Optimally solving banks' legacy problems

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Abstract

We characterize policy interventions directed to minimize the cost to the deposit guarantee scheme and the taxpayers of banks with legacy problems. Non-performing loans (NPLs) with low and risky returns create a debt overhang that induces bank owners to forego profitable lending opportunities. NPL disposal and provisioning requirements can restore the incentives to undertake new lending but, as they force bank owners to absorb losses, can also make them prefer the bank being liquidated. For severe legacy problems, combining those NPL-targeted interventions with positive transfers is optimal and involves no conflict between minimizing the cost to the authority and maximizing overall surplus.

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1 Introduction

The size and persistence of the stocks of non-performing loans (NPLs) held by EU banks in the aftermath of the global financial crisis has worried observers and policy makers for many years.[1] Weak legacy assets are widely regarded as a source of vulnerabilities and an obstacle to the recovery of bank lending. Eventually, the Council of the European Union (2017) launched an ambitious action plan to address the issue from multiple fronts, including improving the efficiency of insolvency procedures, promoting active secondary markets for NPLs, and introducing supervisory guidance on banks' management of NPLs. The plan acknowledges the potential role of new instruments such as setting calendars for NPL provisioning, write-off or disposal directly aimed at pushing banks to remove the weak legacy assets from their balance sheets.[2] The debate on the effectiveness of these measures is commonly intertwined with that on the convenience of combining them with some form of state aid, and how to address the issue that decisive action on NPLs might push some banks into liquidation.

This paper offers an analytical contribution to the discussion. We develop a stylized model of a regulated bank with a legacy portfolio of NPLs, access to insured deposit funding, and new profitable lending opportunities. We characterize interventions that minimize the joint cost to the deposit guarantee scheme (DGS) and the taxpayers in circumstances where the size of the NPL problem creates a debt overhang problem that discourages new lending. We find that interventions leading banks to further absorb the losses associated with their NPLs (specifically, NPL disposals if the market for legacy assets is liquid or prudential provisioning requirements when it is not) combined, in the most severe cases, with public transfers can achieve unconstrained optimality. Moreover, these interventions involve no conflict between the cost-minimizing objective of the authority and the full undertaking of profitable lending opportunities.

At some initial date the bank finances a portfolio of loans with a mixture of insured deposits and owners' capital, in proportions constrained by existing capital regulation. At

[1] For initiatives by European authorities to diagnose and addres the problem, see European Banking Authority (2016), European Central Bank (2017), and European Systemic Risk Board (2017).

[2] In the US similar incentives have existed for longer, e.g. in the form of accounting rules that imply the full write-off of bad loans within a limited time horizon and a tax treatment whereby the losses associated with defaulted exposures become tax deductible only once the exposure is written-off.

an interim date, which is the focus of the analysis, a fraction of the initial loans become non-performing, meaning that their payoffs in all future states of the economy will be lower than those of the performing loans. Due to provisioning and capital requirements applied on them, NPLs may require bank owners to recapitalize the bank at the interim date. Yet, if the bank's capital is insufficient to absorb the losses implied by the NPLs in some adverse future states, a bank that retains a large amount of NPLs may end up failing, imposing a cost to the DGS.

The presence of these potentially uncovered losses affects bank owners' decisions in two interrelated manners. First, it makes them reluctant to undertake new good lending, which may need to be partly funded with new own funds. Bank owners anticipate that part of the returns of such investments will effectively go to the DGS in the form of lower losses in case of failure (or a lower probability of failure). This situation is akin to the classical debt overhang problem of Myers (1977) and may preclude new lending. Second, uncovered losses may also make bank owners reluctant to provide the new capital necessary to comply with capital regulation after recognizing the losses implied by the NPLs, since such capital reduces the losses that the DGS would incur if the bank fails. So bank owners may prefer the straight liquidation of the bank rather than complying with the regulatory requirements.[3]

In this context, we consider the problem faced by an authority whose objective is to minimize the cost of the guarantees offered by the DGS and any (other) transfer from taxpayers implied by the intervention.[4] The authority can, in principle, use a wide array of policies that include combinations of mandatory NPL sales, increases in the prudential provisioning of NPLs, changes in the capital requirements on performing and new loans, mandatory new lending amounts, and also monetary transfers to the bank. Once the authority sets its policy, bank owners decide whether to be compliant or to precipitate the liquidation of the bank. We later show that an intervention toolkit that includes solely an NPL-targeted policy implying an increase in loss recognition (via disposal or provisioning requirements applied to NPLs) and public transfers is always sufficient to achieve optimal policies. The transfer

[3] To keep things simple, we assume that liquidation implies no other inefficiency than the loss of the bank's new lending opportunities. Since even under this assumption liquidation is never an outcome of the optimal intervention policies, including additional inefficiencies from bank liquidation would not alter our results.

[4] In practice, most DGSs get funded in normal times with fees paid by the insured banks but such funding is insufficient in crisis times, thus requiring a fiscal backstop. Rather than explicitly discussing the funding of the DGS, we just assume for simplicity that the authority dealing with the bank aims to minimize the joint costs of the NPL problem to the DGS and the taxpayers.

could be reinterpreted as a scheme partly subsidizing the loss recognition, e.g. by means of a publicly sponsored asset management company when banks are required to dispose their NPLs or by a public recapitalization at below market terms when banks are required to increase provisions on their NPLs. Importantly, we distinguish between the case in which the secondary market for NPLs is liquid (allowing the sale of the NPLs at their fundamental value) and the one in which it is not (and NPLs can only be sold at a discount).

We first provide a sufficient condition for the optimality of the general class of interventions. The condition states that an intervention policy is unconstrained optimal if it meets three criteria. First, it either has no cost to the authority or leaves zero rents to the bank owners. Second, it avoids liquidation and leads to the full undertaking of the new lending opportunities. Third, if the market for NPLs is illiquid, the intervention does not include the disposal of any NPL. These criteria build on the intuition that, given that insured depositors are always fully repaid, a reduction of the cost to the authority can only be achieved through an increase in the net value of the bank and/or a reduction in the value bank owners extract from the bank. The net value of the bank is maximized when all its profitable lending opportunities are undertaken, and, should the NPL market be illiquid, no disposal of NPLs is required. Interestingly, optimal policies satisfying these criteria avoid liquidation, even when the cost of the NPL problem to the authority is strictly positive because resolving the bank would only increase such cost.

When NPLs have a liquid market, there are combinations of NPL disposal requirements and transfers that allow to meet the unconstrained optimality criteria. We first show that after disposing of a sufficiently large fraction of their NPLs, the bank owners would always be willing to lend. The reason is that the NPL disposal reduces the DGS subsidy whose loss would otherwise make bank owners reluctant to undertake the new lending. Next we show that if the disposal of the NPLs is so onerous that bank owners would prefer to let the bank being liquidated, the authority can make a positive monetary transfer just big enough to induce compliance (and new lending).[5] Thus, a "stick and carrot" policy that combines NPL disposals with public transfers (that leave zero rents to the bank owners) is optimal.

When the market of NPLs is illiquid, the use of the NPL disposal tool implies additional losses. But there is a manner in which bank owners can be induced to invest without dis-

[5] We identify cases in which NPL disposal is not necessary to prevent the debt overhang problem but still the authority needs to make positive transfers to prevent bank owners from letting the bank being liquidated.

posing of their NPLs: prudential provisioning. We show that forcing further provisioning of the NPLs effectively reduces the subsidy from the DGS associated with them, which in turn reduces bank owners' reluctance to undertake new profitable lending. Thus, suitable combinations of prudential provisioning of NPLs and public transfers allow to achieve unconstrained optimality and the presence of an illiquid NPL market does not increase the overall cost of the legacy problem.

We complement the analysis by discussing a number of additional variations in the policy toolkit. In addition to their practical relevance, these variations help us test the robustness of our key results. First, we consider interventions based on a purchase and assumption (P&A) transaction in which, following the disposal of some of the NPLs of the distressed bank (or their prudential provisioning), its assets (including new lending opportunities) and its liabilities are transferred to a healthier bank. We show that the financial strength of the healthier bank reduces the incentives of the merged bank (relative to the stand-alone distressed bank) to forego new lending and thus the need for NPL disposals or prudential provisioning relative to the baseline stand-alone optimal intervention. This might help rationalize the historical preference of the US Federal Deposit Insurance Corporation (FDIC) and other resolution authorities for this type of interventions. Yet, we find that inducing the voluntary participation of the healthier bank involves the same minimum amount of public transfers and minimum overall cost to the authority as the stand-alone interventions of the baseline setup.

In a second extension, we analyze the case in which the bank is also initially funded with uninsured long-term debt on which the authority may exercise some bail-in power as part of its intervention policy. We show that, when the bank has sufficiently many NPLs, the exercise of such bail-in power is essential to preserve the optimality to the authority of inducing the new lending. Absent that power, interventions inducing new lending would imply an onerous transfer of value from the authority to the long-term debtholders and it might be less costly to liquidate the bank.

In a third extension, we consider an authority that is not allowed to make transfers to the bank, and find that there are cases in which such authority would also push the bank into liquidation (e.g., by imposing a sufficiently large NPL disposal or prudential provisioning requirement). The reason is that, in these cases, it is impossible to induce new lending and allowing the bank to continue would simply increase the option-like liabilities of the DGS.[6] Relative to the optimal unconstrained policy, the expected cost to the authority in these cases increases by exactly the NPV of the forgone lending opportunities. So restrictions on state aid in this setup would backfire and have a cost to the DGS in excess of the money saved by prohibiting the transfers.

Finally, we also briefly discuss other extensions covered in detail in an appendix, including the analysis of the impact of the prospects of intervention on the quality of ex ante lending decisions. We show that the optimal interventions do not aggravate the distortions already due to the presence of deposit insurance.

The paper is organized as follows. Section 2 presents the model. In Section 3 we establish the sufficient conditions for the optimality of public interventions in our setup. Section 4 shows that when the NPL market is liquid, interventions based on NPL disposal requirements and public transfers can attain optimality. Section 5 shows that, if the NPL market is illiquid, replacing the NPL disposal requirement with a prudential adjustment to the provisioning of NPLs may allow the authority to attain optimality at the same cost as if the market were liquid. Section 6 analyzes the possibility of attaining optimality with additional variations in the policy toolkit and in extended environments (presence of uninsured long-term debt and ex ante moral hazard problems by bank owners). Section 7 concludes. Appendix A includes the proofs of the main results. Appendix B provides details on the variations of the baseline model.

Related literature In our setup, bank shareholders face a reluctance to reduce leverage and a debt overhang problem similar to those described in classical corporate finance references. Black and Scholes (1973) and Merton (1974) were first to establish that shareholders might avoid leverage reductions in order to preserve the value of their default option. As in Admati et al. (2018), the conflict may push shareholders to increase leverage over time and, if the firm is required to reduce its leverage, they may prefer selling safer assets first rather than raising new equity.

In Myers (1977), the prospects of appropriation of the returns of new investment by the more senior creditors (debt overhang) causes underinvestment. We analyze a similar problem

[6] As in Merton (1977), the residual riskiness of the NPLs increases the DGS liability relative to the situation in which the position is closed at the interim date through the sale (or provisioning) of the NPLs.

in a context in which a bank is partly funded with insured deposits and subject to capital requirements. And we address the problem from the perspective of an authority which cares about the costs to the DGS and the taxpayers.

In an institutional set-up similar to ours, Bahaj and Malherbe (2017) show the ambiguous impact of capital requirements on the lending incentives. In their setup, adding capital mitigates the debt overhang problem but also reduces the risk-shifting motives for undertaking new lending, hence the ambiguity. Their paper focuses on the implications for lending without addressing the design of optimal policies.

Our paper is also related to papers aimed at characterizing optimal interventions on distressed banks. Most of the existing papers combine asymmetric information on the quality of existing or new assets with gambling incentives. Bruche and Llobet (2013) address the problem of preventing the inefficient rolling-over of bad loans from a mechanism-design social-welfare-maximizing perspective. Their optimal interventions can be interpreted as quantity-dependent subsidies to loan disposal and leave no informational rents to bank owners.[7] In Philippon and Schnabl (2013) asymmetric information on banks' new investment opportunities gives rise to both underinvestment in profitable projects and opportunistic investment in unprofitable risky ones. The optimal policy consists of cash injections (that limit the debt overhang) in exchange for preferred stock and warrants (that limit risk shifting temptations). Diamond and Rajan (2011) show the role of mandatory illiquid asset disposals in a context where the interaction between distressed banks (that hold illiquid assets for gambling reasons) and sound banks (that hoard liquidity in order to profit from fire sales by distressed banks) produces suboptimal investment in good assets.

Relative to these papers, we combine a number of features that make our analysis particularly suitable to describe the problems faced when dealing with NPL problems in Europe: (i) we focus on regulated banks with access to insured deposit funding; (ii) we assume the new lending opportunities are with the same banks that hold the damaged assets; and (iii) we consider policy design from the perspective of minimizing the cost of the NPL problem to the DGS and the taxpayer (as mandated by recent regulatory reforms).

By its discussion on the role of transfers in the optimal interventions, our paper is also related to the literature on the motivations for bank bailouts, including Freixas (1999),

[7] Berglöf and Roland (1995), Aghion et al. (1999), and Mitchell (2001) explore related asymmetric information setups in which banks fail to efficiently restructure their bad loans due to gambling incentives.

Acharya and Yorulmazer (2007), Diamond and Rajan (2012), Farhi and Tirole (2012), and Keister (2016). Most of this literature considers the conflict between the ex-post optimality of bail-outs and their negative ex-ante incentive effects. In our model, bank liabilities are insured to start with and the positive transfers involved in some of the optimal interventions do not leave (additional) rents to bank shareholders and, thus, do not aggravate the ex ante moral hazard problem already generated by the presence of deposit insurance.

2 The model

There are three dates, $t = 0, 1, 2$, a bank, and three classes of agents with a stake at the bank: bank owners, depositors, and an authority. All the agents are risk neutral and have deep pockets and a zero discount rate. Bank owners provide equity funding to the bank and decide its funding and investment policy subject to capital regulation and the intervention policy set by the authority. Depositors provide deposit funding fully insured by a DGS.[8] The authority runs the DGS, fixes some intervention policy at the interim date ($t = 1$), and liquidates the bank, if necessary.

The bank's initial balance sheet At $t = 0$ the bank originates a measure one of ex ante identical loans that pay off at $t = 2$. The bank is initially funded with equity provided by its owners, e_0, and insured deposits, d_0. Regulation establishes a minimum capital requirement at both $t = 0$ and $t = 1$. At $t = 0$, the capital requirement per unit of lending is $\gamma > 0$, and we assume it to be binding, that is, $e_0 = \gamma$ and $d_0 = 1 - \gamma$.[9]

The loan portfolio at the interim date At $t = 1$ a fraction x of the loans deteriorate. We refer to these loans as NPLs, as opposed to the rest, called performing loans. The type of each loan is public information. A performing loan pays B_s at $t = 2$, where s is the state of the economy at that date, which can be high ($s = H$) or low ($s = L$), with probabilities μ and $1 - \mu$, respectively. An NPL pays Q_s in each final state s. At $t = 1$, the capital requirement per unit of principal still equals γ for performing loans but becomes $\phi > \gamma$

[8] We will treat bank owners and depositors as a single representative agent of each class, even if referring to them in plural.

[9] In Appendix B.5 we endogenize bank owners' capital structure decision at $t = 0$ and show the optimality of these choices.

for NPLs, reflecting the combined impact of provisioning standards and regulatory capital requirements. We make the following assumption:

Assumption 1 $Q_L < 1 - \phi < Q_H < 1 - \gamma < B_L < B_H$.

The assumption fixes the relationship between loan returns and capital requirements: (i) the return of each type of loan is higher in state H than in state L; (ii) the capital required on performing loans is enough to fully cover their potential losses in state L ($\gamma > 1 - B_L$); (iii) the interim capital requirement on NPLs is not sufficient to fully absorb the losses realized on these loans in state L ($\phi < 1 - Q_L$); and (iv) the interim capital requirement on NPLs is sufficient to fully absorb their losses in state H ($\phi > 1 - Q_H$), but the initial capital requirement is not ($\gamma < 1 - Q_H$). Properties (i)-(iii) are essential for the presence of NPLs to create a potential debt overhang problem, while property (iv) is only imposed for simplicity.[10]

New lending opportunities At $t = 1$ the bank has the opportunity to originate an additional measure y of ex ante identical loans with the same capital requirement at $t = 1$ and payoff structure at $t = 2$ as the existing performing loans. Moreover, we assume that the new loans have positive NPV, that is,

Assumption 2 $E[B_s] > 1$.

Finally, the following assumption on the probability μ of reaching state H at $t = 2$ ensures that, despite the positive NPV of the new loans, bank owners with too many NPLs might not find optimal to undertake new lending:

Assumption 3 $\mu < \frac{\gamma}{B_H - 1 + \gamma}$.

Public intervention policies Due to the presence of NPLs, the DGS may face disbursements associated with either the insolvency of the bank at $t = 2$ or its liquidation at $t = 1$ (if its owners do not comply with the regulatory requirements at that date). At $t = 1$, the authority adopts an intervention policy directed to minimize the overall expected cost of the

[10] Specifically, all the results in the paper would hold if the assumption $1 - \phi < Q_H < 1 - \gamma$ were replaced with the weaker assumption $E[Q_s] < 1 - \gamma$.

DGS liabilities and any (other) public funds involved in dealing with the bank. To focus the discussion, we assume for the time being that the authority can ask the bank to dispose of a fraction $\alpha \in [0,1]$ of its NPLs, increase the capital requirement on retained NPLs to $\tilde{\phi} \geq \phi$, and make a transfer $T \geq 0$ to the bank conditional on bank owners' compliance with all requirements. We assume that the NPLs can be disposed of at $t = 1$ by selling them at a market price q that is equal to or lower than their fundamental value, that is, $q \leq E[Q_s]$. We say that the NPL market is liquid if $q = E[Q_s]$, and illiquid if $q < E[Q_s]$.

We will later show that the focus on the $(\alpha, \tilde{\phi}, T)$ toolkit implies no loss of generality in the sense that the access to any other tools would not allow to reduce the overall expected cost of the legacy problem to the authority.

The bank's response After observing an intervention policy $(\alpha, \tilde{\phi}, T)$, the bank decides at $t = 1$ whether to comply with it or not. If compliant, the bank decides how much new lending $I \in [0, y]$ to undertake and some funding decisions $(\Delta e, d_1)$ compatible with capital requirements, where Δe is the net contribution made by bank owners at $t = 1$ (equity injections, if positive, or dividend payments, if negative) and d_1 are the deposits of the bank at $t = 1$, which may differ from d_0.

Liquidation A bank not compliant with the intervention is liquidated. Liquidation means that the bank owners obtain a zero payoff and the bank loses its new lending opportunities. The DGS pays back the deposits and appropriates the existing bank assets, whose expected payoff at $t = 2$ is not affected by the liquidation. Hence, if the residual net worth of the bank is negative, the DGS makes a loss in expectation, while if it is positive, the DGS makes a profit.[11] Thus, in this setup, the only surplus loss associated with liquidation is the NPV of the bank's new lending opportunities.

3 Sufficient condition for optimality

In this section we establish a sufficient condition for the optimality to the authority of interventions of a broader class than the $(\alpha, \tilde{\phi}, T)$ interventions that she deploys in the

[11] It is possible to prove that in either case bank owners take decisions that avoid liquidation whenever the residual net worth under liquidation is non-negative, so that the DGS never makes a profit in expectation.

baseline setup. We will later show that there are interventions of the baseline class that satisfy such condition and hence allow the authority to reach *unconstrained* optimality.

Unconstrained intervention policies allow the authority to set at $t = 1$ an NPL disposal requirement α, a public transfer T, the size of the new lending undertaken by the bank $I \in [0, y]$, and its new capital structure as characterized by its owners' net capital injection Δe and the new deposit amount d_1.[12] An unconstrained intervention policy is thus described by a tuple $(\alpha, T, I, \Delta e, d_1)$. The authority's choices must satisfy the bank's *sources and uses of funds* equality, which says

$$q\alpha x + T + \Delta e + (d_1 - d_0) = I. \tag{1}$$

The left hand side (LHS) of this equation includes the bank resources from the sale at price q of a fraction α of its NPLs, the public transfer T, the net equity injection by bank owners, and the variation in bank deposits between $t = 0$ and $t = 1$ (where the last two can be negative). The right hand side (RHS) accounts for the use of those funds in undertaking the new lending I.

As in the baseline setup, bank owners have the option not to comply with the intervention policy set by the authority, in which case the bank is liquidated, bank owners obtain a zero payoff, and no new lending takes place.

For a given intervention $(\alpha, T, I, \Delta e, d_1)$, the total continuation value of the assets of a compliant bank at $t = 1$ is given by:

$$V = E[Q_s](1 - \alpha)x + E[B_s](1 - x + I), \tag{2}$$

which takes into account that the bank ends $t = 1$ with NPLs and performing loans amounting to $(1 - \alpha)x$ and $1 - x + I$, respectively. This total continuation value can be decomposed as

$$V = E + D - G, \tag{3}$$

where E is the continuation value of bank owners' equity stake, $D = d_1$ is the continuation value of the insured deposits, and G is the expected cost of the guarantees offered by the

[12] In this design, we allow the authority to override the minimum capital requirements γ and ϕ, while in the baseline interventions the authority can only increase the capital requirements on NPLs (that is, the interventions $(\alpha, \tilde{\phi}, T)$ must satisfy $\tilde{\phi} \geq \phi$).

DGS on those deposits. The variables in (3) are generally functions of x, α, I, and d_1, but we will omit these arguments when there is no risk of confusion.[13]

The present value of the compliant bank to its owners at $t = 1$ net of the equity injection Δe is then

$$\Pi = -\Delta e + E. \tag{4}$$

Analogously, the expected cost of the NPL problem to the authority is

$$C = G + T. \tag{5}$$

Using the sources and uses of funds equality in (1), the total continuation value of bank assets in (2) and its decomposition in (3), we have

$$\begin{aligned} \Pi - C &= -\Delta e + E - T - G \\ &= V - D - (I - q\alpha x - (d_1 - d_0)) \\ &= (E[Q_s]x + E[B_s](1 - x) - d_0) + (E[B_s] - 1)\, I - (E[Q_s] - q)\, \alpha x, \end{aligned} \tag{6}$$

which uses the definitions of Π in (4) and C in (5). This equation says that the difference between the net value of the bank to its owners and the cost of the bank to the authority equals the sum of the net overall value of the bank at the beginning of $t = 1$ (understood as the value of its initial loans net of the initial obligations vis-à-vis the depositors) and the NPV of the lending undertaken at $t = 1$, minus the loss of value due to the disposal of NPLs when their market is illiquid (that is, $q < E[Q_s]$). Importantly, the only elements of $(\alpha, T, I, \Delta e, d_1)$ that affect $\Pi - C$ are the amount of new lending I and, with an illiquid NPL market, the disposal requirement α. The other elements of the intervention policy affect the distribution of the total continuation value of bank assets between bank owners (Π) and the authority (C), but not such total continuation value.

From (6), the cost of the bank to the authority, conditional on the bank being subsequently compliant, can be expressed as

$$C = \Pi - (E[B_s] - 1)\, I + (E[Q_s] - q)\, \alpha x - (E[Q_s]x + E[B_s](1 - x) - d_0)\,. \tag{7}$$

[13] The transfer T and the net equity injection Δe affect the resources available for the bank to undertake its lending at $t = 1$ as captured by (1) but have no direct effect on the overall continuation value of the bank and its components as expressed in (3).

The equation has three ceteris paribus implications. First, an increase in Π leads to an increase in C. The reason is that, for given values of the bank's assets and initial deposits, a higher value to the bank owners can only come from an increase in the subsidies G associated with the guarantees provided by the DGS. Second, an increase in the profitable lending undertaken by the bank at $t = 1$ leads to a reduction in the cost of the bank to the authority, which would benefit from the positive NPV of the new lending. Finally, when the NPL market is illiquid, an increase in the disposal requirement leads to an increase in C. This is because, for a constant Π, the expected bank asset value reduction implied by the NPL disposal in an illiquid market would be borne by the authority in the form of a higher C.

The expression for C in (7) was derived assuming the bank complies with the intervention at $t = 1$, but is easy to check that it must also hold under non-compliance. In such case, the liquidation of the bank implies $\Pi = I = \alpha = 0$.

So, in general, in order to minimize the cost C, the authority should aim to simultaneously (i) minimize the net value of the bank to its owners, Π, (ii) maximize the undertaking of the new lending opportunities, I, and (iii) avoid the disposal of NPLs if their market is illiquid. The following result builds on this intuition to establish a sufficient condition for the optimality of public interventions on NPLs:

Lemma 1 (*Sufficient condition for optimality*) *Let $(\alpha, T, I, \Delta e, d_1)$ be an intervention policy that:*

C1. Leads to a zero cost of the NPL problem to the authority ($C = 0$) or a zero net value of the bank to its owners ($\Pi = 0$).

C2. Induces the bank to comply and undertake all its new lending ($I = y$).

C3. Does not impose any disposal of NPLs if their market is illiquid ($\alpha = 0$ if $q < E[B_s]$).

Then $(\alpha, T, I, \Delta e, d_1)$ is an optimal intervention policy.

The intuition for this sufficient condition is as follows. Obviously, if a policy has zero cost to the DGS and the taxpayers then it is optimal. Consider the alternative case with $C > 0$. From (7), the only way to reduce C is to reduce bank owners' net value Π, to increase new lending I, or to reduce NPL disposals in an illiquid market. But, if the policy already leads to $\Pi = 0$, maximum new lending ($I = y$) and no NPL disposal if the NPL market is illiquid ($\alpha = 0$ if $q < E[B_s]$), then there is no room for reducing C any further. In fact, an additional reduction in Π would lead bank owners to opt for non-compliance (liquidation), in which case the new lending would not be undertaken and the NPL problem to the authority would increase.

Two implications of this lemma are worth noting at this point. First, intervention policies satisfying the sufficient condition for optimality avoid bank liquidation even when the overall cost of the NPL problem to the DGS and the taxpayers is strictly positive. Intuitively, liquidating the bank in those cases would make such cost strictly larger because it would preclude using the NPV of the new lending opportunities to reduce the expected liabilities of the DGS. Second, if a baseline intervention of the class $(\alpha, \tilde{\phi}, T)$ satisfies conditions C1-C3, then it is optimal in an unconstrained sense.

4 Optimal interventions with a liquid NPL market

In this section we consider the case in which NPLs can be sold in a liquid market, $q = E[B_s]$. We use the optimality condition in Lemma 1 to show that there exist policies of the class $(\alpha, \tilde{\phi}, T)$ satisfying $\tilde{\phi} = \phi$ which are optimal. For brevity, we refer to this subclass of policies as (α, T) interventions. These policies trivially satisfy condition C3 in Lemma 1 when the NPL market is liquid.

We split the analysis of this case in three steps. First, we study how the disposal requirement α and the decision on new lending I affect the cost of the compliant bank to the DGS. Second, we use this to show that a sufficiently large NPL disposal requirement induces full new lending if the bank complies. Finally, we analyze the bank's decision between complying and being liquidated, and identify the cases in which the *minimal* optimal intervention policy includes a positive transfer in order to induce compliance and avoid liquidation.[14]

4.1 The cost of deposit guarantees under compliance

Suppose that the bank subject to an (α, T) intervention complies and decides to undertake new lending I under funding decisions $(\Delta e, d_1)$ that satisfy (1). The bank also needs to

[14] By *minimal* we understand the optimal policy of the class (α, T) with the lowest T and, conditional on that, with the lowest α.

satisfy its regulatory capital requirement at $t = 1$, which can be written as

$$(1-\alpha)x + (1-x) + I - d_1 \geq \phi(1-\alpha)x + \gamma(1-x+I), \tag{8}$$

where the LHS is the bank's available regulatory capital (the book value of its assets after disposing of a fraction α of the NPLs and undertaking the new lending I minus the book value of its debt liabilities d_1) and the RHS is the bank's required regulatory capital, as determined by the fractional requirement ϕ on its retained NPLs plus the fractional requirement γ on its old and new performing loans.[15]

By standard arguments, it is (weakly) optimal for the limited liability bank owners to choose the maximum amount of insured deposit funding d_1 compatible with capital regulation, since this maximizes the subsidy associated with the deposit guarantee.[16] Thus, the optimal deposit base of the compliant bank is

$$d_1 = (1-\phi)(1-\alpha)x + (1-\gamma)(1-x+I), \tag{9}$$

which is independent from the transfer T.[17]

Assumption 1 implies that a bank that satisfies its capital requirements at $t = 1$ is always solvent in state H. Instead, the bank might be insolvent in state L. In state L, performing loans generate a capital surplus $\delta_B = B_L - 1 + \gamma > 0$, while NPLs generate a capital surplus $\delta_Q = Q_L - 1 + \phi < 0$ (that is, a capital deficit). Using the expression for d_1 in (9), the liabilities of the DGS in state L can be written as

$$\left(-\delta_Q(1-\alpha)x - \delta_B\,(1-x+I)\right)^+ . \tag{10}$$

So the DGS incurs a strictly positive cost in state L if the capital deficit associated with NPLs exceeds the capital surplus of the performing loans in such state (in which case the cost equals the difference between the two amounts).

[15] The expression in the RHS means that the assets of the bank after $t = 1$ are exclusively made of performing and non-performing loans; in other words, the cash received from the NPL sale $q\alpha x$, the transfer T, and the new equity injection Δe are fully used in financing the new lending and repaying some deposits (whenever $d_1 < d_0$), as implied by (1).

[16] Assumption 1 implies that, if the bank only has performing and new loans, it is solvent in the two aggregate states and thus obtains no subsidy on its deposits. By a continuity argument, the same is true when the measure of NPLs x is small. In those cases, the Modigliani-Miller result on capital structure indifference applies. But this implies that choosing the maximum admissible value of d_1 is also optimal in this case.

[17] From (1) we have that, for a given level of new lending I, the transfer T reduces one-by-one the additional net funding Δe that bank owners have to provide at $t = 1$.

From (10) we can derive the following lemma.

Lemma 2 (*Expected cost of deposit guarantees*) *The expected cost of the deposit guarantees enjoyed by a compliant bank subject to an intervention* (α, T) *that undertakes new lending* I *is given by*

$$G(\alpha, I|x) = \begin{cases} 0, & \text{if } \alpha \geq \alpha_{solv}(I, x), \\ (1-\mu)\left[(\delta_B - (1-\alpha)\delta_Q)\, x - \delta_B(1+I)\right], & \text{if } \alpha < \alpha_{solv}(I, x), \end{cases} \tag{11}$$

where $\alpha_{solv}(I, x) \equiv \left(1 - \frac{\delta_B}{-\delta_Q}\frac{1-x+I}{x}\right)^{+} \in [0, 1)$ *is the minimum fractional NPL disposal which makes the bank solvent in all states. The function* $G(\alpha, I|x)$ *is decreasing in* α *and* I*, and increasing in* x*, and strictly so if* $G(\alpha, I|x) > 0$*. The threshold* $\alpha_{solv}(I, x)$ *is decreasing in* I *and increasing in* x*, and strictly so if* $\alpha_{solv}(I, x) > 0$*.*

The lemma states that, conditional on compliance, NPL disposal reduces the expected cost of the bank to the DGS. The NPL sale forces bank owners to absorb the underlying losses at $t = 1$, removing the subsidy $-\delta_Q > 0$ per unit of NPLs that the bank would otherwise obtain from the DGS in the bad state. Once the disposed fraction of NPLs exceeds the threshold $\alpha_{\text{solv}}(I, x)$, the expected cost of the deposit guarantees to the DGS becomes zero. New lending reduces both the cost of the guarantees and the minimum NPL disposal for which such cost becomes zero because the new loans contribute a capital surplus δ_B in the bad state which partly or fully offsets the deficit caused by retained NPLs.

4.2 NPL disposal and optimal lending by the compliant bank

We now turn to the analysis of the decision on new lending of the compliant bank and describe how it depends on the NPL disposal requirement. Under a given (α, T), bank owners choose the amount of new lending I that maximizes the net value that they extract from the bank, Π. Taking into account that the overall expected cost of the compliant bank to the authority is $C = G(\alpha, I|x) + T$, we can use (6) to obtain the following expression for Π:

$$\Pi(\alpha, T, I|x) = E[Q_s]x + E[B_s](1-x) - (1-\gamma) + (E[B_s] - 1)I + G(\alpha, I|x) + T. \tag{12}$$

The dependence of Π on the fraction of NPLs that the bank is required to dispose of, α, comes entirely through the cost of the bank to the DGS, G. As established in Lemma 2, the marginal impact of α on G is (weakly) negative so that bank owners never have a strict preference for NPL disposal.[18]

From (12) and using (11), we can find the marginal effect of new lending on the net value of the bank to its owners:

$$\frac{\partial \Pi(\alpha, T, I|x)}{\partial I} = \begin{cases} E[B_s] - 1, & \text{if } \alpha \geq \alpha_{\text{solv}}(I, x), \\ E[B_s] - 1 - (1-\mu)\delta_B, & \text{if } \alpha < \alpha_{\text{solv}}(I, x). \end{cases} \tag{13}$$

Thus, if the compliant bank is always solvent at $t = 2$ (that is, if $\alpha \geq \alpha_{\text{solv}}(I, x)$), then I marginally increases Π by exactly the NPV of the new loans—bank owners appropriate all the marginal gains from the new investment. In contrast, if the bank is not solvent in the bad state ($\alpha < \alpha_{\text{solv}}(I, x)$), part of the marginal NPV generated by the new loans contributes to reduce the expected cost of the bank to the DGS—this explains the term $(1-\mu)\delta_B$ subtracted from the marginal profitability of the new lending.

It is a matter of algebraic manipulation from (13) to show that, under Assumption 3, $(1-\mu)\delta_B$ is large enough to make $\partial \Pi(\alpha, T, I|x)/\partial I < 0$ if $\alpha < \alpha_{\text{solv}}(I, x)$.[19]

Let us consider the non-trivial case in which $\alpha < \alpha_{\text{solv}}(0, x)$, so that if the compliant bank chooses $I = 0$, it will not be solvent in state L. From (13), owners' net value $\Pi(\alpha, T, I|x)$ is V-shaped with respect to the new lending I. For low I, $\Pi(\alpha, T, I|x)$ decreases with I but the capital surplus associated with the new loans brings the bank closer to solvency in state L. So, for high enough I, the bank becomes solvent in the two states ($\alpha \geq \alpha_{\text{solv}}(I, x)$ is satisfied) and further increases in I increase $\Pi(\alpha, T, I|x)$. Thus bank owners find optimal to either undertake all the feasible new lending, $I = y$, or no new lending at all, $I = 0$. Undertaking the maximal amount of new lending is optimal to bank owners if and only if $\Pi(\alpha, T, y|x) \geq \Pi(\alpha, T, 0|x)$, which using (12) is equivalent to

$$(E[B_s] - 1)y \geq G(\alpha, 0|x) - G(\alpha, y|x),$$

which is independent from the transfer T. But if bank owners find optimal to lend $I = y$ then the bank is solvent in the two states and $G(\alpha, y|x) = 0$. Hence

$$(E[B_s] - 1)y \geq G(\alpha, 0|x) \tag{14}$$

[18] So the implicit assumption that a compliant bank does not dispose of more NPLs than those required by the authority is without loss of generality.

[19] The interested reader can find the characterization of the optimal intervention policies when Assumption 3 does not hold and the discussion of the role played by NPL disposal in that context in Appendix C of an older version of this paper published as CEPR Discussion paper No. DP13718.

is a necessary and sufficient condition for the compliant bank to be willing to undertake new lending after the NPL disposal. This condition says that for bank owners to undertake new lending its NPV must exceed the forgone DGS subsidy.

From (14) we can derive the next formal result.

Lemma 3 (*New lending decision of the compliant bank*) *Consider a bank that complies with an intervention* (α, T)*. Let* $\alpha_{lend}(x,y) \in [0,1)$ *be zero if* $(E[B_s]-1)y \geq G(0,0|x)$ *and the solution to*

$$(E[B_s]-1)y = G(\alpha_{lend}(x,y),0|x), \tag{15}$$

otherwise. Then:

1. *If* $\alpha < \alpha_{lend}(x,y)$ *the bank does not undertake any new lending,* $I = 0$*.*
2. *Otherwise, the bank undertakes all its new lending,* $I = y$*. Besides, in this case, the bank becomes solvent in the two states.*

Finally, $\alpha_{lend}(x,y) > 0$ *for* $x > \frac{\delta_B}{\delta_B - \delta_Q}$ *and* $y < \frac{G(0,0|x)}{E[B_s]-1}$*, and* $\alpha_{lend}(x,y)$ *is increasing in* x *and decreasing in* y*, and strictly so if* $\alpha_{lend}(x,y) > 0$*.*

The lemma shows that there are circumstances in which a sufficiently large fractional NPL disposal requirement is necessary (and sufficient) for the compliant bank to undertake its new lending. Otherwise a debt overhang problem as in Myers (1977) arises. Figure 1 depicts the region of (x, y) space in which a disposal requirement $\alpha_{\text{lend}}(x,y) > 0$ is necessary to induce new lending by the compliant bank. A sufficiently large disposal of NPLs forces the bank owners to absorb the corresponding losses, removing the deposit insurance subsidy that makes the option to simply stick to the legacy assets more attractive than the undertaking of the new lending. The minimum fractional NPL disposal that induces new lending is strictly positive when the amount of NPLs in the bank is large and the size of the new lending opportunities is small. Moreover, such fraction is increasing in the amount of NPLs initially held by the bank and decreasing in the size of the new lending opportunities.

4.3 The potential optimality of positive transfers

Building on prior results, in this subsection we show that there exist interventions of the class (α, T) that satisfy the unconstrained optimality conditions in Lemma 1 and characterize the minimal optimal intervention. According to Lemma 1, when the NPL market is liquid an intervention that induces full new lending and either has a zero cost to the authority or reduces the net value of the bank to its owners to zero is optimal.

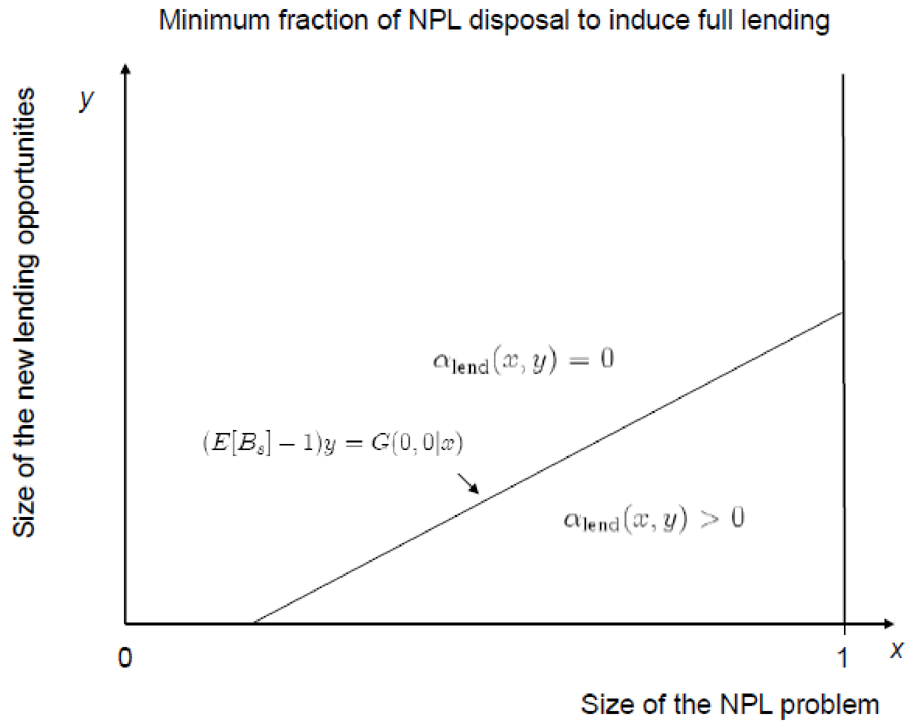

Figure 1

Let (α, T) be an intervention policy that leads the compliant bank to undertake its new lending opportunities in full. Lemma 3 implies that $\alpha \geq \alpha_{\text{lend}}(x, y)$ and that, after undertaking new lending in full, the bank is solvent in the two states at $t = 2$. Taking into account that non-compliance leads to liquidation (and a zero residual payoff), bank owners will prefer compliance to liquidation if and only if $\Pi(\alpha, T, y|x) \geq 0$, that is,

$$E[Q_s]x + E[B_s](1 - x) - (1 - \gamma) + T - (E[B_s] - 1)y \geq 0. \tag{16}$$

In the polar case in which all the initial loans were non-performing ($x = 1$), this condition would simplify to

$$(E[B_s] - 1)y + T \geq 1 - \gamma - E[Q_s], \tag{17}$$

whose RHS is strictly positive under Assumption 1. So, if the size y of the new lending opportunities is small and the public transfer T is zero, (17) will not be satisfied. Thus, for x close to one and y close to zero, the authority cannot induce the bank to undertake investment using NPL disposal as the only tool. The reason is that NPL disposal affects in a conflicting manner the two decisions the bank takes at $t = 1$, namely whether to comply or not, and, conditional on the former, how much new lending to undertake. NPL disposal reduces the subsidy received from the DGS, which reduces the debt overhang problem and incentivizes new lending should the bank be compliant. Yet, the reduction in the deposit insurance subsidy comes hand in hand with bank owners' need to inject funds in order to cover the NPL losses, which gives incentives to the limited liability protected bank owners to let the bank be liquidated at the interim date.

The following proposition, which constitutes the core result of this section, characterizes the minimal optimal intervention policy of the class (α, T), showing that positive public transfers solve the aforementioned conflict, when it appears:

Proposition 1 (*Optimal intervention with a liquid NPL market*) *There exists an intervention of the class (α, T) that is optimal. The minimal optimal intervention is $(\alpha^*(x, y), T^*(x, y))$ with*

$$\begin{aligned} \alpha^*(x,y) &= \alpha_{lend}(x,y), \text{ and} \\ T^*(x,y) &= \left(1 - \gamma - E[Q_s]x - E[B_s](1-x) - (E[B_s]-1)y\right)^+ . \end{aligned} \tag{18}$$

Moreover, the expected cost of the NPL problem to the authority is just $T^*(x, y)$, which is increasing in x and decreasing in y, and strictly so when $T^*(x, y) > 0$. The following regions in the (x, y) space exist and have positive measure

1. *$\alpha^*(x, y) = 0$ and $T^*(x, y) = 0$: for low x relative to y.*

2. *$\alpha^*(x, y) > 0$ and $T^*(x, y) > 0$: for high x and low y.*

3. *$\alpha^*(x, y) = 0$ and $T^*(x, y) > 0$: for high x and medium y.*

4. *$\alpha^*(x, y) > 0$ and $T^*(x, y) = 0$: for medium x and low y, provided ϕ is close to γ.*

Finally, an intervention policy $(\alpha'(x, y), T'(x, y))$ is optimal if and only if $\alpha'(x, y) \geq \alpha^*(x, y)$ and $T'(x, y) = T^*(x, y)$.

The proposition shows that, when the NPL market is liquid, an authority having fractional NPL disposal requirements and public transfers as intervention tools can always achieve unconstrained optimality. Moreover, optimal policies induce full new lending and, despite having been designed with the microprudential objective of minimizing the cost of the bank to the DGS and the taxpayers, also achieve the macroprudential goal of maximizing overall social surplus.

The proposition characterizes how the minimal optimal intervention of the class (α, T) depends on the amount of NPLs held by the bank prior to the intervention, x, and the size of the bank's new lending opportunities, y. As illustrated in Figure 2, the proposition identifies four different regions:

- Region I. When x is low relative to y, there is no need for intervention and the cost to the authority is zero. Intuitively, the debt overhang problem is so weak that the bank has incentives to lend even without NPL disposal, and the returns at $t = 2$ are sufficient to make the bank solvent in the two states.

- Region II. When x is large and y is small, some positive forced disposal of NPLs is necessary in order to induce the bank to lend. Moreover, to induce compliance the intervention needs to be complemented with positive public transfers.[20]

- Region III. There always exists a third region in which x is large and y is intermediate in which the authority needs to make a positive transfer to prevent bank owners from preferring that their bank is liquidated but NPL disposal is not necessary. The reason is that the new equity funding needed to comply with capital requirements (specifically those on the retained NPLs) is, conditional on compliance, large enough to restore the incentives to lend. However, with $T = 0$ bank owners would prefer liquidation to the injection of the required equity.

- Region IV. There can also be a fourth region in which the minimal optimal intervention only requires a positive disposal requirement. In this region, x is in a medium range while y is small. Opposite to Region III, here in the absence of intervention, the bank would inject (or retain) sufficient equity to avoid liquidation but would not undertake

[20] In the absence of intervention ($\alpha = T = 0$), the bank in this region would continue without undertaking new lending if x is not too large and opt for liquidation otherwise.

new lending. This region exists if the capital requirement on NPLs, ϕ, is not much larger than that on performing loans, γ.

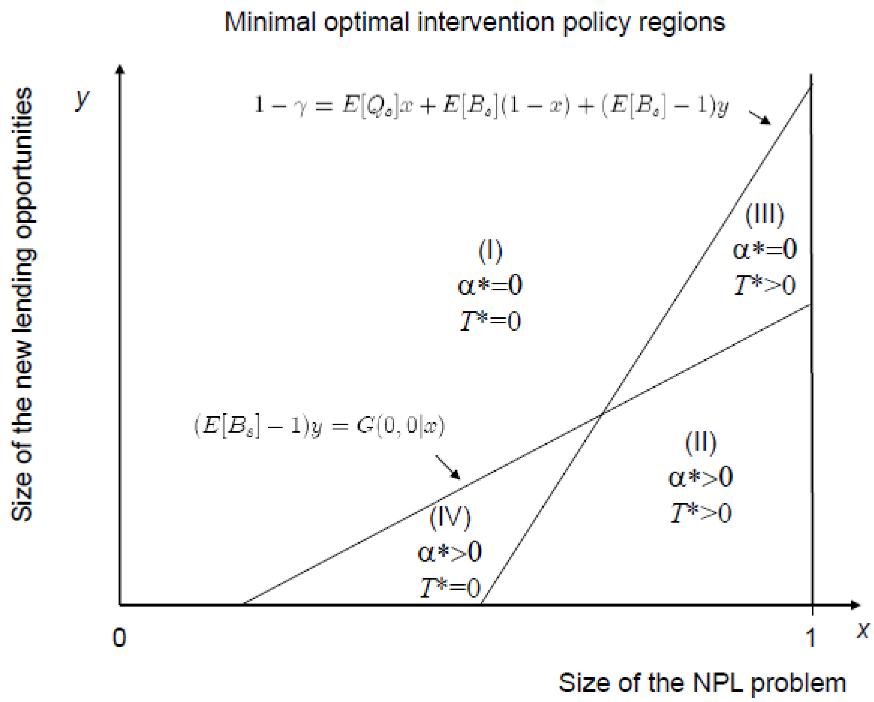

Figure 2

5 Optimal intervention with an illiquid NPL market

In this section, we consider the case in which the NPL market is illiquid, that is, $q < E[Q_s]$. This underpricing might reflect that the available investors lack the necessary skills or capabilities (e.g., local presence) to extract as much recovery value from NPLs as the originating banks. It might also reflect that the investors with the capability to manage these assets most efficiently have higher opportunity cost of funds.[21] The results below show that (α, T) interventions are generally no longer able to reach optimality because any NPL

[21] In an extended setup in which there were heterogeneity in the ex post performance of specific NPL portfolios and some buyers were privately informed about such performance, the underpricing might also be due to a winner's curse problem in the market for NPL portfolios, as in Rock (1986).

disposal reduces the net value of the bank and hence what can be divided between bank owners' net continuation value Π and the overall cost to the authority C (recall equation (7)). We find that the authority can nevertheless achieve optimality using the subclass of $(\alpha, \tilde{\phi}, T)$ interventions with $\alpha = 0$, which we refer to as $(\tilde{\phi}, T)$ interventions or, as justified below, interventions based on prudential provisioning. We show below that, in the presence of an illiquid NPL market, it is optimal for the authority to replace NPL disposals with an increase in the capital requirements or provisions applied on the retained NPLs and that such substitution does not increase the overall cost of the legacy problem relative to the case in which the NPL market were liquid.

5.1 Prudential provisioning

Our preferred interpretation for interventions involving $\tilde{\phi} \geq \phi$ is prudential provisioning. International Accounting Standards (as well as the Generally Accepted Accounting Principles applied in the US) require banks to provision their NPLs on an expected lifetime basis. In our model such losses amount to $1 - E[Q_s]$ per unit of initial book value of the loans, which in accounting jargon implies that each unit of NPLs would have a net carrying value (initial book value minus provisions) equal to $E[Q_s]$. Overriding accounting standards, bank supervisors may impose prudential adjustments to the provisioning of NPLs on the basis of a more conservative valuation of their recovery value, say $q' \leq E[Q_s]$ per unit. This will force the bank to deduct the valuation difference of its retained NPLs, $(E[Q_s]-q')(1-\alpha)x$, from its available regulatory capital. Additionally, depending on the approach to capital requirements under which the bank operates, NPLs may also be subject to capital requirements, say in the form of a requirement γ' imposed on the gross carrying amount of the retained NPLs, $(1 - \alpha)x$.[22]

With these ingredients, the constraint imposed by provisioning and capital requirements at $t = 1$ could be formally described as

$$(1 - \alpha)x + (1 - x) + I - d_1 - (1 - q')(1 - \alpha)x \geq \gamma'(1 - \alpha)x + \gamma(1 - x + I), \quad (19)$$

where the LHS is the available regulatory capital at $t = 1$ (gross book asset value minus

[22] For instance, under the advanced IRB of Basel III (Basel Committee on Banking Supervision, 2017), the capital requirement on a defaulted exposure is given by the difference between some estimated adverse-scenario loss-given-default (LGD) and the bank's best estimate of the expected loss, where the latter is supposed to be covered by provisions.

deposit liabilities minus provisions) and the RHS is the required regulatory capital (required capital on retained NPLs plus required capital on performing and new loans). This constraint can be rewritten as

$$(1-\alpha)x + (1-x) + I - d_1 \geq ((1-q') + \gamma')(1-\alpha)x + \gamma(1-x+I), \tag{20}$$

which for $\tilde{\phi} = (1-q') + \gamma'$ is analogous to the constraint (8) used in our baseline analysis (but with $\tilde{\phi}$ replacing ϕ) and supports our claim that $\tilde{\phi}$ in the interventions discussed below captures the combined effect of prudential provisioning, accounting standards, and capital requirements.[23]

5.2 Replacing NPLs disposals with prudential provisioning

Building on this interpretation, consider an authority using interventions of the class $(\tilde{\phi}, T)$ with $\tilde{\phi} \geq \phi$. The purpose of this subsection is to show that there exist $(\tilde{\phi}, T)$ interventions that satisfy the sufficient condition for optimality in Lemma 1. By design, these interventions satisfy condition C3 in such lemma, so the analysis can follow the same steps conducted in Section 4 when the main tool was an NPL disposal requirement. It is easy to show that (i) increasing $\tilde{\phi}$ reduces the cost of the compliant bank to the DGS, (ii) a sufficiently large $\tilde{\phi}$ induces full new lending by the compliant bank, and (iii) public transfers are sometimes necessary to ensure that bank owners prefer compliance to the liquidation of the bank.

Analogously to (10), the expression for the cost incurred by the DGS in state L for a compliant bank subject to an NPL provisioning requirement $\tilde{\phi}$ is now

$$\left(-\tilde{\delta}_Q(\tilde{\phi})x - \delta_B\left(1-x+I\right)\right)^+. \tag{21}$$

where $\tilde{\delta}_Q(\tilde{\phi}) = (1 - Q_L - \tilde{\phi})$ is the capital surplus associated with the NPLs in the bad state after the prudential provisioning $\tilde{\phi}$ is imposed. In comparison with (10), this expression replaces $\delta_Q(1-\alpha)$ with $\tilde{\delta}_Q(\tilde{\phi})$ and any effect achieved by rising α above zero in (10) can be replicated here by increasing $\tilde{\phi}$ above ϕ. So prudential provisioning has the same power to reduce the overall capital deficit associated with the NPLs as NPL disposal requirements in the world with a liquid NPL market. This makes it equally effective too in encouraging the bank to undertake its new lending as the NPL disposal requirement was in the case

[23] Under this interpretation, Assumption 1 holds insofar as $Q_L < q' - \gamma' \leq Q_H$, and in particular holds for $q' = E[Q_s]$ (expected loss provisioning) and $\gamma' = 0$ (no additonal capital requirement on NPLs).

of a liquid NPL market. This explains our capacity to obtain the next result building on arguments used in the liquid-market case.

Proposition 2 (*Optimal intervention with an illiquid NPL market*) *There exists an intervention of the class* $(\tilde{\phi}, T)$ *that is optimal. The minimal optimal intervention of this class is* $(\tilde{\phi}^*, T^*)$ *with* $\tilde{\phi}^* = (1-\alpha^*)\phi + \alpha^*(1-Q_L)$, *where* (α^*, T^*) *denotes the minimal optimal intervention of the class* (α, T) *in the liquid NPL market scenario covered by Proposition 1. In addition, there exist parameter values such that no intervention of the class* (α, T) *is optimal.*

So, when the NPL market is illiquid, the combination of prudential provisioning and public transfers allows to achieve optimality. Intuitively, the prudent provisioning of NPLs reduces bank owners' gambling motive to forego new lending as effectively as the disposal of NPLs when the NPL market is liquid. The two tools reduce the value of the implied DGS guarantees and thus address the debt overhang problem. But, when the NPL market is illiquid, provisioning has the advantage of preserving the overall value of the bank assets. Thus, for a given overall cost to the authority C, prudential provisioning makes the absorption of the losses associated with the NPLs less onerous to bank owners (less detrimental to their net continuation value Π) than an NPL disposal requirement.

In fact, the asset value reduction implied by the NPL disposal renders (α, T) interventions sometimes unable to achieve optimality. This is because once interventions push bank owners to obtain zero net continuation value (with less than that they would just prefer to have their bank liquidated), the asset value reduction associated with NPL disposal would be borne by the authority, in the form of a larger public transfer T than if prudential provisioning is used.[24] A final implication of Proposition 2 is that, while the illiquidity of the NPL market renders prudential provisioning preferable to NPL disposal, it does not increase the overall cost of the legacy problem to the authority.

[24] For some parameter configurations, there exist (α, T) interventions with $\alpha > 0$ that remain optimal with an illiquid NPL market. Specifically, in the leftmost part of region IV in Figure 2, the value reduction implied by NPL disposals can be accomodated by reducing Π while keeping the cost to the authority at $C = 0$. This does not contradict the sufficient (yet not necessary) conditions for optimality provided in Lemma 1.

6 Alternative forms of intervention and robustness

In this section we explore the possibility of implementing optimal interventions with tools different from the NPL disposal and provisioning requirements combined with public transfers analyzed in prior sections. We show that common bank resolution tools such as P&A transactions whereby a healthy bank absorbs assets and liabilities of a troubled bank may allow optimality to be reached with a smaller NPL disposal or provisioning requirement. We also show that, in the presence of uninsured long-term debt, optimal interventions may have to rely on the partial bail in of such debt, and discuss optimal interventions in the case in which public transfers cannot be used. Finally, we show the robustness of the results to some forms of moral hazard by bank owners.

In the interest of space, the discussion will focus on how some of the relevant expressions in the baseline model get modified in each of the extensions, relegating a more detailed presentation of the analysis to Appendix B. In addition, we assume throughout the section that the NPL market is liquid, $q = E[Q_s]$, and consider baseline interventions based on an NPL disposal requirement. This is without loss of generality as all the analysis would hold for an illiquid NPL market after substituting references to NPL disposal requirements with prudential provisioning as described in Section 5.

6.1 Purchase and assumption interventions

Bank resolution frequently involves P&A transactions. These transactions preserve the continuity of some of the relationships of the distressed bank with its customers under the umbrella of the absorbing bank, and may be accompanied by several forms of financial support from the authority to the purchaser (from cash transfers to loss-sharing agreements).[25] In this section we discuss how the optimal interventions would change in the presence of a healthier bank to which the authority can transfer the assets of the distressed bank, including the not yet undertaken new lending opportunities, and its liabilities. Relative to the baseline interventions considered above, P&A interventions may reduce the need for NPL disposals but not the need for public transfers and the overall cost to the authority.

[25] P&A transactions, with or without assistance, are the most frequent method of resolution employed by the FDIC as receiver of federally insured depository institutions in the US. Under Dodd-Frank Act the FDIC is also the Orderly Resolution Authority for systemically important financial institutions whose bankruptcy might pose a threat to US financial stability. See White and Yorulmazer (2014) for additional details.

Assume that at $t = 1$ there exists a second bank with z units of performing loans and $(1-\gamma)z$ units of deposits. For simplicity, assume that it has no new lending opportunities.[26] We also assume that, if this "strong" bank purchases our "weak" bank of prior sections, the new merged institution preserves access to the lending opportunities of the weak bank.

Formally, a P&A intervention policy in this setting can be described by a tuple (α, T_s, T_w) consisting of: (i) an NPL disposal requirement α on the weak bank that is followed by the transfer to the strong bank of the remaining loans and all the deposits of the weak bank as well as the NPL disposal revenues; (ii) a transfer $T_s \geq 0$ from the authority to the owners of the strong bank; and (iii) a transfer $T_w \geq 0$ from the owners of the strong bank to the initial owners of the weak bank. For simplicity, we assume that the owners of the strong bank keep the entire ownership of the merged bank so that T_w is all the value that the owners of the weak bank extract from it.[27]

For a given P&A intervention (α, T_s, T_w), let $I \leq y$ be the new lending undertaken by the merged bank, Π_s be the present value of the strong bank to its owners at $t = 1$ (net of any equity injection or dividend payment at that date), Π_w be the value that the owners of the weak bank obtain from the intervention, and C the cost of the NPL problem to the authority (which amounts to the transfer T_w and the expected cost of guaranteeing the deposits of the merged bank). By definition, $\Pi_w = T_w$.

We assume that the authority cannot force the strong bank to participate in the P&A intervention, which implies

$$\Pi_s \geq \overline{\Pi}_s = E[B_s]z - (1-\gamma)z, \tag{22}$$

where $\overline{\Pi}_s$ is the net value of the strong bank to its owners if the two banks do not merge.

The cost C of the NPL problem to the authority in (7) can be adapted to this extended setup as follows:

$$C = \left(\Pi_s - \overline{\Pi}_s\right) + \Pi_w - \left(E[B_s] - 1\right) I - \left(E[Q_s]x + E[B_s](1-x) - d_0\right), \tag{23}$$

where only the terms corresponding to bank owners' values change. From (22) and $\Pi_w \geq 0$, the analogous to Lemma 1 states that a P&A intervention that induces $I = y$ and that, if it

[26] Or the bank has already undertaken its new loans and variable z includes them.

[27] Implementations in which T_w were paid by giving to the owners of the weak bank an ownership stake in the strong bank would be equivalent.

leads to $C > 0$, then it involves $\Pi_w = 0$ and $\Pi_s = \overline{\Pi}_s$, is optimal. This implies, in particular, that P&A intervention policies cannot reduce the cost of the weak bank to the authority relative to that obtained in the baseline setup.

Analogously to (10), the expression for cost of the deposit guarantees on the merged bank in state L is

$$(-\delta_Q(1-\alpha)x - \delta_B(1-x+I+z))^+, \tag{24}$$

which includes a new term in z that reflects how the performing loans in the initial balance sheet of the strong bank contribute to offset the capital deficit associated with the NPLs of the weak bank. The implied reduction in the value of the deposit guarantees improves the incentives of the merged bank (relative to the weak bank) to undertake the new lending, and reduces the minimum fractional NPL disposal needed to induce lending (possibly to zero if the acquiring bank is sufficiently strong).

Thus, optimal P&A interventions may involve a lower minimum NPL disposal requirement than in our baseline setup. However, the owners of the strong bank will have to be compensated for the contribution of their good loans to reducing G, which means that eventually the transfer paid to the strong bank owners T_s will have to be as large as the transfer T^* of the baseline interventions. See Appendix B.1 for further details.

6.2 Long-term debt and the need for bail-in arrangements

In the baseline model all the bank debt consists of deposits insured by the DGS and we find that it is always optimal to avoid liquidation and to induce the bank to undertake all its new lending opportunities. We have interpreted such result as implying the alignment between the microprudential objective of minimizing the cost of the safety net and the macroprudential objective of maximizing aggregate welfare. In this section we extend the model to allow for the presence of some outstanding long-term (LT) debt at the interim date and show that the two objectives remain aligned provided that the LT debt is *bailinable* (as it is, e.g., in the EU bank resolution regime). Otherwise, minimizing the cost of the legacy problem to the authority may sometimes be incompatible with inducing new lending, as in these cases LT debtholders would appropriate a disproportionate fraction of its value, in the detriment of the authority.

Consider that the capital structure of the bank at $t = 0$ includes, in addition to insured deposits and owners' equity, some uninsured LT debt that promises a repayment h_0 at $t = 2$. We assume, as in the baseline version of the model, that the bank's overall leverage equals the maximum compatible with regulation, $d_0 + h_0 = 1 - \gamma$, and that LT debt is junior to insured deposits.

The authority's objective is still to minimize the cost of the bank to the DGS and the taxpayer. Moreover, we assume that, if not in conflict with such main goal, the authority prefers policies that maximize aggregate welfare or, equivalently, that maximize new lending.[28] Finally, in addition to setting a NPL disposal requirement α and a transfer T, we allow the authority to bail in the LT debt as part of its intervention policy. We model this policy tool as the capacity to fix a new promised repayment $h_1 \leq h_0$ for the LT debt.[29]

An intervention policy is thus described by a tuple (α, h_1, T) with $h_1 \leq h_0$. Realistically, we assume that the bail-in of LT debt must satisfy two institutional conditions.[30] First, it must respect the seniority of debt relative to equity, meaning that if $h_1 < h_0$ then the intervention policy must induce a net value for the bank owners equal to zero. Second, it must satisfy the no-creditor-worse-off criterion, meaning that the value of the new promise h_1 on LT debt induced by the intervention policy, denoted with H, must satisfy

$$H \geq H_{\text{res}}, \tag{25}$$

where $H_{\text{res}} = \min\left(\max\left(xE[Q_s] + (1-x)E[B_s] - d_0, 0\right), h_0\right)$ is the value of the outstanding promise h_0 if the bank were liquidated at $t = 1$.[31]

For a given new investment level I, the expression for the cost C of the bank to the authority analogous to (7) can be written as:

$$C = \Pi + H - \left(E[B_s] - 1\right) I - \left(E[Q_s]x + E[B_s](1-x) - d_0\right). \tag{26}$$

From (25), the analogous to Lemma 1 states that an intervention policy that induces $I = y$ and that, if it leads to $C > 0$, then it also leads to $\Pi = 0$ and $H = H_{\text{res}}$, is optimal. The last

[28] In the baseline model this subsidiary objective is implied by the primary objective, as stated in Lemma 1. In this extension, the subsidiary objective constraints the authority to choose policies that avoid liquidation and/or remove the debt overhang problem whenever they imply no additional cost to the authority.

[29] In the EU, under the Bank Recovery and Resolution Directive (BRRD), authorities have the power (and sometimes the obligation) to bail-in some of the debt liabilities of a failing bank in the context of its resolution.

[30] All the results in this extension are also valid without these institutional constraints but we introduce them to emphasize that the results are compatible with standard constraints on authorities' bail-in powers.

[31] The expression for H_{res} takes into account that insured deposits are senior to LT debt.

condition ensures that the LT debtholders appropriate no value from the new lending, since from (26) this would imply an increase in the cost of the bank to the authority.

As in the baseline model, the analysis in Appendix B.2 shows that there are policies (α, h_1, T) compatible with the sufficient optimality conditions, that is, optimal intervention policies avoid liquidation, induce full new lending, and require some bail-in of LT debt if public transfers are realized. However, inducing new lending is not always optimal when the authority is not empowered to restructure LT debt. In fact, the public transfer needed to induce new lending may not minimize the cost to the authority because a too large fraction of the NPV of the new lending is appropriated by the LT debtholders.[32]

6.3 Interventions without public transfers

In this section we analyze the optimal intervention policy when the use of public transfers at $t = 1$ is not permitted. Here $T = 0$ and the authority can only use the NPL disposal requirement α to minimize the only component of the authority's cost in this case: the expected cost of the bank's deposit guarantees, G.

Consider the interesting case in which the optimal intervention in the baseline setup features $T^* > 0$, which happens when the fraction of NPLs in the bank is high and the size of new lending opportunities is medium or low (regions II and III in Proposition 1). In this situation the authority cannot anymore avoid liquidation and induce new lending at the same time. So it has to decide between the two.

In order to (attempt to) avoid liquidation, the authority would have to set the NPL disposal requirement α below the full lending threshold $\alpha^* = \alpha_{\text{lend}}(x, y)$. However, as shown in Appendix B.3, the reduction in α might not be able to induce the bank to be compliant but, if it is, by definition the net value of the bank to its owners must be positive. Since the bank does not undertake new lending the expression for the cost $C = G$ for the authority in (7) implies that the rise in bank owners' value Π must come at the expense of an increase in the DGS liability G. But then the authority would prefer any disposal policy that leads to liquidation.[33]

[32] Thus, there may be a conflict between microprudential and macroprudential goals, as discussed by Alessandri and Panetta (2015), among others. To the best of our knowledge, we are the first to provide a model in which the possibility to bail-in uninsured debt helps resolve this conflict.

[33] It can be shown that, simultaneously, there could be *exactly one* optimal policy that avoids liquidation. This policy would also involve no lending and leave bank owners with a zero net continuation value, so it

Compared to the optimal (α, T) policies, the cost of the NPL problem to the authority increases by exactly the NPV of the lending opportunities which are foregone when the bank is liquidated. Moreover, the funds that the DGS must contribute to fully repay the deposits of the liquidated bank exceed the size of the transfer $T^* > 0$ associated with the optimal (α, T) policy. Altogether, these results suggest that restrictions to direct state aid when dealing with a NPL problem may backfire, leading to increases in the cost of the bank to the DGS that exceed the money saved to the taxpayers.

6.4 Robustness to moral hazard

We have shown that optimal interventions in the baseline model induce full new lending and may include the use of public transfers to avoid liquidation. In this section we briefly discuss the robustness of our results to the presence of two standard moral hazard problems that might affect bank owners' incentives regarding the quality of the lending decisions made by the bank.

First, we have so far assumed that the new lending opportunities of the bank are socially valuable, so that the authority gains from inducing the bank to lend (using the associated NPV to reduce the cost of the legacy problem). In the extension developed in Appendix B.4, the bank has also access to some riskier but socially unvaluable lending opportunities (that is, a gambling investment opportunity). Having NPLs makes bank owners less willing to undertake the good new lending and more willing to undertake the gambling investment. We show that (α, T) interventions are still able to achieve optimality, but the presence of the gambling opportunity forces the authority to impose a larger NPL disposal requirement. Interestingly, the overall cost to the authority is the same as in the baseline model.

Second, optimal intervention policies include positive transfers to banks that have a sufficiently large fraction of NPLs at the interim date. This raises the concern that the expectation of these transfers might increase bank owners' incentives to take excessive risk at the initial date. In Appendix B.5, we address this issue by allowing bank owners to exert some unobservable effort at the initial date that increases the likelihood that their loans remain performing at the interim date. We show that, from an ex ante perspective, the moral

would be payoff-equivalent to liquidation for both bank owners and the authority. The policy would exhibit $\alpha < \alpha^*$ and would exist only if x is not too high. A detailed description of this equivalent optimal constrained intervention is omitted for brevity.

hazard problem at loan origination is not aggravated by the anticipation of optimal policies which involve positive transfers relative to the case in which the bank expects interventions with no transfers. The reason is that whenever optimal policies of the class (α, T) feature $T^* > 0$, these positive transfers, the forced NPLs sales, the induced new lending, and the existing capital regulation end up implying equity contributions by the bank owners such that they obtain exactly the same zero net continuation payoffs that they would have obtained if the bank were liquidated. So optimal interventions featuring $T^* > 0$ can be interpreted as a recovery plans with full dilution of pre-existing equity, issuance of new equity among possibly new shareholders, and some injection of public funds.

7 Conclusion

Damaged legacy assets compromise banks' solvency and constitute a contingent liability for deposit guarantee schemes. Besides, they may be an obstacle to the origination of new socially valuable lending by the affected banks. Recent policy initiatives to address the NPL problem in the EU consider the possibility of using supervisory guidance on NPL disposal or stringent calendars for the full provisioning and write-off of damaged loans. These tools are directed to induce banks to dispose of their NPLs and/or to ensure the existence of loss absorbing capacity against the worst realizations of the returns on retained NPLs. In the US, accounting and regulatory practices encouraging the quick disposal or full write-off of bad loans, as well as the frequent use of P&A transactions to resolve weak banks, have a longer tradition.

We have provided a simple analytical framework in which these instruments can be part of an optimal intervention policy regarding legacy problems among regulated banks. Compulsory NPL sales when the markets for these assets are liquid, or prudential provisioning of NPLs when they are not, force bank owners to increase the absorption of losses associated with the legacy assets and to give up more of the option-like subsidy associated with the access to insured deposit funding. If banks comply with the requirements implied by these policies, the obstacle to the undertaking of profitable new lending can be removed.

Sometimes bank owners may prefer that their bank gets liquidated rather than assuming the burden of the intervention. The analysis reveals that in such cases a policy aimed at minimizing the joint cost of the legacy problem to the DGS and the taxpayers should combine tough requirements on NPL disposal or provisioning with the minimal transfers needed to avoid liquidation. We show that, when optimally designed, these "stick and carrot" policies avoid bank liquidation, induce new lending, and do not leave rents to the bank owners. Quite intuitively, the NPV of the new lending undertaken by the unliquidated bank once freed from the legacy problem contributes to reduce the cost of the problem to the DGS and the taxpayers.

The results in the paper have a number of relevant implications for the design of policy interventions to deal with banks' legacy problems. First, both the minimal fraction of NPLs that each bank should be required to dispose of (or the prudential provisioning adjustment applied to NPLs) and the transfers that its owners may have to receive in exchange are increasing in each bank's initial fraction of NPLs. So, instead of a one-size-fits-all approach, these results suggest the convenience of more decisive interventions (along both the stick and the carrot dimensions) on banks with more severe problems.

Second, we find that when banks have a substantial fraction of their funding in the form of uninsured long-term debt, the partial restructuring of this debt using bail-in provisions is an intervention tool that valuably complements compulsory NPL disposals and public transfers. Such restructuring avoids an excessive appropriation of the NPV of the new lending by the long-term debtholders ensuring that inducing new lending is a feature of the policies aimed to minimize the cost of the legacy problem to the DGS and the taxpayers.

Finally, the analysis reveals potential shortcomings associated with the existence of limitations to the involvement of public funds in the solution of legacy problems. When the legacy problems are more severe, prohibiting the transfers associated with the optimal interventions increases the expected cost to the DGS in excess of the forbidden transfers. The intuition is that without such transfers it may no longer be possible to induce new lending among the affected banks and the NPV of such lending would have reduced the expected cost of the bank to the DGS. Importantly, the optimal interventions that we have characterized leave no rents to bank shareholders so they are not a source of moral hazard problems such as those typically alluded to justify no bail-out provisions.

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Appendix

A Proofs

This appendix contains the proofs of the propositions included in the body of the paper.

Proof of Lemma 1 We first prove that there is no policy leading to a strictly negative cost to the authority. Suppose on the contrary that the policy $(\alpha, T, I, \Delta e, d_1)$ induces a negative cost $C < 0$. Let Π be the net value of the bank to its owners under such intervention. The only way in which $C < 0$ can arise is if the bank is not compliant with the policy, so that it is liquidated at $t = 1$ at a gain to the DGS. This can only happen when the bank asset value exceeds the repayments due to depositors, that is, when

$$E[Q_s]x + E[B_s](1-x) > 1-\gamma, \tag{27}$$

Denote by Π' and C' the value to the bank owners and the cost to the authority if the bank were compliant. Equations (6) and (27), and the fact that if the bank is not liquidated at $t = 1$ then $C' \geq 0$, imply that $\Pi' > 0$. But then letting the bank being liquidated would not be optimal to its owners. So it is not possible to have interventions leading to $C < 0$.

Suppose now $(\alpha, T, I, \Delta e, d_1)$ satisfies the conditions in the lemma and is not optimal. Let the net value to bank owners and the cost to the authority under such policy be again denoted by Π and C, respectively. If $C = 0$ then, taking into account the previous result that the cost can never be strictly negative, we have that the policy is optimal.

Suppose instead that $C > 0$. Then, to satisfy the sufficient conditions stated in the lemma, we must have $\Pi = 0$. Let $(\alpha^*, T^*, I^*, \Delta e^*, d_1^*)$ be an optimal policy and let C^*, Π^*, and $\widetilde{I} \in \{0, I^*\}$ denote the payoffs and lending decision induced by this policy (where $\widetilde{I} = 0$ would mean that the bank is not compliant in which case $\Pi^* = 0$). For $(\alpha, T, I, \Delta e, d_1)$ not to be optimal, we should have $C^* < C$. But then we would have

$$\begin{aligned}(E[Q_s]x + E[B_s](1-x) - d_0) + (E[B_s]-1)\,\widetilde{I} - (E[Q_s]-q)\,\alpha^* x = \Pi^* - C^* \geq \Pi - C^* \\ > \Pi - C = (E[Q_s]x + E[B_s](1-x) - d_0) + (E[B_s]-1)\,y - (E[Q_s]-q)\,\alpha x \\ = (E[Q_s]x + E[B_s](1-x) - d_0) + (E[B_s]-1)\,y.\end{aligned} \tag{28}$$

where we have sequentially used equality (6), $\Pi^* \geq \Pi = 0$, $C^* < C$, equality (6) again, and that condition C3 in the lemma implies that $(E[Q_s]-q)\,\alpha x = 0$. However, (28) implies $\widetilde{I} > y$, which cannot happen.■

Proof of Lemma 2 The compliant bank deposit base d_1 is given by (9). Assumption 1 then implies that the bank repays entirely its deposits in state H and its cost to the DGS is

$$\begin{aligned} G(\alpha, I|x) &= (1-\mu)\left(d_1 - Q_L(1-\alpha)x + B_L(1-x+I)\right)^+ \\ &= (1-\mu)\left((\delta_B - (1-\alpha)\delta_Q)\,x - \delta_B(1+I)\right)^+. \end{aligned}$$

Let $\alpha_{\mathrm{solv}}(I,x)$ be defined as the solution to

$$(\delta_B - (1-\alpha_{\mathrm{solv}}(I,x))\delta_Q)\,x - \delta_B(1+I) = 0$$

whenever it is non-negative, and $\alpha_{\mathrm{solv}}(I,x) = 0$ otherwise. It is a matter of algebraic manipulation to check that the resulting analytical expression for $\alpha_{\mathrm{solv}}(I,x)$ coincides with that stated in the lemma.■

Proof of Lemma 3 We have argued in the main text preceding the lemma that the compliant bank finds optimal to undertake its new lending in full if (14) holds, and not to undertake any new lending otherwise. Lemma 2 shows that $G(\alpha,0|x)$ is decreasing in α and strictly so if $G(\alpha,0|x) > 0$. Moreover, for $\alpha \in [\alpha_{\mathrm{solv}}(0,x),1)$, we have that $G(\alpha,0|x) = 0$. The results in the lemma follow immediately from these properties.■

Proof of Proposition 1 Let (α^*,T^*) be the intervention policy described as the minimal unconstrained optimal one in the statement of the proposition, and Π^* and C^* be the associated net expected payoff to bank owners and cost to the authority, respectively. From Lemma 3 we have that the intervention induces full new lending conditional on the bank being compliant and that the bank is solvent in the two states after undertaking its new lending. Moreover, by the way T^* is defined in (18), inequality (16) is satisfied so the bank indeed finds optimal to be compliant and lend in full. Also, by construction if $T^* > 0$ then inequality (16) is binding, which means that $\Pi^* = 0$. We deduce that $C^* = T^*$ and we conclude that either $C^* = 0$ or $C^* = T^* > 0$ in which case $\Pi^* = 0$. Hence (α^*,T^*) satisfies the two sufficient criteria for optimality in Lemma 1, so the policy is unconstrained optimal.

We now proceed to characterize the unconstrained optimal interventions of the class (α,T). The same arguments as those conducted above imply that any policy (α',T^*) with $\alpha' \geq \alpha^*$ also satisfies the criteria for optimality in Lemma 1 and is thus unconstrained optimal.

Let (α',T') be an unconstrained optimal policy and let Π',C' be the associated net expected payoff to bank owners and cost to the authority, respectively. By the optimality assumption we have $C' = T^*$.

Suppose that $\alpha' \geq \alpha^*$. Then the policy induces a compliant bank to undertake full new lending and to be solvent in the two states. If the bank decides to be compliant, then the cost for the authority is $C' = T'$, which implies $T' = T^*$. If the bank decides not to be compliant, then $\Pi' = 0$ and from (7) we have

$$C' = 1 - \gamma - E[Q_s]x - E[B_s](1-x).$$

Comparing this expression with the expression for T^* in (18), we have that $C' = T^*$ is equivalent to

$$1 - \gamma = E[Q_s]x - E[B_s](1-x) \Leftrightarrow C' = 0. \tag{29}$$

Suppose that following the intervention (α', T'), the bank complies and undertakes some positive lending $I > 0$. Let Π'' be the payoff for the bank owners under such sequence of actions. From (6) we have that

$$\Pi'' = (E[Q_s]x + E[B_s](1-x) - (1-\gamma)) + (E[B_s] - 1)\, I + T' > 0,$$

where in the inequality we have used (29), $T' \geq 0$ and $I > 0$. But under $\Pi'' > 0 = \Pi'$ the bank would find optimal to be compliant.

Suppose that $\alpha' < \alpha^*$. This in particular requires $\alpha^* > 0$ and implies that the optimal policy (α', T') does not induce lending (regardless of compliance or not). Using this, and that $\Pi' \geq 0$, we have from (7) that

$$C' \geq 1 - \gamma - E[Q_s]x - E[B_s](1-x).$$

From the definition of T^* in (18), the inequality implies that $C' = T^*$ is again equivalent to (29), so that the inequality needs to be binding, that is $\Pi' = 0$. The same argument as above then leads to a contradiction, so that $\alpha' < \alpha^*$ cannot be part of an unconstrained optimal policy.

We conclude that a policy (α', T') is unconstrained optimal if and only if $\alpha' \geq \alpha^*$ and $T' = T^*$, which in particular implies that (α^*, T^*) is the minimal unconstrained optimal policy of the class (α, T).

We now proceed to show the existence of the four regions in the (x, y) space mentioned in the proposition. Let us define the following lines in the (x, y) space:

$$\begin{aligned} y_{\alpha^*=0}(x) &= \frac{(1-\mu)\left[(\delta_B - \delta_Q)\, x - \delta_B\right]}{E[B_s] - 1}, \\ y_{T^*=0}(x) &= \frac{1 - \gamma + (E[B_s] - E[Q_s])\, x - E[B_s]}{E[B_s] - 1}. \end{aligned} \tag{30}$$

From Lemmas 2 and 3, and the definition of (α^*, T^*) in (18), we have that for $x, y \geq 0$

$$\begin{aligned} \alpha^* &= 0 \text{ if and only if } y \geq y_{\alpha^*=0}(x), \\ T^* &= 0 \text{ if and only if } y \geq y_{T^*=0}(x). \end{aligned} \tag{31}$$

We also have that

$$y_{\alpha^*=0}(0) < 0, \;\; y_{T^*=0}(0) < 0,$$

which, from (31), implies that region I exists and has positive measure. Moreover, we have that

$$\begin{aligned} 0 < \frac{dy_{\alpha^*=0}}{dx} &= \frac{(1-\mu)\,(\delta_B - \delta_Q)}{E[B_s] - 1} = \frac{(1-\mu)\,(B_L + \gamma - (Q_L + \phi))}{E[B_s] - 1} \\ &< \frac{(1-\mu)\,(B_L - Q_L)}{E[B_s] - 1} < \frac{E[B_s] - E[Q_s]}{E[B_s] - 1} = \frac{dy_{T^*=0}}{dx}, \end{aligned} \tag{32}$$

where he have used $\delta_B = B_L - 1 + \gamma$, $\delta_Q = Q_L - 1 + \phi$, $B_H > Q_H$, and $\phi > \gamma$. The inequality implies that (x, y) belongs to region I for low x relative to y.

In addition, we have that

$$\begin{aligned} y_{T^*=0}(1) &= \frac{1-\gamma-E[Q_s]}{E[B_s]-1} \geq \frac{(1-\mu)(1-\gamma-Q_L)}{E[B_s]-1} \\ &> \frac{(1-\mu)(1-\phi-Q_L)}{E[B_s]-1} = \frac{-(1-\mu)\delta_Q}{E[B_s]-1} = y_{T^*=0}(1) > 0. \end{aligned}$$

The inequality implies that there exist both region II and III and that have positive measure. Moreover, the inequality implies (x, y) belongs to region II for high values of x and low values of y, and belongs to region III for high values of x and medium values of y.

Finally, let $x_{\alpha^*=0}$ and $x_{T^*=0}$, be defined as

$$\begin{aligned} y_{\alpha^*=0}(x_{\alpha^*=0}) &= 0 \Leftrightarrow (1-\mu)\left[(\delta_B - \delta_Q)\, x_{\alpha^*=0} - \delta_B\right] = 0, \\ y_{T^*=0}(x_{T^*=0}) &= 0 \Leftrightarrow 1-\gamma + (E[B_s] - E[Q_s])\, x_{T^*=0} - E[B_s] = 0. \end{aligned}$$

From inequality (32) we have that region IV exists if and only if $x_{\alpha^*=0} < x_{T^*=0}$. For $\phi \to \gamma$ we have that

$$\begin{aligned} 1-\gamma + (E[B_s] - E[Q_s])\, x_{T^*=0} - E[B_s] = 0 = (1-\mu)\left[(\delta_B - \delta_Q)\, x_{\alpha^*=0} - \delta_B\right] = \\ = (1-\mu)\left[(1-\phi-Q_L)\, x_{\alpha^*=0} - (1-\gamma-B_L)(1-x_{\alpha^*=0})\right] = \\ = (1-\mu)\left[1-\gamma-Q_L x_{\alpha^*=0} - B_L(1-x_{\alpha^*=0})\right] > \\ > E\left[1-\gamma-Q_s x_{\alpha^*=0} - B_s(1-x_{\alpha^*=0})\right] = \\ = 1-\gamma + (E[B_s] - E[Q_s])\, x_{\alpha^*=0} - E[B_s], \end{aligned} \tag{33}$$

where in the last inequality we have used that

$$1-\gamma-Q_L x_{\alpha^*=0} - B_L(1-x_{\alpha^*=0}) = 0 \Rightarrow 1-\gamma-Q_H x_{\alpha^*=0} - B_H(1-x_{\alpha^*=0}) < 0.$$

Looking at the expressions at the extremes of the chain of inequalities in (33), we conclude that $x_{\alpha^*=0} < x_{T^*=0}$.■

Proof of Proposition 2 The first statement in the proposition is an immediate consequence of the arguments in subsection 5.1 and the comparison between (10) and (21).

Consider values of the parameters such that the minimal optimal intervention with a liquid NPL market, (α^*, T^*), which is given by expression (18) in Proposition 1, satisfies $\alpha^* > 0$ and $T^* > 0$. Let $(\tilde{\phi}^*, T^*)$ denote the unconstrained optimal intervention defined in the first part of this proposition. Let C^* be the cost for the authority induced by this policy, which satisfies $C^* = T^*$. Consider any possible (α, T) intervention and denote by I the new lending it induces and by Π, C the payoffs it leads to for the bank owners and the authority, respectively. Crucially, $\alpha^* > 0$ implies that we cannot have $\alpha = 0$ and $I = y$ at the same time. Using this, equations (7) and (18), and the fact that $\Pi \geq 0$, we have the following sequence of inequalities:

$$C = \Pi - (E[B_s] - 1)\, I + (E[Q_s] - q)\, \alpha x - (E[Q_s]x + E[B_s](1 - x) - (1 - \gamma))$$
$$> 1 - \gamma - E[Q_s]x - E[B_s](1 - x) - (E[B_s] - 1)y = T^* = C^*.$$

Since $C > C^*$, the intervention (α, T) is not unconstrained optimal.■

B Details on extensions

B.1 Purchase and assumption interventions

The next proposition states formally the results discussed in Section 6.1:

Proposition 3 (*Optimal P&A interventions*) *Let (α^*, T^*) be the minimal optimal intervention policy in the baseline model and suppose that there is a strong bank with $z > 0$ units of performing loans. The minimal optimal P&A intervention $(\widetilde{\alpha}, \widetilde{T}_s, \widetilde{T}_w)$ satisfies $\widetilde{\alpha} \leq \alpha^*$, with $\widetilde{\alpha} < \alpha^*$ if $\alpha^* > 0$, $\widetilde{T}_s = T^*$, and $\widetilde{T}_w = 0$ if $\widetilde{T}_s > 0$. Besides, $\widetilde{\alpha}$ is decreasing in z, and $\widetilde{\alpha} = 0$ if z is sufficiently large.*

Proof From (23), the optimality condition analogous to Lemma 1 states that P&A intervention policy (α, T_s, T_w) is optimal if it induces full new lending and, if it involves a positive cost to the authority, then $\Pi_w = 0$ and $\Pi_s = \overline{\Pi}_s$. For given α and I, from (24) we have that the cost for the DGS of the compliant merged bank is:

$$\widetilde{G}(\alpha, I|x) = (1 - \mu)\left(-\delta_Q(1 - \alpha)x - \delta_B\left(1 - x + I + z\right)\right)^+. \tag{34}$$

The minimum NPL disposal requirement inducing full new lending, $\widetilde{\alpha}_{\text{lend}}(x, y, z)$, is given by:

$$(E[B_s] - 1)y = \widetilde{G}(\widetilde{\alpha}_{\text{lend}}(x, y, z), 0|x). \tag{35}$$

Comparing these expressions to those in (11) and (15), we have that

$$\widetilde{\alpha}_{\text{lend}}(x, y, z) \leq \alpha_{\text{lend}}(x, y) \text{ and}$$
$$\widetilde{\alpha}_{\text{lend}}(x, y, z) < \alpha_{\text{lend}}(x, y) \text{ if } \alpha_{\text{lend}}(x, y) > 0.$$

Moreover, (34) and (35) imply that $\widetilde{\alpha}_{\text{lend}}(x, y, z)$ is decreasing in z, and equal to zero if z is sufficiently large. Finally, the expression for Π_w an be obtained from (23). As in the baseline model, a merged bank that finds optimal to undertake new lending will be solvent in state L. The results in the proposition then immediately follow using the same arguments leading to Proposition 1 in Section 4.3.■

B.2 Long-term debt and the need for bail-in arrangements

The next proposition states formally the results discussed in Section 6.2:

Proposition 4 (*Optimal policy with uninsured LT debt*) *When the bank has outstanding uninsured LT debt and the authority has bail-in powers on it, then optimal intervention policies induce the undertaking of all new profitable lending and impose some bail-in on LT debt whenever they involve public transfers. In contrast, if the authority does not have bail-in powers, in some cases optimal policies do not induce new lending (and could lead to liquidation).*

Proof Suppose the authority has bail-in powers. The proof of the proposition follows closely the sequence of intermediate results in the baseline model in Section 3 and 4. For the sake of brevity, we only sketch them here highlighting the main differences and new arguments.

Let (α, h_1, T) be an intervention policy compliant with the LT debt bail-in rules. From the expression for the cost of the bank for the authority in (26), we deduce that if (α, h_1, T) satisfies the two following properties then it is an optimal policy. First, it induces full new lending, $I = y$. Second, if it leads to a positive cost to the authority, $C > 0$, then bank owners' net continuation value is zero, $\Pi = 0$, and the continuation value of LT debt is equal to that under bank liquidation, $H = H_{\text{res}}$. In addition, if (α, h_1, T) meets the two criteria then any optimal intervention policy meets them as well.

Given the policy set by the authority, a compliant bank that undertakes new lending I will always find (weakly) optimal to raise as much deposit funding as allowed by the regulatory environment, so that

$$d_1 + h_1 = (1 - \phi)(1 - \alpha)x + (1 - \gamma)(1 - x + I). \tag{36}$$

Comparing to (9) we have that the overall notional amount of the debt issued by the bank coincides with that in the baseline model and in particular is affected by the intervention policy only through α. As a result, the intervention policy induces full lending by a compliant bank if and only if $\alpha \geq \alpha_{\text{lend}}(x, y)$ and, in that case, the bank is solvent in the two states, so that $H = h_1$.

Consider the policy (α^*, h_1, T) with $\alpha^* \geq \alpha_{\text{lend}}(x, y), h_1 \leq h_0$. Taking into account that the policy induces full lending by a compliant bank and the bank to be solvent in the two states, we have from (26) that the net continuation value for the bank owners under compliance would be:

$$\Pi(\alpha^*, h_1, T) = (E[B_s] - 1)\, I + (E[Q_s]x + E[B_s](1 - x) - d_0) - h_1 + T. \tag{37}$$

If $\Pi(\alpha^*, h_0, 0) \geq 0$ then the intervention policy $(\alpha^*, h_0, 0)$ meets the (extended) optimality conditions so that any optimal policy avoids liquidation and leads to full lending. Moreover any other optimal policy (α', h_1', T') must have $h_1' = h_0, T' = 0$. Otherwise from (37) we would have that $\Pi(\alpha', h_1', T') > 0$ and the policy would either not meet the optimality conditions or the LT debt bail-in rules. The proposition is thus satisfied.

Suppose that $\Pi(\alpha^*, h_0, 0) < 0$ and let us distinguish two cases.

i) $\Pi(\alpha^*, H_{\text{res}}, 0) \geq 0$. In this case, we have that there exists a unique $h_1^* \in [H_{\text{res}}, h_0]$ such that $\Pi(\alpha^*, h_1^*, 0) = 0$. By construction, the intervention policy satisfies the LT debt bail-in rules, leads to full lending and zero cost for the authority. Then the (extended) optimality conditions imply that $(\alpha^*, h_1^*, 0)$ is optimal and that any other optimal policy must meet the optimality conditions. From (37) we easily deduce that any other optimal policy (α', h_1', T') must have $h_1' = h_1^*$, $T' = 0$, and the proposition is satisfied.

ii) $\Pi(\alpha^*, H_{\text{res}}, 0) < 0$. Let us define $h_1^* = H_{\text{res}}, T^* = -\Pi(\alpha^*, H_{\text{res}}, 0) > 0$. Then by construction the intervention policy (α^*, h_1^*, T^*) meets the LT debt bail-in rules and the (extended) optimality conditions so that it is optimal and any other optimal policy meets those conditions. Taking into account that the LT debt bail-in rules impose the lower bound $h_1 \geq h_1^* = H_{\text{res}}$ we easily deduce from (37) that any other optimal policy (α', h_1', T') must have $h_1' = h_1^*$, $T' = T^*$, and the proposition is satisfied.

Suppose the authority does not have bail-in powers so that intervention policies are described by the pair (α, T). We are going to show that there exist values of the bank's balance sheet parameters such that not inducing new lending by the bank strictly reduces the cost of the bank for the authority relative to any policy that induces it.

Consider the limit case with $h_0 = 1-\gamma$, $d_0 = 0$, $x = 1$, and $y > 0$ such that $(E[B_s] - 1)\, y < 1 - \gamma - E[Q_s]$. Suppose the authority sets a policy with $T = 0$. Then, since the bank has no deposits, the cost for the authority of such policy is zero. We have from (37) that if a policy (α, T) induces full lending then it must necessarily satisfy $T > 0$ and thus has a strictly higher cost than any policy with no transfers.■

B.3 Interventions without public transfers

The next proposition states formally the results discussed in Section 6.3:

Proposition 5 (*The cost of prohibiting public transfers*) *Whenever the optimal intervention of the class (α, T) features $T^* > 0$, then optimal interventions of the class $(\alpha, T = 0)$ lead to liquidation. Moreover, the the overall expected cost for the authority in the optimal interventions of the class $(\alpha, T = 0)$ are increased by $(E[B_s] - 1)y$ relative to those under the optimal intervention of the class (α, T).*

Proof Consider a constrained authority that must set $T = 0$. Its objective would then reduce to minimize the cost of the bank to the DGS, G, and its only tool would be the fractional NPL disposal requirement, α. Denote (α^*, T^*) the minimal (unconstrained) optimal policy, which is described in Proposition 1. Suppose that $T^* > 0$. Then Lemma 1 implies that the net continuation value of the bank to its owners under (α^*, T^*) satisfies $\Pi^* = 0$.

Let α be an intervention policy and denote $\Pi(\alpha)$ bank owners' net value if the bank is compliant with it, and $G(\alpha)$ the cost of the bank to the DGS under the optimal decision of the bank under such policy.

Suppose that $\alpha \geq \alpha^*$. We have that

$$\Pi(\alpha) = \Pi^* - T^* < 0, \tag{38}$$

where we have used (12) and the fact that after disposing of a fraction α^* (or $\alpha \geq \alpha^*$) of its NPLs the bank finds optimal to undertake its new lending in full and, conditional on that, it is solvent in the two states. But then under α the bank finds optimal not to be compliant, implying

$$G(\alpha) = 1 - \gamma - E[Q_s]x - E[B_s](1 - x), \tag{39}$$

which does not depend on the exact value of $\alpha \geq \alpha^*$ and we can hereafter refer to as $G(\alpha^*)$. Using (18) and (38), we deduce that

$$G(\alpha^*) = (E[B_s] - 1)y + T^* > 0. \tag{40}$$

Suppose that $\alpha < \alpha^*$. If the bank finds optimal to opt for liquidation then its cost to the DGS is given by (39) and hence equals $G(\alpha^*) > 0$. If instead the bank finds optimal to be compliant, it will not undertake any new lending, which means

$$\Pi(\alpha) = E[Q_s]x + E[B_s](1 - x) - (1 - \gamma) + G(\alpha) \geq 0,$$

or, equivalently,

$$G(\alpha) \geq 1 - \gamma - E[Q_s]x - E[B_s](1 - x) = G(\alpha^*),$$

which means that setting $\alpha \geq \alpha^*$ and pushing the bank into liquidation is less costly. This concludes the proof that any policy that leads to liquidation is optimal for the constrained authority.

Moreover, taking into account that the cost for the DGS under any optimal constrained policy is equal to $G(\alpha^*)$, equation (40) implies that the cost of the bank to the DGS under the constrained optimal policy exceeds the transfer T^* associated with the minimal unconstrained optimal policy.■

B.4 Risk-shifting in new lending opportunities

The focus of the baseline model is on the optimal policies to deal with the legacy problems of a bank with new profitable lending opportunities. Yet, one of the concerns when dealing with banks in distress is their incentive to gamble, that is, to undertake risky investments with the purpose of benefiting from risk shifting. In this section we show that the main results of the paper remain valid when the bank has a risk-shifting opportunity provided the authority rises the NPL disposal requirement enough to remove bank owners' gambling temptation.

In the baseline model the bank at $t = 1$ has the opportunity to undertake up to y units of lending with a payoff structure equal to that of performing loans. We refer to this positive NPV investment as "good lending." We now assume that, as an alternative, the bank could undertake up to y units of "risky lending" with return $\widetilde{B}_s$ in state s, where $\widetilde{B}_L < B_L$, $\widetilde{B}_H > B_H$, and $E[\widetilde{B}_s] < E[B_s]$.

To streamline the presentation, consider the polar case with $\widetilde{B}_L = 0$ and $\widetilde{B}_H$ that satisfies:

Assumption 4 $1 < \mu\widetilde{B}_H + (1-\mu)(1-\gamma) < E[B_s]$.

The precise implications of this assumption will become clear below but it essentially requires that the expected payoff of the risk-shifting opportunity is sufficiently lower than that of good lending but not too low. The authority is assumed to observe the amount of new lending I but not bank owners' choice between good and risky lending. Finally, the capital requirement per unit of new lending remains γ.

Suppose that a bank newly created at $t = 1$ could decide between the two lending opportunities subject to the per unit capital requirement γ. From Assumption 1, bank owners' expected net present value per unit of lending for each of the lending opportunities would be

$$\begin{array}{ll} \text{Good lending:} & E[B_s] - 1. \\ \text{Risk-shifting:} & \mu\widetilde{B}_H + (1-\mu)(1-\gamma) - 1. \end{array} \tag{41}$$

Notice that the value of risky lending to bank owners includes the expected payoff of the investment, $\mu\widetilde{B}_H$, and the expected value of its associated deposit guarantees, $(1-\mu)(1-\gamma)$. Yet, Assumption 4 implies that a "newly" created bank would strictly prefer good lending to risky lending, and risky lending to no lending at all. A bank without legacy assets would thus maximize the overall value of its investments and cause no costs to the authority. As we show next this is not the case in the presence of legacy problems.

Consider a bank with a fraction x of NPLs and the two competing lending opportunities with an overall maximum size y. Since the expected payoff of good lending is higher than that of risky lending, the sufficient optimality condition in Lemma 1 can be adapted to this setting by replacing new lending with new *good* lending. Suppose the authority sets an intervention policy (α, T). To analyze the bank's compliance and new lending decision, denote by $\widetilde{G}(\alpha, I|x)$ and $\widetilde{\Pi}(\alpha, T, I|x)$ the expected cost of the bank to the DGS and the net value of the bank to its owners if the bank is compliant an undertakes I units of risky lending. The expressions for these variables, which are analogous to those in (11) and (12), are given by:

$$\widetilde{G}(\alpha, I|x) = (1-\mu)\left[(\delta_B - (1-\alpha)\delta_Q)\, x - \delta_B + (1-\gamma)I\right]^+, \tag{42}$$

$$\widetilde{\Pi}(\alpha, T, I|x) = E[Q_s]x + E[B_s](1-x) - (1-\gamma) + (\mu\widetilde{B}_H - 1)I + \widetilde{G}(\alpha, I|x) + T. \tag{43}$$

Notice that the term $(1-\gamma)I$ in the expression for $\widetilde{G}(\alpha, I|x)$ accounts for the expected subsidy per unit of risky lending received from the DGS in state L. The analogous term in $\widetilde{G}(\alpha, I|x)$ was $-\delta_B I < 0$, and accounted for the fact that new good lending reduces the expected subsidy from the DGS, which was the reason why the bank might find optimal not to undertake good lending in the baseline model.

From (42), (43) and Assumption 4 we have that

$$\frac{\partial\widetilde{\Pi}(\alpha, T, I|x)}{\partial I} \geq \mu\widetilde{B}_H - 1 - (1-\mu)(1-\gamma) > 0, \tag{44}$$

which means that a compliant bank will never pass up the opportunity to lend because such option is dominated by the full undertaking of risky lending. Hence, an intervention policy induces a compliant bank to undertake good lending if and only if it makes it preferable to risky lending. We next show that that is not always the case under the minimal optimal policies of the baseline model.

In fact, let x, y be such that $\alpha_{\text{lend}}(x, y) > 0$. By the definition of $\alpha_{\text{lend}}(x, y)$ in Lemma 3 and using (44) we have that:

$$\widetilde{\Pi}(\alpha_{\text{lend}}(x,y), T, y|x) > \widetilde{\Pi}(\alpha_{\text{lend}}(x,y), T, 0|x) = \Pi(\alpha_{\text{lend}}(x,y), T, 0|x) = \Pi(\alpha_{\text{lend}}(x,y), T, y|x), \tag{45}$$

which means that for an NPL disposal requirement $\alpha_{\text{lend}}(x, y)$ a compliant bank strictly prefers risky lending. This implies that, if $\alpha^* > 0$, the minimal optimal intervention policies in Proposition 1 do not induce good lending.

Recall that from (41) and the second inequality in Assumption 4 we have that a bank with no legacy portfolio would find strictly optimal to undertake good lending. Using that $\delta_B > 0$, it is a matter of simple algebra to check that this implies that:

$$\widetilde{\Pi}(\alpha = 1, T, y|x) < \Pi(\alpha = 1, T, y|x). \tag{46}$$

The inequality says that a compliant bank that disposes its entire portfolio of NPLs finds strictly optimal to undertake good lending. The intuition is that after the disposal of all the NPLs, the only legacy loans in the bank portfolio are performing ones, which have a capital surplus in state L and, if anything, strengthen bank owners' incentives to undertake good lending relative to those of a bank with no legacy loans.

From (45), (46). and the results in Proposition 1 we obtain that:

Proposition 6 (*Optimal policies with risk-shifting possibilities*) *Let (α^*, T^*) be the minimal optimal intervention policy in the baseline model and suppose that the bank has the opportunity to undertake some competing new risky lending. The minimal optimal intervention policy $(\widetilde{\alpha}^*, \widetilde{T}^*)$ in this economy satisfies*

$$\begin{aligned} \alpha^* &\leq \widetilde{\alpha}^* < 1 \text{ and } \widetilde{\alpha}^* > \alpha^* \text{ if } \alpha^* > 0, \\ \widetilde{T}^* &= T^*, \end{aligned}$$

and induces the same expected cost to the authority as in the baseline model.

Proof Results immediately from Lemma 3, Proposition 1, the extended version of the sufficient optimality condition in Lemma 1 for the economy with risk-shifting opportunities, and equations (45) and (46).■

The proposition states that, while the presence of risk-shifting opportunities does not increase the cost of the legacy problem to the authority, preventing risk shifting may force the authority to impose a larger NPL disposal requirement than in the baseline setup.

B.5 Initial loan monitoring decisions

We have thus far taken both the bank's deposit choice d_0 at $t = 0$ and its fraction of NPLs at $t = 1$ as given. In this section we analyze how the two are determined by the bank owners' optimal decision regarding capital structure and (unobservable) loan monitoring at $t = 0$. We find that bank owners want the bank to use as much deposits funding as allowed by regulation to maximize expected subsidies from the authority, which justifies our assumption $d_0 = 1 - \gamma$ in the baseline analysis. Besides, we show that moral hazard problems on loan monitoring are not aggravated by the use of positive transfers in optimal intervention policies relative to interventions designed to avoid public transfers at the interim date. Interestingly, a policy of no intervention when legacy problems arise may aggravate moral hazard problems on loan monitoring and make legacy problems more likely.

We assume that at $t = 0$ the bank chooses the initial amount of deposits d_0 subject to the regulatory constraint $d_0 \leq 1 - \gamma$. After investing in one unit of loans, monitoring can reduce the likelihood that the loans become non-performing at $t = 1$. Specifically, bank owners by choosing a monitoring level $m \in [0, 1]$ at at a private cost $c(m)$ at $t = 0$ make the fraction of NPLs in the bank at $t = 1$ be $x_1 = 0$ with probability m and $x_1 = x > 0$ otherwise. To focus on the interesting situation in which optimal intervention policies may feature positive transfers, we assume:

Assumption 5 $E[Q_s]x + E[B_s](1 - x) + (E[B_s] - 1)y < 1 - \gamma.$

In addition, we assume that bank owners' disutility cost of monitoring is increasing and convex, and satisfies the Inada-type conditions that guarantee a unique interior solution in m:

Assumption 6 $c(0) = 0,\ c'(0) = 0,$ *and* $c'(1) > x\left(E[B_s] - E[Q_s]\right).$

The efficient monitoring level satisfies the first order condition:

$$c'(m^{FB}) = \left(E[B_s] - E[Q_s]\right)x. \tag{47}$$

When considering their choice of d_0 and m at $t = 0$, bank owners anticipate that, for each possible d_0, if $x_1 = x$ at $t = 1$, the authority will set the optimal intervention policy given such x. Such optimal intervention policies can be found by simply replacing $1-\gamma$ with d_0 in the expression for the optimal public transfer in (18), since Proposition 1 has been derived for $d_0 = 1-\gamma$ but is valid for any d_0.

Since optimal policies induce full investment at the interim date and make the bank solvent in the two states at $t = 2$, we can write the initial net value of the bank to its owners as a function of d_0 and m as:

$$\begin{aligned}\Pi_0(d_0, m) &= -(1-d_0) - c(m) + m\left(E[B_s] + (E[B_s]-1)y - d_0\right) + \\ &\quad +(1-m)\left(E[Q_s]x + E[B_s](1-x) + (E[B_s]-1)y - d_0\right)^+ \end{aligned} \tag{48}$$

The intuition for the net value expression is as follows. The first two terms capture the initial equity contribution by the bank owners and the disutility cost from monitoring loans, respectively. The third and fourth terms are owners' net continuation value at $t = 1$ conditional on $x_1 = 0$ and $x_1 = x$, respectively. The fourth term also captures that if the optimal intervention policy features a positive transfer (which happens when $d_0 > E[Q_s]x + E[B_s](1-x) + (E[B_s]-1)y$) then the net continuation value of the bank for its owners is zero.

From (48), the bank's optimal monitoring level under a given choice of d_0, $m^*(d_0)$, satisfies the following first order condition:

$$c'(m^*(d_0)) = \min\left(\left(E[B_s] - E[Q_s]\right)x, E[B_s] + (E[B_s]-1)y - d_0\right), \tag{49}$$

whose comparison with (47) implies that

$$m^*(d_0) < m^{FB} \text{ iff } d_0 > \bar{d} = E[Q_s]x + E[B_s](1-x) + (E[B_s]-1)y. \tag{50}$$

We have thus that monitoring gets reduced relative to its efficient level when the bank's initial deposits are sufficiently large.[34] The reason is that for $d_0 > \bar{d}$, the optimal intervention policy under $x_1 = x$ involves a positive transfer to bank owners.

Since the bank chooses d_0 at $t = 0$ in order to maximize $\Pi_0(d_0, m^*(d_0))$, the envelope theorem implies

$$\frac{\mathrm{d}\Pi_0(d_0, m^*(d_0))}{\mathrm{d}d_0} = \begin{cases} 0 & \text{if } d_0 < E[Q_s]x + E[B_s](1-x) + (E[B_s]-1)y \\ 1 - m^*(d_0) & \text{if } E[Q_s]x + E[B_s](1-x) + (E[B_s]-1)y < d_0 \leq 1-\gamma \end{cases}.$$

Thus, for low d_0, marginal changes in d_0 do not affect the net value of the bank for its owners. In contrast, for high d_0, a marginal increase in d_0 rises one by one the subsidy that bank owners receive from the DGS when $x_1 = x$ and, thus, increases the continuation value that they extract from the bank in proportion to the probability $1 - m^*(d_0)$ of such outcome. If

[34] Under Assumption 5, there exist $d_0 \leq 1-\gamma$ satisfying the necessary and sufficient condition for inefficient monitoring in (50).

$\gamma < 1 - \bar{d}$, maximizing such subsidy pushes the bank to optimally choose $d_0 = 1 - \gamma$ and, subsequently, $m = m^*(1 - \gamma) < m^{FB}$ (by (50)).

Prior arguments might suggest that the transfers involved in optimal intervention policies are the cause of the inefficiently low monitoring, but this is not the case. To see this, suppose the authority liquidates a bank whenever $T^* > 0$ and suppose that the bank anticipates at $t = 0$ this new intervention rule. Since by design $T^* > 0$ is just enough to avoid the bank liquidation and leaves a zero net continuation value to the bank owners when $x_1 = x$, the initial net value of the bank to its owners as a function of d_0 and m under the new intervention rule would still be given by (48). Hence neither the bank's optimal capital structure nor its monitoring choice would be changed.[35]

Besides, if the authority does not intervene (that is, sets $\alpha = T = 0$), although still liquidates the bank if it does not comply with capital regulation at $t = 1$, bank owners might obtain rents at the expense of the DGS by not undertaking new lending at that date. When that is the case, monitoring incentives get further reduced. The overall conclusion is that, in absence of intervention, the monitoring level can be strictly lower (and is never higher) than under the optimal intervention policies described in the paper.

The next result formalizes our discussion above:

Proposition 7 (*Initial capital structure and monitoring decisions*) *Under the expectation of an optimal intervention at the interim date, bank owners choose as much initial deposits d_0 as compatible with regulation, $d_0 = 1 - \gamma$, and an inefficiently low monitoring level, $m^* < m^{FB}$. Liquidating the bank whenever the optimal intervention policy includes positive transfers would not modify these decisions. In contrast, not intervening at the interim date might reduce (and will never increase) the monitoring level.*

Proof Only the last statement in the proposition has not been proven in the main text preceding it. Consider a situation in which the authority does not intervene, that is where $\alpha = 0, T = 0$. The initial net value of the bank to its owners as a function of d_0 and m can be written as the following expression which is analogous to that in (48):

$$\begin{aligned}\widehat{\Pi}_0(d_0, m) \;=\;& -(1 - d_0) - c(m) + m\left(E[B_s] + (E[B_s] - 1)y - d_0\right) \\ & +(1-m)\left(E[Q_s]x + E[B_s](1-x) + \max\left((E[B_s]-1)y, G(0,0|x)\right) - d_0\right)^+, \quad (51)\end{aligned}$$

where $G(0, 0|x)$ denotes the expected value of the guarantee on deposits if the bank is compliant and does not invest and whose expression is given in Lemma 2. The last term

[35] The moral hazard problem that leads to having $m = m^*(1 - \gamma) < m^{FB}$ in this context is therefore not caused by the intervention with which the authority solves the NPL problem ex post. In fact, for a given value of $d_0 > \bar{d}$, it is not even caused by the presence of a guarantee on bank deposits but by the unobservability of the monitoring decision m. Yet the guarantee on bank deposits explains why the bank chooses $d_0 = 1 - \gamma > \bar{d}$ in the first place. If deposits were uninsured and priced according to the choice of m implied by $m^*(d_0)$, the bank owners would choose $d_0 \leq \bar{d}$ and this would lead to $m = m^{FB}$.

takes into account that when $x_1 = x$, a compliant bank finds optimal to undertake new lending only if $(E[B_s] - 1)y > G(0,0|x)$ and that in such a case the bank is solvent in the two states at $t = 2$. That term also captures that the bank owners have the option not to be compliant, let the bank be liquidated and obtain zero value.

For a given d_0, we have that the bank's optimal monitoring level, $\widehat{m}^*(d_0)$, is given by:

$$c'(\widehat{m}^*(d_0)) = \min\left((E[B_s]\text{-}E[Q_s])\,x - (G(0,0|x)\text{–}(E[B_s]\text{–}1)y)^+\,, E[B_s] + (E[B_s]\text{-}1)y - d_0\right). \tag{52}$$

Comparing with (49) we have that $\widehat{m}^*(d_0) \leq m^*(d_0)$ for all d_0 and

$$\widehat{m}^*(d_0) < m^*(d_0) \text{ iff } G(0,0|x) > (E[B_s] - 1)y \text{ and } d_0 < xE[Q_s] + (1-x)\,E[B_s] + G(0,0|x). \tag{53}$$

Using the envelope theorem we have that

$$\frac{\mathrm{d}\widehat{\Pi}_0(d_0, \widehat{m}^*(d_0))}{\mathrm{d}d_0} = \begin{cases} 0 & \text{if } d_0 < E[Q_s]x\text{+}E[B_s]\,(1\text{-}x)\text{+}\max\left((E[B_s]\text{–}1)y, G(0,0|x)\right) \\ 1\text{–}m^*(d_0) & \text{if } E[Q_s]x\text{+}E[B_s]\,(1\text{–}x)\text{+}\max\left((E[B_s]\text{–}1)y, G(0,0|x)\right) < d_0 \end{cases}, \tag{54}$$

Recall that x satisfies Assumption 5. Let us distinguish two cases:

i) $1 - \gamma \leq E[Q_s]x + E[B_s](1-x) + G(0,0|x)$

The inequality and Assumption 5 imply that $(E[B_s]-1)y < G(0,0|x)$. We will show later that there exist x, y such that *i)* and Assumption 5 can be satisfied, but for the time being let us assume that for some given x, y the two hold. Then (54) implies that $\frac{\mathrm{d}\widehat{\Pi}_0(d_0,\widehat{m}^*(d_0))}{\mathrm{d}d_0} = 0$ for all $d_0 \leq 1-\gamma$ and d_0 is undetermined. Yet, from (49) and (52) we have for any d_0 that

$$\begin{aligned} c'(\widehat{m}^*(d_0)) &= (E[B_s]\text{–}E[Q_s])\,x + (E[B_s]\text{–}1)y - G(0,0|x) < E[B_s] + (E[B_s]\text{–}1)y - (1\text{–}\gamma) \\ &= \min\left((E[B_s]\text{–}E[Q_s])\,x, E[B_s] + (E[B_s]\text{–}1)y - (1\text{–}\gamma)\right) = c'(m^*(1\text{–}\gamma)), \end{aligned}$$

and thus $\widehat{m}^*(d_0) < m^*(1-\gamma)$.

ii) $1 - \gamma > E[Q_s]x + E[B_s](1-x) + G(0,0|x)$

Then (54) implies that $\frac{\mathrm{d}\widehat{\Pi}_0(d_0,\widehat{m}^*(d_0))}{\mathrm{d}d_0} > 0$ for d_0 sufficiently close to $1-\gamma$ and the bank finds optimal to choose $d_0 = 1-\gamma$. Besides, from (49) and (52) we have that $\widehat{m}^*(1-\gamma) = m^*(1-\gamma)$.

We have thus far proven that a no intervention policy never increases loan monitoring relative to that induced by the optimal intervention policies described in the baseline model. The only remaining thing to prove is that for some values of the parameters loan monitoring strictly decreases when there are no interventions. In order to prove that, it suffices to show that for ϕ sufficiently close to γ there exist pairs x, y such that Assumption 5 and condition *i)* above are satisfied.

Suppose ϕ is very close to γ. From Assumption 1, we have that $E[Q_s] < 1-\gamma < E[B_s]$ which implies that there exists $x' \in (0,1)$ such that

$$E[Q_s]x' + E[B_s](1-x') = 1-\gamma. \tag{55}$$

Since ϕ is very close to γ the deposits d_1 at $t = 1$ of a compliant bank with a fraction x' of NPLs that does not undertake new lending opportunities satisfy $d_1 \simeq 1 - \gamma$. We must necessarily have from (55) that $G(0, 0|x') > 0$ because the expected payoff of the bank loans equals the notional value of its deposits and bank loans are risky. Hence using the continuity of the function $G(0, 0|x)$, we have that for x slightly below x' the following inequality is satisfied

$$E[Q_s]x + E[B_s](1 - x) < 1 - \gamma < E[Q_s]x + E[B_s](1 - x) + G(0, 0|x).$$

Choosing y low enough we obtain a pair x, y that satisfies Assumption 5 and condition *i)*.■