Entrepreneurial Innovation, Patent Protection, and Industry Dynamics *

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Abstract

We assess the effects of intellectual property (IP) protection in a dynamic model in which the value of IP engendered by innovative entrepreneurs gets eroded by subsequent imitators and innovators, and imitation has pro-competitive effects. We find that welfare and innovation are maximized with zero protection against further innovation and, conditional on this, with full protection against imitation. However, if some protection against innovation is unavoidable, allowing for some imitation may be socially beneficial. These results are robust to endogenizing imitation and get reinforced when entrepreneurs are financially constrained.

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1 Introduction

The proliferation of patents in highly technological markets makes entry of new firms difficult, among other reasons, because of the risk of infringing some patents. One example is the market for smartphones, in which producers are entangled in endless legal battles.[1] Some practitioners doubt about the effectiveness of the patent system in generating the right incentives to innovate and refer to this problem as the "tragedy of the anti-commons," describing strategic patenting and patent stacking as obstacles to innovation (Heller and Eisenberg (1998)).

Patent proliferation has been spurred by the strong protection of innovators' intellectual property rights (IP), especially in the United States. It has been argued that the creation of a unique Court of Appeals of the Federal Circuit in 1982, as well as the 1984's Semiconductors Act and the extension of patent duration to 20 years, strengthened the protection of IP. However, whether these reforms have really promoted innovation is theoretically and empirically controversial.[2] In fact, some quantitative assessments indicate that these reforms may have been detrimental to innovation (Levin et al. (1985), Hall and Ziedonis (2001)).

In this paper we study the effects of the protection of intellectual property rights in a tractable industry dynamics model. We focus on the effects on the speed of innovation and on social welfare in markets in which entrants face uncertainty on whether their product might infringe some of the existing IP rights. We consider an industry made up of a continuum of business niches where each niche can be thought of as a distinct product. The successful developers of improved versions of each product contribute to welfare and appropriate temporary monopoly profits like in a standard quality ladder model with limit pricing (Grossman and Helpman (1991)). These temporary monopolies are based on the protection granted by IP and are threatened by the entry of the developers of even better versions of the product (innovators) as well as imitators. Prospective entrants,

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[1]See "The Great Patent Battle," *The Economist*, 10/21/2010.
[2]See Gallini (2002) for a review of the reforms and their effect on patenting activity.

due to the uncertainty on which niche they will occupy, anticipate that in markets in which more incumbents hold patents conflicts with them will be more inevitable and the resulting profits will tend to be lower; this discourages their entry. We find that, if feasible, incumbents should be protected against imitation but not against genuine innovation.

The success of the innovators is compromised by the competition coming from other contemporaneous innovators and by the opposition of incumbent monopolists, who use their IP to fight the entrants.[3] We assume that incumbents exert lower effective resistance to entry in non-monopolized business niches than in monopolized ones. This assumption captures three complementary mechanisms. First, with competing incumbents, it should be easier and cheaper for the entrant to warrant her entry by obtaining a license for one of the substitute technologies. Second, the fact that imitation has previously succeeded in the niche may signal that the patent of the previous monopolist was invalid or had expired, in which case his resistance to the new entrant might also lack legal support. Finally, to the extent that court damages due to patent infringement tend to be related to foregone profits, the entrant may expect to reach a more favorable settlement with incumbents when the pre-entry profits in the niche are low.[4] For simplicity, we assume that IP protection is just ineffective in non-monopolized niches.

An important feature of the model is that entrants are uncertain about the specific niche that their product may challenge and whether it will be occupied by a patent holder or not. We believe that these sources of uncertainty, while somewhat overlooked by the literature, are very important in practice. First, innovation activities often engender products with uses different from those originally intended by their developers. Second, developers may pursue products intended to occupy an empty market niche but, because innovation takes time, they may find that, by the time the product is available, the niche has been filled by a faster or luckier developer. Finally, innovators may not be aware of

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[3]We assume that the strength of IP protection affects the incumbents' probability of expelling the innovators and imitators who challenge their business niches. This modeling allows us to abstract from the traditional distinction between patent length and patent breadth (Scotchmer (2004)).

[4]The evidence in Cockburn and MacGarvie (2011) for the software industry is consistent with these views.

all of the intellectual property potentially connected to their products and target niches that they hoped to find vacant.

Strengthening IP protection in our setup involves dynamic trade-offs that give rise to the main findings in the paper. First, stronger protection against future innovation implies a larger expected duration of the monopoly obtained by the developers of IP who enter successfully, but also a stronger protection of the incumbents and, hence, a higher hurdle for subsequent entrants. Reducing the hurdle for successful entry and lengthening the duration of the monopoly rights granted are substitute forms of rewarding a potential innovator. However, we find that, due to discounting, the former is a more effective means of increasing the steady-state rate of innovation and our measure of social welfare than the latter. So the socially optimal level of protection against further innovation is zero.

Second, stronger protection against imitation lengthens the expected duration of the monopoly obtained by successful innovators but has no direct effect on innovators' entry hurdle. It has, however, an indirect effect due to the fact that imitation reduces the steady-state fraction of business niches monopolized by IP holders and, hence, the effective opposition faced by subsequent innovators. We show that if the protection that IP grants against further innovation is chosen optimally (i.e. equals zero), then innovation and welfare monotonically grow with the protection against imitation. Otherwise, the innovation rate and social welfare may well be maximized at intermediate levels of protection against imitation.

The literature on the optimal protection of innovators reaches different conclusions depending on the assumptions on how firm-specific is the process of engendering an innovation. When only one firm can come up with a given innovation, firms typically underinvest in R&D because they do not internalize all the social returns of their investment. In this strand of the literature, started with Nordhaus (1969) and summarized by Hopenhayn et al. (2012), patent protection arises as a way for firms to internalize a larger part of the surplus from the innovation. In contrast, when several firms may compete to

obtain a similar innovation, producing a patent race situation, as in Loury (1979), patent protection may be excessive. Firms have incentives to invest in R&D in order to be the first to obtain the innovation and do not internalize that they erode the profitability of the investments undertaken by their competitors.

Our paper shares with the second branch of this literature the feature that, at a given point in time, all entrants compete for accessing a scarce number of market niches. However, our result that patent protection against further innovation is undesirable does not arise from the patent race component of the model but from its dynamic part. We find that the advantages of encouraging innovation by protecting an innovator once it is already in the market are offset by the erosion of his profits as a prospective entrant. Thus, our analysis is novel in emphasizing dynamic industry equilibrium effects that establish bidirectional relationships between the blanket of overlapping IP claims and the innovation process.

Equilibrium analysis is a feature of papers such as Aghion et al. (2001) or O'Donoghue and Zweimüller (2004), which analyze IP protection from the perspective of endogenous growth models.[5] The first paper studies R&D competition in a quality ladder model where each good is sold by two firms. These firms have a productivity level that they may improve by investing in R&D or by imitating the leader. The authors show that protecting leaders against this imitation has an inverse U-shaped effect on growth, but the logic behind their finding is different from ours. Imitation helps backward firms to catch up with the leaders, which may try to elude the ensuing competition by increasing their innovation effort. This makes imitation potentially good for innovation. With too much imitation, though, leadership is too short-lived and the net effect becomes negative.

O'Donoghue and Zweimüller (2004) study the effect on growth of various aspects of patent policy (i.e. leading breadth and patentability requirements) in a general equilibrium context. Their paper emphasizes effects associated with demand and the reallocation

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[5]Papers in this tradition also include Grossman and Lai (2004), who study the effect of IP on international trade, and Boldrin and Levine (2006), who analyze the optimal degree of IP protection when the size of the economy grows.

of resources across sectors which are typically neglected in partial equilibrium models. Our work focuses on the dynamic industry-level effects rather than general-equilibrium consequences.

Our baseline model formalizes imitation as an exogenous random arrival process but can be extended to make imitation the result of a costly and risky entry decision similar to that of innovators. The model can also be extended to analyze the situation in which innovators face financial constraints. Our key results are robust to both extensions.

The rest of the paper proceeds as follows. Section 2 introduces our baseline industry dynamics setup. Section 3 analyzes its equilibrium and steady-state properties. Section 4 explores the welfare implications of IP protection and discusses optimal IP policies. Section 5 describes the extension in which imitation is endogenous. Section 6 incorporates financial constraints in the analysis. Section 7 concludes. The Appendix contains all proofs.

2   The Model

This section describes an infinite horizon, discrete-time model of an industry. Agents are risk-neutral and have an intertemporal discount factor $\beta < 1$. The industry consists of a measure-one continuum of business niches. Each niche can be interpreted as the market for a different product.[6] At each date t there is a proportion $x_t \in [0, 1]$ of niches monopolized by producers protected by an active patent. Monopolists during their incumbency earn a per period profit flow of $a > 0$.[7]

Active patents become worthless whenever their niche is successfully occupied either by an imitator or by the holder of a patent for a superior substitute of the product. At each date t, each monopolized niche is challenged by at least an imitator with a probability

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[6]This simplification allows us to abstract from cross-product competition and to focus on competition related with concomitant and future entry into each niche.

[7]In Section 4 we interpret the introduction of newer products in terms of a quality ladder model with limit pricing (Grossman and Helpman (1991)) in which a is the quality improvement of each successful innovation.

$\delta > 0$, which is exogenous and independent across niches.[8] Patents allow incumbents to fight imitation and preserve their monopoly in a challenged niche with probability λ_1. When this protection fails, the niche becomes Bertrand competitive and firms make zero profits.

The entry of innovators occurs once the imitation process is completed. Each niche (monopolized or not) is challenged by the holder of a new patent with an endogenous probability q_t. In monopolized niches, an incumbent patent holder challenged by an entrant preserves its position with probability λ_2. When this protection fails, the entrant replaces the incumbent as the monopolist of that niche and the new patent joins the stock of active patents.

Under these assumptions, the value of an active patent at date t (that is, the present value of the monopoly profits that it yields), can be recursively written as

$$v_t = a + \beta[1 - (1 - \lambda_1)\delta][1 - (1 - \lambda_2)q_{t+1}]v_{t+1}, \tag{1}$$

where the two terms in square brackets represent the probability at date $t + 1$ of surmounting the entry of imitators and innovators, respectively. The law of motion of the stock of active patents, x_t, can be written as

$$x_t = [1 - (1 - \lambda_1)\delta]x_{t-1} + \{1 - [1 - (1 - \lambda_1)\delta]x_{t-1}\}q_t. \tag{2}$$

The first term in the right hand side of this expression accounts for the niches that, being monopolized at $t - 1$, remain monopolized after the entry of imitators at t; the second term accounts for non-monopolized niches that become monopolized by successful innovators at t. Notice that when these innovators enter previously monopolized niches they simply replace previously active patents with new ones, keeping the size of the stock x_t unchanged.

The probability of innovative entry q_t is determined as follows. At each date there is an infinite number of potential innovative entrepreneurs who may attempt to engender

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[8]In Section 5 we extend the model to endogenize the entry of imitators.

and develop an innovation. Innovating is risky and involves a *non-pecuniary entry cost* $\Phi > 0$ and a *pecuniary development cost* normalized to one. These costs are incurred one period before the potential new product is generated. The distinction between the pecuniary and non-pecuniary part of the overall cost $1 + \Phi$ is immaterial in the baseline model but becomes useful when imitation is endogenized (Section 5) and when innovators face financial constraints (Section 6).

Innovators have to overcome the competition of simultaneous developers of new products and the opposition of the incumbent monopolists. We capture the first of these risks in the form of congestion, as in the literature on search frictions.[9] If $e_t \in [0, \infty)$ denotes the mass of innovations developed between dates $t - 1$ and t, we postulate that each of them becomes the challenging product of a niche with an identical and independent probability $1/(1 + e_t)$.[10] Accordingly, the probability of success goes to one as the measure of simultaneously developed innovations goes to zero, and to zero as the measure of potential entrants goes to infinity. Also, like in a reduced-form patent race among symmetric contestants, the probability of success of any given innovation declines with the number of competing innovations.[11]

For consistency, the probability of innovative entry in a given niche q_t must equal the product of the number of innovations subject to development at that date, e_t, and the probability with which each of them gives rise to a challenging product, $1/(1 + e_t)$. So we must have $q_t = e_t/(1 + e_t)$, which is increasing in e_t. For brevity, we will refer to q_t as the *innovation flow*.

An innovator that challenges an empty or competitive niche becomes its monopolist with probability one. In contrast, in an already monopolized niche, becoming the new

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[9]See Mortensen (1982) and Pissarides (1985) for classical examples in labor economics.

[10]This specific formulation is adopted for analytical tractability and satisfies the standard properties of matching technologies. Of course, coordination and congestion problems could be modeled in many other ways. For example, the explicitly probabilistic *urn-ball process* postulated by the literature on random matching would imply a success probability of $[1 - \exp(-e)]/e$ for each innovation. Our formulation is simply more tractable.

[11]Opposite to classical models in the patent-race literature (e.g., Loury (1979) and Lee and Wilde (1980)), we abstract from the timing of innovation.

monopolist entails overcoming the opposition from the incumbent (based on a legal dispute on patent rights), which we assume to occur with probability $1 - \lambda_2$. Thus, using the equality $1/(1 + e_t) = 1 - q_t$ to rewrite the probability with which a given innovation becomes the successful challenger of a niche, an innovator's probability of success in becoming a monopolist at date t can be written as

$$p_t = \{1 - \lambda_2[1 - (1 - \lambda_1)\delta]x_{t-1}\}(1 - q_t), \tag{3}$$

where $[1 - (1 - \lambda_1)\delta]x_{t-1}$ is the fraction of niches that, taking into account the prior entry of imitators at date t, remain monopolized when innovators reach them.

Finally, for the mass of innovations subject to development at any date t, e_t, to be finite, the net gain from entering and developing an innovation must be zero or strictly negative:

$$-(1 + \Phi) + \beta p_t v_t \leq 0. \tag{4}$$

Using (3) to substitute for p_t, (4) can be rewritten as

$$-(1 + \Phi) + \beta\{1 - \lambda_2[1 - (1 - \lambda_1)\delta]x_{t-1}\}(1 - q_t)v_t \leq 0, \tag{5}$$

to which we will refer as the *free-entry* inequality. If this inequality is strict, no innovations are developed in the corresponding date and the innovation flow q_t must be zero. To guarantee this, we impose a last equilibrium condition,

$$q_t[-(1 + \Phi) + \beta\{1 - \lambda_2[1 - (1 - \lambda_1)\delta]x_{t-1}\}(1 - q_t)v_t] = 0, \tag{6}$$

to which we will refer as the *complementary slackness* condition.

3 Equilibrium

In this section we define the dynamic equilibrium of the industry and analyze its steady-state properties. Equilibrium conditions determine three key endogenous variables at each date t: the innovation flow q_t, the stock of active patents x_t, and the value of a patent v_t.

Definition 1. *Given an initial stock of active patents x_0, an equilibrium is a sequence of non-negative triples (x_t, v_t, q_t), for $t = 1, ...\infty$, that satisfy the valuation equation (1), the law of motion (2), the free-entry inequality (5), and the complementary slackness condition (6).*

When the innovation flow q_t is strictly positive along the equilibrium sequence, the set of equilibrium conditions described in Definition 1 can be reduced to a bidimensional non-linear system of first-order difference equations in x_t and v_t. Specifically, equation (2) can always be used to produce an expression for q_t in terms of x_{t-1} and x_t. Moreover, having $q_t > 0$ requires (5) to hold with equality, by (6). The substitution of the expression for q_t in such equality and in (1), respectively, yields the two difference equations of the following reduced system in x_t and v_t:

$$\beta(1 - x_t)\frac{1 - \lambda_2(1 - \psi)x_{t-1}}{1 - (1 - \psi)x_{t-1}}v_t - (1 + \Phi) = 0, \tag{7}$$

$$\beta(1 - \psi)\frac{1 - (1 - \lambda_2)x_t - \lambda_2(1 - \psi)x_{t-1}}{1 - (1 - \psi)x_{t-1}}v_t - v_{t-1} + a = 0, \tag{8}$$

where $\psi \equiv (1 - \lambda_1)\delta$ denotes the *effective imitation risk* (or probability with which each patent holder is replaced by an imitator in a given period).

Importantly, if $\psi > 0$, the above system describes the dynamics of equilibrium in the neighborhood of any steady-state equilibrium (SS) with a strictly positive stock of active patents, $x_{ss} > 0$, because compensating its attrition due to imitation requires a positive innovation flow $q_{ss} > 0$. Specifically, we must have

$$q_{ss} = \frac{\psi x_{ss}}{1 - (1 - \psi)x_{ss}}, \tag{9}$$

by (2). When equations (7) and (8) are evaluated in a steady-state equilibrium with

$x_t = x_{t-1} = x_{ss}$ and $v_t = v_{t-1} = v_{ss}$ for all t, we obtain

$$\beta(1 - x_{ss})\frac{1 - \lambda_2(1 - \psi)x_{ss}}{1 - (1 - \psi)x_{ss}}v_{ss} - (1 + \Phi) = 0, \tag{10}$$

$$\left[1 - \beta(1 - \psi)\frac{1 - (1 - \lambda_2\psi)x_{ss}}{1 - (1 - \psi)x_{ss}}\right]v_{ss} - a = 0. \tag{11}$$

After solving (10) and (11), the steady-state innovation flow q_{ss} can be obtained using (9).

The next lemma provides a necessary and sufficient condition for the existence of a (unique and locally stable) steady-state equilibrium with a strictly positive stock of active patents.

Lemma 1. *There exists a steady-state equilibrium with $x_{ss} > 0$ if and only if*

$$\frac{\beta a}{1 - \beta(1 - \psi)} \geq 1 + \Phi. \tag{12}$$

This equilibrium is unique, locally stable, and exhibits monotonic convergence in the state variable x_t and saddle-path convergence in the jump variable v_t.

The steady-state stock of active patents x_{ss} and the steady-state value of a patent v_{ss} can be described as the coordinates of the intersection between two curves (see Figure 1): the *free-entry curve* defined by equation (10) and the *present-value curve* defined by (11). The free-entry curve describes an increasing relationship between x_{ss} and v_{ss} which reflects that, when the stock of active patents is larger, the developers of new products are more likely to find opposition from incumbents and, thus, less likely to enter successfully. So a larger (after entry) value of patents v_{ss} is necessary to encourage innovators to innovate. The present-value curve describes the negative relationship between x_{ss} and v_{ss} implied by equation (11), which expresses the value of a patent as a discounted sum of the per-period monopoly profits a. Intuitively, as shown in (9), a larger x_{ss} implies a larger innovation flow q_{ss}, which in turn increases the risk that a patent becomes worthless and, hence, erodes v_{ss}.
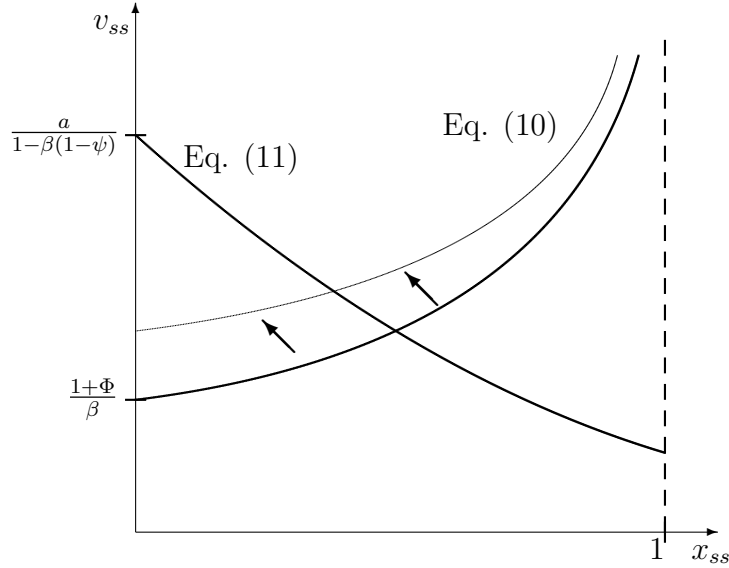
Figure 1: Characterization of the steady state. It is easy to study the comparative-statics of most parameters. Here, we illustrate the effect of an increase in Φ.

Graphically, the existence condition in Lemma 1 is equivalent to requiring that the intercept of the free-entry curve (10) in Figure 1 is lower than the intercept of the present-value curve (11) so that both curves intersect once. Economically, condition (12) guarantees that a single innovator ($e_t = 0$) facing no opposition from incumbent monopolists ($x_t = 0$) would make positive entry profits.

Figure 1 is also useful to perform comparative statics regarding the effects of most parameters on x_{ss} and v_{ss}. The next proposition summarizes these effects.

Proposition 1. *In a steady-state equilibrium with $x_{ss} > 0$, the effects of marginal changes in the parameters on the steady-state variables x_{ss}, v_{ss}, and q_{ss} have the signs shown in the following table:*

	a	β	Φ	ψ	λ_2
x_{ss}	+	+	−	−	−
v_{ss}	+	+	+	−	+
q_{ss}	+	+	−	?	−

In the next two subsections we first comment on the effects of each parameter on x_{ss}

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and v_{ss}, and then discuss the effects on q_{ss}. The third subsection contains a brief note on transitional dynamics.

3.1   Determinants of the stock and value of active patents

The monopoly rents a and the discount factor β have, for standard reasons, positive effects on the value v_{ss} and the steady-state stock x_{ss} of active patents. Higher innovation costs, measured by Φ, make the equilibrium flow of innovation less intense. This reduces the stock of active patents x_{ss}, continuously eroded by imitation, and increases the expected duration of the monopoly associated with each active patent and, thus, v_{ss}. The rise in v_{ss} allows entering innovators to be compensated for the larger entry cost.

As one could expect, a lower probability that an innovator is replaced (either because the effective imitation risk ψ is lower or the IP protection against further innovation λ_2 is larger) increases the value v_{ss} of each active patent. However, imitation risk and innovation risk have opposite implications for x_{ss}. Lowering ψ merely expands the expected valuable life of each patent, resulting in a higher x_{ss}. But the protection against innovative entry λ_2 has the additional effect of weakening the incentives for new innovators to enter. As Proposition 1 shows, the latter effect dominates because, from the perspective of a potential entrant, the future protection granted by a larger λ_2 is discounted vis-a-vis the extra hurdle to current entry that it imposes.[12] This insight explains the social undesirability of increasing λ_2 that we establish in Section 4.

3.2   Determinants of the innovation flow

We now turn to the effects of parameters on the steady-state innovation flow q_{ss}. Equation (9) establishes a positive relationship between q_{ss} and the steady-state stock of active patents x_{ss} because, intuitively, innovation in the steady state must be sufficiently large to compensate the attrition in the stock of active patents due to imitation. This connection makes q_{ss} to move in the same direction as x_{ss} in response to changes in most parameters.

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[12]This effect can be appreciated by noting that the expression in equation (27) (in the proof of Proposition 1) would become positive if the discount factor β were greater than one.

The exception arises when we consider the impact of the effective imitation risk ψ on q_{ss} because this parameter has an additional direct effect on (9). So we have

$$\frac{dq_{ss}}{d\psi} = \frac{\partial q_{ss}}{\partial \psi} + \frac{\partial q_{ss}}{\partial x_{ss}}\frac{\partial x_{ss}}{\partial \psi}.$$

The direct effect $\partial q_{ss}/\partial \psi$ is positive because imitation reinforces the need to compensate for the attrition in x_{ss}. In contrast, the indirect effect is negative because the imitation risk reduces x_{ss}, and the fall in x_{ss} reduces q_{ss} as explained before (i.e. we have $\partial x_{ss}/\partial \psi < 0$ and $\partial q_{ss}/\partial x_{ss} > 0$). This gives rise to a generally ambiguously-signed overall effect. Economically, the ambiguity is explained by the fact that, while imitation facilitates entry by increasing the fraction of competitive niches, it also erodes the expected profits of a successful innovator.

It can be shown that when the protection against genuine innovation, λ_2, is close to zero, the second effect dominates, so entry monotonically decreases with ψ. Numerical simulations, however, show that, for larger values of λ_2, the direct effect may dominate when ψ is low, as illustrated by the solid curve in Figure 2. In those cases, the innovation flow is maximized at some interior value of the effective imitation risk ψ. These results imply non-trivial trade-offs for our discussion below on the socially optimal level of protection against imitation λ_1 (recall that $\psi \equiv (1 - \lambda_1)\delta$, where δ is the flow of entry of imitators) and its link to the socially optimal level of protection against innovation λ_2.

3.3 Transitional dynamics starting from too many patents

This subsection provides a brief note on the case in which equilibrium dynamics is not characterized by equations (7) and (8) because entry is zero at some dates. The absence of entry for a few periods may be a feature along part of the path of transition towards steady state when the initial stock of active patents, say x_0, is well above the steady-state value x_{ss}. How will the steady-state be reached in such a situation?

If the predetermined proportion of monopolized niches x_{t-1} is very large, v_t many be very low for a few periods, making (5) hold with strict inequality, in which case entry is
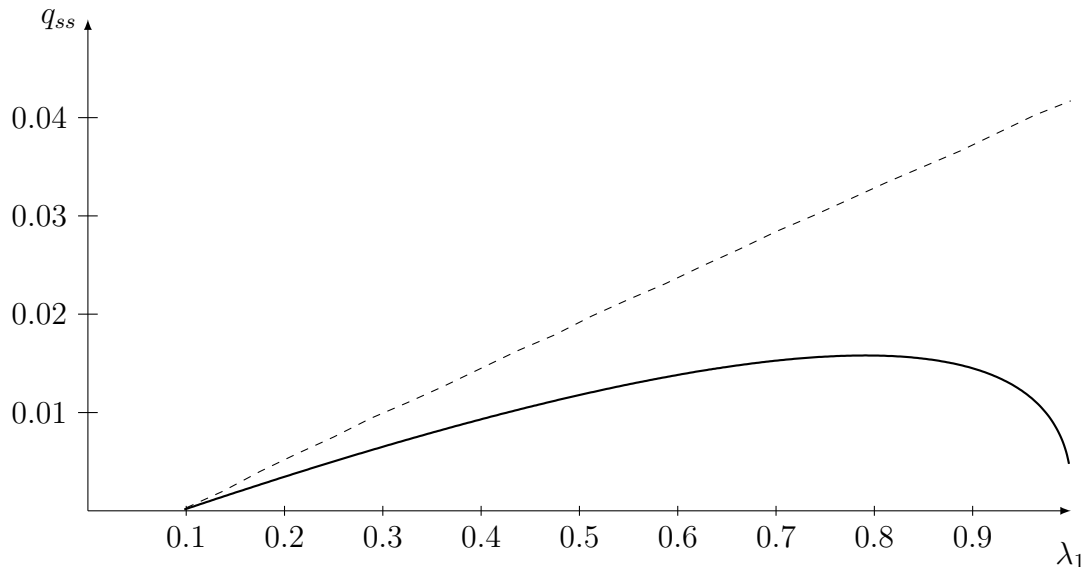
Figure 2: Steady-state innovation and imitation risk. This graph depicts q_{ss} as a function λ_1. The underlying parameter values are $a = 0.1$, $\beta = 0.96$, $\delta = 0.05$, and $\Phi = 0.15$. The solid and dashed curves correspond to the cases $\lambda_2 = 0.5$ and $\lambda_2 = 0$, respectively.

not profitable and we have $q_t = 0$ by (6). The absence of entry and the attrition of x_t due to imitation will tend to increase over time the profitability of innovating, up to a point in which $q_t > 0$ is compatible with (6), and the dynamics of the system is again describable by (7) and (8).[13]

4 Welfare Effects of IP Protection

In order to perform a meaningful welfare analysis, we need to formalize the demand side of the industry. We do this along the lines of a standard quality ladder model with limit pricing. In particular, we specify demand and the welfare measure as in the sequential innovation setup of Hopenhayn et al. (2006). Social welfare in this model is equal to consumers' surplus since innovators break even in expectation and, thus, obtain no surplus in equilibrium.

We assume that there is a unit mass of infinitely-lived homogeneous consumers willing to buy at most one unit of the product from each niche $j \in [0, 1]$ at each date t. Utility is

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[13]In this transition with zero entry for some periods, the reduction in the stock of active patents due to imitation will typically lead to a situation with $x_t > x_{ss} > (1 - \psi)x_t$ just one period before reaching the steady state.

additive across goods and dates, the intertemporal discount factor is $\beta < 1$, and the net utility flow from buying good j at price P_{jt} is $U_{jt} = A_{jt} - P_{jt}$, where A_{jt} is the quality of the good. The successful entry of an innovation in a given niche improves the quality of the best good available in that niche by a units, while the successful entry of an imitator in the niche makes the production technology of the best quality good freely available to the imitator. Finally, we assume, for simplicity, that production costs are zero.

How are goods priced in each niche? How does consumers' utility evolve over time? Notice that active monopolists are always able to charge a price $P_{jt} = a$ that captures the full quality advantage of their product vis-a-vis the best competing product. Innovation does not always *immediately* increase consumers' net utility flow. Specifically, when an innovator enters a non-monopolized niche, consumers enjoy the greater quality of the new good but also pay a higher price, so their net utility gain is zero. The increase in consumers' net utility occurs later, when the monopolized niche experiences the entry of a competitor of either equal quality (an imitator) or greater quality (an innovator). Consumers enjoy then an extra surplus of a per period for all periods ahead either because of the smaller price (zero) paid for the same good (after imitation) or because they pay the same price for a higher quality good (after innovation).

In this setup, consumers' net utility in the steady-state equilibrium grows linearly over time. As a stationary social welfare measure, following Hopenhayn et al. (2006), we use the present value of consumers' *incremental* net utility flows due to the imitation and innovation completed in a typical date,

$$W_{ss} = \{\psi + (1 - \psi)(1 - \lambda_2)q_{ss}\}x_{ss}\frac{a}{1 - \beta}. \tag{13}$$

To explain (13), notice that utility additions only occur over the measure x_{ss} of monopolized business niches and are associated with either imitation, which occurs at rate ψ over those niches, or innovation, which occurs at rate $(1 - \lambda_2)q_{ss}$ over the remaining proportion $1 - \psi$. Both entry processes imply a perpetual addition with discounted value of $a/(1-\beta)$ to consumers' net utility flow.

From (13) we can decompose the total effect of any model parameter θ on social welfare in up to a direct effect and two indirect effects channeled through the steady-state variables x_{ss} and q_{ss}:

$$\frac{dW_{ss}}{d\theta} = \frac{\partial W_{ss}}{\partial \theta} + \frac{\partial W_{ss}}{\partial x_{ss}}\frac{dx_{ss}}{d\theta} + \frac{\partial W_{ss}}{\partial q_{ss}}\frac{dq_{ss}}{d\theta}, \qquad (14)$$

where $\partial W_{ss}/\partial x_{ss} = W_{ss}/x_{ss} > 0$ and $\partial W_{ss}/\partial q_{ss} = (1-\psi)(1-\lambda_2)x_{ss}a/(1-\beta) > 0$. The next proposition builds on this decomposition to establish the global welfare effect of the main parameters.

Proposition 2. *Social welfare increases in the incremental value of innovation a and the discount factor β, and decreases in the innovation cost Φ and the protection of IP against further innovation λ_2. The effect of the effective imitation risk ψ is, in general, ambiguous.*

The positive effects of a, β, and Φ are self-explanatory once we recall the effects of these parameters on the steady-state flow of innovation and the stock of active patents, and notice that their direct effect on our welfare measure is either of the same positive sign as the indirect effects (a and β) or zero (Φ).

The generic ambiguity of the welfare effect of $\psi \equiv (1-\lambda_1)\delta$ can be illustrated using numerical examples like those in Figure 3, where the dashed line, generated with $\lambda_2 = 0$, is monotonically increasing in λ_1 while the solid curve, generated with $\lambda_2 = 0.5$, has an inverted-U shape with an interior welfare-maximizing λ_1. The potential non-monotonicity is due to the combination of various forces. First, restricting imitation has a *negative* direct effect on social welfare because it slows down the process whereby consumers attain the price reductions associated with the successful imitation of products sold in monopolized niches. Second, there is a *positive* effect channeled through x_{ss} because the protection of incumbents against imitators contributes to sustain a larger stock of active patents. Finally, there is a per se *ambiguous* effect channeled through q_{ss} which was already illustrated in Figure 2: Imitation has a potentially non-monotonic effect on the innovation
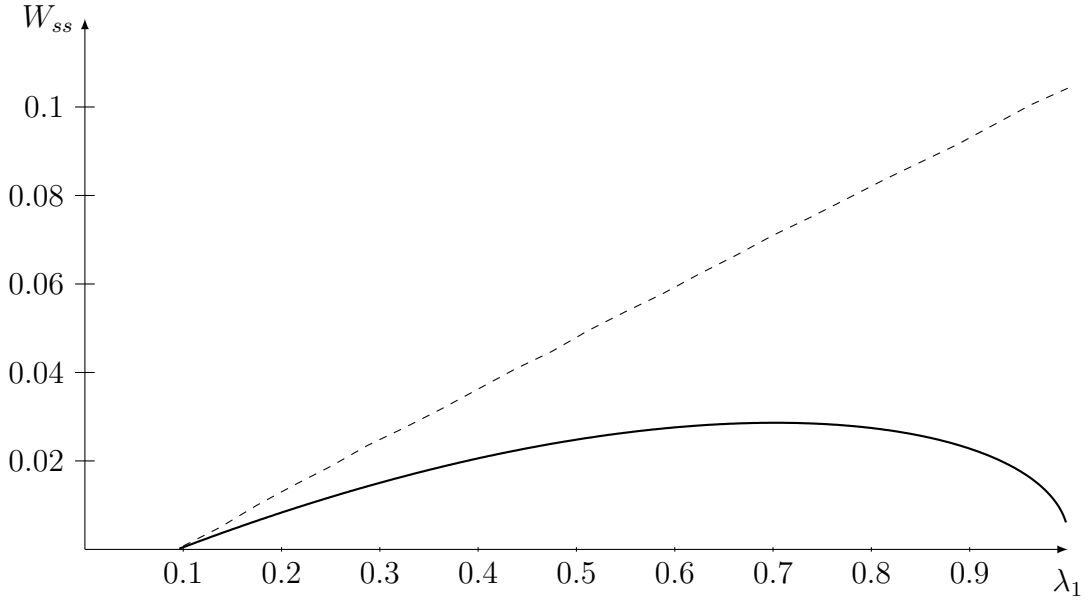
16

Figure 3: Steady-state welfare and imitation risk. This graph depicts W_{ss} as a function of λ_1. The solid and dashed curves correspond to the cases $\lambda_2 = 0.5$ and $\lambda_2 = 0$, respectively. Other parameters are set as in Figure 2.

flow because, on the one hand, it frees up niches from protected incumbents, facilitating entry, but, on the other, it erodes the value of incumbency and, hence, the incentives to enter.

In the case of the protection of IP against further innovation, λ_2, there is no ambiguity. Proposition 2 shows that increasing λ_2 reduces welfare: It has a negative direct effect because, for a given flow of innovative entry q_{ss}, a higher λ_2 implies a lower rate of *successful* entry. And it also has negative welfare effects channeled through the increase in the proportion of monopolized niches x_{ss} and the reduction of the innovation flow q_{ss} (recall Proposition 1). Interestingly, the ambiguity about the effects of imitation risk on welfare disappears when the protection of IP against further innovation is optimally set.

Proposition 3. *If the protection of IP against innovation is set at its socially optimal value of zero ($\lambda_2 = 0$), then it is socially optimal to grant innovators maximum protection against imitation ($\lambda_1 = 1$). In this social optimum, the innovation flow q_{ss} is also maximized.*

For suboptimal levels of protection against innovation ($\lambda_2 > 0$), the optimal protection
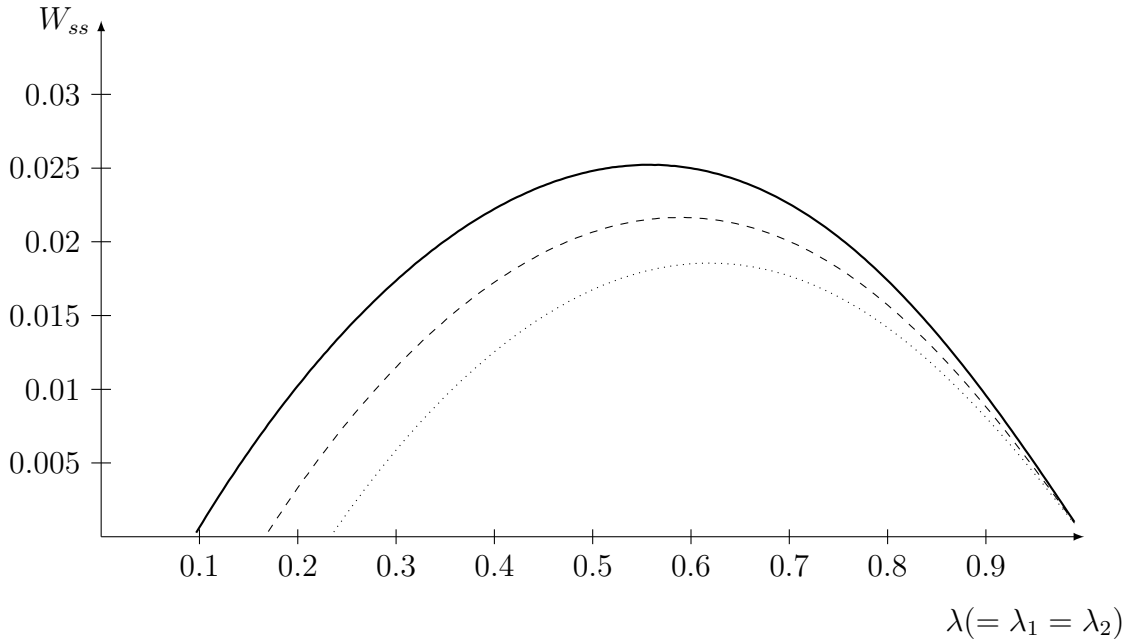
17

Figure 4: Steady-state welfare and IP protection for $\lambda_1 = \lambda_2 = \lambda$. The solid curve corresponds to the baseline economy with $\Phi = 0.15$. The dashed and dotted curves correspond to $\Phi = 0.2$ and $\Phi = 0.25$, respectively. Other parameters are set as in Figures 2 and 3.

against imitation is not necessarily maximal ($\lambda_1 = 1$). As illustrated by the solid line in Figure 3 (which uses the same parameterization as Figure 2), the socially optimal value of λ_1 can be interior and, in this case, it will generally be lower than the value of λ_1 that maximizes the innovation flow q_{ss}.[14] This occurs because a reduction of λ_1 in the margin, in spite of reducing innovation, has a positive effect on the speed at which innovative products become cheaper to consumers.

The discussion on the socially optimal level of protection against imitation becomes specially relevant when the social planner cannot control λ_1 and λ_2 independently, say because it is legally difficult to clearly distinguish between imitation and innovation, and the patent statute may hinder both at the same time, making λ_1 and λ_2 comove in positively correlated manner. What will happen then? Figure 4 provides an example. Its solid line corresponds to a parameterization that imposes $\lambda_1 = \lambda_2 = \lambda$ while the rest of the parameters take the same values as in previous figures.[15] In this case, the overall

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[14]When W_{ss} is concave in λ_1, like in the solid curve of Figure 3, it is possible to prove that $dq_{ss}/d\lambda_1 > 0$ at the (welfare maximizing) point in which $dW_{ss}/d\lambda_1 = 0$.

[15]Qualitatively, the results would be similar if λ_2 were made an arbitrary differentiable and increasing

degree of IP protection reproduces the type of inverted-U shaped effect on social welfare that appeared in the solid line of Figure 3. Intuitively, social welfare is maximized at a level of protection intermediate between the protection that innovation should receive against innovation ($\lambda_2 = 0$) and against imitation ($\lambda_1 = 1$), if they could be controlled separately.

5   Endogenous Imitation

In this section we endogenize the intensity of the imitation threat, so far captured by the exogenous constant probability δ with which a niche monopolized by the holder of an active patent is challenged by an imitator. Since in general this probability may not be constant over time, we will denote it by δ_t. We assume, using a parallelism with the entry of innovators, that there is an infinite supply of potential imitators whose entry is subject to congestion and cannot be targeted to a specific market niche.

We further assume that an imitator that enters a competitive niche obtains zero profits, while entry in a monopolized niche yields a per-period profit $\varepsilon \in (0, a)$ until the niche experiences further imitation or innovation.[16] This assumption captures the idea that, only in this last case, the quality improvement relative to the challenged product is sufficient to provide a (small) positive profit to the imitator.[17] Opposite to innovators, the temporary monopoly position of successful imitators is assumed to feature no legal protection against any form of future entry.

We assume that imitation entails the non-pecuniary entry cost Φ but does not require any development investment. As in the case of innovation, entry occurs one period after incurring the entry cost. Finally, to model congestion in the imitation process, we assume, as in the case of innovative entry, that if the flow of imitators in period t is e_t^i, the

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function of λ_1.

[16]Imitators could also enter an empty niche, if there were any, in which case imitation would not be possible. But we ignore this possibility because it will never occur in and around the steady state.

[17]Competitive niches are those where at least an imitator has been successful in the past, so our assumption is consistent with the idea that the returns to subsequently successful imitation are declining in the length of the imitating chain. Our results would be robust to allowing subsequent imitation to yield profits which are positive but smaller than ε.

proportion of niches challenged by imitators is

$$\delta_t = \frac{e_t^i}{1 + e_t^i}. \tag{15}$$

Hence, the probability that an imitator succeeds in entering a monopolized niche can be written as

$$p_t^i = \frac{1}{1 + e_t^i} x_{t-1}(1 - \lambda_1)(1 - q_t) = x_{t-1}(1 - \lambda_1)(1 - \delta_t)(1 - q_t), \tag{16}$$

where the factors entering after the first equality are the probabilities of succeeding among simultaneous imitations, being assigned to a monopolized niche, succeeding in court against the established monopolist, and not being replaced by an innovator before the end of the entry period. The last equality is written using (15).

The present value of the profits of a successful imitator, v_t^i, can be found from

$$v_t^i = \varepsilon + \beta(1 - \delta_{t+1})(1 - q_{t+1})v_{t+1}^i, \tag{17}$$

where the discounting of future profits differs with respect to the case of an innovator (equation (1)) in that an imitator lacks IP protection. Finally, the free-entry condition for an imitator is

$$\beta p_t^i v_t^i - \Phi \leq 0, \tag{18}$$

and the complementary slackness condition for the imitation flow imposes $\delta_t(\beta p_t^i v_t^i - \Phi) = 0$, so that δ_t is zero if the net present value of imitative entry is negative.

With these elements in place, we can state the following result:

Proposition 4. *Extending the conditions (9)-(11) that describe the steady-state equilibrium of the model to the case in which imitation is endogenous only requires replacing ψ with $(1 - \lambda_1)\delta_{ss}$ and adding*

$$\delta_{ss} = \frac{(1 - x_{ss})[\beta\varepsilon(1 - \lambda_1)x_{ss} - (1 - \beta)\Phi]}{\Phi(1 - \lambda_1)x_{ss} + [\beta\varepsilon(1 - \lambda_1)x_{ss} + \beta\Phi](1 - x_{ss})}, \tag{19}$$

to the system of equilibrium conditions.

Notice that (19) describes a relationship between the steady-state flow of imitative entry and the proportion of niches monopolized by successful innovators. This relationship adds an *imitation curve* to the free-entry curve and the present-value curve that allowed us to solve for x_{ss} and v_{ss} in the baseline model (recall Figure 1).[18] The extended system of steady-state conditions is still recursive in that (10), (11), and (19) allow to first solve for x_{ss}, v_{ss} and δ_{ss}, and then use (9) to find q_{ss}.

Social welfare W_{ss} can be computed in a way similar to what has been described for the baseline model (equation (13)). The only difference is that now the turnover associated with the entry of imitators or innovators in niches monopolized by imitators also contributes to welfare, which introduces a new term in the expression for W_{ss}, which becomes

$$
\begin{aligned}
W_{ss} \;=\; & \{(1 - \lambda_1)\delta_{ss} + [1 - (1 - \lambda_1)\delta_{ss}](1 - \lambda_2)q_{ss}\}x_{ss}\frac{a}{1 - \beta} + \\
& + \; [\delta_{ss} + (1 - \delta_{ss})q_{ss}]\,(1 - x_{ss})\frac{(1 - \lambda_1)\delta_{ss}x_{ss}(1 - q_{ss})}{q_{ss} + \delta_{ss}(1 - q_{ss})}\frac{\varepsilon}{1 - \beta}.
\end{aligned} \tag{20}
$$

The first term captures, as before, the gains originated when niches monopolized by previously successful innovators experience the entry of new innovators or imitators, and the new second term captures the additional gains originated when niches monopolized by previously successful imitators are successfully challenged by either an innovator or a second imitator.[19]

The effects of changes in the IP protection parameters, λ_1 and λ_2, are discussed next. We start with the protection against innovation, λ_2, because in this case the effects are simpler to summarize. Figure 5 describes the variables δ_{ss}, x_{ss}, q_{ss}, and W_{ss} as functions

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[18]Numerical examples show that (19) describes an inverse U-shaped relationship between x_{ss} and δ_{ss}. When x_{ss} is low, the returns from imitation are small, since the probability that a firm occupies a previously monopolized niche is small. When x_{ss} is very large, however, the steady-state innovation flow q_{ss} is large, which reduces the expected duration of the span for which an imitator reaps profits of ε, so again the expected returns from imitation are small.

[19] The new term is proportional to the discounted value of the permanent quality improvement ε brought by each imitator. The square brackets in the second term contain the probability with which one of those niches experiences imitative or innovative entry. The term $1 - x_{ss}$ accounts for the proportion of niches not monopolized by innovators and the fractional factor corresponds to the proportion of those niches which are occupied by unchallenged imitators in steady state.
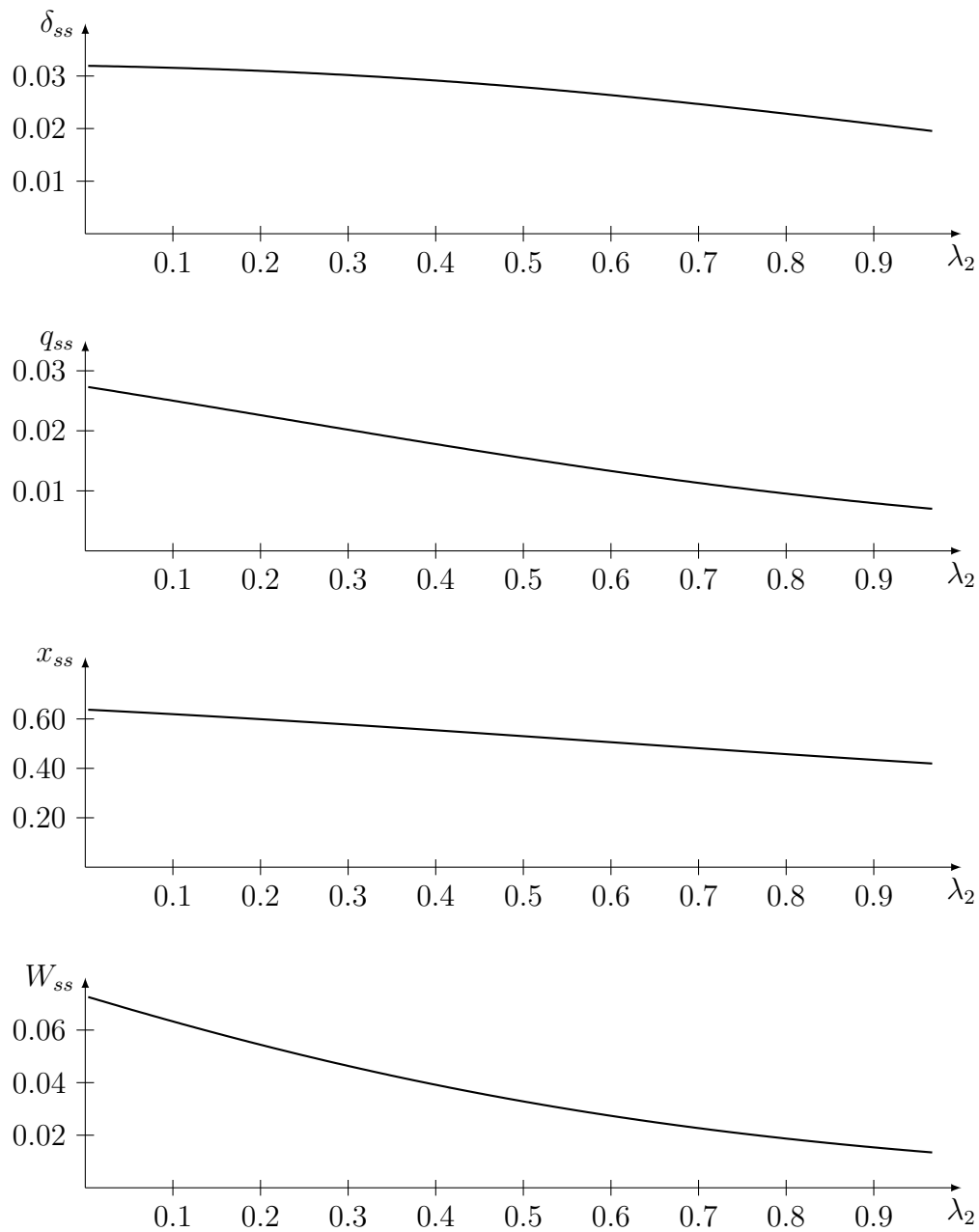
Figure 5: Endogenous imitation and IP protection against innovation. The various panels depict the steady-state values of imitation, innovation, proportion of monopolized niches, and welfare as functions of λ_2. The underlying parameters are $a = 0.1$, $\beta = 0.96$, $\varepsilon = 0.05$, and $\Phi = 0.15$. The protection against imitation is $\lambda_1 = 0.5$.

of λ_2. It shows that increasing λ_2 is detrimental to the imitation flow δ_{ss}, while all other variables behave qualitatively as in the baseline model, including social welfare W_{ss} and the innovation flow q_{ss} which are both maximized with $\lambda_2 = 0$.

Figure 6 shows the effects of varying the protection against imitation, λ_1, and, as in Figures 2 and 3, we depict simultaneously the cases with $\lambda_2 = 0$ (the dashed curves) and $\lambda_2 = 0.5$ (the solid curves). Importantly, the key result from the baseline model that, if λ_2 is set equal to its socially optimal value of zero, then welfare is maximized under the maximal protection against imitation ($\lambda_1 = 1$) is confirmed. It is also the case that the combination $(\lambda_1, \lambda_2) = (1, 0)$ maximizes the innovation flow. However, for $\lambda_2 > 0$, the maximum values of welfare and innovation may be reached, as in the baseline model, for values of λ_1 compatible with positive levels of imitation.

Figure 6 also contains information about the non-trivial interactions between imitation and innovation implied by the model. For instance, the effect of λ_1 on the imitation flow δ_{ss} is non-monotonic in the curve generated under $\lambda_2 = 0$. More generally, for a large range of (low) values of λ_1, increasing λ_1 causes very little impact on δ_{ss}. What is behind is an effect that static models cannot capture. Increasing the protection of IP against imitation increases the steady-state proportion of business niches monopolized by holders of active patents, x_{ss}, and this makes the entry of imitators *more* rather than *less* attractive. For sufficiently high values of λ_1, the discouragement of imitation due to the low prospects of successfully entering those niches dominates the appeal of challenging them with greater probability. Then δ_{ss} quickly declines to zero. Simultaneously, the steady-state proportion of monopolized niches becomes one, because there is no longer attrition due to imitation but only turnover due to innovation.[20]

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[20]Describing steady-state once the limit $x_{ss} = 1$ is reached requires considering an alternative writing of the state-state equations of the model, since q_{ss} can no longer be recovered using (9). Interestingly, equations (3)-(5) do no longer depend on λ_1 if $x_{t-1} = 1$ and $\delta = 0$. This allows to analytically solve for the equilibrium values of v_t and q_t. Such q_t corresponds to the value of q_{ss} in the flat section of the corresponding curves in Figure 6.
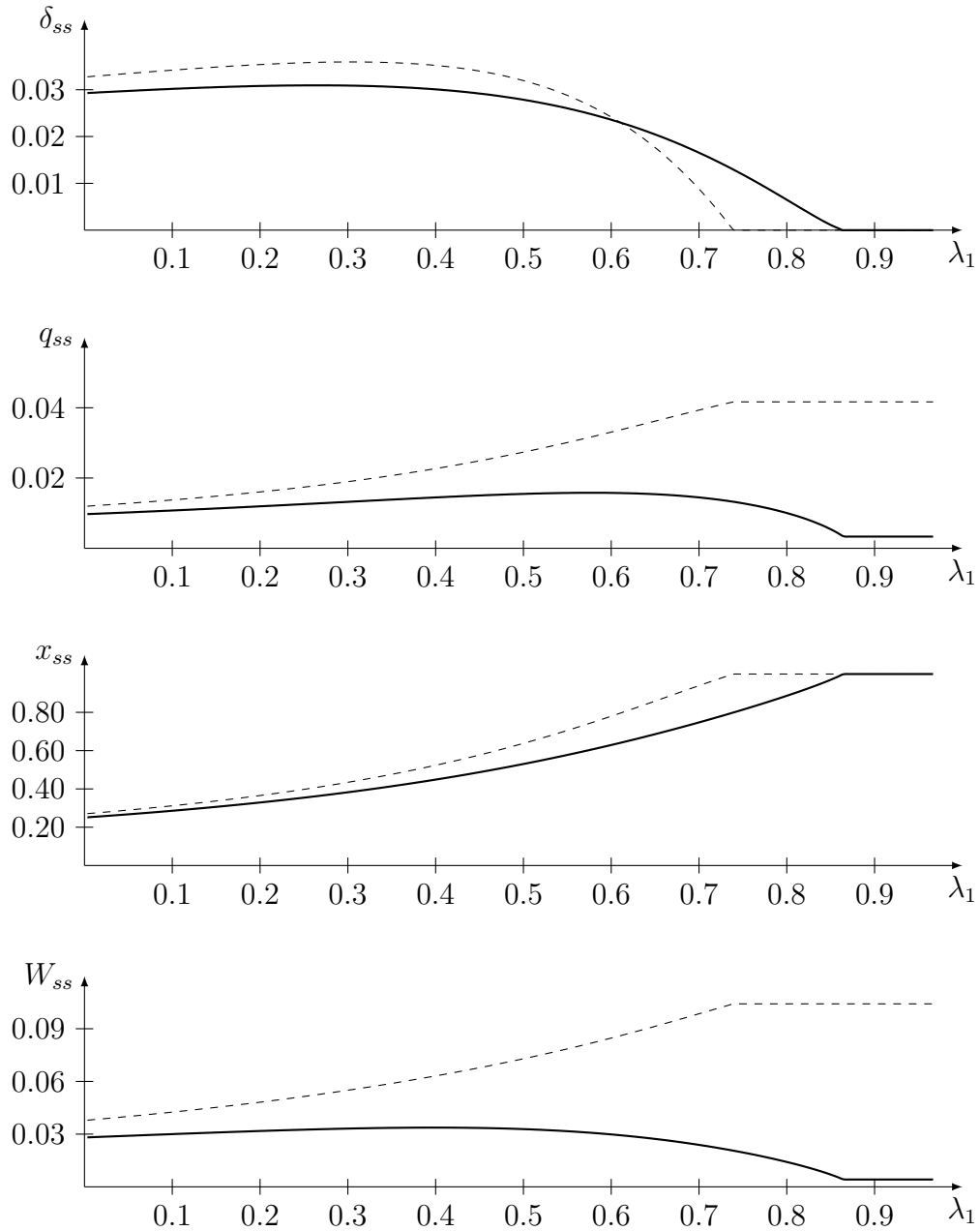
Figure 6: Endogenous imitation and IP protection against imitation. The various panels depict the steady-state values of imitation, innovation, proportion of monopolized niches, and welfare as functions of λ_1. The solid and dashed curves correspond to the cases with $\lambda_2 = 0.5$ and $\lambda_2 = 0$, respectively. Other parameters are set as in Figure 5.

6   Innovation with External Financing Frictions

In this section we extend the model to consider the effects of frictions in the external financing of innovation. The existing literature on the financing of innovative start-ups acknowledges the importance of financial frictions and the way in which access to informal sources of capital (friends and relatives, business angels) and venture capital compensates for the lack the collateral typically required for the access to bank loans. However, the connection between financial constraints and IP protection has not been explicitly explored.[21] So this section is an attempt to connect two essentially divorced literature traditions.

To introduce financial frictions in our baseline model, we assume that the potential entrepreneurial innovators are penniless and subject to limited liability. Each innovator can incur the non-pecuniary cost Φ and engender an invention without any external support. However, the full development of the invention by the innovator ("in-house development") would require her to incur a pecuniary cost (normalized to one) which has to be externally financed.

To rationalize the partial in-house development of the innovation (and the use of licensing as a remedy to financial constraints), we assume that the invention can be developed into a potentially marketable new product using a measure-one continuum of alternative development paths. For simplicity, we assume that only one of these paths can lead to a new marketable product at t, and ex-ante all paths are equally likely to lead to such a product. The new product will give to its developer the chance to occupy a niche in the industry as in the baseline version of the model. So a successful developer has a probability p_t of obtaining a monopoly position with value v_t.

For each path, the innovator has a choice between in-house development and the licensing to an outside developer with deep pockets. When the development of a path is

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[21]Most papers consider the traditional partial equilibrium setup of corporate finance and focus on understanding specific features of startup financing such as the staging of finance (Gompers (1995) and Neher (1999)), the use of convertible securities (Casamatta (2003) and Schmidt (2003)), or venture capital contracting (Repullo and Suarez (2004)). Some papers, including Holmstrom and Tirole (1997), Inderst and Muller (2004), and Michelacci and Suarez (2004), examine the equilibrium implications of financial constraints, but do not discuss IP protection.

licensed the pecuniary cost is assumed to be $1 + c$, where $c > 0$ reflects some friction in the transferring of the relevant technology to the licensee.[22]

The external financing of in-house development is affected by a moral hazard problem as in Holmstrom and Tirole (1997). Specifically, the innovator has an unobservable choice between two development effort levels that determine the probability of success in the development of the paths under her management (conditional on one of the paths being the one that leads to the marketable product). To simplify the notation, we assume that such probability of success is one under high effort and zero under low effort, but under the latter the innovator obtains a non-verifiable private benefit of $b > 0$ per path under her development.[23]

We assume that parameter values are such that $\beta p_t v_t > b$ at all dates, so that exerting higher effort is first-best optimal. However, an innovator that pledges a sufficiently large fraction of future profits to external financiers may be tempted to exert low effort, rendering the financing deal unfeasible.

When licensing a path, the innovator fully relinquishes its development to a licensee who, if successful, appropriates the profits generated by the new product. We assume that there is a large pool of potential licensees with deep pockets (e.g. incumbent firms with internal funds) so that the development of the paths licensed to them involves no moral hazard. By virtue of competition, these licensees pay royalties to the innovator equal to the whole expected net present value of the external development of each path.

In order to guarantee that licensing can ameliorate the moral hazard problem of the innovator, we assume $b > c$. The following lemma summarizes the (partial equilibrium) outcomes of external financing problem that we have just set:

Lemma 2. *In the setup with external financing frictions, if $\beta p_t v_t - 1 \geq b$, entering innovators develop their inventions fully in-house, obtaining a net payoff $\beta p_t v_t - (1 + \Phi)$.*

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[22]The cost c may capture the cost of acquiring some relevant know-how that the innovator already possesses as well as the legal costs associated with licensing the corresponding development path.

[23]The same results hold if the success probability under low effort is $1 - \Delta$, with $\Delta \in (0, 1)$. All final equations are the same except for the fact that b must be replaced by b/Δ.

If $c \leq \beta p_t v_t - 1 < b$, they out-license a fraction

$$\alpha_t = \frac{b - (\beta p_t v_t - 1)}{b - c} \tag{21}$$

of the development paths and develop the remaining fraction in-house, obtaining a net payoff $(1 - \alpha_t)b - \Phi$. Finally, if $\beta p_t v_t - 1 < c$, developing the innovation is unfeasible.

The parameters related to the moral hazard problem, b, and the technology transfer cost, c, play a crucial role in the partial equilibrium results shown in this lemma. When the net present value of diligent in-house development, $\beta p_t v_t - 1$, is larger than b, full in-house development is feasible and, hence, optimal. When it is smaller than b but larger than c, licensing becomes part of the second-best solution. If this present value is smaller than c, the development of the innovation becomes nonviable.[24] In the case with non-trivial licensing, the optimal licensed fraction α_t is increasing in b and c, and decreasing in $\beta p_t v_t - 1$.

The following proposition shows that there is a mapping between our baseline model and this setup for the intermediate case ($c \leq \beta p_t v_t - 1 < b$) in which external financing frictions alter but do not impede the development of entrepreneurial innovations.

Proposition 5. *In the setup with external financing frictions, if $b > \Phi$, then entering innovators out-license a fraction $\alpha = 1 - \frac{\Phi}{b}$ of the development paths of their inventions. Around the steady state, the equilibrium conditions and the existence condition (12) are the same as in the baseline model except in that the entry cost parameter Φ has to be replaced by $\hat{\Phi} = \Phi + (1 - \frac{\Phi}{b})c$.*

As in the baseline version of the model, innovators reap all the present value of the inventions, net of pecuniary and non-pecuniary costs. However, the net present value appropriated by the innovator is smaller because of the technology-transfer costs $c\alpha_t$. Innovators' free entry condition makes the licensing decision in equilibrium equal to the

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[24] So $\beta p_t v_t - 1 < c$ can only possibly occur in periods with no innovative entry. This situation cannot occur in a steady state with $x_{ss} > 0$ but it might occur in the transition to such a steady state if the industry starts with some pre-determined x_{t-1} sufficiently larger than x_{ss}. See footnote 13.

same constant $\alpha = 1 - \frac{\Phi}{b}$ in all periods with positive entry, so the effective transfer costs become $\left(1 - \frac{\Phi}{b}\right) c$. It turns out that these costs enter the innovator's problem in exactly the same way as Φ in the baseline model. Thus, all results go through if Φ is replaced by $\hat{\Phi} = \Phi + (1 - \frac{\Phi}{b})c$, which is increasing in Φ, b, and c.

The following result is a corollary of Propositions 1 and 2:

Proposition 6. *In the setup with external financing frictions, if $b > \Phi$, an increase in the severity of the frictions (i.e. increasing b or c) produces a reduction in the steady-state levels of innovative entry q_{ss}, the mass of active patents x_{ss}, and social welfare W_{ss}, while it increases the technology transfer costs $(1 - \frac{\Phi}{b})c$ and the profits from incumbency v_{ss} of each innovator.*

Regarding the socially desirable levels of IP protection, the conclusions of Propositions 2 and 3 remain valid. Yet, financial constraints have the potential to modify the trade-offs concerning the optimal degree of protection against imitation (or the overall level of IP protection) when the level of protection against innovation is not zero (or cannot be set separately).

To keep the discussion brief, we focus on the effects of changing Φ in the scenario with $\lambda_1 = \lambda_2 = \lambda$ already explored in Figure 4. The solid line corresponds to the previously discussed baseline case without financial constraints (in which $a = 0.1$, $\beta = 0.96$, $\delta = 0.05$ and $\Phi = 0.15$). In terms of the extended model, such case can be understood as a situation with $b \leq \Phi$, where by Lemma 2 the financial constraints are not binding and the innovator does not need to license her innovation ($\alpha = 0$). The dashed and dotted curves correspond to the cases with $\hat{\Phi} = 0.2$ and $\hat{\Phi} = 0.25$, respectively. For a reference technology transfer cost of $c = 0.1$, these values of $\hat{\Phi}$ would correspond to situations with $b = 0.3$ ($\alpha = 0.5$) and $b \to \infty$ ($\alpha = 1$), respectively.

In addition to illustrating the detrimental welfare effect of financial constraints, Figure 4 reveals that tightening the constraints, when binding, increases the socially optimal value of the overall IP protection parameter λ. The reason why this effect occurs is different

from what transpires of a typical static analysis. Remember that in our model increasing λ_2 alone discourages innovation (and reduces welfare), so it must be that tightening the financial constraints increases the net relative advantages of protecting patent holders against imitation.

Tighter financial constraints reduce per se the steady state fraction of monopolized niches and, hence, the hurdle for innovative entry coming from the opposition of the incumbents. This makes the traditional patent value enhancing effect of fighting imitation relatively more important, explaining why the socially optimal value of λ increases.

7 Concluding Remarks

Innovation is considered key to industry dynamics. Entry, exit, and innovation are complex interrelated phenomena in every industry, and especially so in the youngest and more technology-intensive industries. Many of these industries rely on IP as the source of temporary monopoly power that allows the successful innovators to obtain a return for their previous R&D investments. IP protection, however, is a double cutting edge knife for the dynamics of innovative industries, as the protection of incumbent innovators may be an obstacle to the success of novel innovators.

This paper contributes to the growing literature that analyzes the role of IP protection by embedding it in an industry dynamics setting in which innovation and imitation are different, interrelated processes modeled along similar lines. We find that welfare and innovation are maximized with zero protection against further innovation and, conditional on this, with full protection against imitation. However, if some protection against innovation is unavoidable, allowing for some imitation may be socially beneficial. These results are robust to endogenizing imitation and get reinforced when entrepreneurs are financially constrained.

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Appendix

Proof of Lemma 1: This proof has two parts. First we discuss the uniqueness and existence of a SS equilibrium. Then we discuss the local stability of the SS equilibrium.

Existence and uniqueness of the SS equilibrium: For brevity we eliminate the subscripts from x_{ss} and v_{ss} and rewrite (10) and (11) abstractly as:

$$
\begin{aligned}
f_1(x, v; \theta) &= 0, & (22) \\
f_2(x, v; \theta) &= 0, & (23)
\end{aligned}
$$

where θ is the vector of parameters of the model. We will save on notation by referring to a single parameter $\psi \equiv (1 - \lambda_1)\delta$ rather than δ and λ_1 separately.

To establish the sign of the monotonic relationship between x_{ss} and v_{ss} in each of the equations, notice that

$$
\frac{\partial f_1}{\partial v} = \beta(1 - x)\frac{1 - \lambda_2(1 - \psi)x}{1 - (1 - \psi)x} > 0,
$$

and

$$
\frac{\partial f_1}{\partial x} = \beta v \frac{\lambda_2(1 - \psi)[2x - (1 - \psi)x^2] - (1 - \lambda_2)\psi - \lambda_2}{[1 - (1 - \psi)x]^2}.
$$

The numerator in the last expression is increasing in x and, hence, maximum at $x = 1$, but if we evaluate the numerator at $x = 1$ we obtain

$$
-\lambda_2\psi^2 - (1 - \lambda_2)\psi < 0,
$$

so $\frac{\partial f_1}{\partial x} < 0$ for all x. This implies that (10) defines an upward slopping curve in (x_{ss}, v_{ss}) space. Moreover, it is immediate to check that v_{ss} goes to infinity as x_{ss} approaches one.

As for (11), it can be verified that

$$
\frac{\partial f_2}{\partial x} = \frac{\beta\psi(1 - \psi)(1 - \lambda_2)}{1 - (1 - \psi)x}v > 0
$$

and

$$
\frac{\partial f_2}{\partial v} = 1 - \beta(1 - \psi)\frac{1 - (1 - \lambda_2\psi)x}{1 - (1 - \psi)x} > 0,
$$

so (11) describes a downward slopping curve. Obviously, the upward and downward sloping curves just described can intersect at most once and such an intersection, if it exists, defines the unique SS equilibrium. Since (10) has a vertical asymptote at $x = 1$, the necessary and sufficient condition for existence of the SS equilibrium is that the intercept of (10), $a/[1 - \beta(1 - \psi)]$, is lower than the intercept of (11), $(1 + \Phi)/\beta$, which explains condition (12).

Stability of the SS equilibrium: To analyze the local stability of the system around steady state, we proceed to log-linearize (7) and (8) around the SS point (v, x). Log-linearizing (7) yields

$$
-\frac{1}{1 - x}dx_t + \frac{(1 - \psi)(1 - \lambda_2)}{[1 - (1 - \lambda_2\psi)x][1 - (1 - \psi)x]}dx_{t-1} + \frac{1}{v}dv_t = 0.
$$

Log-linearizing (8) yields

$$
-\frac{1 - \lambda_2}{1 - (1 - \lambda_2\psi)x}dx_t + \frac{1}{v}dv_t - \frac{1}{v - a}dv_{t-1} + \frac{(1 - \psi)(1 - \lambda_2)(1 - x)}{[1 - (1 - \lambda_2\psi)x][1 - (1 - \psi)x]}dx_{t-1} = 0
$$

These expressions can be written as the following system of equations

$$dx_t - \frac{1-x}{v}dv_t = \frac{(1-\psi)(1-\lambda_2)(1-x)}{[1-(1-\lambda_2\psi)x][1-(1-\psi)x]}dx_{t-1},$$

$$-\frac{(1-\lambda_2)v}{(1-(1-\lambda_2\psi)x)}dx_t + dv_t = -\frac{(1-\psi)(1-\lambda_2)(1-x)v}{[1-(1-\lambda_2\psi)x][1-(1-\psi)x]}dx_{t-1} + \frac{v}{v-a}dv_{t-1},$$

or in matrix form as

$$\begin{bmatrix} 1 & w_{12} \\ w_{21} & 1 \end{bmatrix} \begin{bmatrix} dx_t \\ dv_t \end{bmatrix} = \begin{bmatrix} z_{11} & 0 \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} dx_{t-1} \\ dv_{t-1} \end{bmatrix} \tag{24}$$

where

$$
\begin{aligned}
w_{11} &= 1, & z_{11} &= \frac{(1-\psi)(1-\lambda_2)(1-x)}{[1-(1-\lambda_2\psi)x][1-(1-\psi)x]} > 0, \\
w_{12} &= -\frac{1-x}{v} < 0, & z_{12} &= 0, \\
w_{21} &= -\frac{(1-\lambda_2)v}{1-(1-\lambda_2\psi)x} < 0, & z_{21} &= -\frac{(1-\psi)(1-\lambda_2)(1-x)v}{[1-(1-\lambda_2\psi)x][1-(1-\psi)x]} < 0, \\
w_{22} &= 1, & z_{22} &= \frac{v}{v-a} > 1.
\end{aligned}
$$

Pre-multiplying both sides of (24) by the inverse of matrix W and pre-multiplying both sides of by it, the system becomes

$$\begin{bmatrix} dv_t \\ dx_t \end{bmatrix} = Y \begin{bmatrix} dv_{t-1} \\ dx_{t-1} \end{bmatrix},$$

with

$$Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \equiv \frac{1}{1-w_{12}w_{21}} \begin{bmatrix} z_{11} - w_{12}z_{21} & -w_{12}z_{22} \\ -w_{21}z_{11} + z_{21} & z_{22} \end{bmatrix}.$$

The two eigenvalues, μ_1 and μ_2, of matrix Y can be found as the solutions to the equation

$$\det(Y - \mu I) = 0$$

where I is the identity matrix of rank 2. Proving saddle-path convergence towards SS in the log-linearized system amounts to showing that, in absolute value, one of the eigenvalues of matrix Y is greater than 1 and the other is less than 1. We will further show that both eigenvalues are positive.

Since the function $D(\mu) \equiv \det(Y - \mu I)$ describes a parabola that tends to infinity when μ tends to both plus and minus infinity, then showing that $D(0) > 0 > D(1)$ would be enough for our proof. Consider first the sign of

$$D(0) = \det(Y) = \frac{z_{11}z_{22}}{1-w_{21}w_{12}}.$$

Clearly, $z_{11}z_{22} > 0$, so proving that $D(0) > 0$ boils down to showing that

$$1 - w_{12}w_{21} = 1 - \frac{1-x}{v}\frac{(1-\lambda_2)v}{1-(1-\lambda_2\psi)x} > 1 - \frac{(1-\lambda_2)(1-x)}{1-(1-\lambda_2\psi)x} = \frac{\lambda_2[1-(1-\psi)x]}{1-(1-\lambda_2\psi)x} \in (0,1). \tag{25}$$

Now, as for

$$D(1) = \det(Y - I) = \frac{(y_{11}-1)(y_{22}-1) - y_{12}y_{21}}{(1-w_{21}w_{12})^2},$$

notice that we can ignore the denominator and prove the negativity of

$$
\begin{aligned}
(y_{11}-1)(y_{22}-1) - y_{12}y_{21} &= [z_{11}-w_{12}z_{21}-(1-w_{12}w_{21})][z_{22}-(1-w_{12}w_{21})] - w_{12}w_{21}z_{11}z_{22} + w_{12}z_{22}z_{21} \\
&= (1-w_{12}w_{21})[w_{12}(z_{21}-w_{21}) - (z_{11}-1)(1-z_{22})].
\end{aligned}
$$

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We already know, from (25), that $(1 - w_{12}w_{21}) > 0$. Moreover, from the expressions above, we clearly have $w_{12} < 0$, $1 - z_{22} < 0$, and

$$z_{21} - w_{21} = \frac{(1 - \lambda_2)\psi v}{[1 - (1 - \lambda_2\psi)x][1 - (1 - \psi)x]} > 0.$$

It only remains to show that $z_{11} - 1 < 0$, where

$$z_{11} - 1 = \frac{-\psi - (1 - \psi)\lambda_2 + 2\lambda_2(1 - \psi)x - \lambda_2(1 - \psi)^2 x^2}{[1 - (1 - \lambda_2\psi)x][1 - (1 - \psi)x]}.$$

The denominator of this expression is clearly positive, while the numerator is maximized at $x = 1$. But at $x = 1$ the denominator becomes $-\psi[1 - \lambda_2(1 - \psi)] < 0$, so the denominator must be negative for all x. □

Proof of Proposition 1: For the sake of brevity, we will refer to the equilibrium equations using the same notation as in the proof of Lemma 1.

Effect of a: The parameter a only operates through equation (11). It is immediate that $\frac{\partial f_2}{\partial a} = -1 < 0$. As a result, increases in a shift upward the curve defined by (11) in Figure 1, resulting in an increase in the SS values of v and x.

Effect of β: The effect of β on x is immediate, since an increase in β produces an upward shift in the curve defined by (10) and a downward shift in the curve defined by (11) in Figure 1. Regarding the effect on v, let us implicitly define $x_2(v; \theta)$ from the equation $f_2(x_2(v; \theta), v; \theta) = 0$, recalling that f_2 is the left hand side of (11). Also, define

$$g(v; \theta) \equiv f_1(x_2(v; \theta), v; \theta), \tag{26}$$

so that v_{ss} solves $g(v; \theta) = 0$. Using the Implicit Function Theorem, it is enough for the result to show that g is increasing in v and decreasing in β. With respect to the first,

$$\frac{\partial g}{\partial v} = \frac{\partial f_1}{\partial v} + \frac{\partial f_1}{\partial x} x_2'(v; \theta) > 0$$

since $\partial x_2/\partial v = -(\partial f_2/\partial x)/(\partial f_2/\partial v) < 0$. Regarding the second, we obtain

$$\frac{\partial g}{\partial \lambda_2} = -\frac{\lambda_2 v\{[v - a - \beta(1 - \psi)v]^2 + \beta^2\psi(1 - \psi)(1 - \lambda_2)v^2\}}{(1 - \psi)[v - a - \beta(1 - \psi)v]^2} < 0,$$

where $\psi \equiv (1 - \lambda_1)\delta$, as already defined in the proof of Lemma 1.

Effect of Φ: The parameter Φ only operates through equation (10). It is immediate that $\frac{\partial f_1}{\partial \Phi} = -1 < 0$. As a result, increases in Φ shift upward the curve defined by (10) in Figure 1, resulting in an increase in v and a decrease in x.

Effect of δ and λ_1: Because of the way δ and λ_1 enter all expressions, the effects of these two parameters are colinear, but with the opposite sign. For brevity, we will refer to the effect of λ_1 only. Similarly to the case of β, the effect of increasing λ_1 on x is immediate from the upward shift of the curve defined by (10) and the downward shift of the curve defined by (11). Regarding the effect on v_{ss}, and using the function g defined in (26) above, it is enough to show that $\partial g/\partial \lambda_1 < 0$. In particular, this derivative can be written as

$$\frac{\partial g}{\partial \lambda_1} = \delta\frac{v[a - (1 - \beta)v]\{-(v - a)[a - (1 - \beta)v] + \beta v[\beta v\lambda_2^2(1 - \psi)^2 - (v - a)(1 + \lambda_2 - 2\varphi\lambda_2)]\}}{(1 - \psi)^2 v[v - a + \beta(1 - \varphi\lambda_2)v]^2}$$

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Notice that $x \in [0, 1]$ and $v \in [a/[1 - \beta(1 - \psi)], a/[1 - \beta(1 - \psi)\lambda_2]]$, so $a - (1 - \beta)v > 0$. Moreover, the last term in the expression in curly brackets will be negative as long as

$$v \geq \frac{a}{1 - \dfrac{\beta\lambda_2^2(1 - \psi)^2}{1 + \lambda_2 - 2\varphi\lambda_2}},$$

which is true since

$$\frac{a}{1 - \dfrac{\beta\lambda_2^2(1 - \psi)^2}{1 + \lambda_2 - 2\varphi\lambda_2}} < \frac{a}{1 - \beta\lambda_2(1 - \psi)} < v.$$

Effect of λ_2: The effect on v is immediate, since an increase in λ_2 entails an upward shift of the two curves depicted in Figure 1. Regarding the effect on x, define $v_2(x; \theta)$ from the equation $f_2(x, v_2(x; \theta); \theta) = 0$, recalling that f_2 is the left hand side of (11). Also, define

$$h(x; \theta) \equiv f_1(x, v_2(x; \theta); \theta),$$

so that x_{ss} solves $h(x; \theta) = 0$. Using the Implicit Function Theorem, it is enough for the result to show that h is decreasing in both x and λ_2. With respect to the first,

$$\frac{\partial h}{\partial x} = \frac{\partial f_1}{\partial x} + \frac{\partial f_1}{\partial v}\frac{\partial v_2}{\partial x} < 0$$

since $\partial v_2/\partial x = -(\partial f_2/\partial v)/(\partial f_2/\partial x) < 0$. Regarding the second, we obtain

$$\frac{\partial h}{\partial \lambda_2} = -\beta(1 - x)x(1 - \psi)\frac{(1 - \beta)[1 - (1 - \psi)x]a}{\{1 - (1 - \psi)x - \beta(1 - \psi)[1 - (1 - \lambda_2\psi)x]\}^2} < 0. \tag{27}$$

$\square$

Proof of Proposition 2: Direct inspection of (13) and the results in Proposition 1 allow us to construct the following table:

	a	β	Φ	ψ	λ_2
$\dfrac{\partial W_{ss}}{\partial \theta}$	$+$	$+$	0	$+$	$-$
$\dfrac{\partial W_{ss}}{\partial x_{ss}}\dfrac{dx_{ss}}{d\theta}$	$+$	$+$	$-$	$-$	$-$
$\dfrac{\partial W_{ss}}{\partial q_{ss}}\dfrac{dq_{ss}}{d\theta}$	$+$	$+$	$-$	$?$	$-$
$\dfrac{dW_{ss}}{d\theta}$	$+$	$+$	$-$	$?$	$-$

The last row sums up the various partial effects and constitutes the proof of the results stated in the proposition. $\square$

Proof of Proposition 3: From Proposition 1 and 2 it is immediate that $\lambda_2 = 0$ leads, for any given value of λ_1 to the highest innovation flow and the maximum welfare. Making $\lambda_2 = 0$ in (10) and (11) simplifies the system of equations and yields the following explicit solution for x_{ss} and v_{ss}:

$$v_{ss} = a + (1 - \psi)(1 + \Phi), \tag{28}$$

$$x_{ss} = \frac{\beta[a + (1 - \psi)(1 + \Phi)] - (1 - \psi)(1 + \Phi)}{\beta[a + (1 + \Phi)(1 - \psi)] - (1 + \Phi)}. \tag{29}$$

After some algebra, it is possible to write q_{ss} and W_{ss} as

$$q_{ss} = 1 - \frac{1 + \Phi}{\beta \left[a + (1 + \Phi)(1 - \psi)\right]}, \tag{30}$$

$$W_{ss} = \left\{1 - \frac{1 + \Phi}{\beta \left[a + (1 + \Phi)(1 - \psi)\right]}\right\} \frac{a}{1 - \beta}, \tag{31}$$

where taking the derivative with respect to ψ directly leads to

$$\frac{\partial q_{ss}}{\partial \psi} = -\frac{(1 + \Phi)^2}{\beta \left[a + (1 + \Phi)(1 - \psi)\right]^2} < 0,$$

$$\frac{\partial W_{ss}}{\partial \psi} = -\frac{(1 + \Phi)^2}{\beta \left[a + (1 + \Phi)(1 - \psi)\right]^2} \frac{a}{1 - \beta} < 0,$$

that is, welfare and the innovation flow increase with the effective imitation risk ψ. Thus the socially optimal (and innovation maximizing) value of λ_1 is zero. $\square$

Proof of Proposition 4: In the steady state, imitative entry must be positive so the free entry condition (18) must hold with equality. Using (16) to substitute for p_t^i, imitators' free entry condition in steady state can be written as

$$\beta x_{ss}(1 - \lambda_1)(1 - \delta_{ss})(1 - q_{ss})v_{ss}^i = \Phi,$$

while the steady-state version of (17) implies

$$[1 - \beta(1 - \delta_{ss})(1 - q_{ss})]v_{ss}^i = \varepsilon.$$

Now, solving for v_{ss}^i in the first of the previous equations and substituting the result into the second yields

$$\frac{1 - \beta(1 - \delta_{ss})(1 - q_{ss})}{\beta x_{ss}(1 - \lambda_1)(1 - \delta_{ss})(1 - q_{ss})}\Phi = \varepsilon. \tag{32}$$

But in steady state we have, as in (9),

$$q_{ss} = \frac{(1 - \lambda_1)\delta_{ss}x_{ss}}{1 - [1 - (1 - \lambda_1)\delta_{ss}]x_{ss}},$$

which can be substituted into (32) to get a relationship between δ_{ss} and x_{ss} only. The solution for δ_{ss} in that expression yields (19). $\square$

Proof of Lemma 2: In the setup just described, it is optimal for the innovator to undertake as much development as financially feasible in-house. Since the development technology is linear, the division of licensed paths across (one or more) licensees is irrelevant and the licensing decision can be simply summarized by the total proportion of out-licensed paths, $\alpha_t \in [0,1]$ Licensing helps the innovator solve her financial problem in two ways: first, by reducing the scale of the in-house development problem and, thus, the implied financing needs to $1 - \alpha_t$, and, second, by allowing her to use the royalty proceeds $T_t = \alpha_t[\beta p_t v_t - (1 + c)]$ to cover internally some of those needs.[25]

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[25]Due to this revenue, the licensing of the paths that the entrepreneur does not develop in-house, if feasible, clearly dominates the alternative of leaving some paths undeveloped.

If the innovator pledges to her financiers a part $R_t \leq v_t$ of the discounted value of the conditional-on-success profits of any product arising from the paths left for in-house development, her incentive compatibility condition for exerting high effort can be written as

$$(1 - \alpha_t)\beta p_t(v_t - R_t) \geq (1 - \alpha_t)b, \tag{33}$$

while the individual rationality condition of the competitive financiers is

$$(1 - \alpha_t)\beta p_t R_t \geq (1 - \alpha_t) - T_t. \tag{34}$$

Obviously, it will always be optimal for the innovator to choose R_t so as to make (34) hold with equality. But then, using the resulting equality together with the expression for T_t to substitute for R_t in (33), we can conclude that a licensing decision $\alpha_t \in [0, 1]$ is feasible if and only if

$$\beta p_t v_t - 1 - c\alpha_t \geq (1 - \alpha_t)b, \tag{35}$$

where the left hand side is the total net present value appropriated by the innovator if she chooses high effort, and the right hand side is what she could get by choosing low effort.

Clearly, (35) is easier to satisfy with larger values of α_t insofar as $c < b$, as we have assumed. The optimal value of α_t is the lowest number in the range $[0, 1]$ that satisfies (35), if it exists. Notice that when $\beta p_t v_t - 1 \geq b$, the incentive compatibility constrained written in (33) holds for $\alpha_t = 0$ and, thus, the first-best allocation (full in-house development) is feasible and, therefore optimal, yielding net gains from innovative entry equal to $\beta p_t v_t - (1 + \Phi)$. When $c \leq \beta p_t v_t - 1 < b$, there always exists a unique $\alpha_t \in (0, 1)$ for which (33) holds with equality. Any other feasible α would be larger and, from the arguments given in the text, suboptimal. Profits under α_t can be computed as

$$\beta p_t v_t - 1 - c\alpha_t - \Phi = (1 - \alpha_t)b - \Phi, \tag{36}$$

where the last equality arises, again, from (33). □

Proof of Proposition 5: From (36), the net gains from entry in the case where licensing occurs in equilibrium can be rewritten as $\beta p_t v_t - (1 + \Phi) - c\alpha_t$. Therefore, when the external financing frictions lead to licensing, the counterpart of the free-entry conditions (4) and (6) are

$$\begin{aligned}
\beta p_t v_t - (1 + \Phi) - c\alpha_t &\leq 0, \\
q_t[\beta p_t v_t - (1 + \Phi) - c\alpha_t] &= 0.
\end{aligned}$$

Moreover, when there is positive entry in a given period, the first equation holds with equality and together with (21), it pins down the value of the equilibrium licensing decision to a constant:

$$\alpha_t = \alpha^* = 1 - \frac{\Phi}{b} > 0.$$

Replacing this expression for α_t in (36), we can rewrite the net gain from entry as

$$\beta p_t v_t - (1 + \Phi) - \left(1 - \frac{\Phi}{b}\right)c = \beta p_t v_t - (1 + \hat{\Phi}),$$

where $\hat{\Phi} = \Phi + (1 - \frac{\Phi}{b})c$, which is increasing in Φ, b, and c. Hence, all the results and conditions obtained for the baseline model are valid for the case with external financing frictions if the original parameter Φ is replaced by $\hat{\Phi}$. □