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JEL Classification: D83, E24, J41, L11

Keywords: Dual Labor Markets, Temporary Contracts, Firm Dynamics, Unemployment

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Dual Labor Markets and the Equilibrium Distribution of Firms*

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May 30, 2024

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1 Introduction

Many labor markets are characterized by a two-tier system, with the co-existence of open-ended contracts (OECs), protected by large termination costs, and fixed-term contracts (FTCs) of short duration. While extensive research has examined the impact of labor market duality on workers, its effects on firm choices and outcomes remain largely unexplored. In this paper, we analyze how different firms use each type of contract, and how their hiring and firing choices affect the allocation of workers across firms, firm entry and exit, and the equilibrium distribution of firms. This allows us to assess the effects of dual labor markets on the aggregate productivity of countries and to study the macroeconomic effects of policies aimed at reducing the share of temporary workers in the economy.

We start by documenting new facts about the heterogeneous use of FTCs across firms. To do so, we exploit Spanish administrative firm-level data for the period 2004-2019. The case of Spain is of particular interest because its labor market duality is very stark, with a high incidence of FTCs and a strong employment protection for OECs.[1] We uncover three important facts. First, there is a large degree of heterogeneity in the usage of FTCs across firms. For instance, among firms with 11 to 50 employees, the average temporary share is 25%, but the firms at the 10th and 90th percentiles of the distribution employ nearly 0% and 68% of workers under FTCs, respectively. Second, although the use of FTCs varies greatly across narrowly-defined industries, provinces, and time, most of the variation occurs due to firm-specific factors within industry, province, and time period, of which time-invariant unobserved firm characteristics play an overwhelmingly relevant role. And third, exploiting the panel dimension of our data, we find that the *within-firm* variation uncovers a positive correlation between firm size and the share of temporary workers, while the *between-firm* variation shows a negative relationship between firm size and the share of temporary workers.

Next, we write a model of firm dynamics with search-and-matching frictions and a two-tier labor market structure to understand these facts and the macroeconomic consequences of dual labor markets. As in Kaas and Kircher (2015) and Schaal (2017), a set of multi-worker firms operating a decreasing returns-to-scale technology can direct unemployed workers by posting (and committing to) dynamic long-term contracts. As in Gavazza, Mongey and Violante (2018) and Carrillo-Tudela, Gartner and Kaas (2023), firms may improve their hiring prospects by exerting higher recruiting effort. In our model, firms may simultaneously post two types of contract, open-ended and fixed-term, which differ in terms of recruiting costs, the rate at which workers separate from the firm, and the ability of the worker to accumulate human capital on the job. We place a large emphasis on firm heterogeneity. Firms differ ex-ante in a *permanent* technology type and ex-post in persistent and *transitory* productivity, in the number of workers, and in the skill composition of their workforce. A firm's technology type determines the permanent component of firm productivity, as well as the relative productivity of workers of different human capital levels. Human capital is accumulated

[1] In 2019, the share of workers under an FTC was 26.3% in Spain, the second highest among OECD countries, just behind the 27.0% in Chile and ahead of the 24.4% in South Korea, 21.8% in Poland, 20.3% in Portugal, or 20.3% in Netherlands (see https://data.oecd.org/emp/temporary-employment.htm).

within the firm and is firm-specific. These two properties serve to give a value to worker stability (better achieved through OECs) while keeping a tractable framework where unemployed workers remain identical. In addition, there is firm exit and entry. Firms exit the economy if they are hit by a destruction shock, or if they lose or fire their last remaining worker, which generates endogenous firm selection. A free-entry condition pins down the aggregate measure of operating firms. In equilibrium, unemployed workers remain ex-ante indifferent between applying for a job under either contract, as well as between the firms that offer them, because less ex-post attractive offers are posted in tighter markets. Firms wishing to expand quickly can either offer higher value to workers (attracting longer queues), exert higher recruiting effort, or both. These choices, when aggregated across firms, lead to endogenously different matching efficiencies across OE and FT labor markets, which is key to fit the data on aggregate labor market flows by contract type.

We calibrate the model to match cross-sectional features of our firm-level data as well as to the aggregate flows into and out of unemployment by contract type. The calibration delivers two key results: (i) high-skilled workers are more productive in high-type firms, implying positive assortative matching between firms and workers; and (ii) recruiting costs are higher for OECs, implying that, for a given market tightness, there are on average more matches per unit of time in the FT market than in the OE market. These two calibration results explain the main empirical facts that we documented in the data. First, because of (i), high-type firms use OECs more extensively to retain a larger share of their workers and enhance their chances of accumulating human capital on the job. In the cross-section of firms, this generates the inverse *between-firm* relationship between firm size and the temporary share that we see in the data. Second, because of (ii), as firms of given productivity grow toward their optimal size, they accumulate relatively more FT workers, which generates the positive *within-firm* relationship between firm size and the temporary share that we see in the data. This is because firms face a trade-off between the lower costs of attracting workers to FTCs (due to the higher job-filling rates for given labor market tightness) and the higher worker turnover of FT positions. This trade-off is solved differently by different firms: because of decreasing returns to scale in production, the larger the firm is for a given productivity level, the lower its opportunity cost of leaving a vacancy unfilled. This implies that firms choose a higher temporary share as they grow toward their optimal size.

The calibrated economy exhibits two additional features that will be consequential for our counterfactuals. First, labor market frictions imply a significant degree of misallocation of workers between firms, with output losses equal to 3.9% relative to an economy where the marginal product of each type of worker is equalized across all existing firms. This happens because there is dispersion of employment across identical firms and because the most productive firms do not get enough employment and operate with a suboptimal level of worker skills. Second, there is substantial firm selection: the share of highly productive firms is 26% among entrants and 40% among incumbents. This selection happens through the higher exit rate of the less productive firm types, which on average operate with lower levels of employment and higher shares of temporary workers.[2]

[2] In an extension, we study a version of our economy that generates selection upon entry.

In the last part of the paper, we use the model for counterfactual analysis with the goal to assess the effects of dual labor markets on the aggregate economy. We do so by comparing the effects of different policy tools aimed at reducing the share of workers employed under FTCs. In our economy, changes in the menu of contracts change the hiring and firing behavior of firms, shaping firm growth, firm entry and exit, and the equilibrium distribution of firms and workers.[3]

We start by changing the maximum duration of FTCs. This is a key characteristic of dual labor markets, as in all countries there is a legal (and relatively short) maximum duration that a worker can stay at the firm under an FTC. Reducing the maximum duration is also a common policy tool for countries that want to limit the use of FTCs (i.e., Spain in its 2022 labor market reforms). The effects of a reduction on the maximum duration of FTCs are *ex ante* ambiguous. On the one hand, reducing the maximum duration of FTCs brings flexibility gains, as firms are less likely to pay firing costs when they get hit by negative productivity shocks. On the other hand, it brings turnover losses, as firms are more likely to pay hiring costs to keep their employment level constant. Given the short duration of FTCs in Spain, in our calibrated economy the second effect dominates, making FTCs less attractive to firms when their maximum duration is reduced. Firms react by hiring proportionately more from the OE market (through higher recruiting effort and the offer of better contracts) and increasing the promotion rate of their incumbent FT workers. This leads to a sharp decline in the aggregate temporary share. However, the policy comes with the cost of an increase in unemployment because there is a decline in the unemployment-to-employment flow that more than offsets the mild decrease in the employment-to-unemployment flow.

Moreover, the policy leads to a small increase in aggregate productivity. We show, through a decomposition, that this is because the productivity losses that stem from increased misallocation of workers across firms (i.e., from firms having less flexibility to adjust employment to shocks as they use fewer FTCs) are slightly lower than the gains due to (i) better firm selection (i.e. worse firm types being relatively more damaged and exiting more), and (ii) increased human capital accumulation from longer worker tenure in the firm. All in all, aggregate output and welfare decline.

As an alternative policy, we introduce a tax on the use of FTCs, mirroring several policies in place in France, Portugal, and Spain. When calibrated to produce the same reduction in the aggregate share of FTCs as our baseline policy of shortening FTC duration, the effect of the tax on aggregate productivity is quantitatively similar. However, the negative effects on unemployment, output, and welfare are less severe, mostly due to the effects on job destruction being less severe. As a result, the tax policy leads to lower worker turnover, lower unemployment, and higher output and welfare than placing limits on the duration of FTCs. Finally, we show that an outright ban on FTCs would be suboptimal: even though FTCs are an impediment to human capital accumulation within the firm,

[3]This is consistent with recent empirical evidence showing that changes in various labor market policies may affect the equilibrium distribution of workers and firms. For instance, Dustmann, Lindner, Schönberg, Umkehrer and vom Berge (2021), Luca and Luca (2019), and Chava, Oettl and Singh (2019) show that changes in minimum wages in Germany or the U.S. reallocate workers to larger and more productive firms, or increase the exit rates of smaller and less productive firms. Likewise, Carry (2022) shows that changes in the minimum workweek regulation in France led to distributional effects among both firms and workers.

they provide flexibility to firms who need to readjust employment in response to adverse shocks. In net, welfare would decrease slightly (by 0.31%) if FTCs were banned completely.

In sum, these various policy exercises show that, through the lens of our calibrated model, restricting the use of FTCs does not typically lead to productivity losses as commonly thought: although the allocation of workers across firms worsens, this is always offset by a better selection of firms in equilibrium. However, restricting the use of FTCs is still a bad idea, because it leads to an increase in unemployment and a fall in total output and welfare.

Related literature There is a large literature studying the effects of the duality of employment contracts on the labor market outcomes of workers, the average unemployment rate, the volatility of employment, and the dynamics of labor market flows. Blanchard and Landier (2002), Cahuc and Postel-Vinay (2002), Bentolila, Cahuc, Dolado and Le Barbanchon (2012), and Sala, Silva and Toledo (2012) study the effect of dual labor markets in models with search and matching frictions *à la* Mortensen and Pissarides (1994). In these models, search is random and firms do not choose which type of contracts to offer. If they did, they would hire all new workers in FTCs as in Costain, Jimeno and Thomas (2010) because, from the firm's side, the flexibility of FTCs dominates OECs.[4] Hence, these models are not designed to understand the differential choices of FTCs vs OECs across firms. Furthermore, because firms can only hire one worker, they cannot be used to link contract choices to firm dynamics. Our paper contributes to this literature on both of these fronts.

Several papers provide arguments for the coexistence of FTCs and OECs. In a similar framework as the papers above, Cahuc, Charlot and Malherbet (2016b) allow for firms to be heterogeneous in their expected job duration and to choose the type of contract associated to their vacancies. Firms prefer to use FTCs for jobs of short expected duration (in order to save on firing costs) and OECs for jobs of long expected duration (to save on vacancy posting costs). In the context of directed search models, Berton and Garibaldi (2012) argue that an advantage of OECs over FTCs for firms is that the vacancy-filling rate will be higher in equilibrium when posting OECs as more job-seekers will self-select into the OE market, which offers them higher-value jobs ex-post. Our model features a similar equilibrium logic. However, by properly parameterizing the hiring cost function of the OE and FT markets, we can obtain vacancy-filling rates that are larger in the FT than in the OE markets, a feature that is needed to match the large gap in labor market flows between contract types in the data. Other explanations for the coexistence of OECs and FTCs are that duality diminishes on-the-job search and hence allows firms to retain high-quality workers (as in Cao, Shao and Silos (2013)), and that FTCs can be used by firms to overcome financial constrains (as in Caggese and Cuñat (2008)).

We also relate to a macro literature studying the equilibrium dynamics of multi-worker firms in the context of frictional labor markets. We use a directed search framework with dynamic long-term contracts in the spirit of Kaas and Kircher (2015) and Schaal (2017). We adapt this framework to a continuous-time setting with slow-moving state transitions similar to Roldan-Blanco and Gilbukh

[4]Still, the endogenous firing decision in these models allows to make separations contingent on contract type and hence the aggregate share of FTCs is not undetermined in equilibrium.

(2021), and extend it to incorporate segmented labor markets, different ex-ante firm types, and double-sided ex-post heterogeneity. An alternative approach in the literature has been to assume random search and study firm dynamics in the context of decreasing returns and Nash bargaining (as in Elsby and Michaels (2013) or Acemoglu and Hawkins (2014)), or in settings with on-the-job search and a variety of wage-setting protocols (e.g. Moscarini and Postel-Vinay (2013), Coles and Mortensen (2016), Bilal, Engbom, Mongey and Violante (2022), Gouin-Bonenfant (2022), Elsby and Gottfries (2022), Audoly (2023), and Gulyas (2023)). These models have proved very successful for non-dual markets. Our contribution to this literature is to provide a quantitative model for the firm and aggregate labor market dynamics of tiered markets.

On the empirical side, several papers have studied the effects of employment protection legislation in the context of dual labor markets. For example, Daruich, Di Addario and Saggio (2023) show that relaxing constraints on FTCs relative to OECs in Italy failed to increase overall employment, while Cahuc, Carry, Malherbet and Martins (2022) show that a policy intended to restrict the use of FTCs by new establishments of large firms in Portugal did not increase the number of permanent contracts and ended up decreasing employment in large firms. We complement these studies by arguing that changing the duration of FTCs yields significant effects on aggregate productivity and employment that mask various selection and reallocation forces.

Finally, even though dual labor markets are typically associated to European economies due to specific legislation, recent papers have documented a *de facto* duality in the U.S. labor market as well (e.g. Gregory, Menzio and Wiczer (2022) and Ahn, Hobijn and Şahin (2023)). Furthermore, Gulyas (2023) documents a similar between-firm and within-firm variation of firm size and the share of low-wage workers in Germany, a country with an extremely low share of temporary employment. Given this, we view our framework as potentially useful for a wider set of economies.

Outline The rest of the paper is organized as follows. Section 2 describes the data and our main empirical findings. Section 3 present the model and Section 4 discusses the estimation of its parameters. Section 5 describes our various policy experiments. Section 6 offers concluding remarks. Proofs and additional results, tables, and figures can be found in the Online Appendix.

2 Empirical Findings

Data We use annual data for Spain from the *Central de Balances Integrada* (CBI) dataset, a comprehensive and unbalanced panel of confidential firm-level balance-sheet data compiled and processed by the *Central de Balances*, a department within Banco de España.[5] This dataset covers the quasi-universe of Spanish firms, including large and small firms as well as privately held and publicly traded firms. Among many other items from the balance sheet of firms, the data provide information on total employment and the type of employment contract. We use data for the period 2004-2019. We restrict our sample to firms observed for at least 5 years and whose average employment over the

[5] See Almunia, López-Rodríguez and Moral-Benito (2018) for details.

Table 1: Temporary share, descriptive statistics

Firm size (employment)	Share of firms (%)	Mean	Distribution of firm-level share of temporary workers p10	p25	p50	p75	p90	p95
Total	**100**	**0.181**	**0.000**	**0.000**	**0.027**	**0.294**	**0.591**	**0.800**
1-10	77.65	0.164	0.000	0.000	0.000	0.250	0.541	0.776
11-50	19.04	0.250	0.000	0.031	0.163	0.391	0.677	0.825
51-100	1.78	0.255	0.000	0.034	0.160	0.393	0.701	0.861
101-200	0.99	0.237	0.000	0.029	0.147	0.361	0.645	0.833
201-500	0.30	0.222	0.000	0.026	0.137	0.329	0.589	0.796

Note: Selected moments of the distribution of the share of workers under temporary contracts. Each row corresponds to one firm size category, each column to a different moment of the distribution. p10 to p95 refer to percentiles 5 to 95 of the distribution. CBI data pooled over all years.

period is at least one worker. After some cleaning, we keep data for 7,153,669 firm-year observations, corresponding to 705,879 different firms. Remarkably, the average share of FTCs in our sample aggregates very well to the aggregate temporary share from the labor force survey data (see Figure E.1 in the Online Appendix E). Our companion paper, Auciello-Estévez, Pijoan-Mas, Roldan-Blanco and Tagliati (2023), describes the data in more detail and expands on some of the empirical results presented below.

Cross-sectional distribution In our sample, the average share of temporary workers across firms is 18.1%, while the median share is 2.7% (see first row in Table 1). This gap reflects a highly right-skewed distribution. Part of this skewness is due to a large incidence of very small firms (1 or 2 workers) with no temporary workers. However, when we look at the temporary share within firm size bins, the highly right-skewed distribution is still apparent (see rows 2 to 6 in Table 1). For instance, within the subset of firms between 11 and 50 workers, the average share of temporary workers is 25.0%, and the share of temporary workers at the 25th, 50th and 90th percentiles is 3.1%, 16.3%, and 67.7%, respectively. Finally, in Table 1 we also observe that the use of temporary workers tends to increase with firm size —a feature that we will discuss in more detail later— and that most of Spanish firms (more than 95%) have 50 workers or less.

Aggregate determinants Next, in order to understand the heterogeneity in the use of temporary contracts across firms, we propose a regression of the type:

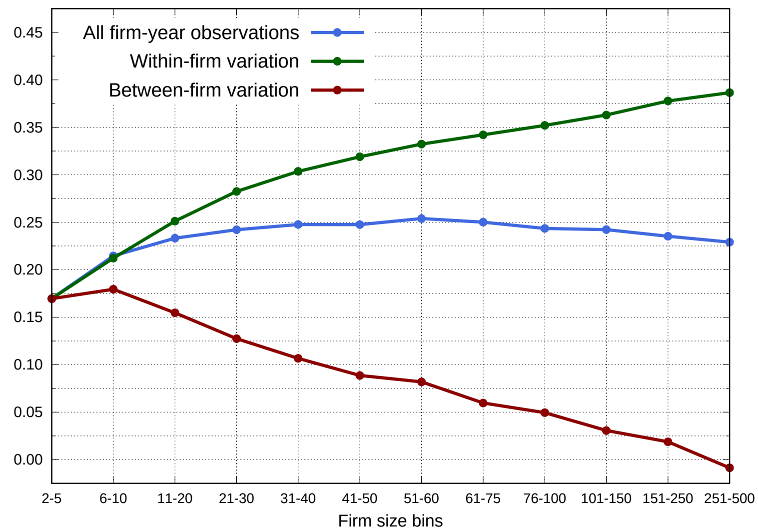
$$TempSh_{ft} = \left(\alpha_i + \alpha_p + \alpha_t\right) + \alpha_f + \mathbf{X}_{ft}\boldsymbol{\beta} + \epsilon_{ft} \tag{1}$$

where $TempSh_{ft}$ is the share of temporary workers of firm f at time t; α_i, α_p, and α_t, are 4-digit industry, province, and year fixed effects; α_f are firm fixed effects; and $\mathbf{X}_{ft}$ is a vector of covariates.[6]

We find that the aggregate fixed effects (α_i, α_p, α_t) are important. The temporary share differs widely across sectors (ranging from 7.7% in "*Real estate activities*" to 43.1% in "*Employment ac-*

[6]We index each data point by ft instead of $ipft$ since there is no variation in industry i and (almost) no variation in province p at the firm level.

Figure 1: Temporary share, by firm size: within- and between-firm variation.



Notes: The green line reports the coefficients of the size dummies of a regression of temporary share that controls for aggregate and firm-level fixed effects. The red line reports the firm fixed effects of the same regression against dummies of average firm size. The blue line reports the size dummies of a regression of temporary share that controls for aggregate but not firm-level fixed effects.

tivities"), across provinces (ranging between 11.5% in Barcelona to 38.8% in Huelva), and over time (the temporary share is strongly procyclical, ranging from 23.9% in 2006 to 15.8% in 2012). However, the R^2 of regression (1) with only the aggregate fixed effects $(\alpha_i, \alpha_p, \alpha_t)$ is 16%. That is, 84% of the variation in the usage of FTCs remains within industry, province, and time period. In particular, firm fixed effects explain nearly half of the overall variation: the R^2 of regression (1) with aggregate fixed effects increases from 16% to 62% with the inclusion of α_f into the specification.

Temporary contracts and firm size To uncover the firm-level determinants of temporary employment, we run regression (1) with size-bin dummies (2-5 employees, 6-10, 11-20, 21-30, ...) in the vector $\mathbf{X}_{ft}$ together with the aggregate and firm-level fixed effects. As shown in Figure 1, there is a complex relationship between the use of temporary contracts and firm size. The green line represents the estimated $\boldsymbol{\beta}$ coefficients in the regression with both aggregate and firm fixed effects included, and it captures an increasing within-firm variation between the temporary share and firm size. The red line plots the estimated α_f against the (time-series) average of employment of each firm, and it captures a declining between-firm variation between the temporary share and firm size. Both lines are scaled to deliver the same value for the size bin of 2-5 workers. For comparison, the blue line represents the estimated $\boldsymbol{\beta}$ coefficients in a regression that does not include firm fixed effect, and hence it does not distinguish between within- and between-firm variation. The within-firm variation suggests that firms make use of FTCs to grow or decline in size: at the firm level, FTCs are useful to help employment track productivity or demand changes. The between-firm variation suggests that there are important technology differences across firms, whereby firms that are (permanently) larger prefer a lower share of temporary contracts, while firms that are (permanently) smaller prefer a higher share of temporary contracts.

Taking stock Our empirical analysis shows that most variation in the temporary share across firms is explained by firm-specific factors, and not by industry, province, or year effects. Firms use temporary contracts when they grow or decline (within-firm, the temporary share *increases* with employment), but larger firms make less use of temporary contracts (between-firm, the temporary share *declines* with employment). With these results in mind, in the next section we write a firm-dynamics model with a dual labor market structure which we calibrate to replicate these and other features of the data to study the implications of labor market duality for the aggregate economy.

3 Model

3.1 Environment

Time is continuous and infinite. We consider a stationary economy that is populated by a mass of workers with fixed unit measure and an endogenous measure of firms. Both firms and workers are risk-neutral and infinitely-lived, and share a common time discount rate, $\rho > 0$.[7]

Technology A firm has a type $\varphi \in \Phi \equiv \{\varphi_1, \ldots, \varphi_{k_\varphi}\}$ that is kept fixed throughout its life. There are two possible worker skill levels, high (H) and low (L). Let $n_j = 0, 1, 2, \ldots$ denote the number of workers with skill $j \in \mathcal{J} \equiv \{H, L\}$ in a firm. All firms operate the following decreasing returns to scale production function:

$$y(n_H, n_L, z, \varphi) = e^{z+\zeta(\varphi)} \Big(\omega(\varphi) n_H^\alpha + \big(1 - \omega(\varphi)\big) n_L^\alpha \Big)^{\frac{\nu}{\alpha}}, \tag{2}$$

where $\nu \in (0, 1)$ governs the degree of decreasing returns to scale, $\alpha < 1$ governs the elasticity of substitution of workers of different skill levels,[8] $\omega(\varphi) \in (0, 1)$ measures the relative productivity of high-skilled workers for a firm of permanent type $\varphi \in \Phi$, and $\zeta(\varphi) > 0$ denotes the permanent component of firm productivity for a firm of type $\varphi \in \Phi$. The random variable $z \in \mathcal{Z} \equiv \{z_1 < \cdots < z_{k_z}\}$ stands for the idiosyncratic and transitory productivity of the firm, which follows a continuous-time Markov chain with intensity rates $\lambda(z'|z)$.[9]

Firms hire workers by posting two types of contract, *fixed-term* (FT) and *open-ended* (OE), indexed by $i \in \mathcal{I} \equiv \{FT, OE\}$. A worker is therefore uniquely identified by its type, $(i, j) \in \mathcal{I} \times \mathcal{J}$.

Skill accumulation All starting jobs are in low-skilled positions and, as in Ljungqvist and Sargent (2007), workers can access high-skilled positions through skill upgrades that arrive on the job.

[7] Throughout the paper, our convention on notation is the following: for some generic object, we reserve a roman font A, or Greek letters α, for generic variables and deep parameters; the calligraphic font $\mathcal{A}$ is for (countable) sets; the arrow notation $\vec{a}$ is for vectors; the bold font $\mathbf{A}$ is for value functions; and the typeset font $\mathtt{A}$ is for measures of agents.

[8] The elasticity of substitution is $\frac{1}{1-\alpha} > 0$. Skill types are complements in production if $\alpha < \nu$. In this case, the production function is supermodular, i.e. $\partial^2 Y / \partial n_H \partial n_L > 0$, $\forall z, \varphi$.

[9] The transition rates satisfy, $\forall z \in \mathcal{Z}$: $\lambda(z|z) \leq 0$; $\lambda(z'|z) \geq 0$, $\forall z' \neq z$; $\sum_{z' \in \mathcal{Z}} \lambda(z'|z) = 0$; and $\sum_{z' \in \mathcal{Z}} \lambda(z'|z) < +\infty$.

The skills are lost upon displacement. These assumptions make human capital firm-specific.[10] We assume that only OE workers can upgrade to high-skilled jobs. In particular, a low-skill OE worker transitions to a high-skill job with intensity $\tau > 0$ (a parameter), with the opposite transition (skill obsolescence) being impossible. This assumption captures the idea that workers under FTCs do not have time to accumulate skills due to their short tenure at the firm.[11] An alternative interpretation is that worker training by the firm is much more likely to be offered to workers under OECs than under FTCs due to the higher turnover of the latter type of worker.[12] This gives an important motive to offer OECs to workers: it is the only way to ensure that a fraction of workers know the functioning of the firm and can be entrusted with tasks requiring a higher skill.

Worker transitions Firms may choose to promote their FT workers into an OEC, with the opposite transition being illegal. Firms choose a promotion rate p for each one of their FT workers (if any), which carries a promotion cost of $\xi n_{FT} p^{\vartheta}$ units of the firm's output, with $\xi > 0$ and $\vartheta > 1$. When an FT worker is promoted into an OEC, her job description does not change (i.e. FT workers remain low-skill upon promotion). One can think of the promotion costs as administrative or legal costs for contract conversion, as extra screening cost before conversion, or even as training costs (given that all converted FT workers are exposed to a future skill upgrade).

Firms may lose workers for three different reasons: (i) because of an exogenous firm exit shock, with intensity $s^F \geq 0$, dissolving the firm entirely and sending all of its workers into unemployment; (ii) because the contract expires, at rate $s_i^W \geq 0$ for each contract type $i \in \mathcal{I}$; or (iii) because the firm endogenously decides to fire workers. For the latter case, the firm must choose a per-worker firing rate $\delta_{ij} \geq 0$ for each worker type $(i, j) \in \mathcal{I} \times \mathcal{J}$, which carries a layoff cost equal to $\chi n_{ij} \delta_{ij}^{\psi}$ units of the firm's output, with $\chi > 0$ and $\psi > 1$.[13] This cost is meant to capture expenses associated to laying off workers, such as administrative expenses and legal costs.[14]

Potential entrants In the event that a firm loses all of its workers, it exits the market and becomes a so-called *potential entrant*. In order to post a contract, potential entrants must incur a flow cost $\kappa > 0$ and, upon entry, draw a permanent type $\varphi \in \Phi$ and an initial idiosyncratic productivity $z \in \mathcal{Z}$

[10] Firm-specific human capital allows to simplify the model because it makes all unemployed individuals identical. This assumption is supported by an empirical literature highlighting the importance of firm-specific human capital for worker wage growth (see e.g. Topel (1991), Dustmann and Meghir (2005), or Buchinsky, Fougère, Kramarz and Tchernis (2010)).

[11] In our calibrated model, a newly hired FT worker only has an average of half a year before its contract is terminated, which is little time to acquire firm-specific skills. If we allowed workers under FTCs to accumulate human capital, in equilibrium only a tiny fraction of them would do so, and this would come at a high cost in terms of extra state space.

[12] Indeed, several papers show that having a temporary contract diminishes the probability of receiving on-the-job training (e.g. Alba-Ramirez (1994), Dolado, Felgueroso and Jimeno (2000), Bratti, Conti and Sulis (2021), Cabrales, Dolado and Mora (2017)). Additional evidence of this mechanism is that wage returns to experience are much larger for experience years accumulated under OECs than FTCs (see e.g. Garcia-Louzao, Hospido and Ruggieri (2023)).

[13] Notice that, mirroring Spanish law, we do not allow firing cost parameters to depend explicitly on contract type, implying that differences in firing costs across firms will be an endogenous outcome of the model.

[14] The cost, however, does not include pure transfers between the employer and the worker (severance payments). A severance payment in this model would be a pure lump-sum transfer which would have no affect on the equilibrium allocation, as the optimal contract maximizes the joint surplus of the firm and all of its workers.

from some π_φ and π_z distributions, respectively.[15] Firms enter with one worker, whether it is under an OEC or an FTC, which they attract with the same search-and-matching technology as that of operating firms. We describe this technology next.

Search and matching Search is directed. Every instant of time, a firm (i) opens one vacancy of each type $i \in \{FT, OE\}$, (ii) chooses the terms of the offered contracts, and (iii) chooses the recruiting intensity rates (v_{OE}, v_{FT}) for each vacancy.[16] We assume that a recruiting intensity $v_i > 0$ entails a recruiting cost of $A_i v_i^\varsigma$ units of the firm's output, where $A_i > 0$ and $\varsigma > 1$ are parameters. The recruiting cost shifter $A_i > 0$ is potentially contract-specific.[17]

Denote the employment vector of a firm by $\vec{n} \equiv (n_{OEH}, n_{OEL}, n_{FT}) \in \mathcal{N}$, where we write $\mathcal{N}$ for the set of all triplets of integers excluding the zero vector, i.e. $\{(0,0,0)\} \notin \mathcal{N}$. Let $(\vec{n}_t^{t+s}, z_t^{t+s})$ be the full history of possible firm states between dates t and $t+s$. At any t and for all $s > 0$, a contract of type $i \in \mathcal{I}$ for skill type $j \in \mathcal{J}$ offered by a firm of permanent type $\varphi \in \Phi$ is a set of complete state-dependent sequences of wages $w_{ij}(\vec{n}_t^{t+s}, z_t^{t+s}; \varphi)$, recruiting intensities $v_i(\vec{n}_t^{t+s}, z_t^{t+s}; \varphi)$, firing rates $\delta_{ij}(\vec{n}_t^{t+s}, z_t^{t+s}; \varphi)$ and, only for workers employed under FTCs, intensities $p(\vec{n}_t^{t+s}, z_t^{t+s}; \varphi)$ of promotion into an OEC, conditional on no worker separation and firm survival.

We assume the following commitment structure. On the worker's side, workers may forfeit their contract and quit the firm, but in that case they must go back to unemployment (and consume flow utility $b > 0$), from where they can regain employment. On the firm's side, by contrast, there is full commitment to both the contract type as well as to the contractual terms, which cannot be revised or renegotiated for the duration of the match. Therefore, contracts must always comply with the firm's initial promises. Moreover, we assume that the firm cannot discriminate between workers with the same contract type and job description, i.e. all n_{ij} workers of type (i, j) obtain the same contract (though, of course, their individual employment histories may differ).

Given these assumptions, a distinct labor market segment (or "submarket") is indexed by (i) the contract's type $i \in \{FT, OE\}$, and (ii) the long-term value that the worker can expect to obtain from it, denoted W. Each worker can simultaneously search in at most one submarket (i, W), and each firm can simultaneously post only one offer W of each contract type i. We denote by $\mathtt{V}_i(W)$ the intensity-weighted measure of vacancies in submarket (i, W).[18] Likewise, we denote

[15] Note that this implies that entrants cannot choose their type. See Section 5.1.4 for a model extension where they do.

[16] This implies that firms wishing to grow fast must increase the filling rate of vacancies as they cannot do it by raising the number of vacancies. We make this assumption for tractability, but Gavazza *et al.* (2018) and Carrillo-Tudela *et al.* (2023) show that, in practice, the vacancy yield is much more important for firm growth than the vacancy rate.

[17] This could be due to several reasons. For instance, there might be a tougher screening process for incoming OE workers, as these workers will eventually be assigned to high-skilled tasks, which may require more talent or specific knowledge, making hiring under this contract more costly for the same recruiting intensity. Another interpretation is that the value of properly screening candidates is larger for OE workers, who expected to stay longer with the firm. Finally, in Spain there are intermediary companies (*empresas de trabajo temporal*) connecting workers seeking and firms offering FTCs, easing the matching process and lowering the costs of recruiting FT workers.

[18] Precisely, the effective measure of vacancies in submarket (i, W) is defined as $\mathtt{V}_i(W) \equiv \int_{\Omega_i(W)} v_{if} \mathrm{d}f$, where v_{if} is the recruiting intensity of firm f for contract $i \in \{FT, OE\}$, and $\Omega_i(W)$ is the set of firms that offer value W for contract i. See further details in Online Appendix A.5.

by $\mathtt{U}_i(W)$ the measure of unemployed workers applying to the same submarket (i, W). Then, the measure of matches in market segment (i, W) is equal to $\mathtt{M}(\mathtt{V}_i(W), \mathtt{U}_i(W))$, where $\mathtt{M} : \mathbb{R}^2_+ \to \mathbb{R}_+$ is a constant-returns-to-scale matching function whose parameters are common across all submarkets.

As there is a continuum of firms within each submarket and each firm faces the same hiring frictions, by the law of large numbers an individual firm exerting recruiting effort v in submarket (i, W) obtains $v\eta\big(\theta_i(W)\big)$ new hires, where $\theta_i(W) \equiv \mathtt{V}_i(W)/\mathtt{U}_i(W)$ is the effective market tightness and $\eta : \theta \mapsto \mathtt{M}(1, \theta^{-1})$ is the aggregate job-filling rate per effective vacancy. Notice $\eta(\theta)$ is constant across all firms facing the same market tightness θ. Therefore, an individual firm's job-filling rate is a function of this aggregate meeting rate η, which is determined in equilibrium but which the firm takes as given, as well as the recruiting effort v, which is a choice variable for the firm. Similarly, we denote by $\mu(\theta) = \theta\eta(\theta)$ the aggregate job-finding rate in a submarket with tightness θ.[19]

Recursive contracts Because contracts are large and complex objects, we focus on their recursive formulation. We focus on a Markov Perfect Equilibrium, in which contracts are only functions of the firm's state. A firm's state is (i) its permanent type $\varphi \in \Phi$, (ii) its employment vector and productivity $(\vec{n}, z) \in \mathcal{N} \times \mathcal{Z}$, and (iii) the set $\vec{W} = (W_{OEH}, W_{OEL}, W_{FT}) \in \mathbb{R}^3_+$ of outstanding values that the firm promised to its incumbent workers. Then, a recursive contract is defined by:[20]

$$\vec{C}_{ij} \equiv \Big\{ w_{ij}, v_i, \delta_{ij}, p, W'_{ij}(\vec{n}', z') \Big\}.$$

For each worker $(i, j) \in \mathcal{I} \times \mathcal{J}$, a contract includes a wage w_{ij}, a recruiting effort v_i, a per-worker layoff rate δ_{ij}, a promotion rate p for FT workers, and a continuation promise $W'_{ij}(\vec{n}', z')$ for each new possible set of states $(\vec{n}', z')$ of the firm. The new state vector, conditional on firm survival, is

$$(\vec{n}', z') \in \left\{ \begin{array}{l} (n_{OEH},\ n_{OEL} + 1,\ n_{FT},\ z),\ (n_{OEH},\ n_{OEL},\ n_{FT} + 1,\ z), \\ (n_{OEH} + 1,\ n_{OEL} - 1,\ n_{FT},\ z) \\ (n_{OEH} - 1,\ n_{OEL},\ n_{FT},\ z),\ (n_{OEH},\ n_{OEL} - 1,\ n_{FT},\ z),\ (n_{OEH},\ n_{OEL},\ n_{FT} - 1,\ z), \\ (n_{OEH},\ n_{OEL} + 1,\ n_{FT} - 1,\ z), \\ \big\{ (n_{OEH},\ n_{OEL},\ n_{FT},\ z'), : z' \in \mathcal{Z} \big\} \end{array} \right\},$$

depending on which type of transition the firm has in the next stage (including hiring, promotion, separation, skill upgrades and productivity shocks, respectively). Henceforth, we will make use of the short-hand notation $\vec{n}^+_{ij} \equiv (n_{ij} + 1, \vec{n}_{-(ij)})$, $\vec{n}^-_{ij} \equiv (n_{ij} - 1, \vec{n}_{-(ij)})$, $\vec{n}^p \equiv (n_{OEH}, n_{OEL} + 1, n_{FT} - 1)$, and $\vec{n}^\tau \equiv (n_{OEH} + 1, n_{OEL} - 1, n_{FT})$, to denote the various size transitions, where $\vec{n}_{-(ij)} \equiv \vec{n} \backslash \{n_{ij}\}$, for any $i \in \{FT, OE\}$ and $j \in \{H, L\}$.[21]

[19] These rates satisfy standard properties: $\eta(\theta)$ is decreasing and convex, $\mu(\theta)$ is increasing and concave, and the usual Inada conditions apply, namely $\lim_{\theta \to 0} \mu(\theta) = \lim_{\theta \to +\infty} \eta(\theta) = 0$, and $\lim_{\theta \to +\infty} \mu(\theta) = \lim_{\theta \to 0} \eta(\theta) = +\infty$.

[20] To alleviate notation, we do not index recursive contracts explicitly by $(n, z, \varphi, \vec{W})$.

[21] As $\vec{n}$ may contain more than one instance of the same element (e.g. the firm has five FE workers and five OEH workers), the symbols $\cup$ and $\backslash$ represent *multiset* union and difference operators, respectively, meaning $\{a, b\} \cup \{b\} = \{a, b, b\}$ instead of $\{a, b\} \cup \{b\} = \{a, b\}$, and $\{a, b, b\} \backslash \{b\} = \{a, b\}$ instead of $\{a, b, b\} \backslash \{b\} = \{a\}$.

3.2 Equilibrium

3.2.1 Unemployed Worker's Problem

Unemployed workers consume a flow utility $b > 0$ while searching in the labor market. Search is directed toward the submarket (i, W) that offers the most profitable expected return for workers. Thus, the value of unemployment is $\mathbf{U} = \max_{(i,W)} \{U_i(W)\}$, where $U_i(W)$ solves the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\rho U_i(W) = b + \mu\big(\theta_i(W)\big) \max\Big\{W - U_i(W), 0\Big\}. \tag{3}$$

As workers prefer the most profitable offers, when unemployed they must remain indifferent ex-ante between all of those offers which they decide to apply to. Therefore, the following complementary slackness condition must hold:

$$\forall (i, W) \in \mathcal{I} \times \mathbb{R}_+ : \quad U_i(W) \leq \mathbf{U}, \text{ with equality if, and only if, } \mu\big(\theta_i(W)\big) > 0.$$

This condition states that a submarket must maximize the value of remaining unemployed, or else it is never visited by workers. Imposing this condition into equation (3) we find:

$$\theta(W) = \mu^{-1}\left(\frac{\rho\mathbf{U} - b}{W - \mathbf{U}}\right). \tag{4}$$

Equation (4) defines the equilibrium function that maps promised values to market tightness, for any given value of unemployment $\mathbf{U}$. Market tightness is decreasing in W (more attractive contracts for workers ex-post attract more workers per job posting ex-ante), and increasing in $\mathbf{U}$ (a better outside option for workers makes jobs relatively less attractive ex-ante).[22] Market tightness, however, does not depend explicitly on the contract type i because of the indifference condition and the fact that the value of a job is summarized only by W. However, in equilibrium, differences in primitives across contracts will lead different firms to offer different values W for different contracts $i = OE, FT$, giving rise to heterogeneity in aggregate job-filling and job-finding rates by contract type.

3.2.2 Joint Surplus Problem

Next, we characterize the optimal contract menu chosen by firms. For each worker of type (i, j) employed by the firm, this requires finding the vector $\vec{C}_{ij} = \{w_{ij}, v_i, \delta_{ij}, p, W'_{ij}(\vec{n}', z')\}$ that maximizes firm value. As the firm has commitments with its pre-existing workers, this problem is subject to a promise-keeping constraint. As workers do not commit, there is also a worker-participation constraint. We write the firm's and employed worker's value functions in Online Appendix A.1.

As it turns out, however, we can find the optimal contract from a simpler and equivalent problem,

[22] To save on notation, we write $\theta(W)$ instead of $\theta(W, \mathbf{U})$, but the reader should bear in mind that $\mathbf{U}$ is an endogenous object that we solve for in equilibrium, but one which both firms and workers take as given when making decisions.

which involves maximizing the *joint surplus* of the match, that is, the sum of the firm's value and the values of all of its workers. For a match between a firm of type φ in state $(\vec{n}, z, \vec{W})$ and its workers, we define the joint surplus as:

$$\mathbf{\Sigma}(\vec{n}, z, \varphi) \equiv \mathbf{J}\left(\vec{n}, z, \varphi, \vec{W}\right) + \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} n_{ij} W_{ij}, \tag{5}$$

where $\mathbf{J}(\cdot)$ denotes the value of the firm, and recall that $\vec{W} \equiv \{W_{ij}\}$ are the outstanding promised values. As discussed in detail below, the joint surplus is independent of promised values and, anticipating this result, on the left-hand side of equation (5) we have written $\mathbf{\Sigma}(\vec{n}, z, \varphi)$ instead of $\mathbf{\Sigma}(\vec{n}, z, \varphi, \vec{W})$. In Online Appendix A.1, we then show that $\mathbf{\Sigma}(\vec{n}, z, \varphi)$ solves the HJB equation:[23]

$$\begin{aligned}(\rho + s^F)\mathbf{\Sigma}(\vec{n}, z, \varphi) = \max_{p, \{v_i, \delta_{ij}, W'_{iL}(\vec{n}^+_{iL}, z)\}} \Bigg\{ & \underbrace{\mathbf{S}(\vec{n}, z, \varphi)}_{\text{Flow surplus}} + \underbrace{\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} n_{ij}(\delta_{ij} + s^W_i)\Big(\mathbf{\Sigma}(\vec{n}^-_{ij}, z, \varphi) - \mathbf{\Sigma}(\vec{n}, z, \varphi)\Big)}_{\text{Worker separation and firing}} \\ & + \underbrace{\sum_{i \in \mathcal{I}} v_i \eta\Big(W'_{iL}(\vec{n}^+_{iL}, z)\Big)\Big(\mathbf{\Sigma}(\vec{n}^+_{iL}, z, \varphi) - \mathbf{\Sigma}(\vec{n}, z, \varphi)\Big)}_{\text{Hiring an FT or OEL worker}} + \underbrace{n_{FT}\, p\Big(\mathbf{\Sigma}(\vec{n}^p, z, \varphi) - \mathbf{\Sigma}(\vec{n}, z, \varphi)\Big)}_{\text{Promotion of FT workers}} \\ & + \underbrace{n_{OEL}\tau\Big(\mathbf{\Sigma}(\vec{n}^\tau, z, \varphi) - \mathbf{\Sigma}(\vec{n}, z, \varphi)\Big)}_{\text{Skill upgrade of OEL workers}} + \underbrace{\sum_{z' \in \mathcal{Z}} \lambda(z'|z)\Big(\mathbf{\Sigma}(\vec{n}, z', \varphi) - \mathbf{\Sigma}(\vec{n}, z, \varphi)\Big)}_{\text{Productivity shock}} \Bigg\},\end{aligned} \tag{6}$$

subject to $W'_{iL}(\vec{n}^+_{iL}, z) \geq \mathbf{U}$, $\forall i \in \mathcal{I}$, a worker participation constraint that entices workers to remain matched by promising them more utility that their outside option.[24] Equation (6) states that the joint surplus is composed of the flow surplus, plus the changes in joint surplus value due to worker separation, firing, hiring, promoting, skill upgrading, and productivity shocks, where:[25]

$$\begin{aligned}\mathbf{S}(\vec{n}, z, \varphi) \equiv & \underbrace{y(\vec{n}, z, \varphi)}_{\text{Firm's output}} + \underbrace{\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} n_{ij}(\delta_{ij} + s^W_i + s^F)\mathbf{U}}_{\text{Workers' outside options}} - \underbrace{\sum_{i \in \mathcal{I}} A_i v_i^\varsigma}_{\text{Recruiting costs}} \\ & - \underbrace{\xi n_{FT} p^\vartheta}_{\text{Promotion costs}} - \underbrace{\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \chi n_{ij} \delta_{ij}^\psi}_{\text{Firing costs}} - \underbrace{\sum_{i \in \mathcal{I}} v_i \eta\Big(W'_{iL}(\vec{n}^+_{iL}, z)\Big) W'_{iL}(\vec{n}^+_{iL}, z)}_{\text{Expected value delivered to new hires}}.\end{aligned} \tag{7}$$

Equipped with this formulation, we arrive at our main equivalence result (for the proof, see Online Appendix A.1):

[23]Throughout we will write the job-filling rate as $\eta(W(\cdot)) \equiv \eta(\theta(W(\cdot)))$, with $\theta(W)$ defined in equation (4).

[24]Notice that the promise-keeping constraint does not appear as a constraint in the joint surplus problem because definition (5) imposes that it holds with equality in equilibrium. See Online Appendix A.1 for the formal argument.

[25]In words, the flow surplus is the sum of the firm's flow output and worker's outside options in case of separation (first line of (7)), net of four types of costs: recruiting costs, promotion costs, firing costs, and the costs of having to deliver the promised value in case of successful hiring (in expected terms).

Proposition 1 *The firm's and joint surplus problems are equivalent.*

This result is reminiscent of other directed search models with multi-worker firms and long-term contracts (e.g. Schaal (2017)). As in those models, the firm's choices guarantee that the joint surplus is maximized because the contract space is complete, all agents have linear utilities, and these utilities are transferable. Thus, the set of optimal contracts can be found in two stages. In the first stage, we take first-order conditions of problem (6) to find the optimal recruiting intensity, job-filling rates, firing rates, and promotion rates for each job and each contract. Then, in the second stage, we find the set of promised utilities $\vec{W}$ that implement this allocation. We detail these two stages below.

Stage 1 Taking first-order conditions of problem (6) for $W'_{iL}(\vec{n}^+_{iL}, z, \varphi)$ yields:

$$\left.\frac{\partial \eta(W)}{\partial W}\right|_{W'_{iL}(\vec{n}^+_{iL},z,\varphi)} W'_{iL}(\vec{n}^+_{iL}, z, \varphi) + \eta\left(W'_{iL}(\vec{n}^+_{iL}, z, \varphi)\right) = \left(\boldsymbol{\Sigma}(\vec{n}^+_{iL}, z, \varphi) - \boldsymbol{\Sigma}(\vec{n}, z, \varphi)\right) \left.\frac{\partial \eta(W)}{\partial W}\right|_{W'_{iL}(\vec{n}^+_{iL},z,\varphi)}, \tag{8}$$

showing that a firm in state $(\vec{n}, z, \varphi)$ equates the expected marginal cost (left-hand side) to the expected marginal benefit (right-hand side) of promising value $W'_{iL}(\vec{n}^+_{iL}, z, \varphi)$ to new hires. In particular, the marginal cost is given by the increase in the joint surplus that goes to the new hires and the marginal benefit is given by the joint surplus that the new hires will produce.

The optimal recruiting, firing and promotion rates for a firm in state $(\vec{n}, z, \varphi)$ satisfy, respectively:

$$v_i(\vec{n}, z, \varphi) = \left[\eta\left(W'_{iL}(\vec{n}^+_{iL}, z, \varphi)\right)\left(\frac{\boldsymbol{\Sigma}\left(\vec{n}^-_{ij}, z, \varphi\right) - \boldsymbol{\Sigma}(\vec{n}, z, \varphi) - W'_{iL}(\vec{n}^+_{iL}, z, \varphi)}{\varsigma A_i}\right)\right]^{\frac{1}{\varsigma-1}}, \tag{9}$$

$$\delta_{ij}(\vec{n}, z, \varphi) = \left[\frac{\boldsymbol{\Sigma}\left(\vec{n}^-_{ij}, z, \varphi\right) - \boldsymbol{\Sigma}(\vec{n}, z, \varphi) + \mathbf{U}}{\psi\chi}\right]^{\frac{1}{\psi-1}}, \tag{10}$$

$$p(\vec{n}, z, \varphi) = \left[\frac{\boldsymbol{\Sigma}\left(\vec{n}^p, z, \varphi\right) - \boldsymbol{\Sigma}(\vec{n}, z, \varphi)}{\vartheta\xi}\right]^{\frac{1}{\vartheta-1}}. \tag{11}$$

In all three cases, firms equate marginal costs to marginal gains, given by the corresponding changes in the joint surplus value. We note that, despite not having a fixed cost in production, there is still endogenous firm exit. This will happen whenever the joint surplus $\boldsymbol{\Sigma}(\vec{n}, z, \varphi)$ of one-worker firms is below the value of unemployment $\mathbf{U}$ of its worker, in which case δ_{ij} will be positive.[26] This situation is more likely to happen for low values of z and for less productive firm types φ.

Stage 2 The equilibrium promised values that implement these policies can be found by ensuring that the resulting surplus is distributed across agents in a way that maximizes firm profits while keeping ex-ante promises at every point of the state space, as explained in Online Appendix A.1. In practice, this allows us to construct the whole sequence of promised values through an iterative

[26]Note that the joint surplus $\boldsymbol{\Sigma}\left(\vec{n}^-_{ij}, z, \varphi\right)$ of losing the last worker is zero due to the free entry condition, see below.

procedure: the promised value of a firm with size vector $(n_{ij}, \vec{n}_{-(ij)})$ must coincide with the optimal upsize policy (for a type-ij worker) of a firm with size vector $(n_{ij}-1, \vec{n}_{-(ij)})$, as well as with the optimal downsize policy of a firm with size vector $(n_{ij}+1, \vec{n}_{-(ij)})$.

For the special case of $(n_{ij}, \vec{n}_{-(ij)}) = (1, \vec{0})$, promised values must, in turn, be consistent with the optimal choices of potential entrants. These firms have no workers and perceive a value $\mathbf{J}^e$, with:

$$\rho \mathbf{J}^e = -\kappa + \sum_{\varphi \in \Phi} \pi_\varphi(\varphi)\, \widetilde{\mathbf{J}}^e(\varphi), \tag{12}$$

where $\widetilde{\mathbf{J}}^e(\varphi)$ is the expected value of entry for firms of type φ defined by:

$$\widetilde{\mathbf{J}}^e(\varphi) \equiv \sum_{z \in \mathcal{Z}} \pi_z(z) \sum_{i \in \mathcal{I}} \left[\max_{W_i} \left\{ \eta(W_i) \left(\mathbf{J}\left(\vec{n}^e_{iL}, z, \varphi, \{W_i\} \right) - \mathbf{J}^e \right) \text{ s.t. } W_i \geq \mathbf{U} \right\} \right], \tag{13}$$

where we have used the notation $\vec{n}^e_{ij} \equiv (n^e_{ij}, \vec{n}^e_{-(ij)}) = (1, \vec{0})$. Taking first-order conditions pins down the initial promise by entering firms and, by the iterative procedure outlined above, the entire sequence of promised values. We assume free entry into the labor market, i.e. we allow the aggregate measure of firms to freely adjust in equilibrium. Thus, in an equilibrium with positive firm entry, we must have $\mathbf{J}^e = 0$, which pins down the average labor market tightness of the economy.

3.2.3 Closing the Model

To close the characterization of the equilibrium, we must find the steady state distribution of firms and workers. The law of motion for the measure of firms in each state, $\mathtt{f}_t(\vec{n}, z, \varphi)$, is characterized by a set of Kolmogorov forward equations, which we provide in full in Online Appendix A.2. This appendix also shows how to obtain the measure of operating firms, $\mathtt{F}$, as well as the unemployment rate, $\mathtt{U}$, in a stationary equilibrium in which $\frac{\partial}{\partial t}\mathtt{f}_t(\vec{n}, z, \varphi) = 0$.

4 Estimation

This section describes how we bring the model to the data. We start by describing our data sources and several details about the model parameterization. Next, we discuss our calibration strategy (Sections 4.1 and 4.2), and validate it with a global identification exercise and a set of non-targeted moments (Section 4.3). Finally, we explore some features of the calibrated economy, with a special emphasis on the sources of static employment misallocation (Section 4.4).

Data sources We compute moments from the *Central de Balances* (CBI) firm-level data introduced in Section 2. We focus on a sub-sample of firms with at most 60 workers, representing 97.3% of firms from the full sample.[27] These data come at the yearly frequency. For our variables of interest

[27]All of the empirical facts that we established in Section 2 for the full sample also hold for this sub-sample. For instance, we still find that the share of temporary workers increases (respectively, decreases) in firm size when looking at

(firm size, share of temporary workers, and value added per worker), we regress out the aggregate fixed effects (industry, time, province) to ensure that differences across firms do not reflect these other factors, which are not in the model. We also use aggregate data on worker flows into and out of employment by contract type from *Encuesta de Población Activa* (EPA), the Spanish labor force survey. These data come at a quarterly frequency.

Parameterization First, we set the model period to one quarter to match the EPA time frequency, as the CBI data (at a yearly frequency) is used for stock variables. Second, the dynamics of the productivity shock z is represented by the vector of values $\{z_1, \ldots, z_{k_z}\}$ and the $k_z(k_z-1)$ matrix of intensity rates $\{\lambda(z'|z)\}$. As this is a potentially large number of parameters, we recover them from the discretization of an Ornstein-Uhlenbeck diffusion process for idiosyncratic productivity (in logs):

$$d\ln(z_t) = -\rho_z \ln(z_t)dt + \sigma_z dB_t, \tag{14}$$

where B_t is a Wiener process, and (ρ_z, σ_z) are positive persistence and dispersion parameters.[28] Third, we choose a standard Cobb-Douglas specification for the matching function: $\mathtt{M}(\mathtt{V},\mathtt{U}) = \mathtt{V}^{\gamma}\mathtt{U}^{1-\gamma}$, where $\gamma \in (0,1)$ is the matching elasticity, implying meeting rates $\mu(\theta) = \theta^{\gamma}$ for the worker, and $\eta(\theta) = \theta^{\gamma-1}$ for the firm. This functional form leads to convenient analytical representations for the promised value and the job-filling rate (see Online Appendix A.3). Finally, to avoid a problem of state space dimensionality, we set $(N_{OEH}, N_{OEL}, N_{FT}) = (30, 15, 15)$, which imposes an upper bound of 60 workers per firm, and we assume $k_\varphi = 2$ firm types and $k_z = 5$ productivity levels.[29]

4.1 Externally Set Parameters

Given the parameterization described above, we have 24 parameters to calibrate. Of these, 9 parameters, namely $\mathbf{p}_{\text{ext}} \equiv (\rho, \gamma, \rho_z, \sigma_z, \varsigma, \psi, \vartheta, \tau, \zeta(\varphi_1))$, are set externally (see Table 2). We fix the discount rate to $\rho = 0.0123$, corresponding to an annualized discount rate of $(1+\rho)^4 - 1 \approx 5\%$. For the matching elasticity, we choose $\gamma = 0.5$, a standard value in the literature (e.g. Petrongolo and Pissarides (2001)).[30] The productivity parameters (ρ_z, σ_z) introduced in equation (14) are calibrated to match a yearly autocorrelation of firm-level TFP of 0.81 and a yearly volatility of 0.34. We take these values from Ruiz-García (2021), which estimates an AR(1) process for firm-level TFP using Spanish firm-level balance sheet data from CBI, the same data source that we use in our empirical analysis. As we explain in Online Appendix D.1, these targets imply that $\rho_z = 0.0513$

within-firm (respectively, between-firm) variation. See Figure E.2 in the Online Appendix.

[28] Particularly, we recover the $\{\lambda(z'|z)\}$ intensity rates and $\{z_i\}_{i=1}^{k_z}$ productivity levels from discretizing this process using the Euler-Maruyama and Tauchen (1986) methods (details in Online Appendix D.1). For the entrant firms' productivity distribution π_z we take the ergodic distribution associated with the (calibrated) Markov chain implied by equation (14).

[29] Given these choices, the support of the state space has about 80,000 points and there are about 6 billion potential transitions between states, making the calculation of the invariant distribution of firms computationally challenging. Online Appendix A.2 discusses this issue and how we get around it.

[30] This value is routinely used in models estimated to U.S. data, but it has also been used for European labor markets, specifically in models of dual labor markets (e.g. Thomas (2006), Costain *et al.* (2010) and Bentolila *et al.* (2012)).

Table 2: Externally Set Parameters

Parameter		**Value**	**Target/Source**
ρ	Discount rate	0.0123	5% annual discount rate
γ	Matching elasticity	0.5000	Petrongolo and Pissarides (2001)
ρ_z	Mean-reversion in productivity	0.0513	Ruiz-García (2021)
σ_z	Productivity dispersion	0.1833	Ruiz-García (2021)
ς	Recruiting cost curvature	2.0000	Linear marginal cost of recruiting
ψ	Firing cost curvature	2.0000	Linear marginal gain of promoting
ϑ	Promotion cost curvature	2.0000	Linear marginal gain of firing
τ	Rate of skill upgrade	0.1250	Baley *et al.* (2023)
$\zeta(\varphi_1)$	Permanent productivity φ_1 firms	1.0000	Normalization (without loss)

Note: Set of externally calibrated parameters. See Section 4.1 for details.

and $\sigma_z = 0.1833$. We set the cost curvature parameters of the recruiting, firing, and promotion technologies to $\varsigma = \psi = \vartheta = 2$, so that the recruiting intensity, layoff, and promotion rates are linear in the corresponding net surplus changes.[31] We set the skill conversion rate to $\tau = 1/8$, such that the average duration before a skill upgrade is 8 quarters. This follows from Baley, Figueiredo, Mantovani and Sepahsalari (2023), who show that the returns to (occupational) experience are concave and almost exhausted after two years. Finally, we normalize the permanent productivity component of type-φ_1 firms to $\zeta(\varphi_1) = 1$, which comes without loss as our economy exhibits size-neutrality —only the *relative* size $\zeta(\varphi_2)/\zeta(\varphi_1)$ matters, and this ratio will be calibrated internally.

4.2 Internally Set Parameters

The parameters $\mathbf{p}_{\text{int}} \equiv \Big(\kappa, \zeta(\varphi_2), \nu, \omega(\varphi_1), \alpha, \omega(\varphi_2), \chi, A_{OE}, A_{FT}, s^W_{OE}, s^W_{FT}, \xi, \pi_\varphi, s^F, b\Big)$, are calibrated internally. As the joint estimation of these 15 parameters and the assignment of firms to types φ is numerically unfeasible, we use the two-step procedure in Bonhomme, Lamadon and Manresa (2022). First, we assign individual firms to types by use of some statistics from the data without explicitly solving the model. Then, we estimate the model parameters by the Simulated Method of Moments conditional on this assignment, using the algorithm described in Online Appendix D.2.[32]

4.2.1 Assigning Firms to Permanent Types

The model predicts that firms of different technology types φ differ ex-post in their size, due to $\zeta(\varphi)$, and in their temporary share conditional on firm size, due to $\omega(\varphi)$. In the data, we

[31] We do this for symmetry with the linearity in the hiring policy when $\gamma = 0.5$ (equation (A.27) in the Online Appendix).
[32] The parameters included in $\mathbf{p}_{\text{int}}$ are chosen to minimize:

$$\Big(\vec{M}^{\text{data}} - \vec{M}^{\text{mod}}(\mathbf{p}_{\text{ext}}, \mathbf{p}_{\text{int}})\Big)^\top \mathcal{W}^{-1} \Big(\vec{M}^{\text{data}} - \vec{M}^{\text{mod}}(\mathbf{p}_{\text{ext}}, \mathbf{p}_{\text{int}})\Big), \tag{15}$$

where $\vec{M}^{\text{data}}$ is a vector of moments from the data (conditional on the assignment of firms to types), $\vec{M}^{\text{mod}}(\mathbf{p}_{\text{ext}}, \mathbf{p}_{\text{int}})$ is the model counterpart, and $\mathcal{W}$ is a diagonal matrix of weights containing the squares of the data moments in the diagonal.

Table 3: Assignment of firms in the data to φ types

	Firm size (average, in # workers)	**Temporary share** (average, in %)	**Share of firms** (total, in %)
Firms of type φ_1	9.8	4.3	19.7
Firms of type φ_2	6.8	23.2	80.3
All firms	7.4	19.5	100.0

Note: Results from classifying firms in the data into the two permanent types of the model. See Section 4.2.1 for details.

have documented a between-firm negative correlation between employment and temporary share (Figures 1 and E.2). Therefore, we classify firms into two different technology types with the aim of reproducing this relationship. First, we regress the temporary share against dummies of firm size and unobserved firm fixed effects and we keep the estimated firm fixed effects. These can be thought of as capturing the "permanent temporary share" of each firm. Second, we take the time series average of firm size for each firm. This can be thought of as the "permanent size" of each firm. Third, we group all firms into 50 groups (2% of the population each) based on the "permanent temporary share", from smallest to largest. Fourth, we compute the average "permanent temporary share" and the average "permanent size" in each group. Finally, we run a k-means algorithm with these two variables across all 50 groups to create only two groups, which are our two types (φ_1, φ_2).

The results of this procedure are in Table 3. The first type (labeled φ_1), represent a smaller share of firms (19.7%), make a moderate use of temporary contracts (4.3% of the firm's employment, on average), and are larger in size (9.8 employees). The remainder share of firms (80.3%), those of type φ_2, make ample use of temporary contracts (23.2%), and are smaller (6.8 employees).

4.2.2 Targeted Moments and Model Fit

Once we have classified firms in the data into the model's types, we select various moments to identify our parameters. In this section, we discuss intuitively how each parameter is identified by each moment. Section 4.2.3 will verify these intuitions with a formal identification exercise.

Average and relative firm size We target the average firm size and the relative size of firms of different types, which identifies κ and $\zeta(\varphi_2)/\zeta(\varphi_1)$ respectively. First, average firm size equals the ratio of the employment rate, $\mathtt{E}$, to the measure of operating firms, $\mathtt{F}$. This is mainly affected by the entry cost parameter, κ. To see this, note that changes in κ monotonically shifts the firm's expected value of entry through the free entry condition. In equilibrium, the expected value of entry decreases monotonically with the average labor market tightness $\mathtt{F}/(1-\mathtt{E})$ because it determines the average probability of hiring. The steady state employment rate $\mathtt{E}$ is pinned down by the aggregate labor market flows, which are either direct calibration targets or implied by our calibration targets (see details below). Hence, κ determines the mass of operating firms $\mathtt{F}$ and through that average firm size, $\mathtt{E}/\mathtt{F}$. Our target for average firm size is 7.35 and our economy delivers 7.01 (see Table 4).

Second, the *relative* firm size between φ_2 and φ_1 firm types is driven by the ratio of their permanent productivities, $\zeta(\varphi_2)/\zeta(\varphi_1)$. Because $\zeta(\varphi_1)$ is normalized to 1, this pins down $\zeta(\varphi_2)$. In the data, average firm size of φ_2 firms is a fraction 0.693 of the average size of φ_1 firms. The calibration predicts a relative size of 0.650, and pins down $\zeta(\varphi_2) = 0.8760$ (see Table 4).

Productivity and temporary share by firm characteristics Next, we want the model to be consistent with the observed relationship between (i) firm productivity and firm characteristics, and (ii) temporary share and firm characteristics. These relationships should help identify the parameters in the production function, $(\nu, \alpha, \{\omega(\varphi)\})$, and the costs of firing workers, χ.

To see this, let $n = n_H + n_L$ denote the total number of workers in a firm, our observable measure of size in the data. Using equation (2), we can write log output per worker as:

$$\ln\left(\frac{y(n_H, n_L, z, \varphi)}{n}\right) = z + \zeta(\varphi) - (1-\nu)\ln(n) + \frac{\nu}{\alpha}\ln\left(\omega(\varphi)\left(\frac{n_H}{n}\right)^{\alpha} + (1-\omega(\varphi))\left(\frac{n_L}{n}\right)^{\alpha}\right). \tag{16}$$

If we could observe all the variables of this equation, a non-linear least squares regression could recover the degree of decreasing returns to scale ν from the partial effect of firm size on firm productivity. Moreover, we could recover the relative productivity of high and low skilled workers by firm type, $\omega(\varphi)$, and the elasticity of substitution between the two, $\frac{1}{1-\alpha}$, from the partial effect of changes in the skill composition of the firm on firm productivity. However, in our CBI data we observe neither firm productivity z nor the share of high-skilled workers, n_H/n.

To circumvent this issue, we follow an indirect inference strategy and consider a simplified version of the above equation that can be estimated both in the data and in the model. In particular, we compute a second-order expansion of equation (16), we measure output per worker as the ratio of value added (VA_{it}) to employment (Emp_{it}), we proxy the skill rate with the temporary share (as they are strongly negatively correlated in the model but the former is unobserved), and we send the firm temporary TFP component z to the error term. The resulting regression is:

$$\ln\left(\frac{VA_{it}}{Emp_{it}}\right) = \text{constant} + \beta_0^A\,\mathbf{1}[\varphi_i = \varphi_2] + \beta_1^A\ln(Emp_{it}) + \beta_2^A TempSh_{it} + \beta_3^A TempSh_{it}^2 + \epsilon_{it}^A, \tag{17}$$

for firm i at time t, where $\mathbf{1}[\varphi_i = \varphi_2]$ is an indicator variable for permanent firm type. We run this regression by OLS both in the model and the data, and we target the regression coefficients β_1^A (which helps identify ν), and β_2^A and β_3^A (which help to jointly identify $\omega(\varphi_1)$, $\omega(\varphi_2)$ and α). Note that we pool firms of different types together but control for firm type φ by including the fixed effect $\mathbf{1}[\varphi_i = \varphi_2]$, which eliminates endogeneity concerns due to $\zeta(\varphi)$.

Next, we aim to match the effect of firm size and firm type on the temporary share, which should help identify $\omega(\varphi_2)/\omega(\varphi_1)$ and χ. To do so, we follow again an indirect inference approach and run the following OLS regression both in the model and the data:

$$TempSh_{it} = \beta_0^B\,\mathbf{1}[\varphi_i = \varphi_2] + \sum_{\ell=1}^{N_{\text{bins}}}\beta_\ell^B\,\mathbf{1}[Emp_{it} \in \text{SizeBin}_\ell] + \epsilon_{it}^B. \tag{18}$$

The β_0^B coefficient indicates the differential choice of temporary share by firms of different type (and same size), and should roughly pin down the ratio $\omega(\varphi_2)/\omega(\varphi_1)$, as $\omega(\varphi)$ determines the relative demand of skill and hence of temporary workers in firms of type φ. The $\{\beta_\ell^B\}$ coefficients, in turn, indicate the partial effect of firm size on the temporary share within a firm type, capturing the within-firm variation in Figure E.2.[33] This moment should inform about the firing cost shifter, χ, as this parameter drives the share of each worker type that a firm intends to keep in equilibrium.

Panel A of Table E.1 in Online Appendix E shows the empirical coefficients for regression (17). We find that conditional on type, larger firms are associated with slightly higher productivity ($\beta_1^A = 0.081$). This is the result of the positive correlation between firms size (n) and transitory firm productivity (z) in the cross-section, which dominates the direct negative effect of decreasing returns to scale that we obtain in the calibration. However, as it can be seen in Figure E.3 of Online Appendix E, lower values of ν lead to lower (even negative) values of β_1^A, as implied by the more severe decreasing returns to scale. We also find that, conditional on type, a larger temporary share is associated with a lower firm productivity ($\beta_2^A < 0$ and $\beta_3^A < 0$), consistent with the idea that low-skilled workers, which tend to have FTCs, are less productive. Finally, we also find that low types are, on average, less productive ($\beta_0^A < 0$), which is not surprising given the results of our classification approach described in Table 3.

Panel B of Table E.1, in turn, shows the empirical coefficients of regression (18). The β_0^B coefficient equals 0.201 in the data, which says that, conditional on firm size, φ_2 firms (the less productive ones) use a higher fraction of temporary contracts, 20.1 percentage points more. On the other hand, the $\{\beta_\ell^B\}$ coefficients are monotonically increasing in size bins ℓ, confirming that the within-firm positive relationship between firm size and temporary employment that we saw in Figures 1 and E.2 also holds when we condition on firm type instead of individual firm fixed effects.

In Panel B of Table 4, we report the calibrated model's fit on this regression evidence. In our indirect inference approach, we obtain $\nu = 0.881$, showing moderate decreasing returns to scale. Because temporary contracts are more prevalent among less skilled workers, this recovers $\omega(\varphi_1) = 0.645 > 0.316 = \omega(\varphi_2)$. Moreover, we find $\alpha = 0.542$, which delivers a considerably high elasticity of substitution between worker skill types, $\frac{1}{1-\alpha} = 2.183$. Then, we pin down a value for χ using $\beta_2^B - \beta_1^B$ (equal to 0.032 in the data), which captures the intensity by which the share of temporary workers changes within firm type, on average, when firms transition from the first to the second size bin. We choose to match the gap between these coefficients because the first two size bins comprise the vast majority of firms in our data. Given these parameter values, the calibrated model matches all of these coefficients well.

Worker stocks and flows Next, with the idea to identify $(A_{FT}, A_{OE}, s_{FT}^W, s_{OE}^W, \xi)$, we target the employment-to-unemployment (EU) and unemployment-to-employment (UE) quarterly flow rates by contract type from the Spanish labor force survey (EPA), as well as the average rate of temporary

[33] In practice, we build size bins using 5-worker increments. As our sample contains firms with up to 60 employees, this implies $N_{\text{bins}} = 12$ size bins: 1-5, 6-10, 11-15, and so on up to 56-60.

employment. In the data, we have a 3-state Markov chain (unemployed, employed with FTC, and employed with OEC), with 6 independent transitions. This system pins down two steady-state ratios: the temporary share and the unemployment rate. Consistent with the model, we set the OE-to-FT rate to zero, and target four of the remaining five transitions (leaving the FT-to-OE rate free) plus the average temporary share (leaving the unemployment rate free). In the model we compute the same workers flows, see Online Appendix A.4 for details.

Which parameters are informed by these five targets? First, the two UE rates are primarily affected by the recruiting cost parameters (A_{OE}, A_{FT}), as these act as shifters for the number of matches that take place in each market per unit time, given market tightness. In the data, UE transitions are far more frequent among FT workers (18.77% quarterly) than among OE workers (2.79%). The calibrated model matches these rates closely (18.41% and 2.52%, respectively) with $A_{FT} = 0.0099$ and $A_{OE} = 1.8510$. This means that the model rationalizes the difference in UE rates by making OE hires more costly: for given market tightness, there are more matches in the FT market per unit of time for every match that takes place in the OE market. These differences in recruiting costs endogenously imply that matching efficiency is much higher in the FT labor market.[34]

Secondly, the EU rates are most directly affected by the worker exogenous separation rates, (s^W_{FT}, s^W_{OE}). In the data, transitions into unemployment are far more common in the FT market (12.85% quarterly) than in the OE market (1.38%). These EU rates are closely matched by the calibrated model (11.92% and 1.46%, respectively). The calibration delivers that the separation rate of FT workers is high (equal to 0.46, i.e. an average duration of about 6 and a half months on the job, conditional on no endogenous separations) and that OE jobs are virtually never destroyed exogenously (s^W_{OE} is estimated to be nearly equal to zero). The exogenous separations in FTCs may be driven by voluntary quits or by regulations on the maximum duration of these contracts. The latter will be our preferred interpretation in our policy analysis of Section 5.

Finally, the promotion cost parameter ξ determines the flow of workers being promoted from FTCs to OECs. If we were to target this flow, then together with the other four flows described above we would uniquely pin down the unemployment rate and the temporary share. To identify ξ, we therefore target the average temporary share in the EPA data (21.9%).[35] Our calibrated model matches the average temporary share (21.8%), and implies an unemployment rate of 14.9%.[36]

Other moments First, the probability of entering as φ_1-type, $\pi_\varphi(\varphi_1)$, is pinned down by the share of firms of this type among all operating firms in the stationary solution of the model.[37] In the calibration, we obtain $\pi_\varphi(\varphi_1) = 26\%$, which is substantially lower than the 40.2% of φ_1-type firms

[34] Online Appendix A.5 describes how to compute aggregate matching efficiency in market $i = OE, FT$, which we denote Γ_i. In the calibrated economy, we find a large gap: $\Gamma_{FT}/\Gamma_{OE} \approx 46$.

[35] Recall that the EPA numbers are always slightly higher than the firm-level ones (see Figure E.1). We choose the EPA number to be consistent with the fact that our targets for the worker flow rates have been computed using these data.

[36] This is in the middle of the unemployment figures observed in Spain during our period of analysis (2004-2019). In the data, the unemployment rate obtains its lowest value in 2007 (at 8.2%) and its highest value in 2013 (at 26.1%).

[37] In the model, we compute $\mathtt{F}_{\varphi_1}/\mathtt{F}$, where $\mathtt{F}_{\varphi_1} \equiv \sum_i \sum_j \sum_z \mathtt{f}(\{n_{ij}\}, z, \varphi_1)$, and $\mathtt{f}(\{n_{ij}\}, z, \varphi_1)$ is the stationary measure of type-φ_1 firms of productivity z with n_{ij} workers of skill $j = L, H$ in contracts of type $i = FT, OE$.

Table 4: Internally Estimated Parameters and Model Fit

Parameter		**Value**	**Moment**	**Model**	**Data**	**Source**
Panel A. *Absolute and relative firm size*						
κ	Fixed firm entry cost	823.38	Average firm size	7.01	7.35	CBI
$\zeta(\varphi_2)$	Permanent productivity of φ_2 firms	0.876	Relative size of φ_2 firms	0.650	0.693	CBI
Panel B. *Productivity and temporary share by firm characteristics*						
ν	Returns-to-scale parameter	0.881	β_1^A coefficient, eq. (17)	0.078	0.081	CBI
$\omega(\varphi_1)$	Productivity H workers in φ_1 firms	0.645	β_2^A coefficient, eq. (17)	-0.047	-0.104	CBI
α	Substitutability between worker types	0.542	β_3^A coefficient, eq. (17)	-0.086	-0.092	CBI
$\omega(\varphi_2)$	Productivity H workers in φ_2 firms	0.316	β_0^B coefficient, eq. (18)	0.260	0.201	CBI
χ	Firing cost shifter	0.110	$(\beta_2^B - \beta_1^B)$ gap, eq. (18)	0.041	0.032	CBI
Panel C. *Worker stocks and flows*						
A_{OE}	Recruiting cost shifter (OECs)	1.851	UE rate, OE (quarterly)	0.025	0.028	EPA
A_{FT}	Recruiting cost shifter (FTCs)	0.0099	UE rate, FT (quarterly)	0.184	0.188	EPA
s_{OE}^W	OEC destruction rate	2×10^{-5}	EU rate, OE (quarterly)	0.015	0.014	EPA
s_{FT}^W	FTC destruction rate	0.460	EU rate, FT (quarterly)	0.119	0.129	EPA
ξ	Promotion cost shifter	0.735	Temporary share	0.218	0.219	EPA
Panel D. *Other moments*						
$\pi_\varphi(\varphi_1)$	Probability of entering as a φ_1 type	0.260	Share of φ_1-type firms	0.402	0.197	CBI
s^F	Firm destruction rate	0.0075	Firm entry rate (annual)	0.075	0.075	INE
b	Employment opportunity cost	1.480	Leisure to output p.w.	0.819	0.800	.

Note: The model period is one quarter. The table reports the values of the parameters estimated internally by minimizing the criterion function in equation (15). UE and EU rates in the data are averages over HP-filtered quarterly series from the EPA over the period 2005Q1-2018Q4 (data before 2005 is unavailable), see Appendix A.4 for details. *Data sources:* "CBI" means our sub-sample from the Central de Balances Integrada data; "INE" means data from the Instituto Nacional de Estadística; "EPA" means data from the Encuesta de Población Activa.

in the stationary distribution, indicating higher survival probabilities for the high-type firms.

Second, to pin down s^F we target the entry rate of firms. In the data, firm entry rates fluctuated around 7.5% annually after the Great Recession (García-Perea, Lacuesta and Roldan-Blanco (2021)). In the model, the inflow of firms equals its outflow, so we compute the firm entry rate as the ratio of entrants to the total measure of operating firms.[38] This gives $s^F = 0.0075$ (see Table 4, Panel D).

Finally, we calibrate the opportunity cost of employment, b, to match that the income flow from unemployment represents 80% of the average worker productivity (following Costain *et al.* (2010)). The calibrated model delivers $b = 1.48$ and predicts a value of 81.9% for our target.

4.2.3 Global Identification Results

To validate the identification of the 15 internally estimated parameters, we run the following exercise. For each parameter-moment pair established in the text (see Table 4 for the summary), we allow for quasi-random variation in all remaining parameters and solve the model for each

[38] Formally, $\frac{\mathrm{F}^e}{\mathrm{F}} \sum_{\varphi} \pi_\varphi(\varphi) \Big\{ \sum_{z^e} \pi_z(z^e) \sum_i \eta_i \big(W_{iL}'(\vec{n}_{iL}^e, z^e, \varphi) \big) \Big\}$, where F^e is the measure of potential entrants.

such parameter configuration.[39] As a result, for each level of the identified parameter we obtain a whole distribution for the targeted moment. In Figure E.3 of Online Appendix E we plot, for each parameter-moment pair, the median of this distribution (black dots) and the inter-quartile range (shaded area), together with the empirical target (dashed line).

We consider that a parameter is well-identified by the moment when (i) the distribution changes across different values of the parameter; (ii) the rate of this change is high; (iii) the inter-quartile range of the moment's distribution is narrow throughout the support for the parameter; (iv) the empirical target falls within the inter-quartile range.[40] Because all the remaining parameters are not fixed but instead are varying in a quasi-random fashion within a wide support, this method gives us a global view of identification. Figure E.3 shows that the parameters are well-identified by their corresponding moments along most of the aforementioned criteria.

4.3 Non-Targeted Moments

Next, we validate the calibration results by confronting the model against non-targeted moments. We focus on moments related to features of the cross-sectional distribution of firms against firm size, age, temporary employment and employment growth.[41] Figure E.4 in Online Appendix E shows the distribution of firms and employment by firm size and firm age, in the data and in the calibrated model. Along the size dimension, the model aligns well with the data, especially in terms of firm shares, even though nothing from these distributions was used as calibration targets. On the age dimension, there is a slightly larger share of young firms in the model compared to the data, and these firms hold too much employment, but the model correctly predicts the remaining parts of the age distribution, both in terms of firm shares as well as employment shares.

Second, we validate the calibrated model's predictions on firm employment growth.[42] Figure E.5 compares the distributions of yearly employment growth for total employment, FT employment and OE employment, in the data versus the calibrated model. The calibrated model correctly predicts the dispersion in all three distributions, an indication that the parameters of our idiosyncratic productivity process are well identified. The model is remarkably able to replicate the symmetry in the distribution of growth rates for all types of employment, and the share of firms that obtain their

[39]More precisely, we use the following procedure, inspired by Daruich (2023). First, we set wide enough bounds for each parameter from the $\mathbf{p}_{\text{int}}$ parameter vector. Then, we pick quasi-random realizations from the resulting hypercube using a Sobol sequence, which successively forms finer uniform partitions of the parameter space. Finally, for each parameter combination, we solve the model and store the relevant moments. To implement this last step, we use a high-performance computer, which allows us to parallelize the numerical solution and saves us a large amount of computational time. In total, we solve the model 1.3 million parameter combinations.

[40]Criterion (i) implies that the moment is globally sensitive to variation in the parameter; (ii) gives an idea of how strong this sensitivity is; (iii) measures how much other parameters matter to explain variation in the moment; and (iv) implies that the empirical target is not an outlier occurrence.

[41]To compute these moments in the model, we must rely on simulated data. We simulate 10,000 firms over 400 quarters (i.e. 100 years). We partition each quarter into 2,000 sub-periods to ensure that Poisson rates approximate probabilities well. To allow for convergence, we report results computed using only the last year of our simulated data.

[42]For any firm i, year-on-year employment growth is always computed as $g_{i,t} = \frac{Emp_{i,t+1} - Emp_{i,t}}{\frac{1}{2}(Emp_{i,t+1} + Emp_{i,t})}$, where $Emp_{i,t}$ denotes total, FT, or OE employment. This measure, which we borrow from Davis, Haltiwanger and Schuh (1998), is appealing because it accounts for entry ($g_{i,t} = 2$) and exit ($g_{i,t} = -2$) of firms, treating them symmetrically.

first, or lose their last, FT worker (i.e. the spikes at 2 and –2 in the middle plot of Figure E.5).

4.4 Misallocation

Before proceeding to our policy exercises, we study the extent of worker misallocation generated by the search and matching frictions and the dual labor market structure. Online Appendix B.1 derives the allocation of workers that would result from (i) maximizing output without being subject to the constraints imposed by the search frictions of the market economy, but (ii) taking the allocation of firms across productivity bins (φ, z) as given. In this Benchmark economy the marginal product of either type of labor is equalized across all firms. This means that (i) the allocation of workers and skills is identical between firms of the same productivity class (φ, z); (ii) more productive firms (higher φ or higher z) employ more workers (due to decreasing returns to scale); and (iii) the allocation of worker skills to firms is increasing in φ and independent from z because the skill share parameter $\omega(\varphi)$ is increasing in φ and independent from z.

The competitive equilibrium allocation differs from this benchmark in two ways. First, within firm productivity class (z, φ), firms in the competitive equilibrium display different amounts of workers and skills. Second, the average amount of workers and skills allocated across firms of the same productivity class (z, φ) in the competitive equilibrium differs from the Benchmark allocation. Both of these translate into output losses. Quantitatively, we find that our calibrated economy produces 3.9% less output per worker than the Benchmark economy with the same level of employment and the same distribution of firms, showing that search and matching frictions in Spain are quite damaging for welfare. Of this loss, 40.3% is due to the between productivity class (z, φ) misallocation, and the remainder is due to the within productivity class (z, φ) component. Figure E.6 in Online Appendix B.2 shows the underlying heterogeneity in misallocation across productivity types (z, φ).

5 The Macroeconomic Implications of Dual Labor Markets

We are ready to quantify the macroeconomic effects of dual labor markets and the impact of policies that regulate FTCs. We study three distinct policies: limiting the maximum duration of FTCs (Section 5.1), taxing the use of FTCs (Section 5.2), and banning the use of FTCs (Section 5.3).

5.1 Reducing the Maximum Duration of FTCs

A key characteristic of dual labor markets is the (relatively short) time limit that workers can spend in a firm under an FTC. When the limit is reached, firms are forced to convert the contract into an OEC or let the worker go. Reducing the maximum duration of FTCs is also a common policy tool for countries that want to limit the use of FTCs, as in the 2022 labor market reforms implemented in Spain. In this Section, we explore the consequences of changes in the legal maximum duration of FTCs. To do so, we solve for a series of economies in which we vary the exogenous separation rate for FT workers, s^W_{FT}, such that average duration for FTCs moves between one month and 4 years (in the calibrated economy, the average duration is 6.5 months). We leave all other parameters unchanged at their calibrated values, and compare across steady-state solutions. The results for this series of exercises are reported in Figure 2. Table 5 provides exact numbers for the two policies at the extreme, one with one month FTC duration and another one with 4 years duration.

A policy that reduces the legal duration of FTCs from the baseline duration down to one month (Column (A) in Table 5 and the left-most point in each panel of Figure 2) achieves the intended effect of reducing the overall temporary share of the economy (see Panel (f)), from 21.8% to 0.9%. The policy also increases aggregate productivity, although only modestly by 0.26%. However, this policy would come at the expense of a large reduction in output (Panel (r)) via an increase in the unemployment rate (Panel (q)) that is larger than the increase in aggregate productivity. All in all, limiting FTC duration to one month would reduce welfare by 0.87% (Panel (t)).[43] In what follows, we discuss these results in more detail.

5.1.1 Aggregate Employment and its Composition

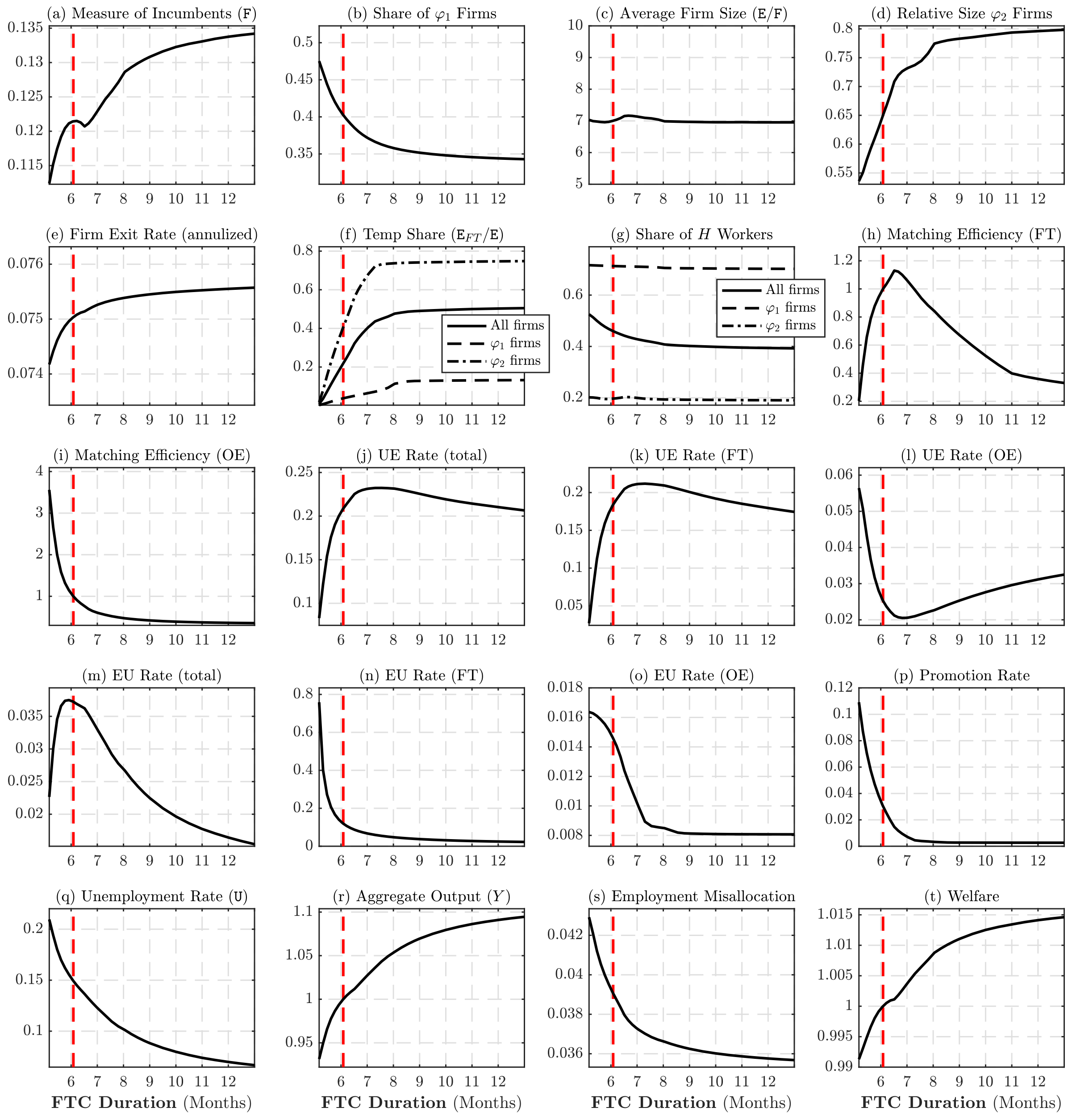
In the model, firms face a trade-off between the opportunity costs (in lost output) from the high worker turnover of workers under FTCs, and the higher recruiting costs of workers under OECs. For given firm productivity (φ, z), firms prefer a higher share of FTCs as they become larger because their marginal product of labor declines and so does the opportunity cost of unfilled vacancies. When FTCs expire more quickly, the turnover of workers with FTCs increases and this trade-off is tilted in favor of OE hiring. As firms' recruiting efforts for OE workers increase, the matching efficiency in this market raises (Panel (i)), while it decreases in the FT market (Panel (h)).

This intuition helps explain the policy effects on aggregate worker flows. On the one hand, the aggregate EU flow declines (Panel (m)) as FTC duration declines from 6.5 months. This is the result of the strong decline in the share of temporary workers, which more than offsets the mechanical increase in the EU flow of FTCs and the very small increase in the EU flow of OECs (Panels (n) and (o)). On the other hand, the aggregate UE flow also declines (Panel (j)). As now firms increasingly prefer to hire through the OE margin, job-finding rates decline in the FT market (Panel (k)), increase slightly in the OE market (Panel (l)), and promotion rates increase sharply (Panel (p)), with an overall decline in the UE flow due to the lower hiring needs of firms as the job destruction rate declines. The combination of declining job-filling rates and declining job-destruction rates lead to an increase in unemployment (Panel (q)), from 14.9% in the baseline to 21.0% in the one-month FTC duration policy.[44] Finally, as firms now prefer to employ more workers under an OEC, and because only OE workers can accumulate human capital on the job, the share of high-skill workers in the economy increases substantially, from 46.0% in the baseline to 52.5% under the policy (Panel (g)).

[43] To compute welfare, we use equation (A.37) in Online Appendix A.6. In words, welfare equals the present discounted value of production net of firing, recruiting, and promotion costs over all operating firms, plus the value of home production across all unemployed workers, net of the total entry costs paid by all potential entrant firms.

[44] The dynamics of unemployment are written in the system of equations (A.29a)-(A.29d) of the Online Appendix.

Figure 2: Effects of FTC duration on selected equilibrium variables.



Notes: This figure shows the effects of reducing the maximum duration of FTCs on a number of macroeconomic aggregates of interest. For all panels, the horizontal axis represents $1/s^W_{FT}$, and is measured in quarters. The plots shows different stationary solutions of the model, keeping all parameters fixed at their baseline calibration values except for s^W_{FT}. The red dashed vertical line shows the expected duration in the baseline calibration. Aggregate output, matching efficiency, and welfare are all normalized to one at the baseline calibration. For the computation of EU and UE rates, see Online Appendix A.4.

Table 5: Effects of changes in average FTC duration on macro aggregates

	(A)	(B)	(C)
	Short duration (1 month)	*Baseline* (6.5 months)	*Long duration* (4 years)
Measure of operating firms	0.112	0.121	0.134
... Share of type-φ_1 firms	47.5 %	40.2 %	34.3 %
Average firm size	7.03	7.01	6.95
... Relative size φ_2 firms	0.54	0.65	0.80
Firm entry rate (annualized)	7.4 %	7.5 %	7.6 %
Average temporary share	0.9 %	21.8 %	50.4 %
... within φ_1 firms	0.3 %	5.2 %	13.2 %
... within φ_2 firms	1.8 %	32.9 %	74.8 %
Share of H workers	52.5 %	46.0 %	39.3 %
... within φ_1 firms	71.7 %	71.3 %	70.2 %
... within φ_2 firms	20.2 %	19.7 %	19.0 %
Matching efficiency (FT)	0.28	1.40	0.48
Matching efficiency (OE)	0.11	0.03	0.01
UE rate (total)	8.3 %	20.9 %	20.8 %
... UE rate (FT)	2.7 %	18.4 %	17.7 %
... UE rate (OE)	5.6 %	2.5 %	3.2 %
EU rate (total)	2.3 %	3.7 %	1.6 %
... EU rate (FT)	75.9 %	11.9 %	2.4 %
... EU rate (OE)	1.6 %	1.5 %	0.8 %
Promotion rate	10.9 %	3.0 %	0.3 %
Unemployment rate	21.0 %	14.9 %	6.8 %

Notes: This table shows the effects of reducing the maximum duration of FTCs on a number of macroeconomic aggregates of interest. Column (B) corresponds to the baseline calibration; in column (C), we set $s^W_{FT} = 1/16$ so that FTCs expire on average after 16 quarters, or 4 years; in column (A) we set $s^W_{FT} = 3$, so that FTCs expire on average after 1 month. UE, EU and promotion rates are quarterly figures, whereas the firm entry rate is an annual figure. For the computation of EU and UE rates, see Online Appendix A.4.

5.1.2 The Measure of Firms and its Composition

As a byproduct of the changing firm incentives described above, there are also important consequences for the total measure of operating firms and the distribution of firms across productivity levels. First, with FTCs being less useful for firms there is a reduction in the value of firm incumbency, which reduces firm entry and the total measure of operating firms $\mathtt{F}$ (Panel (a)). As there is a decrease in $\mathtt{F}$ but the employment rate $\mathtt{E} = 1 - \mathtt{U}$ also declines, the net effect on firm size is ambiguous. Quantitatively, Panel (c) shows that firms barely change size across different policies. Second, under the policy reform, there are strong selection effects, with a decrease in the turnover of firms (Panel (e)), a higher share in equilibrium of firms of the more productive permanent type (Panel (b)), and an increase in the relative size of those firms (Panel (d)). Because firms of the less productive type φ_2 rely more on FT workers, the worsening of these contracts damages these firms relatively more, inducing a higher exit risk among them.

5.1.3 Aggregate Productivity

Next, we turn to aggregate productivity. The understand the policy effects on this margin, we provide a novel decomposition. In fact, our decomposition formula should be general to models of firm dynamics featuring decreasing returns to scale in production, firm selection, and worker misallocation. Let us start by defining aggregate output as

$$Y \equiv \sum_{n_H=0}^{+\infty} \sum_{n_L=0}^{+\infty} \sum_{z \in \mathcal{Z}} \sum_{\varphi \in \Phi} \Big[y(n_H, n_L, z, \varphi)\, \mathtt{f}(n_H, n_L, z, \varphi) \Big], \tag{19}$$

where recall that $\mathtt{f}(n_H, n_L, z, \varphi) \geq 0$ is the measure of operating firms that are of type φ, have productivity z, and employ $n_H \equiv n_{OEH}$ high-skill workers and $n_L \equiv n_{FT} + n_{OEL}$ low-skill workers. The production function $y(\cdot)$ is given in equation (2). Our object of interest is aggregate productivity, defined as output per worker, $Y/\mathtt{E}$.

Now, to decompose aggregate productivity, we need to introduce some new notation. First, let $n \equiv n_H + n_L$ be total firm employment. Second, let $\widehat{n} \equiv n/(\mathtt{E}/\mathtt{F})$ be the total employment of a firm relative to the average firm size, so that if $\widehat{n} > 1$ (respectively, $\widehat{n} < 1$) the firm is larger (respectively, smaller) than the average firm in the economy. And third, let $h \equiv n_H/n$ be the skill share of the firm. Let $\widehat{\mathcal{N}}$ and $\mathcal{H}$ denote the supports of $\widehat{n}$ and h which, because of our assumptions, are countable sets.[45] Then, in Online Appendix B.3 we show that aggregate output per worker can be written as

$$\frac{Y}{\mathtt{E}} = \underbrace{\left(\frac{\mathtt{F}}{\mathtt{E}}\right)^{1-\nu}}_{\substack{\text{Firm size}\\ \text{component}}} \left[\sum_{z \in \mathcal{Z}} \sum_{\varphi \in \Phi} \underbrace{\frac{\mathtt{F}_{z,\varphi}}{\mathtt{F}}}_{\substack{\text{Firm}\\ \text{selection}\\ \text{component}}} \underbrace{\left(\sum_{\widehat{n} \in \widehat{\mathcal{N}}} \sum_{h \in \mathcal{H}} y^h_{z,\varphi}(\widehat{n}, h)\, \mathtt{g}_{z,\varphi}(\widehat{n}, h) \right)}_{\text{Worker reallocation component}} \right] \tag{20}$$

In these equations, $\mathtt{F}_{z,\varphi} \geq 0$ is the measure of operating firms of productivity type (z, φ), so that $\sum_z \sum_\varphi \mathtt{F}_{z,\varphi} = \mathtt{F}$; we define $y^h_{z,\varphi}(\widehat{n}, h) \equiv \widehat{n}^\nu y\,(h, 1-h, z, \varphi)$, with $y(\cdot)$ given by equation (2); and $\mathtt{g}_{z,\varphi}(\widehat{n}, h) \in (0, 1)$, which is defined precisely in equation (B.6) of the Online Appendix, is the fraction of firms of productivity type (z, φ) that have relative employment $\widehat{n}$ and skill share h.

Equation (20) shows that we can decompose aggregate productivity changes into changes in three distinct components. The first term, the *firm size component*, reflects the effect of firm size on aggregate productivity. This term captures productivity gains due to increasing the number of firms per worker, which increases aggregate productivity due to the existence of decreasing returns to scale ($\nu < 1$). Our results (second row in Table 6) show that the *firm size* channel has a negligible impact on productivity when the duration of FTCs decreases. Indeed, panel (c) of Figure 2 shows that the measure of operating firms and the employment rate move almost one for one and yield negligible changes on firm size.

The second term, the *firm selection component*, reflects the effects of firm composition on

[45] For a given $\mathtt{E}/\mathtt{F}$, $\widehat{n}$ takes values in a countable set because n_H and n_L are both integers. Moreover, h takes values in the subset of rational numbers contained in the unit interval, i.e. $\mathcal{H} = \mathbb{Q} \cap [0, 1]$ (for instance, if $n = 1$, then $h \in \{0, 1\}$; if $n = 2$, then $h \in \{0, 1/2, 1\}$; if $n = 3$, then $h \in \{0, 1/3, 2/3, 1\}$; and so on).

Table 6: Effects of changes in average FTC duration on productivity, misallocation and welfare

	(A)	(B)
	Short duration (1 month)	*Long duration* (4 years)
Change in output per worker, of which:	0.26 %	-0.23 %
(a) *Firm size channel*	-0.05 %	0.09 %
(b) *Firm selection channel*	4.91 %	-4.37 %
(c) *Reallocation channel*, of which:	-5.77 %	2.90 %
... between-firm component	-5.62 %	2.59 %
... within-firm component	-0.70 %	0.75 %
Change in output	-6.86 %	9.29 %
Output loss from misallocation (in levels), of which:	4.29 %	3.57 %
... share due to between-firm misallocation	39.52 %	60.15 %
Change in welfare	-0.87 %	1.44 %

Notes: This table shows the effect of the policy, expressed in percentage changes with respect to the baseline calibration of 6.5 months FTC duration (with the exception of the output loss from misallocation and the share of it that is due to between-firm misallocation, which are expressed in levels). Welfare is computed as in equation (A.37), see Online Appendix A.6.

aggregate productivity. Changes in the shares of firms in each productivity bin lead to productivity gains if those firms that are more productive become more abundant in the economy. This channel works through firm exit: to the extent that the policy change lowers the survival probability of the less productive firms, aggregate productivity will rise.[46] We find that the firm selection effect contributes a 4.91% increase in aggregate productivity in response to the policy change (third row in Table 6). Indeed, as FTC duration decreases, the new distribution of firms features a higher share of φ_1 firms, as seen in Panel (b) of Figure 2.

The third term, the *worker reallocation component*, captures the effects of employment reallocation *within* and *across* firms of a given productivity class (φ, z). Through this channel, aggregate productivity improves in response to a policy change when the relative allocation of workers across firms improves, either through the reallocation of total employment (relative to average firm size) or through a reallocation of human capital within the firm. The probability mass function $g_{z,\varphi}(\widehat{n}, h)$ measures how firms are distributed in the space of relative sizes and human capital. Changes in this distribution command aggregate productivity gains or losses through employment reallocation dynamics. With the reduction in the duration of FTCs, the allocation of employment across firms worsens: the reallocation channel, in isolation, would lead to a 5.77% decrease if FTCs expired after one month (fourth row in Table 6).

To understand what drives the worker reallocation channel margin, we split the joint probability $g_{z,\varphi}(\widehat{n}, h)$ into the product of conditional and marginal probabilities:

$$g_{z,\varphi}(\widehat{n}, h) = g^{A}_{z,\varphi}(h|\widehat{n})g^{B}_{z,\varphi}(\widehat{n}). \tag{21}$$

The first term, $g^{A}_{z,\varphi}(h|\widehat{n})$, reflects a *within-firm* reallocation component, as it captures how the skill

[46]See Section 5.1.4 for a model extension where there is also selection upon entry.

composition of workers changes within firms of the same productivity (z, φ) and same relative size $\widehat{n}$. The second term, $g^B_{z,\varphi}(\widehat{n})$, reflects a *between-firm* reallocation component, as it captures how the relative number of workers $\widehat{n}$ changes across firms of different productivities (z, φ). In our exercise, we find that the worsening in productivity due to the worker reallocation channel is predominantly due to the between-firm component (fifth and sixth rows in Table 6). Intuitively, when FTCs become of shorter duration, firms lose flexibility to adjust their total number of employees in response to productivity shocks. This is the classic misallocation effect due to an increase in firing costs as in Hopenhayn and Rogerson (1993). Instead, the within-firm component brings productivity gains due to the increase in human capital for given firm size due to the decline in worker turnover.

Finally, we can explore how the effects of the policy are heterogeneous across firms of different productivity classes, (z, φ). We relegate the results and the discussion to Online Appendix B.2.2, where we emphasize heterogeneous effects on employment misallocation.

5.1.4 Extension: Selection Upon Entry

As we have seen, a quantitatively important margin to evaluate the effects of policy on aggregate productivity is the firm selection channel. In the baseline model, firm entry is fully undirected, in the sense that potential entrants know neither their type φ nor their initial productivity shock z when they pay the entry cost κ. This means that changes in the economic environment leading to relative changes in the value of entry for different types of firms do not change the composition of entrants. Thus, all selection occurs on the exit margin.

In this Section we explore how the effects of shortening the duration of FTCs change when we allow for selection of entrants. To do so, we assume that potential entrants can choose their type φ. We also assume that the technology choice φ entails a flow entry cost $\kappa(\varphi) - \varepsilon_\varphi$, where $\kappa(\varphi)$ is common to all entrants and ε_φ is idiosyncratic. Finally, we assume that the common component $\kappa(\varphi)$ is known before entry but the actual realization of ε_φ is not. This captures uncertainty about the actual costs of entry that leads to smooth changes in the share of entrants of each type. Under these assumptions, the ex-ante value of entry $\mathbf{J}^e$, common to all entrants, is

$$\mathbf{J}^e = \mathbb{E}\left[\max_{i \in \{FT,OE\}} \left\{ \mathbf{J}^e(\varphi_i) - \kappa(\varphi_i) + \varepsilon_{\varphi_i} \right\}\right] \tag{22}$$

where the expectation is taken over all possible realizations of ε_{φ_1} and ε_{φ_2}. Free entry requires that $\mathbf{J}^e = 0$. The idea is that a more productive technology yields higher value of entry $\mathbf{J}^e(\varphi)$ but it also entails a higher entry cost $\kappa(\varphi)$, perhaps due to the investments in more productive technologies. We make this problem tractable with the assumption of Extreme Value distributions for ε_{φ_1} and ε_{φ_2}. Full details can be found in Online Appendix C.

In this Appendix, we show that for any chosen elasticity of entry composition $\pi_\varphi(\varphi_1)/\pi_\varphi(\varphi_2)$ to the differential value of entry $\mathbf{J}^e(\varphi_1) - \mathbf{J}^e(\varphi_2)$, there is a pair of entry costs $\big(\kappa(\varphi_1), \kappa(\varphi_2)\big)$ that delivers the exact same model outcomes as the calibrated economy with undirected entry. This is useful because it implies that we do not need to recalibrate the economy with the directed entry.

Instead, we chose arbitrarily an elasticity of one, obtain the $\kappa(\varphi_1)$ and $\kappa(\varphi_2)$ that deliver the same model outcomes as the baseline, and perform the policy reform of varying the duration of FTCs.

We find that when we shorten the contract duration of FTCs to 1 month, the share of φ_1 firms increases more than in the case of undirected entry: up to 65.5% instead of 47.5% (see Table E.2 in the Online Appendix). This is by construction, and the exact amount depends on the chosen elasticity of entry composition to the differential value of entry.

The consequences of a larger share of φ_1 firms in the economy are important. First, there is an overall smaller mass of operating firms, as φ_1 firms have a higher demand of workers and crowd out other firms by increasing average labor market tightness and lowering the value of entry for all firms. This results in average firm size increasing from 7.01 to 7.78 workers. Second, the share of workers with an FTC declines more (to 0.6% instead of 0.9%) as φ_1 firms demand fewer FT workers. This implies that the EU flow also declines more (to 1.8% instead of 2.3%), leading to both higher human capital accumulation (60.4% of high-skilled workers instead of 52.5%) and lower unemployment rate (17.8% instead of 21.0%). Third, output per worker barely changes but there are substantial changes in its different components (see Table E.3 in the Online Appendix). The *firm size channel* leads to productivity losses of 1.24% due to the reduction in the number of operating firms. The *firm selection channel* leads to productivity gains as large as 12.38% (due to the larger increase in φ_1 firms), which compares to 4.91% productivity gains in the economy without directed entry. However, the *worker reallocation channel* leads to productivity losses of 14.42%, much larger than the 5.77% losses in the economy without directed entry. This worsening of the worker reallocation is partly due to (i) the fact that φ_1 firms, by using a lower share of FT workers, are less flexible in growing and shrinking with productivity shocks, and in part also due to (ii) the fact that φ_2 firms experience productivity losses as their share of skilled workers grows above their optimal size.

Finally, due to the lower increase in unemployment, the output loss is halved (3.48% instead of 6.86%) and welfare increases slightly (instead of falling) due to the combination of lower output loss and the saving on the recurrent recruiting costs associated to the FTCs.

5.2 Taxing the Use of FTCs

An alternative policy used to limit the temporary share is the direct taxation of the stock of workers employed under FTCs. This type of policy is currently in place in countries such as France, Portugal, and Spain (see Cahuc, Benghalem, Charlot, Limon and Malherbet (2016a)). We study the effect of such a policy by introducing a linear tax on the number of FT workers, so that the flow of firm net revenues becomes $y(\vec{n}, z, \varphi) - \tau_{FT} n_{FT}$. Proceeds are rebated lump sum to ensure that the tax is resource-neutral (i.e. change in welfare only comes from changes in firm policies and ensuing changes in the distribution, and not from the fact that the economy has more or fewer resources).

We pick the tax rate τ_{FT} to produce the same reduction in the aggregate share of FTCs as in the economy where the maximum duration of FTCs is limited to 1 month, and we report the results of these two economies in Online Appendix Tables E.4 and E.5. The effect of this tax on aggregate productivity is very similar to the effect of lowering the maximum duration of FTCs. Firm selection is unchanged and so is the misallocation of workers. However, the effects on unemployment, output, and welfare differ. In particular, there is much less job destruction and worker turnover with a tax on the use of FTCs than with a reduction on the maximum duration of FTCs: the EU rate becomes 1.8% in the economy with a tax on FTCs and 2.3% in the economy that limits the maximum duration of FTCs. These changes mainly come from the EU rate of FT workers and are only partly offset by higher hiring in the economy with limits in the duration of FTCs: the UE rate becomes 7.0% in the tax economy and 8.3% in the economy that limits the maximum duration of FTCs. As a consequence, the unemployment rate grows to 19.6% in the tax economy, below the 21.6% rate in the economy that limits the duration of FTCs.

Finally, because aggregate productivity hardly varies across policies, the effects on aggregate output and welfare are dominated by the effects on total employment. Output falls by 5.2% in the tax economy, compared to 6.86% in the economy that limits the maximum duration of FTCs. For welfare, these figures are -0.65% and -0.85% respectively. We hence conclude this Section by noting that taxes on the stock of workers employed under FTCs are preferable to placing stricter limits to the maximum duration of these contracts, as this latter policy generates more job destruction.

5.3 Outright Ban on the Use of FTCs

To conclude, we explore the consequences of banning the use of FTCs altogether. To implement this counterfactual, we make FTCs identical to OECs by (i) setting $s_{FT}^{W} = s_{OE}^{W}$ and $A_{FT} = A_{OE}$, (ii) allowing FTCs to acquire skill $j = H$ at the same rate τ as OECs, and (iii) set the promotion cost shifter ξ to infinity. This implies that, as in the benchmark economy, every instant of time every firm posts two vacancies; however, different from the benchmark economy, the two vacancies are identical to each other and equivalent to the OEC vacancy of the dual markets economy.

The aggregate consequences of banning FTCs are reported in Column (B) of Online Appendix Tables E.4 and E.5. We find that banning FTCs is bad for the economy. Output per worker increases slightly but unemployment grows more, leading to overall output and welfare losses equal to 3.18% and 0.31%. FTCs are useful to firms because they give them flexibility to adjust employment to changing productivity. This is because of both their lower cost of recruiting effort and their short fixed duration. At the same time, they are bad because they help less productive firm types to survive. Our results show that when banning FTCs, the productivity losses due to the worker reallocation channel are similar to the productivity gains of the firm selection channel. However, unemployment rises from 14.9% in the benchmark economy up to 17.8% under the ban. This is mainly due to the large drop in the UE rate (20.9% to 7.5%), which is the result of a fall in aggregate matching efficiency resulting from the disappearance of FTCs.

6 Conclusion

Many labor markets are characterized by a dual structure, whereby firms are allowed to offer both open-ended contracts (OECs) with large termination costs, and fixed-term contracts (FTCs) of short duration. Using rich administrative balance-sheet data for Spain (2004-2019), we document that there exists a high degree of heterogeneity in the use of FTCs between firms. An overwhelming majority of the cross-sectional variation in the temporary share is due to firm-level characteristics. In particular, though larger firms exhibit lower shares of temporary employment when comparing between firms within the same sector, region and time period, this relationship is reversed when exploiting within-firm variation.

We build and calibrate a model of firm dynamics with search and matching frictions to explore the macroeconomic implications of dual labor markets. To match the micro-level facts, our calibrated model grants the permanently larger and more productive firms with a stronger preference for high-skill workers. Workers accumulate firm-specific human capital on the job, and this rationalizes that these firms rely less on temporary employment in the cross-section of firms as worker turnover damages human capital accumulation. On the other hand, within-firm variation is driven by the trade-off between higher worker stability and higher hiring costs of workers under OECs. Firms that are far from their optimal size feature a large marginal product of labor and are especially damaged by worker turnover, so they are willing to pay the higher recruiting costs of OECs.

In our main quantitative exercise, we find that reducing the average duration of FTCs is effective in lowering the share of temporary employment in the economy and to increase aggregate productivity through strong firm selection effects. However, this policy is ill-advised as it increases unemployment and decreases overall welfare via the misallocation of employment both within and across firms. Taxing firms for their use of FTCs would lead similar results, but with lower unemployment increases and hence lower welfare losses. All in all, our paper emphasizes that the firm side is an important dimension to consider when quantifying the aggregate implications of labor market duality, a phenomenon which is pervasive in both emerging and developed economies.

Finally, we note that the core of our analysis features selection of firms due to differential exit after policy reforms. In a robustness exercise, we show that adding selection upon entry, whereby potential entrants choose their technology and this choice responds to the labor market regulations, may affect our results. In this case, the effects of a reduction in the duration of FTCs on aggregate productivity would not change much: the better selection of firms is partly offset by the larger misallocation of workers due to the use of OECs by these firms. However, the losses due to increased unemployment would be much lower and so would the total output losses. This is again due to the lower use of FTCs by the more productive firms. Overall, there could be small welfare gains due to the saving in recruiting costs associated to FTCs. It is therefore a matter of interest to better understand the choice of technology by entrants, and to measure how this choice is affected by changes in the labor market conditions.

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Dual Labor Markets and the Equilibrium Distribution of Firms

by Josep Pijoan-Mas and Pau Roldan-Blanco

Appendix Materials (for Online Publication Only)

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A Additional Theoretical Results

A.1 Firm's and Joint Surplus Problem Equivalence

In order to show the equivalence, first we write the employed worker's and the firm's problem.

Employed Worker's Problem Given a menu of contracts $\vec{C} = \{w_{ij}, v_i, \delta_{ij}, p, W'_{ij}(\vec{n}', z') : (i,j) \in \mathcal{I} \times \mathcal{J}\}$ currently paid by the firm, the value of a worker of type (i,j) satisfies the following HJB equation:

$$\begin{aligned}
\rho \mathbf{W}_{ij}(\vec{n}, z, \varphi; \vec{C}) \; &= \; w_{ij} + (\delta_{ij} + s_i^W + s^F)\Big(\mathbf{U} - \mathbf{W}_{ij}(\vec{n}, z, \varphi; \vec{C})\Big) \\
\textit{Same type co-worker separates:} \quad & + (n_{ij} - 1)(\delta_{ij} + s_i^W)\Big(W'_{ij}(\vec{n}_{ij}^-, z) - \mathbf{W}_{ij}(\vec{n}, z, \varphi; \vec{C})\Big) \\
\textit{Different type co-worker separates:} \quad & + \sum_{(i',j') \neq (i,j)} n_{i'j'}(\delta_{i'j'} + s_{i'}^W)\Big(W'_{ij}(\vec{n}_{i'j'}^-, z) - \mathbf{W}_{ij}(\vec{n}, z, \varphi; \vec{C})\Big) \\
\textit{Firm hires with either contract:} \quad & + \sum_{i' \in \mathcal{I}} v_{i'} \eta \Big(W'_{i'L}(\vec{n}_{i'L}^+, z)\Big)\Big(W'_{ij}(\vec{n}_{i'L}^+, z) - \mathbf{W}_{ij}(\vec{n}, z, \varphi; \vec{C})\Big) \\
\textit{FT worker is promoted:} \quad & + n_{FT}\, p \Big(W_{ij}^p(\vec{n}^p, z) - \mathbf{W}_{ij}(\vec{n}, z, \varphi; \vec{C})\Big) \\
\textit{Skill upgrade for OE low-types:} \quad & + n_{OEL} \tau \Big(W_{ij}^\tau(\vec{n}^\tau, z) - \mathbf{W}_{ij}(\vec{n}, z, \varphi; \vec{C})\Big) \\
\textit{Productivity shock:} \quad & + \sum_{z' \in \mathcal{Z}} \lambda(z'|z)\Big(W'_{ij}(\vec{n}, z') - \mathbf{W}_{ij}(\vec{n}, z, \varphi; \vec{C})\Big). \qquad \text{(A.1)}
\end{aligned}$$

The right-hand side of this equation incorporates the different sources of value for the employed worker. On the first line, the worker gets a wage w_{ij}, and the option value from separation into unemployment (second additive term), either because she gets laid off (at rate δ_{ij}), or her contract expires (at rate s_i^W), or because the firm exits (at rate s^F). The second and third lines include the change in value from the separation of a co-worker that was employed under the same contract and job type (second line), or under the other contract type, or job type, or both (third line). The fourth line is the change in value due to the firm hiring an additional worker for a low-type job with a contract of type i'. The fifth and sixth lines of equation (A.1) refer to the change in value when either a promotion or skill accumulation take place, where we have defined:

$$W_{ij}^p(\vec{n}^p, z) \equiv \begin{cases} \dfrac{1}{n_{FT}}\Big(W'_{OEL}(\vec{n}^p, z) + (n_{FT} - 1)W'_{FT}(\vec{n}^p, z)\Big) & \text{if } i = FT, \; \forall j \in \{L, H\} \\ W'_{OE,j}(\vec{n}^p, z) & \text{if } i = OE, \; \forall j \in \{L, H\} \end{cases}$$

$$W_{ij}^\tau(\vec{n}^\tau, z) \equiv \begin{cases} W'_{FT}(\vec{n}^\tau, z) & \text{if } i = FT, \; \forall j \in \{L, H\} \\ \dfrac{1}{n_{OEL}}\Big(W'_{OEH}(\vec{n}^\tau, z) + (n_{OEL} - 1)W'_{OEL}(\vec{n}^\tau, z)\Big) & \text{if } (i,j) = (OE, L) \\ W'_{OEH}(\vec{n}^\tau, z) & \text{if } (i,j) = (OE, H) \end{cases}$$

Finally, the last line of equation (A.1) includes the change in value due to a productivity shock.

Firm's Problem An operating firm of type $\varphi \in \Phi$ in state $(\vec{n}, z, \vec{W})$ must choose a menu of contracts $\vec{C}_{ij} = \{w_{ij}, v_i, \delta_{ij}, p, W'_{ij}(\vec{n}', z')\}$ for each $(i,j) \in \mathcal{I} \times \mathcal{J}$, where recall that $\vec{W} = \{W_{ij}\}$ denotes the set outstanding promises to its current workers. Let $\mathbf{J}(\vec{n}, z, \varphi, \vec{W})$ be the value of this firm. The HJB equation is:

$$\begin{aligned}
\rho \mathbf{J}(\vec{n}, z, \varphi, \vec{W}) \;=\; & \max_{\{w_{ij}, v_i, \delta_{ij}, p, W'_{ij}(\vec{n}', z')\}} \Bigg\{ y(\vec{n}, z, \varphi) - \xi n_{FT} p^{\vartheta} + s^F \Big(\mathbf{J}^e - \mathbf{J}(\vec{n}, z, \varphi, \vec{W}) \Big) \\
\textit{Wage bill, firing and recruiting costs:} \quad & + \sum_{i \in \mathcal{I}} \Bigg[\sum_{j \in \mathcal{J}} \Bigg(- w_{ij} n_{ij} - \chi n_{ij} \delta_{ij}^{\psi} - A_i v_i^{\varsigma} \\
\textit{Workers type } (i,j) \textit{ separate:} \quad & + n_{ij}(\delta_{ij} + s_i^W) \Big(\mathbf{J}(\vec{n}_{ij}^{-}, z, \varphi, \vec{W}'(\vec{n}_{ij}^{-}, z)) - \mathbf{J}(\vec{n}, z, \varphi, \vec{W}) \Big) \Bigg) \\
\textit{Hiring under contract } i\textit{:} \quad & + v_i \eta \Big(W'_{iL}(\vec{n}_{iL}^{+}, z) \Big) \Big(\mathbf{J}(\vec{n}_{iL}^{+}, z, \varphi, \vec{W}'(\vec{n}_{iL}^{+}, z)) - \mathbf{J}(\vec{n}, z, \varphi, \vec{W}) \Big) \Bigg] \\
\textit{FT workers promoted:} \quad & + n_{FT} p \Big(\mathbf{J}(\vec{n}^{p}, z, \varphi, \vec{W}'(\vec{n}^{p}, z)) - \mathbf{J}(\vec{n}, z, \varphi, \vec{W}) \Big) \\
\textit{Skill upgrade for OE low-types:} \quad & + n_{OEL} \tau \Big(\mathbf{J}(\vec{n}^{\tau}, z, \varphi, \vec{W}'(\vec{n}^{\tau}, z)) - \mathbf{J}(\vec{n}, z, \varphi, \vec{W}) \Big) \\
\textit{Productivity shock:} \quad & + \sum_{z' \in \mathcal{Z}} \lambda(z'|z) \Big(\mathbf{J}(\vec{n}, z', \varphi, \vec{W}'(\vec{n}, z')) - \mathbf{J}(\vec{n}, z, \varphi, \vec{W}) \Big) \Bigg\}, \qquad \text{(A.2)}
\end{aligned}$$

where $\boldsymbol{J}^e$ is a firm's value of having no workers (satisfying equation (12)). Problem (A.2) is subject to two constraints:

$$\mathbf{W}_{ij}(\vec{n}, z, \varphi; \vec{C}) \;\geq W_{ij}, \qquad \text{(A.3a)}$$

$$W'_{ij}(\vec{n}', z') \;\geq \mathbf{U}, \quad \forall (\vec{n}', z'), \qquad \text{(A.3b)}$$

for all $(i, j) \in \mathcal{I} \times \mathcal{J}$. Constraint (A.3a) is a *promise-keeping* constraint: the firm must deliver an expected value to each worker (left-hand side) that is no lower than the outstanding promise (right-hand side). This constraint is in place because of the firm's initial commitment to the posted contracts. Constraint (A.3b) is a *worker-participation* constraint: for every possible future state $(\vec{n}', z')$, the value that each worker obtains cannot be lower than the outside option. This constraint is in place because workers do not commit, and must therefore be enticed to remain matched.

With these equations in place, we are now ready to prove the equivalence result (Proposition 1), which we spell more precisely as follows:

Proposition 1 *The firm's and joint surplus problems are equivalent, in the following sense:*

1. *For any set of contracts $\vec{C}$ that solves problem (A.2)-(A.3a)-(A.3b), the subset*

$$\vec{C}_{\Sigma} \equiv \Big\{ v_i, \delta_{ij}, p, W'_{iL}(\vec{n}_{iL}^{+}, z) \Big\}_{(i,j) \in \mathcal{I} \times \mathcal{J}} \subset \vec{C}$$

   solves problem (6).

2. *Conversely, if $\vec{C}_{\Sigma}$ constitutes a solution to problem (6), then there exists a unique set of wages and continuation promises $\vec{C}_W \equiv \{ w_{ij}, W'_{ij}(\vec{n}_{ij}^{-}, z), W'_{ij}(\vec{n}^{p}, z), W'_{ij}(\vec{n}^{\tau}, z), \{ W'_{ij}(\vec{n}, z') \}_{z' \in \mathcal{Z}} \}_{(i,j) \in \mathcal{I} \times \mathcal{J}}$ such that $\vec{C}_W \cup \vec{C}_{\Sigma}$ solves*

problem (A.2)-(A.3a)-(A.3b).

Proof of Proposition 1 Let $\vec{C} = \{\vec{C}_{ij}\}$, with $\vec{C}_{ij} = \left\{\overline{w}_{ij}, \overline{v}_i, \overline{\delta}_{ij}, \overline{p}, \overline{W}'_{ij}(\vec{n}', z')\right\}$, denote a given policy. Then, we can re-write the problem of a type-φ firm in (A.2)-(A.3a)-(A.3b) as follows:

$$\mathbf{J}(\vec{n}, z, \varphi, \vec{W}) = \max_{\vec{C}} \widetilde{\mathbf{J}}\left(\vec{n}, z, \varphi, \vec{W} \,\middle|\, \vec{C}\right), \quad \text{subject to} \begin{cases} \mathbf{W}_{ij}\left(\vec{n}, z, \varphi; \vec{C}\right) \geq W_{ij}, & \forall (i,j) \\ \overline{W}'_{ij}(\vec{n}', z') \geq \mathbf{U}, & \forall (\vec{n}', z'), \ \forall (i,j) \end{cases}$$

where:

$$\begin{aligned} \widetilde{\mathbf{J}}\left(\vec{n}, z, \varphi, \vec{W} \,\middle|\, \vec{C}\right) \equiv \frac{1}{\overline{\rho}\left(\vec{n}, z, \varphi | \vec{C}\right)} \Bigg\{ & y(\vec{n}, z, \varphi) - \xi n_{FT} \overline{p}^{\vartheta} + \sum_{i \in \mathcal{I}} \Bigg[- A_i \overline{v}_i^{\varsigma} + \sum_{j \in \mathcal{J}} \Bigg(- \overline{w}_{ij} n_{ij} - \chi n_{ij} \overline{\delta}_{ij}^{\psi} \\ & + n_{ij} \left(\overline{\delta}_{ij} + s_i^W\right) \mathbf{J}(\vec{n}_{ij}^-, z, \varphi, \vec{W}'(\vec{n}_{ij}^-, z)) \Bigg) + \overline{v}_i \eta \left(\overline{W}'_{iL}(\vec{n}_{iL}^+, z)\right) \mathbf{J}(\vec{n}_{iL}^+, z, \varphi, \vec{W}'(\vec{n}_{iL}^+, z)) \Bigg] \\ & + \overline{p} n_{FT} \mathbf{J}(\vec{n}^p, z, \varphi, \vec{W}'(\vec{n}^p, z)) + n_{OEL} \tau \, \mathbf{J}(\vec{n}^{\tau}, z, \varphi, \vec{W}'(\vec{n}^{\tau}, z)) \\ & + \sum_{z' \in \mathcal{Z}} \lambda(z'|z) \mathbf{J}(\vec{n}, z', \varphi, \vec{W}'(\vec{n}, z')) \Bigg\} \end{aligned} \tag{A.4}$$

and where we have defined:

$$\overline{\rho}\left(\vec{n}, z, \varphi | \vec{C}\right) \equiv \rho + s^F + n_{FT} \overline{p} + n_{OEL} \tau + \sum_{i \in \mathcal{I}} \left[\overline{v}_i \eta \left(\overline{W}'_{iL}(\vec{n}_{iL}^+, z)\right) + \sum_{j \in \mathcal{J}} n_{ij} \left(\overline{\delta}_{ij} + s_i^W\right) \right]$$

as the effective discount rate of the firm. By monotonicity of preferences, if $\vec{C}$ is optimal, then the promise-keeping constraint (A.3a) must hold with equality: $\mathbf{W}_{ij}\left(\vec{n}, z, \varphi; \vec{C}\right) = W_{ij}$, $\forall (i,j) \in \mathcal{I} \times \mathcal{J}$.[47] Imposing this into equation (A.1) allows us to solve for wages:

$$\begin{aligned} \overline{w}_{ij} = \overline{\rho}\left(\vec{n}, z, \varphi | \vec{C}\right) W_{ij} - \Bigg[& (\overline{\delta}_{ij} + s_i^W + s^F) \mathbf{U} + (n_{ij} - 1) \left(\overline{\delta}_{ij} + s_i^W\right) \overline{W}'_{ij}(\vec{n}_{ij}^-, z) + \sum_{(i',j') \neq (i,j)} n_{i'j'} (\overline{\delta}_{i'j'} + s_{i'}^W) \overline{W}'_{ij}(\vec{n}_{i'j'}^-, z) \\ & + n_{FT} \overline{p} \left(\mathbf{1}[i = FT] \frac{\overline{W}'_{OEL}(\vec{n}^p, z) + (n_{FT} - 1) \overline{W}'_{FT}(\vec{n}^p, z)}{n_{FT}} + \mathbf{1}[i = OE] \overline{W}'_{OE,j}(\vec{n}^p, z) \right) \\ & + n_{OEL} \tau \left(\mathbf{1}[(i,j) = (OE, L)] \frac{\overline{W}'_{OEH}(\vec{n}^{\tau}, z) + (n_{OEL} - 1) \overline{W}'_{OEL}(\vec{n}^{\tau}, z)}{n_{OEL}} + \mathbf{1}[(i,j) \neq (OE, L)] \overline{W}'_{ij}(\vec{n}^{\tau}, z) \right) \\ & + \sum_{i' \in \mathcal{I}} \overline{v}_{i'} \eta \left(\overline{W}'_{i'L}(\vec{n}_{i'L}^+, z)\right) \overline{W}'_{ij}(\vec{n}_{i'L}^+, z) + \sum_{z' \in \mathcal{Z}} \lambda(z'|z) \overline{W}'_{ij}(\vec{n}, z') \Bigg] \end{aligned} \tag{A.5}$$

[47] Suppose not. Then, the firm could increase its value by offering a combination of flow and continuation payoffs to the workers that would yield lower value to them and still comply with the firm's initial promises, a contradiction with $\vec{C}$ being optimal.

where $\mathbf{1}[\cdot]$'s are indicator functions. Define the joint surplus under policy $\vec{C}$ as:

$$\widetilde{\mathbf{\Sigma}}\left(\vec{n}, z, \varphi, \vec{W} \middle| \vec{C}\right) \equiv \widetilde{\mathbf{J}}\left(\vec{n}, z, \varphi, \vec{W} \middle| \vec{C}\right) + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} n_{ij} W_{ij} \tag{A.6}$$

Likewise, define the *maximized* joint surplus as:

$$\mathbf{\Sigma}(\vec{n}, z, \varphi, \vec{W}) \equiv \max_{\vec{C}} \left\{ \widetilde{\mathbf{\Sigma}}\left(\vec{n}, z, \varphi, \vec{W} \middle| \vec{C}\right), \text{ such that } \overline{W}'_{ij}(\vec{n}', z') \geq \mathbf{U}, \ \forall(\vec{n}', z'), \ \forall(i,j) \right\} \tag{A.7}$$

Plugging (A.5) inside (A.4) and collecting terms:

$$\begin{aligned}
\underbrace{\widetilde{\mathbf{J}}\left(\vec{n}, z, \varphi, \vec{W} \;\middle|\; \vec{C}\right) + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} n_{ij} W_{ij}}_{=\,\widetilde{\mathbf{\Sigma}}\left(\vec{n},z,\varphi,\vec{W}|\vec{C}\right)} = \frac{1}{\overline{\rho}\left(\vec{n}, z, \varphi | \vec{C}\right)} \Bigg\{ & y(\vec{n}, z, \varphi) - \xi n_{FT} \overline{p}^{\vartheta} - \sum_{i\in\mathcal{I}} A_i \overline{v}_i \varsigma \\
& + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} \left(n_{ij}(\overline{\delta}_{ij} + s_i^W + s^F)\mathbf{U} - \chi n_{ij} \overline{\delta}_{ij}^{\psi} \right) \\
& + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} n_{ij}(\overline{\delta}_{ij} + s_i^W) \underbrace{\left(\mathbf{J}(\vec{n}_{ij}^-, z, \varphi, \vec{W}'(\vec{n}_{ij}^-, z)) + (n_{ij} - 1)\overline{W}'_{ij}(\vec{n}_{ij}^-, z) + \sum_{(i',j')\neq(i,j)} n_{i'j'} \overline{W}'_{i'j'}(\vec{n}_{i'j'}^-, z) \right)}_{=\,\mathbf{\Sigma}\left(\vec{n}_{ij}^-, z, \varphi, \vec{W}'(\vec{n}_{ij}^-, z)\right)} \\
& + \underbrace{\sum_{i\in\mathcal{I}} \left[\sum_{j\in\mathcal{J}} n_{ij} \left(\sum_{i'\in\mathcal{I}} \overline{v}_{i'} \eta\left(\overline{W}'_{i'L}(\vec{n}_{i'L}^+, z)\right) \overline{W}'_{ij}(\vec{n}_{i'L}^+, z) \right) + \overline{v}_i \eta\left(\overline{W}'_{iL}(\vec{n}_{iL}^+, z)\right) \mathbf{J}(\vec{n}_{iL}^+, z, \varphi, \vec{W}'(\vec{n}_{iL}^+, z)) \right]}_{[*]} \\
& + n_{FT} \overline{p} \underbrace{\left(\mathbf{J}(\vec{n}^p, z, \varphi, \vec{W}'(\vec{n}^p, z)) + (n_{FT} - 1)\overline{W}'_{FT}(\vec{n}^p, z) + (n_{OEL} + 1)\overline{W}'_{OEL}(\vec{n}^p, z) \right)}_{=\,\mathbf{\Sigma}\left(\vec{n}^p, z, \varphi, \vec{W}'(\vec{n}^p, z)\right)} \\
& + n_{OEL} \tau \underbrace{\left(\mathbf{J}(\vec{n}^\tau, z, \varphi, \vec{W}'(\vec{n}^\tau, z)) + (n_{OEL} - 1)\overline{W}'_{OEL}(\vec{n}^\tau, z) + (n_{OEH} + 1)\overline{W}'_{OEH}(\vec{n}^\tau, z) \right)}_{=\,\mathbf{\Sigma}\left(\vec{n}^\tau, z, \varphi, \vec{W}'(\vec{n}^\tau, z)\right)} \\
& + \sum_{z'\in\mathcal{Z}} \lambda(z'|z) \underbrace{\left(\mathbf{J}(\vec{n}, z', \varphi, \vec{W}'(\vec{n}, z')) + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} n_{ij} \overline{W}'_{ij}(\vec{n}, z') \right)}_{=\,\mathbf{\Sigma}\left(\vec{n}, z', \varphi, \vec{W}'(\vec{n}, z')\right)} \Bigg\}
\end{aligned} \tag{A.8}$$

where the terms in blue follow from definitions (A.6) and (A.7). The term labeled $[*]$ can be written as follows:

$$[*] = \sum_{i\in\mathcal{I}} \left[\sum_{j\in\mathcal{J}} n_{ij} \left(\sum_{i'\in\mathcal{I}} \overline{v}_{i'} \eta\left(\overline{W}'_{i'L}(\vec{n}_{i'L}^+, z)\right) \overline{W}'_{ij}(\vec{n}_{i'L}^+, z) \right) + \overline{v}_i \eta\left(\overline{W}'_{iL}(\vec{n}_{iL}^+, z)\right) \mathbf{J}(\vec{n}_{iL}^+, z, \varphi, \vec{W}'(\vec{n}_{iL}^+, z)) \right]$$

$$= \sum_{i\in\mathcal{I}} \overline{v}_i \eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL}, z)\right)\left[\mathbf{J}(\vec{n}^+_{iL}, z, \varphi, \vec{W}'(\vec{n}^+_{iL}, z)) + n_{iL}\overline{W}'_{iL}(\vec{n}^+_{iL}, z) + \sum_{(i',j')\neq(i,L)} n_{i'j'}\overline{W}'_{i'j'}(\vec{n}^+_{iL}, z)\right]$$

$$= \sum_{i\in\mathcal{I}} \overline{v}_i \eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL}, z)\right)\left[\mathbf{J}(\vec{n}^+_{iL}, z, \varphi, \vec{W}'(\vec{n}^+_{iL}, z)) + (n_{iL}+1)\overline{W}'_{iL}(\vec{n}^+_{iL}, z) + \sum_{(i',j')\neq(i,L)} n_{i'j'}\overline{W}'_{i'j'}(\vec{n}^+_{iL}, z) - \overline{W}'_{iL}(\vec{n}^+_{iL}, z)\right]$$

$$= \sum_{i\in\mathcal{I}} \overline{v}_i \eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL}, z)\right)\mathbf{\Sigma}\left(\vec{n}^+_{iL}, z, \varphi, \vec{W}'(\vec{n}^+_{iL}, z)\right) - \sum_{i\in\mathcal{I}} \overline{v}_i \eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL}, z)\right)\overline{W}'_{iL}(\vec{n}^+_{iL}, z)$$

Putting this back into equation (A.8), we find:

$$\begin{aligned}\widetilde{\mathbf{\Sigma}}\left(\vec{n}, z, \varphi, \vec{W}\middle|\vec{C}\right) \;=\; \frac{1}{\overline{\rho}\left(\vec{n}, z, \varphi|\vec{C}\right)}\Bigg\{ & y(\vec{n}, z, \varphi) - \xi n_{FT}\overline{p}^{\vartheta} - \sum_{i\in\mathcal{I}} A_i \overline{v}_i^{\varsigma} + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\Bigg(n_{ij}(\overline{\delta}_{ij} + s_i^W + s^F)\mathbf{U} - \chi n_{ij}\overline{\delta}_{ij}^{\psi} \\ & - \overline{v}_i\eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL}, z)\right)\overline{W}'_{iL}(\vec{n}^+_{iL}, z) + n_{ij}(\overline{\delta}_{ij} + s_i^W)\mathbf{\Sigma}\left(\vec{n}^-_{ij}, z, \varphi, \vec{W}'(\vec{n}^-_{ij}, z)\right) \\ & + \overline{v}_i\eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL}, z)\right)\mathbf{\Sigma}\left(\vec{n}^+_{iL}, z, \varphi, \vec{W}'(\vec{n}^+_{iL}, z)\right)\Bigg) + n_{FT}\overline{p}\mathbf{\Sigma}(\vec{n}^p, z, \varphi, \vec{W}'(\vec{n}^p, z)) \\ & + n_{OEL}\tau\mathbf{\Sigma}(\vec{n}^\tau, z, \varphi, \vec{W}'(\vec{n}^\tau, z)) + \sum_{z'\in\mathcal{Z}}\lambda(z'|z)\mathbf{\Sigma}(\vec{n}, z', \varphi, \vec{W}'(\vec{n}, z'))\Bigg\}\end{aligned} \tag{A.9}$$

Note that the right-hand side of (A.9) is independent of $\vec{W}$ and w, so we can omit these as arguments of the $\mathbf{\Sigma}$ function in equation (A.9), and further simplify the equation into:

$$\begin{aligned}\widetilde{\mathbf{\Sigma}}\left(\vec{n}, z, \varphi\middle|\vec{C}_\Sigma\right) \;=\; \frac{1}{\overline{\rho}\left(\vec{n}, z, \varphi|\vec{C}\right)}\Bigg\{ & y(\vec{n}, z, \varphi) - \xi n_{FT}\overline{p}^{\vartheta} - \sum_{i\in\mathcal{I}} A_i \overline{v}_i^{\varsigma} + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\Bigg(n_{ij}(\overline{\delta}_{ij} + s_i^W + s^F)\mathbf{U} - \chi n_{ij}\overline{\delta}_{ij}^{\psi} \\ & - \overline{v}_i\eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL}, z)\right)\overline{W}'_{iL}(\vec{n}^+_{iL}, z) + n_{ij}(\overline{\delta}_{ij} + s_i^W)\mathbf{\Sigma}\left(\vec{n}^-_{ij}, z, \varphi\right) + \overline{v}_i\eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL}, z)\right)\mathbf{\Sigma}\left(\vec{n}^+_{iL}, z, \varphi\right)\Bigg) \\ & + n_{FT}\overline{p}\mathbf{\Sigma}(\vec{n}^p, z, \varphi) + n_{OEL}\tau\mathbf{\Sigma}(\vec{n}^\tau, z, \varphi) + \sum_{z'\in\mathcal{Z}}\lambda(z'|z)\mathbf{\Sigma}(\vec{n}, z', \varphi)\Bigg\}\end{aligned} \tag{A.10}$$

Thus, out of the full set $\vec{C} = \left\{\overline{w}_{ij}, \overline{v}_i, \overline{\delta}_{ij}, \overline{p}, \overline{W}'_{ij}(\vec{n}', z')\right\}_{(i,j)\in\mathcal{I}\times\mathcal{J}}$, only the elements

$$\vec{C}_\Sigma \equiv \left\{\overline{v}_i, \overline{\delta}_{ij}, \overline{p}, \overline{W}'_{iL}(\vec{n}^+_{iL}, z)\right\}_{(i,j)\in\mathcal{I}\times\mathcal{J}} \subset \vec{C}$$

matter for the joint surplus. The optimal contract is then:

$$\vec{C}^*_\Sigma = \arg\max_{\vec{C}_\Sigma}\left\{\widetilde{\mathbf{\Sigma}}\left(\vec{n}, z, \varphi\middle|\vec{C}_\Sigma\right) \text{ s.t. } W'_{iL}(\vec{n}^+_{iL}, z) \geq \mathbf{U},\ \forall i \in \mathcal{I}\right\} \tag{A.11}$$

Wages $\{\overline{w}_{ij}\}_{(i,j)\in\mathcal{I}\times\mathcal{J}}$ are given by equation (A.5), while the remaining continuation values are chosen to be consistent with the solution to problem (A.11). Summing up: by expressing the firm's problem in terms of continuation promises, we have shown that the optimal contract must maximize the joint surplus. Conversely, for any combination of continuation promises that maximizes the joint surplus, there is a unique wage and set of outstanding promises that maximizes the firm's value subject to the promise-keeping constraint. ■

A.2 Equilibrium Distributions of Firms and Workers

A.2.1 Preliminaries

Let $\mathtt{f}_t(\vec{n}, z, \varphi) > 0$ be the measure of type-φ firms with employment vector $\vec{n} = \{n_{ij}\} \in \mathcal{N}$ and idiosyncratic productivity $z \in \mathcal{Z}$ at time $t \in \mathbb{R}_+$. Further, let $\mathtt{F}_t \equiv \sum_{\vec{n}\in\mathcal{N}} \sum_{z\in\mathcal{Z}} \sum_{\varphi\in\Phi} \mathtt{f}_t(\vec{n}, z, \varphi)$ be the total measure of operating firms, and let $\mathtt{F}^e_t > 0$ be the measure of potential entrants. Both $\mathtt{F}_t$ and $\mathtt{F}^e_t$ are endogenous objects, and are determined in equilibrium to be consistent with the sorting patterns of firms and workers.

Firms in state $(\vec{n}, z, \varphi)$ seek to hire new workers of type (i, L) in market segment $W'_{iL}(\vec{n}^+_{iL}, z, \varphi)$ for each contract $i \in \mathcal{I}$. Let $\theta_i(\vec{n}^+_{iL}, z, \varphi) \equiv \theta\Big(W'_{iL}(\vec{n}^+_{iL}, z, \varphi)\Big)$ denote the equilibrium tightness in the submarket where firms of type $(\vec{n}, z, \varphi)$ look to hire an additional (low-skill) worker under contract i. By equation (4), we have

$$\theta_i(\vec{n}^+_{iL}, z, \varphi) = \mu^{-1}\left(\frac{\rho\mathbf{U} - b}{W'_{iL}(\vec{n}^+_{iL}, z, \varphi) - \mathbf{U}}\right) \tag{A.12}$$

By definition, market tightness must adjust to guarantee that:

$$\mathtt{u}_{it}(\vec{n}^+_{iL}, z, \varphi)\theta_i(\vec{n}^+_{iL}, z, \varphi) = v_i(\vec{n}, z, \varphi)\mathtt{f}_t(\vec{n}, z, \varphi) \tag{A.13}$$

for all t, where $\mathtt{u}_{it}(\vec{n}^+_{iL}, z, \varphi)$ is the measure of unemployed workers looking to be hired in a type-i contract by a firm in state $(\vec{n}, z, \varphi)$, and $v_i(\vec{n}, z, \varphi)$ denotes this firm's recruiting effort.[48] As the job-filling rate for an individual firm in state $(\vec{n}, z, \varphi)$ is $v_i(\vec{n}, z, \varphi)\eta\left(\theta_i(\vec{n}^+_{iL}, z, \varphi)\right)$, and there exist $\mathtt{f}_t(\vec{n}, z, \varphi)$ firms with this identical job-filling rate, equation (A.13) simply states that, in equilibrium, the total number of vacancies created by firms in a given state equals the total number of vacancies found by workers in the corresponding submarket, or

$$\underbrace{\mu\Big(\theta_i(\vec{n}^+_{iL}, z, \varphi)\Big)}_{\substack{\text{Job-finding rate}\\ \text{per worker}}} \underbrace{\mathtt{u}_{it}(\vec{n}^+_{iL}, z, \varphi)}_{\text{\# job seekers}} = \underbrace{v_i(\vec{n}, z, \varphi)\eta\Big(\theta_i(\vec{n}^+_{iL}, z, \varphi)\Big)}_{\text{Job-filling rate per firm}} \underbrace{\mathtt{f}_t(\vec{n}, z, \varphi)}_{\text{\# firms}} \tag{A.14}$$

Next, denote by $\mathtt{e}_{ij,t}(\vec{n}, z, \varphi)$ the measure of workers of type (i, j) employed by firms of type $(\vec{n}, z, \varphi)$ at time t. The assignment of workers to firms satisfies:

$$\mathtt{e}_{ij,t}(\vec{n}, z, \varphi) = n_{ij}\mathtt{f}_t(\vec{n}, z, \varphi) \tag{A.15}$$

for every state. The unit measure of workers must be either matched with a firm or unmatched and searching. This gives us a formula for the unemployment rate, $\mathtt{U}_t = 1 - \mathtt{E}_t$, where:

$$\mathtt{E}_t = \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} \sum_{z\in\mathcal{Z}} \sum_{\varphi\in\Phi} n_{ij}\mathtt{f}_t(\{n_{ij}\}, z, \varphi). \tag{A.16}$$

[48] Recall that each firm posts one vacancy of type i at each time t, so v_i can be interpreted both as the recruiting effort of an individual firm, as well as the *effective* measure of vacancies per firm.

A.2.2 Kolmogorov Forward Equations

Next, we write down the dynamics of the distribution of firms, which satisfy a set of Kolmogorov forward equations. The law of motion for the measure of firms in state $(\vec{n}, z, \varphi)$ is given by

$$\begin{aligned}\frac{\partial \mathtt{f}_t(\vec{n},z,\varphi)}{\partial t} + o(\vec{n},z,\varphi)\mathtt{f}_t(\vec{n},z,\varphi) = &\sum_{i\in\mathcal{I}} v_i(\vec{n}^-_{iL},z,\varphi)\eta\Big(\theta_i(\vec{n},z,\varphi)\Big)\mathtt{f}_t(\vec{n}^-_{iL},z,\varphi) \\ &+ \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}(n_{ij}+1)\Big(\delta_{ij}(\vec{n}^+_{ij},z,\varphi)+s^W_i\Big)\mathtt{f}_t(\vec{n}^+_{ij},z,\varphi) \\ &+ (n_{FT}+1)p(\vec{n}^-_p,z,\varphi)\mathtt{f}_t(\vec{n}^-_p,z,\varphi) \\ &+ (n_{OEL}+1)\tau\mathtt{f}_t(\vec{n}^-_\tau,z,\varphi) \\ &+ \sum_{\widehat{z}\neq z}\lambda(z|\widehat{z})\mathtt{f}_t(\vec{n},\widehat{z},\varphi), \end{aligned} \tag{A.17}$$

with $\theta_i(\cdot)$ given by equation (A.12), and

$$o(\vec{n},z,\varphi) \equiv s^F + \sum_{i\in\mathcal{I}} v_i(\vec{n},z,\varphi)\eta\Big(\theta_i(\vec{n}^+_{iL},z,\varphi)\Big) + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} n_{ij}\Big(\delta_{ij}(\vec{n},z,\varphi)+s^W_i\Big) + n_{FT}p(\vec{n},z,\varphi) + n_{OEL}\tau + \sum_{\widehat{z}\neq z}\lambda(\widehat{z}|z)$$

is the outflow rate. The right-hand side of equation (A.17) gives all the inflows into state $(\vec{n}, z, \varphi)$. Inflows come from firms with $\vec{n}^-_{iL} \in \{(n_{OEH}, n_{OEL}, n_{FT}-1), (n_{OEH}, n_{OEL}-1, n_{FT})\}$ that hire a worker with contract i, firms with $\vec{n}^+_{ij} \equiv (n_{ij}+1, \vec{n}_{-(ij)})$ that fire a type-(i,j) worker, firms with $\vec{n}^-_p \equiv (n_{OEH}, n_{OEL}-1, n_{FT}+1)$ that promote an FT worker into an OEC, firms with $\vec{n}^-_\tau \equiv (n_{OEH}-1, n_{OEL}+1, n_{FT})$ for whom a low-skill OE worker has a skill upgrade, and firms at $\vec{n} = (n_{ij}, \vec{n}_{-(ij)})$ that transition into productivity z from some $\widehat{z} \neq z$.[49]

On the other hand, the dynamics of potential entrants are given by

$$\frac{\partial \mathtt{F}^e_t}{\partial t} + \mathtt{F}^e_t \sum_{\varphi\in\Phi}\sum_{z^e\in\mathcal{Z}}\pi_\varphi(\varphi)\pi_z(z^e)\sum_{i\in\mathcal{I}}\eta\Big(\theta_i(\vec{n}^e_{iL},z^e,\varphi)\Big) = s^F\mathtt{F}_t + \sum_{\varphi\in\Phi}\sum_{z\in\mathcal{Z}}\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\Big(\delta_{ij}(\vec{n}^e_{ij},z,\varphi)+s^W_i\Big)\mathtt{f}_t(\vec{n}^e_{ij},z,\varphi) \tag{A.18}$$

where, on the right-hand side, the first additive term are inflows from exiting firms and the second term are inflows from firms with $\vec{n}^e_{ij} \equiv (n_{ij}, \vec{n}_{-(ij)}) = (1, \vec{0})$ that lose their last worker. On the left-hand side, the second additive term collects outflows from successful entrants.

A.2.3 Finding the Stationary Distribution

To find the stationary distribution, we impose $\frac{\partial \mathtt{f}_t(\vec{n},z,\varphi)}{\partial t} = \frac{\partial \mathtt{F}^e_t}{\partial t} = 0$ into equations (A.17) and (A.18), and solve for the resulting system of equations. In the numerical implementation, we solve for the *share*, not the measure, of firms in each state $s \equiv (n_{OEH}, n_{OEL}, n_{FT}, z, \varphi)$. States take values in $\overline{\mathcal{N}}_{OEH} \times \overline{\mathcal{N}}_{OEL} \times \overline{\mathcal{N}}_{FT} \times \{z_1, \ldots, z_{k_z}\} \times \{\varphi_1, \varphi_2\}$, where we denote $\overline{\mathcal{N}}_{ij} \equiv \{0, 1, 2, \ldots, N_{ij}\}$, for some positive integers $\{N_{ij}\}$. Then, there is one inactive state, where $n_{OEH} = n_{OEL} = n_{FT} = 0$, which we label $s = 0$, and $S \equiv \big[(N_{OEH}+1)\cdot(N_{OEL}+1)\cdot(N_{FT}+1)-1\big]\cdot k_z \cdot 2$ active states. Denote by $\phi_s \in [0,1]$ the share of firms in state $s = 1, \ldots, S$, such that $\sum_{s=1}^S \phi_s = 1 - \phi_0$, where $\phi_0 > 0$ is the share of potential entrant firms. Stacking all of these shares into the

[49] Inflows from hiring must be multiplied by $\pi_z(z)\pi_\varphi(\varphi)$ whenever they come from successful entrants, i.e. for $(n_{iL}, n_H) = (1, 0)$, $\forall i$.

column vector $\vec{\phi} \equiv [\phi_0, \phi_1, \ldots, \phi_s]^\top$, we have a system of $S+1$ non-linear equations, which in matrix form reads:

$$\mathbf{G}^\top \vec{\phi} = \vec{0} \tag{A.19}$$

The object $\mathbf{G}$ is a $(S+1)$-dimensional infinitesimal generator matrix, which registers inflows in the diagonal and outflows in the off-diagonal, such that:

$$\mathbf{G} = \begin{pmatrix} -\sum_{s\neq 0} t_{0,s} & t_{0,1} & t_{0,2} & \ldots & t_{0,S} \\ t_{1,0} & -\sum_{s\neq 1} t_{1,s} & t_{1,2} & \ldots & t_{1,S} \\ t_{2,0} & t_{2,1} & -\sum_{s\neq 2} t_{2,s} & \ldots & t_{2,S} \\ \vdots & \vdots & \ddots & \vdots & \\ t_{S,0} & t_{S,1} & t_{S,2} & \ldots & -\sum_{i\neq S} t_{S,i} \end{pmatrix}$$

The transition rates $t_{s,s'}$ are built using the optimal policies, following the laws of motion stated in equations (A.17) and (A.18). To solve for $\vec{\phi}$, we write system (A.19) as $(\mathbf{G}^\top + \mathbf{I})\vec{\phi} = \vec{\phi}$, where $\mathbf{I}$ is a $(S+1)$-dimensional identity matrix. This means that $\vec{\phi}$ can be computed as the eigenvector of $\mathbf{G}^\top + \mathbf{I}$ that is associated with the unit eigenvalue. We exploit this fact to find our invariant distribution.

In our calibration, we have $S = 79,350$, so $\mathbf{G}$ and $\mathbf{I}$ are very large matrices. However, as the vast majority of transitions are illegal, $\mathbf{G}$ has many zero entries, so in practice we define $(\mathbf{G}, \mathbf{I})$ as sparse matrices to be able to save on computing time and memory.

A.2.4 Aggregate Measures of Agents

Having found the invariant distribution, we make the following normalization:

$$\widetilde{\mathtt{f}}_s \equiv \frac{\phi_s}{1-\phi_0}$$

for each state $s = 1, \ldots, S$ corresponding to a point $(n_{OEH}, n_{OEL}, n_{FT}, z, \varphi)$ in the state space. In words, $\widetilde{\mathtt{f}}_s$ is the share of firms in state s relative to all operating firms, i.e. $\widetilde{\mathtt{f}}(\vec{n}, z, \varphi) \equiv \mathtt{f}(\vec{n}, z, \varphi)/\mathtt{F}$. To proceed, we use identity (A.15) to compute:

$$\widetilde{\mathtt{e}}_{ij}(\vec{n}, z, \varphi) = n_{ij}\widetilde{\mathtt{f}}(\vec{n}, z, \varphi)$$

That is, $\widetilde{\mathtt{e}}_{ij} \equiv \mathtt{e}_{ij}/\mathtt{F}$ is the measure of workers of type (i,j) employed in firms of type $(\vec{n}, z, \varphi)$, as a share of the total measure of operating firms. From this we can find $\widetilde{\mathtt{E}}_{ij} \equiv \mathtt{E}_{ij}/\mathtt{F} = \sum_{\vec{n}\in\mathcal{N}} \sum_z \sum_\varphi \widetilde{\mathtt{e}}_{ij}(\vec{n}, z, \varphi)$, i.e. the total measure of employed individuals of type (i,j) per firm, and $\widetilde{\mathtt{E}} \equiv \mathtt{E}/\mathtt{F} = \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} \widetilde{\mathtt{E}}_{ij}$, i.e. the average firm size.

To save on notation, let $\mathtt{f}_i^v(\vec{n}, z, \varphi) \equiv v_i(\vec{n}, z, \varphi)\mathtt{f}(\vec{n}, z, \varphi)$ denote the aggregate measure of effective vacancies available for contract i in a given state $(\vec{n}, z, \varphi)$. Similarly, let $\widetilde{\mathtt{f}}_i^v(\vec{n}, z, \varphi) \equiv \mathtt{f}_i^v(\vec{n}, z, \varphi)/\mathtt{F}$. Using (A.13), we know:

$$\mathtt{u}_{OE}(n_{OEH}, n_{OEL} + 1, n_{FT}, z, \varphi) = \mathtt{F}\,\frac{\widetilde{\mathtt{f}}_{OE}^v(n_{OEH}, n_{OEL}, n_{FT}, z, \varphi)}{\theta_{OE}(n_{OEH}, n_{OEL} + 1, n_{FT}, z, \varphi)}$$
$$\mathtt{u}_{FT}(n_{OEH}, n_{OEL}, n_{FT} + 1, z, \varphi) = \mathtt{F}\,\frac{\widetilde{\mathtt{f}}_{FT}^v(n_{OEH}, n_{OEL}, n_{FT}, z, \varphi)}{\theta_{FT}(n_{OEH}, n_{OEL}, n_{FT} + 1, z, \varphi)}$$

To make progress, in the first equation we add up both sides across all $(n_{OEH}, n_{FT}, z, \varphi)$, as well as over $n_{OEL} = 1, \ldots, N_{OE}$ (i.e. omitting $n_{OEL} = 0$). Similarly, in the second equation we add across all $(n_{OEH}, n_{OEL}, z, \varphi)$, as well as over $n_{FT} = 1, \ldots, N_{FT}$ (i.e. omitting $n_{FT} = 0$). That is:

$$\sum_{n_{OEH}} \sum_{n_{OEL} \neq 0} \sum_{n_{FT}} \sum_{z} \sum_{\varphi} \mathtt{u}_{OE}(n_{OEH}, n_{OEL} + 1, n_{FT}, z, \varphi) = \mathtt{F} \left(\sum_{n_{OEH}} \sum_{n_{OEL} \neq 0} \sum_{n_{FT}} \sum_{z} \sum_{\varphi} \frac{\widetilde{\mathtt{f}}^{v}_{OE}(n_{OEH}, n_{OEL}, n_{FT}, z, \varphi)}{\theta_{OE}(n_{OEH}, n_{OEL} + 1, n_{FT}, z, \varphi)} \right) \tag{A.20}$$

$$\sum_{n_{OEH}} \sum_{n_{OEL}} \sum_{n_{FT} \neq 0} \sum_{z} \sum_{\varphi} \mathtt{u}_{FT}(n_{OEH}, n_{OEL}, n_{FT} + 1, z, \varphi) = \mathtt{F} \left(\sum_{n_{OEH}} \sum_{n_{OEL}} \sum_{n_{FT} \neq 0} \sum_{z} \sum_{\varphi} \frac{\widetilde{\mathtt{f}}^{v}_{FT}(n_{OEH}, n_{OEL}, n_{FT}, z, \varphi)}{\theta_{FT}(n_{OEH}, n_{OEL}, n_{FT} + 1, z, \varphi)} \right) \tag{A.21}$$

Developing the left-hand side of (A.20) and (A.21) yields:

$$\begin{aligned}
\sum_{n_{OEH}} \sum_{n_{OEL} \neq 0} \sum_{n_{FT}} \sum_{z} \sum_{\varphi} \mathtt{u}_{OE}(n_{OEH}, n_{OEL} + 1, n_{FT}, z, \varphi) &= \mathtt{U}_{OE} - \sum_{n_{OEH}} \sum_{n_{FT}} \sum_{z} \sum_{\varphi} \mathtt{u}_{OE}(n_{OEH}, 1, n_{FT}, z, \varphi) \\
&= \underbrace{1 - \mathtt{E} - \mathtt{U}_{FT}}_{=\mathtt{U}_{OE}} - \sum_{z} \sum_{\varphi} \left(\frac{\mathtt{F}^{e}}{\theta_{OE}(0,1,0,z,\varphi)} + \sum_{(n_{OEH}, n_{FT}) \neq (0,0)} \frac{\mathtt{f}^{v}_{OE}(n_{OEH}, 0, n_{FT}, z, \varphi)}{\theta_{OE}(n_{OEH}, 1, n_{FT}, z, \varphi)} \right) \\
&= 1 - \mathtt{U}_{FT} - \mathtt{F} \left[\widetilde{\mathtt{E}} + \sum_{z} \sum_{\varphi} \left(\frac{\widetilde{\mathtt{F}}^{e}}{\theta_{OE}(0,1,0,z,\varphi)} + \sum_{(n_{OEH}, n_{FT}) \neq (0,0)} \frac{\widetilde{\mathtt{f}}^{v}_{OE}(n_{OEH}, 0, n_{FT}, z, \varphi)}{\theta_{OE}(n_{OEH}, 1, n_{FT}, z, \varphi)} \right) \right]
\end{aligned}$$

and

$$\begin{aligned}
\sum_{n_{OEH}} \sum_{n_{OEL}} \sum_{n_{FT} \neq 0} \sum_{z} \sum_{\varphi} \mathtt{u}_{FT}(n_{OEH}, n_{OEL}, n_{FT} + 1, z, \varphi) &= \mathtt{U}_{FT} - \sum_{n_{OEH}} \sum_{n_{OEL}} \sum_{z} \sum_{\varphi} \mathtt{u}_{FT}(n_{OEH}, n_{OEL}, 1, z, \varphi) \\
&= \mathtt{U}_{FT} - \sum_{z} \sum_{\varphi} \left(\frac{\mathtt{F}^{e}}{\theta_{FT}(0,0,1,z,\varphi)} + \sum_{(n_{OEH}, n_{OEL}) \neq (0,0)} \frac{\mathtt{f}^{v}_{FT}(n_{OEH}, n_{OEL}, 0, z, \varphi)}{\theta_{FT}(n_{OEH}, n_{OEL}, 1, z, \varphi)} \right) \\
&= \mathtt{U}_{FT} - \mathtt{F} \left[\sum_{z} \sum_{\varphi} \left(\frac{\widetilde{\mathtt{F}}^{e}}{\theta_{FT}(0,0,1,z,\varphi)} + \sum_{(n_{OEH}, n_{OEL}) \neq (0,0)} \frac{\widetilde{\mathtt{f}}^{v}_{FT}(n_{OEH}, n_{OEL}, 0, z, \varphi)}{\theta_{FT}(n_{OEH}, n_{OEL}, 1, z, \varphi)} \right) \right]
\end{aligned}$$

respectively. Substituting these back into equations (A.20) and (A.21) yields:

$$\begin{aligned}
1 - \mathtt{U}_{FT} - \mathtt{F} &\left[\widetilde{\mathtt{E}} + \sum_{z \in \mathcal{Z}} \sum_{\varphi \in \Phi} \left(\frac{\widetilde{\mathtt{F}}^{e}}{\theta_{OE}(0,1,0,z,\varphi)} + \sum_{(n_{OEH}, n_{FT}) \neq (0,0)} \frac{\widetilde{\mathtt{f}}^{v}_{OE}(n_{OEH}, 0, n_{FT}, z, \varphi)}{\theta_{OE}(n_{OEH}, 1, n_{FT}, z, \varphi)} \right) \right] \\
&= \mathtt{F} \left(\sum_{n_{OEH}} \sum_{n_{OEL} \neq 0} \sum_{n_{FT}} \sum_{z \in \mathcal{Z}} \sum_{\varphi \in \Phi} \frac{\widetilde{\mathtt{f}}^{v}_{OE}(n_{OEH}, n_{OEL}, n_{FT}, z, \varphi)}{\theta_{OE}(n_{OEH}, n_{OEL} + 1, n_{FT}, z, \varphi)} \right)
\end{aligned}$$

and

$$\mathtt{U}_{FT} - \mathtt{F} \left[\sum_{z \in \mathcal{Z}} \sum_{\varphi \in \Phi} \left(\frac{\widetilde{\mathtt{F}}^{e}}{\theta_{FT}(0,0,1,z,\varphi)} + \sum_{(n_{OEH}, n_{OEL}) \neq (0,0)} \frac{\widetilde{\mathtt{f}}^{v}_{FT}(n_{OEH}, n_{OEL}, 0, z, \varphi)}{\theta_{FT}(n_{OEH}, n_{OEL}, 1, z, \varphi)} \right) \right]$$

$$= \mathtt{F}\left(\sum_{n_{OEH}}\sum_{n_{OEL}}\sum_{n_{FT}\neq 0}\sum_{z\in\mathcal{Z}}\sum_{\varphi\in\Phi}\frac{\widetilde{\mathtt{f}}^{v}_{FT}(n_{OEH},n_{OEL},n_{FT},z,\varphi)}{\theta_{FT}(n_{OEH},n_{OEL},n_{FT}+1,z,\varphi)}\right)$$

Solving for $\mathtt{F}$ on each equation yields:

$$\mathtt{F} = \frac{1-\mathtt{U}_{FT}}{\widetilde{\mathtt{E}}+\widetilde{\mathtt{U}}_{OE}} \qquad \text{and} \qquad \mathtt{F} = \frac{\mathtt{U}_{FT}}{\widetilde{\mathtt{U}}_{FT}} \tag{A.22}$$

respectively, where we have defined:

$$\widetilde{\mathtt{U}}_{OE} \equiv \sum_{z\in\mathcal{Z}}\sum_{\varphi\in\Phi}\Bigg[\frac{\widetilde{\mathtt{F}}^{e}}{\theta_{OE}(0,1,0,z,\varphi)} + \sum_{(n_{OEH},n_{FT})\neq(0,0)}\frac{\widetilde{\mathtt{f}}^{v}_{OE}(n_{OEH},0,n_{FT},z,\varphi)}{\theta_{OE}(n_{OEH},1,n_{FT},z,\varphi)} + \sum_{n_{OEH}}\sum_{n_{OEL}\neq 0}\sum_{n_{FT}}\frac{\widetilde{\mathtt{f}}^{v}_{OE}(n_{OEH},n_{OEL},n_{FT},z,\varphi)}{\theta_{OE}(n_{OEH},n_{OEL}+1,n_{FT},z,\varphi)}\Bigg]$$

and

$$\widetilde{\mathtt{U}}_{FT} \equiv \sum_{z\in\mathcal{Z}}\sum_{\varphi\in\Phi}\Bigg[\frac{\widetilde{\mathtt{F}}^{e}}{\theta_{FT}(0,0,1,z,\varphi)} + \sum_{(n_{OEH},n_{OEL})\neq(0,0)}\frac{\widetilde{\mathtt{f}}^{v}_{FT}(n_{OEH},n_{OEL},0,z,\varphi)}{\theta_{FT}(n_{OEH},n_{OEL},1,z,\varphi)} + \sum_{n_{OEH}}\sum_{n_{OEL}}\sum_{n_{FT}\neq 0}\frac{\widetilde{\mathtt{f}}^{v}_{FT}(n_{OEH},n_{OEL},n_{FT},z,\varphi)}{\theta_{FT}(n_{OEH},n_{OEL},n_{FT}+1,z,\varphi)}\Bigg]$$

Solving for $\mathtt{U}_{FT}$ from equation (A.22) gives

$$\mathtt{U}_{FT} = \frac{\widetilde{\mathtt{U}}_{FT}}{\widetilde{\mathtt{E}}+\widetilde{\mathtt{U}}_{OE}+\widetilde{\mathtt{U}}_{FT}} \tag{A.23}$$

From this, we can finally obtain the aggregate measure of operating firms as

$$\mathtt{F} = \left(\widetilde{\mathtt{E}}+\widetilde{\mathtt{U}}_{OE}+\widetilde{\mathtt{U}}_{FT}\right)^{-1} \tag{A.24}$$

Equation (A.24) reflects a feasibility condition: the sum of the measures of all employed and unemployed workers must equal one, the size of the labor force. Once we have the measure of operating firms $\mathtt{F}$, we can compute all the remaining aggregates. Namely, (i) the total measure of potential entrants is $\mathtt{F}^{e} = \phi_0\mathtt{F}$; (ii) the employment and unemployment rates are given by $\mathtt{E} = \widetilde{\mathtt{E}}\mathtt{F}$ and $\mathtt{U} = 1-\mathtt{E}$; (iii) the aggregate temporary share is given by $\mathtt{E}_{FT}/\mathtt{E}$; and (iv) the aggregate share of skilled workers (or "skill share") is $\mathtt{E}_{OEH}/\mathtt{E}$.

A.3 Cobb-Douglas Matching Function

Assume $\mathtt{M}(\mathtt{V},\mathtt{U}) = \mathtt{V}^{\gamma}\mathtt{U}^{1-\gamma}$. Using equation (8), some algebra shows:

$$W'_{iL}(\vec{n}^{+}_{iL},z,\varphi) = \gamma\mathbf{U} + (1-\gamma)\Big(\mathbf{\Sigma}(\vec{n}^{+}_{iL},z,\varphi) - \mathbf{\Sigma}(\vec{n},z,\varphi)\Big) \tag{A.25}$$

This expression is intuitive: the continuation promise $W'_{iL}(\vec{n}^{+}_{iL},z)$ for a new worker under contract i is a weighted average of the worker's outside option, $\mathbf{U}$, and the marginal net joint surplus gain from the hire,

$\mathbf{\Sigma}(\vec{n}^+_{iL}, z, \varphi) - \mathbf{\Sigma}(\vec{n}, z, \varphi)$. The object $(1-\gamma)$ gives the share of the overall gains (net of the outside option) that accrue to the new worker. On the other hand, and using the definition of joint surplus (equation (5)), the firm obtains the following change in value from hiring:

$$\mathbf{J}\Big(\vec{n}^+_{iL}, z, \varphi, \vec{W}'(\vec{n}^+_{iL}, z, \varphi)\Big) - \mathbf{J}\big(\vec{n}, z, \varphi, \vec{W}\big) = \underbrace{\gamma\Big(\mathbf{\Sigma}(\vec{n}^+_{iL}, z, \varphi) - \mathbf{\Sigma}(\vec{n}, z, \varphi) - \mathbf{U}\Big)}_{\text{New surplus, shared with new hire}} + \underbrace{\sum_{i' \in \mathcal{I}} \sum_{j \in \mathcal{J}} n_{i'j}\Big(W'_{i'j}(\vec{n}, z, \varphi) - W'_{i'j}(\vec{n}^+_{iL}, z, \varphi)\Big)}_{\text{Transfer of value between firm and pre-existing workers}} \tag{A.26}$$

The firm's marginal gain in value is composed of two terms. On the one hand, the firm absorbs the share γ of the total net gain in joint surplus that is not absorbed by the new hire. On the other hand, since we assume that all workers within the firm that are employed under the same contract must earn the same value, there must be a transfer of value between the firm and all its pre-existing workers (for *both* contract types) after hiring takes place. This is captured by the second additive term. Therefore, in equilibrium, firms must strike a balance between the surplus they extract from new hires and the surplus they extract from their pre-existing workers when a hire takes place.

The aggregate job-filling rate for type-φ firms in state $(\vec{n}, z)$ can then be written as follows:

$$\eta(\vec{n}, z, \varphi) = \left[(1-\gamma)\frac{\mathbf{\Sigma}(\vec{n}^+_{iL}, z, \varphi) - \mathbf{\Sigma}(\vec{n}, z, \varphi) - \mathbf{U}}{\rho\mathbf{U} - b}\right]^{\frac{1-\gamma}{\gamma}} \tag{A.27}$$

Given our choice of $\gamma = 0.5$ (discussed below), equation (A.27) implies that the job-filling rate is linear in the ratio of the marginal net gain in joint surplus from a new match that accrues to a worker to the expected value of that worker's search. Therefore, in our calibrated model, conditional on a recruiting policy firms can expect to find workers at a rate that is proportional to the returns that the job delivers ex-post.

Finally, with Cobb-Douglas matching the free-entry condition (equations (12)-(13)), written in terms of joint surplus, reads:

$$\kappa = \gamma\left(\frac{1-\gamma}{\rho\mathbf{U} - b}\right)^{\frac{1-\gamma}{\gamma}} \left[\sum_{\varphi \in \Phi} \pi_\varphi(\varphi) \sum_{z^e \in \mathcal{Z}} \pi_z(z^e) \sum_{i \in \mathcal{I}} \Big(\mathbf{\Sigma}(\vec{n}^e_{iL}, z^e, \varphi) - \mathbf{U}\Big)^{\frac{1}{\gamma}}\right] \tag{A.28}$$

Note that the expected value of a single-worker firm is monotonically decreasing in the value of unemployment, because for higher $\mathbf{U}$ both the job-filling rate and the ex-post gains in match surplus that accrue to the firm are lower. This ensures that the equilibrium value of unemployment, $\mathbf{U}$, is unique.

A.4 Worker Flow Rates

In the data We compute the EU and UE quarterly rates by type of contract using data from the *Encuesta de Población Activa* (EPA), the Labor Force Survey compiled by the *Instituto Nacional de Estadística* (INE), the Spanish national statistical agency. The data come at the quarterly frequency for the period 2006Q1-2019Q4. Denote by $UE^{i,\text{data}}_{t,t+1}$ the U-to-E flow from quarter t to $t+1$ into a contract of type $i = OE, FT$, and similarly for $EU^{i,\text{data}}_{t,t+1}$. E-to-E flows from an FTC into an OEC are denoted by $EE^{FtoO,\text{data}}_{t,t+1}$. Labor market rates are then defined as follows:

$$\widehat{UE}^{\text{data}}_{i,t} \equiv \frac{\sum UE^{i,\text{data}}_{t,t+1}}{\sum U^{\text{data}}_t} \qquad \text{and} \qquad \widehat{EU}^{\text{data}}_{i,t} \equiv \frac{\sum EU^{i,\text{data}}_{t,t+1}}{\sum E^{i,\text{data}}_t}$$

where $\sum$ denotes the sum of sample weights for all observations in that category, $\sum U^{\text{data}}_t$ is the number of unemployed at time t, and $\sum E^{i,\text{data}}_t$ is the number of employed in contract type i at time t. Similarly, the promotion rate in the data is computed as follows:

$$\widehat{EE}^{\text{data}}_{FtoO,t} \equiv \frac{\sum EE^{FtoO,\text{data}}_{t,t+1}}{\sum E^{FT,\text{data}}_t}$$

where $E^{FT,\text{data}}_t$ is the stock of FT workers in quarter t. The values reported in Table 4 are time-series averages over the HP-filtered quarterly series $\left\{\widehat{UE}^{\text{data}}_{i,t}, \widehat{EU}^{\text{data}}_{i,t}, \widehat{EE}^{\text{data}}_{FtoO,t}\right\}_{t=2005Q2}^{2019Q4}$.

In the model As we do not have empirical flows for within-firm skill upgrades, we categorize workers into four employment status: high-skill and low-skill employed with an OEC, employed with an FTC, and unemployed. The following set of equations describes flows between these states through the lens of the model:

$$\begin{aligned}
\frac{\partial \mathtt{E}_{OEH}}{\partial t} &= \sum_{\vec{n}}\sum_{z}\sum_{\varphi}\left\{\tau \mathtt{e}_{OEL}(\vec{n},z,\varphi) - \left(\delta_{OEH}(\vec{n},z,\varphi) + s^W_{OE} + s^F\right)\mathtt{e}_{OEH}(\vec{n},z,\varphi)\right\} \\
\frac{\partial \mathtt{E}_{OEL}}{\partial t} &= \sum_{\vec{n}}\sum_{z}\sum_{\varphi}\Bigg\{p(\vec{n},z,\varphi)\mathtt{e}_{FT}(\vec{n},z,\varphi) + \mu\left(\theta_{OE}(\vec{n}^+_{OEL},z,\varphi)\right)\mathtt{u}_{OE}(\vec{n}^+_{OEL},z,\varphi) \\
&\qquad - \left(\delta_{OEL}(\vec{n},z,\varphi) + s^W_{OE} + s^F\right)\mathtt{e}_{OEL}(\vec{n},z,\varphi) - \tau\mathtt{e}_{OEL}(\vec{n},z,\varphi)\Bigg\} \\
\frac{\partial \mathtt{E}_{FT}}{\partial t} &= \sum_{\vec{n}}\sum_{z}\sum_{\varphi}\Bigg\{\mu\left(\theta_{FT}(\vec{n}^+_{FT},z,\varphi)\right)\mathtt{u}_{FT}(\vec{n}^+_{FT},z,\varphi) \\
&\qquad - \left(\delta_{FT}(\vec{n},z,\varphi) + s^W_{FT} + s^F\right)\mathtt{e}_{FT}(\vec{n},z,\varphi) - p(\vec{n},z,\varphi)\mathtt{e}_{FT}(\vec{n},z,\varphi)\Bigg\} \\
\frac{\partial \mathtt{U}}{\partial t} &= \sum_{\vec{n}}\sum_{z}\sum_{\varphi}\Bigg\{\sum_{j=L,H}\left(\delta_{OE,j}(\vec{n},z,\varphi) + s^W_{OE} + s^F\right)\mathtt{e}_{OE,j}(\vec{n},z,\varphi) + \left(\delta_{FT}(\vec{n},z,\varphi) + s^W_{FT} + s^F\right)\mathtt{e}_{FT}(\vec{n},z,\varphi) \\
&\qquad - \sum_{i=OE,FT}\mu\left(\theta_i(\vec{n}^+_{iL},z,\varphi)\right)\mathtt{u}_i(\vec{n}^+_{iL},z,\varphi)\Bigg\}
\end{aligned}$$

Each of these equations collect inflows (terms with a positive sign) outflows (terms with a negative sign). A more compact way of writing the dynamical system above is:

$$\frac{\partial \mathtt{E}_{OEH}}{\partial t} = -\lambda_{EU_{OEH}}\mathtt{E}_{OEH} + \lambda_{EE_{LH}}\mathtt{E}_{OEL} \tag{A.29a}$$

$$\frac{\partial \mathtt{E}_{OEL}}{\partial t} = -\left(\lambda_{EU_{OEL}} + \lambda_{EE_{LH}}\right)\mathtt{E}_{OEL} + \lambda_{EE_{FtoO}}\mathtt{E}_{FT} + \lambda_{UE_{OE}}\mathtt{U} \tag{A.29b}$$

$$\frac{\partial \mathtt{E}_{FT}}{\partial t} = -\left(\lambda_{EU_{FT}} + \lambda_{EE_{FtoO}}\right)\mathtt{E}_{FT} + \lambda_{UE_{FT}}\mathtt{U} \tag{A.29c}$$

$$\frac{\partial \mathtt{U}}{\partial t} = \lambda_{EU_{OEH}}\mathtt{E}_{OEH} + \lambda_{EU_{OEL}}\mathtt{E}_{OEL} + \lambda_{EU_{FT}}\mathtt{E}_{FT} - \left(\lambda_{UE_{OE}} + \lambda_{UE_{FT}}\right)\mathtt{U} \tag{A.29d}$$

where we have defined the following average intensities:

$$\lambda_{EU_{OE,j}} \equiv \frac{EU_{OE,j}}{\mathtt{E}_{OE,j}} \qquad \lambda_{EE_{FtoO}} \equiv \frac{EE_{FtoO}}{\mathtt{E}_{FT}} \qquad \lambda_{UE_{OE}} \equiv \frac{UE_{OE}}{\mathtt{U}}$$

$$\lambda_{EU_{FT}} \equiv \frac{EU_{FT}}{\mathtt{E}_{FT}} \qquad \lambda_{UE_{FT}} \equiv \frac{UE_{FT}}{\mathtt{U}} \qquad \lambda_{EE_{LH}} \equiv \frac{EE_{LH}}{\mathtt{E}_{OEL}}$$

with

$$EU_{OE,j} \equiv \sum_{\vec{n}}\sum_{z}\sum_{\varphi}\left(\delta_{OE,j}(\vec{n},z,\varphi) + s^{W}_{OE} + s^{F}\right)\mathtt{e}_{OE,j}(\vec{n},z,\varphi) \qquad EE_{FtoO} \equiv \sum_{\vec{n}}\sum_{z}\sum_{\varphi} p(\vec{n},z,\varphi)\mathtt{e}_{FT}(\vec{n},z,\varphi)$$

$$UE_{OE} \equiv \sum_{\vec{n}}\sum_{z}\sum_{\varphi}\mu\left(\theta_{OE}(\vec{n}^{+}_{OEL},z,\varphi)\right)\mathtt{u}_{OE}(\vec{n}^{+}_{OEL},z,\varphi) \qquad EU_{FT} \equiv \sum_{\vec{n}}\sum_{z}\sum_{\varphi}\left(\delta_{FT}(\vec{n},z,\varphi) + s^{W}_{FT} + s^{F}\right)\mathtt{e}_{FT}(\vec{n},z,\varphi)$$

$$UE_{FT} \equiv \sum_{\vec{n}}\sum_{z}\sum_{\varphi}\mu\left(\theta_{FT}(\vec{n}^{+}_{FT},z,\varphi)\right)\mathtt{u}_{FT}(\vec{n}^{+}_{FT},z,\varphi) \qquad EE_{LH} \equiv \sum_{\vec{n}}\sum_{z}\sum_{\varphi}\tau\mathtt{e}_{OEL}(\vec{n},z,\varphi)$$

for $j = H, L$. Note, in particular, that since all firms face the same rate of skill upgrade, we have that $\lambda_{EE_{LH}} = \tau$. We can write system (A.29a)-(A.29d) in vector-matrix form as follows:

$$\frac{\partial}{\partial t}\begin{bmatrix}\mathtt{E}_{OEH}\\ \mathtt{E}_{OEL}\\ \mathtt{E}_{FT}\\ \mathtt{U}\end{bmatrix} = \begin{pmatrix} -\lambda_{EU_{OEH}} & \lambda_{EE_{LH}} & 0 & 0\\ 0 & -(\lambda_{EU_{OEL}} + \lambda_{EE_{LH}}) & \lambda_{EE_{FtoO}} & \lambda_{UE_{OE}}\\ 0 & 0 & -(\lambda_{EU_{FT}} + \lambda_{EE_{FtoO}}) & \lambda_{UE_{FT}}\\ \lambda_{EU_{OEH}} & \lambda_{EU_{OEL}} & \lambda_{EU_{FT}} & -(\lambda_{UE_{OE}} + \lambda_{EU_{FT}}) \end{pmatrix}\begin{bmatrix}\mathtt{E}_{OEH}\\ \mathtt{E}_{OEL}\\ \mathtt{E}_{FT}\\ \mathtt{U}\end{bmatrix}$$

Setting the right-hand side to the zero vector and solving the resulting system of linear equations will give us the stationary measures in the reduced-form model. Notice that, by construction, the resulting stocks coincide exactly, by construction, with the ones derived from firm-level flows in Online Appendix A.2.4.

Using these results, we can now construct flow rates. As the model is set in continuous time, we must produce discrete-time approximations in order to have numbers that can be compared to the ones from the quarterly data. For this, we compute for each contract type $i = OE, FT$:

$$\widehat{UE}_i^{\text{model}} = \frac{1 - e^{-UE_i dt}}{\mathtt{U}} \qquad \text{and} \qquad \widehat{EU}_i^{\text{model}} = \frac{1 - e^{-\sum_{j=L,H} EU_{ij} dt}}{\sum_{j=L,H}\mathtt{E}_{ij}}$$

In these ratios, in the numerator we have transformed instantaneous Poisson rates into quarterly probabilities by setting $dt = 1/4$.[50] For the overall UE and EU rates, we compute:

$$\widehat{UE}_{\text{total}}^{\text{model}} = \frac{1 - e^{-\sum_i UE_i dt}}{\mathtt{U}} \qquad \text{and} \qquad \widehat{EU}_{\text{total}}^{\text{model}} = \frac{1 - e^{-\sum_i\sum_j EU_{ij} dt}}{\sum_i\sum_j \mathtt{E}_{ij}}$$

[50] In particular, the numerator is the probability that there is *at least* one transition (i.e. one or more transitions) within a given quarter, which we compute as the complementary probability of no transitions.

Similarly, to obtain the promotion rate at the quarterly frequency in the model, we compute:

$$\widehat{EE}_{FtoO}^{\text{model}} = \frac{1 - e^{-EE_{FtoO}dt}}{\mathtt{E}_{FT}}$$

For estimation purposes, we treat $\widehat{UE}_i^{\text{model}}$, $\widehat{EU}_i^{\text{model}}$ and $\widehat{EE}_{FtoO}^{\text{model}}$ as the direct model counterparts of $\widehat{UE}_i^{\text{data}}$, $\widehat{EU}_i^{\text{data}}$ and $\widehat{EE}_{FtoO}^{\text{data}}$, respectively.

A.5 Aggregate Matching Function and Aggregate Matching Efficiency

This appendix shows that our model with recruiting intensity generates endogenous differences in aggregate matching efficiency between contracts.[51] Recall that a submarket is a pair (i, W), where $i \in \{FT, OE\}$ is the contract type and W is the promised utility. At any given time, a firm f posts one vacancy of each contract type i and chooses the intensity v_{if} with which to recruit into it. Assuming a Cobb-Douglas matching function, the number of matches in market (i, W) is $\mathtt{M}_i(W) \equiv \left(\mathtt{V}_i(W)\right)^{\gamma}\left(\mathtt{U}_i(W)\right)^{1-\gamma}$, where $\mathtt{U}_i(W)$ is the measure of unemployed workers seeking employment in submarket (i, W), and

$$\mathtt{V}_i(W) \equiv \int_{\Omega_i(W)} v_{if} \mathrm{d}f \tag{A.30}$$

is the effective measure of vacancies in submarket (i, W), where $\Omega_i(W)$ is the set of firms that offer value W for contract i. The total number of matches that occur for contract i, denoted $\mathtt{M}_i$, can then be found by aggregating $\mathtt{M}_i(W)$ across all promised values W. This can be written as

$$\mathtt{M}_i \equiv \int \mathtt{M}_i(W)\mathrm{d}W = \Gamma_i \mathtt{V}_i^{\gamma}\mathtt{U}_i^{1-\gamma}, \tag{A.31}$$

where $\mathtt{V}_i = \mathtt{F} + \mathtt{F}^e$ is the number of *actual* vacancies posted for contract i (equal to the sum of operating firms and potential entrants, as each such firm posts exactly one vacancy of each type), $\mathtt{U}_i \equiv \int \mathtt{U}_i(W)dW$ is the total number of job seekers for contract i, and Γ_i is the *aggregate matching efficiency* for contract i, defined by

$$\Gamma_i \equiv \int \left(\frac{\mathtt{V}_i(W)}{\mathtt{V}_i}\right)^{\gamma}\left(\frac{\mathtt{U}_i(W)}{\mathtt{U}_i}\right)^{1-\gamma} dW. \tag{A.32}$$

Notice that in a version of the model without endogenous recruiting intensity (e.g. $v_{if} = v$), differences in aggregate matching efficiency between markets would emerge only from the fact that firms and unemployed workers do not distribute uniformly across markets (as search is directed).[52] With recruiting intensity, however, there is an added component affecting matching efficiency, stemming from underlying differences in recruiting costs across contracts, which operates over and above differences in promised values.

To see how directed search and recruiting costs both generate differences in matching efficiency, note that the *aggregate* job-finding and job-filling rates in contract i can be written as

$$\frac{\mathtt{M}_i}{\mathtt{U}_i} = \Gamma_i\Theta_i^{\gamma} \qquad \text{and} \qquad \frac{\mathtt{M}_i}{\mathtt{V}_i} = \Gamma_i\Theta_i^{\gamma-1}, \tag{A.33}$$

[51] The intuitions are similar to those developed in Gavazza *et al.* (2018) and Carrillo-Tudela *et al.* (2023). Relative to the former, we explore aggregate matching intensity in the context of directed search. Relative to the latter, we introduce dual labor markets.

[52] Indeed, in such an environment, we would have $\Gamma_i = 1$ if search was random.

respectively, where $\Theta_i \equiv \mathtt{V}_i/\mathtt{U}_i$ is the *aggregate market tightness* for contracts of type i. Therefore, we can compute the aggregate UE rate (the fraction of the unemployed that find a job with contract i) by

$$UE_i \equiv \frac{\mathtt{M}_i}{\mathtt{U}} = \Gamma_i \Theta_i^{\gamma} \frac{\mathtt{U}_i}{\mathtt{U}}, \tag{A.34}$$

where $\mathtt{U} \equiv \sum_i \mathtt{U}_i$ is the total pool of unemployed workers. In the data, $UE_{FT} > UE_{OE}$. Note

$$\frac{UE_{FT}}{UE_{OE}} = \frac{\Gamma_{FT}}{\Gamma_{OE}} \left(\frac{\mathtt{U}_{FT}}{\mathtt{U}_{OE}}\right)^{1-\gamma}. \tag{A.35}$$

As OECs offer better deals ex-post, we have $\mathtt{U}_{FT} < \mathtt{U}_{OE}$. Therefore, in order to obtain $UE_{FT} > UE_{OE}$, it must be that $\Gamma_{FT} > \Gamma_{OE}$. In the calibration, this will be the result of recruiting costs being sufficiently low for FTCs compared to OECs, for given recruiting intensity (i.e. $A_{FT} < A_{OE}$).

In the stationary equilibrium of the model, all firms in a given state $(\vec{n}, z, \varphi)$ deliver the same promised value and choose identical recruiting effort $v_i(\vec{n}, z, \varphi)$. Using (A.13), this allows us to write equation (A.32) as

$$\Gamma_i = \sum_{\vec{n}} \sum_{z} \sum_{\varphi} \left[\underbrace{\frac{v_i(\vec{n}, z, \varphi)}{\mathtt{V}_i}}_{\text{Relative recruiting intensity}} \left(\underbrace{\frac{\theta_i(\vec{n}, z, \varphi)}{\Theta_i}}_{\text{Relative market tightness}} \right)^{1-\gamma} \mathtt{f}(\vec{n}, z, \varphi) \right]. \tag{A.36}$$

Equation (A.36) says that the aggregate matching efficiency in contract i is determined by two components: (i) the relative recruiting intensity in each market, and (ii) the relative degree of market tightness in each market (which results from the fact that search is directed). How these translate into matching efficiency depends on the distribution of firms $\{\mathtt{f}(\vec{n}, z, \varphi)\}$ and on the matching elasticity, γ.

A.6 Aggregate Welfare

We compute welfare in the stationary equilibrium as the present discounted sum of the value of production net of firing, recruiting, and promotion costs over all operating firms, plus the value of home production b across all unemployed workers, net of the total entry costs paid by all potential entrant firms, that is:

$$\mathscr{W} = \frac{1}{\rho} \Bigg\{ -\mathtt{F}^e \kappa + \mathtt{U} b + \sum_{\vec{n} \in \mathcal{N}} \sum_{\vec{z} \in \mathcal{Z}} \sum_{\varphi \in \Phi} \mathtt{f}(\vec{n}, z, \varphi) \times \dots \tag{A.37}$$

$$\dots \times \left[y(\vec{n}, z, \varphi) - \xi n_{FT} \left[p(\vec{n}, z, \varphi)\right]^{\vartheta} - \sum_{i \in \mathcal{I}} \left(A_i \left[v_i(\vec{n}, z, \varphi)\right]^{\varsigma} + \sum_{j \in \mathcal{J}} \chi n_{ij} \left[\delta_{ij}(\vec{n}, z, \varphi)\right]^{\psi} \right) \right] \Bigg\}$$

B Productivity and Misallocation

B.1 Output-Maximizing Benchmark Allocation of Workers to Firms

Consider being able to freely allocate workers to firms in order to maximize output without being constrained by the search frictions present in the competitive equilibrium, but taking as given the distribution of firms across productivity types (z, φ). In this section we show that, in such a benchmark allocation of workers, the marginal product of each type of labor is equalized across firms. Hence, the allocation of high and low skilled workers is identical across firms of the same productivity type, (z, φ).

We explore two cases under which workers are allocated to firms of different productivities: (i) a case in which the measures of employed workers by skill type are fixed but total employment is not (Section B.1.1); and (ii) a case in which the total measure of employed workers is fixed but its split between skill types is not (Section B.1.2). This latter case is the one we discuss in the main text, and below in Section B.2. Once again, in both cases we assume that $\mathtt{F}_{z,\varphi}$, the distribution of firms across productivity types (z, φ), is taken as given.

B.1.1 Benchmark Allocation of Workers when the Employment Measures by Skill are Fixed

For a given measure of operating firms $\mathtt{F}_{z,\varphi}$ and given measures of high skilled and low skilled workers $\mathtt{E}_H$ and $\mathtt{E}_H$, we can obtain the first-best allocation of workers $n_H^*(z, \varphi)$ and $n_L^*(z, \varphi)$ across skill types (z, φ) as the solution to the following problem:

$$\max_{\{n_H(z,\varphi), n_L(z,\varphi)\}} \sum_{z\in\mathcal{Z}} \sum_{\varphi\in\Phi} y\Big(n_H(z,\varphi), n_L(z,\varphi), z, \varphi\Big)\mathtt{F}_{z,\varphi} \qquad \text{s.t.} \begin{cases} \sum_{z\in\mathcal{Z}} \sum_{\varphi\in\Phi} n_H(z,\varphi)\mathtt{F}_{z,\varphi} = \mathtt{E}_H \\ \sum_{z\in\mathcal{Z}} \sum_{\varphi\in\Phi} n_L(z,\varphi)\mathtt{F}_{z,\varphi} = \mathtt{E}_L \end{cases}$$

The FOC are given by $\partial y(n_H, n_L, z, \varphi)/\partial n_H = \lambda_H$ and $\partial y(n_H, n_L, z, \varphi)/\partial n_L = \lambda_L$, where $\lambda_H \geq 0$ and $\lambda_L \geq 0$ are the Lagrange multipliers associated to each constraint. Combining these two equations gives us the ratio of each type of worker,

$$\frac{n_H(z,\varphi;\lambda_H,\lambda_L)}{n_L(z,\varphi;\lambda_H,\lambda_L)} = \left(\frac{\omega(\varphi)}{1-\omega(\varphi)}\frac{\lambda_L}{\lambda_H}\right)^{\frac{1}{1-\alpha}} \tag{B.1}$$

which varies by φ but not by z, and varies by the aggregate relative scarcity of each type of worker given by λ_L/λ_H. Hence, all firms of the same type φ obtain the same ratio of high and low skilled workers. Let $\mathtt{E}_H(\varphi) \equiv \sum_z n_H^*(z,\varphi)\mathtt{F}_{z,\varphi}$ and $\mathtt{E}_L(\varphi) \equiv \sum_z n_L^*(z,\varphi)\mathtt{F}_{z,\varphi}$ be the total measures of high- and low-skilled workers allocated to firms of type φ. Then, using equation (B.1), the skill ratio at firms of type φ is given by

$$\frac{\mathtt{E}_H(\varphi)}{\mathtt{E}_L(\varphi)} = \left(\frac{\omega(\varphi)}{1-\omega(\varphi)}\frac{\lambda_L}{\lambda_H}\right)^{\frac{1}{1-\alpha}}$$

and aggregating over firms of type φ it must be that

$$\frac{\mathtt{E}_H}{\mathtt{E}_L} = \sum_{\varphi\in\Phi} \frac{\mathtt{E}_H(\varphi)}{\mathtt{E}_L(\varphi)}\frac{\mathtt{F}_\varphi}{\mathtt{F}} = \sum_{\varphi\in\Phi} \left(\frac{\omega(\varphi)}{1-\omega(\varphi)}\frac{\lambda_L}{\lambda_H}\right)^{\frac{1}{1-\alpha}} \frac{\mathtt{F}_\varphi}{\mathtt{F}}$$

where $\mathtt{F}_\varphi \equiv \sum_z \mathtt{F}_{z,\varphi}$. Therefore,

$$\left(\frac{\lambda_H}{\lambda_L}\right)^{\frac{1}{1-\alpha}} = \frac{\mathtt{E}_L}{\mathtt{E}_H} \sum_{\varphi\in\Phi} \left(\frac{\omega(\varphi)}{1-\omega(\varphi)}\right)^{\frac{1}{1-\alpha}} \frac{\mathtt{F}_\varphi}{\mathtt{F}}$$

Hence, using equation (B.1):

$$n_{HL}^*(\varphi) \equiv \frac{n_H^*(z,\varphi)}{n_L^*(z,\varphi)} = \frac{\left(\frac{\omega(\varphi)}{1-\omega(\varphi)}\right)^{\frac{1}{1-\alpha}}}{\sum_{\varphi'\in\Phi}\left(\frac{\omega(\varphi')}{1-\omega(\varphi')}\right)^{\frac{1}{1-\alpha}}\frac{\mathtt{F}_{\varphi'}}{\mathtt{F}}}\left(\frac{\mathtt{E}_H}{\mathtt{E}_L}\right)$$

Therefore, the skill share is a function of a firm's permanent type, but not of its idiosyncratic productivity: $h^*(\varphi) \equiv \frac{n_H^*(z,\varphi)}{n_H^*(z,\varphi)+n_L^*(z,\varphi)} = \left[1+\frac{1}{n_{HL}^*(\varphi)}\right]^{-1}$. Next, using the FOC of say n_H we obtain the employment demand:

$$n(z,\varphi;\lambda_H) = \left[\frac{\nu}{\lambda_H}\omega(\varphi)\Big(h^*(\varphi)\Big)^{\alpha-1}\left(\omega(\varphi)\Big(h^*(\varphi)\Big)^{\alpha}+(1-\omega(\varphi))\Big(1-h^*(\varphi)\Big)^{\alpha}\right)^{\frac{\nu}{\alpha}-1}e^{z+\zeta(\varphi)}\right]^{\frac{1}{1-\nu}} \tag{B.2}$$

To characterize $n^*(z,\varphi)$ we need to get rid of λ_H. To do so, we aggregate equation (B.2):

$$\mathtt{E}_H+\mathtt{E}_L = \left(\frac{\nu}{\lambda_H}\right)^{\frac{1}{1-\nu}}\sum_{z\in\mathcal{Z}}\sum_{\varphi\in\Phi}\left[\omega(\varphi)\Big(h^*(\varphi)\Big)^{\alpha-1}\left(\omega(\varphi)\Big(h^*(\varphi)\Big)^{\alpha}+(1-\omega(\varphi))\Big(1-h^*(\varphi)\Big)^{\alpha}\right)^{\frac{\nu}{\alpha}-1}e^{z+\zeta(\varphi)}\right]^{\frac{1}{1-\nu}}\mathtt{F}_{z,\varphi}$$

which gives us the total employment of firm (z,φ):

$$n^*(z,\varphi) = \frac{(\mathtt{E}_H+\mathtt{E}_L)\left[\omega(\varphi)\Big(h^*(\varphi)\Big)^{\alpha-1}\left(\omega(\varphi)\Big(h^*(\varphi)\Big)^{\alpha}+(1-\omega(\varphi))\Big(1-h^*(\varphi)\Big)^{\alpha}\right)^{\frac{\nu}{\alpha}-1}e^{z+\zeta(\varphi)}\right]^{\frac{1}{1-\nu}}}{\sum_{z'\in\mathcal{Z}}\sum_{\varphi'\in\Phi}\left[\omega(\varphi')\Big(h^*(\varphi')\Big)^{\alpha-1}\left(\omega(\varphi')\Big(h^*(\varphi')\Big)^{\alpha}+(1-\omega(\varphi'))\Big(1-h^*(\varphi')\Big)^{\alpha}\right)^{\frac{\nu}{\alpha}-1}e^{z'+\zeta(\varphi')}\right]^{\frac{1}{1-\nu}}\mathtt{F}_{z',\varphi'}}$$

B.1.2 Benchmark Allocation of Workers when the Employment Measures by Skill are Not Fixed

We can also characterize the output-maximizing allocation when the measure of operating firms $\mathtt{F}_{z,\varphi}$ and total employment $\mathtt{E}$ are given, but the split of $\mathtt{E}$ into $\mathtt{E}_H$ and $\mathtt{E}_L$ is kept free. This would imply solving the problem:

$$\max_{\{n_H(z,\varphi),n_L(z,\varphi)\}}\sum_{z\in\mathcal{Z}}\sum_{\varphi\in\Phi}y(n_H,n_L,z,\varphi)\mathtt{F}_{z,\varphi} \qquad \text{s.t.} \sum_{z\in\mathcal{Z}}\sum_{\varphi\in\Phi}n_H(z,\varphi)\mathtt{F}_{z,\varphi}+\sum_{z\in\mathcal{Z}}\sum_{\varphi\in\Phi}n_L(z,\varphi)\mathtt{F}_{z,\varphi} = \mathtt{E}_H+\mathtt{E}_L$$

Compared to our case above, we now only have one constraint, and therefore a single Lagrange multiplier, λ. The ratio of FOCs already gives the ratio of skilled to unskilled workers:

$$\frac{n_H^{**}(z,\varphi)}{n_L^{**}(z,\varphi)} = \left(\frac{\omega(\varphi)}{1-\omega(\varphi)}\right)^{\frac{1}{1-\alpha}} \tag{B.3}$$

which again implies that the skill share is only a function of permanent productivity. Then, following the same derivations as above, we obtain the total employment of a firm of type (z, φ):

$$n^*(z,\varphi) = \frac{(\mathtt{E}_H + \mathtt{E}_L)\left[\omega(\varphi)\Big(h^*(\varphi)\Big)^{\alpha-1}\left(\omega(\varphi)\Big(h^*(\varphi)\Big)^{\alpha} + \big(1-\omega(\varphi)\big)\Big(1-h^*(\varphi)\Big)^{\alpha}\right)^{\frac{\nu}{\alpha}-1} e^{z+\zeta(\varphi)}\right]^{\frac{1}{1-\nu}}}{\sum_{z'\in\mathcal{Z}}\sum_{\varphi'\in\Phi}\left[\omega(\varphi')\Big(h^*(\varphi')\Big)^{\alpha-1}\left(\omega(\varphi')\Big(h^*(\varphi')\Big)^{\alpha} + \big(1-\omega(\varphi')\big)\Big(1-h^*(\varphi')\Big)^{\alpha}\right)^{\frac{\nu}{\alpha}-1} e^{z'+\zeta(\varphi')}\right]^{\frac{1}{1-\nu}} \mathtt{F}_{z',\varphi'}} \tag{B.4}$$

B.2 Misallocation

B.2.1 Misallocation in the Baseline Calibration

The top two panels of Figure E.6 describe the allocation of the total number of workers across firm types φ (left vs. right panels) and transitory productivity states z (different crosses within each panel) for the competitive equilibrium (henceforth, "CE") of the calibrated economy (black schedule) and the Benchmark (red schedule) allocations, respectively. We see that the CE allocates too few workers to the most productive firms: among the high types (φ_1), the market allocates an average of 20.5 workers to the highest productivity firms (z_5), while these same firms should employ on average 25.3 in an economy in which total output is maximized. This misallocation reflects the mean-reversion of productivity shocks and the rigidity of OECs. It is efficient for very productive type-φ_1 firms to employ many high-skilled workers. However, the good shocks may turn into bad shocks in the future, which in the CE would make it very hard for firms to get rid of redundant workers hired under OECs.

The bottom two panels in Figure E.6 describe the allocation of human capital (the ratio of high skilled workers to all workers) across firms. As discussed in the main text, the Benchmark's allocation is invariant in z and increases in φ. The CE fails to allocate the right amount of human capital across firms. In particular, while it does allocate more human capital to type-φ_1 firms, the allocation of human capital to productive type-φ_1 firms is way lower than the output-maximizing one. Type-φ_2 firms, by contrast, get a similar amount of human capital in both the CE and Benchmark. This uncovers and important static inefficiency of the menu of contracts available in the dual labor market economy. In particular, OE contacts are too rigid for type-φ_1 firms. For these firms, it would be efficient to face less worker turnover such that their employees have time to acquire more human capital. The tool to achieve this worker stability would be to employ more workers under OECs and less workers under FTCs. But OE workers are too rigid and expensive to undo whenever bad productivity shock arise, which leads to an inefficient provision of human capital.

B.2.2 Misallocation and Heterogeneous Effects in the Policy Counterfactual

Figure E.7 shows the percentage changes in total employment, the skill share, and the temporary share for firms of different permanent types and productivities when moving from the baseline calibration to the policy that limits the duration of FTCs to 1 month. When possible (i.e. for total employment and the skill share), we also show the policy response in the output-maximizing Benchmark economy (red lines).

First, we see that all low-type (φ_2) firms reduce employment. However, among them, highly productive (z_5) firms reduce it more than they should and unproductive (z_1) firms do not reduce it enough. In a nutshell, the patterns of between-firm misallocation that we identified in the top panels of Figure E.6 get exacerbated. Second, though all firms increase their share of skilled workers, the adjustment is much more dramatic among low-type firms. However, as we saw in the bottom panels of Figure E.6, it is precisely the high-type firms that are furthest from their optimal allocation of human capital. Therefore, though beneficial for the overall levels of skill in the economy, the policy fails to sufficiently reallocate high-skill workers into the right firms. Finally, consistent with these results, all firms reduce their share of temporary employment, although the decrease is particularly pronounced among low-type firms (whose levels of temporary employment are overall much higher).

This changes in the allocation of workers across and within firms are reflected in changes in aggregate misallocation (see Table 6). Our results reveal that, as FTC duration decreases, all sources of misallocation worsen: the output loss goes from 3.91% in the baseline to 4.29% in the short duration policy, and total output declines by 6.86%. Interestingly, the contribution to these output losses by between-firm misallocation decreases as FTCs become of shorter duration. Namely, a policy that would make FTCs shorter would increase all forms of employment misallocation, but would hurt particularly strongly the allocation of workers between firms of the same type. Therefore, the declining output losses that we see as FTC duration increases are due to a better allocation of workers between firms of the same type.

B.3 Productivity Decomposition

In order to decompose aggregate productivity into the different parts shown in equation (20), we first need to transform equation (19) in two steps.

First, note that the measure of firms in state (n_H, n_L, z, φ) can always be written as $\mathtt{f}(n_H, n_L, z, \varphi) = \mathtt{F}_{z,\varphi}\widetilde{\mathtt{f}}_{z,\varphi}(n_H, n_L)$, where $\widetilde{\mathtt{f}}_{z,\varphi}(n_H, n_L)$ denotes the share of operating firms of type (z, φ) with employment vector (n_H, n_L), and $\mathtt{F}_{z,\varphi} \equiv \sum_{n_H} \sum_{n_L} \mathtt{f}(n_H, n_L, z, \varphi)$ is the measure of firms in state (z, φ), so that $\sum_z \sum_\varphi \mathtt{F}_{z,\varphi} = \mathtt{F}$. Then, we can write equation (19) as:

$$\frac{Y}{\mathtt{E}} = \frac{\mathtt{F}}{\mathtt{E}} \left[\sum_{z \in \mathcal{Z}} \sum_{\varphi \in \Phi} \frac{\mathtt{F}_{z,\varphi}}{\mathtt{F}} \left(\sum_{n_H=0}^{+\infty} \sum_{n_L=0}^{+\infty} y(n_H, n_L, z, \varphi) \widetilde{\mathtt{f}}_{z,\varphi}(n_H, n_L) \right) \right]$$

Second, given that the production function $y(n_H, n_L, z, \varphi)$ is homogeneous of degree ν in n_H and n_L, we can rewrite it as a function of total employment, $n \equiv n_H + n_L$, and the skill share in the firm, $h \equiv n_H/n$, so that

$$y(n_H, n_L, z, \varphi) = \underbrace{n^\nu y(h, 1-h, z, \varphi)}_{\equiv y^h(n,h,z,\varphi)}$$

Notice that, because of the discreteness of the state space, h takes values in the subset of rational numbers contained in the unit interval: if $n = 1$, then $h \in \mathcal{H}_1 \equiv \{0, 1\}$; if $n = 2$, then $h \in \mathcal{H}_2 \equiv \{0, 1/2, 1\}$; if $n = 3$, then $h \in \mathcal{H}_3 \equiv \{0, 1/3, 2/3, 1\}$; and so on. In general, we have $h \in \mathcal{H} \equiv \bigcup_{n \in \mathcal{N}} \mathcal{H}_n = \mathbb{Q} \cap [0, 1]$, where $\mathbb{Q}$ is the set of all rational numbers.

To identify the size effect, instead of firm employment n we will use employment relative to average firm size, $\widehat{n} \equiv n/(\mathtt{E}/\mathtt{F})$, such that

$$y^h(n, h, z, \varphi) = \left(\frac{\mathtt{E}}{\mathtt{F}}\right)^\nu y^h_{z,\varphi}(\widehat{n}, h)$$

Notice that, for given $\mathtt{E}/\mathtt{F}$, normalized firm employment $\widehat{n}$ is a random variable with discrete support, denoted

$\widehat{\mathcal{N}}$. Given this, aggregate productivity $Y/\mathtt{E}$ can be written as:

$$\frac{Y}{\mathtt{E}} = \left(\frac{\mathtt{F}}{\mathtt{E}}\right)^{1-\nu} \left[\sum_{z\in\mathcal{Z}} \sum_{\varphi\in\Phi} \frac{\mathtt{F}_{z,\varphi}}{\mathtt{F}} \left(\sum_{\widehat{n}\in\widehat{\mathcal{N}}} \sum_{h\in\mathcal{H}} y^{h}_{z,\varphi}(\widehat{n},h)\mathtt{g}_{z,\varphi}(\widehat{n},h)\right)\right] \tag{B.5}$$

where

$$\mathtt{g}_{z,\varphi}(\widehat{n},h) \equiv \sum_{n_H=0}^{+\infty} \sum_{n_L=0}^{+\infty} \widetilde{\mathtt{f}}_{z,\varphi}(n_H,n_L)\, \mathbf{1}\left[\left(\frac{n_H+n_L}{\mathtt{E}/\mathtt{F}} = \widehat{n}\right) \wedge \left(\frac{n_H}{n_H+n_L} = h\right)\right] \tag{B.6}$$

is the share of firms of type (z,φ) that have relative size $\widehat{n}$ and skill share h. This is equivalent to the result shown in equation (20).

Finally, the joint probability $\mathtt{g}_{z,\varphi}(\widehat{n},h)$ can be written as:

$$\mathtt{g}_{z,\varphi}(\widehat{n},h) = \underbrace{\mathtt{g}^{A}_{z,\varphi}(h|\widehat{n})}_{\substack{\text{Within}\\ \text{component}}}\ \underbrace{\mathtt{g}^{B}_{z,\varphi}(\widehat{n})}_{\substack{\text{Between}\\ \text{component}}}$$

for some conditional probability mass function $\mathtt{g}^{A}_{z,\varphi}(h|\widehat{n})$ and some marginal probability mass function $\mathtt{g}^{B}_{z,\varphi}(\widehat{n})$. On the one hand, $\mathtt{g}^{A}_{z,\varphi}(h|\widehat{n})$ denotes the share of (z,φ) that have skill share h conditional on having relative size $\widehat{n}$, so its contribution quantifies the within-firm, across worker component of productivity. On the other hand, $\mathtt{g}^{B}_{z,\varphi}(\widehat{n})$ denotes the share of firms of type (z,φ) that are of relative size $\widehat{n}$, regardless of their skill share, so its contribution quantifies the between-firm component of productivity.

C Model Extension with Directed Entry

Using extreme value shocks, the directed entry problem in Section 5.1.4 obtains convenient closed-form solutions. In particular, let the shocks be distributed Gumbel, $\varepsilon_{\varphi} \sim G(\mu_{\varepsilon},\sigma_{\varepsilon})$. Then, one can obtain: (i) an expression for the value of entry (equation (22)) as

$$\mathbf{J}^{e} = \sigma_{\varepsilon} \ln\left(\sum_{\varphi\in\Phi} \exp\left(\frac{\mathbf{J}^{e}(\varphi)-\kappa(\varphi)}{\sigma_{\varepsilon}}\right)\right) + \mu_{\varepsilon} + \sigma_{\varepsilon}\gamma_{\varepsilon} \tag{C.1}$$

where γ_{ε} is Euler's constant, and (ii) an expression for the fraction of entrants of each type as

$$\pi_{\varphi}(\varphi) = \frac{\exp\left(\frac{\mathbf{J}^{e}(\varphi)-\kappa(\varphi)}{\sigma_{\varepsilon}}\right)}{\sum_{\varphi'\in\Phi} \exp\left(\frac{\mathbf{J}^{e}(\varphi')-\kappa(\varphi')}{\sigma_{\varepsilon}}\right)} \tag{C.2}$$

To understand how selection upon entry works, note that the fraction of firms choosing technology φ increases with the value $\mathbf{J}^{e}(\varphi)$ of entering with technology φ, and that how much selection changes with the change of value is decreasing in the variance parameter σ_{ε}. In particular, for the case with only two values of φ one can easily write,

$$\frac{d \ln \left(\frac{\pi_\varphi(\varphi_1)}{\pi_\varphi(\varphi_2)} \right)}{d \big(\mathbf{J}^e(\varphi_1) - \mathbf{J}^e(\varphi_2) \big)} = \frac{1}{\sigma_\varepsilon} \tag{C.3}$$

Thus, $1/\sigma_\varepsilon$ gives us the semi-elasticity of the odds ratio of entering as a φ_1 type to the gain in value of entry as a type φ_1. The *actual* entry elasticity is:

$$\mathcal{E}_{\text{entry}} \equiv \frac{d \ln \left(\frac{\pi_\varphi(\varphi_1)}{\pi_\varphi(\varphi_2)} \right)}{d \ln \left(\mathbf{J}^e(\varphi_1) - \mathbf{J}^e(\varphi_2) \right)} = \frac{\mathbf{J}^e(\varphi_1) - \mathbf{J}^e(\varphi_2)}{\sigma_\varepsilon} \tag{C.4}$$

To implement directed entry in practice, first we normalize the location parameter to $\mu_\varepsilon = -\sigma_\varepsilon \gamma_\varepsilon$. This is a way to undo the effect of σ_ε on the ex-ante value of entry, $\mathbf{J}^e$.[53] Second, to recover the $\kappa(\varphi_1)$ and $\kappa(\varphi_2)$ parameters, we use equation (C.1) and the free entry condition $\mathbf{J}^e = 0$ to write:

$$0 = \ln \left(\sum_{\varphi \in \Phi} \exp \left(\frac{\mathbf{J}^e(\varphi) - \kappa(\varphi)}{\sigma_\varepsilon} \right) \right)$$

Then using equation (C.2) for $\pi_\varphi(\varphi_1)$, we obtain:

$$\kappa(\varphi) = \mathbf{J}^e(\varphi) - \sigma_\varepsilon \ln \left(\pi_\varphi(\varphi) \right),$$

for $\varphi \in \{\varphi_1, \varphi_2\}$. That is, the average cost of entry of each technology is given by the value of entry of that technology (which can be computed with equation (13)), the parameter σ_ε, and the calibrated entry fraction of firms of each type.

While the model with directed entry calibrated as explained above delivers the same model outcomes as the baseline model with random entry, the counterfactuals are going to differ. In particular, as equation (C.3) shows, counterfactuals that change the value of entry with each technology differently, $\mathbf{J}^e(\varphi_1) - \mathbf{J}^e(\varphi_2)$, will change the number of entrants with each technology in the model with random entry.

D Numerical Appendix

D.1 Idiosyncratic Productivity

In the model, idiosyncratic productivity z is governed by a continuous-time Markov chain (CTMC), with associated infinitesimal generator matrix:

$$\mathbf{\Lambda}_z = \begin{pmatrix} -\sum_{j \neq 1} \lambda_{1j} & \lambda_{12} & \dots & \lambda_{1k} \\ \lambda_{21} & -\sum_{j \neq 2} \lambda_{2j} & \dots & \lambda_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{k1} & \lambda_{k2} & \dots & -\sum_{j \neq k} \lambda_{kj} \end{pmatrix}$$

[53] When σ_ε is large, the probability of getting a high draw of ε_φ increases, and this increases the expected value function. Setting $\mu_\varepsilon = -\sigma_\varepsilon \gamma_\varepsilon$ removes this effect.

where $\lambda_{ij} > 0$ is short-hand for $\lambda(z_j|z_i)$, $z_i, z_j \in \mathcal{Z}$. For the numerical implementation, we recover these rates by assuming an Ornstein-Uhlenbeck (OU) process for z_t (in logs):

$$d\ln(z_t) = -\rho_z \ln(z_t)dt + \sigma_z dB_t$$

where B_t is a Wiener process, and $\rho_z, \sigma_z > 0$.[54] This is a continuous-time process defined on a continuous support. To simulate such a process, and draw a one-to-one mapping between the (ρ_z, σ_z) parameters and the $\{\lambda_{ij}\}$ rates, we use the following steps:

1. First, we approximate the process in discrete time. For a given time interval $[0, T] \subset \mathbb{R}_+$, we partition the space into M subintervals of equal length $dt > 0$, i.e. $\mathcal{T} \equiv \{0 = t_0 < t_1 < \cdots < t_M = T\}$ with $t_{m+1} - t_m = dt$ and $dt = T/M$. As the model is calibrated at the quarterly frequency, dt represents a quarter. Then, we approximate the OU process using the Euler-Maruyama method:

$$\ln(z_m) = (1 - \rho_z dt)\ln(z_{m-1}) + \sigma_z \sqrt{dt}\, \varepsilon^z_m, \qquad \varepsilon^z_m \sim \text{i.i.d. } \mathcal{N}(0,1) \tag{D.1}$$

   for each $m = 1, \ldots, M$. From Ruiz-García (2021), we know that the autocorrelation coefficient and the dispersion in firm-level TFP in Spain is 0.81 and 0.34 at an annual frequency. Therefore, for persistence, we set $\rho_z = 1 - (0.81)^{1/4} = 0.0513$ for our quarterly calibration. Moreover, we compute a quarterly figure for dispersion from the yearly dispersion parameter as $\sigma_z = 0.34 \left(\sum_{q=1}^{4} (0.81)^{(q-1)/2}\right)^{-1/2} = 0.1833$.

2. To estimate the discrete-time AR(1) process (D.1), we use the Tauchen (1986) method. The outcome of this method is a transition probability matrix $\mathbf{\Pi}_z = (\pi_{ij})$, where π_{ij} denotes the probability of a z_i-to-z_j transition in the $\mathcal{T}$ space.

3. For the mapping from $\{\pi_{ij}\}$ transition probabilities to $\{\lambda_{ij}\}$ intensity rates, we use that any CTMC with generator matrix $\mathbf{\Lambda}_z$ maps into a discrete-time Markov chain with transition matrix $\mathbf{\Pi}_z(t)$ at time t in which holding times between arrivals are independently and exponentially distributed, so that $\mathbf{\Pi}_z(t) = e^{\mathbf{\Lambda}_z t}$. Then, we can solve for $\{\lambda_{ij}\}$ to obtain:

$$\lambda_{ij} = \begin{cases} -\frac{\pi_{ij}}{1-\pi_{ii}} \frac{\ln(\pi_{ii})}{dt} & \text{for } i \neq j \\ \frac{\ln(\pi_{ii})}{dt} & \text{otherwise} \end{cases}$$

D.2 Stationary Solution Algorithm

We solve the model on a grid $\overline{\mathcal{N}}_{OEH} \times \overline{\mathcal{N}}_{OEL} \times \overline{\mathcal{N}}_{FT} \times \mathcal{Z} \times \Phi$, where $\overline{\mathcal{N}}_{ij} \equiv \{0, 1, 2, \ldots, N_{ij}\}$, $(i, j) \in \mathcal{I} \times \mathcal{J}$, for sufficiently large positive integer N_{ij}. In practice, we use $(N_{OEH}, N_{OEL}, N_{FT}) = (30, 15, 15)$.

Step 0. Set $\iota = 0$. Choose a guess $\mathbf{U}^{(0)} > b/\rho$.

Step 1. At iteration $\iota \in \{0, 1, 2, \ldots\}$, given a guess $\mathbf{U}^{(\iota)}$, use Value Function Iteration to solve for the object $\mathbf{\Sigma}^{(\iota)} \in \overline{\mathcal{N}}_{OEH} \times \overline{\mathcal{N}}_{OEL} \times \overline{\mathcal{N}}_{FT} \times \mathcal{Z} \times \Phi$:[55]

[54] In levels, this is a diffusion of the type $dz_t = \mu(z_t)dt + \sigma(z_t)dB_t$, with $\mu(z) = z\left(-\rho_z \ln(z) + \frac{\sigma_z^2}{2}\right)$ and $\sigma(z) = \sigma_z z$.

[55] To arrive at this expression, we have used results (8) and (A.27) into equation (A.10).

$$\boldsymbol{\Sigma}^{(\iota)}(\vec{n},z,\varphi) = \frac{1}{\rho^{(\iota)}(\vec{n},z,\varphi)}\Bigg\{y(\vec{n},z,\varphi) - \xi n_{FT}\left[p^{(\iota)}(\vec{n},z,\varphi)\right]^{\vartheta} - \sum_{i\in\mathcal{I}} A_i\left[v_i^{(\iota)}(\vec{n},z,\varphi)\right]^{\varsigma}$$

$$+ \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\left[n_{ij}\left(\delta_{ij}^{(\iota)}(\vec{n},z,\varphi) + s_i^W + s^F\right)\mathbf{U}^{(\iota)} - \chi n_{ij}\left[\delta_{ij}^{(\iota)}(\vec{n},z,\varphi)\right]^{\psi} + n_{ij}\left(\delta_{ij}^{(\iota)}(\vec{n},z,\varphi) + s_i^W\right)\boldsymbol{\Sigma}^{(\iota)}(\vec{n}_{ij}^{-},z,\varphi)\right]$$

$$+ \sum_{i\in\mathcal{I}}\left[v_i^{(\iota)}(\vec{n},z,\varphi)\eta^{(\iota)}(\vec{n},z,\varphi)\max\left(\boldsymbol{\Sigma}^{(\iota)}(\vec{n}_{iL}^{+},z,\varphi) - \mathbf{U}^{(\iota)}, \gamma\left(\boldsymbol{\Sigma}^{(\iota)}(\vec{n}_{iL}^{+},z,\varphi) - \mathbf{U}^{(\iota)}\right) + (1-\gamma)\boldsymbol{\Sigma}^{(\iota)}(\vec{n},z,\varphi)\right)\right]$$

$$+ n_{FT}p^{(\iota)}(\vec{n},z,\varphi)\boldsymbol{\Sigma}^{(\iota)}(\vec{n}^{p},z,\varphi) + n_{OEL}\tau\,\boldsymbol{\Sigma}^{(\iota)}(\vec{n}^{\tau},z,\varphi) + \sum_{z'\in\mathcal{Z}}\lambda(z'|z)\boldsymbol{\Sigma}^{(\iota)}(\vec{n},z',\varphi)\Bigg\}$$

for each $\varphi \in \Phi$, where:

$$\delta_{ij}^{(\iota)}(\vec{n},z,\varphi) \equiv \left[\frac{1}{\psi\chi}\left(\mathbf{U}^{(\iota)} + \boldsymbol{\Sigma}^{(\iota)}(\vec{n}_{ij}^{-},z,\varphi) - \boldsymbol{\Sigma}^{(\iota)}(\vec{n},z,\varphi)\right)\right]^{\frac{1}{\psi-1}}$$

$$p^{(\iota)}(\vec{n},z,\varphi) \equiv \left[\frac{1}{\vartheta\xi}\left(\boldsymbol{\Sigma}^{(\iota)}(\vec{n}^{p},z,\varphi) - \boldsymbol{\Sigma}^{(\iota)}(\vec{n},z,\varphi)\right)\right]^{\frac{1}{\vartheta-1}}$$

$$v_i^{(\iota)}(\vec{n},z,\varphi) \equiv \left[\left(\frac{\gamma}{\varsigma A_i}\right)\eta^{(\iota)}(\vec{n},z,\varphi)\left(\boldsymbol{\Sigma}^{(\iota)}(\vec{n}_{iL}^{+},z,\varphi) - \boldsymbol{\Sigma}^{(\iota)}(\vec{n},z,\varphi) - \mathbf{U}^{(\iota)}\right)\right]^{\frac{1}{\varsigma-1}}$$

$$\eta^{(\iota)}(\vec{n},z,\varphi) \equiv \left[\left(\frac{1-\gamma}{\rho\mathbf{U}^{(\iota)} - b}\right)\left(\boldsymbol{\Sigma}^{(\iota)}(\vec{n}_{iL}^{+},z,\varphi) - \boldsymbol{\Sigma}^{(\iota)}(\vec{n},z,\varphi) - \mathbf{U}^{(\iota)}\right)\right]^{\frac{1-\gamma}{\gamma}}$$

$$\rho^{(\iota)}(\vec{n},z,\varphi) \equiv \rho + s^F + n_{FT}p^{(\iota)}(\vec{n},z,\varphi) + n_{OEL}\tau + \sum_{i\in\mathcal{I}}\left[v_i^{(\iota)}(\vec{n},z,\varphi)\eta^{(\iota)}(\vec{n},z,\varphi) + \sum_{j\in\mathcal{J}} n_{ij}\left(\delta_{ij}^{(\iota)}(\vec{n},z,\varphi) + s_i^W\right)\right]$$

Step 2. Use a non-linear equation solver to find $\mathbf{U}^{*}$ as the solution to the free-entry condition:

$$-\kappa + \sum_{\varphi\in\Phi}\pi_{\varphi}(\varphi)\cdot\mathbf{J}_e^{(\iota)}(\varphi;\mathbf{U}^{*}) = 0$$

where

$$\mathbf{J}_e^{(\iota)}(\varphi;\mathbf{U}^{*}) \equiv \gamma\left(\frac{\rho\mathbf{U}^{*} - b}{1-\gamma}\right)^{\frac{\gamma-1}{\gamma}}\left[\sum_{z^e\in\mathcal{Z}}\pi_z(z^e)\left(\sum_{i\in\mathcal{I}}\left(\boldsymbol{\Sigma}^{(\iota)}(\vec{n}_{iL}^{e},z^e,\varphi) - \mathbf{U}^{*}\right)^{\frac{1}{\gamma}}\right)\right]$$

is the value of entering with type φ, with $\vec{n}_i^e = (n_i^e, \vec{n}^e_{-(ij)}) = (1,\vec{0})$.

Step 3. Compute

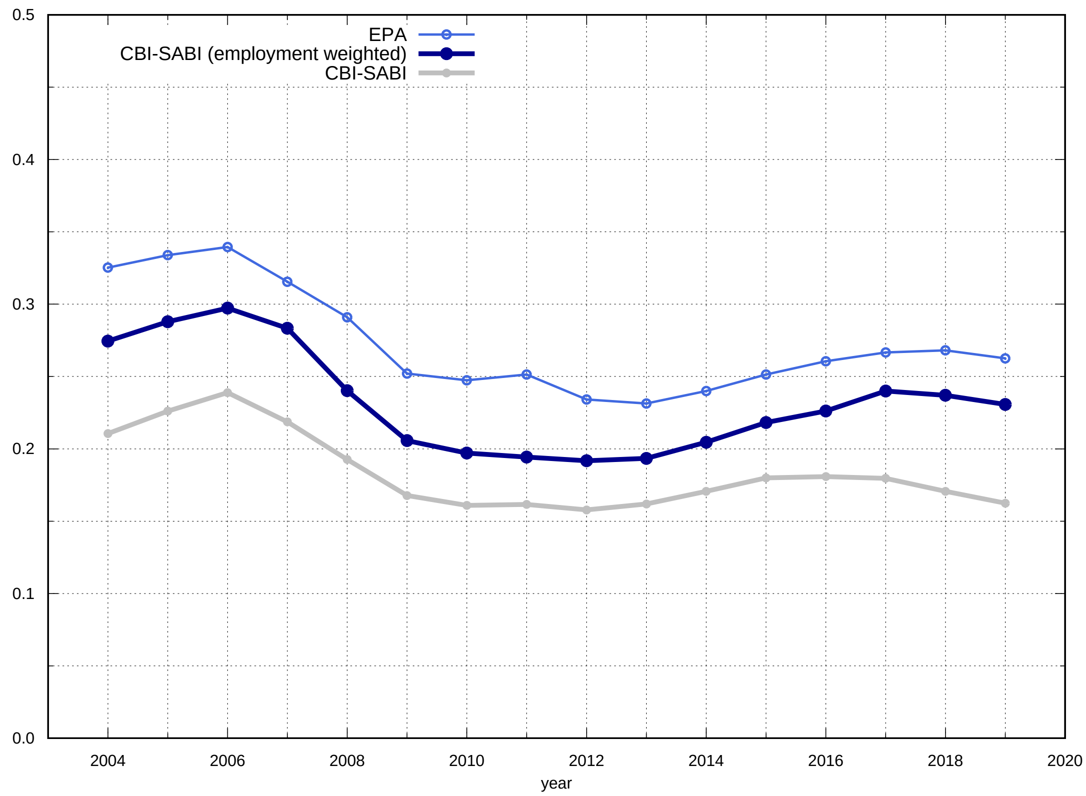
$$\epsilon^{(\iota)} \equiv \left|\frac{\mathbf{U}^{(\iota)} - \mathbf{U}^{*}}{\mathbf{U}^{*}}\right|$$

Proceed to Step 4 if $\epsilon^{(\iota)} < \varepsilon$ for some small tolerance $\varepsilon > 0$. Otherwise, set $\mathbf{U}^{(\iota+1)} = \varrho\cdot\mathbf{U}^{*} + (1-\varrho)\cdot\mathbf{U}^{(\iota)}$, where $\varrho \in (0,1]$ is a dampening parameter, and go back to Step 1 with $[\iota] \leftarrow [\iota+1]$.

Step 4. Find the distribution and aggregate measures of agents as described in Online Appendix A.2.

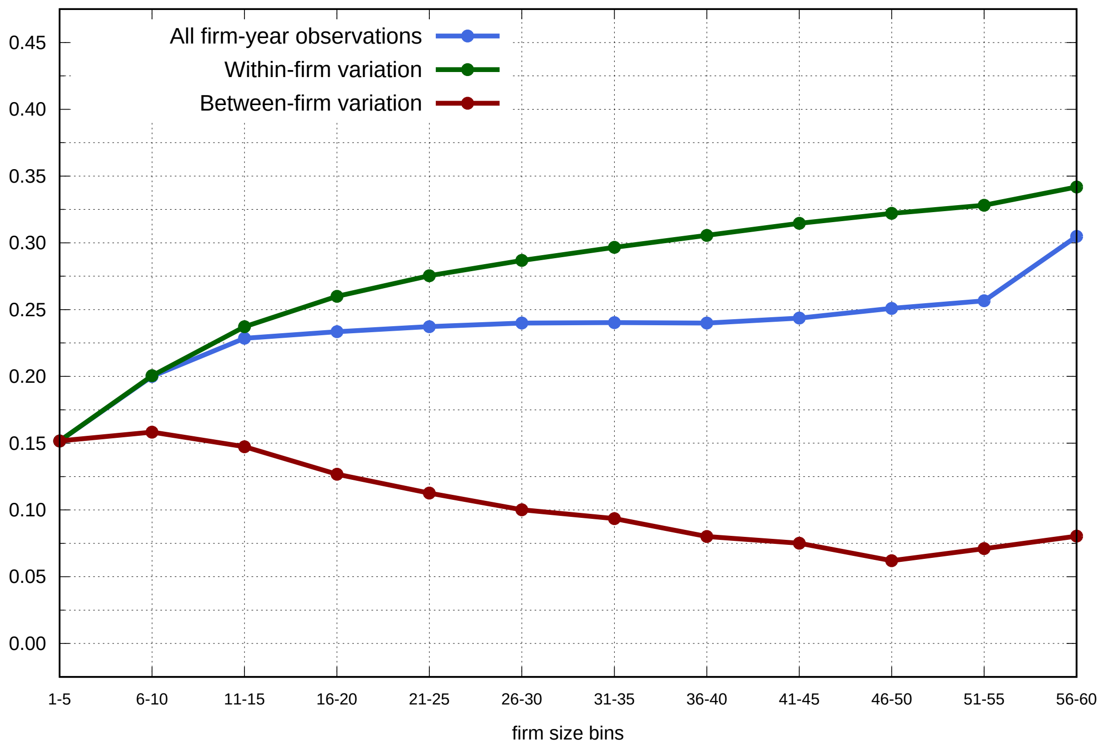
E Additional Figures and Tables

Figure E.1: Aggregate temporary share over time



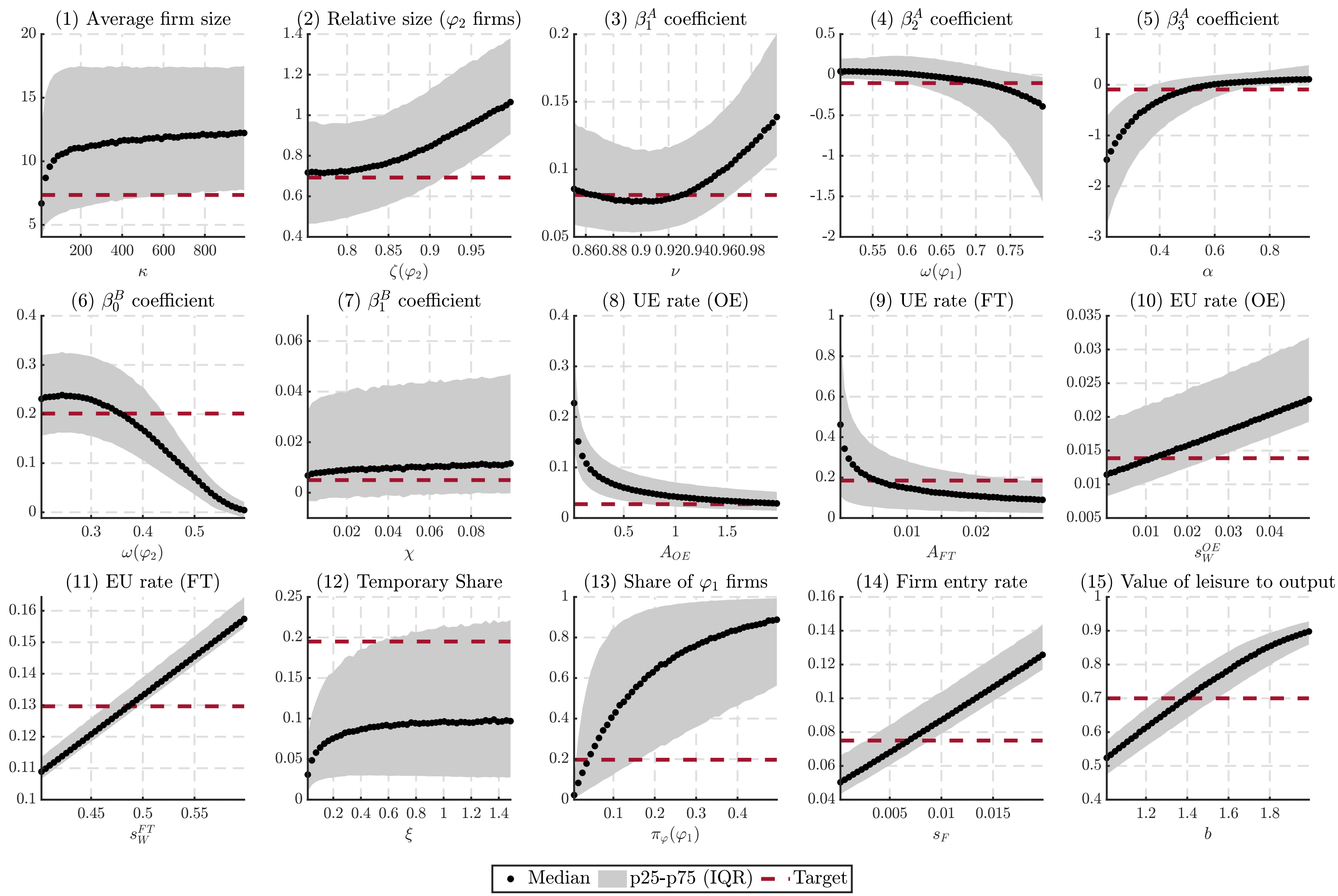
Notes: Light blue line: share of temporary workers in the Encuesta de Población Activa (the Spanish Labor Force Survey); grey line: average share of temporary workers across firms in our firm-level data; dark blue line: employment-weighted average share of temporary workers across firms in our firm-level data, which corresponds to the average share of temporary workers in the economy.

Figure E.2: Temporary share, by firm size, in the sub-sample.



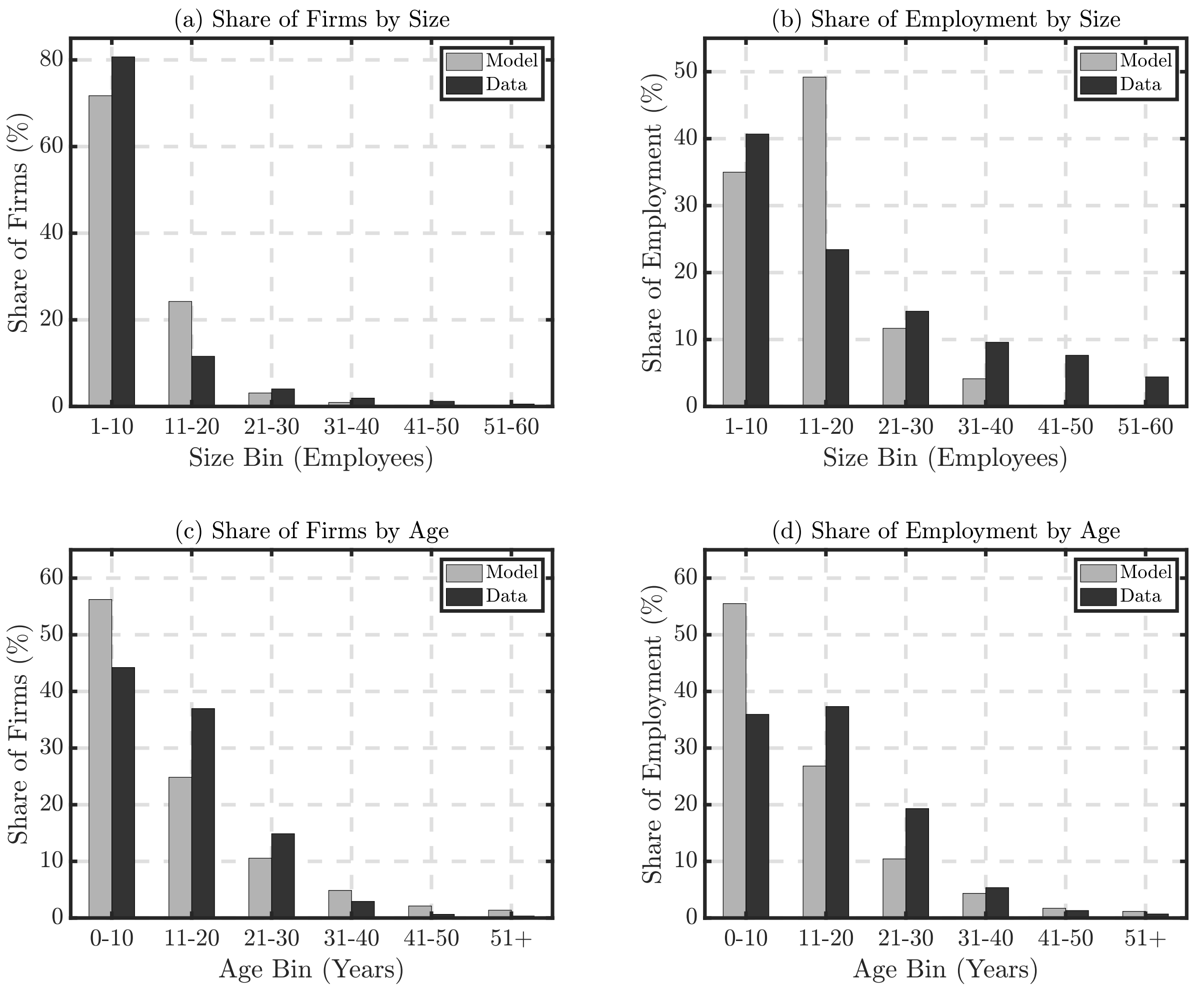
Notes: This figure is the analogue of Figure 1 in the main text, but for the sub-sample used to calibrate the model.

Figure E.3: Global identification results.



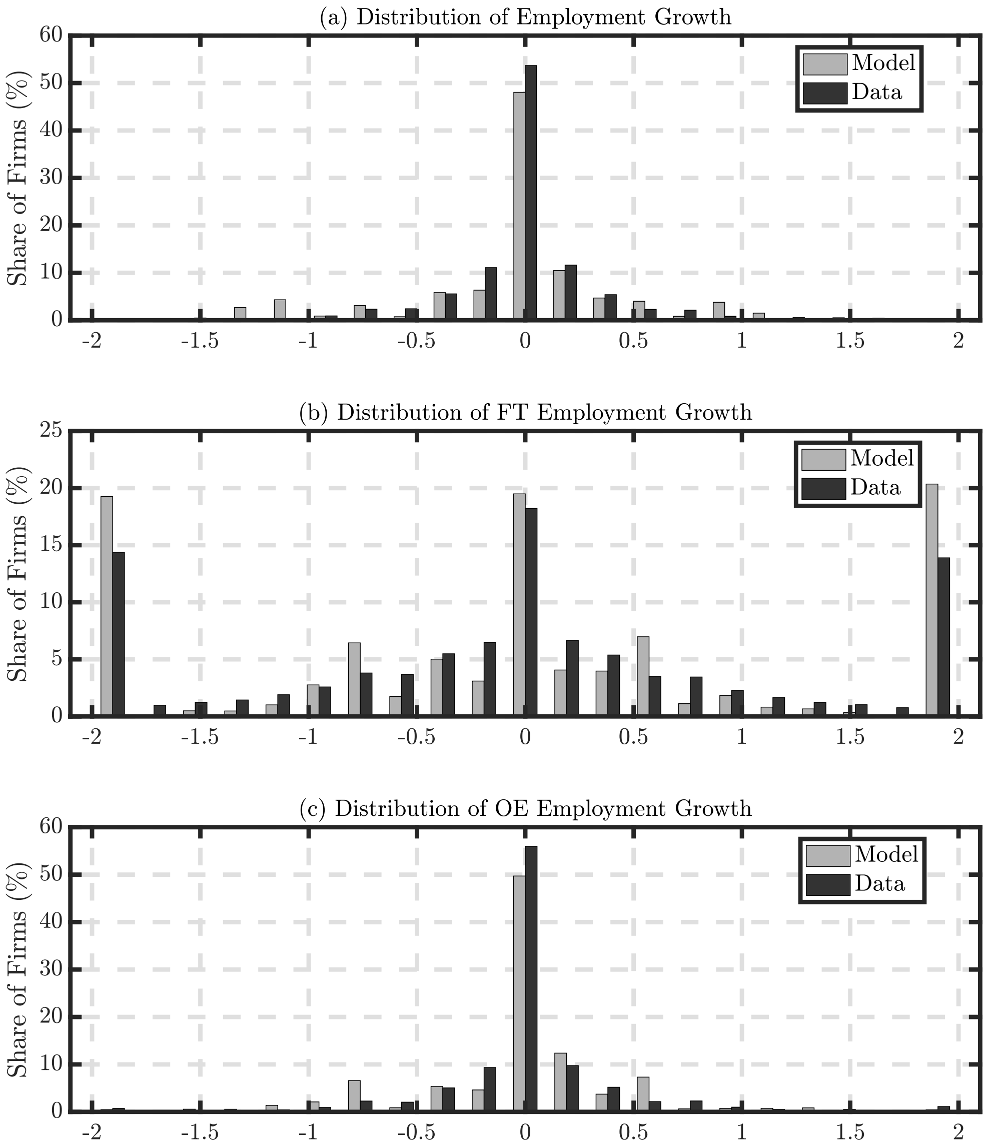
Notes: Global identification results based on approximately 1.3 million model solutions. The black dots are the median of the distribution of each targeted moment for given value of the chosen parameter, generated from underlying random variation in all other parameters. The shaded areas are the 25th and 75th percentiles. The red dashed horizontal line is the data target.

Figure E.4: Non-Targeted Moments: Distribution of Firms and Employment by Size and Age. Model versus Data.



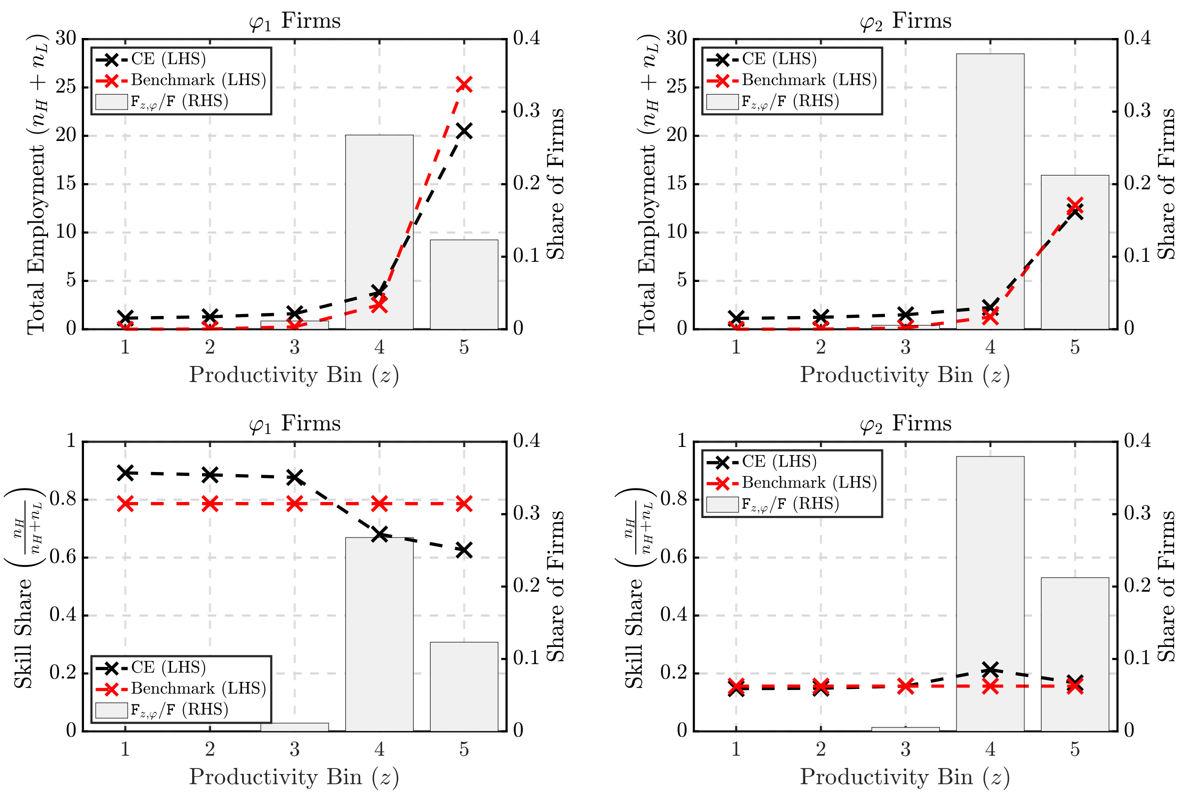
Notes: This figure plots the distribution of firms and employment by firm size (total number of employees) and firm age (in years), in the data and in the calibrated model.

Figure E.5: Non-Targeted Moments: Distribution of Yearly Employment Growth Rates. Model versus Data.



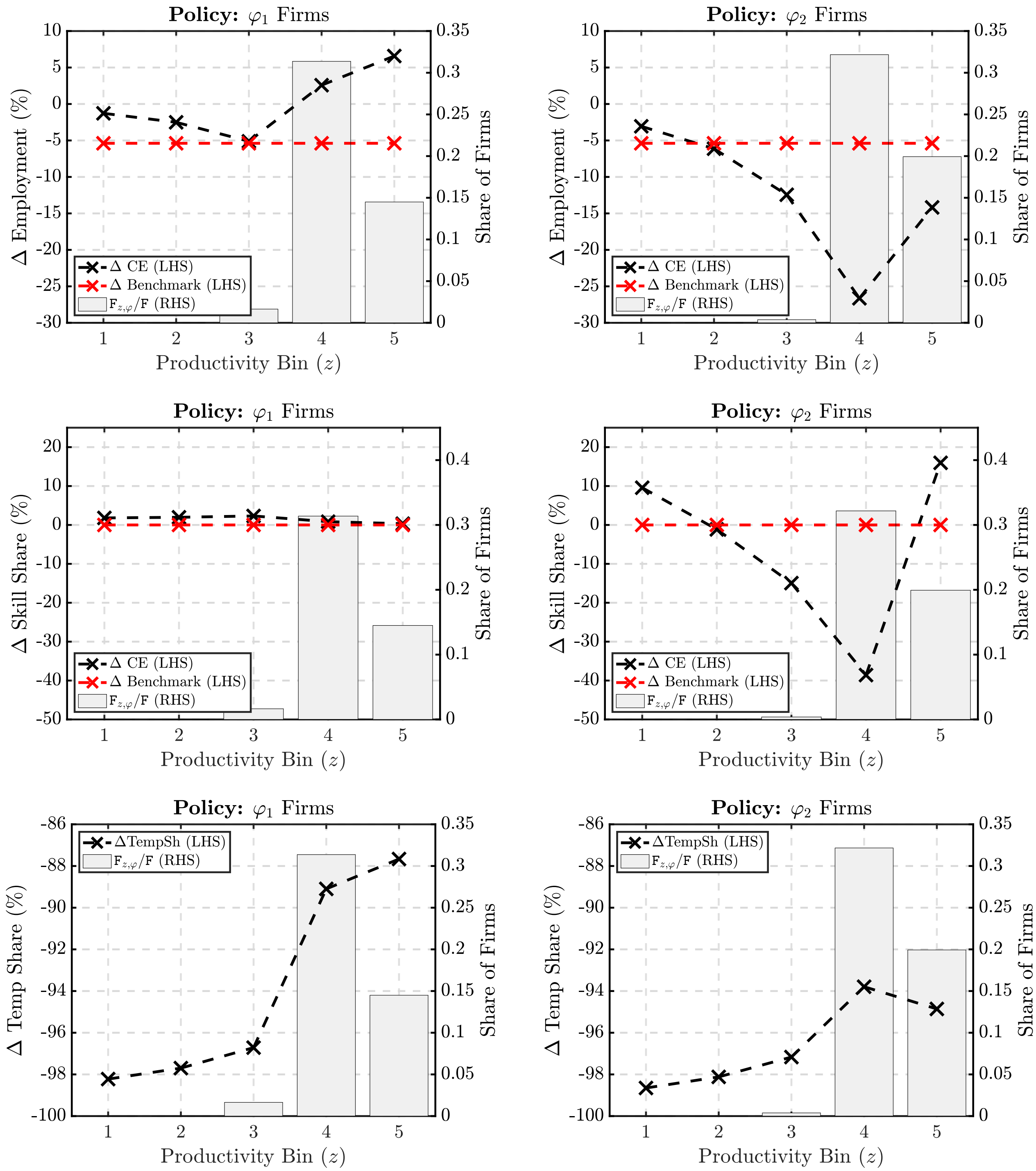
Notes: This figure plots the distribution of employment growth rates for total employment (top), FT employment (middle) and OE employment (bottom), in the data and in the calibrated model.

Figure E.6: Misallocation Across Productivity Classes in the Baseline Calibration.



Notes: This figure compares the allocation of employment (top row) and of the skill share (bottom row) for firms of different z's (x-axis) and φ's (left and right panels), in the competitive equilibrium (CE) and the Benchmark's solutions. For the CE, we provide the average employment within each productivity group. For the Benchmark, we provide the employment level of the corresponding productivity group. The magnitudes can be read from the left-hand axes. We also plot the stationary distribution of firms across productivity levels in the CE (right-hand axes).

Figure E.7: Heterogeneous effects by productivity groups of shortening FTC duration.



Notes: These figures show the percentage change in employment (top row), the skill share (middle row) and the temporary share (bottom row) for high-type (left column) and low-type (right column) firms and for each transitory productivity bin (bins on the horizontal axis) when moving from the baseline duration of FTCs to a policy that limits FTCs to 1 month. When available (i.e. for total employment and the skill share), we also show the percentage change for the Benchmark allocation that maximizes total output.

Table E.1: Regression evidence in the data

A. Productivity: $\ln(VA_{it}/Emp_{it})$			**B. Temporary share:** $TempSh_{it}$		
β_0^A	$\mathbf{1}[\varphi_i = \varphi_2]$	-0.003	β_0^B	$\mathbf{1}[\varphi_i = \varphi_2]$	0.201
β_1^A	$\ln(Emp_{it})$	0.081	β_1^B	$Emp_{it} \in [1, 5)$	0.005
β_2^A	$TempSh_{it}$	-0.104	β_2^B	$Emp_{it} \in [5, 10)$	0.037
β_3^A	$TempSh_{it}^2$	-0.092	β_3^B	$Emp_{it} \in [10, 15)$	0.080
			β_4^B	$Emp_{it} \in [15, 20)$	0.099
			β_5^B	$Emp_{it} \in [20, 25)$	0.113
			β_6^B	$Emp_{it} \in [25, 30)$	0.122
			β_7^B	$Emp_{it} \in [30, 35)$	0.129
			β_8^B	$Emp_{it} \in [35, 40)$	0.135
			β_9^B	$Emp_{it} \in [40, 45)$	0.142
			β_{10}^B	$Emp_{it} \in [45, 50)$	0.150
			β_{11}^B	$Emp_{it} \in [50, 55]$	0.156
			β_{12}^B	$Emp_{it} \in [55, 60]$	0.190
# observations		6,316,320	# observations		6,664,229
R^2		0.01	R^2		0.14

Notes: Results, in the firm-level data, from regression (17), in Panel A, and regression (18), in Panel B. Variables are all net of aggregate FE (sector, region, and province). All coefficients are significant at the 1% level.

Table E.2: Effects of changes in average FTC duration on macro aggregates when entry is directed

	(A) *Short duration* (1 month)	(B) *Baseline* (6.5 months)	(C) *Long duration* (1 years)
Measure of operating firms	0.106	0.121	0.140
... Share of type-φ_1 firms	65.5 %	40.2 %	19.0 %
Average firm size	7.78	7.01	6.68
... Relative size φ_2 firms	0.53	0.65	0.80
Firm entry rate (annualized)	7.4 %	7.5 %	7.5 %
Average temporary share	0.6 %	21.8 %	60.9 %
... within φ_1 firms	0.3 %	5.2 %	13.3 %
... within φ_2 firms	1.8 %	32.9 %	74.8 %
Share of H workers	60.4 %	46.0 %	30.7 %
... within φ_1 firms	71.7 %	71.3 %	70.2 %
... within φ_2 firms	20.1 %	19.7 %	19.1 %
Matching efficiency (FT)	0.21	1.40	0.57
Matching efficiency (OE)	0.09	0.03	0.01
UE rate (total)	8.0 %	20.9 %	24.5 %
... UE rate (FT)	2.4 %	18.4 %	22.1 %
... UE rate (OE)	5.6 %	2.5 %	2.4 %
EU rate (total)	1.8 %	3.7 %	1.7 %
... EU rate (FT)	76.4 %	11.9 %	2.3 %
... EU rate (OE)	1.3 %	1.5 %	0.9 %
Promotion rate	11.1 %	3.0 %	0.2 %
Unemployment rate	17.8 %	14.9 %	6.3 %

Notes: This table is the same as Table 5 in the main text, but for the economy with directed entry and an entry elasticity of one. For the description of the extension of the model with directed entry, see Online Appendix C.

Table E.3: Effects of changes in average FTC duration on productivity, misallocation and welfare when entry is directed

	(A) *Short duration* (1 month)	(B) *Long duration* (4 years)
Change in output per worker, of which:	-0.08 %	-0.49 %
(a) *Firm size channel*	-1.24 %	0.57 %
(b) *Firm selection channel*	12.38 %	-11.36 %
(c) *Reallocation channel*, of which:	-14.42 %	6.06 %
... between-firm component	-14.33 %	5.68 %
... within-firm component	-9.11 %	4.25 %
Change in output	-3.48 %	9.64 %
Output loss from misallocation (in levels), of which:	4.49 %	3.12 %
... share due to between-firm misallocation	44.81 %	59.84 %
Change in welfare	0.05 %	0.88 %

Notes: This table is the same as Table 6 in the main text, but for the economy with directed entry and an entry elasticity of one. For the description of the extension of the model with directed entry, see Online Appendix C.

Table E.4: Effects on macro aggregates of various alternative policies

		(A)	(B)	(C)
	Baseline calibration	Short duration	Ban on FTCs	Linear Tax on FT Employment
Measure of operating firms	0.121	0.112	0.120	0.114
... Share of type-φ_1 firms	40.2 %	47.5 %	45.7%	47.5 %
Average firm size	7.01	7.03	6.91	7.05
... Relative size φ_2 firms	0.65	0.54	0.54	0.54
Firm entry rate (annualized)	7.5 %	7.4 %	7.4 %	7.4 %
Average temporary share	21.8 %	0.9 %	.	0.9 %
... within φ_1 firms	5.2 %	0.3 %	.	0.4 %
... within φ_2 firms	32.9 %	1.8 %	.	1.6 %
Share of H **workers**	46.0 %	52.5 %	51.5 %	52.6 %
... within φ_1 firms	71.3 %	71.7 %	71.7 %	71.7 %
... within φ_2 firms	19.7 %	20.2 %	19.7%	20.4 %
Matching efficiency (FT)	1.40	0.28	0.42	0.04
Matching efficiency (OE)	0.03	0.11	0.04	0.09
UE rate (total)	20.9 %	8.3 %	7.5 %	7.0 %
... UE rate (FT)	18.4 %	2.7 %	.	0.9 %
... UE rate (OE)	2.5 %	5.6 %	.	6.1 %
EU rate (total)	3.7 %	2.3 %	1.7 %	1.8 %
... EU rate (FT)	11.9 %	75.9 %	.	16.7 %
... EU rate (OE)	1.5 %	1.6 %	.	1.6 %
Promotion rate	3.0 %	10.9 %	.	12.9 %
Unemployment rate	14.9 %	21.0 %	17.8 %	19.6 %

Notes: This table shows the effects of various policies on a number of macroeconomic aggregates of interest. The tax rate in column (C) have been chosen to ensure that the effect of the policy on the average temporary share is the same as the baseline policy discussed in the paper, in which the duration of FTCs decreases to 1 month.

Table E.5: Effects on productivity, misallocation and welfare of various alternative policies

	(A)	(B)	(C)
	Short duration	Ban on FTCs	Linear Tax on FT Employment
Change in output per worker, of which:	0.26 %	0.24 %	0.24 %
(a) *Firm size channel*	-0.05 %	0.17 %	-0.07 %
(b) *Firm selection channel*	4.91 %	2.70 %	4.95 %
(c) *Reallocation channel*, of which:	-5.77 %	-3.56 %	-5.77 %
... between-firm component	-5.62 %	-3.32 %	-5.64 %
... within-firm component	-0.70 %	0.87 %	-0.84 %
Change in output	-6.86 %	-3.18 %	-5.24 %
Output loss from misallocation (in levels), of which:	4.29 %	4.22 %	4.29 %
... share due to between-firm misallocation	39.52 %	37.87 %	39.64 %
Change in welfare	-0.87 %	-0.31 %	-0.65 %

Notes: This table shows the effect of various policies, expressed in percentage changes with respect to the baseline calibration (with the exception of the output loss from misallocation and the share of it that is due to between-firm misallocation, which are expressed in levels). Welfare is computed as in equation (A.37), see Online Appendix A.6. The taxes rates in columns (B) and (C) have been chosen to ensure that the effect of the policy on the average temporary share is the same as the baseline policy discussed in the paper, in which the duration of FTCs decreases to 1 month.