Supplemental Material

GDP Solera
The Ideal Vintage Mix

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[Link to the paper]

SM.A Proof of identification results

SM.A.1 Proposition 1

Proof. Let f_w denote the spectrum of a time series $\{w_t\}$. Identification of the autocovariance function of $\{w_t\}$ is equivalent to identification of f_w. Therefore, an alternative statement for proposition 1 is that under assumption 1, if $N > 1$, $f_{\Delta x}$ and $f_{v_1}, \dots, f_{v_I}$ are nonparametrically identified from $f_{\Delta y}$. To understand why, let us write

$$f_{\Delta y}(\lambda) = 1_{M\times M} f_{\Delta x}(\lambda) + |1 - e^{i\lambda}|^2 \operatorname{diag}\Big(f_{v_1}(\lambda), \dots, f_{v_N}(\lambda)\Big), \quad 0 \le \lambda \le 2\pi.$$

If E_i is the $M_i \times M$ matrix such that $y_{it} = E_i y_t$, we get $E_{i_1} f_{\Delta y}(\lambda) E'_{i_2} = 1_{M_{i_1}\times M_{i_2}} f_{\Delta x}(\lambda)$ for $i_1 \neq i_2$, where the pair i_1, i_2 exists only if $N > 1$. With $f_{\Delta x}$ pinned down, one then recovers

$$f_{v_i}(\lambda) = |1 - e^{i\lambda}|^{-2} E_i [f_{Dy}(\lambda) - 1_{M\times M} f_{Dx}(\lambda)] E'_i,$$

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dealing with the removable singularity at $\lambda = 0$ by using that each entry f_{v_i} is holomorphic over the unit circle. □

It follows from the proof of proposition 1 that if in addition to $N > 1$ we have $M_i > 1$ for at least one i, the model imposes overidentifying restrictions and is, therefore, testable. This is the case in our empirical analysis. If the spectra $f_{\Delta x}, f_{v_1}, \ldots, f_{v_N}$ belong to a particular parametric class, an alternative approach to testing the overidentifying restrictions would rely on dynamic specification tests, as in Fiorentini and Sentana (2019).

SM.A.2 Proposition 2

Proof. By condition (b) in the proposition, $D_{i_1} = E_{i_1}\delta_t$ and $D_{i_2} = E_{i_2}\delta_t$ are time-invariant. By assumption 1 and condition (a), moreover, $E_{i_1}v_t$ and $E_{i_2}v_t$ are uncorrelated at all lags and leads. Consequently,

$$E_{i_1} f_{\Delta y}(\lambda) E'_{i_2} = D_{i_1} f_{\Delta x}(\lambda) D'_{i_2}, \quad 0 \leq \lambda \leq 2\pi.$$

Now, by condition (c), we have that $\text{rank}(D_{i_1}) = \text{rank}(D_{i_2}) = C$. Therefore,

$$f_{\Delta x} = (D'_{i_1} D_{i_1})^{-1} D'_{i_1} f_{\Delta y} D_{i_2} (D'_{i_2} D_{i_2})^{-1}.$$

Identification of $f_{v_1}, \ldots, f_{v_N}$ then follows by an argument analogous to the one in the proof of proposition 1. □

SM.B Details of estimation and filtering

Our objective is to conduct inference on parameters θ and latent variables $x_1, \ldots, x_T$. As we mentioned in section 3, a Bayesian approach offers a convenient option to perform estimation and filtering, integrating sampling and signal-extraction uncertainty in a unified, conceptually natural framework. Moreover, our model lends itself to stable and efficient algorithms, exploiting a Gibbs sampler for estimation and the Durbin and Koopman (2002) algorithm for signal extraction.

This appendix discusses the details. Subsection SM.B.1 casts our model in state-space form, while subsection SM.B.2 presents the family of priors we use. In turn, subsection SM.B.3 lays out the Gibbs sampler algorithm to draw from the posterior distribution of parameters and latent variables, and subsection SM.B.4 explains how we implement filtering in a way that automatically accounts for sampling uncertainty. Finally, subsection SM.B.5 reports the posterior distributions from our full-sample estimates.

SM.B.1 State-space representation of the model

The parameter vector of the model is $\theta = (\theta_x, \theta_1, \ldots, \theta_N)$. Given θ, we can cast equations (2), (3) and (4) in state-space form as

$$
\begin{aligned}
y_t &= H_t X_t, \\
X_t &= C(\theta) + F(\theta) X_{t-1} + G(\theta) U_t, \\
U_t &\overset{iid}{\sim} N(0_{(C+M)\times 1}, I_{C+M}), \\
\text{where} \qquad X_t &= (x_t, x_{t-1}, v_{1t}, \ldots, v_{Nt})', \\
U_t &= (\varepsilon_{xt}, \varepsilon_{1t}, \ldots, \varepsilon_{Nt})', \\
H_t &= \left(\delta_t \;\; 0_{M\times C} \;\; I_M\right), \\
C(\theta) &= \begin{pmatrix} \mu_x \\ 0_{C\times 1} \\ 0_{M\times 1} \end{pmatrix}, \\
F(\theta) &= \operatorname{diag}\left(\begin{bmatrix} I_C + \operatorname{diag}(\rho_x) & -\operatorname{diag}(\rho_x) \\ I_C & 0_{C\times C} \end{bmatrix}, \operatorname{diag}(\rho_1), \ldots, \operatorname{diag}(\rho_N) \right), \\
G(\theta) &= \operatorname{diag}\left(\operatorname{Ch}(\Sigma_x), \operatorname{Ch}(\Sigma_1), \ldots, \operatorname{Ch}(\Sigma_N)\right).
\end{aligned}
$$

For the initial condition we have $X_1 \sim N(\tilde{\mu}_{X_1}, \tilde{\Sigma}_{X_1})$ which are compatible with both μ_{x_1}, Σ_{x_1} and the covariance-stationarity of $v_{1t}, \ldots, v_{Nt}$.

This linear state-space representation with Gaussian errors is important as it implies that $X_1, \ldots, X_T, U_1, \ldots, U_T$ will be jointly normally distributed conditional on $y_1, \ldots, y_T, \theta$, so that we can rely on the algorithm of Durbin and Koopman (2002) to efficiently simulate the conditional distribution of the latent variables given the observables.

SM.B.2 Prior distributions

We specify $N+1$ independent priors for $\theta_x, \theta_1, \ldots, \theta_N$. The family of priors we describe is fairly standard and permits a simple implementation of the Gibbs sampler when the priors are conjugate conditional on the latent variables. It can also accommodate a flat prior for certain values of the hyperparameters.

Specifically, for the parameters of the signals process we use

i) $\Sigma_x \sim \mathcal{W}_C^{-1}(d_x S_x, d_x)$, where $\mathcal{W}^{-1}(dS, S)$ is the inverse Wishart distribution with mean $S/(d - \dim(S) - 1)$ and degrees-of-freedom parameter d, and

ii) $\beta_x = (\mu_x', \rho_x')'|\Sigma_x \sim N(b_x, R_x^{-1} \otimes \Sigma_x)$.

The hyperparameters S_x and b_x control the prior location of Σ_x and β_x, while d_x and R_x govern the informativeness of the prior distributions. In particular, high values of d_x and R_x produce tight priors while $d_x = 0$ and $R_x = 0_{2\times 2}$ yield a flat prior over Σ_x and β_x.

For the parameters of the measurement errors processes $v_i, i = 1, \ldots, N$, we use

i) $\Sigma_i \sim \mathcal{W}_{M_i}^{-1}(d_i S_i, d_i)$, and

ii) $\rho_i|\Sigma_i \sim N(b_i, R_i^{-1} \otimes \Sigma_i)$.

The same considerations we made for S_x, b_x, d_x, R_x above apply to S_i, b_i, d_i, R_i too.

In our empirical analysis we calibrate the location hyperparameters using the estimates of the two-measurement model in Almuzara, Fiorentini, and Sentana (2023). Specifically, we choose the following values:

a) $b_x = (m_x 1_{1\times C}, r_m 1_{1\times C})'$ and $R_x = \text{diag}\{10, 30\}$. This mean that in effect, we shrink our model towards a diagonal VAR. We set $m_x = 0.375$ and $r_m = 0.5$.

b) $S_x = \varsigma_x[w_x 1_{C\times C} + (1 - w_x)I_C]$ and $d_x = 20 + C$. In other words, we shrink our model towards one in which the covariance matrix of innovations to x_t has a common-factor structure. We set $\varsigma_x = 0.57$ and $w_x = 0.8$.

c) $b_i = r_i 1_{M_i\times 1}$ and $R_i = 10$. We calibrate $r_{\text{GDE}} = 0$ for GDE and $r_{\text{GDI}} = 0.8$ for GDI.

d) $S_i = \varsigma_i[w_v 1_{M_i\times M_i} + (1 - w_v)I_{M_i}]$ and $d_v = 20 + M_i$. We set $\varsigma_i = 0.1(1 + r_i)$ and $w_v = 0.2$.

SM.B.3 Estimation algorithm

Let $p(\cdot)$ denote a generic density (with respect to an appropriate dominating measure), and define $y = (y_1, \dots, y_T)$ and $X = (X_1, \dots, X_T)$. Although the prior $p(\theta)$ and the likelihood $p(y|\theta)$ are readily available, with the latter being an output of the Kalman filter applied to the state-space representation of the model, the posterior $p(\theta|y)$ is not. Bayesian estimation can instead be performed via Markov Chain Monte Carlo (MCMC), an algorithm that effectively draws a Markov chain $\{\theta^s\}_{s\geq 1}$ whose simulated unconditional distribution approximates the desired posterior.

A convenient approach to MCMC in our model is Gibbs sampling, which draws from the posterior of a block of variables or parameters conditional on previous draws from the other blocks in a sequential manner. The algorithm updates unknowns by drawing iteratively from the following distributions:

(1) $p(X|\theta, y)$: using the state-space representation of the model, X is obtained from the simulation smoother proposed by Durbin and Koopman (2002).

(2) $p(\theta_x|\theta_1, \dots, \theta_N, X, y)$: first notice that $\Delta x_1, \dots, \Delta x_T$ are sufficient for θ_x, i.e.,

$$p(\theta_x|\theta_1, \dots, \theta_N, X, y) = p(\theta_x|\Delta x_1, \dots, \Delta x_T)\,,$$

and because of the conjugacy of the prior we recover μ_x, ρ_x, Σ_x from

(i) $\Sigma_x|\mu_x, \rho_x, \Delta x_1, \dots, \Delta x_T \sim \mathcal{W}_C^{-1}(\tilde{d}_x\tilde{S}_x, \tilde{d}_x)$, where

$$\tilde{d}_x = d_x + T - 1,$$

$$\tilde{d}_x \tilde{S}_x = d_x S_x + \sum_{t=2}^{T} (\Delta x_t - \mu_x - \text{diag}(\rho_x)\Delta x_{t-1})(\Delta x_t - \mu_x - \text{diag}(\rho_x)\Delta x_{t-1})' \,;$$

(ii) $\beta_x | \Sigma_x, \Delta x_1, \ldots, \Delta x_T \sim N(\tilde{b}_x, \tilde{R}_x^{-1})$, where

$$\tilde{R}_x = R_x \otimes \Sigma_x^{-1} + \sum_{t=2}^{T} \begin{pmatrix} I_C \\ \text{diag}(\Delta x_{t-1}) \end{pmatrix} \Sigma_x^{-1} \begin{pmatrix} I_C & \text{diag}(\Delta x_{t-1}) \end{pmatrix},$$

$$\tilde{R}_x \tilde{b}_x = (R_x \otimes \Sigma_x^{-1}) b_x + \sum_{t=2}^{T} \begin{pmatrix} \Sigma_x^{-1} \Delta x_t \\ \text{diag}(\Delta x_{t-1}) \Sigma_x^{-1} \Delta x_t \end{pmatrix}.$$

(3) $p\left(\theta_i \middle| \theta_x, (\theta_j)_{j \neq i}, X, y\right)$ for each i: first notice that $(v_{i1}, \ldots, v_{iT})$ are sufficient for θ_i, i.e.,

$$p\left(\theta_i \middle| \theta_x, (\theta_j)_{j \neq i}, X, y\right) = p(\theta_i | v_{i1}, \ldots, v_{iT}) \,,$$

and because of the conjugacy of the prior we recover ρ_i, Σ_i from

(i) $\Sigma_i | \rho_i, v_{i1}, \ldots, v_{iT} \sim \mathcal{W}_{M_i}^{-1}(\tilde{d}_i \tilde{S}_i, \tilde{d}_i)$, where

$$\tilde{d}_i = d_i + T - 1,$$

$$\tilde{d}_i \tilde{S}_i = d_i S_i + \sum_{t=2}^{T} \left(v_{it} - \text{diag}(\rho_i) v_{i,t-1}\right)^2 ;$$

(ii) $\rho_i | \Sigma_i, v_{i1}, \ldots, v_{iT} \sim N(\tilde{b}_i, \tilde{R}_i^{-1})$, where

$$\tilde{R}_i = R_i \otimes \Sigma_i^{-1} + \sum_{t=2}^{T} \text{diag}(v_{i,t-1}) \Sigma_i^{-1} \text{diag}(v_{i,t-1}),$$

$$\tilde{R}_i \tilde{b}_i = (R_i \otimes \Sigma_i^{-1}) b_i + \sum_{t=2}^{T} \text{diag}(v_{i,t-1}) \Sigma_i^{-1} v_{it}.$$

Flat priors can be implemented by setting the hyperparameters values $d_x = 0_{C \times 1}$, $R_x = 0_{3 \times 3}$, $d_i = 0_{M_i \times 1}$, and $R_i = 0_{2 \times 2}$, which despite generating improper priors still lead to a well-defined algorithm and a proper posterior.

SM.B.4 Filtering

Signal extraction of x_t is a natural by-product of our estimation procedure. The latent variable draws we obtain in step (1) from iteration over the Gibbs sampler algorithm

$(X^s)_{s\geq 1}$ have the desired distribution $p(X|y)$. Moreover, the Gibbs sampler already integrates estimation uncertainty because

$$p(X\big|y) = \int_{\Theta} p(X\big|\theta, y)\ p(\theta\big|y)\ d\theta,$$

where Θ denotes the parameter space.

It is worth noting that while $p(X\big|\theta, y)$ is a normal density, X given y need not be normal once θ is integrated out. In particular, Var $(x_t\big|y)$ may depend on the data through the posterior density of θ, in contrast to Var $(x_t\big|\theta, y)$, which is constant in y.

The Markov chain $(X^s, \theta^s)_{s\geq 1}$ is all we need to approximate by simulation the posterior distribution of the different objects of interest that we study in the empirical section.

SM.B.5 Posterior distributions

We estimate our model using the prior described above running the Gibbs sampler for 200,000 iterations with a burn-in of 100,000 and a thinning of 1 every 5 iterations. The result is a Markov chain $(X^s, \theta^s)_{s=1}^S$, with $S = 20,000$ and low autocorrelation across draws that by all accounts appears to have converged. Our empirical analysis is based on it.

Here we present a summary of the posterior estimates.

Parameter	Prior Median	Posterior Median	Prior 90%-Probability	Posterior 90%-Probability	MC s.e.
$\mu_x{}^{(1)}$	0.367	0.420	[−0.042, 0.783]	[0.324, 0.518]	0.0004
$\mu_x{}^{(2)}$	0.367	0.430	[−0.048, 0.780]	[0.338, 0.522]	0.0004
$\mu_x{}^{(3)}$	0.369	0.418	[−0.050, 0.777]	[0.328, 0.511]	0.0004
$\mu_x{}^{(4)}$	0.367	0.440	[−0.046, 0.779]	[0.350, 0.534]	0.0004
$\mu_x{}^{(5)}$	0.369	0.446	[−0.043, 0.781]	[0.358, 0.539]	0.0004
$\rho_x{}^{(1)}$	0.497	0.290	[0.263, 0.737]	[0.183, 0.394]	0.0005
$\rho_x{}^{(2)}$	0.499	0.292	[0.263, 0.735]	[0.190, 0.391]	0.0005
$\rho_x{}^{(3)}$	0.500	0.307	[0.262, 0.737]	[0.205, 0.405]	0.0005
$\rho_x{}^{(4)}$	0.500	0.292	[0.262, 0.741]	[0.191, 0.389]	0.0005
$\rho_x{}^{(5)}$	0.498	0.289	[0.262, 0.733]	[0.188, 0.385]	0.0004
$\Sigma_x{}^{(1,1)}$	0.590	0.293	[0.366, 1.034]	[0.234, 0.371]	0.0003
$\Sigma_x{}^{(2,1)}$	0.470	0.245	[0.262, 0.861]	[0.195, 0.311]	0.0003
$\Sigma_x{}^{(3,1)}$	0.468	0.253	[0.263, 0.864]	[0.200, 0.320]	0.0003
$\Sigma_x{}^{(4,1)}$	0.469	0.250	[0.263, 0.864]	[0.199, 0.317]	0.0003

$\Sigma_x{}^{(5,1)}$	0.469	0.249	[0.265, 0.865]	[0.199, 0.313]	0.0003
$\Sigma_x{}^{(1,2)}$	0.470	0.245	[0.262, 0.861]	[0.195, 0.311]	0.0003
$\Sigma_x{}^{(2,2)}$	0.590	0.277	[0.364, 1.032]	[0.224, 0.343]	0.0003
$\Sigma_x{}^{(3,2)}$	0.470	0.250	[0.264, 0.864]	[0.199, 0.314]	0.0003
$\Sigma_x{}^{(4,2)}$	0.471	0.247	[0.263, 0.866]	[0.198, 0.310]	0.0003
$\Sigma_x{}^{(5,2)}$	0.470	0.245	[0.263, 0.862]	[0.198, 0.307]	0.0002
$\Sigma_x{}^{(1,3)}$	0.468	0.253	[0.263, 0.864]	[0.200, 0.320]	0.0003
$\Sigma_x{}^{(2,3)}$	0.470	0.250	[0.264, 0.864]	[0.199, 0.314]	0.0003
$\Sigma_x{}^{(3,3)}$	0.592	0.294	[0.367, 1.036]	[0.238, 0.366]	0.0003
$\Sigma_x{}^{(4,3)}$	0.469	0.259	[0.265, 0.866]	[0.208, 0.325]	0.0003
$\Sigma_x{}^{(5,3)}$	0.468	0.256	[0.266, 0.866]	[0.207, 0.321]	0.0003
$\Sigma_x{}^{(1,4)}$	0.469	0.250	[0.263, 0.864]	[0.199, 0.317]	0.0003
$\Sigma_x{}^{(2,4)}$	0.471	0.247	[0.263, 0.866]	[0.198, 0.310]	0.0003
$\Sigma_x{}^{(3,4)}$	0.469	0.259	[0.265, 0.866]	[0.208, 0.325]	0.0003
$\Sigma_x{}^{(4,4)}$	0.591	0.286	[0.367, 1.032]	[0.234, 0.355]	0.0003
$\Sigma_x{}^{(5,4)}$	0.469	0.257	[0.267, 0.867]	[0.208, 0.320]	0.0003
$\Sigma_x{}^{(1,5)}$	0.469	0.249	[0.265, 0.865]	[0.199, 0.313]	0.0003
$\Sigma_x{}^{(2,5)}$	0.470	0.245	[0.263, 0.862]	[0.198, 0.307]	0.0002
$\Sigma_x{}^{(3,5)}$	0.468	0.256	[0.266, 0.866]	[0.207, 0.321]	0.0003
$\Sigma_x{}^{(4,5)}$	0.469	0.257	[0.267, 0.867]	[0.208, 0.320]	0.0003
$\Sigma_x{}^{(5,5)}$	0.590	0.283	[0.369, 1.030]	[0.232, 0.350]	0.0003
$\rho_{GDE}{}^{(nc1)}$	−0.002	0.780	[−0.210, 0.211]	[0.691, 0.855]	0.0005
$\rho_{GDE}{}^{(nc2)}$	−0.002	0.779	[−0.210, 0.208]	[0.693, 0.851]	0.0005
$\rho_{GDE}{}^{(nc3)}$	−0.001	0.777	[−0.208, 0.210]	[0.690, 0.849]	0.0005
$\rho_{GDE}{}^{(nc4)}$	−0.001	0.615	[−0.210, 0.209]	[0.496, 0.722]	0.0006
$\rho_{GDE}{}^{(nc5)}$	0.000	0.386	[−0.211, 0.209]	[0.240, 0.525]	0.0008
$\rho_{GDE}{}^{(nc6)}$	−0.002	0.300	[−0.210, 0.204]	[0.145, 0.456]	0.0009
$\rho_{GDE}{}^{(c1)}$	−0.003	0.358	[−0.210, 0.210]	[0.168, 0.547]	0.0013
$\rho_{GDE}{}^{(c2)}$	0.000	0.369	[−0.207, 0.208]	[0.195, 0.547]	0.0012
$\rho_{GDE}{}^{(c3)}$	−0.002	0.389	[−0.213, 0.210]	[0.216, 0.558]	0.0012
$\rho_{GDE}{}^{(c4)}$	−0.002	0.380	[−0.211, 0.208]	[0.223, 0.536]	0.0010
$\rho_{GDE}{}^{(c5)}$	0.002	0.370	[−0.212, 0.212]	[0.213, 0.530]	0.0010
$\Sigma_{GDE}{}^{(nc1,nc1)}$	0.152	0.206	[0.095, 0.268]	[0.156, 0.275]	0.0003
$\Sigma_{GDE}{}^{(nc2,nc1)}$	0.029	0.159	[−0.025, 0.099]	[0.113, 0.221]	0.0003
$\Sigma_{GDE}{}^{(nc3,nc1)}$	0.029	0.139	[−0.026, 0.100]	[0.095, 0.197]	0.0003
$\Sigma_{GDE}{}^{(nc4,nc1)}$	0.029	0.077	[−0.024, 0.100]	[0.039, 0.125]	0.0002
$\Sigma_{GDE}{}^{(nc5,nc1)}$	0.029	0.034	[−0.025, 0.098]	[0.001, 0.069]	0.0002
$\Sigma_{GDE}{}^{(nc6,nc1)}$	0.029	0.015	[−0.025, 0.099]	[−0.017, 0.049]	0.0002
$\Sigma_{GDE}{}^{(c1,nc1)}$	0.030	0.033	[−0.025, 0.101]	[−0.005, 0.074]	0.0003
$\Sigma_{GDE}{}^{(c2,nc1)}$	0.030	0.032	[−0.025, 0.099]	[−0.004, 0.069]	0.0003
$\Sigma_{GDE}{}^{(c3,nc1)}$	0.030	0.033	[−0.024, 0.100]	[−0.003, 0.072]	0.0003
$\Sigma_{GDE}{}^{(c4,nc1)}$	0.029	0.030	[−0.025, 0.100]	[−0.002, 0.065]	0.0002
$\Sigma_{GDE}{}^{(c5,nc1)}$	0.030	0.028	[−0.024, 0.100]	[−0.003, 0.061]	0.0002
$\Sigma_{GDE}{}^{(nc1,nc2)}$	0.029	0.159	[−0.025, 0.099]	[0.113, 0.221]	0.0003
$\Sigma_{GDE}{}^{(nc2,nc2)}$	0.152	0.185	[0.095, 0.265]	[0.138, 0.248]	0.0003
$\Sigma_{GDE}{}^{(nc3,nc2)}$	0.029	0.143	[−0.025, 0.100]	[0.100, 0.201]	0.0003

$\Sigma_{GDE}{}^{(nc4,nc2)}$	0.030	0.075	[−0.024, 0.099]	[0.038, 0.122]	0.0002
$\Sigma_{GDE}{}^{(nc5,nc2)}$	0.029	0.033	[−0.025, 0.100]	[0.001, 0.067]	0.0002
$\Sigma_{GDE}{}^{(nc6,nc2)}$	0.030	0.020	[−0.025, 0.100]	[−0.011, 0.053]	0.0002
$\Sigma_{GDE}{}^{(c1,nc2)}$	0.029	0.034	[−0.025, 0.100]	[−0.002, 0.074]	0.0003
$\Sigma_{GDE}{}^{(c2,nc2)}$	0.030	0.033	[−0.025, 0.100]	[−0.001, 0.070]	0.0003
$\Sigma_{GDE}{}^{(c3,nc2)}$	0.029	0.035	[−0.024, 0.100]	[0.001, 0.072]	0.0003
$\Sigma_{GDE}{}^{(c4,nc2)}$	0.030	0.031	[−0.024, 0.100]	[0.000, 0.065]	0.0002
$\Sigma_{GDE}{}^{(c5,nc2)}$	0.029	0.030	[−0.024, 0.100]	[0.001, 0.062]	0.0002
$\Sigma_{GDE}{}^{(nc1,nc3)}$	0.029	0.139	[−0.026, 0.100]	[0.095, 0.197]	0.0003
$\Sigma_{GDE}{}^{(nc2,nc3)}$	0.029	0.143	[−0.025, 0.100]	[0.100, 0.201]	0.0003
$\Sigma_{GDE}{}^{(nc3,nc3)}$	0.152	0.176	[0.094, 0.265]	[0.132, 0.235]	0.0003
$\Sigma_{GDE}{}^{(nc4,nc3)}$	0.029	0.079	[−0.025, 0.098]	[0.043, 0.124]	0.0002
$\Sigma_{GDE}{}^{(nc5,nc3)}$	0.029	0.031	[−0.025, 0.100]	[0.001, 0.063]	0.0002
$\Sigma_{GDE}{}^{(nc6,nc3)}$	0.029	0.021	[−0.025, 0.098]	[−0.009, 0.053]	0.0002
$\Sigma_{GDE}{}^{(c1,nc3)}$	0.029	0.034	[−0.026, 0.100]	[−0.001, 0.071]	0.0003
$\Sigma_{GDE}{}^{(c2,nc3)}$	0.029	0.033	[−0.026, 0.099]	[0.001, 0.069]	0.0002
$\Sigma_{GDE}{}^{(c3,nc3)}$	0.029	0.036	[−0.025, 0.099]	[0.003, 0.071]	0.0003
$\Sigma_{GDE}{}^{(c4,nc3)}$	0.029	0.032	[−0.025, 0.100]	[0.003, 0.065]	0.0002
$\Sigma_{GDE}{}^{(c5,nc3)}$	0.029	0.032	[−0.025, 0.099]	[0.003, 0.063]	0.0002
$\Sigma_{GDE}{}^{(nc1,nc4)}$	0.029	0.077	[−0.024, 0.100]	[0.039, 0.125]	0.0002
$\Sigma_{GDE}{}^{(nc2,nc4)}$	0.030	0.075	[−0.024, 0.099]	[0.038, 0.122]	0.0002
$\Sigma_{GDE}{}^{(nc3,nc4)}$	0.029	0.079	[−0.025, 0.098]	[0.043, 0.124]	0.0002
$\Sigma_{GDE}{}^{(nc4,nc4)}$	0.152	0.166	[0.095, 0.263]	[0.125, 0.221]	0.0003
$\Sigma_{GDE}{}^{(nc5,nc4)}$	0.029	0.059	[−0.024, 0.097]	[0.031, 0.094]	0.0002
$\Sigma_{GDE}{}^{(nc6,nc4)}$	0.030	0.036	[−0.025, 0.099]	[0.009, 0.068]	0.0002
$\Sigma_{GDE}{}^{(c1,nc4)}$	0.030	0.038	[−0.024, 0.097]	[0.007, 0.073]	0.0002
$\Sigma_{GDE}{}^{(c2,nc4)}$	0.030	0.037	[−0.025, 0.099]	[0.009, 0.070]	0.0002
$\Sigma_{GDE}{}^{(c3,nc4)}$	0.030	0.040	[−0.025, 0.101]	[0.012, 0.073]	0.0002
$\Sigma_{GDE}{}^{(c4,nc4)}$	0.029	0.039	[−0.024, 0.099]	[0.013, 0.070]	0.0002
$\Sigma_{GDE}{}^{(c5,nc4)}$	0.030	0.038	[−0.024, 0.100]	[0.013, 0.068]	0.0002
$\Sigma_{GDE}{}^{(nc1,nc5)}$	0.029	0.034	[−0.025, 0.098]	[0.001, 0.069]	0.0002
$\Sigma_{GDE}{}^{(nc2,nc5)}$	0.029	0.033	[−0.025, 0.100]	[0.001, 0.067]	0.0002
$\Sigma_{GDE}{}^{(nc3,nc5)}$	0.029	0.031	[−0.025, 0.100]	[0.001, 0.063]	0.0002
$\Sigma_{GDE}{}^{(nc4,nc5)}$	0.029	0.059	[−0.024, 0.097]	[0.031, 0.094]	0.0002
$\Sigma_{GDE}{}^{(nc5,nc5)}$	0.152	0.089	[0.095, 0.264]	[0.067, 0.120]	0.0001
$\Sigma_{GDE}{}^{(nc6,nc5)}$	0.029	0.036	[−0.025, 0.099]	[0.018, 0.060]	0.0001
$\Sigma_{GDE}{}^{(c1,nc5)}$	0.029	0.026	[−0.025, 0.101]	[0.008, 0.049]	0.0001
$\Sigma_{GDE}{}^{(c2,nc5)}$	0.030	0.026	[−0.025, 0.100]	[0.009, 0.048]	0.0001
$\Sigma_{GDE}{}^{(c3,nc5)}$	0.030	0.028	[−0.024, 0.101]	[0.011, 0.051]	0.0001
$\Sigma_{GDE}{}^{(c4,nc5)}$	0.029	0.029	[−0.025, 0.099]	[0.012, 0.050]	0.0001
$\Sigma_{GDE}{}^{(c5,nc5)}$	0.030	0.027	[−0.025, 0.099]	[0.011, 0.048]	0.0001
$\Sigma_{GDE}{}^{(nc1,nc6)}$	0.029	0.015	[−0.025, 0.099]	[−0.017, 0.049]	0.0002
$\Sigma_{GDE}{}^{(nc2,nc6)}$	0.030	0.020	[−0.025, 0.100]	[−0.011, 0.053]	0.0002
$\Sigma_{GDE}{}^{(nc3,nc6)}$	0.029	0.021	[−0.025, 0.098]	[−0.009, 0.053]	0.0002
$\Sigma_{GDE}{}^{(nc4,nc6)}$	0.030	0.036	[−0.025, 0.099]	[0.009, 0.068]	0.0002
$\Sigma_{GDE}{}^{(nc5,nc6)}$	0.029	0.036	[−0.025, 0.099]	[0.018, 0.060]	0.0001

$\Sigma_{GDE}{}^{(nc6,nc6)}$	0.152	0.077	[0.095, 0.266]	[0.058, 0.104]	0.0001
$\Sigma_{GDE}{}^{(c1,nc6)}$	0.029	0.028	[−0.026, 0.099]	[0.010, 0.050]	0.0001
$\Sigma_{GDE}{}^{(c2,nc6)}$	0.030	0.029	[−0.025, 0.098]	[0.013, 0.049]	0.0001
$\Sigma_{GDE}{}^{(c3,nc6)}$	0.029	0.033	[−0.025, 0.101]	[0.017, 0.054]	0.0001
$\Sigma_{GDE}{}^{(c4,nc6)}$	0.029	0.033	[−0.025, 0.099]	[0.018, 0.054]	0.0001
$\Sigma_{GDE}{}^{(c5,nc6)}$	0.029	0.034	[−0.025, 0.100]	[0.019, 0.054]	0.0001
$\Sigma_{GDE}{}^{(nc1,c1)}$	0.030	0.033	[−0.025, 0.101]	[−0.005, 0.074]	0.0003
$\Sigma_{GDE}{}^{(nc2,c1)}$	0.029	0.034	[−0.025, 0.100]	[−0.002, 0.074]	0.0003
$\Sigma_{GDE}{}^{(nc3,c1)}$	0.029	0.034	[−0.026, 0.100]	[−0.001, 0.071]	0.0003
$\Sigma_{GDE}{}^{(nc4,c1)}$	0.030	0.038	[−0.024, 0.097]	[0.007, 0.073]	0.0002
$\Sigma_{GDE}{}^{(nc5,c1)}$	0.029	0.026	[−0.025, 0.101]	[0.008, 0.049]	0.0001
$\Sigma_{GDE}{}^{(nc6,c1)}$	0.029	0.028	[−0.026, 0.099]	[0.010, 0.050]	0.0001
$\Sigma_{GDE}{}^{(c1,c1)}$	0.153	0.081	[0.095, 0.267]	[0.059, 0.115]	0.0002
$\Sigma_{GDE}{}^{(c2,c1)}$	0.029	0.034	[−0.025, 0.100]	[0.016, 0.059]	0.0001
$\Sigma_{GDE}{}^{(c3,c1)}$	0.029	0.034	[−0.025, 0.100]	[0.017, 0.059]	0.0001
$\Sigma_{GDE}{}^{(c4,c1)}$	0.029	0.033	[−0.026, 0.101]	[0.016, 0.056]	0.0001
$\Sigma_{GDE}{}^{(c5,c1)}$	0.029	0.032	[−0.026, 0.099]	[0.016, 0.055]	0.0001
$\Sigma_{GDE}{}^{(nc1,c2)}$	0.030	0.032	[−0.025, 0.099]	[−0.004, 0.069]	0.0003
$\Sigma_{GDE}{}^{(nc2,c2)}$	0.030	0.033	[−0.025, 0.100]	[−0.001, 0.070]	0.0003
$\Sigma_{GDE}{}^{(nc3,c2)}$	0.029	0.033	[−0.026, 0.099]	[0.001, 0.069]	0.0002
$\Sigma_{GDE}{}^{(nc4,c2)}$	0.030	0.037	[−0.025, 0.099]	[0.009, 0.070]	0.0002
$\Sigma_{GDE}{}^{(nc5,c2)}$	0.030	0.026	[−0.025, 0.100]	[0.009, 0.048]	0.0001
$\Sigma_{GDE}{}^{(nc6,c2)}$	0.030	0.029	[−0.025, 0.098]	[0.013, 0.049]	0.0001
$\Sigma_{GDE}{}^{(c1,c2)}$	0.029	0.034	[−0.025, 0.100]	[0.016, 0.059]	0.0001
$\Sigma_{GDE}{}^{(c2,c2)}$	0.152	0.075	[0.095, 0.266]	[0.055, 0.104]	0.0001
$\Sigma_{GDE}{}^{(c3,c2)}$	0.029	0.036	[−0.026, 0.100]	[0.019, 0.059]	0.0001
$\Sigma_{GDE}{}^{(c4,c2)}$	0.029	0.035	[−0.026, 0.100]	[0.018, 0.057]	0.0001
$\Sigma_{GDE}{}^{(c5,c2)}$	0.029	0.034	[−0.025, 0.100]	[0.018, 0.056]	0.0001
$\Sigma_{GDE}{}^{(nc1,c3)}$	0.030	0.033	[−0.024, 0.100]	[−0.003, 0.072]	0.0003
$\Sigma_{GDE}{}^{(nc2,c3)}$	0.029	0.035	[−0.024, 0.100]	[0.001, 0.072]	0.0003
$\Sigma_{GDE}{}^{(nc3,c3)}$	0.029	0.036	[−0.025, 0.099]	[0.003, 0.071]	0.0003
$\Sigma_{GDE}{}^{(nc4,c3)}$	0.030	0.040	[−0.025, 0.101]	[0.012, 0.073]	0.0002
$\Sigma_{GDE}{}^{(nc5,c3)}$	0.030	0.028	[−0.024, 0.101]	[0.011, 0.051]	0.0001
$\Sigma_{GDE}{}^{(nc6,c3)}$	0.029	0.033	[−0.025, 0.101]	[0.017, 0.054]	0.0001
$\Sigma_{GDE}{}^{(c1,c3)}$	0.029	0.034	[−0.025, 0.100]	[0.017, 0.059]	0.0001
$\Sigma_{GDE}{}^{(c2,c3)}$	0.029	0.036	[−0.026, 0.100]	[0.019, 0.059]	0.0001
$\Sigma_{GDE}{}^{(c3,c3)}$	0.153	0.076	[0.095, 0.267]	[0.056, 0.104]	0.0001
$\Sigma_{GDE}{}^{(c4,c3)}$	0.029	0.041	[−0.024, 0.099]	[0.024, 0.063]	0.0001
$\Sigma_{GDE}{}^{(c5,c3)}$	0.030	0.039	[−0.025, 0.101]	[0.024, 0.062]	0.0001
$\Sigma_{GDE}{}^{(nc1,c4)}$	0.029	0.030	[−0.025, 0.100]	[−0.002, 0.065]	0.0002
$\Sigma_{GDE}{}^{(nc2,c4)}$	0.030	0.031	[−0.024, 0.100]	[0.000, 0.065]	0.0002
$\Sigma_{GDE}{}^{(nc3,c4)}$	0.029	0.032	[−0.025, 0.100]	[0.003, 0.065]	0.0002
$\Sigma_{GDE}{}^{(nc4,c4)}$	0.029	0.039	[−0.024, 0.099]	[0.013, 0.070]	0.0002
$\Sigma_{GDE}{}^{(nc5,c4)}$	0.029	0.029	[−0.025, 0.099]	[0.012, 0.050]	0.0001
$\Sigma_{GDE}{}^{(nc6,c4)}$	0.029	0.033	[−0.025, 0.099]	[0.018, 0.054]	0.0001
$\Sigma_{GDE}{}^{(c1,c4)}$	0.029	0.033	[−0.026, 0.101]	[0.016, 0.056]	0.0001

${\Sigma_{GDE}}^{(c2,c4)}$	0.029	0.035	[−0.026, 0.100]	[0.018, 0.057]	0.0001
${\Sigma_{GDE}}^{(c3,c4)}$	0.029	0.041	[−0.024, 0.099]	[0.024, 0.063]	0.0001
${\Sigma_{GDE}}^{(c4,c4)}$	0.151	0.071	[0.095, 0.267]	[0.053, 0.096]	0.0001
${\Sigma_{GDE}}^{(c5,c4)}$	0.030	0.040	[−0.025, 0.100]	[0.025, 0.062]	0.0001
${\Sigma_{GDE}}^{(nc1,c5)}$	0.030	0.028	[−0.024, 0.100]	[−0.003, 0.061]	0.0002
${\Sigma_{GDE}}^{(nc2,c5)}$	0.029	0.030	[−0.024, 0.100]	[0.001, 0.062]	0.0002
${\Sigma_{GDE}}^{(nc3,c5)}$	0.029	0.032	[−0.025, 0.099]	[0.003, 0.063]	0.0002
${\Sigma_{GDE}}^{(nc4,c5)}$	0.030	0.038	[−0.024, 0.100]	[0.013, 0.068]	0.0002
${\Sigma_{GDE}}^{(nc5,c5)}$	0.030	0.027	[−0.025, 0.099]	[0.011, 0.048]	0.0001
${\Sigma_{GDE}}^{(nc6,c5)}$	0.029	0.034	[−0.025, 0.100]	[0.019, 0.054]	0.0001
${\Sigma_{GDE}}^{(c1,c5)}$	0.029	0.032	[−0.026, 0.099]	[0.016, 0.055]	0.0001
${\Sigma_{GDE}}^{(c2,c5)}$	0.029	0.034	[−0.025, 0.100]	[0.018, 0.056]	0.0001
${\Sigma_{GDE}}^{(c3,c5)}$	0.030	0.039	[−0.025, 0.101]	[0.024, 0.062]	0.0001
${\Sigma_{GDE}}^{(c4,c5)}$	0.030	0.040	[−0.025, 0.100]	[0.025, 0.062]	0.0001
${\Sigma_{GDE}}^{(c5,c5)}$	0.153	0.069	[0.095, 0.267]	[0.051, 0.092]	0.0001
${\rho_{GDI}}^{(nc1)}$	0.799	0.793	[0.525, 1.074]	[0.695, 0.881]	0.0005
${\rho_{GDI}}^{(nc2)}$	0.800	0.779	[0.521, 1.078]	[0.679, 0.871]	0.0005
${\rho_{GDI}}^{(nc3)}$	0.802	0.817	[0.527, 1.077]	[0.721, 0.905]	0.0004
${\rho_{GDI}}^{(nc4)}$	0.802	0.849	[0.523, 1.076]	[0.771, 0.924]	0.0004
${\rho_{GDI}}^{(nc5)}$	0.802	0.851	[0.527, 1.076]	[0.779, 0.920]	0.0004
${\rho_{GDI}}^{(c1)}$	0.801	0.839	[0.522, 1.078]	[0.739, 0.935]	0.0006
${\rho_{GDI}}^{(c2)}$	0.801	0.920	[0.524, 1.077]	[0.862, 0.976]	0.0003
${\rho_{GDI}}^{(c3)}$	0.802	0.881	[0.526, 1.078]	[0.824, 0.937]	0.0003
${\rho_{GDI}}^{(c4)}$	0.798	0.890	[0.529, 1.078]	[0.837, 0.941]	0.0003
${\rho_{GDI}}^{(c5)}$	0.802	0.896	[0.523, 1.076]	[0.847, 0.944]	0.0002
${\Sigma_{GDI}}^{(nc1,nc1)}$	0.265	0.279	[0.163, 0.464]	[0.209, 0.374]	0.0004
${\Sigma_{GDI}}^{(nc2,nc1)}$	0.052	0.186	[−0.044, 0.174]	[0.127, 0.263]	0.0003
${\Sigma_{GDI}}^{(nc3,nc1)}$	0.052	0.053	[−0.041, 0.172]	[0.007, 0.107]	0.0003
${\Sigma_{GDI}}^{(nc4,nc1)}$	0.050	0.019	[−0.046, 0.175]	[−0.029, 0.072]	0.0003
${\Sigma_{GDI}}^{(nc5,nc1)}$	0.051	0.033	[−0.045, 0.171]	[−0.015, 0.086]	0.0003
${\Sigma_{GDI}}^{(c1,nc1)}$	0.051	0.035	[−0.042, 0.172]	[−0.016, 0.088]	0.0003
${\Sigma_{GDI}}^{(c2,nc1)}$	0.051	0.035	[−0.044, 0.174]	[−0.014, 0.088]	0.0003
${\Sigma_{GDI}}^{(c3,nc1)}$	0.051	0.037	[−0.044, 0.174]	[−0.014, 0.092]	0.0003
${\Sigma_{GDI}}^{(c4,nc1)}$	0.051	0.037	[−0.045, 0.176]	[−0.014, 0.093]	0.0003
${\Sigma_{GDI}}^{(c5,nc1)}$	0.051	0.033	[−0.045, 0.174]	[−0.018, 0.089]	0.0003
${\Sigma_{GDI}}^{(nc1,nc2)}$	0.052	0.186	[−0.044, 0.174]	[0.127, 0.263]	0.0003
${\Sigma_{GDI}}^{(nc2,nc2)}$	0.266	0.271	[0.165, 0.465]	[0.208, 0.356]	0.0004
${\Sigma_{GDI}}^{(nc3,nc2)}$	0.051	0.040	[−0.042, 0.178]	[−0.001, 0.087]	0.0002
${\Sigma_{GDI}}^{(nc4,nc2)}$	0.052	0.022	[−0.043, 0.174]	[−0.020, 0.068]	0.0002
${\Sigma_{GDI}}^{(nc5,nc2)}$	0.052	0.035	[−0.041, 0.174]	[−0.008, 0.082]	0.0002
${\Sigma_{GDI}}^{(c1,nc2)}$	0.051	0.033	[−0.045, 0.172]	[−0.015, 0.084]	0.0003
${\Sigma_{GDI}}^{(c2,nc2)}$	0.052	0.033	[−0.042, 0.176]	[−0.014, 0.083]	0.0003
${\Sigma_{GDI}}^{(c3,nc2)}$	0.051	0.035	[−0.042, 0.175]	[−0.013, 0.085]	0.0003
${\Sigma_{GDI}}^{(c4,nc2)}$	0.052	0.035	[−0.043, 0.175]	[−0.012, 0.086]	0.0003
${\Sigma_{GDI}}^{(c5,nc2)}$	0.053	0.030	[−0.042, 0.175]	[−0.017, 0.080]	0.0003
${\Sigma_{GDI}}^{(nc1,nc3)}$	0.052	0.053	[−0.041, 0.172]	[0.007, 0.107]	0.0003

${\Sigma_{GDI}}^{(nc2,nc3)}$	0.051	0.040	[−0.042, 0.178]	[−0.001, 0.087]	0.0002
${\Sigma_{GDI}}^{(nc3,nc3)}$	0.266	0.167	[0.166, 0.463]	[0.128, 0.220]	0.0002
${\Sigma_{GDI}}^{(nc4,nc3)}$	0.051	0.055	[−0.043, 0.173]	[0.022, 0.094]	0.0002
${\Sigma_{GDI}}^{(nc5,nc3)}$	0.051	0.047	[−0.044, 0.173]	[0.015, 0.084]	0.0002
${\Sigma_{GDI}}^{(c1,nc3)}$	0.051	0.040	[−0.044, 0.174]	[0.008, 0.075]	0.0002
${\Sigma_{GDI}}^{(c2,nc3)}$	0.051	0.041	[−0.045, 0.176]	[0.009, 0.077]	0.0002
${\Sigma_{GDI}}^{(c3,nc3)}$	0.051	0.045	[−0.043, 0.172]	[0.013, 0.083]	0.0002
${\Sigma_{GDI}}^{(c4,nc3)}$	0.051	0.047	[−0.042, 0.173]	[0.014, 0.084]	0.0002
${\Sigma_{GDI}}^{(c5,nc3)}$	0.051	0.047	[−0.041, 0.176]	[0.014, 0.084]	0.0002
${\Sigma_{GDI}}^{(nc1,nc4)}$	0.050	0.019	[−0.046, 0.175]	[−0.029, 0.072]	0.0003
${\Sigma_{GDI}}^{(nc2,nc4)}$	0.052	0.022	[−0.043, 0.174]	[−0.020, 0.068]	0.0002
${\Sigma_{GDI}}^{(nc3,nc4)}$	0.051	0.055	[−0.043, 0.173]	[0.022, 0.094]	0.0002
${\Sigma_{GDI}}^{(nc4,nc4)}$	0.266	0.174	[0.165, 0.463]	[0.135, 0.228]	0.0002
${\Sigma_{GDI}}^{(nc5,nc4)}$	0.052	0.093	[−0.043, 0.174]	[0.060, 0.136]	0.0002
${\Sigma_{GDI}}^{(c1,nc4)}$	0.051	0.061	[−0.044, 0.173]	[0.029, 0.099]	0.0002
${\Sigma_{GDI}}^{(c2,nc4)}$	0.052	0.065	[−0.042, 0.176]	[0.033, 0.103]	0.0002
${\Sigma_{GDI}}^{(c3,nc4)}$	0.051	0.076	[−0.041, 0.175]	[0.043, 0.116]	0.0002
${\Sigma_{GDI}}^{(c4,nc4)}$	0.051	0.078	[−0.042, 0.174]	[0.045, 0.118]	0.0002
${\Sigma_{GDI}}^{(c5,nc4)}$	0.051	0.082	[−0.043, 0.174]	[0.049, 0.121]	0.0002
${\Sigma_{GDI}}^{(nc1,nc5)}$	0.051	0.033	[−0.045, 0.171]	[−0.015, 0.086]	0.0003
${\Sigma_{GDI}}^{(nc2,nc5)}$	0.052	0.035	[−0.041, 0.174]	[−0.008, 0.082]	0.0002
${\Sigma_{GDI}}^{(nc3,nc5)}$	0.051	0.047	[−0.044, 0.173]	[0.015, 0.084]	0.0002
${\Sigma_{GDI}}^{(nc4,nc5)}$	0.052	0.093	[−0.043, 0.174]	[0.060, 0.136]	0.0002
${\Sigma_{GDI}}^{(nc5,nc5)}$	0.265	0.173	[0.166, 0.465]	[0.134, 0.224]	0.0002
${\Sigma_{GDI}}^{(c1,nc5)}$	0.050	0.075	[−0.043, 0.173]	[0.044, 0.114]	0.0002
${\Sigma_{GDI}}^{(c2,nc5)}$	0.051	0.081	[−0.042, 0.174]	[0.049, 0.119]	0.0002
${\Sigma_{GDI}}^{(c3,nc5)}$	0.051	0.099	[−0.042, 0.174]	[0.066, 0.139]	0.0002
${\Sigma_{GDI}}^{(c4,nc5)}$	0.052	0.102	[−0.041, 0.174]	[0.070, 0.143]	0.0002
${\Sigma_{GDI}}^{(c5,nc5)}$	0.052	0.106	[−0.044, 0.174]	[0.074, 0.147]	0.0002
${\Sigma_{GDI}}^{(nc1,c1)}$	0.051	0.035	[−0.042, 0.172]	[−0.016, 0.088]	0.0003
${\Sigma_{GDI}}^{(nc2,c1)}$	0.051	0.033	[−0.045, 0.172]	[−0.015, 0.084]	0.0003
${\Sigma_{GDI}}^{(nc3,c1)}$	0.051	0.040	[−0.044, 0.174]	[0.008, 0.075]	0.0002
${\Sigma_{GDI}}^{(nc4,c1)}$	0.051	0.061	[−0.044, 0.173]	[0.029, 0.099]	0.0002
${\Sigma_{GDI}}^{(nc5,c1)}$	0.050	0.075	[−0.043, 0.173]	[0.044, 0.114]	0.0002
${\Sigma_{GDI}}^{(c1,c1)}$	0.265	0.147	[0.166, 0.465]	[0.109, 0.199]	0.0002
${\Sigma_{GDI}}^{(c2,c1)}$	0.051	0.079	[−0.044, 0.176]	[0.048, 0.120]	0.0002
${\Sigma_{GDI}}^{(c3,c1)}$	0.051	0.087	[−0.042, 0.173]	[0.054, 0.128]	0.0002
${\Sigma_{GDI}}^{(c4,c1)}$	0.051	0.087	[−0.043, 0.170]	[0.055, 0.128]	0.0002
${\Sigma_{GDI}}^{(c5,c1)}$	0.051	0.090	[−0.044, 0.173]	[0.056, 0.132]	0.0002
${\Sigma_{GDI}}^{(nc1,c2)}$	0.051	0.035	[−0.044, 0.174]	[−0.014, 0.088]	0.0003
${\Sigma_{GDI}}^{(nc2,c2)}$	0.052	0.033	[−0.042, 0.176]	[−0.014, 0.083]	0.0003
${\Sigma_{GDI}}^{(nc3,c2)}$	0.051	0.041	[−0.045, 0.176]	[0.009, 0.077]	0.0002
${\Sigma_{GDI}}^{(nc4,c2)}$	0.052	0.065	[−0.042, 0.176]	[0.033, 0.103]	0.0002
${\Sigma_{GDI}}^{(nc5,c2)}$	0.051	0.081	[−0.042, 0.174]	[0.049, 0.119]	0.0002
${\Sigma_{GDI}}^{(c1,c2)}$	0.051	0.079	[−0.044, 0.176]	[0.048, 0.120]	0.0002
${\Sigma_{GDI}}^{(c2,c2)}$	0.267	0.147	[0.165, 0.460]	[0.110, 0.199]	0.0002

$\Sigma_{GDI}^{(c3,c2)}$	0.051	0.094	[−0.042, 0.175]	[0.062, 0.137]	0.0002
$\Sigma_{GDI}^{(c4,c2)}$	0.052	0.095	[−0.043, 0.178]	[0.062, 0.137]	0.0002
$\Sigma_{GDI}^{(c5,c2)}$	0.051	0.097	[−0.043, 0.176]	[0.065, 0.140]	0.0002
$\Sigma_{GDI}^{(nc1,c3)}$	0.051	0.037	[−0.044, 0.174]	[−0.014, 0.092]	0.0003
$\Sigma_{GDI}^{(nc2,c3)}$	0.051	0.035	[−0.042, 0.175]	[−0.013, 0.085]	0.0003
$\Sigma_{GDI}^{(nc3,c3)}$	0.051	0.045	[−0.043, 0.172]	[0.013, 0.083]	0.0002
$\Sigma_{GDI}^{(nc4,c3)}$	0.051	0.076	[−0.041, 0.175]	[0.043, 0.116]	0.0002
$\Sigma_{GDI}^{(nc5,c3)}$	0.051	0.099	[−0.042, 0.174]	[0.066, 0.139]	0.0002
$\Sigma_{GDI}^{(c1,c3)}$	0.051	0.087	[−0.042, 0.173]	[0.054, 0.128]	0.0002
$\Sigma_{GDI}^{(c2,c3)}$	0.051	0.094	[−0.042, 0.175]	[0.062, 0.137]	0.0002
$\Sigma_{GDI}^{(c3,c3)}$	0.264	0.167	[0.164, 0.463]	[0.128, 0.218]	0.0002
$\Sigma_{GDI}^{(c4,c3)}$	0.051	0.118	[−0.043, 0.173]	[0.084, 0.161]	0.0002
$\Sigma_{GDI}^{(c5,c3)}$	0.051	0.120	[−0.041, 0.174]	[0.086, 0.164]	0.0002
$\Sigma_{GDI}^{(nc1,c4)}$	0.051	0.037	[−0.045, 0.176]	[−0.014, 0.093]	0.0003
$\Sigma_{GDI}^{(nc2,c4)}$	0.052	0.035	[−0.043, 0.175]	[−0.012, 0.086]	0.0003
$\Sigma_{GDI}^{(nc3,c4)}$	0.051	0.047	[−0.042, 0.173]	[0.014, 0.084]	0.0002
$\Sigma_{GDI}^{(nc4,c4)}$	0.051	0.078	[−0.042, 0.174]	[0.045, 0.118]	0.0002
$\Sigma_{GDI}^{(nc5,c4)}$	0.052	0.102	[−0.041, 0.174]	[0.070, 0.143]	0.0002
$\Sigma_{GDI}^{(c1,c4)}$	0.051	0.087	[−0.043, 0.170]	[0.055, 0.128]	0.0002
$\Sigma_{GDI}^{(c2,c4)}$	0.052	0.095	[−0.043, 0.178]	[0.062, 0.137]	0.0002
$\Sigma_{GDI}^{(c3,c4)}$	0.051	0.118	[−0.043, 0.173]	[0.084, 0.161]	0.0002
$\Sigma_{GDI}^{(c4,c4)}$	0.265	0.173	[0.165, 0.466]	[0.135, 0.221]	0.0002
$\Sigma_{GDI}^{(c5,c4)}$	0.052	0.131	[−0.044, 0.174]	[0.097, 0.174]	0.0002
$\Sigma_{GDI}^{(nc1,c5)}$	0.051	0.033	[−0.045, 0.174]	[−0.018, 0.089]	0.0003
$\Sigma_{GDI}^{(nc2,c5)}$	0.053	0.030	[−0.042, 0.175]	[−0.017, 0.080]	0.0003
$\Sigma_{GDI}^{(nc3,c5)}$	0.051	0.047	[−0.041, 0.176]	[0.014, 0.084]	0.0002
$\Sigma_{GDI}^{(nc4,c5)}$	0.051	0.082	[−0.043, 0.174]	[0.049, 0.121]	0.0002
$\Sigma_{GDI}^{(nc5,c5)}$	0.052	0.106	[−0.044, 0.174]	[0.074, 0.147]	0.0002
$\Sigma_{GDI}^{(c1,c5)}$	0.051	0.090	[−0.044, 0.173]	[0.056, 0.132]	0.0002
$\Sigma_{GDI}^{(c2,c5)}$	0.051	0.097	[−0.043, 0.176]	[0.065, 0.140]	0.0002
$\Sigma_{GDI}^{(c3,c5)}$	0.051	0.120	[−0.041, 0.174]	[0.086, 0.164]	0.0002
$\Sigma_{GDI}^{(c4,c5)}$	0.052	0.131	[−0.044, 0.174]	[0.097, 0.174]	0.0002
$\Sigma_{GDI}^{(c5,c5)}$	0.266	0.181	[0.165, 0.463]	[0.143, 0.230]	0.0002

TABLE SM.B.1. Prior and posterior distribution of parameters of the model

SM.C Model comparison and robustness

Despite its parsimony, the model we use in our empirical analysis remains high-dimensional: it contains 277 parameters that we estimate using a cross-section of $N = 21$ measurements over $T = 157$ periods. Moreover, many of the NT potential observations are missing or unavailable. To discipline our estimates, we use a prior that applies shrinkage towards a simpler model with a single-factor structure on the innovations. In addition, one may consider a more flexible model in which Δx_t follows an unrestricted VAR instead of a diagonal one. A natural question is whether imposing or relaxing these constraints leads to an improved model fit.

For that reason, we compare the baseline specification in the main body of the paper against three variants that: (1) impose the single factor structure on the different innovations, (2) relax the diagonal VAR restriction on Δx_t, and (3) combine both. We thank an anonymous referee for suggesting our exploration of these alternative specifications. Subsection SM.C.1 presents the estimated models while subsection SM.C.2 reports their marginal likelihoods, thereby allowing us to produce posterior odds against our baseline model.

It appears that the covariance matrix Σ_x is in fact well approximated by a single-factor structure while Σ_{GDE} and Σ_{GDI} are not. Consequently, it is perhaps not surprising that the additional flexibility in the covariance matrix of innovations, particularly for the measurement errors, leads to substantial improvements in model fit. In contrast, flexibility in the autoregressive structure of Δx_t does not lead to improvements. Reassuringly, the estimates of economic activity that we obtain from the different alternatives are very similar, possibly because the rich cross-section that we use attenuates the role of those modelling choices on filtering.

In turn, subsection SM.C.3 shows that our baseline model is capable of approximating well the autocovariance function of a selection of statistical discrepancies, providing additional evidence of adequate model fit. Next, in subsection SM.C.4 we compare our baseline model with a non-nested alternative that further decomposes the signals x_t into trend and cycle components. Finally, subsection SM.C.5 investigates the sensitivity of our model to the assumption of block-independence between GDE and GDI measurement errors.

SM.C.1 Alternative specifications

We consider three alternatives to our baseline, which we label model M0:

(M1) Δx_t is a diagonal VAR(1) process with a single factor structure in its innovations,

$$\Delta x_t = \mu_x + \operatorname{diag}(\rho_x)\,\Delta x_{t-1} + \left[\lambda_x \eta_{xt} + \operatorname{diag}(\sigma_x)\varepsilon_{xt}\right],$$
$$\eta_{xt} \overset{iid}{\sim} N(0,1) \text{ independent of } \varepsilon_{xt} \overset{iid}{\sim} N(0_{C\times 1}, I_C),$$

while v_{it} follows another diagonal VAR(1) model with a single factor structure in its innovations,

$$v_{it} = \operatorname{diag}(\rho_i) v_{i,t-1} + \left[\lambda_i \eta_{it} + \operatorname{diag}(\sigma_i)\varepsilon_{it}\right],$$
$$\eta_{it} \overset{iid}{\sim} N(0,1) \text{ independent of } \varepsilon_{it} \overset{iid}{\sim} N(0_{M_i\times 1}, I_{M_i}).$$

In this case, the prior distribution is

$$1/\sigma_x^2 \sim \text{Gamma}(d_x/2, 2/(d_x w_x \varsigma_x)),$$
$$\begin{pmatrix} \mu_x \\ \rho_x \\ \lambda_x \end{pmatrix} \Bigg| \sigma_x \sim N\left(\begin{pmatrix} b_x \\ \sqrt{(1-w_x)\varsigma_x} 1_{C\times 1} \end{pmatrix}, \operatorname{diag}\{R_x^{-1}, d_x\} \otimes \operatorname{diag}(\sigma_x^2)\right),$$
$$1/\sigma_i^2 \sim \text{Gamma}(d_i/2, 2/(d_i w_i \varsigma_i)),$$
$$\begin{pmatrix} \rho_i \\ \lambda_i \end{pmatrix} \Bigg| \sigma_i \sim N\left(\begin{pmatrix} b_i \\ \sqrt{(1-w_v)\varsigma_i} 1_{C\times 1} \end{pmatrix}, \operatorname{diag}\{R_i^{-1}, d_v\} \otimes \operatorname{diag}(\sigma_i^2)\right),$$

with $d_x, w_x, \varsigma_x, b_x, R_x, d_i, w_i, \varsigma_i, b_i, R_i$ as in our baseline model.

(M2) Δx_t is an unrestricted VAR(1) process with an unrestricted covariance matrix for its innovations,

$$\Delta x_t = \mu_x + \rho_x \Delta x_{t-1} + \text{Ch}(\Sigma_x)\varepsilon_{xt},$$
$$\varepsilon_{xt} \overset{iid}{\sim} N(0_{C\times 1}, I_C),$$

while v_{it} follows a diagonal VAR(1) model, also with an unrestricted covariance matrix for its innovations

$$
\begin{aligned}
v_{it} &= \operatorname{diag}(\rho_i) v_{i,t-1} + \operatorname{Ch}(\Sigma_i)\varepsilon_{it}, \\
\varepsilon_{it} &\overset{iid}{\sim} N(0_{M_i\times 1}, I_{M_i}).
\end{aligned}
$$

The prior distribution now is

$$
\begin{aligned}
\Sigma_x &\sim \mathcal{W}^{-1}(d_x S_x, d_x), \\
\begin{pmatrix} \mu_x \\ \operatorname{vec}(\rho_x) \end{pmatrix} \Bigg| \Sigma_x &\sim N\left(\begin{pmatrix} m_x 1_{C\times 1} \\ r_m \operatorname{vec}(I_C) \end{pmatrix}, \bar{R}_x^{-1} \otimes \Sigma_x \right), \\
\Sigma_i &\sim \mathcal{W}^{-1}(d_i S_i, d_i), \\
\rho_i \Big| \Sigma_i &\sim N\left(b_i, R_i^{-1} \otimes \Sigma_i\right),
\end{aligned}
$$

where $\bar{R}_x = \operatorname{diag}\{10, 30 I_C\}$ and $d_x, S_x, m_x, r_x, d_i, S_i, b_i, R_i$ as in our baseline model.

(M3) Δx_t is an unrestricted VAR(1) with a single factor structure in its innovations,

$$
\begin{aligned}
\Delta x_t &= \mu_x + \rho_x \Delta x_{t-1} + \left[\lambda_x \eta_{xt} + \operatorname{diag}(\sigma_x)\varepsilon_{xt}\right], \\
\eta_{xt} &\overset{iid}{\sim} N(0,1) \text{ independent of } \varepsilon_{xt} \overset{iid}{\sim} N(0_{C\times 1}, I_C),
\end{aligned}
$$

while v_{it} is a diagonal VAR(1) model with a single factor structure in its innovations,

$$
\begin{aligned}
v_{it} &= \operatorname{diag}(\rho_i) v_{i,t-1} + \left[\lambda_i \eta_{it} + \operatorname{diag}(\sigma_i)\varepsilon_{it}\right], \\
\eta_{it} &\overset{iid}{\sim} N(0,1) \text{ independent of } \varepsilon_{it} \overset{iid}{\sim} N(0_{M_i\times 1}, I_{M_i}).
\end{aligned}
$$

As one would expect, the prior distribution is

$$
\begin{aligned}
1/\sigma_x^2 &\sim \operatorname{Gamma}(d_x/2, 2/(d_x w_x \varsigma_x)), \\
\begin{pmatrix} \mu_x \\ \operatorname{vec}(\rho_x) \\ \lambda_x \end{pmatrix} \Bigg| \sigma_x &\sim N\left(\begin{pmatrix} m_x 1_{C\times 1} \\ r_m \operatorname{vec}(I_C) \\ \sqrt{(1-w_x)\varsigma_x} 1_{C\times 1} \end{pmatrix}, \operatorname{diag}\{\bar{R}_x^{-1}, d_x\} \otimes \operatorname{diag}(\sigma_x^2) \right), \\
1/\sigma_i^2 &\sim \operatorname{Gamma}(d_i/2, 2/(d_i w_i \varsigma_i)),
\end{aligned}
$$

$$\begin{pmatrix} \rho_i \\ \lambda_i \end{pmatrix} \Bigg| \sigma_i \sim N\left(\begin{pmatrix} b_i \\ \sqrt{(1-w_v)\varsigma_i} 1_{C\times 1} \end{pmatrix}, \text{diag}\{R_i^{-1}, d_v\} \otimes \text{diag}(\sigma_i^2) \right),$$

with $d_x, w_x, \varsigma_x, m_x, r_m, d_i, w_i, \varsigma_i, b_i, R_i$ as in our baseline model and $\bar{R}_x$ as in M2.

SM.C.2 Model comparison and estimates of economic activity

It is straightforward to adapt the Gibbs sampler algorithm we discussed in subsection SM.B.3 to estimate models M1, M2 and M3. For each model we run the Gibbs sampler for 200,000 draws, with burn-in of 100,000 and thinning of 1 in 5. We use the method proposed by Chib (1995) to recover the marginal log likelihood of each model from the Gibbs output. Results are reported in table SM.C.1.

TABLE SM.C.1. Marginal likelihoods for alternative models

	Model	Marginal log-likelihood	Numerical SE
M0	Diagonal ρ_x + Unrestricted Σ's	-2,121.12	(0.074)
M1	Diagonal ρ_x + Factor structure on Σ's	-2,188.08	(0.053)
M2	Unrestricted ρ_x + Unrestricted Σ's	-2,129.64	(0.071)
M3	Unrestricted ρ_x + Factor structure on Σ's	-2,214.69	(0.060)

NOTES. Marginal log-likelihood and numerical standard errors computed with the method of Chib (1995)

According to the marginal likelihood values, our baseline model (M0) is ranked first followed by M2, which relaxes the diagonal VAR restriction of M0. Estimates from model M2 for the off-diagonal elements of the VAR matrix ρ_x (available in the replication code) show that those are hardly different from zero — the 90%-probability intervals for each of them invariably contain zero. The same holds in model M3. Therefore, relaxing the diagonal restrictions on ρ_x, which adds a lot of parameters, explains the lower marginal likelihoods of model M2 compared to M0 and M3 compared to M1.

On the other hand, the models that impose a single factor structure on Σ_x, Σ_{GDE} and

Σ_{GDI} do significantly worse. Nevertheless, it appears that Σ_x is well approximated by a single-factor structure. Specifically, the estimates in table SM.B.1 suggest that the posterior median of Σ_x has the form $(0.25 \times 1_{C\times C} + 0.05 \times I_C)$, so that the single common factor explains most of the variation across signals.The posterior medians of the five eigenvalues of Σ_x, split between large and small eigenvalues, are (90%-credible intervals in parentheses)

- 1.295 (1.057–1.602),
- 0.049 (0.040–0.062), 0.037 (0.031–0.045), 0.030 (0.026–0.036), 0.024 (0.020–0.029).

It is clear that there is a single dominant eigenvalue followed by four relatively small ones, consistently with a single-factor structure. In this respect, Table SM.B.1 also suggests that the common shock to the different signals is more important than their specific components in explaining the variance of their innovations, as one would expect from the strong cross-sectional dependence between the different comprehensive revisions observed in Figure 1.

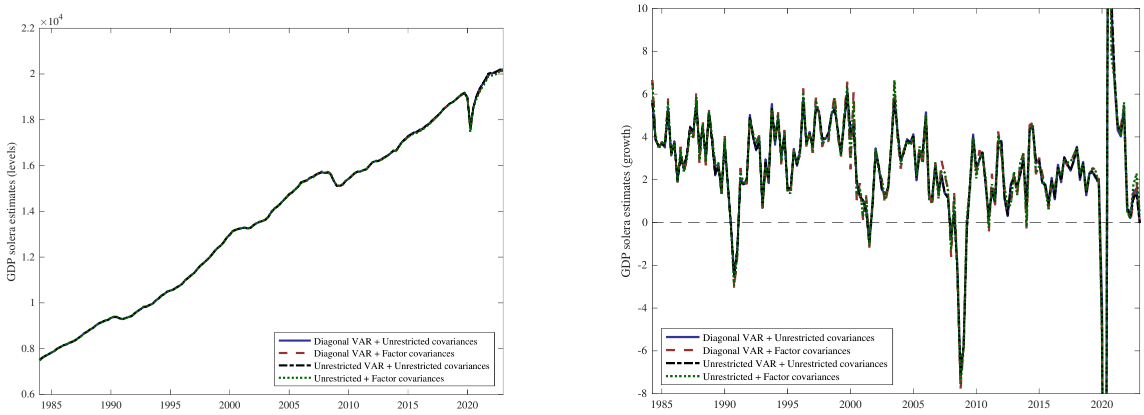
In contrast, this is not the case for either Σ_{GDE} or Σ_{GDI}. In fact, the eleven eigenvalues of Σ_{GDE} are

- 0.638 (0.462–0.886), 0.217 (0.157–0.308), 0.104 (0.081–0.137), 0.067 (0.056–0.082),
- 0.054 (0.047–0.064), 0.045 (0.039–0.053), 0.039 (0.034–0.045), 0.033 (0.029–0.038), 0.029 (0.026–0.033), 0.025 (0.022–0.029), 0.021 (0.018–0.025),

while the ten eigenvalues of Σ_{GDI} are

- 0.829 (0.633–1.095), 0.411 (0.314–0.546), 0.159 (0.129–0.204), 0.117 (0.097–0.142), 0.091 (0.077–0.108), 0.075 (0.065–0.088),
- 0.063 (0.055–0.073), 0.054 (0.047–0.062), 0.045 (0.039–0.052), 0.037 (0.031–0.043).

These estimates suggest that quite a few factors would be needed to adequately capture the cross-sectional dependence of the innovations in the measurement errors in GDE and GDI.



(a) Estimates of GDP in levels

(b) Estimates of GDP growth rates

FIGURE SM.C.1. **Comparison of GDP estimates from alternative models**. Panel (a) reports the posterior median estimates of the level of GDP defined as $\exp(x_{Ct})$ while panel (b) reports the posterior median estimates of the quarterly annualized growth rate of GDP defined as $4\Delta x_{Ct}$.

Despite the large differences in goodness of fit, all of the alternatives we considered have similar implications for the smoothed estimates of economic activity, both in levels and growth rates. Figure SM.C.1 shows this for x_{Ct} (left panel) and Δx_{Ct} (right panel).

Differences across models are negligible and mostly concentrated around the most recent periods, as one would expect. One possible explanation is that the relatively low correlation across measurement errors allows cross-sectional averaging to play a bigger role and attenuates the importance of those assumptions in estimating the latent state variables. This is consistent with the weights we report in table 2, which show that lag and lead quarters receive small weights in the optimal filter.

SM.C.3 The autocorrelation of the statistical discrepancies

Statistical discrepancies appear mean-reverting but, as can be seen in figure SM.C.2, they can be quite persistent. Our model, though, can accommodate a fair amount of persistence because it postulates AR(1) measurement errors. To assess to what extent this is so, in this subsection we look at the autocorrelation function of a selection of five statistical discrepancies defined as the log difference between GDE and GDI estimates for various estimates. Specifically, given that there is no first estimate of GDI, we focus on the second and third early estimates, and the first, second and third annual estimates.

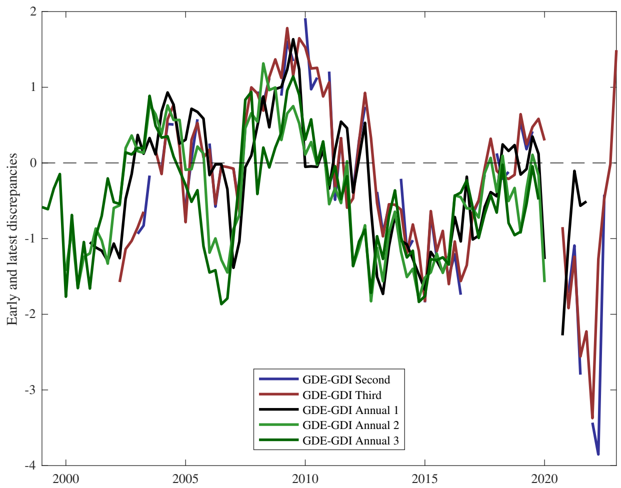
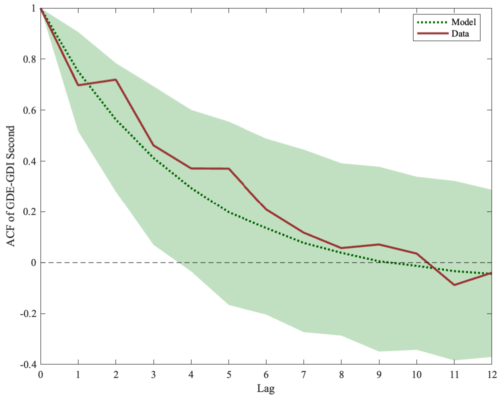
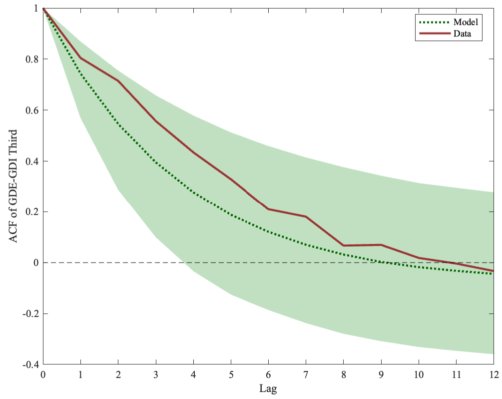


FIGURE SM.C.2. **Statistical discrepancies for early and latest estimates in the BEA data.**

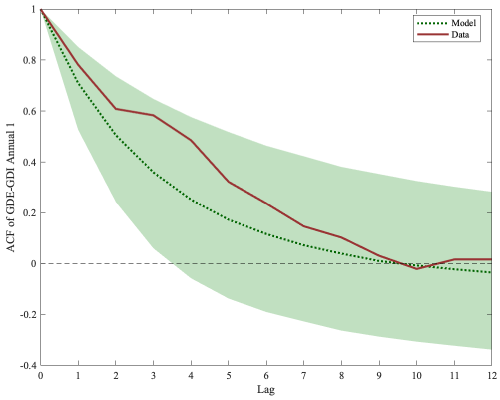
The autocorrelation functions estimated from the data are reported in figure SM.C.3 below for lags 0 to 12, which correspond to three years of quarterly observations. We treat missing observations (e.g., second GDI estimates for every fourth quarter) as missing at random. We compare them with the autocorrelations implied by our model for the contemporaneous difference between measurement errors for the m^{th} measurement $v^m_{GDEt} - v^m_{GDIt}$, which correspond to an ARMA(2,1) process with AR roots given by the corresponding roots of v^m_{GDEt} and v^m_{GDIt}, and an MA coefficient that depends on the serial correlation of those measurement errors and the relative variances of their innovations. As can be seen, the model-based autocorrelations seem to match their empirical counterparts very well.



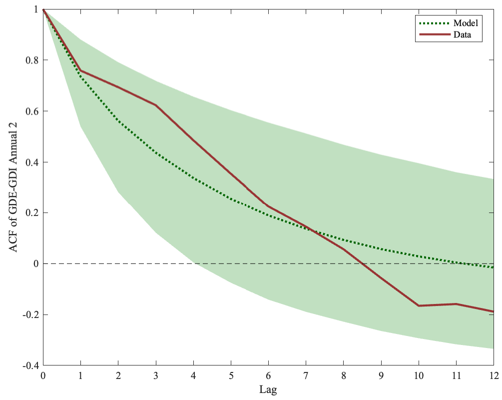
(a) Second estimate



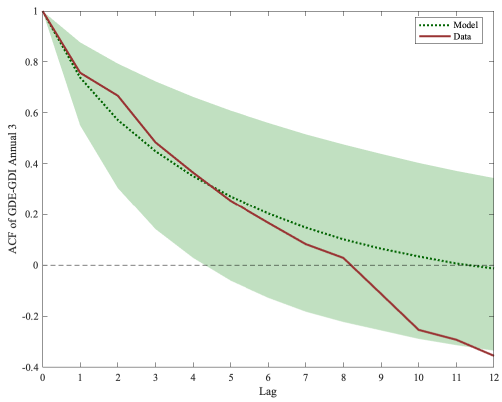
(b) Third estimate



(c) Annual estimate 1



(d) Annual estimate 2



(e) Annual estimate 3

FIGURE SM.C.3. **Fit of serial dependence of early and latest statistical discrepancies** In each panel, the solid red line indicates the sample autocorrelation function computed from BEA data, the dotted green line is the posterior median estimate of the autocorrelation function and the shaded area is a 90%-posterior probability interval pointwise for each lag.

SM.C.4 A non-nested alternative: Trend-Cycle model

As we showed in proposition 2, the autocovariance function of Δx_t is non-parametrically identified under mild assumptions. We decided to model Δx_t as a diagonal VAR(1) with unrestricted error covariance matrix for its innovations because it offers a fairly parsimonious way to fit the serial and cross-sectional dependence we see in the data, as discussed in sections SM.C.2 and SM.C.3. One implication of our model is that each element of x_t has a unit root and, therefore, admits a Beveridge and Nelson (1981) decomposition into trend and cycle components. For that reason, in this subsection we draw inspiration from Morley, Nelson, and Zivot (2003) and explicitly decompose each x_t into trend and cycle components. We thank an anonymous referee for suggesting this possibility.

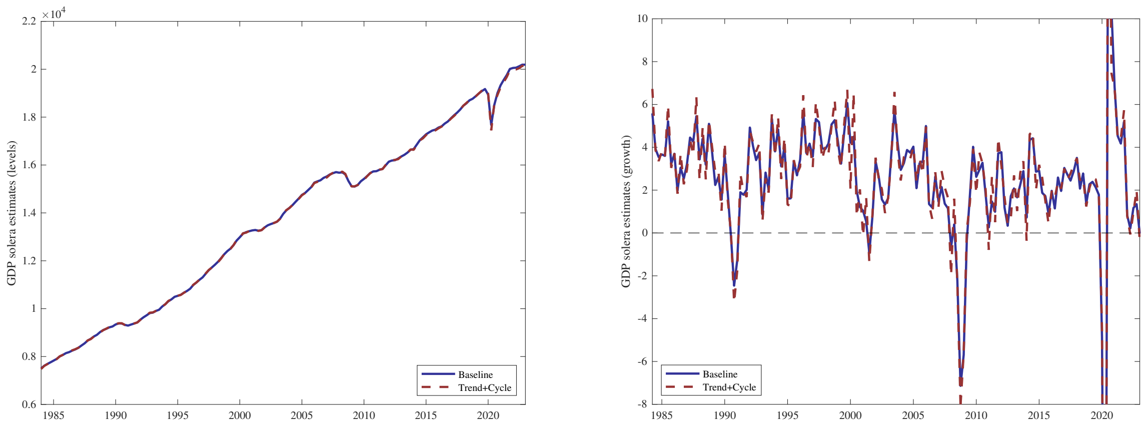
Specifically, we assume that

$$
\begin{aligned}
x_t &= \tau_t + c_t, \\
\tau_t &= \mu + \tau_{t-1} + [\lambda_\tau \eta_{\tau t} + \sigma_\tau \varepsilon_{\tau t}], \\
c_t &= \mathrm{diag}(\phi_{c1}) c_{t-1} + \mathrm{diag}(\phi_{c2}) c_{t-2} + [\lambda_{c\tau}\eta_{\tau t} + \lambda_c \eta_{ct} + \mathrm{diag}(\sigma_{c\tau})\varepsilon_{\tau t} + \mathrm{diag}(\sigma_c)\varepsilon_{ct}].
\end{aligned}
$$

We use a factor structure in the innovations of the trend τ_t and the cycle c_t for two reasons. First, as mentioned in subsection SM.C.2, a single-factor structure seems appropriate for the vector of signals x_t. Second, it allows us to introduce in a relatively simple manner correlation between the shocks to the trend and cyclical components. This is important because Morley et al. (2003) argue that differences in estimates of the trend component of GDE from state-space approaches and Nelson-Beveridge decompositions can be reconciled by allowing for these correlations.

With a convenient choice of Gaussian-inverse Wishart priors, we can adapt the Gibbs sampler algorithm of our baseline model to handle this alternative. We can also obtain marginal likelihoods with the approach of Chib (1995). This procedure yields a marginal likelihood for the trend-cycle model of -2,490,01 (numerical SE = 0.072), which is below that of our baseline model. A potential explanation for why richer cyclical dynamics do not improve the model fit as measured by the marginal likelihood may be that in Morley et al. (2003) the decomposition is applied to GDE data directly rather than to x_t. To some extent, the presence of measurement errors in our model plays a role similar to the inclusion of a cyclical component. As Morley et al. (2003), though, we find negative correlation between shocks to trend and cycle components, and conjugate roots in the AR(2) polynomials of each entry of c_t. At the same time, our estimates indicate more persistence in c_t than theirs, as one would expect in the presence of measurement errors. Detailed results for this trend-cycle model are available in the replication material.

Importantly, Figure SM.C.4 shows that the estimates for the level and growth rates of GDP implied by both models are very close. This is reassuring because our main goal is estimate x_t from y_t.



(a) Estimates of GDP in levels

(b) Estimates of GDP growth rates

FIGURE SM.C.4. **Comparison of GDP estimates from alternative models**. Panel (a) reports the posterior median estimates of the level of GDP defined as $\exp(x_{Ct})$ while panel (b) reports the posterior median estimates of the quarterly annualized growth rate of GDP defined as $4\Delta x_{Ct}$.

SM.C.5 Correlation across expenditure and income measures

There are two reasons for imposing zero correlation between the shocks to the true GDP and the GDI and GDE measurement errors. Primarily, it allows us to achieve non-parametric identification. And second, the empirical evidence of a strong cyclical pattern in the statistical discrepancy is inconclusive (see, Nalewaik (2010) and the subsequent discussion, as well as footnote 17 in Almuzara et al. (2023)).

Still, we have re-estimated a generalised version of our model in which we allow for non-zero correlation between the shocks to the common factor of the innovation of the signals and the common factors in the innovations to the measurement errors of the expenditure and income measures of GDP. In a set up with multiple measurements, this assumption is analogous to the one made in Aruoba, Diebold, Nalewaik, Schorfheide, and Song (2016). Given that the (non-parametric) identification information for those correlations must necessarily come from prior information, we use a grid of degenerate priors ranging from 0 to 30% to assess the sensitivity of our smoothed estimates to the values of this parameter.

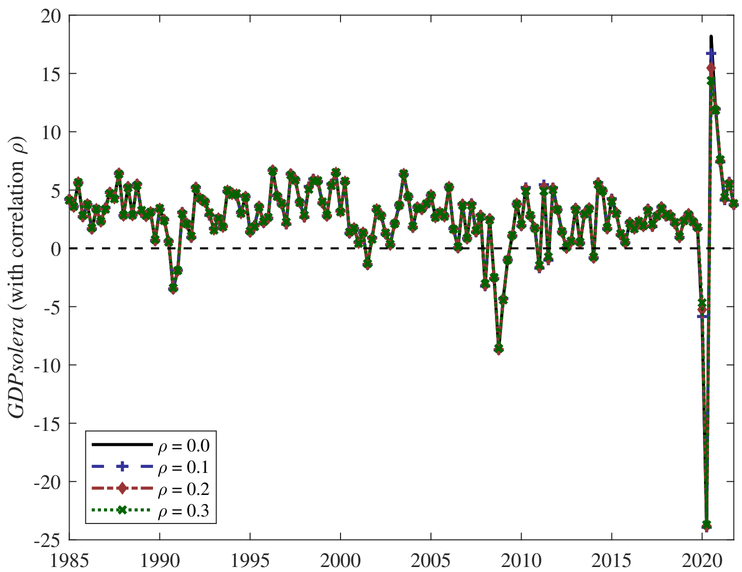


FIGURE SM.C.5. ***GDPsolera*** estimated using data until January 2022 allowing for correlation between shocks to different measurement errors.

As can be seen in Figure SM.C.5, the posterior means of our estimates are hardly affected, except in the third quarter of 2020. Therefore, the identifying assumption of zero correlation does not seem to significantly affect *GDPsolera* or any of the conclusions from our empirical analysis.

SM.D Data and release schedule for GDP estimates

TABLE SM.D.1. GDE and GDI release schedule for the period 2016Q1-2018Q2.

Release Month	Estimate	GDE		GDI	
		New	Updated	New	Updated
Jan 2017	Advance	2016Q4			
Feb 2017	Second		2016Q4		
Mar 2017	Third		2016Q4	2016Q4	
Apr 2017	Advance	2017Q1			
May 2017	Second		2017Q1	2017Q1	
Jun 2017	Third		2017Q1		2017Q1
Jul 2017	Advance	2017Q2	2014Q1-2016Q4		2014Q1-2016Q4
Aug 2017	Second		2017Q2	2017Q2	
Sep 2017	Third		2017Q2		2017Q2
Oct 2017	Advance	2017Q3			
Nov 2017	Second		2017Q3	2017Q3	
Dec 2017	Third		2017Q3		2017Q3
Jan 2018	Advance	2017Q4			
Feb 2018	Second		2017Q4		
Mar 2018	Third		2017Q4	2017Q4	
Apr 2018	Advance	2018Q1			
May 2018	Second		2018Q1	2018Q1	
Jun 2018	Third		2018Q1		2018Q1
Jul 2018	Advance	2018Q2	1947Q1-2017Q4	2018Q2	1947Q1-2017Q4

NOTES. [*] Annual update, [**] Comprehensive update, [†] 13 quarters, i.e. last 3 years

SM.E Implications of L^2-optimality

Consider the following model for the release process. For each type of estimate i and quarter t, the statistical office collects inputs $\iota_{it}^1, \ldots, \iota_{it}^{J_{it}}$ – such as sectoral surveys – on which the estimates y_{it}^m are based. The objective of this appendix is to show that if the statistical office produced estimates with the objective of minimizing their expected square loss (i.e., the L^2-distance between the estimate and x_t), the optimal signal-extraction rule implicit in the Kalman smoother would exclusively map x_t to its most recent release.

For ease of exposition, we abstract from comprehensive revisions by setting $C = 1$, and assume that the measurements in levels are covariance stationary. Fix i and t and let $\sigma(\cdot)$ denote a (generated) σ-algebra. We will assume that (i) there are integers $\{J_{it}^m\}_{m=1}^{M_i}$ such that $J_{it}^m \leq J_{it}^{m+1}$ and y_{it}^m is measurable with respect to the increasing sequence of information sets $\mathcal{I}_{it}^m = \sigma\{\iota_{it}^1, \ldots, \iota_{it}^{J_{it}^m}\}$ for all m, and (ii) the statistical office minimizes $L^2(y_{it}^m - x_t) = \mathbb{E}\left[|y_{it}^m - x_t|^2\right]$. Assumption (i) allows for data on past and future periods to be included among the time-t inputs. From (i) we obtain $\mathcal{I}_{it}^m \subset \mathcal{I}_{it}^{m+1}$ for all m, and from (ii),

$$y_{it}^m = \mathbb{E}\left[x_t \middle| \mathcal{I}_{it}^m\right], \quad m = 1, \ldots, M_i.$$

Let $\tilde{\mathcal{I}}_{it}$ be a σ-algebra such that $\tilde{\mathcal{I}}_{it} \subset \mathcal{I}_{it}^m$ for all m. For example, if the time-t inputs include all the data needed to construct past measurements, $\tilde{\mathcal{I}}_{it}$ may be the σ-algebra generated by all past measurements. With a slight abuse of notation,

$$\mathbb{E}\left[x_t \middle| y_{it}^1, \ldots, y_{it}^m, \tilde{\mathcal{I}}_{it}\right] = \mathbb{E}\left[\mathbb{E}\left[x_t \middle| \mathcal{I}_{it}^m\right] \middle| y_{it}^1, \ldots, y_{it}^m, \tilde{\mathcal{I}}_{it}\right] = \mathbb{E}\left[y_{it}^m \middle| y_{it}^1, \ldots, y_{it}^m, \tilde{\mathcal{I}}_{it}\right] = y_{it}^m,$$

by the law of iterated expectations. In simple words, if measurements minimize expected square loss, all measurements of x_t but the most recent one contain no useful information to extract x_t.

To provide further intuition, consider the following:

Example. Suppose we have two measurements of x,

$$y_1 = x + v_1,$$

$$y_2 = x + v_2.$$

Suppose $x \sim N(\mu, \omega^2)$ and $(v_1, v_2) \sim N(0_{2\times 1}, \Sigma)$, with x stochastically independent of the v's. The best estimate of x given available y's is a linear combination of the y's where the coefficients are given by the usual regression formula (covariance divided by variance). An estimate $\hat{x}$ is best if it minimizes the mean of $(x - \hat{x})^2$.

Imagine y_1 is released first. The best estimate $\hat{x}_1$ of x given y_1 is

$$\hat{x}_1 = \mathbb{E}\left[x \middle| y_1\right] = (1 - \alpha)\mu + \alpha y_1,$$
$$\alpha = \frac{\mathrm{Cov}(x, y_1)}{\mathrm{Var}\,(y_1)} = \frac{\omega^2}{\omega^2 + \sigma_{11}}.$$

Next month, y_2 is released. The best estimate $\hat{x}_2$ of x given y_1, y_2 is

$$\hat{x}_2 = \mathbb{E}\left[x \middle| y_1, y_2\right] = (1 - \beta_1 - \beta_2)\mu + \beta_1 y_1 + \beta_2 y_2,$$
$$\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \left[\begin{pmatrix} \mathrm{Var}\,(y_1) & \mathrm{Cov}(y_1, y_2) \\ \mathrm{Cov}(y_1, y_2) & \mathrm{Var}\,(y_2) \end{pmatrix}\right]^{-1} \begin{pmatrix} \mathrm{Cov}(x, y_1) \\ \mathrm{Cov}(x, y_2) \end{pmatrix}$$
$$= \frac{\omega^2}{\Delta} \begin{pmatrix} \mathrm{Cov}(y_2 - y_1, y_2) \\ \mathrm{Cov}(y_2 - y_1, y_1) \end{pmatrix} = \frac{\omega^2}{\Delta} \begin{pmatrix} \sigma_{22} - \sigma_{12} \\ \sigma_{11} - \sigma_{12} \end{pmatrix},$$

where Δ is the determinant of the above covariance matrix.

We can now ask two questions:

(1) Why is it that in general $\hat{x}_1 \neq y_1$?

To have $\hat{x}_1 = y_1$ we would need $\sigma_{11} = 0$, in which case the signal extraction problem will be irrelevant. Otherwise, it is always optimal to apply some smoothing, here pulling estimates towards the unconditional mean or in a dynamic setup, towards a conditional mean.

(2) When can we disregard y_1 and use just the latest vintage y_2?

This is equivalent to asking under which conditions $\beta_1 = 0$. We can do so if the latest vintage is uncorrelated to the latest revision error, i.e., $\text{Cov}(y_2 - y_1, y_2) = 0$.

Intuition One analogy that may help is mean-variance portfolio analysis. If we identify μ with the return to the safe asset and y_1, y_2 with those of the risky assets, then pulling estimates towards the mean is like using a safe asset to improve the risk-return trade-off of a portfolio, thereby addressing question (1).

Regarding question (2), if y_2 is in the mean-variance frontier for risky assets and y_1 is any other risky asset, then we must have that $\text{Cov}(y_2, y_2 - y_1) = 0$. Otherwise, it would be possible to construct a portfolio that combines the two risky assets with the same expected return but lower risk. In econometrics, this is called the *Hausman principle*: if one estimator $\hat{\theta}_2$ is the most efficient within a class of unbiased estimators Θ, then $\text{Cov}\left(\hat{\theta}_2 - \hat{\theta}_1, \hat{\theta}_2\right) = 0$ for any other $\hat{\theta}_1 \in \Theta$.

By induction, if we have M vintages $y_1, \dots, y_M$ ordered from oldest to newest, the condition for ignoring $y_1, \dots, y_{M-1}$ and only using y_M in filtering x out is that $\text{Cov}(y_M, y_M - y_i) = 0$ for all $i = 1, \dots, M-1$. These covariance restrictions are precisely the conditions that we test empirically in section 4.2.3.

SM.F News and noise model

Consider a setup in which $N = 1$, so that we can omit the subindex indicating type, which would be 1, $M = M_1 = 3$, and there is a single comprehensive version of GDP, so that $C = 1$. Suppose the data follows the news-noise model of Jacobs and van Norden (2011) and Jacobs, Sarferaz, Sturm, and van Norden (2022):

$$\Delta y_t = \begin{pmatrix} \Delta y_t^1 \\ \Delta y_t^2 \\ \Delta y_t^3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Delta \tilde{y}_t + \begin{pmatrix} \nu_t^1 \\ \nu_t^2 \\ \nu_t^3 \end{pmatrix} + \begin{pmatrix} \zeta_t^1 \\ \zeta_t^2 \\ \zeta_t^3 \end{pmatrix} = 1_{3\times 1} \Delta \tilde{y}_t + \nu_t + \zeta_t,$$

where ν_t^m and ζ_t^m are news and noise components. News are defined by the condition that $\text{Cov}\left(\nu_t^m, \Delta\tilde{y}_t + \nu_t^{m'}\right) = 0$ for all $m' \leq m$, while noise must satisfy $\text{Cov}(\zeta_t^m, \Delta\tilde{y}_t + \nu_t^m) = 0$. These, however, are not enough to pin down a unique decomposition of y_t in terms of $\tilde{y}_t, \nu_t, \zeta_t$ and we will further impose $\zeta_t^1, \zeta_t^2, \zeta_t^3$ are uncorrelated with each other.

To simplify the argument, we will assume that (i) $\Delta\tilde{y}_t + \nu_t^3$ follows an AR(1) process and (ii) ν_t and ζ_t are uncorrelated over time. Moreover, we note that the news-noise model is typically applied to measurements of GDP growth, as opposed to our model, which focuses on the level.

The goal is to understand how the news-noise model maps to ours, namely

$$\begin{pmatrix} y_t^1 \\ y_t^2 \\ y_t^3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} x_t + \begin{pmatrix} v_t^1 \\ v_t^2 \\ v_t^3 \end{pmatrix} = 1_{3\times1} x_t + v_t.$$

We can write

$$\Delta y_t = \begin{pmatrix} \Delta y_t^1 \\ \Delta y_t^2 \\ \Delta y_t^3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} (\Delta\tilde{y}_t + \nu_t^3) + \begin{pmatrix} (\nu_t^2 - \nu_t^3) + (\nu_t^1 - \nu_t^2) \\ (\nu_t^2 - \nu_t^3) \\ 0 \end{pmatrix} + \begin{pmatrix} \zeta_t^1 \\ \zeta_t^2 \\ \zeta_t^3 \end{pmatrix},$$

where $\nu_t^1 - \nu_t^2, \nu_t^2 - \nu_t^3, \zeta_t^1, \zeta_t^2, \zeta_t^3$ are mutually orthogonal white noise processes. If we set

$$\begin{aligned}
\Delta x_t &= \Delta\tilde{y}_t + \nu_t^3, \\
\Delta v_t^1 &= (\nu_t^2 - \nu_t^3) + (\nu_t^1 - \nu_t^2) + \zeta_t^1, \\
\Delta v_t^2 &= (\nu_t^2 - \nu_t^3) + \zeta_t^2, \\
\Delta v_t^3 &= \zeta_t^3,
\end{aligned}$$

we obtain a particular case of our model in which, not surprisingly, $\rho = 1_{3\times1}$. Measurement error are therefore white noise in first differences with a particular variance matrix,

$$\text{Var}\left(\Delta v_t\right) = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} & 0 \\ \Sigma_{12} & \Sigma_{22} & 0 \\ 0 & 0 & \Sigma_{33} \end{pmatrix}.$$

If we give Δv_t the factor structure in (4) (again maintaining $\rho = 1_{M\times 1}$),

$$\Delta v_t = \begin{pmatrix} \Delta v_t^1 \\ \Delta v_t^2 \\ \Delta v_t^3 \end{pmatrix} = \begin{pmatrix} \lambda^1 \\ \lambda^2 \\ \lambda^3 \end{pmatrix} \eta_t + \begin{pmatrix} \sigma^1 \varepsilon_t^1 \\ \sigma^2 \varepsilon_t^2 \\ \sigma^3 \varepsilon_t^3 \end{pmatrix} = \lambda \eta_t + \mathrm{diag}(\sigma)\varepsilon_t,$$

with $\eta_t \overset{iid}{\sim} N(0,1)$, $\varepsilon_t \overset{iid}{\sim} N(0_{3\times 1}, I_3)$ and η_t independent of ε_t, the news-noise model implies the restriction $\lambda_3 = 0$. The rest of the parameters, $\lambda^1, \lambda^2, \sigma^1, \sigma^2, \sigma^3$, can be recovered from Var (Δv_t).

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