Abstract

We claim that the stock market encourages business creation, innovation, and growth by allowing the recycling of "informed capital". Due to incentive and information problems, start-ups face larger costs of going public than mature firms. Sustaining a tight relationship with a monitor (bank, venture capitalist) allows them to finance their operations without going public until profitability prospects are clearer or incentive problems are less severe. However, the earlier young firms go public, the quicker monitors' informed capital is redirected towards new start-ups. Hence, when informed capital is in limited supply, factors that lower the costs for start-ups to go public encourage business creation. Technological spill-overs associated with business creation and thick market externalities in the young firms segment of the stock market provide prima facie cases for encouraging young firms to go public.

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1 Introduction

Over the past two decades, the US venture capital industry has been remarkably active in the financing of young innovative companies, and this has gone together with an unprecedented growth in the size, liquidity, and value of Nasdaq, the stock market where most start-ups go public. This paper digs out some of the theoretical linkages between the roles that stock markets and expert financiers such as venture capitalists play in the financing of new businesses, studying their interactions with business creation, innovation, and economic growth.

There is wide consensus that venture capitalists, as well as some banks when involved in tight relationships with the firms that they finance, have special value for start-ups. They use their expertise, reputation, and wealth (in brief, their informed capital) in order to monitor the activities of entrepreneurs that, due to incentive problems, find difficulties in raising funds from the public. It has been argued that the stock market facilitates the recycling of informed capital by allowing the sufficiently mature companies to go public and the monitors to redirect their resources towards new start-ups. We bring this argument to general equilibrium, showing its implications for business creation and growth. The result is a theory of financial development in which informed capital and the stock market play distinct but complementary roles.

We consider an economy where start-ups are developed by entrepreneurs who are liquidity constrained. We postulate that, due to incentive and information problems, start-ups face larger costs of accessing the stock market than mature companies.

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2 The monitoring role of special classes of financiers has been emphasized by the literature on financial intermediation, including Diamond (1991), Rajan (1992), and Holmstrom and Tirole (1997).
3 The recycling role of the stock market is documented, among others, by Black and Gilson (1998), Lin and Smith (1998), and Gompers and Lerner (1999).
4 Rajan and Zingales (1998) document that young US companies are much more dependent on external finance than their mature counterparts.
5 The costs of going public include flotation costs, the underpricing at the initial public offering (IPO), and any other cost associated with establishing management control systems that work effectively under disperse ownership (see Pagano et al., 1998, for a review). In Sections 2 and 8 we...
Their alternative is to establish a tight relationship with a monitor (i.e. a bank or a venture capitalist) and not to go public until profitability prospects are clearer or incentive problems are less severe.

Monitors' informed capital is, however, in limited supply. Our preferred motivation for this is that monitoring skills are scarce because they relate to experience which is hard to accumulate. The limitation can also be due to constraints to monitors' capacity to raise external funds. Either way, informed capital ends up earning scarcity rents, so start-ups must choose between paying these rents and incurring the costs of going public. As the equilibrium rents obtained by informed capital are positively related to the number of entrepreneurs who decide to start-up a new business, when the economic environment becomes more favorable to entrepreneurship, the rents increase, start-ups decide to go public earlier, and the size of the stock market for young companies endogenously increases.

In our economy, businesses are created when entrepreneurs and monitors get matched after a process of search. In equilibrium the business creation rate is directly related to the number of entrepreneurs that search for informed capital and the amount of informed capital available for funding them. The number of searching entrepreneurs is increasing in the profitability of the start-ups, while the amount of available informed capital increases when its recycling speeds up (because of greater market liquidity, lower costs of going public or greater profitability of the start-ups).

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6 On January 25th 1997, The Economist wrote: “The main problem is not a lack of investment opportunities, but a shortage of people expert enough to spot them. Because venture capitalists spend so much time with the companies they invest in, they tend to finance just a few firms a year each” (p.21). For formal evidence, see Cumming and MacIntosh (2001).

7 In Holmstrom and Tirole (1997), monitors suffer from an incentive problem that requires them to finance a fraction of each monitored project with their own wealth, which is limited. Arguably, monitors’ accumulation of wealth is bounded by life cycle considerations and risk aversion. Gompers and Lerner (1998) document the importance of past performance and reputation in venture capitalists’ fundraising.

8 Inderst and Müller (2002) consider an environment similar to ours where informed capital earns no rents.
With this basic mechanism in place, we first study the efficiency of the equilibrium allocation and then develop two extensions. In the first extension we model the connection between business creation and growth. In the second, we analyze the effects of liquidity (or other “thick market”) externalities which generate a strategic complementarity between the going public decisions of young firms.

The efficiency results are driven by the fact that entrepreneurs and monitors set the terms of their relationships through bargaining once the entrepreneurs have already incurred some costs to create their businesses. Consequently, entrepreneurs decide whether to start up their businesses motivated by rewards that do not necessarily equal the value of their marginal contribution to firm creation. Thus their starting-up decisions are generally inefficient. In particular, if monitors' bargaining power is too high (low), the number of entrepreneurs that start-up their businesses is too low (high). Interestingly, as entrepreneurs' rewards approach the socially efficient ones, the value of informed capital and, thus, the incentives for young firms to go public increase. This suggests that institutions such as banking regulation and competition policy, which influence monitors' ability to appropriate rents, may affect business creation and stock market development.

Our model contributes to the literature on financial development by stressing that the stock market facilitates the recycling of informed capital, which generates a complementarity between two modes of financing that are typically regarded as substitutes. In our economy, the stock market promotes growth through business creation rather than savings. Specifically, we assume that the innovations introduced by successful young firms generate technological spill-overs on future firms and, thus, feed the rate of technological progress. Technological progress, in turn, raises the

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9 See Levine (1997) for a survey of the financial development literature.
10 This is consistent with Levine and Zervos (1998) who find that bank development and stock market liquidity are strongly related to productivity growth but not to savings.
pro...tability of new businesses and the value of informed capital, so it encourages..rms to go public early. But, then, the rate of business creation rises, spill-overs boost technological progress, and a virtuous circle is completed. By the same logic, however, the economy may get trapped in a vicious low-growth, slow-recycling circle. In this case, encouraging young ..rms to go public can increase welfare.

In the presence of liquidity externalities (or, equivalently, economies of scale in market monitoring or in investment banking) which make the net gains from going public increasing in the number of similar ..rms listed in the stock market, ..rms' going public decisions are strategic complements and multiple equilibria may emerge.\textsuperscript{12} Liquidity externalities, together with technological spill-overs, provide a prima facie case for policies directed to encourage ..rms to go public.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 characterizes individual ..rm behavior. Section 4 analyzes equilibrium. Section 5 discusses the results on ef ciency. Section 6 contains the extension on growth. Section 7 deals with liquidity externalities. In the concluding section we discuss the main empirical implications of our analysis.

2 The model

We consider an economy in continuous time where there is just one ..nal good, which is the numeraire.

2.1 Agents

There are three classes of agents, entrepreneurs, monitors, and investors, in continuous masses of size \( E; M; \) and \( I; \) respectively. All of them are in..nitely lived and maximize the expected present value of their income stream net of the relevant utility costs.

\textsuperscript{12}Pagano (1993) introduces this type of “thick market” externalities.
The entrepreneurs have a subjective discount rate $\frac{1}{2}$ and are able to develop one business project per unit of time. The monitors also have a discount rate $\frac{1}{2}$ and can monitor one entrepreneur per unit of time. Finally, the investors have a discount rate $r < \frac{1}{2}$ and are endowed with some exogenous flow of income which is large enough to guarantee that their supply of funds is, on the relevant range, perfectly elastic at the rate $r$:

The difference between $r$; that will be the market interest rate, and $\frac{1}{2}$ is intended to capture the fact that the securities placed in hands of (a large number of) investors tend to have greater liquidity and integrate into better diversified portfolios than those privately held by the entrepreneurs and their monitors.

2.2 Technologies

At every instant $t$, a mass $N$ of business projects is randomly allocated among the entrepreneurs not involved in any other project. Each project becomes a firm if one unit of funds is invested. An unfunded project is lost for ever unless its entrepreneur incurs a maintenance (utility) cost $c$ per unit of time.

There are up to three stages in a firm’s life. In the initial start-up stage ($s = 1$), no income is generated and there is uncertainty on whether the firm will turn out be successful or unsuccessful. Each firm has a probability $\theta$ of being successful and a probability $1 - \theta$ of being unsuccessful. When this uncertainty gets resolved, the firm enters the development stage ($s = 2$) during which no income is yet produced. At the final maturity stage ($s = 3$) a successful firm yields a constant income flow $y > 0$ per unit of time, while an unsuccessful firm yields no income. Any firm can be liquidated at any point in its life at a constant liquidation value $Q \cdot 1$.

The transition from one stage to the next requires the entrepreneur’s unobservable effort. Specifically, if the entrepreneur complies, the transitions from $s = 1$ to $s = 2$ (i.e., discovering whether the firm is successful or unsuccessful) and from $s = 2$ to
s = 3 (i.e., reaching maturity) occur at Poisson arrival rates λ and θ; respectively. If instead the entrepreneur shirks, he obtains a flow of unverifiable private benefits b· ½Q but no transition takes place. At maturity, the entrepreneur's unobservable effort plays no role.

2.3 Financing modes

We assume that the mass of entrepreneurs, E; is large relative to the flow of new business projects, N; so that the probability that an entrepreneur receives a project is small enough (relative to the difference between his discount rate ½ and the market interest rate r) to dissuade him from accumulating wealth. Hence entrepreneurs always require external funding for creating a business.

The access to external funding is, however, obstructed by the previously described moral hazard problem. Specifically, we assume that the private benefits flow b is so large that the claim on the firm's maturity value that the entrepreneur should retain in order to comply is incompatible with properly compensating his financiers —the Appendix provides a formal statement of this assumption. Hence the access to external funding requires a solution to the moral hazard problem. We consider two alternatives: informed capital and going public.

2.3.1 Informed capital

The first solution is that the entrepreneur establishes a relationship with a monitor who, in addition to contributing the required funds, can use her expertise, reputation, and dedication in order to guarantee that the entrepreneur complies. As in

13 The assumption b· ½Q ensures that unsuccessful firms are liquidated as soon as they are discovered to be so.

14 Given the difference between ½ and r; monitors want to commit as little wealth as possible to their activity. We assume that the monitors have some wealth with which to get started and guarantee the availability of funds for their new projects by forming coalitions (such as banks of venture capital funds) that pool together and, thus, diversify away the (idiosyncratic) risks involved in a large number of firms.
Holmstrom and Tirole (1997), the idea is that monitoring reduces the benefits from shirking, which thus becomes unattractive to the entrepreneur. Hereafter we refer to each monitor’s monitoring capacity as her unit of informed capital.

We assume that the market for informed capital is subject to search frictions so it takes time for entrepreneurs to match with a suitable monitor and vice versa. Following Pissarides (1990), we model the flow of viable matches using a matching function $h(e; m)$ whose arguments denote the masses of searching entrepreneurs and searching monitors, respectively. We assume that this function is homogeneous of degree one, increasing, concave, and continuously differentiable. Its homogeneity allows us to write the Poisson rate at which an entrepreneur finds a suitable monitor as

$$q(\mu) = \frac{h(e; m)}{e} = h(1; \frac{1}{\mu});$$

which is decreasing in the ratio $\frac{e}{m}$. Analogously, the Poisson rate at which a free monitor finds a suitable entrepreneur can be written as $\mu q(\mu)$; which is increasing in the ratio $\mu$. This ratio can be naturally interpreted as an index of informed capital scarcity: the larger the number of searching entrepreneurs per available monitor, the slower (quicker) an entrepreneur (monitor) will find a suitable monitor (entrepreneur).

In order to guarantee that the equilibrium value of $\mu$ is interior, we assume that

$$\lim_{\mu \to 0} q(\mu) = \lim_{\mu \to 1} \mu q(\mu) = 1 \quad \text{and} \quad \lim_{\mu \to 1} q(\mu) = \lim_{\mu \to 0} \mu q(\mu) = 0: \quad (1)$$

During their search (that we denote as stage $s = 0$), entrepreneurs incur the maintenance cost $c$ per unit of time, while we assume for simplicity that monitors’ search cost is zero.\footnote{Under the configuration of parameters on which we focus below, the maintenance cost $c$ is important to regulate entrepreneurs’ entry in the market for informed capital.} After a match, the entrepreneur and the monitor bargain on the contract that establishes the compensation of each party and the conditions for the termination of the relationship. For simplicity we consider contracts that can be contingent on whether the firm turns out to be successful or unsuccessful and
the arrival of maturity, but not on the date at which the corresponding contingency occurs. We assume a generalized Nash bargaining solution in which entrepreneurs' and monitors' bargaining powers are \( \bar{\bar{x}} \) and \( 1 - \bar{\bar{x}} \), respectively.

2.3.2 Going public

The second solution to the entrepreneur's financing problem is to adopt some mechanism of management control (accounting, auditing, corporate governance, etc.) that makes the private benefit flow verifiable and, thereby, guarantees that the entrepreneur does not shirk. To keep things simple, we model the introduction of this mechanism as an instantaneous restructuring that entails an unrecoverable fixed cost \( F \) and allows the firm to be sold to the investors, that is, to go public.\(^{16}\)

Beyond its literal interpretation, \( F \) may encompass (the present value of) any cost incurred by a non-mature firm because of going public before its maturity. For instance, the shorter track record of a non-mature firm may in practice imply greater uncertainty on its value, worsen the IPO winner's curse problem, and lead to greater underpricing.\(^{17}\) It might also be the case that the transparency required to go public at that stage leads to the disclosure of proprietary information from which competitors can benefit at the detriment of the firm — an effect that can be particularly damaging for young innovating firms.\(^{18}\) Finally, \( F \) might comprise the value losses due to agency problems that persist after the young firm goes public. For simplicity, the components of going public costs which are common to mature and non-mature firms

\(^{16}\) Depending on the exact mechanism (or combination of them), the cost \( F \) may represent the present value of a stream of fees to external auditors, the wages of the accountants, the remuneration of the board of directors, etc.

\(^{17}\) Indeed, Habib and Ljungqvist (2001) nd that, after controlling for other IPO characteristics, the underpricing decreases significantly with firm age. The nding by Pagano et al. (1998) that firm's age is a significant predictor of the probability of going public is also consistent with the view that the cost of going public is on average greater for young firms than for mature firms.

\(^{18}\) See Bhattacharya and Chiesa (1995) for a model in which transparency destroys firm value.
are normalized to zero.\(^{19}\)

In order to ensure that informed capital has a role to play in the economy, we assume that investors’ valuation of a firm that goes public at the start-up stage, \(R_1\); is insufficient to cover its total financing requirements, inclusive of the restructuring cost, \(1 + F\). So start-ups have no option but to rely on informed capital. This condition can be expressed as

\[
F > F_1 \times \frac{[\theta R_2 + (1 - \theta)Q_i]}{1 + \theta} 1; \tag{2}
\]

since \(R_1\) can be obtained from the asset pricing formula

\[
r R_1 = \frac{[\theta R_2 + (1 - \theta)Q_i]}{1 + \theta} R_3;
\]

where \(R_2\) is the investors’ value of a successful firm in the development stage, while \(R_3\) is the investors’ value of the successful firm in the maturity stage.\(^{20}\)

Under (2), a non-mature firm will have to delay going public at least until discovering that it is successful.

2.4 Summing up

Entrepreneurs who decide to develop their projects must first search for a monitor, obtain from her the funds to get started, and then devote their effort to the discovery of whether the firm will be successful. Unsuccessful firms are liquidated. Successful firms, instead, face a non-trivial choice between two alternative solutions to the moral hazard problem which affects them until maturity: either staying under the surveillance of their monitor or restructuring their management control mechanisms and going public. After maturing, the moral hazard problem is naturally solved and firms that maintain relationships with monitors definitely go public.

\(^{19}\)This includes the going-public costs caused by agency problems that extend beyond maturity (for instance, free cash flow problems a la Jensen (1986)).

\(^{20}\)The expression for \(R_2\) emerges from the asset pricing formula \(r R_2 = \frac{1}{\theta} (R_3 \times R_2)\):
The model focuses on the terms under which ...rms ...rst access and then abandon informed capital inancing. In Section 3 we characterize the contract between an entrepreneur and a monitor who have just matched. At that point, the values of their outside options are taken as given. In Section 4, we determine the equilibrium value of these outside options as well as the remaining endogenous variables. The model has the property that key endogenous variables such as the index of informed capital scarcity, the value of informed capital, and ..rms' going public decisions jump to their steady state values instantaneously. Thus we exclude time indices from all but the variables with transitional dynamics. To guide the reader, Table 1 provides a legend for the main symbols used throughout the paper.

3 Entrepreneur-monitor relationships

After an entrepreneur and a monitor get matched, they sign a contract which establishes the conditions for terminating their relationship and distributing the revenue generated up to that point. The only non-trivial termination decision is whether a non-mature ..rm should or should not go public once it is discovered to be successful. Since monitoring solves the moral hazard problem, the division of revenue plays a pure distributional role and can be implemented, without loss of ecience, through a constant sharing rule. Hence, we represent the entrepreneur-monitor contract by a pair \((\ell; f)\); where \(\ell \in [0; 1]\) denotes the entrepreneur’s share in the revenue generated throughout the relationship and \(f \in [0; 1]\) denotes the probability that an acknowledged successful ..rm goes public without waiting till maturity, i.e., its going public decision.

The contract \((\ell; f)\) determines the entrepreneur’s and the monitor’s value of the relationship at its various stages. At the start-up stage, the entrepreneur’s value of the relationship, \(U_1\); solves

\[
\frac{1}{\ell} U_1 = \ell f R_2 + (1 - f) U_2 + (1 - \ell) \ell Q - U_1; \quad (3)
\]
Table 1: Legend

Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>mass of entrepreneurs</td>
</tr>
<tr>
<td>M</td>
<td>mass of monitors</td>
</tr>
<tr>
<td>I</td>
<td>mass of investors</td>
</tr>
<tr>
<td>N</td>
<td>flow of new projects</td>
</tr>
<tr>
<td>r</td>
<td>investors’ discount rate</td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>entrepreneurs’ and monitors’ discount rate</td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td>entrepreneurs’ bargaining power</td>
</tr>
<tr>
<td>c</td>
<td>maintenance cost of an unfunded project</td>
</tr>
<tr>
<td>$\lambda_{s}$</td>
<td>Poisson rate at which a ..rm discovers whether it is successful</td>
</tr>
<tr>
<td>$\lambda_{\theta}$</td>
<td>Poisson rate at which a successful ..rm matures</td>
</tr>
<tr>
<td>$\theta$</td>
<td>probability that a ..rm is successful</td>
</tr>
<tr>
<td>y</td>
<td>income flow of a mature successful ..rm</td>
</tr>
<tr>
<td>Q</td>
<td>liquidation value of a ..rm</td>
</tr>
<tr>
<td>F</td>
<td>cost of the restructuring required to go public before maturity</td>
</tr>
<tr>
<td>$\frac{1}{4}$</td>
<td>importance of the technological spill-overs</td>
</tr>
</tbody>
</table>

Revenues and values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{s}$</td>
<td>revenue generated by selling a (successful) ..rm in stage s</td>
</tr>
<tr>
<td>$U_{s}$</td>
<td>entrepreneurs’ value (of a relationship) in stage s</td>
</tr>
<tr>
<td>$V_{s}$</td>
<td>monitors’ value (of a relationship) in stage s</td>
</tr>
<tr>
<td>$F_{s}$</td>
<td>a (successful) ..rm’s shadow value of going public in stage s</td>
</tr>
<tr>
<td>$s$</td>
<td>stage (0=search, 1=start-up, 2=development, 3=maturity)</td>
</tr>
</tbody>
</table>

Contract and aggregate variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\beta}$</td>
<td>entrepreneurs’ share in relationships’ revenue</td>
</tr>
<tr>
<td>$f$</td>
<td>probability that a non-mature successful ..rm goes public</td>
</tr>
<tr>
<td>$\mu$</td>
<td>index of informed capital scarcity</td>
</tr>
<tr>
<td>$n$</td>
<td>rate of business creation</td>
</tr>
<tr>
<td>$m_{s}$</td>
<td>stock of informed capital in stage s</td>
</tr>
<tr>
<td>$g$</td>
<td>rate of technological progress</td>
</tr>
<tr>
<td>p</td>
<td>mass of publicly-traded non-mature successful ..rms</td>
</tr>
</tbody>
</table>
which equals the instantaneous return from being in the relationship to the expected capital gains associated with the discovery of whether the rm is successful or not. Notice that $R_2 \cdot F$ is the net revenue raised when a rm turned out to be successful goes public, $U_2$ is the entrepreneur’s value of continuing the relationship after the rm turns out to be successful, and $Q$ is the liquidation value which is recovered once the rm turns out to be unsuccessful. The continuation value $U_2$ can be obtained from the equation

$$\frac{1}{2}U_2 = \frac{1}{2}(R_3 \cdot U_2);$$  \hspace{1cm} (4)

which reflects that, at maturity, the successful rm can be sold to the investors at the price $R_3 < \frac{R}{2}$.

Analogously, the monitor’s value of the relationship at the start-up stage, $V_1$, solves

$$\frac{1}{2}V_1 = \frac{1}{2}f \circ f [(1 - \circ) (R_2 \cdot F) + V_0] + \circ (1 - f) V_2 + (1 - \circ) [(1 - \circ) Q + V_0] \cdot V_1;$$  \hspace{1cm} (5)

whose interpretation is symmetric to that of (3) except for the terms in $V_0$, which capture the gains from recycling the monitor’s unit of informed capital whenever the relationship breaks up. The equation

$$\frac{1}{2}V_2 = \frac{1}{2}[(1 \cdot \circ) R_3 + V_0 \cdot V_2]$$  \hspace{1cm} (6)

gives the monitor’s value of continuing the relationship (up to maturity) after the rm turns out to be successful, $V_2$.

In the bargaining on $(\circ; f)$, the entrepreneur’s outside option is the value of an unfunded investment project, $U_0$; while the monitor’s outside option is the sum of the unit of funds required to start up the project and the value of her unit of informed capital, $V_0$. Under the postulated generalized Nash bargaining solution, the contract solves the following program:

$$\max_{(\circ; f) \in [0,1] \times [0,1]} (U_1 \cdot U_0) - (V_1 \cdot V_0) \cdot 1;$$  \hspace{1cm} (7)
where $U_0$ and $V_0$ are taken as given. Equations (3)-(6) imply that \( \frac{\partial U}{\partial \Phi} = i \frac{\partial V}{\partial \Phi} \), which allows us to rewrite the first order condition for the choice of $\Phi$ as

\[
U_1 = U_0 + \bar{S}; \tag{8}
\]

or as

\[
V_1 = 1 + V_0 + (1 - \bar{S}) S; \tag{9}
\]

where $S = (U_1 + V_1) - (1 + U_0 + V_0)$ is the surplus of the relationship at its inception.

It follows from (7), (8), and (9) that the entrepreneur and the monitor will agree on the going public decision $f$ that maximizes $S$.

To obtain an expression for $S$ that shows the effect of the going public decision $f$, we first add up (3) and (5), using (4) and (6). Grouping together the terms in $U_1 + V_1$ and then adding and subtracting constants so as to isolate $S$ on the left hand side yields

\[
S = \frac{\delta}{\gamma^2} f (R_2 - F + V_0) + \frac{\delta}{1 + \gamma^2} (R_3 + V_0) + (1 - \delta) (Q + V_0) (1 + U_0 + V_0); \tag{10}
\]

The discount factor $\frac{\delta}{\gamma^2}$ accounts for the fact that no revenue is generated until the firm enters the development stage; the additional discount factor $\frac{1}{1 + \gamma^2}$ appears because if a successful firm does not go public, its sale (and the recycling of the unit of informed capital) is delayed up to maturity.

Maximizing (10) with respect to $f$ identifies a critical value,

\[
F_2 \gtrsim \frac{1}{1 + \gamma^2} (\frac{1}{\gamma^2} r) R_2 + \frac{\delta}{\gamma^2} V_0; \tag{11}
\]
such that:

Proposition 1 If $F \leq F_2$, successful firms choose to go public at the development stage, otherwise they go public at the maturity stage.

The critical value $F_2$ represents the shadow value of going public for a non-mature successful firm. It adds up the liquidity/diversification gain generated by selling its
stream of future income to investors (who have a lower discount rate than entre-
preneurs and monitors) and the recycling gain associated with freeing the unit of
informed capital of value $V_0$ at the development rather than at the maturity stage.
Both gains are inversely related to the Poisson rate $\lambda$ at which the transition from
the development stage to the maturity stage occurs.

By using Proposition 1 and equation (10), we can write the (maximized) surplus
of the entrepreneur-monitor relationship as

$$S = \frac{\epsilon}{\sqrt{\pi}} (R_2 + V_0 + \min F; F_2 g) + (1 - \epsilon)(Q + V_0) (1 + U_0 + V_0); \quad (12)$$

which will be useful in the analysis below.

4 Equilibrium

An equilibrium is an index of informed capital scarcity $\mu \in [0; 1)$ and a contract
$(\theta, f) \in [0; 1] \times [0; 1]$ governing each entrepreneur-monitor relationship, such that no
privately profitable business opportunity remains unexploited. In order to emphasize
the importance of the recycling of informed capital in times in which business oppor-
tunities are very abundant, we focus on the situation in which the flow of business
opportunities, $N;$ is large relative to the stock of the informed capital, $M$. In this
situation, the value of an unfunded investment project, $U_0;$ is zero and only a fraction
of the entrepreneurs who receive projects decide to search for a monitor. When so,
the rate of business creation is ultimately constrained by the stock of informed capital
and the value of one unit of informed capital, $V_0;$ is strictly positive.

To characterize the unique equilibrium of this economy, we first reduce the different
equilibrium conditions to a single equation that determines $\mu$ and, recursively, $V_0$.

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21 This situation may arise during an unanticipated technological revolution: business opportunities flourish but informed capital is in limited supply. Even if informed capital could be accumulated, it may remain scarce for a long time if the mass of unfunded projects exhibits a large growth rate. In an economy similar to ours, Inderst and Müller (2002) analyze the case where business creation is constrained by the number of entrepreneurs.
Then we use our results in Section 3 to characterize the equilibrium contract. Finally, we write down the dynamics of the stock of informed capital and the masses of rms in each stage of their life cycle, and compute the steady state rate of business creation.

4.1 The equilibrium value of informed capital

First we write $V_0$ in terms of $\mu$: To do this, notice that the value of an unfunded project, $U_0$, solves

$$\frac{1}{2}U_0 = c + q(\mu) (U_1 - U_0);$$

(13)

since a searching entrepreneur incurs a maintenance cost $c$ per unit of time and matches with a monitor at a Poisson arrival rate $q(\mu)$, in which case he starts a relationship of value $U_1$. Combining (8) and (13) with the fact that $U_0 = 0$, we get

$$-q(\mu)S = c;$$

(14)

On the other hand, the value of a free unit of informed capital, $V_0$, solves

$$\frac{1}{2}V_0 = \mu q(\mu) (V_1 - V_0 - 1);$$

(15)

since a searching monitor matches with an entrepreneur at a Poisson arrival rate $\mu q(\mu)$, in which case she invests one unit of funds and her informed capital in a relationship of value $V_1$: Combining (9) with (14) and (15), we obtain

$$V_0(\mu) = \frac{1}{\mu} \left( \frac{1}{2} q(\mu) \right);$$

(16)

which establishes an intuitive (linearly) increasing relationship between the value and the scarcity of informed capital.

We next express $F_2$ and $S$ as functions of $\mu$: By substituting (16) into (11) it immediately follows that

$$F_2(\mu) = \frac{1}{1 + \gamma_2 [\gamma_2 q(\mu) R_2 + \frac{1}{2} q(\mu)]};$$

(17)
But then substituting this expression into (12) and combining (9) and (15) with $U_0 = 0$; we obtain

$$S(\mu) = \left[ R_2 + \min F_2(\mu) + (1 - \mu) q(\mu) \right] \mu \left( + \frac{\mu}{2} \right). \tag*{(18)}$$

Since $F_2(\mu)$ and $\mu q(\mu)$ are both strictly increasing in $\mu$, we can ensure that the function $S(\mu)$ is positive and, then, strictly decreasing for all $\mu$ by assuming that $\left[ R_2 + F + (1 - \mu) q \right] \mu > \frac{\mu}{2}$.

Finally, substituting (18) into (14) we obtain an equation in $\mu$ which we hereafter call entrepreneurs’ free entry condition:

$$-q(\mu) S(\mu) = c; \tag*{(19)}$$

whose unique solution is the equilibrium value of the index of informed capital scarcity $\mu$.\footnote{Since $q(\mu)$ is strictly decreasing, $q(\mu) S(\mu)$ is strictly decreasing in $\mu$. Moreover, it is continuous and satisfies $\lim_{x \to -1} q(x) S(x) = 0$ and $\lim_{x \to 0} q(x) S(x) = 1$; by (1). Hence there is a unique $\mu \in (0; 1)$ for which (19) holds.} To explain (19), recall that entrepreneurs earn no rents in equilibrium since the supply of informed capital is small relative to the number of entrepreneurs receiving projects. Thus the equilibrium value of $\mu$ is obtained by equating entrepreneurs’ expected return from searching, $-q(\mu) S(\mu)$, with the maintenance cost $c$ that they incur while searching. The former is decreasing in $\mu$ because the scarcity of informed capital reduces both the probability of finding a monitor, $q(\mu)$; and (as a result of the increased value of informed capital) the surplus, $S(\mu)$. In general, factors that encourage entrepreneurs to develop their projects (such as an increase in the profitability of new businesses) raise the equilibrium value of $\mu$. Specifically, using (18), we obtain:

**Proposition 2** The equilibrium value of the index of informed capital scarcity $\mu$ is increasing in $\mu$; $\mu$, $\mu q(\mu)$ and decreasing in $\frac{\mu}{2}; r; F$; and $c$.\footnote{Since $q(\mu)$ is strictly decreasing, $q(\mu) S(\mu)$ is strictly decreasing in $\mu$. Moreover, it is continuous and satisfies $\lim_{x \to -1} q(x) S(x) = 0$ and $\lim_{x \to 0} q(x) S(x) = 1$; by (1). Hence there is a unique $\mu \in (0; 1)$ for which (19) holds.}
Given (16), the parameters that increase (decrease) \( \mu \) generally increase (decrease) also the equilibrium value of informed capital \( V_0(\mu) \). The only exceptions are \( c \) whose direct effect on \( V_0(\mu) \) differs in sign from that conducted through \( \mu \). For example, increasing the entrepreneurs' bargaining power \( \bar{\sigma} \) has a negative direct effect on \( V_0(\mu) \) since monitors are left with a lower share of the surplus. But increasing \( \bar{\sigma} \) also leads further entrepreneurs to undertake their projects, which increases the scarcity of informed capital and, hence, pushes \( V_0(\mu) \) up. This general equilibrium effect dominates for small \( \bar{\sigma} \); while the direct effect dominates for large \( \bar{\sigma} \): the result is an inverted U-shaped relationship between \( \bar{\sigma} \) and \( V_0(\mu) \).

### 4.2 The equilibrium contract

By Proposition 1, the equilibrium going public decision \( f \) can be obtained by comparing the shadow value of going public in the development stage, \( F_2(\mu) \); with the restructuring cost, \( F \): The various parameters of the model may affect this difference directly and through their impact on the equilibrium value of informed capital \( V_0(\mu) \).

In some cases both effects go in the same direction, yielding clear-cut results:

**Proposition 3** High values of \( \gamma; \beta; \gamma; \beta; y; \beta; \gamma; \beta; \) and low values of \( r; F; \beta; \gamma; \beta; \gamma; \beta; \) and \( c \) make successful firms more likely to go public before maturity.

Changes in parameters that raise both the liquidity/diversification gain from going public and the recycling gain invite firms to go public early. In some cases, however, the gains move in opposite directions or their sign is ambiguous. For example, an increase in entrepreneurs' and monitors' discount rate \( \frac{1}{2} \) increases the liquidity/diversification gain but reduces the profitability of new businesses and thus the recycling gain. Conversely, an increase in the Poisson rate at which a firm matures \(^1\) raises profitability and the value of informed capital, but also reduces the opportunity cost of waiting till maturity, so the liquidity/diversification gain falls while the effect on the recycling gain is ambiguous.
In order to determine the unique equilibrium value of the entrepreneur’s share in revenue, \( \bar{\alpha} \) notice that (8) together with the free entry condition \( U_0 = 0 \) implies \( U_1 = \bar{S}(\mu) \): Together with the equilibrium going public decision, this expression can be substituted into (3) in order to solve for \( \bar{\alpha} \). Essentially, \( \bar{\alpha} \) must leave the entrepreneur with a share \( \bar{\alpha} \) of the surplus of his relationship with the monitor. Since the value of the monitor’s outside option is strictly positive, the surplus \( S(\mu) \) is always smaller than the present value of the revenue generated throughout the relationship, so we always have \( \bar{\alpha} < \bar{\alpha} \): Interestingly, when \( F = F_2(\mu) \); the contract with \( f = 0 \) features a smaller share than the contract with \( f = 1 \): So the entrepreneur’s share tends to be higher if the firm goes public early, since from the monitor’s perspective the quicker recycling of her informed capital is a substitute for pecuniary rewards. Other comparative statics results on \( \bar{\alpha} \) are ambiguous and we omit their discussion for brevity.

4.3 The steady state rate of business creation

We next derive the relationship between the stock of informed capital, \( M \); and the steady state rate of business creation, \( n \). This rate is important because a fraction \( \delta \) of the start-ups eventually become mature successful ..rms, so the steady state pool of productive ..rms and, consequently, aggregate income grow at (linear) rates \( \delta n \) and \( \delta y_n \) respectively.

At any date \( t \); the flow of new businesses is

\[
\dot{n}_t = \mu q(\mu) m_\alpha;
\]  

where \( m_\alpha \) is the mass of searching monitors (henceforth, the stock of free informed capital) and \( \mu q(\mu) \) is the rate at which the match with entrepreneurs. The stock of free informed capital can be determined as the difference between \( M \) and the masses of ..rms which rely on informed capital ..nancing either in the start-up stage, \( m_{\text{st}} \), or
in the development stage, \( m_{2t} \):

\[
m_{0t} = M \cdot m_{1t} \cdot m_{2t}:
\]

The evolution of \( m_{1t} \) is driven by the creation of the new start-ups and the exit, at rate \( \gamma \); of those that reach the development stage:

\[
m_{1t} = n_{t} \cdot m_{1t}:
\]

Analogously, \( m_{2t} \) is increased by the flow of firms that turn out to be successful but do not go public, and decreased by the exit, at rate \( \beta \); of the firms that reach maturity:

\[
m_{2t} = \beta \cdot (1 - f) \cdot m_{1t} \cdot m_{2t}:
\]

The steady state values of \( m_{0t}; m_{1t}; m_{2t} \), and \( n_{t} \), can be obtained by setting \( m_{2t} = m_{1t} = 0 \) in equations (22) and (23) and solving for them after using (20) and (21). The resulting steady-state rate of business creation is:

\[
n = \mu q(\mu) m_{0} = \frac{\mu q(\mu) \cdot M}{\gamma + [1 + \gamma \cdot (1 - f)\mu q(\mu)]}:
\]

Thus, the rate of business creation equals the product of the stock of free informed capital, \( m_{0} \), and the rate at which it gets reused, \( \mu q(\mu) \) (which is increasing in \( \mu \)). In steady state, \( m_{0} \) is a constant fraction of the total supply of informed capital \( M \) and depends positively on the speed at which informed capital exits ongoing relationships (so it is increasing in \( \gamma \); \( \beta \) and \( f \); and decreasing in \( \gamma \)) and negatively on the rate at which it gets reused, \( \mu q(\mu) \). So \( \mu q(\mu) \) enters twice and with opposite signs in (24); nevertheless, the rate of business creation is overall increasing in \( \mu \) (since business creation is, precisely, what makes \( m_{0} \) depend negatively on \( \mu \)).

Clearly, parameters whose direct impact on the last term in (24) has the same sign as their total impact on \( f \) and \( \mu \) have unambiguous effects on the business creation rate \( n \). In particular, Propositions 2 and 3 imply that:
Proposition 4 The steady-state rate of business creation is increasing in \( \zeta \); \( y \); \( Q \); and \( M \); and decreasing in \( r \); \( F \); and \( c \).

Other parameters have ambiguous effects on \( n \). For example, the probability that a start-up is successful, \( \delta \); increases profitability and leads to an increase in \( \mu \) but its overall effect can be negative if \( f = 0 \) because having a greater fraction of successful .rms which only go public at maturity may depress \( m_0 \). Similarly, the ambiguous (or non-monotonic) effects of \( \bar{t} \); \( \bar{\theta} \) and \( \bar{\eta} \) on the going public decision \( f \) induce similarly ambiguous (or non-monotonic) effects on \( n \).

We conclude this section by comparing the steady-state rate of business creation \( n \) with those that would emerge if either non-mature .rms did not suffer a moral hazard problem (and hence informed capital were redundant), \( n_0 \); or if informed capital were needed but its access were not subject to search frictions, \( n \). In the .rst case, we would simply have \( n = N \). With the moral hazard problem but without search frictions, the immediate re-employment of informed capital after it gets freed would imply \( n = [1 + \frac{\zeta}{\varphi}(1 + f)]^1 \cdot M \); where \( f \) denotes the going public decision of non-mature successful .rms in such an economy. Clearly, the equilibrium value of informed capital in this economy is always greater than in one with search frictions, so necessarily \( f^\wedge > f \). Thus we have \( n < n < n \), which implies that both the moral hazard problem and the search frictions have a negative cumulative impact on .rm creation.

5 Efficiency

In this section we analyze the efficiency of the equilibrium allocation. Since the welfare of the population of investors is invariant to the equilibrium allocation, we denote...
social welfare $W$ as the discounted value of the aggregate income flows of monitors and entrepreneurs net of any relevant utility cost. The moral hazard problem and the search frictions that affect the financing of start-ups are taken as given. At any point in time, the state of the economy is fully summarized by the masses of start-ups, $m_{1t}$, and non-mature successful firms, $m_{2t}$, that are financed with informed capital. Without loss of generality, we consider time invariant allocations described by the index of informed capital scarcity $\mu$ and the going public decision $f$.\footnote{Considering time invariant values of $\mu$ and $f$ implies no loss of generality since neither the equilibrium (as seen in the previous section) nor the social optimum (as can be deduced from the analysis below) involve values of $\mu$ and $f$ that depend on $m_{1t}$ and $m_{2t}$:}

We can implicitly define the social welfare function $W(m_{1t}; m_{2t}; \mu; f)$ as the solution to the equation

$$\frac{\partial W}{\partial m_{1t}} = R(f)m_{1t} + \gamma m_{2t} \left[ c\mu + \mu q(\mu) \right] (M_{1} m_{2t} - m_{1t}) + W_{1} m_{1t} + W_{2} m_{2t}$$ \hspace{1cm} (25)

where

$$R(f) = \left[ \gamma f \left( R_{2} - F \right) + (1 - \gamma) Q \right]$$ \hspace{1cm} (26)

$W_{1} = \frac{\partial W}{\partial m_{1t}}$; and $W_{2} = \frac{\partial W}{\partial m_{2t}}$; while $m_{1t}$ and $m_{2t}$ are described by (22) and (23), respectively. The first term in the right hand side of (25) accounts for the instantaneous net flow of revenue generated by start-ups as they enter the development stage, the second for that generated by successful firms that remain private as they reach maturity, and the third for the outflows associated with entrepreneurs' search costs and the investment required by the new start-ups; the two final terms reflect the welfare gains derived from the time variation in $m_{1t}$ and $m_{2t}$, respectively. To explain (26), notice that the first term in brackets reflects the revenue associated with the possibility that a start-up turns out to be successful and goes public, while the second reflects the proceeds from its liquidation if it turns out to be unsuccessful.

In the Appendix we provide explicit expressions for the partial derivatives of $W$ with respect to $\mu$ and $f$. In order to evaluate these derivatives at the equilibrium
allocation, it is useful to define

\[
\frac{\partial}{\partial \mu} \left( q(\mu) + \mu q'(\mu) \right)
\]

which gives the elasticity of the number of matches between entrepreneurs and monitors with respect to the number of searching entrepreneurs. We show that, at the equilibrium allocation, the sign of \( \frac{\partial W}{\partial \mu} \) coincides with that of \( \frac{\partial}{\partial \mu} \) so the equilibrium ratio of searching entrepreneurs to searching monitors \( \mu \) is socially efficient only with \( \frac{\partial}{\partial \mu} = -\); which is generally not the case. In contrast, the sign of \( \frac{\partial W}{\partial f} \) coincides with that of \( F_2(\mu) \) \( F \); which by Proposition 1 is, precisely, positive when \( f = 1 \) and negative when \( f = 0 \); so the equilibrium \( f \) is socially efficient (for given \( \mu \)). Therefore:

Proposition 5 If in equilibrium \( \frac{\partial}{\partial \mu} = - \); then welfare cannot increase by marginally distorting the equilibrium allocation \((\mu, f)\). Otherwise, welfare increases if entrepreneurs’ entry decisions are marginally distorted so as to increase \( \mu \) if \( \frac{\partial}{\partial \mu} > - \) and decrease \( \mu \) if \( \frac{\partial}{\partial \mu} < - \):

This result shows that the equilibrium allocation is vulnerable to the search-related inefficiencies first pointed out by Hosios (1990). In a Walrasian environment, competitive prices would make each entrepreneur undertaking a project appropriate the value of his marginal contribution to the generation of surplus, \( \frac{\partial}{\partial \mu} q(\mu) S(\mu) \), while under Nash bargaining he obtains \( -q(\mu) S(\mu) \); since \( \frac{\partial}{\partial \mu} \) and \( - \) need not coincide, the entry decisions determined by (19) are generally not socially efficient. For example, when monitors are “too strong” \( - < \frac{\partial}{\partial \mu} \), entrepreneurs develop an inefficiently low number of projects.

Our next result shows that these inefficiencies eventually translate into a lower value of informed capital and may cause the underdevelopment of the stock market:
Proposition 6 As $\bar{\mu}$ approaches $\mu$, the equilibrium value of informed capital $V_0$ and, thus, the shadow value of the stock market for non-mature successful firms $F_2(\mu)$ increase.

In other words, economies where the distribution of bargaining power leads to a more efficient allocation of resources will value more the recycling role of the stock market. Thus, the emergence of markets for young companies like Nasdaq may depend on factors such as the extent to which monitors' informational monopolies are legally protected or to which monitors compete for entrepreneurs by publicly pre-committing to the terms of their future financial contracts.\(^{25}\)

6 Growth

In this section we analyze the interactions between business creation and growth. Recent historical experience suggests that start-ups play an important role in technological innovation.\(^{26}\) Theorists have pointed out various reasons why new businesses may be better innovators than mature companies.\(^{27}\) The growth literature emphasizes the importance of technological spill-overs in spreading the benefits from innovation. We incorporate these aspects into the model by first allowing for an (exogenous) rate of technological progress and then endogenizing it by assuming that the maturity of successful firms produces innovations that increase the productivity of subsequent firms.

\(^{25}\) The results obtained in a labor market context by, among others, Acemoglu and Shimer (1999) imply that efficiency would prevail if monitors competed for entrepreneurs by posting the terms of their financial contracts.

\(^{26}\) Hobijn and Jovanovic (1999) document that the main winners of the IT revolution have been some newly created firms rather than the incumbent ones.

\(^{27}\) Aghion and Tirole (1994) consider the holdup problem that affects an innovator and the potential user of the innovation. They show that, when the incentives of the former are important, the optimal solution involves making him the owner of his innovation, that is, creating a new firm. Greenwood and Jovanovic (1999) provide additional reasons.
6.1 The effect of growth on financial development

In order to incorporate technological progress, we assume that all relevant quantities in the life of a firm are scaled up by a factor $X_t$ that identifies the state of the technology at time $t$ and grows at a constant exponential rate $g = \frac{X_t}{X_{t-1}} < r$. Thus, at time $t$, a searching entrepreneur incurs a cost $cX_t$; creating a firm requires an investment of $X_t$, its liquidation yields $Q_t = QX_t$, the private benefit flow that the entrepreneur can obtain from shirking is $b_t = bX_t$, and the cost of going public is $F_t = FX_t$. Analogously, if a successful firm reaches maturity at time $t$, its output is $y_t = yX_t$ from that time onwards.

Let $k_t$ denote the density of successful firms that reach maturity at time $t$. Then, by standard arguments, this economy has a balanced-growth equilibrium path where aggregate output is

$$O_t = \frac{Z}{1 - \int_1^\infty yX_s k_s ds} = \frac{\theta}{g} e^{gt};$$

which grows at rate $g$, while both the index of informed capital scarcity $\mu$ and the contract $(\bar{\gamma}; f)$ are constant over time. As in Section 4, we can reduce the different equilibrium conditions to a single equation that uniquely determines $\mu$ and then obtain $f$ recursively.

Specifically, the value at time $t$ of the surplus of a relationship in which firm type is unknown is given by the product of $X_t$ and the quantity

$$S(\mu; g) = \frac{1}{1 + \frac{1}{4} g} \left( \frac{1}{1 - \frac{1}{4} g} \right) R_2 (g) + \frac{1}{4} (1 - \frac{1}{4} g) Q_2 (\mu; g),$$

where

$$F_2 (\mu; g) = \frac{1}{1 + \frac{1}{4} g} \left( \frac{1}{1 - \frac{1}{4} g} \right) R_2 (g) + \frac{1}{4} (1 - \frac{1}{4} g) c'$$

We are implicitly assuming that all activities have either a direct or an opportunity cost in terms of some limited resources (say, labor) whose growing prices make all relevant costs grow at the same rate as $X_t$. For example, if the restructuring required before flotation is labor intensive, $F_1$ can be interpreted as the cost of the restructuring in terms of hours and $X_t$ as the hourly wage rate in terms of the numeraire, which any growth model would predict to grow as the economy grows.
and $R_2(g) = \frac{1}{1+g}$. After scaling up by $X_t$, these quantities have the same interpretation as our previous variables $S(\mu), F_2(\mu)$; and $R_2$, respectively, from which they only differ in that $r$ has been replaced by $r + g$ and $\frac{1}{2}$ by $\frac{1}{4}$. Consequently the three variables are now increasing functions of $g$.

As before, the equilibrium value of the index of informed capital scarcity $\mu$ is the unique solution to entrepreneurs’ free-entry condition:

$$\bar{q}(\mu) S(\mu, g) = c$$ (29)

while the going public decision of non-mature successful firms is determined by the rule

$$f = \frac{1}{2} \quad \text{if} \quad F \cdot F_2(\mu, g);$$
$$0 \quad \text{otherwise},$$ (30)

analogous to that described in Proposition 1.

The upward sloping curve $FE$ in Figure 1 represents the relationship between $\mu$ and $g$ implied by (29). For given $\mu$, increasing the rate of technological progress $g$ raises the surplus $S(\mu, g)$ and, thus, the incentive for entrepreneurs to develop their projects. Hence, in order to satisfy entrepreneurs’ free entry condition the value of $\mu$ must rise when $g$ rises.

Figure 1 also depicts the downward sloping schedule $F_2(\mu, g) = F$ below and above which firms set $f = 0$ and $f = 1$; respectively. By (28), a higher rate of technological progress makes firms more likely to go public early both because $g$ directly increases the liquidity/diversification gain and the recycling gain and because the rise in $\mu$ further increases the latter. The consequences for the rate of business creation can be immediately derived from (24) so as to obtain the following result:

**Proposition 7** An increase in the rate of technological progress $g$ increases the index of informed capital scarcity $\mu$, the value of informed capital $V_0$, and the likelihood that successful firms go public before maturity $f$. As a result, the business creation rate $n$ is increasing in $g$.
6.2 Endogenous growth

We now endogenize the rate of technological progress by considering a positive technological externality related to the success of new businesses: we assume that $g$ is proportional to the density of successful firms that reach maturity, $k_t$. Along the balanced-growth path we have $k_t = \dot{n}$ so, by (24), the rate of technological progress $g$ is

$$g = \frac{3/2}{\nu} n = \frac{3/2}{\nu} \mu q(\mu) M \left[ 1 + \frac{1}{q}(1 - f) \right] \mu q(\mu);$$

(31)

where $3/2$ measures the importance of the technological spill-overs. To guarantee that $g < r$; we assume that $3/2, M < r$: Equations (29)-(31) characterize the balanced-growth equilibrium of the model.

When informed capital becomes scarcer, it gets matched more quickly, which

---

29 This modelling of technological externalities follows, among others, Caballero and Jäger (1993) and Aghion and Howitt (1998).
raises the business creation rate and, through the spill-overs, the rate of technological progress. So for each value of \( f \); equation (31) describes a positive relationship \( TE_f \) between \( \mu \) and \( g \); As Figure 1 shows, \( TE_1 \) always yields above \( TE_0 \) since a quicker recycling of informed capital allows to sustain, for each \( \mu \); a larger rate of business creation. Moreover, both curves are continuous, pass through the origin, and are bounded above by the line \( g = \frac{3}{2}, M \); Hence \( TE_0 \) and \( TE_1 \) cross the \( FE \) curve at least once, like at points A and B; respectively, in Figure 1.

Intersections such as A and B provide the candidate equilibria. To constitute an equilibrium, however, the underlying going public decisions must satisfy (30), that is, the intersection must occur on the solid segment of the corresponding \( TE_f \) curve. In Figure 1, both A and B are equilibria. Point B identifies a high-growth equilibrium where informed capital is relatively scarcer than in the low-growth equilibrium at point A. In the low-growth equilibrium A; the economy suffers a financial underdevelopment trap: the growth rate is low because the stock market does not provide enough recycling of informed capital, which in turn occurs because the low growth rate makes start-ups little profitable and, thus, depresses the value of informed capital.

The comparative statics of these equilibria can be analyzed by noting that Propositions 2 and 3 have immediate implications for how changes in parameters move horizontally the curves \( FE \) and \( F_2(\mu g) = F \); respectively. On the other hand, (31) shows that the \( TE_f \) curves move upwards with \( \frac{3}{2}, \gamma, \theta \); and \( M \) (and \( TE_0 \) also with \( \theta \)). The results can be summarized as follows:

**Proposition 8** The equilibrium growth rate is increasing in \( \frac{3}{2}, \gamma, \delta \); \( Q \); and \( M \); while it is decreasing in \( r \); \( F \); and \( c \).

To illustrate the mechanics of these results, consider the effects of a reduction in the restructuring cost \( F \); When \( F \) decreases, the section of \( FE \) that stands in the \( f = 1 \) region rotates towards the right and the \( F_2(\mu g) = F \) curve moves towards the left,
while $T E_0$ and $T E_1$ remain unchanged. So the high-growth equilibrium $B$ moves up along $T E_1$, while the low-growth equilibrium $A$ remains unchanged. Moreover, if $F$ continues decreasing, the $f = 1$ region further expands up to, eventually, absorb point $A$, thus leaving the high-growth equilibrium $B$ as the only equilibrium. Summing up, lowering $F$ leads to a greater scarcity of informed capital and, sometimes, to a quicker recycling of informed capital, and both effects help sustaining a higher rate of technological progress $g$.

In the presence of technological externalities, the private shadow value of going public $F_2(\mu; g)$ is lower than the social one, so there may be situations in which encouraging firms to go public can increase welfare. Consider, for instance, the polar situation in which $F$ is just above $F_2(\mu; g)$ so that all non-mature successful firms are choosing $f = 0$: With a small subsidy, a government might induce these firms to go public, favoring the recycling of informed capital and boosting business creation and growth (both over the transition path and in the new steady state). Since the effect on growth is additional to the effects that arise in the benchmark model (where going public decisions are socially efficient), the subsidy would increase welfare. In practical terms, this means that governments may want to encourage the recycling of informed capital in industries characterized by large technological spill-overs and in which the scarcity of informed capital is perceived to constrain firm creation.

7 Liquidity externalities

We broadly refer to liquidity externalities as the increase in the net gains from going public that certain firms enjoy if the number of similar firms listed in the stock market increases. These externalities may emerge for various reasons. First, the access to a larger set of similar listed companies may allow investors to better diversify idio-
syncratic risk or to economize on the costs of gathering information about them.\footnote{See Pagano (1993) and Subrahmanyam and Titman (1999) for the microfoundations of these mechanisms.}

Second, with a larger number of similar companies around, investors can better distinguish between the management-specific and the sector-specific factors behind firm performance and, thereby, implement more effective management control systems.\footnote{The monitoring role of market investors is analyzed by Holmstrom and Tirole (1993). Acemoglu and Zilibotti (1999) stress that relative performance evaluations may improve managerial incentives.}

Finally, with a larger number of similar IPOs, investment banks can take advantage of scale economies and experience gains in information processing and in price setting.\footnote{Benveniste et al. (2001) offer evidence in this respect.}

Formally, we capture the presence of liquidity externalities by assuming that the cost $F$ is a decreasing function, $F(p)$, of the mass of publicly-traded non-mature successful firms, $p$. To simplify the discussion, we further assume that $F(p) = 1$ and $F(\, \circ M \rightarrow ) > F_0$.

Following similar steps to those that led to equation (24), we obtain that the steady state value of $p$ is given by

$$p(\mu; f) = \frac{\frac{-1}{\mu} \cdot \mu q(\mu) M}{\mu + 1 + \frac{-1}{\mu} (1 - f) \cdot \mu q(\mu)}$$

which is increasing in both $\mu$ and $f$. For a given going public decision $f$, equation (19) and the condition $F(p(\mu; f)) = F$ \footnote{Equation (32) characterize the candidate steady state allocations in the $\mu, f$ space. These allocations are indeed an equilibrium if $f$ is fixed according to Proposition 1.}

Two remarks can be made. First, there may be multiple equilibria. In particular, there always exists an equilibrium in which the non-mature successful firms do not go public until maturity ($f = 0$) since, if this is the case, we have $p = 0$, the cost $F$ goes to infinity, and going public only at maturity is indeed privately optimal. However,
there may also be an equilibrium in which the non-mature successful .rms go public early \((f = 1)\): when they do so, \(p\) is large, the externality makes \(F\) low, and going public early becomes indeed privately optimal.\(^{33}\) The two resulting equilibria are Pareto-ranked: welfare is larger in the equilibrium with \(f = 1\). If the economy is stuck in the equilibrium with \(f = 0\), reducing the private cost of going public might help unblock the situation, lead to the equilibrium with \(f = 1\), and improve welfare.\(^{34}\)

Second, some form of government support to IPOs may be desirable not just as a means to ensure that agents coordinate in an equilibrium with \(f = 1\); but also to bring such an equilibrium into existence. Differently from the benchmark model, the private value of going public for a non-mature successful .rm is below its social value since the .rm does not internalize the positive effect of its decision on the cost of going public of other .rms. So, even without a coordination problem, .rms tend to go public later than what would be socially efficient.

8 Conclusions

We have analyzed the implications of .rms' going public decisions for business creation and growth. In our model, young .rms face a trade-off between the liquidity, diversification, and recycling gains of going public and the costs due to getting listed before reaching maturity. The earlier .rms go public, the quicker the informed capital which they use gets recycled for the financing of new .rms. This mechanism creates a linkage between the factors that determine .rms' going public decisions (the costs of going public, the liquidity of the stock market, and the value of informed capital) and aggregate variables such as the rate of business creation, the size of the stock market, and, eventually, the rate of economic growth.

\(^{33}\)Formally, this is the case if under \(f = 1\); the value of \(\mu\) that solves (19) and (32) satisfies \(F(p(\mu;1)) < F_2(\mu)\):

\(^{34}\)An explicit evaluation of the gains from moving from an equilibrium with \(f = 0\) to one with \(f = 1\) is, however, complicate since, opposite to the baseline version of the model, the endogenous variables \(\mu\) and \(f\) would be functions of the state variables of the system, \(u_t; d_t;\) and \(p_t\):
Consistent with the U.S. experience during the IT revolution, we predict that, when the profitability of business opportunities increases, firms tend to go public earlier so as to more quickly recycle their (more valuable) informed capital. This can explain the recently observed reduction in the average age at which firms go public in the U.S. and, consequently, the rise in the number of IPOs and the consolidation of Nasdaq as a market for young firms.\(^{35}\)

Our analysis also uncovers various factors which might lie behind cross-country differences in going public patterns and, according to our model, be the cause of deeper differences in economic performance.\(^{36}\) First, several legal and financial institutions may produce significant differences in the cost of the restructuring that young firms must undertake before going public. The rule of law, the efficiency of the judicial system, the statutory protection of minority shareholders, and the existence of listing and accounting standards which suit the peculiarities of young firms can reduce the costs of guaranteeing managerial compliance in the absence of informed capital.\(^{37}\) In addition, going public at an early stage may entail the disclosure of information that competitors and tax authorities can use to the detriment of the firm, especially if the property rights of young firms are badly protected (say, because patent law is poorly enforced) and the tax system is little effective in levying taxes on privately held firms.\(^{38}\)

\(^{35}\)Jovanovic and Rosseau (2001) document the shortening in the average time to the IPO and show that this phenomenon is typical of technological revolutions led by new firms. The data in Fama and French (2001) also indicates that the firms behind the rise in new listings registered in the U.S. after 1977 are younger (in terms of growth opportunities, earnings, and dividends) than their predecessors.

\(^{36}\)Over the last two decades the US economy has outperformed the European economy in terms of both the adoption of new technologies (OECD, 1994) and employment growth (Acemoglu, 2001). Our model establishes a linkage between these facts and the evidence found by Pagano et al. (1998), Planell (1995), and Rydqvist and Hogholm (1995) that the typical newly listed company is older and larger in Italy, Spain, and Sweden, respectively, than in the US.

\(^{37}\)La Porta et al. (1997) document the positive effect of the rule of law, the efficiency of the judicial system, and the statutory protection of minority shareholders on the number of IPOs per capita and on stock market capitalization. Dyck and Zingales (2002) study the effect of these institutions on managerial control rents.

\(^{38}\)According to Pagano et al. (1998), going public increases the annual tax bill of Italian companies.
The lack of entrepreneurial entry induced by an inadequate balance of bargaining power in the market for informed capital provides an alternative source of cross-country variation in the going public decision of young companies. We have shown that, if monitors appropriate too much of the surplus of new firms, entrepreneurs' incentives to create them and, thus, the value of informed capital get depressed; in such a situation, there are few new firms and they go public late.

Lastly, IPO activity can slacken if technological and liquidity externalities leave the economy trapped in equilibria where either the low growth rate or the large costs of going public depress entrepreneurs' incentives to develop their businesses and to lead them public early. In situations like these, policies directed to encourage IPOs may induce more favorable dynamics and increase welfare.

To conclude, a brief comment on our modelling of the supply of informed capital. Our results are robust to the introduction of a positively sloped supply of informed capital. In such a case, fundamentals that affect the profitability of an entrepreneur-monitor relationship would affect not only entrepreneurs' incentives to develop their projects but also monitors' incentives to be active. Insofar as the induced supply of informed capital does not turn out to be perfectly elastic (and any heterogeneous cost for monitors to become active would ensure this), factors that favor recycling will continue to stimulate business creation. The same would happen in a model where informed capital could be accumulated, provided that the economy evolves along a balanced growth path where the demand for informed capital grows at the same rate as its supply and, hence, the value of informed capital remains positive.  

\[39\]

\footnote{This would be like in Sussman and Zeira (1995), where banks increase their lending capacity as the economy grows.}
Appendix

Impossibility of going public without restructuring

To rule out the possibility that non-mature firms go public without previously restructuring their management control mechanisms, it suffices to guarantee that, even if a firm were known to be successful, its direct financing by investors would not be feasible. Consider a non-mature successful firm in which the entrepreneur has a share \( \alpha \) in the value of the firm at its maturity, \( Y \); and investors have the remaining share \( 1 - \alpha \). The entrepreneur's value from running the firm, \( \frac{1}{2} \), is then given by

\[
\frac{1}{2} = \max \left\{ b \left( \frac{1}{2} \alpha Y + \frac{1}{2} D \right), \frac{1}{2} \alpha \right\},
\]

where \( b \) and \( \frac{1}{2} \alpha Y \) are the instantaneous expected returns from shirking and complying, respectively. The latter exceeds the former and, thus, complying is incentive compatible if and only if

\[
\alpha \frac{1 + b \alpha}{1 Y} \leq \frac{1}{2} \frac{1 + r}{1 Y},
\]

(33)

The entrepreneur will be able to finance his firm if and only if the value of the investors' share, \( D \), exceeds one. If (33) holds, \( D \) solves

\[
rD = 1 \left[ (1 - \alpha) Y + \frac{1}{2} D \right];
\]

so having \( D > 1 \) requires

\[
\frac{1}{2} > \frac{1 + \frac{1}{2} Y}{1 Y};
\]

(34)

Clearly, if \( b \) is sufficiently large, (33) and (34) are incompatible. In particular, if

\[
\frac{b}{\frac{1}{2}} > \frac{1 + \frac{1}{2} Y}{1 + \frac{1}{2} Y} \frac{1 + r}{1 + \frac{1}{2} Y},
\]

(35)

direct financing by the investors is not feasible.

Results on efficiency

In this section we prove our results on efficiency. We start obtaining the dynamics of the costate variables \( W_1 \) and \( W_2 \) that appear in (25). Time indices are omitted, for brevity. Partially deriving (25) with respect to \( m_1 \) and \( m_2 \) we obtain

\[
\frac{1}{2} W_1 = R(\beta) + \left[ \mu + \mu q(\mu) \right] + W_{11} m_1 + W_{12} m_2 + W_1 \frac{m_1}{\theta m_1} + W_2 \frac{m_2}{\theta m_2};
\]

33
\[ 3W_2 = Y + [\mu + \mu q(\mu)] + W_{12}m_1 + W_{22}m_2 + W_1 \frac{\partial m_1}{\partial \mu} + W_2 \frac{\partial m_2}{\partial \mu}; \]

We can now substitute \( \frac{dW_1}{dt} \) for \( W_{11}m_1 + W_{12}m_2 \) and \( \frac{dW_2}{dt} \) for \( W_{12}m_1 + W_{22}m_2 \); and use (22) and (23) to obtain the partial derivatives of \( m_1 \) and \( m_2 \): Solving for \( W_1 \) and \( W_2 \) and collecting terms leads to the following linear system of differential equations in \( W_1 \) and \( W_2 \):

\[
\begin{align*}
    W_1 &= [\mu + \frac{1}{2} + \mu q(\mu)] W_1 \frac{(1 + f)}{1 + \frac{1}{2} [\mu + \mu q(\mu)]} \left[ \mu + \mu q(\mu) \right]; \\
    W_2 &= \mu q(\mu) W_1 + (1 + \frac{1}{2} W_2) Y + [\mu + \mu q(\mu)];
\end{align*}
\]

This system is globally unstable so \( W_1 \) and \( W_2 \) are two jump variables that must satisfy the conditions \( W_1 = W_2 = 0 \) at every point in time: Using (36) and (37) this implies

\[
\begin{align*}
    W_1 &= \frac{\omega (1 + f) [\mu + \mu q(\mu)] + (1 + \frac{1}{2} \mu q(\mu)] + \frac{\omega (1 + f)}{1 + \frac{1}{2} [\mu + \mu q(\mu)]} \left[ \mu + \mu q(\mu) \right]}{\left[ \mu + \mu q(\mu) \right]}; \\
    W_2 &= \frac{\mu q(\mu) + \mu q(\mu)] + \mu q(\mu) \left[ \mu + \mu q(\mu) \right]}{\left[ \mu + \mu q(\mu) \right]};
\end{align*}
\]

1. Going public decision  
   We want to prove that under the equilibrium value of \( \mu \) the equilibrium going public decision \( f \) maximizes \( W \): From (25), the derivative of \( W \) with respect to \( f \) is

\[
\frac{\partial W}{\partial f} = \frac{\omega (1 + f) \left[ \mu + \mu q(\mu) \right]}{\left[ \mu + \mu q(\mu) \right]};
\]

which, from (39), has the same sign as

\[
B(\mu) = \left[ \mu + \frac{1}{2} \mu q(\mu) \right] \left[ \frac{1}{1 + \frac{1}{2} r} \right] R_2 + \mu q(\mu) \left[ \mu + \mu q(\mu) \right];
\]

This expression does not depend on \( f \) so \( W \) is maximized at \( f = 1 \) if \( B(\mu) \cdot 0 \) and at \( f = 0 \) if \( B(\mu) \cdot 0. \) We will prove that in equilibrium the sign of \( B(\mu) \) coincides with that of \( F_2(\mu) \); which, by Proposition 1, yields the result.

Notice that from (16) and (17) that, in equilibrium, we have

\[
\frac{1}{1 + \frac{1}{2} r} R_2 = F_2(\mu) \left[ 1 + \frac{1}{2} \right];
\]

There are two possible cases:
(i) $F_2(\mu) : F$: Then $\min F; F_2(\mu)g = F$ and, by (18) and (26), we have

$$R(1) = \left[+ \frac{1}{2} (1 - \mu) \right] \mu q(\mu) S(\mu) + \left[+ \frac{1}{2} \right]:$$

Then, using (19), we can write

$$R(1) = \left[+ \frac{1}{2} (1 - \mu) \right] \mu q(\mu) \frac{c}{q(\mu)} + \left[+ \frac{1}{2} \right]:$$  \hspace{1cm} (43)

With (42) and (43) we can substitute in (41) for $R(1)$ and $\frac{1}{2} (1 - \mu) R_2$; respectively, and obtain:

$$B(\mu) = \left(1 + \frac{1}{2} \right) \left[+ \frac{1}{2} \right] \mu q(\mu) [F_2(\mu) \mid F];$$

whose sign indeed coincides with that of $F_2(\mu) \mid F$.

(ii) $F_2(\mu) < F$: Then $\min F; F_2(\mu)g = F_2(\mu)$ and, by (18) and (26), we have

$$R(1) = \left[+ \frac{1}{2} (1 - \mu) \right] \mu q(\mu) S(\mu) + \left[+ \frac{1}{2} \right] + \left[+ \frac{1}{2} \right]:$$

Then, using (19), we can write

$$R(1) = \left[+ \frac{1}{2} (1 - \mu) \right] \mu q(\mu) \frac{c}{q(\mu)} + \left[+ \frac{1}{2} \right]:$$  \hspace{1cm} (44)

With (42) and (44) we can substitute in (41) for $R(1)$ and $\frac{1}{2} (1 - \mu) R_2$; respectively, and obtain:

$$B(\mu) = \left(1 + \frac{1}{2} \right) \left[+ \frac{1}{2} \right] \mu q(\mu) + \left[+ \frac{1}{2} \right]:$$

whose sign also coincides with that of $F_2(\mu) \mid F$.

2. Informed capital scarcity

We want to evaluate the effect on $W$ of marginally changing $\mu$ in a steady state equilibrium. A marginal change in $\mu$ may have a direct impact on $W$ as well as an indirect impact through $f$. However, the change in $f$ will only occur if $F_2(\mu) = F$; in which case the result in Part 1 implies $\frac{\partial W}{\partial \mu} = 0$. Hence the partial derivative $\frac{\partial W}{\partial \mu}$ suffices to evaluate the overall effect of changing $\mu$. From (25) we .nd that

$$\frac{\partial W}{\partial \mu} = \left(1 + \mu q(\mu) \right) \left(1 + M \right) \left(1 + \mu q(\mu) \right):$$  \hspace{1cm} (45)

We will .rst prove that in equilibrium

$$W_1 = 1 + \frac{c}{q(\mu)}:$$  \hspace{1cm} (46)

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To show this, start with the case where $F_2(\mu) > F$; so $f = 1$: Then (46) can be immediately obtained by evaluating $W_1$ using (38) and (43). In the case where $F_2(\mu) < F$; we have $\min F; F_2(\mu) g = F$ and $f = 0$: Equations (18) and (26) imply then that

$$R(0) = [\xi + \frac{1}{2} + (1 - \bar{\mu}) \mu q(\mu)] S(\mu) + (\xi + \frac{1}{2} i - \mu g(\mu) [R_2 + F_2(\mu)];$$

We can use (17) to substitute for $F_2(\mu)$ and (19) to substitute for $S(\mu)$: Plugging the resulting expression in (38) so as to evaluate $W_1$ at $f = 0$ yields, after some algebra, (46).

Finally, we can substitute (46) into (45) to obtain that in a steady state ($m_1 = m_2 = 0$):

$$\frac{\partial W}{\partial \mu} = \frac{c}{t^2} (M_i m_1 m_2)[\xi (\mu) - \bar{\mu}];$$

where the sign is that of $\xi (\mu) - \bar{\mu}$: Hence, if $\xi (\mu) > \bar{\mu}$ welfare cannot increase by marginally distorting $\mu$; however, if $\xi (\mu) > \bar{\mu}$, increasing $\mu$ will increase $W$; while if $\xi (\mu) < \bar{\mu}$; decreasing $\mu$ will increase $W$.

3. Bargaining power, welfare, and the value of informed capital

We want to show that in a steady state the sign of the effects of a marginal change in $\bar{\mu}$ on both $W$ and $V_0$ is given by $\xi (\mu) - \bar{\mu}$: So changes in $\bar{\mu}$ that increase (decrease) $W$ also increase (decrease) $V_0$:

(i) Effect on $W$ A marginal change in $\bar{\mu}$ may impact $W$ through $\mu$ as well as through $f$. However, a change in $f$ will only occur if $F_2(\mu) = F$; in which case the result in Part 1 implies $\partial W / \partial f = 0$. Hence only the exact effect matters. The continuity of (18) and (19) in $\mu$ and $\bar{\mu}$ implies that $\mu$ varies continuously with $\bar{\mu}$: If $F_2(\mu) \neq F$; a marginal change in $\bar{\mu}$ does not change $f$ so we have

$$\frac{dW}{d\bar{\mu}} = \frac{\partial W}{\partial \mu} \frac{d\mu}{d\bar{\mu}};$$

In Part 2 we have already shown that in a steady state equilibrium $\partial W / \partial \mu$ has the same sign as $\xi (\mu) - \bar{\mu}$: Moreover, differentiating equations (18) and (19) with respect to $\mu$ and $\bar{\mu}$; one can check that

$$\frac{d\mu}{d\bar{\mu}} = \frac{[-(\xi + \frac{1}{2} \mu q(\mu)] \mu q(\mu) g}{[\xi + \frac{1}{2} \mu q(\mu)] \mu g(\mu) g} > 0;$$

when $F_2(\mu) > F$; and

$$\frac{d\mu}{d\bar{\mu}} = \frac{[-(\bar{\mu} + \frac{1}{2} \mu q(\mu)] \mu q(\mu) g}{[\bar{\mu} + \frac{1}{2} \mu q(\mu)] \mu g(\mu) g} > 0;$$

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when $F_2(\mu) < F$: Hence in both cases the sign of $dW = d^{-}$ is that of $\dot{\gamma}(\mu) i^{-}$. This also implies that, even at the non-differentiability point where $F_2(\mu) = F$, $W$ is increasing in $^{-}$ if $\dot{\gamma}(\mu) > ^{-}$ and decreasing if $\dot{\gamma}(\mu) < ^{-}$.

(ii) Effect on $V_0$. First notice that the continuity of $\mu$ in $^{-}$ together with (16) implies that $V_0$ varies continuously with $^{-}$: Yet there is a non-differentiability point at $F_2(\mu) = F$: At any other point, the effect on $V_0$ of a change in $^{-}$ can be measured by differentiating (16):

$$\frac{dV_0}{d^{-}} = i \frac{c\mu}{\frac{1}{2}^2} + \frac{(1 i^{-}) c}{\frac{1}{2}} \frac{d\mu}{d^{-}};$$

When $F_2(\mu) > F$; (49) implies

$$\frac{dV_0}{d^{-}} = \frac{c\mu(1 + \frac{1}{2})}{(1 + \frac{1}{2}[1 i^{-}\dot{\mu}]) + (1 i^{-}) \mu q(\mu)}$$

whereas when $F_2(\mu) < F$; (50) implies

$$\frac{dV_0}{d^{-}} = \frac{c\mu(1 + \frac{1}{2})}{(1 + \frac{1}{2}[1 i^{-}\dot{\mu}]) + (1 i^{-}) \mu q(\mu)} \frac{\dot{\gamma}(\mu) i^{-}}{\frac{1}{2}^2};$$

Hence in both cases the sign of $dV_0 = d^{-}$ coincides with that of $\dot{\gamma}(\mu) i^{-}$. This also implies that, even at the non-differentiability point where $F_2(\mu) = F$, $V_0$ is increasing in $^{-}$ if $\dot{\gamma}(\mu) > ^{-}$ and decreasing if $\dot{\gamma}(\mu) < ^{-}$.
References


