Assessing the Procyclicality of Expected Credit Loss Provisions*

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Abstract

The Global Financial Crisis has pushed accounting standards to shift from an incurred loss approach to an expected credit loss approach (ECL) to loan loss provisions. The ECL approach adopted by IFRS 9 and the incoming update of US GAAP implies a more timely recognition of credit losses but also a greater responsiveness to changes in aggregate conditions, which raises procyclicality concerns. This paper develops a recursive model for the assessment of the implications of different provisioning approaches for banks' profits and regulatory capital. Its application to a portfolio of European corporate loans suggests that the new standards will sizably increase the deterioration of profits and regulatory capital at the beginning of downturns, potentially contributing to a contraction in credit supply at that point in time.

Keywords: credit loss allowances, expected credit losses, incurred losses, rating migrations, procyclicality.

JEL codes: G21, G28, M41

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1 Introduction

The delayed recognition of credit losses under the prevailing incurred loss (IL) approach has been argued to have contributed to the severity of the Global Financial Crisis. Specifically, provisioning "too little, too late" might have prevented banks from being more cautious in good times and reduced transparency and the pressure for prompt corrective action in bad times (see FSF, 2009). In an attempt to promote more forward-looking provisioning practices, the G-20 called in 2009 for a shift to an expected credit loss (ECL) approach. As a result, both the International Accounting Standards Board (IASB) and the US Financial Accounting Standards Board (FASB) have developed important reforms, namely IFRS 9 and an update of US GAAP, scheduled to come into force in 2018 and 2021, respectively.[1] With some differences, both reforms coincide in adopting an ECL approach.

While there is a general perception that the new ECL approach will make an overall positive contribution to financial stability (see, e.g., ESRB, 2017), there are concerns that the point-in-time nature of the estimates of ECLs determining the size of the new loan loss provisions might end up making them more procyclical than their predecessors.[2] In particular, under both IFRS 9 and US GAAP, the implied ECLs are intended to represent best unbiased estimates of the discounted credit losses expected to emerge over some specified horizons. In the case of IFRS 9, such horizon varies from one year (stage 1 exposures) to the residual lifetime of the instruments (stage 2 and 3 exposures), depending on whether their credit quality has not or has deteriorated relative to the point at which the instrument was initially recognized. Instead, the so-called current expected credit loss (CECL) model envisaged by US GAAP opts for using the residual lifetime horizon for all exposures. The procyclicality concern is that an adverse change in aggregate conditions (e.g. from expansion to contraction or from normal to crisis times) leads to a sudden increase in ECLs precisely at the point in which the economy is weakening. The fear is that individual banks' reactions to such an increase (or to its impact on their profits and regulatory capital) cause or amplify a credit crunch and end up producing negative feedback effects in the evolution of the economy.

This paper develops a recursive model with which to assess the cyclical implications of

[1] See IASB (2014) and FASB (2016) for details.
[2] See, for instance, Barclays (2017). Cohen and Edwards (2017) reach a different conclusion.

the new ECL approaches (IFRS 9 and CECL) relative to those of the less forward-looking approaches used so far by banks (IL and the one-year expected loss approach behind the internal-ratings based approach to capital regulation). The model contains the minimal ingredients needed to assess the average levels and the dynamics of the allowances associated with a given loan portfolio and their implications for the profit or loss (P/L) and the common equity Tier 1 (CET1) of the bank holding such portfolio. The model is calibrated to analyze the behavior of a typical portfolio of European corporate loans over the business cycle and to compare the cyclical behavior of impairment allowances, P/L, and CET1 across the various impairment measurement approaches.[3]

Difficulties for modeling impairment allowances under the new approaches include the need to project expected credit losses over horizons longer than one year and, in the case of IFRS 9, having to keep track not only of the current credit quality (or rating) of a given loan but also of its credit quality at origination and its effective contractual interest rate (used for discounting). Some of these features introduce high dimensionality to the state space required to describe the evolution over time of a loan portfolio in a compact way. In general, a cohort of loans of a given rating, even if assumed to be composed of ex ante identical loans with the same effective contractual rate and to have a credit quality that evolves according to a cohort-independent rating migration matrix, would have to be distinguished from a cohort of loans originated with different effective contractual rates, even if their origination rating were the same.

Ideally, one would like to characterize the performance of alternative provisioning methods in a setup where the pricing of the loans and the dynamics of the composition of the portfolio of loans of a representative holder (say, a bank) could be endogenously established in a way consistent with the background assumptions regarding the ratings-migration matrix, the loss-given-default parameters, and the maturity of the loans, as well as the evolution of aggregate risk and its impact on the previous parameters. Ideally, one would like to be explicit about the newly originated loans that enter the portfolio, possibly replacing the loans that mature or are resolved.

[3] Of course, real-world banks hold a variety of loan portfolios as well as non-loan assets, so the case of a bank fully specialized in European corporate loans must be interpreted as a "laboratory case" with which to run controlled experiments on the effects of the alternative provisioning methods.

In a stationary situation without aggregate risk, one would like to be able to obtain the ergodic distribution of loans over the categories relevant for the measurement of their credit loss allowances under alternative provisioning methods. One would also like to be able to characterize the dynamic response of the system to shocks that either perturb punctually the composition of the loan portfolio (like the unanticipated once-and-for-all shocks commonly analyzed in macroeconomic theory) or affect more recurrently, in the form of systematic aggregate risk, the dynamics of the system. Besides, one would like to keep the model just rich enough to be suitable for calibration, i.e. for providing a tentative quantitative (and not only qualitative) assessment of the implications of different provisioning methods.

We achieve all this using a recursive ratings-migration model which is highly tractable thanks to a rather compact description of possible credit risk categories.[4] In the version with aggregate risk, tractability is maintained by describing the economic cycle as a two-state Markov chain. A largely simplifying shortcut is the modeling of loan maturity as random (as in Leland and Toft, 1996), which prevents us from having to keep track of loan vintages.[5]

We calibrate the versions of the model with and without aggregate risk in order to match the characteristics of a typical portfolio of corporate loans issued by European banks. In the version with aggregate risk, we use evidence of the sensitivity of migration matrices and credit loss parameters to business cycles, as in Bangia et al. (2002). The results point to relevant differences between the compared methods regarding both the level of the allowances and their dynamic responses to shocks. The new more forward-looking methods, IFRS 9 and CECL, imply significantly larger impairment allowances and sharper on-impact responses to negative shocks than the old IL and one-year expected loss approaches.

Under the current calibration of the model with aggregate risk, the arrival of a typical recession implies on-impact increases in IFRS 9 and CECL provisions equivalent to about a third of a bank's fully loaded capital conservation buffer (CCB) or, equivalently, about twice

[4]See Trueck and Rachev (2009) for an overview of ratings-migration models and Gruenberger (2012) for an early application of them to the analysis of IFRS 9.

[5]Instead, in the version with aggregate risk, we need to keep track of the aggregate state of the economic cycle in which the loans are originated, since this affects the interest rate relevant for the discounting of their ECLs.

as large as those implied by the IL approach.[6] Thus the differential impact under the new approaches is sizeable but still suitably absorbable if banks' CCB is sufficiently loaded when the shock hits. As we show, the arrival of a contraction that is anticipated to be more severe or longer than average will tend to produce sharper responses. By contrast, if the arrival of a contraction can be anticipated one period in advance, its impact will be significantly smaller.

Given the difficulty to predict changes in the cyclical position of the economy, these results suggest that, unless some regulatory filters offset or smooth away the cyclical impact of impairment allowances on CET1, more forward-looking provisioning methods may mean that banks experience more sudden falls in regulatory capital right at the beginning of contractionary phases of the credit or business cycle. Banks can, of course, try to prepare for this by holding higher precautionary capital buffers during good times. Alternatively, they may adjust, when the time comes, by cutting dividends or by issuing new equity. However, ample anecdotal evidence and some formal empirical evidence indicate that, when confronted with such choices, banks undertake at least part of the adjustment by reducing their risk-weighted assets (RWAs), for example by cutting the origination of new loans or rebalancing towards safer ones.[7]

Therefore, the impact of the more cyclically-sensitive IFRS 9 and, to a lesser extent, CECL provisions on the supply of credit just at the beginning of downturns might be significantly larger than that of the preceding approaches. Negative feedback effects might then follow, very much via the same type of mechanisms extensively discussed in the literature on the procyclical effects of risk-sensitive bank capital requirements.[8] We cannot then rule out that, contrary to its intended purpose, ECL methods, through this specific channel, amplify

[6] Under Basel III, banks' reporting earnings must retain them until reaching a CCB (or buffer of capital on top of the regulatory minimum) equivalent to 2.5% of their risk-weighted assets.

[7] See, for example, Mésonnier and Monks (2015), and Gropp et al. (2016), as well the references therein. The evidence in the latter papers is consistent with average bank responses to the ESRB Questionnaire on Assessing Second Round Effects that accompanied the EBA stress test in 2016. The questionnaire examined the way in which banks would expect to reestablish their desired levels of capitalization after exiting the adverse scenario.

[8] Contributions to the literature on the procyclical effects of capital requirements include Kashyap and Stein (2004) and Repullo and Suarez (2013). Jiménez et al (2017) document the countercyclicality associated with the Spanish statistical provisions, with results suggesting that the effects of changes in capital pressure on credit are significantly more pronounced in recessions that in expansions.

rather than reduce the cyclicality of credit supply.

From a normative perspective, this potential shortcoming of the new ECL provisions should be weighed against the benefits of creating provisions for future credit losses earlier and more cautiously, which include having financial statements that reflect the weakness or strength of the reporting institutions in a more timely and reliable way.[9] Offering a comprehensive normative assessment of the new provisions exceeds the scope of this paper. Yet, our quantification of the potential procyclical effects of IFRS 9 and CECL provisions can be useful in the context of current discussions on the adjustments that may need to be made to microprudential regulation or macroprudential policies in the light of the new accounting standards (see, for example, BCBS, 2016, and ESRB, 2017).

The paper is organized as follows. Section 2 describes the baseline model without aggregate risk. Section 3 develops formulas for measuring impairment losses under the various provisioning approaches and for assessing their effects on P/L and CET1. Section 4 explores the effect of an ad hoc shock to the credit quality of bank loans in the calibrated version of the baseline model. Section 5 presents and calibrates the model with aggregate risk and uses it to analyze the response to the arrival of a typical recession under the various measures. Having looked at banks operating under the internal-ratings based (IRB) approach to capital requirements as a benchmark, Section 6 analyzes the results in the case of a bank operating under the standardized approach. Section 7 describes several extensions. Section 8 discusses the macroprudential implications of the results. Section 9 concludes the paper.

2 Baseline model without aggregate risk

This section develops a simple recursive model of a bank's loan portfolio. The model is based on ten assumptions that fully describe the elements needed to understand the dynamics of loan origination, rating migration, default, maturity, and pricing at origination of the loans that make up the loan portfolio. The tree in Figure 1 summarizes the contingencies

[9] Laeven and Majnoni (2003) and Huizinga and Laeven (2012) document bank provisioning practices during economic slowdowns and their implications for financial stability. Beatty and Liao (2011) document that banks recognizing loan losses in a timelier manner experience lower reductions in lending during contractionary periods. Bushman and Williams (2012, 2015) document the link between a timely and decisive recognition of loan losses and banks' risk profiles. For a review of the literature on loan loss provisions and their interaction with bank regulation, see BSBC (2015).

over the life of a loan (variables on each branch describe the relevant marginal conditional probabilities).

Model assumptions:

1. In each date t, existing loans belong to one of three credit rating categories: standard (j=1), substandard (j=2) or non-performing (j=3). We denote the measure of loans that belong to each category as x_{jt}.

2. In each date t, the bank originates a continuum of standard loans of measure $e_{1t} > 0$, with a principal normalized to one and a constant interest payment per period equal to c. In the language of IFRS 9, c is the effective contractual interest rate at which future expected losses must be discounted. In the dynamic analysis below, we will assume a steady flow of entry of new loans $e_{1t} = e_1$ at each t.

3. Each loan's exposure at default (EAD) is constant and equal to one up to maturity.

4. Loans mature randomly and independently. Specifically, loans rated j=1, 2 mature at the end of each period with a constant probability δ_j.[10] This implies that, conditional on remaining in rating j, a loan's expected life span is of $1/\delta_j$ periods. By the law of large numbers, the fraction of loans of a given rating j that mature at the end of each period is δ_j. In steady state, this produces a stream of maturity cash flows very similar to those that would emerge with a portfolio of perfectly-staggered loans with identical deterministic maturities at origination.

5. In the case of non-performing loans (NPLs, identified by j=3), δ_3 represents the independent per period probability of a loan being resolved, in which case it pays back a fraction $1 - \lambda$ of its principal and exits the portfolio. The constant λ is therefore the loss rate at resolution, which coincides with the expected loss given default (LGD) in the baseline model.

6. Each loan rated j=1, 2 at t that matures at t+1 defaults independently with probability PD_j. Maturing loans that do not default pay back their principal of one plus interest c.

[10] Allowing for $\delta_1 \neq \delta_2$ may help capture the possibility that longer maturity loans get early redeemed with different probabilities depending on their credit quality.

Each defaulted loan is resolved within the same period with an independent probability $\delta_3/2$.[11] Otherwise, it enters the stock of NPLs (j=3).

7. Each loan rated j=1, 2 at t that does not mature at $t+1$ goes through one of the following exhaustive possibilities:

   (a) Default, which occurs independently with probability PD_j. As in the case when a maturing loan defaults, a non-maturing loan that defaults is resolved within the same period with probability $\delta_3/2$, yielding $1-\lambda$. Otherwise, it enters the stock of non-performing loans (j=3).

   (b) Migration to rating $i \neq j$ (i=1,2), which occurs independently with probability a_{ij}. In this case the loan pays interest c and continues for one more period with its new rating.

   (c) Staying in rating j, which occurs independently with probability $a_{jj} = 1 - a_{ij} - PD_j$. In this case the loan pays interest c and continues for one more period with its previous rating.

8. NPLs (j=3) pay no interest and never return to the performing categories. They accumulate in category j=3 up to their resolution.[12]

9. The contractual interest rate c is established at origination as in a perfectly competitive environment with risk-neutral banks that face an opportunity cost of funds between any two periods equal to a constant r. The originating bank is assumed to hold the loans up to their maturity.[13]

[11] We divide δ_3 by two to reflect the fact that, if loans default uniformly during the period between t and t+1, they will, on average, have just half a period to be resolved. The model can trivially accommodate alternative assumptions on same-period resolutions.

[12] For calibration purposes, it is possible to account for potential gains from the unmodelled interest accrued while in default or from returning to performing categories by adjusting the loss rate λ.

[13] In the language of IFRS 9, this implies that the loans satisfy the "business model" condition required for basic lending assets to be measured at amortized cost.

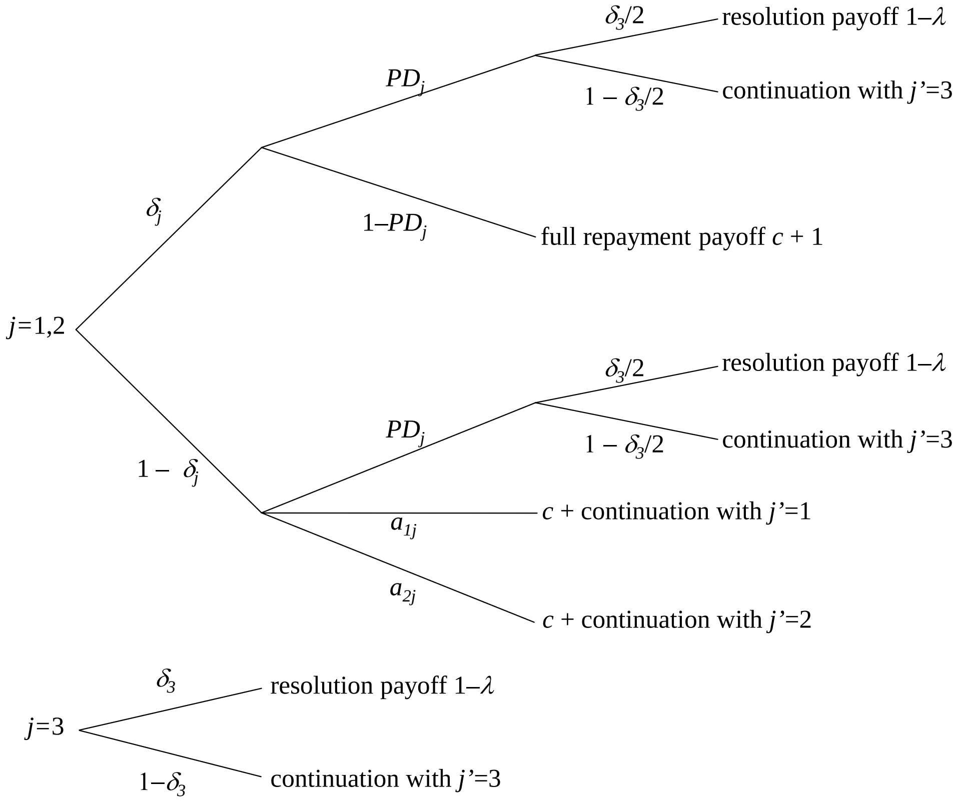

Figure 1. Possible transitions of a loan rated j
Possible contingencies between two dates and their implications for payoffs and continuation value.

10. Finally, one period corresponds to a calendar year, and dates t, $t+1$, $t+2$, etc. denote year-end accounting reporting dates (so "period t" ends at "date t").

In the version of the model with aggregate risk that we present in Section 5, we will allow all the parameters in the tree depicted in Figure 1 to vary with the aggregate state of the economy.

2.1 Portfolio dynamics without aggregate risk

The baseline model has no aggregate risk. By the law of large numbers, the evolution of the loans **in** each rating can be represented by the following difference equation:

$$x_t = Mx_{t-1} + e_t \tag{1}$$

where

$$x_t = \begin{pmatrix} x_{1t} \\ x_{2t} \\ x_{3t} \end{pmatrix} \tag{2}$$

is the vector that describes the loans in each rating category j=1,2,3;

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} = \begin{pmatrix} (1-\delta_1)a_{11} & (1-\delta_2)a_{12} & 0 \\ (1-\delta_1)a_{21} & (1-\delta_2)a_{22} & 0 \\ (1-\delta_3/2)PD_1 & (1-\delta_3/2)PD_2 & (1-\delta_3) \end{pmatrix} \tag{3}$$

accounts for the migrations across categories of the non-matured, non-defaulted loans, and

$$e_t = \begin{pmatrix} e_{1t} \\ 0 \\ 0 \end{pmatrix} \tag{4}$$

accounts for the new loans originated at each date, which we write reflecting the fact that, at origination, all loans have rating j=1.

2.2 Steady state portfolio without aggregate risk

If the amount of newly originated loans is equal at all dates ($e_t = e$ for all t), the loan portfolio will asymptotically converge to a time-invariant or steady-state portfolio x^* that can be obtained as the vector that solves:

$$x = Mx + e \Leftrightarrow (I - M)x = e, \tag{5}$$

that is,

$$x^* = (I - M)^{-1}e. \tag{6}$$

We use this portfolio as a benchmark when measuring provisions under different approaches in the absence of aggregate risk.

3 Measuring impairment losses

In this section we derive formulas for the measurement of the impairments generated by the previously described loan portfolio under different approaches. We also discuss how to endogenously determine a contractual loan rate c consistent with our assumptions on the competitive pricing of loans at origination. Finally, we provide formulae for assessing the impact of impairment measurement on the bank's P/L and CET1.

3.1 Incurred losses

Under the narrowest interpretation, allowances measured on an incurred loss basis (that is, upon clear evidence of impairment) are restricted to the losses associated with existing NPLs. With this criterion, provisions at year t under the IL approach would be

$$IL_t = \lambda x_{3t}, \tag{7}$$

since the loss rate λ is the expected LGD of the bank's NPLs at date t. Note that, under our assumptions, the losses associated with loans defaulted between dates $t-1$ and t which are resolved within such period, $\lambda(\delta_3/2)(PD_1 x_{1t-1} + PD_2 x_{2t-1})$, do not enter IL_t and therefore will be directly recorded in the P/L of year t.

3.2 Discounted one-year expected losses

For consistency with the measure of expected losses applied to stage 1 exposures under IFRS 9, we define the one-year discounted expected losses as

$$EL_t^{1Y} = \lambda \left[\beta(PD_1 x_{1t} + PD_2 x_{2t}) + x_{3t}\right] \tag{8}$$

where $\beta = 1/(1+c)$ is the discount factor based on the contractual interest rate c. Accordingly, for loans performing at t (rated j=1, 2), impairment allowances are computed as the discounted expected losses due to default events expected to occur within the immediately incoming year. They are therefore forward-looking, but the forecasting horizon is limited to one year. Instead, for NPLs (j=3), the default event has already happened and the allowances equal the expected LGD of the loans, exactly as in IL_t.

Roughly speaking, EL_t^{1Y} coincides with the notion of expected losses prescribed for regulatory purposes for banks following the IRB approach to capital requirements.[14] In matrix notation, which will be useful when comparing the different impairment allowance measures later on, EL_t^{1Y} can also be expressed as

$$EL_t^{1Y} = \lambda\left(\beta b x_t + x_{3t}\right), \tag{9}$$

[14] Differences between the BCBS prescriptions on expected losses for IRB banks and our definition of EL_t^{1Y} include the absence of discounting ($\beta = 1$) and the preference for using through-the-cycle (rather than point-in-time) PDs, as well as the use LGD parameters that reflect a distressed liquidation scenario rather than a central scenario. To simplify the analysis, we abstract from all these differences when taking EL_t^{1Y} as a proxy of IRB banks' regulatory provisions.

where

$$b = (PD_1, PD_2, 0). \tag{10}$$

3.3 Discounted lifetime expected losses

The definition for discounted lifetime expected losses arises if, instead of considering the discounted expected losses due to default events expected to occur within the immediately incoming year, we consider defaults events expected to occur over the whole residual lifetime of the loans:

$$EL_t^{LT} = \lambda b \left(\beta x_t + \beta^2 M x_t + \beta^3 M^2 x_t + \beta^4 M^3 x_t + ... \right) + \lambda x_{3t}, \tag{11}$$

This measure reflects the fact that the losses expected from currently performing loans at any future year $t + \tau$, with $\tau = 1, 2, 3...$ can be found as $\lambda b M^{\tau-1} x_t$, where b contains the relevant one-year-ahead PDs (see (10)) and $M^{\tau-1} x_t$ gives the projected composition of the portfolio at each future year $t + \tau - 1$. It also reflects that the allowance for NPLs again equals the expected LGD of the affected loans.

Roughly speaking, EL_t^{LT} coincides with the notion of CECL adopted by FASB for the incoming update of US GAAP.[15] Equation (11) can also be expressed as

$$EL_t^{LT} = \beta \lambda b (I + \beta M + \beta^2 M^2 + \beta^3 M^3 + ...) x_t + \lambda x_{3t}, \tag{12}$$

where the parenthesis is the infinite sum of a geometric series of matrices, which can be found as

$$B = (I - \beta M)^{-1}. \tag{13}$$

Thus, using matrix notation, we can write EL_t^{LT} as

$$EL_t^{LT} = \lambda \left(\beta b B x_t + x_{3t} \right), \tag{14}$$

Since obviously $B \geq I$, we have $EL_t^{LT} \geq EL_t^{1Y}$.

[15] Opposite to IFRS 9, under the update of US GAAP, the discount factor β will not be based on the effective contractual interest rate of the loan, but on a reference risk-free rate. However, to limit the number of features producing differences between the various compared approaches, we will use the same value of β (the one prescribed by IFRS 9) throughout the analysis.

3.4 Impairment allowances under IFRS 9

As already mentioned, IFRS 9 adopts, for performing loans, a mixed-horizon approach that combines the one-year and lifetime expected loss approaches described above. Specifically, the allowances for loans that have not suffered a significant increase in credit risk since origination ("stage 1" loans or, in our model, the loans in x_{1t}) must equal their one-year expected losses, while the allowances for performing loans with deteriorated credit quality ("stage 2" loans or, in our model, the loans in x_{2t}) must equal their lifetime expected losses. Finally, for NPLs ("stage 3" loans or, in our model, the loans in x_{3t}), the allowance simply equals the (non-discounted) expected LGD, as under any of the other approaches.

Combining the formulas obtained in (9) and (14), the impairment allowances under IFRS 9 can be described as

$$EL_t^{IFRS9} = \lambda\beta b \begin{pmatrix} x_{1t} \\ 0 \\ 0 \end{pmatrix} + \lambda\beta bB \begin{pmatrix} 0 \\ x_{2t} \\ 0 \end{pmatrix} + \lambda x_{3t}, \tag{15}$$

which, together with $EL_t^{LT} \geq EL_t^{1Y}$, implies $EL_t^{1Y} \leq EL_t^{IFRS9} \leq EL_t^{LT}$.

3.5 Loan rates under competitive pricing

Taking advantage of the recursivity of the model, we can obtain the bank's ex-coupon value of loans rated j at any given date, v_j, by solving the following system of Bellman-type equations:

$$v_j = \mu\left[(1 - PD_j)c + (1 - PD_j)\delta_j + PD_j(\delta_3/2)(1-\lambda) + m_{1j}v_1 + m_{2j}v_2 + m_{3j}v_3\right], \tag{16}$$

for j=1, 2, and

$$v_3 = \mu\left[\delta_3(1-\lambda) + (1-\delta_3)v_3\right], \tag{17}$$

where $\mu = 1/(1+r)$ is the discount factor of the risk neutral bank. Intuitively, the square brackets in (16) and (17) contain the payoffs and continuation value that a loan rated j=1, 2 or j=3, respectively, will produce in the contingencies that, in each case, can occur one period ahead (weighted by the corresponding probabilities).[16]

[16] For calibration purposes, the discount rate r does not need to equal the risk-free rate. One might adjust the value of r to reflect the marginal weighted average costs of funds of the bank or even an extra element capturing (in reduced form) a mark-up applied on that cost if the bank is not perfectly competitive.

In (16), the first term in the square brackets represents the interest that a loan currently rated j=1, 2 will pay at the next due date if it continues to perform. The second term captures the terminal value obtained if the loan matures without defaulting. The third term accounts for the terminal value recovered if the loan defaults and is resolved within the period. The fourth and fifth terms reflect the continuation value obtained if the loan does not mature and receives (or retains) rating 1 or 2, respectively, for the next period. The last term measures the continuation value obtained if the loan defaults but is not resolved within the period, thus becoming an NPL.

In (17), the first term in the square brackets represents the terminal value recovered if an NPL is resolved within the period. The last term reflects the continuation value of the NPL if it remains unresolved at $t + 1$.

Perfect competition implies that the value of extending a loan of size one rated as standard (j=1) at origination must equal the value of its principal (one), $v_1 = 1$, so that the bank obtains zero net present value from its origination. Solving for c in this equation delivers the endogenous contractual interest rate that, consistently with the prescription of IFRS 9, will enter the discount factor $\beta = 1/(1 + c)$ used in the various expectation-based impairment measures established above.

3.6 Implications for P/L and CET1

To explore the implications of impairment measurement for the dynamics of the P/L account and for CET1, we need to make further assumptions regarding the bank **holding** the loan portfolio discussed so far and its capital structure. To simplify the discussion, we abstract from bank failure and assume that the bank's only assets at the end of any period t are the loans described by vector x_t and that its liabilities **consist** exclusively of (i) one-period risk-free debt d_t that promises to pay interest r per period, (ii) impairment allowances a_t computed under one of the measurement approaches described above (so $a_t = IL_t, EL_t^{1Y}, EL_t^{LT}, EL_t^{IFRS9}$), and (iii) CET1 denoted by k_t. This means that the bank's balance sheet at the end of any period t can be described as

$$\begin{array}{c|c} x_{1t} & d_t \\ x_{2t} & a_t \\ x_{3t} & k_t \end{array} \tag{18}$$

with the law of motion of x_t described by (1) and the law of motion of k_t given by

$$k_t = k_{t-1} + PL_t - \text{div}_t + \text{recap}_t, \tag{19}$$

where PL_t is the result of the P/L account at the end of period t, $\text{div}_t \geq 0$ are cash dividends paid at the end of period t, and $\text{recap}_t \geq 0$ are injections of CET1 at the end of period t. Under these assumptions, the dynamics of d_t can be recovered residually from the balance sheet identity, $d_t = \Sigma_{j=1,2,3} x_{jt} - a_t - k_t$.

The result of the P/L account can in turn be written as

$$PL_t = \left\{ \sum_{j=1,2} \left[c(1-PD_j) - \frac{\delta_3}{2} PD_j \lambda \right] x_{jt-1} - \delta_3 \lambda x_{3t-1} \right\} - r \left(\sum_{j=1,2,3} x_{jt-1} - a_{t-1} - k_{t-1} \right) - \Delta a_t, \tag{20}$$

where the first term contains the income from performing loans net of realized losses on defaulted loans resolved during period t, the second term is the interest paid on d_{t-1}, and the third term is the variation in credit loss allowances between periods $t-1$ and t.

To model dividends, div_t, and equity injections, recap_t, in a simple manner, we assume that the bank manages the evolution of its CET1 using a simple sS-rule based entirely on existing capital regulations.[17] Specifically, current Basel III prescriptions include the minimum capital requirements and the so-called capital conservation buffer (CCB). Minimum capital requirements force the bank to operate with a CET1 of at least $\underline{k}_t$, while the CCB requires the bank to retain profits, whenever feasible, until reaching a fully loaded buffer

[17] This rule can be rationalized as the one that minimizes the equity capital committed to support the loan portfolio and is consistent with the view that banks find equity financing more costly than debt financing. However, for simplicity, we effectively consider the limit case where the excess cost of equity financing goes to zero (so that, for instance, the loan pricing equations described above do not depend on the bank's capital structure). Additionally, the working of the sS rule proposed here implicitly assumes the absence of fixed costs associated with the rasing of new equity. If such costs were to be introduced, the optimal rule would imply, as in Fischer, Heinkel, and Zechner (1989), discrete recapitalizations to an endogenous level within the bands if the lower band were to be otherwise passed.

equal to 2.5% of its RWAs. This means that a bank with positive profits must accumulate them until its CET1 reaches a level $\overline{k}_t = 1.3125\underline{k}_t$.[18]

Thus, we assume the bank's dividends and equity injections to be determined as

$$\text{div}_t = \max[(k_{t-1} + PL_t) - 1.3125\underline{k}_t, 0] \tag{21}$$

and

$$\text{recap}_t = \max[\underline{k}_t - (k_{t-1} + PL_t), 0], \tag{22}$$

respectively.

Minimum capital requirement under the IRB approach For portfolios operated under the IRB approach, the IRB formula specified in BCBS (2004, paragraph 272) establishes that the regulatory capital requirement must be

$$\underline{k}_t^{IRB} = \sum_{j=1,2} \gamma_j x_{jt}, \tag{23}$$

with

$$\gamma_j = \lambda \frac{1 + [(1/\delta_j) - 2.5] m_j}{1 - 1.5 m_j} \left[\Phi \left(\frac{\Phi^{-1}(PD_j) + \text{cor}_j^{0.5} \Phi^{-1}(0.999)}{(1 - \text{cor}_j)^{0.5}} \right) - PD_j \right], \tag{24}$$

where $m_j = [0.11852 - 0.05478 \ln(PD_j)]^2$ is a maturity adjustment coefficient, $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal distribution, and cor_j is a correlation coefficient fixed as $\text{cor}_j = 0.24 - 0.12(1 - \exp(-50 PD_j))/(1 - \exp(-50))$.[19]

Minimum capital requirement under the standardized (SA) approach For banks or portfolios operated under the SA approach, the regulatory minimum capital requirement applicable to loans to corporations without an external rating is just 8% of the exposure net of its "specific provisions" (a regulatory concept related to impairment allowances). Assuming that all the loans in x_t correspond to unrated borrowers and that all the impairment allowances a_t qualify as specific provisions, this implies that

$$\underline{k}_t^{SA} = 0.08 \left(\sum_{j=1,2,3} x_{jt} - a_t \right). \tag{25}$$

[18] Under Basel III, RWAs equal 12.5 (or 1/0.08) times the bank's minimal required capital $\underline{k}_t$. Thus a fully loaded CCB amounts to a multiple $0.025 \times 12.5 = 0.3125$ of $\underline{k}_t$.

[19] In (24) we measure the maturity of performing loans as the expected residual maturity $1/\delta_j$ implied by our formulation.

Formulas (23) and (25) will allow us to assess the impact of different impairment measurement methods on the dynamics of PL_t, k_t, div_t, and recap_t under each of the approaches to capital requirements.

It is important to notice that, as a first approximation, our analysis abstracts from the existence of "regulatory filters" dealing with the implications of possible discrepancies between "accounting" and "regulatory" provisions and their effects on "regulatory capital." In this sense, our assessment can be seen as an evaluation of the impact of accounting rules on bank capital dynamics in the extreme event that bank capital regulators accept the new accounting provisions (and the resulting accounting capital) as provisions (and available capital) for regulatory purposes as well.[20]

4 An initial quantitative exploration

4.1 Steady state calibration

The model described so far features a relatively small number of parameters. Table 1 describes their value under a parameterization intended to represent a typical portfolio of corporate loans issued by EU banks. Given the absence of detailed publicly available microeconomic information on such a portfolio, the calibration relies on matching aggregate variables taken from recent European Banking Authority (EBA) reports and European Central Bank (ECB) statistics using rating migration and default probabilities consistent with the Global Corporate Default reports produced by Standard & Poor's (S&P) over the period 1981-2015.[21]

Banks' discount rate r is fixed at 1.8% so as to obtain a contractual loan rate c equal to 2.54%, which is very close to the 2.52% average interest rate of new corporate loans made by

[20] In the case of banks operating under the IRB approach, the current regulatory regime (which might be revised to accommodate the expected credit loss approaches in accounting) relies on a notion of one-year expected losses similar to EL_t^{1Y}, say REL_t^{1Y}. If REL_t^{1Y} exceeds the accounting allowances, a_t, the difference, $REL_t^{1Y} - a_t$, must be subtracted from CET1. By contrast, if $REL_t^{1Y} - a_t < 0$, the difference can be added back as Tier 2 capital up to a maximum of 0.6% of the bank's credit RWAs. In the case of SA banks, there is a filter for general provisions (which, for simplicity, we assume to be zero in our analysis), which can be added back as Tier 2 capital up to a maximum of 1.25% of credit RWAs.

[21] We use reports equivalent to S&P (2016) published in years 2003 and 2005-2016, which provide the relevant information for each of the years between 1981 and 2015.

Euro Area banks in the period from January 2010 to September 2016.[22] The probabilities of default (PDs) and yearly probabilities of migration across our standard and substandard categories are extracted from S&P rating migration data using the procedure described in Appendix A. These probabilities are consistent with the alignment of our standard category (j=1) with ratings AAA to BB in the S&P classification and our substandard category (j=2) with ratings B to C.

Table 1
Calibration of the model without aggregate risk

Banks' discount rate	r	1.8%
Yearly probability of migration 1 → 2 if not maturing	a_{21}	7.37%
Yearly probability of migration 2 → 1 if not maturing	a_{12}	6.29%
Yearly probability of default if rated j=1	PD_1	0.85%
Yearly probability of default if rated j=2	PD_2	7.29%
Loss given default	λ	36%
Average time to maturity if rated j=1	$1/\delta_1$	5 years
Average time to maturity if rated j=2	$1/\delta_2$	5 years
Yearly probability of resolution of NPLs	δ_3	44.6%
Newly originated loans per period (all rated j=1)	e_1	1

In a nutshell, we reduce the 7×7 rating-migration probabilities and the seven PDs in S&P data to the 2×2 migration probabilities and two PDs that appear in matrix M (equation (3)) by calculating weighted averages that take into account the steady-state composition that the loan portfolio would have under its 7-ratings representation. To obtain this composition, we assume that loans have an average duration of 5 years (or δ_1=δ_2=0.2) as in Table 1; that they have a rating BB at origination, and that they then evolve (through improvements or deteriorations in their credit quality before defaulting or maturing) exactly as in our model, but with the seven non-default rating categories in the original S&P data.

Under these assumptions, we obtain an average yearly PD for our standard and substandard categories of 0.9% and 7.3%, respectively. As shown in Table 2, given the composition of the "reduced" steady-state portfolio, the average annual loan default rate equals 1.9%, which is below the average 2.5% PD for non-defaulted corporate exposures that EBA (2013,

[22] We use the Euro area (changing composition), annualised agreed rate/narrowly defined effective rate on euro-denominated loans other than revolving loans and overdrafts, and convenience and extended credit card debt, made by banks to non-financial corporations (see http://sdw.ecb.europa.eu/quickview.do?SERIES_KEY=124.MIR.M.U2.B.A2A.A.R.A.2240.EUR.N)

Figure 12) reports for the period from the first half of 2009 to the second half of 2012 for a sample of EU banks operating under the IRB approach.

Table 2
Endogenous variables under the no-aggregate-risk calibration
(IRB bank, all variables in **percentages**)

Yearly contractual loan rate, c	2.54
Steady-state portfolio shares (percentage of total loans)	
Standard loans, $x_1^*/(\Sigma_{j=1,2,3}x_j^*)$	81.29
Substandard loans, $x_2^*/(\Sigma_{j=1,2,3}x_j^*)$	15.53
NPLs, $x_3^*/(\Sigma_{j=1,2,3}x_j^*)$	3.18
Average yearly PD on non-defaulted loans, $(\Sigma_{j=1,2}PD_jx_j^*)/(\Sigma_{j=1,2}x_j^*)$	1.88
Average yearly PD on total loans, $(\Sigma_{j=1,2}PD_jx_j^* + x_3^*)/(\Sigma_{j=1,2,3}x_j^*)$	5.00
Steady-state allowances (percentage of total loans):	
Incurred losses	1.14
One-year expected losses	1.78
Lifetime expected losses	4.64
IFRS 9 allowances	2.67
Stage 1 allowances	0.24
Stage 2 allowances	1.28
Stage 3 allowances	1.14
IRB capital requirement for standard loans, γ_1	7.57
IRB capital requirement for substandard loans, γ_2	12.86
IRB minimum capital requirement (percentage of total loans), $\underline{k}$	8.15
IRB minimum capital requirement + CCB (percentage of total loans), $\overline{k}$	10.70

The LGD parameter λ is set equal to 36%, which roughly matches the average LGD on corporate exposures that the EBA (2013, Figures 11 and 13) reports for the period from the first half of 2009 to the second half of 2012 for the same sample as above. Finally, we set δ_3 equal to 44.6% in order to produce a steady-state fraction of NPLs consistent with the 5% average PD, including defaulted exposures that the EBA (2013, Figure 10) reports for the earliest period in its study, namely the first half of 2008.[23] This value of δ_3 implies an average time to resolution for NPLs of 2.24 years, which is very close to the 2.42-year average duration of corporate insolvency proceedings across EU countries documented by the EBA

[23] We take this observation, right before experiencing the full negative impact of the Global Financial Crisis, as the best proxy in the data for the model's steady state. As shown in Table 2, with this procedure, we obtain a 3.2% share of defaulted exposures in the steady state portfolio, right inbetween the 2.5% and 4.4% reported by the EBA (2013, Figure 8) for corporate loans in the second half of 2008 and the first half of 2009, respectively.

(2016, Figure 13).

Finally, the assumed flow of newly originated loans, e_1=1, only provides a normalization and solely affects the size of the steady-state loan portfolio.

The second section of Table 2 reports the size of the credit impairment allowances in steady state using each of the measurement methods that we compare. The third section reports the IRB capital requirements, the implied overall minimum capital requirement ($\underline{k}$) and the minimum requirement plus CCB ($\overline{k}$) that we use to model the dynamics of CET1.[24] The various impairment measures are ranked as predicted above. While for the considered portfolio, impairment allowances under IFRS 9 (EL_t^{IFRS9}) more than double those associated with the IL approach (IL_t), the incoming CECL approach (EL_t^{LT}) almost quadruples them. Note that the rise in allowances associated with IFRS 9 comes mostly from stage 2 loans in spite of the fact that these loans only represent a modest 15.5% in the loan portfolio.

4.2 Analyzing the impact of a solvency shock

The following thought experiment represents a first look into the implications of the model for how the various credit impairment measures respond to shocks that erode the expected credit quality of a loan portfolio. Suppose that the loan portfolio is at its steady-state composition at an initial date t=–1. Suppose also that at t=0 the system is hit by a large, unexpected once-and-for-all shock that renders an extra 35% of the standard-quality loans of the previous date substandard (instead of remaining standard for one more period), so that their rating migrations, typically driven by a_{11} and a_{21}, become punctually driven by $a'_{11} = a_{11} - 0.35$ and $a'_{21} = a_{21} + 0.35$ respectively. Formally, this means perturbing m_{11} and m_{21} to $m'_{11} = (1 - \delta_1)(a_{11} - 0.35)$ and $m'_{21} = (1 - \delta_1)(a_{21} + 0.35)$ for one period.[25]

From t=1 onwards, the system simply follows its own dynamics, according to the parameters described in Table 1, without further shocks. Notice, however, that the presence of an abnormally high amount of substandard loans will make the effects of the initial shock persistent over time. This can be seen in Panel A of Figure 2, which depicts the evolution of

[24] To keep the analysis focused, we first discuss the case of IRB banks, postponing the comparison with SA banks until Section 6.

[25] This shock might represent a drastic change in the prospects of an industry in which the loan portfolio is initially concentrated: for instance, the sudden end of a construction boom in a portfolio over-concentrated in loans to real estate developers.

NPLs in this thought experiment.

The results regarding the evolution of the various impairment measures over the same time span appear in Panel B of Figure 2. Impairment allowances IL_t, EL_t^{1Y}, EL_t^{LT}, and EL_t^{IFRS9} are reported as a percentage of the total initial loans. The levels of the series at t=–1 reflect the different sizes of the various measures in steady state.

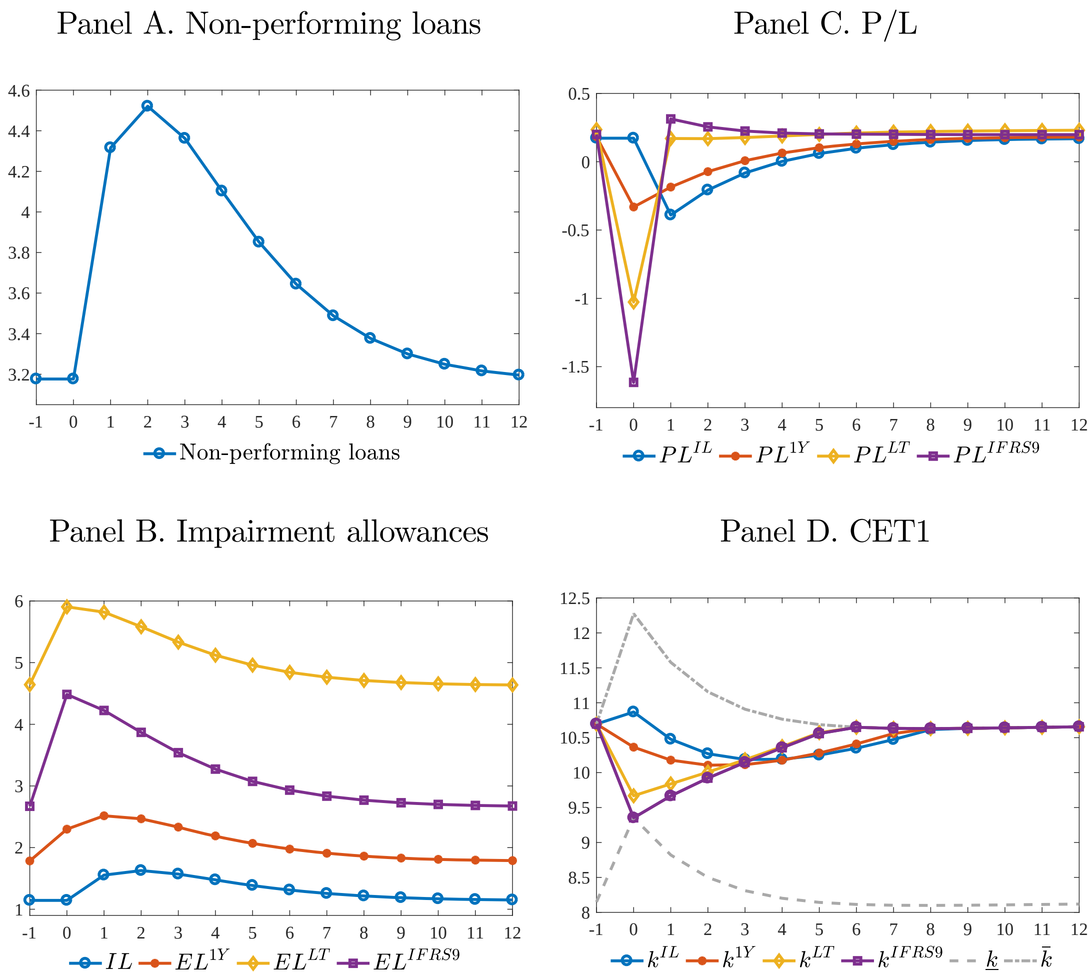

Figure 2. Effects of a negative shock to credit quality
Responses to an unexpected once-and-for-all shock to credit quality
(IRB bank, **as a percentage** of initial exposures)

The results shown in Figure 2 for t=0,1,2... are equivalent to a typical impulse response function in macroeconomic analysis. When the shock hits at t=0, all measures except IL_t, which, given its backward-looking nature, reacts with a delay of one period, move upwards for one or two periods before entering a pattern of exponential decay, driven by maturity, defaults, migration of substandard loans back to the standard category, and the continued origination of new standard-quality loans.[26]

The responses of EL_t^{1Y} and, when it comes, IL_t to the shock are much smaller than those of the forward-looking measures with longer projection horizons. Interestingly, the on-impact response of EL_t^{IFRS9} (which increases by about 1.9 percentage points of initial exposures) exceeds that of EL_t^{LT} (which increases by about 1.3 percentage points), reflecting the so-called "cliff effect" associated with the change in provisioning horizon when exposures shift from stage 1 to stage 2. In contrast, EL_t^{1Y} increases by barely 0.5 percentage points at its peak (at t=1) and IL_t increases by roughly 0.4 percentage points at its peak (at t=2).

The implications of the various impairment measures for P/L are described in Panel C of Figure 2. Essentially, each measure spreads the (same final) impact of the shock on P/L over time in a different manner. EL_t^{IFRS9} and, to a lesser extent, EL_t^{LT} front-load the impact of the shock to the extent that P/L becomes very negative on impact, but then positive and even above normal for a number of periods afterwards. With EL_t^{1Y} (and IL_t), P/L is affected much less (and with a delay) on impact but remains negative for several periods. Interestingly, the measure which allows P/L to return to normal the soonest in this experiment is EL_t^{LT}.

Panel D of Figure 2 shows the implications for an IRB bank's CET1. Before the shock hits, at t=–1, the bank is assumed to have its fully-loaded CCB, implying a buffer on top of the minimum required capital of more than 2.5% of total assets. The change in the bands $\underline{k}$ and $\overline{k}$ reflected in the figure is the result of the change in RWAs following the deterioration in the composition of the loan portfolio. The differences in the effects of the alternative measures on CET1 are dramatic, essentially mirroring their impact on P/L.

In the case of IFRS 9, an abnormal extra shift of 35% of the loans from j=1 to j=2 at t=0 implies consuming the CCB in that very year and having to raise a (small) amount of new equity. Using the alterative measures, including CECL (EL_t^{LT}), no equity issuance is required and the return to normal capital levels occurs solely via earnings retention.

[26] Variations of the experiment that simultaneously shut down or reduce origination of new loans for a number periods can be easily performed without losing consistency. Experiencing lower loan origination after t=0 delays the process of reversion to the steady state but does not qualitatively affect the results.

Of course, whether or not there is a need for recapitalization under the various impairment measures in this thought experiment depends on the ad hoc size of the initial shock, so far fixed at 35% for purely illustrative reasons. However, the (weak) order of the recapitalization needs that each measurement method would imply happens to be invariant to the size of the shock. This can be seen in Figure 3, which shows the cumulative capital issuance needs implied by a shock like this under each measure (vertical axis) as a function of the additional fraction of standard loans that the shock converts into substandard (horizontal axis).

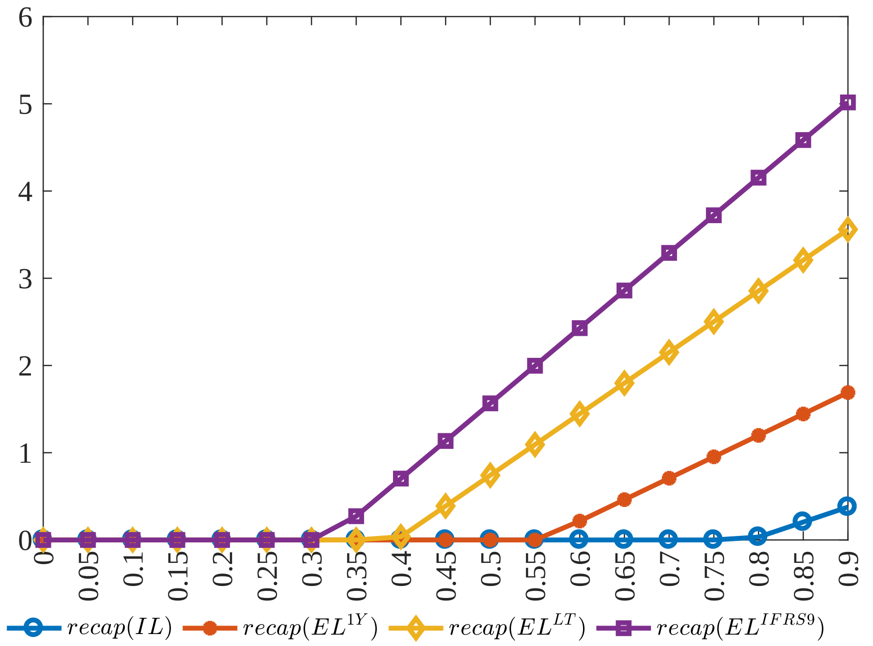

Figure 3. Recapitalization needs and the size of the shock
X-axis: fraction of standard loans abnormally turning substandard
Y-axis: cumulative recapitalization needs
(IRB bank, as a percentage of initial exposures)

5 Adding aggregate risk

The most natural way to incorporate aggregate risk in the model is to consider an aggregate state variable s_t whose evolution affects the key parameters governing portfolio dynamics and credit losses in the model. To keep things simple, we assume that s_t follows a Markov chain with two states s=1,2 and time-invariant transition probabilities $p_{s's}$ =Prob($s_{t+1} = s'|s_t = s$). In this representation, s=1 could identify expansion or quiet periods, while s=2 could identify contraction or crisis periods.[27]

In Appendix B we extend the model and the formulae for the calculation of impairment allowances to accommodate the case in which the parameters determining the (expected) maturity of the loans, default probabilities, rates of migration across ratings, probabilities of being resolved when in default, loss rates upon resolution, and the origination of new loans between any dates t and $t+1$ may vary with the arrival state s_{t+1}.

An approach that allows us to keep the analysis recursive as in the baseline model is to expand the vectors describing loan portfolios so that components describe "loans originated in state z, currently in state s and rated j", for each possible (z, s, j) combination, instead of just "loans rated j". In parallel, we expand the transition matrices describing the dynamics of these portfolios to reflect the possible transitions of the aggregate state and their impact on all the relevant parameters. The need to keep track of the state at origination z comes from the IFRS 9 prescription that future credit losses of each loan must be discounted using the effective contractual interest rate, which now varies with the aggregate state at origination and is denoted c_z.

5.1 Calibration with aggregate risk

Table 3 describes the calibration of the model with aggregate risk. As explained further in section A.3 of Appendix A, we allow for state-variation in the probabilities of loans migrating across rating categories and into default in a way consistent with the historical correlation between those variables (as observed in S&P rating-migration data) and the US business cycle as dated by the National Bureau of Economic Research (NBER).[28] The dynamics of the aggregate state as parameterized in Table 3 imply that the average duration of expansion and contraction periods is 6.75 years and 2 years, respectively, meaning that the system spends about 77% of the time in state s=1. Expansions are characterized by significantly smaller PDs among both standard and substandard loans than contractions. During a contraction, the probability of standard loans being downgraded (or, under IFRS 9, moved into stage 2) is almost double than during an expansion and the probability of substandard loans recovering

[27]For an empirical ratings-migration model in which macroeconomic conditions are represented in this manner, see Bangia et al. (2002).

[28]See http://www.nber.org/cycles.html.

standard quality (or returning to stage 1) is reduced by about one-third.

To keep the potential sources of cyclical variation under control, we maintain the parameters determining the effective maturity of performing loans, the speed of resolution of NPLs, the LGD, and the flow of entry of new loans as time invariant (and equal to their values in the calibration without aggregate risk).

Table 3
Calibration of the model with aggregate risk

Parameters without variation with the aggregate state			
Banks' discount rate	r	1.8%	
Persistence of the expansion state (s=1)	p_{11}	0.852	
Persistence of the contraction state (s=2)	p_{22}	0.5	

Parameters that may possibly vary with the aggregate state		If $s' = 1$	If $s' = 2$
Yearly probability of migration 1 → 2 if not maturing	a_{21}	6.16%	11.44%
Yearly probability of migration 2 → 1 if not maturing	a_{12}	6.82%	4.47%
Yearly probability of default if rated j=1	PD_1	0.54%	1.91%
Yearly probability of default if rated j=2	PD_2	6.05%	11.50%
Loss given default	λ	36%	36%
Average time to maturity if rated j=1	$1/\delta_1$	5 years	5 years
Average time to maturity if rated j=2	$1/\delta_2$	5 years	5 years
Yearly probability of resolution of NPLs	δ_3	44.6%	44.6%
Newly originated loans per period (all rated j=1)	e_1	1	1

5.2 Cyclicality of the various impairment measures

Table 4 reports unconditional means, standard deviations, and means conditional on each aggregate state for a number of endogenous variables. The variation in the aggregate state causes a significant variation in the composition of the bank's loan portfolio. Not surprisingly, in the contraction state, substandard and non-performing loans represent a larger share of the portfolio, and the overall realized default rate is more than double than in the expansion state. As in previous sections, we focus the analysis of the implications for CET1 on the case of IRB banks, leaving the comparison with the case of SA banks for Section 6.

The mean relative sizes of the various impairment allowances are essentially the same as obtained for the case without aggregate risk. Interestingly, impairments measured under IFRS 9 are the most volatile, followed closely by the lifetime CECLs of the new US GAAP rules. The least volatile provisions are those computed under the IL approach.

Table 4
Endogenous variables under the aggregate risk calibration
(IRB bank, percentage of mean exposures unless indicated)

			Conditional means	
	Mean	St. Dev.	Expansions	Contractions
Yearly contractual loan rate c (%)			2.52	2.62
Share of standard loans (%)	81.35	3.48	82.68	76.85
Share of substandard loans (%)	15.46	1.90	14.59	18.42
Share of NPLs (%)	3.19	1.05	2.73	4.73
Realized default rate (% of performing loans)	1.89	0.90	1.36	3.43
Impairment allowances:				
Incurred losses	1.15	0.38	0.98	1.70
One-year expected losses	1.79	0.50	1.55	2.60
Lifetime expected losses	4.65	0.59	4.36	5.63
IFRS 9 allowances	2.67	0.62	2.38	3.66
Stage 1 allowances	0.24	0.05	0.22	0.33
Stage 2 allowances	1.28	0.21	1.18	1.63
Stage 3 allowances	1.15	0.38	0.98	1.70
IRB minimum capital requirement	8.15	0.07	8.14	8.19
IRB minimum capital requirement + CCB	10.69	0.09	10.68	10.74

The decomposition by stage shown for IFRS 9 reveals that allowances associated with NPLs, followed by those associated with substandard loans, are those that contribute most to cross-state variation in impairment allowances. However, NPLs are treated in the same way by all measures, which means that the differing volatilities of the various measures must stem from the treatment of standard loans (which is the same in EL^{1Y} and EL^{IFRS9}, but is different in IL and EL^{LT}) and stage 2 loans (which is the same in EL^{LT} and EL^{IFRS9}, but is different in IL and EL^{1Y}) or from the cyclical shift of loans across stages 1 and 2 (under EL^{IFRS9}).

5.3 Impact on the cyclicality of P/L and CET1

Table 5 summarizes the impact of the various impairment measurement approaches on P/L and CET1 in the case of an IRB bank. The unconditional mean of P/L differs across provisioning methods, reflecting that different levels of provisions imply *de facto* different levels of debt financing for the same portfolio and, hence, different amounts of interest expense. Confirming what one might expect after observing the volatility ranking of the impairment measures in Table 4, P/L is significantly more volatile under the more forward-looking EL^{LT} and EL^{IFRS9} than under EL^{1Y} or IL. EL^{IFRS9} (IL) is clearly the impairment measure producing a higher (lower) volatility of P/L across aggregate states.

Table 5
Endogenous variables
under the aggregate-risk calibration
(IRB bank, percentage of mean exposures unless otherwise indicated)

	IL	EL^{1Y}	EL^{LT}	EL^{IFRS9}
P/L				
Unconditional mean	0.16	0.17	0.23	0.19
Conditional mean, expansions	0.35	0.41	0.49	0.46
Conditional mean, contractions	-0.46	-0.61	-0.66	-0.71
Standard deviation	0.34	0.43	0.51	0.50
CET1				
Unconditional mean	10.20	10.19	10.25	10.17
Conditional mean, expansions	10.38	10.43	10.53	10.46
Conditional mean, contractions	9.55	9.32	9.28	9.16
Standard deviation	0.76	0.76	0.71	0.77
Probability of dividends being paid (%)				
Unconditional	49.53	51.79	56.38	53.93
Conditional, expansions	64.20	67.11	73.07	69.89
Conditional, contractions	0	0	0	0
Dividends, if positive				
Conditional mean, expansions	0.35	0.36	0.42	0.38
Conditional mean, contractions	–	–	–	–
Probability of having to recapitalize (%)				
Unconditional	2.34	2.86	2.34	3.41
Conditional, expansions	0	0	0	0
Conditional, contractions	10.26	12.50	10.22	14.94
Recapitalization, if positive				
Conditional mean, expansions	–	–	–	–
Conditional mean, contractions	0.42	0.40	0.34	0.38

The more forward-looking impairment measures are the ones that make the bank, on average, more CET1-rich in expansion states and less CET1-rich in contraction states; that is, those that render CET1 more procyclical in this sense. In any case, the reported quantitative differences for this variable are not huge, in part because under our assumptions on the bank's management of its CET1, the range of variation in CET1 under any of the impairment measures is limited by the regulation-determined bands of the sS-rule described in equations (21) and (22). As explained above, the bank adjusts its CET1 to remain within those bands by paying dividends or raising new equity.

Thus, a complementary way to assess the potential procyclicality associated with each impairment measure is to look at the frequency and size (conditional on them being strictly positive) of dividends and recapitalizations. Quite intuitively, under all measures we obtain that dividend distributions only occur (if at all) during periods of expansion, while recapitalizations only occur (if at all) during periods of contraction.

Relative to EL^{1Y}, the usage of EL^{IFRS9} implies an increase from 12% to 15% in the probability that the bank needs to be recapitalized during periods of contraction (mirrored by a more modest increase from 67% to 70% in the probability of dividends being paid during periods of expansion).[29] Instead, the usage of EL^{LT} reduces both the probability of having to recapitalize the bank in a contraction and the conditional size of the recapitalization needs. This striking difference (despite the similar standard deviation of P/L) is largely due to the higher mean value of P/L implied by the lower leverage kept by the bank under CECL.

5.4 Effects of the arrival of a contraction

Using the same layout as in Figure 2, Figure 4 shows the effects of the arrival of a contraction at t=0 (that is, the realization of s_0=2) after having spent a long enough period in the expansion state (that is, having had s_t=1 for sufficiently many dates prior to t=0). From t=1 onwards the aggregate state follows the Markov chain calibrated in Table 3, thus making the trajectories followed by the variables depicted in Figure 4 stochastic. The figure depicts the average trajectories resulting from simulating 10,000 paths.

The fact that the trajectories depicted are average trajectories is important for interpreting Figure 4. For example, in Panel D, the average trajectory of CET1 lies within the

[29] However, these effects become counterbalanced by the fact that, when strictly positive, the average size of the recapitalizations needed (and dividends paid) under EL^{IFRS9} is slightly lower than that under EL^{1Y}.

average bands of the sS-rule that determines its management, but this does not mean that the bank does not need to recapitalize (or does not pay dividends) after the initial shock. Actually, many of the actual trajectories are upward and touch the upper band for paying dividends (e.g. if the contraction ends and does not return) and several are downward and force the bank to recapitalize (e.g. if the contraction lasts a long time or another contraction follows soon after an initial recovery).

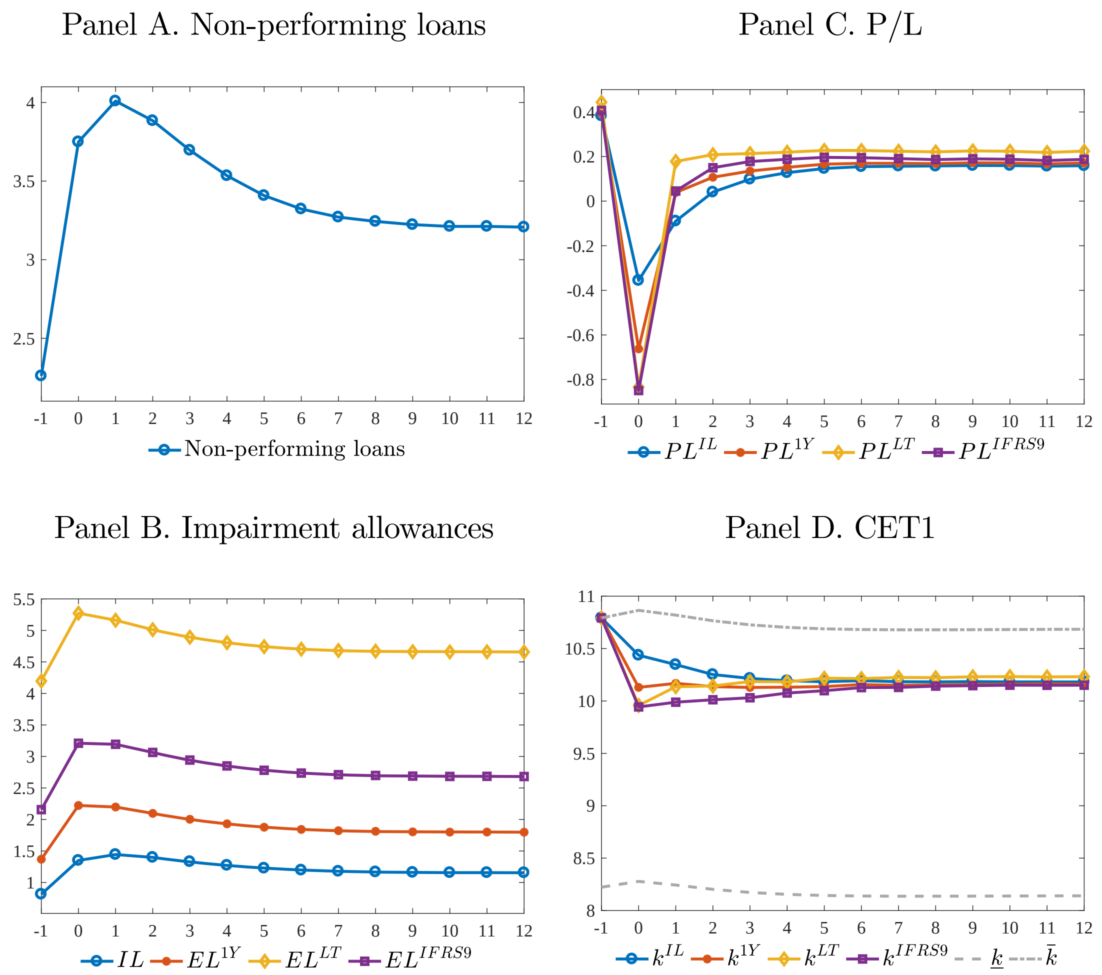

Figure 4. Effects of the arrival of a contraction
Average responses to the arrival of s=2 after a long period in s=1
(IRB bank, as a percentage of average exposures).

To illustrate the difference between the average and the realized trajectories, Figure 5 shows 500 simulated trajectories for CET1 under EL^{1Y} and EL^{IFRS9}. Under IFRS 9, it takes four consecutive years in the contraction state (s=2) for a bank to deplete its CCB and require a recapitalization. By contrast, under the one-year expected loss approach, the CCB would be used up only after five years in the contraction state.

Intuitively, the closer the average trajectory for CET1 is to the lower band in Panel D of Figure 4, the more likely it is that the bank needs to raise new equity following the arrival of the contraction state. This explains why, as reported in Table 4, the probability of the bank needing to be recapitalized in the contraction state is higher under EL^{IFRS9} than under any of the other three approaches.

Panel A. CET1 under EL^{1Y} Panel B. CET1 under EL^{IFRS9}

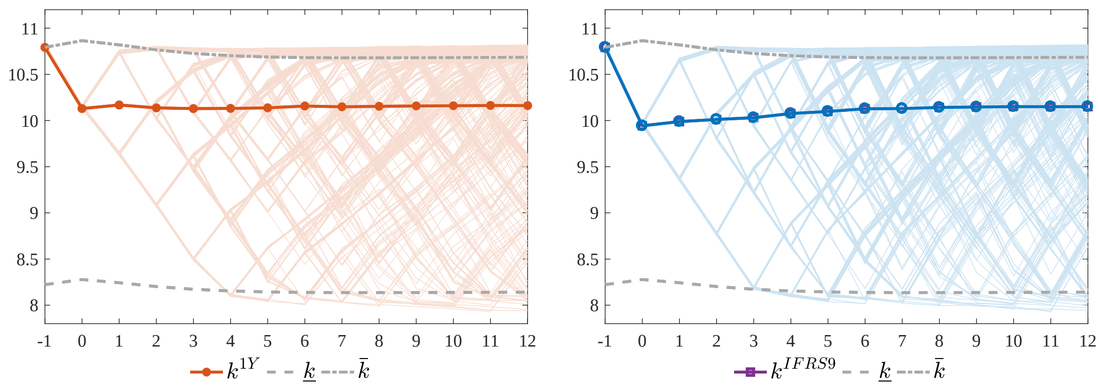

Figure 5. CET1 after the arrival of a contraction (IRB bank)
500 simulated trajectories of CET1 under EL^{1Y} and EL^{IFRS9}
in response to the arrival of s=2 after **a long period** in s=1
(IRB bank, **as a percentage** of average exposures)

6 The case of SA banks

Capital requirements for banks following the standardized approach (SA banks) apply to exposures net of specific provisions and, hence, are sensitive to how those provisions are computed. Thus, Table 6 includes the same variables as Table 5 for IRB banks together with details on the minimum capital requirement implied by each of the impairment measurement methods. Except for the minimum capital requirement and the implied size of a fully-loaded CCB, all the other variables in Table 4 are equally valid for IRB and SA banks.

Table 6
Endogenous variables
under SA capital requirements
(SA bank, as a percentage of mean exposures unless otherwise indicated)

	IL	EL^{1Y}	EL^{LT}	EL^{IFRS9}
P/L				
Unconditional mean	0.15	0.16	0.20	0.17
Conditional mean, expansions	0.34	0.39	0.46	0.44
Conditional mean, contractions	-0.46	-0.62	-0.69	-0.73
Standard deviation	0.34	0.43	0.51	0.50
Minimum capital requirement				
Unconditional mean	7.72	7.57	6.88	7.36
Conditional mean, expansions	7.72	7.56	6.88	7.35
Conditional mean, contractions	7.74	7.58	6.89	7.37
Standard deviation	0.14	0.17	0.18	0.19
CET1				
Unconditional mean	9.70	9.50	8.68	9.23
Conditional mean, expansions	9.88	9.76	8.97	9.54
Conditional mean, contractions	9.04	8.61	7.67	8.19
Standard deviation	0.83	0.83	0.77	0.85
Probability of dividends being paid (%)				
Unconditional	51.32	52.95	59.08	53.20
Conditional, expansions	66.53	68.64	76.59	68.96
Conditional, contractions	0	0	0	0
Dividends, if positive				
Conditional mean, expansions	0.32	0.33	0.35	0.35
Conditional mean, contractions	–	–	–	–
Probability of having to recapitalize (%)				
Unconditional	2.36	2.67	2.67	2.94
Conditional, expansions	0	0	0	0
Conditional, contractions	10.33	11.70	11.68	12.88
Recapitalization, if positive				
Conditional mean, expansions	–	–	–	–
Conditional mean, contractions	0.40	0.30	0.36	0.40

The results in Table 6 are qualitatively very similar to those described for an IRB bank in Table 5, with some quantitative differences that are worth commenting on. It turns out that, in our calibration, an SA bank holding exactly the same loan portfolio as an IRB bank would be able to support it with lower average levels of CET1 (between 48 basis points and

157 basis points lower, depending on the impairment measurement method). Therefore, in a typical year, our SA bank features *de facto* higher leverage levels, and hence higher interest expenses than its IRB counterpart. This explains why its P/L is slightly lower than that of an IRB bank. This difference explains most of the level differences which can be seen in the remaining variables in Table 6.

When comparing impairment measurement methods in the case of an SA bank, the differences are very similar to those observed in Table 5 for IRB banks. The higher state-dependence of the more forward-looking measures explains the higher cross-state differences in CET1, dividends and probabilities of needing capital injections under such measures. As for IRB banks, the differences associated with IFRS 9 relative to either the incurred loss approach or the one-year expected loss approach are significant, but not huge.

To facilitate the comparison of the relevant differences between SA banks and IRB banks, Table 7 contains a selection of variables from previous Tables 4, 5 and 6. For brevity, the discussion focuses on banks adopting IFRS 9. The selection is based on the assumption that the relevant impairment allowances for an SA bank prior to the adoption of IFRS 9 are those of the incurred loss method, IL, while for an IRB bank the one-year expected loss method, EL^{1Y}, is applied, as required by regulation. The results point to IFRS 9 having an extremely similar quantitative impact on SA banks and IRB banks, in terms of both the means and the cyclical sensitivity of the relevant variables.

This is further confirmed by Figure 6, which shows the counterpart of Figure 5 for a bank operating under the SA approach. It depicts 500 simulated trajectories for CET1 under IL and EL^{IFRS9}. As in Figure 5, it takes four consecutive years in the contraction state (s=2) for an SA bank under IFRS 9 to use up its CCB and require a recapitalization, while under the incurred loss method, the CCB would be fully depleted only after (roughly) five years in the contraction state.[30]

[30] In this case, the dashed lines that delimit the band within which CET1 evolves are averages across simulated trajectories, since the sizes of the minimum capital requirement and the minimum capital requirement plus the fully-loaded CCB depend on the size of the corresponding provisions.

Table 7
SA banks vs IRB banks:
Highlighted differences
(as a percentage of mean exposures unless otherwise indicated)

	SA bank		IRB bank	
	IL	EL^{IFRS9}	EL^{1Y}	EL^{IFRS9}
P/L				
Unconditional mean	0.15	0.17	0.17	0.19
Standard deviation	0.34	0.50	0.43	0.50
Minimum capital requirement				
Unconditional mean	7.72	7.36	8.15	8.15
Standard deviation	0.14	0.19	0.07	0.07
CET1				
Unconditional mean	9.70	9.23	10.19	10.17
Standard deviation	0.83	0.85	0.76	0.77
Probability of dividends being paid (%)				
Unconditional	51.32	53.20	51.79	53.93
Conditional, expansions	66.53	68.96	67.11	69.89
Dividends, if positive				
Conditional mean, expansions	0.32	0.35	0.36	0.38
Probability of having to recapitalize (%)				
Unconditional	2.36	2.94	2.86	3.41
Conditional, contractions	10.33	12.88	12.50	14.94
Recapitalization, if positive				
Conditional mean, contractions	0.40	0.40	0.40	0.38

Panel A. CET1 under IL Panel B. CET1 under EL^{IFRS9}

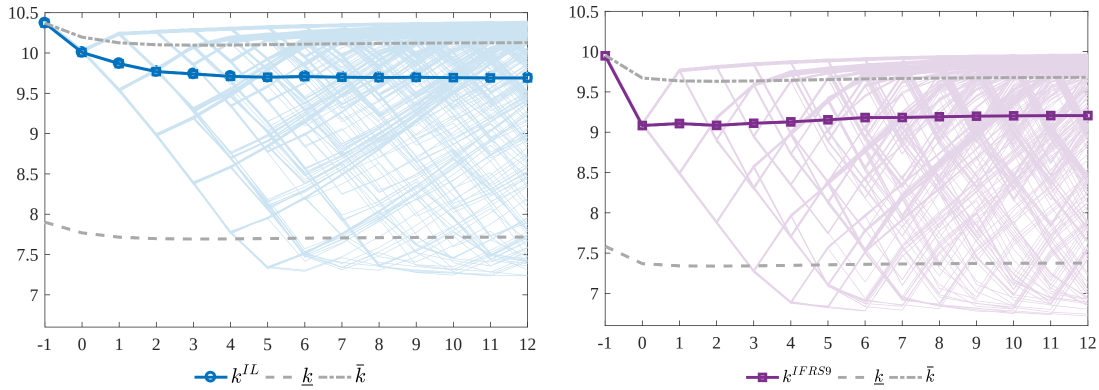

Figure 6. CET1 after the arrival of a contraction (SA bank)
500 simulated trajectories of CET1 under IL and EL^{IFRS9}
in response to the arrival of s=2 after a long period in s=1
(SA bank, as a percentage of average exposures)

7 Extensions

7.1 Especially severe crises

In this section, we explore whether the severity of crises and the potential anticipation of a particularly severe crisis make a difference in terms of our assessment of the cyclicality of the new more forward-looking provisioning methods (IFRS 9 and CECL) vis-a-vis the prior less forward-looking measures (incurred losses and one-year expected losses). For brevity, we again focus on IRB banks and on the comparison of one of the more forward looking approaches. IFRS 9, with just one of the alternatives, namely the one-year expected loss approach (the one so far prescribed by regulation for IRB banks).

7.1.1 Unanticipatedly long crises

We first explore what happens with the dynamic responses analyzed in the benchmark calibration with aggregate risk when we condition them on the realization of the contraction state s=2 for four consecutive periods starting from t=0. So, as in the analysis shown in Figure 4, we assume that the bank starts at t=–1 with the portfolio and impairment allowances resulting from having been in the expansion state (s=1) for a long enough period, and that at t=0 the aggregate state switches to contraction (s=2).

In Figure 7, we compare the average response trajectories already shown in Figure 4 (where, from t=1 onwards, the aggregate state evolves stochastically according to the Markov chain calibrated in Table 3) with trajectories conditional on remaining in state s=2 for at least up to date t=3 (four years).[31]

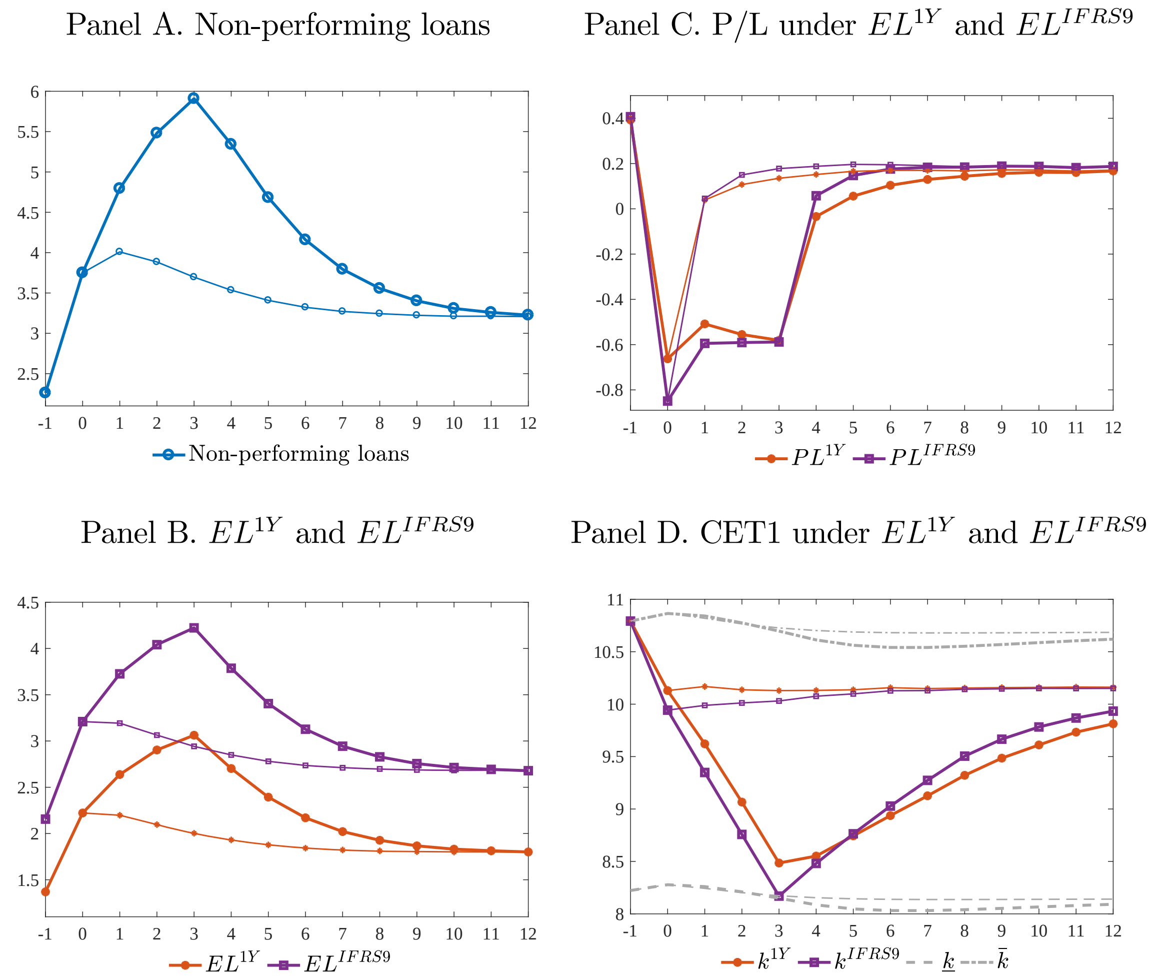

Figure 7. Unanticipatedly long crises
Average responses to the arrival of s=2 when the contraction is unanticipatedly "long" (thick lines) rather than "average" (thin lines) (IRB bank, as a percentage of average exposures)

[31] In the conditional trajectories, the aggregate state is again assumed to evolve according to the calibrated Markov chain from t=4 onwards.

When a crisis is longer than expected, the largest differential impact of EL^{IFRS9} relative to EL^{1Y} still happens in the first year of the crisis (t=0), since EL^{IFRS9} front-loads the expected beyond-one-year losses of the stage 2 loans. In years two to four of the crisis (t=1,2,3) the differential impact of IFRS 9 (compared to one-year) expected losses on P/L lessens before it switches sign (after t=5). In the first years of the crisis, EL^{IFRS9} leaves CET1 closer to the recapitalization band and, in the fourth year (t=3), the duration of the crisis forces the bank to recapitalize only under EL^{IFRS9}. However, EL^{IFRS9} supports a quicker recovery of profitability and, hence, CET1 after t=5.

7.1.2 Anticipatedly long crises

We now turn to the case in which crises can be anticipated to be long from their outset. To study this case, we extend the model to add a third aggregate state that describes "long crises" (s=3) as opposed to "short crises" (s=2) or "expansions" (s=1). To streamline the analysis, we make s=2 and s=3 have exactly the same impact on credit risk parameters as prior s=2 in Table 3, and keep the impact of s=1 on credit risk parameters also exactly the same as in Table 3. The only difference between states s=2 and s=3 is their persistence, which determines the average time it takes for a crisis period to come to an end. Specifically, we consider the following transition probability matrix for the aggregate state:

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} = \begin{pmatrix} 0.8520 & 0.6348 & 0.250 \\ 0.1221 & 0.3652 & 0 \\ 0.0259 & 0 & 0.750 \end{pmatrix}, \quad (26)$$

which implies an average duration of four years for long crises (s=3), 1.6 years for short crises (s=2), and the same duration as in our benchmark calibration for periods of expansion (s=1). The parameters in (26) are calibrated to make s=3 to occur with an unconditional frequency of 8% (equivalent to suffering an average of two long crises per century) and to keep the unconditional frequency of s=1 at the same 77% as in our benchmark calibration.

In Figure 8 we compare the average response trajectories that follow the entry in state s=2 (thin lines) or state s=3 (thick lines) after having spent a sufficiently long period in state s=1. Therefore, the figure illustrates the average differences between entering a "normal" short crisis or a "less frequent" long crisis at t=0. Both EL^{1Y} and EL^{IFRS9} behave differently across short and long crises from the very first period, since even the one-year ahead loss projections behind EL^{1Y} factor in the lower probability of a recovery at t=1 under s=3 than under s=2. However, EL^{IFRS9} additionally takes into account the losses further into the future associated with the stage 2 loans. Hence, the differential rise on impact experienced by EL^{IFRS9} is higher than that experienced by EL^{1Y}.

This difference also explains the larger initial impact of IFRS 9 on P/L and CET1. As a result, at the onset of an anticipatedly long crisis, EL^{IFRS9} pushes CET1 closer to the recapitalization band and the difference with respect to EL^{1Y} increases. Quantitatively, however, the effect on CET1 is still moderate, using up on impact less than half of the fully loaded CCB. Of course, later on in the long crisis, EL^{IFRS9} results, on average, in a quicker recovery of profitability and CET1 than EL^{1Y}.

As a quantitative summary of the implications of an anticipatedly long crisis, the following table reports the unconditional yearly probabilities of the bank needing equity injections, under each of the impairment measures compared, in the baseline model with aggregate risk and in the current extension:

	IL	EL^{1Y}	EL^{LT}	EL^{IFRS9}
Baseline model	2.34%	2.86%	2.34%	3.41%
Model with anticipatedly long crises	3.28%	3.78%	4.23%	4.52%

Note that the anticipatedly long crises significantly increase the probability of having to recapitalize banks under the CECL approach (EL^{LT}), making it closer to the one obtained under IFRS 9.

7.2 Better foreseeable crises

We now consider the case in which some crises can be foreseen one year in advance. **Similar** to the treatment of long crises in the previous subsection, we formalize this by introducing a third aggregate state, s=3, which describes normal or expansion states in which a crisis (transition to state s'=2) is expected in the next year with a larger than usual probability. So we make s=3 identical to s=1 in all respects (that is, the way it affects the PDs, rating migration probabilities, and LGDs of the loans, et cetera) except in the probability of switching to aggregate state $s' = 2$ in the next year.

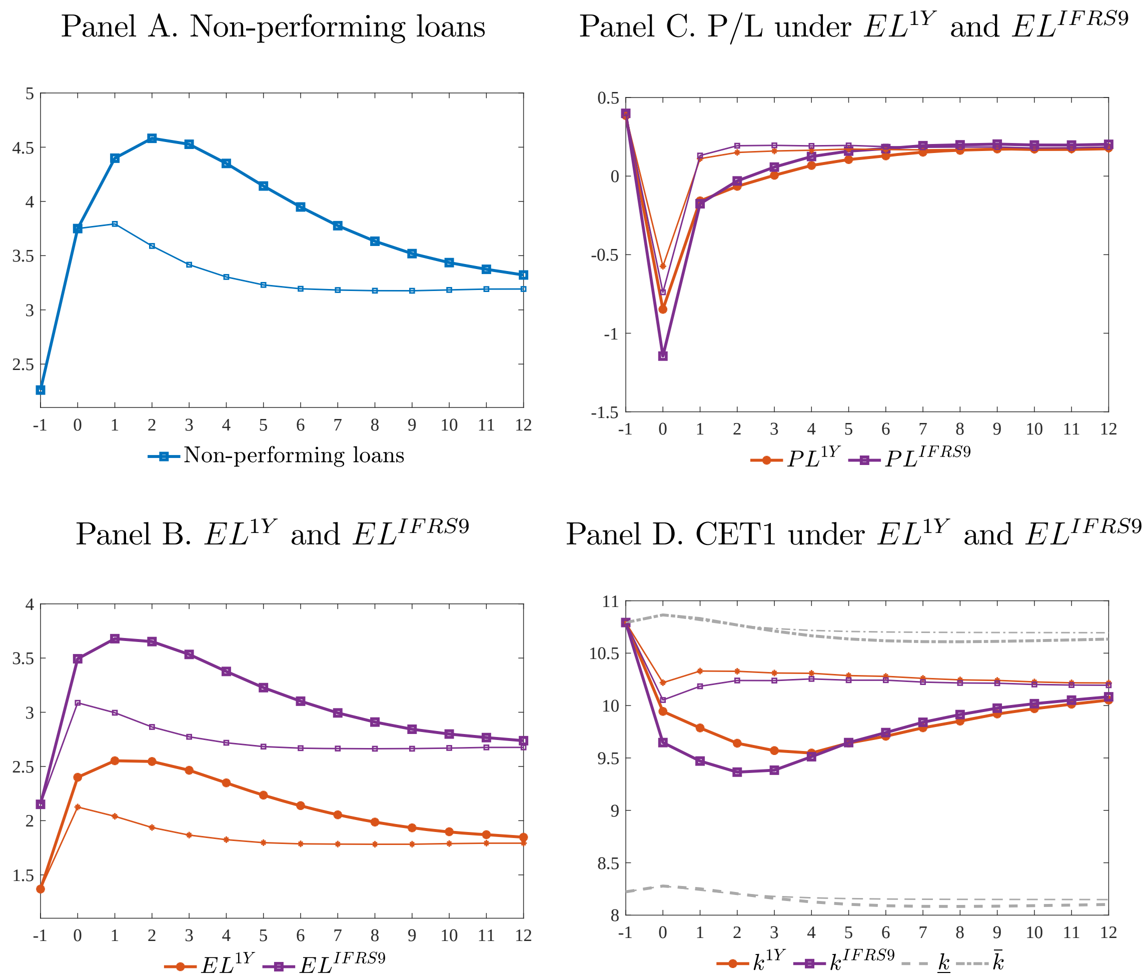

Figure 8. Anticipatedly long crises
Average responses to the arrival of a contraction at t=0 when it is anticipated to be "long" (s'=3, thick lines) rather than "normal" (s'=2, thin lines) (IRB bank, as a percentage of average exposures)

To streamline the analysis, we look at the case in which s=3 is followed by s'=1 with probability one and assume that half of the crisis are preceded by $s = 3$ (while the other half are preceded, as before, by $s = 1$, which means that they are not seen as coming). Adjusting the transition probabilities to imply the same relative frequencies and expected durations of non-crisis versus crisis periods as the baseline calibration in Table 3, the matrix of state transition probabilities used for this exercise is

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} = \begin{pmatrix} 0.8391 & 0.5 & 0 \\ 0.0740 & 0.5 & 1 \\ 0.0869 & 0 & 0 \end{pmatrix}.$$

The thick lines in Figure 9 show the average response trajectories to the arrival of the pre-crisis state s'=3 at t=–1 after having spent a long time in the normal state s=1. We compare EL^{1Y} and EL^{IFRS9} and include, using thin lines, the results of the baseline model (regarding the arrival of s'=2 at t=0 having been in s=1 for a long period). The results confirm the notion that being able to better anticipate the arrival of a crisis helps to considerably soften its impact on impairment allowances, P/L, and CET1.

Finally, as in the previous extension, the following table reports the unconditional yearly probabilities of the bank needing equity injections under each of the impairment measures compared, in the baseline model with aggregate risk and in the current extension. Indeed, crises that are better anticipated imply a lower yearly probability that the bank needs an equity injection. Yet, the ranking of the various approaches in terms of this variable remains the same as in the baseline model, with IFRS 9 performing the worst:

	IL	EL^{1Y}	EL^{LT}	EL^{IFRS9}
Baseline model	2.34%	2.86%	2.34%	3.41%
Model with better foreseeable crises	1.84%	1.99%	1.54%	2.66%

7.3 Other possible extensions

In this section, we briefly describe additional extensions that the model could accommodate at some cost in terms of notational, computational, and calibration complexity.

Multiple standard and substandard ratings Adding more rating categories within the broader standard and substandard categories would essentially imply expanding the dimensionality of the vectors and matrices described in the baseline model and in the aggregate-risk extension. If loans were assumed to be originated in more than just one category, the need to keep track of the (various) contractual interest rates for discounting purposes means we would need to expand the dimensionality of the model further. Alternatively, an equivalent and potentially less notationally cumbersome possibility would be to consider the same number of portfolios as different-at-origination loans and to aggregate across them the impairment allowances and the implications for P/L and CET1.

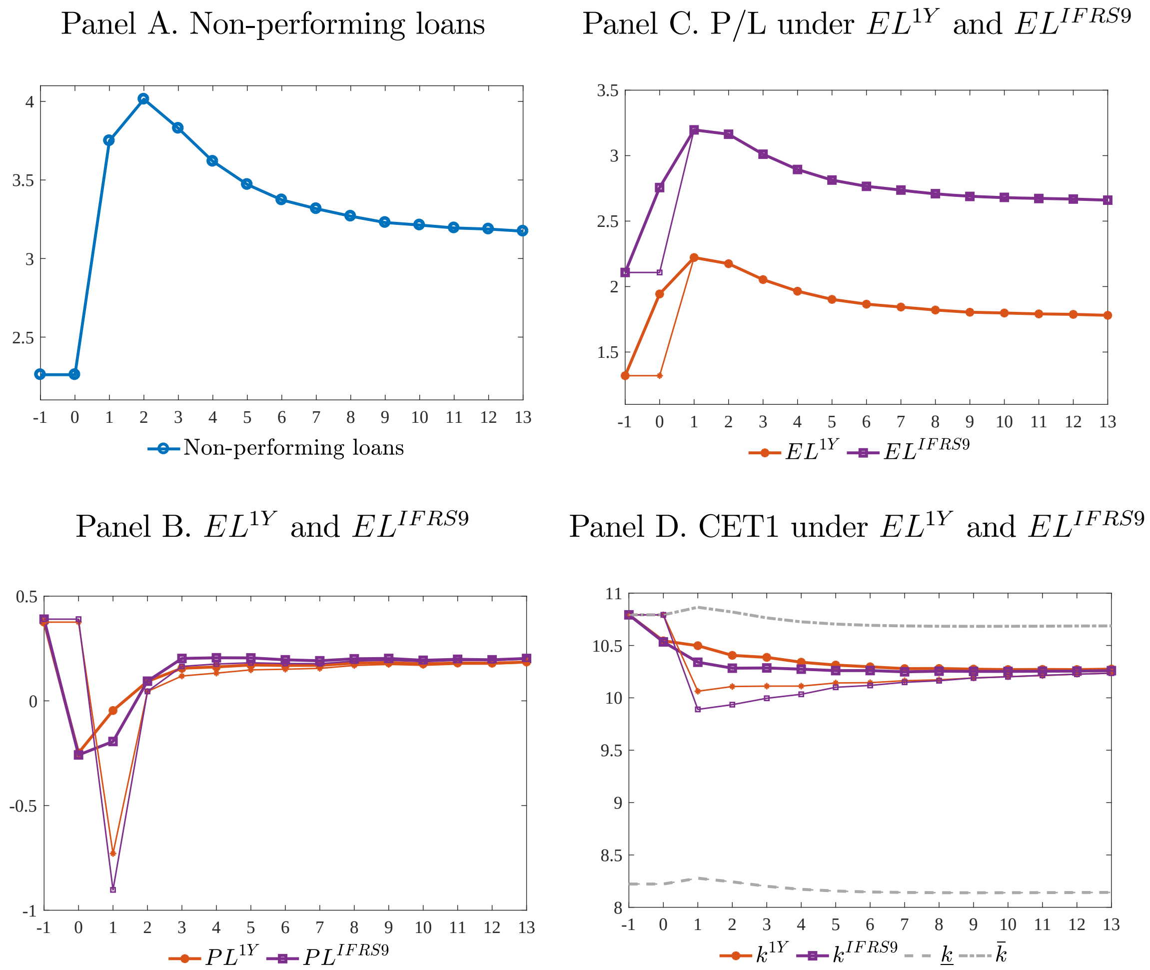

Figure 9. Better foreseeable crises
Average responses to the arrival of pre-crisis state at t=–1 after long in s=1 (thick lines). Thin lines describe the arrival of s=2 at t=0 in the baseline model (IRB bank, as a percentage of average exposures)

Relative criterion for credit quality deterioration This extension would be a natural further development of the previous one and only relevant for the assessment of IFRS 9. Under IFRS 9, the shift to the lifetime approach ("stage 2") for a given loan is supposed to be applied not when an absolute substandard rating is attained, but when the deterioration in terms of the rating at origination is significant in relative terms, for example because the rating has fallen by more than two or three notches. This distinction is relevant if operating under a ratings scale that is finer than the one we have used in our analysis. As in the case with the above-mentioned multiple standard and substandard ratings, keeping the analysis recursive under the relative criterion for treating loans as "stage 1" or "stage 2" loans in IFRS 9 would require considering as many portfolios as different-at-origination loan ratings and writing expressions for impairment allowances that impute lifetime expected losses to the components of each portfolio for which the current rating is significantly lower than the initial rating.

8 Macroprudential implications

What are the implications of these results with regard to the potential procyclical effects linked to the various provisioning methods? Are the provisions associated with IFRS 9 and CECL more procyclical than their predecessors? Answering these questions is difficult. Even in the absence of offsetting regulatory filters or sufficient excess capital buffers, a fall in CET1 that reduces a bank's CCB (and hence forces it to cancel its dividends), or even leads to it requiring equity issuance in order to continue complying with the minimum capital requirement, does not necessarily imply that credit supply will contract. This will depend on to what extent the bank dislikes cancelling dividends and, if the CCB is get fully depleted, on how quickly or cheaply the bank can raise new capital. Our simulations are produced as if there were no concerns or imperfections on these two fronts. Otherwise, the bank might opt for reducing its lending. If this process occurs at an economy-wide level (e.g. in response to an aggregate shock), the contractionary effects on aggregate credit supply might be significant, potentially causing negative second-round effects on the system (e.g. by weakening aggregate demand or damaging inter-firm credit chains) and likely causing even larger default rates among the surviving loans.

These feedback effects –although theoretically and empirically difficult to assess– are at the heart of the motivation for the macroprudential approach to financial regulation.[32] Sim-

[32] As put by Hanson, Kashyap and Stein (2011, p. 5), "in the simplest terms, one can characterize the macroprudential approach to financial regulation as an effort to control the social costs associated with excessive balance sheet shrinkage on the part of multiple financial institutions hit with a common shock."

ilar to discussions on the potential procyclical effects of Basel capital requirements (Kashyap and Stein, 2004, and Repullo and Suarez, 2013), there are multiple factors that will determine whether or not ECL methods will add procyclicality to the system. For example, even if it causes a contraction in credit supply when a negative shock hits the economy, such a contraction may be lower than the contraction in credit demand, which may also be negatively affected by the shock. Moreover, banks may react to IFRS 9 and CECL by choosing to have larger voluntary capital buffers in the first place. Besides, the negative effects of an additional contraction in credit supply may be counterbalanced by the advantages of an earlier recognition of loan losses (e.g. by precluding forbearance or the continuation of dividend payments during the initial stages of a crisis), including the possibility that they could enable banks to return to sound financial health more quickly.

Despite all these considerations, recent evidence (including Mésonnier and Monks, 2015, Gropp et al., 2016, and Jiménez et al., 2017) suggests that banks tend to accommodate sudden increases in capital requirements or other regulatory buffers (or, similarly, falls in available regulatory capital) by reducing risk-weighted assets, most typically bank lending, which has significant impact on the real economy. While the size of the additional procyclical losses of regulatory capital implied by our results is not alarming, it is significant enough to warrant further macroprudential attention.

Fortunately, there is a broad range of policies that might help to address the procyclical effects of IFRS 9 and CECL if deemed necessary. One possibility is to rely on the existing regulatory buffers and, specifically, on the countercyclical capital buffer (CCyB), possibly after a suitable revision of its guidance. The national macroprudential authorities could proactively use the CCyB to offset undesirable credit supply effects. This would involve setting the CCyB at a level above zero in expansionary or normal times, so as to have the capacity to partly or fully release it if, and when, the change in aggregate conditions leads to a sudden increase in provisioning needs. This use of this macroprudential tool could be combined with internal and external stress tests as a means to gauge the importance of the variation in impairment allowances associated with adverse scenarios, guarantee the sufficiency of the micro- and macroprudential buffers, and allow for remedial policy action if required.

9 Conclusions

We have described a simple recursive model for the assessment of the level and cyclical implications of the new ECL approaches to impairment allowances under IFRS 9 and the incoming update of US GAAP. We have calibrated the model to represent a portfolio of corporate loans of an EU bank. We have compared the old incurred loss approach, the one-year expected loss approach (used to establish the regulatory provisions of IRB banks), the lifetime expected loss approach behind the CECL of US GAAP, and the mixed-horizon expected loss approach of IFRS 9.

Our results suggest that the loan loss provisions implied by IFRS 9 and the CECL approach will rise more suddenly than their predecessors when the cyclical position of the economy switches from expansion to contraction (or if banks experience a shock that sizably damages the credit quality of their loan portfolios). This implies that P/L and, without the application of regulatory filters, CET1 will decline more severely at the start of those episodes.

While the early and decisive recognition of forthcoming losses may have significant advantages (e.g. in terms of transparency, market discipline, inducing prompt supervisory intervention, etc.), it may also imply, via its effects on regulatory capital, a loss of lending capacity for banks at the very beginning of a contraction (or in the direct aftermath of a negative credit-quality shock), potentially contributing, through feedback effects, to its severity. With this concern in mind, the quantitative results of the paper suggest that the arrival of an average recession might imply on-impact losses of CET1 twice as large as those under the incurred loss approach and equivalent to about one third of the fully-loaded CCB of the analyzed bank. While this loss is significantly smaller than the amount that would deplete a fully-loaded CCB and thus presumably manageable in most circumstances, it would be advisable for macroprudential authorities to monitor the developments on this front (e.g. through stress testing) and to stand ready to take compensatory measures (e.g. the release of the CCyB), if the potential negative impact on credit supply makes it necessary.

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Appendices

A Calibration details

A.1 Migration and default rates for our two non-default states

We calibrate the migration and default probabilities of our two non-default loan categories using S&P rating migration data with a finer rating partition. Specifically, let the 7×7 matrix $\widetilde{A}$ describe yearly migrations across the seven non-default ratings in the main S&P classification, namely AAA, AA, A, BBB, BB, B and CCC/C. Under our convention, each element $\tilde{a}_{ij}$ of this matrix denotes a loan's probability of migrating to S&P rating i from S&P rating j, and the yearly probability of default corresponding to S&P rating j can be found as $\widetilde{PD}_j = 1 - \sum_{i=1}^{7} \tilde{a}_{ij}$.[33] We obtain $\widetilde{A}$ by averaging the yearly matrices provided by S&P global corporate default studies covering the period from 1981 to 2015:

$$\widetilde{A} = \begin{pmatrix} 0.8960 & 0.0054 & 0.0005 & 0.0002 & 0.0002 & 0.0000 & 0.0007 \\ 0.0967 & 0.9073 & 0.0209 & 0.0022 & 0.0008 & 0.0006 & 0.0000 \\ 0.0048 & 0.0798 & 0.9161 & 0.0463 & 0.0034 & 0.0026 & 0.0022 \\ 0.0010 & 0.0056 & 0.0557 & 0.8930 & 0.0626 & 0.0034 & 0.0039 \\ 0.0005 & 0.0007 & 0.0044 & 0.0465 & 0.8343 & 0.0618 & 0.0112 \\ 0.0003 & 0.0009 & 0.0017 & 0.0082 & 0.0809 & 0.8392 & 0.1390 \\ 0.0006 & 0.0002 & 0.0002 & 0.0013 & 0.0079 & 0.0432 & 0.5752 \end{pmatrix}, \tag{A.1}$$

which implies

$$\widetilde{PD}^T = (0.0000, 0.0002, 0.0005, 0.0023, 0.0100, 0.0493, 0.2678).$$

In order to calibrate our model, we want to collapse the above seven-state Markov process to the two-state one specified in our model. We want to obtain its 2×2 transition probability matrix, which we denote A, and the implied probabilities of default in each state, $PD_j = 1 - \sum_{i=1}^{2} a_{ij}$ for j=1,2. To collapse the seven-state process into the two-state process, we assume that the S&P states 1 to 5 (AAA, AA, A, BBB, BB) correspond to our state 1 and S&P states 6 to 7 (B, CCC/C) to our state 2. We also assume that all the loans originated by the bank belong to the BB category, so that the vector representing the entry of new loans in steady state under the S&P classification is $\widetilde{e}^T = (0, 0, 0, 0, 1, 0, 0)$. Under these assumptions, we produce an average PD for the steady state portfolio of 1.88%, slightly below the 2.5% average PD on non-defaulted exposures of reported by the EBA (2013, Figure 12) for the period from the first half of 2009 to the second half of 2012 for a sample of EU banks using the IRB approach.

[33] We have reweighted the original migration rates in S&P matrices to avoid having "non-rated" as a possible migration.

The steady state portfolio under the S&P classification can be found as $z^* = [I_{7\times 7} - \widetilde{M}]^{-1}\widetilde{e}$, where the matrix $\widetilde{M}$ has elements $\widetilde{m}_{ij} = (1-\delta_j)\widetilde{a}_{ij}$ and δ_j is the independent probability of a loan rated j maturing at the end of period t. For the calibration we set δ_j=0.20 across all categories, so that loans have an average maturity of five years. The "collapsed" steady state portfolio x^* associated with z^* has $x_1^* = \sum_{j=1}^{5} z_j^*$ and $x_2^* = \sum_{j=6}^{7} z_j^*$.

For the collapsed portfolio, we construct the 2×2 transition matrix M (that accounts for loan maturity) as

$$M = \begin{pmatrix} \frac{\sum_{j=1}^{5}\sum_{i=1}^{5}\widetilde{m}_{ij}z_j^*}{x_1^*} & \frac{\sum_{j=6}^{7}\sum_{i=1}^{5}\widetilde{m}_{ij}z_j^*}{x_2^*} & 0 \\ \frac{\sum_{j=1}^{5}\sum_{i=6}^{7}\widetilde{m}_{ij}z_j^*}{x_1^*} & \frac{\sum_{j=6}^{7}\sum_{i=6}^{7}\widetilde{m}_{ij}z_j^*}{x_2^*} & 0 \\ (1-\delta_3/2)PD_1 & (1-\delta_3/2)PD_2 & (1-\delta_3) \end{pmatrix}, \tag{A.2}$$

where the probabilities of default for the collapsed categories are found as

$$PD_1 = \frac{\sum_{j=1}^{5}\widetilde{PD}_j z_j^*}{x_1^*},$$

and

$$PD_2 = \frac{\sum_{j=6}^{7}\widetilde{PD}_j z_j^*}{x_2^*}.$$

Putting it in words, we find the moments describing the dynamics of the collapsed portfolio as weighted averages of those of the original distribution, with the weights determined by the steady state composition of the collapsed categories in terms of the initial categories.

A.2 Calibrating defaulted loans' resolution rate

The yearly probability of resolution of NPLs δ_3 is calibrated to match the 5% average probability of default including defaulted exposures ($PDID$) that the EBA (2013, Figure 10) reports for the second half of 2008. In the model the value of such probability in steady state can be computed as

$$PDID = \frac{PD_1 x_1^* + PD_2 x_2^* + x_3^*}{\sum_{j=1}^{3} x_j^*}.$$

Solving for x_3^* we find

$$x_3^* = \frac{PD_1 x_1^* + PD_2 x_2^* - (x_1^* + x_2^*)PDID}{PDID - 1}. \tag{A.3}$$

It should be noted that the dynamic system in (1) allows us to compute x_1^* and x_2^* independently from δ_3, so the law of motion of NPLs evaluated at the steady state implies

$$x_3^* = (1-\delta_3/2)PD_1 x_1^* + (1-\delta_3/2)PD_2 x_2^* + (1-\delta_3)x_3^*$$

or

$$\delta_3 = \frac{2(PD_1 x_1^* + PD_2 x_2^*)}{PD_1 x_1^* + PD_2 x_2^* + 2x_3^*}. \tag{A.4}$$

Finally, we can evaluate (A.4) using x_1^*, x_2^* and the value of x_3^* found in (A.3).

A.3 State contingent migration matrices

In the model described in Appendix B, we capture aggregate risk through an aggregate state variable $s_t \in \{1, 2\}$ that follows a Markov chain with a time-invariant transition matrix. We calibrate the state contingent migration matrices $M(1)$ and $M(2)$ of such a version of the model following a procedure analogous to that which results in M in (A.2) but starting from state-contingent versions, $\widetilde{A}(1)$ and $\widetilde{A}(2)$, of the 7×7 migration matrix $\widetilde{A}$ in (A.1). As described in A.1, we can go from each $\widetilde{A}(s)$ to the maturity adjusted matrix $\widetilde{M}(s)$ with elements $\widetilde{m}_{ij}(s) = (1 - \delta_j)\widetilde{a}_{ij}$ and then find the elements of $M(s)$ as weighted averages of the elements of $\widetilde{M}(s)$. To keep things simple, we use the same unconditional weights as in (A.2), implying

$$M(s) = \begin{pmatrix} \frac{\sum_{j=1}^{5}\sum_{i=1}^{5}\widetilde{m}_{ij}(s)z_j^*}{x_1^*} & \frac{\sum_{j=6}^{7}\sum_{i=1}^{5}\widetilde{m}_{ij}(s)z_j^*}{x_2^*} & 0 \\ \frac{\sum_{j=1}^{5}\sum_{i=6}^{7}\widetilde{m}_{ij}(s)z_j^*}{x_1^*} & \frac{\sum_{j=6}^{7}\sum_{i=6}^{7}\widetilde{m}_{ij}(s)z_j^*}{x_2^*} & 0 \\ (1-\delta_3(s)/2)PD_1(s) & (1-\delta_3(s)/2)PD_2(s) & (1-\delta_3(s)) \end{pmatrix}$$

where

$$PD_1(s) = \frac{\sum_{j=1}^{5}\widetilde{PD}_j(s)z_j^*}{x_1^*},$$

$$PD_2(s) = \frac{\sum_{j=6}^{7}\widetilde{PD}_j(s)z_j^*}{x_2^*},$$

with $\widetilde{PD}_j(s) = 1 - \sum_{i=1}^{7}\tilde{a}_{ij}(s)$.

We calibrate $\widetilde{A}(1)$ and $\widetilde{A}(2)$ exploring the business cycle sensitivity of S&P yearly migration matrices previously averaged to find $\widetilde{A}$. We identify state s=1 with normal or expansion years and s=2 with crisis or contraction years. We use the years identified by the NBER as the start of the recession to identify the entry in state s=2 and assume that each of the contractions observed in the period from 1981 to 2015 lasted exactly two years. This is consistent with the NBER dating of US recessions except for the recession started in 2001, to which the NBER attributes a duration of less than one year. However, the behavior of corporate ratings migrations and defaults around such recession does not suggest it was shorter for our purposes than the other three. To illustrate this, Figure A1 depicts the time series of two of the elements of the yearly default rates $\widetilde{PD}_j$ and migration matrices $\widetilde{A}$ whose cyclical behavior is more evident: (i) the default rate among BB exposures ($\widetilde{PD}_5$) and (ii)

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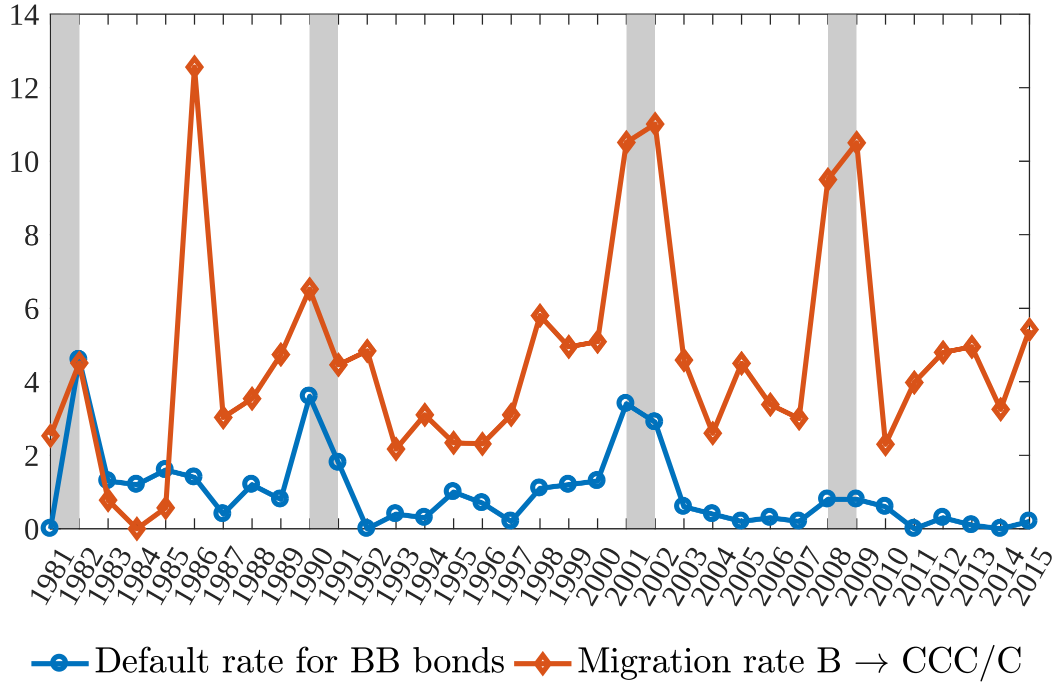

Figure A.1. Sensitivity of default and migrations rates to aggregate states
Selected yearly S&P default and downgrading rates. Grey bars identify 2-year periods following the start of NBER recessions

In light of this, we estimate $\widetilde{A}(2)$ by averaging the yearly migration matrices of years 1981, 1982, 1990, 1991, 2001, 2002, 2008 and 2009, and $\widetilde{A}(1)$ by averaging all the remaining ones. This leads to

$$\widetilde{A}(1) = \begin{pmatrix} 0.8923 & 0.0057 & 0.0005 & 0.0002 & 0.0002 & 0.0000 & 0.0000 \\ 0.1012 & 0.9203 & 0.0209 & 0.0023 & 0.0007 & 0.0003 & 0.0000 \\ 0.0039 & 0.0668 & 0.9228 & 0.0500 & 0.0036 & 0.0025 & 0.0027 \\ 0.0010 & 0.0058 & 0.0495 & 0.8939 & 0.0668 & 0.0036 & 0.0043 \\ 0.0007 & 0.0002 & 0.0040 & 0.0429 & 0.8484 & 0.0679 & 0.0117 \\ 0.0000 & 0.0009 & 0.0020 & 0.0084 & 0.0680 & 0.8511 & 0.1548 \\ 0.0000 & 0.0002 & 0.0001 & 0.0009 & 0.0059 & 0.0360 & 0.5860 \end{pmatrix},$$

implying

$$\widetilde{PD}(1)^T = (0.0000, 0.0001, 0.0002, 0.0014, 0.0063, 0.0386, 0.2405),$$

and

$$\widetilde{A}(2) = \begin{pmatrix} 0.9087 & 0.0044 & 0.0003 & 0.0005 & 0.0002 & 0.0000 & 0.0030 \\ 0.0786 & 0.8632 & 0.0209 & 0.0014 & 0.0013 & 0.0017 & 0.0000 \\ 0.0077 & 0.1237 & 0.8936 & 0.0340 & 0.0026 & 0.0027 & 0.0009 \\ 0.0010 & 0.0050 & 0.0767 & 0.8899 & 0.0482 & 0.0028 & 0.0024 \\ 0.0000 & 0.0022 & 0.0057 & 0.0587 & 0.7865 & 0.0411 & 0.0095 \\ 0.0013 & 0.0007 & 0.0008 & 0.0076 & 0.1245 & 0.7988 & 0.0858 \\ 0.0027 & 0.0002 & 0.0006 & 0.0025 & 0.0143 & 0.0676 & 0.5389 \end{pmatrix},$$

implying

$$\widetilde{PD}(2)^T = (0.0000, 0.0005, 0.0014, 0.0054, 0.0224, 0.0853, 0.3596).$$

Finally, we set p_{12} =Prob($s_{t+1} = 1|s_t = 2$) equal to 0.5 so that contractions have an expected duration of two years, and p_{21} =Prob($s_{t+1} = 2|s_t = 1$) equal to 0.148 so that expansion periods have the same average duration as the ones observed in our sample period, (35-8)/4=6.75 years.

B The model with aggregate risk

In this appendix we present the equations of the benchmark model with aggregate risk. We capture the latter by introducing an aggregate state variable that can take two values $s_t \in \{1, 2\}$ at each date t and follows a Markov chain with time-invariant transition probabilities $p_{s's}$ =Prob($s_{t+1} = s'|s_t = s$). The approach can be trivially generalized to deal with a larger number of aggregate states.

In order to measure expected losses corresponding to default events in any future date t, we have to keep track of the aggregate state in which the loans existing at t were originated, z=1,2, the aggregate state at time t, s=1, 2, and the credit quality or rating of the loan at t, j=1, 2, 3. Thus, it is convenient to describe (stochastic) loan portfolios held at any date t as vectors of the form

$$y_t = \begin{pmatrix} x_t(1,1,1) \\ x_t(1,1,2) \\ x_t(1,1,3) \\ x_t(1,2,1) \\ x_t(1,2,2) \\ x_t(1,2,3) \\ x_t(2,1,1) \\ x_t(2,1,2) \\ x_t(2,1,3) \\ x_t(2,2,1) \\ x_t(2,2,2) \\ x_t(2,2,3) \end{pmatrix}, \tag{B.1}$$

where component $x_t(z, s, j)$ denotes the measure of loans at t that were originated in aggregate state z, are in aggregate state s and have rating j.[34]

Our assumptions regarding the evolution and payoffs of the loans between any date t and $t + 1$ are as follows. Loans rated j=1, 2 at t mature at $t + 1$ with probability $\delta_j(s')$, where s' denotes the aggregate state at $t + 1$ (unknown at date t). In the case of NPLs (j=3), $\delta_3(s')$ represents the independent probability of a loan being resolved, in which case

[34] Along a specific history (or sequence of aggregate states), for any z and j, the value of $x_t(z, s, j)$ will equal 0 whenever $s_t \neq s$.

it pays back a fraction $1 - \widetilde{\lambda}(s')$ of its unit principal and exits the portfolio. Conditional on s', each loan rated j=1, 2 at t which matures at $t+1$ defaults independently with probability $PD_j(s')$, being resolved within the period with probability $\delta_3(s')/2$ or entering the stock of NPLs (j=3) with probability $1-\delta_3(s')/2$. Maturing loans that do not default pay back their principal of one plus the contractual interest c_z, established at origination.

Conditional on s', each loan rated j=1, 2 at t which does not mature at $t+1$ goes through one of the following exhaustive possibilities:

1. Default, which occurs independently with probability $PD_j(s')$, and in which case one of two things can happen: (i) it is resolved within the period with probability $\delta_3(s')/2$; or (ii) it enters the stock of NPLs (j=3) with probability $1 - \delta_3(s')/2$.

2. Migration to rating $i \neq j$ (i=1,2), in which case it pays interest c_z and continues for one more period; this occurs independently with probability $a_{ij}(s')$.

3. Continuation in rating j, in which case it pays interest c_z and continues for one more period; this occurs independently with probability

$$a_{jj}(s') = 1 - a_{ij}(s') - PD_j(s').$$

B.1 Portfolio dynamics under aggregate risk

Under aggregate risk, the dynamics of the loan portfolio between any dates t and $t+1$ is no longer deterministic, but driven by the realization of the aggregate state variable at $t+1$, s_{t+1}. To describe the dynamics of the system compactly, let the binary variable $\xi_{t+1} = 1$ if $s_{t+1} = 1$ and $\xi_{t+1} = 0$ if $s_{t+1} = 2$. The dynamics of the system can be described as

$$y_{t+1} = G(\xi_{t+1})y_t + g(\xi_{t+1}),$$

where

$$G(\xi_{t+1}) = \begin{pmatrix} \begin{pmatrix} \xi_{t+1}M(1) & \xi_{t+1}M(1) \\ (1-\xi_{t+1})\,M(2) & (1-\xi_{t+1})\,M(2) \end{pmatrix} & 0_{6\times 6} \\ 0_{6\times 6} & \begin{pmatrix} \xi_{t+1}M(1) & \xi_{t+1}M(1) \\ (1-\xi_{t+1})\,M(2) & (1-\xi_{t+1})\,M(2) \end{pmatrix} \end{pmatrix},$$

$$g(\xi_{t+1})^T = \begin{pmatrix} \xi_{t+1}e_1(1), & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & (1-\xi_{t+1})\,e_1(2), & 0, & 0 \end{pmatrix},$$

$$\xi_{t+1} = \begin{cases} 1 & \text{if } u_{t+1} \in [0, p_{1s_t}], \\ 0 & \text{otherwise,} \end{cases}$$

$$s_{t+1} = \xi_{t+1} + 2\left(1 - \xi_{t+1}\right),$$

u_{t+1} is an independently and identically distributed uniform random variable with support $[0,1]$, $e_1(s')$ is the (potentially different across states s') measure of new loans originated at $t+1$, and $0_{6\times 6}$ denotes a 6×6 matrix full of zeros.

B.2 Incurred losses

Incurred losses measured at date t would be those associated with NPLs that are part of the bank's portfolio at date t. Thus, the incurred losses reported at t would be given by

$$IL_t = \sum_{z=1,2} \sum_{s=1,2} \lambda(s) x_t(z, s, 3),$$

where $\lambda(s)$ is the expected LGD on an NPL conditional on being at state s in date t. This can be more compactly expressed as

$$IL_t = \widehat{b} y_t, \qquad \text{(B.2)}$$

where $\widehat{b} = (0, 0, \lambda(1), 0, 0, \lambda(2), 0, 0, \lambda(1), 0, 0, \lambda(2))$.

The expected LGD conditional on each current state s can be found as functions of the previously specified primitives of the model (state-transition probabilities, probabilities of resolution of the defaulted loans in subsequent periods, and loss rates $\widetilde{\lambda}(s')$ suffered if resolution happens in each of the possible future states s') by solving the following system of recursive equations:

$$\lambda(s) = \sum_{s'=1,2} p_{s's} \left[\delta_3(s') \widetilde{\lambda}(s') + (1 - \delta_3(s')) \lambda(s') \right], \qquad \text{(B.3)}$$

for s=1, 2.

B.3 Discounted one-year expected losses

Based on the loan portfolio held by the bank at t, provisions computed on the basis of the discounted one-year expected losses add to the incurred losses written above the losses stemming from default events expected to occur within the year immediately following. Since a period in the model is one year, the corresponding allowances are given by

$$EL_t^{1Y} = (b_\beta + \widehat{b}) y_t, \qquad \text{(B.4)}$$

where $b_\beta = (\beta_1 b, \beta_2 b)$, $\beta_z = 1/(1 + c_z)$, and $b = (b_{11}, b_{12}, 0, b_{21}, b_{22}, 0)$, with

$$b_{sj} = \sum_{s'=1,2} p_{s's} PD_j(s') \left\{ [\delta_3(s')/2] \, \widetilde{\lambda}(s') + [1 - \delta_3(s')/2] \, \lambda(s') \right\}, \qquad \text{(B.5)}$$

for j=1, 2. The coefficients defined in (B.5) attribute one-year expected losses to loans rated j=1, 2 in state s by taking into account their PD and LGD over each of the possible states s' that can be reached at $t + 1$, where the corresponding s' are weighted by their probability of occurring given s. The losses associated these one-year ahead defaults are discounted using the contractual interest rate of the loans, c_z, as set at their origination. In Section B.6, we derive an expression for the endogenous value of such rate under our assumptions on loan pricing. As for the loans that are already non-performing (j=3) at date t, the term $\widehat{b}y_t$ in (B.4) implies attributing their conditional-on-s LGD to them, exactly as in (B.2).

B.4 Discounted lifetime expected losses

Impairment allowances computed on an lifetime-expected basis imply taking into account not just the default events that may affect the currently performing loans in the next year, but also those occurring in any subsequent period. Building on prior notation and the same approach explained for the model without aggregate risk, these provisions can be computed as

$$\begin{aligned} EL_t^{LT} &= b_\beta y_t + b_\beta M_\beta y_t + b_\beta M_\beta^2 y_t + b_\beta M_\beta^3 y_t + ... + \widehat{b}y_t \\ &= b_\beta(I + M_\beta + M_\beta^2 + M_\beta^3 + ...)y_t + \widehat{b}y_t \\ &= b_\beta(I - M_\beta)^{-1} y_t + \widehat{b}y_t = (b_\beta B_\beta + \widehat{b})y_t, \end{aligned} \tag{B.6}$$

with

$$M_\beta = \begin{pmatrix} \beta_1 M_p & 0_{6\times 6} \\ 0_{6\times 6} & \beta_2 M_p \end{pmatrix},$$

$$M_p = \begin{pmatrix} p_{11}M(1) & p_{12}M(1) \\ p_{21}M(2) & p_{22}M(2) \end{pmatrix},$$

$$M(s') = \begin{pmatrix} m_{11}(s') & m_{12}(s') & 0 \\ m_{21}(s') & m_{22}(s') & 0 \\ (1-\delta_3(s')/2)PD_1(s') & (1-\delta_3(s')/2)PD_2(s') & (1-\delta_3(s')) \end{pmatrix},$$

and $m_{ij}(s') = (1 - \delta_j(s'))a_{ij}(s')$.

B.5 Discounted expected losses under IFRS 9

As already mentioned, IFRS 9 adopts a hybrid approach that combines the one-year-ahead and lifetime approaches described above. Specifically, it applies the one-year-ahead measurement to loans whose credit quality has not increased significantly since origination. For us, these are the loans with j=1, namely those in the components $x_t(z, s, 1)$ of y_t. By contrast, it considers the lifetime expected losses for loans whose credit risk has significantly increased since origination. For us, these are the loans with j=2, namely those in the components $x_t(z, s, 2)$ of y_t.

As in the case without aggregate risk, it is convenient to split vector y_t into a new auxiliary vector

$$\hat{y}_t = \begin{pmatrix} x_t(1,1,1) \\ 0 \\ 0 \\ x_t(1,2,1) \\ 0 \\ 0 \\ x_t(2,1,1) \\ 0 \\ 0 \\ x_t(2,2,1) \\ 0 \\ 0 \end{pmatrix},$$

which contains the loans with j=1, and the difference

$$\tilde{y}_t = y_t - \hat{y}_t,$$

which contains the rest.

Combining the formulas obtained in (B.4) and (B.6), loan loss provisions under IFRS 9 can be compactly described as follows:[35]

$$EL_t^{IFRS9} = b_\beta \hat{y}_t + b_\beta B_\beta \tilde{y}_t + \widehat{b} y_t. \tag{B.7}$$

B.6 Determining the initial loan rate

Taking advantage of the recursivity of the model, for given values of the contractual interest rates c_z of the loans originated in each of the aggregate states z=1,2, one can obtain the ex-coupon value of a loan originated in state z, when the current aggregate state is s and their current rating is j, $v_j(z,s)$, by solving the system of Bellman-type equations given by:

$$\begin{aligned} v_j(z,s) &= \mu \sum_{s'=1,2} p_{s's} \Big[(1 - PD_j(s'))c_z + (1 - PD_j(s'))\delta_j(s') + PD_j(s')(\delta_3(s')/2)(1 - \widetilde{\lambda}(s')) \\ &\qquad + m_{1j}(s')v_1(z,s') + m_{2j}(s')v_2(z,s') + m_{3j}(s')v_3(z,s') \Big], \end{aligned} \tag{B.8}$$

for $(z,s,j) \in \{1,2\} \times \{1,2\} \times \{1,2\}$, and

$$v_j(z,s) = \mu \sum_{s'=1,2} p_{s's} [\delta_3(s')(1 - \widetilde{\lambda}(s')) + (1 - \delta_3(s'))v_3(z,s')],$$

for $(z,s,j) \in \{1,2\} \times \{1,2\} \times \{3\}$.

Under perfect competition and using the fact that all loans are assumed to be of credit quality j=1 at origination, the interest rates c_z can be found as those that make $v(z,z,1) = 1$ for z=1,2, respectively.

[35] These definitions clearly imply $EL_t^{IFRS9} = EL_t^{LT} - b_\beta(B_\beta - I)\hat{y}_t \leq EL_t^{LT}$ and $EL_t^{IFRS9} = EL_t^{1Y} + b_\beta(B_\beta - I)\tilde{y}_t \geq EL_t^{1Y}$.

B.7 Implications for P/L and CET1

By trivially extending the formula derived for the case without aggregate risk, the result of the P/L account with aggregate risk can be written as

$$\begin{aligned} PL_t \;=\; & \sum_{z=1,2}\left\{\sum_{j=1,2}\left[c_z(1\text{–}PD_j(s_t))\text{–}\frac{\delta_3(s_t)}{2}PD_j(s_t)\widetilde{\lambda}(s_t)\right]x_{t\text{-}1}(z,s_t,j)\text{–}\delta_3(s_t)\widetilde{\lambda}(s_t)x_{t\text{-}1}(z,s_t,3)\right\} \\ & \text{–}r\left(\sum_{z=1,2}\sum_{j=1,2,3}x_{t\text{-}1}(z,s_t,j)\text{–}a_{t\text{-}1}\text{–}k_{t\text{-}1}\right)\text{–}\Delta a_t, \end{aligned} \tag{B.9}$$

which differs from (20) in the dependence on the aggregate state at the end of period t, s_t, of a number of the relevant parameters affecting the default, resolution, and loss upon resolution of the loans.

With the same logic as in the baseline model, dividends and equity injections are now determined by

$$\text{div}_t = \max[(k_{t-1} + PL_t) - 1.3125\underline{k}_t, 0] \tag{B.10}$$

and

$$\text{recap}_t = \max[\underline{k}_t - (k_{t-1} + PL_t), 0]. \tag{B.11}$$

Finally, for IRB banks, the minimum capital requirement is now given by[36]

$$\underline{k}_t^{IRB} = \sum_{j=1,2} \gamma_j(s_t)x_{jt}, \tag{B.12}$$

and

$$\gamma_j(s_t)\text{=}\lambda(s_t)\frac{1\text{+}\left[\left(\sum_{s'}p_{s's_t}\frac{1}{\delta_j(s')}\right)\text{–}2.5\right]\overline{m}_j}{1\text{–}1.5\overline{m}_j}\left[\Phi\left(\frac{\Phi^{-1}(\overline{PD}_j)+\text{cor}_j^{0.5}\Phi^{-1}(0.999)}{(1\text{–}\text{cor}_j)^{0.5}}\right)\text{–}\overline{PD}_j\right], \tag{B.13}$$

where $\overline{m}_j = [0.11852 - 0.05478\ln(\overline{PD}_j)]^2$ is a maturity adjustment coefficient, $\overline{\text{cor}}_j$ is a correlation coefficient fixed as $\overline{\text{cor}}_j = 0.24 - 0.12(1 - \exp(-50\overline{PD}_j))/(1 - \exp(-50))$, and

$$\overline{PD}_j = \sum_{i=1,2} \pi_i PD_j(s_i) \tag{B.14}$$

is the through-the-cycle PD for loans rated j (with π_i denoting the unconditional probability of aggregate state i). Equation (B.14) implies assuming that the bank follows a strict through-the-cycle approach to the calculation of capital requirements (which avoids adding cyclicality to the system through this channel).[37]

[36] For SA banks, the equation for the minimum capital requirements in (25) remains valid.

[37] A point-in-time approach would imply setting $\overline{PD}_j(s_t) = \sum_{s'} p_{s's_t}PD_j(s')$ instead of $\overline{PD}_j$ in (B.13).