Bank Capital Forbearance*

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Abstract

We analyze the strategic interaction between undercapitalized banks and a supervisor who may intervene by preventive recapitalization. Supervisory forbearance emerges because of a commitment problem, reinforced by fiscal costs and constrained capacity. Private incentives to comply are lower when supervisors have lower credibility, especially for highly levered banks. Less credible supervisors (facing higher cost of intervention) end up intervening more banks, yet producing higher forbearance and systemic costs of bank distress. Importantly, when public intervention capacity is constrained, private recapitalization decisions become strategic complements, leading to equilibria with extremely high forbearance and high systemic costs of bank failure.

Keywords: bank supervision; bank recapitalization; forbearance.

JEL Classification: G21, G28

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1 Introduction

Supervisors have the critical task of assessing bank resilience to ensure timely corrective action when required. This process is fraught with challenges. Even when supervisors identify a capital shortfall, they cannot force shareholders to contribute additional capital. Private incentives to recapitalize are particularly poor in overleveraged banks since compliance implies a net transfer to creditors. Since banks’ default imposes large costs on the economy, addressing capital shortfalls requires a public backup. Yet injecting public funds in private banks as a precautionary measure is unpopular and fiscally costly, especially in difficult economic circumstances. We introduce these elements in a conceptual framework able to address two key questions: How does supervisory credibility affect bank private recapitalization choices? How do cost of intervention and limits on public intervention capacity affect supervisory choices?

Previous work on supervisory incentives explained an inadequate or delayed intervention as reflecting a desire to hide weak supervisory skills or bad decisions (Boot and Thakor, 1993; Morrison and White, 2013). Supervisory forbearance may also result from the desire to avoid any disclosure that may trigger a panic (Walther and White, 2016; Chan and van Wijnbergen, 2017). We focus on supervisory forbearance driven by limited credibility of the threat of intervention. The prospect of insufficient corrective action undermines banks’ incentives to privately recapitalize. To counteract weak private incentives, a supervisor may end up choosing a higher level of public intervention than under stronger supervisory commitment, and suffer higher fiscal and systemic costs of bank distress. When this mechanism is combined with a hard budget or capacity constraint on public intervention (as during a sovereign debt crisis), banks’ private recapitalization decisions become strategic complements and can lead to self-fulfilling equilibrium with very high forbearance and severe systemic costs.

One historical example of forbearance causing deepened distress by delaying intervention is the S&L bank default wave in the 1980s. Many small banks undermined by rising interest rates were allowed to keep gambling on their mortgage loan portfolios, leading to defaults and massive losses for the US deposit insurance system (Degennaro and Thompson, 1996). Some analysts also saw an instance of forbearance in the first stress test run by the Euro-
pean Banking Authority (EBA) in 2010. The stress test run by the US authorities in 2009, where many banks were challenged to significantly recapitalize under the threat of partial nationalization, raised the opposite perception. Our interpretation is that a key difference was the existence of a credible commitment to intervene at the necessary scale in case of private inaction. The experiences witnessed in Iceland as well as Greece, Ireland, Portugal and Spain during the Great Recession and perhaps, more recently, Italy too suggest that a prompt preventive intervention may at times exceed available fiscal capacity. In such circumstances supervisors may be pressed to allow some risky banks to operate with insufficient capital.

Formally, we consider a game between banks and a supervisor with discretionary powers to publicly recapitalize banks in trouble. The game starts when, in the context of a crisis, both banks and the supervisor learn the number of banks in distress. The supervisor interacts with shareholders in banks subject to debt overhang by first requiring a private recapitalization. The recapitalization solves the debt overhang problem and forces shareholders to give up their limited liability put option (as advocated by Bhattacharya and Nyborg (2011)). Noncompliant banks may be recapitalized publicly. As bank shareholders value control, they prefer not to be intervened by a forced public recapitalization. However, they are aware that the supervisor’s decision will balance the systemic costs of bank default against the excess cost of public funds, as well as some political and reputational costs, so eventually may not be willing to intervene all noncompliant banks. Thus, for some banks the supervisor may choose to do nothing (capital forbearance).

Capital forbearance implies that some banks remain undercapitalized and shareholders retain control at times of poor incentives, which causes some expected social losses. In the baseline model, distressed banks’ individual decisions on whether to privately recapitalize are

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1 At that point, the EBA estimated that “2.5 billion [euro] would have corrected the capital shortfall by the banks that failed the test, at a time when market estimates suggested 300 billion euro, an amount that proved much more accurate” (Onando and Resti, 2011). On a more anecdotal basis, Dexia Bank was reported to have a capital ratio of over 10% in July 2011, and collapsed three months later.

2 While forbearance is more typically associated with the supervisory management of distressed banks in countries with weak fiscal positions, empirical evidence in Bian et al. (2017) points to postponed bank intervention ahead of elections also in Germany.

3 For simplicity we consider a situation without asymmetries of information in which all troubled banks are identical. We also abstract from the role of market discipline by assuming that bank is funded with fully insured deposits.
strategic substitutes, since when more banks restore their solvency there is less pressure on the supervisor to intervene to limit the potential losses. With fewer public recapitalizations, there is a higher chance that an undercapitalized bank benefits from supervisory forbearance. This reduces each troubled bank’s marginal incentive to privately recapitalize.

The analysis of this equilibrium and several extensions of the model yields predictions consistent with the evidence on the economics and political economy of supervisory interventions and systemic crises. First, we show that the supervisor facing higher political and reputational costs of intervention paradoxically ends up intervening more banks in equilibrium. The intuition is that a higher supervisory cost reduces the intervention threat for each level of private recapitalization. Banks react to this by recapitalizing less privately. Given a low level of compliance, the supervisor is compelled to publicly recapitalize more banks in order to limit the systemic costs of bank default. Yet the increase in preventive intervention does not fully compensate for private non-compliance, thus resulting in greater forbearance and a higher systemic loss. Empirically, it has been shown that political concerns play an important role in delaying government interventions (Brown and Dinç, 2005; Liu and Ngo, 2104; Bian et al., 2017). Relatedly, more captured supervisors encounter higher failure rates among the banks in their jurisdiction (Agarwal et al., 2014).

Second, similar effects occur when the supervisor faces a higher cost of public funds: its threat of intervention is also softer, promoting weaker bank compliance. Despite the rise in publicly intervened banks, more banks are left undercapitalized and their failure causes larger systemic costs. This is consistent with the fact that countries with more fiscally constrained governments face more severe banking crises (Reinhart and Rogoff, 2011).

Third, a rise in banks’ leverage has two opposite effects on the equilibrium level of forbearance. On the one hand, leverage directly increases the cost of a private recapitalization, reducing banks’ incentives to comply. On the other, it increases the severity of the systemic costs of supervisory inaction and hence the supervisor’s propensity to intervene, which in turn pushes banks towards compliance. We find that the indirect effect dominates when leverage is low. Once leverage exceeds a certain level, private inaction grows with it. Eventually, very high leverage increases equilibrium forbearance, thus magnifying the systemic costs due to bank failure. This is consistent with the evidence that advanced economies with
a higher growth in bank liabilities (relative to GDP) tend to experience more severe banking crises (Schularick and Taylor, 2012).  

Finally, in an important extension of the model, we analyze how the interaction between private and public recapitalization decisions changes if the supervisor faces a hard constraint to its recapitalization capacity. A fiscal budget constraint may arise when banking distress is combined with a sovereign crisis. Such a constraint produces a too-many-to-recapitalize context where banks’ private recapitalization decisions become strategic complements (similarly to banks’ risk-taking decisions in Acharya and Yorulmazer 2007). Once the constraint binds, a supervisor cannot react to a rise in the number of non-compliant banks by recapitalizing more of them and supervisory forbearance rises quickly, further discouraging private recapitalizations. This gives rise to the possibility of a second high-forbearance equilibrium in which banks coordinate in low recapitalization strategies, producing an exhaustion of the supervisor’s intervention capacity, and a deeper systemic crisis. This extension provides a novel supervisory-based channel for a critical sovereign-bank nexus even without deposit insurance nor bank holdings of sovereign debt.

From a policy perspective, the highlighted results have relevant implications. The commitment problem affecting the credibility of the supervisor’s intervention threat might be addressed with a proper institutional design. The effect of leverage on forbearance implies that ex ante capital regulation can serve as a partial substitute for ex post commitment to intervene troubled banks. Finally, our results on the dramatic consequences potentially associated with limits to intervention capacity provide a rationale for the preventive value of having a strong public backstop for bank resolution. This reinforces the credibility of the public recapitalization threat, with beneficial effects on private compliance.

**Related literature.** Our paper contributes to the literature on supervisory interventions on banks. The earlier studies focus on closure rules for banks in distress as a tool to reduce deposit insurance liabilities (Acharya and Dreyfus, 1989; Allen and Saunders, 1993; Fries et al., 1997). In these settings private willingness to supply capital is assumed to be

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4See also de Mooij et al. (2013) for direct evidence on the effect of bank leverage on the odds of a systemic crisis.

5Elements of such design may include having an independent supervisor, with a clear mandate focused on avoiding future systemic costs, isolated from political interference and yet with financial capacity to intervene on many banks.
exogenous and the supervisor decides on bank closure, which in practice is a rare form of intervention except for very small banks. Our work focuses on recapitalization at an interim stage of bank distress, looking at the interaction between private recapitalization decisions and the prospect of supervisory intervention.

In our model, poor commitment induces an insufficient degree of private recapitalization relative to what is socially optimal. The issue of time inconsistent policy, first analyzed by Kydland and Prescott (1977) has been studied extensively in the literature on monetary policy (Barro and Gordon, 1983). As in our context, the inability to commit leads to an inefficient private response that may be quite costly to reverse. In the banking context, time inconsistency problems regarding policy interventions have been identified as a root cause of bank runs and delayed responses in the analysis of suspension of convertibility (Ennis and Keister, 2010) and application of bail-in provisions (Keister and Mitkov, 2017).

Our setup incorporates a realistic supervisory trade-off between early intervention costs (including deadweight losses from taxation) and the spillover effects associated with future bank defaults (Bhattacharya et al., 1998). Philippon and Schnabl (2013) consider this trade-off from a normative perspective: characterizing policies that optimally deal with informational asymmetries regarding banks’ exposure to debt overhang. Similarly to our setup, the regulator takes into account the deadweight losses of taxation and the negative externality of default. When an undercapitalized bank forgoes profitable lending, it worsens other banks’ debt overhang via increased borrowers’ defaults. On the other hand, government intervention generates free-riding as well as opportunistic participation. Optimal interventions feature public equity injections with voluntary participation, which leads to self-selection of the weaker banks into the program. We abstract from informational asymmetries and focus on the interaction between banks and the supervisor in the absence of commitment to an optimal intervention rule.

Shapiro and Skeie (2015) also study government interventions in the presence of fiscal costs, in a sequential bailout problem. The supervisor’s cost of injecting capital into a bank is private information, so an earlier bailout decision reveals information about future choices. Since there is a trade-off between the cost of a run and the effect of a bailout on moral hazard, a low cost supervisor may choose not to bail out a bad bank in order to signal
toughness and reduce subsequent risk-taking. In our model, even under full information regarding supervisory capacity, the prospect of public intervention affects banks’ decisions at an earlier stage.

Colliard and Gromb (2017) studies how a single bank renegotiates its debt under asymmetric information, in the shadow of a potential government intervention and show how it may affect the delay in negotiations. In some circumstances, making the government commit not to interfere speeds up the workout process, improving efficiency.

Considering a situation of diffused bank distress establishes a connection between our paper and the literature on bailout externalities (Perotti, 2002; Perotti and Suarez, 2002; Acharya and Yorulmazer, 2007; Fahri and Tirole, 2012). In Acharya and Yorulmazer (2007), the ex-post choice of the supervisor to bail out failing banks is affected by a too-many-to-fail problem which gives rise to strategic complementarities in banks’ risk decisions: it encourages banks to be more correlated so as to fail together and increase the chance of benefiting from a bailout. In Fahri and Tirole (2012), an accommodating interest rate policy also generates strategic complementarities in banks’ risk choices. The too-many-to-fail result resembles conceptually our too-many-to-recapitalize outcome in the presence of a hard limit to the supervisor’s public recapitalization capacity. A public commitment to a resolution policy that rewards solvent banks (Perotti and Suarez, 2002) or punishes weak banks (Walther and White, 2016) may positively affect ex ante risk incentives. In the current setup, the lack of commitment to intervene on undercapitalized banks is a source of excessively low private recapitalization, high capital forbearance, and high systemic costs.

**Outline of the paper.** The rest of the paper is organized as follows. Section 2 describes the baseline model. Section 3 characterizes the equilibrium of the baseline game between weak banks and the supervisor, discusses its comparative statics, and elaborates on the predictions regarding the effect of bank leverage on the equilibrium outcomes. Section 4 develops the extension in which the supervisor faces a too-many-to-recapitalize problem. Section 5 provides a discussion of the welfare implications of our results and the inefficiency derived from the lack of supervisory commitment to intervene. Section 6 concludes the paper with a summary of the main empirical and policy implications of our analysis. All proofs are in the Appendix.
2 The model

We consider a game played between a bank supervisor and some banks damaged by a solvency shock. There are three relevant dates \( t = 0, 1, 2 \), and all agents are risk neutral.

2.1 Banks

At \( t = 0 \) a mass \( \phi \) of banks are damaged by a solvency shock. The banks are owned and managed in the interest of their initial shareholders, who discount future payoffs at a rate normalized to zero. Damaged banks, unless they are recapitalized at \( t = 1 \), face a significant probability of failing at \( t = 2 \), causing some systemic costs that they do not internalize.

Damaged banks can prevent their failure through a recapitalization at \( t = 1 \). A *private recapitalization* has a net cost \( c \) to the initial owners (relative to the non-recapitalization benchmark). This cost reflects an implicit positive transfer to preexisting debtholders, since levered banks give up on a part of their “Merton put” on risky bank assets.\(^6\) A *public recapitalization* results from a supervisory intervention and is assumed to involve the dilution of the pre-existing equity to the extent needed for it to be as financially costly to the initial shareholders as a private recapitalization. In addition, a public recapitalization is assumed to involve a loss of control for them and, hence, an overall cost \( c + \Delta \), where \( \Delta > 0 \) denotes the underlying control rents (as, e.g., in Grossman and Hart, 1988). Therefore, a public recapitalization can be interpreted as an instance of punishment to the initial owners for their refusal to comply with the private recapitalization request.

2.2 The supervisor

At \( t = 0 \), aware of the solvency shock, the supervisor identifies the damaged banks through some supervisory review process or stress test exercise. To prevent the failure of these banks, the supervisor can ask, but not force, each of them to privately recapitalize.

At \( t = 1 \) each damaged bank can choose to comply \( (r = 1) \) or not \( (r = 0) \) with the supervisory request. We denote as \( m \) the number of damaged banks that comply. Compliant

\(^6\)Shareholders’ reluctance to recapitalize a levered firm has its roots in long recognized conflicts of interest between shareholders and debtholders (Jensen and Meckling, 1976; Myers, 1977); see Admati et al. (2018) for an interesting restatement.
banks cause a zero net cost to the supervisor.

If a damaged bank refuses to privately recapitalize, the supervisor can still prevent its failure in the bad state by undertaking a public recapitalization. A public recapitalization implies some net cost to the supervisor equal to $\tau_0 + (\tau_1/2)n$ per intervened bank, where $n$ is the overall mass of punished banks, and $\tau_0 > 0$ and $\tau_1 \geq 0$ are parameters. This cost captures both the excess cost of public funding (e.g., due to the need to increase distortionary taxation or issue expensive government debt at $t = 1$) and the political and reputational costs associated with the intervention (including personal losses related to incidence of supervisory capture). Thus, the supervisor’s overall early intervention cost associated with a mass $n$ of banks intervened at $t = 1$ is $\tau_0 n + (\tau_1/2)n^2$.

The alternative course of action on a damaged bank that refuses to privately recapitalize is forbearance, that is, leaving the bank likely to fail at $t = 2$ and its owners unpunished at $t = 1$. Under this alternative the supervisor suffers a cost. The expected cost of bank failure equals to $(\lambda/2)(\phi - m - n)$ per forborne bank, where $\lambda > 0$ is a parameter. Thus, forbearance produces overall systemic costs $(\lambda/2)(\phi - m - n)^2$, whose convexity reflects that the marginal systemic cost associated with bank failure at $t = 2$ is increasing in the mass of failing banks.$^7$

The supervisor chooses the measure $n \in [0, \phi - m]$ of banks to intervene at $t = 1$ so as to minimize the sum of the expected early intervention cost and the expected systemic costs of its decision.

### 2.3 Sequence of events

The sequence of events is the following:

- At $t = 0$, the supervisor identifies the mass $\phi$ of damaged banks and calls them to recapitalize.

$^7$We set up the cost of forbearance similarly to the cost of early intervention but abstracting from the linear component (the term in $\tau_0$ in such cost). It is easy to show that as long as the linear component in the cost of forbearance were lower than $\tau_0$, our results would remain unchanged. Otherwise, even a very small mass of failing banks at $t = 2$ would produce systemic costs exceeding the cost of a public recapitalization at $t = 1$, leading the supervisor to recapitalize all undercapitalized banks publicly irrespectively of their mass. In such regime, the implicit recapitalization threat is so strong that an equilibrium without forbearance can always be sustained.
At $t = 1$, there are two stages:

- Stage 1. Damaged banks simultaneously decide whether to comply ($r = 1$) or not ($r = 0$); the resulting measure of compliant banks is $m$.

- Stage 2. The supervisor intervenes a measure $n$ of the mass $\phi - m$ of non-compliant banks; a measure $\phi - m - n$ of banks are forborne.

At $t = 2$, aggregate uncertainty realizes; in some bad state the forborne banks fail, causing systemic costs.

The following table summarizes the variables and payoffs relevant for the damaged banks and the supervisor in the sequential game played at $t = 1$.

<table>
<thead>
<tr>
<th>Bank-level outcome</th>
<th>Affected mass of banks</th>
<th>Per bank cost to bank owners</th>
<th>Overall cost to supervisor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comply, no intervention</td>
<td>$m$</td>
<td>$c$</td>
<td>0</td>
</tr>
<tr>
<td>Not comply, intervention</td>
<td>$n$</td>
<td>$c + \Delta$</td>
<td>$\tau_0 n + (\tau_1/2)n^2$</td>
</tr>
<tr>
<td>Not comply, forbearance</td>
<td>$\phi - m - n$</td>
<td>0</td>
<td>$(\lambda/2)(\phi - m - n)^2$</td>
</tr>
</tbody>
</table>

To focus the analysis on the interesting cases in which supervisory interventions emerge in equilibrium we adopt the following assumption:

**Assumption 1** $\lambda \phi > \tau_0$.

As it will become clear below, Assumption 1 is the necessary and sufficient condition for the supervisor to be willing to intervene a positive mass of non-compliant banks in the polar case in which no damaged bank complies.

### 2.4 No active supervision benchmark

It is trivial to see that in the absence of a supervisor being able to act on damaged banks, no damaged bank would privately recapitalize. In other words, damaged banks would simply gamble for survival, remaining exposed to failure at $t = 2$, and bearing a cost $0$ (rather than the cost $c$ of a private recapitalization). The supervisor’s cost in this situation ($m = n = 0$) would be $(\lambda/2)\phi^2$. This benchmark provides a lower bound to the cost incurred by banks and an upper bound to the cost incurred by the supervisor in the full game.
3 Analysis of the supervisory game

This section first characterizes the equilibrium of the sequential game played between the damaged banks and the supervisor. Then we discuss the comparative statics of such equilibrium and the way in which the results can be extended to investigate the interaction between banks’ leverage and forbearance.

3.1 Equilibrium

The game played by the damaged banks and the supervisor at \( t = 1 \) can be solved by backward induction.

In the second stage, the supervisor decides on the mass of banks to publicly recapitalize after having observed some mass of privately recapitalizing banks \( m \) in the first stage. The supervisor’s reaction function is given by

\[
N(m) = \arg \min_{0 \leq n \leq \phi - m} \tau_0 n + (\tau_1/2)n^2 + (\lambda/2)(\phi - m - n)^2.
\] (1)

Solving the first order condition of the implied minimization and taking into account that \( N(m) \) must be non-negative, the supervisor’s reaction function can be written as

\[
N(m) = \max \left\{ \frac{\lambda(\phi - m) - \tau_0}{\lambda + \tau_1}, 0 \right\},
\] (2)

which is piece-wise linear as depicted in Figure 1. Specifically, \( N(m) \) is positive and lower than \( \phi \) at \( m = 0 \) and decreasing until it reaches a value of zero at \( m = (\lambda \phi - \tau_0)/\lambda < \phi \).

In the first stage of the game, banks simultaneously decide whether to comply \( (r = 1) \) or not \( (r = 0) \). Given some expectation about the value of \( m \), a damaged bank would be indifferent between the two alternatives if and only if

\[
c = \frac{N(m)}{\phi - m}(c + \Delta),
\] (3)

where the left hand side (LHS) is bank owners’ cost of compliance, \( c \), and the right hand side (RHS) is the product of the bank’s probability of being publicly recapitalized, \( N(m)/(\phi - m) \), and bank owners’ cost of such intervention, \( (c + \Delta) \). Importantly, for \( m < (\lambda \phi - \tau_0)/\lambda \), we have, using (2),

\[
d \left( \frac{N(m)}{\phi - m} \right)/dm = -\frac{\lambda \tau_0}{(\lambda + \tau_1)(\phi - m)^2} < 0,
\]
Figure 1: The supervisor’s reaction function

which means that the public recapitalization threat hanging on a marginal non-compliant bank weakens as more banks comply. This leads to the following result.

**Lemma 1.** Banks’ private recapitalization decisions are strategic substitutes over the range

\[ m < \frac{(\lambda \phi - \tau_0)}{\lambda} \]  

in which the public recapitalization threat is not zero.

In such range, if more banks recapitalize privately, the supervisor reduces the public recapitalization threat so banks face a higher chance of benefiting from supervisory forbearance. This boosts a marginal bank’s payoff from non-compliance and discourages it from privately recapitalizing, explaining the strategic substitutability between banks’ decisions.

To depict this indifference condition in the same space as the supervisor’s reaction function in Figure 1, define

\[ I(m) = \frac{c}{c+\Delta}(\phi - m) \]  

(4)

as the solution of (3) in \( n \). Then, for any \( m < \phi \), a damaged bank strictly prefers to comply for \( n > I(m) \), not to comply for \( n < I(m) \), and is indifferent between the two for \( n = I(m) \).
Figure 2: Equilibrium with privately recapitalizing banks

Figure 2 depicts $I(m)$ in conjunction with the supervisor’s reaction function $N(m)$.

Given the strategic substitutability between damaged banks’ decisions to comply, the symmetric equilibrium of the game may involve the use of mixed strategies by the banks in the first stage (and a probabilistic threat of public recapitalization on the non-complying banks in the second stage). The following proposition describes the unique symmetric subgame perfect Nash equilibrium (SPNE) of the game.

**Proposition 1.** The game between the damaged banks and the supervisor has a unique

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8 We present a case with $I(0) = c/(c + \Delta) < N(0) = (\lambda \phi - \tau_0)/(\lambda + \tau_1)$, so that the two curves intersect once on the downward sloping section of $N(m)$. Note that $I(m)$ is downward sloping, with $I(0) = c\phi/(c + \Delta) < \phi$ and $I(\phi) = 0$. Importantly, the point $(\phi, 0)$ does not belong to the indifference line because, for $n = 0$, (3) cannot hold for any $m$. So $I(m)$ and $N(m)$ do not further intersect at $(\phi, 0)$.
symmetric SPNE with \((m, n) = (m^*, n^*)\), where

\[
(m^*, n^*) = \begin{cases} 
(0, \frac{\lambda_0 - \tau_0}{\lambda + \tau_1}), & \text{if } \left(\frac{\lambda}{\lambda + \tau_1} - \frac{c}{c + \Delta}\right) \phi - \frac{\tau_0}{\lambda + \tau_1} \leq 0, \\
\left(\phi - \frac{\tau_0(c + \Delta)}{\lambda \Delta - \tau_1 c}, \frac{\tau_0 c}{\lambda \Delta - \tau_1 c}\right), & \text{if } \left(\frac{\lambda}{\lambda + \tau_1} - \frac{c}{c + \Delta}\right) \phi - \frac{\tau_0}{\lambda + \tau_1} > 0.
\end{cases}
\] (5)

Depending on the relative importance of systemic costs to private recapitalization and early intervention costs, there are two regimes, one corresponding to a situation where some damaged banks privately recapitalize (as in Figure 2) and another one in which no bank recapitalizes privately.

The emergence of the regime with positive bank compliance \((m^* > 0)\) occurs when banks face a relatively low private cost of compliance and there is a sufficiently high threat of intervention. Banks’ cost of compliance are low relative to the cost of risking an intervention whenever bank owners’ control rents \((\Delta)\) are large and the private recapitalization cost \((c)\) is small. Quite intuitively, the threat of intervention is high when the supervisor finds early intervention not so costly \((\tau_1\) is low), but the systemic costs of bank failure are sufficiently high \((\lambda\) is high) and there is a large mass of damaged banks \((\phi\) is large).\(^9\)

The following corollary summarizes the implications of Proposition 1 for the mass of forborne banks.

**Corollary 1.** The level of forbearance implied by the unique symmetric SPNE of the game between the damaged banks and the supervisor is given by

\[
\phi - m^* - n^* = \begin{cases} 
\frac{\tau_0 + \tau_1 \phi}{\lambda + \tau_1}, & \text{if } \left(\frac{\lambda}{\lambda + \tau_1} - \frac{c}{c + \Delta}\right) \phi - \frac{\tau_0}{\lambda + \tau_1} \leq 0, \\
\frac{\tau_0 \Delta}{\lambda \Delta - \tau_1 c}, & \text{if } \left(\frac{\lambda}{\lambda + \tau_1} - \frac{c}{c + \Delta}\right) \phi - \frac{\tau_0}{\lambda + \tau_1} > 0.
\end{cases}
\] (6)

In the regime where no bank complies, the mass of intervened banks grows, but less than one-by-one, with \(\phi\). The rise in the marginal cost of early intervention discourages the supervisor from offsetting the increase in the mass of damaged banks and forbearance rises.

In the regime with strictly positive bank compliance \((m^* > 0)\), the public recapitalization threat (as measured by \(N(m^*)/(\phi - m^*)\)) stays at a constant level and the rise in the mass

\(^9\)In fact, if \(\lambda/(\lambda + \tau_1) - c/(c + \Delta) \leq 0\), then the regime with \(m^* = 0\) prevails irrespectively of the value of \(\phi\). Otherwise, there is always a large enough value of \(\phi\) above which the equilibrium features \(m^* > 0\).
of damaged banks is accommodated with an equal rise in the mass of privately recapitalized banks. These outcomes are sustained on the implicit (and credible) threat that, if the mass of privately recapitalized banks grew less than one-to-one with \( \phi \), the supervisor would intervene with greater intensity and banks would strictly prefer to comply than to be exposed to the risk of intervention.

### 3.2 Comparative statics

This section discusses how changes in the various model parameters affect the equilibrium described above. For brevity, we focus on the most interesting regime in which bank compliance \( m^* \) is strictly positive.\(^{10}\) The following proposition summarizes the results regarding the impact of model parameters on public recapitalizations (\( n^* \)) and capital forbearance (\( \phi - m^* - n^* \)). Its proof provides also details on the impact of the parameters on \( n^* \) and the condition required for having an equilibrium with \( m^* > 0 \).

**Proposition 2.** *In the regime with a strictly positive mass of privately recapitalizing banks, the mass of publicly recapitalized banks and capital forbearance increase with the supervisor’s intervention costs (\( \tau_0 \) and \( \tau_1 \)) and with banks’ private recapitalization costs (\( c \)), and decrease with the systemic cost of bank failure (\( \lambda \)) and bank owners’ control rents (\( \Delta \)).*

The most intriguing of these results are the effects of the supervisor’s intervention cost parameters on the equilibrium level of supervisory intervention. Perhaps paradoxically, a supervisor whose interventions are more costly imposes a weaker recapitalization threat on damaged banks, thus discouraging them from complying, and ends up intervening more banks. However, this increase in public interventions is not enough to compensate for the lower level of bank compliance. As a result, such a supervisor exhibits higher capital forbearance.\(^{11}\)

\(^{10}\) In the regime with \( m^* = 0 \), the mass of publicly recapitalized banks and the mass of forborne banks respond exclusively to the costs faced by the supervisor, as shown in the corresponding parts of (5) and (6). The supervisor recapitalizes more (forbears less) when its early intervention cost (positively associated with parameters \( \tau_0 \) and \( \tau_1 \)) is lower, the systemic costs of leaving banks undercapitalized (\( \lambda \)) are higher, and the mass of damaged banks (\( \phi \)) is larger.

\(^{11}\) Intuitively, at the prior level of bank compliance \( m^* \), the intervention threat \( N(m^*) \) would be too low to induce compliance. Restoring banks’ indifference requires a lower level of compliance \( m^* \) so that the supervisor responds with a higher \( n^* \).
The next proposition uncovers additional results whereby the effects of intervention costs on public recapitalization and forbearance are reinforced when banks’ private recapitalization costs increase.

**Proposition 3.** In the regime with a strictly positive mass of privately recapitalizing banks, a higher private recapitalization cost $c$ reinforces the effect of the costs of early supervisory intervention on public recapitalizations and bank capital forbearance. Formally, the cross derivatives of both $n^*$ and $\phi - m^* - n^*$ with respect to either $\tau_0$ and $c$ or $\tau_1$ and $c$, are all positive.

These reinforcement effects mean that, ceteris paribus, in a situation in which the costs of supervisory intervention are higher (e.g. because of having supervisors with stronger political biases or more prone to supervisory capture), the same increase in banks’ private recapitalization cost (e.g. because of a poor legal protection of investors’ rights, a less developed market for seasoned equity offerings, or greater reluctance to give up the valuable Merton put associated with limited liability) would end up producing a higher rise in public recapitalizations and bank capital forbearance.

### 3.3 The effects of bank leverage on capital forbearance

In this subsection we extend the model to analyze the effects of bank leverage. We interpret bank leverage as a direct determinant of the size of the capital deficits $d$ that damaged banks would have to cover by recapitalizing at $t = 1$ and, if not doing it, of the shortfalls emerging at $t = 2$ if they default. According to this logic, we now make the cost of privately recapitalizing a bank not just $c$ but $cd$, directly proportional to the size of its leverage or capital deficit $d$. Likewise, we make the overall cost of publicly recapitalizing a mass $n$ of banks not just a quadratic function of $n$ but of the capital deficits $dn$ that it has to cover: $\tau_0 dn + (\tau_1/2)(dn)^2$. Finally, we make the systemic costs associated with the likely failure of undercapitalized banks at $t = 2$ not just a quadratic function of $\phi - m - n$ but of the overall shortfall $d(\phi - m - n)$ featured by those banks: $(\lambda/2)[d(\phi - m - n)]^2$.

In this extended setup, we can easily reproduce prior results regarding the equilibrium of the game played between the damaged banks and the supervisor obtaining closed-form
expressions that allow to examine the impact of leverage on its outcomes. The following proposition summarizes the results regarding public interventions and capital forbearance.

Proposition 4. In the regime with a strictly positive mass of privately recapitalizing banks, the mass of publicly recapitalized banks increases with bank leverage, while capital forbearance is U-shaped related to it, with a minimum at \( \bar{d} = \lambda \Delta / (2 \tau_1 c) \).

By increasing the cost of a private recapitalization, leverage \( d \) reduces banks’ incentives to comply. This is a direct effect of leverage on bank compliance. At the same time, leverage also increases the systemic costs of leaving banks undercapitalized and, thus, increases the supervisor’s propensity to intervene. As a result, public recapitalizations \( n^* \) rise. This produces an indirect positive effect on banks’ compliance, so the overall effect on bank compliance \( m^* \) is, at first sight, ambiguous. In the proof of the proposition we show that the direct (indirect) effect dominates when \( d \) is above (below) some threshold, producing an inverted U-shaped relationship between \( d \) and \( m^* \). This pattern makes capital forbearance first decreasing and then increasing with leverage.

Quite intuitively, the threshold \( \bar{d} \) above which bank leverage increases forbearance is higher whenever bank owners’ cost of a public intervention (\( \Delta \)) and the systemic costs of bank failure (\( \lambda \)) are larger, since those encourage directly or indirectly bank compliance. By the same token, the threshold is lower whenever banks’ cost of a private recapitalization (\( c \)) and the supervisor’s cost of intervention (\( \tau_1 \)), which discourage compliance, are larger.

4 Adding a limit to supervisory capacity

In this section, we analyze the important implications of introducing an upper limit to the mass of banks that the supervisor is able to early recapitalize. Such constraint might arise for two main reasons.

First, bank interventions require funding, most frequently in the form of an increase in the level of debt of a supervisory agency, if not directly the government. If the authorities in charge have a weak reputation regarding their commitment or capability to intervene at the required level (North, 1993) or the economic climate surrounding the banking crisis is sufficiently adverse, investors may fear that the devised intervention will not preclude the
possibility of a fully fledged bank and sovereign crisis in the near future. Afraid of that, they may limit their funding to an amount that only allows the recapitalization of a mass $n_0$ of troubled banks.$^{12}$

A second, related possibility is that, if investors are not as well informed as the supervisor on banks’ capital needs, the size of the supervisory interventions may signal the severity of a banking crisis (e.g. the size of $\phi$ in our baseline model). Then it might happen that exceeding a critical level of intervention raises doubts on, e.g., the sufficiency of the deposit insurance fund (or the capacity of the supporting government to repay the insured deposits in full)$^{13}$. This might trigger a bank run, in spite of the existence of deposit insurance (Dang et al., 2017). So $n_0$ can also be interpreted as the maximum level of early supervisory intervention compatible with not triggering a run at $t = 1$.

### 4.1 The too-many-to-recapitalize problem

If the supervisor’s capacity to publicly recapitalize banks at $t = 1$ is limited to a maximum mass of $n_0$ banks, its reaction function becomes

$$N_0(m) = \min\{N(m), n_0\}, \quad (7)$$

where $N(m)$ is the unconstrained reaction function defined in (2). For parameter values and values of $m$ low enough to make $N(m) > n_0$, the constrained reaction function $N_0(m)$ is no longer sensitive to $m$. This makes the public recapitalization threat represented by the probability $n/(\phi - m) = N_0(m)/(\phi - m)$ which is overall increasing (rather than decreasing) in $m$. Then banks’ private recapitalization decisions become strategic complements over such range, opening the possibility of of having multiple equilibria.

Figure 3 depicts a situation in which the baseline game (without a constrained supervisor) features an equilibrium $(m^*, n^*)$ with $m^* > 0$ and in which the newly added capacity constraint $n_0$ takes a value in the interval $(n^*, I(0))$. In this situation, the interior equilibrium discussed in reference to Figure 2 coexists with two other SPNE: (i) a corner equilibrium

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$^{12}$If, on the contrary, institutions are strong, the devised intervention is credible and/or government’s financing capacity is plentiful, authorities may face no constraint to their intervention capacity.

$^{13}$This problem can be especially pronounced in banking jurisdictions in which the relevant agencies operate without a credible fiscal backstop.
with \( m = 0 \) and \( n = n_0 \), and (ii) a second interior equilibrium with \( m = I^{-1}(n_0) \) and \( n = n_0 \). Importantly, the second interior equilibrium, opposite to the other two, is not stable.\(^{14}\)

Using Figure 3 as a reference, it is easy to infer the various equilibrium configurations that may emerge depending on the size of \( n_0 \) and the equilibrium of the unconstrained game. The following proposition summarizes the conditions under which the second equilibrium with extreme forbearance arises.\(^{15}\)

**Proposition 5.** Under circumstances that would otherwise allow to sustain an equilibrium with \( m^* > 0 \) privately recapitalizing banks, the presence of a constraint \( n_0 < I(0) = c\phi/(c+\Delta) \) to the supervisor’s public recapitalization capacity implies the existence of an equilibrium with

\(^{14}\)The second interior equilibrium is not stable in the sense that it is not robust to having an excess mass \( \varepsilon \to 0 \) of damaged banks arbitrarily deviating to either \( r = 0 \) or \( r = 1 \). Shall that happen, all other damaged banks would want to deviate to \( r = 0 \) or \( r = 1 \), respectively. This suggests that the play of the game would converge to the (stable) equilibria with \( m = 0 \) or \( m = m^* \), respectively. This evidences the strategic complementarity between banks’ decisions in the neighborhood of the second interior equilibrium.

\(^{15}\)For completeness, the proof of the proposition also discusses the case in which the equilibrium of the unconstrained game features a zero mass of privately recapitalizing banks.
extreme forbearance. Over the range $n_0 \in [N(m^*), I(0))$, with

$$N(m^*) = \frac{\tau_0 c}{\lambda \Delta - \tau_1 c} > 0,$$

the extreme forbearance equilibrium coexists with the equilibrium of the unconstrained game. For $n_0 \in [0, N(m^*))$, only the equilibrium with extreme forbearance exists. The multiple equilibria range is increasing in the mass of damaged banks $\phi$.

Thus, the emergence of a too-many-to-recapitalize problem may turn banks’ private recapitalization decisions strategic complements, giving rise to the possibility of a pure strategy equilibrium with no private recapitalizations (and extreme forbearance). For intermediate values of $n_0$, such equilibrium coexists with the mixed strategy equilibrium of the baseline setup. Intuitively, the larger mass of damaged banks $\phi$, the lower the value of $n_0$ for which banks’ coordination in non-compliance can lead to the exhaustion of intervention capacity, and hence to the existence of the equilibrium with extreme forbearance.

The result regarding the impact of $\phi$ on the sustainability of an equilibrium with extreme forbearance suggests a channel through which banks might ex ante favor the emergence of such equilibrium by adopting correlated risky investment strategies at an earlier (unmodeled) stage of the game. Banks’ common exposures to systemic risk might guarantee that, in a crisis, the mass of damaged banks is large enough to exhaust the supervisor’s intervention capacity in case no one recapitalizes privately, and induce the equilibrium with extreme forbearance. So, if the supervisor has a limited intervention capacity, ex ante and ex post moral hazard by banks can reinforce each other in a particularly perverse manner, dramatically increasing the severity of systemic crises.

The following result further elaborates on the circumstances potentially leading to extreme forbearance.

**Proposition 6.** Under circumstances that would otherwise allow to sustain an equilibrium with $m^* > 0$ privately recapitalizing banks, the range of values of the supervisor’s capacity constraint $n_0$ for which only the equilibrium with extreme forbearance exists is increasing in the private recapitalization cost ($c$) and the supervisor’s cost of intervention ($\tau_0$ and $\tau_1$). Moreover, the effects of these two costs on the length of such range reinforce each other.
Thus, similarly to what happens in the unique equilibrium of the baseline unconstrained game when \( m^* > 0 \) (Proposition 3), a higher cost of private recapitalization for banks reinforces the effect of the supervisory intervention cost on the incidence of forbearance – here in the form of expanding the range of parameters for which the only stable equilibrium features extreme forbearance.

4.2 Implications for the sovereign-bank nexus

The too-many-to-recapitalize problem provides a novel channel linking banks to the strength of public finances, thus speaking to the literature on the nexus between sovereign risk and bank risk. One of the implications of our analysis is that damaged banks operating in the country with a more fiscally constrained government face a lower threat of supervisory intervention. As a consequence, their incentives to restore their solvency problems by themselves (via private recapitalization) also weaken, effectively calling the supervisor to intervene more. However, once the supervisor exhausts the available funds, it can no longer respond to a further reduction in private recapitalizations by intervening more banks, and an equilibrium with extreme lack of bank compliance and extreme supervisory forbearance emerges.

In such equilibrium, public interventions are not enough to compensate for private inaction and the economy remains exposed to experiencing a high bank failure rate and large future systemic costs, with their obvious impact on future government finances. Although the link between banking sector vulnerabilities and the fiscal weakness of the government has been extensively discussed in the literature (Dell’Ariccia et al., 2018), our model provides a new mechanism where the link (and the possibility of catastrophic outcomes during a crisis) does not come from government guarantees or banks’ holdings of sovereign debt (Acharya et al., 2014; Brunnermeier et al., 2016; Leonello, 2018) but from the importance of the supervisor’s capacity to undertake precautionary recapitalizations at an early stage and the strategic interaction between the supervised banks.

5 The welfare cost of lack of supervisory commitment

In this section, we discuss the welfare implication of our results. We define welfare in a utilitarian manner, abstracting from redistributational effects. In our model, the costs incurred
by banks’ initial owners in case of a private or public recapitalization are redistributive in nature. Those include losses due to the reduction of their net equity-like payoffs (in favor of debtholders or the safety net) when their banks’ leverage is reduced and their cash flow and control rights are diluted in favor of new shareholders (possibly including the government). In contrast, the costs taken into account by the supervisor are a mixture of redistributional costs (such as the political and reputational costs of an early intervention) and net deadweight losses (such as the costs of the distortionary taxation necessary to finance an early intervention or the systemic costs of the future failure of undercapitalized banks). Thus we will measure welfare as

\[ W = -(1 - \alpha)[\tau_0 n + (\tau_1/2)n^2] - (\lambda/2)(\phi - m - n)^2, \]  

where \( \alpha \in [0, 1] \) is a parameter that accounts for the fraction of the supervisor’s early intervention costs that corresponds to political and reputational costs. For \( \alpha = 0 \), social welfare \( W \) is simply equal to the negative of the costs that the supervisor minimizes when deciding on public recapitalizations at \( t = 1 \). For \( \alpha > 0 \), the early intervention costs that enter \( W \) are smaller than those entering the supervisor’s objective function, implying the the supervisor would tend to intervene, for each given \( m \), less than what a benevolent social planner acting at that time would do. The social planner would impose a tougher public recapitalization threat on banks, leading to an equilibrium with more private recapitalization and less forbearance. Moreover, as stated in the following proposition, in the regime with \( m^* > 0 \), the equilibrium induced by the planner would also feature less public recapitalizations, thus improving welfare through the reduction in both early intervention costs and systemic costs.

**Proposition 7.** Under circumstances that would otherwise allow to sustain an equilibrium with \( m^* > 0 \) privately recapitalizing banks, if public recapitalizations are decided by a planner that disregards the political and reputational costs of early interventions, private recapitalizations and welfare increase, while public recapitalizations and forbearance decrease.

However, the political and reputational costs that lead the supervisor to be softer than socially desirable at the time of deciding on early intervention are not the only reason whereby the outcome of our supervisory game is generally inefficient. A second reason is that the supervisor decides on such interventions once damaged banks’ recapitalizations have been
decided (and, thus, taking $m$ as given). If the supervisor could commit to a *public recapitalization rule* $N_0(m)$ fixed at (or before) $t = 0$, a proper choice of $N_0(m)$ could lead to higher welfare. In fact, the following proposition shows that it would be possible to attain a first best allocation in which all damaged banks comply with the required recapitalization.

**Proposition 8.** If the supervisor could commit to follow a public recapitalization rule, the rule

$$N_0(m) = \frac{c + \varepsilon}{c + \Delta} (\phi - m),$$

for any $\varepsilon > 0$, would uniquely implement the first best allocation in which all banks recapitalize privately ($m = \phi$), no public recapitalizations occur ($n = 0$) and forbearance is zero ($\phi - m - n = 0$).

Intuitively, a rule such as (10) sets the implicit public recapitalization threat for each $m$ to a level that makes each individual damaged bank to strictly prefer compliance (recall (3)), thus inducing an equilibrium with $m = \phi$. The contingency of the rule on $m$ also implies $n = 0$ and, trivially, no forbearance, thus pushing welfare $W$ to its maximum possible level of zero.

However, the true challenge for welfare maximization in this setup is to overcome the time inconsistency problem that might lead any supervisor or social planner to renege from a rule such as (10) once at $t = 1$. In theory, writing such rule in the law or charter regulating the behavior of the supervisor (and demanding personal liability to those making its decisions in case of violating it) might solve the problem. In practice, however, the changing nature of the circumstances in which supervisor may be called to intervene (or, in model terms, uncertainty about the values of parameters such as $\phi, c$, or $\Delta$ at the time the law is written) might prevent the usage of rules and leave its early intervention decisions subject to discretion.

As in the literature on time inconsistency in the context of, e.g., monetary policy, delegating the relevant decisions to authorities with specific biases (relative to the social planner) can be valuable. Our prior analysis suggests that delegation to authorities attributing low costs to early public interventions and high costs to future systemic problems might help. The relative weighting of both costs by the authorities might be shaped by their own subjective preferences, as well as by the contents of their mandates, their career prospects, and
the details of their compensation schemes.\textsuperscript{16}

6 Concluding remarks

We have analyzed the interplay between banks and a supervisor after the latter discovers that a significant mass of banks may turn insolvent unless they get properly recapitalized. Our analysis disentangles the strategic interaction between the banks requested to private recapitalize and a supervisor with the discretionary power to undertake a public recapitalization (or nationalization) of noncompliant banks. We discuss the determinants of the credibility of the underlying public recapitalization threat and its impact on banks’ private recapitalization decisions, the resulting level of supervisory forbearance, and its implications for the systemic costs due to bank failure.

Our analysis offers a number of testable predictions and several important policy implications. First, it predicts that, ceteris paribus, when supervisors face higher political or reputational costs of early intervention, banks’ private recapitalizations will be less frequently observed, public recapitalizations will be more frequently observed, and the systemic costs of bank distress will be larger, evidencing a larger level of forbearance. This latter prediction is consistent with Agarwal et al. (2014), which shows that higher leniency of state supervisors is related to higher bank failure rates and, eventually, more costly bail-outs (lower repayment rates on the financial assistance funds received from the government).

Second, we also predict that the incidence of public recapitalizations of damaged banks will tend to be higher in economies with more highly levered banks, while the overall incidence of capital forbearance (i.e. the mass of banks that remain undercapitalized after the intervention) will be non-monotonically related to bank leverage: decreasing with it when leverage is small but increasing with it beyond some point.

Third, the results predict that economies facing a higher cost of early intervention or a limit to their capacity to publicly recapitalize banks at some early stage are more exposed to suffer from a negative feedback loop whereby the softer threat of public intervention

\textsuperscript{16}Analyzing the design of an optimal compensation scheme for the supervisor exceeds the scope of this paper since it would require adopting a less reduced-form approach to the elements that enter its payoff function in our setup.
discourages banks from privately recapitalizing, increases or exhausts the usage of the supervisor’s intervention capacity, and rises the resulting forbearance and systemic costs. Our analysis uncovers a too-many-to-recapitalize problem that can bring a novel dimension of the sovereign-bank nexus: if the prospect of a deep systemic crisis further limits (e.g. via a fiscal channel) the capacity of the supervisor to early intervene and encourages banks to coordinate in high risk, low compliance strategies, the implied self-fulfilling logic can lead to equilibria with extreme levels of forbearance and deeply disturbed government finances.

Our findings also have important policy implications. The comparative statics of the private recapitalization cost and the analysis of the role of leverage suggest that policies reducing the importance of the Merton’s put (such as controls on leverage and risk taking by banks) or facilitating the undertaking of leverage reduction transactions (e.g. by reducing the informational and agency frictions behind equity issuance costs) can reduce the need for public intervention on trouble banks and the levels of forbearance. While not explicitly considered in our analysis, the ex ante issuance of securities such as contingent convertible debt (CoCos) that provide for the equivalent to a private recapitalization without the need to resort on the discretionary decision of bank owners (or giving them a cheap alternative to effectively recapitalize their banks) would also help.¹⁷

Finally, the analysis of the too-many-to-recapitalize problem, the possibility of having a supervisory-based sovereign-bank nexus, and the welfare losses associated with the lack of commitment of the supervisor to a tough early intervention policy have implications for the institutional design of the supervisory agencies in charge of early intervention on troubled banks: the importance of their independence and clear mandates, their internal governance (to avoid giving excessive weight to the political and reputational costs of intervention), and the access to credible and sufficiently sizable funding capacity.

¹⁷When the bank is recapitalized through the conversion of CoCos into equity, the initial owners’ stake in the bank may also get diluted, both directly and via the reduction in the Merton’s put. However, CoCos might be designed to either not make conversion a discretionary choice of bank owners or imply a lower cost to them than a seasoned equity offering with the same leverage-reduction effect.
Appendix: Proofs

Proof of Lemma 1

The result follows trivially from the discussion provided in the main text.

Proof of Proposition 1

To prove the results in Proposition 1, it is useful to examine the position of the indifference line \( n = I(m) \) relative to the supervisor's reaction function \( n = N(m) \), already depicted in Figure 1.

For \( (\lambda/(\lambda+\tau_1) - (c/(c+\Delta)))\phi - \tau_0/((\lambda+\tau_1) \leq 0 \), we have \( I(0) \geq N(0) \), which means that damaged banks' indifference line lies everywhere above the supervisor's reaction function, as depicted in Figure 4. Then at all points on the supervisor's reaction function banks prefer not to comply. But then the unique symmetric SPNE must involve \( m = 0 \) and the supervisor's best response to such first stage outcome, that is, \( n = N(0) = (\lambda\phi-\tau_0)/((\lambda+\tau_1) \), as indicated in Figure 4.

![Equilibrium without privately recapitalizing banks (\( m^* = 0 \))](image)

**Figure 4:** Equilibrium without privately recapitalizing banks
For \((\lambda/(\lambda + \tau_1) - (c/(c + \Delta)))\phi - \tau_0/(\lambda + \tau_1) > 0\), we have \(I(0) < N(0)\), which, given the form of the curves \(N(m)\) and \(I(m)\) guarantees a single crossing between them in the section of the supervisor’s reaction function where \(N(m) > 0\). This situation is depicted in Figure 2. In this case the unique symmetric SPNE of the game involves the values of \((m, n)\) at such intersection, \((m^*, n^*)\). To show that such point is a SPNE, notice that lying on \(n = I(m)\) means that bank owners are indifferent between privately recapitalizing or not, and, hence, \(m^*\) can be sustained as the result of damaged banks playing an uncorrelated symmetric mixed strategy in which they privately recapitalize with probability \(p^* = m^*/\phi\). Simultaneously, lying also on \(n = N(m)\) means that by publicly recapitalizing a mass \(n^*\) of the non-compliant banks, the supervisor is playing a best response to damaged banks’ actions in the first stage. Finally, the uniqueness of this equilibrium comes from the fact that, for values of \(m\) strictly lower than \(m^*\), we have \(N(m) > I(m)\), which means that banks would prefer \(r = 1\), which is incompatible with sustaining \(m < \phi\), while, for values of \(m\) strictly higher than \(m^*\), we have \(N(m) < I(m)\), which means that banks would prefer \(r = 0\), which is incompatible with sustaining \(m > 0\).

**Proof of Proposition 2**

Table A1 summarizes the comparative statics of the equilibrium in which there is a strictly positive mass of privately recapitalizing banks. In the table, signs +, – or = indicate whether increasing the parameter indicated in the first column of the table increases, decreases, or does not change the endogenous variable indicated in the heading of each column. In a slight abuse of terminology, the table assimilates variations in the “likelihood” of having an equilibrium with \(m^* > 0\): All the results arise immediately from partially differentiating the closed-form expressions of the relevant equilibrium variables.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Likelihood of equilibrium w. (m^* &gt; 0)</th>
<th>Intervention threat w. (n^<em>/(\phi - m^</em>))</th>
<th>Recapitalized banks: Privately (m^*)</th>
<th>Publicly (n^*)</th>
<th>Forborne banks (\phi - m^* - n^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damaged banks (\phi)</td>
<td>+</td>
<td>=</td>
<td>+</td>
<td>=</td>
<td>=</td>
</tr>
<tr>
<td>Systemic cost (\lambda)</td>
<td>+</td>
<td>=</td>
<td>+</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Intervention cost (\tau_0)</td>
<td>–</td>
<td>=</td>
<td>–</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Intervention cost (\tau_1)</td>
<td>–</td>
<td>=</td>
<td>–</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Owners’ private recap cost (c)</td>
<td>–</td>
<td>+</td>
<td>–</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Owners’ control rent (\Delta)</td>
<td>+</td>
<td>–</td>
<td>+</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

The effects of changing \(\lambda, c,\) and \(\Delta\) go in the natural direction, in the sense that (i) they encourage the player(s) directly suffering the corresponding cost to take actions that reduce...
the incidence of such cost, and (ii) produce partially offsetting changes in the actions of the opposing player(s). In the case of \( c \) and \( \Delta \), the direct effects (on \( m^* \)) dominate the indirect ones (on \( n^* \)) as for the final variation in forbearance \( (\phi - m^* - n^*) \). In the case of \( \lambda \), the indirect effect (on \( m^* \)) dominates.

**Proof of Proposition 3**

First, we find the effect of \( \tau_0 \) on the mass of publicly recapitalized banks \( n^* \) and on forbearance \( \phi - m^* - n^* \) by just deriving in (5) and (6), respectively, using the expression that corresponds to the case in which the the equilibrium mass of compliant banks is positive:

\[
\frac{\partial n^*}{\partial \tau_0} = \frac{c}{\lambda \Delta - \tau_1 c} > 0, \tag{11}
\]

\[
\frac{\partial (\phi - m^* - n^*)}{\partial \tau_0} = \frac{\Delta}{\lambda \Delta - \tau_1 c} > 0. \tag{12}
\]

Next, we further derive the above expression with respect to \( c \):

\[
\frac{\partial^2 n^*}{\partial \tau_0 \partial c} = \frac{\lambda \Delta}{(\lambda \Delta - \tau_1 c)^2} > 0, \tag{13}
\]

\[
\frac{\partial^2 (\phi - m^* - n^*)}{\partial \tau_0 \partial c} = \frac{\tau_1 \Delta}{(\lambda \Delta - \tau_1 c)^2} > 0, \tag{14}
\]

whose signs mean that a rise in \( c \) reinforces the effects of \( \tau_0 \) on \( n^* \) and \( \phi - m^* - n^* \).

Proceeding similarly to find the effects of \( \tau_1 \) on \( n^* \) and \( \phi - m^* - n^* \), we obtain:

\[
\frac{\partial n^*}{\partial \tau_1} = \frac{\tau_0 \Delta}{(\lambda \Delta - \tau_1 c)^2} > 0, \tag{15}
\]

\[
\frac{\partial (\phi - m^* - n^*)}{\partial \tau_1} = \frac{\tau_0 \Delta c}{(\lambda \Delta - \tau_1 c)^2} > 0, \tag{16}
\]

and the cross-derivatives:

\[
\frac{\partial^2 n^*}{\partial \tau_1 \partial c} = \frac{2 \tau_0 c \lambda \Delta}{(\lambda \Delta - \tau_1 c)^3} > 0, \tag{17}
\]

\[
\frac{\partial^2 (\phi - m^* - n^*)}{\partial \tau_1 \partial c} = \frac{2 \tau_0 \Delta c^2}{(\lambda \Delta - \tau_1 c)^3} > 0, \tag{18}
\]

which imply that a rise in \( c \) also reinforces the effects of \( \tau_1 \) on \( n^* \) and \( \phi - m^* - n^* \).

**6.1 Proof of Proposition 4**

In the extended game, the supervisor’s reaction function is defined by

\[
N_d(m) = \arg \min_{0 \leq n \leq \phi - m} \tau_0 dn + (\tau_1/2)(dn)^2 + \lambda/2(d(\phi - m - n))^2, \tag{19}
\]

27
which is solved for
\[
N_d(m) = \max \left\{ \frac{\lambda d(\phi - m) - \tau_0}{(\lambda + \tau_1)d}, 0 \right\}. \tag{20}
\]

An individual bank’s indifference condition regarding the choice between privately recapitalizing or not is now given by the line \( n = I_d(m) \) with
\[
I_d(m) = \frac{cd}{cd + \Delta}(\phi - m). \tag{21}
\]

Exploring (20) and (21), it becomes clear that leverage \( d \) increases the propensity of the supervisor to intervene (that is, would shift its reaction function outwards in a figure similar to Figure 2) and reduces banks’ incentives to comply (that is, would shift their indifference condition inwards in a figure similar to Figure 2).

Following the same steps as in the proof of Proposition 1, the unique symmetric SPNE of the extended game can be found to be
\[
(m^*, n^*) = \begin{cases} 
(0, \frac{\lambda \phi d - \tau_0}{(\lambda + \tau_1)d}), & \text{if } \left( \frac{\lambda}{\lambda + \tau_1} - \frac{cd}{cd + \Delta} \right) \phi - \frac{\tau_0}{(\lambda + \tau_1)d} \leq 0, \\
(\phi - \frac{\tau_0(cd + \Delta)}{(\Delta - \tau_1 cd)d}, \frac{\tau_0 \tau_1 c^2}{\lambda \Delta - \tau_1 cd}), & \text{if } \left( \frac{\lambda}{\lambda + \tau_1} - \frac{cd}{cd + \Delta} \right) \phi - \frac{\tau_0}{(\lambda + \tau_1)d} > 0.
\end{cases} \tag{22}
\]

Focusing on the regime in which the mass of privately recapitalizing banks \( m^* \) is strictly positive, it is immediate to find that
\[
\frac{\partial n^*}{\partial d} = \frac{\tau_0 \tau_1 c^2}{(\lambda \Delta - \tau_1 cd)^2} > 0,
\]
and also
\[
\phi - m^* - n^* = \frac{\tau_0 \Delta}{(\lambda \Delta - \tau_1 cd)d},
\]
which implies
\[
\frac{\partial (\phi - m^* - n^*)}{\partial d} = -\frac{\tau_0 \Delta (\Delta \lambda - 2\tau_1 cd)}{(\lambda \Delta - \tau_1 cd)^2 d^2},
\]
which is strictly positive if and only if \( d > d^* = \frac{\Delta \lambda}{2\tau_1 c} \). So capital forbearance is first increasing and then decreasing in \( d \).

To better understand the roots of the non-monotonicity in the effect of leverage on forbearance, notice that
\[
\frac{\partial m^*}{\partial d} = -\frac{\tau_0 [\tau_1 c^2 d^2 + 2\tau_1 \Delta cd - \lambda \Delta^2]}{(\lambda \Delta - \tau_1 cd)^2 d^2},
\]
which is strictly positive if and only if
\[
\tau_1 c^2 d^2 + 2\tau_1 \Delta cd - \lambda \Delta^2 < 0. \tag{23}
\]
The quadratic equation \(\tau_1 c^2 d^2 + 2\tau_1 \Delta cd - \lambda \Delta^2 = 0\) has two roots, one of which is negative. The other root is

\[
\hat{d} = \frac{\Delta}{c} \left( \sqrt{1 + \lambda/\tau_1} - 1 \right) > 0.
\]

Thus the inequality (23) is true if and only if \(d < \hat{d}\), which means that the mass of privately recapitalizing banks is first increasing and then decreasing with \(d\). Given that \(n^*\) is monotonically increasing in \(d\), this inverted U-shaped relationship between \(d\) and \(m^*\) helps explains the U-shaped relationship between \(d\) and forbearance as well as the fact that \(\tilde{d} > \hat{d}\). ■

**Proof of Proposition 5**

The results follow from the taxonomy of the possibilities that may emerge in the constrained game when the unconstrained game features some privately recapitalizing banks, that is, \(m^* > 0\) and, hence, \(n^* = N(m^*)\). The situation depicted in Figure 3 corresponds with case 3 in the following list:

1. If \(n_0 < N(m^*)\), the constrained game has just one equilibrium with \((m, n) = (0, n_0)\).

2. If \(n_0 = N(m^*)\), the constrained game features two equilibria: one with \((m, n) = (0, n_0)\) and the same equilibrium \((m^*, N(m^*))\) as the unconstrained game.

3. If \(n_0 \in (N(m^*), (c/(c + \Delta))\phi]\), the constrained game features three equilibria: one with \((m, n) = (0, n_0)\), the same equilibrium \((m^*, N(m^*))\) as the unconstrained game, and a third (unstable) one with \((I^{-1}(n_0), n_0)\).

4. If \(n_0 > (c/(c + \Delta))\phi\), the constrained game has just the same equilibrium as the unconstrained game.

When the unconstrained game features \(m^* = 0\) and, hence, \(n^* = N(0)\) (as illustrated in Figure 4), the possibilities that emerge when the supervisor faces a capacity constraint \(n_0\) are only two:

1. For \(n_0 < N(0)\), the constrained game has just one equilibrium with \((m, n) = (0, n_0)\) (which involves larger forbearance than the equilibrium of the unconstrained game).

2. For \(n_0 \geq N(0)\), the constrained game has just the same equilibrium \((0, N(0))\) as the unconstrained game. ■

**Proof of Proposition 6**

According to Proposition 5, the range of values of \(n_0\) over which only the equilibrium with extreme forbearance exists is \([0, N(m^*))\). It is immediate to see that \(\partial N(m^*)/\partial c > 0\), \(\partial N(m^*)/\partial \tau_i > 0\), and \(\partial^2 N(m^*)/\partial (\tau_i \partial c) > 0\) for \(i = 0, 1\), which proves the result. ■

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Proof of Proposition 7

In the game in which public recapitalizations were decided, for each given $m$, by a social planner who disregards the political and reputational costs of its interventions, the equilibrium would be given by:

$$ (m^{SP}, n^{SP}) = \begin{cases} 
(0, \frac{\lambda}{\lambda + (1-\alpha)\tau_1} \phi), & \text{if } \left(\frac{\lambda}{\lambda + (1-\alpha)\tau_1} \frac{c}{c+\Delta}\right) \phi - \frac{(1-\alpha)\tau_0}{\lambda + (1-\alpha)\tau_1} \leq 0, \\
(\phi - \frac{(1-\alpha)\tau_0(c+\Delta)}{\lambda\Delta - (1-\alpha)\tau_1\Delta c}, \frac{(1-\alpha)\tau_0 c}{\lambda\Delta - (1-\alpha)\tau_1\Delta c}), & \text{if } \left(\frac{\lambda}{\lambda + (1-\alpha)\tau_1} \frac{c}{c+\Delta}\right) \phi - \frac{(1-\alpha)\tau_0}{\lambda + (1-\alpha)\tau_1} > 0,
\end{cases} $$

which implies a level of forbearance given by

$$ \phi - m^{SP} - n^{SP} = \begin{cases} 
\frac{(1-\alpha)\tau_0 + (1-\alpha)\tau_1 \phi}{\lambda + (1-\alpha)\tau_1}, & \text{if } \left(\frac{\lambda}{\lambda + (1-\alpha)\tau_1} \frac{c}{c+\Delta}\right) \phi - \frac{(1-\alpha)\tau_0}{\lambda + (1-\alpha)\tau_1} \leq 0, \\
\frac{(1-\alpha)\tau_0 \Delta}{\lambda\Delta - (1-\alpha)\tau_1\Delta c}, & \text{if } \left(\frac{\lambda}{\lambda + (1-\alpha)\tau_1} \frac{c}{c+\Delta}\right) \phi - \frac{(1-\alpha)\tau_0}{\lambda + (1-\alpha)\tau_1} > 0.
\end{cases} $$

And it is immediate to show that in the regime with $m^{SP} > 0$, we have $\partial m^{SP}/\partial \alpha > 0$, $\partial n^{SP}/\partial \alpha < 0$, and $\partial(\phi - m^{SP} - n^{SP})/\partial \alpha < 0$. Moreover, the condition for the prevalence of such regime gets relaxed as $\alpha$ rises. Altogether, this implies that in circumstances in which the equilibrium of the supervisory game features $m^* > 0$, the equilibrium in which the social planner decides on $n$ features $m^{SP} > m^*$, $n^{SP} < n^*$, and $\phi - m^{SP} - n^{SP} < \phi - m^* - n^*$.

Proof of Proposition 8

The first best (FB) choice of $m$ and $n$ would emerge from solving

$$ \max_{m \geq 0, n \geq 0} W, \\ \text{s.t.:} \quad m + n \leq \phi, $$

which, using (9), clearly yields the solution $m^{FB} = \phi$ and $n^{FB} = 0$, under which welfare is $W^{FB} = 0$. In contrast, the outcome of the supervisory game without commitment when the equilibrium features a strictly positive mass of privately recapitalizing banks is

$$ W^* = -\frac{(1-\alpha)\tau_0^2 c[\lambda\Delta - (\tau_1/2)c] + (\lambda/2)\tau_0^2 \Delta^2}{(\lambda\Delta - \tau_1 c)^2} < 0, $$

which is obtained by simply plugging in (9) the expressions for $m^*$ and $n^*$ provided in (5).

Now, consider the case in which the supervisor can commit to decide on public recapitalizations using the rule (10), for $\varepsilon \geq 0$. To show that the first best can be implemented, notice that if a damaged bank expects the mass of privately recapitalized banks to be $m$ and
the subsequent decision of the supervisor to be \( n = N_0(m) \), its owners’ payoff from privately recapitalizing would be \(-c\), while their expected payoff from not doing so would be

\[
-\frac{N_0(m)}{\phi - m}(c + \Delta) = -(c + \varepsilon) < -c,
\]

which means that privately recapitalizing is indeed a best response (and strictly so for \( \varepsilon > 0 \)). So an equilibrium with \( m = \phi \) and \( n = N_0(\phi) = 0 \) can be sustained (and for \( \varepsilon > 0 \) this equilibrium is unique).
References


