The Optimal Scope of the Royalty Base in Patent Licensing*

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Abstract

There is considerable controversy among legal scholars about the relative merits of using the total value of the product as a base for a royalty in licensing contracts, as opposed to the value of the component that the technology contributes to. In this paper we make use of the fact that these two royalty bases are equivalent to using ad-valorem and per-unit royalties, respectively. We analyze the welfare implication of the two rules abstracting from implementation and practicability considerations. We show that ad-valorem royalties tend to lead to lower prices, particularly in the context of successive monopolies. They benefit upstream innovators but not necessarily hurt downstream producers. This benefit increases when there are multiple innovators contributing complementary technologies, as it is typical of Standard-Setting Organizations. When we endogenize the investment decisions, we show that a sufficient condition for ad-valorem royalties to improve social welfare is that enticing more upstream investment is optimal. Our findings contribute to explain why in practice most licensing contracts include royalties based on the value of the product.

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1 Introduction

The licensing of a patented technology is one of the most important sources of revenue for many innovators, particularly, when they do not participate in the production in the final market. A licensing contract typically includes a royalty payment that comprises two components: a *royalty rate* and a *royalty base*. Most attention in the economic literature has been devoted to the optimal determination of the royalty rate. Much less work has been done in studying the scope of the royalty base. This scope can be determined in two principal ways. One option is the value of the sales of the entire product that incorporates the patented technology. Alternatively, the scope of the royalty base can be associated to the value of the component of the infringing product.[1]

The royalty rates associated to these royalty bases are denoted as *ad-valorem* and *per-unit* rates, respectively. The former consists of a payment comprising a percentage of the value of the sales of the product. The latter corresponds to a constant payment based on the number of units sold. An example can help clarify the relationship between these two royalty bases and rates. Consider a producer that sells a good at a price $p = \$100$. This product comprises a component which embeds the patented technology of an upstream innovator. An ad-valorem rate is a percentage s of that price for each unit sold. Therefore, the total income from this royalty corresponds to a portion s of the revenue from the sale of this product, $s \times p \times Q$, where Q is the quantity sold. If, instead, the royalty applies to the value of the component, say \$10, the royalty rate could be either a percentage of this value, say 20%, or just a fixed amount per-unit sold, in this case $r = \$2$. Both a percentage of the value of the component and a per-unit royalty are equivalent in this last case as they do not depend on the price of the final product.[2] Nevertheless, establishing a per-unit royalty rate

[1] These royalty bases receive in the legal literature the name of *entire market value rule* and the *apportionment rule*, respectively

[2] In other words, if the value of the component is v, a percentage rate t of this value leads to a royalty

is easier, since it avoids specifying the value of the component in the contract to set the rate. As a result, the total revenue of the licensor would be in this case the per-unit royalty times the number of units sold, $r \times Q$.

It has been often understood among practitioners and legal scholars that in a world without legal frictions, ad-valorem royalties would yield the same market outcomes than per-unit royalties if they were adjusted appropriately.[3] In the previous example, this would imply that an ad-valorem royalty of $s = 2\%$ would be equivalent to a per-unit royalty of $r = \$2$. In this paper we show that such an equivalence is false. We find that in most circumstances, ad-valorem royalties yield market outcomes that are welfare superior to those resulting from the use of per-unit royalties, both in terms of lower prices and stronger incentives for firms to innovate.

In the main part of the paper we analyze the decision of an upstream innovator that licenses its technology to a pure downstream producer. This innovator holds all the bargaining power when the choosing the royalty rate. Abstracting from the incentives to innovate – that is, assuming that all research and development have been successfully carried out – we show that ad-valorem royalties favor the innovator whereas per-unit royalties tend to benefit the producer. Importantly, the resulting price in the final market is never higher under ad-valorem royalties. The reason is that ad-valorem royalties impose a royalty tax on the downstream mark-up, reducing the profitability of price increases – in the example, 2% of the increase in price is transferred to the innovator –. As a result, they typically make the double-marginalization problem less severe, generating lower distortions in the final market.

payment equal to $s \times v \times Q$ and a per-unit rate r would result in a royalty payment of $r \times Q$. If $r = t \times v$, then both royalty payments are identical. Note that in both cases the royalty payment is independent of p, the price of the final product.

[3] As stated by the U.S. Court of Appeals for the Federal Circuit "there is nothing inherently wrong with using the market value for the entire product for the infringing component or feature, so long as the multiplier accounts for the proportion of the base represented by the infringing component or feature." Lucent Techs., Inc. v. Gateway, Inc., 580 F.3d 1301, 1338-39 (Fed. Cir. 2009).

Only under an isoelastic demand function prices are identical under both licensing schemes. Even in that case, however, ad-valorem royalties lead to lower prices when we allow for other realistic features, such as a more balanced allocation of bargaining power among the innovator and the producer.

This insight is intimately related to a classical result in the indirect taxation literature. Suits and Musgrave (1953), citing even earlier insights in Wicksell (1896), study the least distortionary way to raise revenue from consumption taxes. They show that, when there is market power, ad-valorem taxes (such as the Value-Added Tax or VAT) are preferred to per-unit taxes, as they allow to raise the same revenue through lower final prices. The main difference with our case is that in our setup the innovator maximizes profits (rather than minimize market distortions) but is still interested in lower prices since they generate more sales and with them higher revenues.

Such a connection has not been made in the licensing literature before. This is striking since the proper definition of the royalty base has been a controversial topic in recent years, placing it at the core of many legal cases, particularly related to Standard Setting Organizations (SSOs). SSOs are forums in which firms like technology developers and implementers who produce the final goods discuss the specifications of standards for complex products such as mobile phones or WI-FI enabled devices. Hundreds of patents of many innovators typically cover the technologies necessary to sell products that comply with the standard.

Traditionally these patents have been licensed under ad-valorem royalties. However, courts have recently ruled against this practice and in favor of per-unit royalties, particularly in the context of SSOs. Their claim has been that ad-valorem royalties are subject to more frictions (Sherry and Teece, 1999). Since both royalty bases are argued to be otherwise equivalent, these frictions makes them unappealing in most situations compared to per-unit royalties. This position has been summarized by the U.S. Court of Appeals to the Federal

Circuit (CAFC) in the Ericsson Inc. v. D-Link Systems Inc. case[4] in stating that "[t]he principle, applicable specifically to the choice of a royalty base, is that, where a multi-component product is at issue and the patented feature is not the item which imbues the combination of the other features with value, care must be taken to avoid misleading the jury by placing undue emphasis on the value of the entire product" and the "reliance on the entire market value [using ad-valorem royalties] might mislead the jury, who may be less equipped to understand the extent to which the royalty rate would need to do the work in such instances." As a result, unless "the patented feature drives the demand for the entire multi-component product" ad-valorem royalties should be avoided. This recommendation has been recently incorporated in the updated patent policy of the Institute of Electrical and Electronics Engineers (IEEE), an association that sponsors many of the most relevant standards. On February 2, 2015 the Department of Justice (DOJ) issued a Business Review Letter praising IEEE for this change,[5] generating a controversy among academics and practitioners.[6]

Our paper shows that this move from ad-valorem to per-unit royalties proposed by courts and sponsored by SSOs and antitrust authorities is likely to lead to higher final prices which ought to be traded off with the lower legal frictions that they might entail. Furthermore, we show that these negative welfare consequences are going to be more significant precisely in those contexts in which many firms contribute technologies that are combined in the same product. In particular, in the second part of the paper we show that, once we introduce several upstream innovators that provide complementary technologies, an additional force appears. As it is well known, the interaction of several licensors creates a classical problem of Cournot complements, known in this context as *royalty stacking*: by requiring a large royalty, innovators reduce the quantity that the final good producer sells, creating a negative

[4] 773 F.3d 1201, 1227 (Fed. Cir. 2014).

[5] See `http://www.justice.gov/atr/public/busreview/311470.pdf`

[6] See Teece (2015) or Sidak (2015).

externality on all the rest of the innovators. As a result, prices are higher than those that would emerge from the profit maximizing behavior of an upstream monopolist that holds all technologies.[7] The model shows that the royalty-stacking problem is more severe under per-unit royalties that under ad-valorem ones.

Finally, our model allows us to study the incentives for firms to innovate as a result of the royalty base used, under the current patent system. This is an important dimension that has been mostly ignored in the literature and in previous legal debates which implicitly assume that all technologies are already present. We model a first stage in which both the innovator and the producer simultaneously must make an investment. The innovator invests in the R&D necessary to create the technology, while the producer invests in the implementation of the technology in a final product that can be marketed. Since the choice of the royalty base affects the allocation of profits among the firms operating in different production stages, it will also affect the incentives to invest and, consequently, social welfare.

In the context of an upstream and downstream monopoly, ad-valorem royalties are usually superior from an ex-ante perspective. By allocating profits to the innovator they spur upstream investment at the expense of producer's incentives. However, because they mitigate the double-marginalization problem, total surplus is higher, which might partially compensate the decrease of investment incentives of the producer. It is only when the upstream innovator has a cost of R&D substantially lower than the implementation cost of the downstream producer that per-unit royalties may engender more incentives to innovate and increase social welfare.

When we consider multiple innovators with complementary technologies, numerical results indicate that ad-valorem royalties have the potential to foster not only the investment of those innovators but also of the producer that implements these technologies into final

[7]Spulber (2014) shows that this distortion does not arise when innovators follow quantity-setting strategies.

goods. The reason is that by increasing upstream profits, ad-valorem royalties increase the productivity of the investment of all parties, generating a positive feedback loop. As a result, the positive effect on social welfare of the lower prices that these royalties engender is reinforced by the higher investment incentives.

1.1 Literature Review

By comparing the welfare implications of ad-valorem and per-unit royalties, we contribute to the well-established literature on licensing contracts. This literature has typically focused on the trade-offs between fixed fees – understood as payments independent of the quantity produced – and per-unit royalties. In vertical relationships, Kamien and Tauman (1986) among others have shown that fixed fees are superior to per-unit royalties even when there are several downstream producers to which the technology can be licensed.[8] Surprisingly, this literature has paid very little attention to ad-valorem royalties. This is striking since the empirical evidence shows that these royalties are prevalent. For example, in a sample of 278 contracts, Bousquet et al. (1998) showed that 225 include royalties but only in 9 of them these royalties were paid per unit sold.

Our results suggest that ad-valorem royalties tend to spur more innovation and lead to lower final prices, which explains their popularity. Few contributions in the literature have studied the trade-off between both types of royalties. Bousquet et al. (1998) compare ad-valorem and per-unit royalties in combination with fixed fees in the case of vertical relationships like the ones we consider here. They show that when there is uncertainty regarding the demand, typical of product innovations, ad-valorem royalties in combination with fixed fees are more effective for risk-sharing. In contrast, in the case of cost uncertainty, typical of process innovations, the ranking between the two royalty schemes is far less clear.

[8] Of course, the previous results do not hold when we consider market frictions. Hernández-Murillo and Llobet (2006), for example, have shown that royalties can be optimal when asymmetric information considerations are taken into account.

Other papers have analyzed different trade-offs involving the two royalty bases and, in particular, their implications for raising rival's costs (Salop and Scheffman, 1983) when a vertically integrated firm licenses its technology to downstream competitors. San Martín and Saracho (2010) show that under Cournot competition ad-valorem royalties constitute a more effective commitment to soften downstream competition, raising the final price. However, Colombo and Filippini (2012) show that the opposite is true when downstream firms compete in prices. Our model abstracts from the previous effects by assuming that innovators do not have downstream operations. Although this is a limitation of our work, it is also a relevant situation in as much as the court cases discussed earlier and the proposed patent reforms concern vertically disaggregated firms such as large innovators or non-practicing entities. Furthermore, the conflicting results of the previous two papers indicate that it would be difficult to obtain robust additional insights by introducing vertical integration into the model.

Our work is also related to the literature that studies the optimal reward for complementary technologies in patent pools or standard-setting organizations. As in our paper, Gilbert and Katz (2011) study the incentives for innovators to carry out R&D to uncover the complementary technologies that are embedded in complex products. Firms choose which technologies to pursue. They show that the optimal payoff from innovation must counterbalance two forces. On the one hand, firms cannot appropriate all the return from the innovation, leading to underinvestment. On the other hand, for each innovation firms engage in a patent race, leading to overinvestment. An important conclusion is that, even in the case of perfectly complementary innovations, an equal division of surplus among innovators is unlikely to be optimal, since it encourages firms to obtain either only one or all innovations. Instead, here we focus on the interaction between upstream innovators and downstream producers. Since we assume that innovators are identical and do not choose which technologies to pursue, equal division among them is optimal in our context. Furthermore, the lack of a patent race component always leads to underinvestment, resulting from the lack of appropriability of all the returns from the innovation.

The paper proceeds as follows. In section 2 we introduce the benchmark model that includes an upstream innovator and a downstream producer and we allow for different allocations of bargaining power. Section 3 studies the case of multiple upstream innovators. Section 4 briefly discusses some extensions of the model. Section 5 concludes discussing some policy implications. All proofs are relegated to the appendix.

2 The Benchmark Model

Consider the market for a new product. Its development requires the participation of two firms. An upstream innovator, denoted as U, uncovers the basic technology that is required for the product. Development also requires a downstream producer that adapts the technology and creates the final product that can be marketed. This firm is denoted as D and faces a demand function $D(p)$, given a price p.

We treat the investment decisions of these firms symmetrically. Each firm exerts effort e_s, for $s = U, D$. Efforts are complementary in the development of the final product. In particular, we assume that the upstream technology is successful with probability e_U and the downstream producer can adapt it successfully with probability e_D, so that the final product can be marketed with probability $e_U e_D$. Firms face an increasing and convex cost of effort $C(e_s) = \frac{1}{2}e_s^2$, for $s = U, D$.

Research effort may engender technologies that have alternative uses beyond the product considered. These uses lead to profits $\pi_0^U > 0$ for the innovator if its research effort succeeds and $\pi_0^D > 0$ if the final producer succeeds. These profits can originate, for example, from different applications of the technology developed upstream or from its use in other products that the downstream producer may already sell.[9]

The innovator offers a licensing contract to the final good producer to allow the sale of the good. The amount to be paid is determined as the result of Nash bargaining. We denote the bargaining power of the innovator and the producer by γ and $1-\gamma$, respectively, for $\gamma \in [0,1]$. Since the contribution of both parties is essential for the product to be marketable, the outside option in the negotiation of each of the parties is set to 0 and in case of disagreement they only obtain the profits from other uses, π_0^s, for $s = U, D$. Once the payment has been agreed on the producer chooses the price for the final good and incurs in a marginal cost of production $c \geq 0$.

As discussed in the introduction, these licensing payments will involve a royalty that the producer will have to pay according to the royalty base specified in the contract. We will compare the two relevant ones. The first base consists of the units sold, $q = D(p)$, and the producer will pay a *per-unit* royalty depending on them. The second base consists of the total gross revenue, $pD(p)$, and the producer will pay a proportion of it in the form of an *ad-valorem* royalty.

To summarize, the timing of the model is as follows. In the first stage both firms choose simultaneously their level of effort. If effort leads to a successful product, in the second stage firms negotiate the licensing agreement. In the last stage, the final price is set by the downstream producer.[10] Notice that the structure of the model implies that contracts are incomplete. Effort is not ex-ante contractible. Furthermore, although firms may set ex-ante the type of contract to be used (the royalty base), the specific royalty rate is chosen only

[9] As we discuss later, differences in these outside profits have effects that are isomorphic to differences in the cost of effort. In particular, $\pi_0^D > \pi_0^U$ will lead to implications equivalent to a lower marginal costs of effort for the downstream producer.

[10] Notice that we do not allow the negotiation of the royalty to be related to the price that will be set in the final market. This assumption is consistent with standard licensing contracts. Furthermore, in a context in which a producer engages in bilateral negotiations with several innovators who own patents that need to be licensed, writing contracts that determine the final price would not be compatible with innovators having different interests.

after the value of the innovation has been uncovered.[11]

In the next subsections we characterize the subgame perfect equilibrium of the game. We start by comparing the equilibrium prices under both royalty bases. We then proceed to study how the incentives to innovate are affected by the royalty base.

2.1 Equilibrium Royalties and Prices

The price that maximizes profits for the producer depends on the royalty base used. Under per-unit royalties the producer incurs in a marginal cost of $c + r$, where we denote as r the per-unit royalty payment that the innovator receives for each unit sold. Profit maximization implies that the producer will charge an optimal price $p^*(r)$ increasing in r. A higher per-unit royalty is passed through a higher final price for the product. The innovator obtains total profits equal to the per-unit royalty rate times the quantity sold which depends on the final price, $rD(p^*(r))$.

We denote the ad-valorem royalty as s. This means that the producer retains a proportion $1-s$ from the revenue of each unit sold. With some abuse of notation we denote the optimal price as $p^*(s)$. As in the case of per-unit royalties, this price is increasing in the royalty rate but for a different reason. When s increases from each unit sold the producer incurs in the same cost of production c but obtains a lower revenue. This lower profitability entices the firm to produce less and raise the price. The innovator receives a share s of the total revenue of the producer, $sp^*(s)D(p^*(s))$.

In order to isolate the different effects at work in the model, we start with the case in which the innovator has all the bargaining power, $\gamma = 1$. This case corresponds to the situation in which the innovator chooses the royalty rate. The next result shows that under very weak regularity assumptions over the demand, ad-valorem royalties always lead

[11] This kind of incompleteness is common in the literature on profit sharing and used in papers like Romano (1994) in the context of retail price maintenance contracts. In standard setting environments, Fair, Reasonable and Non-Discriminatory (FRAND) commitments are well described by this timing.

to lower (or equal) prices than per-unit ones. In other words the double-marginalization problem typical of vertical relations like the one assumed in this model is less severe under ad-valorem royalties.

Proposition 1. *Set $\gamma = 1$ and assume that $D(p)$ is a twice-continuously differentiable demand function with a price elasticity $\eta(p)$ increasing in p. Then, under successive monopolies,*

1. *if an ad-valorem royalty and a per-unit one lead to the same final price, the ad-valorem royalty results in higher innovator's profits.*

2. *The ad-valorem royalty that maximizes innovator's profits leads to a lower final price than the per-unit royalty that maximizes innovator's profits.*

First notice that the result holds for a large family of demand functions, the isoelastic one being a limiting case. It includes typical functions like the linear demand and, more generally, log-concave demand functions, a standard class of demand specifications which guarantee that the problem of the producer is well-behaved.

The first part of the proposition shows that for a given final price an ad-valorem royalty allows the upstream innovator to extract a larger share of the surplus from the relationship with the downstream producer. Remarkably, although this result has never been stated in the context of licensing contracts, it is the counterpart of a classical result in public finance. Early literature on indirect taxation compared taxes based on the units sold (per-unit) or the value of sales (ad-valorem). In the context of a market monopolist Suits and Musgrave (1953) showed that contingent on raising the same revenue, ad-valorem taxes turn out to be less distorting and are, therefore, superior from a social stand-point.

To interpret this result it is useful to review how the trade-off of the producer when choosing the optimal price is resolved as a function of the royalty rate. As mentioned before, an increase both in the per-unit and the ad-valorem royalty leads to an increase in the optimal price but through a different mechanism. A higher per-unit royalty increases the marginal cost by the same amount independently of the quantity produced, $D(p)$. A higher ad-valorem royalty, however, has an impact on each unit sold that depends on the current price. Since the revenue of the firm is concave in the price, any increase beyond the monopoly one implies a larger than linear drop in revenue.

Since the cost of raising the price under ad-valorem royalties is increasing in the price, the downstream producer chooses not to pass through as much of an increase in the royalty rate as in the case of per-unit royalties where the drop in income is proportional to the number of units sold. This implies that the innovator can choose an ad-valorem royalty that extracts a higher share of the total surplus while implementing the same price in the final market that could be achieved under per-unit royalties.

The second part of the proposition characterizes the profit maximizing ad-valorem royalty rate and it shows that it induces a lower final price for the product.[12] When the innovator increases the royalty rate a trade-off arises between the capacity to extract more surplus and the reduction of that surplus as a result of a higher price. Under ad-valorem royalties both the innovator and the producer care about maximizing total revenues $p(s)D(p(s))$, from which each firm obtains a portion. This implies that the incentives are better aligned and the innovator does not want to set a royalty too high because that would cause a significant price increase and depress the share of the surplus that it can extract $sp(s)D(p(s))$. Under per-unit royalties the innovator does not internalize such an effect and induces a higher price in order to extract a higher revenue from each unit sold.

The previous result also indicates that under ad-valorem royalties the innovator is better off. Notice that because the equilibrium price is lower consumer surplus also increases. The

[12] Gaudin and White (2014) derive a counterpart of this result in the context of indirect taxation and show that the result is robust to other assumptions on downstream competition.

effect on the final producer, however, arises from the combination of two opposing forces. On the one hand, ad-valorem royalties raise total profits because they lead to a lower equilibrium price which, irrespective of the royalty base, it is above the monopoly price due to double-marginalization. On the other hand, the innovator retains a higher proportion of these profits.

As an illustration of the previous effects, in Figure 1 we numerically show the case when demand is linear, $D(p) = 1 - p$. In the top three graphs we reproduce the equilibrium prices and profits that arise from the previous expressions. The case when $c = 0$ is particularly illuminating of the different effects at work. Under per-unit royalties a higher income for the innovator comes at the expense of a higher perceived cost by the downstream producer leading to a price above the monopoly one. Under ad-valorem royalties, however, there is no conflict between the innovator and the producer since both firms are interested in maximizing total revenue. Hence, no matter how large is s the downstream producer always chooses the monopoly price.[13] In other words, there is no pass-through of a higher ad-valorem royalty into a higher price. Hence, it is optimal for the innovator to choose $s = 1$. This case constitutes an extreme illustration of the different forces that operate under the two royalty bases in Proposition 1. As the marginal cost increases, the double-marginalization effect becomes more relevant under ad-valorem royalties, narrowing the gap between the resulting price and the one that emerges under per-unit royalties. An implication of this result is that in many technological products for which fixed costs are the main cost component, as opposed to variable production costs, ad-valorem royalties are specially beneficial for consumers.

The figure also shows that upstream profits are higher under ad-valorem royalties while

[13]This result holds, of course, for any demand function since the downstream producer maximizes

$$\max_{p} \,(1 - s)pD(p),$$

leading to an optimal price p^* equal to the monopoly price and, thus, independent of the royalty paid.

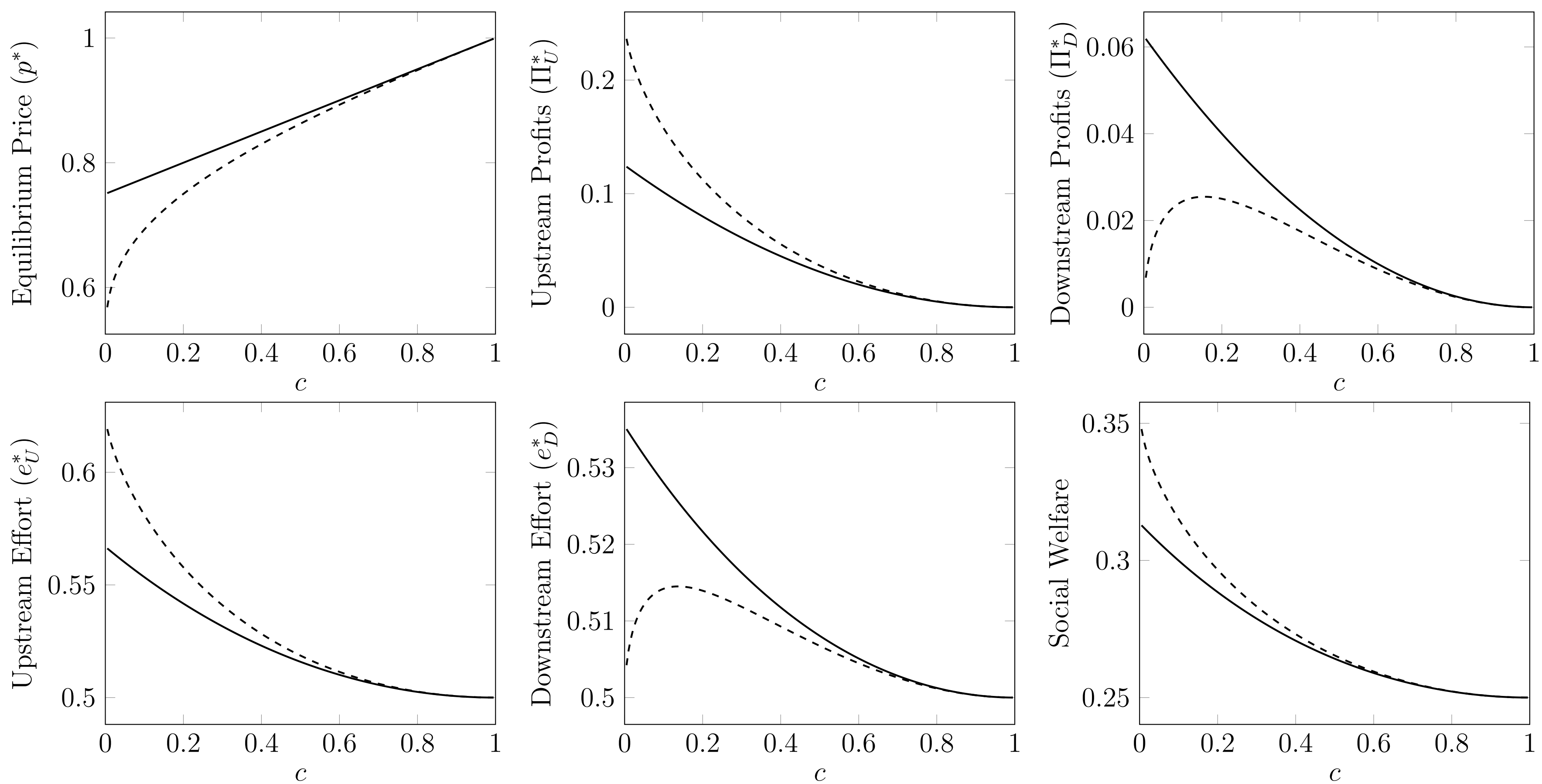

Figure 1: Equilibrium prices, ex-post profits, effort levels, and social welfare with per-unit (solid line) and ad-valorem (dashed line) royalties for changes in the marginal cost c. Benchmark parameters are $\pi_0^U = \pi_0^D = 1$.

the opposite is true for the downstream producer. An interesting result, however, is that ad-valorem royalties generate profits for the producer that are not monotonic in the cost. Using the intuition discussed before about the price, when the marginal cost is low, the innovator can charge a high royalty without distorting much the price. As marginal cost increases, however, this royalty must be decreased in order to keep the price low so that revenues are not excessively depressed, increasing downstream profits.

It is specially interesting to consider the case of the isoelastic demand function $D(p) = p^{-\eta}$ with $\eta > 1$. As Proposition 1 states, this is a corner case and, indeed, in the next proposition we show that for $\gamma = 1$ prices are identical regardless of whether the innovator chooses per-unit or ad-valorem royalties. Profits, however, are different in each case.

Lemma 1. *Consider an isoelastic demand function $D(p) = p^{-\eta}$ with $\eta > 1$ and set $\gamma = 1$. Then,*

1. *the equilibrium price is the same under per-unit and ad-valorem royalties, and*
2. *innovator profits are higher under ad-valorem royalties, and consequently, producer profits are higher under per-unit ones.*

Furthermore, the profits of the innovator under ad-valorem royalties coincide with the profits of the producer under per-unit royalties.

The fact that the equilibrium price is identical under per-unit and ad-valorem royalties implies that the two effects that delivered the second part of Proposition 1 cancel out here. Remember that the optimal royalty rate arises from balancing out the capacity to extract surplus with the reduction of this surplus when prices increase. In the general case, under ad-valorem royalties the innovator chooses to lower the royalty rate – compared to the one that would lead to the same price under per-unit royalties – as a way to lower the final price and increase total revenues. This increase partially benefits from the lower elasticity associated to a lower price and, thus, the larger demand boost. When demand elasticity is constant this last effect does not exist and the incentives to lower the price are reduced to the point that they cancel out with the loss in profits resulting from the lower royalty.

Obviously, if the final price is the same in equilibrium, aggregate profits must also be the same. However, the way they are split is different in each case. The innovator benefits more from ad-valorem royalties. This result is consistent with the first part of Proposition 1 and it would explain why in the court cases we have seen in recent years patent holders defend ad-valorem royalties whereas implementers claim in favor of per-unit ones. Interestingly, the profits of one firm in the case of per-unit royalties coincide with the profits of the other under ad-valorem royalties. This property will become useful in the next section when we discuss the investment decision of both firms.

Our setup also allows us to understand whether different allocations of the bargaining power have different effects on per-unit and ad-valorem royalties. Remember that we assume that the two parties split the surplus according to Nash Bargaining. This means that the royalty maximizes $\Pi_U^{\gamma}\Pi_D^{1-\gamma}$, where Π_U and Π_D are the profits of the upstream innovator and downstream producer, respectively.

In order to be able to compare ad-valorem and per-unit royalties in the context of the model we need to isolate the effect of different allocations of bargaining power from the generic effects discussed in Proposition 1. This turns out to be easy if we focus on the case of an isoelastic demand where, as we saw in the previous lemma, the price is the same in both cases if $\gamma = 1$. The next proposition shows that this equivalence is not, however, a generic result for $\gamma \neq 1$.

Proposition 2. *Under an isoelastic demand function, for any bargaining power $\gamma \in (0, 1)$, per unit royalties lead to strictly higher prices than ad-valorem royalties. For $\gamma = 0$ and $\gamma = 1$ the price is independent of the royalty base.*

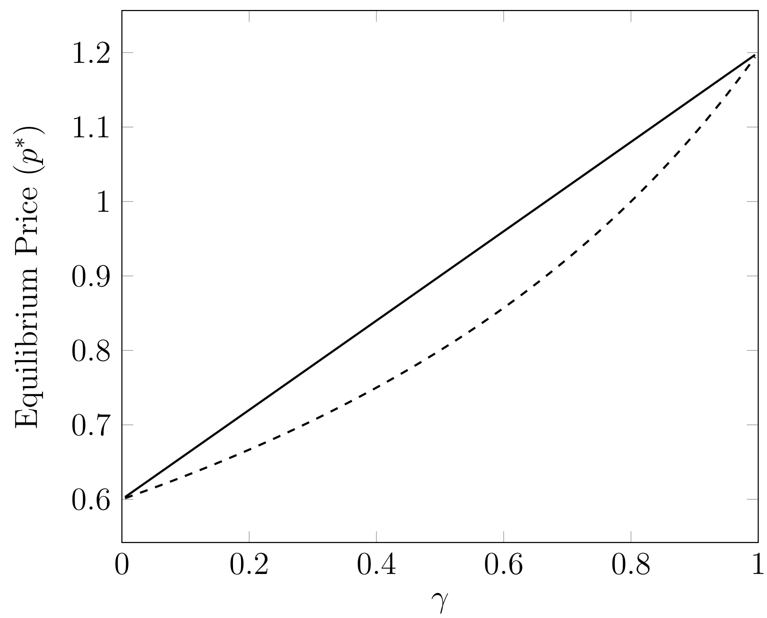

Figure 2: Final price under per-unit (solid line) and ad-valorem (dashed line) royalties given bargaining power γ. The parameter values used are $c = 0.3$, $\eta = 2$.

It turns out that the equilibrium price is the same under both royalty bases only when $\gamma = 1$, as stated in Lemma 1, or $\gamma = 0$. This last case arises from the fact that when all the bargaining power is allocated downstream the producer wants to set the minimal possible royalty, $r^* = s^* = 0$, resulting in the monopoly price in both cases. Interestingly, in the intermediate situation, when $\gamma \in (0, 1)$, the price is systematically higher under per-unit royalties, exacerbating the double-marginalization problem. This difference is illustrated in Figure 2. The reason is that both parties internalize a part of the double-marginalization problem that the vertical relationship generates, and ad-valorem royalties are more effective in aligning the incentives of innovator and producer, as both take into account the effect on the price in the negotiation to a larger extent.

Having characterized the impact on equilibrium downstream prices of alternative royalty base regimes, we now move to the first stage in which firms choose their investment efforts simultaneously. Throughout the rest of the paper we will make the following two assumptions:

Assumption 1. *Demand is isoelastic, $D(p) = p^{-\eta}$, with $\eta > 1$.*

Assumption 2. *The innovator holds all bargaining power, $\gamma = 1$.*

These assumptions introduce analytic tractability and allow us to disentangle the effects of ad-valorem and per-unit royalties analyzed in Proposition 1 from those that will emerge once we consider innovation incentives or other market structures. These assumptions also mean that the positive effects that ad-valorem generate through lower prices will be undervalued.

2.2 First-Stage Effort

In the first stage firms simultaneously choose their investment. As in the previous section, denote the generic expression for profits of the upstream innovator and downstream producer resulting from the royalty and posterior pricing stages as Π_U and Π_D, respectively.

The producer chooses effort e_D to maximize profits from the new product as well as profits from the alternative use of the innovation, net of the cost of this effort. That is, the optimal effort arises from

$$\max_{e_D} \; e_U e_D \Pi_D + e_D \pi_0^D - \frac{1}{2} e_D^2. \tag{1}$$

In a symmetric way, the level of effort e_U that maximizes profits for the innovator can be obtained from

$$\max_{e_U} \; e_U e_D \Pi_U + e_U \pi_0^U - \frac{1}{2} e_U^2. \tag{2}$$

The previous expressions for profits imply that investment (or effort) is more beneficial when profits are higher both from the sale of the product but also from other uses that it might have. The effort of both firms is complementary in the success of the product considered, but independent in connection with the alternative uses of their investments.

The next result is, under Assumption 1 and 2, a direct consequence of the symmetry in profits of both firms under per-unit and ad-valorem royalties.

Proposition 3. *Under an isoelastic demand function and $\gamma = 1$, ex-ante social welfare is higher with ad-valorem royalties if and only if $\pi_0^D \geq \pi_0^U$.*

In order to understand the previous result it is useful to start with the case in which both firms obtain the same profits from the alternative uses of the innovation. Remember that the producer benefits more from the production of the good under per-unit royalties and the innovator benefits more under ad-valorem royalties. Thus, when $\pi_0^D = \pi_0^U$, under per-unit royalties a higher proportion of the investment is carried out by the producer, whereas under ad-valorem royalties a higher proportion is carried out by the upstream innovator. However, from Lemma 1, since profits of the innovator under one royalty scheme are identical to the profits of the producer under the other, the total probability of success, $e_U e_D$, is preserved in both cases leading to identical social welfare in both situations, given that the final price is the same.

Raising π_0^D above π_0^U leads to an increase in the investment of the producer. In equilibrium, due to the complementarity of efforts, the investment of the innovator also increases, albeit by an smaller amount. Facing a convex cost of effort, the producer who is already making a large investment is not affected as much as the innovator by the higher proportion of profits that per-unit royalties allocate downstream. Thus, a switch to ad-valorem royalties increases the probability of success, since it boosts the incentives of the innovator who faces a low marginal cost of effort, while having a smaller negative effect on the producer.[14]

The same results could be obtained if we assumed that outside profits were the same for both firms but the marginal costs of effort were different. In particular, if the cost of effort was lower for the producer so that it would naturally tend to invest more, ad-valorem royalties would be optimal, as they would spur innovator's effort without taxing much producer's effort. For example, if the marginal costs of effort were $C'_U(e_U) = a_U + e_U$

[14]The results would be unchanged if we used consumer or producer surplus as our welfare measure since the price is the same under both royalty bases and so is surplus.

and $C'_D(e_D) = a_D + e_D$, ad-valorem royalties would lead to higher welfare if and only if $a_D \leq a_U$.

The case studied here, satisfying Assumptions 1 and 2, simplifies the exposition but it is easy to see that dropping these assumptions would make ad-valorem royalties superior under more general conditions. In order to see the effects of relaxing Assumption 1, consider the case of the linear demand. The lower part of Figure 1 includes the simulation of effort decisions and ex-ante social welfare assuming that outside profits are identical, $\pi_0^D = \pi_0^U$. Although per-unit royalties spur the investment of the downstream producer and ad-valorem royalties spur the investment of the innovator, the two effects are not of the same magnitude when the demand is not isoelastic. The probability of success is higher under ad-valorem royalties since the double-marginalization effect is smaller when the elasticity of demand $\eta(p)$ is increasing in p and, thus, total profits are higher in this case. The combination of the higher overall incentives to innovate together with a lower price makes social welfare higher under ad-valorem royalties.

Consider now the case in which not all bargaining power is in the hands of the upstream innovator and $\pi_0^D = \pi_0^U$. We know that, at the other extreme, when $\gamma = 0$, ad-valorem and per-unit royalties coincide and they allocate all profits to the producer making both royalty bases equivalent. When the bargaining power of the innovator increases two forces emerge. On the one hand, the lower price and higher total profits that ad-valorem royalties entail feed back into higher overall incentives to invest. On the other hand, ad-valorem royalties tend to allocate more profits to the innovator distorting the incentives of the producer to exert effort. The first force dominates when γ is low, since as we can see in Figure 2 price differences increase as more bargaining power is allocated to the innovator. A lower price increases the marginal return to effort by expanding market size. When γ is low the innovator is likely to choose a low investment due to the low profits and, so, the marginal cost of effort is low. When γ is high, however, price differences shrink as γ increases and the second effect dominates, reducing the advantage of ad-valorem royalties due to the decrease in producer's effort. As Proposition 3 indicates, in the limit, when $\gamma = 1$, the probability of success is the same in both cases.

The previous discussion is also helpful in order to understand the consequences of enlarging the family of contracts we consider. Suppose, in particular, that in our benchmark model we allow for fixed fees, understood as payments that the producer makes independently of the quantity sold (alone or in combination with royalties). In that case, both parties would agree on a contract that includes a 0 royalty together with a fixed fee that splits total surplus according to each firm's bargaining power. As a result, the first force dominates since no double marginalization would take place and total surplus is maximized.

The second force, however, might also become more significant in this case. In particular, suppose that $\gamma = 1$ so that the innovator has all the bargaining power. It is still the case that total surplus is higher under fixed fees than under royalties (either per-unit or ad-valorem). Nevertheless, in that case, the equilibrium fixed fee would extract all profits from the producer who, in anticipation, would choose to exert minimum effort. Thus, in instances in which the effort of the party that holds little bargaining power is important, fixed fees will lead to lower profits and lower incentives to innovate.[15]

3 Multiple Upstream Innovators

In recent years the debate about the royalty base has gained relevance due to disputes over royalty rates in the case of patents related to Standard Setting Organizations (SSOs). Court decisions like the ones discussed in the introduction have generally argued in favor of

[15]Even in less extreme cases, the bad risk-sharing properties of fixed fees makes them unattractive as a way to split surplus in licensing agreements. In practice, they are often observed in combination with royalties but for different reasons. For example, they arise in settlements as a way for licensees to compensate patent holders for previous infringements.

moving from ad-valorem to per-unit royalties. Remarkably, in those decisions the number of innovators that contribute the technologies that are embedded in a product has not been considered relevant. In this section we show that it is precisely in the context of SSOs that ad-valorem royalties are most preferable and the move towards per-unit ones is most harmful.

In this section we extend the previous framework by assuming that there are $N_U > 1$ components necessary for the final product, each one researched by a different innovator. Component i is researched by upstream firm i, for $i = 1, ..., , N_U$. We assume that these components are symmetric so that the cost for innovator i of exerting effort e_U^i is $C(e_U^i) = \frac{1}{2}(e_U^i)^2$. Similarly, the probability of success is symmetric among innovators and equal to

$$E(e_D, e_U^1, .., e_U^{N_U}) = \left(\sum_{i=1}^{N_U} (e_U^i)^\alpha \right)^{\frac{1}{\alpha}} e_D,$$

where $\alpha \in [0, 1]$ measures the degree of independence of the different components. When α is high the effect of effort of innovator i on the productivity of the effort of innovator j is small. As α decreases innovations become more complementary. A low value of α is consistent with the existence of SSOs which aim to coordinate the firms that provide technologies for a product.[16] As before, we assume that the alternative uses of each component lead to profits $\pi_0^U > 0$ for each innovator. In order to abstract from the effects discussed in the previous sections favoring ad-valorem royalties we conduct our analysis under Assumptions 1 and 2.

In the first stage firms simultaneously choose their research effort. The unique downstream producer chooses e_D to maximize

$$\max_{e_D} \ E(e_D, e_U^1, .., e_U^{N_U}) \times \Pi_D + e_D \pi_0^D - \frac{1}{2} e_D^2.$$

Innovator i chooses e_U^i to maximize,

$$\max_{e_U^i} \ E(e_D, e_U^1, .., e_U^{N_U}) \times \Pi_U + e_U^i \pi_0^U - \frac{1}{2}(e_U^i)^2.$$

[16]Nevertheless, we assume that all components are required for the final product, as in the case of Standard Essential Patents (SEPs). In section 4 we discuss the effects of relaxing this assumption and allowing for innovations to be substitutes.

The same kind of complementarity discussed in the case of one innovator operates here between different innovators. Keeping profits from marketing the product constant, increases in N_U raise the probability of success of the innovation and thus the incentives of all parties to invest. Of course, profits in the final stages of the game will change as N_U increases, affecting this result. We now analyze the direction of these changes and, in particular, how the profits of the innovators and the producer are shaped under the different bases for the royalty rate as N_U increases. As in the case of one innovator, we start by comparing equilibrium prices and profits under per-unit and ad-valorem royalties.

3.1 Equilibrium Royalties and Prices

With per-unit royalties the producer has a marginal cost $c + R$ where $R \equiv \sum_i^{N_U} r_i$. That is, the producer only cares about the total royalty charged by all innovators. That marginal cost determines the producer's profit-maximizing price. This is a standard problem that has been extensively discussed in the literature, starting in Shapiro (2001), and its implications are well-known. Innovators choose their royalty rate to maximize their own profits without internalizing the fact that the higher is their royalty the lower will be not only their sales but also the sales of the other innovators. This negative externality implies that innovators will choose a royalty that will be too high, even higher than the one that an upstream monopolist would decide if it owned all innovations. This effect has been denoted as *royalty stacking* and it is typical of all markets in which firms sell complementary products.

Under ad-valorem royalties the total royalty rate is again $S \equiv \sum_i^{N_U} s_i$. As in the previous case, the producer cares only about the total royalty paid and the proportion of the revenue that it can keep, $(1 - S)pD(p)$. This means that the royalty stacking effect that we discussed in the case of per-unit royalties will also arise here. As the next proposition shows, however, the strength of this effect differs in both cases.

Proposition 4. *Consider the case in Assumptions 1 and 2. Suppose that innovation efforts have been successful. Under ad-valorem royalties the equilibrium price in the final market is always lower. Furthermore, innovators' profits are higher under ad-valorem royalties whereas producer's profits are higher under per-unit royalties.*

Some of the results obtained in the one innovator case are preserved here. In particular, it still true that ad-valorem royalties tend to favor innovators, compared to the producer, whereas the opposite is true for per-unit royalties. An important difference, however, is that ad-valorem royalties lead to lower downstream prices even under an isoelastic demand function. Thus, ex-post consumer surplus is always higher under ad-valorem royalties.

For the reasons discussed in Proposition 1 if under per-unit and ad-valorem royalties the price in the final market were the same innovators would obtain higher profits under the latter. As in the case of one innovator, the optimal royalty trades off the effect on the final price and total surplus with the part of the surplus that the firm can appropriate. We argued that ad-valorem royalties allocated a larger proportion of the surplus to the innovator and the marginal gains from raising the royalty were low, reducing the equilibrium distortions.

The only exception was the case of an isoelastic demand where the innovator had more incentives to raise the royalty because this generated less of an effect on total revenues. The result in this proposition shows that when $N_U > 1$ even with an isoelastic demand we obtain a lower equilibrium price under ad-valorem royalties. The reason is that, compared to the case of one innovator, increasing the royalty rate is less profitable since it generates a distortion that reduces total surplus but the proportion of the total revenue it can appropriate is smaller since total revenue is divided among all innovators.

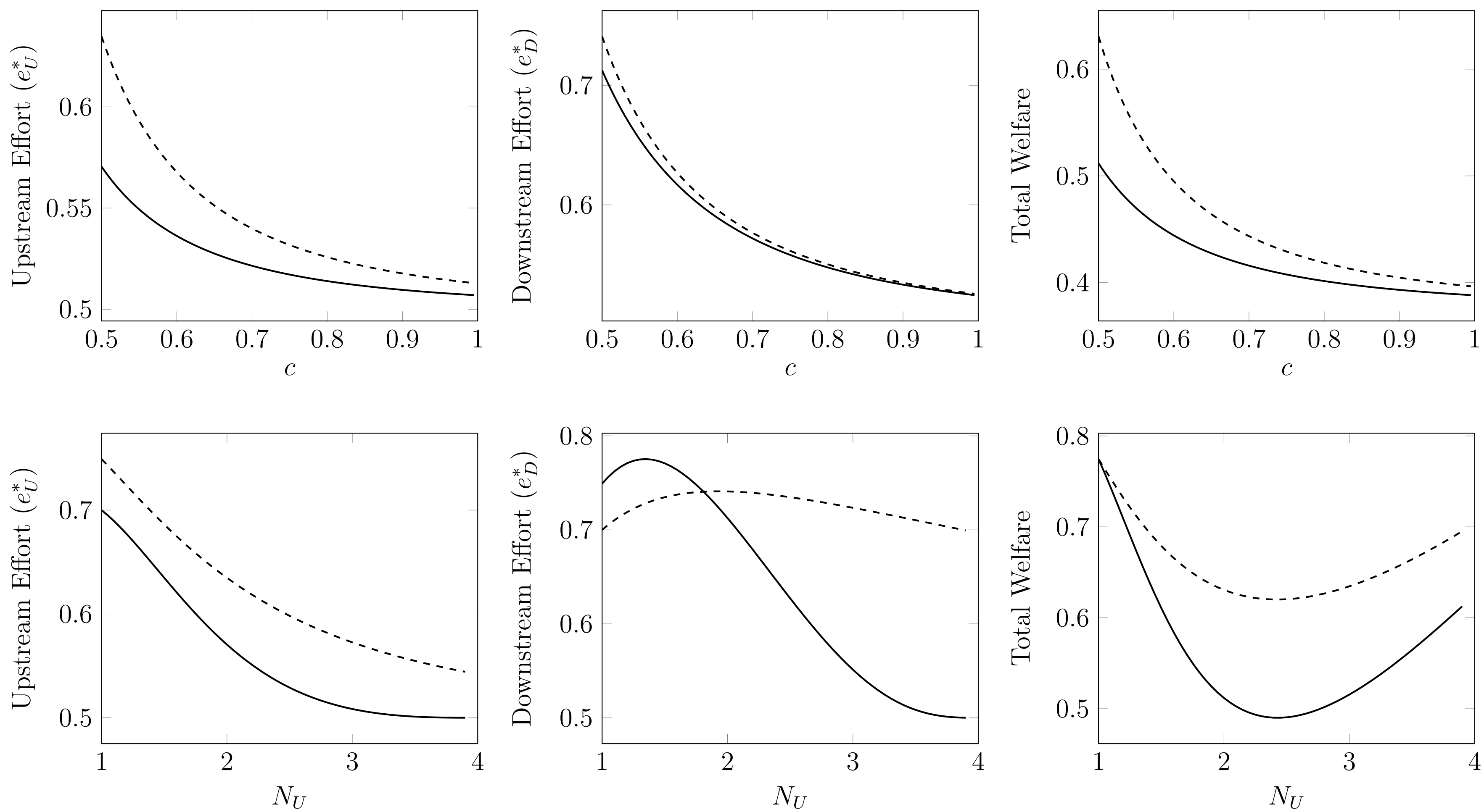

Figure 3: Effort by upstream innovators, the downstream producer, and total welfare under the use of per-unit (solid line) and ad-valorem (dashed line) royalties for changes in the marginal cost c (above) and the number of upstream developers, N_U (below). The baseline parameter values are $\pi_0^D = \pi_0^U = 0.5$, $\alpha = 0.7$, $N_U = 2$, $\eta = 4$, and $c = 0.5$

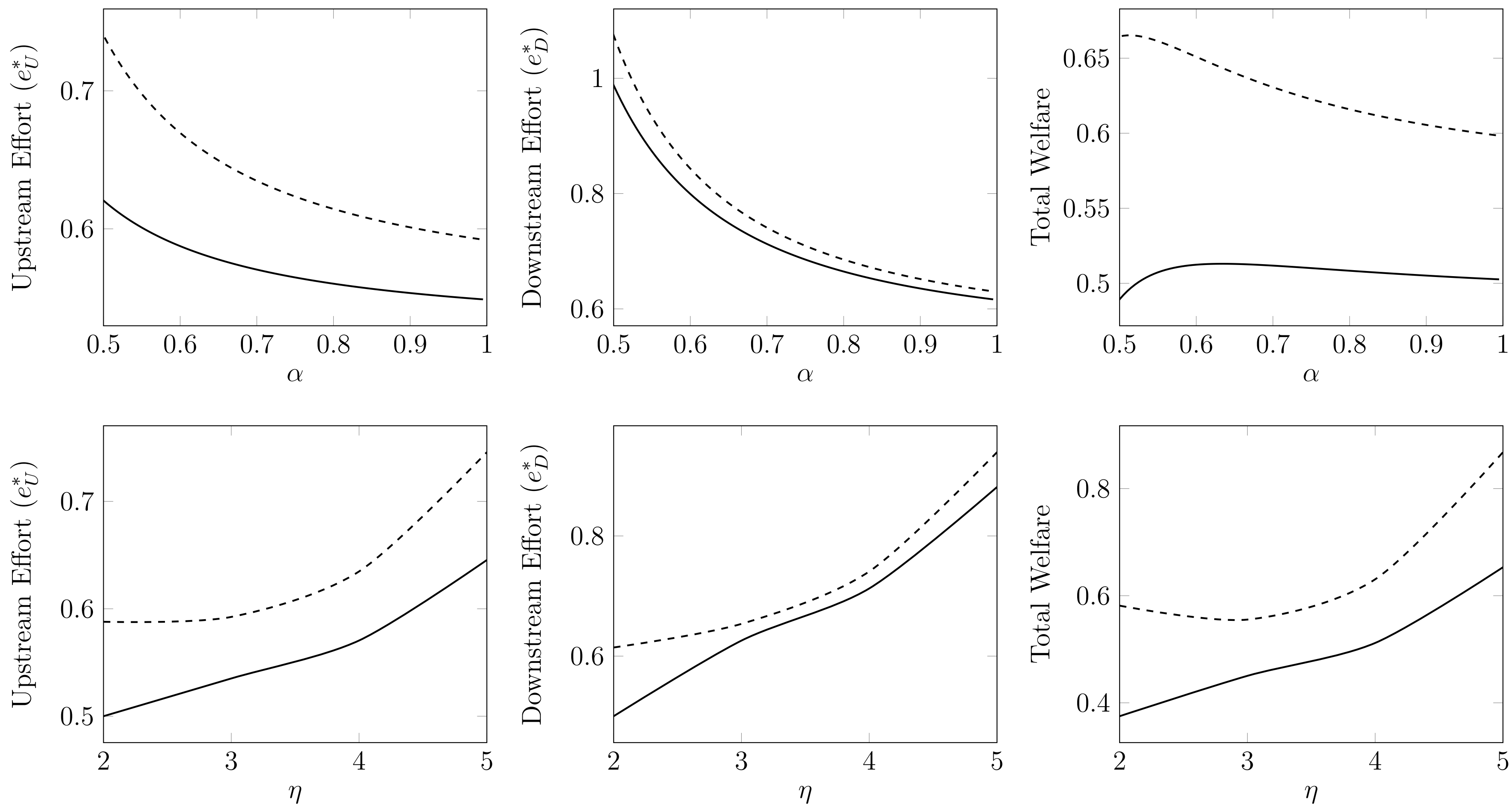

Figure 4: Effort by upstream innovators, the downstream producer, and total welfare under the use of per-unit (solid line) and ad-valorem (dashed line) royalties for changes in the substitubility between innovations α (above), and the elasticity of the demand, η (below). The baseline parameter values are $\pi_0^D = \pi_0^U = 0.5$, $\alpha = 0.7$, $N_U = 2$, $\eta = 4$, and $c = 0.5$

3.2 First-Stage Effort

In order to understand ex-ante incentives, we need to rely on numerical analysis. In Figures 3 and 4 we provide an example of the effects of the different parameters of the model over the equilibrium decisions in the first stage of the model, upstream and downstream effort, together with social welfare.

The results indicate that ad-valorem royalties typically translate into higher investment by upstream innovators. This effect is particularly important for low values of c and α. In the case of the former, as in the one innovator case, the result stems from the fact that the double-marginalization effect under ad-valorem royalties is small when c is low, implying a large optimal royalty s^* and large upstream profits which spur investment. For the latter, notice that low values of α imply that the investment of different innovators are more complementary. As a result, the higher profits that ad-valorem royalties imply are reinforced by the increased investment of other innovators.

In the case of the downstream producer the equilibrium effect on investment is the combination of two forces. On the one hand, ad-valorem royalties lead to lower downstream profits contingent on success, as illustrated in Proposition 4, which feed back into lower incentives to invest. On the other hand, as mentioned in the previous paragraph, innovators invest more and, due to the complementarity of investments assumption, the marginal productivity of downstream investment raises. The numerical results indicate that the second force typically dominates and downstream investment also raises under ad-valorem royalties. The only exception corresponds to the case in which $N_U < 2$. As we know from the previous section, when there is only one innovator under ad-valorem royalties producer profits are lower and so are the incentives to innovate.

Total welfare is generally higher under ad-valorem royalties. This result is due to two reasons. First, ad-valorem royalties lead to lower final prices, favoring consumers. Second, they typically generate an increase in total investment and in the resulting probability of product success. The difference in total surplus is particularly large when we consider more complementary components and the impact of ad-valorem royalties over the incentives to invest by innovators is highest.

4 Extensions

As we discuss next, the results in this paper are robust to numerous changes in the model. Here we discuss two that are developed in more detail in the working paper version.

4.1 Weakly Complementary Components

The implicit assumption of the previous section was that the success of all components was necessary for a product to be marketable. In the context of some SSOs, however, this might not always be the case. The standard often includes components that are optional and downstream producers might decide whether to license them or not. This reduces the market power of innovators as their royalty demands might exclude their technology from the standard.

In the working paper version of this work we address this case by assuming that each innovation allows the producer to reduce the marginal cost of production but none of them is essential for the final product. Innovations have a varying degree of complementarity (or even substitubility) in terms of cost reductions and we show that the main result of the paper, namely, the lower price that ad-valorem royalties entail remains unchanged.

We show, however, that an additional effect emerges. When complementarity of innovations is low the total royalty stack is small and so is the price. As the complementarity increases both the total royalty and the equilibrium price increase for the usual reasons. However, when complementarity is sufficiently high, the price decreases because of the significant reductions in the cost that more technologies generate, inducing a decrease in the price that compensates for the increased royalty rate. While ad-valorem royalties always lead to lower prices, the difference with per-unit ones is particularly important for intermediate values of complementarity, precisely when the royalty stack is largest.

This result also suggests that in the case of products that embed numerous technologies and for which the complementarities are very important, such as the ones typically coordinated by SSOs, the endogenous decisions of which innovators conform the standard might mitigate the royalty stacking problem.

4.2 Downstream Competition

The main driver of our model is the different pass-through that the ad-valorem and per-unit royalty rates induce in the final market. When there is perfect competition in the final market per-unit and ad-valorem royalties are identical since they generate the same pass-through, at least when there is a unique innovator. If we consider that N_D identical firms compete in quantities, we can transition between those extreme cases in a monotonic way.

Once we endogenize the effort decisions in the first stage the results do not change qualitatively from those described in our benchmark model. In the case of several innovators with complementary components we showed that ad-valorem royalties, by allocating higher profits upstream, generated more effort due to the feedback loop between the complementary decisions of these innovators. When there are multiple downstream producers, however, allocating profits downstream through per-unit royalties does not generate such a positive feedback. The reason is that producers sell substitute products. As a result, an increase in downstream profits generates an increase in the profits from being a successful monopolist in the market but also a lower probability that the firm becomes a monopolist and can benefit from that increase in profits. This second force limits the investment incentives of the producers.

5 Concluding Remarks

In this paper we have shown that, under many circumstances, ad-valorem royalties which are based on the value of sales yield superior outcomes from both consumer welfare and total welfare standpoints than per-unit royalty rates, based on the value of the components of the infringing product that are covered by the patented technology. In our analysis, ad-valorem royalties are better from the consumer welfare and total welfare perspectives for two reasons. First, they mitigate the double marginalization problem that naturally arises in technology markets characterized by market power at the licensing and manufacturing levels of the vertical chain. This is because innovators internalize to a greater extent the impact on the product price of an increase in their royalty rate when this royalty is based on firm revenues. Second, investments by both innovators and producers are typically greater under ad-valorem royalties because overall industry profits, and hence the incentives to innovate, are greater when the double marginalization problem is less severe. This effect is stronger, making ad-valorem royalties even more attractive, when bargaining power is distributed between both parties or when there are multiple innovators licensing complementary technologies. Because ad-valorem royalties shift profits upstream, they partially compensate the tendency to underinvest by owners of complementary technologies.

Our results can be used to develop normative implications. Courts have emphasized the advantages of using per-unit royalties, for example, as a way to prevent royalty payments that are considered *too* high when the royalty rate applies to the price of the product. We show that these frictions need to be weighted against the effects that mandating per-unit royalties would have, particularly in the context of SSOs and their standard essential patents. Although it is in those environments in which the frictions that courts are trying to prevent might become more apparent, it is also in those environments in which ad-valorem royalties might help deliver a lower royalty stack, more incentives to innovate and, as a result, greater consumer and social welfare.

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A Proofs

Lemma A.1. *A twice continuously differentiable profit function* $\Pi(p) = (p-c)D(p)$ *is quasiconcave if the elasticity of the demand* $\eta(p) = -\frac{D'(p)p}{D(p)}$ *is increasing in* p.

Proof of Lemma A.1: A single variable function $\Pi(p)$ is quasiconcave if (i) it is either always increasing or always decreasing in p or (ii) if there exists a p^* such that $\Pi(p)$ is increasing for $p < p^*$ and decreasing for $p > p^*$.

Let's assume, towards a contradiction that neither of the two previous conditions is true. In that case, at least one of the solutions to the first-order condition must characterize a minimum. This first-order condition can be written as the standard Lerner index

$$\frac{p^* - c}{p^*} = \frac{1}{\eta(p^*)}$$

which shows that when $\eta(p)$ is increasing in p there can be at most one solution, since the left-hand side is always increasing while the right-hand side is always decreasing.

We now show that this unique critical value p^* always defines a maximum. In order to do that, notice that

$$\frac{\partial \eta}{\partial p} = -\frac{D'(p)D(p) + pD''(p)D(p) - D'(p)^2}{D^2(p)}.$$

This expression is positive if and only if

$$D''(p) \leq \frac{D'(p)^2}{D(p)} - \frac{D'(p)}{p},$$

We can now compute

$$\Pi''(p^*) = D''(p^*)(p^* - c) + 2D'(p^*) < D'(p^*)\left(1 - \frac{1}{\eta(p^*)}\right) = D'(p^*)\frac{p^* - c}{p^*} < 0,$$

where first inequality comes from the upper bound on $D''(p)$ originating from the previous expression. Thus, the first-order condition determines a maximum and the profit function is quasi-concave. □

Lemma A.2. *Assume that* $D(p)$ *is a twice-continuously differentiable demand function with a price elasticity* $\eta(p)$ *increasing in* p. *Then, under per-unit royalties the optimal price has an upper bound*

$$p^*_{pu} \leq \frac{(c + r^*)^2}{c}.$$

Proof of Lemma A.2: Using Lerner's rule we have that under per-unit royalties the optimal price is determined as

$$\left(1 - \frac{1}{\eta(p^*_{pu})}\right) p^*_{pu} = c + r, \tag{A.1}$$

where we have made explicit the dependency of the demand elasticity η on p. Using the Implicit Function Theorem we have that

$$\frac{\partial p^*_{pu}}{\partial r} = \frac{1}{\left(1 - \frac{1}{\eta(p)}\right) + \frac{p}{\eta(p)^2}\eta'(p)} > 0. \tag{A.2}$$

In the first stage the innovator chooses the royalty to maximize $rD(p^*_{pu}(r))$, resulting in a first-order condition

$$D(p^*_{pu}(r^*)) + r^* D'(p^*_{pu}(r^*))\frac{\partial p^*_{pu}}{\partial r} = 0.$$

Solving for r^* and after replacing (A.2) we have that

$$r^* = -\frac{D(p^*_{pu}(r^*))}{D'(p^*_{pu}(r^*))\frac{\partial p^*_{pu}}{\partial r}} = \frac{p^*_{pu}\left(1 - \frac{1}{\eta(p^*_{pu})}\right) + \frac{(p^*_{pu})^2}{\eta(p^*_{pu})^2}\eta'(p^*_{pu})}{\eta(p^*_{pu})}.$$

Substituting r^* in the first-order condition for the downstream producer, (A.1), and rearranging terms we obtain

$$\left(1 - \frac{1}{\eta(p^*_{pu})}\right)^2 p^*_{pu} = c + \frac{(p^*_{pu})^2}{\eta(p^*_{pu})^3}\eta'(p^*_{pu}).$$

We can now replace the left-hand side expression by $\frac{(c+r)^2}{cp^*_{pu}}$ and rearrange terms to obtain

$$p^*_{pu} = \frac{(c + r^*)^2}{c + \frac{(p^*_{pu})^2}{\eta(p^*_{pu})^3}\eta'(p^*_{pu})} \leq \frac{(c + r^*)^2}{c}$$

where the last inequality arises from $\eta'(p) \geq 0$. □

Proof of Proposition 1: For the first part, take a per-unit royalty r. The profit maximizing price for the downstream producer satisfies

$$\left(1 - \frac{1}{\eta}\right) D(p^*_{pu}) = c + r, \tag{A.3}$$

whereas with ad-valorem royalties the same price could be reached with a royalty s such that

$$(1 - s)\left(1 - \frac{1}{\eta}\right) D(p^*_{av}) = c. \tag{A.4}$$

Notice that to simplify notation we have dropped the argument in the demand elasticity η. From Lemma A.1 these first-order conditions are necessary and sufficient. Hence, the two royalty schemes lead to the same price, $p^*_{pu} = p^*_{av} = p^*$ if and only if

$$r = \frac{s}{1-s}c.$$

Upstream profits under per-unit royalties are

$$\Pi_{U,pu}(r) = rD(p^*) = s\frac{c}{1-s}D(p^*) < sp^*D(p^*) = \Pi_{U,av}(s),$$

where the last inequality comes from the fact that $(1-s)p^* > c$

For the second part, take the optimal per-unit royalty r^* and the equilibrium price $p^*_{pu} = p^*$. As shown before, this same price could be induced under an ad-valorem royalty $\hat{s} = \frac{r}{c+r}$. Given the concavity of the profits of the innovator it is enough to show that the derivative of the profits of the upstream producer evaluated at $\hat{s}$ is positive. In particular,

$$\frac{\partial \Pi_{U,av}}{\partial s}(\hat{s}) = p^*D(p^*) + \hat{s}\left[pD'(p^*) + D(p^*)\right]\frac{\partial p^*}{\partial s}.$$

Using the Implicit Function Theorem on (A.3) and (A.4) and using the fact that under $\hat{s}$ the equilibrium price is the same we have that

$$\frac{\partial p^*_{av}}{\partial s} = \frac{\left(1-\frac{1}{\eta}\right)D(p^*)}{1-\hat{s}}\frac{\partial p^*}{\partial r} = \frac{c+r^*}{1-\hat{s}}\frac{\partial p^*}{\partial r}.$$

Replacing in the previous first-order condition together with the expression for $\hat{s}$ and the fact that $p^*D'(p^*) + D(p^*) = (c+r)D'(p^*)$ from the optimality of p^* we have that

$$\frac{\partial \Pi_{U,av}}{\partial s}(\hat{s}) = p^*_{av}D(p^*_{av}) + \frac{r^*(c+r^*)^2}{c}D'(p^*)\frac{\partial p^*}{\partial r}.$$

Finally, notice that the first-order condition that pins down r^* implies that $D'(p^*)\frac{\partial p^*}{\partial r} = \frac{D(p^*)}{r^*}$ and replacing we have that

$$\frac{\partial \Pi_{U,av}}{\partial s}(\hat{s}) = \left[p^*_{av} - \frac{(c+r^*)^2}{c}\right]D(p^*_{av}) < 0$$

where the last inequality comes from Lemma A.2. □

Proof of Lemma 1: Consider an isoelastic demand function $D(p) = p^{-\eta}$ with $\eta > 1$. Given a per-unit royalty r the monopoly price corresponds to $p^*(r) = \frac{(c+r)\eta}{\eta-1}$. Replacing in the profit function of the innovator we have that the optimal per-unit royalty corresponds to $r^* = \frac{c}{\eta-1}$. The final price can be computed as $p^*_{pu} = c\left(\frac{\eta}{\eta-1}\right)^2$, where pu stands for per-unit royalties. Profits become

$$\Pi^*_{U,pu} = \frac{(\eta-1)^{2\eta-1}}{\eta^{2\eta}} c^{1-\eta},$$
$$\Pi^*_{D,pu} = \frac{(\eta-1)^{2(\eta-1)}}{\eta^{2\eta-1}} c^{1-\eta},$$

implying that per-unit royalties allocate a higher share of the total surplus to the downstream producer, $\Pi^*_{D,pu} > \Pi^*_{U,pu}$.

Under ad-valorem royalties the monopoly price that the producer sets is equal to $p^*(s) = \frac{\eta c}{(1-s)(\eta-1)}$. Replacing in the profit function of the innovator we have that the equilibrium royalty rate corresponds to $s^* = \frac{1}{\eta}$. The equilibrium price is equal to $p^*_{av} = c\left(\frac{\eta}{\eta-1}\right)^2$. Thus, $p^*_{pu} = p^*_{av}$. However, profits are different. In particular,

$$\Pi^*_{U,av} = \frac{(\eta-1)^{2(\eta-1)}}{\eta^{2\eta-1}} c^{1-\eta},$$
$$\Pi^*_{D,av} = \frac{(\eta-1)^{2\eta-1}}{\eta^{2\eta}} c^{1-\eta}.$$

That is, $\Pi^*_{U,av} = \Pi^*_{D,pu}$ (and, thus, $\Pi^*_{D,av} = \Pi^*_{U,pu}$). □

Proof of Proposition 2: Contingent on successful development and given a royalty r, calculations included in the proof of Lemma 1 imply that the final price is $p^*(r) = (c+r)\frac{\eta}{\eta-1}$. Profits correspond to $\Pi_U(r) = r(p^*(r))^{-\eta}$ and $\Pi_D(r) = (p^*(r) - (c+r))p^*(r)^{-\eta}$. Hence, the equilibrium royalty results from

$$r^* = \arg\max_r \; \Pi_U(r)^\gamma \Pi_D(r)^{1-\gamma},$$

or $r^* = \frac{\gamma c}{\eta-1}$. The equilibrium price becomes $p^*_{pu} = c\frac{\eta(\eta+\gamma-1)}{(\eta-1)^2}$, increasing in γ.

Under ad-valorem royalties, given a royalty s, the final price is $p^*(s) = \frac{c\eta}{(1-s)(\eta-1)}$. Profits can be written as $\Pi_U(s) = s(p^*(s))^{1-\eta}$ and $\Pi_D(r) = ((1-s)p^*(s) - c)p^*(s)^{-\eta}$. The equilibrium royalty results from

$$s^* = \arg\max_s \; \Pi_U(s)^\gamma \Pi_D(s)^{1-\gamma},$$

or $s^* = \frac{\gamma}{\eta}$. The final price is then $p^*_{av}(s) = c\frac{\eta^2}{(\eta-1)(\eta-\gamma)}$, again, increasing in γ.

The comparison of the two prices delivers the result. □

Proof of Proposition 3: Given that prices in both cases are identical, $p_{av}^M = p_{pu}^M = p^M$, consumer surplus contingent on success is also the same and we denote it as CS. Consider a given level of profits for the innovator and producer Π_U and Π_D, respectively. The social welfare function can be written as

$$W(\Pi_D, \Pi_U, \pi_0^U, \pi_0^D) = e_U^* e_D^* (CS + \Pi_U + \Pi_D) + e_U^* \pi_0^U + e_D^* \pi_0^D - \frac{(e_u^*)^2}{2} - \frac{(e_D^*)^2}{2}, \tag{A.5}$$

where e_U^* and e_D^* are the equilibrium levels of effort arising from (1) and (2),

$$e_U^* = \frac{\pi_0^D \Pi_U + \pi_0^U}{1 - \Pi_U \Pi_D}, \tag{A.6}$$

$$e_D^* = \frac{\pi_0^U \Pi_D + \pi_0^D}{1 - \Pi_U \Pi_D}, \tag{A.7}$$

if $\Pi_U \Pi_D < 1$ and $e_U^* = e_D^* = \infty$ otherwise. This latter case can be safely ignored since changes in the royalty base would have no effect on effort.

Notice that $CS + \Pi_U + \Pi_D$ is constant in both cases since $\Pi_{U,pu}^* + \Pi_{D,pu}^* = \Pi_{U,av}^* + \Pi_{D,av}^*$. Furthermore, equilibrium effort levels are the same if $\pi_0^U = \pi_0^D$, implying that the expected total surplus is the same in both cases.

Let $\pi_0^U \geq \pi_0^D$ and consider the following problem:

$$\max_{\lambda} \tilde{W}(\lambda) = \max_{\lambda} \lambda W_{pu}(\pi_0^U, \pi_0^D) + (1 - \lambda) W_{av}(\pi_0^U, \pi_0^D)$$

where $W_{pu}(\pi_0^U, \pi_0^D) \equiv (\Pi_{D,pu}, \Pi_{U,pu}, \pi_0^U, \pi_0^D)$ and $W_{av}(\pi_0^U, \pi_0^D) \equiv (\Pi_{D,av}, \Pi_{U,av}, \pi_0^U, \pi_0^D)$ correspond to welfare under per-unit and ad-valorem royalties. It is enough to show that $\tilde{W}$ is supermodular in λ and π_0^u and this implies that

$$\frac{\partial W}{\partial \pi_0^U}(\Pi_D, \Pi_U, \pi_0^U, \pi_0^D) - \frac{\partial W}{\partial \pi_0^U}(\Pi_U, \Pi_D, \pi_0^U, \pi_0^D) > 0$$

with $\Pi_D > \Pi_U$ and $\Pi_U + \Pi_D$ and $\Pi_U \Pi_D$ constant. The first term (and by symmetry the second) can be computed as

$$\frac{\partial W}{\partial \pi_0^U}(\Pi_D, \Pi_U, \pi_0^U, \pi_0^D) = \frac{\pi_0^D(\Pi_U \Pi_D + 1) + 2\pi_0^U \Pi_D}{(1 - \Pi_U \Pi_D)^2} CS + \frac{\pi_0^D(\Pi_U^2 \Pi_D + \Pi_D) + \pi_0^U(\Pi_D \Pi_U + \Pi_D)}{(1 - \Pi_U \Pi_D)^2}.$$

The difference among the two corresponds to

$$(\Pi_D - \Pi_U)\left[\frac{2\pi_0^U}{(1 - \Pi_U \Pi_D)^2} CS + \frac{\pi_0^D(1 - \Pi_U \Pi_D) + \pi_0^U(\Pi_U + \Pi_D)}{(1 - \Pi_U \Pi_D)^2}\right] > 0.$$

$\square$

Proof of Proposition 4: Under per-unit royalties, the optimal downstream price is equal to $p^*(R) = \frac{\eta}{\eta-1}(c+R)$. Innovator $i = 1, ..., N_U$ chooses the royalty rate r_i to maximize

$$\max_{r_i} \; r_i \left(p^*(R)\right)^{-\eta}.$$

Focusing on a symmetric equilibrium in which $r_i = r^*$ for all i we have that

$$r^* = \frac{c}{\eta - N_U} \text{ and } p^*_{pu} = \frac{\eta^2}{(\eta-1)(\eta-N_U)}c,$$

which are defined only if $\eta > N_U$. This royalty is increasing in N_U. Profits can be computed as

$$\begin{aligned}
\Pi^*_{U,pu} &= \frac{(\eta-N_U)^{\eta-1}(\eta-1)^{\eta}}{\eta^{2\eta}}c^{1-\eta}, \\
\Pi^*_{D,pu} &= \frac{(\eta-N_U)^{\eta-1}(\eta-1)^{\eta-1}}{\eta^{2\eta-1}}c^{1-\eta},
\end{aligned}$$

which are decreasing in N_U due to the previous effect.

Under ad-valorem royalties the optimal price corresponds to $p^M(S) = \frac{\eta}{(1-S)(\eta-1)}c$. Innovator i chooses s_i in the first stage to maximize

$$\max_{s_i} \; s_i \left(p^*(S)\right)^{1-\eta}.$$

Focusing on a symmetric equilibrium in which $s_i = s^*$ for all i we have that

$$s^* = \frac{1}{\eta + N_U - 1} \text{ and } p^*_{av} = \frac{\eta(\eta+N_U-1)}{(\eta-1)^2}c,$$

where as before the price is increasing in N_U. Profits can be computed as

$$\begin{aligned}
\Pi^*_{U,av} &= \frac{(\eta-1)^{2\eta-2}}{\eta^{\eta-1}(\eta+N_U-1)^{\eta}}c^{1-\eta}, \\
\Pi^*_{D,av} &= \frac{(\eta-1)^{2\eta-1}}{\eta^{\eta}(\eta+N_U-1)^{\eta}}c^{1-\eta}.
\end{aligned}$$

The ordering of the prices arises from the comparison between p^*_{pu} and p^*_{av}, which is strict when $N_U > 1$. Regarding profits, notice that

$$\frac{\Pi^*_{D,pu}}{\Pi^*_{U,pu}} = \frac{\eta}{\eta-1} = \frac{\Pi^*_{U,av}}{\Pi^*_{D,av}},$$

proving the second part of the result, since $\frac{\eta}{\eta-1} > 1$. $\square$