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On the State of the Union

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An overlapping generations model of marriage and divorce is constructed to analyze family structure and intergenerational mobility. Agents differ by sex, marital status, and human capital. Single agents meet in a marriage market and decide whether to accept or reject proposals to wed. Married couples must decide whether to separate or not. Parents invest in their children depending on their wherewithal. A simulated version of the theoretical prototype can generate an equilibrium with a significant number of female-headed families and a high degree of persistence in income across generations. To illustrate the model's mechanics, the effects of two antipoverty policies, namely child support and welfare, are investigated.

I. Introduction

On what basis do people choose to get married and divorced? How do they decide on the amount of time and resources to invest in their children? To what extent do the making and breaking of couples influence intergenerational mobility? How do different types of anti-

Helpful comments from Sherwin Rosen, Aloysius Siow (who was a referee), and another referee are gratefully acknowledged. As the final touches were being placed on the first draft of this paper, S. Rao Aiyagari was taken from the profession. His career was just beginning to wax. Who knows how brilliant the light would have been that shone from him? It is an honor to have worked with Rao. He is missed. It is hoped that some of his spirit lives on here.

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poverty programs influence matching, divorce, investment in children, and intergenerational mobility? The task here will be to develop a prototypical general equilibrium search model of marriage and divorce in which agents differ by sex and human capital levels to address these questions. Agents of the opposite sex meet in a marriage market and decide whether to accept a match as a mate. The utility from a marriage arises from love, the consumption of home-produced goods, and human capital investment in children. Each period, children from the oldest generation enter the marriage market with the human capital levels that they obtained during their childhood; simultaneously, adults from the oldest generation die. The implications of this model for the marital status of the population and for intergenerational mobility are analyzed.

The analysis follows the view of Becker (1991) that marriage is a partnership for joint production and consumption. The emphasis here, though, is on the importance of competitive forces determining the formation and dissolution of families from a general equilibrium perspective. The inquiry here treads in the steps of Mortensen's (1988) search-theoretic model of marriage and divorce; see Weiss (1997) for an excellent discussion of the various approaches that can be taken to marriage and divorce. It is not new that there is a relationship between inequality and the level of human capital investment by parents in their children, and that such investment may lead to persistence of fortune among generations of the same family. This has been studied by Loury (1981) and Becker and Tomes (1993). These studies on human capital investment, however, use one-sex models in which a single parent makes decisions on how much to invest in his or her children. Yet marriage market dynamics can have important implications for economic inequality and for the persistence of economic status. The need to integrate marriage into the analysis of intergenerational mobility has already been noted by Becker and Tomes. The goal here is to fill in this gap.

Furthermore, a dynamic general equilibrium model of marriage and intergenerational mobility is a natural tool that could be used to analyze some very important public policy issues, such as child support payments and welfare. On the one hand, a policy such as welfare helps female-headed households by providing them with badly needed resources, and this reduces poverty. On the other hand, the U.S. welfare system has long been criticized for promoting the dissolution of the family by sponsoring female headship. To illustrate the theoretical prototype's workings, the effects of child support payments and welfare are analyzed. While these experiments show potential uses for the model, the prototype is still very crude. It needs to be improved in many ways—some of which are discussed—

before any serious policy analysis can be done. Future generations of this type of model may some day serve this purpose.

Some empirical motivation.—Why are the questions posed at the outset interesting to ask? Consider some observations about marriage, divorce, and intergenerational mobility in the United States. First, since the 1970s, only about 65 percent of adults are married at a given time. The rest are either single, divorced, or widowed. The fraction of people that are married has been falling over time. Second, as a consequence, about 17.5 percent of households with children are headed by a single female.

Third, this would not be a matter for concern but for the fact that children from single-parent families are less likely to be successful than children living with two parents. A recent study by McLanahan and Sandefur (1994) shows that children living in single-parent households are more likely than children from two-parent families to drop out of high school (25 percent vs. 15 percent), to be idle (29 vs. 19 percent), and to experience teen births (31 vs. 14 percent) and are less likely to go to college (48 vs. 51 percent, if they complete high school). What economic factors might be important in accounting for these differences? In 1995, the median income for female-headed families with children was about one-third of the median income for married couples with children. Moreover, 32.4 percent of all female-headed families were below the poverty line; the same figure for married couples was 5.6 percent. Sandefur (1996) calculates that 52 percent of all female heads with children were participating in the Aid to Families with Dependent Children (AFDC) program in 1992. In fact, Moffitt (1992) notes that most exits and entrances into welfare are associated with changes in family structure, and not with changes in labor market circumstances. He suggests that ‘‘a model of marital search would be a more accurate descriptor of AFDC entry and exit than a wage-search model of the type employed in the job-search literature’’ (p. 26).

Fourth, economic well-being is quite persistent across generations. Stokey (1998) reviews several studies trying to figure out the fraction of a father’s relative position that his son inherits. For several indicators of economic success, the persistence coefficients are in the range of 0.4–0.5, and even higher.

II. Economic Environment

Consider an economy populated by two groups of agents, females and males. At any point in time, the female and male populations consist of a continuum of children and a continuum of adults. Each adult is indexed by a productivity level. Let x denote the type (productivity) of an adult female and z denote the type (productivity) of an adult male. Assume that x and z are contained in the sets $\mathscr{X} = \{x_1, x_2, \ldots, x_n\}$ and $\mathscr{Z} = \{z_1, z_2, \ldots, z_n\}$. An adult lives for two periods. Each adult female has two children attached to her throughout the two periods of her life. Assume that one of these children is female and the other is male. Children become adults after they have been raised by their parents for two periods.

At the beginning of each period, there exists a marriage market for single agents. Any single agent can take a draw from this market. Agents are free to accept or reject a mate as they desire. If a single agent accepts a marriage proposal, he or she is married for the current period. Otherwise, the agent is single and can take a new draw at the beginning of the next period. Similarly, at the beginning of each period, married agents decide whether to remain married or get divorced. Note that a divorced agent can never remarry, given the assumption of a two-period time horizon (since it takes one period to draw a new match). Furthermore, suppose for technical convenience that agents can match with (and therefore marry) only people of the same generation or age.

Agents are endowed with one unit of (nonsleeping) time in each period. Females must split this time between work, child care, and leisure, whereas males divide their time between work and leisure. Married agents derive utility from love, the public consumption of household goods, human capital investment in their children, and leisure. The child care time spent by the mother, together with the level of family consumption, determines the human capital obtained by a child. Parents treat their children equally. Single males do not worry about the human capital of their children, so they care only about their own consumption of goods and leisure. Single females do care about the investment in their children and must allocate their nonleisure time between work and child care. After two periods with their mother, children are each endowed with a productivity level that depends on the human capital investment received throughout their childhood. Each period the oldest adult males and females are replaced by the oldest children who enter into the marriage market.

A. *Preferences*

Females have the following utility function:

$$F(c, e, 1 - l - t) \equiv U(c) + \delta_1 V(e) + \delta_2 R(1 - l - t)$$
$$\equiv \ln c + \delta_1 \ln e + \delta_2 \ln(1 - l - t).$$

Here c is the consumption of household production, which is a public good for the family, and e is the level of human capital investment in children. Females allocate l units of their time for work and t units of it for child care. The utility function for married males is described by

$$M(c, e, 1 - n) \equiv U(c) + \theta_1 V(e) + \theta_2 R(1 - n)$$
$$\equiv \ln c + \theta_1 \ln e + \theta_2 \ln(1 - n).$$

Males spend n units of their time working. They do not spend time on child care. The parameters δ_1 and θ_1 reflect the differences across females and males in altruism toward their children. A single male does not realize a utility flow from the level of human capital investment in his children. His utility function is

$$M(c, 0, 1 - n) \equiv U(c) + \theta_2 R(1 - n) \equiv \ln c + \theta_2 \ln(1 - n).$$

One interpretation is that he no longer cares about his offspring. Another, more charitable, one is that he no longer enjoys the benefit from living with them.

B. Household Production

Household production for a married couple is given by

$$c = Y(l, n; x, z, \gamma) \equiv (xl + zn) - \gamma.$$

For a single female the household production function is

$$c = D(l; x) \equiv xl,$$

whereas for a single male it is described by

$$c = S(n; z) \equiv zn.$$

The functions Y, D, and S have a clear interpretation under the parameterization above. The variables x and z can be thought of as the market wages for type x females and type z males. The parameter γ represents the quality of the match between a male and a female. Let $\gamma \in \mathcal{G} = \{\gamma_1, \gamma_2, \ldots, \gamma_m\}$ be a discrete random variable distributed in line with the distribution function $\Gamma(\gamma_h) = \Pr[\gamma = \gamma_h]$. This variable is drawn immediately *after* entry into marriage and may be negative (love) or positive (hate) in value.

C. Transmission of Human Capital

Human capital investment in children is given by

$$e = Q(t, c) \equiv t^{\alpha} c^{1-\alpha}, \qquad (1)$$

which transforms the child care time of the mother (the father's time does not matter in the parameterization above) and the amount of the home-produced goods into human capital investment. Recall that children are nurtured for two periods. At the end of every period the children of the oldest generation enter into the marriage market as single adults. The productivity levels for females and males are drawn from the distributions

$$\Xi(x_i | e_{-2} + e_{-1}) = \Pr[x = x_i | e_{-2} + e_{-1}] \tag{2}$$

and

$$\Lambda(z_j | e_{-2} + e_{-1}) = \Pr[z = z_j | e_{-2} + e_{-1}], \tag{3}$$

where e_{-1} and e_{-2} indicate the human capital investment during the two periods of an agent's childhood. The distribution functions Ξ and Λ are stochastically increasing in $e_{-2} + e_{-1}$ in the sense of first-order stochastic dominance. Thus higher human capital investment in children by parents increases the likelihood that children will be successful in life. Let the conditional distribution Ξ be represented by a discrete approximation, à la Tauchen (1986), to a lognormal distribution with mean $\mu_{x|e}$ and standard deviation $\sigma_{x|e}$. Similarly, suppose that Λ is also given by a discrete approximation to a lognormal with mean $\mu_{z|e}$ and standard deviation $\sigma_{z|e}$. These conditional means are given by

$$\mu_{x|e} = \epsilon(e_{-2} + e_{-1}),$$
$$\mu_{z|e} = c_z + \epsilon(e_{-2} + e_{-1}),$$

where ϵ is the parameter governing the technology that maps human capital investment into productivity levels.

After the first period of adulthood the productivity levels for females and males evolve according to the following transition functions:

$$X(x_j | x_i) = \Pr[x' = x_j | x = x_i]$$

and

$$Z(z_j | z_i) = \Pr[z' = z_j | z = z_i].$$

Again, in line with Tauchen (1986), let X and Z be discrete approximations to the stochastic processes

$$\ln x' = (1 - \rho_x)\mu_x + \rho_x \ln x + \sigma_x\sqrt{1 - \rho_x^2}\xi \quad \text{with } \xi \sim N(0, 1),$$
$$\ln z' = (1 - \rho_z)\mu_z + \rho_z \ln z + \sigma_z\sqrt{1 - \rho_z^2}\zeta \quad \text{with } \zeta \sim N(0, 1).$$

III. Decision Making

A. Household Activity—Married Agents

When a female of type x and a male of type z are matched, they each decide how to allocate their time across its various uses given the optimal choices of their partner. Consider the married female's problem first. Suppose that the function $n = N^m(x, z, \gamma)$ gives her mate's labor supply. Then a type x female who is married to a type z male solves the following problem:

$$F^m(x, z, \gamma) = \max_{l,t} F(c, e, 1 - l - t) \qquad \text{P(1)}$$

subject to

$$c = Y(l, N^m(x, z, \gamma); x, z, \gamma)$$

and

$$e = Q(t, c).$$

Let the decision rules for work and child care effort, l and t, that solve this problem be represented by $l = L^m(x, z, \gamma)$ and $t = T^m(x, z, \gamma)$.

Similarly, let $n = N^m(x, z, \gamma)$ represent the decision rule that obtains from the married male's problem:

$$M^m(x, z, \gamma) = \max_{n} M(c, e, 1 - n) \qquad \text{P(2)}$$

subject to

$$c = Y(L^m(x, z, \gamma), n; x, z, \gamma)$$

and

$$e = Q(T^m(x, z, \gamma), c).$$

Observe that decisions within a family are determined noncooperatively by the maximizing behavior of agents in a Nash equilibrium. Here $F^m(x, z, \gamma)$ and $M^m(x, z, \gamma)$ give the equilibrium utility levels obtained in a marriage between a type x female and a type z male. Denote the equilibrium level of human capital investment in a two-parent family by

$$e = E^m(x, z, \gamma) = Q(T^m(x, z, \gamma), Y(L^m(x, z, \gamma), N^m(x, z, \gamma); x, z, \gamma)).$$

B. Household Activity—Single Agents

A single type x female will solve the following problem:

$$F^s(x) = \max_{l,t} F(c, e, 1 - l - t) \qquad \text{P(3)}$$

subject to

$$c = D(l, x)$$

and

$$e = Q(t, c).$$

Let the utility-maximizing work and child care effort levels that solve this problem be represented by $l = L^s(x)$ and $t = T^s(x)$. Denote the equilibrium level of human capital investment in a single-parent family by $e = E^s(x)$.

Finally, the maximized utility of a single male is given by the following problem:

$$M^s(z) = \max_{n} M(c, 0, 1 - n) \qquad \text{P(4)}$$

subject to

$$c = S(n; z).$$

Let $n = N^s(z)$ be the optimal work decision for a single male.

C. Search

Let the odds of drawing a single age j female of type x_i in the marriage market be represented by

$$\Phi_j(x_i), \quad \text{where } \Phi_j(x_i) \geq 0 \ \forall \ x_i \text{ and } \sum_{i=1}^{n} \Phi_j(x_i) = 1,$$

and the odds of meeting a single age j male of type z_i be denoted by

$$\Omega_j(z_i), \quad \text{where } \Omega_j(z_i) \geq 0 \ \forall \ z_i \text{ and } \sum_{i=1}^{n} \Omega_j(z_i) = 1.$$

In equilibrium the distributions of two-period-old males and females that will be around next period in the marriage market, or Φ_2' and Ω_2', will depend on the distributions of one-period-old males and females that are around this period, or Φ_1 and Ω_1. Express this dependence by $(\Phi_2', \Omega_2') = \mathbf{P}(\Phi_1, \Omega_1)$. A key step in the analysis will be to compute such matching probabilities.

Now, consider an age i couple indexed by (x, z, γ). Both parties face a decision: should they choose married or single life for the period? Let the female's expected lifetime utility associated with this match in marriage be denoted by $W_i(x, z, \gamma)$ and her expected lifetime utility from single life be represented by $G_i(x; \cdot)$. Clearly, a married female will *want* to remain married if and only if $W_i(x, z, \gamma) \geq G_i(x; \cdot)$; otherwise, it is in her best interest to get a divorce. Equally as clearly, a single female will *desire* to marry if and only if $\sum_h \Gamma(\gamma_h) W_i(x, z, \gamma_h) \geq G_i(x; \cdot)$; otherwise, she will go it alone. Similarly, let the male's expected lifetime utility from married life be given by $H_i(x, z, \gamma)$ and the value of being single be $B_i(z; \cdot)$. A married male would wish to remain so if and only if $H_i(x, z, \gamma) \geq B_i(z; \cdot)$, whereas a single male will like to marry if and only if $\sum_h \Gamma(\gamma_h) H_i(x, z, \gamma_h) \geq B_i(z; \cdot)$.

Define the indicator functions $I_i^s(x, z; \cdot)$ and $J_i^s(x, z; \cdot)$, summarizing the matching decisions for single age i males and females, by

$$I_1^s(x, z; \Phi_1, \Omega_1) = \begin{cases} 1 & \text{if} \sum_{h=1}^{m} \Gamma(\gamma_h) H_1(x, z, \gamma_h) \geq B_1(z; \Phi_1, \Omega_1) \\ 0 & \text{otherwise,} \end{cases}$$

$$I_2^s(x, z) = \begin{cases} 1 & \text{if} \sum_{h=1}^{m} \Gamma(\gamma_h) H_2(x, z, \gamma_h) \geq B_2(z) \\ 0 & \text{otherwise,} \end{cases} \qquad \text{P(5)}$$

$$J_1^s(x, z; \Phi_1, \Omega_1) = \begin{cases} 1 & \text{if} \sum_{h=1}^{m} \Gamma(\gamma_h) W_1(x, z, \gamma_h) \geq G_1(x; \Phi_1, \Omega_1) \\ 0 & \text{otherwise,} \end{cases}$$

$$J_2^s(x, z) = \begin{cases} 1 & \text{if} \sum_{h=1}^{m} \Gamma(\gamma_h) W_2(x, z, \gamma_h) \geq G_2(x) \\ 0 & \text{otherwise.} \end{cases}$$

Note that the accept/reject decisions for the young, unlike the ones for the old, will depend on the type distributions for young agents. The reason for this will become evident soon. Likewise, let the indicator functions $I_2^m(x, z, \gamma)$ and $J_2^m(x, z, \gamma)$ define the matching decisions for married two-period-old males and females so that

$$I_2^m(x, z, \gamma) = \begin{cases} 1 & \text{if } H_2(x, z, \gamma) \geq B_2(z) \\ 0 & \text{otherwise,} \end{cases} \qquad \text{P(6)}$$

$$J_2^m(x, z, \gamma) = \begin{cases} 1 & \text{if } W_2(x, z, \gamma) \geq G_2(x) \\ 0 & \text{otherwise.} \end{cases}$$

The value function for a one-period-old married female appears as

$$W_1(x_i, z_j, \gamma_h) = F^m(x_i, z_j, \gamma_h) + \beta \sum_{k=1}^{n} \sum_{l=1}^{n}$$

$$\max\{W_2(x_k, z_l, \gamma_h) I_2^m(x_k, z_l, \gamma_h), G_2(x_k)\} X(x_k | x_i) Z(z_l | z_j), \quad \text{V(1)}$$

where β is the discount factor. Observe that $X(x_k|x_i)Z(z_l|z_j)$ is the probability that a married couple will move from state (x_i, z_j) to state (x_k, z_l). The female would like to remain married if $W_2(x_k, z_l, \gamma_h) \geq G_2(x_k)$ and get a divorce otherwise. Remaining married is feasible, however, only if it is mutually agreeable or $I_2^m(x_k, z_l, \gamma_h) = 1$. Therefore, the value of being married to a young female depends on the values that her husband will derive from married and single lives when old. This dependence is expressed through $I_2^m(x_k, z_l, \gamma_h)$, as defined by P(6). Note that $W_2(x_k, z_l, \gamma_h)$ and $G_2(x_k)$ are defined trivially by $W_2(x_k, z_l, \gamma_h) = F^m(x_k, z_l, \gamma_h)$ and $G_2(x_k) = F^s(x_k)$. Likewise, the value function for a one-period-old married male is

$$H_1(x_i, z_j, \gamma_h) = M^m(x_i, z_j, \gamma_h) + \beta \sum_{k=1}^{n} \sum_{l=1}^{n}$$

$$\max\{H_2(x_k, z_l, \gamma_h) J_2^m(x_k, z_l, \gamma_h), B_2(z_l)\} X(x_k | x_i) Z(z_l | z_j), \quad \text{V(2)}$$

where $H_2(x_k, z_l, \gamma_h) = M^m(x_k, z_l, \gamma_h)$ and $B_2(z_l) = M^s(z_l)$.

The recursion for a one-period-old single type x_i female is

$$G_1(x_i; \Phi_1, \Omega_1) = F^s(x_i) + \beta \sum_{k=1}^{n} \sum_{l=1}^{n}$$

$$\max\left\{\sum_{h=1}^{m} \Gamma(\gamma_h) W_2(x_k, z_l, \gamma_h) I_2^s(x_k, z_l), G_2(x_k)\right\}$$

$$\times X(x_k | x_i) \Omega_2'(z_l), \quad \text{V(3)}$$

with $(\Phi_2', \Omega_2') = \mathbf{P}(\Phi_1, \Omega_1)$. Here $X(x_k|x_i)\Omega_2'(z_l)$ gives the probability that a single female of type x_i will transit to a productivity level of x_k and meet a single male of type z_l. Note that the value of being a young single female today depends on the availability of males tomorrow, or on Ω_2'. This in turn depends on the distributions of one-period-old males and females that are around this period, or on Φ_1 and Ω_1 through $\mathbf{P}$.[1] This explains the dependence of J_1^s on Φ_1 and Ω_1. The analogous recursion for a male is

$$B_1(z_j; \Phi_1, \Omega_1) = M^s(z_j) + \beta \sum_{k=1}^{n} \sum_{l=1}^{n} \max\left\{ \sum_{h=1}^{m} \Gamma(\gamma_h) H_2(x_k, z_l, \gamma_h) J_2^s(x_k, z_l), B_2(z_l) \right\} \times Z(z_l | z_j) \Phi_2'(x_k), \quad \mathrm{V}(4)$$

with $(\Phi_2', \Omega_2') = \mathbf{P}(\Phi_1, \Omega_1)$.

D. *Discussion*

Some discussion about the decisions facing agents may be in order. To begin with, where do the gains from marriage accrue from? These gains underlie an agent's decision to accept or reject a mate, as determined by P(5) and P(6). There are three gains from marriage. First, consumption in the household is a public good. By marrying, a couple can pool their incomes and obtain a greater level of consumption. Thus there are economies of scale in household consumption. Second, marriage may yield utility per se (love) if a good match ($\gamma < 0$) is generated. It is true that a young couple could suffer from a bad match ($\gamma > 0$). Young adults view these events asymmetrically, however, since they always have the option of dissolving a bad match through a divorce when old. The option to divorce works to generate positive expected utility from a marriage for a young adult. Third, males enjoy utility from having children around only when they are married.

Within a marriage, husband and wife play a noncooperative Nash game. That is, parties each choose their time allocations taking as given the decision of their spouse. Note that males' or females' choices about their time allocations—and hence implicitly about their consumption and investment in children—are static in nature, as is readily deduced by the forms of P(1), P(2), P(3), and P(4). This greatly simplifies the analysis. This property would be destroyed if a couple arrived at their decisions cooperatively via Nash bargaining. Now, the time allocation decisions would depend on each party's threat point, or the values of being single as given by $G_i(x; \cdot)$

[1] Since an adult lives for only two periods, there is no possibility of meeting a new mate after the second period of life. Hence, there is no need to enter the type distributions into W_2, H_2, G_2, B_2, etc.

and $B_i(z; \cdot)$. For a young agent this would immediately bring in a dynamic element to the problem.

The static nature of the time allocation decisions would also be lost if adults could borrow or lend on a capital market, or if working in the market today influences one's productivity tomorrow—a concern women may face when deciding whether or not to stay at home and look after the kids. Additionally, an adult's momentary utility depends on the level of human capital investment in his or her children, and not the son's and daughter's expected utilities. This simplifies the analysis. In principle, an offspring's expected lifetime utility could be written as a function of the levels of human capital investment over his or her childhood. But knowing this function would amount to knowing the solution to a young agent's dynamic optimization problem, and this does not have the simple separable form $V(e) = \ln e$.[2]

IV. Stationary Equilibrium

How are the odds of meeting a single age j, type x female, $\Phi_j(x)$, or a single age j, type z male, $\Omega_j(z)$, determined in stationary equilibrium? To begin with, consider the odds of meeting a two-period-old single woman or man of a given type in the marriage market next period. Denote these probabilities by $\Phi_2'(x)$ and $\Omega_2'(z)$. Clearly, the key step in determining these odds is calculating the number of young adults of each type that remain unmarried from the current period. This will depend on the number of agents of each type in the current period, Φ_1 and Ω_1, and the accept/reject decision rules describing their marriage decisions, J_1^s and I_1^s. Hence, Φ_2' and Ω_2' are determined by an operator, $\mathbf{P}_2$, of the form

$$(\Phi_2', \Omega_2') = \mathbf{P}_2(\Phi_1, \Omega_1, J_1^s, I_1^s).$$

Next, what are the odds of meeting a one-period-old single agent of a given type in the marriage market, or $\Phi_1(x)$ and $\Omega_1(z)$? Now, young adults today were born two periods ago. So this must depend on the stocks of young adults that were around then (i.e., the current generation's parents), or on $\Phi_{1,-2}$ and $\Omega_{1,-2}$.[3] The young adults around today could have come from many different family backgrounds. Some could have been raised throughout their childhood with two parents and others with a single parent, and some could

[2] There are some subtleties here, too. When parents care about the utilities of their offspring, they may not want to invest equally in their sons and daughters (see Siow and Zhu 1998).

[3] Let y_{-j} represent the quantity y some j periods ago.

have experienced at adolescence either a family breakup or a marriage of their parent. The number of agents in each category will depend on their parents' accept/reject decision rules, $J^s_{1,-2}$, $I^s_{1,-2}$, $J^m_{2,-1}$, $J^s_{2,-1}$, $I^m_{2,-1}$, and $I^s_{2,-1}$. Furthermore, the number of young adults of each type will be influenced by their parents' child investment decision rules, E^m and E^s.[4] Therefore, Φ_1 and Ω_1 are determined by an operator, $\tilde{\mathbf{P}}_1$, that can be written as

$$(\Phi_1, \Omega_1) = \tilde{\mathbf{P}}_1(\Phi_{1,-2}, \Omega_{1,-2}, J^s_{1,-2}, I^s_{1,-2}, J^m_{2,-1}, J^s_{2,-1}, I^m_{2,-1}, I^s_{2,-1}, E^m, E^s).$$

In a steady state, $\Phi_1 = \Phi_{1,-1} = \Phi_{1,-2}$ and $\Phi'_2 = \Phi_2$ and so forth, so that the two expressions above can be written more simply as

$$(\Phi_2, \Omega_2) = \mathbf{P}_2(\Phi_1, \Omega_1, J^s_1, I^s_1) \tag{4}$$

and

$$(\Phi_1, \Omega_1) = \mathbf{P}_1(J^s_1, I^s_1, J^m_2, J^s_2, I^m_2, I^s_2, E^m, E^s). \tag{5}$$

(Explicit expressions for [4] and [5] are given in the Appendix.) On the one hand, observe from P(5), P(6), and V(1)–V(4) that to compute J^s_1 and I^s_1 requires knowing Φ_2 and Ω_2. On the other hand, to calculate Φ_2 and Ω_2, one needs to know J^s_1, I^s_1, J^s_2, J^m_2, I^m_2, and I^s_2, as shown by (4) and (5). This is a classic fixed-point problem. It is solved here numerically.

It is now time to take stock of the situation so far.

Definition 1. A stationary matching equilibrium is a set of allocation rules, $L^m(x, z, \gamma)$, $T^m(x, z, \gamma)$, $N^m(x, z, \gamma)$, $L^s(x)$, $T^s(x)$, $N^s(z)$, $I^m_2(x, z, \gamma)$, $J^m_2(x, z, \gamma)$, $I^s_1(x, z; \Phi_1, \Omega_1)$, $I^s_2(x, z)$, $J^s_1(x, z; \Phi_1, \Omega_1)$, $J^s_2(x, z)$, $E^m(x, z, \gamma)$, and $E^s(x)$, and matching probabilities, $\Phi_1(x)$, $\Phi_2(x)$, $\Omega_1(z)$, and $\Omega_2(z)$, such that the following conditions hold:

1. The functions $L^m(x, z, \gamma)$, $T^m(x, z, \gamma)$, $E^m(x, z, \gamma)$, and $N^m(x, z, \gamma)$ describe an equilibrium for a married couple, or satisfy problems P(1) and P(2).
2. The functions $L^s(x)$, $T^s(x)$, and $E^s(x)$ solve the single female's household problem P(3).
3. The function $N^s(z)$ solves the single male's household problem P(4).
4. Single agents' accept/reject choices $I^s_1(x, z; \Phi_1, \Omega_1)$, $I^s_2(x, z)$, $J^s_1(x, z; \Phi_1, \Omega_1)$, and $J^s_2(x, z)$ are described by P(5) in conjunction with V(1), V(2), V(3), and V(4).
5. Married agents' accept/reject choices $I^m_2(x, z, \gamma)$ and $J^m_2(x, z, \gamma)$ are described by P(6).

[4] Given the forms of problems P(1), P(2), and P(3), the functions E^m and E^s do not change over time.

6. The matching probabilities $\Phi_1(x)$, $\Phi_2(x)$, $\Omega_1(z)$, and $\Omega_2(z)$ are governed by the stationary distributions described by (4) and (5).

While not much can be said about the model at a general level, a feel for the forces at play can be gleaned by solving it numerically and conducting comparative statics exercises. This is the subject of the next section.

V. A Numerical Example

In order to solve the model numerically, values must be assigned to the model's various parameters.[5] Table 1 lists the parameter values used. Note that at this time very little is known about the appropriate choice of parameter values, or functional forms, to use in a model such as this. Given that the primary interest here is to illustrate the mechanics of the model developed, these parameter values are picked to find a benchmark equilibrium that displays several features of interest. These features will be discussed now. Beforehand, note that the benchmark equilibrium presupposes that a divorced male must pay 10 percent of his current income in child support to his former spouse. Furthermore, single women who do not work are eligible to receive a welfare payment amounting to 22 percent of average income in the economy. These two policies are discussed in more detail later on.

The marital status of the population is shown in table 2. At any point in time, about 22.5 percent of people are not married, either because they have never married or are divorced. The matching shock plays an important role in generating divorce in the second period of life. When the variance of the matching shock is set to zero (leaving its mean value unchanged), the percentage of divorces falls from 9.1 to 3.8 percent. Some people will still choose to divorce, either because the extra income generated from a marriage cannot cover the fixed cost (note that the mean value of the match shock is positive) or because they can do better on welfare.[6]

Figure 1 shows the matching set for young agents in the model. Recall that a marriage occurs when the product of the male and female indicator functions returns a value of one; otherwise no marriage occurs. As can be seen, nobody wants to marry a mate with low

[5] The algorithm used to compute the competitive equilibrium under study is detailed in Aiyagari, Greenwood, and Guner (1999).

[6] It is interesting that letting divorced males realize utility from their children increases the number of divorces in the model. Now there is less of a utility cost from divorce.

TABLE 1

Parameter Values

Tastes	$\delta_1 = .5, \delta_2 = .9, \theta_1 = .1, \theta_2 = .7, \beta = .67$
Technology	$\alpha = .4, \epsilon = 4.2, c_z = 4.2$
Stochastic structure	$\mu_{x\|e} = \epsilon e, \mu_{z\|e} = c_z + \epsilon e, \sigma_{x\|e} = .4, \sigma_{z\|e} = .4$
	$\rho_x = .7, \rho_z = .7$
	$\Gamma(\gamma_1) = .5, \Gamma(\gamma_2) = .5, \gamma_1 = 2.6, \gamma_2 = 0$
	$n = 17, m = 2$
Policy variables	$a = .10, w = .22, \tau = .03$

TABLE 2

Marital Status
(Percentage Distribution)

	Young	Old
Married	72	83
Single	28	8
Divorced	. . .	9

productivity—the exception being very rich males, who will marry any woman. These people are unattractive to the opposite sex. The fact that women tend to select the best men has been discussed in the labor economics literature. For example, Cornwell and Rupert (1997) find that married men earn more than unmarried ones; this is often called the marriage premium. They argue that the same traits that make a man attractive to a woman, such as ability, ambition, dependability, determination, and honesty, are also valued by employers. To an outside observer, marriage would be a signal, so to speak, of the quality of a man. In any event, this type of selection effect is a natural outcome within the context of a bilateral search model.

In contrast, consider a world in which men and women face no search frictions when finding their first mate. Here household production is maximized by choosing a mate with the highest productivity. Again, some people may still choose to remain single, however, because they do better on welfare. Now, only 2.6 percent of people fail to marry when young. These agents are the worst types. Not surprisingly, there is perfect assortative mating among the married population. Approximately 9.9 percent of people divorce when old because of bad match quality shocks or changes in types.[7]

[7] As in the benchmark model, agents are free to either remain married or divorce, as their best interest dictates. As before, divorced agents cannot remarry because it takes a period to rematch. If they could, a large percentage of people would rematch in the second period, either because of a bad match quality shock or because of type changes. For this reason the frictionless model is not very realistic.

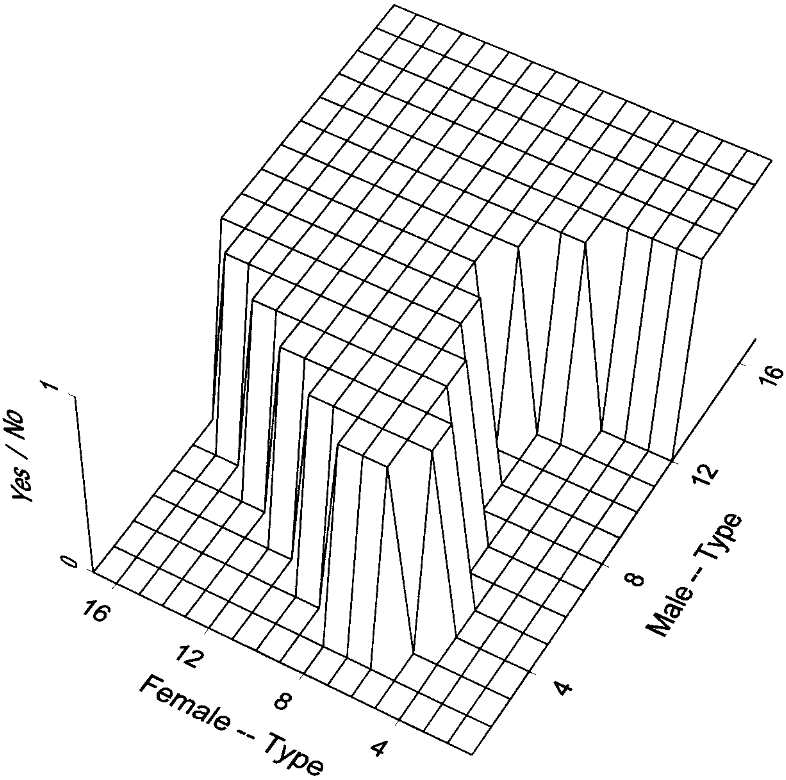

Fig. 1.—Matching set for young agents

There is considerable income inequality in the benchmark equilibrium. The wage distributions for males and females are approximately lognormal with a standard deviation for wages of 52 percent.[8] The standard deviations for male and female expected lifetime incomes are about 40 and 36 percent. As one would expect, family income is lowest for unmarried females. This occurs for two reasons: first, these are single-income families, and second, they tend to be at the lower end of the productivity distribution. Family income for females by marital status is given in table 3. Unmarried females have about 31–47 percent of their married counterparts' family incomes.

In the model, married females spend more time with their children and less time working than either single or divorced females do (see table 4). This is not surprising since a two-parent family can

[8] In the equilibrium that was constructed, males earn about 38 percent more on average than females.

TABLE 3

Female Family Income

	Young	Old
Married	1.00	.96
Single	.33	.31
Divorced	. . .	.47

Note.—Income is expressed relative to a young married female's income.

TABLE 4

Female Time Allocations (%)

	Married	Single	Single (Welfare)	Divorced	Divorced (Welfare)
Market work	22.5	54.0	0	53.0	0
Child care	14.0	8.0	18.0	9.0	18.0

rely on the income that the male brings into the household. The mother in a two-parent family then uses some of her freed-up time to invest in her children, from which both parents realize utility. Note that married females enjoy more leisure. A welfare mother spends no time working but invests the most time in her children. She also enjoys the most leisure.[9]

How much intergenerational income mobility is there in the model? The correlation between parents' and their offspring's lifetime family income is about .53 for sons and .32 for daughters, showing a fairly high degree of persistence in income across generations. Children who come from low-income families suffer from a lack of human capital investment. They, too, then tend to be poor when they grow up. This occurs for two reasons. First, the low level of human capital investment leads to low productivity levels for these individuals. Thus they can earn less in the labor market. Second, the low productivity levels make them relatively unattractive mates

[9] It is interesting that allowing for a variable labor supply dramatically affects the equilibrium number of marriages, other things equal. Suppose that market work for working men and that for working women are held fixed at their mean levels. Still assume that women on welfare do not work. Fix child care time for women at its mean level. The equilibrium number of marriages rises by 19 percentage points to 97 percent. The question is, Why? First, divorce is much less attractive for a woman. Single women who are not on welfare work the most in the benchmark equilibrium (see table 4). They cannot make up now for the lost family income by working more. Marriage is now more attractive for men. Married women work the least in the benchmark equilibrium, and their husbands would like them to work more. They do now.

TABLE 5

EFFECTS OF CHILDHOOD HISTORY ON FEMALE INCOME

	CHILDHOOD HISTORY			
	$m \to m$ (1)	$m \to s$ (2)	$s \to m$ (3)	$s \to s$ (4)
Expected wage	1.00	.71	.67	.48
Expected family income	1.00	.85	.82	.68

NOTE.—The numbers are reported relative to col. 1.

in the marriage market. Females in this category are much more likely to spend part of their adult lives parenting their children alone. Hence the cycle goes on.[10]

The impact of childhood history on the expected earnings for a female is presented in table 5. Consider the fate of a girl who spends all of her life in a single-parent family vis-à-vis one who grows up in a two-parent household. The child from the broken home can expect to realize, when she grows up, about two-thirds of the family income of the child from an intact one.

In general, multiple equilibria may be a problem in two-sided search models, a point highlighted in Burdett and Coles (1997) and Burdett and Wright (1998). Indeed, for the parameterization employed, one other equilibrium was found. This is an equilibrium in which everybody marries in the first period. There is no reason to remain single in the first period if everyone else is getting married in the first period. This transpires because there is no option value to waiting; there will be no eligible mates next period. In this equilibrium some agents still get divorced, though, because their match is poor in that the couple drew a bad value for γ. It is interesting that average expected income and lifetime utility are higher in this equilibrium than in the benchmark one.[11] Income inequality is lower, too. This equilibrium is not stable, however, in the sense that when the economy is started off from a variety of other initial distributions

[10] The extent of persistence in the type shock plays an important role in a young agent's decision to marry or not. It can now be explained why. To this end, suppose that type shocks are permanent. Therefore, an agent's type does not change over time. Now, 99 percent of young agents remain single! Since type is permanent, there is less incentive to marry a low type: there is no chance that he or she can improve. So the value of waiting to find a new mate increases. There are now more single women in equilibrium, and consequently, there is less investment in children. This, in turn, causes the long-run quality of young adults to suffer. The drop in the quality of the mating pool then leads to fewer marriages.

[11] Average expected lifetime income is 27 percent higher in the equilibrium in which everybody gets married in the first period.

(some extremely close to the equilibrium in which everyone marries), it always converges to the *benchmark* equilibrium.

To illustrate the mechanics of the prototype general equilibrium search model of marriage and divorce developed here, two policy experiments will now be conducted. Policy makers have tried to protect the welfare of children by making divorced fathers pay child support and by providing state aid to destitute single mothers. Each of these policies will be examined in turn.

A. *Child Support*

How does child support work in the model? Each divorced mother would now receive the fraction a of her former spouse's income as child support. Thus the budget constraint for a divorced female of type x who was married to a man whose current income is $S(n, z)$ would now be

$$c = D(l, x) + aS(n, z),$$

whereas that of her ex-spouse appears as

$$c = (1 - a)S(n, z).$$

Note that with the introduction of child support the current income of her ex-husband becomes a relevant state variable for a divorced female. Likewise, for a single male in the second period of his life, it will matter whether or not he was married in the first period.

The direct effect of child support is, of course, to increase the living standards of children living in single-parent families. Their mothers now have more resources to invest in them. There are indirect effects as well. First, the necessity of paying child support makes divorce less attractive to males: in the model, males are the party most likely to walk from a marriage. Second, the uplifting effect that child support has on investment in children from single-parent families makes them better mates in the marriage market. This reduces the incidence of divorce when these children grow up.

Raising child support from 10 to 15 percent improves the model economy's long-run health. Lifetime earnings for a child raised in a family that suffered through a divorce rise by about 5.3 percent. More is invested in these children. Furthermore, the number of children living in a family that has experienced a divorce drops by about one percentage point. There are now three percentage points fewer children living with a single parent in equilibrium because divorce has been dissuaded and the quality of the mating pool has improved. Both of these effects lead to a 5.6 percent increase in average lifetime earnings for males and females taken as whole. As can be seen from figure 2, the income distributions for the economy with a 15 percent child support rate stochastically dominate the ones that occur when the economy has a 10 percent rate. Expected lifetime utility increases by about 8.1 percent, when measured in consumption units.[12]

It may seem paradoxical that males are better off in the equilibrium with a higher rate of child support. Child support acts as a tax on male divorcees. Furthermore, a divorced male also realizes no utility from his offspring. So, at first glance, it may appear that males should be worse off from the higher rate of child support. To understand the mechanisms at work, it pays to artificially decompose the experiment into short- and long-run effects. For the short-run effects, consider the impact on males when the distribution of young agents is held fixed; that is, the induced changes in human capital investments by parents are not allowed to affect the type distribution of children. Now, indeed, males do suffer a slight loss in expected income and utility. Lifetime expected income for males falls by about 0.1 percent. The small size of this number should not be surprising. First, the rate of child support was raised by only five percentage points. Second, only about 9 percent are divorced in the initial equilibrium. Third, a divorced male will pay child support for only one-half of his life, and this will be discounted by a factor of 0.7. Note that $[0.05 \times 0.7 \times (0.09/2)] \times 100$ percent $= 0.16$ percent, a number not too far off from that obtained.

Hence, almost all the effects in the experiment derive from the improvement in the long-run quality of young adults. On this, clearly there is a large externality present in the model. Parents do not value human capital development in their children in the same way that their children will value it themselves.[13] The appropriate way to

[12] Think about computing the expected lifetime utility for a person who will be randomly thrown into the economy above in line with the stationary distributions Φ_1 and Ω_1. The reported number refers to the fraction by which an agent's consumption would have to be raised in each state of the world in the benchmark economy to make him or her as well off as in the new situation.

[13] Consider the world in which men and women face no search frictions when finding their first mate, which was discussed earlier. Here almost everybody is married in the first period. Hence, very few children are raised by a single mother. Expected lifetime income and welfare are about 14.0 and 20.5 percent higher in this economy, as compared with the benchmark one. Still, it is not perfect. To understand why, note that children would always like their father's type to be as high as possible. This is not always true for their mother's type. In the model, a father contributes to a child's development solely through the income he brings home. A mother also contributes to a child's development through the time she spends on nurture. So children with a father from the upper end of the distribution may prefer a mother of a type lower than their father was matched with since she will spend more time with the children. Thus a parent's decision about a mate, or his or her work-effort decision, may not be in a child's best interest.

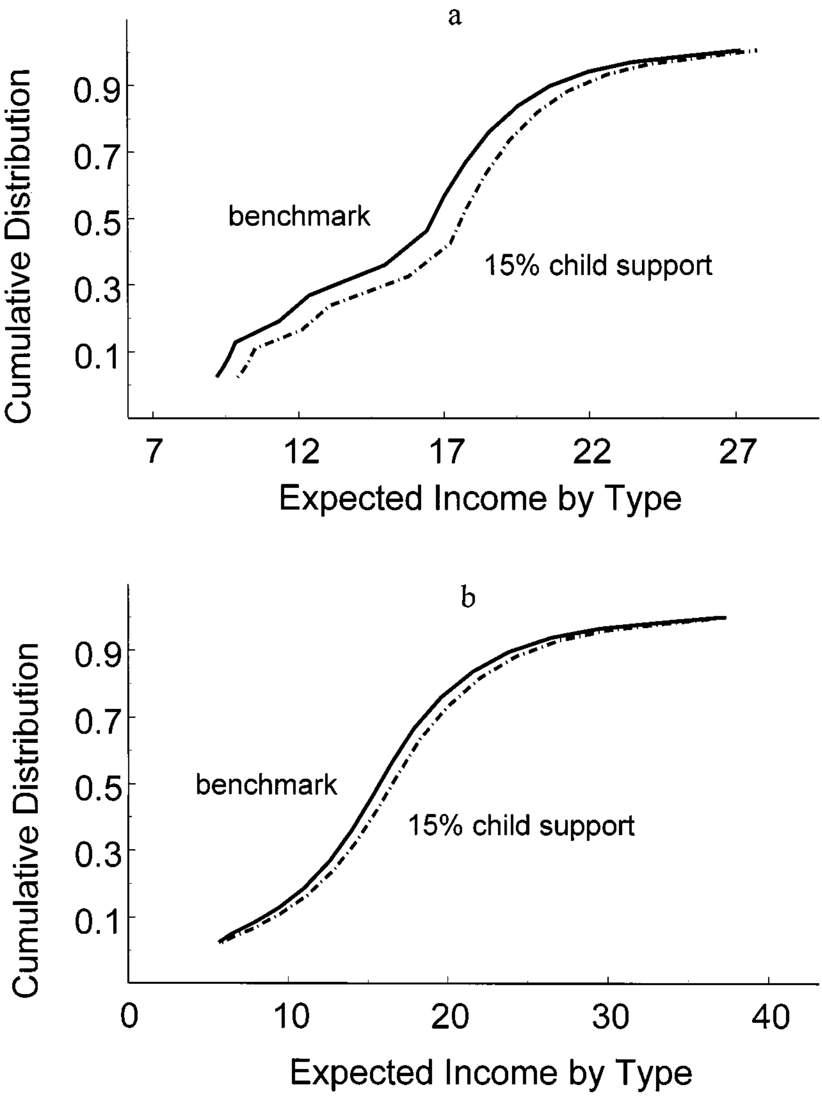

FIG. 2.—Income distributions, child support experiment: *a*, females; *b*, males

model how parents care about their children is very much an open question. Additionally, little is known about the human capital transmission mechanism from parents to children, or the form of (1)–(3). Reliable statements about the welfare effects of child support will have to wait until progress is made on such issues.

B. Welfare

In the benchmark equilibrium, any single mother is eligible for a welfare payment, provided that she does not work. This payment is set to equal the fraction w of the average level of income in the benchmark economy. Thus a single mother faces a choice about whether she should work or collect welfare. Welfare payments made to single mothers are financed by income taxes levied on the rest of the population. The rate of income taxation is τ. The level of benefits is set to equal 22 percent of average income in the benchmark economy.[14] A 3 percent rate of income taxation is needed to finance this.

The effects of welfare are multifaceted. First, on the positive side, welfare can be thought of as an insurance program against the vagaries of life, here divorce and out-of-wedlock births.[15] Second, welfare allows a single mother to spend more time with her children. Otherwise, this woman would have to work to support her family, which takes time away from her offspring. But welfare has negative aspects. First, some women may choose to stop working in order to gain leisure.[16] This leads to a drop in household income. The extra time made available for child rearing may not compensate for this. Second, the availability of welfare may make marriage less attractive to women since it raises the value of being single.[17] Third, the higher

[14] This number is roughly in line with the average amount of benefits received from AFDC and food stamps.

[15] In fact, as Cubeddu and Ríos-Rull (1997, app. 3) document, the risk of divorce and out-of-wedlock births in the United States is high, and their economic consequences large. For instance, a married woman aged between 25 and 29 faces a 12 percent chance of becoming divorced by age 30–34. The odds that a single woman without children in this age group will be stuck raising children alone in the next five years are 16 percent. Additionally, Duncan and Hoffman (1985) calculate that female income drops by more than 40 percent in the year following a divorce. Even five years after a divorce, if a female does not remarry, her income remains about 40 percent of its predivorce level. Presumably agents could save to self-insure against such risks, a possibility not allowed in the current model in order to ease the computational burden. Such possibilities would reduce the benefits of welfare. Cubeddu and Ríos-Rull analyze the effects that marital risk may have on aggregate savings.

[16] Moffitt (1992) notes that the welfare system has generated nontrivial work disincentives (but not of the magnitude needed to explain female poverty).

[17] The connection between welfare and family structure is not well understood. While recent empirical work does find a positive association between the number of single mothers and AFDC benefits, the size of the effect is not large enough to explain the postwar rise in female headship.

rate of income taxation reduces the income available to a married couple, and this again reduces the attractiveness of marriage. Which effects dominate is a quantitative question (and the answer obtained could obviously hinge on the particular structure employed). The economy with welfare will now be compared to one without it.

Welfare allows single mothers to spend more time with their children, at least in the model. Welfare mothers spend about 18 percent of their time on child care, as opposed to the 8 percent spent by single mothers in the economy with welfare. As a result, the level of human capital in children from single-parent families increases. These children are better off. The lifetime utility distribution for women is plotted in figure 3. There are fewer suffering women in an economy with welfare. This increase in the utility of the lower strata of women comes about primarily from a gain in leisure. This can be gleaned from the *after-tax* income distribution for women. The after-tax income distribution for women in an economy without welfare stochastically dominates the one for the economy with welfare, as figure 3 illustrates. The number of single mothers in the economy moves up with the introduction of welfare. Approximately 19.9 percent of children live with a single mother in the economy without welfare as opposed to 22.5 percent in the benchmark economy. Overall, this rise in single parenthood in conjunction with higher labor income taxes operates to lower both the average levels of after-tax income and lifetime utility in the economy by about 8.7 and 5.8 percent (the latter measured in consumption units). The after-tax income and lifetime utility distributions for males in the economy without welfare stochastically dominate those obtained in the benchmark economy.

The weight on female leisure, δ_2.—The deleterious effects of welfare derive from the mother's incentive to capture leisure by going on public assistance. This suggests that the impact that welfare has on the economy could be sensitive to the weight on leisure in the utility function. To address this conjecture, consider an economy in which females place more weight on leisure. Specifically, let δ_2 now equal 0.925 (as opposed to 0.9 in the benchmark economy). The number of women on welfare now rises by about 5.5 percentage points (from 15.7 to 21.2 percent). Somewhat surprisingly, working mothers spend a little more time working (because of the higher tax rate of 3.7 percent that is needed to finance welfare) and a little less with their children.

Now compare the economy with and without welfare, as before. With the advent of welfare, the number of single mothers now rises by a much larger 7.3 percentage points (as opposed to 2.6 previously). Expected income and utility fall by 19.0 and 23.0 percent

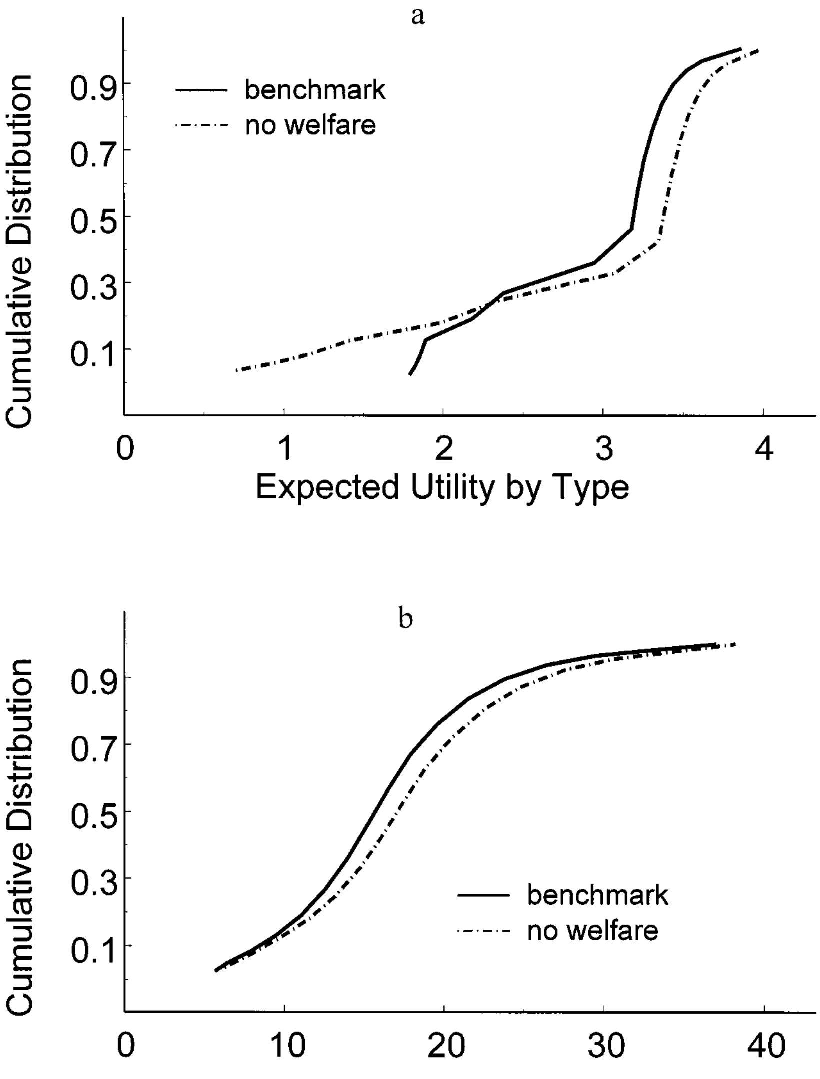

FIG. 3.—Utility and income distributions for females, welfare experiment: *a*, expected utility; *b*, expected income.

with the introduction of welfare. This experiment makes it clear that precise information about key parameters, such as δ_2, will be needed for conducting policy analysis. Therefore, a key step in the evolution of dynamic general equilibrium models of marriage and divorce will be determining appropriate parameter values to use. Perhaps parameters such as δ_2 could be estimated from time-use data. Clearly, this is required before any serious policy analysis can be done.

1. Transitional Dynamics

While in the long run the economy is better off without welfare, in the sense that expected lifetime utility is higher, rescinding welfare could have painful effects in the short run. So, what does the transition path look like when one moves from the benchmark equilibrium with welfare to the new steady state without it? The welfare gain for each generation of young women along the transition path is plotted in figure 4. As can be seen, a young woman's expected lifetime utility drops by about 5.5 percent (measured in consumption units) initially. It takes at least 15 periods (and a period here is 10 years) before women can expect to be as well off under the new regime as under the old one.[18] And the gains, since they occur well off into the future, will be discounted heavily. Males are better off along the transition path, however, in that each generation realizes a higher level of expected lifetime utility than in the benchmark economy. Even the initial generations gain about a 5 percent increase in welfare. Income inequality worsens initially. As figure 4 illustrates, the number of people at the low end of the income distribution rises quite dramatically when welfare is first removed. Thus taking transitional dynamics into account may significantly alter the welfare effects of public policy.

VI. Conclusion

A family's rung on the economic ladder is quite persistent across generations. Divorce is usually associated with a significant drop in the material well-being of a woman. Children from single-parent families are less likely to be successful than ones living with two parents. To address these observations, a prototype overlapping generations model of marriage, divorce, and investment in children is constructed. In the model there are males and females, who may differ from one another according to their marital status and level of hu-

[18] Since the number of single agents falls immediately following the elimination of welfare, there may be some unhappily married females in the short run.

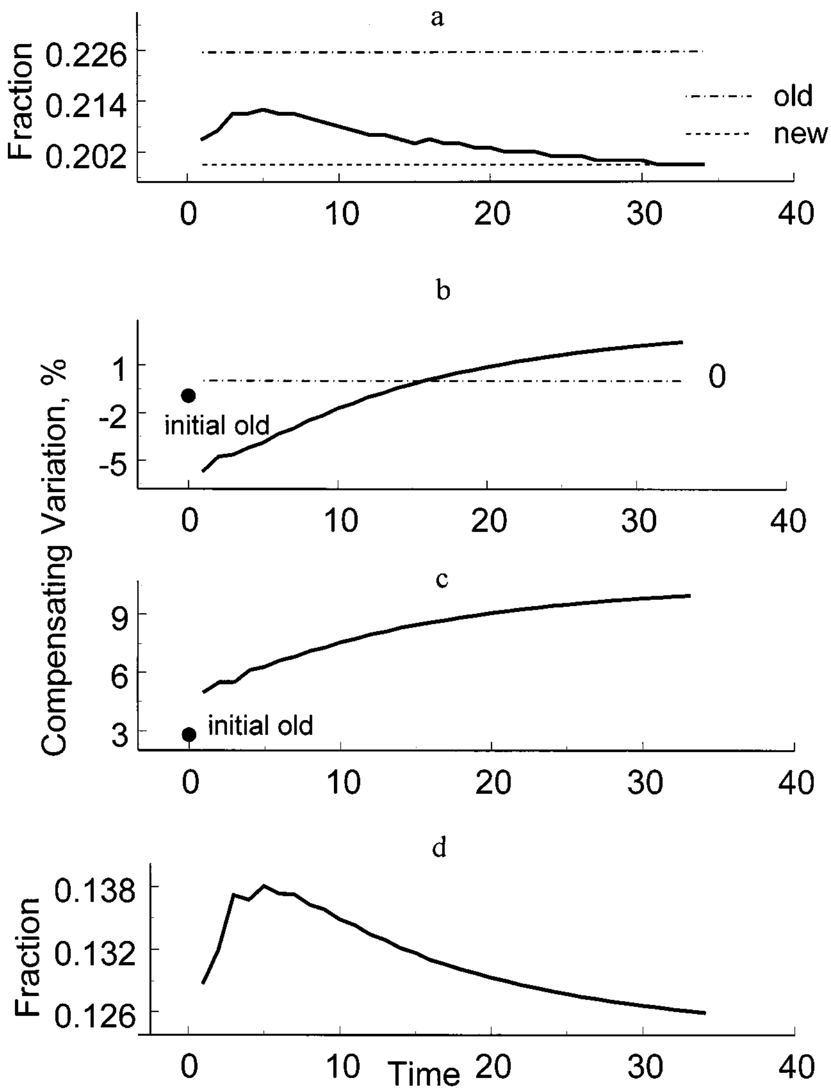

FIG. 4.—Transitional dynamics, welfare experiment: *a*, number of single agents; *b*, welfare—females; *c*, welfare—males; *d*, income distribution, lowest three types.

man capital. Each period single agents meet in a marriage market. They must decide whether to accept or reject offers to wed. Likewise, married agents decide whether to remain married or to divorce. In the equilibrium modeled, most individuals are reluctant to marry a mate from the lower end of the distribution. Hence, people in the middle and upper end segments of the distribution tend to marry others in this range, and consequently, there is some degree of assortative mating. Each period parents invest, according to their means, in their children. This leads to inertia in intergenerational income mobility. The model can generate an equilibrium in which a significant number of children live in a single-parent home and there is a substantial degree of intergenerational persistence in income.

To illustrate the workings of the model, two policy experiments are tried: child support and welfare. Child support has two effects: it increases the living standard for children living in single-parent families and it discourages fathers from abandoning their families. An increase in child support results in more marriages, fewer divorces, and fewer single-parent families. In the experiment run, it unambiguously lifts up society's income distribution, in the sense that the new income distribution stochastically dominates the old one. Welfare allows single parents to spend more time with their children, which is good for their offspring's human capital development. It encourages women to choose single life and to withdraw from the labor force in order to gain leisure, however, at least in the experiment conducted. As a consequence, welfare is found to increase the well-being of children from single-parent families, but it also leads to fewer marriages, a higher number of divorces, and a greater incidence of single-parent families. While the equilibrium distribution of women's utilities is better at the low end, it is worse everywhere else, and it has a lower average value.

Additionally, the model suggests that the transitional dynamics associated with policy changes may take a long time to work themselves through the system. While in the long run a woman's expected lifetime utility may be higher in the economy without welfare, in the short run (which may be agonizingly long), this need not be the case, as is illustrated. Finally, the numbers reported in the experiments are presented to illustrate how a model such as this works and what it can be used for. They are not intended to do service in public policy debates. The numerical results may well be sensitive to the parameter values imposed and functional forms adopted. A key step in the development of models such as this will be pinning down an appropriate parameterization to use. Furthermore, the structure of the theoretical prototype developed here is still crude, as will now be discussed.

There are many potential ways to improve the primitive nature of the framework used here. And any serious policy analysis would demand improvements. First, more periods could be added to the framework. This may be important for two reasons. Turnover in the marriage market may be sensitive to the number of periods there are. An individual could be more likely to remain single or to divorce if he or she believes that there will be lots of opportunities to find another mate. Also, at any point in time, most people in the United States do not have dependent children. A natural way to do this in the model is to extend the time horizon to include periods without children, such as retirement. This may moderate the welfare gains from family policy. Second, adding savings could make the framework more interesting. As discussed, the risks of divorce are large. Individuals could self-insure against its consequences by accumulating assets. This possibility may lower the welfare gains from public policy aimed at reducing the deleterious effects of divorce. Whether allowing for tangible wealth will promote or dissuade marriage is hard to tell in advance. On the one hand, the presence of tangible wealth makes divorce more attractive since it eases its burden; on the other hand, this may make marriage more attractive because it is less costly to dissolve.[19] Third, other models of household decision making may describe the behavior of families more accurately. Perhaps, for example, a husband and wife arrive at their decisions via Nash bargaining, or they care about a child's welfare as opposed to the level of human capital investments they make. Fourth, a fertility decision could be added. It is natural to believe that the decisions to marry and have offspring are connected. This also may moderate the welfare gains from public policy since any resources directed to families may be partially dissipated through larger family size (see Knowles 1999).[20]

Appendix

Steady-State Matching Probabilities

How are the odds of meeting a single age j, type x female, $\Phi_j(x)$, or a single age j, type z male, $\Omega_j(z)$, determined in stationary equilibrium? To begin with, consider the odds of meeting a two-period-old single agent of a given type in the marriage market, or $\Phi_2(x)$ and $\Omega_2(z)$. This depends on the number of single agents who remain unmarried from the previous period. How many are there? To answer this, consider the problem of a one-period-old female of type x_i. She will draw z_j from the marriage market with probability $\Omega_1(z_j)$. If $I_1^s(x_i, z_j; \cdot)J_1^s(x_i, z_j; \cdot) = 1$, she will marry z_j. The conditional

[19] These considerations may affect males and females differently, too. Plus the effect on marriage is likely to depend on how communal assets get split up at divorce, how much is lost in litigation, etc.

[20] Individuals with high levels of human capital tend to marry later than individuals with low levels. Modeling the schooling decision for young adults and the timing of marriage and births may bear some fruit.

probability that a one-period-old type x_i female will be married is therefore given by

$$\sum_{j=1}^{n}\Omega_1(z_j)I_1^s(x_i, z_j; \cdot)J_1^s(x_i, z_j; \cdot).$$

The conditional odds that this woman will remain single are

$$1 - \sum_{j=1}^{n} \Omega_1(z_j)I_1^s(x_i, z_j; \cdot)J_1^s(x_i, z_j; \cdot).$$

Thus the numbers of married and single one-period-old type x_i females are given by

$$\Phi_1(x_i)\sum_{j=1}^{n}\Omega_1(z_j)I_1^s(x_i, z_j; \cdot)J_1^s(x_i, z_j; \cdot)$$

and

$$\Phi_1(x_i)\left[1 - \sum_{j=1}^{n}\Omega_1(z_j)I_1^s(x_i, z_j; \cdot)J_1^s(x_i, z_j; \cdot)\right].$$

Given this supply of one-period-old single females, the quantity of two-period-old type x_k single females will be

$$\sum_{i=1}^{n} X(x_k|x_i)\Phi_1(x_i)\left[1 - \sum_{j=1}^{n}\Omega_1(z_j)I_1^s(x_i, z_j; \cdot)J_1^s(x_i, z_j; \cdot)\right].$$

The odds of drawing a two-period-old type x_k female in the marriage market will therefore be

$$\Phi_2(x_k) = \frac{\sum_{i=1}^{n} X(x_k|x_i)\Phi_1(x_i)\left[1 - \sum_{j=1}^{n}\Omega_1(z_j)I_1^s(x_i, z_j; \cdot)J_1^s(x_i, z_j; \cdot)\right]}{\sum_{k=1}^{n}\sum_{i=1}^{n} X(x_k|x_i)\Phi_1(x_i)\left[1 - \sum_{j=1}^{n}\Omega_1(z_j)I_1^s(x_i, z_j; \cdot)J_1^s(x_i, z_j; \cdot)\right]}. \tag{A1}$$

The analogous formula for $\Omega_2(z_l)$ is

$$\Omega_2(z_l) = \frac{\sum_{l=1}^{n} Z(z_l|z_j)\Omega_1(z_j)\left[1 - \sum_{i=1}^{n}\Phi_1(x_i)I_1^s(x_i, z_j; \cdot)J_1^s(x_i, z_j; \cdot)\right]}{\sum_{l=1}^{n}\sum_{j=1}^{n} Z(z_l|z_j)\Omega_1(z_j)\left[1 - \sum_{i=1}^{n} \Phi_1(x_i)I_1^s(x_i, z_j; \cdot)J_1^s(x_i, z_j; \cdot)\right]}. \tag{A2}$$

It is easy to see that (A1) and (A2) describe a mapping of the form (4).

Next, what are the odds of meeting a one-period-old single agent of a given type in the marriage market, or $\Phi_1(x)$ and $\Omega_1(z)$? Note that the formulas for $\Phi_2(x_k)$ and $\Omega_2(z_l)$ above depend on $\Phi_1(x_i)$ and $\Omega_1(z_j)$. To determine these probabilities, let $\Upsilon^{mm}(x_i, z_j, x_k, z_l, \gamma_h)$ represent the number of young men or women who grew up their entire life with married parents who transited from state (x_i, z_j) to (x_k, z_l) and experienced the match quality γ_h. In similar fashion, let $\Upsilon^{ss}(x_i, x_k)$ denote the number of males or females who grew up with a single mother whose life went from x_i to x_k, and $\Upsilon^{ms}(x_i, z_j, x_k, \gamma_h)$ the number who suffered a mid-childhood breakup, and so forth. Now, denote the educational input by a married family of type (x_i, z_j, γ_h) by $E^m(x_i, z_j, \gamma_h)$ and that of a single mother of type x_i by $E^s(x_i)$. Then it is easy to see that the number of young women of type x_r is given by

$$\begin{aligned}\Phi_1(x_r) = &\sum_{i,j,k,l,h} \Xi[x_r|E^m(x_i, z_j, \gamma_h) + E^m(x_k, z_l, \gamma_h)]\Upsilon^{mm}(x_i, z_j, x_k, z_l, \gamma_h)\\ &+ \sum_{i,k} \Xi[x_r|E^s(x_i) + E^s(x_k)]\Upsilon^{ss}(x_i, x_k)\\ &+ \sum_{i,j,k,h} \Xi[x_r|E^m(x_i, z_j, \gamma_h) + E^s(x_k)]\Upsilon^{ms}(x_i, z_j, x_k, \gamma_h)\\ &+ \sum_{i,k,l,h} \Xi[x_r|E^s(x_i) + E^m(x_k, z_l, \gamma_h)]\Upsilon^{sm}(x_i, x_k, z_l, \gamma_h).\end{aligned} \tag{A3}$$

Clearly,

$$\begin{aligned}\Omega_1(z_r) = &\sum_{i,j,k,l,h} \Lambda[z_r|E^m(x_i, z_j, \gamma_h) + E^m(x_k, z_l, \gamma_h)]\Upsilon^{mm}(x_i, z_j, x_k, z_l, \gamma_h)\\ &+ \sum_{i,k} \Lambda[z_r|E^s(x_i) + E^s(x_k)]\Upsilon^{ss}(x_i, x_k)\\ &+ \sum_{i,j,k,h} \Lambda[z_r|E^m(x_i, z_j, \gamma_h) + E^s(x_k)]\Upsilon^{ms}(x_i, z_j, x_k, \gamma_h)\\ &+ \sum_{i,k,l,h} \Lambda[z_r|E^s(x_i) + E^m(x_k, z_l, \gamma_h)]\Upsilon^{sm}(x_i, x_k, z_l, \gamma_h).\end{aligned} \tag{A4}$$

So all that remain to be specified are Υ^{mm}, Υ^{ss}, and so forth. They are determined in line with[21]

$$\begin{aligned}\Upsilon^{mm}(x_i, z_j, x_k, z_l, \gamma_h) \equiv{}& \Phi_1(x_i)\Omega_1(z_j)\Gamma(\gamma_h)I_1^s(x_i, z_j; \cdot)J_1^s(x_i, z_j; \cdot)\\ &\times I_2^m(x_k, z_l, \gamma_h)J_2^m(x_k, z_l, \gamma_h)X(x_k|x_i)Z(z_l|z_j),\end{aligned}$$

[21] With child support in the model, $\Upsilon^{ms}(x_i, z_j, x_k, \gamma_h)$ will become $\Upsilon^{ms}(x_i, z_j, x_k, z_l, \gamma_h)$, indicating the fact that ex-husband z_j will pay child support from his current income nz_l.

$$\Upsilon^{ss}(x_i, x_k) \equiv \Phi_1(x_i)\left[1 - \sum_{j=1}^{n} \Omega_1(z_j) I_1^s(x_i, z_j; \cdot) J_1^s(x_i, z_j; \cdot)\right]$$
$$\times X(x_k|x_i)\left[1 - \sum_{l=1}^{n} I_2^s(x_k, z_l) J_2^s(x_k, z_l) \Omega_2(z_l)\right],$$
$$\Upsilon^{ms}(x_i, z_j, x_k, \gamma_h) \equiv \Phi_1(x_i)\Omega_1(z_j)\Gamma(\gamma_h) I_1^s(x_i, z_j; \cdot) J_1^s(x_i, z_j; \cdot)$$
$$\times X(x_k|x_i)\left[1 - \sum_{l=1}^{n} I_2^m(x_k, z_l, \gamma_h) J_2^m(x_k, z_l, \gamma_h) Z(z_l|z_j)\right], \tag{A5}$$
$$\Upsilon^{sm}(x_i, x_k, z_l, \gamma_h) \equiv \Phi_1(x_i)\left[1 - \sum_{j=1}^{n} \Omega_1(z_j) I_1^s(x_i, z_j; \cdot) J_1^s(x_i, z_j; \cdot)\right]$$
$$\times I_2^s(x_k, z_l) J_2^s(x_k, z_l) \Gamma(\gamma_h) X(x_k|x_i) \Omega_2(z_l).$$

Finally, note that the mapping given by (5) implicitly describes equations (A3) and (A4) taken together with (A5).

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