compromised by the possibility of runs. Specifically, a good bank that invests only in risky assets ($C = 0$) is *fundamentally solvent* at termination ($a_g > B$), and its assets are worth more if continued than if early liquidated ($a_g > q_g$). In contrast, a bad bank that invests only in risky assets is *fundamentally insolvent* at termination (since $B > a_b$), and its assets are worth more if early liquidated ($q_b > a_b$). By continuity, these properties remain true when the bank holds liquidity ($C > 0$) insofar as C is not very large.

To formally guarantee this, we assume that C is always such that

$$a_g(1-C) + C \geq B, \tag{2}$$

so that the good bank is fundamentally solvent, but

$$q_g(1-C) + C < D, \tag{3}$$

so that it is vulnerable to runs. For further use, let $\bar{C}$ be the highest value in the interval $[0, 1]$ compatible with Eq. (2).

If the shock hits the bank at $t = 0$, debtholders' decisions regarding the exercise of their put options become non-trivial. If they start exercising their put options, the bank will begin consuming its cash holdings. Once the bank runs out of cash, the LLR decides whether to support it ($\xi = 1$) or not ($\xi = 0$). If the bank gets supported, all the residual debtholders are paid D.[13] Otherwise, the bank is forced into liquidation and its liquidation value gets proportionally divided among the residual debtholders.

When the bank is hit by the shock at $t = 0$, a process of potential revelation of the true quality of its illiquid assets starts. We assume that the arrival of public news revealing such

[13] In equilibrium, the LLR will only assist the bank when the quality of its assets remains unknown at the time of the intervention. To justify why debtholders recover just the early repayment D (rather than the termination payoff B), we can assume that market signals about the quality of illiquid assets become uninformative after the LLR intervenes and that debtholders, afraid of getting less than D at maturity, keep exercising their put options until they get rid of all their debt.

quality follows a Poisson process with intensity λ. The speed of arrival of information implied by λ and the speed of the run implied by δ determine the buying time effectiveness of cash holdings. As shown below, under our assumptions, learning that the illiquid assets are good at any time before the cash gets exhausted leads the crisis to self-resolve because the LLR is willing to support the bank if it runs out of cash (since $a_g > q_g$) and, in anticipation of this, debtholders no longer find optimal to redeem their debt. So the bank (efficiently) continues with its illiquid assets until termination. In contrast, when the news is bad, exercising their puts is a dominant strategy for debtholders. The bank eventually runs out of cash, the LLR does not support it (since $a_b < q_b$), and the illiquid assets end up (efficiently) liquidated.

The decision of the LLR is less trivial when the quality of the illiquid assets remains uncertain after the bank runs out of cash. In this scenario, the LLR has to decide by comparing the *expected* continuation value of the illiquid assets, $\bar{a} = \mu a_g + (1-\mu)a_b$, and their *expected* liquidation value, $\bar{q} = \mu q_g + (1-\mu)q_b$. Thus, the bank is supported if and only if

$$\mu > \bar{\mu} = \frac{q_b - a_b}{(a_g - q_g) + (q_b - a_b)}. \tag{4}$$

We will refer to the *strong bank* case and *weak bank* case depending on whether (4) holds or not. In both cases, the continuation vs. liquidation decision made by the LLR in the absence of news about asset quality is, with some probability, less efficient than the one attainable if the news had arrived on time.

In the baseline case analyzed below, emergency lending is assumed to be made at the zero risk-free rate. In Section 7.1 we extend the analysis to the case in which the LLR lends on an expected break-even basis. We show that in that case having $\mu > \bar{\mu}$ is not a sufficient condition to guarantee support, implying that some *modestly strong* banks (with $\mu \in (\bar{\mu}, \hat{\mu})$ for some $\hat{\mu} \in (\bar{\mu}, 1)$) would be inefficiently liquidated.

2.3 *Strategy for the analysis*

To simplify the exposition, the core of our analysis focuses on the case in which the realization of the shock at $t = 0$ gives rise to an *early run (ER) equilibrium*: the situation in which debtholders start exercising their puts from $t = 0$. After establishing conditions that guarantee the existence of this equilibrium, we will discuss the impact of the bank's liquidity C and the expected intervention of the LLR on equilibrium outcomes.[14]

We will then move backwards, to discuss the trade-offs regarding the choice of the liquidity holdings C at $t = -1$ from the perspective of both the LLR (ex ante social welfare) and the initial owners (ex ante total market value of the bank). The choice of C is observable at $t = -1$ prior to the issuance of the bank's debt, meaning that bank owners when deciding on C take into account the impact of this variable on the issuance value of debt and the residual value of equity.

To keep the analysis simple, we first treat the capital structure of the bank, as defined by D, b, and δ, as exogenously given. In Section 6 we assume that the initial owners have some wealth $w < 1$ they can use to provide (inside) equity funding $k \leq w$ to their bank, financing the rest of the initial assets, $1 - k > 0$, with the puttable debt described above. This allows us to endogenize D as a result of initial owners choice of k and to discuss the role of capital requirements.[15]

3 Solving the model

As the model will be solved by backward induction, it is convenient to start analyzing what happens when news reveal the type of the bank's illiquid assets during a run (i.e. prior to the exhaustion of the bank's cash); in the remaining terminal nodes, the situation is trivial and will be described in due course. It is also convenient to get familiar with the role of Poisson processes in helping us obtain expressions for the time span during which the bank can resist a run that starts at $t = 0$, and for the probability that news arrive prior to the point in which its cash gets exhausted.

[14] An alternative *late run equilibrium* configuration in which debtholders only start exercising their put options when further news confirm that the illiquid assets of the bank are bad is discussed in Section A of the Online Appendix. We show that, for some parameter values, liquidity standards may help sustain such equilibrium. Intuitively, liquidity standards reduce investors' incentives to run early by increasing their prospect of recovering value out of their debt claims when the bank's assets turn out to be damaged.

[15] Endogenizing b and δ would require attributing some value to the puttability of bank debt. The literature offers abundant rationales for each of these features, but capturing them here in a fully structural way would blur the essence of our contribution.

3.1 *News and ex post efficiency*

We want to show that the arrival of good news during a run stops the run, whereas the arrival of bad news implies that the bank gets liquidated once it fully consumes its cash. This implies that the arrival of news induces ex post efficient outcomes regarding the continuation vs. liquidation of the bank's illiquid assets.

Let the good news arrive at some date $t > 0$ when the residual fraction of bank debtholders is n_t and the available cash is $C_t = C - (1 - n_t)D \geq 0$ (which reflects that a fraction $1 - n_t$ of the initial debt has been canceled using cash). Then, if the run stops at t, the terminal value of assets is $a_g(1 - C) + [C - (1 - n_t)D]$, while the residual debt promises to pay n_tB at termination. Now, we can establish the following chain of inequalities:

$$a_g(1 - C) + [C - (1 - n_t)D] \geq B - (1 - n_t)D = n_tB + (1 - n_t)bD > n_tB, \tag{5}$$

where the first inequality follows from (2) (we are just subtracting the consumed cash from both sides of it) and the second inequality follows from having $b > 0$.

Eq. (5) means that, insofar as the bank can accommodate the run using its cash, the bank with good assets remains fundamentally solvent and a Nash equilibrium in which residual debtholders do not exercise their put options is sustainable after the good news. Specifically, waiting to be paid $B = (1+b)D$ at termination rather than recovering D prior to termination is a best response for any individual debtholder who expects no other debtholder to exercise her put.[16]

Upon the arrival of bad news, the situation is straightforward. The inequalities contained in Eq. (1) imply that the bank with bad assets is insolvent both if early liquidated and if continued, and irrespectively of the available cash or the fraction of residual debtholders. Moreover, all agents anticipate that the LLR will not support the bank. Debtholders with

[16] In the absence of a LLR, a second subgame perfect Nash equilibrium might also exist, based on the self-fulfilling prophecy that debtholders' run continues and the good bank is forced to liquidate its assets. This is because, as in e.g. Diamond and Dybvig (1983), liquidating the illiquid assets produces insolvency. However, in our setup the possibility of such an equilibrium is removed by the expectation that, if the occasion arrived, the LLR would support the bank whose assets are known to be good. Eventually, then, the run stops as soon as the good news arrive, the LLR intervention is unneeded on the equilibrium path, and the bank can preserve any cash available when the news arrive.

the opportunity to put their debt before the bank exhausts its cash find it optimal to recover D because, as shown in detail in Section 3.3, the payoff to residual debtholders at liquidation is lower than D. These results are summarized in the following proposition:

Proposition 1 *The arrival of good news during the early run stops the run, allowing the bank to continue up to termination. In contrast, the arrival of bad news does not stop the run and leads to the full liquidation of the bank once its cash gets exhausted.*

3.2 *How long will the bank resist a run?*

Suppose debtholders start exercising their puts immediately after the shock realizes at date 0 and assume that no good news arrive that interrupt the run. Let n_t denote as before the fraction of debtholders who have not exercised their put options by an arbitrary date $t \geq 0$. Since the opportunities to exercise the puts arrive among debtholders as independent Poisson processes with intensity δ, the dynamics of n_t is driven by

$$\dot{n}_t = -\delta n_t, \tag{6}$$

with the initial condition $n_0 = 1$. Integrating in Eq. (6) implies $n_t = \exp(-\delta t)$. So the bank will exhaust its cash at the date τ such that $(1 - n_\tau)D = C$, that is, when

$$[1 - \exp(-\delta\tau)]D = C. \tag{7}$$

Solving for τ yields the following result:

Proposition 2 *Once a run starts, the bank can resist it without assistance for a maximum time span of length*

$$\tau \equiv -\frac{1}{\delta}\ln(\frac{D-C}{D}), \tag{8}$$

which is greater than zero for $C > 0$, increasing in C, and decreasing in δ and D.

3.3 *How much is recovered when the bank gets liquidated?*

The bank is liquidated when its cash gets exhausted and the LLR does not support it ($\xi = 0$). At liquidation, the value of bank assets is $q_i(1 - C)$, where $i = g, b$ denotes their quality, and the fraction of residual debtholders is $n_\tau = \exp(-\delta\tau) = (D - C)/D$ as explained above. So the amount recovered by each residual debtholder, conditional on asset quality i, can be written as

$$Q_i = \frac{q_i (1 - C)}{n_\tau} = \frac{q_i (1 - C)}{D - C} D < D, \tag{9}$$

where the last inequality follows from Eq. (1) and (3).

Thus the payoff received by the fraction $1 - n_\tau = 1 - \exp(-\delta\tau) = C/D$ of debtholders who manage to recover D prior to liquidation is strictly larger than the payoffs of those trapped at the bank when liquidated. This explains why the former will prefer to exercise their put options whenever the probability that the bank will be liquidated is sufficiently high.

In the context of a run, whether debtholders manage to get paid D or Q_i is just a matter of luck. Then, from the perspective of the date at which the run starts, the expected payoffs accruing to debtholders, conditional on the quality of the illiquid assets being i and the bank being liquidated, can be computed as a weighted average of each of the outcomes:

$$[1 - \exp(-\delta\tau)]D + \exp(-\delta\tau)Q_i = C + q_i (1 - C), \tag{10}$$

which, quite intuitively, equals the total value of bank's assets conditional on liquidation.[17]

4 The early run equilibrium

We define the *early run equilibrium* as the subgame perfect Nash equilibrium of the game that starts after the bank gets hit by a shock at $t = 0$ in which, unless and until good news stop the run, all debtholders exercise their put options as soon as they have the opportunity to do so. In this equilibrium, the logic pushing debtholders to take D whenever possible is that D is higher than the expected value of waiting for the next occasion, if any, to get back D, for the end of the run or for the liquidation of the bank, whichever comes first.

Let $V_t^{ER}(C)$ denote a residual debtholder's value of not exercising the put option at date $t \in [0, \tau]$ when the bank's *initial* cash holding is C, when no news have yet revealed the

[17] Eq. (10) obtains directly from Eq. (7) and (9).

quality of the illiquid assets and when, in all subsequent opportunities, residual debtholders are assumed to exercise their puts unless good news stop the run. Having $V_t^{ER}(C) \leq D$ for all $t \in [0, \tau]$ means that recovering D, if having the occasion to do so, is a debtholder's best response to the strategies followed by the subsequent players in the game (debtholders who have not yet canceled their debt and the LLR if called upon to act). Thus,

Proposition 3 *An early run equilibrium is sustainable if and only if a residual debtholder's value of not putting her debt at some date t during an early run satisfies $V_t^{ER}(C) \leq D$ for all $t \in [0, \tau]$.*

As shown in detail in the proof of the following proposition, to find out the expression for $V_t^{ER}(C)$, it is convenient to think of it as the weighted average, using weights μ and $1-\mu$, of the expected payoffs that a debtholder not exercising her put option at date t would obtain conditional on the illiquid assets of the bank being good and bad, respectively. The result is the following:

Proposition 4 *A residual debtholder's value of not putting her debt at some date $t \in [0, \tau]$ during an early run can be written as follows*

$$\begin{aligned} V_t^{ER}(C) &= D + \mu[1 - \exp(-(\delta+\lambda)(\tau - t))]\frac{\lambda}{\delta+\lambda}(B - D) \\ &\quad - \exp(\delta t)\{\mu \exp(-\lambda(\tau - t))[D - C - q_g(1 - C)] + (1-\mu)[D - C - q_b(1 - C)]\} \\ &\quad + \xi \exp(\delta t)\exp(-\lambda(\tau - t))[D - C - \bar{q}(1 - C)]. \end{aligned} \tag{11}$$

Eq. (11) reflects that the holder of one unit of debt during an early run does not always recover D. Specifically, its second term says that if the assets are good and the news come on time, the debtholder recovers B instead of D. The third term says that, if the debtholder gets trapped at the bank and the illiquid assets end up liquidated, her payment is lower than D. The sub-term multiplied by μ reflects that, if the illiquid assets are good, liquidation only happens if no news arrive prior to date τ (and no LLR support is received at τ). The sub-term multiplied by $1 - \mu$ reflects that, in contrast, a bad bank not supported by the LLR will get liquidated irrespectively of the possible arrival of news prior to date τ. Finally, the last term in Eq. (11) captures the gains, relative to the liquidation payoffs that we have just described, associated with receiving LLR support ($\xi = 1$) at date τ.

The reasoning that may lead to having $V_t^{ER}(C) \leq D$ is a combination of what explains why a debtholder might find it profitable to recover D even if no other debtholders were trying to subsequently recover D (a *fundamental run*), the logic of a *dynamic run* a la He and Xiong (2012) (where each debtholder's incentive to run is reinforced by the fear that, if subsequent debtholders are also early runners, the bank will be consuming its cash and the chances to recover D at a later date will be declining), and distortions to that logic that come from the potential support received from the LLR.

In the weak bank case ($\mu \leq \bar{\mu}$), debtholders anticipate that the uninformed LLR will not support the bank ($\xi = 0$) and the He and Xiong (2012) effect unambiguously reinforces debtholders' incentives to run. It is easy to check that, in this case, $V_t^{ER}(C)$ is decreasing in t. So having $V_t^{ER}(C) \leq D$ for all $t \in [0, \tau]$ only requires having $V_0^{ER}(C) \leq D$.

However, in the strong bank case ($\mu > \bar{\mu}$), the expectation of support from the uninformed LLR ($\xi = 1$) creates a countervailing effect: the bank is more likely to be supported the closer the bank is to exhaust its cash (since this makes less likely the potential revelation that its assets are bad). Due to the third term in Eq. (11), the expectation of being supported increases as time passes, making $V_t^{ER}(C)$ increasing when t approaches τ. For simplicity we will focus the core of our analysis on parameter configurations for which having $\mu > \bar{\mu}$ is compatible with having $V_t^{ER}(C) \leq D$ for all $t \in [0, \tau]$, so that the ER equilibrium exists. Alternative configurations of equilibrium are discussed in Section A of the Online Appendix.

5 Welfare and optimal liquidity holdings

Assessing ex ante welfare in the early run equilibrium, $W_{-1}^{ER}(C)$, is equivalent to properly accounting for the returns that the bank's initial assets produce over the various future paths that the bank can follow. Building on the analysis that led us to obtain an expression for $V_t^{ER}(C)$ in Proposition 4, we obtain the following result:

Proposition 5 *The ex ante welfare associated with the early run equilibrium is*

$$
\begin{aligned}
W_{-1}^{ER}(C) \quad = \quad & C + \{[1 - \varepsilon(1-\mu)]a_g + \varepsilon(1-\mu)q_b\}(1-C) \\
& - \varepsilon \exp(-\lambda\tau)[\mu(1-\xi)(a_g - q_g) + (1-\mu)\xi(q_b - a_b)](1-C), \qquad (12)
\end{aligned}
$$

where $\exp(-\lambda\tau) = [(D-C)/D]^{\lambda/\delta}$ *by (8).*

The first two terms in Eq. (12) represent the returns that the bank generates in a full information scenario in which its illiquid assets are continued or liquidated according to the ex post most efficient rule (that is, depending on whether they are good or bad, respectively). The third term represents the deadweight losses due to the uninformed nature of the decision made by the LLR when the bank exhausts its cash at date τ and no news on the quality of the illiquid assets has been received. In our model, consistent with Bagehot's doctrine, LLR support ($\xi = 1$) is welfare enhancing if the illiquid assets are good and welfare reducing if they are bad. But, in the absence of news about asset quality by date τ, the LLR decision involves either type I error (good assets are liquidated) or type II error (bad assets are not liquidated). As reflected in Eq. (12), type I error occurs, with a cost proportional to $a_g - q_g > 0$, in the weak bank case ($\xi = 0$), while type II error occurs, with cost proportional to $q_b - a_b > 0$, in the strong bank case ($\xi = 1$).

Is there a social value to postponing the LLR support decision? The quick answer is yes. To see this, consider a notional ceteris paribus increase in τ. Such change would reduce the absolute size of the third term of $W_{-1}^{ER}(C)$ (which is negative) and, thus, be good for welfare. Intuitively, it would increase the probability that news arrive prior to date τ and reduce the type I or II errors potentially associated with the otherwise uninformed decision of the LLR. The right answer, however, requires an important qualification: In our setup, τ can only be increased by increasing C, which implies forgoing part of the bank's investment in illiquid assets, which is its only potential source of strictly positive net present value.[18]

[18] Mathematically, τ could also be reduced, without affecting other terms in (12), by reducing D or δ, which we are treating as exogenously fixed parameters for the time being. Setting $C = D = 0$ or $C = \delta = 0$ in the current model would trivially maximize $W_{-1}^{ER}(C)$ but this is because, to keep things simple, we are not explicitly modeling the gains from financing the bank with debt or from making bank debt redeemable.

To formally analyze the dependence of $W_{-1}^{ER}(C)$ with respect to C, it is convenient to rewrite it as

$$W_{-1}^{ER}(C) = C + A_H(1-C) - A_L\left(\frac{D-C}{D}\right)^{\lambda/\delta}(1-C) \tag{13}$$

where $A_H = [1-\varepsilon(1-\mu)]a_g + \varepsilon(1-\mu)q_b$, $A_L = \varepsilon[\mu(1-\xi)(a_g - q_g) + (1-\mu)\xi(q_b - a_b)]$, and $((D-C)/D)^{\lambda/\delta}$ replaces $\exp(-\lambda\tau)$. Intuitively, A_H can be interpreted as the *fundamental per-unit value* of illiquid assets at $t=-1$ (the gross expected return that they would generate under efficient full-information decisions on continuation vs. liquidation), which must be greater than one for the investment in the bank to be a source of social surplus.[19] A_L are the per-unit differential losses on illiquid assets incurred due to the uninformed decision of the LLR, if it happens.

We can prove the following result:

Proposition 6 *The ex ante welfare associated with the early run equilibrium,* $W_{-1}^{ER}(C)$*, is a strictly concave function of* C*, which, depending on parameters, may be increasing or decreasing at* $C=0$*. If it is decreasing at* $C=0$*,* $W_{-1}^{ER}(C)$ *is maximized at* $C^*=0$*. If it is strictly increasing at* $C=0$*,* $W_{-1}^{ER}(C)$ *reaches a maximum over the interval* $[0,\bar{C}]$ *at some unique* $C^*>0$*.*

As shown in the proof of the proposition, having strictly positive optimal cash holdings, $C^*>0$, requires the net present value of the assets of the bank under the liquidation policy induced with $C=0$, which is $A_H - A_L - 1$, to be small relative to the losses, A_L, that can be avoided by having enough time to obtain the relevant information during a run. It also requires that the effectiveness of cash holdings as a means for gaining the relevant information (which at $C=0$ is directly proportional to $\lambda/(\delta D)$) is large enough. Quite intuitively this says that, ceteris paribus, liquidity holdings make more sense in situations in which the rate of arrival of information during a run is high relative to the rate at which debt gets canceled.

[19] Otherwise, $W_{-1}^{ER}(C)$ would be trivially maximized at $C=1$, where $W_{-1}^{ER}(1)=1$.

5.1 *Total market value and the need for liquidity standards*

Before we present numerical examples showing that the welfare-maximizing liquidity holdings C^* can be strictly positive, it is worth clarifying the relation between ex ante welfare $W_{-1}^{ER}(C)$ and the ex ante total market value of the bank, $TMV_{-1}^{ER}(C)$, which would be the driver of the decision on C of the bank's initial owners in the absence of regulation.

At $t = -1$ the initial owners place the bank's debt and equity among investors and start up the bank, thus appropriating the difference between the market value of the securities sold to investors or retained as their own investment, $TMV_{-1}^{ER}(C)$, and the required unit of investment as a profit. Both debt and equity are assumed to be competitively priced by the risk neutral investors under each choice of C by the bank, the (given) values of the parameters D, b, and δ that describe the puttable debt contract, and the anticipated course of events in subsequent stages of the game. Equityholders anticipate that they will receive the part of the total expected cash flows generated by bank assets which are not owed to the debtholders or, if applicable, to the LLR.

In the weak bank case ($\xi = 0$), the LLR never intervenes on the equilibrium path and, thus, $TMV_{-1}^{ER}(C)$ is made of the expected value of exactly the same cash flows taken into account when computing $W_{-1}^{ER}(C)$; the capital structure simply divides such value among security holders. Therefore, we have $TMV_{-1}^{ER}(C) = W_{-1}^{ER}(C)$, which means that the initial owners fully internalize the net social gains associated with their choice of C. So in the weak bank case there is no obvious rationale for imposing C^* by means of regulation.[20]

In the strong bank case (where $\xi = 1$), things are different because, when the early run takes place and no news arrive prior to date τ, the LLR intervenes, providing a net value transfer of $(D - C) - a_b(1 - C) > 0$ to investors if the illiquid assets of the bank are bad.[21] This value transfer is appropriated by debtholders, who incorporate it in the valuation of

[20] Of course we could always argue, as in Diamond and Kashyap (2016), that debtholders might have difficulties in directly assessing the bank's liquidity position, justifying a monitoring role for the supervisor. This is because shareholders might be tempted to opportunistically distribute C as a dividend at some point after $t = -1$.

[21] Specifically, the LLR advances $D - C$ at the zero risk-free rate at $t = \tau$ and only recovers $a_b(1 - C)$ at termination. In contrast, if the assets are good, having $a_g(1 - C) > D - C$, by (2) and $D < B$, guarantees the full repayment of the emergency lending.

the debt at $t = -1$ and, hence, in $TMV_{-1}^{ER}(C)$.

Encompassing the two cases, the total market value of the bank can be written as

$$TMV_{-1}^{ER}(C) = W_{-1}^{ER}(C) + \xi\varepsilon \exp(-\lambda\tau)(1-\mu)[(D-C) - a_b(1-C)], \tag{14}$$

where the term multiplied by ξ contains the expected net subsidy received by a strong bank. This term is decreasing in C, both because cash prolongs the time τ over which the strong bank can resist a run (increasing the likelihood that LLR does not have to intervene) and because the value transfer received in case of intervention is decreasing in C (since $a_b < q_b < 1$). Hence, the marginal value of liquidity holdings is lower for the owners of the strong bank than for an ex ante social welfare maximizer.

In fact, as shown in the proof of the following proposition, when $\xi = 1$, the decline with C of the value transfer term related to LLR support exceeds the increase with C of the information gains included in $W_{-1}^{ER}(C)$ (the last term in Eq. (13)). This implies that, in the strong bank case, $TMV_{-1}^{ER}(C)$ is strictly decreasing in C.

Proposition 7 *The ex ante total market value of the bank associated with the early run equilibrium, $TMV_{-1}^{ER}(C)$, coincides with $W_{-1}^{ER}(C)$ in the weak bank case ($\xi = 0$), while it is strictly larger than $W_{-1}^{ER}(C)$ and strictly decreasing in C in the strong bank case ($\xi = 1$).*

The fact that $TMV_{-1}^{ER}(C)$ is strictly decreasing in C when $\xi = 1$ has the important implication that if $C^* > 0$, it will be socially optimal to impose a regulatory liquidity requirement of the form $C \geq C^*$, which will be binding in equilibrium. Intuitively, the initial owners of a strong bank anticipate that LLR support will be granted if the bank exhausts its cash prior to the revelation of the quality of its illiquid assets. And they foresee that the payoffs to security holders in such a situation are better than in the alternative situation in which the quality of the illiquid assets is discovered on time, so they choose the lowest possible liquidity.

The total market value of the bank at $t = -1$ can be broken down into the issuance value of debt and the issuance value of equity. In particular, by first principles, the value of debt at $t = -1$ can be written as

$$V_{-1}^{ER}(C) = (1-\varepsilon)(1+b)D + \varepsilon V_0^{ER}(C), \tag{15}$$

which uses the fact that debtholders get the full repayment $B = (1 + b)D$ at termination if the bank is not hit by a shock at $t = 0$, and they obtain expected payments equal to $V_0^{ER}(C)$ otherwise. From here, the value of equity at $t = -1$, $E_{-1}^{ER}(C)$, can be found as the residual:

$$E_{-1}^{ER}(C) = TMV_{-1}^{ER}(C) - V_{-1}^{ER}(C). \tag{16}$$

5.2 *Numerical examples*

In this subsection we introduce some parameterizations under which the socially optimal liquidity holdings C^* are interior. Such parameterizations appear in Table 1 together with the implied C^*. We will use variations of these examples when we endogenize the bank's capital structure below and also in the Online Appendix (when we discuss the possible existence of equilibria other than the early run equilibrium and when we analyze numerically the determinants of C^*). For concreteness, one can take one month as the relevant unit of time and interpret the parameters and results accordingly, but the values of the parameters are purely illustrative, so the results only tell about qualitative properties of the model.

Table 1

Numerical examples. This table reports the values of the model parameters in the baseline "strong bank" and "weak bank" examples discussed in the main text and the Online Appendix. C^* is the socially optimal value of the liquidity holdings in each example (interpretable as a fraction of the bank's total initial assets, which have been normalized to one).

	ε	μ	a_g	a_b	q_g	q_b	λ	δ	b	D	C^*
Strong bank	0.2	0.50	1.2	0.5	0.80	0.70	2.5	0.167	0.01	1	0.0492
Weak bank	0.2	0.25	1.2	0.5	0.80	0.70	2.5	0.167	0.01	1	0.0608

The above parameters imply $\bar{\mu} = 1/3$ and, hence, allow to obtain a strong bank example or a weak bank example by merely fixing the value of μ above or below that threshold. For instance, with $\mu = 0.5$, the bank is strong and its socially optimal liquidity holdings C^* are roughly 5% of total assets (which would allow the bank to resist a run for), although its owners would choose $C = 0$ unless liquidity standards force them to do otherwise. Instead, with $\mu = 0.25$ the bank is weak and it is in the best interest of its owners to choose the socially optimal liquidity holdings C^*, which in this case are about 6% of total assets, without the need for regulation. The numerical comparative statics of C^* obtained around these examples are reported in Section B of the Online Appendix.[22]

6 Adding capital

Up to now we assumed the bank's capital structure was exogenously fixed at $t = -1$. This simplified the analysis, but it precluded us from considering the role of capital regulation. In this section, we modify our model to investigate the interplay between the capital and liquidity regulations. To that end, we assume that the owners of the bank have a fixed endowment of w at $t = -1$, out of which they decide to contribute $k \leq w$ to finance the bank, so that $1 - k$ needs to be funded externally. External funding is in the form of debt as the one captured in the baseline model. Regarding such debt, we endogenize D but keep treating b and δ as exogenous parameters.

As in, e.g., Holmström and Tirole (1997), we link the bank's solvency to its capital structure through a moral hazard problem affecting some costly managerial actions that determine the probability μ with which the illiquid assets remain good when the bank is hit by the solvency shock at $t = 0$. Specifically, we assume that bank owners decide the unobservable value of μ at $t = -1$, right after the bank has decided on how much to invest in cash C and on how much of owners' wealth they contribute as equity funding k. Thus, when bank owners decide on μ, the debt D needed to raise $1 - k$ externally has already been issued.

The choice of μ implies a private non-pecuniary cost to bank owners $\Psi(\mu)$, with $\Psi' > 0$ and $\Psi'' > 0$. For concreteness, the following functional form is explored in the numerical examples:

$$\Psi(\mu) = \psi \left(\frac{\mu}{1-\mu} \right)^2, \tag{17}$$

with $\psi > 0$, which guarantees solutions for μ in the interior of the $[0, 1]$ interval. Taking this cost into account requires modifying the expression for the social and private value of the

[22] As discussed in the Online Appendix, for some of the parameters, the sign of such dependence can be established analytically, but for other there are interesting non-monotonicities (for instance, C^* is first increasing and then decreasing in μ, δ, and λ), which are showed and explained there.

bank. This involves subtracting $\Psi(\mu)$ from W_{-1}^{ER} in Eq. (12) and (13), so that

$$W_{-1}^{ER}\mid_{new}= W_{-1}^{ER}\mid_{old} -\Psi(\mu); \tag{18}$$

with this modification, the expressions for TMV_{-1}^{ER} in Eq. (14) and E_{-1}^{ER} in Eq. (16), already based on W_{-1}^{ER}, would still be valid.[23]

In this extended version of the model, any choice (C, k) of liquidity and capital at $t = -1$ (either by the bank owners or by the regulator) induces a sequential game with three subsequent moves: (i) bank owners set optimally the value of D (and thus $B = (1+b)D$) required to attract funds $1-k$ from debtholders, (ii) bank owners choose (unobservable) μ, and (iii) if relevant, the LLR decides whether to support or not the bank ξ. Let $(D(C, k), \mu^*(C, k), \xi(C, k))$ denote the subgame perfect Nash equilibrium (SPNE) of this game. Such equilibrium can be found as the tuple compatible with the conditions reflecting the relevant players' best response in each stage. In stage (i) bank owners will choose the minimal D that satisfies

$$V_{-1}^{ER} = 1 - k, \tag{19}$$

under the anticipated subsequent choices of μ^* and ξ. In stage (ii) bank owners will set

$$\mu^* = \arg\max_{\mu} E_{-1}^{ER}, \tag{20}$$

where D and the LLR's subsequent choice of ξ are taken as given.[24] In stage (iii), the LLR will choose $\xi = 1$ if and only if it believes the bank to be strong, that is, if $\mu^* > \bar{\mu}$.

Our main interest is to analyze situations where such SPNE involves strong banks ($\xi = 1$). In practical terms, we can fix $\xi = 1$, solve for the candidate equilibrium in (D, μ^*), and check that $\mu^* > \bar{\mu}$. Likewise, to explore a SPNE with weak banks, we can fix $\xi = 0$, solve for the candidate equilibrium in (D, μ^*), and check that $\mu^* \leq \bar{\mu}$.

The final objective of the discussion is to clarify which value of (C, k) would maximize the social ex ante value of the bank, W_{-1}^{ER}, or the private ex ante value of the bank to its

[23] As in the analysis of the baseline model, we focus the discussion on the situation in which, when the bank suffers a solvency shock, an early run starts. So in all of the examples used in the discussion below we will assume and verify that the condition established in Proposition 3 ($V_t^{ER} \leq D$ for all $t \in [0, \tau]$) holds for the whole range of values of C and k explored in the analysis.

[24] Notice that the unobservability of μ to the LLR effectively makes (ii) and (iii) part of a simultaneous-move subgame between the bank and the LLR.

owners, TMV_{-1}^{ER}. Comparing the socially and privately optimal choices of (C,k) will clarify the need or not for liquidity and capital regulation.

Importantly, we will assume that regulation does not alter the emergence of an ER equilibrium if the solvency shock realizes at $t=0$. It is immediate to see that for sufficiently large values of k, the combination of low leverage (low D) and a high probability of preserving good asset quality after the shock (high μ) would make the bank "supersolvent" (in the language of Rochet and Vives 2004) so the ER equilibrium would no longer be sustained. In that case, debtholders would keep rolling over their debt at least until the news on asset quality clarify whether the bank is solvent or not. Hence, for highly capitalized banks, the risk of an ER disappears and the informational role of liquidity standards that we emphasize no longer exists.[25]

6.1 *Strong bank results*

Fig. 1 was produced with the parameters in Table 1, excluding μ and D which are now endogenous, and setting the parameter of the private cost of improving asset quality as $\psi = 0.005$.[26] Under this parameterization, the bank is strong over the whole depicted range of values of (C,k). As reflected in the μ panel of the figure, bank owners' choose values of μ above $\bar{\mu}$, which is the level indicated in the 3D graph by the rectangle depicted at the bottom. Quite intuitively, μ is increasing in k as a larger share of owners' financing reduces the bank's leverage and increases insiders' incentives to guarantee that bank assets are resilient to the solvency shock—a standard skin-in-the-game effect.

In contrast, μ is slightly decreasing in C due to the combined impact of two forces that, in this case, push in the same direction. The first and more straightforward is that C reduces the share of illiquid assets held by the bank, which other things equal diminishes owners' incentives to invest in increasing μ. The second force operates through the cost of debt financing. As shown in the $D/(1-k)$ panel, increasing the liquidity holdings C increases

[25] In our formulation k is limited by the initial endowment of bank owners w, so we can defend that the ER equilibrium arises if such endowment is small. This can be thought of as a reduced form for deeper informational or agency frictions limiting the bank's capacity to raise (inside) equity financing.

[26] This choice of ψ makes the endogenous values of μ and D under $(C,k)=(0,0)$ approximately equal to those used in Table 1 for the strong bank baseline example.

quite significantly the promised repayments D needed to raise any given amount of funding $1-k$ at $t=-1$. Debtholders anticipate that a strong bank endowed with more liquidity gives more time to the LLR to discover, in the event of a run, the quality of its illiquid assets. This increases the chances that the bank ends up resolved (rather than blindly supported), in which case they will experience losses. The higher cost of debt acts as a reverse skin-in-the-game effect on bank owners' incentives regarding μ.

Since μ is below its first best level in this extension, the overall negative effect of C on μ identifies a novel second best cost of liquidity standards, one that further deteriorates the bank's fundamental solvency and reinforces the costs behind the existence of a limited socially optimal level of C in the baseline model. Indeed, as reflected in the W_{-1}^{ER} panel, the social surplus generated by the bank is unambiguously increasing in k—the maximum inside-equity-participation principle common to moral hazard models with risk neutrality applies here—but its relation with C has an inverted U-shape.

By arguments already exposed when bankers' only choice was C, the strong bank needs both liquidity and solvency regulation. As reflected in the TMV_{-1}^{ER} panel, the owners of a strong bank would maximize their wealth at $t=-1$ by choosing $(C,k)=(0,0)$ even if their wealth $w>0$ would allow them to provide some equity financing. Intuitively, TMV_{-1}^{ER} is decreasing in both C and k because of the distortions associated with the prospect of benefiting from blind LLR support in a run.[27]

As in the baseline model, lowering C increases the chances of such support. Since LLR support makes debt financing effectively subsidized, bank owners in this example prefer debt financing to equity financing in spite of the advantages of the latter with regards to the moral hazard problem that affects their choice of μ. Regulation in this framework should therefore force bank owners to contribute their wealth to the financing of the bank by setting a minimum capital standard of $k=w$ and accompany it with a liquidity standard that fixes the liquidity holdings which are optimal for that level of k.

[27] In the $D-V_0^{ER}$ panel, we verify that for the whole range of depicted values of (C,k) the necessary condition for an early run, $D>V_0^{ER}$, holds. We have also numerically checked the corresponding condition for all $t\in(0,\tau)$.

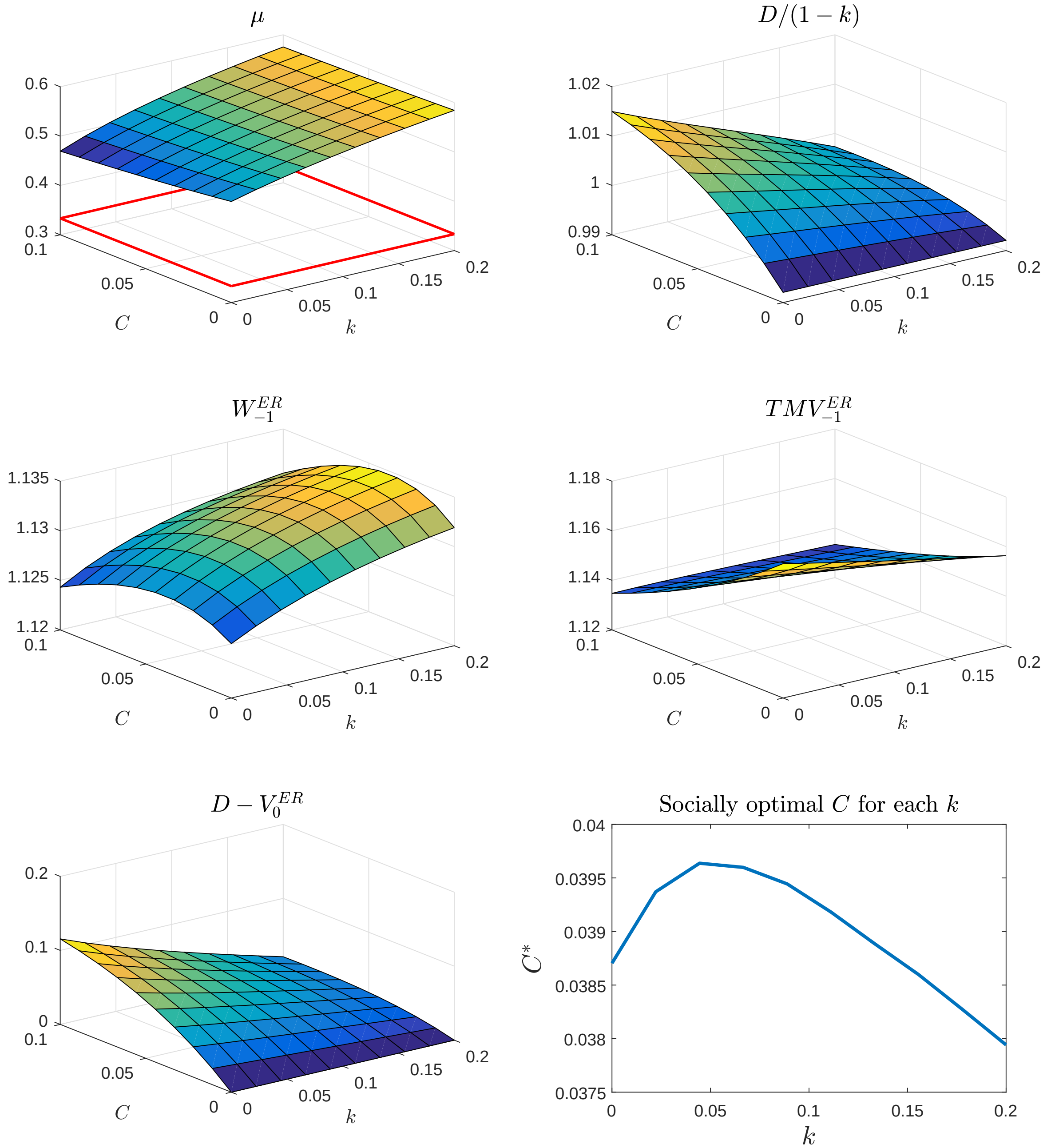

Fig. 1. Strong bank example of the extended model. Based on the parameterization explained in the text, which makes the bank strong, the tridimensional panels in this figure represent each of the model variables indicated at the top of each panel as a function of the bank's liquidity holdings C and capital ratio k. The bidimesional panel depicts the liquidity holdings C^* that maximize the social surplus generated by the bank, W^{ER}, for each given value of the capital ratio k.

Those liquidity standards are shown in the $C(k)$ panel of Fig. 1, which is an immediate by-product of the W_{-1}^{ER} panel. In this example, the optimal liquidity standards are first increasing and then decreasing in k, so capital standards work at the margin as first a complement and then a substitute for liquidity standards. This non-monotonic relation is the result of the combination of several forces that operate in opposite directions. One immediate implication of increasing k is to reduce leverage (D), which reduces the intensity of the cash outflows during the run and, hence, increases the time over which any given amount of C allows the bank to resist the run. This increases the effectiveness of C as an "information buying" device but also the need for it, producing per se what might be a non-monotonic effect on $C(k)$. Additionally, rising k reduces the moral hazard problem with respect to μ. This makes the strong bank stronger, which reduces the probability of making a type II error when supporting the bank without knowing the quality of its illiquid assets. This in turn diminishes the informational value of C and contributes to the decline of $C(k)$ for higher values of k. Notice, however, that in the decreasing section of $C(k)$ capital standards are a rather imperfect substitute for liquidity standards: in this example, increasing k from 5% to 20% only reduces the optimal liquidity holdings $C(k)$ by about 15 basis points.

6.2 *Weak bank results*

Fig. 2 was also produced with the parameters in Table 1, again excluding the now endogenous μ and D, and setting the parameter of the private cost of improving asset quality as $\psi = 0.035$ (seven times bigger than in the strong bank example above). Under this parameterization, the bank is weak over the whole depicted range of values of (C, k). As shown in the μ panel, bank owners' choose values of μ below $\bar{\mu}$, which is the level indicated by the rectangle depicted at the top of the 3D graph.

As in the strong bank case, μ is increasing in k due to the standard skin-in-the-game effect. Opposite to the strong bank case, however, μ is increasing in C over the depicted range. While it is still the case that a larger C reduces the share of assets affected by the choice of μ, there are now two other effects operating in the direction of increasing μ. First, since the uninformed LLR does not support a weak bank, increasing C increases the chances that the illiquid assets are discovered to be good during the run (in which case debtholders end up repaid in full and equityholders obtain a strictly positive terminal payoffs) and, with it, bank owners' incentives to increase the likelihood that assets remain good in a crisis.[28] Second, the burden of debt repayments, as reflected in the $D/(1-k)$ panel is now slightly decreasing in C, since debtholders discount the larger probability of being fully repaid and, even if not, recovering a higher liquidation value on the bank's illiquid assets.

Explaining the remaining panels of Fig. 2 is easier than for Fig. 1, since in the case of the weak bank, as in our baseline model, the absence of distortions associated with the prospect of subsidized LLR support makes bank owners' objective function, TMV_{-1}^{ER}, equivalent to the full social surplus generated by the bank, W_{-1}^{ER}, rendering liquidity and capital regulation unnecessary.[29] As reflected in the corresponding panels, both measures are increasing in k –the maximum inside-equity-participation principle applies– and are related to C in an inverted U-shape manner.

In this setup, bank owners (or a social planner) would set $k = w$ and accompany this capital ratio with the associated socially and privately optimal liquidity holdings $C(k)$ which appear depicted in the corresponding panel of Fig. 2. In this weak bank example, $C(k)$ has qualitatively the same hump shape as in the strong bank example but exhibits complementarity between k and C over a wider range of values of k. This is because, opposite to the strong bank case, the increase in μ which follows from the reduction of the moral hazard problem makes the weak bank stronger and, hence, increases the type I error potentially incurred when denying support to a bank whose asset quality remains unknown when the LLR is forced to decide. This makes the informational value of C increase with k. Eventually, however, the leverage reduction effect dominates and $C(k)$ decreases with k.

[28] In fact, with $C = 0$ bank owners choose $\mu = 0$, because, without time for asset quality to be revealed, the weak bank gets immediately resolved and they obtain no payoff irrespective of their prior choice of μ.

[29] The same caveat as in Footnote 20 applies here and could be extended to the need to verify that the bank's leverage remains at its initially set level.

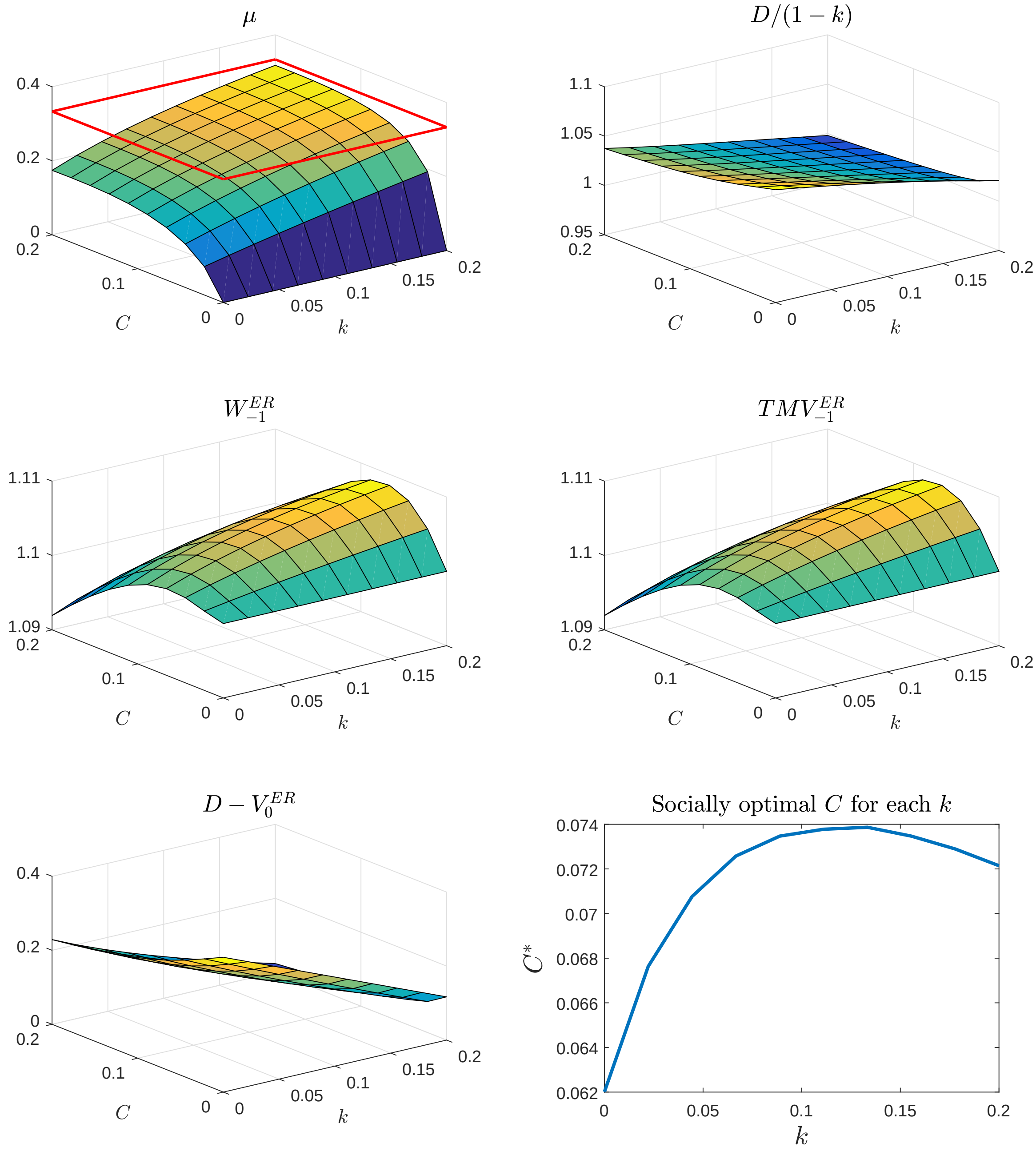

Fig. 2. Weak bank example of the extended model. Based on the parameterization explained in the text, which makes the bank weak, the tridimensional panels in this figure represent each of the model variables indicated at the top of each panel as a function of the bank's liquidity holdings C and capital ratio k. The bidimesional panel depicts the liquidity holdings C^* that maximize the social surplus generated by the bank, W^{ER}, for each given value of the capital ratio k.

7 Further extensions

In this section, we present the results of two additional extensions to our baseline model (with exogenous D and μ). Section 7.1 discusses the implications of forcing the LLR to lend on an expected break-even basis. Section 7.2, in turn, considers the possibility of using temporary LLR support instead of liquidity holdings for the purpose of buying time for the arrival of information.[30]

7.1 *Fair pricing of LLR support*

What are the implications of forcing the LLR to lend on an expected break-even basis, i.e. at terms that imply no subsidization of supported banks? In the baseline model, banks that exhaust their cash prior to the discovery of the quality of their illiquid assets get supported ($\xi = 1$) if and only if, conditional on the probability μ that the assets are good, their continuation yields higher overall expected value than liquidation ($\bar{a} > \bar{q}$). This means that banks are only supported in the strong bank case, $\mu > \bar{\mu}$. Importantly, the LLR lends at the (zero) risk-free rate, implying that its support involves an expected subsidy, as reflected in the second term of the RHS of Eq. (14). That subsidy is the source of the discrepancy between the social and the private value of the bank, $W_{-1}^{ER}(C)$ and $TMV_{-1}^{ER}(C)$, respectively, and the reason for imposing liquidity requirements on a strong bank.

Suppose alternatively that the LLR is obliged to lend at terms that imply no expected subsidy or tax. LLR support implies advancing $D - C$ to a bank that, if its assets are good, will yield $a_g(1 - C) > D - C$ at termination, while, if its assets are bad, will yield $a_b(1 - C) < D - C$. Thus, the feasibility of break-even LLR support requires the existence of $F \leq a_g(1 - C)$ such that

$$\mu F + (1 - \mu)a_b(1 - C) = D - C, \tag{21}$$

where F is the repayment due to the LLR. It is immediate to see that the existence of such

[30] In Section C of the Online Appendix, we provide a discussion of two additional issues: (i) bankers' incentives to produce information on their condition, and (ii) a variation of our model in which what needs to be discovered is not the quality of the bank's assets but the potential systemic importance of its failure.

F requires

$$\mu \geq \hat{\mu} \equiv \frac{(D - C) - a_b(1 - C)}{(a_g - a_b)(1 - C)}, \tag{22}$$

where, using the assumption in Eq. (1), one can easily prove that $\hat{\mu} \in (\bar{\mu}, 1)$. Therefore, getting support on a break-even basis is unfeasible for *modestly strong* banks with $\mu \in (\bar{\mu}, \hat{\mu})$ and feasible for *sufficiently strong* banks with $\mu \in [\hat{\mu}, 1]$.

Thus, if the LLR lends on an expected break-even basis, $\xi = 1$ can only occur for $\mu \in [\hat{\mu}, 1]$. Formally, the (conditional on ξ) expressions for $V_t^{ER}(C)$ and $W_{-1}^{ER}(C)$ remain valid, but the absence of subsidies now implies $TMV_{-1}^{ER}(C) = W_{-1}^{ER}(C)$ for all values of the parameters. These modifications of the baseline analysis have several important implications:

1. The incentives of bank owners and the social planner with respect to C are fully aligned for all values of μ. Thus, regulatory liquidity requirements would no longer be needed.

2. Differently from the case in which LLR lending occurs at the risk-free rate, there is an additional range $(\bar{\mu}, \hat{\mu})$ of values of the probability that assets are good for which the early run will lead to liquidation if the quality of bank assets remains unknown at $t = \tau$. However, in this range liquidating banks involves a net efficiency loss (the cost of higher type I error exceeds the gain from lower type II error). Thus, the baseline arrangement that combines subsidized LLR support and liquidity requirements is superior in ex ante welfare terms.

3. For $\mu \in (\bar{\mu}, \hat{\mu})$, the arrangement based on break-even LLR support will lead banks to voluntary hold liquidity higher than C^*.[31] This is because the above-mentioned efficiency losses can be reduced, on expectation, by "buying additional time" for the information on asset quality to possibly arrive during the run.

[31] To see this, assume a situation in which the baseline arrangement implies an interior socially optimal amount of liquidity $C^* > 0$. Then, C^* will satisfy the first order condition $\partial W_{-1}^{ER}/\partial C = 0$ and the second order condition $\partial^2 W_{-1}^{ER}/\partial C^2 < 0$, where for $\mu \in (\bar{\mu}, \mu^*)$ the baseline arrangement implies $\xi = 1$. Now, consider the effect of a marginal decrease in the prospect of receiving support, $d\xi < 0$. We can assess the impact on the optimal liquidity choice by differentiating the first order condition:

$$\underbrace{\frac{\partial^2 W_{-1}^{ER}}{\partial C^2}}_{-} dC^* + \frac{\partial^2 W_{-1}^{ER}}{\partial C \partial \xi} \underbrace{d\xi}_{-} = 0,$$

where one can check that $\partial^2 W_{-1}^{ER}(C^*)/\partial C \partial \xi < 0$ for $\mu > \bar{\mu}$. Thus dC^* must be positive.

If the reason for adopting a break-even LLR arrangement is the financing of the subsidy that arises when the LLR lends at the (zero)risk free rate, the society could opt for some form of liquidity insurance arrangement in which banks pay ex ante for the expected cost of the support that they may receive if an early run happens. Liquidity insurance cum liquidity requirements would then be an ex ante break-even arrangement superior to the one based on forbidding the LLR to lend on an ex post subsidized basis.

7.2 *Buying time through temporary LLR support*

In our baseline model, LLR support becomes irreversible once granted. This irreversibility might have various causes. For example, unmodeled political or reputational considerations might make it too costly to acknowledge that a previously supported bank is no longer considered solvent. The political cost might also come from the implied unequal treatment of debtholders who manage to exercise their puts before support is canceled and those who do not. Yet another reason could be that, under LLR support, the information about the quality of bank assets ceases to arrive (or becomes too noisy), e.g. because market participants no longer have incentives to discover it or because the relevant market prices get distorted by the presence of LLR support.[32]

In this section, we examine the implications of relaxing the assumption of irreversibility. For tractability, we focus on the case in which the "buying time" role played by liquidity holdings gets fully replaced by some temporary support from the LLR. Specifically, we consider the case in which banks carry no liquid assets ($C = 0$) and the LLR supports them as soon as an early run starts, but with the goal to just wait until the quality of illiquid assets is unveiled. We assume that like in Proposition 1, if assets are good, the run self-resolves and LLR support does not need to continue, while if assets are bad, support is withdrawn so as to (efficiently) force the bank into resolution. However, to make the comparison with the baseline liquidity-based arrangement non-trivial, we assume that there is a probability $\pi > 0$ that the temporary support becomes permanent, implying that the bank with bad assets is inefficiently continued up to termination.[33]

[32] See Acharya and Thakor (2016) for an explicit model of this channel.

[33] Having $\pi > 0$ might also reflect that the information arriving about the quality of assets of a supported

In this new setup, we could derive an expression for debtholders' value of not putting their debt at $t \geq 0$ (conditional on the quality of assets not having yet been revealed) similar to $V_t^{ER}(C)$ but adapted to the case in which the bank carries no cash ($C = 0$) and gets temporary LLR support from $t = 0$. Call it $\hat{V}_t^{ER}$. Assuming, that (as is common) the LLR is senior to other debtholders, for values of π sufficiently lower than one, we would get $\hat{V}_t^{ER} < D$ for all $t \geq 0$ confirming the sustainability of an early run that starts at $t = 0$. Intuitively, the early run would occur because debtholders would fear that by the time the quality of the illiquid assets gets revealed, there is a high enough chance that the bank is bad and gets resolved, and they receive much less than D.[34]

The ex ante welfare associated with the operation of this arrangement can be written as the expected value of the payoffs extracted from bank assets in each of the possible final states:

$$\hat{W}_{-1}^{ER} = [1 - \varepsilon(1-\mu)]a_g + \varepsilon(1-\mu)\left[q_b - \pi(q_b - a_b)\right], \tag{23}$$

where the first term contains the payoffs extracted when there is no run or the run ends with the good bank unresolved, while the second term shows the value extracted in the run when the bank is bad (which is affected by the probability $\pi > 0$ of mistakenly leaving the bad bank unresolved).

We can establish whether the arrangement based on temporary LLR support dominates or not the baseline arrangement examined before by comparing the expressions for $\hat{W}_{-1}^{ER}$ in Eq. (23) and $W_{-1}^{ER}(C)$ in Eq. (12). In fact, we can extract some general lessons by looking at a few polar scenarios:

- Consider a strong bank which is not subject to liquidity requirements. In that case, under the baseline arrangement, the bank chooses $C = 0$ (implying $\tau = 0$) and, in an

bank is noisier than that received about the quality of assets of a non-supported bank—our insights would be very similar if the noise could also lead to the inefficient liquidation of good assets. Having $\pi > 0$ can also capture the possibility that information arrives so late that it is no longer possible to extract the value from an "early" liquidation of bad assets.

[34] They may receive much less than D not just because $q_b < D$, by the assumption in Eq. (1), but also because, in the event of liquidation, the debt with the LLR is senior to that with the residual debtholders.

early run, gets supported for sure, $\xi = 1$. So the implied ex ante welfare is

$$W_{-1}^{ER}(0)_{|\xi=1} = [1 - \varepsilon(1-\mu)]a_g + \varepsilon(1-\mu)a_b < \hat{W}_{-1}^{ER}, \tag{24}$$

for all $\pi < 1$, since $a_b < q_b$, by Eq. (1). Thus, for a strong bank not subject to liquidity requirements, the arrangement based on temporary LLR support would be strictly superior for all $\pi < 1$.

- Consider now an either weak or strong bank that holds the socially optimal amount of liquidity C^* under the baseline arrangement. In the polar case with $\lambda \to \infty$ (i.e. when information arrives arbitrarily close to $t = 0$) we have $C^* \to 0$ and

$$\lim_{\lambda\to\infty} W_{-1}^{ER}(C^*) = [1 - \varepsilon(1-\mu)]a_g + \varepsilon(1-\mu)q_b > \hat{W}_{-1}^{ER}, \tag{25}$$

for all $\pi > 0$. Thus, by continuity, provided that information arrives at a sufficiently high rate and banks hold the socially optimal liquidity buffers, the baseline arrangement is strictly superior.

- Finally, consider again an either weak or strong bank that holds the socially optimal amount of liquidity C^* under the baseline arrangement. In the alternative polar case with $\lambda \to 0$ (i.e. when information arrives arbitrarily slowly) we also have $C^* \to 0$, but in this case

$$\lim_{\lambda\to 0} W_{-1}^{ER}(C^*) = [1 - \varepsilon(1-\mu)]a_g + \varepsilon(1-\mu)q_b - \varepsilon[\mu(1-\xi)(a_g - q_g) + (1-\mu)\xi(q_b - a_b)].$$

Hence, we can distinguish two subcases. If the bank is strong ($\xi = 1$), we have

$$\lim_{\lambda\to 0} W_{-1}^{ER}(C^*) = [1 - \varepsilon(1-\mu)]a_g + \varepsilon(1-\mu)a_b < \hat{W}_{-1}^{ER}, \tag{26}$$

as in Eq. (24). If the bank is weak ($\xi = 0$), we have

$$\lim_{\lambda\to 0} W_{-1}^{ER}(C^*) = (1-\varepsilon)\, a_g + \varepsilon\mu q_g + \varepsilon(1-\mu)q_b,$$

which is strictly lower than $\hat{W}_{-1}^{ER}$ if and only if

$$\pi < \frac{\mu}{1-\mu}\frac{a_g - q_b}{q_b - a_b}. \tag{27}$$

In fact, $\frac{\mu}{1-\mu}\frac{a_g-q_b}{q_b-a_b} \leq 1$ if and only if the bank is weak. Thus, by continuity, we can generally establish that, if information arrives at a sufficiently low rate (and, in the weak bank case, Eq. (27) holds), the temporary support arrangement is strictly superior to the baseline arrangement.

All in all, the message is that the dominance of one arrangement over the other crucially depends on the comparison between the risk of irreversibility of (or poorer quality of information under) temporary LLR support (as measured by π) and the "buying time" effectiveness of liquidity holdings (as measured by, e.g., λ).[35]

8 Conclusions

We provided in this paper a novel rationale for banks' liquidity standards, one that builds on the idea that liquidity buffers make banks capable to deal with debt withdrawals for some time before they have to seek support from the LLR. This ability to wait before seeking LLR support is valuable because it allows for the release of information on the bank's financial condition that is useful for the LLR's decision on whether to grant support. Specifically, it generally improves the efficiency of the decision regarding the continuation of the bank as a going concern or its liquidation. Importantly, as we show in the main extension to our model, capital regulation can contribute to reinforce the bank's fundamental solvency but is unable to fully mimic the informational role of liquidity standards during a run.

We considered several other extentions to our model, but we still left out some questions that would seem fruitfull for future research. For example, we assumed that the arrival of information on the bank's financial condition following a shock is exogenous. However, in general the nature and the speed at which information on the bank's financial condition is produced and disclosed is endogenous and depends on the entity responsible for this activity. Further, the bank may not have the proper incentives to disclose that information in a timely

[35] Parameter π in the above formulation might also capture the costs due to "stigma effects" associated with (early) LLR support. Specifically, stigma might be rationalized as the result of investors becoming massively aware of bank trouble and accelerating the speed of the run (He and Manela 2016). Such acceleration might increase the probability of arriving to a point in which the illiquid assets can no longer be orderly liquidated and authorities must choose between disorderly liquidation or a full bail-out.

manner. This provides a rationale for entrusting an agency with the authority to produce information about the bank's financial condition. Importantly, this information would have to be made available not only to the LLR but also to the bank's investors, as it is key for their decision to rollover their debt. Since the disclosure of information affects the LLR's incentives and those of investors differently, it would seem useful to investigate which agency or agencies should have authority to gather and disclose information on banks' financial condition in real time.[36]

[36] See Kahn and Santos (2006) for a model in which differences in regulatory agencies' mandates induce agencies to hold information from their counterparts.

Appendix: Proofs

Proof of Propositions 1-3 These propositions follow directly from simple algebra and the arguments that precede their statement in the main text.■

Proof of Proposition 4 We structure this proof in three parts. First we find expressions for a debtholder's value of not exercising the put option at some $t \in [0, \tau]$ conditional on bank assets being bad and good, respectively. Then, we put together the corresponding unconditional value of not exercising the put at t so as to arrive to Eq. (11).

Part I. Value of not exercising the put conditional on assets being bad We can compute this value as the weighted average over two possible courses of events:

1. News arrive prior to date τ. Since news arrival is a Poisson process with intensity λ, the time span to the arrival of (the next) news, say x, follows and exponential distribution with parameter λ. Thus, the probability that news arrive prior to date τ can be computed as $\Pr(x \leq \tau - t)) = 1 - \exp(-\lambda(\tau - t))$. If news about the bad quality of the illiquid assets arrive prior to date τ, the bank ends up liquidated at date τ. Some lucky debtholders will recover D prior to τ and the remaining ones will obtain $Q_b < D$ at liquidation. Since the arrival of the chance to recover D follows a Poisson process with intensity δ, the probability of having a chance to recover D prior to liquidation is $1 - \exp(-\delta(\tau - t))$, so the expected payoff over this course of events can be written as

$$\begin{aligned}[1 - \exp(-\delta(\tau - t))] D + \exp(-\delta(\tau - t))Q_b &= D - \exp(\delta t) \exp(-\delta\tau)(D - Q_b) \\ &= D - \exp(\delta t)[D - C - q_b(1 - C)], \quad (28)\end{aligned}$$

where the last equality is obtained using Eq. (10) for $i = b$.

2. News do not arrive prior to date τ. This happens with probability $\exp(-\lambda(\tau - t))$. When the bank runs out of cash and the quality of its assets remains unknown, the LLR decides to support the bank ($\xi = 1$) if the bank is strong ($\bar{a} > \bar{q}$) and not to support it ($\xi = 0$) if it is weak ($\bar{a} \leq \bar{q}$). So debtholders with the opportunity to exercise their puts prior to date τ will obtain D, while the remaining ones will obtain $\xi D + (1 - \xi)Q_b$, and the expected payoffs over this course of events can be written as

$$\begin{aligned}&[1 - \exp(-\delta(\tau - t))] D + \exp(-\delta(\tau - t)) [\xi D + (1 - \xi)Q_L] \\ &= D - (1 - \xi) \exp(-\delta(\tau - t))(D - Q_L) \\ &= D - (1 - \xi) \exp(\delta t)[D - C - q_b(1 - C)], \quad (29)\end{aligned}$$

where $\exp(-\delta(\tau - t))$ is, as above, the probability of not having the chance to recover D prior to date τ, and we also use Eq. (10) to re-express the term in $(D - Q_L)$ in the last equality.

Putting together these results, the value of not exercising the put for a residual debtholder at date t conditional on the illiquid assets being bad can be written as

$$V_t^{ER}(C)_{|i=b} = D - [1 - \xi \exp(-\lambda(\tau - t))] \exp(\delta t) [D - C - q_b(1 - C)], \tag{30}$$

where the term multiplied by ξ captures the contribution of the *subsidy* associated with LLR support in the strong bank case.

Part II. Value of not exercising the put conditional on assets being good The simplest way to obtain an expression for $V_t^{ER}(C)$ conditional on assets being good is also to look at how events may unfold for a typical debtholder who retains her debt at $t = 0$. We can distinguish three mutually exclusive courses of events:

1. The debtholder gets the chance to put her debt and obtain D prior to the arrival of news and prior to the exhaustion of the bank's cash. So the debtholder receives D.

2. The news arrive prior to the debtholder having the opportunity to put her debt and prior to the exhaustion of the bank's cash. So the debtholder obtains B by waiting up to termination, since the crisis self-resolves.

3. The bank runs out of cash prior to the debtholder having the opportunity to put her debt and prior to the arrival of news. So the debtholder obtains $\xi D + (1 - \xi)Q_g$.

Thus, using the fact that the payment associated with the exhaustion of cash will occur at date τ if none of the other relevant events occurs before that date, and the independent nature of the Poisson processes driving the arrival of these events, we can write:

$$\begin{aligned} V_t^{ER}(C)_{|i=g} &= [1 - \exp(-(\delta + \lambda)(\tau - t))] \left(\frac{\delta}{\delta + \lambda} D + \frac{\lambda}{\delta + \lambda} B \right) + \\ & \quad \exp(-(\delta + \lambda)(\tau - t)) [\xi D + (1 - \xi)Q_g]. \end{aligned} \tag{31}$$

The factors $1–\exp(-(\delta+\lambda)\tau)$ and $\exp(–(\delta+\lambda)\tau)$ are explained by the fact that if two Poisson processes arrive independently with intensities δ and λ, the arrival of the first of them is a Poisson process with intensity $\delta + \lambda$, and the corresponding span to such an arrival follows an exponential distribution with parameter $\delta + \lambda$. So $\exp(-(\delta + \lambda)\tau)$ is the probability that no first event occurs by date τ and $1 - \exp(-(\delta + \lambda)\tau)$ is the probability that at least one event arrives. The factors $\delta/(\delta + \lambda)$ and $\lambda/(\delta + \lambda)$ describe the probabilities with which the first event is the option to exercise the put and the arrival of (good) news, respectively.

Isolating D and using Eq. (10) to write $\exp(-\delta(\tau-t))(D-Q_g)$ as $\exp(\delta t)[D-C-q_b(1-C)]$, we obtain

$$\begin{aligned} V_t^{ER}(C)_{|i=g} &= D + [1-\exp(-(\delta+\lambda)(\tau-t))]\frac{\lambda}{\delta+\lambda}(B-D) \\ &\quad -(1-\xi)\exp(-\lambda(\tau-t))\exp(\delta t)[D-C-q_g(1-C)], \end{aligned} \tag{32}$$

which reflects that, conditional on bank assets being good, the residual debtholders at time t do not always end up recovering D during the early run. They gain the additional amount $B-D>0$ if the good news arrive on time (so that they can wait until termination) and they incur an additional expected loss $\exp(\delta t)[D-C-q_g(1-C)]$ if the bank is weak ($\xi=0$) and runs out of cash prior to the revelation of the quality of its assets.

Part III. Unconditional value of not exercising the put in an early run Putting together the expressions in Eq. (30) and (32), we obtain the unconditional value of one unit of residual bank debt during an early run as reported in Eq. (11).■

Proof of Proposition 5 Ex ante welfare can be calculated as the expected value of the overall asset returns that the bank generates over all the possible courses of events, which can be described as follows:

1. No shock occurs at $t=0$. This occurs with probability $1-\varepsilon$. The bank assets are good and never liquidated. The bank generates returns $C+a_g(1-C)$.

2. The shock occurs at $t=0$ and the run starts. This occurs with probability ε.

   (a) The illiquid assets are bad. This happens with (conditional) probability $1-\mu$.

      i. News arrive prior to date τ. This occurs with (conditional) probability $1-\exp(-\lambda\tau)$. The bank ends up liquidated, so its overall asset returns are $C+q_b(1-C)$.

      ii. News do not arrive prior to date τ This occurs with (conditional) probability $\exp(-\lambda\tau)$. The bank ends up liquidated in the weak bank case ($\xi=0$) and continued in the strong bank case ($\xi=1$), so its overall asset returns are $C+[q_b-\xi(q_b-a_b)](1-C)$.

   (b) The illiquid assets are good. This happens with (unconditional) probability μ.

      i. News arrive prior to date τ. This occurs with (conditional) probability $1-\exp(-\lambda\tau)$. The bank continues up to termination, so its overall asset returns are $C+a_g(1-C)$.

ii. News do not arrive prior to date τ. This occurs with (conditional) probability $\exp(-\lambda\tau)$. The bank ends up liquidated in the weak bank case ($\xi = 0$) and continued in the strong bank case ($\xi = 1$), so its overall asset returns are $C + [q_g + \xi(a_g - q_g)](1 - C)$.

Putting together these payoffs and after some algebra, we obtain the expression reported in Eq. (12).■

Proof of Proposition 6 From Eq. (13), it is a matter of algebra to check that the first and second derivatives of $W_{-1}^{ER}(C)$ with respect to C can expressed as

$$\frac{dW_{-1}^{ER}(C)}{dC} = -(A_H - 1) + A_L \left(\frac{D - C}{D}\right)^{\lambda/\delta} \left[1 + \frac{\lambda(1 - C)}{\delta(D - C)}\right], \tag{33}$$

$$\frac{d^2W_{-1}^{ER}(C)}{dC^2} = -A_L \left(\frac{D - C}{D}\right)^{\lambda/\delta} \frac{\lambda}{\delta^2(D - C)^2}[\delta(D - 1) + \delta(D - C) + \lambda(1 - C)], \tag{34}$$

where the sign of the first is ambiguous, while the sign of the second is strictly negative. So $W_{-1}^{ER}(C)$ is strictly concave in C. If it is strictly increasing at $C = 0$, i.e.

$$\frac{\lambda}{\delta D} A_L > A_H - A_L - 1, \tag{35}$$

then $W_{-1}^{ER}(C)$ must reach a maximum over the interval $[0, \bar{C}]$ at some point $C^* > 0$. Such point must be unique because $W_{-1}^{ER}(C)$ is strictly concave in C. By the same token, if Eq. (35) does not hold, $W_{-1}^{ER}(C)$ reaches its maximum at $C^* = 0$.■

Proof of Proposition 7 Most of the results in this proposition are proven by the arguments already included in the main text, prior to the proposition. It remains to be shown that $TMV_{-1}^{ER}(C)$ is strictly decreasing in C when $\xi = 1$. To see this, let us rewrite the expression in Eq. (14) using Eq. (13) and $\exp(-\lambda\tau) = ((D - C)/D)^{\lambda/\delta}$:

$$\begin{aligned} TMV_{-1}^{ER}(C) &= C + A_H(1 - C) - A_L \left(\frac{D - C}{D}\right)^{\lambda/\delta} (1 - C) \\ &\quad + \xi\varepsilon \left(\frac{D - C}{D}\right)^{\lambda/\delta} (1 - \mu)[(D - C) - a_b(1 - C)]. \end{aligned} \tag{36}$$

But with $\xi = 1$, we have $A_L = \varepsilon(1 - \mu)(q_b - a_b)$, so the last two terms of the above expression can be grouped together, yielding

$$\begin{aligned} TMV^{ER}_{-1}(C) &= C + A_H(1-C) + \varepsilon(1-\mu)\left(\frac{D-C}{D}\right)^{\lambda/\delta}[(D-C) - a_b(1-C) \\ &\quad -(q_b - a_b)(1-C)] \\ &= C + A_H(1-C) + \varepsilon(1-\mu)\left(\frac{D-C}{D}\right)^{\lambda/\delta}[(D-C) - q_b(1-C)], \quad (37) \end{aligned}$$

which is strictly decreasing in C since $A_H > 1$ and $q_b < 1$.■

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