

Interest Rates and Credit Risk*

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Abstract

This paper explores the effects of shifts in interest rates on corporate leverage and default in the context of a dynamic model in which the link between leverage and default risk comes from the lower incentives of overindebted entrepreneurs to guarantee firm survival. The need to finance new investment pushes firms' leverage ratio above some state-contingent target towards which firms gradually adjust through earnings retention. The response to interest rate rises and cuts is both asymmetric and heterogeneously distributed across firms. Our results help rationalize some of the evidence regarding the *risk-taking channel* of monetary policy.

Keywords: interest rates, short-term debt, search for yield, credit risk, firm dynamics.

JEL Classification: G32, G33, E52.

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1 Introduction

The experience surrounding the Great Recession of 2008-2009 has extended the idea that long periods of low interest rates may have a cost in terms of financial stability. Many observers believe that, by keeping interest rates “too low for too long,” monetary policies contributed to aggregate risk-taking in the years leading to the crisis. Concepts such as *search for yield* (Rajan, 2005) and *risk-taking channel of monetary policy* (Borio and Zhu, 2012) have been coined to describe conjectures regarding the possibility that financial institutions respond to low real interest rates by accepting higher risk exposures.

Formal empirical evidence on the relationship between interest rates and credit risk is recent and still scarce. Ioannidou et al. (2009), Jimenez et al. (2013), and Maddaloni and Peydró (2011) provide evidence that reductions in interest rates are followed by a deterioration of bank lending standards, an increase in lending volumes, and, with some additional lag, abnormally high default rates among the granted loans.¹ However, several other papers support the more traditional view that high short-term interest rates increase credit risk, at least in the short run and certainly for the most indebted borrowers.²

This paper challenges some of the interpretations given to the recent evidence by analyzing the effects of interest rates shifts on corporate leverage and default in the context of a simple infinite horizon model of firm financing. As other contributions in the field, the model emphasizes the role of incentive problems in limiting firms’ access to outside funding. The main novelty of our analysis is that we explicitly deal with the non-linearities that make the response to (random) interest rate shifts heterogeneous across firms and asymmetric across interest rate cuts and rises.

Our analysis is related to the analyses of Holmstrom and Tirole (1997), Repullo and Suarez (2000), and Bolton and Freixas (2006), which consider agency problems between firms and their financiers and represent monetary policy as an exogenous shift in investors’ opportunity cost of funds. It is also related to the macroeconomic literature on the credit

¹See also Altunbas et al. (2010).

²Jacobson et al. (2013) find that short-term interest rates have a significant positive impact on default rates among Swedish firms, and that the effect is stronger in sectors characterized by high leverage. In a clinical study, Landier et al. (2011) argue that interest rate rises started in 2004 could have pushed a major US subprime mortgage originator into making riskier loans.

channel of monetary policy, including Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999), in which the funding of successive cohorts of firms is affected by informational frictions. Differently from these papers, firms in our model solve a genuinely dynamic financing problem with full awareness of the possibility of future random shifts in interest rates.

Our paper fits in the strand of the dynamic corporate finance literature that studies firms' capital structure decisions in an infinite horizon setup (Cooley and Quadrini, 2001, Hennessy and Whited, 2005, Cooley and Quadrini, 2006, Hackbarth et al., 2006, Morellec et al., 2012) without dealing with the more complex issue of optimal dynamic contracts (Clementi and Hopenhayn, 2006, DeMarzo and Fishman, 2007). The impact of interest rate shifts on leverage is explicitly analyzed by Cooley and Quadrini (2006), but without discussing the asymmetric response to interest rate rises and cuts, and in a framework in which default never occurs in equilibrium.

Our model is characterized by four key features. First, firms obtain outside financing using short-term debt contracts.³ Second, their relationship with outside financiers is affected by a moral hazard problem. Entrepreneurs make repeated unobservable decisions on private benefit taking that have a negative impact on their firms' survival probability; *ceteris paribus*, if a firm's continuation value declines, its entrepreneur takes more private benefits, and the firm's survival probability declines. Third, the short-term riskless interest rate (which determines outside financiers' opportunity cost of funds) follows a Markov chain. Finally, the model features endogenous exit (firms that fail) and entry (entrepreneurs who decide whether to start up new firms).

Like in other papers in the literature, we assume that entrepreneurs discount the future at a higher rate than outside financiers, which provides a *prima facie* case for the outside financing of their firms even in the long run.⁴ Entering entrepreneurs start up penniless but can retain earnings in order to reduce their firms' leverage. Entrepreneurs running non-

³The assumption regarding the sole use of short-term debt can be relaxed in several ways. One alternative that we consider as an extension is to allow firms to issue a combination of short-term debt and perpetual debt when they are created.

⁴Entrepreneurs' larger discounting can be justified as a result of unmodeled tax distortions that favor outside debt financing, the existence of alternative profitable investment opportunities for entrepreneurial wealth, or as a reduced form for the discount for undiversifiable idiosyncratic risk that would emerge if entrepreneurs were risk averse.

surviving firms leave the economy forever and the (endogenous) entry of novel entrepreneurs prevents the measure of active firms from going to zero.⁵

The model delivers leverage ratios and default probabilities that are maximum immediately after undertaking the new investment and then, other things equal, decrease as firms gradually move towards their target leverage ratios through earnings retention.⁶ Overindebted firms have lower continuation value and, thus, greater incentives to extract private benefits and put their continuation at risk. Moreover, under the equilibrium levels of leverage that the model generates, non-surviving firms always default on their debts, producing an intuitive correlation between leverage and credit risk.⁷

Our baseline results regarding the effects of interest rate shifts on leverage and default risk can be organized around three main themes:

1. Asymmetries. Interest rate cuts and interest rate rises have asymmetric effects on leverage and default because, given the need to rely on retained earnings for leverage reductions, it is easier to adjust leverage up (following interest rate cuts) than down (following rises). In fact, a rise in interest rates may imply a temporary increase in leverage and credit risk across all firms because of the extra cost of outstanding debt.⁸

2. Heterogeneity. The response of firms to interest rate shifts is heterogeneous because not all firms are equally levered when a shift occurs. Interest rate rises produce effects of identical sign but heterogeneous intensity across firms (leading first to an increase in leverage and default and then to declines as firms converge to their new lower leverage targets), while the effect of interest rate cuts varies in sign across firms (those at target leverage respond by immediately increasing their leverage and default, but overindebted firms speed up their convergence to target leverage, reducing their default rate during the transition).

⁵As a convenient way of capturing congestion (or aggregate decreasing returns to scale) in finding a profitable investment opportunity, we assume that the costs of entry of potential entrants is increasing in the number of already existing firms.

⁶See Flannery and Rangan (2006) and the references therein for evidence regarding firms' gradual adjustment towards target capital structure.

⁷See Molina (2005) for a recent restatement of the importance of leverage as a determinant of default probabilities.

⁸The empirical literature on the effects of monetary policy shocks on real output have also found asymmetries (e.g. Cover, 1992, or Weise, 1999), typically attributed to nominal rigidities. To the best of our knowledge, the potential asymmetric effects of interest rate shocks on corporate leverage and default have not been directly analyzed at either an individual firm level or the aggregate level.

3. Aggregate effects. Under our baseline parameterization, we find that both interest rate rises and interest rate cuts produce short run increases in the default rate. The short-run effect of interest rate rises is explained by the increase in effective leverage across all firms, before all of them adjust to the new lower target. Instead, the leverage-increasing effect of an interest rate cut on less levered firms dominates the opposite effect on overindebted firms.⁹

Several features of the model that help keep it transparent about the underlying mechanism (fixed firm size, ex ante homogeneity, lack of initial entrepreneurial wealth) imply leaving aside potentially important phenomena regarding firms' life cycle (growth, learning about firms' types, dynamic selection based on the survival of the fittest, etc.). Moreover, the macroeconomic environment is solely captured through the exogenous process driving interest rate shifts, which excludes aggregate feedback effects. Yet the richness of the response detected with this exercise suggests several candidate explanations for recent empirical results about the link between monetary policy and credit risk.

First, the asymmetries and heterogeneity in the response to interest rate shifts might explain the ambiguous or mixed signs of the aggregate effects found in linear regression models. Our analysis points to the explicit consideration of non-linearities and the complexities of aggregation as a fruitful strategy for future research.

Second, our results provide a possible rationalization of empirical findings sometimes interpreted as evidence in favor of the existence of *search for yield* incentives among financial institutions or of a *risk-taking channel* of monetary policy. Interestingly, our rationalization is based on the dynamic extension of a pretty canonical moral hazard model of the relationship between firms and their outside financiers (see Tirole, 2005), rather than relying on irrational agents (e.g. Shleifer and Vishny, 2010) or on procyclical accounting, risk-management and managerial compensation practices (e.g. Adrian and Shin, 2010).

The paper is organized as follows. Section 2 describes the model. Section 3 studies the case without aggregate uncertainty regarding interest rates, for which we can obtain analytical results. Section 4 extends the characterization of equilibrium to the case with aggregate uncertainty, presents the baseline parameterization, and the quantitative results,

⁹As discussed in Section 4.3, due to the presence of nonlinearities, some of the aggregate effects of changes in interest rates (especially the short-term effects of rate cuts) change sign when we move away from our baseline parameterization.

including a battery of robustness checks on the baseline results. Section 5 discusses the consistency of our results with existing evidence, and elaborates on their macroprudential implications. Section 6 concludes. Appendix A contains proofs, Appendix B presents the details of the model with aggregate uncertainty, and Appendix C develops an extension in which firms are allowed to issue long-term debt at the start-up date.

2 The model

We consider an infinite-horizon discrete-time economy in which dates are indexed by $t = 0, 1, \dots$ and there are two classes of risk-neutral agents, entrepreneurs and financiers.

2.1 Model ingredients

There many competitive financiers with deep pockets and the opportunity to invest their funds at a riskless interest rate r_t during the period between dates t and $t + 1$. This interest rate realizes at the beginning of each period and follows a Markov chain with two possible values r^L and r^H , with $r^L \leq r^H$ and a time-invariant transition probability matrix $\Pi = \{\pi_{ij}\}_{i=L,H}^{j=L,H}$, where $\pi_{ij} = \Pr[r_{t+1} = r^j \mid r_t = r^i]$.

A new cohort of entrepreneurs, made up of a continuum of individuals, is born at each date t . Entrepreneurs are potentially infinitely-lived and their time preferences are characterized by a constant discount factor $\beta < 1/(1+r^H)$ which gives them incentives to rely on financiers' funding even if they are able to self-finance their firms.

Entrepreneurs are born penniless and with a once-in-a-lifetime investment opportunity. By incurring a non-pecuniary idiosyncratic entry cost θ at date t , each novel entrepreneur can find a project with which to start a new firm. The distribution of θ across novel entrepreneurs is described by a measure function $F(\theta, n_t)$ which depends negatively on the measure n_t of firms operating in the economy at date t . This means that the costs of finding a project are increasing (in the sense of first order stochastic dominance) in the measure of active firms. This provides an equilibrating mechanism for the entry of new firms.

Each project requires an unpostponable initial investment normalized to one and the continuous management by its founding entrepreneur. Each project operative at date t has a probability p_t of remaining productive at date $t + 1$. A productive project generates a

positive cash flow y per period. With probability $1 - p_t$ the project *fails*, in which case it yields a liquidation cash flow L (out of the residual value of the initial investment) and disappears forever. The discretionary liquidation of the project at any date also yields L .

The probability p_t is determined by an unobservable decision of the entrepreneur which also affects the flow of private benefits $u(p_t)$ that he extracts from his project in between dates t and $t + 1$. For simplicity, we assume the private benefits to accrue at $t + 1$ independently of whether the project fails, and $u(\cdot)$ to be decreasing and concave, and with enough curvature to guarantee interior solutions in p_t .¹⁰ Thus, entrepreneurs can appropriate a larger (but marginally decreasing) flow of private benefits by lowering their projects' probability of continuation. This creates a moral hazard problem in the relationship with financiers.

Insofar as a project remains productive, the entrepreneur can raise outside financing in the form of *short-term debt*. Such debt is described as a pair (b_t, R_{t+1}) that specifies the amount borrowed, b_t , and the repayment obligation after one period, R_{t+1} , to which we will henceforth refer as the firm's level of leverage.¹¹ If the firm fails to repay R_{t+1} at $t + 1$, the project is liquidated and the financiers obtain the whole liquidation value L . Otherwise, the firm decides on its short-term debt for the next period subject to its *uses and sources of funds* constraint:

$$c_{t+1} + R_{t+1} = y + b_{t+1}, \quad (1)$$

where c_{t+1} denotes the entrepreneur's dividend. We impose $c_{t+1} \geq 0$ to reflect that the entrepreneur is assumed to begin with no wealth and immediately consume his dividend. This implies that he never has own wealth with which to recapitalize his firm.

2.2 Discussion of the ingredients

We have described r_t as the riskless interest rate at which financiers invest their wealth at the margin, but other interpretations are possible: r_t might also reflect financiers' time-preference parameter, an international risk-free rate relevant for a small open economy, or a

¹⁰A sufficient condition to guarantee this would be having $u'(0) = 0$ and $u'(1) = -\infty$. The specification of $u(\cdot)$ used in the numerical analysis below does not satisfy these properties but we check that the resulting solutions are always interior.

¹¹Since the initial assets (pecuniary investment) of the firm is one, R_{t+1} can also be interpreted as the book-value debt-to-asset ratio with which the firm arrives at date $t + 1$.

convolution of exogenous factors that determine financiers' required expected rate of return at a given date.¹² The two-state Markov process is a simple way to capture uncertainty regarding the evolution of this variable and is useful to fix intuitions.

The assumption that entrepreneurs have larger discount rates than their financiers is common in the literature (e.g. Cooley and Quadrini, 2001). It provides a *prima facie* case for entrepreneurs' to keep relying on external financing. The assumption may reflect the net tax advantages of debt financing if outside financing takes the form of debt and inside financing takes the form of equity (see Hennessy and Whited, 2005). Alternatively, it may capture that entrepreneurs require a return on the wealth invested in their own projects larger than the returns required by financiers with presumably better diversified and more liquid portfolios.

The focus on short-term debt is also standard in the literature (e.g. Kiyotaki and Moore, 1997, and Cooley and Quadrini, 2001) and is adopted mainly for tractability: It makes the state variable that describes the firm's capital structure unidimensional. Allowing for debts with arbitrary maturities would unbearably multiply the dimensions of the state space. In Appendix C, we extend the model to allow firms to also issue long-term debt (consols) at the start-up date.

3 The case without aggregate uncertainty

This section focuses on firms' dynamic financing decisions in the economy in which the riskless interest rate r_t is constant ($r^L = r^H = r$). The analytical results obtained here will help understand the logic of the numerical results that arise in the economy with random fluctuations in r_t .

3.1 Firms' dynamic financing problem

Without fluctuations in the risk-free rate, the only state variable relevant for the optimal dynamic financing problem of a firm that remains productive is the repayment obligation R derived from the debt issued in the prior date. Such a firm has a cash flow y from its prior

¹²For instance, r_t might be a reduced-form for the risk-adjusted required rate of return that risk-averse financiers demand on the securities issued by the entrepreneurs.

period of operation and has to decide on the debt issued in the current date, (b, R') , where, to avoid using time subscripts, we identify the state variable for the next date as R' . The dividend resulting from this decision is $c = y + b - R \geq 0$.

To take into account the moral hazard problem derived from the unobservability of the entrepreneur's choice of the continuation probability p , we extend the description of the debt contract to the triple (p, b, R') and establish the connection between p and the pair (b, R') through an incentive compatibility condition. Assuming that the firm that remains productive is always able to obtain financing for one more period, its dynamic optimization problem can be stated as follows:

$$v(R) = \max_{p, b, R'} (y + b - R) + \beta[u(p) + pv(R')] \quad (2)$$

subject to

$$p = \arg \max_{x \in [0,1]} u(x) + xv(R'), \quad (3)$$

$$pR' + (1 - p)L = b(1 + r), \quad \text{and} \quad (4)$$

$$y + b - R \geq 0, \quad (5)$$

where $v(\cdot)$ is the entrepreneur's value function. The right hand side in (2) reflects that the entrepreneur maximizes his dividend in the decision date plus the discounted value of the private benefits $u(p)$ received in the next date plus the discounted expected continuation value after one period. Continuation value to the entrepreneur is $v(R')$ if the firm remains productive and zero otherwise.

Equation (3) is the entrepreneur's incentive compatibility condition which entails a trade-off between the utility derived from private benefits $u(p)$ (decreasing in p) and the expected continuation value $pv(R')$ (increasing in p). Since we have assumed that the curvature of $u(p)$ is enough to guarantee interior solutions for p , the first order condition

$$u'(p) + v(R') = 0 \quad (6)$$

can be conveniently used to replace (3).

Equation (4) is the participation constraint of the financiers: It says that the effective repayments received after one period (R' in case of continuation and L in case of failure)

must imply an expected return r on the extended funding b .¹³ The third and last constraint (5) imposes the non-negativity of the entrepreneur's dividend.

In order to reduce the stage maximization problem in (2) to one with a single decision variable and a single constraint, we establish the following intuitive result. All proofs are in Appendix A.

Lemma 1 *The value function $v(R)$ is strictly decreasing for all leverage levels R for which refinancing is feasible.*

With $v(R') < 0$ and $u''(p) < 0$, (6) implicitly defines a function $p = P(R')$, with $P'(R') < 0$. Using it to substitute for p and solving for b in (4) yields:

$$b = B(R') \equiv \frac{1}{1+r}[L + P(R')(R' - L)]. \quad (7)$$

This function has a slope

$$B'(R') = \frac{1}{1+r}[P(R') + P'(R')(R' - L)] < \frac{1}{1+r} \quad (8)$$

which, assuming $P(L) > 0$, is positive for R' close to L , but may become negative for larger values of R' . Hence, as in other corporate finance problems, the relationship between debt value b and the promised repayment R' may be non-monotonic, although only the upward sloping section(s) of $B(R')$ would be relevant when solving (2). To avoid complications related to discontinuities that may emerge if $B(R')$ has several local maxima, we adopt the following assumption:

Assumption 1 *$B(R')$ is single-peaked, with a maximum at some \bar{R} .*

This assumption is always satisfied under the parameterizations explored in the numerical part of the paper.¹⁴

Using the function $B(R')$ (which embeds (3) and (4)), the stage problem in (2) can be compactly expressed as:

$$\max_{R'} y + B(R') - R + \beta[u(P(R')) + P(R')v(R')] \quad (9)$$

¹³We have written it directly as an equality since, rather than leaving a surplus to the financiers, entrepreneurs would trivially prefer to pay such surplus as a contemporaneous dividend to themselves.

¹⁴Expressing the assumption in terms of primitives is difficult, since $P(R')$ involves the value function $v(R')$, which can only be found by solving the optimization problem.

$$\text{s.t.: } y + B(R') - R \geq 0, \quad (10)$$

where the only decision variable is R' and the only constraint is a rewritten version of (5).

Our first proposition establishes the key properties of the solution to this problem.

Proposition 1 *The solution to the equation $B'(R^*) + \beta P(R^*)v'(R^*) = 0$, if it exists, defines a unique target leverage level $R^* \leq \bar{\bar{R}}$ such that:*

1. *For $R \leq [0, \bar{R}]$, where $\bar{R} \equiv y + B(R^*)$, the non-failing firm issues new debt $R' = R^*$, current debtholders get fully paid back, and the entrepreneur's dividend is $y + B(R^*) - R$.*
2. *For $R \in (\bar{R}, y + B(\bar{\bar{R}})]$, the firm issues new debt $R' = B^{-1}(R - y) \in (R^*, \bar{\bar{R}}]$, current debtholders get fully paid back, and the dividend is zero.*
3. *For $R > y + B(\bar{\bar{R}})$, the firm cannot be refinanced, defaults on its current debt, and is liquidated.*

3.2 Implied dynamics

The analysis of the dynamics of a non-failing firm's leverage can be described with reference to a phase diagram whose elements emanate directly from Proposition 1. Figure 1 describes a situation in which non-failing firms reach in finite time a unique stable steady state with some leverage level $R = R^*$.

The thicker curve represents the mapping of a non-failing firm's repayment obligation R onto its repayment obligation in the following date, R' , according to the firm's optimal refinancing decisions. By Proposition 1, such curve is piece-wise defined by the horizontal line $R' = R^*$ which corresponds to the target leverage ratio and the upward sloping curve implicitly defined by the boundary of the refinancing constraint (10). This boundary can be described as $R' = B^{-1}(R - y)$, which is directly connected to (the inverse of) the upward-sloping section of the single-peaked $B(R')$ schedule. This curve has a slope strictly larger than one (because $B'(R') < 1$) and is defined for $R \in [y, y + B(\bar{\bar{R}})]$; for $R < y$, any $R' > 0$ satisfies (10). Satisfying (10) requires choosing $R' \geq B^{-1}(R - y)$. Hence, given the relative position of this curve and the horizontal line $R' = R^*$, it is obvious that reaching the target

section it is instable. So the horizontal intersection (if it exists) identifies the only stable steady state in the refinancing problem. The feasibility of a non-explosive financing path for the firm requires that the upper intersection occurs at some $\widehat{R} \geq 1 + y$ because financing the initial investment is equivalent to having to refinance an initial obligation of 1 without the cash y resulting from a prior period of successful operation.¹⁵

The following proposition summarizes the implications of this discussion.

Proposition 2 *The firm's optimal financing path is non-explosive if and only if $\min\{\widehat{R}, \overline{\overline{R}}\} \geq \max\{1 + y, \overline{R}\}$, where \widehat{R} is defined by $y + B(\widehat{R}) - \widehat{R} = 0$. The non-explosive optimal financing path is unique and converges in finite time to the steady state associated with the target leverage level R^* .*

The prior discussion identifies two types of situations depending of the relative position of $1 + y$ and \overline{R} . If $\overline{R} > 1 + y$ (and $\min\{\widehat{R}, \overline{\overline{R}}\} \geq \overline{R}$), the firm reaches its target leverage level R^* from the start (possibly paying an initial dividend to the entrepreneur) and, insofar as it does not fail, remains there forever, rolling over its debt and paying a dividend $y - R^*$ to the entrepreneur at the end of every period. Otherwise, as illustrated in Figure 1, the firm starts assuming some repayment obligation $R_0 > R^*$ such that $B(R_0) = 1$ and pays no initial dividend. Afterwards, insofar as it does not fail, gradually reduces its leverage until it reaches the target leverage R^* . At that point, it starts rolling over its debt and paying dividends $y - R^*$.

3.3 Comparative statics and dynamics

In the next proposition, we establish the properties of the stable steady-state.

Proposition 3 *The steady-state target leverage level R^* is strictly larger than L , increasing in y and L , and decreasing in r , whereas its dependence with respect to β is ambiguous. The implied probability of default, $1 - p^*$, is strictly positive, increasing in L , and decreasing in y and β , whereas its dependence with respect to r is ambiguous.*

¹⁵When the relevant curve only intersects the 45-degree line at R^* , having a non-explosive financing path for the firm requires $B(\overline{\overline{R}}) \geq 1$.

The main rationale for firms to remain levered in the long run is the difference between β and $1/(1+r)$. However, optimal leverage is limited by the moral hazard problem which makes p , ceteris paribus, decreasing in R . The resolution of the trade-off between the fundamental value of leverage and its incentive-related costs drives most of the results, including the effects of varying β and r (and the ambiguity of some of the signs).

When the riskless rate r increases, target leverage decreases because the fundamental value of leverage decreases at the same time as the incentive-related cost of leverage increases because of the reduction in the firm's continuation value. This last effect explains why, in spite of reducing leverage, a higher r does not necessarily lead to a lower probability of default $1 - p^*$.¹⁶ When the entrepreneurs' discount factor β increases the same logic leads to mirror-image implications: the fundamental value of leverage decreases but the value of continuation increases and, hence, incentives improve, creating a force that pushes for higher leverage. So in this case, the probability of default falls, but target leverage may increase or decrease.

Increasing the success cash flow y increases continuation value and, hence, the entrepreneur's incentives to avoid default. This allows the firm to increase its target leverage R^* while actually reducing the default probability $1 - p^*$. The effects of the liquidation value L on target leverage and the associated default are more intriguing. Making L larger increases the firm's continuation value, but makes it so by reducing the cost of outside funding. This increases optimal leverage, up to a point in which the associated default probability is unambiguously higher.¹⁷

We now turn to analyzing the effects of the parameters on the upward sloping section of the phase diagram. This will tell us about the repayment obligation R' and the associated default probability $1 - p(R')$ of those firms whose current leverage is still too high to reach the target leverage level R^* .

Proposition 4 *For $R > \bar{R}$, the firm's optimal refinancing, if feasible, leads to an above-target leverage level R' which is increasing in R and r , and decreasing in β , y , and L .*

¹⁶Yet, in the neighborhood of our baseline calibration, the effect due to the reduction in leverage dominates, making the default risk associated with target leverage decreasing in r .

¹⁷If empirically, large L corresponds to economies in which the collateral value of the firms' real assets is high, this result provides a rationale for the view that higher real asset values induce larger "risk taking."

The implied probability of default, $1 - p(R')$, is strictly positive, and responds in the same direction as R' to changes in parameters.

There is a sharp contrast between the effects of some parameters on target leverage and on the effective leverage of the firms that are above their target. Specifically, increasing the interest rate r increases the leverage and the default risk of the overindebted firms (whereas increasing β , y , or L has the opposite effects).

Figure 2 summarizes the implications of a shift in r for the dynamics of firms' financing. The dark phase diagram corresponds to a higher r than the bright phase diagram. With a larger r , target leverage is lower, but the leverage needed to refinance prior debt (or the initial investment) is larger for all $R > \bar{R}$, making the two phase diagrams cross at a single point.

Comparing the two phase diagrams allows us to analyze the effects of an unanticipated, once-and-for-all *increase* in r . Firms already at their prior target leverage (i.e. at the R^* associated with the bright curve) can immediately start a process of leverage reduction leading to the new target (the R^* associated with the dark curve), and perhaps reach the new target in a single period. In contrast, start-up firms and other overindebted firms face a temporary increase in leverage, experience a slower convergence to target leverage, and suffer higher default rates in their transition to the new target. In sum, a rise in r reduces the target leverage of all firms but may temporarily increase the leverage and default of some (perhaps all) firms.

The effects of *decreasing* r are also heterogeneous across firms and asymmetric from those of increasing r . Following a reduction in r , firms at the prior target leverage (as well as any other firm below the new target leverage) can reach the new steady state immediately, because increasing leverage does not require time. Instead, firms with leverage sufficiently higher than the new target will merely accelerate their convergence to the new target, experiencing lower default rates in the transition.

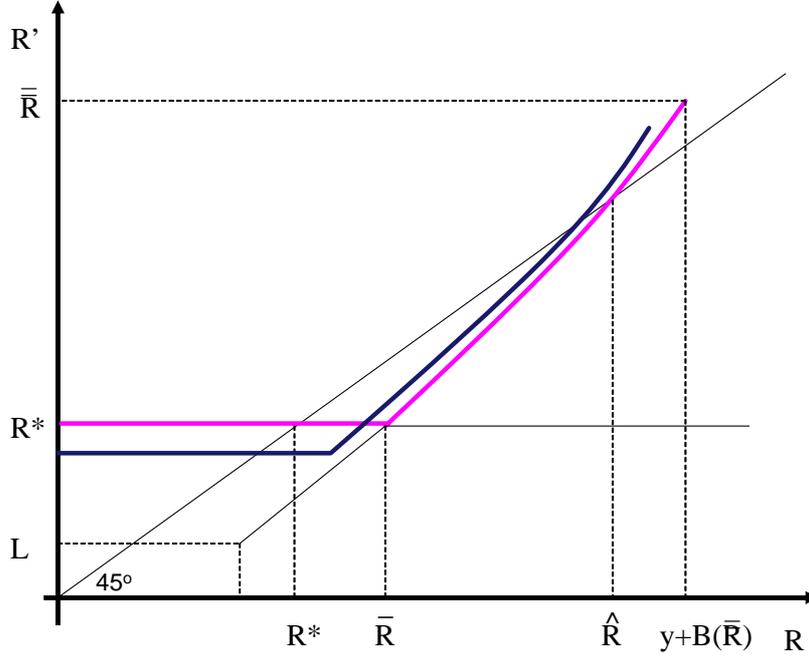


Figure 2. Effects of unexpected once-and-for-all changes in r
This figure plots the repayment obligation R' that results from the optimal refinancing decision of non-failing firm whose current repayment obligation is R . The dark line correspond to a situation in which financiers' required rate of return is larger than in the situation corresponding to the bright line.

Importantly, the heterogeneity in the cross-sectional response to a shift in r , as well as the asymmetry across rises or declines in financiers' opportunity cost of funds, will also arise in the economy with explicit (rationally anticipated) aggregate uncertainty about r . The macroprudential implications of these results are discussed in Section 5.

3.4 Equilibrium entry

To close the characterization of equilibrium dynamics in the model without aggregate uncertainty, we discuss the determination of the equilibrium measure of active firms n_t and the law of motion of the measure of active firms over possible leverage levels.

Proposition 5 *Assume $\bar{R} < 1 + y \leq \hat{R}$ and denote by $\Psi_t(R)$ the measure of firms that operate at date t with a promised repayment for next period lower than or equal to R . Then:*

1. *Entering firms start with a leverage level $R_0 > R^*$.*
2. *The value of a just-entered firm (and hence the entry cost of the marginal entrant) is increasing in β , y , and L , and decreasing in r .*
3. *The measure of firms active at date t can be found as the unique solution to the equation*

$$n_t = \int_0^\infty P(R)d\Psi_{t-1}(R) + F(v(1+y), n_t). \quad (11)$$

4. *The law of motion for $\Psi_t(R)$ can be described as*

$$\Psi_t(R) = \int_0^\infty \mathbb{I}(R'(x) \leq R)P(x)d\Psi_{t-1}(x) + \mathbb{I}(R_0 \leq R)F(v(1+y), n_t). \quad (12)$$

Proposition 5 focuses on the most interesting case in which entering firms start with a leverage level R_0 above the target level R^* . Results 1 and 2 build on our prior analysis of a firm's problem. Result 2 is based on the fact that the value of a just entered firm can be expressed as $v(1+y)$ and uses properties of the value function. Result 3 reflects that the measure of active firms at t equals the measure of firms active at $t-1$ that do not fail plus the measure of entrants (entrepreneurs with $\theta \leq v(1+y)$). Finally, the law of motion of the distribution of firms' leverage is written taking into account the dynamics of leverage at continuing firms (determined by $R'(R)$) and the initial leverage R_0 of the entrants.

4 The case with aggregate uncertainty

Extending the optimal dynamic financing problem stated in (2)-(5) to the case in which the riskless rate r_t follows a Markov chain with two possible values, r^L and r^H , with $r^L < r^H$, requires several adaptations. First, we write the value function as $v(R, r)$ rather than $v(R)$ to add the interest rate as a relevant descriptor of the aggregate state of the economy. By the same token, the (expected) continuation value of the non-failing firm, previously written as $v(R')$, is now written as $E[v(R', r') | r]$, where r' denotes the interest rate one period ahead and the expectation is based on the information available at the decision date.¹⁸

¹⁸ r_t is the only aggregate state variable in the model which is relevant for the decision problem of non-failing firms. The measure of firms in operation n_t is also an aggregate state variable and it is relevant for the entry decisions of novel entrepreneurs. However, n_t only affects the distribution of the non-pecuniary entry costs. This feature greatly simplifies the analysis.

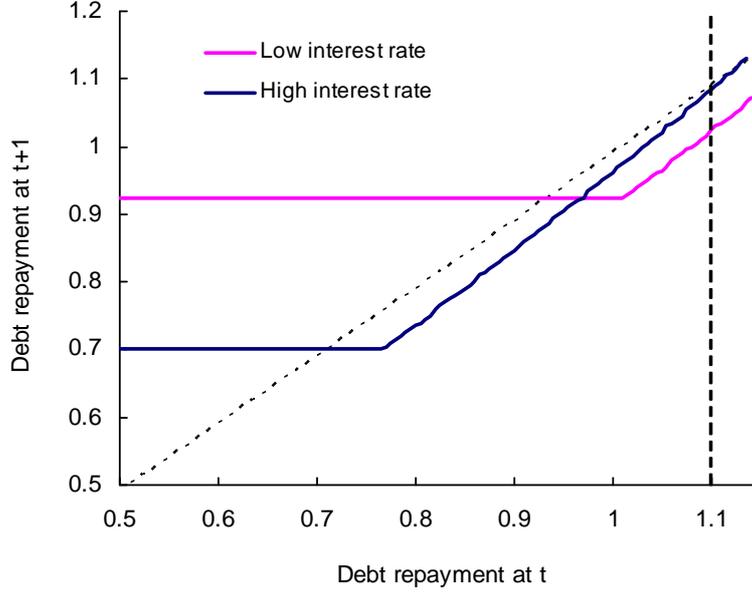


Figure 3. Leverage dynamics with aggregate uncertainty

This figure is the counterpart of Figure 1 for the economy with aggregate uncertainty. The results correspond to the economy whose parameters are described in Table 1. Each curve represents the phase diagram that summarizes the optimal refinancing decision (future leverage) of non-failing firms (vertical axis) conditional on their current leverage (horizontal axis) and the prevailing interest rate. The optimal policy for periods of low (high) interest rates appears in bright (dark) color. Differences in target leverage across states are quite sizable. The vertical dark line helps identifying (on the corresponding curve) the leverage with which entering firms start up in each state.

Objects such as $P(R')$, implicitly defined in (6), and $B(R')$, defined in (7), have to be replaced by $P(R', r)$ and $B(R', r)$ to reflect their dependence, through $E[v(R', r') | r]$, on the value of r at the decision date. Other adaptations of the problem are fully described in Appendix B.

In this new scenario, firms have a different target leverage $R^*(r)$ for each value of r , and the dynamics of leverage when the target cannot be reached also varies with r . The counterpart to the phase diagram depicted in Figure 1 is now a couple of curves of the form $R'(R, r)$. Figure 3 is generated under the baseline parameterization explained below. Target leverage and leverage dynamics under $r = r^L$ (r^H) are described by the bright (dark) phase

diagram.¹⁹ The parameterization implies sizable differences in target leverage across interest rate states (the intercept of each curve) and, for a start-up firm, a much longer transition to target leverage under high interest rates.

The dynamics of the distribution of firms' leverage in this case are affected by the randomly fluctuating r_t . Hence, the economy does not converge to a time-invariant distribution of leverage levels, but we can still define a *stationary equilibrium* characterized by an *invariant law of motion* for the distribution of firms' leverage $\Psi_t(R)$:

Definition 1 *A stationary equilibrium in this economy is an invariant law of motion for the distribution of firms' leverage $\Psi_t(R)$ and a policy function $R'(R, r)$, mapping firms' current debt R and the aggregate state variable r into their next-period leverage R' , such that:*

- *Incumbent entrepreneurs solve their dynamic refinancing problem optimally.*
- *Novel entrepreneurs decide optimally on entry.*
- *The law of motion for $\Psi_t(R)$ is*

$$\Psi_t(R) = \int_0^\infty \mathbb{I}(R'(x, r_t) \leq R) P(x) d\Psi_{t-1}(x) + \mathbb{I}(R'(1+y, r_t) \leq R) F(v(1+y, r_t), n_t), \quad (13)$$

where the measure of active firms satisfies $n_t = \lim_{R \rightarrow \infty} \Psi_t(R)$.

The structure of the model allows us to solve for equilibrium in a conveniently recursive manner. Obtaining $R'(R, r)$ involves solving the dynamic programming problem of continuing firms, which is independent of n_t and $\Psi_t(R)$. We address this using value function iteration methods. The dynamics of $\Psi_t(R)$ can be tracked using (13), which takes into account the continuation probabilities and refinancing decisions (conditional on survival) of the firms operating in the previous date, and the initial leverage of the entering firms. Intuitively, the economy fluctuates around the (two) steady states that would be reached if each of the (two) possible interest rates remained fixed for sufficiently many dates.

Because aggregate uncertainty increases the number of analytically ambiguous-to-sign effects, we will perform the rest of analysis numerically, under a specific parameterization of the model. This will additionally inform us about the order of magnitude of the effects

¹⁹Notice that the relative positions of these state-contingent phase diagrams are qualitatively identical to those of the phase diagrams in Figure 2, which corresponded to two economies with different time-invariant values of r .

already identified in our previous discussions and aggregate effects which can only be found by taking the cross-sectional dynamics of leverage into account.

4.1 Baseline parameterization

To proceed, we adopt the following functional forms for the private benefits function,

$$u(p_t) = \mu_0 - \mu_1 p_t^\alpha, \text{ with } \mu_0, \mu_1 > 0, \text{ and } \alpha > 1, \quad (14)$$

and the density of the distribution of entry costs,

$$f(\theta, n_t) = \frac{1}{n_t} \exp\left(-\frac{1}{n_t}\theta\right). \quad (15)$$

The function described in (14) does not satisfy $u'(0) = 0$ and $u'(1) = -\infty$ but we check that the equilibrium values of p_t are always interior under our choice of parameters. The function in (15) implies that entry costs are exponentially distributed and increase in the sense of first-order stochastic dominance when the measure of operating firms, n_t , increases.

Under (14) and (15), the model has ten parameters: four related to interest rate dynamics (r^H , r^L , π_{HH} , and π_{LL}), the entrepreneurs' discount factor (β), the per period cash flow of operating firms (y), the liquidation value of failing firms (L), and the three parameters of the private benefit function (μ_0 , μ_1 , and α). We consider that a model period corresponds to a year, which is consistent with firms issuing one-year debt and revising their financing policies on an annual basis.

Table 1 describes the values of the parameters used to obtain our numerical results. We aim to illustrate the working of the model under reasonably realistic parameter values but we do not purport to offer quantitative predictions for any specific economy because the model is too stylized to do so. We think the model is most suitable to represent small or medium size firms not too distant from their start-up date (or other firms after a major expansion) and, hence, we take issuers of speculative-grade corporate debt as a reference group. When feasible, we choose parameter values consistent with standard practice and US evidence.

As for interest rate dynamics, we estimate a symmetric Markov chain with data on US real interest rates, 1984-2009, taken from Thomson Datastream. This yields $r^L = 0.23\%$, $r^H =$

3.42% and $\pi_{LL} = \pi_{HH} = 0.9293$, thus capturing rather large and long-lasting fluctuations in the opportunity cost of funds (with state shifts occurring on average every 14 years).²⁰

The discount factor β embeds a measure of entrepreneurs' required expected rate of return on their own wealth, $1/\beta - 1$. Following other papers on entrepreneurial financing, we set $\beta = 0.9386$ so as to match the average real rate of return historically observed in the US stock market, which is of about 6.5%.

We set the per period cash flow y equal to 10% of the initial pecuniary investment (normalized to one). Given the mean value of the equilibrium marginal cost of entry in each state, this cash flow implies an average yearly return of 5.04% and 5.51% on the total initial investment of a *marginal* project started in the low and high interest rate states, respectively.

Table 1
Parameter values

This table describes the parameters used to generate our baseline numerical results. The first block of parameters refers to interest rate dynamics. The second block refers to entrepreneurs' preferences and investment technologies. Empirical sources and other criteria for the setting of these parameters within a realistic range are described in the main text.

Description	Parameter	Value
High-state interest rate	r^H	3.42%
Low-state interest rate	r^L	0.23%
State persistence parameter	$\pi_{LL} = \pi_{HH}$	0.9293
Entrepreneurs' discount factor	β	0.9386
Net cash flow	y	0.10
Liquidation value of assets	L	0.40
Intercept parameter - private benefit function	μ_0	0.1629
Scale parameter - private benefit function	μ_1	0.1684
Elasticity parameter - private benefit function	α	9.1053

The liquidation value L and the parameters of the private benefit function, μ_0 , μ_1 , and α , are set so as to best approximate four calibration targets expressed in terms of the average

²⁰Thus, the current calibration captures swings between the high-rates of the Volcker era, the rather low-rates of the Greenspan era, etc. One could extend the analysis to the case in which interest rates follow a Markov process with $n > 2$ states. But the focus on $n = 2$ helps reporting the results and fixing intuitions. The constraint $\pi_{LL} = \pi_{HH}$ is adopted to guarantee that the asymmetries obtained in the results are not due to the asymmetry of the Markov chain.

values of some variables among firms that have reached their target leverage at some point in their own history. We take this reference group in order to focus on the type of firm that might correspond to a typical speculative-grade corporate debt issuer in Standard and Poor’s database, from which we take several reference statistics.²¹

The first variable we consider is the *default rate* (computed in the model as $1 - p$), for which we set a target of 3%, which roughly corresponds to the long-term average of the yearly speculative-grade default rate reported by Standard and Poor’s (2008, Charts 11 and 12) and is sufficiently high to produce some action on the firm entry and exit side of the model.

The second variable is the *interest-to-income ratio* (computed as $(R' - b)/y$), which we take as a flow-based measure of leverage.²² We set a target of 25%, which is the average leverage ratio observed in the sample of firms from 18 developed economies studied by Halling and Zechner (2011) and is roughly consistent with the median EBITDA-to-interest coverage ratio of 3.4 among BB issuers in the period 1998-2000 reported by Standard and Poor’s (2003).

The third variable is the *recovery rate* (expressed as L/b), for which we set a target of 50%, which is consistent with the average “discounted recovery rate” for defaulted bank debt and bonds that Standard and Poor’s (2008, Chart 22) reports for the period 1987-2007.

The fourth variable is the *private-benefit-to-income ratio* (expressed as $u(p)/y$), for which we set a (highly tentative) target of 35%, which implies that entrepreneurs in successful firms that have ever reached their target leverage obtain about one third of their rewards per period in the form of private benefits (and the rest as equilibrium dividends).²³

²¹Standard and Poor’s (2008, Chart 10) documents that about 50% of issuers in their database that belong to industries other than utilities, real estate, insurance, and financial institutions get speculative grade ratings. We think that speculative-grade issuers are the ones among which the moral hazard problems captured by the model are most relevant.

²²Stock-based measures such as the debt-to-book-asset ratio or the debt-to-market-value ratio would force us to take a position on whether the non-pecuniary entry costs θ appear in the book value of assets or whether the private benefits component of the value of the firm to the entrepreneur, $v(R, r)$, is part of the firm’s market value.

²³Direct evidence on the importance of private benefits in entrepreneurial firms does not exist. The estimates in Dyck and Zingales (2004) and Albuquerque and Schroth (2010), based on premia observed in block trades, point to a much lower importance in publicly traded corporations.

4.2 Results under the baseline parameterization

Table 2 contains a summary of some of the moments generated by the model under the parameterization that we have just described. It reports both *unconditional* moments and moments conditional on low and high interest rates.

Table 2
Simulated moments

Given the parameterization in Table 1, we simulate 1,000 economies for 300 periods and pick only the 25 last periods. Reported moments are calculated by averaging the corresponding moment unconditionally or in each state of the economy. A large proportion of firms are at target leverage a large proportion of time, so the moments for the overall population of firms (first row in the first four blocks) and firms at target leverage (second row) are generally similar. The third row reports the average within the firms that are away from their target leverage. The fourth row refers to start-up firms, which constitute the vast majority of off-target firms.

	Unconditional mean	Mean under r^L	Mean under r^H
Average default rate (%)	3.37	3.53	3.21
Firms at target leverage	2.99	3.47	2.39
Off-target firms	4.66	4.25	5.06
Start-up firms	5.86	4.67	7.04
Average interest-to-income ratio (%)	29.86	20.85	38.80
Firms at target leverage	24.66	20.28	30.10
Off-target firms	43.45	27.73	59.02
Start-up firms	57.33	31.92	82.52
Average recovery rate (%)	46.53	42.95	50.09
Firms at target leverage	49.45	43.24	57.14
Off-target firms	41.05	40.02	42.07
Start-up firms	37.75	38.65	36.87
Average private benefits-to-income (%)	39.30	41.57	37.06
Firms at target leverage	35.13	40.91	27.95
Off-target firms	53.36	49.52	57.17
Start-up firms	65.27	54.04	76.41
Average leverage level	0.8522	0.9305	0.7759
Firms at target leverage	0.8245	0.9250	0.7000
Off-target firms	0.9674	0.9971	0.9381
Start-up firms	1.0601	1.0350	1.0850
Initial value (=marginal entry cost)	0.9004	0.9859	0.8149
Measure of active firms	4.9378	4.9720	4.9040

Additionally to the averages over the whole population of active firms, the table shows the averages for firms at target leverage (the vast majority of firms most of the periods), off-target firms (all the rest), and start-up firms (the most important cluster among off-target firms). Given their importance in the model, the last block of Table 2 reports moments referred to firms' leverage levels (R'), the initial value of start-up firms ($v(1 + y, r)$), and the measure of active firms (n_t).

The unconditional averages of the first four variables for firms at target level are, non-surprisingly, close to the corresponding calibration targets set for mature firms. The variation of most variables across firms and across interest-rate states is quite sizable. Of course, conditional on a particular value of r , debt repayment obligations and interest-to-income ratios positively comove in the cross section of firms, meaning that both variables capture well firms' relative leverage at a given point in time. However, it is worth noting that, across interest rate states, the two variables move in opposite directions which means that low rates make high leverage compatible with having low interest-to-income ratios and implies that this last variable is a poor proxy for default risk in the time-series dimension.

Figure 4 describes the paths of debt repayment obligations R' (top panel, in absolute value) and the default rate $1 - p$ (bottom panel, in percentage points) for firms started up in periods with low (bright curve) and high (dark curve) interest rates. The solid curves show the average conditional trajectories and the dashed curves delimit the range of two standard deviations around them.

The figure reflects firms' financial life cycle (start-ups approach target leverage gradually from above via earnings retention) and the stochastic nature of the economy (where interest rates shifts modify target leverage and force firms to make further adjustments over time). This explains the slow convergence of the average conditional trajectories of each variable to the corresponding unconditional means.²⁴ The two standard deviations bands give an idea of the extent to which firms' individual leverage and default experiences may depend on the evolution of interest rates during their lifetime.

²⁴Under our baseline calibration, without further shifts in interest rates, start-ups would reach target leverage in 2 years with the low interest rate and in 5 with the high one. This makes a mean adjustment period of 3.5 years, which is not incompatible with the ranges of average adjustment speeds estimated by Flannery and Rangan (2006) in a partial adjustment model.

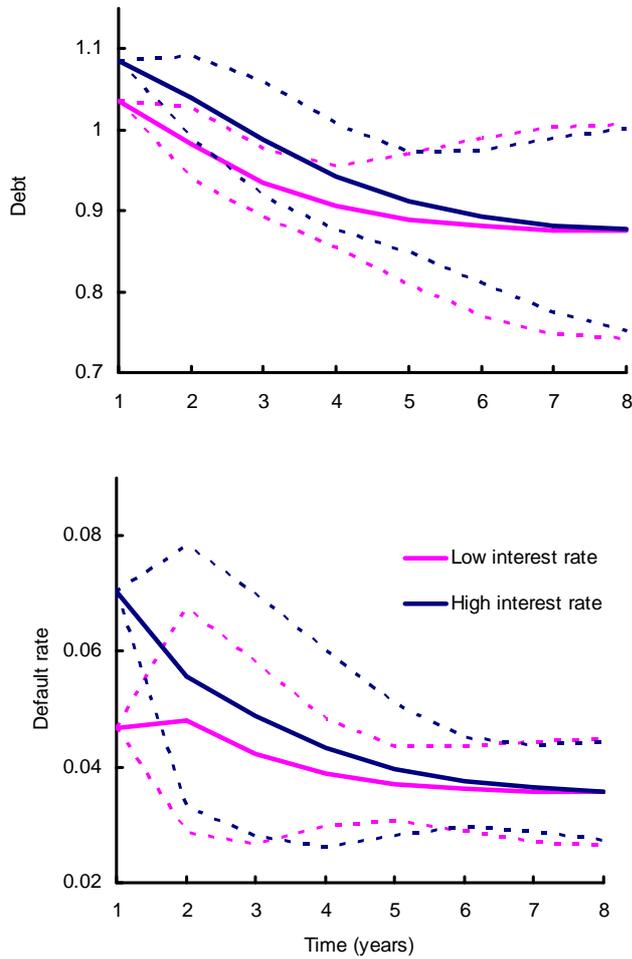


Figure 4. Evolution of leverage and default after investing

Figure 5 shows the average variation in default rates (in percentage points) after an increase (left panel) and a decrease (right panel) in the interest rate. The variation is computed as the deviation from the mean of simulated counterfactual paths in which the interest rate does not shift in year 0 (but may stochastically shift in any later year). The various curves represent the average effects for firms that are hit when they are at their target leverage (dark solid line), when they are away from such target (dashed line), and when they are just starting up (light solid line).

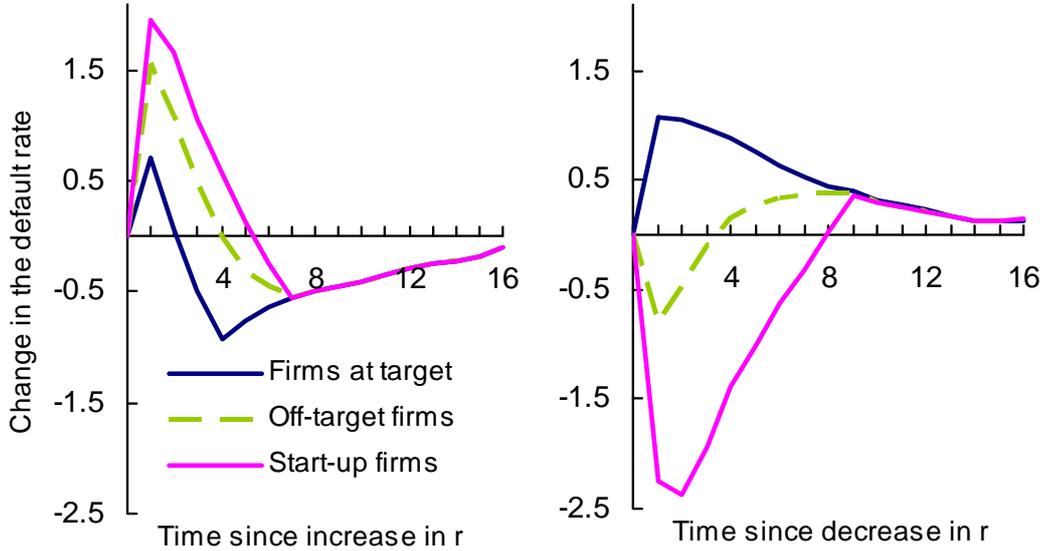


Figure 5. Individual effects of changes in the interest rate

Following an interest rate rise, all firms get into effectively excessive leverage (and higher default) before they adjust to the new, lower target leverage. And this short-term effect is stronger and longer lasting for the initially more indebted firms. In contrast, following an interest rate cut, firms formerly at target leverage can immediately adjust to the new, higher target leverage (which explains the spike in their default rate), whereas for the more indebted firms the cut implies lowering the burden of excessive leverage, a quicker convergence to the new target leverage, and a dramatic reduction in the default rate in the transition to it.

Figure 6 shows the implications of aggregating these heterogeneous and asymmetric effects across the whole population of active firms. It depicts the variation in the aggregate default rate (in percentage points) after an increase (dark line) and a decrease (light line) in the interest rate. As before, the variation is computed as the deviation from a counterfactual path in which the interest rate does not shift in year 0.

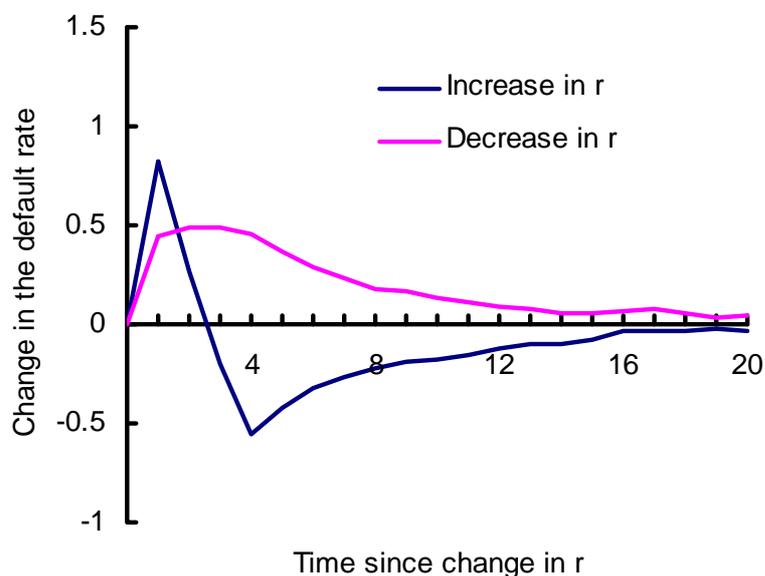


Figure 6. Aggregate effects of changes in the interest rate

Somewhat surprisingly, in the shortest run any shift in the risk-free rate has a negative impact on the aggregate default rate. The large default-increasing effect after a rise follows naturally from the aggregation the increases depicted in the left panel of Figure 5. The effect after a cut occurs because the default-increasing effect on (the large mass of) target firms dominates the default-decreasing effect on the (smaller mass) of off-target firms. Over longer horizons, moving to higher (lower) rates reduces (rises) both aggregate leverage and aggregate default.

Although it is tempting to interpret these results as informative about the potential impact of monetary policy shocks on default rates, one must take into account that the random interest rate shifts of our baseline calibration are larger and less frequent than the shocks explored in the macroeconomic literature. Comparing to that literature would require considering a lower range of variation of interest rates across states and/or a higher frequency of interest rate shifts, which is what we do in some of the robustness checks discussed in the next subsection.

4.3 Robustness of the results

In order to better understand the forces behind our baseline results and check their robustness, we recompute the aggregate effects of interest rate shifts under alternative values of each parameter. Specifically, we change one parameter (or combination of parameters) at a time and investigate the range of variation of such parameter (or combination of parameters) delimited by its logical bounds and either the value for which starting-up the firm is unfeasible (one extreme) or the value for which the start-up reaches its target leverage from the very first period (the other extreme). The columns in Figures 7-10 show the effects of moving the corresponding parameter in each of the directions up to near the limits of the commented range.

Interest rate variation across states The top row in Figure 7 considers the effects of changing the size of the variation in interest rates across states (for a constant mean rate). With smaller (larger) variation in r across states, interest rate shifts produce smaller (larger) aggregate effects on default rates because both target leverage and the overindebtedness of off-target firms vary less (more) across states. The short-run effect of an interest rate rise happens to be the most responsive to this element of the parameterization.

Interest rate persistence The second row in Figure 7 explores the effect of changing the persistence of each interest-rate state. When the persistence is low ($\pi = 0.50$), the stochastic convergence of the conditional mean default rate to the unconditional mean is very quick so only the short-term effects remain significant. Moreover, the short-run effect of an interest rate rise becomes stronger, while the short-term effect of an interest rate decline changes sign (because target leverage differs less across states and the debt-relief effect on overindebted firms becomes dominant). In this scenario, model predictions are consistent with the VAR evidence on the effects of monetary policy shocks on default risk, e.g. Bernanke et al. (1999), which suggests that credit spreads fall in response to expansionary shocks.

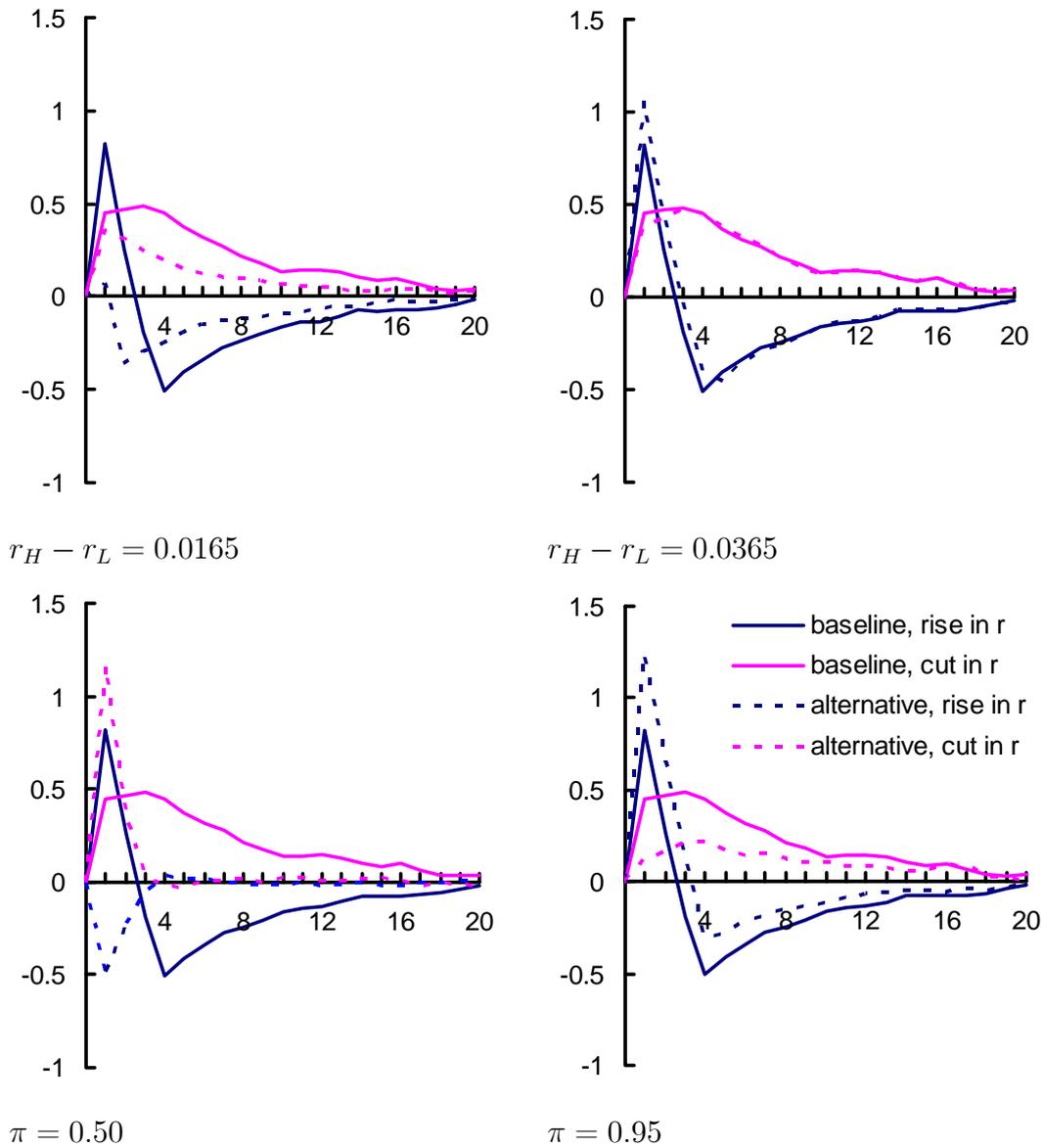


Figure 7. Robustness checks (1/4)

When the persistence of each state becomes very large ($\pi = 0.95$), the short-run effects of an interest rate rise get also reinforced (because the prospects of facing a high interest rate over a long horizon depresses the continuation value and, hence, the incentives of start-up firms), while the long-run effects become weaker (because entrants over successive periods face a similarly worsened overindebtedness problem). Higher persistence dampens the effects

of an interest rate cut because, although low rates still lead firms to target higher leverage, the associated default rate is not as high thanks to the positive effect of enhanced continuation values on incentives.²⁵

Entrepreneurs' discount factor The top row of Figure 8 shows the effects of making the entrepreneur further away (left panel) or closer (right panel) to their financiers in terms of intertemporal discounting. Entrepreneurs' high discounting is the key rationale for their maintained use of outside financing in the long run, in spite of the moral hazard cost. As suggested by Propositions 3 and 4, a lower discount *factor* β leads to higher default rates (because firms may take on more debt or, even if not, their continuation value becomes lower) and the results in the left panel suggest that the effect of interest rates shifts on the (very high) default rates of overindebted firms may become dominant in the aggregate. This also explains why an interest rate rise does not reduce the long-run default rate any more and why the effects of an interest rate cut change sign (due to the strong debt-relief effects on overindebted firms). In contrast, the effects obtained under a higher β are just a quantitatively stronger version of the baseline effects, which are driven by what happens with mature firms.

Net cash flow The second row of Figure 8 considers alternative values of the net cash flow of successful projects, y . Qualitatively the implications of moving y are very similar to the implications of moving β . The impact of both variables on continuation values explains that when they are low, investment projects get close to the feasibility boundary, and the evolution of the high default rates of start-ups dominates the evolution of the aggregate default rate. When they are high, firms easily reach their target leverage and the dominant effects behave qualitatively like the ones identified in Figure 5 for target firms.

²⁵This suggests that empirical situations in which interest rates are kept "too low for too long" might correspond, in model terms, with situations in which the short-term default-increasing effect of interest rates rises is abnormally strong, whereas the default-increasing effects of interest rate cuts is not necessarily too strong.

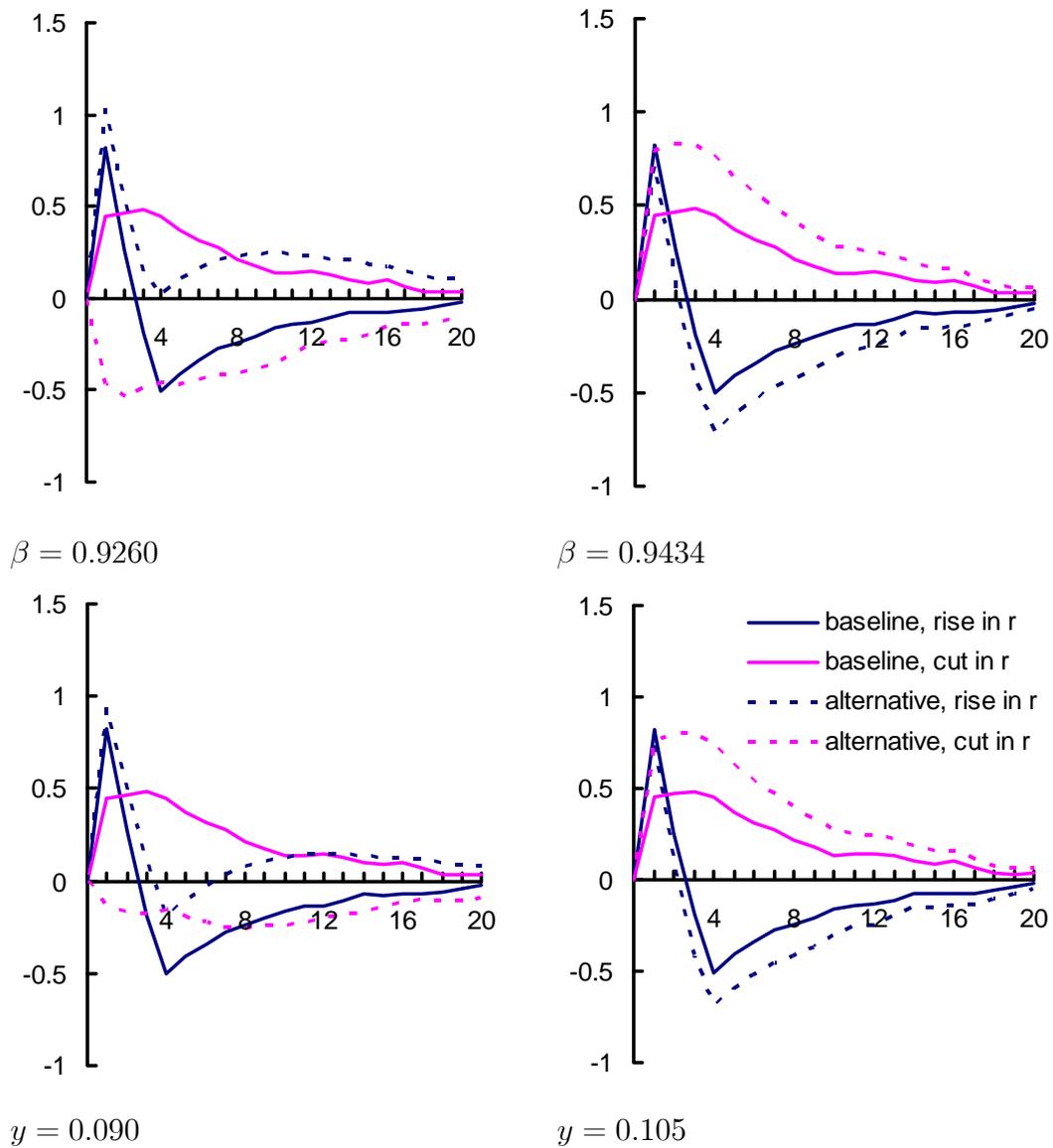


Figure 8. Robustness checks (2/4)

Liquidation value of assets The liquidation value of assets L has importance for lenders' recoveries when a firm fails, which other things equal reduces the repayments R that successful firms must make to their lenders. But this possible reduction in R has a positive feedback effect on incentives to which firms respond by increasing their target leverage and the associated default rate (Proposition 3). For off-target firms, the effects are the opposite,

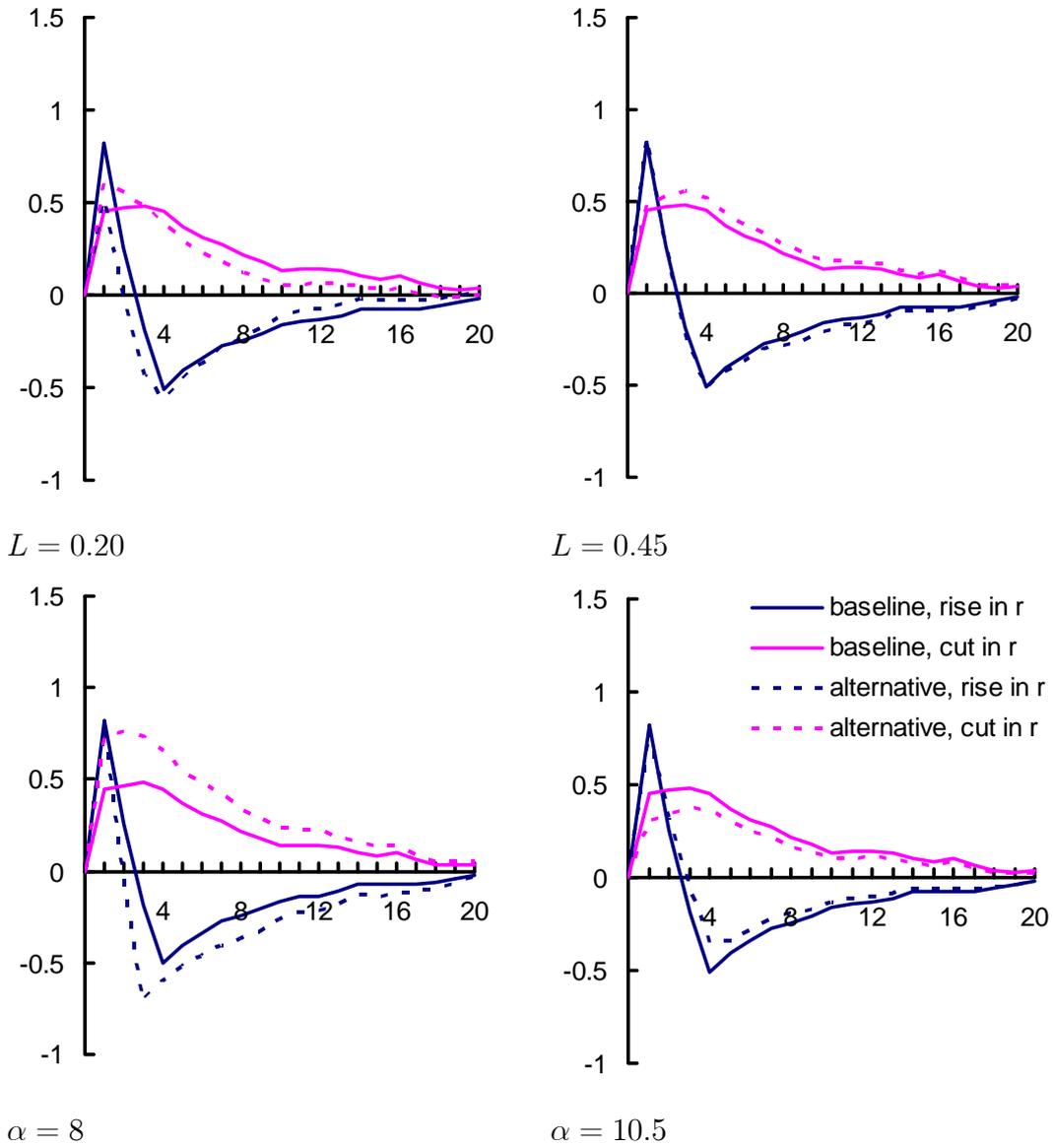


Figure 9. Robustness checks (3/4)

a larger L means lower effective leverage and lower default (Proposition 4). The first row of Figure 9 suggest that the two opposite effects largely compensate each other, making the impact of L on the aggregate effects of interest rate shifts small and slightly dominated by what happens among firms at target leverage.

Elasticity parameter – Private benefit function Parameter α measures the sensitivity of the private benefits $u(p)$ to the entrepreneur’s choice of the success probability p . Additionally, for any sequence of choices of p , a larger α depresses the associated flow of private benefits, so it also depresses the continuation value that determines entrepreneurs’ incentives to survive. Both ways, a larger α worsens the the moral hazard problem and pushes entrepreneurs lower leverage targets. The results in the bottom row of Figure 9 suggest that the choice of higher (lower) leverage targets makes them effectively more (less) reactive to changes in interest rates.²⁶

Intercept and scale parameters – private benefit function Figure 10 summarizes the effects of moving the parameters μ_0 and μ_1 of the private benefit function. We comment on both parameters jointly because the effects of increasing (decreasing) μ_0 are, qualitatively, the mirror image of those of decreasing (increasing) μ_1 . Apparently, the main effect of moving any of them is altering the level of private benefits and, through them, entrepreneurs’ continuation value and incentives.

When incentives worsen (i.e. μ_0 is lower or μ_1 is higher), what happens with off-target firms has a stronger weight on the evolution of the aggregate default rate, up to the extent that, in the depicted figures, an interest rate decline no longer increases the default rate, because the debt-relief effect on overindebted firms dominates the effect on firms that are at target leverage. When incentives improve (i.e. μ_0 is higher or μ_1 is lower), the effects get modified very much in the same direction as when α is lower (see the left bottom panel of Figure 9).

²⁶The other force behind the baseline results—what happens with the default rate of overindebted firms—is less affected by α because start-up leverage is much less dependent on α .

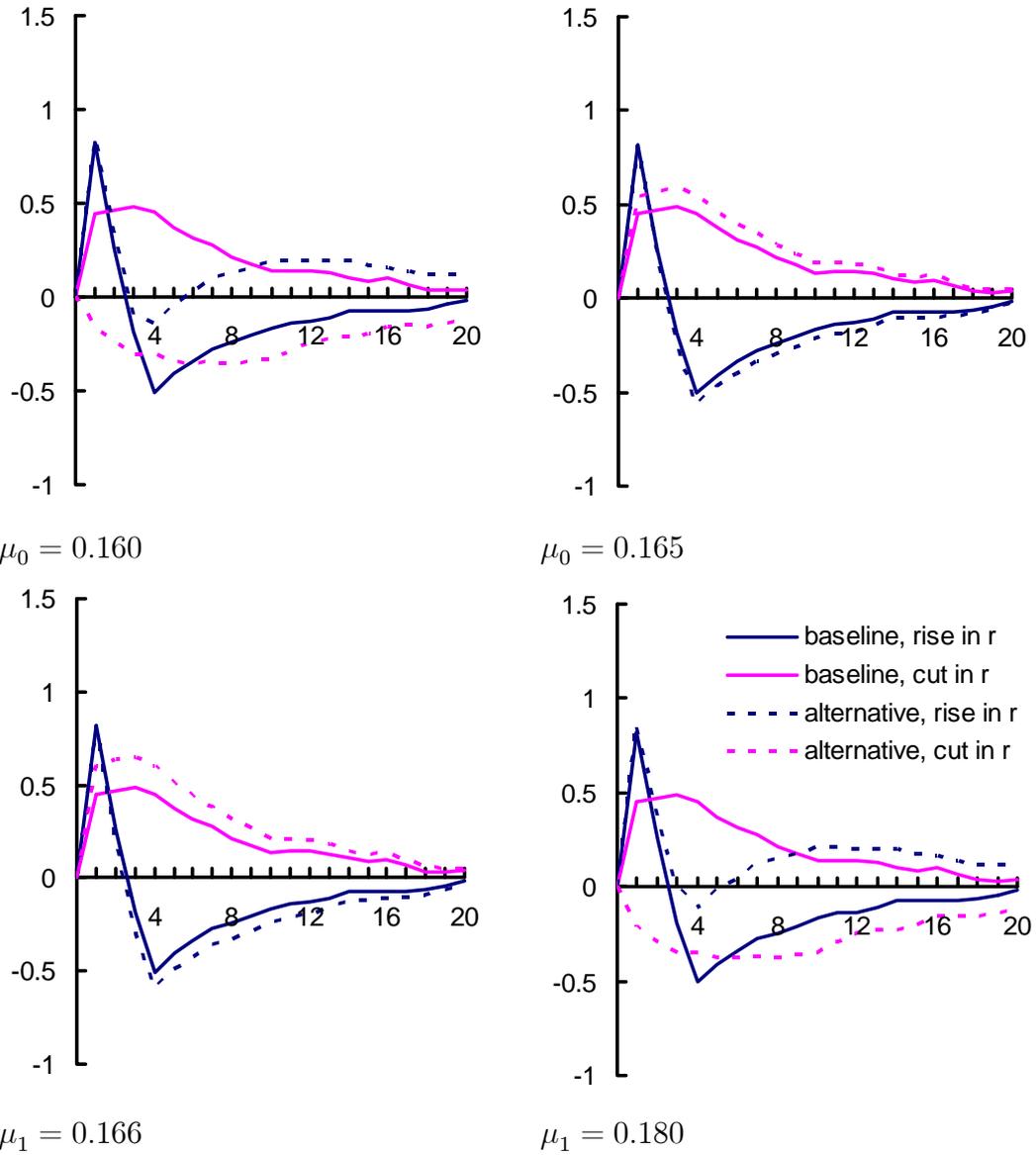


Figure 10. Robustness checks (4/4)

5 Discussion

5.1 Consistency with the empirical corporate finance literature

In this section we discuss the consistency of the main predictions of our analysis with existing evidence on firms' financial life cycle in particular and on firms' capital structure decisions more generally.

The link to the literature on firms' life cycle must be made with caution because our model contains assumptions that leave aside many potentially important factors affecting firms' life cycle. First, our firms' size is fixed and all the investment occurs at the start-up period, whereas real world firms would most typically start small and grow over time. Second, our firms are ex ante equal, except for the interest rate state faced when they start up, so they face no explicit process of learning whereby firm quality gets gradually revealed, reputation or market share build up, or a market discipline mechanism leading to, say, the survival of the fittest.²⁷ Lastly, our entrepreneurs are penniless when they start up, so 100% of their firms' initial funding needs are covered with outside funding. These simplifying assumptions guarantee internal consistency and help understand the mechanisms behind the results, but make it difficult to directly test the predictions of the model.²⁸

Most empirical work on firms' life cycles focuses on how firm size and firm survival evolves with firm age, but does not necessarily account for the importance or evolution of financial factors. Other papers look at the capital structure of young firms but do not pay attention to survival probabilities. So the discussion requires putting together many separate pieces.

Our model's feature regarding firms' gradual approach to some target leverage and the decline, *ceteris paribus*, in their probability of default along such process is consistent with the higher survival rate of older firms documented by Evans (1987) and Audretsch (1991), among others. It is also consistent with the evidence on the effects of firm age (controlling

²⁷However, one can think of our endogenous survival probabilities, affected by the moral hazard problem between entrepreneurs and their financiers, as a reduced form for the influence of finance-related incentives on that process.

²⁸To widen the applicability of implications of our analysis, one might argue that what is described as entry of start-ups in the model actually refers to a major expansion decision of previously established firms. Yet, this reinterpretation would only be strictly valid if there were zero extra inside funding available and if the new investment project were undertaken and funded through a separate limited liability entity. The model would then identify the *ceteris paribus* effects of interest rate shifts on the leverage and default probabilities associated with the funding of these new investments.

for size) on job destruction cited by Cooley and Quadrini (2001). According to Angelini and Generale (2008), financial factors are among the list of factors contributing to age effects, but the list also includes non-financial factors such as product life cycles (Agarwal and Gort, 2002) and idiosyncratic profitability shocks (Warusawitharana, 2013).

A recent paper by Robb and Robinson (2012) documents that external debt financing is much more important for young firms than commonly thought. In their words: “funding from formal debt dwarfs funding from friends and family. The average amount of bank financing is seven times greater than the average amount of insider-financed debt (...). Even among firms that rely on inside debt, the average amount of outside debt is nearly twice that of inside debt.” Huynh and Petrunia (2010) study the evolution of leverage and exit rates along the first years of existence of manufacturing firms in Canada and, consistently with the predictions of our model, find that the median leverage ratio strongly declines with age and that surviving firms tend to have lower leverage ratios than exiting firms.

Looking at a wider sample of firms but without a life-cycle perspective, Zingales (1998) offers evidence on the importance of leverage, in addition to individual efficiency, as a determinant of the probability of firm survival after a sector specific shock. The strong link between leverage and default risk has been recently confirmed by Molina (2005). This link also appears in Jacobson et al. (2013), whose evidence about the lower default probabilities of dividend-paying firms (i.e., under our interpretation, firms that have already reached their target leverage) is consistent with our predictions.

On these broader grounds, our model is also consistent with Flannery and Rangan (2006), Kayhan and Titman (2007) and Byoun (2008), who document the importance of history as a determinant of firms’ capital structure and their tendency to move towards target debt ratios over time. It is also consistent with DeAngelo et al. (2006), who document that dividend payments are much more frequent when retained earnings account for a large proportion of total equity, whereas they fall to near zero if equity is contributed rather than earned.²⁹

To the best of our knowledge, the few papers that analyze the effect of interest rates on default probabilities (already referred to in the Introduction) do not explicitly condition

²⁹Our model considers short-term debt as the only source of outside funding, but a similar moral hazard problem with implications for firms’ survival would emerge if our entrepreneurs were receiving their outside funding in the form of contributed equity.

the interest rate effects on firm leverage and do not allow for asymmetric effects of interest rises and cuts. The richness of the effects that the model produces both under its baseline calibration and under some of the alternative parameterizations discussed in subsection 4.3 make it hard to use effects estimated in linear regression models as a test of the validity of its predictions.

Consistent in spirit to our claim about the importance of taking potential asymmetries into account, Korajczyk and Levy (2003) and Halling et al. (2011) report differences in firms' target leverage (and in the speed of adjustment to the target) across expansions and recessions (and across different types of firms). In order to rationalize this evidence one could think of a variation of our model in which fluctuations in aggregate productivity or aggregate demand are captured by introducing Markov state dynamics in firms' output conditional on success (currently described by the constant y).³⁰

5.2 Macprudential implications

If we think of monetary policy as a possible determinant of what the model describes as r_t —financiers' opportunity cost of funds—then our results have implications for recent discussions on the macroprudential role of monetary policy. First of all, they provide a call for caution against conclusions reached in static models, in models without firm heterogeneity, in linear regression models, and whenever the empirical data contains relatively limited exogenous variation in the empirical counterpart of r_t . Those analyses may miss important aspects of the effects of interest rates on corporate leverage and credit risk.

According to our analytical results, lower interest rates imply larger target leverage but may or may not lead to lower default rates, specially at the aggregate level. Lower rates make the lending to the highest-levered firms safer because they reduce these firms' effective debt burden, but they may make mature firms riskier because mature firms will go for higher leverage. Indeed, under our baseline parameterization of the model, the aggregate default rate is on average higher when the interest rate is lower.³¹ So our results are consistent with

³⁰Firms would react to a switch to the low output state by reducing their target leverage but would experience high leverage and high default risk in the transition to their new target. A switch back to the high output state would increase target leverage but the transitional effects would not be symmetric to (but shorter and less intense than) those found in the switch to the low output state.

³¹However, as some of the robustness-check results show, this effect tends to change sign when incentive

the observation that having interest rates “too low for too long” may imply larger *risk taking* (Borio and Zhu, 2012). However, we also find that interest rate rises will tend to produce a temporary rise in default rates, which means that the macroprudential use of monetary policy (e.g. if authorities care about negative externalities caused by high default rates) requires a deep knowledge of the situation and involves difficult intertemporal trade-offs.

Finally, in a model like ours, absent negative externalities associated with a higher default rate, the equilibrium levels of leverage and default (sometimes interpreted as proxies of lenders’ risk taking, especially when they are banks) cannot be properly deemed excessive from a normative perspective because there would be no welfare gains from constraining entrepreneurs’ leverage decisions.³² Moreover, in our model, some of the usual proxies for risk taking tend to be larger when r_t is lower, but the model per se provides no rationale to advocate keeping interest rates artificially high so as to induce lower leverage and lower default rates. This means that extracting normative macroprudential prescriptions would require identifying and quantifying negative externalities associated with the equilibrium levels of leverage and default produced by a model like ours.

6 Conclusions

Monetary policies leading to a long period of (too) low interest rates have been mentioned as a possible factor contributing to the genesis of the Great Recession of 2008-2009. Wondering about possible theoretical underpinnings of such a view, this paper develops a dynamic corporate financing model with the explicit goal of examining the impact of shifts in the risk-free interest rate on corporate leverage and credit risk. Firms’ financing problem is affected by two key frictions: a moral hazard between entrepreneurs and their outside financiers and entrepreneurial wealth constraints. Interest rates enter the problem by determining outside financiers’ opportunity costs of funds. The endogenous link between leverage and default risk comes from the lower incentives of overindebted entrepreneurs to guarantee the survival

problems are severe enough and the debt-relief effect on overindebted firms becomes dominant.

³²As in other models with financial constraints (e.g. Lorenzoni, 2008), aggregate welfare measures may increase using redistributive policies that reduce the constraints of the agents for which they are more binding. In this economy, one such policy might consist of subsidizing over-indebted firms to speed up their convergence to target leverage. But more than constraining firms’ leverage, this policy would consist on transferring them funds that they would find optimal to use to reduce their leverage.

of their firms.

The need to finance new investment with outside funding pushes firms' leverage ratio above some state-contingent target towards which they gradually adjust through earnings retention. The dynamic response of leverage and default to cuts and rises in interest rates is both asymmetric (because it is easier to adjust to a higher target leverage than to a lower one) and heterogeneously distributed across firms (because interest rates affect the burden of outstanding leverage, which differs across firms). We find that, for some parameterizations, both interest rate rises and interest rate cuts increase the aggregate default rate in the short-run. Instead, higher rates tend to produce lower default rates in the longer run because they induce lower target leverage across all firms. These results help rationalize some of the empirical evidence regarding the so-called risk-taking channel of monetary policy and are relevant to discussions on the potential macroprudential role of monetary policy.

Appendix A: Proofs

Proof of Lemma 1 The domain of the value function is made of the values of R for which (2) has a solution. If (2) does not have a solution, the firm gets liquidated and pays a final dividend of zero. We could extend the definition of the value function for these values of R by making $v(R) = 0$ but in this extended domain the function is constant rather than strictly increasing. So we will constrain attention to the domain in which refinancing is feasible. Hence, consider any two values of the state variable, R_1 and R_2 with $R_1 < R_2$ for which (2) has a solution and denote the optimal decisions made under R_i as (p_i, b_i, R'_i) . Notice that if (p_2, b_2, R'_2) is a feasible solution under R_2 , then it is also a feasible solution under R_1 because the only effect of $R_1 < R_2$ on the constraints is to make (5) less stringent. However, the value of the objective function under R_1 when evaluated at the solution (p_2, b_2, R'_2) together with the definition of $v(R_1)$ implies:

$$v(R_1) \geq (y + b_2 - R_1) + \beta[u(p_2) + pv(R'_2)] = (R_2 - R_1) + v(R_2) > v(R_2),$$

which proves the result. ■

Proof of Proposition 1 The Lagrangian of this problem is:

$$\mathcal{L} = y + B(R') - R + \beta[u(P(R')) + P(R')v(R')] + \lambda[y + B(R') - R], \quad (16)$$

where $\lambda \geq 0$ is the Lagrange multiplier associated to the inequality constraint. By Kuhn-Tucker conditions, the constraint may or may not be binding. If (10) is not binding, then $\lambda = 0$ and the solution for R' satisfies the following FOC:

$$B'(R') + \beta\{[u'(P(R')) + v(R')]P'(R') + P(R')v'(R')\} = B'(R') + \beta P(R')v'(R') = 0, \quad (17)$$

where we have used the fact that $[u'(P(R')) + v(R')] = 0$ by (6). Interestingly, (17) does not involve R , so it yields the same unconstrained solution $R' = R^*$ for all the values of R in which (10) can be satisfied, that is, for all $R \leq \bar{R} \equiv y + B(R^*)$. Notice that in (17) the fact that $v'(R') < 0$ for all R' implies that $B'(R^*) > 0$, which means (as advanced above) that R^* must be in an upward sloping section of $B(R')$ and we must have $R^* \leq \bar{R}$.

For $R > \bar{R}$, (10) is binding, then $\lambda \geq 0$ and the solution involves choosing the lowest $R' > R^*$ that satisfies $B(R') = R - y$. Thus, this solution must also be in the upward sloping section of $B(R')$. With a single-peaked $B(R')$ the inverse of its upward sloping section is well defined, so with slight abuse of notation we can describe the candidate solution as $R' = B^{-1}(R - y)$. The existence of this candidate solution requires $B^{-1}(R - y) \leq \bar{R}$, i.e. $R \leq y + B(\bar{R})$.

The remaining statements contained in the proposition can be immediately derived from these results. ■

Proof of Proposition 3 The results stated in this proposition come from the comparative statics (total differentiation) of a reduced system of equations in R^* and p^* and the exogenous parameters which describes the steady state. To obtain such a reduced system, consider first the first order condition (17) satisfied by R^* :

$$B'(R^*) - \beta P(R^*) = 0. \quad (18)$$

We want to find out a compact expression of $B'(R^*)$.

Particularizing (7) and (6), we have

$$B(R^*) = \frac{1}{1+r} [L + P(R^*)(R^* - L)], \quad (19)$$

and

$$u'(P(R^*)) + v(R^*) = 0. \quad (20)$$

It directly follows from deriving in (19) with respect to R^* that

$$B'(R^*) = \frac{1}{1+r} [P(R^*) + P'(R^*)(R^* - L)]. \quad (21)$$

On the other hand, deriving in (20) with respect to R^* we have:

$$u''(P(R^*))P'(R^*) + v'(R^*) = 0.$$

But applying the Envelop Theorem on (9) for $R = R^*$ and taking into account that the constraint of the stage problem is not binding at that point makes it obvious that $v'(R^*) = -1$, so we can write

$$P'(R^*) = \frac{1}{u''(P(R^*))},$$

which substituted into (21) yields

$$B'(R^*) = \frac{1}{1+r} \left[P(R^*) + \frac{1}{u''(P(R^*))} (R^* - L) \right],$$

which plugged into equation (18) and, after some reordering, allows us to write such equation as:

$$[1 - \beta(1+r)]P(R^*) + \frac{1}{u''(P(R^*))} (R^* - L) = 0. \quad (22)$$

Since $\beta(1+r) < 1$ and $u''(\cdot) < 0$, satisfying this condition requires $R^* > L$, which means that the firm will default on its repayment obligation when it fails. The probability of failure

is $1 - p^*$, which is strictly positive under our assumption that $\lim_{p \rightarrow 1} |u'(p)|$ is sufficiently large.

Now, using the notation $v^* = v(R^*)$, $p^* = P(R^*)$, and $b^* = B(R^*)$, we can characterize the steady state as the solution to the system of the following four equations with four unknowns (namely p^* , b^* , R^* and v^*):

$$v^* = y + b^* - R^* + \beta[u(p^*) + p^*v^*], \quad (23)$$

$$-[1 - \beta(1 + r)]p^*u''(p^*) - (R^* - L) = 0, \quad (24)$$

$$b^* = \frac{1}{1 + r}[L + p^*(R^* - L)], \quad (25)$$

$$u'(p^*) + v^* = 0. \quad (26)$$

In this system, (23), (24), (25), and (26) are the counterparts of (2), (22), (19), and (20), respectively.

Finally, using (25) to substitute for b^* in (23), and (26) to also substitute for v^* in (23), the system simplifies to:

$$H_1(\underset{+}{p^*}, \underset{+}{R^*}; \underset{-}{\beta}, \underset{+}{r}, \underset{-}{y}, \underset{-}{L}) \equiv -(1 - \beta p^*)u'(p^*) - y + (1 - \frac{p^*}{1 + r})R^* - \frac{1 - p^*}{1 + r}L - \beta u(p^*) = 0,$$

$$H_2(\underset{+}{p^*}, \underset{-}{R^*}; \underset{-}{\beta}, \underset{-}{r}, \underset{-}{L}) \equiv -[1 - \beta(1 + r)]p^*u''(p^*) - (R^* - L) = 0,$$

where the only unknowns are p^* and R^* , and the signs below each of the arguments of the functions H_1 and H_2 indicate the sign of the corresponding partial derivative. The remaining results in the proposition come from fully differentiating this system with respect to p^* , R^* , and the exogenous parameters. Signing most of the results is immediate. Only a few require some cumbersome algebra that we skip for brevity. To sign some of the partial derivatives, it is important to remember two properties implicit in our formulation: (i) R^* is necessarily in the upward slopping section of $B(R^*)$, so we must have $B'(R^*) \geq 0$, which implies $-p^*u''(p^*) \geq (R^* - L)$, (ii) the (iso-elastic) form of $u(\cdot)$ adopted in (14) satisfies $-pu'''(p)/u''(p) = 2 - \alpha$ and, hence, is smaller than 1 for all $\alpha > 1$. ■

Proof of Proposition 4 The results stated in this proposition come from the comparative statics (total differentiation) of a reduced system of equations that, for each given value of $R > \bar{R}$, gives the value $R' = B^{-1}(R - y)$ that describes the upward slopping section of the phase diagram depicted in Figure 1, as well as the continuation probability $p = P(R')$ associated with that constrained level of leverage. Going back to the definitions of $B(R')$

and $P(R')$ in terms of more fundamental ingredients of the model, it is convenient to think of the relevant p and R' as determined by the system of three equations with three unknowns (with $b = B(R')$ as the third unknown) given by the binding constraints of the original optimization problem in (2):

$$u'(p) + v(R') = 0, \quad (27)$$

$$L + p(R - L) - (1 + r)b = 0, \quad (28)$$

$$y + b - R = 0, \quad (29)$$

where (27) is (3) written as in (6), (28) corresponds to (4), and (29) is the binding version of (5). Clearly we can use (29) to substitute for b in (28), and arrive to a two-equation system that can be written as:

$$J_1(\underset{-}{p}, \underset{-}{R'}; \underset{+}{\beta}, \underset{-}{r}, \underset{+}{y}, \underset{+}{L}) \equiv u'(\underset{-}{p}) + V(\underset{-}{R'}; \underset{-}{\beta}, \underset{+}{r}, \underset{-}{y}, \underset{+}{L}) = 0, \quad (30)$$

$$J_2(\underset{+}{p}, \underset{+}{R'}; \underset{-}{R}, \underset{-}{r}, \underset{+}{y}, \underset{+}{L}) \equiv L + p(R - L) - (1 + r)(R - y) = 0, \quad (31)$$

where we have written $v(R') = V(R'; \beta, y, L, r)$ to make clear that the continuation value of the firm, as determined by (2) is a function of the state variable as well as all the underlying parameters. The signs below each of the arguments of J_1 and J_2 indicate the sign of the corresponding partial derivatives, most of which are immediate to obtain. The partial derivatives of the function $V(R'; \beta, y, L, r)$ can be obtained using (2), the Envelop Theorem, and the fact that any parameter that tightens a binding constraint will reduce the value resulting from the maximization.

Equations describe two curves and the constrained optimal refinancing decision given R is determined by the intersection between them. To obtain the results stated in the proposition, notice that (30) implicitly defines an decreasing relationship between p and R' which can be compactly described as

$$p = P(\underset{-}{R'}; \underset{+}{\beta}, \underset{-}{r}, \underset{+}{y}, \underset{+}{L}), \quad (32)$$

and substituted into (31) to obtain:

$$K(\underset{+}{R'}; \underset{+}{\beta}, \underset{-}{R}, \underset{-}{r}, \underset{+}{y}, \underset{+}{L}) \equiv J_2(P(R'; \beta, r, y, L), R'; R, r, y, L) = 0. \quad (33)$$

The signs of the partial derivatives shown in (33) can be determined using the fact that having a strictly increasing relationship between R and R' as in the phase diagram in Figure

1 (or, equivalently, having R' in the upward slopping section of $B(R')$) necessarily requires

$$\frac{\partial J_2}{\partial p} \frac{\partial P}{\partial R'} + \frac{\partial J_2}{\partial R'} > 0.$$

Intuitively, this condition implies that relevant intersection between the two downward slopping relationships described by (30) and (31) always occurs at a point in which the curve associated with (31) is steeper than the curve described by (30), and this happens to help signing all the comparative static results in an unambiguous manner. The results for R' come directly from (33), whereas those for the default probability $1 - p$ come from combining the direct effects shown in (32) with those channeled through R' , which if anything reinforce the direct ones.

Finally notice that the statement that $R' > L$ stems from the fact that, for $R > \bar{R}$ we have $R' > R^*$ and we have already proved in Proposition 3 that $R^* > L$. Similarly, we have $1 - p(R') > 1 - p(R^*)$ and we have already established in Proposition 3 that $1 - p(R^*) > 0$. ■

Proof of Proposition 5 As explained when presenting Figure 1, having $\bar{R} < 1 + y \leq \hat{R}$ implies that entering firms start with a leverage level $R_0 = R'(1 + y) > R^*$ (Result 1), where $R'(x)$ represents the leverage of non-failing firms as a function of their previous leverage (i.e. the phase diagram in the figure). Indeed, the financing needs of start-up firm are equivalent to those of a continuing firm which had to refinance a repayment obligation $1 + y$, so the value of a just-entered firm can be expressed as $v(1 + y)$. And, the entry flow is formed by candidate entrants with an entry cost $\theta \leq v(1 + y)$. By Lemma 1, the value function is strictly decreasing, and, as explained in the proof of Proposition 4, it is also increasing in the parameters β , y , and L , and decreasing in r . Moreover, it is easy to check that $\partial v / \partial R + \partial v / \partial y > 0$, so $v(1 + y)$ is strictly increasing in y . Hence, the value of a just-entered firm and the entry cost of the marginal entrant are increasing in β , y , and L , and decreasing in r (Result 2).

Given the assumed distribution of θ , the entry flow can be expressed as $e_t = F(v(1 + y), n_t)$, which explains the second term in the right hand side of (11). The first term gives the measure of firms active at $t - 1$ that continue in operation at t , which is obtained by integrating the survival probability conditional on each leverage level, $P(R)$, over the distribution of leverage at $t - 1$. The right hand side of (11) is a continuous and strictly decreasing function of n_t . Moreover, it is strictly positive at 0 so it necessarily has a unique and strictly positive fixed point which identifies the equilibrium value of n_t (Result 3).

Finally, the law of motion for $\Psi_t(R)$ (Result 4) reflects that (i) active firms with leverage x at $t - 1$ survive with probability $P(x)$, in which case they choose $R'(x)$ at t , and (ii) the measure $e_t = F(v(1 + y), n_t)$ of entering firms start with leverage R_0 . ■

Appendix B: Equations with aggregate uncertainty

In this appendix we briefly extend the key equations of the model without aggregate uncertainty to the case in which r follows a Markov chain. Following convention, r' denotes the value of the interest rate one period ahead. Assuming, as in the case without aggregate uncertainty, that the firm that remains productive is always able to find financing for one more period, its optimal dynamic financing problem when prior leverage is R and financiers' current opportunity cost of funds is r can be stated in the following recursive terms:

$$v(R, r) = \max_{p, b, R'} (y + b - R) + \beta E[u(p) + pv(R', r') | r] \quad (34)$$

subject to

$$p = \arg \max_{x \in [0, 1]} u(x) + xE[v(R', r') | r], \quad (35)$$

$$pR' + (1 - p)L = b(1 + r), \quad \text{and} \quad (36)$$

$$y + b - R \geq 0. \quad (37)$$

This is a straightforward extension of the problem stated in equations (2)-(5) above.

The first order condition for an interior solution to the maximization embedded in (35) is

$$u'(p) + E[v(R', r') | r] = 0, \quad (38)$$

which extends (6) and can be conveniently used to replace (35).

Since the value function satisfies $\partial v(R', r') / \partial R' < 0$ and $u''(p) < 0$, (38) implicitly defines a function $p = P(R', r)$, with $\partial P(R') / \partial R' < 0$. Using it to substitute for p and solving for b in (36) yields:

$$b = B(R', r) \equiv \frac{1}{1 + r} [L + P(R', r)(R' - L)]. \quad (39)$$

Using this function $B(R')$ (which embeds (35) and (36)), the stage problem in (34) can be compactly expressed as:

$$\max_{R'} y + B(R', r) - R + \beta \{u(P(R', r)) + P(R', r)E[v(R', r') | r]\} \quad (40)$$

$$\text{s.t.: } y + B(R', r) - R \geq 0, \quad (41)$$

where the only decision variable is R' and the only constraint is a rewritten version of (37).

To reproduce results similar to those found in Proposition 1, notice that if (41) is not binding, the first order condition of the stage problem simplifies to $\partial B(R^*, r)/\partial R' + \beta P(R^*, r)E[\partial v(R', r')/\partial R'|r] = 0$, which implicitly defines a unique target leverage $R^*(r)$ for each r . Such target can be reached at the optimizing date if and only if $R \leq [0, \bar{R}(r)]$, where $\bar{R}(r) \equiv y + B(R^*(r), r)$. For the interval $R \in (\bar{R}(r), y + B(\bar{\bar{R}}(r), r)]$, where $\bar{\bar{R}}(r)$ is the value of R' that maximizes $B(R', r)$ given r , the firm uses the above-target leverage $R'(R, r)$ implicitly defined by (41) when it holds with equality. For $R > y + B(\bar{\bar{R}}(r), r)$, the firm cannot be refinanced and is liquidated. In a slight abuse of notation, we will refer to the policy function emerging from this optimization as $R'(R, r)$, in the understanding that $R'(R, r) = R^*(r)$ for $R \leq [0, \bar{R}(r)]$, i.e. when target leverage can be reached.

Similarly to Proposition 5, if $1 + y \in (\bar{R}(r), y + B(\bar{\bar{R}}(r), r)]$, then firms entering when financiers' opportunity cost of funds is r will start with a leverage level $R_0(r) > R^*(r)$. The entry flow will be formed by all candidate entrants with an entry cost $\theta \leq v(1 + y, r)$ and the dynamics of the measure of active firms can be represented as

$$n_t = \int_0^\infty P(R, r_{t-1}) d\Psi_{t-1}(R) + F(v(1 + y, r_t), n_t), \quad (42)$$

which implicitly defines n_t as a function of the distribution of leverage across the previous cohort of operating firms and the interest rates prevailing at $t - 1$ and t . This equation generalizes (11). The generalization of the law of motion of $\Psi_t(R)$ is provided in (13).

Appendix C: The model with long-term debt

In the baseline model, short-term debt is the only source of external funding. In this appendix we present an extended version of the model in which firms' can additionally issue long-term debt when started up. Long-term debt is modeled as a perpetual bond that is sold at a price D_0 and promises a constant coupon $z < y$ per period while the firm is alive. For simplicity, we assume that in case of liquidation L is still entirely distributed among the holders of short-term debt. In this case, the presence of long-term debt modifies the problem formulated in (34)-(37) in a rather trivial way: It is as if the cash flow contingent on continuation in the baseline problem were $y - z$ rather than y . So, like in (40)-(41), we can write

$$v(R, r; z) = \max_{R'} y - z + B(R', r; z) - R + \beta \{u(P(R', r; z)) + P(R', r; z)E[v(R', r'; z)|r]\} \quad (43)$$

$$\text{s.t.: } y - z + B(R', r; z) - R \geq 0, \quad (44)$$

where, to reflect the new time-invariant capital structure decision, we add z as an argument in the functions involved. These functions are otherwise defined like in the baseline model. The solution to this problem yields the policy function $R'(R, r; z)$ that describes the short-term leverage decision of a non-failing firm. This policy function has the same form as in the baseline model, making R' equal to some target short-term leverage $R^*(r; z)$ if (44) is not binding, and equal to the value implicitly defined by the lowest solution to $y - z + B(R', r; z) - R = 0$, otherwise.

Let $D(R, r; z)$ denote the ex-coupon valuation of the long-term debt of a continuing firm which carries short-term leverage R from the previous date and faces a date in which financiers' opportunity cost of funds is r . This value obeys the following recursive expression:

$$D(R, r; z) = \frac{1}{1+r} P(R'(R, r; z), r; z) \{z + E[D(R'(R, r; z), r'; z)|r]\}. \quad (45)$$

Finally, assuming that there exist no z that would allow firms to start up with short-term leverage equal to their target leverage, $R^*(r; z)$, the problem solved by the firm at its start-up date can be formulated as follows:

$$\begin{aligned} \max_{D_0, z} \quad & v(1 + y - z - D_0, r; z) \\ \text{st.:} \quad & D_0 = D(1 + y - z - D_0, r; z), \end{aligned}$$

which will deliver a solution $(D_0(r), z(r))$, dependent on the interest rate r prevailing at the start-up date. To explain this formulation, notice that if the firm sells its long-term debt at the price D_0 , then it will have to start up with a short-term leverage equivalent to that of a continuing firm whose repayment obligations coming from the previous date were $1 + y - z - D_0$, where 1 reflects the need to undertake the initial investment, y reflects the lack of the cash flow coming from a prior period of operation, and, differently from the baseline model, z credits for the absence of a coupon to pay on (prior) long-term term, and D_0 credits for the funding covered with the sale of long-term debt. The constraint of the problem implicitly describes all the pairs (D_0, z) consistent with the valuation formula (45) and with the short-term financing needs of the start up, as represented by the previous analogy.

When we solve this extension of the model under the same parameter values as in Table 1, it turns out that firms find it optimal not to issue any long-term debt at their creation, which means that our focus on short-term debt implies no loss of generality for such a parameterization.

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