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We compare various bank capital regulation regimes using a dynamic equilibrium model of relationship lending in which banks are unable to access the equity markets every period and the business cycle determines loans' probabilities of default. Banks hold endogenous capital buffers as a precaution against shocks that impair their future lending capacity. We find that Basel II is more procyclical than Basel I but makes banks safer, and it is generally superior in welfare terms. For high social costs of bank failure, the optimal capital requirements are higher but less cyclically varying, like those currently targeted by Basel III. (*JEL* G21, G28, E44)

Discussions on the procyclical effects of bank capital requirements went to the top of the agenda for regulatory reform following the financial crisis that started in 2007.<sup>1</sup> The argument wherein these effects may occur is well known. In recessions, losses erode banks' capital, while risk-based capital requirements, such as those in Basel II (see Basel Committee on Banking Supervision (BCBS) 2004), become higher. If banks cannot quickly raise sufficient new capital, their lending capacity falls and a credit crunch may follow. Yet, correcting the potential contractionary effect on credit supply by relaxing capital requirements in bad times may increase bank failure probabilities precisely when, because of high loan defaults, they are largest. The conflicting goals at stake explain why some observers (e.g., regulators with an essentially microprudential perspective) think that procyclicality is a necessary evil, whereas others with a

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The declaration of the G20 Washington Summit of November 14–15, 2008, called for the development of "recommendations to mitigate procyclicality, including the review of how valuation and leverage, bank capital, executive compensation, and provisioning practices may exacerbate cyclical trends." See also Brunnermeier et al. (2009) and Kashyap, Rajan, and Stein (2008).

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more macroprudential perspective think that it should be explicitly corrected. Basel III (BCBS 2010) seems like a compromise between these two views. It reinforces the quality and quantity of the minimum capital required to banks, but also establishes that part of the increased requirements be in terms of mandatory buffers—a capital preservation buffer and a countercyclical buffer—that are intended to be built up in good times and released in bad times.

This paper constructs a model that captures the key trade-offs in the debate. The model is simple enough to allow us to trace back the effects to a few basic mechanisms. Yet, for the comparison between regulatory regimes (and the characterization of the capital requirements that maximize social welfare) we rely on numerical methods. In our calibration we use U.S. data for the period prior to the financial crisis that started in 2007.

We find that, in spite of inducing banks to hold voluntary capital buffers that are larger in expansions than in recessions, banks' supply of credit is significantly more procyclical under the risk-based requirements of Basel II than under the flat requirements of Basel I.<sup>2</sup> However, Basel II reduces banks' probabilities of failure, especially in recessions. For this reason, it dominates Basel I in terms of welfare, except for small values of the social cost of bank failure—a parameter with which we capture the externalities that justify the public concern about bank solvency. Moreover, for intermediate values of the social cost of bank failure, Basel II implies a cyclical variation in the capital requirements very similar to that of the socially optimal ones. For larger values of the social cost of bank failure, optimal capital charges should be higher than those in Basel II, but their cyclical variation should be comparatively lower. This suggests that, from the lens of our model and for sufficiently large values of the social cost of bank failure, the reforms introduced by Basel III constitute a move in the right direction.

Our model is constructed to highlight the primary microprudential role of capital requirements (containing banks' risk of failure and, thus, deposit insurance payouts and other social costs due to bank failures) as well as their potential procyclical effect on the supply of bank credit. A number of features of the model respond to the desire to keep transparency about the basic trade-offs. We model the business cycle as a Markov process with two states (expansion and recession), and we abstract from demand-side fluctuations and feedback effects, which could be captured in a fuller macroeconomic model that might embed ours as a building block.

Bank borrowers are overlapping generations of entrepreneurs who demand loans for two consecutive periods. Banks are managed in the interest of their risk-neutral shareholders (providers of their equity capital). Consistent with the view that relationship banking makes banks privately informed about their borrowers, we assume that (1) borrowers become dependent on the banks

<sup>&</sup>lt;sup>2</sup> Basel I (BCBS 1988) established a requirement in terms of capital to risk-weighted assets and classified assets in four broad categories. All corporate loans (as well as consumer loans) were in the top risk category.

with whom they first start a lending relationship, and (2) banks with ongoing relationships have no access to the equity market. The first assumption captures the lock-in effects caused by the potential lemons problem faced by banks when a borrower is switching from another bank.<sup>3</sup> The second assumption captures the implications of these informational asymmetries for the market for seasoned equity offerings, which can make the dilution costs of urgent recapitalizations prohibitively costly.<sup>4</sup>

The combination of relationship lending and the inability of banks with ongoing relationships to access the equity market establishes a natural connection between the capital shortages of some banks and the credit rationing of some borrowers at a given date. It also ensures that two necessary conditions for capital requirements to have aggregate procyclical effects on credit supply are satisfied: Some banks must find it difficult to respond to their capital needs by issuing new equity, and some borrowers must be unable to avoid credit rationing by switching to other sources of finance.<sup>5</sup>

For simplicity, the market for loans to newly born entrepreneurs is assumed to be perfectly competitive and free from capital constraints. Each cohort of new borrowers is funded by banks that renew their lending relationships, have access to the equity market, and hence face no binding limits to their lending capacity.

An important feature of our analysis, distinct from many papers in the literature, is that we allow banks in their first lending period to raise more capital than needed to just satisfy the capital requirement. The existence of voluntary capital buffers has been frequently mentioned as an argument against the prediction of most static models that capital requirements will be binding and as a factor mitigating their procyclical effects. We find, however, that the equilibrium buffers (of up to 3.8% in the expansion state under Basel II) are not sufficient to neutralize the effects of the arrival of a recession on the supply of credit to bank-dependent borrowers (which falls by 12.6% on average in the baseline Basel II scenario).

Other papers wherein endogenous capital buffers emerge as a result of an explicit dynamic optimization problem are Estrella (2004), Peura and Keppo (2006), Elizalde and Repullo (2007), and Zhu (2008). Estrella (2004) considers an individual bank whose dividend policy and equity raising processes are subject to quadratic adjustment costs in a context in which loan losses follow a second-order autoregressive process and bank failure is costly. He shows

<sup>&</sup>lt;sup>3</sup> See Boot (2000) for a survey of the relationship banking literature. Several papers explicitly analyze the costs of switching lenders under asymmetric information (e.g., Sharpe 1990) as well as the trade-offs behind the possible use of multiple lenders as a remedy to the resulting lock-in effects (e.g., Detragiache, Garella, and Guiso 2000). We implicitly assume that these alternatives are very costly.

<sup>&</sup>lt;sup>4</sup> This argument is in line with the logic of Myers and Majluf (1984) and is also subscribed to by Bolton and Freixas (2006). An alternative explanation for banks' reluctance to raise new equity when their capital position is impaired is the debt overhang problem (see Myers 1977 and Hanson, Kashyap, and Stein 2011).

<sup>&</sup>lt;sup>5</sup> These conditions have been noted by Blum and Hellwig (1995) and parallel the conditions in Kashyap, Stein, and Wilcox (1993) for the existence of a bank lending channel in the transmission of monetary policy.

#### The Review of Financial Studies / v 0 n 0 2012

that the optimal capital decisions of the bank change significantly with the introduction of a value-at-risk capital constraint. Peura and Keppo (2006) consider a continuous-time model in which raising bank equity takes time. A supervisor checks at random times whether the bank complies with a minimum capital requirement and the bank may hold capital buffers to reduce the risk of being closed for holding insufficient capital when audited. Similarly, the banks in Elizalde and Repullo (2007) may hold economic capital in excess of their regulatory capital to reduce the risk of losing their valuable charter in case of failure. Zhu (2008) adapts the model of Cooley and Quadrini (2001) to the analysis of banks with decreasing returns to scale, minimum capital requirements, and linear equity-issuance costs. Assuming ex ante heterogeneity in banks' capital positions, the paper finds that for poorly capitalized banks, risk-based capital requirements increase safety without causing a major increase in procyclicality, whereas for well-capitalized banks, the converse is true.

Our analysis is simpler along the dynamic dimension than most of the papers mentioned above. However, differently from them, we construct an equilibrium model of relationship banking with endogenous loan rates and a focus on the implications of capital requirements for aggregate bank lending, bank failure probabilities, and social welfare. In this sense, our paper is also related to recent attempts to incorporate bank capital frictions and capital requirements into macroeconomic models. Van den Heuvel (2008) assesses the aggregate steady-state welfare cost of capital requirements in a setup in which deposits provide unique liquidity services to consumers. Meh and Moran (2010), Gertler and Kiyotaki (2010), and Martinez-Miera and Suarez (2012), among others, consider models in which aggregate bank capital is a state variable whose dynamics is constrained by the evolution of the limited wealth of the bankers. In most of these papers, bank capital requirements are binding at all times, although some papers, like Gerali et al. (2010), induce the existence of buffers by postulating that the deviation from some ad hoc target capital ratio involves a quadratic cost. The procyclical effects of capital requirements is the focus of attention in Angelini et al. (2010), where there are no loan defaults or bank failures, making their model silent on an important aspect of the relevant welfare trade-offs, and in Brunnermeier and Sannikov (2011), where requirements take the form of value-at-risk constraints on a trading book and risk comes from variation in asset prices.

Our paper is complementary to prior contributions focused on the design of capital regulation under the new macroprudential perspective. Daníelsson et al. (2001), Kashyap and Stein (2004), Gordy and Howells (2006), Saurina and Trucharte (2007), and, more recently, Brunnermeier et al. (2009) and Hanson, Kashyap, and Stein (2011) discuss the potential importance of the procyclical effects of risk-based capital requirements and elaborate, mostly qualitatively, on the pros and cons of the various options for their correction.

The list of policy options is long and includes (1) smoothing the inputs of the regulatory formulas by promoting the use of through-the-cycle estimates of the probabilities of default (PDs) and losses-given-default (LGDs) that feed them (see Catarineu-Rabell, Jackson, and Tsomocos 2005), (2) smoothing or cyclically adjusting the output of the regulatory formulas (see Repullo, Saurina, and Trucharte 2010), (3) forcing the building up of buffers when cyclically sensitive variables, such as bank profits and credit growth, are high (see Committee of European Banking Supervisors (CEBS) 2009 and BCBS 2010), (4) adopting countercyclical provisioning (see Burroni et al. 2009), (5) exercising regulatory discretion with countercyclical goals in mind, and (6) relying on contingent convertibles and other forms of capital insurance (see Kashyap, Rajan, and Stein 2008). As with most other papers in the literature, our model is too stylized to formally capture the differences between these proposals and hence to inform the comparison between them (which is largely driven by legal, accounting, and political economy issues potentially affecting their effectiveness, predictability, manipulability, risk of capture, and cost of implementation). Our analysis is more informative on the level and degree of cyclical adjustment of the capital requirements that regulators should target to impose in one way or another.

Empirical studies focused on the impact of bank regulation on bank capital decisions, and the supply of credit, are abundant but often inconclusive as they are plagued with problems of endogeneity and poor identification. Because of the Lucas critique, the results from reduced-form analyses of the dynamics of bank capital buffers under specific regulatory regimes cannot be extrapolated for the assessment of new regimes.<sup>6</sup> Yet, the relevance of banks' capital constraints for determining the supply of credit is documented, among others, by Bernanke and Lown (1991), who examine credit supply in the years after the introduction of Basel I, by Ivashina and Scharfstein (2010), who show that after the demise of Lehman Brothers poorly capitalized banks, and by Aiyar, Calomiris, and Wieladek (2012), who document sizable loan supply effects following discretionary shifts in the level of capital requirements in the United Kingdom from 1998 to 2007.

The rest of the paper is organized as follows. Section 1 presents the model. In Section 2, we analyze the capital decision of a representative bank, define the equilibrium, and provide the comparative statics of equilibrium loan rates and capital buffers. In Section 3, we discuss the calibration. Section 4 reports the quantitative results concerning loan rates, capital buffers, credit rationing, and bank failure probabilities under the various regulatory regimes. In Section 5, we compare these regimes in terms of social welfare and characterize the optimal capital requirements. Section 6 discusses the robustness of the results to changes in some of the key assumptions of the model. Section 7 summarizes the main findings and concludes. The Appendix gathers the proofs of the analytical

<sup>&</sup>lt;sup>6</sup> Existing empirical work includes Ayuso, Pérez, and Saurina (2004), Lindquist (2004), Bikker and Metzemakers (2007), and Berger et al. (2008).

results and shows the relationship between the single common risk factor model used in the calibration and the Basel II formula for capital requirements.

# 1. The Model

Consider a discrete-time infinite-horizon economy with three classes of riskneutral agents: entrepreneurs, investors, and banks. Entrepreneurs finance their investments by borrowing from banks. Investors provide funds to the banks in the form of deposits and equity capital. Banks channel funds from investors to entrepreneurs. There is also a government that insures bank deposits and imposes minimum capital requirements on banks.

## 1.1 Entrepreneurs

Entrepreneurs belong to overlapping generations whose members remain active for up to two periods (three dates). Each generation is made up of a measure-one continuum of ex ante identical and penniless individuals. Entrepreneurs born at a date *t* have the opportunity to undertake a sequence of two independent one-period *investment projects* at dates *t* and *t*+1. Each project requires a unit investment and yields a pledgeable return 1+a if it is successful, and  $1-\lambda$  if it fails, where a > 0 and  $0 < \lambda < 1$ .

All projects operating from date *t* to date *t* + 1 have an identical probability of failure  $p_t$ . The outcomes of these projects exhibit positive but imperfect correlation, so their aggregate *failure rate*  $x_t$  is a continuous random variable with support [0, 1] and cumulative distribution function (cdf)  $F_t(x_t)$  such that the probability of project failure satisfies

$$p_t = E_t(x_t) = \int_0^1 x_t \, dF_t(x_t). \tag{1}$$

For simplicity, we consider the case in which the history of the economy up to date *t* only affects  $F_t(x_t)$  (and thus  $p_t$ ) through an observable *state variable*  $s_t$  that can take two values, *l* and *h*, and follows a Markov chain with transition probabilities

$$q_{ss'} = \Pr(s_{t+1} = s' | s_t = s), \text{ for } s, s' = l, h.$$
(2)

Moreover, we assume that the cdfs corresponding with the two states,  $F_l(\cdot)$  and  $F_h(\cdot)$ , are ranked in the sense of first-order stochastic dominance so that  $p_l < p_h$ . Thus, states *l* and *h* may be interpreted as states of expansion (low business failure) and recession (high business failure), respectively.

## **1.2 Investors**

At each date t, there is a large number of investors willing to supply banks with deposits and equity capital in a perfectly elastic fashion at some required rate of return. The required interest rate on bank deposits (which are assumed to be insured by the government) is normalized to zero. In contrast, the required

expected return on bank equity is  $\delta \ge 0$ . This excess *cost of bank capital*  $\delta$  is intended to capture in reduced-form manner distortions (such as agency costs of equity) that imply a comparative disadvantage of equity financing relative to deposit financing (and in addition to deposit insurance).<sup>7</sup>

# 1.3 Banks

Banks are infinitely lived and competitive intermediaries specialized in channeling funds from investors to entrepreneurs. Following the literature on relationship banking, we assume that each entrepreneur relies on a sequence of one-period loans granted by the single bank from which the first loan is obtained. Setting up the relationship with the entrepreneur involves a *setup cost*  $\mu$ , which is subtracted from the bank's first-period revenues.<sup>8</sup> Finally, for simplicity, we abstract from the possibility that part of the second-period investment be internally financed by the entrepreneur.<sup>9</sup>

Banks are funded with insured deposits and equity capital, but access to the latter is affected by an important imperfection: Although banks renewing their portfolio of lending relationships can unrestrictedly raise new equity, recapitalization is impossible for banks with ongoing lending relationships. Our goal here is to capture in a simple way the long delays or high dilution costs that a bank with opaque assets in place may face when arranging an equity injection.<sup>10</sup>

Banks are managed in the interest of their shareholders, who are protected by limited liability. A *capital requirement* obliges them to keep a capital-toloans ratio of at least  $\gamma_s$  when the state of the economy is *s*. This formulation encompasses several regulatory scenarios that will be compared below: a laissez-faire regime with no capital requirements ( $\gamma_l = \gamma_h = 0$ ), a regime with flat capital requirements, such as those of Basel I (which for corporate loans

<sup>&</sup>lt;sup>7</sup> Further to the reasons for the extra cost of equity financing offered by the corporate finance literature, Holmström and Tirole (1997) and Diamond and Rajan (2000) provide agency-based explanations specifically related to banks' monitoring role. For the positive results of the paper,  $\delta$  may also be interpreted as the result of debt tax shields, but in this case it should not constitute a deadweight loss in the normative analysis (see Admati et al. 2011).

<sup>&</sup>lt;sup>8</sup> This cost might include personnel, equipment, and other operating costs associated with the screening and monitoring functions emphasized in the literature on relationship banking.

<sup>&</sup>lt;sup>9</sup> This simplification is standard in relationship banking models (e.g., Sharpe 1990 or Von Thadden 2004). Moreover, if entrepreneurs' first-period profits are small relative to the required second-period investment, the quantitative effects of relaxing this assumption would be small.

<sup>&</sup>lt;sup>10</sup> These costs are typically attributed to asymmetric information. If banks learn about their borrowers after starting a lending relationship (like in Sharpe 1990) and borrower quality is asymmetrically distributed across banks, the market for seasoned equity offerings (SEOs) is likely to be affected by a lemons problem (like in Myers and Majluf 1984). Specifically, after a negative shock, banks with lending relationships of poorer quality will be more interested in issuing equity at any given price. So, the prices at which new equity can be raised may be unattractive to banks with higher-quality relationships and, in sufficiently adverse circumstances, the market for SEOs may collapse.

sets a requirement of Tier 1 capital of  $\gamma_l = \gamma_h = 4\%$ ), and a regime with riskbased capital requirements, such as those of Basel II or Basel III (where the cyclical variation in the inputs of the regulatory formula implies  $\gamma_l < \gamma_h$ ).<sup>11</sup>

# 1.4 Government policies and social welfare

The government performs two tasks in this economy. First, it insures bank deposits (raising lump-sum taxes to cover the cost of repaying depositors in case of bank failure). Second, it imposes minimum capital requirements on banks.

In the normative analysis below, we will assess the welfare implications of the various regulatory scenarios, taking into account possible negative externalities associated with bank failures, which will be assumed to imply a social cost equal to a proportion c of the initial assets of the failed banks.<sup>12</sup> Specifically, given that investors (depositors and bank shareholders) in equilibrium will break even in expected net present value terms over their relevant investment horizons, we will measure *social welfare* as the sum of the expected residual income flows obtained by entrepreneurs from their investment projects minus the expected cost of deposit insurance payouts and the expected social cost of bank failures.

# 2. Equilibrium Analysis

In this section, we characterize banks' equilibrium capital and lending decisions and derive some comparative statics results on equilibrium loan rates and capital buffers.

## 2.1 Banks' optimization problem

We assume that entrepreneurs born at date t obtain their first-period loans from banks that can unrestrictedly raise capital at this date. This is consistent with the assumption that banks with ongoing lending relationships face capital constraints and allows us to analyze the banking industry as if it were made of overlapping generations of banks that operate for two periods, specialize in loans to their contemporaneous entrepreneurs, and can only issue equity when they start operating.

Consider a representative bank that lends a first unit-size loan to the measure one continuum of entrepreneurs born at date t, possibly refinances them at date t+1, and ends its activity at date t+2. Denote the states of the economy at dates t and t+1 by s and s', respectively. At date t the bank raises  $1-k_s$  deposits and

<sup>&</sup>lt;sup>11</sup> The precise Basel formula that makes  $\gamma_s$  an increasing function of the PD of the loans (the probability of project failure  $p_s$ ) is described in Section 3.

<sup>&</sup>lt;sup>12</sup> The externalities commonly identified in the literature include the disruption of the payment system, the erosion of confidence on similar banks and the rest of the financial system, the deterioration of public finances derived from the cost of resolving or supporting banks in trouble, the fall in economic activity associated with a potential credit crunch, and the damage to the general economic climate (see Laeven and Valencia 2008, 2010).

 $k_s$  capital, with  $k_s \ge \gamma_s$  to satisfy the capital requirement (and possibly keeping a buffer  $k_s - \gamma_s > 0$  to better accommodate shocks that may impair its lending capacity in the second period). The bank invests these funds in a unit portfolio of first-period loans whose interest rate  $r_s$  will be determined endogenously, but is taken as given by the perfectly competitive bank.<sup>13</sup>

At date t+1 the bank obtains revenue  $1+r_s$  from the fraction  $1-x_t$  of performing loans (those extended to entrepreneurs with successful projects) and  $1-\lambda$  from the fraction  $x_t$  of defaulted loans, and incurs the setup cost  $\mu$ . So, its assets are worth  $1+r_s-x_t(\lambda+r_s)-\mu$ , whereas its deposit liabilities are  $1-k_s$  (because the deposit rate has been normalized to zero). Thus, the *net* worth (or available capital) of the bank at date t+1 is

$$k'_{s}(x_{t}) = k_{s} + r_{s} - x_{t}(\lambda + r_{s}) - \mu, \qquad (3)$$

where  $x_t$  is a random variable whose conditional cdf is  $F_s(x_t)$ .

The entrepreneurs that started at date *t* demand a second unit-size loan at date t + 1.<sup>14</sup> Because they are dependent on the bank at this stage, their demand is inelastic. Thus, the *second-period loan rate* will be *a*, assigning all the pledgeable return from the investment in the period to the bank.

To comply with capital regulation, funding all second-period projects at date t+1 would require the bank to have an amount of capital equal to  $\gamma_{s'}$ , where s' is the state of the economy at that date. There are three cases to consider. First, if  $k'_s(x_t) < 0$ , the bank fails, the deposit insurer liquidates the bank and repays the depositors, and the entrepreneurs dependent on the bank cannot invest. Second, if  $0 \le k'_s(x_t) < \gamma_{s'}$ , the bank's available capital cannot support funding all of the second-period projects, so some entrepreneurs are credit rationed. Third, if  $k'_s(x_t) \ge \gamma_{s'}$ , the bank can fund all of the second-period projects and, on top of that, pay a dividend  $k'_s(x_t) - \gamma_{s'}$  to its shareholders at date t+1.<sup>15</sup>

Which case obtains depends on the realization of the default rate  $x_t$ . Using the definition (3) of  $k'_s(x_t)$ , it is immediate to show that the bank fails when  $x_t > \hat{x}_s$ , where

$$\widehat{x}_s = \frac{k_s + r_s - \mu}{\lambda + r_s}.$$
(4)

The bank has *insufficient lending capacity* (and rations credit to some of the second-period projects) when  $\widehat{x}_{ss'} < x_t \leq \widehat{x}_s$ , where

$$\widehat{x}_{ss'} = \frac{k_s + r_s - \mu - \gamma_{s'}}{\lambda + r_s}.$$
(5)

<sup>&</sup>lt;sup>13</sup> This corresponds to the idea that entrepreneurs can shop around for their first-period loans before becoming locked in for their second-period loans.

<sup>&</sup>lt;sup>14</sup> This includes entrepreneurs that defaulted on their initial loans because, under our assumptions, such default does not reveal any information about their second-period projects.

<sup>&</sup>lt;sup>15</sup> Because entrepreneurs born at date *t* + 1 borrow from banks that can raise equity at that date, the bank lending to entrepreneurs born at date *t* can use the excess capital either to pay a dividend to its shareholders or to reduce the deposits to be raised at this date. However, with deposit insurance and an excess cost of bank capital  $\delta \ge 0$ , the second alternative is strictly suboptimal.

And, the bank has *excess lending capacity* (and pays a dividend to its shareholders) when  $x_t \leq \hat{x}_{ss'}$ .

The following proposition provides an expression of the net present value for the shareholders of a bank that can raise capital at date *t*. Because the result follows quite directly from the sequence of definitions that it contains, we will omit its proof, replacing it with the brief explanation given below.

**Proposition 1.** The net present value for the shareholders of a representative bank that in state *s* has capital  $k_s$  and faces an interest rate  $r_s$  on its unit of initial loans is

$$v_s(k_s, r_s) = \frac{1}{1+\delta} E_t[v_{ss'}(x_t)] - k_s,$$
(6)

where

$$v_{ss'}(x_t) = \begin{cases} \pi_{s'} + k'_s(x_t) - \gamma_{s'}, & \text{if } x_t \le \widehat{x}_{ss'}, \\ \pi_{s'} \frac{k'_s(x_t)}{\gamma_{s'}}, & \text{if } \widehat{x}_{ss'} < x_t \le \widehat{x}_s, \\ 0, & \text{if } x_t > \widehat{x}_s, \end{cases}$$
(7)

is the conditional equity value at date t + 1, inclusive of dividends, and

$$\pi_{s'} = \frac{1}{1+\delta} \int_0^1 \max\{\gamma_{s'} + a - x_{t+1}(\lambda + a), 0\} dF_{s'}(x_{t+1})$$
(8)

is the discounted gross return that equity earns on each unit of loans made at date t + 1.

The operator  $E_t(\cdot)$  in (6) takes into account the uncertainty at date *t* about both the state of the economy at date t+1 (which affects  $\gamma_{s'}$  and  $\pi_{s'}$ ) and the default rate  $x_t$  of initial loans (which determines the capital  $k'_s(x_t)$  available at t+1). Expected future payoffs in (6) and (8) are discounted at the shareholders' required expected return  $\delta$ . The three expressions in the right-hand side of (7) correspond to the three cases mentioned above. With excess lending capacity, the bank funds all of the second-period projects, which yields a discounted gross return  $\pi_{s'}$ , and pays a dividend  $k'_s(x_t) - \gamma_{s'}$ . With insufficient lending capacity, the bank funds a fraction  $k'_s(x_t)/\gamma_{s'}$  of the second-period projects, which yields a discounted gross return  $\pi_{s'}k'_s(x_t)/\gamma_{s'}$ . Finally, in case of bank failure, the shareholders get a zero payoff.<sup>16</sup>

The representative bank that first lends to a generation of entrepreneurs in state *s* takes the initial loan rate  $r_s$  as given and chooses its capital  $k_s$ 

<sup>&</sup>lt;sup>16</sup> As specified in (8),  $\pi_{s'}$  is obtained by integrating with respect to the probability distribution of the default rate  $x_{t+1}$  the net worth that the bank generates at date t+2 out of each unit of lending at date t+1. The expression in the integrand of (8) is identical to (3), except for the fact that the bank's capital is  $\gamma_{s'}$ , the loan rate is *a*, the setup cost  $\mu$  has already been incurred, and shareholders' limited liability is taken into account using the max operator.

so as to maximize  $v_s(k_s, r_s)$  subject to the requirement  $k_s \ge \gamma_s$  insofar as the resulting value is not negative. If it were negative, shareholders would prefer not to operate the bank. To guarantee that operating the bank is profitable, we henceforth assume that the following sufficient condition holds.

## **Assumption 1.** $v_s(\gamma_s, a) \ge 0$ and $\pi_s - \gamma_s \ge 0$ for s = l, h.

This assumption states that making loans at a rate equal to the project's net success return a while satisfying the capital requirement with equality constitutes a nonnegative net present value investment for the bank's shareholders in the two lending periods.

The following result characterizes the initial capital decision of the bank.

**Proposition 2.** The capital decision  $k_s$  of a representative bank that in state s faces an interest rate  $r_s$  on its unit of initial loans always has a solution, which may be interior or at the corner  $k_s = \gamma_s$ . When the solution is interior, the probability that in the next period the bank ends with excess lending capacity in the low default state s' = l and rations credit in the high default state s' = h is strictly positive.

The existence of a solution follows directly from the fact that  $v_s(k_s, r_s)$  is continuous in  $k_s$ , for any given interest rate  $r_s$ . We show in the Appendix that the function  $v_s(k_s, r_s)$  is neither concave nor convex in  $k_s$ , and its maximization with respect to  $k_s$  may have interior solutions or corner solutions with  $k_s = \gamma_s$ .<sup>17</sup> The intuition for the positive probability that (in an interior solution) the bank ends with excess lending capacity in state s' = l and rations credit in state s' = h is the following. If in the two possible states at date t + 1 the bank had a probability one of finding itself with excess lending capacity, then it would have an incentive to reduce its capital at date t + 1 the bank had a probability one of finding itself with insufficient lending capacity, then it would have an incentive to increase its capital at date t to relax its capital constraint at date t + 1.

### 2.2 Equilibrium

To define an equilibrium, it only remains to describe how the loan rate  $r_s$  applicable to lending relationships starting in state *s* is determined. Under perfect competition, the pricing of initial loans must be such that the net present value of the representative bank for its shareholders is zero under its optimal capital decision. Were it negative, no bank would extend these loans. Were it

<sup>&</sup>lt;sup>17</sup> Note that because the function  $v_s(k_s, r_s)$  is not concave in  $k_s$ , there may be multiple optimal values of  $k_s$  corresponding to any  $r_s$ .

The Review of Financial Studies / v 0 n 0 2012

 Table 1

 Comparative statics of the initial loan rate  $r_s^*$ 

	z	=	а	λ	μ	δ	γı	$\gamma_h$	$q_{sh}$
$\frac{dr_s^*}{dz}$			—	+	+	+	+	+	+

positive, banks would have an incentive to expand the scale of their activities. Hence, in each state of the economy s = l, h we must have

$$v_s(k_s^*, r_s^*) = 0,$$
 (9)

for

$$k_s^* = \arg\max_{k_s \ge \gamma_s} v_s(k_s, r_s^*).$$
(10)

An *equilibrium* is a sequence of pairs  $\{(k_t, r_t)\}$  describing the capital-to-loans ratio  $k_t$  of the banks that can issue equity at date t and the interest rate  $r_t$  on their initial loans, such that each pair  $(k_t, r_t)$  satisfies (9) and (10) for  $s = s_t$ , where  $s_t$  is the state of the economy at date t. The following result proves the existence of an equilibrium.

**Proposition 3.** There exists a unique  $r_s^*$  that satisfies equilibrium conditions (9) and (10).

The uniqueness of  $r_s^*$  follows from the fact that, for each initial state *s*, the net present value of the bank is an overall continuous and increasing function of  $r_s$  (after taking into account how the capital decision  $k_s$  varies with  $r_s$ ). Moreover, such function is negative for sufficiently low values of  $r_s$  and, by Assumption 1, nonnegative when  $r_s$  equals *a*, which guarantees the existence of a unique solution.

# 2.3 Comparative statics

Table 1 summarizes the comparative statics of the equilibrium *initial loan* rate  $r_s^*$ , which are derived in the Appendix. The table shows the sign of the derivative  $dr_s^*/dz$  obtained by differentiating (9) with respect to a parameter denoted generically by z.

The effects of the various parameters on  $r_s^*$  are inversely related to their impact on bank profitability. Other things equal, the success return *a* impacts positively on the profitability of continuation lending; the loss given default  $\lambda$  affects negatively the profitability of both initial lending (directly) and continuation lending (directly and by reducing the availability of capital in the second period); the setup cost  $\mu$  has a similar negative effect, with no direct effect on the profitability of continuation loans; the cost of bank capital  $\delta$ increases the cost of making loans in both periods; the capital requirements  $\gamma_l$ and  $\gamma_h$  increase the burden of capital regulation in the corresponding initial or continuation state; finally, in any regulatory regime with  $\gamma_l \leq \gamma_h$ , the probability

Comparativ	Comparative statics of the initial capital $k_s^*$ (in an interior equilibrium)												
		z.	=	а	λ	$\mu$	δ	Υl	$\gamma_h$	$q_{sh}$			
$\frac{\partial k_s^*}{\partial z}$	(direct effect)			+	?	+	-	?	?	?			
$\frac{\partial k_s^*}{\partial r_s}  \frac{dr_s^*}{dz}$	(loan rate effect)			+	-	-	-	-	-	-			
$\frac{dk_s^*}{dz}$	(total effect)			+	?	?	-	?	?	?			

Table 2Comparative statics of the initial capital  $k_s^*$  (in an interior equilibrium)

of ending in the high default state  $q_{sh}$  decreases the profitability of continuation lending because in state *h* loan losses are higher and the capital requirement is not lower than in state l.<sup>18</sup>

Table 2 summarizes the comparative statics of the equilibrium *initial capital*  $k_s^*$  chosen by the representative bank in an interior solution. As further explained in the Appendix, we decompose the total effect of the change in any parameter z in a *direct effect*, for constant  $r_s^*$ , and a *loan rate effect*, due to the change in  $r_s^*$ . Because  $k_s$  and  $r_s$  are substitutes in providing the bank with sufficient capital for its continuation lending (see the expression for  $k'_s(x_t)$  in (3)), it turns out that  $\partial k_s^* / \partial r_s$  is negative, implying that the signs of the loan rate effects are the opposite to those in Table 1.

For the parameters *a* and  $\delta$ , the direct effects and the loan rate effects point in the same direction, so the total effect can be signed: Higher profitability of continuation lending and lower costs of bank capital encourage banks to increase self-insurance against default shocks that threaten their continuation lending. For the setup cost  $\mu$ , the direct and the loan rate effects have unambiguous but opposite signs, so the total effect is ambiguous. The positive direct effect comes from the fact that  $\mu$  subtracts to the bank's continuation lending capacity exactly like  $k_s$  adds to it (see again (3)).

The direct effects on  $k_s$  of parameters  $\lambda$ ,  $\gamma_l$ ,  $\gamma_h$ , and  $q_{sh}$  have ambiguous signs. Increasing any of these parameters reduces the profitability of continuation lending (and the value of holding excess capital in the initial lending period) but impairs the expected capital position of the bank when such lending has to be made (so the prospects of ending up with insufficient capital increase). This means that the profitability of continuation lending and the need for selfinsurance move in opposite directions. This ambiguity extends to the total effects.

The details of the relevant analytical expressions suggest that the shape of the distributions of default rates matter for the determination of the unsigned effects, which could only be assessed either empirically or by numerically solving the model under some realistic parameterization. In the rest of the paper, we resort to the second alternative.

<sup>&</sup>lt;sup>18</sup> Obviously, the probability of ending up in the low default state  $q_{sl} = 1 - q_{sh}$  has the opposite effect.

# 3. Calibration

This section presents the parameterization under which we derive our quantitative results. We start by specifying the distributions of the default rate in each state,  $F_l(x_t)$  and  $F_h(x_t)$ , as well as the capital regulation regimes, determining  $\gamma_l$  and  $\gamma_h$ , that will be compared. Finally, we discuss the values given to the parameters of the model: the projects' success return *a* and loss given default  $\lambda$ , the cost of setting up a lending relationship  $\mu$ , the excess cost of bank capital  $\delta$ , the transition probabilities  $q_{ss'}$  for s, s' = l, h, and the parameters in the distributions specified for the default rate. In the calibration, one period is one year.

# 3.1 Default rate distributions

We assume that the probability distributions of the loan default rate x are those implied by the *single common risk factor* model of Vasicek (2002), which was the model used to provide a value-at-risk foundation to the capital requirement formulas of Basel II (see Gordy 2003). As shown in the Appendix, this model implies

$$F_{s}(x) = \Phi\left(\frac{\sqrt{1-\rho_{s}} \Phi^{-1}(x) - \Phi^{-1}(p_{s})}{\sqrt{\rho_{s}}}\right),$$
(11)

for s = l, h, where  $\Phi(\cdot)$  is the cdf of a standard normal random variable and  $\rho_s \in (0, 1)$  is a parameter that measures the dependence of individual defaults on the common risk factor (and thus determines the degree of correlation between loan defaults). With this formulation, the distribution of the default rate in state *s* is fully parameterized by the probability of default  $p_s$  and the correlation parameter  $\rho_s$ .<sup>19</sup>

# 3.2 Regulatory regimes

The quantitative analysis in the paper is based on the assumption that the empirical counterpart of the equity capital that appears in the model (and to which the capital requirements  $\gamma_l$  and  $\gamma_h$  refer to) is what Basel regulations define as Tier 1 capital (essentially, common equity). Both Basel I and Basel II established (1) an overall requirement in terms of the sum of Tier 1 and Tier 2 capital (where the latter included substitutes of common equity with lower loss-absorbing capacity, such as convertible and subordinated debt), and (2) the additional requirement that at least half of the required capital had to take the (presumably more expensive) form of Tier 1 capital. However, the regulatory response to the financial crisis that started in 2007, known as Basel III, has upgraded the role of the second requirement after assessing that only (the core of) Tier 1 capital is truly capable of protecting banks against insolvency

<sup>&</sup>lt;sup>19</sup> It is easy to show that increases in  $p_s$  produce a first-order stochastic dominance shift in the distribution of x, and increases in  $\rho_s$  produce a mean-preserving spread in the distribution of x.

(see BCBS 2010). Consistent with this view, we will focus on Tier 1 capital requirements, but we will incorporate an adjustment to capture the incidence of the overall Tier 1 + Tier 2 requirement on banks' cost of funding.

The positive part of our quantitative analysis considers three capital regulation regimes. In the *laissez-faire regime*, a purely theoretical benchmark, we set  $\gamma_h = \gamma_l = 0$ . In the *Basel I regime* we set  $\gamma_h = \gamma_l = 0.04$ , which corresponds with the minimum Tier 1 capital requirement on all nonmortgage credit to the private sector set by the Basel Accord of 1988 (i.e., one-half of the overall 8% requirement of Tier 1 + Tier 2 capital).

The Tier 1 capital requirements of the *Basel II regime* are obtained by dividing by two the overall requirements of Tier 1 + Tier 2 capital given by the Basel II formula.<sup>20</sup> For corporate exposures of a one-year maturity, this implies<sup>21</sup>

$$\gamma_{s} = \frac{\lambda}{2} \Phi\left(\frac{\Phi^{-1}(p_{s}) + \sqrt{\rho(p_{s})} \Phi^{-1}(0.999)}{\sqrt{1 - \rho(p_{s})}}\right),$$
(12)

where

$$p(p_s) = 0.12 \left( 2 - \frac{1 - e^{-50p_s}}{1 - e^{-50}} \right).$$
(13)

The term  $\rho(p_s)$  reflects the way in which Basel regulators calibrated the correlation parameter  $\rho_s$  in (11) as a decreasing function of the probability of default  $p_s$ . The rationale for this assumption is that, in the cross-section, riskier firms are typically smaller firms for which idiosyncratic risk factors are more important than the common risk factor, so their defaults are less correlated with each other. Because this argument does not apply to the timeseries dimension on which we focus, we will parameterize  $\rho_s$  as a constant  $\rho$  equal to the weighted average of  $\rho(p_s)$  for s = l, h, where the weights are the unconditional probabilities of each state s.<sup>22</sup>

In addition to the three regimes compared in the positive part of our analysis, in the normative part we will characterize an *optimal minimum capital regime* in which the capital requirements  $\gamma_l$  and  $\gamma_h$  are set to maximize our measure of social welfare.

## 3.3 Parameter values

Table 3 describes our baseline parameterization of the model. The value of the success return a determines the interest rate of second-period loans (measured

<sup>&</sup>lt;sup>20</sup> The formula has an explicit value-at-risk interpretation: Given the distribution of the default rate in (11), it requires Tier 1 + Tier 2 capital sufficient to cover loan losses with a confidence level of 99.9%.

<sup>&</sup>lt;sup>21</sup> See BCBS (2004, paragraph 272). The full Basel II formula incorporates an adjustment factor that is increasing in the maturity of the loan, and equals one for a maturity of one year. Also, Basel II distinguishes between expected losses, equal to  $\lambda p_s$ , which should be covered with general loan loss provisions, and the remaining part of the charge,  $\lambda(\gamma_s - p_s)$ , which should be covered with capital. However, from the perspective of our analysis, provisions are just another form of equity capital, so the distinction between these components is immaterial to our calculations.

<sup>&</sup>lt;sup>22</sup> These probabilities are  $\phi_l = (1 - q_{hh})/(2 - q_{ll} - q_{hh})$  and  $\phi_h = (1 - q_{ll})/(2 - q_{ll} - q_{hh})$ , respectively.

The Review of Financial Studies / v 0 n 0 2012

Table 3 Baselii	3 ne paramo	eter value	5					
а	λ	$\mu$	δ	Pl	$p_h$	$q_{ll}$	$q_{hh}$	ρ
0.04	0.45	0.03	0.08	0.010	0.036	0.80	0.64	0.174

as a spread over the risk-free deposit rate, which has been normalized to zero). Standard statistical sources do not provide banks' marginal lending and borrowing interest rates. A common approach is to proxy them with implicit average rates obtained from accounting figures. According to the FDIC Statistics on Banking, *Total interest income* of all U.S. commercial banks was, on average, 5.74% of *Earning assets* in the precrisis years 2004–2007, whereas *Total interest expense* was 2.32% of *Total liabilities*. This implies an average net interest margin of 3.42%.<sup>23</sup> Adding *Service charges on deposit accounts,* which were 0.55% of *Total deposits*, produces an average intermediation margin of 3.97% on deposit-funded activities during the referred period. This justifies our choice of a = 0.04.

Parameter  $\lambda$  determines the loss given default (LGD) of the loans to projects that fail. We take the value  $\lambda = 0.45$  from the Basel II "foundation Internal Ratings-Based (IRB) approach" for unsecured corporate exposures, which was calibrated in line with industry estimates of this parameter.<sup>24</sup>

The value of the setup  $\cot \mu$  is hard to establish directly from the data because its empirical counterpart is included in the broader category of noninterest expense in banks' accounts. In the FDIC Statistics on Banking, the ratio of *Total noninterest expense* of all U.S. commercial banks to *Total assets* for years 2004–2007 has an average of 3.97%.<sup>25</sup> The role of  $\mu$  in the model is to reduce the profitability of bank lending to have realistic initial loan rates. Taking  $\mu = 0.03$ , we obtain first-period loan spreads (over the risk-free deposit rate) of about 100 basis points in the low default state.

For the calibration of the excess cost of bank capital  $\delta$ , we take into account that the regulatory regimes that we compare are described in terms of minimum requirements of Tier 1 capital. However, Basel I and Basel II also required the total amount of Tier 1 + Tier 2 capital to be at least twice as much as the minimum requirement of Tier 1 capital. Instead of considering this second requirement and explicitly modeling the two classes of capital and the frictions possibly affecting each of them, we take a shortcut and make  $\delta$  equal to two times the reference estimate of banks' excess cost of equity financing.<sup>26</sup>

<sup>&</sup>lt;sup>23</sup> The data are available at www2.fdic.gov/SDI/SOB/.

 $<sup>^{24}~</sup>$  The implications of allowing for cyclical variation in  $\lambda$  will be discussed in Section 6.

<sup>&</sup>lt;sup>25</sup> This number is just by coincidence equal to the intermediation margin calculated above.

<sup>&</sup>lt;sup>26</sup> Because the cost  $\delta$  also applies to the capital buffers held on top of the regulatory requirements, our strategy implicitly assumes that Tier 1 capital buffers are matched with buffers of Tier 2 capital of the same size.

To set a reference estimate for  $\delta$ , one may follow the literature on entrepreneurial financing, which commonly assumes a spread between the rates of return required by entrepreneurs and those required by their lenders.<sup>27</sup> Carlstrom and Fuerst (1997) and Gomes, Yaron, and Zhang (2003), among others, set the spread at 5.6%, whereas Iacoviello (2005) opts for a more conservative 4%.<sup>28</sup> An alternative approach, proposed by Van den Heuvel (2008), is to attribute the spread between the costs of banks' equity and deposit funding to the unique liquidity services associated with deposits. He compares the average return on subordinated bank debt (which counts as Tier 2 capital for regulatory purposes, but has the same tax advantages as standard debt) with the average net return of deposits. He finds a spread of 3.16% that can be considered a lower bound estimate of the cost of Tier 1 capital because its main component, common equity, presumably involves larger informational and agency costs than subordinated debt. Given that the various candidate estimates fluctuate around a midvalue of 4%, we set  $\delta = 2 \times 0.04 = 0.08$ .

Under the default rate distributions in (11) and with a state-invariant correlation parameter  $\rho$ , the only parameters of the model subject to Markov chain dynamics are the probabilities of default  $p_l$  and  $p_h$ . To set them, we look at the Special Report "Commercial Banks in 1999" of the Federal Reserve Bank of Philadelphia, that offers data on the experience of U.S. commercial banks during the 1990s.<sup>29</sup> In years around the 1990–1991 recession, the aggregate ratio of *Nonperforming loans* to *Total loans* was slightly above 3%, declined to slightly above 2% in 1993, and remained below 1.5% (with a downward trend) for the rest of the decade. Against this background, the choices in Table 3 ( $p_l$ =0.01 and  $p_h$ =0.036) are fine-tuned so as to imply that the unconditional mean of the Tier 1 capital requirements of Basel II (i.e., the weighted average of the values  $\gamma_l$ =3.2% and  $\gamma_h$ =5.5% obtained from (12) and (13), where the weights are the unconditional probabilities of each state) equals 4%, exactly as in the Basel I regime. This will allow us to attribute the differences in results across these regulatory regimes to a cyclical rather than a level effect.

We set the transition probabilities of the Markov process,  $q_{ll}$  and  $q_{hh}$ , so as to produce expected durations of  $(1 - q_{ll})^{-1} = 5$  years for the low default state and

<sup>&</sup>lt;sup>27</sup> Most papers in the capital structure tradition (e.g., Hennessy and Whited 2007) focus on the net tax disadvantages of equity financing (vis-à-vis debt financing), an aspect of the differential cost of equity funding that does not constitute a deadweight loss from a social welfare perspective (see Admati et al. 2011) and from which we wish to abstract to facilitate the normative analysis in Section 6 below.

<sup>&</sup>lt;sup>28</sup> The spreads found in the entrepreneurial financing literature may be interpreted as a reduced-form discount for the lack of diversification or liquidity associated with entrepreneurs' equity stakes. If extended to outside equity stakes, such discount might reflect differential monitoring costs that shareholders must incur to tackle potential conflicts with managers (e.g., to enforce proper accounting, auditing, and governance). With our formulation we abstract from the fact that, in a world with risk-averse investors, the risk premium component of  $\delta$  might change with banks' capital structure because of the standard logic of the Modigliani-Miller theorem (see Admati et al. 2011).

<sup>&</sup>lt;sup>29</sup> See www.philadelphiafed.org/files/bb/bbspecial.pdf. Similar reports for years after 1999 confirm the overall picture but offer the information with a breakdown (large banks vs. small banks) that does not make the numbers directly comparable.

 $(1-q_{hh})^{-1}=2.8$  years for the high default state.<sup>30</sup> These durations are derived from the analysis of the annual ratio of *Net loan and lease charge-offs* to *Gross loans and leases* for the FDIC-insured commercial banks over the period 1969– 2004.<sup>31</sup> After detrending the series using the standard HP-filter for annual data, we find twenty below-average yearly observations in four complete low default phases (implying an average duration of 20/4=5 years) and fourteen aboveaverage observations in five complete high default phases (implying an average duration of  $14/5 \simeq 2.8$  years).<sup>32</sup>

Finally, as explained above, we set the value of the correlation parameter,  $\rho = 0.174$ , equal to the weighted average of the values of  $\rho(p_s)$  obtained from (13), where the weights are the unconditional probabilities of each state *s* ( $\phi_l = 0.643$  and  $\phi_h = 0.357$ ).

## 4. Quantitative Results

This section describes the equilibrium loan rates, capital buffers, credit rationing, and bank solvency that obtain when solving the model using the parameterization described in the previous section. The outcomes presented in the first three panels of Table 4 come from solving the equilibrium equations (9) and (10) in each state. Credit rationing is defined as the proportion of second-period projects that cannot be undertaken because of banks' insufficient lending capacity or failure. In the fourth panel of Table 4, we report the expected credit rationing in state *s'* for each possible sequence of states (*s*, *s'*), which using the notation in Section 2.1 can be formally written as<sup>33</sup>

$$CR_{ss'} = \int_{\widehat{x}_{ss'}}^{\widehat{x}_s} \left[ 1 - \frac{k'_s(x)}{\gamma_{s'}} \right] dF_s(x) + [1 - F(\widehat{x}_s)], \tag{14}$$

where the first term reflects the rationing due to banks' insufficient lending capacity and the second the rationing due to bank failure. Table 4 also reports the unconditional mean of this variable across all possible trajectories of the economy. The fifth panel shows the probabilities of bank failure of first- and second-period banks in each state, as well as their average values across states and the overall average across banks.

#### 4.1 Loan rates

Initial loan rates are always higher in the high default state h, reflecting the need to compensate banks for both a higher probability of default and a lower

<sup>&</sup>lt;sup>30</sup> The expected duration of state s is  $(1-q_{ss})+2q_{ss}(1-q_{ss})+3q_{ss}^2(1-q_{ss})+...=(1-q_{ss})^{-1}$ .

<sup>&</sup>lt;sup>31</sup> The FDIC Historical Statistics on Banking are available at www2.fdic.gov/hsob/index.asp.

<sup>&</sup>lt;sup>32</sup> The observations of 1969 and 2004 belong to censored below-average phases and are not taken into account. The matched durations are consistent with the results in Koopman, Lucas, and Klaassen (2005), who identify a stochastic cycle in U.S. business failure rates with a period of between eight and eleven years.

<sup>&</sup>lt;sup>33</sup> The need to take expectations comes from the fact that credit rationing in a period in which the prevailing state is s' varies with the realization of the default rate in the previous period.

### Table 4

Equilibrium loan rates, capital buffers, credit rationing, and bank solvency under different regulatory regimes (all variables in %)

	Laissez-faire	Basel I	Basel II
Loan rate in state s			
$r_I^*$	0.8	1.3	1.3
$r_l^*$ $r_h^*$	2.5	3.2	3.3
Capital in state s			
$k_I^*$	4.2	6.7	6.9
$k_l^* = k_h^*$	3.4	6.3	6.7
Capital buffer in state s			
$\Delta_l = k_l^* - \gamma_l$	4.2	2.7	3.8
$\Delta_h = k_h^* - \gamma_h$	3.4	2.3	1.2
Expected credit rationing in state $s'$			
(s,s')=(l,l)	3.2	2.4	0.9
(s,s')=(l,h)	3.2	2.4	12.6
(s,s')=(h,l)	17.2	9.3	5.3
(s,s')=(h,h)	17.2	9.3	12.4
Unconditional	8.2	4.9	5.6
Probability of bank failure			
First-period banks, $s = l$	3.17	0.20	0.16
First-period banks, $s = h$	17.15	2.87	2.25
Unconditional	8.16	1.15	0.90
Second-period banks, $s = l$	0.55	0.03	0.05
Second-period banks, $s = h$	10.21	1.50	0.76
Unconditional	4.02	0.56	0.31
Unconditional, all banks	6.09	0.86	0.61

This table reports the results from numerically solving for the equilibrium of the model under the parameterization described in Table 3. Rows labeled "unconditional" show weighted averages based on the unconditional probabilities of each state. Expected credit rationing in state s' is the expected proportion of second-period projects that cannot be undertaken because of banks' insufficient lending capacity or failure. We report its unconditional mean as well as values conditional on the various combinations of the state of the economy in the reference period s' and in the previous period s. When reporting the probabilities of banks failure, "first-period banks" and "second-period banks" refer to banks funding first- and second-period projects, respectively.

prospective profitability of continuation lending (because  $q_{hh} = 0.64 > 0.20 = q_{lh}$ , so the high default state *h* is more likely to occur after state *h* than after state *l*). The loan rates obtained under Basel I and Basel II are virtually identical (but clearly higher than those emerging in the laissez-faire regime) because the average capital effectively used by the representative bank in its two lending periods is very similar in both regimes.<sup>34</sup>

## 4.2 Capital buffers

As shown in the second and third panel of Table 4, the model produces positive capital buffers even in the laissez-faire regime. The rationale for these buffers is to preserve banks' future lending capacity. For instance, the unregulated first-period bank in the low default state l chooses a capital-to-loans ratio of 4.2%

<sup>&</sup>lt;sup>34</sup> Specifically, as further commented below, first-period capital decisions are very similar, whereas second-period capital coincides with the regulatory minimum, whose average across states in Basel II has been set in the calibration equal to the 4% requirement of Basel I. The comparison of loan rates across Basel I and Basel II is in line with previous results obtained in a static framework (Repullo and Suarez 2004).

and does so for the sole purpose of preserving its capacity to make profitable loans in the second period (which, in its case, only requires not failing in the first period). Interestingly, the capital chosen by this bank as a buffer in the high default state h (in spite of the much higher probability of failure) falls to 3.4% because, given the persistence of each state, second-period lending is expected to be less profitable and hence less worthy to preserve.

The buffers are also positive, though not as sizable, in the Basel I and Basel II regimes. Average buffers are very similar in these two regimes, but their cyclical pattern is markedly different. In state *l*, Basel II only requires  $\gamma_l = 3.2\%$  of Tier 1 capital (as opposed to 4% with Basel I), but a first-period bank chooses capital of 6.9% (as opposed to 6.7% with Basel I). In state *h*, Basel II requires  $\gamma_h = 5.5\%$  (as opposed to 4% with Basel I), and a first-period bank chooses capital of 6.7% (as opposed to 6.3% with Basel I). The much larger (smaller) capital buffer chosen in state *l* (state *h*) under Basel II reflects the optimal response to anticipating that if the economy switches to state *h* (state *l*), the capital requirement will significantly increase (decrease), raising the probability that the bank will find itself with insufficient (excess) lending capacity in the next period.

# 4.3 Credit rationing

The results regarding credit rationing allow us to visualize the magnitude of the concern that leads banks to hold capital buffers. In spite of the equilibrium buffers, credit rationing is significant, especially when the economy comes from or ends in a high default state. In the laissez-faire and Basel I regimes, credit rationing does not depend on the arrival state s' (because the capital requirement does not vary across states) but just on bank profits in the previous period, whose distribution depends on the departure state s. Realizations of the default rate that leave banks with insufficient lending capacity are more likely when s = h. Consequently, expected credit rationing in the Basel I (laissez-faire) regime is 2.4% (3.2%) after a low default state period and 9.3% (17.2%) after a high default state period.

In the Basel II regime, the impact of loan defaults on banks' lending capacity is also present, but the overall effects are dominated by the cross-state variation in capital requirements: The two sequences ending with s' = h exhibit the largest credit rationing (slightly above 12%). In contrast, credit rationing in the sequence (s, s') = (l, l) is only 0.9%. Unconditionally, the laissez-faire regime produces the largest credit rationing (8.2%), followed by Basel II (5.6%) and Basel I (4.9%).

Thus, the main difference between Basel I and Basel II lies in the distribution of credit rationing across state sequences: Basel II produces a larger average supply of credit during expansion periods ((l, l) sequences) and at times when the economy exits a recession ((h, l) sequences), but it produces a much lower average supply of credit when the economy enters a recession ((l, h)

sequences) and throughout the recession ((h,h) sequences). In other words, Basel II amplifies the impact of the business cycle on the supply of credit.

## 4.4 Bank failure probabilities

Despite the capital buffers, banks in our economy are more likely to fail in their first than in their second lending period. This is because of the incidence of the setup cost  $\mu$  and the fact that first-period loan rates are competitive, whereas second-period rates are monopoly rates (because of the hold-up problem). Conditional bank failure probabilities are, realistically, closely related to the loan default cycle: In the Basel regimes, banks are between 15 and 50 times more likely to fail in the high than in the low default state. The laissez-faire regime involves an average probability of failure (6% per year) much higher than Basel I (0.86%) or Basel II (0.61%). Basel II implies greater bank solvency than Basel I because it concentrates the protection coming from bank capital in the high default state. In combination with the results on credit rationing, these results point to a nontrivial welfare comparison between the two Basel regimes that we will investigate in Section 5.

# 4.5 Understanding the forces behind the results

To further understand the forces driving banks' equilibrium capital decisions in the first lending period, which are key to the overall results, this section discusses the effects of changing two parameters that play an important role in the underlying optimization. In the interest of space, we focus on the Basel II regime. The first column of Table 5 reproduces the equilibrium outcomes obtained under our baseline parameterization.

The second column shows the results for the scenario in which the excess cost of bank capital is raised from 8% to 9%. This change reduces the profitability of second-period lending and the direct cost of holding a capital buffer in the first period. Banks react by reducing the capital buffer in both states, which produces a strong impact on credit rationing, especially when the economy enters a recession (*CR*<sub>lh</sub> rises from 12.6% to 20.2%). The reduction in capital buffers increases the probability of failure of first-period banks. And, the unconditional probability of bank failure rises from 0.61% to 0.68%.

The third column in Table 5 reports the results obtained when the transition probability  $q_{ll}$  is raised so as to lengthen the expected duration of the low default state from five to six years. This shift decreases the risk that banks raising capital in expansions face higher capital requirements during their second lending period. Banks' equilibrium buffer in the low default state falls from 3.8% to 3.5%, which implies that expected credit rationing rises from 12.6% to 18.2% if the economy switches to the high default state (and from 0.9% to 1.2% if the economy remains in the low default state). The probability of failure of first-period banks in the low default state increases slightly (from 0.16% to 0.20%), but unconditionally banks' average solvency rises because the economy is less likely to visit the state in which the risk of bank failure is the highest.

#### The Review of Financial Studies / v 0 n 0 2012

#### Table 5

Effect of various parameter changes on equilibrium outcomes under Basel II (all variables in %)

	Baseline results	Higher cost of bank capital δ	Higher duration of state <i>l</i>	Lower success return a
Loan rate in state s				
$r_I^*$	1.3	1.4	1.3	2.1
$r_l^*$ $r_h^*$	3.3	3.4	3.3	4.0
Capital in state <i>s</i>				
$\tilde{k}_{I}^{*}$	6.9	6.4	6.7	5.1
$k_l^*$ $k_h^*$	6.7	6.4	6.7	5.5
Capital buffer in state s				
$\Delta_l = k_l^* - \gamma_l$	3.8	3.3	3.5	2.0
$\Delta_h = k_h^* - \gamma_h$	1.2	0.1	1.2	0.0
Expected credit rationing in state $s'$				
(s,s')=(l,l)	0.9	1.3	1.2	2.7
(s,s')=(l,h)	12.6	20.2	18.2	31.2
(s,s')=(h,l)	5.3	6.1	5.3	7.1
(s,s')=(h,h)	12.4	14.3	12.4	16.6
Unconditional	5.6	7.3	5.9	10.1
Probability of bank failure				
First-period banks, $s = l$	0.16	0.22	0.20	0.39
First-period banks, $s = h$	2.25	2.55	2.25	2.96
Unconditional	0.90	1.05	0.85	1.31
Second-period banks, $s = l$	0.05	0.05	0.05	0.09
Second-period banks, $s = h$	0.76	0.76	0.76	1.10
Unconditional	0.31	0.31	0.28	0.46
Unconditional, all banks	0.61	0.68	0.56	0.88

This table has the same structure as Table 4 and reproduces in its first column the equilibrium outcomes obtained in the Basel II regime under the baseline parameterization of the model. The second column reports the Basel II outcomes when the excess cost of bank capital is raised from its baseline value of 8% to 9%. The third column reports the results obtained when changing the baseline value of the transition probability  $q_{II}$  so as to lengthen the expected duration of the low default state from 5 to 6 years (i.e., we set  $q_{II} = 0.833$  rather than  $q_{II} = 0.8$ ). The last column considers a reduction in the projects' success return *a* from 0.04 to 0.03.

The last column considers a reduction in the projects' success return a from 4% to 3%. This change lowers the profitability of second-period loans and hence, as discussed in Section 2.3, banks' incentives to protect their lending capacity by holding capital buffers in the previous period. The shift increases first-period loan rates, reduces excess capital in state h to zero (a corner solution), increases credit rationing and its procyclicality (especially along the sequence (l, h)), and makes banks significantly more likely to fail.

## 5. Welfare Analysis

In our risk-neutral economy, social welfare can be measured by the sum of the expected net present value of the income flows that the various agents extract from the funding or ownership of entrepreneurs' investment projects. These income flows have been already presented in prior sections, with two exceptions that play a key role in the normative results. First, we are going to consider that bank failures cause negative externalities that imply a loss of social welfare equal to a proportion c of the initial assets of the failed banks.

We will present the welfare comparison of the various regulatory regimes and find the welfare maximizing values of the minimum capital requirements  $\gamma_l$  and  $\gamma_h$  for values of *c* ranging from 0% to 60%.<sup>35</sup>

We are also going to consider that entrepreneurs extract more from their investment projects than just the residual part of the pledgeable success return left after repaying the bank loans. This is consistent with corporate finance theories that emphasize the role of control rents (Hart 1995) and show how incentive problems may give rise to (endogenous) fractions of corporate value that cannot be pledged to outside investors (Tirole 2005). We will assume that entrepreneurs appropriate a nonpledgeable return *b* per period whenever their investment projects are developed and succeed. The practical implication of this parameter is to introduce an extra cost associated with credit rationing. Absent direct empirical estimates of this parameter, we set b=a=0.04, which implies that the overall net present value generated by the investment projects is roughly twice as large as if only pledgeable returns were taken into account.<sup>36</sup>

Depositors are fully protected by deposit insurance and receive their required rate of return (normalized to zero) with probability one, so their stake in social welfare is just zero, and we can ignore their payoffs in the welfare calculations. Similarly, by the zero net present value condition (9) that characterizes equilibrium, bank shareholders break even on expectation over any two periods following a recapitalization of their banks, so we can also ignore their payoffs. This leaves us with entrepreneurs (as the projects' residual claimants) and the government (as insurer of bank deposits and internalizer of the social cost of bank failures) as the only two relevant classes of agents with a nontrivial stake in social welfare.

Assuming the entrepreneurs and the government discount their payoffs at the risk-free deposit rate (that we have normalized to zero), it is convenient to think of the social welfare criterion as the expected net present value of the payoffs that accrue to entrepreneurs and the government in connection with the undertaking and funding of the projects of a given cohort of entrepreneurs. In parallel to the expressions used in Section 2, we will provide expressions for welfare and its various components conditional on the states *s* and *s'* faced by the reference cohort of entrepreneurs in their first and second investment periods.

<sup>&</sup>lt;sup>35</sup> Laeven and Valencia (2008, 2010) provide a discussion and an empirical assessment of the social costs of bank crises. They differentiate between the direct costs of bank resolution, the overall deterioration of public finances (measured by the amount of government debt), and the output losses in the recessions normally following a banking crisis. They report the costs (which vary widely across various crisis episodes around the world) as a percentage of gross domestic product (GDP). Translating their numbers into our setup is not direct because the ratio of bank assets to GDP varies significantly over time and across countries. Because the most appropriate choice of *c* is unclear, we consider the range *c* ∈ [0,0.60].

<sup>&</sup>lt;sup>36</sup> Our results for b=0 (available from the authors upon request) suggest that in the absence of a significant nonpledgeable component in projects' returns, the social costs of capital requirements due to credit rationing are overwhelmed by the social cost of bank failures, tilting the welfare balance strongly in favor of a risk-based regime, such as Basel II, but with higher capital requirements in each state.

The Review of Financial Studies / v 0 n 0 2012

Therefore, social welfare over the investment sequence (s, s') can be written as

$$W_{ss'} = U_{ss'} + DI_{ss'} + BF_{ss'}, (15)$$

where

$$U_{ss'} = (1 - p_s)(a - r_s^* + b) + (1 - CR_{ss'})(1 - p_{s'})b$$
(16)

are the expected payoffs of the entrepreneurs over their two investment periods, inclusive of nonpledgeable returns,

$$DI_{ss'} = \int_{\widehat{x}_s}^{1} k'_s(x) dF_s(x) + (1 - CR_{ss'}) \int_{\widehat{x}_{s'}}^{1} [\gamma_{s'} + a - x(\lambda + a)] dF_{s'}(x)$$
(17)

are the (negative) payoffs to the government stemming from its role as insurer of bank deposits, and

$$BF_{ss'} = -c\{1 - F(\widehat{x}_s) + (1 - CR_{ss'})[1 - F(\widehat{x}_{s'})]\}$$
(18)

are the (negative) payoffs due to the social cost of bank failures, and  $\hat{x}_{s'} = (\gamma_{s'} + a)/(\lambda + a)$  is the threshold default rate above which second-period banks fail.

To explain (16), notice that the first term accounts for the payoff of entrepreneurs' first-period projects, which comprise a pledgeable return  $(1 + a) - (1 + r_s^*) = a - r_s^*$  and a non-pledgeable return *b* if their projects succeed. The second term accounts for the non-pledgeable return *b* obtained from second-period projects insofar as they are undertaken (with probability  $1 - CR_{ss'}$ ) and succeed (with probability  $1 - p_{s'}$ ). In (17) the two terms account for expected deposit insurance payouts associated with banks involved in first- and second-period projects, respectively (which are obtained by integrating banks' net worth at the end of the corresponding period over the realizations of the default rate for which they fail). Finally, (18) is the expected social cost of bank failure obtained by multiplying the proportional cost *c* by banks' average assets and their probabilities of failure in each of the two lending periods.

Our measure of social welfare W is the expected value of  $W_{ss'}$  over the four possible sequences (s, s') weighted by their ergodic probabilities. Our optimal capital requirements,  $\gamma_l^*$  and  $\gamma_h^*$ , are the values of  $\gamma_l$  and  $\gamma_h$  that maximize W.<sup>37</sup>

Figure 1 depicts W as a function of the social cost of bank failure c for each of the regulatory regimes examined in Section 4 as well as under the optimal capital requirements. The comparison with the laissez-faire regime shows that capital regulation adds to social welfare even in the polar scenario with c=0, which means that the loss of entrepreneurial surplus associated with credit

<sup>&</sup>lt;sup>37</sup> To avoid computational problems associated with the possible existence of multiple local maxima, we find  $(\gamma_l^*, \gamma_h^*)$  by evaluating *W* over a fine and wide grid of possible values of  $\gamma_l$  and  $\gamma_h$ .

The Procyclical Effects of Bank Capital Regulation

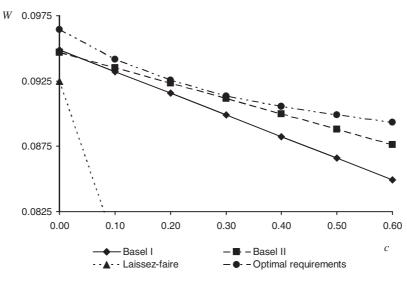


Figure 1

Social welfare versus the social cost of bank failure

This figure depicts our measure of social welfare W as a function of the social cost of bank failure c for each of the four regulatory regimes that we compare: the laissez-faire regime, Basel I, Basel II, and the regime with optimal capital requirements. The underlying parameterization is described in Table 3, and the nonpledgeable return b equals 0.04.

rationing provides a rationale for imposing capital requirements on banks.<sup>38</sup> The social welfare associated with the laissez-faire and the Basel regimes is linearly decreasing in c, with a slope equal to the respective average probability of bank failure (the term that multiplies -c in (18)). Social welfare is lower in Basel II than in Basel I and lower in Basel I than in the laissez-faire regime. In the regime with optimal capital requirements, social welfare is concave in c because  $\gamma_l^*$  and  $\gamma_h^*$  increase with c, offering greater protection against bank failure.

The impact of the welfare cost of credit rationing and the social cost of bank failure explains why Basel I (which, as previously discussed, implies essentially the same average levels of capital and hence the same overall excess cost of bank capital as Basel II) slightly dominates Basel II for very low values of c (less than 5%). The small discrepancy is the net result of the better performance of Basel I in terms of credit rationing and its worse performance in terms of bank failure risk. Opposite to our conjectures at the time of initiating this research

<sup>&</sup>lt;sup>38</sup> In fact, if feasible, banks might compete for first-period loans to entrepreneurs by committing to hold certain amounts of excess capital. Borrowers might accept higher first-period rates in exchange for a lower probability of being credit rationed in the second lending period. With c=0, one could think of bank capital regulation (and its enforcement by supervisors) as an alternative to deal with the commitment problems associated with this market discipline solution. Of course, with c>0, capital regulation would be desirable even if market discipline were feasible.

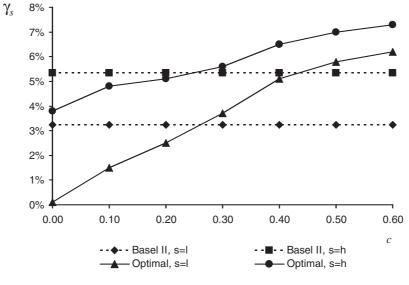


Figure 2

**Optimal capital requirements as a function of** *c* 

This figure depicts the optimal capital requirements for different values of the social cost of bank failure c. The horizontal lines correspond with the (Tier 1) capital requirements of Basel II, which are included to facilite the comparison. The underlying parameterization is described in Table 3, and the nonpledgeable return b equals 0.04.

project, the welfare losses due to credit rationing affect the comparison between Basel I and Basel II very little. This comparison is mainly driven by the social cost of bank failure.

Figure 2 depicts the optimal capital requirements  $\gamma_l^*$  and  $\gamma_h^*$  over the same range of values of *c* as in Figure 1, together with the Basel II requirements  $\gamma_l = 3.2\%$  and  $\gamma_h = 5.5\%$ . The cost of bank capital (relative to the low incidence of credit rationing at the implied equilibrium levels of capital) explains why for low values of *c* (less than 25%), the socially optimal capital requirements would be comparatively much more cyclical and lower in level than the requirements of Basel II. For instance, with *c*=0, the socially optimal  $\gamma_l^*$  is zero, whereas  $\gamma_h^*$  is close to 4%. The picture changes as *c* increases: The goal of reducing banks' probability of failure in state *l* and in preserving their capacity to lend when the economy switches to state *h* makes the optimal capital requirements higher in level and less cyclically varying. Curiously, under our calibration, the optimal level and degree of cyclical dependence of the optimal requirements virtually coincide with those associated with Basel II when the social cost of bank failure amounts to about 25% of bank assets. For larger values of *c*,  $\gamma_l^*$ and  $\gamma_h^*$  are higher but less cyclically varying than their Basel II counterparts.

Figure 2 provides an interesting preliminary assessment of the direction taken in Basel III. As described in BCBS (2010), the new international agreement on regulatory standards reinforces capital regulation by means of

higher requirements of core Tier 1 capital and by complementing them with a capital preservation buffer and a countercyclical buffer. The idea behind these mandatory buffers is to force banks to build up buffers in good times (state l) and release them in bad times (state h). Under the lens of our model, this new regulation would be consistent with a social welfare maximizing choice under the assessment that the social cost of bank failure is at or above the highest levels of c depicted in Figure 2. For these levels of the social cost of bank failure, the optimal capital requirements are higher but have much less cyclical variability than those of Basel II, which suggests that countercyclical add-ons, such as the new mandatory buffers, go in the right direction.<sup>39</sup>

An important insight from Figure 2 is that, insofar as the social cost of bank failure varies across countries, a one-size-fits-all system of capital requirements is suboptimal. For instance, countries with a very big (and hence more systemic) financial sector might face larger negative externalities (per unit of bank assets) when banks fail, in which case their parameter c would be toward the right in the horizontal axis of Figure 2. These countries might benefit from using national discretion to impose (countercyclical) capital surcharges on top of the common minimum standards.

To conclude this section, we consider the effects of changing the nonpledgeable return *b* that entrepreneurs obtain when their projects are successful. Figure 3 depicts the values of the optimal capital requirements  $\gamma_l^*$  and  $\gamma_h^*$  (together with the reference values of their Basel II counterparts) as a function of *b*. To facilitate comparisons, the central value of *b* in this figure is 0.04 (the reference value in Figure 2), and the social cost of bank failure is set at its central value in Figure 2 (*c*=0.3). Over the depicted range, increasing *b* mildly increases the optimal capital requirements, with a somewhat stronger impact on  $\gamma_l^*$  than on  $\gamma_h^*$ , which implies reducing the cyclicality of the requirements. Both adjustments respond to the logic that *b* accounts for a cost associated with credit rationing that the social planner internalizes but banks do not. A higher *b* implies higher but less cyclically varying optimal capital requirements to protect banks' lending capacity, especially when the economy switches from the low default state *l* to the high default state *h*.

Figure 3 can also be read in terms of the limits to the one-size-fits-all principle, applied in this case to a cross-section of potentially heterogeneous borrowers. The pledgeability of the success return of a bank-funded project may depend on the type of business, on the nature of its production technology (e.g., more or less based on tangible assets), on the transparency of its accounts (e.g., on whether it is a publicly traded or a privately held corporation), on bankruptcy law, etc. Lending to businesses characterized by less pledgeability (SMEs?) might require higher but less cyclically varying capital requirements.

<sup>&</sup>lt;sup>39</sup> See Repullo and Saurina (2012) for a critique of the design of the countercyclical buffer of Basel III.

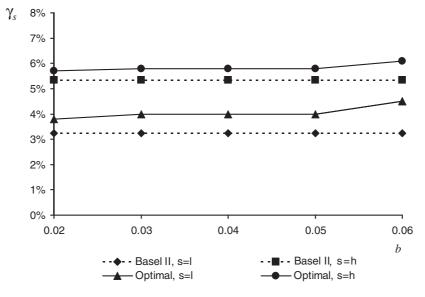


Figure 3

Optimal capital requirements as a function of b

This figure depicts the optimal capital requirements for different values of the nonpledgeable income b that entrepreneurs obtain from their successful projects. The horizontal lines correspond with the (Tier 1) capital requirements of Basel II, which are included to facilite the comparison. The underlying parameterization is described in Table 3, and the social cost of bank failure c equals 0.30.

# 6. Discussion

In this section, we discuss some simplifying features of our model, including the distribution of banks' market power in the first and the second lending periods, the use of short-term loans, and the assumption that banks with ongoing relationships have no access to the equity market. We consider the possible effects of relaxing these assumptions, placing the emphasis on implications regarding capital buffers and credit rationing, which are the most distinctive features of our model.

## 6.1 Competition and market power

In our model, banks are perfectly competitive in the market for first-period loans and act as monopolists in the market for second-period loans. These assumptions guarantee tractability and internal consistency. First, we avoid the complications associated with modeling imperfect competition in the market for initial loans. In such a scenario, loans from different banks would not be perfect substitutes for the initial borrowers, and banks could extract positive surplus from starting lending relationships with specific borrowers, in which case the assumption that all new relationships go to banks that can unrestrictedly raise capital would be less justified. On the one hand, some initial borrowers might find that their most preferred banks suffer capital constraints. On the

other, banks would have to take the value of these prospective relationships into account when deciding their capital buffers. Ignoring these complications does not seem to obviously bias our results in one direction or another, whereas it is clear that adding them would break the simple OLG structure of the model.

As for second-period loans, we have assumed that borrowers are fully locked in to their initial lender. This is consistent with the existence of an (unmodeled) asymmetric information problem (e.g., one that makes borrowers searching for a new bank after one period look like lemons in the refinancing market). If the market for continuation loans were more competitive, the degree of effective competition and the resulting loan rates would vary with the lending capacity of the banks involved. Even in the polar case of perfect competition, banks would be able to appropriate scarcity rents from their (nonrationed) borrowers when rationing emerges in equilibrium.<sup>40</sup>

Relative to the current modeling, a more competitive market for secondperiod loans would entail lower continuation rents for banks, and hence lower incentives to keep capital buffers. Formally, more competition in the market for second-period loans might be captured as a reduction in the projects' pledgeable success return a (the second-period loan rate) accompanied by a matching increase in entrepreneurs's nonpledgeable return b. Banks' equilibrium behavior under given capital requirements depends on a but not on b, so more competition would have the effects qualitatively reflected in the first column of Tables 1 and 2 and quantified (under our baseline parameterization, for a reduction in a from 4% to 3% under Basel II) in the last column of Table 5. The effects are an increase in first-period loan rates, a reduction in capital buffers, higher credit rationing, more procyclicality in credit supply, and greater probabilities of bank failure.

What should be the regulatory response to greater competition? For the change in the last column of Table 5, we have computed the socially optimal capital requirements that would emerge with c=0.30 (the central value of Figure 2) and b=0.05 (i.e., when entrepreneurs appropriate the extra 1% success return that banks lose because of greater competition). The optimal requirements turn out to be 6.0% in the low default state and 7.6% in the high default state (vs. 4.0% and 5.8%, respectively, in the baseline scenario). This suggests, along the same logic as several other results obtained throughout the paper, that higher competition should be accommodated with higher but less cyclically varying capital requirements.

#### 6.2 Short-term loan contracts

We have described the relationship between entrepreneurs and banks as instrumented by a sequence of one-period loans. One might wonder whether

<sup>&</sup>lt;sup>40</sup> Under the assumption of a single common risk factor, either all banks or no bank are capital constrained in the market for second-period loans, so the second-period loan rate would be either the monopoly rate *a* or some breakeven rate that makes continuation lending a zero net present value investment for the banks.

this builds in the "imperfection" that drives our main results. The answer is yes and no. With long-term contracts there might be room for improving over the credit allocation outcomes obtained in our analysis. For example, for given capital buffers, setting higher loan rates in the first period and lower loan rates in the second would reduce the incidence of credit rationing because banks would have more capital to support their second-period lending. But, in the context of our model, long-term contracts pose important commitment problems. In particular, they would have to specify the first- and second-period loan rates and the rationing scheme to use in cases in which the bank ends with insufficient lending capacity (because otherwise the bank might try to renegotiate the terms of the second-period loans by threatening the entrepreneurs with rationing). This means that default rates in the first period would have to be verifiable, and banks would have to be restricted in their ability to pay dividends because lack of capital in the second period might otherwise be strategically used by banks to extract a larger surplus from their locked-in borrowers.

#### 6.3 Imperfect access to the equity market

The assumption that banks with ongoing relationships have no access to the equity market is obviously crucial for our results. With perfect, frictionless access to the equity market in the second lending period, there would be no credit rationing among second-period borrowers, except in the rare event of bank failure (i.e., in which losses are so large that shareholders decide not to recapitalize the bank). Banks in such a context would most likely hold no buffers. Given the ample evidence on capital market imperfections, the key question is whether the specificities of our approach—that ties these imperfections to the informational asymmetries associated with relationship lending—drive the results.

A more general way of capturing capital market imperfections would be to assume that access can occur with some (exogenous) probability  $\upsilon < 1$ . Changes in  $\upsilon$  could then be used to evaluate the marginal effects of the friction on capital buffers and credit rationing. One could also explore situations in which the access to the equity market in the second lending period occurs with some probability  $\upsilon_{s'}$  contingent on the state s' of the economy in that period. This extension would probably reinforce our conclusions about the procyclical effects of risk-based capital requirements, such as those of Basel II. If with some probability banks can access the equity market, they would have lower incentives to keep capital buffers, so depending on parameter values the incidence of credit rationing could be higher.<sup>41</sup> Of course, the procyclicality of bank credit supply would be even stronger if in a situation with  $\upsilon_l > \upsilon_h$ , banks

<sup>&</sup>lt;sup>41</sup> One strong assumption that we make is that banks have frictionless access to the equity market when they renew their stock of lending relationships. This assumption is instrumental in achieving a tractable OLG structure but has the unattractive feature of making first-period borrowers immune to rationing. A modeling alternative would be to assume a structure similar to the one in the popular Calvo (1983) model of staggered price setting, that is, that in each period a fraction of the banks can issue new equity. In this context one would have to discuss the

in good times worked under the (wrong) assumption that the access to equity markets occurs with probability  $v_l$  in all states.

## 6.4 Other extensions

The framework used in this paper could also be extended in a number of other directions. First, we could consider lending relationships that extend over more than two periods. If relationships last for T periods and banks cannot raise equity for the whole length of the relationship, the qualitative results should be very similar to ours. Such a model would, of course, yield richer dynamics, as the effect of a shock would propagate over several periods. Second, we could incorporate cyclically varying demand for loans. One easy way of introducing a downward-sloping aggregate demand for loans would be to assume that entrepreneurs are heterogeneous in their opportunity cost of becoming active in the first period. The time-series variability in projects' success probabilities would tend to produce a larger demand for loans in the low than in the high default state. Further cyclical variability in this demand might be introduced by replacing the current success return a for some  $a_s$ . Finally, one could allow for feedback effects from constrained to unconstrained entrepreneurs by letting  $a_t = a(I_t)$  instead of a, where  $a(I_t)$  is an increasing (and possibly concave) function and  $I_t$  is the aggregate investment at date t. This would capture demand externalities or technological complementarities similar to those studied in endogenous growth theory.

Additional cyclicality might be introduced by allowing for cyclical variation in the loss given default (LGD) parameter  $\lambda$ . Using data on bond defaults, Altman et al. (2005) find that LGDs are positively correlated with default rates, thus suggesting that  $\lambda$  might be higher in state *h* than in state *l*. Moreover, under the so-called "advanced internal ratings-based (IRB) approach" of Basel II, banks must compute their capital requirements taking into account the estimate of  $\lambda$  derived from their internal models. This means that in an advanced IRB regulatory environment capital requirements would exhibit even more cyclicality than in the "foundation IRB" environment that we have considered. Conceptually, the implications of a cyclically varying  $\lambda$  are thus very similar (both within and between regulatory regimes) to those of increasing the amplitude of the cyclical variation in the probability of default parameter  $p_s$ , which would certainly tend to exacerbate the cyclical effects that we find.

## 7. Concluding Remarks

We assess the cyclical effects of bank capital regulation and its implications for the design of optimal capital requirements in the context of a simple OLG model

allocation of the newly born entrepreneurs to the existing banks. Would they demand loans to the recapitalizing banks only? If not, how would the pricing of the new loans be determined, and what would be the effect on banks' incentives to hold excess capital?

#### The Review of Financial Studies / v 0 n 0 2012

with two aggregate default states (identified with expansions and recessions) in which banks may hold capital buffers in excess of the minimum regulatory requirements in anticipation of possible difficulties to raise equity capital in the future. We calibrate the model using U.S. data for the period prior to the financial crisis that started in 2007, and compare a laissez-faire regime with no capital requirements, Basel I, Basel II, and the regime in which capital requirements maximize a social welfare function that incorporates a social cost of bank failure. Although our analysis relies on a number of simplifying assumptions, including abstracting from demand side fluctuations and aggregate feedback effects,<sup>42</sup> we believe that it helps identify some of the key trade-offs for the assessment of the different regulatory regimes and for the design of optimal capital requirements.

We show that the interaction of relationship lending (which makes borrowers dependent on the future lending capacity of their bank) with frictions in banks' access to equity markets (which may constraint their future lending capacity) has the potential to cause significant cyclical swings in the supply of credit. We find that the swings are more pronounced under the risk-based requirements of Basel II than under the flat requirement of Basel I. Specifically, despite Basel II inducing banks to hold larger buffers during expansions (in order to prepare for the rise in capital requirements when entering a recession), the arrival of recessions is likely to produce sizeable credit rationing among borrowers dependent on capital-constrained banks.

However, bank failure probabilities are also related to the state of the business cycle and risk-based requirements, such as those of Basel II are more effective in bringing them down during recessions, when they are larger. This means that the welfare comparison between the two Basel regimes, as well as the design of socially optimal capital requirements, involves nontrivial trade-offs. In fact, we find that Basel II dominates Basel I in welfare terms, except for very low values of the social cost of bank failure. We also find that the optimal capital requirements are lower and more cyclically varying than the requirements of Basel II if the social cost of bank failure is low and higher and less cyclically varying if it is large.

So, conditional on assessing that the social cost of bank failure is large (as the recent crisis might have confirmed), Basel III may be considered a move in the right direction: The new capital conservation and countercyclical buffers would assume the role of making the effective capital requirements faced by banks less sensitive to the cycle (in particular, less prone to produce credit rationing at the arrival of recessions) than the Basel II requirements.

The analysis throws some caveats about the design of capital requirements and about the robustness (or potential biases) of our results. By being explicit about the economic determinants of the socially optimal requirements, we

<sup>&</sup>lt;sup>42</sup> To capture loan demand and aggregate feedback effects, one could embed our model into a fuller macroeconomic setup along the lines suggested in Section 6.4.

uncover the limits of the one-size-fits-all principle. Economies with larger social costs of bank failure (e.g., with a banking sector whose size implies a larger systemic threat) or loans to borrowers with projects that involve a larger fraction of nonpledgeable returns (e.g., because of fiercer competition between lenders or because of lower transparency of their production processes) should face larger but less cyclically varying capital requirements than economies or loans at the other side of the spectrum.

Finally, although a key contribution of the paper is to incorporate the discussion about credit supply effects to the positive and normative analysis of bank capital requirements, there are several reasons to believe that our quantitative estimates of these effects are closer to a lower bound than to an unbiased estimate. As noted above, allowing for cyclical variation in the excess cost of bank capital, in capital market imperfections, or in the loans' loss given default (that we consider constant in the baseline model), as well as for greater competition in the market for continuation loans, would most likely exacerbate the procyclical effects of bank capital requirements, calling for additional countercyclical amendments to the regulation.

## Appendix

## **Proof of Proposition 2**

The existence of a solution to the bank's capital decision problem follows, by the Weierstrass theorem, from the fact that the net present value  $v_s(k_s, r_s)$  is continuous in  $k_s$  for any given interest rate  $r_s$ . The function  $v_s(k_s, r_s)$  may be written as

$$v_{s}(k_{s}, r_{s}) = q_{sl}v_{sl}(k_{s}, r_{s}) + q_{sh}v_{sh}(k_{s}, r_{s}),$$
(A1)

where

$$v_{ss'}(k_s, r_s) = \frac{1}{1+\delta} \left[ \int_0^{\hat{x}_{ss'}} [\pi_{s'} + k'_s(x) - \gamma_{s'}] dF_s(x) + \frac{\pi_{s'}}{\gamma_{s'}} \int_{\hat{x}_{ss'}}^{\hat{x}_s} k'_s(x) dF_s(x) \right] - k_s.$$
(A2)

Using the definitions of  $k'_s(x)$  in (3),  $\hat{x}_s$  in (4), and  $\hat{x}_{ss'}$  in (5), one can establish the following properties of function  $v_{ss'}(k_s, r_s)$ :<sup>43</sup>

(1) For  $k_s \le \mu - r_s$ , we have  $\widehat{x}_{ss'} < \widehat{x}_s \le 0$ , so

$$\frac{\partial v_{ss'}}{\partial k_s} = -1 < 0. \tag{A3}$$

(2) For  $\mu - r_s < k_s \le \mu - r_s + \gamma_{s'}$ , we have  $\widehat{x}_{ss'} \le 0 < \widehat{x}_s$ , so

$$\frac{\partial v_{ss'}}{\partial k_s} = \frac{\pi_{s'}}{(1+\delta)\gamma_{s'}} F_s(\widehat{x}_s) - 1 \leq 0, \text{ and } \frac{\partial^2 v_{ss'}}{\partial k_s^2} = \frac{\pi_{s'} F_s'(\widehat{x}_s)}{(1+\delta)\gamma_{s'}(\lambda+r_s)} > 0.$$
(A4)

<sup>&</sup>lt;sup>43</sup> Note that in (A2) we do not have to worry about values of  $\hat{x}_s$  and  $\hat{x}_{ss'}$  smaller than 0 or greater than 1, because the distribution  $F_s(x)$  has support [0, 1].

The Review of Financial Studies / v 0 n 0 2012

(3) For 
$$\mu - r_s + \gamma_{s'} < k_s < \mu + \lambda + \gamma_{s'}$$
, we have  $0 < \widehat{x}_{ss'} < 1$ , so

$$\frac{\partial v_{ss'}}{\partial k_s} = \frac{1}{(1+\delta)\gamma_{s'}} \left[ \pi_{s'} F_s(\widehat{x}_s) - (\pi_{s'} - \gamma_{s'}) F_s(\widehat{x}_{ss'}) \right] - 1 \leq 0, \tag{A5}$$

and

$$\frac{\partial^2 v_{ss'}}{\partial k_s^2} = \frac{1}{(1+\delta)\gamma_{s'}(\lambda+r_s)} [\pi_{s'}F_s'(\widehat{x}_s) - (\pi_{s'} - \gamma_{s'})F_s'(\widehat{x}_{ss'})] \leq 0.$$
(A6)

(4) For  $\mu + \lambda + \gamma_{s'} \leq k_s$ , we have  $1 \leq \widehat{x}_{ss'} < \widehat{x}_s$ , so

$$\frac{\partial v_{ss'}}{\partial k_s} = \frac{1}{1+\delta} - 1 < 0. \tag{A7}$$

Hence, the function  $v_{ss'}(k_s, r_s)$  is linearly decreasing or strictly convex for  $k_s \le \mu - r_s + \gamma_{s'}$ , linearly decreasing for  $k_s \le \mu + \lambda + \gamma_{s'}$ , and may be increasing or decreasing, and concave or convex for  $\mu - r_s + \gamma_{s'} < k_s < \mu + \lambda + \gamma_{s'}$ . Because  $\gamma_l \le \gamma_h$  implies  $\mu - r_s + \gamma_l \le \mu - r_s + \gamma_h$  and  $\mu + \lambda + \gamma_l \le \mu + \lambda + \gamma_h$ , it follows that the problem  $\max_{\gamma_s \le k_s \le 1} v_s(k_s, r_s)$  cannot have an interior solution with  $k_s \le \mu - r_s + \gamma_l$  or with  $k_s \ge \mu + \lambda + \gamma_h$ . Hence, if there is an interior solution it must involve  $k_s \in (\mu - r_s + \gamma_l, \mu + \lambda + \gamma_h)$ . Because  $\hat{x}_{sl} > 0$  for  $\mu - r_s + \gamma_l < k_s$ , it follows that there is a positive probability  $F_s(\hat{x}_{sl})$  that the bank has excess lending capacity in state l (and given that  $\hat{x}_{sl} > \hat{x}_{sh}$  possibly also in state h). Also, because  $\hat{x}_{sh} < 1$  for  $k_s < \mu + \lambda + \gamma_h$ , it follows that there is a positive probability  $1 - F_s(\hat{x}_{sh})$  that the bank has insufficient lending capacity in state h (and given that  $\hat{x}_{sl} > \hat{x}_{sh}$  possibly also in state l). Finally, assuming that  $\mu + \lambda + \gamma_h < 1$  (in our calibration  $\mu + \lambda + \gamma_h = 0.535$ ), the fact that  $v_s(k_s, r_s)$  is decreasing in  $k_s$  for  $k_s \ge \mu + \lambda + \gamma_h$  implies that there cannot be a solution at the corner  $k_s = 1$ .

#### **Proof of Proposition 3**

By the theorem of the maximum, the function  $v_s(k_s(r_s), r_s)$ , where  $k_s(r_s)$  denotes banks' optimal choice of  $k_s$  given  $r_s$ , is continuous in  $r_s$ . Moreover, we have

$$\frac{dv_s}{dr_s} = \frac{\partial v_s}{\partial k_s} \frac{dk_s}{dr_s} + \frac{\partial v_s}{\partial r_s}.$$
(A8)

When  $k_s(r_s)$  is interior, the first term is zero, by the envelope theorem, and the second is positive, because  $r_s$  has a positive impact on  $k'_s(x_t)$  and consequently on  $v_{ss'}(x_t)$ . When  $k_s(r_s)$  is at the corner  $\gamma_s$ , we have  $dk_s/dr_s = 0$ , whereas the second term is positive as in the previous case. Hence, we have  $dv_s/dr_s > 0$ . Moreover, for sufficiently low interest rates we have  $v_s(k_s(r_s), r_s) < 0$ , whereas for  $r_s = a$ , Assumption 1 implies  $v_s(k_s(r_s), r_s) > 0$ . Hence, we conclude that there is a unique  $r_s^*$  that satisfies  $v_s(k_s(r_s^*), r_s^*) = 0$ .

**Comparative statics of the equilibrium loan rate.** The sign of  $dr_s^*/dz$  for  $z=a,\lambda,\mu,\delta,\gamma_l$ ,  $\gamma_h, q_{sh}$  is obtained by total differentiation of (9):

$$\frac{\partial v_s}{\partial k_s} \frac{dk_s^*}{dz} + \frac{\partial v_s}{\partial r_s} \frac{dr_s^*}{dz} + \frac{\partial v_s}{\partial z} = 0.$$
(A9)

When  $k_s^*$  is interior, the first term in (A9) is zero, by the envelope theorem. Hence, we have

$$\frac{dr_s^*}{dz} = -\left(\frac{\partial v_s}{\partial r_s}\right)^{-1} \frac{\partial v_s}{\partial z}.$$
(A10)

In the proof of Proposition 3 we have noted that  $\partial v_s/\partial r_s > 0$ , so the sign of  $dr_s^*/dz$  is opposite to the sign of  $\partial v_s/\partial z$ . Similarly, when  $k_s^* = \gamma_s$ , we have  $dk_s^*/dz = 0$  for all  $z \neq \gamma_s$ , in which case the

 Table A1

 Effects on the net present value of the bank

	z	=	а	λ	$\mu$	δ	γı	$\gamma_h$	$q_{sh}$
$\frac{\partial v_s}{\partial z}$			+	-	-	_	-	-	_

Table A2 Effects on the marginal value of capital  $z = a + \lambda + \mu + \delta + \nu$ 

	z	=	а	λ	$\mu$	0	Υh	γl	$q_{sh}$
$\frac{\partial^2 v_s}{\partial k_s  \partial z}$			+	?	-	-	?	?	?

first term in (A9) is also zero and (A10) obtains again. Finally, for  $z = \gamma_s$ , we have  $dk_s^*/d\gamma_s = 1$ , which implies

$$\frac{dr_s^*}{l\gamma_s} = -\left(\frac{\partial v_s}{\partial r_s}\right)^{-1} \left(\frac{\partial v_s}{\partial \gamma_s} + \frac{\partial v_s}{\partial k_s}\right),\tag{A11}$$

where  $\partial v_s / \partial k_s \leq 0$ , because otherwise  $k_s^* = \gamma_s$  would not be optimal. With these expressions in mind, the results in Table 1 can be immediately related to the (self-explanatory) signs of the partial derivatives of  $v_s(k_s^*, r_s^*)$  that we summarize in Table A1 (and whose detailed expressions we omit, for brevity).

**Comparative statics of the equilibrium capital.** When the equilibrium capital in state *s* is at the corner  $k_s^* = \gamma_s$ , with  $\partial v_s / \partial k_s < 0$ , marginal changes in any parameter other than  $\gamma_s$  have no impact on  $k_s^*$ , whereas obviously  $dk_s^*/d\gamma_s = 1$ . Thus, in what follows we focus on the more interesting interior solution case.<sup>44</sup>

The sign of  $dk_s^*/dz$  for  $z=a, \lambda, \mu, \delta, \gamma_l, \gamma_h, q_{sh}$  is obtained by total differentiation of the first-order condition  $\partial v_s/\partial k_s = 0$  that characterizes an interior equilibrium:

$$\frac{\partial^2 v_s}{\partial k_s^2} \frac{dk_s^*}{dz} + \frac{\partial^2 v_s}{\partial k_s \partial r_s} \frac{dr_s^*}{dz} + \frac{\partial^2 v_s}{\partial k_s \partial z} = 0, \tag{A12}$$

where  $\partial^2 v_s / \partial k_s^2 < 0$  by the second-order condition, which implies

$$\frac{dk_s^*}{dz} = -\left(\frac{\partial^2 v_s}{\partial k_s^2}\right)^{-1} \left(\frac{\partial^2 v_s}{\partial k_s \partial z} + \frac{\partial^2 v_s}{\partial k_s \partial r_s}\frac{dr_s^*}{dz}\right).$$
(A13)

Hence, the sign of  $dk_s^*/dz$  coincides with the sign of the last term in brackets, which has two components: the direct effect of z on  $k_s^*$  (for constant  $r_s^*$ ) and the loan rate effect (due to the effect of z on  $r_s^*$ ). The signs of the direct effects shown in the first row of Table 2 coincide with the signs of the cross derivatives  $\partial^2 v_s/\partial k_s \partial z$  summarized in Table A2 (whose detailed expressions we omit, for brevity).

The loan rate effects shown in the second row of Table 2 can be easily determined from the results summarized on Table 1 and the fact that  $\partial^2 v_s / \partial k_s \partial r_s$  is negative. To see the latter, one can

<sup>&</sup>lt;sup>44</sup> The case with  $k_s^* = \gamma_s$  and  $\partial v_s / \partial k_s = 0$  is a mixture of both cases because, depending on the sign of the effect of the marginal variation in a parameter, the optimal decision might shift from being at the corner to being interior. A similar complexity may occur if the change in a parameter breaks some underlying indifference between an interior and a corner solution (or between two interior solutions). We will omit the discussion of these cases, for simplicity.

verify that the way  $k_s$  and  $r_s$  enter in  $v_s(k_s, r_s)$  implies  $\partial^2 v_{ss'}/\partial k_s \partial r_s = \partial^2 v_{ss'}/\partial k_s^2$  (see the proof of Proposition 2 for some relevant intermediate expressions). Hence, using (A1) we conclude

$$\frac{\partial^2 v_s}{\partial k_s \partial r_s} = q_{sl} \frac{\partial^2 v_{sl}}{\partial k_s \partial r_s} + q_{sh} \frac{\partial^2 v_{sh}}{\partial k_s \partial r_s} = q_{sl} \frac{\partial^2 v_{sl}}{\partial k_s^2} + q_{sh} \frac{\partial^2 v_{sh}}{\partial k_s^2} = \frac{\partial^2 v_s}{\partial k_s^2} < 0, \tag{A14}$$

where the last inequality follows from the second-order condition for an interior solution in  $k_s$ .

The single common risk factor model and the Basel II formula. The derivation of (11) can be summarized as follows. Suppose that the project undertaken by entrepreneur *i* in a date in which the state of the economy is *s* fails if  $y_i < 0$ , where  $y_i$  is a latent random variable defined by

$$y_i = \alpha_s + \sqrt{\rho_s} \ u + \sqrt{1 - \rho_s} \ \varepsilon_i, \tag{A15}$$

where  $\alpha_s$  is a state-contingent parameter that determines the mean of the latent variable, u is the single common risk factor,  $\varepsilon_i$  is an idiosyncratic risk factor, and  $\rho_s \in (0, 1)$  is a (potentially state-dependent) parameter that determines the extent of correlation in project failures. Suppose further that u and  $\varepsilon_i$  are standard normal random variables, independently distributed from each other and over time, as well as, in the case of  $\varepsilon_i$ , across projects. The probability of failure of the project of entrepreneur i in state s is  $p_s = \Pr(y_i < 0) = \Phi(-\alpha_s)$ , because  $y_i \sim N(\alpha_s, 1)$ , which implies  $\alpha_s = -\Phi^{-1}(p_s)$ .

With a continuum of projects, the failure rate x (the fraction of projects that fail) will only be a function of the realization of the common factor u. Specifically, by the law of large numbers, the effects of the idiosyncratic factors  $\varepsilon_i$  will be diversified away and x will coincide with the probability of failure of a (representative) project conditional on the state of the economy s and the realization of u:

$$x = g_s(u) = \Pr\left(-\Phi^{-1}(p_s) + \sqrt{\rho_s} \ u + \sqrt{1 - \rho_s} \ \varepsilon_i < 0 \ | \ u\right) = \Phi\left(\frac{\Phi^{-1}(p_s) - \sqrt{\rho_s} \ u}{\sqrt{1 - \rho_s}}\right).$$
(A16)

The cdf of the failure rate is  $F_s(x) = \Pr(g_s(u) \le x) = \Pr(u \ge g_s^{-1}(x))$  (because  $g'_s(u) < 0$ ), so using the definition of  $g_s(u)$  and the fact that  $u \sim N(0, 1)$  we get (11).

The idea behind the capital requirements associated with the Basel II formula is to require banks to have enough overall (Tier 1 + Tier 2) capital to cover their one-year-ahead loan losses with a probability of 99.9%. In the context of our model, the implied capital requirement in state *s* is then  $\lambda F_s^{-1}(0.999)$ , where  $\lambda$  is the loss given default and  $F_s^{-1}(0.999)$  is the 99.9% quantile of the distribution of the default rate. To obtain an explicit formula for this quantile, one can invert the cdf of the default rate *x* in state *s* as given by (11). This yields a formula very similar to that in (12) except for two differences. The first is that the term in  $\rho_s$ , which appears in (11), is replaced in (12) by the function  $\rho(p_s)$ . This function was introduced by the Basel regulators on the basis of cross-sectional evidence pointing to default correlation being smaller for exposures with higher probabilities of default. The second difference is that (12) refers to the minimum regulatory requirement of Tier 1 capital only, which is just one half of the minimum overall (Tier 1 + Tier 2) capital requirement established in the Basel II formula.

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