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On the testing of correlated effects with panel data

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Using orthogonal deviations of the variables, correlated effects biases are regarded as misspecification biases due to the exclusion of relevant variables in a standard regression model. Following this approach, Hausman- and Chamberlain-type tests of correlated effects are obtained as Wald tests in an extended model estimated by OLS, and robust generalisations are suggested. Alternative estimators which introduce restrictions in the regression of the effects on the explanatory variables are proposed. Finally, the paper extends the results to dynamic models.

1. Introduction

Testing for the correlation of unobservable individual effects with the right-hand-side variables in panel data regressions is a widespread practice. In static models, the standard procedure is to use a Hausman test [Hausman (1978)] based on the comparison between the within-groups (WG) and the GLS estimators. In this article we regard correlated effects biases as misspecification biases due to the exclusion of relevant variables in a standard regression model. In doing this we follow the work of Mundlak (1978) and Chamberlain (1982) while exploiting a transformation suggested by Arellano and Bover (1990).

Following this approach, in section 2, the Hausman test of correlated effects is obtained as a Wald test in an extended model estimated by OLS. This approach is useful for several reasons. Firstly, it suggests a straightforward generalised test which is robust to heteroskedasticity and autocorrelation of arbitrary forms. Notice that if the errors are heteroskedastic and/or autocorrelated, the standard formulae for the large sample variances of the WG and GLS estimators are not valid. Moreover, WG and GLS cannot be ranked in terms of efficiency so that the variance of the difference between the two does not coincide with

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the difference of variances. Secondly, it clarifies the relationship between the Hausman test and the tests of correlated effects based on the reduced form ('Π matrix') approach of Chamberlain (1982): the Hausman test is a Wald test based on a less general specification of the model under the alternative hypothesis than the Chamberlain-type test. The latter can also be easily calculated as a Wald test in a closely related extended model estimated by OLS. Thirdly, it suggests alternative estimators which introduce restrictions in the regression of the effects on the explanatory variables. These estimators exploit the fact that some partial correlations between the effects and explanatory variables may be zero. This is in contrast with the assumptions of lack of correlation between some of the regressors and the effects used in models of the Hausman–Taylor type [cf. Hausman and Taylor (1981)]. Fourthly, the same procedures can be used in dynamic models, provided the extended model is estimated by instrumental variables instead of least squares. The alternative tests for static models are discussed in section 3, while section 4 discusses the estimators with restricted partial correlations. Finally, section 5 contains the results for dynamic models and section 6 concludes.

2. The model and the Hausman test

The model is given by

$$\mathrm{E}(y_{it} \mid x_i, \eta_i) = x'_{it}\beta + \eta_i, \qquad t = 1, \ldots, T, \quad i = 1, \ldots, N, \tag{1}$$

where x_{it} is a $k \times 1$ vector of explanatory variables, β is the $k \times 1$ vector of coefficients to be estimated, η_i is an unobservable individual effect, and $x_i = (x'_{i1}, \ldots, x'_{iT})'$. The number of time periods T is small, the number of individuals N is large, and the observations are independently distributed over the cross-section.

If we take conditional expectations given x_i alone we have

$$\mathrm{E}(y_{it} \mid x_i) = x'_{it}\beta + \mathrm{E}(\eta_i \mid x_i).$$

The null hypothesis under test is the mean independence of η_i given x_i,[1]

$$\mathrm{H}_0: \quad \mathrm{E}(\eta_i \mid x_i) = 0.$$

[1] Notice that since the conditional expectation in (1) is linear, it coincides with the best linear predictor

$$\mathrm{E}^*(y_{it} \mid x_i, \eta_i) = x'_{it}\beta + \eta_i,$$

hence also

$$\mathrm{E}^*(y_{it} \mid x_i) = x'_{it}\beta + \mathrm{E}^*(\eta_i \mid x_i),$$

so that $\mathrm{E}^*(\eta_i \mid x_i) = 0$ is a sufficient condition for the consistency for β of the least squares regression of y_{it} on x_{it}, given standard regularity conditions.

Let us write the system of T equations for individual i as

$$\mathrm{E}(y_i \mid x_i, \eta_i) = X_i \beta + \eta_i \iota ,$$

where $y_i = (y_{i1}, \ldots, y_{iT})'$, $X_i = (x_{i1}, \ldots, x_{iT})'$ is a $T \times k$ matrix, and ι is a $T \times 1$ vector of ones. Next, we can perform a decomposition between the within-groups and the between-groups variation, transforming the system by means of the nonsingular $T \times T$ transformation matrix

$$H = \begin{bmatrix} A \\ T^{-1}\iota' \end{bmatrix},$$

where A is the $(T-1) \times T$ forward orthogonal deviations operator described in Arellano and Bover (1990). Specifically, the elements of the $(T-1) \times 1$ transformed vector $y_i^* = Ay_i$ are

$$y_{it}^* = \left[\frac{T-t}{T-t+1} \right]^{1/2} \left[y_{it} - \frac{1}{(T-t)} (y_{i(t+1)} + \cdots + y_{iT}) \right],$$

$$t = 1, \ldots, T-1.$$

The operator A has the properties $A\iota = 0$, $AA' = I_{(T-1)}$, and $A'A = Q = I_T - \iota\iota'/T$, where Q is the within-groups operator. The transformed system is

$$\mathrm{E}(y_i^* \mid x_i, \eta_i) = X_i^* \beta, \qquad \mathrm{E}(\bar{y}_i \mid x_i, \eta_i) = \bar{x}_i' \beta + \eta_i ,$$

so that also

$$\mathrm{E}(y_i^* \mid x_i) = X_i^* \beta, \qquad \mathrm{E}(\bar{y}_i \mid x_i) = \bar{x}_i' \beta + \mathrm{E}(\eta_i \mid x_i),$$

in which the variables with bars denote time means and the starred variables denote forward orthogonal deviations, that is, $X_i^* = AX_i$, $\bar{x}_i = X_i'\iota/T$, etc.

With homoskedasticity and absence of serial correlation, that is, if

$$\mathrm{var}(y_i \mid x_i, \eta_i) = \sigma^2 I_T \quad \text{and} \quad \mathrm{var}(\eta_i \mid x_i) = \sigma_\eta^2$$

in which case

$$\begin{aligned} \mathrm{var}(y_i \mid x_i) &= \mathrm{E}(\mathrm{var}(y_i \mid x_i, \eta_i) \mid x_i) + \mathrm{var}(\mathrm{E}(y_i \mid x_i, \eta_i) \mid x_i) \\ &= \sigma^2 I_T + \sigma_\eta^2 \iota\iota' , \end{aligned}$$

it can be readily shown that the conditional variance matrix of the transformed vector $y_i^+ = Hy_i$ is given by

$$\operatorname{var}(y_i^+ \mid x_i) = \sigma^2 \begin{bmatrix} I_{(T-1)} & 0 \\ 0 & 1/\theta^2 T \end{bmatrix}, \tag{2}$$

with $\theta^2 = \sigma^2/(\sigma^2 + T\sigma_\eta^2)$.

We turn to consider the specification of the alternative hypothesis. In the first place, we define the 'Hausman alternative hypothesis' to be

$$\mathrm{H}_1: \quad \mathrm{E}(\eta_i \mid x_i) = \mathrm{E}(\eta_i \mid \bar{x}_i) = \bar{x}_i' \gamma,$$

so that under the null hypothesis we have $\gamma = 0$ and the transformed system under H_1 is

$$\mathrm{E}(y_i^* \mid x_i) = X_i^* \beta, \qquad \mathrm{E}(\bar{y}_i \mid x_i) = \bar{x}_i'(\beta + \gamma) = \bar{x}_i' b. \tag{3}$$

In this setting, OLS applied to the first $(T-1)$ equations gives the WG estimator, OLS applied to the last equation gives the between-groups (BG) estimator, and weighted least squares applied to the complete system under the null gives the GLS estimator:

$$\begin{aligned} \hat{\beta}_{\mathrm{WG}} &= (X^{*\prime} X^*)^{-1} X^{*\prime} y^*, \\ \hat{b}_{\mathrm{BG}} &= (\bar{X}' \bar{X})^{-1} \bar{X}' \bar{y}, \\ \hat{\beta}_{\mathrm{GLS}} &= (X^{*\prime} X^* + \hat{\theta}^2 T \bar{X}' \bar{X})^{-1} (X^{*\prime} y^* + \hat{\theta}^2 T \bar{X}' \bar{y}), \end{aligned} \tag{4a}$$

where $\hat{\theta}^2$ is a consistent estimator of θ^2, $X^* = (X_1^{*\prime}, \ldots, X_N^{*\prime})'$, $y^* = (y_1^{*\prime}, \ldots, y_N^{*\prime})'$, $\bar{X} = (\bar{x}_1, \ldots, \bar{x}_N)'$, and $\bar{y} = (\bar{y}_1, \ldots, \bar{y}_N)'$. Moreover, the large sample variances of WG, BG, and GLS are respectively given by

$$\begin{aligned} V_{\mathrm{WG}} &= \sigma^2 (X^{*\prime} X^*)^{-1}, \\ V_{\mathrm{BG}} &= \sigma^2 (\theta^2 T \bar{X}' \bar{X})^{-1}, \\ V_{\mathrm{GLS}} &= \sigma^2 (X^{*\prime} X^* + \theta^2 T \bar{X}' \bar{X})^{-1}. \end{aligned} \tag{4b}$$

Provided (2) holds, under H_1 $\hat{\beta}_{\mathrm{WG}}$ and $\hat{b}_{\mathrm{BG}}$ are the best linear unbiased estimators of β and b, respectively, and under H_0 $\hat{\beta}_{\mathrm{GLS}}$ is asymptotically equivalent to the (unfeasible) GLS estimator of β.

The Hausman test statistic is given by

$$h = (\hat{\beta}_{\mathrm{GLS}} - \hat{\beta}_{\mathrm{WG}})' (\hat{V}_{\mathrm{WG}} - \hat{V}_{\mathrm{GLS}})^{-1} (\hat{\beta}_{\mathrm{GLS}} - \hat{\beta}_{\mathrm{WG}}), \tag{5}$$

where the hats on the variances denote consistent estimators. Hausman and Taylor (1981) showed that an alternative expression for h can be given in terms of the difference between the BG and the WG estimators:[2]

$$h = (\hat{b}_{\mathrm{BG}} - \hat{\beta}_{\mathrm{WG}})'(\hat{V}_{\mathrm{WG}} + \hat{V}_{\mathrm{BG}})^{-1}(\hat{b}_{\mathrm{BG}} - \hat{\beta}_{\mathrm{WG}}). \tag{6}$$

The statistic h can also be obtained as a Wald statistic of the restriction $\gamma = 0$ from OLS estimates of model (3). That is, we consider the regression

$$\begin{bmatrix} y_i^* \\ \bar{y}_i \end{bmatrix} = \begin{bmatrix} X_i^* & 0 \\ \bar{x}_i' & \bar{x}_i' \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \end{bmatrix} + \begin{bmatrix} v_i^* \\ \bar{u}_i \end{bmatrix}, \tag{7}$$

or in an obvious notation,

$$y_i^+ = W_i \delta + u_i^+ .$$

Clearly, the OLS estimator of β in (7) is the WG estimator $\hat{\beta}_{\mathrm{WG}}$, while the OLS estimator of γ is the difference between $\hat{b}_{\mathrm{BG}}$ and $\hat{\beta}_{\mathrm{WG}}$:

$$\hat{\gamma} = \hat{b}_{\mathrm{BG}} - \hat{\beta}_{\mathrm{WG}} .$$

Therefore the Wald test of the restriction $\gamma = 0$ coincides with (6), and so it is numerically identical to the Hausman statistic. Notice that this test can be regarded as an application of the Chow test under heteroskedasticity, so that the approximations to its finite sample distribution available in the literature apply.

3. Alternative tests for static models

Chamberlain (1982) considered a linear regression of η_i on x_i in order to specify the dependence between the effects and the explanatory variables. We define the 'Chamberlain alternative hypothesis' to be

$$\mathrm{H}_1: \quad \mathrm{E}(\eta_i | x_i) = x_i' \lambda , \tag{8}$$

[2] The fact that the two expressions for the Hausman test are identical is easily verified using

$$V_{\mathrm{GLS}}^{-1} = V_{\mathrm{WG}}^{-1} + V_{\mathrm{BG}}^{-1}, \qquad \hat{\beta}_{\mathrm{GLS}} = V_{\mathrm{GLS}} V_{\mathrm{WG}}^{-1} \hat{\beta}_{\mathrm{WG}} + V_{\mathrm{GLS}} V_{\mathrm{BG}}^{-1} \hat{b}_{\mathrm{BG}} ,$$

which are well-known results due to Maddala (1971). Notice that these results can be immediately obtained from the equations in (4a) and (4b).

where $x_i'\lambda = \sum_{s=1}^{T} x_{is}'\lambda_s$ and λ is a $Tk \times 1$ vector of coefficients. With this specification, the transformed model under the alternative becomes:

$$\mathrm{E}(y_i^* \mid x_i) = \mathrm{X}_i^* \beta, \qquad \mathrm{E}(\bar{y}_i \mid x_i) = \bar{x}_i'\beta + x_i'\lambda .$$

Thus, we consider the OLS regression

$$\begin{bmatrix} y_i^* \\ \bar{y}_i \end{bmatrix} = \begin{bmatrix} X_i^* & 0 \\ \bar{x}_i' & x_i' \end{bmatrix} \begin{bmatrix} \beta \\ \lambda \end{bmatrix} + \begin{bmatrix} v_i^* \\ \bar{u}_i \end{bmatrix}. \tag{9}$$

The OLS estimator of β in this model is still the WG estimator. In addition, notice that in this model and also in model (7), the OLS and the weighted least squares estimators coincide. Here the standard Wald test of $\lambda = 0$ based on OLS estimates of (9) provides an alternative Tk degrees of freedom chi-square test statistic.

The comparison between Hausman and Chamberlain alternatives reveals that the former is a special case of the latter in which $\lambda_1 = \cdots = \lambda_T = \gamma/T$. In effect, the Hausman test is testing the k-moment restrictions

$$\mathrm{E}[\bar{x}_i(\bar{y}_i - \bar{x}_i'\beta)] = 0 ,$$

while the Chamberlain-type test is testing the kT-moment restrictions

$$\mathrm{E}[x_i(\bar{y}_i - \bar{x}_i'\beta)] = 0 ,$$

which amounts to testing $\mathrm{E}(\bar{x}_i\eta_i) = 0$ and $\mathrm{E}(x_i\eta_i) = 0$, respectively, having assumed that (1) holds for all t. If some x's are known to be uncorrelated or only weakly correlated with the effects given the other x's, the power of the test can be improved by excluding those x's from the second block of columns in the extended model. This case is taken up in the next section.

If (2) does not hold because of heteroskedasticity and/or serial correlation, neither of the previous estimators are optimal under H_0 or H_1. However, consistent estimators of the variances of the OLS estimators in models (7) and (9) can be obtained using White's formulae [cf. White (1984)]. For example, for the OLS estimator of δ in model (7) we have

$$\hat{V}_{\delta\delta} = (W'W)^{-1} \left[\sum_{i=1}^{N} W_i'\hat{u}_i^+ \hat{u}_i^{+\prime} W_i \right] (W'W)^{-1} = \begin{bmatrix} \hat{V}_{\beta\beta} & \hat{V}_{\beta\gamma} \\ \hat{V}_{\gamma\beta} & \hat{V}_{\gamma\gamma} \end{bmatrix},$$

where $W = (W_1', \ldots, W_N')'$ and $\hat{u}_i^+$ are OLS residuals. Hence, the generalised test that is robust to heteroskedasticity and autocorrelation is given by

$$h^* = \hat{\gamma}' \hat{V}_{\gamma\gamma}^{-1} \hat{\gamma} . \tag{10}$$

h^* has a large sample chi-square distribution with k degrees of freedom under the null hypothesis $\gamma = 0$. A similar expression can be written down for the kT degrees of freedom test based on the OLS estimator of λ in model (9). Notice that both test statistics can be easily calculated using a standard package.

4. Alternative models with information in levels

Suppose that x_{it} can be partitioned into two subvectors x_{1it} and x_{2it} of dimensions k_1 and k_2, respectively, such that for a Hausman alternative hypothesis,

$$\mathrm{E}(\eta_i | x_i) = \bar{x}'_{1i}\gamma_1 + \bar{x}'_{2i}\gamma_2 ,$$

γ_1 is known to be zero, that is, the partial correlations between η_i and $\bar{x}_{1i}$ vanish relative to the set of variables in x_i (a parallel discussion could be conducted for a Chamberlain alternative in terms of x_{1i} and x_i).

This knowledge may be the result of theoretical considerations or due to previous empirical evidence. An example of this situation would be a λ-constant labour supply equation [see MaCurdy (1981)], in which y_{it} is labour supply, x_{2it} is the real wage, x_{1it} is a vector of observed characteristics which result from allowing variation in some utility parameter, and η_i is a function of the marginal utility of wealth λ, which itself depends on initial assets, future wages, interest rates, and the form of preferences. The variables x_{1it} will also typically be determinants of wages but depending of the specification of preferences may not enter the expression for λ. They would still be correlated with η_i through wages but the partial correlations, once wages have been accounted for, may vanish.

In this case, the transformed model under the alternative can be written as

$$\mathrm{E}(y_i^+ | x_i) = \begin{bmatrix} X_i^* & 0 \\ \bar{x}'_i & \bar{x}'_{2i} \end{bmatrix} \begin{bmatrix} \beta \\ \gamma_2 \end{bmatrix}. \tag{11}$$

Notice that in this model, OLS and weighted LS estimators do not coincide and none of these two estimators of β coincide with the WG estimator. Consistent estimates of the model can be used to obtain a Wald test of lack of correlation of $\bar{x}_{2i}$ with the effects (i.e., a test of $\gamma_2 = 0$) given that $\bar{x}_{1i}$ is uncorrelated. This model is an intermediate situation between the conventional 'fixed effects' and uncorrelated random effects models.

Hausman and Taylor (1981) also considered a class of intermediate models in which $\mathrm{E}(\bar{x}_{1i}\eta_i) = 0$. This is in contrast to our assumption of $\gamma_1 = 0$. The difference between the two models is best seen comparing the form of the resulting estimators. The Hausman–Taylor estimator is

$$\hat{\beta}_{\mathrm{HT}} = (X^{*\prime}X^* + \hat{\theta}^2 T\bar{X}'M_1\bar{X})^{-1}(X^{*\prime}y^* + \hat{\theta}^2 T\bar{X}'M_1\bar{y}), \tag{12}$$

with $M_1 = \bar{X}_1(\bar{X}_1'\bar{X}_1)^{-1}\bar{X}_1'$ [see Arellano and Bover (1990)], whereas the weighted LS estimator of β in model (11) is given by

$$\hat{\beta}_{PC} = (X^{*\prime}X^* + \hat{\theta}^2 T\bar{X}'P_2\bar{X})^{-1}(X^{*\prime}y^* + \hat{\theta}^2 T\bar{X}'P_2\bar{y}), \tag{13}$$

with

$$P_2 = I_N - \bar{X}_2(\bar{X}_2'\bar{X}_2)^{-1}\bar{X}_2'.$$

Moreover, provided (2) holds, the large sample variance of the partially correlated estimator $\hat{\beta}_{PC}$ is given by

$$V_{PC} = \sigma^2(X^{*\prime}X^* + \theta^2 T\bar{X}'P_2\bar{X})^{-1},$$

and note the inequality (in the matrix sense):

$$V_{GLS} \leq V_{PC} \leq V_{WG}.$$

Hausman and Taylor (1981) emphasized the identifiability of observed components of η_i relying on the variables $\bar{x}_{1i}$ as instrumental variables in a dual role. Here we emphasize the efficiency gains that could be obtained moving from WG estimators to estimators that introduce constraints in the regression of the effects on the explanatory variables. In this regard, it is useful to view the problem of correlated effects biases as a problem of omission of relevant explanatory variables in a linear regression model, so that the specification searches are similar to those of finding the relevant set of explanatory variables using standard tests of individual and joint significance in regression analysis.

5. Results for dynamic models

Now consider the model

$$\mathrm{E}(y_{it} - x_{it}'\beta \mid z_i, \eta_i) = \eta_i, \tag{14}$$

where $x_{it} = (y_{i(t-1)}, z_{it}')'$, the sample starts at $t = 0$, z_{it} is a $(k-1)\times 1$ vector and $z_i = (z_{i0}', \ldots, z_{iT}')'$. We wish to test for the mean independence of the individual effects η_i given z_i,

$$\mathrm{E}(\eta_i \mid z_i) = 0.$$

The relevant model under a Hausman alternative in this case is

$$\mathrm{E}\left\{\begin{bmatrix} y_i^* \\ \bar{y}_i \end{bmatrix} - \begin{bmatrix} X_i^* & 0 \\ \bar{x}_i' & \bar{z}_i' \end{bmatrix}\begin{bmatrix} \beta \\ \gamma \end{bmatrix} \,\middle|\, z_i\right\} = 0, \tag{15}$$

where γ is now a $(k-1)\times 1$ parameter vector.

The parameters β and γ in model (15) can still be consistently estimated by instrumental variables using z_i as the instruments. Letting $Z_i = I_T \otimes z_i'$, $Z = (Z_1', \ldots, Z_N')'$, $y^+ = (y_1^{+\prime}, \ldots, y_N^{+\prime})'$, and redefining W_i and W as appropriate in model (15), a GMM estimator of δ is given by

$$\hat{\delta} = (W'ZBZ'W)^{-1} W'ZBZ'y^+ .$$

A one-step estimator can be calculated with $B = (Z'Z)^{-1}$, while a two-step estimator uses $B = (\sum_{i=1}^N Z_i'\hat{u}_i^+ \hat{u}_i^{+\prime} Z_i)^{-1}$, where $\hat{u}_i^+$ are one-step residuals. The large sample variance of the two-step estimator can be consistently estimated by

$$\hat{V}_{\delta\delta} = \left(W'Z \left(\sum_{i=1}^N Z_i'\hat{u}_i^+ \hat{u}_i^{+\prime} Z_i \right)^{-1} Z'W \right)^{-1} .$$

Finally, we can calculate a $(k-1)$ degrees of freedom Wald test of $\gamma = 0$, similar to (10), but which uses $\hat{\gamma}$ and $\hat{V}_{\gamma\gamma}$ as redefined in this section. If in model (15) $\bar{z}_i$ is replaced by z_i and γ by λ, we obtain a dynamic version of (9) from which we can calculate a Chamberlain-type Wald test based on GMM estimators under the alternative hypothesis. Notice that now λ is a $(k-1)(T+1) \times 1$ parameter vector. Moreover, intermediate cases of the type discussed in section 4 can also be considered here.

An alternative procedure is to use a Sargan-difference test (or likelihood-ratio-like test) which would test the additional overidentifying restrictions implied by the model in levels relative to the model in deviations [see Sargan (1988)]. The Sargan-difference test would be testing the same moment restrictions as the Chamberlain-type Wald test. That is, we wish to test the $(k-1) \times (T+1)$ restrictions

$$\mathrm{E}[z_i(\bar{y}_i - \bar{x}_i'\beta)] = 0 .$$

To obtain a Sargan-difference test let us consider the criteria

$$s(\beta) = u'ZBZ'u \quad \text{and} \quad s_o(\beta) = u^{*\prime} Z_o B_o Z_o' u^* ,$$

where $u = (u_1', \ldots, u_N')'$, $u^* = (u_1^{*\prime}, \ldots, u_N^{*\prime})$, $u_i^* = Au_i$, $u_i = y_i - X_i\beta$, the ith row block of Z_o is $I_{(T-1)} \otimes z_i'$, and B and B_o are optimal weighing matrices. Moreover, let $\hat{s}$ and $\hat{s}_o$ be the minimized criteria with respect to β. Then the Sargan difference statistic is given by

$$\hat{s} - \hat{s}_o \underset{a}{\sim} \chi^2_{(k-1)(T+1)} .$$

One advantage of the Wald procedures is that a large sample chi-square statistic can still be obtained on the basis of estimators that do not use the optimal weighing matrices.

A Sargan-difference test has been proposed by Holtz–Eakin (1988) in order to test for the *existence* of individual effects in an autoregressive model. In that case, the instruments are further lags of the dependent variable. Since lagged dependent variables are correlated with the effects by construction, a test of the lack of orthogonality between the errors in levels and lagged dependent variables amounts to a test of the existence of the effects. A Wald test can also be devised for this case on the previous lines. The only difference is that since under the null hypothesis there are different instruments valid for different equations in levels, the between-groups equation needs to be replaced by the complete set of equations in levels in addition to those in orthogonal deviations.

6. Conclusions

This paper exploits the orthogonal deviations transformation of Arellano and Bover (1990) to decompose the T equations of a linear regression with individual effects into two different regressions with uncorrelated errors: a within-groups regression comprising the first $(T-1)$ equations and a between-groups regression consisting of the last equation. In this setting, testing for uncorrelated effects amounts to a 'stability' test of equality of the two regressions. Thus, the Hausman test is obtained as a Wald test based on a particular specification of the alternative hypothesis.

The same strategy is shown to be useful in order to obtain other tests of correlated effects based on different specifications of the alternative hypothesis and for different models. These include Chamberlain alternatives, partially correlated alternatives which introduce restrictions in the regression of the effects on the explanatory variables, and dynamic models. Robust versions of the test statistics in the presence of heteroskedasticity and serial correlation are also presented.

The approach taken in this paper is also useful in order to suggest appropriate estimators for each model under both the null and the alternative hypotheses. In particular, enforcing zero restrictions in the regression of the effects on the independent variables, gives rise to a new estimator which is compared to the model and the estimator studied by Hausman and Taylor (1981).

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