

FLAT TAX REFORMS IN THE U.S.: A BOON FOR THE INCOME POOR

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Summary: In this article we quantify the aggregate, distributional and welfare consequences of two revenue neutral flat-tax reforms using a model economy that replicates the U.S. distributions of earnings, income and wealth in very much detail. We find that the less progressive reform brings about a 2.4 percent increase in steady-state output and a more unequal distribution of after-tax income. In contrast, the more progressive reform brings about a -2.6 percent reduction in steady-state output and a distribution of after-tax income that is more egalitarian. We also find that in the less progressive flat-tax economy aggregate welfare falls by -0.17 percent of consumption, and in the more progressive flat-tax economy it increases by 0.45 percent of consumption. In both flat-tax reforms the income poor pay less income taxes and obtain sizeable welfare gains.

Keywords: Flat-tax reforms; Efficiency; Inequality; Earnings distribution; Income distribution; Wealth distribution.

JEL Classification: D31; E62; H23

*Díaz-Giménez, Universidad Carlos III de Madrid and CAERP <kueli@eco.uc3m.es> and Pijoan-Mas, CEMFI and CEPR <pijoan@cemfi.es>. This paper follows up from the research conducted by Ana Castañeda for her PhD Dissertation and from later work with José-Víctor Ríos-Rull. Ana is no longer in academics and has gracefully declined to sign this paper, which is hers as much as ours. We have benefitted enormously both from her input and from her code. José-Víctor moved on to other projects when we were well into this research. To both of them we are most grateful. Additionally, this research has benefited from comments by assistants to seminars held at CEMFI, Queen Mary, Universidad de Alicante, Universidad Carlos III, Universitat de Barcelona, Universitat Pompeu Fabra and University of Oslo, as well as by attendants to the 12th International Conference on Computing in Economics and Finance. Díaz-Giménez gratefully acknowledges the financial support of the Fundación de Estudios de Economía Aplicada (FEDEA) and of the Spanish Ministerio de Ciencia y Tecnología (Grant SEC2002-004318).

1 Introduction

The debate on fundamental tax reforms has been heating up in recent years as some countries, mostly in Eastern Europe, are starting to adopt flat-tax systems. Since the publication of the seminal work of Hall and Rabushka (1995), academics have been arguing in favor of a simplification of the tax code, a broadening of the tax base and a reduction of marginal taxes.

More recently, academics have simulated the consequences of fundamental tax reforms in models of the U.S. economy. Ventura (1999), for instance, studies a flat-tax reform in a quantitative general equilibrium model of the U.S. economy. He concludes that a flat-tax reform would bring about large gains in output and productivity at the expense of significant increases in inequality. Using a similar methodology, Altig, Auerbach, Kotlikoff, Smetters, and Walliser (2001) also find that a flat-tax reform of the current U.S. income tax system would result in aggregate output gains, and that it would benefit both the very income-poor and the very income-rich at the expense of the middle classes. In this article, we take the discussion one step further, and we simulate two flat-tax reforms in a model economy that replicates most of the features of the current U.S. tax and transfer systems, and that accounts for the aggregate and distributional features of the U.S. economy in much greater detail than previous research.

Our model economy is a version of the neoclassical growth model that combines dynastic and life-cycle features and it is an extension of the model economy described in Castañeda, Díaz-Giménez, and Ríos-Rull (2003). Our households have identical preferences, they are altruistic towards their descendants, and they go through the life-cycle stages of working age and retirement. The duration of their lives and their wages are random, and they make optimal dynamic consumption and labor decisions. The firms in our model economy behave competitively and all prices are flexible.

As shown in Castañeda, Díaz-Giménez, and Ríos-Rull (2003), this class of model economies replicates the U.S. marginal distributions of labor earnings, income and wealth in very much detail. This is a critical feature for the quantitative evaluation of tax reforms because the tax burdens and the incentives to work and save that a tax code creates are very different at different points of the earnings and wealth distributions. Moreover, as Mirrlees (1971) pointed out, the distributional details are fundamental in measuring the trade-offs involved in choosing between efficiency and equality, since both the aggregate and the welfare changes depend crucially on the exact number of households of each type that there are in the economy.

Another distinguishing feature of our model economy is that we replicate the U.S. tax system and the lump-sum part of U.S. transfers in very much detail. Specifically, our bench-

mark model economy includes a personal income tax, a corporate income tax, a payroll tax, a consumption tax and an estate tax. We have designed these taxes to replicate the main properties and to collect the same revenues as the corresponding taxes in the U.S. economy. To simulate the flat-tax reform, we replace our versions of the corporate income tax and the personal income tax with an integrated flat-tax on capital and labor income. To make the average tax rates of the labor income tax progressive, a large amount of labor income is tax-exempt.

We study two revenue-neutral flat-tax reforms that differ in their tax rates and in the amounts of the labor income tax exemption. In the first flat-tax reform the tax rate is 22 percent and the labor income tax exemption is \$16,000 per household. In the second flat-tax reform the tax rate is 29 percent and the labor income tax exemption is \$32,000 per household. For obvious reasons, we call the first reform the less progressive flat-tax reform and we call the second reform the more progressive flat-tax reform.

We find that these two flat-tax reforms have very different steady-state aggregate, distributional and welfare consequences. The less progressive flat-tax reform turns out to be more efficient than the current progressive tax system. Under this reform, steady-state output and labor productivity increase by 2.4 and 3.2 percent, when compared with the corresponding values of the benchmark model economy. In contrast, the more progressive flat-tax reform is less efficient than the current income tax system. Under this reform, steady-state output and labor productivity decrease by -2.6 percent and -1.4 percent. These last results, which we discuss at length in Section 5 below, differ widely from the findings of previous research and from the conventional wisdom about the efficiency of flat taxes.

We also find that both reforms result in significant increases in wealth inequality. The Gini index of wealth in our benchmark model economy is 0.818.¹ In the steady-state of the less progressive flat-tax reform it increases to 0.839, and in the steady state of the more progressive flat-tax reform it increases further to 0.845. However, both reforms have very different distributional implications in other dimensions. For instance, the less progressive flat-tax reform results in more unequal distributions of earnings and, more importantly, of after-tax income (their Gini indexes are 0.615 and 0.524 compared to 0.613 and 0.510 in the benchmark economy). In contrast, and perhaps surprisingly, the distributions of earnings and after-tax income under the more progressive flat-tax reform are more egalitarian (their Gini indexes are 0.610 and 0.497). Therefore, we find that a simple income tax code with

¹According to the 1998 Survey of Consumer Finances, the Gini index of wealth in the U.S. economy is 0.803.

only two parameters (the marginal tax rate and the fixed deduction) may bring about a distribution of after-tax income that is more egalitarian than the one that obtains under the current personal and corporate income taxes.

These results establish that a policymaker who was required to choose between these two reforms would face the classical trade-off between efficiency and equality. In Section 5.6 we use our model economy to ask ourselves which economy she should choose: the more efficient but less egalitarian model economy E_1 , or the less efficient but more egalitarian model economy E_2 ? To quantify the trade-off and to answer this question, we compare the steady-state aggregate welfare of our three model economies using a Benthamite social welfare function. We find that the less progressive tax-reform results in a steady-state welfare loss which is equivalent to -0.17 percent of consumption, and that the more progressive tax-reform results in a steady-state welfare gain which is equivalent to $+0.45$ percent of consumption.² Finally, we compute the individual welfare changes for each household-type and we find that both reforms are a significant boon for the income poor. More precisely, the households in the bottom 40 percent of the after-tax income distribution of the benchmark model economy would be happier in the less progressive flat-tax economy, and the share of happier households increases to an impressive 70 percent in the more progressive flat-tax economy.

A detailed description of the model economy and its calibration, and an intuitive analysis of our findings follows in the ensuing pages.

2 The model economy

2.1 Population and endowment dynamics

Our model economy is inhabited by a measure one continuum of households. The households are endowed with ℓ units of disposable time each period and they are either workers or retirees. Workers face an uninsured idiosyncratic stochastic process that determines the value of their endowment of efficiency labor units. They also face an exogenous and positive probability of retiring. Retirees are endowed with zero efficiency labor units and they face an exogenous and positive probability of dying. When a retiree dies, it is replaced by a working-age descendant who inherits the retiree's estate and, possibly, some of its earning abilities. We use the one-dimensional shock, s , to denote the household's random age and random endowment of

²Economists often argue that welfare comparisons between steady states are not very interesting since they abstract from the often large changes in welfare that occur during the transitions. This particular case, however, may very well be an exception, since we believe that the accounting for the transitions would reinforce our steady state welfare findings. See section 5.6 for a detailed discussion of this issue.

efficiency labor units jointly.

We assume that this process is independent and identically distributed across households, and that it follows a finite state Markov chain with conditional transition probabilities given by $\Gamma = \Gamma(s' | s) = Pr\{s_{t+1} = s' | s_t = s\}$, where s and $s' \in S$. We assume that s takes values in one of two possible J -dimensional sets, that is $S = \mathcal{E} \cup \mathcal{R} = \{1, 2, \dots, J\} \cup \{J+1, J+2, \dots, 2J\}$. When a household draws shock $s \in \mathcal{E}$, it is a worker and its endowment of efficiency labor units is $e(s) > 0$. When a household draws shock $s \in \mathcal{R}$ it is retired, and its endowment of efficiency labor units is $e(s) = 0$. We use the $s \in \mathcal{R}$ to keep track of the realization of s that a worker faced during the last period of its working-life. We use this information to generate the appropriate intergenerational correlation and life-cycle pattern of earnings (see the discussion in Section 3.1.2 below).

This notation allows us to represent every demographic change in our model economy as a transition between the sets \mathcal{E} and \mathcal{R} . When a household's shock changes from $s \in \mathcal{E}$ to $s' \in \mathcal{R}$, we say that it has retired and when it changes from $s \in \mathcal{R}$ to $s' \in \mathcal{E}$, we say that it has died and has been replaced by a working-age descendant. Moreover, this specification of the joint age and endowment process implies that the transition probability matrix Γ controls the demographics of the model economy, by determining the expected durations of the households' working-lives and retirements; the life-time persistence of earnings, by determining the mobility of households between the states in \mathcal{E} ; the life cycle pattern of earnings, by determining how the endowments of efficiency labor units of new entrants differ from those of senior working-age households; and the intergenerational persistence of earnings, by determining the correlation between the states in \mathcal{E} for consecutive members of the same dynasty. In Section 3.1.2 we discuss these issues in greater detail.

2.2 Liquidation of assets

We assume that every household inherits the estate of the previous member of its dynasty at the beginning of the first period of its working-life. More specifically, we assume that retirees exit the economy and are replaced by their working-age descendants when they draw a shock $s' \in \mathcal{E}$ at the beginning of the period. At this moment the deceased household's estate is liquidated, and the household's descendant inherits a fraction $1 - \tau_e(z_t)$ of this estate, where z_t denotes the value of the household's stock of wealth at the end of period t . The remainder is instantaneously and costlessly transformed into the current period consumption good, and it is taxed away by the government.

2.3 Preferences

We assume that households derive utility from consumption, $c_t \geq 0$, and from non-market uses of their time, and that they care about the utility of their descendents as if it were their own utility. Consequently, the households' preferences can be described by the following standard expected utility function:

$$E \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, \ell - h_t) \mid s_0 \right\}, \quad (1)$$

where function u is continuous and strictly concave in both arguments; $0 < \beta < 1$ is the time-discount factor; ℓ is the endowment of productive time; and $0 \leq h_t \leq \ell$ is labor. Consequently, $\ell - h_t$ is the amount of time that the households allocate to non-market activities.

2.4 Production possibilities

We assume that aggregate output, Y_t , depends on aggregate capital, K_t , and on the aggregate labor input, L_t , through a constant returns to scale aggregate production function, $Y_t = f(K_t, L_t)$. Aggregate capital is obtained adding the wealth of every household, and the aggregate labor input is obtained adding the efficiency labor units supplied by every household. We assume that capital depreciates geometrically at a constant rate, δ , and we use r and w to denote the prices of capital and labor before all taxes.

2.5 The government sector

The government in our model economies taxes households' capital income, labor income, consumption and estates, and it uses the proceeds of taxation to make real transfers to retired households and to finance an exogenously given level of government consumption.

Capital income taxes are described by function $\tau_k(y_k)$, where y_k denotes capital income; labor income taxes are described by function $\tau_l(y_a)$, where y_a denotes the labor income tax base; social security contributions paid by firms are described by function $\tau_{sf}(y_l)$, where y_l denotes labor income, and those paid by households are described by function $\tau_{sh}(y_l)$; household income taxes are described by function $\tau_y(y_b)$, where y_b denotes the household income tax base; consumption taxes are described by function $\tau_c(c)$; estate taxes are described by function $\tau_e(z)$; and public transfers are described by function $\omega(s)$. Therefore, in our model economies, a government policy rule is a specification of $\{\tau_k(y_k), \tau_l(y_a), \tau_{sf}(y_l), \tau_{sh}(y_l), \tau_y(y_b), \tau_c(c), \tau_e(z), \omega(s)\}$ and of a process on government consumption, $\{G_t\}$. Since we also assume

that the government must balance its budget every period, these policies must satisfy the following restriction:

$$G_t + Z_t = T_t, \tag{2}$$

where Z_t and T_t denote aggregate transfers and aggregate tax revenues, respectively.

Social security in our model economy takes the form of transfers to retired households financed with a payroll tax. The inclusion of a pay-as-you-go social security system of this kind has important implications for the question that we ask in this article for several reasons. First, it reduces the steady state aggregate capital stock.³ Second, it plays an important role in helping us to replicate the large fraction of households who own zero or very few assets in the United States.⁴ Third, since public pensions are paid as life annuities, it insures the households against the risk of living too long thereby reducing the reasons for saving. Our calibrating procedure allows us to match the observed size of average retirement public pensions ensuring that the motives for saving in our model economy are realistic.

However, in our model economy pensions are independent of contributions and this feature qualifies the precision of our analysis in two ways: first, the overall amount of idiosyncratic risk diminishes because the labor market history does not have implications for retirement benefits; second, we abstract from a potentially important determinant of labor supply, since increasing the hours worked entitles the households to larger pension benefits.⁵

2.6 Market arrangements

We assume that there are no insurance markets for the household-specific shock.⁶ Partly to buffer their streams of consumption against the shocks, the households in our model economy can accumulate wealth in the form of real capital, a_t . We assume that these wealth

³See Samuelson (1975).

⁴See Hubbard, Skinner, and Zeldes (1994).

⁵We make this assumption for two technical reasons: first, discriminating between households according to their past contributions to a social security system requires the inclusion of a second asset-type state variable; and second, in a model with endogenous labor supply, linking pensions to past contributions makes the optimality condition for leisure an intertemporal decision. These two facts result in a very large increase in our computational costs. (See Part C of the Appendix for details on our computational algorithm).

⁶This is a key feature of this class of model worlds. When insurance markets are allowed to operate, our model economies collapse to a standard representative household model, as long as the right initial conditions hold. Cole and Kocherlakota (1997) study economies of this type with the additional characteristic that private storage is unobservable. They conclude that the best achievable allocation is the equilibrium allocation that obtains when households have access to the market structure assumed in this article. We interpret this finding to imply that the market structure that we use here could arise endogenously from certain unobservability features of the environment —specifically, from both the realization of the shock and the amount of wealth being unobservable.

holdings belong to a compact set \mathcal{A} . The lower bound of this set can be interpreted as a form of liquidity constraints or, alternatively, as a solvency requirement.⁷ The existence of an upper bound for the asset holdings is guaranteed as long as the after-tax rate of return to savings is smaller than the households' common rate of time preference. This condition is always satisfied in equilibrium.⁸ Finally, we assume that firms rent factors of production from households in competitive spot markets. This assumption implies that factor prices are given by the corresponding marginal productivities.

2.7 The households' decision problem

The individual state variables are the shock realization s and the stock of assets a .⁹ The Bellman equation of the household decision problem is the following:

$$v(a, s) = \max_{\substack{c \geq 0 \\ z \in \mathcal{A} \\ 0 \leq h \leq \ell}} u(c, \ell - h) + \beta \sum_{s' \in S} \Gamma_{ss'} v[a'(z), s'], \quad (3)$$

$$s.t. \quad c + z = y - \tau + a, \quad (4)$$

$$y = ar + e(s)hw + \omega(s), \quad (5)$$

$$\tau = \tau_k(y_k) + \tau_l(y_l) + \tau_{sf}(y_l) + \tau_{sh}(y_l) + \tau_y(y_b) + \tau_c(c), \quad (6)$$

$$a'(z) = \begin{cases} z - \tau_e(z) & \text{if } s \in \mathcal{R} \text{ and } s' \in \mathcal{E}, \\ z & \text{otherwise.} \end{cases} \quad (7)$$

where function v is the households' common value function. Notice that income, y , includes three terms: capital income, $y_k = ar$, that can be earned by every household; labor income, $y_l = e(s)hw$, that can be earned only by workers; and transfer income, $\omega(s)$, that can be earned only by retirees. The household policy that solves this problem is a set of functions that map the individual state into choices for consumption, end-of-period savings, and hours worked. We denote this policy by $\{c(a, s), z(a, s), h(a, s)\}$.

2.8 Equilibrium

Each period the economy-wide state is a probability measure, x_t , defined over \mathcal{B} , an appropriate family of subsets of $S \times \mathcal{A}$ that counts the households of each type. In the steady-state

⁷Given that leisure is an argument in the households' utility function, this borrowing constraint can be interpreted as a solvency constraint that prevents the households from going bankrupt in every state of the world.

⁸Huggett (1993) and Marcet, Obiols-Homs, and Weil (2003) prove this proposition.

⁹In our model economy there are no aggregate state variables because we abstract from aggregate uncertainty and we restrict our analysis to the steady states of the economies.

this measure is time invariant, even though the individual state variables and the decisions of the individual households change from one period to the next.¹⁰

Definition 1 *A steady state equilibrium for this economy is a household value function, $v(a, s)$; a household policy, $\{c(a, s), z(a, s), h(a, s)\}$; a government policy, $\{\tau_k(y_k), \tau_l(y_a), \tau_{sf}(y_l), \tau_{sh}(y_l), \tau_y(y_b), \tau_c(c), \tau_e(z), \omega(s), G\}$; a stationary probability measure of households, x ; factor prices, (r, w) ; and macroeconomic aggregates, $\{K, L, T, Z\}$, such that:*

- (i) *When households take factor prices and the government policy as given, the household value function and the household policy solve the households' decision problem described in expression (3).*
- (ii) *The firms in the economy behave as competitive maximizers. That is, their decisions imply that factor prices are factor marginal productivities:*

$$r = f_1(K, L) - \delta \quad \text{and} \quad w = f_2(K, L). \quad (8)$$

where K and L denote the aggregate capital and aggregate labor inputs.

- (iii) *Factor inputs, tax revenues, and transfers are obtained aggregating over households:*

$$K = \int a \, dx \quad (9)$$

$$L = \int h(a, s) \, e(s) \, dx \quad (10)$$

$$T = \int [\tau_k(y_k) + \tau_l(y_a) + \tau_{sf}(y_l) + \tau_{sh}(y_l) + \tau_y(y_b) + \tau_c(c)] \, dx + \quad (11)$$

$$\int \mathbf{I}_{s \in \mathcal{R}} \gamma_{s\mathcal{E}} \tau_e(z) \, z(a, s) \, dx \quad (12)$$

$$Z = \int \omega(s) \, dx. \quad (13)$$

where household income, $y(a, s)$, is defined in expression (5); \mathbf{I} denotes the indicator function; $\gamma_{s\mathcal{E}} \equiv \sum_{s' \in \mathcal{E}} \Gamma_{ss'}$; and, consequently, $(\mathbf{I}_{s \in \mathcal{R}} \gamma_{s\mathcal{E}})$ is the probability that a retiree of type s exits the economy. All integrals are defined over the state space $S \times \mathcal{A}$.

- (iv) *The goods market clears:*

$$\int [c(a, s) + z(a, s)] \, dx + G = f(K, L) + (1 - \delta) K. \quad (14)$$

¹⁰See Hopenhayn and Prescott (1992) and Huggett (1993).

(v) *The government budget constraint is satisfied:*

$$G + Z = T \tag{15}$$

(vi) *The measure of households is stationary:*

$$x(B) = \int_B \left\{ \int_{S \times \mathcal{A}} \left[\mathbf{I}_{z=z(a,s)} \mathbf{I}_{s \notin \mathcal{R} \vee s' \notin \mathcal{E}} + \mathbf{I}_{z=[1-\tau_e(z)]z(a,s)} \mathbf{I}_{s \in \mathcal{R} \wedge s' \in \mathcal{E}} \right] \Gamma_{ss'} dx \right\} dz ds' \tag{16}$$

for all $B \in \mathcal{B}$, where \vee and \wedge are the logical operators “or” and “and”. Equation (16) counts the households, and the cumbersome indicator functions and logical operators are used to account for estate taxation. We describe the procedure that we use to compute this equilibrium in Section B of the Appendix.

3 Calibration

To calibrate our model economy, we choose the functional forms and parameters that describe its preferences, technology, government policy and its joint age and endowment of efficiency labor units process. When all is told, this amounts to choosing the forms of seven functions and the values of a total of 42 parameters. To choose the values of these parameters we need 42 calibration targets. Thirty-six of these targets are statistics that describe the relevant features of the U.S. economy and the remaining 6 are normalization conditions.

3.1 Functional forms and parameters

3.1.1 Preferences

Our choice for the households’ common utility function is

$$u(c, l) = \frac{c^{1-\sigma_1}}{1-\sigma_1} + \chi \frac{(\ell-l)^{1-\sigma_2}}{1-\sigma_2} \tag{17}$$

We make this choice because the households in our model economies face very large changes the market value of their time. These changes would have resulted in extremely large variations in hours worked, if we had chosen the standard non-separable preferences. Consequently, to characterize the households’ preferences we must determine the values of five parameters: the four in the utility function and the value of the time discount factor, β .

3.1.2 The joint age and endowment of efficiency labor units process

The joint age and endowment of efficiency labor units process, $\{s\}$, takes values in set $S = \{\mathcal{E} \cup \mathcal{R}\}$, where \mathcal{E} and \mathcal{R} are two J -dimensional sets. Consequently to specify the process completely we must choose the values of $(2J)^2 + J$ parameters, of which $(2J)^2$ correspond to the transition probability matrix on s , Γ , and the remaining J correspond to the endowments of efficiency labor units. However, we impose some additional restrictions on Γ that reduce the number of parameters to $J^2 + J + 4$.

To understand these restrictions better, it helps to consider the following partition of matrix Γ :

$$\Gamma = \begin{bmatrix} \Gamma_{\mathcal{E}\mathcal{E}} & \Gamma_{\mathcal{E}\mathcal{R}} \\ \Gamma_{\mathcal{R}\mathcal{E}} & \Gamma_{\mathcal{R}\mathcal{R}} \end{bmatrix} \quad (18)$$

Submatrix $\Gamma_{\mathcal{E}\mathcal{E}}$ contains the transition probabilities of working-age households that are still of working-age one period later. Since we impose no restrictions on these transitions, to characterize $\Gamma_{\mathcal{E}\mathcal{E}}$ we must choose the values of J^2 parameters.

Submatrix $\Gamma_{\mathcal{E}\mathcal{R}}$ describes the transitions from the working-age states into the retirement states. The value of this submatrix is $\Gamma_{\mathcal{E}\mathcal{R}} = p_{e\varrho}I$, where $p_{e\varrho}$ is the probability of retiring and I is the identity matrix. This is because we assume that every working-age household faces the same probability of retiring and because, to keep track of the earnings ability of retirees, we use only the realization of their last working-age shock. Consequently, to characterize $\Gamma_{\mathcal{E}\mathcal{R}}$ we must choose the value of only one parameter.

Submatrix $\Gamma_{\mathcal{R}\mathcal{E}}$ describes the transitions from the retirement states into the working-age states that take place when a retiree exits the economy and is replaced by a working-age descendant. The rows of this submatrix contain a two parameter transformation of the stationary distribution of $s \in \mathcal{E}$, which we denote $\gamma_{\mathcal{E}}^*$. This transformation allows us to approximate both the life-cycle profile and the intergenerational correlation of earnings. Intuitively, the transformation amounts to shifting the probability mass from $\gamma_{\mathcal{E}}^*$ both towards the first row of $\Gamma_{\mathcal{R}\mathcal{E}}$ and towards its diagonal.¹¹ Consequently, to characterize $\Gamma_{\mathcal{R}\mathcal{E}}$ we must choose the value of the two shift parameters.

Finally, submatrix $\Gamma_{\mathcal{R}\mathcal{R}}$ contains the transition probabilities of retired households that are still retired one period later. The value of this submatrix is $\Gamma_{\mathcal{R}\mathcal{R}} = p_{\varrho\varrho}I$, where $(1 - p_{\varrho\varrho})$ is the probability of exiting the economy. This is because we assume that every retired household

¹¹The definitions of the two shift parameters can be found in Section A of the Appendix and a detailed justification of our procedure can be found in Castañeda, Díaz-Giménez, and Ríos-Rull (2003).

faces the same probability of exit and because the type of retired households never changes. Therefore, to identify this submatrix we must choose the value of only one parameter.

To keep the dimension of process $\{s\}$ as small as possible while still being able to achieve our calibration targets, we choose $J = 4$. Therefore, to characterize process $\{s\}$, we must choose the values of $J^2 + J + 4 = 20$ parameters.¹²

3.1.3 Technology

In the U.S. after World War II, the real wage has increased at an approximately constant rate—at least until 1973—and factor income shares have displayed no trend. To account for these two features of the data, we choose a standard Cobb-Douglas aggregate production function in capital and labor. Therefore, to specify the aggregate technology, we must choose the values of two parameters: the capital income share, θ , and the capital depreciation rate, δ .

3.1.4 Government Policy

Capital income taxes: Our choice for the model economy’s capital income tax function is

$$\tau_k(y_k) = a_1 y_k \tag{19}$$

Of course, in the U.S. economy different types of capital are taxed at different rates and may gain different types of deductions. In order to simplify our model economy we consider just one type of capital good. Consistently, we will calibrate a_1 as an average rate over all capital income.¹³

Payroll taxes: Our choice for the model economy’s payroll tax function is

$$\tau_{sf}(y_l) = \tau_{sh}(y_l) = \begin{cases} a_2 y_l & \text{for } 0 \leq y_l \leq a_3 \\ a_2 a_3 & \text{otherwise} \end{cases} \tag{20}$$

We chose this function because it approximates the shape of the U.S. payroll tax function where the marginal payroll tax rate is a positive constant up to a certain level of labor income and it is zero from that level of income onwards. To replicate the U.S. Social Security tax

¹²Notice that we have not yet imposed that Γ must be a Markov matrix.

¹³The underlying assumption is that in their savings decisions all households in the economy hold the same market portfolio.

code, we assume that the payroll taxes paid by the model economy households and firms are identical.

Household income taxes: Our choice for the model economy’s income tax function is

$$\tau_y(y_b) = a_4 \left[y_b - (y_b^{-a_5} + a_6)^{-1/a_5} \right] \quad (21)$$

where $y_b = y_k + y_l - \tau_k - \tau_{sf}$. This is the function chosen by Gouveia and Strauss (1994) to model the 1989 U.S. effective federal personal income taxes. Notice that both capital income taxes and social security contributions made by firms are excluded from the household income tax base both in the U.S and in our model economy.

Estate taxes: Our choice for the model economy’s estate tax function is

$$\tau_e(z) = \begin{cases} 0 & \text{for } z < a_7 \\ a_8(z - a_7) & \text{otherwise} \end{cases} \quad (22)$$

We chose this function because it replicates the main features of the current U.S. effective estate taxes.¹⁴

Consumption taxes: Our choice for the model economy’s consumption tax function is

$$\tau_c(c) = a_9 c \quad (23)$$

3.2 Targets

The U.S. tax code defines tax bases in annual terms. Since the income tax, the payroll tax and the estate tax are not proportional taxes, the obvious choice for our model period is one year. Moreover, the Survey of Consumer Finances, which is our main micro-data source, is also yearly.

3.2.1 Normalization conditions

First we enumerate the normalization conditions. The household endowment of disposable time is an arbitrary constant and we choose it to be $\ell = 3.2$. We also normalize the endowment of efficiency labor units of the least productive households to be $e(1) = 1.0$. Finally, since

¹⁴See, for example, Aaron and Munnell (1992).

matrix Γ is a Markov matrix, its rows must add up to one. This property imposes four additional normalization conditions on the rows of $\Gamma_{\mathcal{E}\mathcal{E}}$.¹⁵

3.2.2 Macroeconomic and demographic targets

Ratios: We target a capital to output ratio, K/Y , of 3.58; a capital income share of 0.376; and an investment to output ratio, I/Y , of 22.5 percent. We obtain our target value for the capital output share dividing \$288,000, which was average household wealth in the U.S. in 1997 according the 1998 Survey of Consumer Finances, by \$80,376, which was per household Gross Domestic Product according to the Economic Report of the President (2000), U.S. 1997.¹⁶ Our target for the capital income share is the value that obtains when we use the methods described in Cooley and Prescott (1995) and we exclude the public sector from the computations.¹⁷ To calculate the value of our target for I/Y , we define investment as the sum of gross private fixed domestic investment, change in business inventories, and 75 percent of the private consumption expenditures in consumer durables using data for 1997 from the Economic Report of the President (2000).¹⁸

Allocation of time and consumption: We target a value of $H/\ell = 33$ percent for the average share of disposable time allocated to working in the market.¹⁹ For the curvature of consumption we choose a value of $\sigma_1 = 1.5$. This value falls within the range (1–3) that is standard in the literature.²⁰ Finally, we want our model economy to replicate the relative cross-sectional variability of U.S. consumption and hours. To this purpose, we target a value of $cv(c)/cv(h) = 3.5$ for the ratio of the cross-sectional coefficients of variation of these two variables.

The age structure of the population: We target the expected durations of the working-life and retirement of the model economy households to be 45 and 18 years, to replicate the

¹⁵Note that our assumptions about the structure of matrix Γ imply that once submatrix $\Gamma_{\mathcal{E}\mathcal{E}}$ has been appropriately normalized, every row of Γ adds up to one without imposing any further restrictions.

¹⁶We obtained this number dividing the U.S. population quoted for 1997 in Table B-34 of the Economic Report of the President (2000) by the U.S. average household size which was 2.59 according to the 1998 SCF (see Budría, Díaz-Giménez, Quadrini, and Ríos-Rull (2002)).

¹⁷See Castañeda, Díaz-Giménez, and Ríos-Rull (1998) for details about this number.

¹⁸This definition of investment is approximately consistent with the 1998 Survey of Consumer Finances definition of household wealth, which includes the value of vehicles, but does not include the values of other consumer durables.

¹⁹See Juster and Stafford (1991) for details about this number.

²⁰Recent calibration exercises find very similar values for σ_1 . For example, Heathcote, Storesletten, and Violante (2004) report a value of 1.44 and Pijoan-Mas (2006) reports a value of 1.46 for this parameter.

corresponding values for the U.S. economy.

The life-cycle profile of earnings: To replicate the life-cycle profile of earnings we target to the ratio of the average earnings of households between ages 60 and 41 to that of households between ages 40 and 21 in the U.S. economy. According to the Panel Study of Income Dynamics the average value of this statistic was 1.303 in the 1972–1991 period.

The intergenerational transmission of earnings ability: To replicate the intergenerational correlation of earnings of the U.S. economy, we target the cross-sectional correlation between the average life-time earnings of one generation of households and the average life-time earnings of their immediate descendants. Solon (1992) and Zimmerman (1992) measure this statistic for fathers and sons in the U.S. economy, and they report it to be approximately 0.4.

3.2.3 Government Policy

In Table 1 we report the revenues obtained by the combined U.S. Federal, State, and Local Governments for the 1997 fiscal year. To calibrate the model economy tax functions we must allocate the different U.S. economy tax revenue items to the tax instruments of the benchmark model economy. To this purpose, we choose the parameters of the model economy household income tax, capital income tax, estate tax, and payroll taxes so that they collect the revenues levied by the U.S. personal income taxes, corporate profit taxes, estate and gift taxes, and payroll taxes. The remaining sources of government revenues in the U.S. are sales and gross receipts taxes, property taxes, excise taxes, custom duties and fees, and other taxes. Added together, in 1997 these tax instruments collected 7.09 percent of GDP which we allocate to the consumption tax in the model economy.²¹

Total tax revenues in both the U.S in 1997 and in the model amount to 27.52 percent of GDP. Since we require the government budget of our model economy to be balanced, we must allocate these revenues to our two government expenditure items: government consumption and transfers. We target a value for the model economy’s aggregate transfers to output ratio of $Z/Y = 5.21$ percent. This value corresponds to the share of U.S. GDP accounted for by Medicare and two thirds of Social Security transfers in 1997. We make this choice because transfers in our model economies are lump-sum, and Social Security transfers in the U.S.

²¹Since we also target government transfers and government expenditures (see below), the model economy’s consumption tax rate is determined residually to balance the government budget.

Fiscal Year	1997	
	\$Billion	%GDP
Gross Domestic Product (GDP)	8185.20	100.00
Total Federal, State and Local Gvt Receipts	2252.75	27.52
Individual Income Taxes	896.54	10.95
Social Insurance and Retirement	539.37	6.60
Sales and Gross Receipts Taxes	261.73	3.20
Property Taxes	218.83	2.67
Corporate Profit Taxes	216.11	2.64
Excise Taxes	56.92	0.70
Estate and Gift Taxes	19.85	0.24
Custom Duties and Fees	17.93	0.22
Other Taxes	25.47	0.30

Table 1: Federal, State, and Local Government Receipts Source: Tables B78, B81, and B86 of the Economic Report of the President 2000.

economy are mildly progressive. This gives us a residual share for government expenditures to GDP of 22.30 which is our target for the G/Y ratio in our model economy.²² We discuss our choices for the various tax function parameters in the paragraphs below.

Capital income taxes: We choose the capital income tax rate of function (19) so that the revenues collected by this tax in the benchmark model economy match the revenues collected by the corporate profit tax in the U.S. economy.

Payroll taxes: To characterize the payroll tax function described in expression (20), we must determine the values of parameters a_2 and a_3 . In 1997 in the U.S. the payroll tax rate paid by both households and firms was 7.65 percent each and it was levied only on the first \$62,700 of gross labor earnings. This value was approximately equal to 78 percent of the U.S. per household GDP. To replicate this values, in our model economy we make $a_2 = 0.0765$ and $a_3 = 0.78\bar{y}$, where \bar{y} denotes per household output. These choices imply that the payroll tax collections in our model economy are endogenous and they can be interpreted as an overidentification restriction.

Household income taxes: To characterize the income tax function described in expression (21), we must determine the values of parameters a_4 , a_5 and a_6 . Since a_4 and a_5 are

²²Our target for the G/Y ratio is 4.48 percentage points larger than the 17.89 obtained for the Government Expenditures and Gross Investment entry in the NIPA tables. The difference is essentially accounted for by the sum of net interest payments and the deficit (3.58 percent of GDP).

unit-independent, we use the values reported by Gouveia and Strauss (1994) for these parameters, namely, $a_4 = 0.258$ and $a_5 = 0.768$. To determine the value of a_6 , we require that the tax rate levied on average household output in our benchmark model economy is the same as the effective tax rate on per household GDP in the U.S. economy. These choices imply that the household income tax collections in our model economy are endogenous and they can be interpreted as an overidentification restriction.

Estate taxes: To characterize the estate tax function described in expression (22), we must determine the values of parameters a_7 and a_8 . In the U.S. during the 1987–1997 period the first \$600,000 were tax exempt average per household GDP was approximately \$60,000.²³ To replicate this feature of the U.S. estate tax code, we make $a_7 = 10\bar{y}$. Finally, we choose the value of a_8 so that our model economy’s estate tax collections replicate the U.S. economy estate tax collections.

Consumption taxes: We choose parameter a_9 in the consumption tax function described in expression (23) so that the government in the model economy balances its budget. Therefore, the consumption tax collections in our model economy are also endogenous, and they can be interpreted as an additional overidentification restriction.

3.2.4 The distributions of earnings and wealth

The conditions that we have described so far specify a total of 27 targets. Since to solve our model economy we have to determine the values of 42 parameters, we need 15 additional targets. These 15 targets are the Gini indexes and 13 additional points form the Lorenz curves of U.S. earnings and wealth reported in Table 8.²⁴

3.3 Choices: mapping statistics into parameters

The values of some of the model economy parameters are obtained directly either because they are normalization conditions or because they are uniquely determined by one of our targets. In this fashion, we choose $\ell = 3.2$, $e(1) = 1.0$, $\sigma_1 = 1.5$, $\theta = 0.376$, $p_{e\varrho} = 0.022$, $1 - p_{\varrho\varrho} = 0.056$ and we choose four of the transition probabilities of $\Gamma_{\mathcal{E}\mathcal{E}}$. Similarly, $a_3 = 0.0765$ is taken directly from the U.S. payroll tax code, and $a_4 = 0.258$ and $a_5 = 0.768$ are taken

²³See, for example, Aaron and Munnell (1992).

²⁴A detailed discussion of this last, non-standard feature of our calibration procedure can be found in Castañeda, Díaz-Giménez, and Ríos-Rull (2003).

directly from the values estimated by Gouveia and Strauss (1994) for the U.S. economy. To determine the values of the remaining 29 parameters, we solve the system of 29 non-linear equations that results from imposing that the relevant statistics of the model economy should be equal to their corresponding targets. The details of the procedure that we use to solve this system can be found in Section C of the Appendix.

4 Findings: the calibration exercise

In this subsection we discuss our calibration results very briefly. A detailed discussion of the reasons that allow our model economy to account for the main aggregate and distributional statistics of the U.S. economy can be found in Castañeda, Díaz-Giménez, and Ríos-Rull (2003).

Calibrating our model economy requires finding a stochastic process for the endowment of efficiency labor units that allows our benchmark model economy to replicate the targeted aggregate and distributional statistics of the U.S. economy. This process, however, is not to be taken literally, since it represents everything that we do not know about the workings of our economy. An ambitious continuation of this research would be to endogenize at least some of its main features which we now describe.

Table 2: The relative endowments of efficiency labor units, $e(s)$, and the stationary distribution of working-age households, $\gamma_{\mathcal{E}}^*$ (%)

	$s = 1$	$s = 2$	$s = 3$	$s = 4$
$e(s)$	1.00	3.17	9.91	634.98
$\gamma_{\mathcal{E}}^*$ (%)	47.78	37.24	14.91	0.0638

In Table 2 we report the relative endowments of efficiency labor units and the invariant measures of each type of working-age households. We have normalized the endowment of efficiency labor units of workers of type $s = 1$ to be $e(s) = 1$. The endowments of workers of $s = 2$, $s = 3$, and $s = 4$ are, approximately, 3, 10, and 635. This means that, in our model economy, the luckiest workers are 635 times as lucky as the unluckiest ones. The stationary distribution shows that each period 85 percent of the workers are unlucky and draw shocks $s = 1$ or $s = 2$, while one out of every 1,567 workers is extremely lucky and draws shock $s = 4$.

In Table 3 we report the transition probabilities between the working-age states. Every row sums up to 97.78 percent plus or minus rounding errors. This is because the probability

Table 3: The transition probabilities of the process on the endowment of efficiency labor units for working-age households that remain of working-age one period later, $\Gamma_{\mathcal{E}\mathcal{E}}$ (%)

From s	To s'			
	$s' = 1$	$s' = 2$	$s' = 3$	$s' = 4$
$s = 1$	96.15	1.39	0.23	0.009
$s = 2$	1.60	96.00	0.18	0.000
$s = 3$	1.19	0.00	96.56	0.028
$s = 4$	6.63	0.45	6.52	84.18

that a worker retires is 2.22 percent. Table 3 shows that the first three shocks are very persistent. Their expected durations are 25.7, 25.3 and 29.4 years. On the other hand shock $s = 4$ is relatively transitory and its expected duration is only 7.6 years. As far as the transitions are concerned, we find that a worker whose current shock is $s = 1$ is most likely to make a transition to shock $s = 2$ than to any of the other shocks. Likewise, a worker whose current shock is either $s = 2$ or $s = 3$ is most likely to move back to shock $s = 1$. Only very rarely workers whose current shock is either $s = 1$ or $s = 2$ will make a transition to either shock $s = 3$ or shock $s = 4$. Finally, when a worker draws shock $s = 4$, it is most likely that it will draw either shock $s = 3$ or shock $s = 1$ shortly afterwards. We report all the other model economy parameters in Table 4.

In the first two rows of Tables 6 and 7 and in the first two rows of each of the panels of Table 8 we report the statistics that describe the main aggregate and distributional features of the U.S. and the benchmark model economies. These numbers confirm that, overall, our model economy succeeds in replicating the main relevant features of the U.S. economy in very much detail. Naturally, there are some exceptions. For instance, our parsimonious modelling of the life cycle does not allow us to match life-cycle profile of earnings and the intergenerational correlation of earnings simultaneously. Castañeda, Díaz-Giménez, and Ríos-Rull (2003) discuss this issue in detail, and they show that this class of model economies can account for these two statistics one at a time. We are particularly encouraged by the ability of our model economy to replicate the fiscal policy ratios (see Table 7) and the earnings, income and wealth distributions (see Table 8) that constitute the main focus of this article.²⁵

²⁵Recall that the payroll tax collections, the household income tax collections and the income distribution have not been targeted in our calibration exercise.

Table 4: Parameter values for the benchmark model economy

<i>Preferences</i>		
Time discount factor	β	0.930
Curvature of consumption	σ_1	1.500
Curvature of leisure	σ_2	1.119
Relative share of consumption and leisure	χ	1.050
Endowment of discretionary time	ℓ	3.200
<i>Technology</i>		
Capital income share	θ	0.376
Capital depreciation rate	δ	0.050
<i>Age and endowment process</i>		
Probability of retiring	p_{ee}	0.022
Probability of dying	$1 - p_{ee}$	0.056
Life cycle earnings profile	ϕ_1	1.000
Intergenerational persistence of earnings	ϕ_2	0.733
<i>Fiscal policy</i>		
Government consumption	G	0.369
Retirement pensions	ω	0.800
Capital income tax function	a_1	0.146
Payroll tax function	a_2	1.262
	a_3	0.076
	a_4	0.258
Household income tax function	a_5	0.768
	a_6	0.456
	a_7	16.179
Estate tax function	a_8	0.246
	a_9	0.099
Consumption tax function		

5 Findings: The flat tax reforms

We study two fundamental, revenue neutral, tax reforms that are related to the classical reform proposed by Hall and Rabushka (1995). These reforms replace the current personal income tax with a flat tax on all labor income above a large tax-exempt level and the current corporate income taxes with an integrated flat tax on business income. The labor income tax function, $\tau_l(y_a)$, is

$$\tau_l(y_a) = \begin{cases} 0 & \text{for } y_a < a_{10} \\ a_{11}(y_a - a_{10}) & \text{otherwise} \end{cases} \quad (24)$$

where the tax base is labor income net of social security taxes paid by firms, $y_a = y_l - \tau_{sf}(y_l)$, a_{10} is the tax-exempt, and a_{11} is the flat-tax rate. The business income tax is identical to the capital income tax defined in expression (19) above. Since capital and labor income are taxed at the same marginal tax rate, we make $a_1 = a_{10}$. In this class of reforms, the progressivity of direct taxes arises from the fixed deduction on labor income, while capital income taxes are not progressive. Another defining feature of this class of reforms is that they eliminates the double taxation of capital income. Finally, in this article we do not allow firms to expense new investment when calculating the base of the capital income tax.

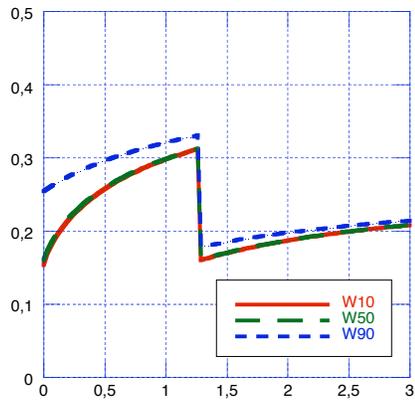
To find the values of the tax parameters of our reformed model economies, we do the following: first we choose the values for the labor income tax-exemptions. In model economy E_1 this value is $a_{10} = 0.3236$ which corresponds to 20 percent of the benchmark model economy per household output or, approximately, \$16,000. In model economy E_2 it is $a_{10} = 0.6472$ which corresponds to 40 percent of the benchmark model economy per household output or, approximately, \$32,000.²⁶ Next we search for the flat tax rates that make the reforms revenue neutral. These tax rates turn out to be $a_1 = a_{10} = 21.5$ percent in model economy E_1 and $a_1 = a_{10} = 29.2$ percent in model economy E_2 . Henceforth we refer to model economy E_1 as the less progressive model economy and to model economy E_2 as the more progressive model economy.

5.1 Taxes, taxes, taxes

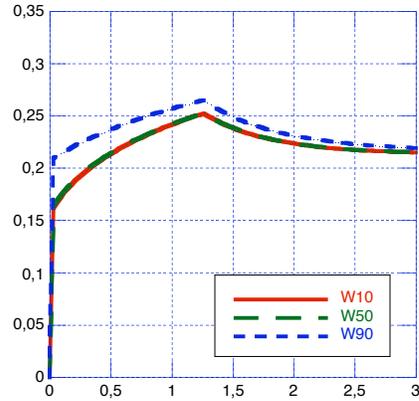
Taxes in the U.S. interact in interesting and perhaps somewhat surprising ways. The current personal income tax, τ_y , is progressive in the classical sense since both its marginal and its average tax rates are increasing in income. In contrast, the current payroll tax is not progressive. In 1997 the marginal payroll tax on labor incomes below \$62,700 was constant and equal

²⁶Ventura (1999) makes the same choices for the values of the tax-exemptions.

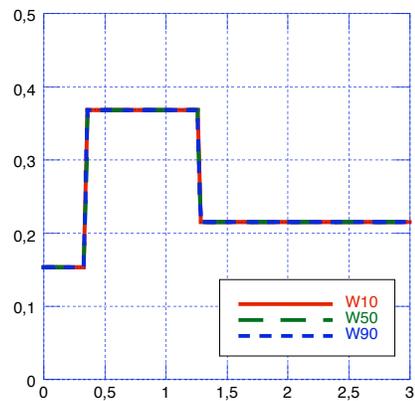
Figure 1: Tax rates on labor income in the benchmark and in the reformed model economies (E_0 , E_1 and E_2)



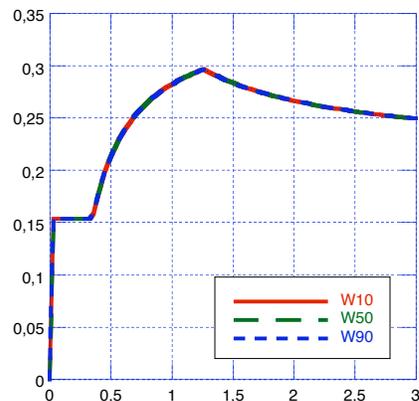
Panel A: Marginal tax rates in E_0



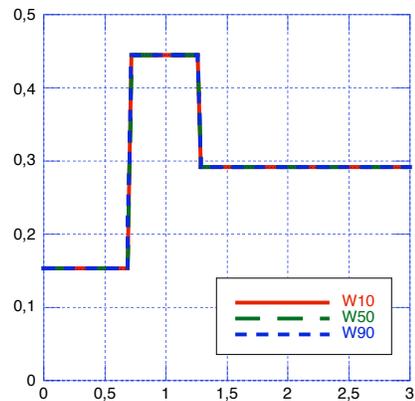
Panel B: Average tax rates in E_0



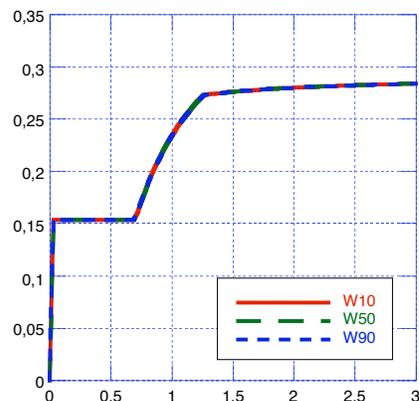
Panel C: Marginal tax rates in E_1



Panel D: Average tax rates in E_1



Panel E: Marginal tax rates in E_2



Panel F: Average tax rates in E_2

to 15.3 percent, and the marginal payroll tax rate on labor incomes above this threshold was zero. Since in our model economies households give up leisure for consumption, and since retirement pensions are independent from contributions, the households are concerned with total effective income taxes considered together, and not in the specific amounts collected with the various taxes separately.

To illustrate the interactions between the current payroll and personal income taxes, in Figure 1 we represent the total effective taxes on labor income in our three model economies. Since the current personal income tax rates depend on both capital and labor income, in each graph we plot the tax rates paid by households whose net worths are \$0, \$9,370, and \$454,120 which puts them in the first, the fifth, and the ninth deciles of the wealth distribution.²⁷

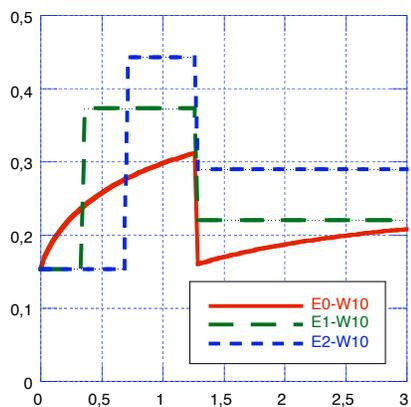
A careful inspection of Figure 1 reveals various features of the current U.S. tax system that we have found both surprising and interesting. First, in the benchmark model economy the shape of the effective marginal labor income tax function faced by different households in the bottom half of the wealth distribution is almost the same. In the flat-tax economies this is obviously the case because marginal tax rates are flat for everyone by design. Therefore, the current tax system is not too different from the flat tax system that we propose, at least as far as its progressivity with respect to wealth is concerned.

Second, if we add together the payroll and the personal income taxes, it turns out that the current labor income taxation, far from being progressive, actually becomes unquestionably regressive.²⁸ Specifically, Panel A of Figure 1 shows that the marginal tax rate on labor income faced by wealth-poor households starts at about 15 percent and it reaches its maximum value of approximately 30 percent when labor income reaches \$62,700. At this income level, the marginal payroll tax rate drops to zero and the total marginal tax on labor income drops back to approximately 15% which is the marginal tax rate paid by households with zero labor income. In the case of very wealthy households, the regressivity of marginal labor income tax rates is even more remarkable: the marginal tax on labor income paid by households who earn \$62,700 is 17%, which is only two-thirds of the 25% paid by equally wealthy households

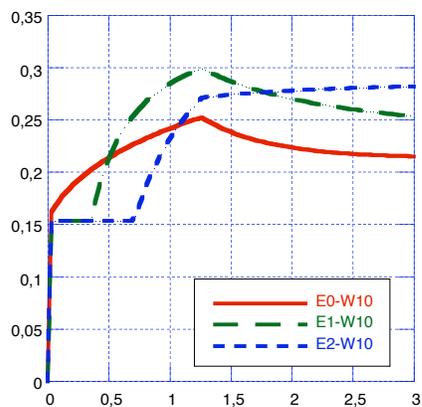
²⁷We have transformed the model economy units into U.S. dollars to give the reader a better sense of the magnitudes involved.

²⁸In this article we use the adjectives “progressive”, “proportional”, and “regressive” because they give us an intuitive description of the shape of the tax functions, but we do not use them to imply any normative judgement about fairness. We do this for two reasons. First, because to evaluate the fairness of a tax instrument, we should evaluate its consequences for the distribution of welfare or, at least, for the distribution of after-tax income; and second because to measure the fairness of a tax instrument, we should use the lifetime tax burden and the lifetime taxable income, and not the tax burden and the taxable income of any single period.

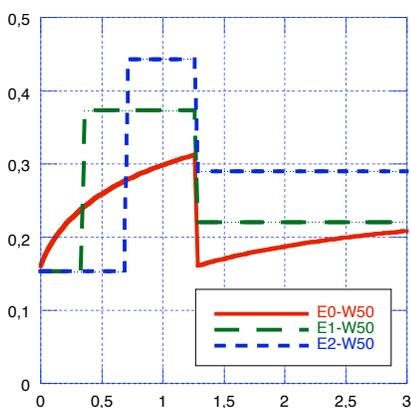
Figure 2: Tax rates on labor income paid by the first decile, the median and the ninth decile of the wealth distribution (W10, W50 and W90)



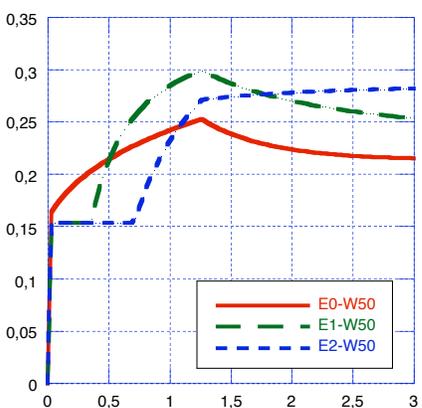
Panel A: Marginal tax rates (W10)



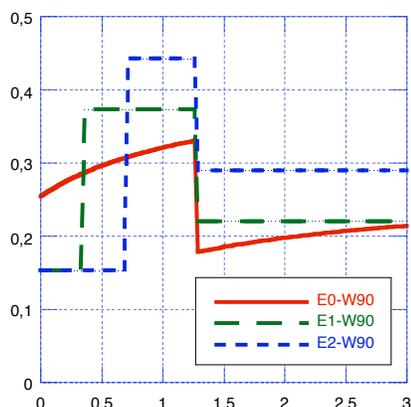
Panel B: Average tax rates (W10)



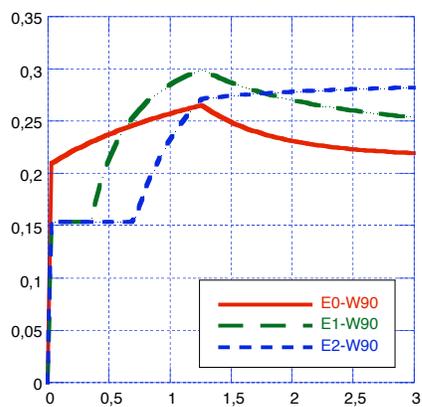
Panel C: Marginal tax rates (W50)



Panel D: Average tax rates (W50)



Panel E: Marginal tax rates (W90)



Panel F: Average tax rates (W90)

who earn zero labor income.²⁹

This interaction between payroll taxes and labor income taxes is also present in the flat tax economies, albeit in a smaller degree (see Panels C and E of Figure 1). In both cases, the marginal income tax rates are step functions, and in both cases the middle labor incomes pay the highest marginal taxes. But, unlike the current system, in both flat tax reforms, the labor income rich pay higher marginal labor income taxes than the labor income poor.

As far as average labor income taxes are concerned, we find that they are progressive only in model economy E_2 (see Panels B, D and F of Figure 1). In the benchmark model economy and in model economy E_1 average taxes peak at the payroll tax income cap and they decrease for higher labor income levels. In contrast, in model economy E_2 , once the payroll tax income cap is reached, the average tax rate increases asymptotically to that economy's flat tax rate.³⁰

To compare the labor income taxes before and after the reform, in Figure 2 we plot the marginal and average tax rates of the three model economies in the same graphs for the three values of wealth mentioned above. We find that, when compared with the current tax system, flat taxes favor the labor income poorest at the expense of all other labor income earners for the three levels of wealth that we consider.

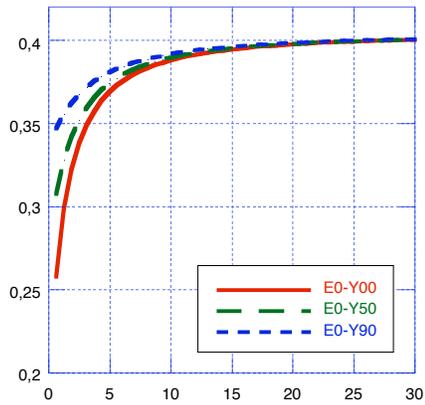
As Figure 3 illustrates, the comparison of capital income taxes is very different. In Panels A and B of that figure we plot the marginal and the average tax rates paid on capital income by the households in the first, fifth and ninth deciles of the income distribution in the benchmark model economy when we consider together the capital income tax and the personal income tax. In this case, since the capital income tax is proportional and uncapped and the personal income tax is progressive, the total average and marginal taxes on capital income are also progressive.

Panels C and D of that same figure illustrate the large role played by the double taxation of capital in the current tax system. Even though the marginal rates of the capital income tax are higher in the two reformed economies (21.5 and 29.2 percent) than in the benchmark model economy (14.6 percent), when we account for the double taxation of capital income this result is reversed. With the exception of low incomes, panel C illustrates that the effective marginal taxes on capital income are higher in the benchmark model economy than in the

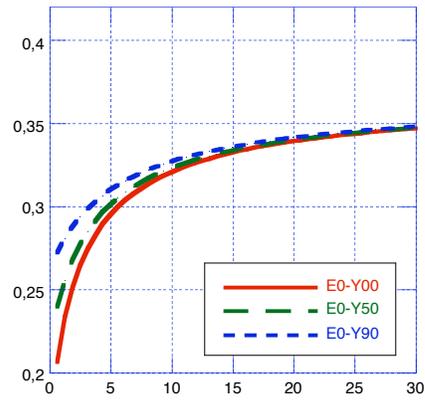
²⁹Notice that the shape of the current payroll tax creates serious technical difficulties when solving the households' decision problem because it implies that the returns to working are increasing in hours at certain points, and therefore the household decision problem becomes non-convex. See Section B of the Appendix for details about this issue.

³⁰This is the case because in model economy E_2 the flat tax rate is higher than the average tax at the payroll tax cap.

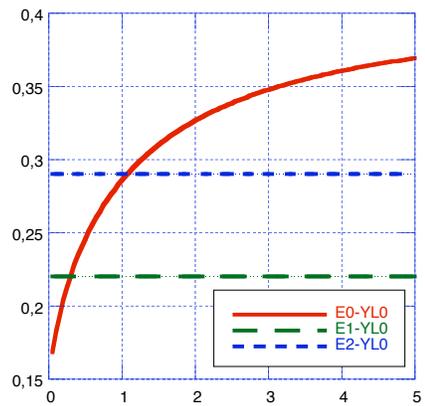
Figure 3: Tax rates on capital income in the benchmark and in the reformed model economies (E_0 , E_1 and E_2)



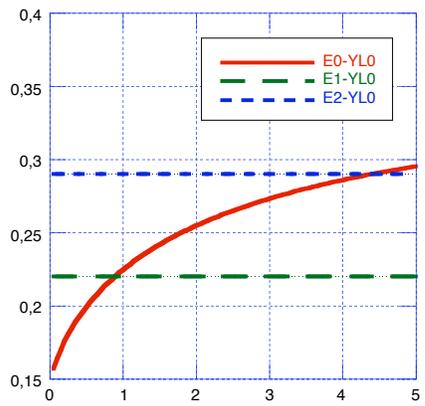
Panel A: Marginal tax rates in E_0



Panel B: Average tax rates in E_0



Panel C: Marginal tax rates (E_i)



Panel D: Average tax rates (E_i)

two reformed economies. Panel D shows that the total average tax rates on capital income display a similar behavior. As we discuss below, these changes in the taxation of capital bring about large changes in the steady-state capital stocks of the model economies.

Table 5: Production, inputs and input ratios in the model economies

	Y	K	L^a	H^b	K/L	L/H	Y/H
E_0	1.62	5.76	0.75	33.70	7.64	2.24	4.80
E_1	1.66	6.16	0.75	33.46	8.19	2.25	4.95
E_2	1.58	5.43	0.75	33.26	7.26	2.25	4.74
$E_1/E_0(\%)$	2.44	6.93	-0.16	-0.69	7.10	0.53	3.15
$E_2/E_0(\%)$	-2.64	-5.72	-0.73	-1.30	-5.03	0.58	-1.35

^aVariable L denotes the aggregate labor input.

^bVariable H denotes the average percentage of the endowment of time allocated to the market.

5.2 Macroeconomic aggregates and ratios

In Tables 5 and 6 we report the main macroeconomic aggregates and ratios of our model economies. We find that the steady-state aggregate consequences of the two flat tax reforms turn out to be very different. While the less progressive reform is expansionary (output increases by 2.4 percent and labor productivity by 3.15 percent), the more progressive reform is contractionary (output decreases by -2.6 percent and labor productivity by -1.4 percent).

The expansion in model economy E_1 output is brought about by an increase in the aggregate capital and a reduction in the aggregate labor input (6.9 percent and slightly less than -0.2 percent). In contrast, the contraction in model economy E_2 is brought about by a reduction both in the aggregate capital and in the aggregate labor input (-5.7 percent -0.7 percent). Since the capital to labor ratio increases in model economy E_1 and decreases in model economy E_2 , in model economy E_1 the steady-state interest rate is lower and the wage rate is higher than in the benchmark economy, and the opposite is true for economy E_2 . We also find out that the changes in labor productivity are the result of large changes in the capital to labor ratio, K/L , which dwarf the changes in the average efficiency of labor, L/H . (see the last two columns of Table 5).

The reasons that justify these results are the following: first, the flat tax reforms reduce the marginal capital income tax rates faced by the wealthy and by the income rich, but they increase the marginal capital incomes tax rates faced by the wealth poor; second, flat tax

Table 6: The values of the targeted ratios and aggregates in the U.S. and in the benchmark model economies

	$C/Y(\%)$	$I/Y(\%)$	$G/Y(\%)$	K/Y	$H^a/\ell(\%)$	$(cv_c/cv_l)^b$	$e_{40/20}$	$\rho(f, s)$
<i>U.S.</i>	54.2	22.5	23.3	3.58	33.3	3.5	1.30	0.40
E_0	59.2	18.0	22.8	3.56	33.7	3.3	1.23	0.14
E_1	59.0	18.8	22.3	3.71	33.5	3.6	1.24	0.15
E_2	59.2	17.4	23.4	3.45	33.3	3.5	1.23	0.16
E_0	59.2	18.0	22.8	3.56	—	—	—	—
E_1/Y_0	60.4	19.2	22.8	3.81	—	—	—	—
E_2/Y_0	57.6	17.0	22.8	3.35	—	—	—	—

^aThis ratio denotes the average share of disposable time allocated to the market.

^bThis statistic is the ratio of the coefficients of variation of consumption and of hours worked.

reforms increase the marginal taxes on labor income paid by every household except the labor income poor. Since the reforms distort every margin and affect different households in different ways, their overall effects can vary significantly from one reform to another. Overall, we find that the flat tax reforms that we consider are expansionary as long as the integrated flat rate is small enough, and that they become contractionary as we increase the labor income tax-exemption.

Both the size and the sign of our results contrast sharply with the findings by Ventura (1999) and Altig, Auerbach, Kotlikoff, Smetters, and Walliser (2001). Like we do, Ventura (1999) considers two flat tax reforms with labor income tax exemptions equal to 20 and 40 percent of per household income in his benchmark economy. Unlike us, he finds both reforms to be expansionary, with steady state output gains in the order of 10 percent.³¹ There are important differences between his model economy and ours. First, Ventura (1999)'s capital income tax allows for the full expensing of investment. Therefore, in his reformed economies capital accumulation is not taxed in the margin and this brings about very large increases in aggregate capital (the capital to labor ratio increases by approximately 20 percent in his two reforms). Second, Ventura (1999) uses the statutory income brackets and tax rates to proxy for the effective personal income tax rates. Therefore, in his benchmark model economy the marginal taxes rates of the personal income tax are higher than ours and the flat tax reforms bring about efficiency gains that are larger. Finally, Ventura (1999) largely understates the

³¹In his reforms the revenue neutral marginal tax rates are 19.1 and 25.2 percent respectively. These tax rates are somewhat lower than our 21.5 and 29.2 percent rates.

concentration of the earnings, income and wealth distribution and this distorts his findings.³²

The comparison of our findings to those of Altig, Auerbach, Kotlikoff, Smetters, and Walliser (2001) is less direct. Altig, Auerbach, Kotlikoff, Smetters, and Walliser (2001) study a sequence of reforms. First, they look at a purely proportional tax on all income; second, they allow for full expensing of new investment, which makes their income tax equivalent to a consumption tax; and third, they add a labor income tax exemption.³³ They find that these three reforms increase aggregate output in the long run. A strictly proportional income tax increases output by 5 percent, allowing for the expensing of new investment increases output by an additional 4 percent and adding a fixed deduction to labor income requires a higher marginal tax rate that brings the output increase back to 4.5 percent. Our model economy differs from Altig, Auerbach, Kotlikoff, Smetters, and Walliser (2001)'s in several important dimensions. First, we allow for earnings and wealth mobility.³⁴ This feature of our model economy should reduce the impact of the reforms because our income process is mean reverting, at least at the dynastic level. Second, we consider uninsurable labor market uncertainty. Third, earnings, income and wealth are more concentrated in our model economy than in theirs. Finally, our households are altruistic towards their descendants and our model economy displays some of the intergenerational correlation of earnings observed in the data. We think that this feature is important because the bequest motive is arguably one of the main determinants of wealth accumulation (see De Nardi (2004), for example). Meaningful evaluations of the distributional consequences of tax reforms require realistic wealth distributions, but this realism should also be achieved through the appropriate margins.

In Table 6 we report additional aggregate statistics of the benchmark and the flat-tax economies. Overall, we find that the changes brought about by the flat-tax reforms are rather small. Not surprisingly, the most noteworthy changes are those in the investment to output ratio which is 18.0 in the benchmark model economy, 18.8 in model economy E_1 and 17.4 in economy in model economy E_2 .

5.3 Fiscal Policy Ratios

In Table 7 we report the main fiscal policy ratios of the model economies. In model economy E_1 the tax revenue to output ratio is smaller than in the benchmark economy, and its government expenditures to output ratio and its transfers to output ratio are reduced accordingly.

³²The benchmark economy in Ventura (1999) displays a gini index for the earnings distribution equal to 0.47 and a gini index for the wealth distribution equal to 0.60.

³³They consider two additional reforms with different tax reliefs for capital holders during the transition.

³⁴Ventura (1999) also models this feature.

Table 7: Fiscal policy ratios in U.S. and in the model economies (%)

	G/Y	Z/Y	T/Y	T_y/Y	T_l/Y	T_k/Y	T_s/Y	T_c/Y	T_e/Y	T_L/Y	T_K/Y	T_Y/Y
<i>U.S.</i>	22.3	5.2	27.5	11.0	–	2.6	6.6	7.1	0.24	–	–	–
E_0	22.8	4.5	27.2	11.6	–	2.9	5.9	6.5	0.37	7.9	5.1	14.5
E_1	22.3	4.3	26.6	–	9.9	4.1	5.8	6.5	0.46	9.9	4.1	14.0
E_2	23.4	4.6	28.0	–	9.3	5.9	5.9	6.5	0.43	9.3	5.9	15.2
E_0	22.8	4.5	27.2	11.6	–	2.9	5.9	6.5	0.37	7.9	5.1	14.5
E_1/Y_0	22.8	4.5	27.3	–	10.1	4.2	5.9	6.6	0.47	10.1	4.2	14.3
E_2/Y_0	22.8	4.5	27.3	–	9.0	5.7	5.8	6.3	0.41	9.1	5.7	14.8

The opposite is the case in model economy E_2 . These results arise because both reforms are revenue neutral, and while aggregate output in model economy E_1 is somewhat larger than in the benchmark economy, in model economy E_2 it is somewhat smaller.

As Hall and Rabushka (1995) had guessed, we find that the labor income tax of the reformed economies collects less revenues than the personal income tax of the benchmark model economy, and these revenue losses are compensated by the higher revenues collected by the capital income tax. Moreover, in our three model economies the revenues collected by the payroll and consumption taxes, and the combined revenues of the total taxes on labor and capital income are very similar.

To make the tax collections of the three model economies comparable, we decompose the personal income taxes of the benchmark model economy into two shares, the share attributable to labor income, which we label T_L , and the share attributable to capital income taxes, which we label T_K .³⁵ In the last three columns of Table 7 we report these two shares and the sum of the labor and capital income taxes which we label T_Y . We find that the two flat-tax reforms bring about increases in the average tax burden on labor income (from 7.9 percent in economy E_0 to 10.1 and 9.1 percent in economies E_1 and E_2). In contrast, the average tax burden on capital income decreases in economy E_1 and increases in economy E_2 (from 5.1 percent to 4.2 and 5.7 percent). If we compare the two reformed economies, we find that economy E_1 places a higher tax burden on labor income and a lower tax burden on capital income than economy E_2 . This is because the higher flat-tax rate of economy E_2 affects both labor and capital income, while the larger tax exemption affects only labor income.

³⁵This decomposition of household income taxes is similar to the one used by Mendoza, Razin, and Tesar (1994).

5.4 Earnings, income and wealth inequality

In Table 8 we describe the earnings, income and wealth inequality in the U.S. and in the model economies. Since the three economies have identical processes on the endowments of efficiency labor units, it is not surprising that the changes in the distribution of earnings are very small. In contrast, before-tax income and, especially, wealth become more unequally distributed under both reforms. This is not surprising since the marginal tax on capital income for the wealthy is lower in the reformed economies than in the benchmark economy. More interestingly, the implications of the reforms for the distribution of after-tax income differ significantly: while after-tax income inequality increases in the less progressive tax reform, it decreases sizably in the more progressive tax reform. It is along this dimension that policymakers truly face the classical trade-off: the gains in efficiency of the less progressive flat tax reforms are obtained at the expense of greater after-tax income inequality.

In particular, in the second panel of Table 8 we report the Gini indexes and some points of the Lorenz curves of the wealth distributions. The flat tax reforms bring about large increases of the Gini index of wealth (from 0.813 in the benchmark economy to 0.839 and 0.845 in the two flat-tax economies) and in the shares of wealth owned by the top quintiles and the top percentile (2.3 and 3.4 percentage points, and 3.2 and 4.9 percentage points). In the last two panels of Table 8 we report the Gini indexes and the Lorenz curves of the income distributions. We find that the changes in before-tax income are rather small. The Gini indexes increase from 0.533 to 0.541 under both reforms, and the changes in the shares earned by the quintiles are minor (less than half a percentage point in every case).

Most of the distributional consequences of the flat-tax reforms occur in the after tax income distribution. In the less progressive reform the Gini index of after-tax income increases from 0.510 to 0.524 and the shares of after-tax income earned by the households in the top quintile and in the top percentile increase by 1.4 and 1.6 percentage points. In contrast, under the more progressive tax reform the Gini index of after-tax income decreases to 0.497 and the share earned by the bottom 60 percent of the distribution increases by 1.2 percentage points. However, in spite of the large progressivity of this reform, the share of after-tax income earned by the income richest still increases by one percentage point.

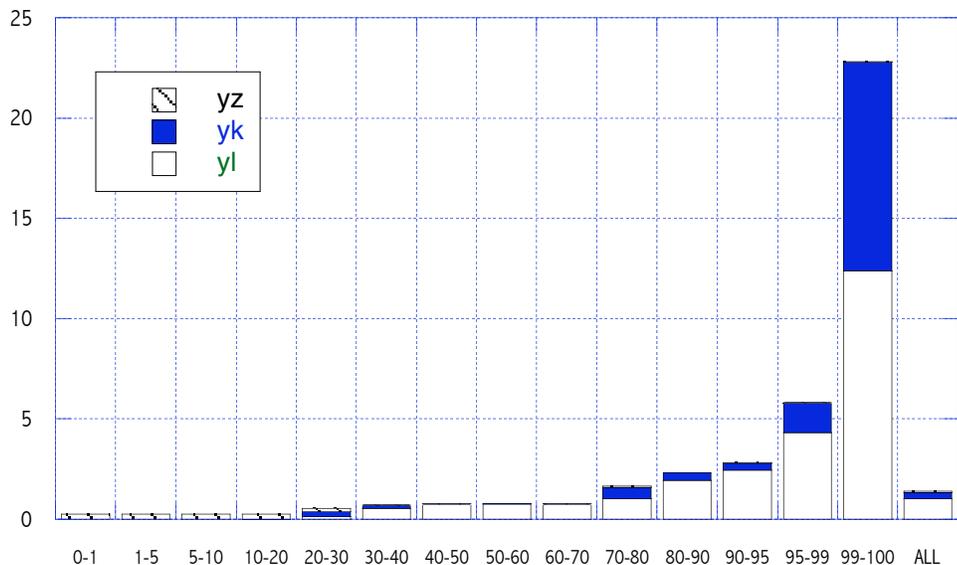
5.5 The distribution of the tax burden

In Table 9 we rank the households according to their income before taxes and after transfers and we report the distribution of the tax burden in our model economies along this dimension.

Table 8: The distributions of earnings, wealth and income in the U.S. and in the model economies

The earnings distributions									
	<i>Gini</i>	<i>Quintiles (%)</i>					<i>Top groups (%)</i>		
<i>Economy</i>		1st	2nd	3rd	4th	5th	90–95	95–99	99–100
<i>U.S.</i>	0.611	−0.4	3.2	12.5	23.3	61.4	12.4	16.4	14.8
E_0	0.613	0.0	4.2	14.4	18.8	62.5	11.8	16.7	15.2
E_1	0.615	0.0	4.2	14.2	18.7	62.9	12.0	16.8	15.0
E_2	0.610	0.0	5.2	13.8	18.2	62.8	11.7	16.9	15.4
The wealth distributions									
	<i>Gini</i>	<i>Quintiles (%)</i>					<i>Top Groups (%)</i>		
<i>Economy</i>		1st	2nd	3rd	4th	5th	90–95	95–99	99–100
<i>U.S.</i>	0.803	−0.4	1.7	5.7	13.4	79.5	12.6	24.0	29.6
E_0	0.818	0.0	0.3	1.5	15.9	82.2	12.6	19.8	34.7
E_1	0.839	0.0	0.1	1.0	14.4	84.5	12.5	20.4	37.9
E_2	0.845	0.0	0.1	1.2	13.0	85.6	12.3	20.7	39.6
<i>The income distributions (before all taxes and after transfers)</i>									
	<i>Gini</i>	<i>Quintiles (%)</i>					<i>Top Groups (%)</i>		
<i>Economy</i>		1st	2nd	3rd	4th	5th	90–95	95–99	99–100
<i>U.S.</i>	0.550	2.4	7.2	12.5	20.0	58.0	10.3	15.3	17.5
E_0	0.533	3.7	8.9	10.9	17.2	59.3	10.1	16.6	16.0
E_1	0.541	3.6	8.5	10.8	17.1	60.0	10.3	16.8	16.2
E_2	0.541	3.7	8.9	10.3	16.9	60.2	10.1	17.0	17.0
<i>The income distributions (after all income taxes and transfers)</i>									
	<i>Gini</i>	<i>Quintiles (%)</i>					<i>Top Groups (%)</i>		
<i>Economy</i>		1st	2nd	3rd	4th	5th	90–95	95–99	99–100
E_0	0.510	4.6	9.3	11.0	17.2	57.9	10.1	16.3	14.9
E_1	0.524	4.7	8.9	10.4	16.7	59.3	10.1	16.7	16.5
E_2	0.497	4.9	9.8	11.4	17.1	56.7	9.5	15.9	15.9

Figure 4: Labor income, capital income, and transfer income in the benchmark model economy



The tax burden under the current tax system. We find that the current distribution of the tax burden almost proportional, in spite of the fact that the personal income tax code is designed to make the current tax system progressive. It is true that the average personal income tax collections are clearly progressive (see the first row of Panel 1 of Table 9). But, with the exception of the average taxes paid by the households in the first quintile and the last percentile, this progressivity all but disappears when we add all income taxes together or when we consider the entire tax system (see the first rows of Panels 6 and 7). The average tax collections of both the capital income taxes and of all taxes on labor income added together (see Panels 2 and 5) display interesting see-saw patterns. The peculiarities of the interaction between the payroll tax and the personal income tax discussed above are some of the reasons that justify this pattern. Other reasons can be found in Figure 4 where we plot the sources of household income for the deciles of the income distribution. As Figure 4 illustrates, the shares of capital income also display the see-saw pattern, since the households in the second quintile, many of them rich retirees, own a disproportionately large share of total capital.

The tax burden in the flat-tax economies. When we compare the distribution of the tax burden before and after the reforms we find the following: first, the income poorest households pay significantly less income taxes in the reformed economies than in the benchmark model economy (see Panel 6 of Table 9); second, in model economy E_1 , the income rich pay

Table 9: Average taxes paid by the quantiles of the distribution of income before taxes and after transfers (%)

	<i>All</i>	<i>Quintiles</i>					<i>Top Quantiles</i>		
	0–100	1st	2nd	3rd	4th	5th	90–95	95–99	99–100
<i>Panel 1: Personal income taxes (τ_y)</i>									
E_0	8.5	4.5	6.7	7.7	9.1	14.6	14.1	17.6	19.5
<i>Panel 2: Capital income taxes ($\tau_k + \tau_{yk}$)</i>									
E_0	3.6	0.4	7.0	0.2	4.7	5.6	3.1	8.5	13.9
E_1	2.9	0.3	6.5	0.1	3.8	3.8	2.0	5.6	9.7
E_2	3.8	0.3	7.2	0.3	6.1	5.3	3.0	7.8	13.9
<i>Panel 3: Labor income taxes ($\tau_l + \tau_{yl}$)</i>									
E_0	6.0	0.0	3.5	7.6	7.0	11.7	12.6	13.0	11.6
E_1	7.7	0.0	3.4	10.7	9.8	14.6	16.2	14.4	11.1
E_2	3.8	0.0	0.0	0.0	3.1	16.1	17.9	17.7	13.8
<i>Panel 4: Consumption taxes (τ_c)</i>									
E_0	9.1	10.1	12.5	7.7	8.5	6.5	5.8	6.7	6.5
E_1	9.1	10.4	12.9	7.4	8.4	6.5	5.6	6.8	7.1
E_2	9.2	10.3	11.8	8.4	9.3	6.2	5.5	6.2	6.6
<i>Panel 5: All labor income taxes ($\tau_l + \tau_{yl} + \tau_s$)</i>									
E_0	14.1	0.0	10.9	22.7	18.4	18.4	19.7	16.3	13.1
E_1	15.9	0.0	11.1	25.9	21.2	21.2	23.1	17.6	12.5
E_2	12.1	0.0	8.4	15.2	14.1	23.0	25.2	20.9	15.1
<i>Panel 6: All income taxes ($\tau_k + \tau_l + \tau_y + \tau_s$)</i>									
E_0	18.8	4.8	19.0	22.9	23.2	24.0	22.8	24.8	27.0
E_1	18.8	0.3	17.7	26.0	25.0	25.0	25.2	23.2	22.2
E_2	16.0	0.3	15.6	15.4	20.3	28.3	28.2	28.8	28.9
<i>Panel 7: All Taxes ($\tau_k + \tau_l + \tau_y + \tau_s + \tau_c + \tau_e$)</i>									
E_0	27.9	14.9	31.5	30.6	31.9	30.7	28.8	31.6	35.5
E_1	28.0	10.6	30.5	33.4	33.8	31.6	30.9	30.1	32.0
E_2	25.3	10.6	27.4	23.9	29.8	34.7	33.9	35.1	38.1

less taxes than the households in the third and fourth income quintiles who foot the largest shares of the tax bill (see Panel 7); third, in model economy E_2 the tax burden is distributed more progressively than in the benchmark economy: the households in the bottom four quintiles bear smaller shares of the total tax burden and the households in the top quintile bear a larger share (see Panel 7); fourth, in both flat tax economies capital income taxes replicate the see-saw pattern of the benchmark economy and average capital income taxes are higher in economy E_2 than in economy E_0 , while in economy E_1 they are lower;³⁶ and fifth the large labor income tax exemption of model economy E_2 generates a very unequal distribution of average labor income tax rates. If we exclude payroll taxes, the bottom 60 percent of the income distribution of model economy E_2 pays no labor income taxes, whereas the average tax rates paid by the households in the top quintile are 4.4 percentage points higher than in the benchmark economy, and 1.5 percentage points higher than in model economy E_1 (see the Panel 3 of Table 9).

5.6 Welfare

In model economy E_1 , the less progressive flat-tax economy, aggregate output, consumption, productivity and leisure are all higher than in model economy E_0 . In contrast, in model economy E_2 , the more progressive flat-tax economy, aggregate output, consumption and productivity are lower, and only aggregate leisure is higher than in model economy E_0 . These results are consistent with the idea that high tax rates and small tax bases are more distortionary than low tax rates and big tax bases, at least on aggregate.

However, it is also true that the after-tax income distribution in model economy E_1 is significantly more unequal than in model economy E_2 . Therefore, a policymaker who had to choose between these two reforms would face the classical trade-off between efficiency and equality. Which economy should she choose: the more efficient but less egalitarian model economy E_1 , or the less efficient but more egalitarian model economy E_2 ? In this section we use a Benthamite social social welfare function to quantify the trade-off between efficiency and equality and to answer this question.³⁷

To carry out the welfare comparisons, we define $v_0(a, s, \Delta)$ as the equilibrium value function of a household of type (a, s) in model economy E_0 whose equilibrium consumption

³⁶This finding is consistent with the fact that in model economy E_1 there is more aggregate capital than in model economy E_0 , while in model economy E_2 there is less aggregate capital.

³⁷Benthamite social welfare functions give identical weights to every household in the economy. Consequently, with concave utility functions, equal sharing is the welfare maximizing allocation. Also notice that in this section we compare the welfare of steady-state allocations and we remain conspicuously silent about the transitions between these steady-states.

allocation is changed by a fraction Δ every period and whose leisure remains unchanged. Formally,

$$v_0(a, s, \Delta) = u(c_0(a, s)(1 + \Delta), \ell - h_0(a, s)) + \beta \sum_{s' \in S} \Gamma_{ss'} v(z_0(a, s), s', \Delta) \quad (25)$$

where $c_0(a, s)$, $h_0(a, s)$ and $z_0(a, s)$ are the optimal decision rules that solve the household decision problem defined in expressions (3–7). Next, we define the welfare gain of living in the steady-state of flat-tax economy E_i (where $i = 1, 2$), as the fraction of additional consumption, Δ_i , that we must give to or take away from the households of the benchmark model economy so that they attain the steady-state welfare of the households in model economy. Formally, Δ_i is the solution to the equation

$$\int v_0(a, s, \Delta_i) dx_0 = \int v_i(a, s) dx_i \quad (26)$$

where v_i and x_i are the equilibrium value function and the equilibrium stationary distribution of households in the flat-tax model economy E_i .

We find that the equivalent variation in consumption for the less progressive flat-tax reform is $\Delta_1 = -0.0017$, and that the equivalent variation in consumption for the more progressive flat-tax reform is $\Delta_2 = 0.0045$. This means that, from a Benthamite point of view, the flat-tax reform with a low tax rate and a small labor income tax exemption results in an aggregate welfare loss that is equivalent to -0.17 percent of consumption, and that the flat-tax reform with a high tax rate and a big labor income tax exemption results in an aggregate welfare gain that is equivalent to 0.45 percent of consumption. This leads us to conclude that, in Benthamite welfare terms, equality wins the trade-off and that, given the choice, a Benthamite social planner would choose the more progressive flat-tax reform. This result is consistent with the findings by Conesa and Krueger (2005), who show that a purely proportional income tax reduces social welfare in spite of increasing aggregate output and consumption by almost nine percent.

Flat-tax reforms are fundamental tax reforms that change every margin of the households' and the firms' decision problems. These changes in the individual behavior of households and firms translate into changes in aggregate allocations and prices, which result in further changes in the individual decisions. Moreover, the solutions to these fundamentally different decision problems generate equilibrium distributions of households that are also fundamentally different. To improve our intuitive understanding of our welfare findings, it is useful to decompose the equivalent variation in consumption discussed above into different components.

To this purpose, we define two auxiliary measures of the equivalent variations in consumption for each reform. First, we compute the equivalent variation in consumption that makes the households indifferent between the benchmark model economy E_0 and the flat-tax economy E_i ignoring the changes in the equilibrium distribution of households. We denote this variation by Δ_i^a , and we define it as follows:

$$\int v_0(a, s, \Delta_i^a; r_0, w_0) dx_0 = \int v_i(a, s; r_i, w_i) dx_0 \quad (27)$$

Notice that in this expression we calculate the aggregate welfare of the flat-tax economy using its equilibrium price vector, (r_i, w_i) , and the equilibrium stationary distribution of the benchmark model economy.

Second, we compute the equivalent variation in consumption that makes the households indifferent between the benchmark model economy E_0 and the flat-tax economy E_i ignoring both the changes in the equilibrium distribution of households and the changes in the size of the economy. We denote this variation by Δ_i^b , and we define it as follows:

$$\int v_0(a, s, \Delta_i^b; r_0, w_0) dx_0 = \int v_i(a, s; r_0, w_0) dx_0 \quad (28)$$

Notice that now we calculate the aggregate welfare of the flat-tax economy using both the equilibrium stationary distribution and the equilibrium price vector of the benchmark model economy.

These two equivalent variations allow us to decompose the total equivalent variation that we have defined in expression (26) above as follows:

$$\Delta_i = \Delta_i^b + (\Delta_i^a - \Delta_i^b) + (\Delta_i - \Delta_i^a) \quad (29)$$

The first term of expression (29) measures the welfare changes that are due to the reshuffling of resources between different households and ignoring the general equilibrium effects and the changes in the distribution of households. The second term measures the welfare changes that are due to the general equilibrium effects only. And the third term measures the welfare gains that are due to the changes in the distribution of households.

In Table 10 we report this decomposition for the two reforms that we study in this article. We find that, when we abstract from the distributional and general equilibrium changes, the less progressive flat-tax reform results in a welfare loss that is equivalent to -0.27 percent of consumption. In contrast, the more progressive flat-tax reform results in a welfare gain that is equivalent to 3.64 percent of consumption. These welfare changes are the direct consequence of the redistribution of the tax burden, and of the new individual allocations of consumption and leisure that the flat-tax systems generate. This result confirms Domeij

Table 10: Decomposing the aggregate welfare changes

<i>Equivalent variations in consumption (%)</i>				
<i>Economy</i>	Δ_i^b	$(\Delta_i^a - \Delta_i^b)$	$(\Delta_i - \Delta_i^a)$	Δ_i
E_1	-0.27	0.75	-0.65	-0.17
E_2	3.64	-0.58	-2.62	0.45

and Heathcote (2004)'s findings who show that shifting the tax burden from labor to capital income brings about sizeable welfare gains because it implies a transfer of resources from the wealth-rich households to the wealth-poor households. This transfer brings about large gains in Benthamite welfare. As we have discussed above, in flat-tax economy E_2 capital income tax collections are higher than in flat-tax economy E_1 and labor income tax collections are lower. Hence, if all other things were to remain equal, the more progressive flat-tax reform would have been significantly better than the less progressive flat-tax reform.

When we consider the general equilibrium effects brought about by the change in prices the sign of the welfare changes is reversed. The less progressive flat-tax reform results in a welfare gain that is equivalent to 0.75 percent of consumption, and the more progressive tax reform results in a welfare loss that is equivalent to -0.58 percent of consumption. These welfare changes are the consequence of the efficiency gains and losses that result from the new aggregate values of consumption and leisure in the flat-tax economies.

Finally, the new equilibrium distributions of the flat-tax model economies result in welfare losses that are equivalent to -0.65 percent of consumption in the less progressive flat-tax reform, and to -2.62 percent of consumption in the more progressive flat-tax reform. These welfare losses arise because both reforms put more households in points in the state space that have a lower utility.

Welfare comparisons between steady-states can be misleading because they ignore the welfare changes that take place during the transitions between the steady-states. These welfare changes may be large and they might even reverse the signs of the steady-state comparisons. However, there are some exceptions to this general rule and this case may very well be one of these exceptions. As we have discussed above, the steady-state aggregate stock of capital is almost seven percent larger in the flat-tax economy E_1 than in the benchmark economy E_0 . Therefore, during the transition from E_0 to E_1 the households will pay the cost of accumulating capital and will, consequently, enjoy less consumption. This implies that accounting for the transition would make the less progressive flat-tax reform even more costly

in welfare terms. In contrast, the steady-state capital stock is almost six percent smaller in flat-tax economy E_2 than economy E_0 . In this case the transition from E_0 to E_2 should be quite a pleasant affair since it allows the households to enjoy more consumption while they are reducing their capital stock. Therefore, we conjecture that accounting for the transition would make the welfare gains of the more progressive flat-tax reform even larger. If these two conjectures are correct, accounting for the transitions would increase the steady-state welfare differences between the two flat-tax reforms.

Finally, we compute the individual welfare changes brought about by the reforms for the various types of households individually. Formally, for each household type $(a, s) \in \mathcal{A} \times S$, we compute the equivalent variation $\Delta_i(a, s)$ as follows:

$$v_0(a, s, \Delta_i(a, s); r_0, w_0) = v_i(a, s; r_i, w_i) \quad (30)$$

This welfare measure is the fraction of additional consumption that we must give to or take away from each household-type of the steady state of the benchmark model economy to make it indifferent between staying in the benchmark model economy forever or being dropped in the steady-state of the flat-tax economy keeping its assets and its household-specific shock. These household-specific welfare measures take into account the general equilibrium effects but they ignore the changes in the equilibrium distributions by construction.

Table 11: Welfare inequality

		<i>Equivalent variation of consumption (%)</i>									
<i>Economy</i>	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	
E_1	0.3	3.0	2.7	0.5	-0.2	-0.5	-0.5	-0.9	-1.1	0.0	
E_2	2.4	4.7	4.5	2.2	4.1	5.1	5.1	-0.4	-2.9	-3.3	

Once we have computed the welfare changes for each household-type, we aggregate them over the deciles of the before-tax income distribution of the benchmark model economy. We report these statistics in Table 11. The table shows that both flat-tax reforms bring about sizable boons for the income-poor. More specifically, it turns out that the households in the bottom 40 percent of the income distribution of the benchmark model economy would be happier in the steady-state of the less progressive flat-tax model economy (the households in the top decile would also be marginally happier), and that this percentage of happier income-poor households increases to an impressive 70 percent in the more progressive flat-tax model economy. On the other hand, the remaining households, who happen to be the income-rich, would be happier under the current tax system.

6 Concluding comments

Hall and Rabushka (1995) claimed that revenue neutral flat-tax reforms would be expansionary: we find that they can be, as long as we choose the appropriate flat-tax rate and labor income tax exemption. Many suspected that flat-tax reforms would increase wealth inequality: we find that indeed they do, but we also find that in steady-state Benthamite welfare terms it matters little. Policy-makers fret that flat-tax reforms will increase the tax rates capital paid on capital income by the wealthy: we find that, by removing the double taxation of capital income, flat-tax reforms can actually reduce the rates of capital income taxes. Public finance economists have puzzled about the trade-off between efficiency and equality: this research shows that flat-tax reforms create a large Robin Hood effect in the reshuffling of the tax burden, and it suggests that, at least in our class of model economies, equality wins the trade-off. Finally, economic folklore has defended progressive taxation on the grounds that it is good for the poor: we find that flat taxes can be better.

One way to continue with this research project is to compute the transitions in our class of model economies. We know that this task is not easy, and that it probably requires a large cluster of parallel computers, but we surely hope that someone will raise to the task of giving it a try.

Appendix

A The definition of parameters ϕ_1 and ϕ_2

Let p_{ij} denote the transition probability from $i \in \mathcal{R}$ to $j \in \mathcal{E}$, let γ_i^* be the invariant measure of households that receive shock $i \in \mathcal{E}$, and let ϕ_1 and ϕ_2 be the two parameters whose roles are described in Section 4.1.2, then the recursive procedure that we use to compute the p_{ij} is the following:

- *Step 1:* First, we use parameter ϕ_1 to shift the probability mass from a matrix with vector $\gamma_{\mathcal{E}}^* = (\gamma_1^*, \gamma_2^*, \gamma_3^*, \gamma_4^*)$ in every row towards its diagonal, as follows:

$$\begin{aligned} p_{51} &= \gamma_1^* + \phi_1 \gamma_2^* + \phi_1^2 \gamma_3^* + \phi_1^3 \gamma_4^* \\ p_{52} &= (1 - \phi_1)[\gamma_2^* + \phi_1 \gamma_3^* + \phi_1^2 \gamma_4^*] \\ p_{53} &= (1 - \phi_1)[\gamma_3^* + \phi_1 \gamma_4^*] \\ p_{54} &= (1 - \phi_1) \gamma_4^* \\ p_{61} &= (1 - \phi_1) \gamma_1^* \\ p_{62} &= \phi_1 \gamma_1^* + \gamma_2^* + \phi_1 \gamma_3^* + \phi_1^2 \gamma_4^* \\ p_{63} &= (1 - \phi_1)[\gamma_3^* + \phi_1 \gamma_4^*] \\ p_{64} &= (1 - \phi_1) \gamma_4^* \\ p_{71} &= (1 - \phi_1) \gamma_1^* \\ p_{72} &= (1 - \phi_1)[\phi_1 \gamma_1^* + \gamma_2^*] \\ p_{73} &= \phi_1^2 \gamma_1^* + \phi_1 \gamma_2^* + \gamma_3^* + \phi_1 \gamma_4^* \\ p_{74} &= (1 - \phi_1) \gamma_4^* \\ p_{81} &= (1 - \phi_1) \gamma_1^* \\ p_{82} &= (1 - \phi_1)[\phi_1 \gamma_1^* + \gamma_2^*] \\ p_{83} &= (1 - \phi_1)[\phi_1^2 \gamma_1^* + \phi_1 \gamma_2^* + \gamma_3^*] \\ p_{84} &= \phi_1^3 \gamma_1^* + \phi_1^2 \gamma_2^* + \phi_1 \gamma_3^* + \gamma_4^* \end{aligned}$$

- *Step 2:* Then for $i = 5, 6, 7, 8$ we use parameter ϕ_2 to shift the resulting probability mass towards the first column as follows:

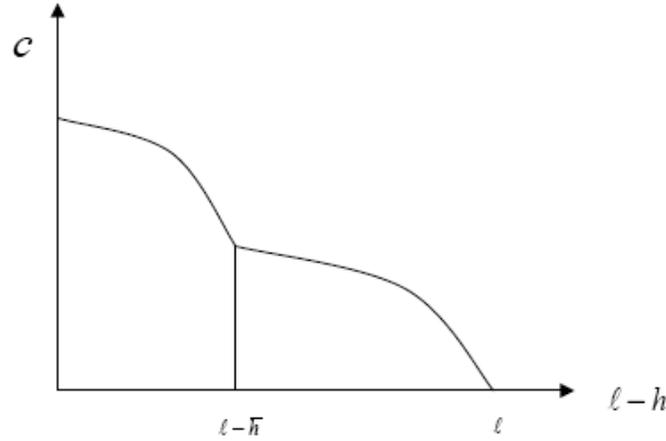
$$\begin{aligned}
p_{i1} &= p_{i1} + \phi_2 p_{i2} + \phi_2^2 p_{i3} + \phi_2^3 p_{i4} \\
p_{i2} &= (1 - \phi_2)[p_{i2} + \phi_2 p_{i1} + \phi_2^2 p_{i4}] \\
p_{i3} &= (1 - \phi_2)[p_{i3} + \phi_2 p_{i4}] \\
p_{i4} &= (1 - \phi_2)p_{i4}
\end{aligned}$$

B Non-convexities

In Section 5.1 and in Figure 1 we establish that in our benchmark model economy, after a point, the total marginal tax on labor income is decreasing. Of course, this makes the marginal tax on work effort also decreasing, and it creates a serious problem when we try to find the optimal household policy. Specifically, given the choice of next period assets, z , the budget set of the contemporaneous labor decision becomes non-convex. In Figure 5 we illustrate this point. Consider pair of individual state variables (a, s) and a choice of end-of-period assets, z . Then, equations (4), (5) and (6) and the boundary constraints on c and h define the consumption possibilities set for c and $\ell - h$. In Figure 5 we plot an example of this set in which $a = 0$. When the household chooses not to work and to enjoy ℓ units of leisure, its consumption is zero. As the household starts to work, its consumption increases albeit at a decreasing rate. This is because of the progressivity of the personal income tax, τ_y , which reduces the after-tax wage of every extra hour of work. Let \bar{h} be the hours of work such that $e(s)\bar{h}w = a_3$. For $h > \bar{h}$ the marginal payroll tax is zero. Therefore the slope of the consumption possibilities set increases discretely at $h = \bar{h}$ and from that point onwards it decreases monotonically as we increase h , again because of the progressivity of τ_y .

This lack of convexity is twice unfortunate. First, because it implies that the first order necessary conditions are no longer sufficient for the optimum, and therefore they do not identify the optimal solution uniquely. In fact there are two points that potentially satisfy the first order conditions, one above and one below the threshold \bar{h} , and only one of these points is the optimal solution. Second, as we change the choice of end-of-period assets, z , the optimal choice of hours becomes discontinuous exactly when we move from a solution at one side of \bar{h} to a solution at the other side of \bar{h} . This is much more troublesome for our computational procedure, and it forces us to use the much more computationally intensive discrete value function iterations instead of Euler equation iterations to solve the household decision problem.

Figure 5: Non-convex constraints



C Computation

As we have mentioned in Section 3, to calibrate our model economy we must solve a system of 29 non-linear equations in 29 unknowns. Actually, we solve a smaller system of 25 non-linear equations in 25 unknowns because the value of government expenditures, G , is determined residually from the government budget, and because three of the tax parameters are functions of our guess for aggregate output. This non-linear system is only the outer loop of our computational procedure because we must also find the stationary equilibrium values of the capital labor ratio, K/L , and of aggregate output, Y , for each vector of unknowns. The details of our computational procedure are the following:

- *Step 1:* We choose a vector of weights, one for each of the 25 non-linear equations. These weights measure the relative importance that we attach to each one of our targets.
- *Step 2:* We guess a value for the 25 unknowns
- *Step 3:* We guess an initial value for aggregate output, Y_0 (which determines the values of the three tax parameters mentioned above).
- *Step 4:* We guess an initial value for the capital labor ratio $(K/L)_0$
- *Step 5:* We compute the decision rules, the stationary distribution of households and the new value of the capital labor ratio, $(K/L)_1$
- *Step 6:* We iterate on K/L until convergence

- *Step 7:* We compute the new value of aggregate output, Y_1 , that results from the converged value of K/L
- *Step 8:* We iterate on Y until convergence
- *Step 9:* We iterate on the 25-dimensional vector of unknowns until we find an acceptable solution to the system of 25 non-linear equations.

To find the solution of the system of 25 non-linear equations in 25 unknowns, we use a standard non-linear equation solver. Specifically, we use a modification of Powell’s hybrid method that is implemented in subroutine DNSQ from the SLATEC package.

To calculate the decision rules, we discretize the state space and we use a refinement of the discrete value function iteration method. Our refinement uses upper bounds and monotonicity to reduce the size of the control space and Howard’s policy improvement algorithm to reduce the number of the searches. The size of our state space is $n_k \times n_s = 681 \times 8 = 5,448$ points. The size of our control space is $n_k \times n_n = 681 \times 201 = 136,881$ points for workers and $n_k = 681$ for retirees. Since the numbers of working-age and retirement states are $n_w = n_r = 4$, the total number of search points is $[(n_k \times n_w) \times (n_k \times n_n)] + [(n_k \times n_r) \times n_k] = 374,718,888$ points.

We approximate the stationary distribution, x^* , with a piecewise linearization of its associated distribution function. The grid for this approximation has 80,000 unequally spaced points which are very close to each other near the origin (see Aiyagari (1994), Huggett (1995) or Ríos-Rull (1998) for details).

To compute the model economy’s distributional and aggregate statistics we compute the integrals with respect to the stationary distribution, x^* . We evaluate these integrals directly using our approximation to the distribution function for every statistic except for those that measure mobility, the earnings life cycle, and the intergenerational correlation of earnings. To compute these three statistics, we use a representative sample of 20,000 households drawn from x^* (see Ríos-Rull (1998) for details).

References

- AARON, H. J., AND A. H. MUNNELL (1992): “Reassessing the role for wealth transfer taxes,” *National Tax Journal*, 45, 119–143.
- AIYAGARI, S. R. (1994): “Uninsured Idiosyncratic Risk, and Aggregate Saving,” *Quarterly Journal of Economics*, 109(3), 659–684.
- ALTIG, D., A. J. AUERBACH, L. J. KOTLIKOFF, K. A. SMETTERS, AND J. WALLISER (2001): “Simulating Fundamental Tax Reform in the United States,” *American Economic Review*, 91(3), 574–595.
- BUDRÍA, S., J. DÍAZ-GIMÉNEZ, V. QUADRINI, AND J.-V. RÍOS-RULL (2002): “New Facts on the U.S. Distribution of Earnings, Income and Wealth,” *Federal Reserve Bank of Minneapolis Quarterly Review*, 26(3), 2–35.
- CASTAÑEDA, A., J. DÍAZ-GIMÉNEZ, AND J.-V. RÍOS-RULL (1998): “Exploring the Income Distribution Business Cycle Dynamics,” *Journal of Monetary Economics*, 42(1).
- (2003): “Accounting for U.S. Earnings and Wealth Inequality,” *Journal of Political Economy*, 111(4), 818–857.
- COLE, H. L., AND N. R. KOCHERLAKOTA (1997): “Efficient Allocations With Hidden Income and Hidden Storage,” Discussion Paper 238, Federal Reserve Bank of Minneapolis, Staff Report.
- CONESA, J. C., AND D. KRUEGER (2005): “On the Optimal Progressivity of the Income Tax Code,” forthcoming, *Journal of Monetary Economics*.
- COOLEY, T. F., AND E. C. PRESCOTT (1995): “Economic Growth and Business Cycles,” in *Frontiers of Business Cycle Research*, ed. by T. F. Cooley, chap. 1. Princeton University Press, Princeton.
- DE NARDI, M. (2004): “Wealth Inequality and Intergenerational Links,” *Review of Economic Studies*, 71(3), 743–768.
- DOMEIJ, D., AND J. HEATHCOTE (2004): “On The Distributional Effects Of Reducing Capital Taxes,” *International Economic Review*, 45(2), 523–554.
- GOUVEIA, M., AND R. P. STRAUSS (1994): “Effective Federal Individual Income Tax Functions: An Exploratory Empirical Analysis,” *National Tax Journal*, 47(2), 317–39.
- HALL, R., AND A. RABUSHKA (1995): *The Flat Tax*. Hoover Institution Press.
- HEATHCOTE, J., K. STORESLETTEN, AND G. VIOLANTE (2004): “The Macroeconomic Implications of Rising Wage Inequality in the US,” Mimeo.
- HOPENHAYN, H., AND E. C. PRESCOTT (1992): “Stochastic Monotonicity and Stationary Distributions for Dynamic Economies,” *Econometrica*, 60(6), 1387–1406.

- HUBBARD, G., J. SKINNER, AND S. ZELDES (1994): “The Importance of Precautionary Motives in Explaining Individual and Aggregate Saving,” *Carnegie-Rochester Conference Series on Public Policy*, 40, 59–125.
- HUGGETT, M. (1993): “The Risk Free Rate in Heterogeneous-Agents, Incomplete Insurance Economies,” *Journal of Economic Dynamics and Control*, 17(5/6), 953–970.
- (1995): “The One-Sector Growth Model with Idiosyncratic Risk,” Federal Reserve Bank of Minneapolis Discussion Paper 105.
- JUSTER, F. P., AND F. P. STAFFORD (1991): “The Allocation of Time: Empirical Findings, Behavioral Models, and Problems of Measurement,” *Journal of Economic Literature*, XXIX(2), 471–522.
- MARCET, A., F. OBIOLS-HOMS, AND P. WEIL (2003): “Incomplete Markets, Labor Supply and Capital Accumulation,” Unpublished manuscript, Universitat Pompeu Fabra.
- MENDOZA, E. G., A. RAZIN, AND L. L. TESAR (1994): “Effective tax rates in macroeconomics Cross-country estimates of tax rates on factor incomes and consumption,” *Journal of Monetary Economics*, 34(3), 297–323.
- MIRRELEES, J. A. (1971): “An Exploration in the Theory of Optimum Income Taxation,” *Review of Economic Studies*, 38(2), 175–208.
- PIJOAN-MAS, J. (2006): “Precautionary Savings or Working Longer Hours?,” *Review of Economic Dynamics*, 9(2), 326–352.
- RÍOS-RULL, J.-V. (1998): “Computing Equilibria in Models with Heterogenous Agents,” in *Computational Methods for the Study of Dynamic Economics*, ed. by R. Marimon, and A. Scott, chap. 9. Oxford University Press.
- SAMUELSON, P. (1975): “Optimum Social Security in a Life-Cycle Growth Model,” *International Economic Review*, 16(3), 539–544.
- SOLON, G. (1992): “Intergenerational Income Mobility in the United States,” *American Economic Review*, 82(3), 393–406.
- VENTURA, G. (1999): “Flat Tax Reform: a Quantitative Exploration,” *Journal of Economic Dynamics and Control*, 23, 1425–1458.
- ZIMMERMAN, D. J. (1992): “Regression Towards Mediocrity in Economic Stature,” *American Economic Review*, 3, 409–429.