Comments on
"Marginal Mean Models for Dynamic Regimes"
by Murphy, van der Laan, Robins, and CPPRG

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This is a very interesting article. I begin by providing a simplified summary of what I understood about it, followed by some generic comments.

Summary

This paper considers the evaluation of a social program using experimental panel data. The program lasts for several periods. Each period an individual-specific treatment regime is chosen depending on need. Evaluation is performed by comparing the averages of some response variable for treatment and control groups at the end of the program.

The goal of the paper is to use the observational data to evaluate counterfactual treatment assignment policies.

To summarize the approach in the paper, let us consider a simplified setting in which there are only two possible treatment levels and two periods.

The complete *severity* (S) and *response* (Y) vector of *latent variables* is given by

$$O_{sr} = (S_0, S_1(0), S_1(1), Y(0,0), Y(0,1), Y(1,0), Y(1,1)) \,.$$

(There is also an auxiliary vector of variables

$$O_{aux} = (V_0, V_1(0), V_1(1)) \,,$$

which plays an important role in the identification strategy followed in the paper, but for simplicity I abstract from this.)

The observed data consists of:

- the *treatment* indicators:

  $$\overline{A}_2 = (A_1, A_2)$$

  where A_t is a $0-1$ binary variable indicating each of the two possible treatment regimes in period t,

- and the *observable severity* and *response*:

  $$(S_0, S_1, Y)$$

  where

  $$S_1 = S_1(0)\,(1-A_1) + S_1(1)A_1$$

$$\begin{aligned} Y &= Y(0,0)\,(1-A_1)\,(1-A_2) + Y(0,1)\,(1-A_1)\,A_2 \\ &\quad + Y(1,0)A_1\,(1-A_2) + Y(1,1)A_1A_2. \end{aligned}$$

Note that the joint probability distribution of O_{sr} and $\overline{A}_2$ can be factorized as:

$$\begin{aligned} &P_{obs}\,(A_1, A_2, O_{sr} \mid S_0) \\ &= \pi_2\,(A_2 \mid A_1, S_0, O_{sr})\,\pi_1\,(A_1 \mid S_0, O_{sr})\,P\,(O_{sr} \mid S_0)\,. \end{aligned}$$

The situation contemplated in the paper is as follows. We have experimental data (i.e. there are observations on individuals belonging to experimental $(D = 1)$ and control $(D = 0)$ groups).

Administration of treatment to those in the experimental group is stochastic, following the (unknown) probability distributions $\pi_2 (A_2 \mid A_1, S_0, O_{sr})$ and $\pi_1 (A_1 \mid S_0, O_{sr})$.

The average effect on the response variable Y of the actual treatment conducted (given initial severity S_0) is therefore given by

$$\beta_{obs} (S_0) = E_{obs} (Y \mid S_0, D = 1) - E_{obs} (Y \mid S_0, D = 0) .$$

The paper, however, is interested in using the data to measure the average effect of a counterfactual treatment policy given by $p_2 (A_2 \mid S_1)$ and $p_1 (A_1 \mid S_0)$. Thus the interest is in calculating expectations from the alternative distribution

$$P_c (A_1, A_2, O_{sr} \mid S_0) = p_2 (A_2 \mid S_1) \, p_1 (A_1 \mid S_0) \, P (O_{sr} \mid S_0) .$$

Namely,

$$\beta_c (S_0) = E_c (Y \mid S_0, D = 1) - E_{obs} (Y \mid S_0, D = 0) ,$$

where for discrete severity and response:

$$\begin{aligned} & E_c (Y \mid S_0, D) \\ = & \sum_{A_2, A_1, O_{sr}} Y p_2 (A_2 \mid S_1) \, p_1 (A_1 \mid S_0) \, P (O_{sr} \mid S_0) . \end{aligned}$$

Identification of $\beta_c(S_0)$ is achieved by assuming *sequential randomization* (given the observed auxiliary variables for the previous periods, which we omitted):

$$\pi_1(A_1 \mid S_0, O_{sr}) = \pi_1(A_1 \mid S_0)$$
$$\pi_2(A_2 \mid A_1, S_0, O_{sr}) = \pi_2(A_2 \mid A_1, S_{1,} S_0).$$

Under such assumption, π_1 and π_2 are identified from the available data.

In practice the quality of the conditioning auxiliary variables will be crucial for the credibility of the assumption.

Moreover,

$$E_c(Y \mid S_0, D) =$$

$$\sum_{A_2, A_1, O_{sr}} \{Y \frac{p_2(A_2 \mid S_1)\, p_1(A_1 \mid S_0)}{\pi_2(A_2 \mid A_1, S_{1,} S_0)\, \pi_1(A_1 \mid S_0)}$$
$$\pi_2(A_2 \mid A_1, S_{1,} S_0)\, \pi_1(A_1 \mid S_0)\, P(O_{sr} \mid S_0)\}$$

$$= E_{obs}\left(W_p\left(\overline{A}_2, \overline{S}_1\right) Y \mid S_0, D\right)$$

where the $W_p\left(\overline{A}_2, \overline{S}_1\right)$ are weights that perform the switch from the counterfactual to the observational distribution:

$$W_p\left(\overline{A}_2, \overline{S}_1\right) = \frac{p_2(A_2 \mid S_1)\, p_1(A_1 \mid S_0)}{\pi_2(A_2 \mid A_1, S_{1,} S_0)\, \pi_1(A_1 \mid S_0)}.$$

To implement this result, the authors consider a parametric model:

$$E_c\left(Y \mid S_0, D\right) = \mu\left(S_0, D; \beta\right).$$

Given the assumptions, a consistent estimator of β can be obtained by solving

$$\frac{1}{n}\sum_{i=1}^{n} W_p\left(\overline{A}_{2i}, \overline{S}_{1i}\right)\frac{\partial\mu\left(S_{0i}, D_i; \beta\right)}{\partial\beta'}\left[Y_i - \mu\left(S_{0i}, D_i; \beta\right)\right] = 0,$$

where $W_p\left(\overline{A}_{2i}, \overline{S}_{1i}\right) = 1$ for controls.

- In practice, $W_p\left(\overline{A}_{2i}, \overline{S}_{1i}\right)$ depends on estimated parameters because $\pi_2\left(A_2 \mid A_1, S_{1,}S_0\right)$ and $\pi_1\left(A_1 \mid S_0\right)$ are parameterized and their coefficients are estimated from data.
- The paper also considers an alternative estimator, which takes into account the efficiency increases that can be derived from exploiting the correlation between the moment conditions above and the score functions for the treatment assignment probabilities.

- The methodology is applied to the evaluation of a program designed to reduce drug-use and conduct disorders in children at risk.
- Treatment consisted on home visiting assignments on a semester basis.
- Severity is an assessment of need (a measure on the quality of family functioning).
- Two response variables were considered: a social health profile and a rating of self esteem.
- There was a quantitative (deterministic) rule for assigning visits, but in practice staff members were allowed to deviate from the rule by taking into account considerations other than the measure of severity.
- The empirical analysis compares the effect of the program as implemented with the counterfactual effect that would have prevailed had the rule been implemented without deviance.

Comments

1) *Sequential randomization conditional on unobserved differences in origin.*

One might expect longitudinal data to help in controlling for time-invariant unobserved heterogeneity. That is, suppose sequential randomization holds given an individual effect η:

$$\pi_t \left(A_t \mid \overline{A}_{t-1}, \overline{L}_{t-1}, O_{sr}, \eta \right) = \pi_t \left(A_t \mid \overline{A}_{t-1}, \overline{L}_{t-1}, \eta \right).$$

Then weights will also depend on the effects: $W_p \left(\overline{A}_2, \overline{S}_1, \eta \right)$, giving rise to a non-linear fixed effects panel data model $E_{obs} \left(W_p \left(\overline{A}_2, \overline{S}_1, \eta \right) Y \mid S_0, D, \eta \right)$.

2) *Dynamic aspects in the evaluation of the program.*

It is surprising that the treatment is dynamic but its evaluation is static. Why not consider dynamic aspects in the evaluation of the program? This seems specially relevant for ongoing programs whose duration is not always pre-specified from the outset.

Questions like the following seem relevant. Is program duration too long or too short? How much of the two-year effect of the program was achieved by the end of the first year? What would be the gain from an additional year? Is it possible that variation in treatment regime from one period to the next is harmful/desirable?

3) *Possible endogenous reaction of subjects to the program.*

A problem arises if individuals are not indifferent to treatment and severity becomes (to some extent) a choice variable (eg. if children dislike FAST visitors, part of their response to the program may be transitory, as an endogenous response in order to reduce the number of visits).

4) *Non-experimental data under multiple policies.*

Suppose a policy is some unemployment benefit or income tax exemption schedule. In these situations control groups are typically not available. But suppose there are exogenous differences in policies across states or countries. Observational policies in states A and B are π_{At} and π_{Bt}. We can directly evaluate the relative average effect $E_A(Y) - E_B(Y)$, but also the relative effect of some counterfactual policy p_t using the assumptions and changes of measure suggested in the paper:

$$E_C(Y) = E_A\left(\prod_t \frac{p_t}{\pi_{At}} Y\right) = E_B\left(\prod_t \frac{p_t}{\pi_{Bt}} Y\right).$$

In these applications, however, endogenous reaction and anticipation to policy rules is usually important and cannot be ignored.