Moral Hazard and Debt Maturity

Gur Huberman
Columbia Business School

Rafael Repullo
CEMFI and CEPR

December 2023

Abstract

We present a model of the determinants of the maturity of a bank's uninsured debt. The bank borrows funds to invest in a long-term project whose riskiness is not verifiable. This moral hazard problem leads to an excessive level of risk. Short-term debt may have a disciplining effect on the bank's risk-shifting incentives, but it may lead to inefficient liquidation. We characterize the conditions under which short- and long-term debt are feasible, and show circumstances under which only short-term debt is feasible and under which short-term debt dominates long-term debt when both are feasible. The results are consistent with key features of the common narrative of the period preceding the 2007-2009 global financial crisis.

JEL Classification: G21, G32

Keywords: Short-term debt, long-term debt, optimal financial contracts, risk-shifting, rollover risk, inefficient liquidation.

We are very grateful to Patrick Bolton, Charles Calomiris, Eddie Dekel, Douglas Diamond, Douglas Gale, Zhiguo He, Martin Hellwig, Paul Kupiec, Nellie Liang, John Moore, Lev Ratnovski, Jean-Charles Rochet, Tano Santos, Martin Schneider, Hyun Shin, Javier Suarez, Xavier Vives, Ansgar Walther, and Mark Westerfield, as well as to numerous seminar audiences, for their valuable comments and suggestions. We would also like to thank Enzo Cerletti, Anatoli Segura, and Manuel Ruiz for their excellent research assistance. Financial support from the FDIC Center for Financial Research and the Spanish Ministry of Science and Innovation is gratefully acknowledged. E-mail: gh16@columbia.edu, repullo@cemfi.es.

1 Introduction

Funding long-term investments with short-term debt risks failure to roll over the debt. Such failure can happen if adverse news about the investments' final payoff arrive at the rollover date, or if short-term lenders have better or more urgent uses for their funds. In that case the investments are liquidated, even when liquidation might be inefficient. *Why then fund long-term investments with short-term rather than long-term debt?*

Following Diamond and Dybvig (1983), a voluminous literature focuses on the lenders' demand for liquidity. This paper is different. In our model the lenders do not face liquidity shocks, but they observe some relevant information on the prospects of the investment that may lead them to withdraw their funding. But if early liquidation is inefficient, the question about using short-term debt remains. Here is where moral hazard enters the picture. Suppose that the borrowers can choose the risk of their investments after the borrowing is arranged. In such situation, they will have an incentive to take excessive risks. We argue that *using short-term may be justified as a way to ameliorate the borrowers' risk-shifting incentives.*

We consider a borrowing firm that has three attributes of a bank. First, it funds itself mostly (in the model only) by issuing debt. Second, it can easily modify the risk profile of its investments.[1] Third, it invests in financial assets, not real assets that can be redeployed to other sectors of the economy, which means that their liquidation value is related to (in the model a fraction of) their expected continuation value. For this reason, we will henceforth refer to the firm as a bank.

A comparison between short- and long-term debt entails the analysis of the optimal decision at the outset of the bank's shareholders. At that point they know that if short-term debt is used, they will have to refinance it. We argue that when there is a moral hazard problem in the choice of risk, the anticipation of refinancing needs may act as a disciplining device that could render short-term debt superior to long-term debt. So the trade-off is between the disciplining benefits of short-term debt and the cost of inefficient liquidation.

[1] Flannery (1994) notes that banks can easily modify the risk profile of their assets, and that contracts preventing such modifications are difficult to write and enforce, which in his view rationalizes the use of short-term debt.

The model has three dates: an initial date where the financing of the investment is arranged in a competitive debt market and its risk is privately chosen by the bank, an interim date where noisy public information about the eventual investment payoff is revealed, and depending on this information the bank may be liquidated, and a final date where investment returns are realized, if the bank was not liquidated at the interim date.

We start analyzing the simpler case of long-term debt, where the information revealed at the interim date is irrelevant. Next we examine short-term debt. In this case, if there is no liquidation at the interim date the bank repays the initial lenders by issuing new short-term debt, which matures at the final date, and if there is liquidation at the interim date the liquidation proceeds go to the initial lenders. Finally, we compare the bank's payoff in the optimal contract with long- and short-term debt, to derive the determinants of the optimal maturity structure.

The main results may be summarized as follows. First, we show that the positive incentive effects of short-term debt only obtain when it is risky, that is, when it implies a positive probability of early liquidation. Second, we show that there are circumstances in which risky short-term debt may be the only way to secure funding and in which risky short-term debt may dominate long-term debt when both are feasible. Thus, costly liquidation of investments may be a feature of the optimal financing contract. Third, we show that using risky short-term debt may involve paying an up-front dividend to the bank shareholders.[2]

To explain the intuition for these results is it useful to refer to Stiglitz and Weiss (1981). They present two models, one based on adverse selection and the other one on moral hazard. In the latter they show how "higher interest rates induce firms to undertake projects with lower probabilities of success but higher payoffs when successful."

In our model higher borrowing costs induce banks to undertake investments with lower probabilities of success but higher payoffs when successful. From this perspective, the difference in risk-shifting incentives between long- and risky short-term debt lies in the relevant

[2] A popular rationale for the use of short-term debt is that it is generally cheaper, because the yield curve tends to have a positive slope. In our setup the (riskless) yield curve is flat. Nevertheless, short-term debt could be (endogenously) cheaper due to its positive incentive effects.

cost of the bank's borrowing. With long-term debt the relevant cost reflects the unconditional probability of success, whereas with risky short-term debt it reflects the probability of success conditional on observing the (good) information that leads to the rollover of the debt at the interim date, because otherwise the bank is liquidated and the shareholders get nothing. Since the unconditional probability of success is, ceteris paribus, lower than the probability of success conditional on observing good news, the bank will have an incentive to choose riskier investments with long-term debt.

However, there is an effect working in the opposite direction, which comes from the fact that the initial lenders will have to be compensated for the potential liquidation costs. We show that the first effect dominates when the profitability of the bank's investments is sufficiently low and the quality of the information revealed at the interim date is sufficiently high.

The intuition for the result that short-term debt only makes a difference when it is risky should now be clear. If the initial short-term debt is safe and therefore is always rolled over, the cost of the long-term debt will be the same as the expected cost of the short-term debt, and so long-term debt will be equivalent to safe short-term debt.

The key role of liquidation at the interim date also explains the result that using short-term debt may involve paying an up-front dividend to the bank shareholders. Such dividend does not make sense in the case of long-term debt, since it increases the amount due to the lenders and consequently worsens the moral hazard problem. But it may be useful in the case of short-term debt in order to guarantee that early liquidation obtains with positive probability.

One important assumption to get the disciplining effects of short-term debt is that the lenders cannot agree to renegotiate down their claim in order to avoid the liquidation costs. This assumption is standard in the literature,[3] and can be justified by assuming that the lenders are dispersed and cannot coordinate on writing down their claim.[4]

[3]See, for example, Rajan (1992), Hart and Moore (1995), and Bolton and Freixas (2000).

[4]Diamond (2004) provides an alternative rationale in terms of an equilibrium model of lender behavior in which "properly structured short-term debt provides incentives for each short-term lender to enforce his contract even when it hurts lenders collectively."

Our results are can be related to events in the run-up to the 2007-2009 global financial crisis. According to Bernanke (2010), "Leading up to the crisis, the shadow banking system, as well as some of the largest global banks, had become dependent on various forms of short-term wholesale funding." Brunnermeier (2009), Shin (2010), and Tirole (2010), among others, share the view that the pre-crisis period was associated with banks increasingly financing their asset holdings with shorter maturity instruments. Other aspects of the common narrative of the pre-crisis period include a decline in profitability, leading to widespread search for yield (Rajan, 2005), and increased opacity of the financial sector's balance sheets (Brunnermeier, 2009).[5] Our results on the determinants of the optimal choice of maturity suggest that these changes are consistent with a shift from long- to risky short-term debt financing.

Literature review Liquidity risk plays a major role in most papers that analyze short-term debt finance. The model presented here is an exception in that the lenders have no demand for liquidity. We focus on the possibility that adverse news about the bank's investment could lead to early liquidation, which happens when its conditional expected payoff is lower than the amount due to the lenders at the interim date. The reason why short-term debt may be useful is that, aware of the possibility of failure to refinance in the future, the bank chooses safer investments.

Liquidity risk is the focus of the seminal paper by Diamond and Dybvig (1983). They show how banks may efficiently insure this risk, but may be subject to runs by demand depositors suspecting that other depositors may want to withdraw their funds, and therefore render the bank illiquid. Our model is closer to the work of Jacklin and Bhattacharya (1988) on informationally-based bank runs. But their focus is very different from ours.

Calomiris and Kahn (1991) provide a rationale for the issuance of demandable debt by banks. In their model, shareholders can abscond with bank assets, which they will have an incentive to do when they learn that investment returns will be low. In this context, it is optimal to use short-term demandable debt, because it gives depositors the option to

[5] See Rajan (2005), Brunnermeier (2009), and Gorton (2010), among others.

force liquidation before the absconding is done. In contrast with this setup, our focus is on the role of short-term debt as a disciplinary device on ex-ante risk-shifting incentives. The related work of Diamond and Rajan (2001) shows how short-term demandable debt allows bank managers to commit to paying depositors ex-post the full return of their relationship loans. In their words, "financial fragility allows liquidity creation."

Theoretical research on the maturity structure of firms' debt includes the seminal work of Diamond (1991). He considers an adverse selection model of a firm's choice of debt maturity in which firms with high credit ratings issue short-term debt and firms with lower credit ratings issue long-term debt. The optimal maturity structure trades off a borrower's preference for short-term debt (due to private information about its future credit rating) against liquidity risk. Rajan (1992) studies a moral hazard model of a firm's choice between a bank and an arm's-length lender. The bank monitors the firm and can lend either short-term or long-term, whereas the arm's-length lender must lend long-term. The choice of financing mode depends crucially on the relative bargaining power of the firm and the bank after they acquire information on the future payoff of the investment.

The closest references to our work are the dynamic capital structure papers by Leland and Toft (1996), Leland (1998), and Cheng and Milbradt (2012) showing the potential benefits of short-term debt in curbing risk-shifting incentives. These papers consider a firm with assets whose unlevered value follows a continuous diffusion process with proportional volatility. In Leland and Toft (1996) the optimal maturity of the debt is derived from balancing its tax advantage against bankruptcy and agency costs (due to the equityholders' ex ante choice of asset risk). Leland (1998) assumes that the firm can choose at any time between a low and a high level of risk and that the risk strategy followed by the firm cannot be precontracted in debt covenants. Cheng and Milbradt (2012) also assume that the firm can adjust the risk ex post focusing on the role of staggered short-term debt in the presence of a coordination problem among creditors that may lead to rollover freezes.

In contrast, the paper by Della Seta et al. (2020) uses a similar setup to conclude that risk-shifting incentives do not arise when debt maturity is sufficiently long. The key difference with earlier research is the introduction of financial frictions that restrict equity issuance to absorb potential losses at rollover dates.

In the light of these contrasting results, our contribution lies in examining the role of short-term debt in a discrete-time agency model in which the different forces at play may be easier to analyze. A key difference with the literature is that in these models the maturity of the debt is set at the outset and it does not vary with the ex post evolution of the firm's investments. In our setup, the choice of short or long maturity depends on the parameters that characterize the environment, in particular the profitability of the investment.

An important simplifying assumption is that the firm (or the bank in our preferred interpretation) is entirely funded with debt, so there is not an endogenous leverage decision.[6] Taking into account the role of (exogenous) capital regulation in banking this does not seem a very restrictive assumption.

The view that short-term debt can act as a disciplining device has been criticized by Admati and Hellwig (2013). They write: "In the years before the financial crisis of 2007-2009, as banks were building up enormous risks, they dramatically expanded the extent of their borrowing, relying in particular on short-term debt." But the point that our paper makes is about the counterfactual, in particular whether in the specific environment in which banks were operating using long-term debt would have led to even higher risk-taking.

As an alternative to the theories of debt maturity based on the disciplinary role of short-term debt, Diamond and He (2014) consider the relationship between debt maturity and debt overhang (the reduced incentive of borrowers to invest because some value accrues to the existing lenders). They examine the idea of Myers (1977) that short-term debt should reduce overhang, and show circumstances under which this is not necessarily the case. Brunnermeier and Oehmke (2013) consider a model in which borrowers cannot commit to a maturity structure, showing that in this case a maturity rat race may lead to extreme reliance on short-term financing.

Finally, we note the connection of our paper with the literature on sovereign debt maturity. For example, Jeanne (2009) shows that short-term debt helps to ensure that debtor

[6] At the same time, given that the bank is funded in a competitive debt market, the market value of its equity will be positive.

countries implement investor-friendly policies, but also makes them vulnerable to crises caused by bad shocks. Thus, the benefits of short-term debt in terms of incentives are traded off against the costs in terms of unwarranted crises.

Structure of the paper Section 2 presents the model. Sections 3 and 4 characterize the optimal contract with long- and short-term debt. Section 5 analyzes the optimal debt maturity structure. Section 6 examines the possible mixing of short- and long-term debt, the consequences of regulating liquidity risk, and the effects of monetary policy. Section 7 contains our concluding remarks. The proofs of the analytical results are in the Appendix.

2 The Model

Consider an economy with three dates ($t = 0, 1, 2$), a risk-neutral *bank*, and a large number of risk-neutral wholesale *lenders*. The bank and the lenders have the same discount rate for both periods that is normalized to zero.

The bank can undertake an indivisible investment that has a unit cost at the initial date $t = 0$ and yields a random payoff at the terminal date $t = 2$ given by

$$R = \begin{cases} R(p), & \text{with probability } p, \\ 0, & \text{with probability } 1 - p, \end{cases} \tag{1}$$

where $R(p) > 0$, $R'(p) < 0$, and $R''(p) \leq 0$. We assume that the probability p of the success payoff $R(p)$ is privately chosen by the bank at $t = 0$, which is the source of the moral hazard problem. Thus, higher risk-taking (lower p) is associated with a higher success payoff.[7]

The expected payoff of the bank's investment $pR(p)$ is maximized at the first-best probability of success

$$p^* = \arg\max_p (pR(p)).$$

The function $pR(p)$ satisfies $(pR(p))'' = 2R'(p) + pR''(p) < 0$, so it is concave. Since $(pR(p))' = R(p) + pR'(p)$ equals $R(0) > 0$ for $p = 0$, we have $p^* > 0$. We further as-

[7] This setup is borrowed from Allen and Gale (2000, Chapter 8) and is essentially the moral hazard model in Stiglitz and Weiss (1981). An alternative approach would be to follow Holmström and Tirole (1997) and assume that the success payoff is fixed and the bank gets private benefits $\Pi(p)$, with $\Pi'(p) < 0$. The two approaches yield similar results.

sume that $R(1) + R'(1) \leq 0$, which implies that $p^* \leq 1$. It then follows that the first-best probability of success p^* is characterized by the first-order condition

$$(p^* R(p^*))' = 0. \tag{2}$$

The bank does not have any capital, so to undertake the investment it has to secure funding by lenders. This funding may come in the form of *long-term debt*, that matures at the terminal date $t = 2$, or *short-term debt*, that matures and has to be rolled over at the interim date $t = 1$.

To introduce some meaningful difference between short- and long-term debt, we assume that at $t = 1$ the lenders observe a *binary public signal* s_j $(j = 0, 1)$ on the future payoff of the bank's investment, and based on this signal they decide whether to refinance the short-term debt. If they do, final payoffs will be obtained at $t = 2$. If they do not, the bank will be liquidated at $t = 1$.

As in Repullo (2005), the signal s_j $(j = 0, 1)$ observed by the lenders at $t = 1$ satisfies

$$\Pr(s_0 \mid R = 0) = \Pr(s_1 \mid R > 0) = q,$$

where parameter $q \in [1/2, 1]$ describes the quality of the lenders' information.[8] It should be noted that the information observed at the interim date $t = 1$ is only about whether the investment will succeed or fail, and not about the particular value $R(p)$ taken by the success payoff.

By Bayes' law, the posterior probabilities of success are

$$p_0 = \Pr(R > 0 \mid s_0) = \frac{(1-q)p}{\Pr(s_0)}, \tag{3}$$

$$p_1 = \Pr(R > 0 \mid s_1) = \frac{qp}{\Pr(s_1)}, \tag{4}$$

where the probabilities of the two signals are

$$\Pr(s_0) = p + q - 2qp, \tag{5}$$

$$\Pr(s_1) = 1 - p - q + 2qp. \tag{6}$$

[8] More generally, we could have two parameters describing the quality of the lenders' information, namely $q_0 = \Pr(s_0 \mid R_0)$ and $q_1 = \Pr(s_1 \mid R_1)$, but this would complicate the notation without any significant change in the results.

For $p \in (0,1)$ and $q \in (1/2,1)$ the posterior probabilities satisfy $p_0 < p < p_1$. For this reason, the states corresponding to observing signals s_0 and s_1 will be called, respectively, the bad and the good state.

If the bank is liquidated at the interim date $t = 1$, the *liquidation value* of the investment is a fraction $\lambda \leq 1$ of its conditional expected payoff, that is $\lambda E(R \mid s)$. Parameter λ is the *recovery rate*. Notice that for any $\lambda < 1$ liquidating the bank at $t = 1$ will be inefficient.[9]

In principle, the bank could raise at $t = 0$ more than the unit of funds required for the investment and pay out the excess as an *up-front dividend* $D \geq 0$. This possibility turns out to be useful in some circumstances discussed below.

An example The linear payoff function

$$R(p) = a(2 - p), \tag{7}$$

with $a > 0$, satisfies the required properties and will be used to derive the numerical results of the paper. Parameter a characterizes the profitability of the bank's investment. For this function we have $(pR(p))' = 2a(1-p)$, which implies $p^* = 1$. Thus, the first-best would be a safe investment with $R(p^*) = a$. In the absence of moral hazard, $a \geq 1$ ensures a non-negative net present value of the investment.

3 Long-term Debt

We start the analysis characterizing the optimal contract with long-term debt. Obviously, in this case the information revealed at the interim date $t = 1$ is completely irrelevant.

A *contract with long-term debt* specifies the dividend D paid up-front and the face value B of the debt maturing at $t = 2$ that the lenders receive in exchange for $1 + D$ funds provided at $t = 0$. Such contract determines the probability of success p chosen by the bank at $t = 0$.

An *optimal contract with long-term debt* is a triple (D_L, B_L, p_L) that solves

$$\max_{(D,B,p)} \left[D + p\left(R(p) - B\right)\right] \tag{8}$$

[9] Note that the liquidation costs $(1-\lambda)E(R \mid s)$ would be avoided if the lenders could renegotiate their claim. We are implicitly assuming that the lenders are dispersed, so such renegotiation is impossible.

subject to the *bank's incentive compatibility constraint*

$$p_L = \arg\max_p \left[p\left(R(p) - B_L\right)\right], \tag{9}$$

and the *lenders' participation constraint*

$$p_L B_L = 1 + D_L. \tag{10}$$

The incentive compatibility constraint (9) characterizes the bank's choice of p given the promised repayment B_L, and the participation constraint (10) ensures that the lenders get the required expected return on their investment. Notice that this constraint must hold with equality, for otherwise the dividend D could be increased without changing B and p, improving the bank's payoff.

The solution to (9) is characterized by the first-order condition

$$(pR(p))' = B. \tag{11}$$

Since $pR(p)$ is concave, the left-hand side of (11) is decreasing in p, which implies that higher values of B are associated with lower values of the probability of success p, that is $dp/dB < 0$. This is the standard risk-shifting effect that obtains under debt finance. Moreover, using the characterization (2) of the first-best probability of success p^*, it follows that $p_L < p^*$, that is the bank will take on more risk than in the first-best.

We next show that raising more than one unit of funds and paying out the excess as an up-front dividend D is not optimal. Suppose to the contrary that $D = pB - 1 > 0$ and consider the effect on the bank's payoff of a change in the face value B, which is

$$\frac{d}{dB}\left[(pB - 1) + p\left(R(p) - B\right)\right] = \frac{d}{dB}\left[pR(p) - 1\right] = B\frac{dp}{dB} < 0,$$

where we have used the first-order condition (11) and the result $dp/dB < 0$. This means that whenever the constraint $D \geq 0$ is not binding, reducing the face value B increases the bank's payoff.

The intuition for this result is that setting $D > 0$ worsens the bank's moral hazard problem. Since the lenders' participation constraint is satisfied with equality, this translates into a lower payoff for the bank.

Solving for B in the participation constraint $pB = 1$ and substituting it into the first-order condition (11) gives the key equation that characterizes the optimal contract with long-term debt

$$H(p) = p\,(pR(p))' = 1. \tag{12}$$

Since $(pR(p))'$ is positive for $0 \leq p < p^*$, with $(p^*R(p^*))' = 0$, it follows that the function $H(p)$ is positive for $0 < p < p^*$, and satisfies $H(0) = H(p^*) = 0$.

The equation $H(p) = 1$ may have no solution, a single solution, or multiple solutions. In the first case, financing the bank with long-term debt is not feasible: the bank's risk-shifting incentives are so strong that the lenders' participation constraint cannot be satisfied. In the second case, the single solution characterizes the optimal contract with long-term debt. And in the third case, note that substituting the participation constraint (10) into the bank's objective function (8) gives $pR(p) - 1$. Since the function $pR(p)$ is increasing in the interval $(0, p^*)$ where the multiple solutions would be located, the optimal contract is characterized by the solution p_L with the highest probability of success. This choice can be implemented by the bank offering the rate $B_L = 1/p_L$ to the lenders. Hence, we have the following result.

Proposition 1 *Financing the bank with long-term debt is feasible if the equation $H(p) = 1$ has a solution, in which case the optimal contract is given by $D_L = 0$, $B_L = 1/p_L$ and*

$$p_L = \max\{p \in (0, p^*) \mid H(p) = 1\}. \tag{13}$$

An example (continued) For the payoff function $R(p) = a(2 - p)$ we have

$$H(p) = 2ap(1 - p), \tag{14}$$

so solving for the optimal contract with long-term debt gives

$$p_L = \frac{1}{2}\left(1 + \sqrt{\frac{a-2}{a}}\right). \tag{15}$$

The term inside the square root will be non-negative if $a \geq 2$. Hence, financing the bank with long-term debt requires that the profitability of the bank's investment be higher than in the absence of moral hazard, which only requires $a \geq 1$. Increases in a increase the probability of success p_L in the optimal contract, bringing it closer to the first-best $p^* = 1$.

4 Short-term Debt

We next analyze the optimal contract with short-term debt. In this case, the quality q of the information revealed at the interim date $t = 1$ is key for the characterization of the optimal contract.

Let M denote the face value of the debt maturing at $t = 1$ that the lenders receive in exchange for $1 + D$ funds provided at $t = 0$, where as before D is the dividend paid up-front to the bank. At $t = 1$ the bank will try to issue new debt, payable at $t = 2$, in order to repay the initial lenders. The face value of this debt will naturally depend on the signal observed at this date. Let N_j denote the face value of the debt maturing at $t = 2$ that the interim lenders receive in exchange for funding the repayment of the initial debt, if it is rolled over in state s_j $(j = 0, 1)$.

The decision to roll over the initial debt depends on the posterior probabilities of success of the investment, p_j $(j = 0, 1)$. As stated in (3) and (4), these probabilities depend on the quality of the signal q, which is known, and the prior probability p, which is not. Hence, the interim lenders will have to decide on the basis of the value $\widehat{p}$ that they conjecture the bank chose at $t = 0$. Let $\widehat{p}_j$ $(j = 0, 1)$ denote the corresponding posterior probabilities, obtained by replacing p by $\widehat{p}$ in (3) and (4).

At $t = 1$ the lenders will roll over the bank's initial debt in state s_j $(j = 0, 1)$ if the conjectured expected value of the bank's investment is greater than or equal to the face value M of the debt to be refinanced, that is if

$$\widehat{E}(R \mid s_j) = \widehat{p}_j R(\widehat{p}) \geq M.$$

In this case, the interim lenders' participation constraint implies that the face value of the debt maturing at $t = 2$ will be

$$\widehat{N}_j = \frac{M}{\widehat{p}_j}. \tag{16}$$

When $\widehat{E}(R \mid s_j) < M$ the initial debt will not be rolled over in state s_j, in which case the initial lenders get the liquidation value $\lambda \widehat{E}(R \mid s_j)$.[10]

[10] We are assuming that no dividend is paid to the bank at the interim date $t = 1$, and that no fresh equity

From here it follows that the *initial lenders' participation constraint* is given by

$$\nu(M,\widehat{p}) = \widehat{\Pr}(s_0)\nu_0 + \widehat{\Pr}(s_1)\nu_1 = 1 + D, \tag{17}$$

where for $j = 0,1$ $\widehat{\Pr}(s_j)$ denotes the probability of state s_j conjectured by the lenders , obtained by replacing p by $\widehat{p}$ in (5) and (6), and

$$\nu_j = \begin{cases} M, & \text{if } \widehat{E}(R \mid s_j) \geq M, \\ \lambda \widehat{E}(R \mid s_j), & \text{otherwise.} \end{cases} \tag{18}$$

Thus, when $\widehat{E}(R \mid s_j) \geq M$ the initial lenders' payoff in state s_j equals the face value M of the initial debt, and when $\widehat{E}(R \mid s_j) < M$ the bank is liquidated in state s_j and the initial lenders get the liquidation value $\lambda \widehat{E}(R \mid s_j)$. As before, the participation constraint (17) is written as an equality because otherwise the dividend D could be increased without changing the bank's incentives, improving its payoff.

Next, the *bank's payoff* is given by

$$\pi(D,M,p,\widehat{p}) = \Pr(s_0)\pi_0 + \Pr(s_1)\pi_1, \tag{19}$$

where for $j = 0,1$

$$\pi_j = \begin{cases} D + \Pr(R > 0 \mid s_j)\max\{R(p) - \widehat{N}_j, 0\}, & \text{if } \widehat{E}(R \mid s_j) \geq M, \\ D, & \text{otherwise.} \end{cases} \tag{20}$$

Thus, when $\widehat{E}(R \mid s_j) \geq M$ the bank's payoff in state s_j equals the up-front dividend D plus the expected continuation payoff net of the repayment to the interim lenders, and when $\widehat{E}(R \mid s_j) < M$ the bank is liquidated in state s_j and it only gets the up-front dividend D.[11]

is injected at this date, possibly coming from saving a positive up-front dividend received at $t = 0$. Both assumptions are made without loss of generality. As we will see below, paying a dividend at $t = 0$ may have a positive effect on the bank's choice of p, but paying a dividend at $t = 1$ entails no incentive effect, since at this point p has already been chosen. And, as we will also see below, whenever the optimal contract with short-term debt entails a positive dividend, the expected value of the bank's investment in the bad state is equal to the face value of the debt to be refinanced, so injecting equity to avoid liquidation in this state will not increase the bank's payoff.

[11]The use of the max operator in (20) is explained by the fact that the lenders' conjectured probability of success $\widehat{p}$ may be different from the actual probability p chosen by the bank. In particular, when $\widehat{p} < p$ it may be the case that $R(p) < \widehat{N}_j$. However, this will never obtain in the optimal contract with short-term debt, in which the lenders' expectations are rational, so $\widehat{p} = p$.

A *contract with short-term debt* specifies the dividend D paid up-front and the face value M of the debt maturing at $t = 1$ that the lenders receive in exchange for $1+D$ funds provided at $t = 0$. Such contract determines the probability of success p chosen by the bank at $t = 0$ and the face value $\widehat{N}_j$ of the interim debt payable to the lenders at $t = 2$, if the initial debt is rolled over in state s_j.

An *optimal contract with short-term debt* is a triple (D_S, M_S, p_S) that solves

$$\max_{(D,M,p)} \pi(D, M, p, \widehat{p})$$

subject to the *bank's incentive compatibility constraint*

$$p_S = \arg\max_p \pi(D_S, M_S, p, \widehat{p}), \tag{21}$$

the *initial lenders' participation constraint*

$$\nu(M_S, \widehat{p}) = 1 + D_S, \tag{22}$$

and the *rational expectations constraint*

$$\widehat{p} = p_S. \tag{23}$$

The incentive compatibility constraint (21) characterizes the bank's choice of p given the promised interim repayment M_S and the rollover decision implied by the lenders' conjecture $\widehat{p}$ of the value of p chosen by the bank. The participation constraint (22) ensures that the initial lenders get the required expected return on their investment. Finally, (23) ensures that lenders' expectations are rational.

There are two possible types of contracts with short-term debt: one in which the initial debt is safe, in the sense that the initial lenders are fully repaid in both states, and another one in which the initial debt is risky, in the sense that the initial lenders are fully repaid in the good state s_1 and the bank is liquidated in the bad state s_0.[12] We next characterize the optimal contracts with safe and risky short-term debt.

[12] We ignore contracts in which there is liquidation in both states, since one can show that they are either not feasible or dominated by one of the other two possible types of contracts.

4.1 Safe short-term debt

When the short-term debt is safe, the initial lenders are fully repaid in both states, so their participation constraint (17) simplifies to

$$\nu(M, \widehat{p}) = M = 1 + D. \tag{24}$$

Using $\Pr(s_0)\Pr(R > 0 \mid s_0) = (1-q)p$ and $\Pr(s_1)\Pr(R > 0 \mid s_1) = qp$ by (3) and (4), the definition (16) of $\widehat{N}_j$, and the expressions of the posterior probabilities $\widehat{p}_0$ and $\widehat{p}_1$ obtained by replacing p by $\widehat{p}$ in (3) and (4), the bank's payoff (19) simplifies to

$$\begin{aligned}
\pi(D, M, p, \widehat{p}) &= D + (1-q)p\left[R(p) - \frac{M}{\widehat{p}_0}\right] + qp\left[R(p) - \frac{M}{\widehat{p}_1}\right] \\
&= D + (1-q)p\left[R(p) - \frac{\widehat{\Pr}(s_0)}{(1-q)\widehat{p}}M\right] + qp\left[R(p) - \frac{\widehat{\Pr}(s_1)}{q\widehat{p}}M\right] \\
&= D + p\left[R(p) - \frac{M}{\widehat{p}}\right].
\end{aligned} \tag{25}$$

Hence, the first-order condition that characterizes the bank's choice of p is

$$(pR(p))' = \frac{M}{\widehat{p}}. \tag{26}$$

Substituting the participation constraint $M = 1 + D$ and the rational expectations constraint $\widehat{p} = p$ into this condition, and using the definition (12) of $H(p)$ gives

$$H(p) = 1 + D. \tag{27}$$

For $D = 0$ this is identical to the condition that characterizes the optimal contract with long-term debt. And for the same incentive reasons as before, there should be no up-front dividend. Therefore, the candidate optimal contract with safe short-term debt is such that $D_S = 0$, $M_S = 1$, and $p_S = p_L$, where p_L is the probability of success in the optimal contract with long-term debt.

However, for this to be an optimal contract with safe short-term debt it must be the case that the initial debt is rolled over in the bad state s_0,[13] which requires

$$E(R \mid s_0) = p_0 R(p_L) \geq 1, \tag{28}$$

[13] Since $E(R \mid s_1) = p_1 R(p) > p_0 R(p) = E(R \mid s_0)$, if the initial debt is rolled over in the bad state s_0 it will also be rolled over in the good state s_1.

where p_0 is given by (3). For $q = 1/2$ the condition reduces to $p_L R(p_L) \geq 1$, which holds if long-term financing is feasible. For $q = 1$ the condition is never satisfied, because $p_0 = 0$. By definition (3), p_0 is decreasing in the quality of the lenders' information q, so there must be an intermediate value of q for which the constraint is satisfied with equality. Hence, we have the following result.

Proposition 2 *Financing the bank with safe short-term debt is feasible if financing the bank with long term debt is feasible and $q \leq q(p_L)$, where p_L is the probability of success in the optimal contract with long-term debt and*

$$q(p) = \frac{p(R(p) - 1)}{1 + p(R(p) - 2)}, \tag{29}$$

in which case $(D_S, M_S, p_S) = (0, 1, p_L)$ is the optimal contract with safe short-term debt.

We conclude that using safe short-term debt does not add anything relative to using long-term debt.[14] Thus, the only possible role of short-term debt is when it is risky.

4.2 Risky short-term debt

When the short-term debt is risky, the initial lenders are fully repaid in the good state s_1, and the bank is liquidated in the bad state s_0.

Using $\widehat{\Pr}(s_0)\lambda \widehat{E}(R \mid s_0) = (1-q)\lambda \widehat{p} R(\widehat{p})$ by (3) and $\widehat{\Pr}(s_1) = 1 - \widehat{p} - q + 2q\widehat{p}$ by (6), the initial lenders' participation constraint (17) becomes

$$\nu(M, \widehat{p}) = (1-q)\lambda \widehat{p} R(\widehat{p}) + (1 - \widehat{p} - q + 2q\widehat{p})M = 1 + D. \tag{30}$$

And using $\Pr(s_1)\Pr(R > 0 \mid s_1) = qp$ by (4), the definition (16) of $\widehat{N}_1$, and the expression of $\widehat{p}_1$ obtained by replacing p by $\widehat{p}$ in (4), the bank's payoff (19) simplifies to

$$\pi(D, M, p, \widehat{p}) = D + qp\left[R(p) - \widehat{N}_1\right] = D + qp\left[R(p) - \frac{1 - \widehat{p} - q + 2q\widehat{p}}{q\widehat{p}}M\right]. \tag{31}$$

Hence, the first-order condition that characterizes the bank's choice of p is

$$(pR(p))' = \widehat{N}_1 = \frac{1 - \widehat{p} - q + 2q\widehat{p}}{q\widehat{p}}M. \tag{32}$$

[14] It should be noted that the short-term debt issued after the rollover of the initial debt is not safe. In both states, the bank pays a premium over the riskless rate to cover the default risk, which is higher in the bad state s_0.

Solving for M in the participation constraint (30), substituting it into this condition, and using the rational expectations constraint $\widehat{p} = p$ and the definition (12) of $H(p)$ gives

$$H(p) = \frac{1}{q}[1 + D - (1-q)\lambda p R(p)]. \tag{33}$$

The remaining question is how do we set the optimal up-front dividend $D \geq 0$. To answer this question we have to introduce the constraint that the initial debt is not rolled over in the bad state s_0, which requires

$$E(R \mid s_0) = p_0 R(p) \leq M, \tag{34}$$

where p_0 is given by (3).[15] Solving for M in the participation constraint (30), substituting it into (34), and using the expression (3) of p_0 and the rational expectations constraint $\widehat{p} = p$, allows us to write this condition in terms of a lower bound for the up-front dividend

$$D \geq \max\left\{ \left[\frac{1}{p+q-2qp} - (1-\lambda)\right](1-q)pR(p) - 1, 0 \right\}. \tag{35}$$

Equation (33) characterizes the values of p and D that satisfy the bank's incentive compatibility constraint and the initial lenders' participation constraint. Condition (35) characterizes the values of p and D for which the initial debt is not rolled over in the bad state s_0. Financing the bank with risky short-term debt will be feasible if equation (33) has a solution for some D that satisfies condition (35). Moreover, this condition may be written with equality, because higher values of D worsen the moral hazard problem, leading to a lower payoff to the bank.

To characterize the optimal contract, note that solving for M in the participation constraint (30), substituting it into the bank's objective function (31), and using the rational expectations constraint $\widehat{p} = p$ implies that the bank's payoff becomes

$$\pi(D, M, p, p) = [q + (1-q)\lambda]pR(p) - 1. \tag{36}$$

Since the function $pR(p)$ is increasing in the interval $(0, p^*)$, the optimal contract with risky

[15] This condition may be written with a weak inequality, because when $E(R \mid s_0) = M$ the bank's stake under continuation is the same as under liquidation (that is, zero).

short-term debt is the feasible contract with the highest probability of success p_S.[16] The up-front dividend D_S is obtained from equation (33) and the face value of the initial debt M_S follows from the participation constraint (30). Hence, we have the following result.

Proposition 3 *Financing the bank with risky short-term debt is feasible if equation* (33) *has a solution for some D that satisfies condition* (35), *in which case the optimal contract is given by*

$$\begin{aligned} D_S &= qH(p_S) + (1-q)\lambda p_S R(p_S) - 1, &(37)\\ M_S &= \frac{1 + D_S - (1-q)\lambda p_S R(p_S)}{1 - p_S - q + 2qp_S}, \textit{ and} &(38)\\ p_S &= \max\left\{p \in (0, p^*) \mid \exists D \text{ that satisfies (33) and (35)}\right\}. &(39) \end{aligned}$$

As in the case of the optimal contract with long-term debt, if there are multiple p's that satisfy the conditions in (39), the optimal contract can be implemented by the bank offering the initial lenders the rate M_S in exchange for funding $1 + D_S$ at $t = 0$.

It is interesting to examine the relationship between the quality of the lenders' information q and the probability of success p_S chosen by the bank in the optimal contract. First, note that when q tends to 1, the first term in the max operator of (35) will be negative, which implies $D_S = 0$. But then, by (33), when q tends to 1, p_S will approach the highest solution to the equation $H(p) = 1$, which by Proposition 1 implies $p_S = p_L < 1$.[17] For lower values of q, the conditional probabilities of success p_0 and p_1 in (3) and (4) get closer to the unconditional probability of success p. Thus, we conjecture that below some critical value $\widehat{q}$, ensuring the liquidation of the bank in the bad state requires worsening the moral hazard problem by paying a positive up-front dividend ($D_S > 0$). This is shown to be the case in the example below.

[16] It should be noted that any $\overline{p}$ that solves (33) for a D that satisfies (35) is such that $\overline{p} < p^*$. To see this, suppose to the contrary that $\overline{p} \geq p^*$. Since the function $H(p)$ satisfies $H(p) \leq 0$ for $p \geq p^*$, it must be the case that $1 + D - (1-q)\lambda\overline{p}R(\overline{p}) \leq 0$, which violates the participation constraint (30).

[17] The result that for $q = 1$ the optimal contract with risky short-term debt is equivalent to the optimal contract with long-term debt follows from the assumption that the failure return of the bank's investment is zero, so advancing the liquidation to $t = 1$ does not have any cost. The same result would obtain for positive failure returns if the recovery rate were $\lambda = 1$.

An example (continued) To compute the optimal contract with risky short-term debt for the payoff function $R(p) = a(2-p)$, let $D_1(p)$ denote the solution for D in equation (33) after substituting the payoff function, which gives the quadratic equation

$$D_1(p) = -a[2q + \lambda(1-q)]p^2 + 2a[q + \lambda(1-q)]p - 1. \tag{40}$$

Condition (35), which ensures that the initial debt is not rolled over in the bad state s_0, written as an equality, gives the function

$$D_2(p) = \max\left\{\left[\frac{1}{p+q-2qp} - (1-\lambda)\right](1-q)ap(2-p) - 1, 0\right\}. \tag{41}$$

By Proposition 3, the optimal contract is found at the intersection with the highest p of these two functions.

This is illustrated in Figure 1 for two values of the quality of the lenders' information, namely $q = 0.9$ and $q = 0.7$, a profitability parameter $a = 2.4$, and a recovery rate $\lambda = 0.9$.

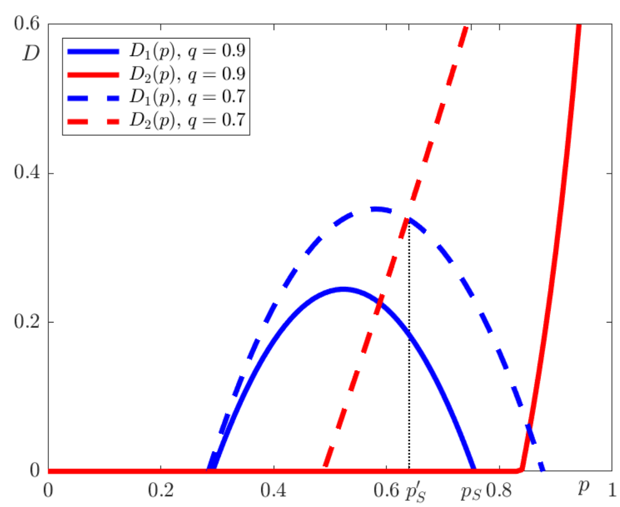

Figure 1. The optimal contract with risky short-term debt

This figure shows the determination of the probability of success in the optimal contract with risky short-term debt for a high value (solid lines) and a low value (dashed lines) of the quality of the lenders' information. In the first case the upfront dividend is zero while in the second it is positive.

For $q = 0.9$ the solid blue line represents the function $D_1(p)$ and the solid red line represents the function $D_2(p)$. For $q = 0.7$ the dashed blue line represents the function $D_1(p)$ and the dashed red line represents the function $D_2(p)$. The probability of success chosen by the bank in the optimal contract is $p_S = 0.76$ when $q = 0.9$, and $p'_S = 0.64$ when $q = 0.7$. For $q = 0.9$ we have $D_S = 0$, i.e. no up-front dividend, while for $q = 0.7$ we have $D'_S = 0.34$, i.e. a positive up-front dividend. As noted above, the intuition for this result is that when the quality of the lenders' information is not sufficiently high, ensuring the liquidation of the bank in the bad state requires worsening the moral hazard problem by paying a positive up-front dividend.

Figure 2 illustrates the relationship between the quality of the lenders' information q (in the horizontal axis) and the probability of success p_S (in the vertical axis) chosen by the bank in the optimal contract with risky short-term debt for a profitability parameter $a = 2.4$ and a recovery rate $\lambda = 0.9$. In the region where $D_S = 0$, that is for $q \in [\widehat{q}, 1]$, the relationship is decreasing, so noisier information improves incentives. In the region where $D_S > 0$, that is

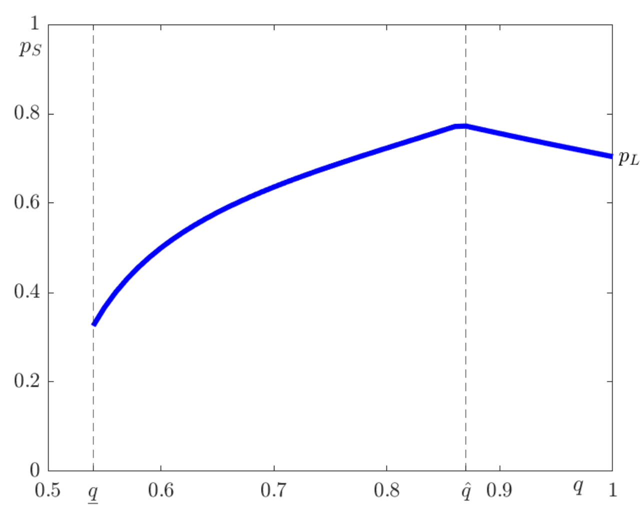

Figure 2. The probability of success with risky short-term debt

This figure shows the relationship between the quality of the lenders' information (in the horizontal axis) and the probability of success (in the vertical axis) chosen by the bank in the optimal contract with risky short-term debt.

for $q \in [\underline{q}, \widehat{q}]$, the relationship is increasing. Below the lower bound $\underline{q}$ the functions $D_1(p)$ and $D_2(p)$ do not intersect, which means that financing the bank with risky short-term debt is not feasible. As noted above, for $q = 1$ the optimal contract with risky short-term debt is equivalent to the optimal contract with long-term debt, so $p_S = p_L$.

The following proposition shows that the results in Figure 2 are general.

Proposition 4 *In the region where the optimal contract is characterized by a zero up-front dividend ($D_S = 0$), for sufficiently high values of the recovery rate λ, the probability of success p_S is decreasing in the quality q of the lenders' information. In the region where the optimal contract is characterized by a positive up-front dividend ($D_S > 0$), the probability of success p_S is always increasing in the quality q of the lenders' information.*

Explaining the intuition for the result that some noise in the lenders' information may improve incentives is key to understand the potential benefits of risky short-term debt. In principle, the result seems surprising: *Why would noise improve incentives?*

To explain it, recall that the first-order condition (32) that characterizes the bank's choice of p is $(pR(p))' = N_1$, where N_1 is the face value of the debt maturing at $t = 2$ that the lenders receive in exchange for M funds provided at $t = 1$ in the good state s_1. Increases in N_1 worsen the bank's moral hazard problem, and consequently reduce the probability of success p chosen by the bank (since $(pR(p))'' < 0$). Therefore, the only way in which adding noise (reducing q) could increase p is by reducing the amount N_1 due to the lenders.

To see why this is the case in the region where $D_S = 0$, use (32) to write the lenders' participation constraint (30) as

$$(1 - q)\lambda pR(p) + qpN_1 = 1. \tag{42}$$

The lenders' payoff has two components: They get N_1 when the initial debt is rolled over and the investment succeeds, that is with probability $\Pr(s_1)\Pr(R > 0 \mid s_1) = qp$, which gives the term qpN_1, and they get the liquidation value of the investment $\lambda E(R \mid s_0)$ when the initial debt is not rolled over, that is with probability $\Pr(s_0)$, which gives the term $\Pr(s_0)\Pr(R > 0 \mid s_0)\lambda R(p) = (1 - q)\lambda pR(p)$. Reductions in q increase this component of the lenders' payoff. If this effect is sufficiently strong, which happens when λ is large, there will be a reduction in N_1, which explains the result $\partial p_S/\partial q < 0$.

In other words, the noise in the lenders' information leads to a type I error (liquidating the bank when the investment would succeed) that increases their payoff in the bad state s_0 and reduces what they require in the good state s_1, leading the bank to choose a higher probability of success.

Summing up, in this section we have characterized the optimal contract with short-term debt, showing that safe short-term debt does not add anything relative to long-term debt, but that risky short-term may ameliorate the bank's risk-shifting incentives. The next section compares the feasibility and optimality of long-term and risky short-term debt.

5 Optimal Bank Financing

This section shows that there are situations in which (i) long-term debt is the only feasible financing instrument, (ii) risky short-term debt is the only feasible financing instrument, (iii) both long-term and risky short-term debt are feasible and the former dominates, and (iv) both long-term and risky short-term debt are feasible and the latter dominates. It then relates the results to features of the common narrative of the 2007-2009 global financial crisis.

For the sake of clarity and brevity, the discussion on optimal bank financing will be conducted in terms of our simple parametric example, based on the linear payoff function $R(p) = a(2-p)$. In particular, we will consider combinations of the profitability of the bank's investment a and the quality of the lenders' information q for which (i)-(iv) obtain. Also, to facilitate stating the results, risky short-term debt will henceforth be referred to as short-term debt.

Let π_L denote the bank's payoff in the optimal contract with long-term debt (whenever it is feasible). The incentive compatibility constraint (9) implies $\pi_L \geq 0$ (since $p = 0$ could have been chosen). Moreover, since $(pR(p))'' < 0$, by (11) we have

$$\frac{d}{dp}\left[p(R(p) - B_L)\right] = (pR(p))' - B_L > (p_L R(p_L))' - B_L = 0,$$

for $p \in [0, p_L)$, which implies

$$\pi_L = p_L R(p_L) - 1 > 0. \tag{43}$$

Thus, the bank gets a strictly positive payoff in the optimal contract with long-term debt.

Let π_S denote the bank's payoff in the optimal contract with short-term debt (whenever it is feasible). The incentive compatibility constraint (21) implies $\pi_S \geq 0$ (since $p = 0$ could have been chosen). Moreover, since $(pR(p))'' < 0$, by (32) we have

$$\begin{aligned} \frac{d}{dp}\left\{p\left[R(p) - \frac{1 - p_S - q + 2qp_S}{qp_S} M_S\right]\right\} &= (pR(p))' - \frac{1 - p_S - q + 2qp_S}{qp_S} M_S \\ &> (p_S R(p_S))' - \frac{1 - p_S - q + 2qp_S}{qp_S} M_S = 0, \end{aligned}$$

for $p \in [0, p_S)$, which by (36) implies

$$\pi_S = [q + \lambda(1 - q)] p_S R(p_S) - 1 > 0. \tag{44}$$

Thus, the bank also gets a strictly positive payoff in the optimal contract with short-term debt.

For long-term debt, the quality of the lenders' information q is irrelevant, and as shown above feasibility only requires that the profitability of the bank's investments satisfies $a \geq 2$. For short-term debt, feasibility requires that the functions $D_1(p)$ and $D_2(p)$ defined in (40) and (41) have an intersection, so feasibility depends on the combination of the profitability of the bank's investment a and the quality of the lenders' information q. Moreover, by our results in Section 4, for lower values of q the optimal contract with short-term debt entails a positive up-front dividend $D_S > 0$.

Figure 3 illustrates the results for a recovery rate $\lambda = 0.9$, with the profitability of the bank's investments a in the horizontal axis and the quality of the lenders' information q in the vertical axis. There are three regions, denoted I (yellow), II (green), and III (blue). Each of these regions has two subregions, denoted a and b. In region Ia both long-term and short-term debt (with a zero up-front dividend) are feasible, and short-term debt dominates long-term debt. In region Ib, where $a < 2$, only short-term debt (with a zero up-front dividend) is feasible. In region IIa both long-term and short-term debt (with a positive up-front dividend) are feasible, and short-term debt dominates long-term debt. In region IIb, where $a < 2$, only short-term debt (with a positive up-front dividend) is feasible. In region IIIa both long-term and short-term debt (with a positive up-front dividend) are feasible, and long-term debt dominates short-term debt. Finally, in region IIIb only long-term debt is feasible.

We can summarize these results as follows. Long-term debt is the optimal financing instrument when the profitability of the bank's investment a is high and the quality of the lenders' information q is low. Conversely, short-term debt is the optimal financing instrument when the profitability of the bank's investment a is low and the quality of the lenders' information q is high.

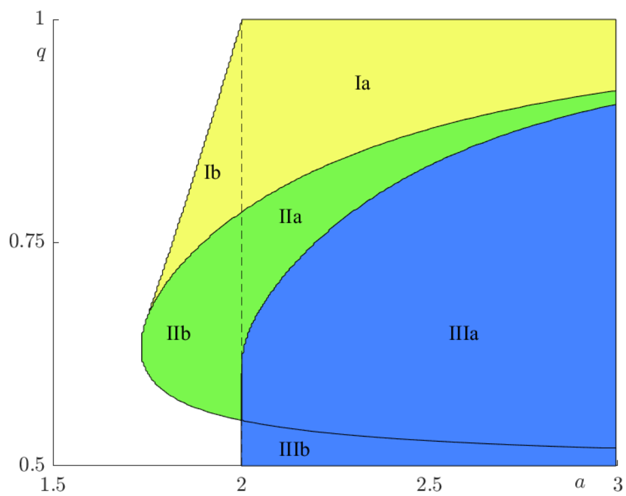

Figure 3. Optimal bank financing

This figure shows the optimal contract for different combinations of the profitability of the bank's investments (in the horizontal axis) and the quality of the lenders' information (in the vertical axis). Short-term debt is optimal in region I (with a zero upfront dividend) and in region II (with a positive upfront dividend). Long-term debt is optimal in region III. In regions Ib and IIb long-term debt is not feasible, while in region IIIb short-term debt is not feasible.

It should be noted that when short-term debt dominates long-term debt, by (43) and (44) we have

$$\pi_S = [q + \lambda(1-q)]p_S R(p_S) - 1 > p_L R(p_L) - 1 = \pi_L,$$

which implies $p_S R(p_S) > p_L R(p_L)$. But since $(pR(p))'$ is positive for $p < p^*$, it follows that $p_S > p_L$. In other words, when short-term is the optimal financing instrument, the bank will choose safer investments. In this sense we may conclude that *risky short-term debt has a disciplining effect on the bank's risk-shifting incentives.*

These results provide a rationale for the shortening of maturities in the run-up to the global financial crisis of 2007-2009. Consider the effects of following simultaneous changes in the banks' environment. First, a reduction in the profitability of investments, leading to widespread search for yield (Rajan, 2005). Second, an increase in opacity, partly driven by securitization, which led to "an opaque web of interconnected obligations" (Brunnermeier, 2009). In terms of our model, these changes correspond to a reduction in the profitability parameter a, and a reduction in the quality of the lenders' information q. Starting with an initial situation in which banks are funded with long-term debt, choosing a point in Region III, such changes may lead to banks moving to a point in Region II, where it is optimal to use short-term debt, possibly with a positive up-front dividend.

Thus, we conclude that our results are consistent with banks shifting to "dangerous forms of debt" (to use the terminology of Tirole, 2003), as well as making high payments to bank shareholders, the empirical counterpart of the positive up-front dividend,[18] as an optimal response to changes in the economic and financial environment. Conversely, as noted by Flannery (1994), "if short-funding bank assets provides important incentive benefits, regulations that limit a bank's ability to employ this funding device may reduce social welfare."

Summing up, we have shown that the positive incentive effects associated with the use of short-term are such that in some circumstances it may either dominate long-term debt or even become the only feasible form of finance. We have also pointed out some parallels between the model predictions and features of the narrative of the period prior to the global financial crisis.

[18] The interpretation could also apply to the high bonuses to bank executives, since it can be argued that such payments are (at least in part) earnings distributions.

6 Extensions

This section discusses three relevant variations of our model dealing with (i) the possible role of combining short- and long-term debt, (ii) the effect of introducing liquidity regulations that limit banks' maturity transformation, and (iii) the role of monetary policy in the choice between short- and long-term debt.

6.1 Mixed debt finance

We have assumed so far that the bank may be funded with either short- or long-term debt. We now consider a situation in which the bank raises a fraction γ of its funding at $t = 0$ by issuing short-term debt, and the remaining $1 - \gamma$ by issuing long-term debt. As before, there are two possible cases, namely the case where the short-term debt is rolled over in both states and the case where the short-term debt is only rolled over in the good state and the bank is liquidated in the bad state. If the bank is liquidated at $t = 1$ or fails at $t = 2$, we will assume that short-term debt is senior to long-term debt.

Let γM denote the face value of the debt maturing at $t = 1$ that the lenders receive in exchange for $\gamma(1 + D)$ funds provided at $t = 0$, and let $(1 - \gamma)B$ denote the face value of the debt maturing at $t = 2$ that the lenders receive in exchange for $(1 - \gamma)(1 + D)$ funds provided at $t = 0$.

When the short-term debt is safe, the initial short-term lenders participation constraint reduces to $M = 1 + D$, and the face value of the debt issued in state s_j is $\widehat{N}_j = \gamma M/\widehat{p}_j$. Using the same derivation as in Section 4.1, the bank's payoff becomes

$$\pi(D, M, B, p, \widehat{p}, \gamma) = D + p\left[R(p) - \frac{\gamma M}{\widehat{p}} - (1 - \gamma)B\right].$$

Hence, the first-order condition that characterizes the bank's choice of p is

$$(pR(p))' = \frac{\gamma M}{\widehat{p}} - (1 - \gamma)B.$$

Substituting into this condition the participation constraint of the short-term lenders $M = 1 + D$, the participation constraint of the long-term lenders $\widehat{p}B = 1 + D$, and the rational expectations constraint $\widehat{p} = p$, and using the definition (12) of $H(p)$ gives $H(p) = 1 + D$. Since, as before, it is optimal to set $D = 0$, we get the same condition (12) that characterizes the optimal contract with long-term debt. Thus, in this case mixed debt does not add anything relative to using long-term debt.

When the short-term debt is risky, the initial short-term lenders' participation constraint becomes

$$\widehat{\Pr}(s_0)\lambda\widehat{E}(R \mid s_0) + \widehat{\Pr}(s_1)\gamma M = \gamma(1 + D),$$

and the long-term lenders' participation constraint becomes

$$\widehat{\Pr}(s_1)\widehat{\Pr}(R > 0 \mid s_1)(1 - \gamma)B = (1 - \gamma)(1 + D),$$

where we have used the assumption that short-term debt is senior to long-term debt. Adding up the two constraints, and using $\widehat{\Pr}(s_0)\lambda\widehat{E}(R \mid s_0) = (1 - q)\lambda\widehat{p}R(\widehat{p})$ by (3) and $\widehat{\Pr}(s_1)\widehat{\Pr}(R > 0 \mid s_1) = q\widehat{p}$ by (4), gives

$$(1 - q)\lambda\widehat{p}R(\widehat{p}) + q\widehat{p}[\widehat{N}_1 + (1 - \gamma)B] = 1 + D,$$

where $\widehat{N}_1 = \gamma M/\widehat{p}_1$. The bank's payoff may be written as

$$\pi(D, M, B, p, \widehat{p}, \gamma) = D + qp\left[R(p) - \widehat{N}_1 + (1 - \gamma)B\right].$$

Hence, the first-order condition that characterizes the bank's choice of p is

$$(pR(p))' = \widehat{N}_1 + (1 - \gamma)B.$$

Substituting the joint participation constraint derived above and the rational expectations constraint $\widehat{p} = p$ into this condition, and using the definition (12) of $H(p)$ gives the same condition (33) that characterizes the optimal contract with risky short-term debt.

As before, to determine the optimal up-front dividend D we have to introduce the constraint that the initial short-term debt is not rolled over in the bad state s_0, which requires $E(R \mid s_0) \leq \gamma M$. Clearly, this constraint becomes tighter as the proportion γ of short-term funding goes down, so financing the bank with a mixture of long-term and risky short-term debt may not be feasible.

We conclude that using a combination of short- and long-term debt is at best equivalent to using only long-term debt or only risky short-term debt, so it does not add anything relative to the cases analyzed above.

6.2 Liquidity regulation

One of the elements of the regulation agreed by the Basel Committee on Banking Supervision in 2010, known as Basel III, is a pair of liquidity standards, called the *Liquidity Coverage Ratio* and the *Net Stable Funding Ratio*. In particular, the former requires banks to have "an adequate stock of unencumbered high-quality liquid assets that can be converted into cash easily and immediately in private markets to meet its liquidity needs for a 30 calendar day liquidity stress scenario."

We next consider the effect of a regulation that requires banks to match the amount of short-term borrowing with liquid assets. To do this we need to introduce a liquid asset, which we assume yields a zero return—the same as the expected return required by investors.

Let γM denote the face value of the debt maturing at $t = 1$ that the lenders receive in exchange for $\gamma(1+C+D)$ funds provided at $t = 0$, and let $(1-\gamma)B$ denote the face value of the debt maturing at $t = 2$ that the lenders receive in exchange for $(1-\gamma)(1+C+D)$ funds provided at $t = 0$, where C denotes the bank's investment in the liquid asset. The liquidity requirement may then be written as $C \geq \gamma M$. Assuming that short-term debt is senior to long-term debt, this implies that the short-term debt will always be safe. The bank's payoff will then be

$$\pi(D, C, M, B, p, \widehat{p}, \gamma) = D + p\left[R(p) + C - \frac{\gamma M}{\widehat{p}} - (1-\gamma)B\right].$$

Hence, the first-order condition that characterizes the bank's choice of p is

$$(pR(p))' = \frac{\gamma M}{\widehat{p}} + (1-\gamma)B - C.$$

Substituting into this condition the participation constraint of the short-term lenders $M = 1 + D + C$, the participation constraint of the long-term lenders $\widehat{p}B = 1 + D + C$, and the rational expectations constraint $\widehat{p} = p$, and using the definition (12) of $H(p)$ gives

$H(p) = 1 + D$. Since, as before, it is optimal to set $D = 0$, we get the same condition (12) that characterizes the optimal contract with long-term debt. We conclude that imposing a liquidity requirement effectively eliminates the possibility of using risky short-term debt. By our previous results, this may imply riskier bank investments.

As an alternative to quantity-based liquidity regulation, it has been suggested (by Perotti and Suarez, 2011, among others) the possibility of using levies on uninsured short-term liabilities, which would operate like Pigouvian taxes. To discuss the effects of such regulation, suppose that we introduce a proportional levy τ on using short-term debt, payable ex-ante (so that to fund a unit investment with short-term debt you have to raise $1 + \tau$).[19] In this setup, safe short-term debt will be clearly dominated by long-term debt, which in the absence of the levy is payoff-equivalent to safe short-term debt. In the case of risky short-term debt the initial lenders' participation constraint (30) becomes

$$(1 - q)\lambda\widehat{p}R(\widehat{p}) + (1 - \widehat{p} - q + 2q\widehat{p})M = 1 + \tau + D.$$

Following the same steps as in Section 4.2, the condition that characterizes the bank's choice of p is now

$$H(p) = \frac{1 + \tau + D - (1 - q)\lambda pR(p)}{q}.$$

Since for any $\tau > 0$ the right-hand side of this expression is greater than the right-hand side of (33), we conclude that the levy will lower the bank's choice of p. Hence, either the levy will have no effect (when long-term debt is optimal), or it will shift the optimal choice of financing from risky short-term debt to long-term debt, or it will not change the bank's choice of risky short-term debt. However, in the last two cases the bank will choose riskier investments.

It should be noted that the preceding results on the negative effects of liquidity regulation are derived for a model that does not incorporate features that have been considered in the literature as possible rationales of such regulation such as the negative effects of asset fire sales at the interim period (as in Eisenbach, 2017) or the role of liquidity buffers as supervisory

[19] As noted by Stein (2012), such levy could be implemented by means of a reserve requirement remunerated below market rates.

tools for buying time to discover the solvency of the bank (as in Santos and Suarez, 2019).

6.3 Monetary policy

We have up to now normalized to zero the expected return required by the lenders. We next consider what happens when this return is a variable r, which may be interpreted as the one-period policy rate set by the central bank.

To characterize the optimal contract with long-term debt it suffices to note that the lenders' participation constraint now becomes $pB = (1+r)^2$. Substituting this expression into the bank's first-order condition (11), and using the definition (12) of $H(p)$ gives

$$H(p) = (1+r)^2 .$$

Hence, an increase in r will lower the bank's choice of p. Moreover, in the case where $R(p) = a(2-p)$ the optimal contract is the same as in the model with $r' = 0$ and $a' = a/(1+r)^2$.

To characterize the optimal contract with risky short-term debt we first note that the initial lenders' participation constraint (30) becomes

$$\frac{(1-q)\lambda\widehat{p}R(\widehat{p})}{1+r} + (1-\widehat{p}-q+2q\widehat{p})M = (1+r)(1+D). \tag{45}$$

The face value of the debt issued in state s_1 is now

$$\widehat{N}_1 = (1+r)\frac{(1-\widehat{p}-q+2q\widehat{p})M}{q\widehat{p}}. \tag{46}$$

Following the same steps as in Section 4.2, one can show that the optimal contract with risky short-term debt is characterized by highest value of p that satisfies

$$\frac{qH(p)+(1-q)\lambda pR(p)}{(1+r)^2} - 1 = \max\left\{\left[\frac{1}{p+q-2qp} - (1-\lambda)\right]\frac{(1-q)pR(p)}{(1+r)^2} - 1, 0\right\}.$$

As before, in the case where $R(p) = a(2-p)$ the optimal contract is the same as in a model with $r' = 0$ and $a' = a/(1+r)^2$.

We then conclude that a tightening (loosening) of monetary policy will always increase (decrease) risk-taking.[20] Thus, in contrast with most discussions on search for yield, which

[20] The positive relationship between banks' funding costs and their portfolio risk should not be surprising, since as noted above it is a straightforward implication of the analysis in Stiglitz and Weiss (1981).

focus on the level of real interest rates, our results indicate that what really matters is the relationship between banks' funding costs and the profitability of their investments.[21] In our parametric example, the choice of p in the optimal contract with both long-term and risky short-term debt depends on the ratio $a/(1+r)^2$, so changes in the profitability parameter a have the opposite effect as changes in the interest rate r. This suggests that *the search for yield phenomenon should be linked to the fall in the spreads between the return of banks' assets and the cost of their liabilities.*

Finally, and in connection with our earlier discussion of the events prior to the global financial crisis, our results also suggest that the increase in policy rates by the Federal Reserve and the European Central Bank in run-up to the crisis might have contributed to a reduction in intermediation spreads and consequently in the maturity of banks' debt.

7 Concluding Remarks

This paper presents a model of the maturity of a bank's uninsured debt. We consider a bank that borrows funds to invest in assets whose riskiness is privately chosen after the funding terms are arranged. This moral hazard problem causes excessive risk-taking. Short-term debt may act as a disciplinary device when the lenders observe some interim signal of the assets' risk, but only when it is not rolled over with positive probability, leading to the bank's early liquidation. We characterize the conditions under which short- and long-term debt are feasible, and show circumstances under which only short-term debt is feasible and under which short-term debt dominates long-term debt when both are feasible. These latter cases obtain when the profitability of the bank's investments is sufficiently low and when the quality of the lenders' information is sufficiently high. We also show that to ensure the credibility of the liquidation threat, the optimal contract with short-term debt may involve raising more than the cost of the investment and paying the difference as an up-front dividend.

[21] One notable exception is Shin (2010): "The phenomenon of search for yield often appears in the late stages of a boom as investors migrate down the asset quality curve as *risk spreads* (our italics) are compressed."

These results provide a rationale for the reported widespread use of wholesale short-term debt before the 2007-2009 global financial crisis. In particular, our model shows that a reduction in the profitability of banks' investments together with an increase in the opacity of banks' balance sheets are consistent with a change in the optimal funding strategy from long- to short-term debt, possibly with an increase in payouts.

It should be noted that the model focuses on the optimal behavior of a single bank, ignoring general equilibrium effects, which could be relevant for policy analysis. For example, as examined by Stein (2012), short-term debt together with correlated shocks may generate fire-sale externalities at the rollover date that may justify liquidity regulation or other public policies. In the context of our model, this means that the liquidation costs would depend on the aggregate maturity structure of banks' debt: more short-term debt would lead to more liquidations and hence lower liquidation values. This is analyzed by Eisenbach (2017) in a model of the choice of debt maturity by a continuum of banks that invest in long-term projects. Using rollover risk as a disciplining device is effective when banks face purely idiosyncratic risks, but with correlated risks it leads to excessive risk-taking in good times and excessive fire sales in bad times.

It should also be noted that the paper entirely focuses on debt finance, abstracting from the possibility of funding the bank with equity. Since equity finance would ameliorate the banks' risk-shifting incentives, adding this possibility would require the introduction of a differential cost of equity–otherwise we would end up with 100% equity. This might require distinguishing between inside and outside shareholders, with inside shareholders being those responsible for the risk-taking decisions, as well as modeling possible conflicts of interest between managers and outside shareholders. Such model could be used to analyze the effect of bank capital regulaiton on debt maturity, a topic that is left for future research.

Appendix

Proof of Proposition 2 The expression for $q(p)$ in (29) is obtained by substituting (3) into (28) and solving for q. To complete the proof of the result, it remains to show that if $q > q(p_L)$, no other solution to the equation $H(p) = 1$, say $\widetilde{p} < p_L$, will satisfy $q \leq q(\widetilde{p})$. Now for $p \in [\widetilde{p}, p_L]$ we have

$$q'(p) = \frac{(1-p)\left(pR(p)\right)' + pR(p) - 1}{\left[1 + p(R(p) - 2)\right]^2} > 0,$$

since the first term in the numerator is positive because $(pR(p))' > 0$ for $p < p^*$, and the second term is greater than 1 because

$$pR(p) \geq \widetilde{p}R(\widetilde{p}) = \int_0^{\widetilde{p}} \left(pR(p)\right)' dp = \int_0^{\widetilde{p}} \frac{1}{\widetilde{p}} dp = 1$$

for $p \in [\widetilde{p}, p_L]$. Hence, we conclude that $q(\widetilde{p}) < q(p_L) < q$, as required. □

Proof of Proposition 4 When $D_S = 0$, p_S is found by solving the equation $D_1(p) = qH(p) + (1-q)\lambda pR(p) - 1 = 0$, which gives

$$\frac{dp_S}{dq} = -\frac{\partial D_1(p_S)/\partial q}{D_1'(p_S)}.$$

Choosing the highest solution to the equation $D_1(p) = 0$ implies $D_1'(p_S) < 0$, so to prove that $dp_S/dq < 0$ it suffices to check whether

$$\frac{\partial D_1(p_S)}{\partial q} = H(p_S) - \lambda p_S R(p_S) = \frac{1}{q}[1 - \lambda p_S R(p_S)] < 0,$$

where we have used the definition of $D_1(p_S)$. Hence, since $p_S R(p_S) > 1$, for sufficiently high values of the recovery rate λ we conclude that $dp_S/dq < 0$.[22]

When $D_S > 0$, p_S is found by solving the equation

$$D_1(p) = qH(p) + (1-q)\lambda pR(p) - 1 = \left[\frac{1}{p + q - 2qp} - (1-\lambda)\right](1-q)pR(p) - 1 = D_2(p),$$

which simplifies to

$$\frac{(2q-1)(p-1)}{qp(p+q-2qp)} - \frac{R'(p)}{R(p)} = 0.$$

[22] In our numerical example, the lower bound for the recovery rate is $\underline{\lambda} = [p_L R(p_L)]^{-1} = 0.46$ for $a = 2.4$, where $p_L = 0.7$ obtains from (15).

Differentiating this condition gives

$$\frac{dp_S}{dq} = \frac{\dfrac{1-p_S}{p_S}\dfrac{p_S(2q-1)^2+2q(1-q)}{[q(p_S+q-2qp_S)]^2}}{\dfrac{2q-1}{q}\dfrac{(2q-1)(1-p_S)^2+1-q}{[p_S(p_S+q-2qp_S)]^2} - \dfrac{R(p_S)R''(p_S)-(R'(p))^2}{(R(p))^2}} > 0,$$

where we have used $R(p) > 0$, $R'(p) < 0$, and $R''(p) \leq 0$ and $q \geq 1/2$. $\square$

References

Admati, A., and M. Hellwig (2013), "Does Debt Discipline Bankers? An Academic Myth about Bank Indebtedness," Rock Center for Corporate Governance at Stanford University Working Paper No. 132.

Allen, F., and D. Gale (2000), *Comparing Financial Systems*, Cambridge, MA: MIT Press.

Basel Committee on Banking Supervision (2010), "Basel III: International Framework for Liquidity Risk Measurement, Standards and Monitoring," Bank for International Settlements, Basel.

Bernanke, B. (2010), "Causes of the Recent Financial and Economic Crisis," Testimony Before the Financial Crisis Inquiry Commission.

Bolton, P., and X. Freixas (2000), "Equity, Bonds, and Bank Debt: Capital Structure and Financial Market Equilibrium under Asymmetric Information," *Journal of Political Economy*, 108, 324-351.

Brunnermeier, M. (2009), "Deciphering the Liquidity and Credit Crunch 2007-2009," *Journal of Economic Perspectives*, 23, 77-100.

Brunnermeier, M., and M. Oehmke (2013), "The Maturity Rat Race," *Journal of Finance*, 68, 483-521.

Calomiris, C., and C. Kahn (1991), "The Role of Demandable Debt in Structuring Optimal Banking Arrangements," *American Economic Review*, 81, 497-513.

Cheng, I.-H, and K. Milbradt (2012), "The Hazards of Debt: Rollover Freezes, Incentives, and Bailouts," *Review of Financial Studies*, 25, 1070-1110.

Della Seta, M., E. Morellec, and F. Zucchi (2020), "Short-term Debt and Incentives for Risk-taking," *Journal of Financial Economics*, 137, 179-203.

Diamond, D. (1991), "Debt Maturity Structure and Liquidity Risk," *Quarterly Journal of Economics*, 106, 709-737.

Diamond (2004), "Committing to Commit: Short-term Debt When Enforcement if Costly," *Journal of Finance*, 59, 1447-1479.

Diamond, D., and P. Dybvig (1983), "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 91, 401-419.

Diamond, D., and Z. He (2014), "A Theory of Debt Maturity: The Long and Short of Debt Overhang," *Journal of Finance*, 69, 719-762.

Diamond, D., and R. Rajan (2001), "Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking," *Journal of Political Economy*, 109, 287-327.

Eisenbach, T. (2017), "Rollover Risk as Market Discipline: A Two-Sided Inefficiency," *Journal of Financial Economics*, 126, 252–269.

Flannery, M. (1994), "Debt Maturity and the Deadweight Cost of Leverage: Optimally Financing Banking Firms," *American Economic Review*, 84, 320-331.

Gorton, G. (2010), *Slapped by the Invisible Hand: The Panic of 2007*, Oxford: Oxford University Press.

Hart, O., and J. Moore (1995), "Debt and Seniority: An Analysis of the Role of Hard Claims in Constraining Management," *American Economic Review*, 85, 567-585.

Holmström, B., and J. Tirole (1997), "Financial Intermediation, Loanable Funds, and the Real Sector," *Quarterly Journal of Economics*, 112, 663-691.

Jacklin, C., and S. Bhattacharya (1988), "Distinguishing Panics and Informationally-Based Bank Runs: Welfare and Policy Implications," *Journal of Political Economy*, 96, 568-592.

Jeanne, O. (2009), "Debt Maturity and the International Financial Architecture," *American Economic Review*, 99, 2135-2148.

Leland, H. (1998), "Agency Costs, Risk Management, and Capital Structure," *Journal of Finance*, 53, 1213-1243.

Leland, H., and K. Toft (1996), "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads," *Journal of Finance*, 51, 987-1019.

Myers, S. (1977), "Determinants of Corporate Borrowing," *Journal of Financial Economics*, 5, 147-175.

Perotti, E., and J. Suarez (2011), "A Pigovian Approach to Liquidity Regulation," *International Journal of Central Banking*, 7, 3-41.

Rajan, R. (1992), "Insiders and Outsiders: The Choice between Informed and Arm's-Length Debt," *Journal of Finance*, 47, 1367-1400.

Rajan, R. (2005), "Has Financial Development Made the World Riskier?," Proceedings of the Jackson Hole Conference organized by the Federal Reserve Bank of Kansas City.

Repullo, R. (2005), "Liquidity, Risk-Taking, and the Lender of Last Resort," *International Journal of Central Banking*, 1, 47-80.

Santos, J., and J. Suarez (2019), "Liquidity Standards and the Value of an Informed Lender of Last Resort," *Journal of Financial Economics*, 132, 351-368.

Shin, H. S. (2010), *Risk and Liquidity*, Oxford: Oxford University Press.

Stein, J. (2012), "Monetary Policy as Financial Stability Regulation," *Quarterly Journal of Economics*, 127, 57-95.

Stiglitz, J., and A. Weiss (1981), "Credit Rationing in Markets with Imperfect Information," *American Economic Review*, 71, 393-410.

Tirole, J. (2003), "Inefficient Foreign Borrowing: A Dual- and Common-Agency Perspective," *American Economic Review*, 93, 1678-1702.

Tirole, J. (2010), "Lessons from the Crisis," in M. Dewatripont, J.-C. Rochet, and J. Tirole (eds.), *Balancing the Banks*, Princeton: Princeton University Press.