Optimal Dynamic Capital Requirements*

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Abstract

We characterize welfare maximizing capital requirement policies in a quantitative macro-banking model with household, firm and bank defaults calibrated to Euro Area data. We optimize on the level of the capital requirements applied to each loan class and their sensitivity to changes in default risk. We find that getting the level right (so that bank failure risk remains contained) is of foremost importance, while the optimal sensitivity to default risk is positive but typically smaller than under Basel IRB formulas. Starting from low levels, savers and borrowers benefit from higher capital requirements. At higher levels, only savers prefer tighter requirements.

Keywords: Macroprudential policy; Bank fragility; Capital requirements; Financial frictions; Default risk.

JEL codes: E3, E44, G01, G21

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1. **Introduction**

This paper examines the optimal calibration of Basel-type dynamic capital requirements.\(^1\) As in Basel II and III, we consider policy rules that make capital charges associated with different types of exposures (mortgages and corporate loans) increasing in borrowers’ anticipated default risk. In our setup bank fragility is key to the operation of bank-related transmission channels. Banks intermediate funds from saving households to borrowing households and entrepreneurs, and all borrowers, including banks, can default on their lenders.

The model features three key distortions. First, banks operate under limited liability and safety net guarantees in the form of insured deposits. Second, uninsured bank debt is not priced according to the individual risk profile of each bank (which is treated as unobservable by the savers) but according to the expected economy-wide bank failure risk. Last, all external financing is subject to costly state verification frictions like in Bernake, Gertler and Gilchrist (1999) (henceforth BGG) and takes the form of uncontingent debt.

Absent regulation, the first two distortions provide an incentive for banks to opt for excessive leverage and excessively loose lending standards. On the other hand, like in other papers in the BGG tradition and, more recently, Gertler and Kiyotaki (2010), internal equity financing is limited by the endogenously accumulated net worth of the borrowing households or the owners of the borrowing firms and banks. In this context, optimal bank capital regulation trades off the distortions and deadweight losses from excessive borrower and bank fragility against the scarcity of borrowers’ and bankers’ net worth and the implications for the levels of investment that they can sustain.

In order to provide quantitative results, we calibrate the model to match first and second moments of key Euro Area macroeconomic and banking data. Differently from related attempts (e.g. Christiano, Motto and Rostagno, 2008; Gerali et al, 2010), in addition to key macroeconomic variables, we also match the moments of banking variables such as capital ratios, write-offs, loan spreads, loan-to-GDP ratios, etc. Importantly, we calibrate the capital requirements on mortgages and corporate loans in a way consistent with the internal ratings based (IRB) approach of Basel regulation, making their level related to the probability of default (PD) of the corresponding loans.

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\(^1\)The focus on Basel-type regulatory tools links our work to the partial equilibrium analysis of the procyclical effects of capital requirements in, for example, Kashyap and Stein (2004) and Repullo and Suarez (2013).
Our analysis yields several interesting policy conclusions. First, we address upfront the potential conflicting interests of savers and borrowers regarding the adequate level of capital for each class of loans and its time variation in response to changes in default risk. We find that, regardless of the Pareto weights given to saving and borrowing households, it is always optimal to impose capital requirements that keep bank defaults, and, consequently, the strength of bank-related amplification channels and their associated deadweight losses, sufficiently low.

Our results also show that increasing capital requirements from their baseline levels is Pareto-improving up to a point and redistributive after that. Starting from high levels of bank default, both savers and borrowers benefit from capital requirements increases due to the reduction in the social costs of bank default. Once bank default is close to zero, the ensuing tightening in lending standards strongly penalizes the borrowers. In contrast, savers continue benefiting mainly due to the higher returns on their bank equity holdings, which offset their share of the welfare losses due to lower bank-funded investment.

Most importantly, we find that it is optimal to make the capital requirements on corporate loans and, especially, mortgages higher in level than under Basel II, but less responsive to (time) variation in default risk than what a point-in-time estimate of the inputs of its IRB formula would imply. While a high PD-sensitivity may help keep banks safe, it amplifies the volatility in lending standards and destabilizes borrowers’ consumption. Thus, we argue that our results as supportive of regulators’ attempts to reinforce banks’ capitalization while ameliorating procyclicality.

Our paper is part of a growing literature which incorporates banking in otherwise standard DSGE models. Our analysis differs from studies such as Curdia and Woodford (2010), Gertler and Kiyotaki (2010), Gerali et al. (2010) and Meh and Moran (2010) in that we provide a normative assessment of capital regulation. The focus on bank fragility is shared with Markovic

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2 Most papers on macroprudential policy abstract from heterogeneity because, under incomplete markets, there is no commonly accepted criterion for the assignment of welfare weights to the different agents. Exceptions include Goodhart et al (2013) and Lambertini, Mendicino and Punzi (2013) who, in discussing loan-to-value limits, show heterogeneous effects of macroprudential policy. Angelini, Neri and Panetta (2014) also consider heterogeneity but they restrict their policy analysis to stabilization goals.

3 Basel II and III recommend banks to feed the IRB formulas with stable through-the-cycle (instead of point-in-time) estimates of the PDs. However, the practical implementation of a through-the-cycle approach is challenging (conceptually and in terms of accountability) and many banks follow a point-in-time approach.

4 Additionally to encouraging the use of through-the-cycle PDs, this could be achieved through a countercyclical capital buffer (CCyB) such as the one introduced by Basel III (which is intended to be built up during upturns, up to a size of 2.5% of risk weighted assets, and to be released during downturns).

Our model shares a number ingredients with Clerc et al. (2015), which we modify and extend in important dimensions: we integrate bank owners and entrepreneurs into the dynasty of saving households, simplifying the welfare analysis; we distinguish between insured and uninsured bank debt, showing the (un)importance of government guarantees for our welfare results; and we allow for the existence of non-bank funded investment. In addition, we properly calibrate the model to the Euro Area data and assess welfare in the fully stochastic economy.

2. Model Economy

We consider an economy populated by two dynasties: patient households (denoted by $s$) and impatient households (denoted by $m$). Households that belong to each dynasty differ in terms of their subjective discount factor, $\beta_m \leq \beta_s$. The total mass of households is normalized to one, of which an exogenous fraction $x_s$ are patient and the remaining fraction $x_m = 1 - x_s$ are impatient. In equilibrium, impatient households borrow.

The patient dynasty consists of three different classes of members, workers, entrepreneurs, and bankers, with measures $x_\varrho$ for $\varrho = w, e, b$, respectively. Workers supply labor to the production sector and transfer their wage income to the household. Entrepreneurs and bankers manage entrepreneurial firms (denoted by $f$) and banks (denoted by $j=M,F$), respectively. They use their scarce net worth to provide equity financing to entrepreneurial firms and banks, respectively, and can transfer their accumulated earnings back to the patient households as dividends or once they retire.\footnote{As in Gertler, Kiyotaki, and Queralto (2012), Brunnermeier and Sannikov (2014), and He and Krishnamurthy (2014), linking equity financing to the limited wealth of some inside owners of firms and banks captures (unmodeled) informational and agency frictions constraining the capability to raise outside equity.} Entrepreneurs and bankers receive consumption insurance from their dynasty, while the firms and banks that they own can individually default on their debts.

The impatient dynasty consists of workers only and its borrowing takes the form of non-
recourse mortgage loans made against a continuum of the individual housing units subject to idiosyncratic return shocks. Similarly to entrepreneurs and bankers, impatient workers receive consumption insurance from their dynasty and can individually default on their mortgages.

We assume two types of competitive banks that finance their loans by raising equity from bankers and debt from patient households. The loans extended to impatient households and the banks extending them are denoted by $M$, while those extended to firms and the banks extending them are denoted by $F$.\footnote{Having banks specialized in each class of loans simplifies their pricing and avoids cross-subsidization effects that would otherwise diminish the benefits of banks’ limited liability.} A fraction $\kappa$ of bank debt are deposits insured by a deposit insurance agency (DIA) funded with lump sum taxes. Banks are subject to capital requirements set by a prudential authority. The next subsections describe some of these ingredients in detail; the rest, including some variable definitions, the most conventional building blocks, and the market clearing conditions, appear in the Online Appendix.

### 2.1 Notation

All borrowers are subject to idiosyncratic shocks $\omega_{i,t+1}$ which are iid across borrowers of class $i \in \{m, f, M, F\}$ and across borrower classes, and follow a log-normal distribution with a mean of one and a stochastic standard deviation $\sigma_{i,t+1}$. We will denote by $F_{i,t+1}(\cdot)$ the distribution function of $\omega_{i,t+1}$ and by $\overline{\omega}_{i,t+1}$ the threshold realization below which a borrower of class $i$ defaults, so that the probability of default of such a borrower is $F_{i,t+1}(\overline{\omega}_{i,t+1})$.\footnote{The subscript $t+1$ in $F_{i,t+1}(\cdot)$, $G_{i,t+1}(\cdot)$, and $\Gamma_{i,t+1}(\cdot)$ reflects the time-varying standard deviation of $\omega_{i,t+1}$.}

Following BGG, it is useful to define the share of final asset value owned by borrowers of class $i$ which end up in default as

$$G_{i,t+1}(\overline{\omega}_{i,t+1}) = \int_0^{\overline{\omega}_{i,t+1}} \omega_{i,t+1} dF_{i,t+1}(\omega_{i,t+1}),$$  \hspace{1cm} (1)

and the expected share of gross final asset value of such a class of borrowers that goes to the lender as

$$\Gamma_{i,t+1}(\overline{\omega}_{i,t+1}) = G_{i,t+1}(\overline{\omega}_{i,t+1}) + \overline{\omega}_{i,t+1}[1 - F_{i,t+1}(\overline{\omega}_{i,t+1})].$$  \hspace{1cm} (2)

Due to the proportional asset repossession cost $\mu_{i}$, the net share of assets that goes to the lender is $\Gamma_{i,t+1}(\overline{\omega}_{i,t+1}) - \mu_{i} G_{i,t+1}(\overline{\omega}_{i,t+1})$ while $(1 - \Gamma_{i,t+1}(\overline{\omega}_{i,t+1}))$ share accrues to the borrowers.
2.2 Households

Dynasties provide consumption risk sharing to their members and are in charge of taking most household decisions. Each dynasty maximizes

\[
E_t \left\{ \sum_{i=0}^{\infty} (\beta \kappa)^{i+1} \left[ \log (c_{\kappa,t+i}) + \lambda_{t+i} \kappa \log (h_{\kappa,t+i}) - \frac{\varphi_{\kappa}}{1+\eta} (l_{\kappa,t+i})^{1+\eta} \right] \right\}
\]

with \( \kappa = s, m \), where \( c_{\kappa,t} \) denotes the consumption of non-durable goods and \( h_{\kappa,t} \) denotes the total stock of housing held by the various members of the dynasty (which is assumed to provide a proportional amount of housing services also denoted by \( h_{\kappa,t} \)), \( l_{\kappa,t} \) denotes hours worked in the consumption good producing sector, \( \lambda_t \) is a housing preference shock that follows an AR(1) process and is common to both dynasties, \( \nu_{\kappa} \) is a housing preference parameter, \( \varphi_{\kappa} \) is a leisure preference parameter, and \( \eta \) is the inverse of the Frisch elasticity of labor supply.

2.2.1 Patient Households

The patient households’ budget constraint is as follows

\[
c_{s,t} + q_{h,t} [h_{s,t} - (1-\delta_{h,t})h_{s,t-1}] + (q_{k,t} + s_t) k_{s,t} + d_t + B_t \leq [r_{k,t} + (1-\delta_{k,t}) q_{k,t}] k_{s,t-1} + w_t l_{s,t} + \tilde{R}_t d_{t-1} + R_{t-1}^{rf} B_{t-1} - T_{s,t} + \Pi_{s,t} + \Xi_{s,t}
\]

where \( q_{h,t} \) is the price of housing, \( \delta_{h,t} \) is the rate at which housing units depreciate, and \( w_t \) is the wage rate. Savers can hold physical capital \( k_{s,t} \) with price \( q_{k,t} \), depreciation rate \( \delta_{k,t} \), and rental rate \( r_{k,t} \), subject to a management cost \( s_t \) which is taken as given by households. \( T_{s,t} \) is a lump-sum tax used by the DIA to ex-post balance its budget, \( \Pi_{s,t} \) are aggregate net transfers of earnings from entrepreneurs and bankers to the household at period \( t \), and \( \Xi_{s,t} \) are profits from firms that manage the capital stock held by the patient households.\(^8\)

Each individual saver \( s \) can also invest in a risk free asset \( B_t \) (in zero net supply) and in a perfectly diversified portfolio of bank debt \( d_t \). The return on such debt has two components. A fraction \( \kappa \) is interpreted as insured deposits that always pay back the promised gross deposit rate \( R_{t-1}^d \). The remaining fraction \( 1 - \kappa \) is interpreted as uninsured debt that pays back the

\(^8\)The Online Appendix provides expressions for \( \Omega_t, T_{s,t}, \Pi_{s,t}, \) and \( \Xi_{s,t} \). The total cost of deposit insurance is assumed to be shared by the patient and impatient households in proportion to their size in the population. We have checked, however, that our welfare results below are qualitatively the same if the whole cost is paid by the patient households.
promised rate $R_{t-1}^d$ if the issuing bank is solvent and a proportion $1 - \kappa$ of the net recovery value of bank assets in case of default.\footnote{One can alternatively interpret $\kappa$ as the fraction of bank debt that will benefit from a government bailout in case of default. This formulation allows us to consider deviations from full bank debt insurance ($\kappa = 1$) without complicating banks’ capital structure decisions.} We assume banks’ individual risk profiles to be unobservable to savers, so that they base their valuation of bank debt on the anticipated credit risk of an average unit of bank debt. The return on bank debt for savers can be written as

$$
\tilde{R}_t^d = R_{t-1}^d - (1 - \kappa) \Omega_t,
$$

where $\Omega_t$ is the average default loss per unit of bank debt which will be defined below.

### 2.2.2 Impatient Households

Impatient households’ budget constraint is different from (4) in that they borrow, do not invest in capital, and do not receive transfers from entrepreneurs/bankers or capital management firms:

$$
c_{m,t} + q_{h,t} h_{m,t} - b_{m,t} \leq w_{l,m,t} + (1 - \Gamma_{m,t}(\bar{w}_{m,t})) R_t^H q_{h,t-1} h_{m,t-1} - T_{m,t},
$$

where $b_{m,t}$ is the overall amount of mortgage lending granted by banks, $R_t^H = (1 - \delta_{h,t}) q_{h,t}/q_{h,t-1}$ is the gross unlevered return on housing, $(1 - \Gamma_{m,t+1}(\bar{w}_{m,t})) R_t^H q_{h,t-1} h_{m,t-1}$ is net housing equity after accounting for the fraction of housing repossessed by the bank on the individual housing units that default on their mortgages, and $T_{m,t}$ is the lump-sum tax through which borrowers contribute to the funding of the DIA.

This formulation posits that individual household members default on their mortgages in period $t$ when the value of their housing units, $\omega_{m,t} R_t^H q_{h,t-1} h_{m,t-1}$, is lower than the outstanding mortgage debt, $R_{t-1}^M b_{m,t-1}$, that is when $\omega_{m,t} \leq \bar{\omega}_{m,t} = x_{m,t-1}/R_t^H$, where $R_t^M$ is the gross rate on the corresponding loan and $x_{m,t-1} = R_{t-1}^M b_{m,t-1}/(q_{h,t-1} h_{m,t-1})$ is a measure of household leverage at $t - 1$.

Importantly, the problem of the borrowing households includes a second constraint, the participation constraint of the bank, which reflects the competitive pricing of the loans that banks are willing to offer for different choices of leverage by the household:

$$
E_t \Lambda_{b,t+1} \left[ (1 - \Gamma_{M,t+1}(\bar{w}_{M,t+1})) (\Gamma_{m,t+1}(\bar{w}_{m,t+1}) - \mu m G_{m,t+1}(\bar{w}_{m,t+1}) R_t^H) \right] q_{h,t} h_{m,t} \geq v_{b,t} \phi_{M,t} b_{m,t}.
$$

This constraint is further explained in subsection 2.3.2.
2.3 Entrepreneurs and Bankers

At any point in time, each previously active entrepreneur \((\varphi = e)\) or banker \((\varphi = b)\) stays active with an independent probability \(\theta_\varphi\) and retires otherwise, becoming a worker \((\varphi = w)\) and transferring her net worth to the patient dynasty. At the same time, a mass \((1 - \theta_\varphi)x_\varphi\) of workers become new agents of class \(\varphi = e, b\), with an initial endowment \(v_{\varphi,t}\) provided by the patient dynasty (which, for simplicity, is modeled as an exogenous fraction \(\chi_\varphi\) of the wealth of retiring bankers). This guarantees that the size of the population of either class of agents remains constant at \(x_\varphi\) while the aggregate accumulated net worth of active entrepreneurs’ and bankers’ remains limited.\(^{10}\)

2.3.1 Entrepreneurs

Entrepreneurs invest their net worth into entrepreneurial firms and solve the following problem

\[
V_{e,t} = \max_{a_t, \text{div}_{e,t}} \{ \text{div}_{e,t} + E_t \Lambda_{s,t+1} [(1 - \theta_e) n_{e,t+1} + \theta_e V_{e,t+1}] \} \tag{8}
\]

\[
a_t + \text{div}_{e,t} = n_{e,t}
\]

\[
n_{e,t+1} = \int_0^\infty \rho_{f,t+1}(\omega) dF_{f,t+1}(\omega) a_t
\]

\[
\text{div}_{e,t} \geq 0
\]

where \(\Lambda_{s,t+1} = \beta_sc_{s,t}/c_{s,t+1}\) is the stochastic discount factor of the patient dynasty, \(n_{e,t}\) is the entrepreneur’s net worth, \(a_t\) is the part of the net worth symmetrically invested in the measure-one continuum of entrepreneurial firms further described below, \(\text{div}_{e,t}\) are dividends that the entrepreneur can pay to the saving dynasty before retirement, and \(\rho_{f,t+1}(\omega)\) is the rate of return on the entrepreneurial equity invested in a firm that experiences a return shock \(\omega\).

As in Gertler and Kiyotaki (2010), we guess that the value function is linear in net worth, \(V_{e,t} = v_{e,t}n_{e,t}\), where \(v_{e,t}\) is the shadow value of entrepreneurial equity. We guess (and verify) that around steady state \(v_{e,t} > 1\), in which case entrepreneurs only pay dividends upon retirement and their stochastic discount factor can be written as \(\Lambda_{c,t+1} = \Lambda_{s,t+1} [1 - \theta_e + \theta_e v_{e,t+1}]\).

\(^{10}\)This is a formally convenient way to capture firms’ and banks’ reluctance and difficulties to cut dividends and/or raise new equity, especially in bad times (see Mésonnier and Monks, 2015, Gropp et al., 2016, and Jiménez et al., 2017, for recent evidence).
Entrepreneurial firms operate across two consecutive dates, say \( t \) and \( t+1 \), and pay out their terminal net worth to entrepreneurs. Each firm takes equity \( a_t \) from entrepreneurs and borrows \( b_{f,t} \) from banks at interest rate \( R_{t}^{F} \) to buy physical capital from capital producers at \( t \). At \( t+1 \), the firm rents the available effective units of capital, \( \omega_{f,t+1}k_{t} \), to capital users and sells the depreciated capital back to capital producers. Each firm solves the following problem:

\[
\max_{k_t,R_{t}^{K}} E_t \Lambda_{e,t+1} (1 - \Gamma_{f,t+1} (\omega_{f,t+1})) R_{t+1}^{K} q_{k,t}k_{f,t} \tag{9}
\]

subject to the participation constraint of its bank

\[
E_t \Lambda_{b,t+1} (1 - \Gamma_{b,t+1} (\omega_{b,t+1})) \tilde{R}_{t+1}^{F} b_{f,t} \geq v_{b,t} \phi_{F,t} b_{f,t} \tag{10}
\]

where \( R_{t+1}^{K} = ((1 - \delta_{k,t+1}) q_{k,t+1} + r_{k,t+1}) / q_{k,t} \) is the gross return on capital, \( b_{f,t} = q_{k,t}k_{f,t} - a_t \) is the loan taken from the bank, and \( \tilde{R}_{t+1}^{F} b_{f,t} = (\Gamma_{f,t+1} (\omega_{f,t+1}) - G_{f,t+1} (\omega_{f,t+1})) R_{t+1}^{K} q_{k,t}k_{f,t} \) is the gross return that the bank obtains from a diversified portfolio of corporate loans. This constraint captures the competitive pricing of bank loans for different leverage choices by the firm. Further details on this constraint are explained in subsection 2.3.2.

### 2.3.2 Bankers

The problem of the representative banker is similar to the problem of the entrepreneurs with the only difference that bankers can invest their net worth \( n_{b,t} \) into equity of two classes \( j \) of competitive banks that extend loans \( b_{j,t} \) to either impatient households (\( j = M \)) or firms (\( j = F \)). For the two classes of banks to receive positive equity from bankers (\( e_{j,t} > 0 \)), the following no-arbitrage (or indifference) condition must hold:

\[
E_t [\Lambda_{b,t+1} \rho_{M,t+1}] = E_t [\Lambda_{b,t+1} \rho_{F,t+1}] = v_{b,t}, \tag{11}
\]

where \( \Lambda_{b,t+1} \) is bankers’ stochastic discount factor, \( \rho_{j,t+1} = \int_{0}^{\infty} \rho_{j,t+1} (\omega) dF_{j,t+1} (\omega) \) is the return of a diversified portfolio consisting of the equity of banks of type \( j \), and \( v_{b,t} \) is the shadow value of bankers’ net worth.

The representative bank of class \( j \) issues equity \( e_{j,t} \) among bankers and debt \( d_{j,t} \) that promises a gross interest rate \( R_{t}^{d} \) among patient households, and uses these funds to provide a continuum of identical loans of total size \( b_{j,t} \). This loan portfolio has a return \( \omega_{j,t+1} \tilde{R}_{t+1}^{j} \), where \( \omega_{j,t+1} \) is a log-normally distributed bank-idiosyncratic asset return shock and \( \tilde{R}_{t+1}^{j} \) denotes the
realized return on a diversified portfolio of loans of class \( j \). Banks live for a period and give back all their terminal net worth, if positive, to bankers next period.

The objective function of the representative bank of class \( j \) is:

\[
NPV_{j,t} = E_t \Lambda_{b,t+1} \max \left[ \omega_{j,t+1} \tilde{R}_{t+1}^{j} b_{j,t} - R_{t}^{d} d_{j,t}, 0 \right] - v_{b,t} e_{j,t},
\]

where the equity investment \( e_{j,t} \) is valued at its opportunity cost \( v_{b,t} \), and the max operator reflects the fact that the bank defaults when its net worth is negative. The bank is subject to the balance sheet constraint, \( b_{j,t} = e_{j,t} + d_{j,t} \), and the regulatory capital constraint, \( e_{j,t} \geq \phi_{j,t} b_{j,t} \), where \( \phi_{j,t} \) is the capital requirement on loans of class \( j \). In equilibrium, the capital requirement binds because partially insured debt financing is always “cheaper” than equity financing.

The threshold value of \( \omega_{j,t+1} \) below which the bank fails is \( \overline{\omega}_{j,t+1} = (1 - \phi_{j,t}) R_{t}^{d} / \tilde{R}_{t+1}^{j} \) and the probability of failure of a bank of class \( j \) is \( \Psi_{j,t+1} = F_{j,t+1}(\overline{\omega}_{j,t+1}) \). Thus, banks’ willingness to invest in loans with returns described by \( \tilde{R}_{t+1}^{j} \) and subject to a capital requirement \( \phi_{j,t} \) requires having

\[
E_t \Lambda_{b,t+1} \left[ 1 - \Gamma_{j,t+1}(\overline{\omega}_{j,t+1}) \right] \tilde{R}_{t+1}^{j} \geq \phi_{j,t} v_{b,t},
\]

which explains the so-called bank participation constraints provided in (7) and (10).

### 2.4 The Prudential Authority

The prudential authority sets the capital requirements on mortgages and corporate loans following simple policy rules:

\[
\phi_{M,t} = \phi_{M} + \tau_{M}(E_t \Psi_{m,t+1} - \Psi_{m}),
\]

\[
\phi_{F,t} = \phi_{F} + \tau_{F}(E_t \Psi_{f,t+1} - \Psi_{f}),
\]

where \( \phi_{j} \) and \( \tau_{j} \) determine the steady state level and the time-varying component of the requirements applied to loans of each class \( j = M, F \). These rules depend on the deviations of the expected default risk of each class of borrowers, \( E_t \Psi_{m,t+1} \) and \( E_t \Psi_{f,t+1} \), from their steady-state values, \( \Psi_{m} \) and \( \Psi_{f} \), in order to capture the way in which the IRB approach of Basel, combined

\[\text{11} This layer of idiosyncratic uncertainty captures the effect of bank-specific limits to diversification of borrowers’ risk (e.g. regional or sectoral specialization or large exposures) or shocks to unmodeled costs (IT, labor, liquidity management) or revenues (fee income, trading gains).

\[\text{12} In that case, the DIA repossesses \( (1 - \mu_{j}) \omega_{j,t+1} \tilde{R}_{t+1}^{j} b_{j,t} \) where \( \mu_{j} \) is an asset repossession cost, pays off insured deposits in full, and pays a fraction \( 1 - \kappa \) of reposed returns to the holders of uninsured debt.
with measures to mitigate its procyclicality (such as the use of through-the-cycle inputs in the IRB formulas or a CCyB), makes capital requirements vary with the expected probability of default (PD) of each class of loans.\footnote{Tax policies such as risk-sensitive deposit insurance premia, subsidies to equity financing, taxes on debt financing or taxes on specific forms of credit might also help control bank fragility or the supply of each class of loans. Without prejudicing the relative merits of these alternatives, we focus on capital requirements because they are the centerpiece of bank regulation in practice.}

### 3. Calibration

The model is calibrated at the quarterly frequency using Euro Area macroeconomic and financial data for the period 2001:1-2014:4. Table 1 reports the calibration targets.\footnote{All data sources and full details on the definition of some variables are described in the Online Appendix.}

In a first stage we set some parameters following convention. The rest of the parameters are found simultaneously so as to minimize a loss function that weights equally the relative distance between the targeted first and second empirical moments and the corresponding (unconditional) moments generated by the second-order approximation of the model. Using a second order approximation is important because aggregate and idiosyncratic shocks interact in determining moments related to borrowers’ default risk such as loans’ interest rate spreads and write-off rates.

The labor disutility parameter $\varphi$, which only affects the scale of the economy, is consequentially normalized to one for both classes of households. Following convention, we set the Frisch elasticity of labor $\eta$ equal to one, the share of capital in the production function $\alpha$ equal to 0.30, physical capital depreciation $\delta_k$ equal to 0.03, and savers’ discount factor $\beta_s$ equal to 0.995. The autoregressive coefficients in the AR(1) processes followed by all shocks are set equal to $\rho=0.90$ and all bankruptcy cost parameters are set equal to $\mu=0.30$.\footnote{Similar values for $\mu$ are used, among others, in Carlstrom and Fuerst (1997), which refers to the evidence in Alderson and Betker (1995), where estimated liquidation costs are as high as 36% of asset value. Among non-listed bank-dependent firms these cost can be expected to be larger than among the highly levered publicly traded US corporations studied in Andrade and Kaplan (1998), where estimated financial distress costs fall in the range from 10% to 23%. Our choice of 30% is consistent with the large foreclosure, reorganization and liquidation costs found in some of the countries analyzed by Djankov et al. (2008).}

Although second stage parameters are set simultaneously, most of them can be clearly linked with one of the target moments. Bankers’ endowment parameter, $\chi_b$, is used to match the median return on average equity (ROAE) in the systemically significant Euro Area banks.\footnote{https://www.bankingsupervision.europa.eu/ecb/pub/pdf/list_of_supervised_entities_20160101en.pdf}
### Table 1: Calibration Targets

<table>
<thead>
<tr>
<th>Description</th>
<th>Definition</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A) Stochastic means</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction of borrowers</td>
<td>$x_m$</td>
<td>0.437</td>
<td>0.437</td>
</tr>
<tr>
<td>Share of insured deposits</td>
<td>$\kappa$</td>
<td>0.54</td>
<td>0.54</td>
</tr>
<tr>
<td>Equity return of banks</td>
<td>$\rho \times 400$</td>
<td>6.734</td>
<td>9.278</td>
</tr>
<tr>
<td>Borrowers housing wealth share</td>
<td>$x_m q_h h_m$</td>
<td>0.525</td>
<td>0.495</td>
</tr>
<tr>
<td>Housing investment to GDP</td>
<td>$I_h/GDP$</td>
<td>0.060</td>
<td>0.062</td>
</tr>
<tr>
<td>HH loans to GDP</td>
<td>$x_m b_m /GDP$</td>
<td>2.120</td>
<td>2.126</td>
</tr>
<tr>
<td>NFC loans to GDP</td>
<td>$x_e b_f /GDP$</td>
<td>1.770</td>
<td>1.746</td>
</tr>
<tr>
<td>Write-off HH loans</td>
<td>$\Upsilon_m \times 400$</td>
<td>0.118</td>
<td>0.205</td>
</tr>
<tr>
<td>Write-off NFC loans</td>
<td>$\Upsilon_f \times 400$</td>
<td>0.650</td>
<td>0.640</td>
</tr>
<tr>
<td>Spread HH loans</td>
<td>$(R^M - R^d) \times 400$</td>
<td>0.821</td>
<td>0.450</td>
</tr>
<tr>
<td>Spread NFC loans</td>
<td>$(R^F - R^d) \times 400$</td>
<td>1.080</td>
<td>1.148</td>
</tr>
<tr>
<td>Capital held by saving households</td>
<td>$k_s / k$</td>
<td>0.220</td>
<td>0.223</td>
</tr>
<tr>
<td><strong>B) Standard deviations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std(House prices)/std(GDP)</td>
<td>$\sigma(q_{h,t}) / \sigma(GDP_t)$</td>
<td>2.668</td>
<td>2.420</td>
</tr>
<tr>
<td>Std(HH loans)/std(GDP)</td>
<td>$\sigma(x_m b_{m,t}) / \sigma(GDP_t)$</td>
<td>2.413</td>
<td>2.943</td>
</tr>
<tr>
<td>Std(NFC loans)/std(GDP)</td>
<td>$\sigma(x_e b_{f,t}) / \sigma(GDP_t)$</td>
<td>3.806</td>
<td>5.757</td>
</tr>
<tr>
<td>Std(Write-offs HH)/std(GDP)</td>
<td>$\sigma(\Upsilon_{m,t}) / \sigma(GDP_t)$</td>
<td>0.012</td>
<td>0.009</td>
</tr>
<tr>
<td>Std(Write-offs NFC)/std(GDP)</td>
<td>$\sigma(\Upsilon_{f,t}) / \sigma(GDP_t)$</td>
<td>0.050</td>
<td>0.027</td>
</tr>
<tr>
<td>Std(Spread HH loans)/std(GDP)</td>
<td>$\sigma(R^M - R^d) / \sigma(GDP_t)$</td>
<td>0.056</td>
<td>0.069</td>
</tr>
<tr>
<td>Std(Spread NFC loans)/std(GDP)</td>
<td>$\sigma(R^F - R^d) / \sigma(GDP_t)$</td>
<td>0.045</td>
<td>0.082</td>
</tr>
<tr>
<td>Std(GDP)</td>
<td>$\sigma(GDP_t) \times 100$</td>
<td>2.310</td>
<td>2.617</td>
</tr>
</tbody>
</table>

Series expressed in Euro amounts are deflated and their log value is linearly detrended before computing standard deviation targets. Targets for ratios and rates of return are found after linearly detrending the original series. Interest rates, equity returns, write-offs, and spreads are reported in annualized percentage points. The standard deviation (std) of GDP is in quarterly percentage points. Abbreviations HH and NFC stand for households and non-financial corporations, respectively, and are used to refer to mortgage and corporate loans in brief form.

We calibrate the share of borrowers in the economy $x_m$ to match the proportion of indebted households in the Euro Area of 44%, as documented in the 2010 ECB Household Finance and Consumption Survey (HFCS).\(^{17}\) The weight on housing in the utility of borrowers $\nu_m$ matches the share of housing held by the indebted and non-indebted households in the Euro Area whereas the one of the savers, $\nu_s$, is normalized to one.\(^{18}\)


\(^{18}\)In terms of the 2010 HFCS, housing wealth is defined as the value of the household’s main residence + other real estate – other real estate used for business activities.
Borrowers’ discount factor $\beta_m$ and new entrepreneurs’ endowment parameter $\chi_e$ help to match the ratios of household (HH) mortgages to GDP and bank loans to non-financial corporations (NFC) to GDP.\textsuperscript{19} The housing depreciation rate $\delta_h$ is calibrated to match the ratio of residential investment to GDP. The share of insured deposits in bank debt $\kappa$ is set to 0.54 in accordance with the evidence by Demirgüç-Kunt, Kane, and Laeven (2014) for EA countries. The capital management cost parameter $\xi$ is pinned down so as to match the share of physical capital directly held by savers in the model with an estimate, based on EA flow of funds data, of the proportion of assets of the NFC sector whose financing is not supported by banks.

The variance of the four idiosyncratic shocks, the housing and capital adjustment cost parameters and the variance of the seven aggregate shocks, including the risk shocks affecting the variance of the idiosyncratic asset return shocks, are mainly useful to match the remaining targets.\textsuperscript{20} We match the average write-off rates and the spreads between the loan rate and the risk free rate for both types of loans. The volatility of the productivity shock helps us to match the volatility of GDP. We also match the volatility of house prices, HH loans, NFC loans, and of the write-offs rates and spreads of each type of loans. As shown in Table 1, we match the targeted moments very closely.

### 3.1 Calibrating the Capital Requirements

Given the sample period used for the calibration, we find the baseline values of the capital requirement level parameters, $\phi_M$ and $\phi_F$, by feeding the regulatory formula of the IRB approach of Basel II with the steady state values of the annual PD of the corresponding loans, $\Psi_m$, and $\Psi_f$.\textsuperscript{21} According to BCBS (2004), these formulas are

$$\phi_M = 0.45 \left[ \Phi \left( \frac{\Phi^{-1}(\Psi_m) + 0.15^{0.5} \Phi^{-1}(0.999)}{(1 - 0.15)^{0.5}} \right) - \Psi_m \right],$$

and, assuming corporate exposures have a one year maturity,\textsuperscript{22}

$$\phi_F = 0.45 \left[ \Phi \left( \frac{\Phi^{-1}(\Psi_f) + \sigma_f^{0.5} \Phi^{-1}(0.999)}{(1 - \sigma_f)^{0.5}} \right) - \Psi_f \right],$$

\textsuperscript{19}To avoid the counterintuitive impact of the resource costs of default on measured output, we define $GDP_t = c_t + I_{h,t} + I_{k,t}$. A more comprehensive definition of aggregate output $Y_t$ is provided in the Online Appendix.

\textsuperscript{20}The model is calibrated assuming that the two types of banks experience a common risk shock, with standard deviation $\sigma_b$, and have the same unconditional default probability.

\textsuperscript{21}Under our calibration mean annual PDs for mortgage and corporate loans are 0.66% and 1.7%, respectively.

\textsuperscript{22}In the euro area 90% of NFC loans are of maturities of one year or less.
where we have fixed the loss-given-default (LGD) parameters to their regulatory value of 0.45 under the “foundation IRB” approach. $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal distribution, and $\varrho_f$ is a correlation parameter that for corporate exposures Basel II mandates to fix as $\varrho_f = 0.24 - 0.12(1 - \exp(-50\Psi_f))/(1 - \exp(-50))$.

Regarding the parameters $\tau_M$ and $\tau_F$ that control the time variation of the capital requirements in the policy rules (14) and (15), we set them equal to zero as under the through-the-cycle approach that regulators postulate.\(^{23}\)

### 3.2 Resulting Parameters

Table 2 reports all the parameter values resulting from our calibration. The preference and technology parameters we find are in line with the values used by other authors. Borrowers’ discount factor falls within the two standard deviation bands estimated by Carroll and Samwick (1997).

The mean standard deviation of the idiosyncratic shocks to housing and entrepreneurial asset returns needed to match the data happen to be much larger than that of the idiosyncratic shocks to bank asset returns. In contrast, the volatility of the aggregate risk shocks is larger for bank asset returns than for the households’ and entrepreneurs’ asset returns. This confirms the view that banks are good at diversifying idiosyncratic risk but highly exposed to aggregate risk. The standard deviations of the productivity shock and housing preference shocks are not too different from what is estimated in other papers.\(^{24}\) The calibrated standard deviations of the housing and capital depreciation shocks are also in line with conventional values.

Using (16) and (17), our baseline calibration yields values of the sectorial capital requirements $\phi_M$ and $\phi_F$ equal to 3.4% and 7.2%, respectively. The calibration also implies an untargeted yearly average bank default rate of 1.53% and a risk free rate of about 2%.\(^{25}\)

---

\(^{23}\)For values of $\tau_M$ and $\tau_F$ that reproduce a strict point-in-time approach (matching the derivatives of the IRB formulas with respect to $\Psi_m$ and $\Psi_f$), extreme fluctuations in the requirements produce unstable credit supply and bank net worth dynamics. This forces the solution of the model into regions of indeterminacy. In the normative analysis we explore ranges of $\Psi_m$ and $\Psi_f$ for which the model solves well.

\(^{24}\)See, e.g. Iacoviello and Neri (2010) and Jermann and Quadrini (2012).

\(^{25}\)This rate does not seem excessive for a period that includes a severe bank crisis. Indeed, Moody’s average yearly expected default frequencies (EDFs) for Euro Area banks in the period 2008-2014 stand well above 2%.
Table 2: Parameters Values

<table>
<thead>
<tr>
<th>Description</th>
<th>Par.</th>
<th>Value</th>
<th>Description</th>
<th>Par.</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) Pre-set parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Housing weight in s utility</td>
<td>( \nu_s )</td>
<td>0.1</td>
<td>HH bankruptcy cost</td>
<td>( \mu_m )</td>
<td>0.3</td>
</tr>
<tr>
<td>Disutility of labor (( \kappa = s, m ))</td>
<td>( \varphi_\kappa )</td>
<td>1</td>
<td>NFC bankruptcy cost</td>
<td>( \mu_f )</td>
<td>0.3</td>
</tr>
<tr>
<td>Frisch elasticity of labor</td>
<td>( \eta )</td>
<td>1</td>
<td>Bank M bankruptcy cost</td>
<td>( \mu_M )</td>
<td>0.3</td>
</tr>
<tr>
<td>Capital share in production</td>
<td>( \alpha )</td>
<td>0.3</td>
<td>Bank F bankruptcy cost</td>
<td>( \mu_F )</td>
<td>0.3</td>
</tr>
<tr>
<td>Capital depreciation</td>
<td>( \delta_k )</td>
<td>0.03</td>
<td>Entrepreneurs’ survival rate</td>
<td>( \theta_e )</td>
<td>0.975</td>
</tr>
<tr>
<td>Savers’ discount factor</td>
<td>( \beta_s )</td>
<td>0.995</td>
<td>Bankers’ survival rate</td>
<td>( \theta_b )</td>
<td>0.975</td>
</tr>
<tr>
<td></td>
<td>Shocks persistence (all ( \phi ))</td>
<td>( \rho_\phi )</td>
<td>0.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B) Calibrated parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction of borrowers</td>
<td>( x_m )</td>
<td>0.437</td>
<td>Share of insured deposits</td>
<td>( \kappa )</td>
<td>0.54</td>
</tr>
<tr>
<td>Borrowers’ discount factor</td>
<td>( \beta_m )</td>
<td>0.971</td>
<td>Entrepreneurs’ endowment</td>
<td>( \chi_e )</td>
<td>0.3666</td>
</tr>
<tr>
<td>Housing weight in m utility</td>
<td>( \nu_m )</td>
<td>0.202</td>
<td>Bankers’ endowment</td>
<td>( \chi_b )</td>
<td>0.1032</td>
</tr>
<tr>
<td>Housing adjustment cost</td>
<td>( \psi_h )</td>
<td>2.422</td>
<td>Capital managerial cost</td>
<td>( \xi )</td>
<td>0.0014</td>
</tr>
<tr>
<td>Housing depreciation</td>
<td>( \delta_h )</td>
<td>0.012</td>
<td>Capital adjustment cost</td>
<td>( \psi_k )</td>
<td>4.567</td>
</tr>
<tr>
<td>Std. productivity shock</td>
<td>( \sigma_z )</td>
<td>0.0316</td>
<td>Std. housing pref. shock</td>
<td>( \sigma_\lambda )</td>
<td>0.061</td>
</tr>
<tr>
<td>Mean std of iid HH shocks</td>
<td>( \sigma_{w_m} )</td>
<td>0.069</td>
<td>Std. housing depr. shock</td>
<td>( \sigma_{\delta_h} )</td>
<td>0.002</td>
</tr>
<tr>
<td>Mean std of iid NFC shocks</td>
<td>( \sigma_{w_f} )</td>
<td>0.399</td>
<td>Std. capital depr. shock</td>
<td>( \sigma_{\delta_k} )</td>
<td>0.002</td>
</tr>
<tr>
<td>Mean std of iid M bank shocks</td>
<td>( \sigma_{w_M} )</td>
<td>0.012</td>
<td>Std. HH risk shock</td>
<td>( \sigma_m )</td>
<td>0.001</td>
</tr>
<tr>
<td>Mean std of iid F bank shocks</td>
<td>( \sigma_{w_F} )</td>
<td>0.027</td>
<td>Std. NFC risk shock</td>
<td>( \sigma_f )</td>
<td>0.039</td>
</tr>
<tr>
<td></td>
<td>Std. banks’ risk shocks</td>
<td>( \sigma_b )</td>
<td>0.059</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The parameters in A) are set to standard values in the literature, whereas those in B) are calibrated to match the data targets. Abbreviations HH and NFC stand for (borrowing) households and non-financial corporations, respectively.

4. Determinants of Bank Lending Standards

The competitive pricing of loans is summarized by the bank’s participation constraints in each of the borrowers’ problems (equations (7) and (10)). These constraints establish the combination of loan rates and borrower leverage that guarantee sufficient returns on the equity funding that bankers provide to banks. For borrowers, each participation constraint is the loan pricing schedule that determines the interest rate they must pay for each given leverage choice.

The solid lines in Figure 1 depict the relevant participation constraints at the steady state of the baseline calibration. We produce the curves in partial equilibrium: with debt funding rates, the shadow value of bankers wealth, and the aggregate determinants of bank and borrower...
default risk fixed at their steady state levels. At low borrower leverage, the loan rate is locally insensitive to leverage because the borrower’s probability of default is essentially zero. As leverage increases further, the default probability begins to rise and the loan interest rate increases to compensate for expected credit losses. From an aggregate perspective, the position of this schedule describes the “lending standards”. When the curve shifts downwards (upwards), banks loosen (tighten) lending standards and credit supply expands (shrinks).

Bank fragility affects lending standards due to the limited liability distortion, which arises because the pricing of bank debt (irrespective of the fraction which is insured) occurs before the bank makes the lending decisions that shape its own risk profile (based on a bank’s expected risk profile rather than its actual lending choices). The effects of this distortion can be understood by examining the comparative statics of the loan pricing schedules in Figure 1. In particular, the dash-and-dotted lines show the loan pricing schedules that emerge under higher values of the standard deviation of the bank-idiosyncratic asset return shocks, $\sigma_M$ and $\sigma_F$, respectively. Lending standards of each type of bank are relaxed because bankers’ limited liability gains are, by force of competition, passed to borrowers in the form of cheaper or riskier loans.

As shown by the dashed lines in Figure 1, raising the capital requirements produces the opposite (partial equilibrium) effects. The bank reduces its leverage and relies on more expensive equity funding. Bank default risk falls and, with it, the implicit safety net subsidy. The result is an increase in the bank’s weighted average cost of funding which is passed on to borrowers in the form of tighter lending standards.

Finally, the dotted lines in Figure 1 describe the effect of an increase in idiosyncratic borrower risk. Higher borrower risk makes the loan supply schedule steeper simply because it rises borrowers’ idiosyncratic default risk, making loans less profitable to the bank.

5. Capital Regulation and Welfare

In the following sections we analyze the welfare and real economy consequences of capital requirement policies in detail. We focus on the impact of policy parameters changes on the welfare of each class of agents and the extent to which capital requirement policies help stabilize the impact of aggregate shocks. Importantly, using a second-order approximation to solve the model allows us to take the effects of aggregate uncertainty into account.
5.1 The Impact of Capital Requirements on Savers and Borrowers

We first analyze the effect of ceteris paribus changes in the level of the capital requirements applicable to each class of loans, \( \phi_M \) and \( \phi_F \), on the welfare of savers and borrowers. We measure such welfare as the expected lifetime utility \( V_{\kappa,t} \) of each of the dynasties, \( \kappa = s, m \), which can be written in a recursive form as:

\[
V_{\kappa,t} = U(c_{\kappa,t}, h_{\kappa,t}, l_{\kappa,t}) + \beta_{\kappa} E_t V_{\kappa,t+1}.
\] (18)

Figure 2 reports the impact on welfare of varying each \( \phi_j \) while keeping the other fixed at its calibrated baseline value. To help understand these effects, Figure 3 shows how key equilibrium variables vary with each of these parameters.

Column A of Figure 2 shows the welfare impact of changing the requirement on mortgage loans \( \phi_M \). Savers’ welfare increases monotonically with \( \phi_M \), whereas borrowers’ welfare first increases and then decreases with it. A higher \( \phi_M \), by reducing bank leverage, reduces the probability of bank failure and, thus, deposit insurance costs and the bank debt spread (see Row A of Figure 3), which, other things equal, is good for both savers and borrowers. Tightening capital requirements also corrects the limited liability distortions and forces banks to use a larger fraction of (more expensive) equity financing, which, other things equal, tightens the supply of loans and is bad for borrowers.

The net effect of lowering debt funding costs while imposing a larger use of scarce equity funding makes credit supply not necessarily decreasing in \( \phi_M \). In fact, at low levels of \( \phi_M \), the sharp decline in bank failure risk and the cost of deposit insurance, together with the small effect on credit supply makes borrowers’ welfare increasing in \( \phi_M \). However, for \( \phi_M \) larger than about 5% (see Figure 3), the probability of failure of mortgage banks is already close to zero, so further increases in \( \phi_M \) do not decrease average default rates, deposit insurance costs or bank debt spreads further. Instead, they tighten the supply of mortgage loans, damaging the borrowers. Meanwhile, savers continue benefiting from increasing this capital requirement as it allows them to appropriate higher returns on bankers’ equity holdings.

The welfare impact of changing the level of the capital requirement applied to corporate loans, \( \phi_F \), is qualitatively similar to that of changing \( \phi_M \) (see Column B of Figure 2) and responds to the same logic (see Row B of Figure 3). The main differences with respect to changing \( \phi_M \) are quantitative. Starting from its higher baseline value of 7.2%, increasing \( \phi_F \)
has a smaller effect on bank failure risk, deposit insurance costs, and the bank debt spread. The smaller size of the latter explains that increasing $\phi_F$ does not have the same initial expansionary effect on total credit and borrowers’ consumption as increasing $\phi_M$. Yet, the reduction in deposit insurance costs and volatility induced by bank fragility (as we further discuss below) are enough to make borrowers’ welfare initially increasing in $\phi_F$. Once the impact on bank failure risk is exhausted, further increases in $\phi_F$ reduce borrower welfare. At that point, savers also experience some losses due to lower bank-funded investment but these are offset by the higher returns on bankers’ equity holdings. More specifically, the increase in $\phi_F$ increases the demand for bank equity, raising its cost and making credit more expensive. This reduces entrepreneurs’ capital accumulation, hurting both borrowers and savers via lower wage income. However, savers receive a larger offsetting benefit in the form of higher transfers from (retiring) bankers. So savers also prefer a higher capital requirement on corporate loans than borrowers.

5.2 Optimized Capital Requirement Rules

We now turn to the normative analysis. What would be the socially optimal choice of the level parameters $\phi_M$ and $\phi_F$, and the PD-sensitivity parameters, $\tau_M$ and $\tau_F$ that appear in the policy rules (14) and (15)? We address this question by identifying the policy parameters that maximize a social welfare function defined as a weighted average of the expected lifetime utility of the two classes of households:

$$\tilde{V}_t \equiv [\zeta V_{s,t} + (1 - \zeta) V_{m,t}]$$

where $\zeta \in [0, 1]$ is the weight on savers’ welfare. Since with heterogenous agents and incomplete markets there is no commonly accepted criterion for the choice of the weights assigned to each agent, we analyze what happens for different values of $\zeta$. This is equivalent to exploring the Pareto frontier that can be reached by optimizing on our capital requirement policy rules.

For each weight $\zeta$, we search over a multidimensional grid with the following dimensions: $\phi_M \in [0.02, 0.2]$, $\phi_F \in [0.05, 0.2]$, and $\tau_j \in [0, 5]$ for $j = M, F$. As seen previously, changing policy parameters can increase the welfare of one of the two classes of agents while decreasing the welfare of the other. Thus, in some cases, maximizing the weighted sum of the welfare of the two groups of agents may generate outcomes that worsen the situation of one of the groups relative to the initially calibrated policy rule. To avoid such a redistributional impact, we
constrain the social welfare maximization problem so as to ensure that the solution constitutes a Pareto improvement relative to the calibrated policy rule.

The solid line in Figure 4 displays the optimal capital requirements for corporate loans and mortgages for each value of $\zeta \in [0, 1]$, the implied average capital-to-asset ratio for banks, and the optimal sensitivities to the PD of each class of loans. In parallel, the solid line in Figure 5 displays the associated welfare gains for savers and borrowers as a function of $\zeta$. We report welfare in terms of a consumption-equivalent measure calculated as the percentage increase in steady state consumption that would make each class of agents’ welfare under the initially calibrated policy equal to their welfare under the optimized policy rule. Larger $\zeta$ increase the welfare gains of the savers and diminishes those of the borrowers.

The welfare gains of both classes of agents are strictly positive for all $\zeta$ lower than about 0.40. Even with $\zeta = 0$ (i.e. when the policymaker only maximizes borrowers’ welfare), savers obtain gains equivalent to a non-negligible 0.5% permanent consumption increase (while borrowers gain the equivalent to a 0.65%). Interestingly, for $\zeta = 0.26$, the optimal policy yields exactly the same consumption equivalent gains (as a percent of their baseline values) for savers and borrowers. Without prejudice of the normative merit of this specific solution, we select it as our benchmark optimized policy and use it below to analyze further properties of the model.26

5.3 Optimal Sectoral Capital Requirement Curves

The implications of the normative analysis are straightforward. Regardless of the weight on savers’ welfare, capital requirements should be higher than in the baseline calibration (Basel II). Putting a higher weight on savers’ welfare leads to higher capital charges on both mortgages and corporate loans. In addition, the higher the weight on savers’ welfare, the larger the optimal sensitivity of capital charges to the time variation in the PD of each class of loans. In other words, savers benefit less than borrowers from containing the cyclicality that the use of point-in-time PDs in the IRB formulas might impose on capital requirements.

We put our results into perspective by comparing the capital requirement curves generated by the Basel IRB formulas, (16) and (17), when fed with values of the corresponding PDs, with the curves associated with our linear policy rules, (14) and (15), under the optimized values

26 See the Online Appendix for details on the sensitivity of the results to changes in some of the calibrated parameters.
of their parameters. The solid lines in each of the panels of Figure 6 describe the IRB curves, which we depict for ranges of PDs centered at their mean values of $\Psi_m$ and $\Psi_f$ in the baseline calibration and covering from minus two to plus two standard deviations around them. The dotted line in each figure describes the optimized policy rule that emerges when all the weight is put on savers’ welfare (that is, for $\zeta = 1$ or, given the binding Pareto improvement constraint, for any $\zeta > 0.40$). In contrast, the dashed line describes the optimized policy rule when all the weight is put on borrowers’ welfare ($\zeta = 0$).

The results imply that for both classes of loans, the average capital charges should be higher and less responsive to PDs than those emerging from a point-in-time implementation of the IRB formula (since the slopes of the optimal schedules are lower than those implied by (16) and (17)). In the case of mortgage loans (panel on the left), borrowers prefer an essentially flat curve, but with an average level about one percentage point higher than in Basel II. Borrowers dislike the time variation induced by the sensitivity of capital requirements to the PD of the loans since it reinforces the cyclical variation in lending standards, which damages their ability to smooth consumption.

Savers’ favorite policy would increase the average charge on mortgages by at least three more percentage points, more than doubling the Basel II level. Interestingly, such a policy involves some sensitivity to time variation in the corresponding PD, but significantly less than under a point-in-time implementation of the IRB formula. At the margin, savers also benefit from countercyclical adjustments such as those brought by the use of through-the-cycle inputs in the IRB formula or the introduction of the countercyclical capital buffer in Basel III.

In the case of corporate loans (panel on the right), savers’ and borrowers’ favorite optimized policies agree even more clearly on having average capital charges higher than under Basel II. Discrepancies regarding the sensitivity of these charges to time variation in the corresponding PD are larger: borrowers prefer no cyclicality, while savers’ preferred policy features roughly the same cyclicality as a point-in-time implementation of the IRB formula.

If one accepts the original IRB as representing the capital requirements considered adequate from a strict microprudential perspective, we can interpret the results in Figure 6 as revealing the time-invariant and time-varying adjustments which should be added from a macroprudential perspective. Consider, for instance, borrowers’ optimal curves. Under a perfect through-the-cycle implementation of the IRB approach, the results imply the need for adding a static buffer.
for mortgage (corporate) loans of about 1 (2) percentage point(s) relative to Basel II.\textsuperscript{27}

If instead the IRB approach is implemented using point-in-time PDs, our results imply a need for further action on the macroprudential front. In good times (when the PDs are at the lowest values in the depicted ranges) the macroprudential buffer should be 4 (7) percentage points higher for mortgage (corporate) loans than in bad times (when PDs are at the highest values in the depicted ranges). In practical terms, this means that good macroprudential policy cannot be conducted without awareness of relevant developments at the microprudential level.

6. Sources of Welfare Gains

This paper explores the role of capital regulation policy in the presence of a rich stochastic structure. The optimal policy from a macroprudential perspective is not ex ante targeted to smooth one particular source of fluctuations. However, we now assess how the several shocks in the model contribute to the welfare gains achieved under the optimal policy. To this purpose, we consider the welfare gains associated to the benchmark optimized policy ($\zeta = 0.26$) and shut down one or several aggregate shocks at a time and re-compute savers’ and borrower’s welfare gains (without re-optimizing on the underlying capital policy). The difference between the welfare gains when all shocks are present and when some shocks are shut down gives us a sense of the shocks whose accommodation matters the most for the welfare gains associated with the optimal capital requirement policy. As some of these shocks are more microeconomic (idiosyncratic) than macroeconomic (aggregate) in nature, the analysis will shed light on the relative importance of the “microprudential” versus “macroprudential” trade-offs in the setting of optimal capital rules.

Table 3 reports the welfare gains of savers and borrowers (in percent of permanent consumption) when all the shocks are present (part (i)) and when when we shut down all or each of the three risk shocks affecting the dispersion of the idiosyncratic shocks that drive default risk (part (ii)). It also shows the welfare gains that remain when we shut down aggregate shocks other than the risk shocks (part (iii)) and when we shut down all sources of aggregate uncertainty (part (iv)). In the last case, the model only contains the idiosyncratic shocks that make each

\textsuperscript{27}Interpreting these extra buffers as similar to the new Capital Conservation Buffer (CCB) of Basel III would be tempting but imprecise, since the CCB is subject to bank idiosyncratic dynamics and interferes with banks’ payout policies while our extra buffers are simple add-ons to the IRB requirements.
Table 3: Welfare Gains

<table>
<thead>
<tr>
<th></th>
<th>Savers</th>
<th>Borrowers</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) All shocks</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>(ii) No risk shocks</td>
<td>0.44</td>
<td>0.15</td>
</tr>
<tr>
<td>- No bank return risk shocks</td>
<td>0.46</td>
<td>0.21</td>
</tr>
<tr>
<td>- No housing return risk shocks</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>- No entrepreneurial capital return risk shocks</td>
<td>0.59</td>
<td>0.51</td>
</tr>
<tr>
<td>(iii) No other shocks</td>
<td>0.60</td>
<td>0.57</td>
</tr>
<tr>
<td>(iv) No aggregate uncertainty</td>
<td>0.43</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Second-order approximation to the welfare gains (% of permanent consumption) associated with the benchmark optimized policy of the full model across experiments in which none, several or all aggregate shocks are shut down. The distance between the welfare gains when all shocks are present and when some shocks are shut down identifies how important the stabilization of each shock is for the overall welfare gains.

class of borrowers potentially default, so capital regulation is restricted to microprudential goals whose implications for welfare we still assess in a full general equilibrium setting.

Two are the main results of this analysis. First, the large reduction in borrowers’ welfare gains in the absence of risk shocks. This means that stabilizing the impact of these shocks (a “macroprudential” dimension) makes a large contribution to borrowers’ welfare. Risk shocks to bank returns on the loan portfolio, bring the largest gains: they contribute by 65% and 23% to borrowers’ and savers’ welfare gains, respectively.\(^{28}\) Dampening risk shocks to entrepreneurial capital returns is important for borrowers (15% of welfare gains) but not for savers. Somewhat surprisingly, mitigating risk shocks to housing returns does not generate large welfare gains.\(^{29}\)

Second, a large fraction of savers’ welfare gains (67%) remain even after aggregate uncertainty is shut down. See part (iv) in Table 3. Thus, the benchmark optimized policy benefits the savers largely through “microprudential” gains. The root of such gains is the way capital requirements prevent bank failure. As bank debt pricing is not sensitive to bank risk at the margin, banks fail to internalize part of the bankruptcy costs caused by high leverage and capital requirements can contribute to reduce the implied deadweight losses. Savers appropriate

\(^{28}\)Aggregate and idiosyncratic shocks account for about 40 and 60 per cent of bank default, respectively. Thus, aggregate shocks are important drivers of bank fragility in the model.

\(^{29}\)Unlike entrepreneurial risk shocks (which transmitted to the economy via capital accumulation and wages), the housing risk shock only affects the borrowers, who, faced with larger default risk, reduce their leverage, putting downward pressure on house prices. Savers, however, increase their housing demand in an offsetting manner and the aggregate effects turn out to be small.
part of the saved resources through the reduction in the taxes needed to cover deposit insurance costs and through the increased returns on bank equity, which are in turn the result of a more restricted supply of credit. Borrowers’ welfare gains in the absence of aggregate uncertainty are more limited (less than 20% of their total gains), because the benefits from lower bank default risk (e.g. lower taxes to cover deposit insurance costs) are offset by the higher cost of bank loans. On net, borrowers benefit mainly from the stabilization of aggregate risk, whereas savers benefit from a better absorption of both aggregate and idiosyncratic risk.

To complete this discussion, Figures 7 and 8 show how the economy reacts to risk shocks to bank asset returns and entrepreneurial capital returns, which are the shocks whose accommodation under the optimized policy makes a larger contribution to welfare gains. Both shocks have a positive impact on bank default probabilities under the baseline calibrated policy (solid lines). The degree of bank fragility, with its impact on the bank debt spread, bankers’ net worth, and, through it, the supply of loans, is key to the transmission of these shocks. The contraction in credit supply is transmitted to the real economy in the form of lower investment, depressed wages, and, eventually, lower consumption and lower GDP.

Under the benchmark optimized policy (dashed lines) both risk shocks are significantly dampened. Higher capital requirements (as well as a lower sensitivity of the requirements to time variation in PDs) make the economy more resilient to these shocks. The optimized policy almost completely offsets the effects of a bank risk shock (Figure 7): the high level of capital requirements, by keeping bank defaults and bankers’ net worth losses close to zero, avoids the contractionary impact of the rise in bank funding costs and the fall in credit supply that would have otherwise occurred. In contrast, the capacity of capital requirement policy to dampen the impact of entrepreneurial risk shocks is more limited (Figure 8). Banks’ solvency gets somewhat better protected with higher capital requirements but such protection does not fully eliminate the contractionary impact of the shock.\footnote{Entrepreneurial risk shocks affect entrepreneurs' default risk and, as shown in Figure 1, banks tighten their lending standards when such risk increases, even if no bank capital has been lost.}

7. Conclusions

This paper examines the optimal calibration of Basel-type dynamic capital requirement rules in the context of a macroeconomic model with banks. The analysis addresses the conflicting
interests of savers and borrowers regarding the adequate level of capital for each class of loans and its time variation in response to changes in loan default risk.

We find that if capital requirements start from low levels, as under the pre-crisis Basel II regime, both savers and borrowers gain from increasing them. Capital requirements should be higher, especially for mortgage loans, and should feature less sensitivity to time-variation in loan PDs compared to using point-in-time PD estimates for the current IRB formulas. Borrowers benefit more than savers from dampening the potential procyclical effects of such IRB formulas.

A close look at the optimized policy rules uncovers that the most important aspect of capital requirement policy is to ensure that bank default is close to zero. Micro- and macroprudential considerations seem aligned in this respect. Having resilient banks minimizes the deadweight costs of bank defaults and shuts down bank-related amplification channels, thus stabilizing the reaction of the economy to aggregate shocks. Our results confirm that capital charges for a typical mortgage should generally be lower than capital charges for a typical corporate loan, but the differences between those charges under the optimized policy rules are lower than those implied by the current IRB formulas.

The parameters that control the extent to which the capital requirements vary in response to time variation in the default risk of the corresponding loans have a less sizeable impact on social welfare since they affect volatilities and, hence, second order terms. Our results suggest that such variation benefits savers at the expense of borrowers because it helps to keep banks safe but destabilizes borrowers’ consumption. Borrowing households dislike the time variation induced by PD-sensitive capital charges because it increases their borrowing costs in states of the world in which their consumption is already low. This finding supports the use of ‘through-the-cycle’ as opposed to ‘point-in-time’ default frequencies when computing bank capital requirements over the cycle. Alternatively, it supports a greater use of the countercyclical capital buffer.

In our analysis we have abstracted from a number of important considerations that would constitute fruitful avenues for future research. Introducing occasionally binding bank capital constraints could allow the examination of non-linearities producing crisis amplification and asymmetries. Modelling discrete equity issuance and dividend payouts would make the model more suitable for the examination of the costs and benefits of restrictions on banks’ payout policies. Assessing the extent to which borrower-based instruments such as loan-to-value or loan-to-income limits could complement capital-based policies would also be very interesting.
References


Figure 1. Determinants of Bank Lending Standards. Bank lending standards at calibrated parameters (solid line), higher standard deviation of the idiosyncratic shocks to loan portfolio returns (dashed-dotted line), higher capital ratios (dashed line), and higher standard deviation of idiosyncratic shocks to borrowers’ asset returns (dotted line). Vertical lines identify borrowers’ leverage under the baseline calibration.

Figure 2. Welfare Impact of Changes in Capital Requirement Levels. Savers’ and borrowers’ welfare are depicted as functions of the policy parameter determining the level of the capital requirements applicable to mortgage loans (column A) and corporate loans (column B). While changing one parameter, we keep the other equal to its calibrated value.
Figure 3. Impact of Changes in Capital Requirement Levels on Key Variables. Stochastic means of key variables as function of the policy parameter determining the level of the capital requirements applicable to mortgage loans (row A) and corporate loans (row B). While changing one parameter, we keep the other equal to its calibrated value. The probability of bank default and the bank debt spread are in annualized percentage terms, the deposit insurance cost is measured as percentage of GDP.

Figure 4. Optimal Dynamic Capital Requirements. Parameters characterizing the welfare maximizing policy rule and the implied average capital-to-asset ratio for each banks are depicted as functions of the weight $\xi$ that the maximized social welfare measure puts on savers’ welfare. The two panels on the left describe the optimized values of the parameters that determine the average level of the capital requirements for mortgage (HH) and Corporate (NFC) loans. The two panels on the right describe the optimized PD sensitivities of the requirements to time changes in the PDs of the corresponding loans.
Figure 5. Welfare Gains. Individual welfare gains implied by the optimal policy corresponding to each value of the weight $\xi$ that the maximized social welfare measure puts on savers’ welfare under the baseline and alternative values of key parameters. The gains are measured in consumption-equivalent terms, as the percentage increase in the consumption of each agent that would make his welfare under the initially calibrated policy rule equal to his welfare under each optimized policy rule.

Figure 6. Basel vs. Optimal Capital Requirements. The solid line depicts the capital requirements (CR) curve implied by the formula of the internal ratings based (IRB) approach of Basel II and III. The dotted line describes the (linear) CR curve implied by savers’ preferred optimized policy ($\xi=1$). The dashed line describes the (linear) CR curve implied by borrowers’ preferred optimized policy ($\xi=0$). These lines are described over the range covering two standard-deviation bands around the stochastic mean of the PD of loans under the corresponding policy.
Figure 7. Impact of Policy on the Transmission of a Bank Risk Shock. Impulse-response functions to a one-standard deviation negative risk shock to bank asset returns under two alternative capital regulation policies: calibrated (solid line) and benchmark optimized (dashed-and-dotted line) policy. The response of the bank default rate, bank debt spread, and the default rates of mortgage and corporate loans are reported in annualized percentage-point deviations from the steady state. All other variables are in percentage deviations from the steady state.

Figure 8. Impact of Policy on the Transmission of an Entrepreneurial Capital Return Risk Shock. Impulse-response functions to a one-standard deviation negative risk shock to entrepreneurial capital returns under two alternative capital regulation policies: calibrated (solid line) and benchmark optimized (dashed-and-dotted line) policy. The response of the bank default rate, bank debt spread, and the default rates of mortgage and corporate loans are reported in annualized percentage-point deviations from the steady state. All other variables are in percentage deviations from the steady state.