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Abstract

We propose specification tests for Gaussian SVAR models identified with short- and long-run restrictions that assess the theoretical justification of the chosen identification scheme by checking the independence of the structural shocks. We consider both moment tests that focus on their coskewness and cokurtosis and contingency table tests with discrete and continuous grids. Our simulations confirm the finite sample reliability of resampling versions of our proposals, and their power against interesting alternatives. We also apply them to two influential studies: Kilian (2009) with short-run restrictions in oil markets and Blanchard and Quah (1989) with long-run ones for the aggregate economy.

JEL Codes: C32, C52, E32, Q41, Q43.

Keywords: Consistent test, coskewness, cokurtosis, independence test, moment tests, oil market, pseudo maximum likelihood estimators, supply and demand shocks.

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1 Introduction

Arguably, modern macroeconometrics definitely broke up with the large linear simultaneous equations models that were the workhorse of central banks and economic affairs ministries' research departments in the 1960s and 70s after the introduction of Structural Vector Autoregressions (SVARs) in Sims' (1980) seminal paper. As is well known, he suggested an empirically credible and flexible framework for modelling interconnected macroeconomic and financial time series that allow researchers to uncover theoretically motivated structural shocks and study their contemporaneous and dynamic impacts on the observed variables. Nevertheless, SVAR analysis hinges critically on the identification of those shocks, which typically involves imposing a set of economic or statistical restrictions. The most popular identification schemes are short- and long-run homogeneity restrictions based on economic theory (see Kilian and Lütkepohl (2017) for a textbook treatment), although more recently, alternative approaches have been entertained, including sign restrictions (Faust (1998) and Uhlig (2005)), time-varying heteroskedasticity (Sentana and Fiorentini (2001) and Rigobon (2003)), external instruments (Mertens and Ravn (2012) and Stock and Watson (2018)), independent non-Gaussian shocks (Lanne, Meitz and Saikkonen (2017) and Gouriéroux, Monfort and Renne (2017)) or narrative event constraints (Antolín-Díaz and Rubio-Ramírez (2018) and Ludvigson, Ma and Ng (2021)). Montiel-Olea et al (2022) and Lewis (2025) provide lucid critical reviews of some of these approaches.

A crucial challenge in the empirical application of SVARs is whether the identification restrictions chosen are compatible with the observed data, for model misspecification can lead to biased inference and misleading policy conclusions. While testing exactly identified models is far from straightforward (see Arellano, Hansen and Sentana (2012)), some of the above identification schemes imply overidentifying restrictions. Specifically, several recent papers have proposed specification tests that assess the validity of identification based on independent non-Gaussian shocks. For example, Amengual, Fiorentini, and Sentana (2022) (AFS22) develop moment tests that check that all the coskewness and cokurtosis coefficients of the non-Gaussian structural shocks are zero. More recently, Amengual, Fiorentini, and Sentana (2024) (AFS24) propose further specification tests that directly examine whether the joint distribution of the shocks coincides with the product of their marginal ones, thereby enabling researchers to verify the appropriateness of their models before interpreting their outputs.

However, a framework for testing the validity of alternative identification schemes in SVARs estimated by Gaussian pseudo maximum likelihood (PML) is lacking. Our paper aims to fill this gap by proposing novel specification tests that carefully adapt the procedures in AFS22 and AFS24 to check the indedepence of the shocks in those models. Although in a Gaussian context,

lack of correlation between the structural shocks is tantamount to their independence, empirical researchers that estimate those models do not usually take the Gaussianity assumption literally, which they often make because of its computational simplicity and robustness properties of the resulting estimators to distributional misspecification. And yet, the theoretical dynamic macroeconomic models that justify SVARs in the first place maintain the assumption that the different structural shocks are stochastically independent. For example, in a standard new-Keynesian model, the monetary policy shock entering through the Taylor rule is not related in any way to the aggregate supply shock affecting the Phillips curve or the demand shock appearing in the IS curve. Therefore, it seems appropriate to tests the independence of the identified shocks in SVARs estimated by Gaussian PML as a way of checking that they adequately reflect a fundamental property of the shocks in the underlying theoretical model, as they should if the chosen identification scheme were valid. Given their preeminence in empirical work, we focus on models identified through short- and long-run restrictions.

In many empirical finance applications of SVARs, the number of observations is sufficiently large for asymptotic approximations to be reliable. In contrast, the limiting distributions of our tests may be a poor guide for the smaller samples typically used in macroeconomic applications. For that reason, we thoroughly study the finite sample size of our tests in several Monte Carlo exercises and discuss resampling procedures that seem to improve their reliability.

Finally, we illustrate our proposed procedures by applying them to two influential studies: Kilian (2009) with short-run restrictions in oil markets, and Blanchard and Quah (1989) with long-run ones for the aggregate economy.

The structure of the remainder of this paper is as follows. Section 2 introduces SVARs and the identification schemes that we consider. Section 3 then describes the different testing procedures that we propose. Next, section 4 presents the results of our extensive simulation studies that evaluate the performance of our proposed tests in finite samples in terms of both size under correct specification and power under various realistic forms of misspecification. Finally, section 5 revisits the two aforementioned empirical applications, followed by our conclusions in section 6. We also include three appendices with additional computational details.

2 Structural vector autoregressions

Consider the following N-variate SVAR process of order p:

$$\mathbf{y}_t = \boldsymbol{\tau} + \sum_{j=1}^{p} \mathbf{A}_j \mathbf{y}_{t-j} + \mathbf{C}\boldsymbol{\varepsilon}_t^*, \quad \boldsymbol{\varepsilon}_t^* | I_{t-1} \sim i.i.d.(\mathbf{0}, \mathbf{I}_N), \tag{1}$$

where I_{t-1} is the information set available at time $t-1$, $\mathbf{C}$ the matrix of impact multipliers and $\boldsymbol{\varepsilon}_t^*$ the structural shocks, which we have normalised to have zero means, unit variances and zero

covariances.

Let $\boldsymbol{\varepsilon}_t = \mathbf{C}\boldsymbol{\varepsilon}_t^*$ denote the reduced form innovations, so that $\boldsymbol{\varepsilon}_t|I_{t-1} \sim i.i.d.\ (\mathbf{0}, \boldsymbol{\Sigma})$ with $\boldsymbol{\Sigma} = \mathbf{C}\mathbf{C}'$. In addition, let $\mathbf{a}' = (\mathbf{a}_1', \ldots, \mathbf{a}_p') = [vec'(\mathbf{A}_1), \ldots, vec'(\mathbf{A}_p)]$, $\mathbf{c} = vec(\mathbf{C})$ and $\boldsymbol{\sigma} = vech(\boldsymbol{\Sigma})$. As is well known, the Gaussian PML estimators advocated by Bollerslev and Wooldridge (1992) among many others remain root-T consistent for the reduced form parameters $\boldsymbol{\theta}' = (\boldsymbol{\tau}', \mathbf{a}', \boldsymbol{\sigma}')$ characterising the first two conditional moments of $\mathbf{y}_t$ irrespective of the degree of asymmetry and kurtosis of the conditional distribution of $\boldsymbol{\varepsilon}_t$, so long as those first two moments are correctly specified and the third and fourth moments of $\boldsymbol{\varepsilon}_t$ are bounded. Nevertheless, a Gaussian likelihood function is only able to identify $\boldsymbol{\Sigma}$, which means the structural shocks $\boldsymbol{\varepsilon}_t^*$ and their loadings in $\mathbf{C}$ are only identified up to an orthogonal transformation. Specifically, we can use the QR matrix decomposition of $\mathbf{C}'$ to relate this matrix to the Cholesky decomposition of $\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_L\boldsymbol{\Sigma}_L'$ as $\mathbf{C} = \boldsymbol{\Sigma}_L\mathbf{Q}$, where $\mathbf{Q}$ is an $N \times N$ orthogonal matrix, which we can model as a function of $N(N-1)/2$ parameters $\boldsymbol{\omega}$ by assuming that $|\mathbf{Q}| = 1$ (see e.g. Golub and van Loan (2013)). While $\boldsymbol{\Sigma}_L$ is identified from the Gaussian log-likelihood, $\boldsymbol{\omega}$ is not.[1] For that reason, empirical researchers need to impose additional restrictions to identify $\mathbf{C}$.

The two most popular identification schemes are the following:

Short-run restrictions: In his empirical application to US money and income, Sims (1980) reordered the elements of $\mathbf{y}_t$ so that he could identify $\mathbf{C}$ with $\boldsymbol{\Sigma}_L$. These so-called short-run restrictions on the impact multipliers impose a recursive causal mechanism regularly used in numerous subsequent empirical applications.

Long-run restrictions: In their application to US aggregate output and unemployment, Blanchard and Quah (1989) assumed that while supply shocks would affect the first variable in the long run, demand shocks would not. Effectively, their restrictions imply that the matrix of long run multipliers $(\mathbf{I}_N - \sum_{j=1}^p \mathbf{A}_j)^{-1}\mathbf{C}$ is lower triangular.

Part of the popularity of these identification schemes is that parameter estimation remains straightforward. In both cases, researchers first estimate the reduced form parameters by Gaussian PML. Specifically, the equation by equation OLS regression of each variable on a constant and p lags of $\mathbf{y}_t$ yields the PMLEs of the intercept and slope parameters $\boldsymbol{\tau}$ and $\mathbf{a}$, respectively. As for the elements of $\boldsymbol{\sigma}$, one simply estimates the covariance matrix of the N OLS residuals with denominator T.

Under standard regularity conditions, the asymptotic distribution of these Gaussian PMLEs will be given by

$$\sqrt{T}(\tilde{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0) \xrightarrow{d} N[\mathbf{0}, \mathcal{C}(\boldsymbol{\phi}_0)],$$

[1] In fact, the underidentification of $\boldsymbol{\omega}$ would persist even if we assumed for estimation purposes that $\boldsymbol{\varepsilon}_t^*$ followed an elliptical distribution or a location-scale mixture of normals.

where

$$\mathcal{C}(\boldsymbol{\phi}_0) = \mathcal{A}^{-1}(\boldsymbol{\phi}_0)\mathcal{B}(\boldsymbol{\phi}_0)\mathcal{A}^{-1}(\boldsymbol{\phi}_0), \tag{2}$$

$\mathcal{A}(\boldsymbol{\phi}_0)$ is the expected value of the Gaussian Hessian and $\mathcal{B}(\boldsymbol{\phi}_0)$ the variance of the Gaussian score. In Appendix A, we provide detailed expressions for these matrices for model (1) under the maintained assumption that the true structural shocks are cross-sectionally independent but not necessarily Gaussian.

Then, researchers numerically compute a simple Cholesky decomposition of either the variance of the reduced form innovations $\boldsymbol{\Sigma}$ in the case of short-run restrictions, or the spectral density matrix at frequency 0

$$\boldsymbol{\Omega} = (\mathbf{I}_N - \textstyle\sum_{j=1}^{p} \mathbf{A}_j)^{-1}\boldsymbol{\Sigma}(\mathbf{I}_N - \textstyle\sum_{j=1}^{p} \mathbf{A}_j)^{-1}$$

in the case of long-run restrictions. As a result, the estimates of $\mathbf{C}$ satisfy either

$$\mathbf{C} = \boldsymbol{\Sigma}_L \tag{3}$$

or

$$\mathbf{C} = (\mathbf{I}_N - \textstyle\sum_{j=1}^{p} \mathbf{A}_j)\boldsymbol{\Omega}_L. \tag{4}$$

Given that both these mappings are diffeomorphisms, it is straightforward to apply the delta method to obtain the asymptotic distribution of the structural parameters $\boldsymbol{\vartheta}' = (\boldsymbol{\tau}', \mathbf{a}', \mathring{\mathbf{c}}')$, where $\mathring{\mathbf{c}}$ are the $N(N+1)/2$ elements of $\mathbf{c}$ that are effectively estimated as free parameters, whose exact nature depends on whether we rely on (3) or (4) for identification purposes. We provide further details in Appendix B.

The distribution of the Gaussian PMLEs of $\boldsymbol{\vartheta}$ is important for our purposes because our misspecification test statistics are based on shocks recovered from the observed variables using the expression

$$\boldsymbol{\varepsilon}_t^*(\boldsymbol{\vartheta}) = \mathbf{C}^{-1}(\mathbf{y}_t - \boldsymbol{\tau} - \textstyle\sum_{j=1}^{p} \mathbf{A}_j \mathbf{y}_{t-j}), \tag{5}$$

so we need to rely on the theory of moments tests in Newey (1985) and Tauchen (1985) to explicitly account for the fact that we work with estimated shocks in deriving the asymptotic covariance matrices of the average influence functions underlying the tests that we describe next.

3 Testing independence

3.1 Independence tests based on integer moments

As is well known, stochastic independence between the elements of a random vector implies lack of correlation between not only the levels but also any set of single-variable measurable

transformations of those elements. Thus, a rather intuitive way of testing for independence of the structural shocks $\boldsymbol{\varepsilon}_t^*$ without considering any specific parametric alternative can be based on influence functions of the form

$$c_{\mathbf{h}}(\boldsymbol{\varepsilon}_t^*) = \prod_{i=1}^{N} \varepsilon_{it}^{*h_i} - \prod_{i=1}^{N} E(\varepsilon_{it}^{*h_i}), \tag{6}$$

where $\mathbf{h} = \{h_1, ..., h_N\}$, with $h_i \in \mathbb{Z}_{0+}$, denotes the index vector characterising a specific product moment. This is precisely the approach followed in AFS22, where particular attention is paid to third and fourth cross-moments. The derivations in that paper, though, extensively exploit the fact that all the elements of the matrix $\mathbf{C}$ can be identified if one exploits the fact that at least $N-1$ structural shocks are non-Gaussian for estimation purposes. Consequently, we need to adapt the expressions not only to reflect that the asymptotic covariance matrix of the estimators in (2) will be different when the shocks are possibly wrongly assumed Gaussian, but, more importantly, also to recognise that effectively, the dimension of $\mathring{\mathbf{c}}$ is smaller than the dimension of $\mathbf{c}$.

3.2 Independence tests based on the distribution function

Unfortunately, tests based on coskewness and cokurtosis suffer from two limitations. First, standard asymptotic theory provides poor finite sample approximations for procedures based on higher-order moments, whose estimates are quite sensitive to outliers. Second, for any choice of $\mathbf{h}$, one can find joint distributions of the shocks for which (6) is zero on average even though the shocks are cross-sectionally dependent, as illustrated by AFS24 in several examples.

To avoid these criticisms, we follow AFS24 and also assess the potential cross-sectional dependence among two or more shocks by directly comparing their joint empirical cumulative distribution function (cdf) to the product of the marginal empirical cdfs. We do so not only for a discrete grid of values of the arguments of the joint cdf, which provides the intuition for our approach, but also for a continuous grid of values using an extension of the continuum of moments inference procedures in Carrasco and Florens (2000), which provides a consistent test, as we briefly discuss in the next two subsections.

3.2.1 Discrete grid tests

The starting point is a test based on the joint probability of events that involve two or more elements of $\boldsymbol{\varepsilon}_t^*$, which should coincide with the product of the marginal probabilities under the null of independence. Specifically, we begin by defining H points, $k_1 < \cdots < k_h < \cdots < k_H$, so that we can then form a partition of the support of ε_{it}^* into $H+1$ segments, namely $k_{h-1} \leq$

$\varepsilon_{it}^* \leq k_h$ for $h = 1, ..., H+1$ after suitably defining $k_0 = -\infty$ and $k_{H+1} = \infty$.[2] We then collect the indices of the shocks involved in the set $M = \{i^1, ..., i^{\text{M}}\}$, where M denotes the cardinality of the set M, so that we can test for pairwise independence (M=2), joint independence of the entire vector of structural innovations (M=N), and any other intermediate situation.

As in AFS24, let us partition the support of ε_{it}^* into the $H+1$ segments discussed above by the sequence of overlapping increments $\varepsilon_{it}^* \leq k_h$ for $h = 1, ..., H+1$. For practical purposes, let us define $P_{ht}^i = 1_{(-\infty, k_h)}(\varepsilon_{it}^*)$ and

$$p_h(\varepsilon_{it}^*) = P_{ht}^i - u_h^i, \tag{7}$$

with $u_h^i = E(P_{ht}^i) = \Pr(\varepsilon_{it}^* \leq k_h) \equiv F_i(k_h)$, as a set of dummy variables and marginal influence functions, respectively, which trivially give rise to the empirical cdf estimator

$$\hat{u}_h^i = \frac{1}{T} \sum_{t=1}^{T} P_{ht}^i. \tag{8}$$

Let us also define the joint influence function

$$p(\boldsymbol{\varepsilon}_t^*; \mathbf{k}, \mathbf{u}) = \prod_{i \in M} P_{h_i t}^i - \prod_{i \in M} u_{h_i}^i - \upsilon(\mathbf{k}, \mathbf{u}), \tag{9}$$

where $\mathbf{k} = (k_{h_{i^1}}, \ldots, k_{h_{i^{\text{M}}}})'$ and $\mathbf{u} = (u_{i^1}, \ldots, u_{i^{\text{M}}})'$, so that $\upsilon(\mathbf{k}, \mathbf{u}) = 0$ under the independence null.

Importantly, the fact that the estimating moment conditions (7) exactly identify the relevant u_h^i's implies that there is no efficiency loss in sequentially estimating the $\upsilon(\mathbf{k}, \mathbf{u})$'s from (9) by replacing the marginal cdfs by their sample counterparts relative to estimating them jointly from (7) and (9), which in turn implies that the non-centrality parameters of corresponding moment tests that impose $\upsilon(\mathbf{k}, \mathbf{u}) = 0$ will coincide (see Propositions 2 and 3 in AFS24 for further details).

This convenient re-interpretation of the usual Pearson test for independence will allow us to extend our tests to a continuous grid in section 3.2.2. At the same time, the choice of the k's will crucially affect power even though it does not affect the (first-order) asymptotic distribution of the test under the null. Absent any prior knowledge about the true marginal distribution of the shocks, it seems sensible to adapt the grid to their empirical counterpart. AFS24 use the following simple way of choosing the partition, which achieves precisely that goal: instead of fixing arbitrarily the grid points at which one evaluate the cdfs of each of the ε_i^*'s, one chose them so that they correspond to specific quantiles of the marginal distributions.

[2] For notational simplicity, we maintain the assumption that the number of intervals and their limits are common across shocks. Although this assumption is plausible when a researcher has no prior views on the marginal distributions of the different standardised shocks, it would be straightforward to relax it.

Specifically, let $k_h^i = \varkappa_i(u_h)$ for each $i \in M$ be the u_h-quantile of ε_{it}^* for $h = 1, ..., H$, with $0 = u_0 < u_1 < \cdots < u_H < u_{H+1} = 1$, $\varkappa_i(0) = -\infty$ and $\varkappa_i(1) = \infty$. We can compute an alternative independence test for the shocks using the same influence function $p(\boldsymbol{\varepsilon}_t^*)$ in (9), but now estimating the marginal quantiles k_h^i for given u_h^i from the exactly identified moment conditions (7) rather than each marginal cdf u_h^i for fixed k_h^i. Intuitively, a moment test based on a collection of such influence functions will effectively assess that the copula linking the different marginal distributions is flat, which corresponds to the independent one.

As in the case of the tests based on higher-order cross-moments of the shocks discussed in section 3.1, we need again to adapt the expressions in AFS24 to reflect the fact that we can only estimate $\mathring{\mathbf{c}}$ rather than $\mathbf{c}$ by Gaussian PML.

The choice of H, though, is crucial for both small sample performance and power considerations even though the asymptotic distribution under the null is always a χ^2 with H^{M} degrees of freedom. Intuitively, too fine a partition relative to the sample size may introduce size distortions because the joint probability of some individual cells will be poorly estimated. Even in large samples, a fine partition will generate substantial correlation between the influence functions, potentially causing numerical instability. Finally, there is also a power trade-off between the size of the non-centrality parameter and the number of degrees of freedom of the limiting distribution. Partly for these reasons, next we discuss tests which do not depend on H.

3.2.2 Independence tests based on a continuous grid

A different problem with the tests discussed in the previous subsection is that they are not consistent for any specific finite partition of the domain of the shocks because one could always find joint distributions such that the probability of each joint interval is exactly the product of the marginal probabilities even though the shocks are stochastically dependent (see again AFS24 for some examples). Consistent tests of independence based on comparing the joint cdf to the product of the marginal cdfs for all possible values of the arguments go back at least to Hoeffding (1948), who considered a Cramér-von Misses type-test based on the integral of the square differences between the joint cdf and the product of the marginal cdfs, and Blum, Kiefer and Rosenblatt (1961), who considered Kolmogorov-Smirnov-type tests based on the maximum absolute discrepancy. However, those tests rely on specific functionals of the difference, while the discrete grid tests that we studied in the previous section also take into account not only the asymptotic variance of the influence functions for each value of the arguments, like an Anderson-Darling (1952) test would do, but more importantly, the covariance between those influence functions for different values of the arguments.

In principle, we could try to find the limiting distribution of our discrete grid tests in a double

asymptotic framework in which the partitions get finer and finer as the sample size increases. However, this is really unnecessary because the influence functions indexed with respect to the arguments of the joint cdf over $\mathbb{R}^{\mathrm{M}}$ give rise to a continuum of moments in an L^2 space. As a result, AFS24 extend Carrasco and Florens (2000) and directly construct a Hansen (1982) overidentifying restrictions-type test based on the same influence functions as in the discrete grid case, but with a covariance operator playing the role of the usual covariance matrix.

Specifically, by transforming ε_{it}^* into its empirical uniform rank

$$\epsilon_{it}^* = \frac{1}{T}\sum\nolimits_{s=1}^{T} 1_{(-\infty,\varepsilon_{it}^*)}(\varepsilon_{is}^*), \tag{10}$$

we can define the marginal and joint influence functions

$$q_{it}(u_i) = 1_{(0,u_i)}(\epsilon_{it}^*) - u_i, \quad \text{and} \tag{11}$$

$$q_t(\mathbf{u}) = \prod_{i\in M} 1_{(0,u_i)}(\epsilon_{it}^*) - \prod_{i\in M} u_i, \tag{12}$$

which are numerically identical to (7) and (9) in the previous section, respectively.

Next, let ϖ be a probability density function with support the unit hypercube. Then, the function $q_t(\mathbf{u})$ may be regarded as a random element of $L^2(\varpi)$, the space of real-valued functions which are square integrable with respect to the density ϖ. For any functions f and g in $L^2(\varpi)$, the inner product on this Hilbert space is defined as $\langle f, g\rangle = \int_{[0,1]^{\mathrm{M}}} f(\mathbf{u})\, g(\mathbf{u})\varpi(\mathbf{u})\, d\mathbf{u}$. By the central limit theorem for *iid* random elements of a separable Hilbert space (see e.g. proof of Theorem 9 in Rackauskas and Suquet (2006)), we have that under independence, $\sqrt{T}\bar{q}_T(\mathbf{u}) \Rightarrow \mathcal{N}(0, K)$ in $L^2(\varpi)$ as T goes to infinity, where $\bar{q}_T(\mathbf{u})$ denotes the sample average of (12) and $\mathcal{N}(0, K)$ a Gaussian process of $L^2(\varpi)$ fully characterised by its covariance operator K, which is an integral operator from $L^2(\varpi)$ to $L^2(\varpi)$ such that

$$(Kf)(\mathbf{u}) = \int_{[0,1]^{\mathrm{M}}} k(\mathbf{u},\mathbf{v}) f(\mathbf{v})\varpi(\mathbf{v})\, d\mathbf{v}, \tag{13}$$

whose kernel $k(\mathbf{u},\mathbf{v}) = E[q_t(\mathbf{u})q_t(\mathbf{v})]$ is given in Proposition 2b of AFS24.

In terms of the eigenvalues λ_{jk} and the complete set of orthonormal eigenfunctions $\boldsymbol{\mu}_{jk}(\mathbf{u})$ of the spectral decomposition of K, the direct analogue to the J test statistic would be written as

$$\sum_j \sum_k \frac{1}{\lambda_{jk}} \left| \left\langle \sqrt{T}\bar{q}_T, \mu_{jk} \right\rangle \right|^2, \tag{14}$$

Unfortunately, this expression will blow up because of the division by the small eigenvalues. As in Carrasco and Florens (2000), AFS24 use Tikhonov regularisation, which leads to

$$\sum_j \sum_k \frac{\lambda_{jk}}{\lambda_{jk}^2 + \alpha} \left| \left\langle \sqrt{T}\bar{q}_T, \mu_{jk} \right\rangle \right|^2, \tag{15}$$

where $\alpha \geq 0$ is a regularisation parameter.

For computational reasons, it is convenient to rewrite the test statistic (15), which uses as eigenvalues and eigenfunctions those of K, in terms of certain matrices and vectors (see Carrasco et al. (2007) for analogous expressions for K under time series dependence). Specifically, we use the following computationally convenient expression for (15):

$$\mathbf{W}'\{\alpha\mathbf{I}_T + [(\mathbf{I}_T - \boldsymbol{\ell}_T\boldsymbol{\ell}_T'/T)\mathcal{D}^2(\mathbf{I}_T - \boldsymbol{\ell}_T\boldsymbol{\ell}_T'/T)]^2\}^{-1}\mathbf{W}, \tag{16}$$

where $\mathbf{W}$ is a $T\times 1$ vector whose t^{th} element is $w_t = \int q_t(\mathbf{u})\,\bar{q}_T(\mathbf{u})\,\varpi(\mathbf{u})\,d\mathbf{u}$, $\mathcal{D}$ is a $T\times T$ matrix whose $(t,s)^{th}$ element is $d_{ts} = \langle q_t, q_s\rangle/T$, and $\boldsymbol{\ell}_T$ is a $T\times 1$ vector of ones. In practice, only $\mathcal{D}$ is needed to compute the test statistic because (16) is equivalent to

$$\boldsymbol{\ell}_T'\mathcal{D}(\mathbf{I}_T - \boldsymbol{\ell}_T\boldsymbol{\ell}_T'/T)\{\alpha\mathbf{I}_T + [(\mathbf{I}_T - \boldsymbol{\ell}_T\boldsymbol{\ell}_T'/T)\mathcal{D}^2(\mathbf{I}_T - \ell_T\ell_T'/T)]^2\}^{-1}(\mathbf{I}_T - \boldsymbol{\ell}_T\boldsymbol{\ell}_T'/T)\mathcal{D}\boldsymbol{\ell}_T, \tag{17}$$

with the analytical expression for the $(t,s)^{th}$ element of the matrix $\mathcal{D}$ provided in Proposition 4 of AFS24.

In addition to the effects of estimating the marginal cdfs of the shocks on the covariance operator, we must again take into account the sampling variability in estimating $\boldsymbol{\vartheta}$. Fortunately, the only difference with the discrete grid case is that the expected Jacobian will now be a function of the values of the arguments of the cdf, and the same will be true of the covariance between the influence functions and the score of the Gaussian PMLE. With this trivial re-interpretation, all we need is to replace $\mathcal{D}$ by $\mathcal{E} = \mathcal{D} + \mathcal{C}$, in (17), where $\mathcal{C}$ is given in equation (27) in AFS24. Intuitively, the only thing they do is to apply the Carrasco and Florens (2000) procedure to the residuals from projecting the relevant influence functions for testing purposes onto the linear span generated by the influence functions defining the marginal cdfs and the scores of the pseudo log-likelihood function for each value of $\mathbf{u}$ (see Khmaladze (1981) for an analogous transformation).

Once again, though, we must adapt the expressions in AFS24 to reflect the fact that Gaussian PML estimates $\mathring{\mathbf{c}}$ instead of all the elements of $\mathbf{c}$.

4 Finite sample properties

4.1 Design and computational details.

To keep CPU time within bounds, we focus on bivariate data generation processes (DGPs) with VAR(1) dynamics. Specifically, we generate samples of size T from the following processes:

$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \boldsymbol{\tau} + \mathbf{A}\begin{pmatrix} y_{1t-1} \\ y_{2t-1} \end{pmatrix} + \mathbf{C}\begin{pmatrix} \varepsilon_{1t}^* \\ \varepsilon_{2t}^* \end{pmatrix}, \tag{18}$$

with

$$\boldsymbol{\tau}^0 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \text{ and } \mathbf{A}^0 = \begin{pmatrix} \frac{1}{2} & -\frac{1}{6} \\ 0 & \frac{1}{3} \end{pmatrix}.$$

As for the true matrix of impact multipliers, we consider three possibilities:

$$\mathbf{C}_I^0 = \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 2 \end{pmatrix}, \quad \mathbf{C}_{II}^0 = \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 2 \end{pmatrix}, \quad \mathbf{C}_{III}^0 = \begin{pmatrix} 1 & \frac{5}{8} \\ \frac{1}{2} & 2 \end{pmatrix}.$$

$\mathbf{C}_I^0$ trivially satisfies the usual short-run restrictions while $\mathbf{C}_{II}^0$ the long-run ones in conjunction with $\mathbf{A}^0$. In contrast, $\mathbf{C}_{III}^0$ satisfies neither. We then combine each of these matrices with the following distributions of the shocks:

DGP 0: $(\varepsilon_{1t}^*, \varepsilon_{2t}^*)$ follows a standardised bivariate Gaussian, i.e., $(\varepsilon_{1t}^*, \varepsilon_{2t}^*) \sim N(\mathbf{0}, \mathbf{I}_2)$.

DGP 1: ε_{1t}^* and ε_{2t}^* follow two independent univariate scale mixture of normals in which the higher variance component has probability $\lambda = \frac{1}{5}$ and the ratio of the two variances is $\varkappa = \frac{1}{6}$, so that $E(\varepsilon_{it}^*)^4 = 6$ for $i = 1, 2$.

DGP 2: ε_{1t}^* and ε_{2t}^* follow two independent univariate location-scale mixture of normals in which the higher variance component has probability $\lambda = \frac{1}{5}$, $\delta = -0.67$ and $\varkappa = 0.18$ so that $E(\varepsilon_{it}^*)^3 = -\frac{3}{4}$ and $E(\varepsilon_{it}^*)^4 = 6$ for $i = 1, 2$.

DGP 3: $(\varepsilon_{1t}^*, \varepsilon_{2t}^*)$ follows a standardised bivariate scale mixture of two zero mean normals with scalar covariance matrices in which the higher variance component has probability $\lambda = \frac{1}{5}$ and the ratio of the two variances is $\varkappa = \frac{1}{6}$.

DGP 4: $(\varepsilon_{1t}^*, \varepsilon_{2t}^*)$ follows a standardised bivariate discrete mixture of two normals with parameters chosen in such a way that the margins are identical, which we achieve by assuming that

$$\boldsymbol{\delta}_1 = \delta(1-\lambda)\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \boldsymbol{\delta}_2 = -\delta\lambda\begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

$$\boldsymbol{\aleph}_1 = \begin{pmatrix} [1-\delta^2\lambda(1-\lambda)]/[\lambda+(1-\lambda)\varkappa] & -\delta^2\lambda(1-\lambda)/[\lambda(1-\varkappa)+\varkappa] \\ -\delta^2\lambda(1-\lambda)/[\lambda(1-\varkappa)+\varkappa] & [1-\delta^2\lambda(1-\lambda)]/[\lambda+(1-\lambda)\varkappa] \end{pmatrix},$$

and $\boldsymbol{\aleph}_2 = \varkappa\boldsymbol{\aleph}_1$ (see Appendix C for details). In particular, we set $\delta = -0.67$ and $\varkappa = 0.18$ with the mixing probability set to $\lambda = \frac{1}{5}$ so that $E(\varepsilon_{it}^*)^3 = -\frac{3}{4}$ and $E(\varepsilon_{it}^*)^4 = 6$ for $i = 1, 2$.

Therefore, we consider three bivariate distributions in which the shocks are truly independent: DGPs 0 to 2, and two in which they are not: DGPs 3 and 4.[3]

In turn, our PML estimation procedure imposes that either $\mathbf{C}$ or $(\mathbf{I} - \mathbf{A})^{-1}\mathbf{C}$ is lower triangular, depending on whether we rely on the short- or long-run restrictions in estimation. In this

[3] Appendix A.4.2 in Amengual, Carrasco and Sentana (2020) provides further details about DGPs 1 to 3.

respect, we impose the short-run restrictions in designs corresponding to $\mathbf{C}_I^0$ and $\mathbf{C}_{II}^0$, and the long-run restrictions in designs corresponding to $\mathbf{C}_{II}^0$ and $\mathbf{C}_{III}^0$.

Finally, we follow AFS22 and AFS24 in using resampling procedures to compute the rejection rates of all the different test statistics that we have described in sections 3.1, 3.2.1 and 3.2.2.

4.2 Simulation results

This subsection summarises the main findings from the Monte Carlo experiments, focusing on the finite-sample size and power properties of the proposed independence tests. We generate $10,000$ samples for the designs under the null and $2,500$ for those under the alternative. For each sample, we also compute p-values based on $B = 99$ resampling replications. We report results separately for short-run and long-run identification schemes and consider both moderate ($T = 250$) and large ($T = 1,000$) sample sizes, which are representative of macroeconomic and empirical finance applications, respectively.

Finite-sample size

Tables 1-4 report empirical rejection frequencies under the null hypothesis of shock independence. Overall, the size performance of the proposed tests is satisfactory across designs, identification schemes, and sample sizes. When asymptotic critical values are used, the rejection rates are generally close to nominal levels, although some mild over-rejection appears for discrete grid tests when $T = 250$, particularly with finer partitions. As expected, these distortions largely disappear as the sample size increases.

Resampling-based critical values substantially improve finite-sample accuracy in small samples. In particular, for $T = 250$, bootstrap versions of the discrete grid tests correct the slight size distortions observed under asymptotic inference, bringing empirical rejection frequencies very close to their nominal counterparts. Tests based on coskewness and cokurtosis exhibit good size even without resampling, reflecting their relatively simple structure.

The continuous grid test also displays stable size properties. Its empirical size is largely insensitive to the choice of the regularisation parameter α over the range of values considered, namely $\alpha \in [1e^{-5}, 1e^{-8}]$. This robustness is especially appealing given the test's generality and consistency properties.

Overall, these results indicate that the proposed tests can be reliably implemented in sample sizes typical of macroeconomic applications, especially when one employs critical values obtained through resampling procedures.

Finite-sample power: short-run restrictions

Tables 5 and 6 report rejection frequencies under alternatives in which the short-run restrictions imposed are wrong. We exclude the Gaussian design (DGP 0) combined with $\mathbf{C}_{II}^0$, for

which asymptotic power is mechanically equal to size due to the joint normality of the shocks.

Several clear patterns emerge. First, all tests display increasing power as the sample size grows from $T = 250$ to $T = 1,000$, confirming the validity of the asymptotic approximations and the consistency of the procedures under fixed alternatives.

Second, tests based on integer moments – particularly those exploiting cokurtosis – tend to be the most powerful against the non-Gaussian alternatives considered. This is true for DGPs 1 and 2, where the shocks are independent but non-Gaussian, and the restrictions on the impact multiplier matrix $\mathbf{C}$ are not satisfied. And it is especially true for DGPs 3 and 4, where dependence arises through mixtures, even when the restrictions on $\mathbf{C}$ are satisfied. In these settings, higher-order moments capture deviations from independence that are not necessarily concentrated in specific regions of the joint distribution.

Discrete grid tests also exhibit non-negligible power, but it is sensitive to the choice of the partition. Coarse grids may fail to detect dependence localised in the tails, while finer grids require larger samples to perform well. This trade-off is particularly evident when $T = 250$.

The continuous grid test delivers more uniform power across alternatives, although it is sometimes less powerful than the integer moment tests in small samples. This behaviour is consistent with its design: by integrating information over the entire joint distribution, the test avoids reliance on specific features of dependence but may sacrifice power against alternatives that are well aligned with particular cross-moments.

Finite-sample power: long-run restrictions

Tables 7 and 8 report rejection frequencies under alternatives when identification is achieved through long-run restrictions. The qualitative patterns largely mirror those obtained under short-run restrictions, although some differences emerge in smaller samples.

Integer moment tests continue to exhibit strong power against alternatives involving cross-sectional dependence, particularly those generated by location-scale mixtures. Their performance remains robust across both sample sizes, indicating that higher-order cross-moments of the recovered shocks remain informative even when identification relies on long-run restrictions.

Discrete grid tests also display satisfactory power for both sample sizes. However, when $T = 250$, their rejection frequencies are somewhat lower than in the corresponding short-run designs. This difference may reflect the greater sampling variability of the estimated structural shocks under long-run identification, which propagates into noisier estimates of joint cell probabilities and reduces power in small samples. As the sample size increases to $T = 1,000$, this effect largely disappears.

The continuous grid test remains well behaved across all designs and sample sizes. Although

its power is occasionally lower than that of integer moment tests in small samples, it increases steadily with T and remains relatively stable across identification schemes. This robustness suggests that the continuous grid test provides a reliable diagnostic when researchers seek a broadly consistent assessment of independence rather than maximum power against specific forms of dependence.

Comparative assessment

Taken together, the Monte Carlo results highlight a clear complementarity among the proposed tests. Integer moment-based statistics offer strong power against a wide range of economically relevant non-Gaussian alternatives and perform well even in moderately small samples. Discrete grid tests provide intuitive diagnostics and competitive power when the number of partitions is chosen judiciously, especially when combined with resampling. Continuous grid tests, while computationally more demanding, deliver consistent inference and stable behaviour across designs, sample sizes, and identification schemes. Consequently, applied researchers may benefit from using the proposed tests jointly rather than in isolation, exploiting their distinct strengths to assess whether identified structural shocks can plausibly be regarded as independent.

5 Empirical applications

5.1 The oil market

In his seminal contribution, Kilian (2009) argues that movements in the real price of oil cannot be interpreted as the outcome of a single underlying disturbance. Instead, they arise from three distinct structural shocks, each with different macroeconomic implications: (i) an oil supply shock arising from exogenous disruptions to crude oil production (such as geopolitical conflicts); (ii) a global aggregate demand shock associated with fluctuations in worldwide economic activity; and (iii) an oil-specific (precautionary) demand shock, reflecting changes in expectations and concerns about future supply availability rather than contemporaneous production conditions.

The recursive ordering adopted by Kilian (2009) provides a simple and economic interpretable identification scheme, grounded in plausible assumptions about which variables can respond contemporaneously within a given month.

Although this framework corrected several earlier misinterpretations of oil price movements, the parsimonious three-variable, recursively identified SVAR model that he considered could still be vulnerable to omitted-variable bias, misspecified contemporaneous interactions, or violations of the assumed independence of structural shocks. These concerns motivate the application of our independence-based specification tests to his estimated model.

We use the original data from Kilian (2009) from the *American Economic Review* reposi-

tory and estimate a trivariate SVAR with twenty-four lags to replicate his empirical setup. We then apply the independence tests described in sections 3.1, 3.2.1 and 3.2.2. For each test, p-values are obtained from 9,999 resampling replications that impose the null hypothesis of shock independence.

The joint tests based on integer moments reject the null of independence with a p-value equal of 0.010. This rejection is driven primarily by the joint cokurtosis component (p-value 0.008), with a weaker contribution from coskewness (p-value 0.065). Examination of the individual moment components reveals that the rejection is almost entirely attributable to dependence between the oil supply shock and the oil-specific (precautionary) demand shock. A plausible explanation is that, within the three-variable SVAR, the oil-specific demand shock may act as a residual category that absorbs speculative behaviour, inventory demand, or other omitted forces. This interpretation is consistent with Kilian and Murphy (2014), who extend the baseline model by explicitly incorporating inventories (or separate storage/speculative demand).

In contrast, the discrete grid test based on a $3 \times 3 \times 3$ partition does not reject the null of independence at the 5% significance level, yielding a p-value of 0.132. Similarly, the continuous grid test does not reject independence for any conventional choice of the regularisation parameter either. The associated p-values range from 0.340 ($\alpha = 10^{-5}$) to 0.413 ($\alpha = 10^{-7}$).

Overall, these results suggest mild but non-negligible departures from independence that are detectable by higher-order cross-moment tests but not by distribution-based methods, highlighting the value of applying multiple diagnostic tools when assessing SVAR identification.

5.2 Aggregate demand and supply

Blanchard and Quah (1989) propose an influential alternative identification strategy for demand and productivity shocks. Their approach exploits long-run restrictions by assuming that aggregate demand shocks have no permanent effects on real economic activity while aggregate supply shocks do. They apply this framework to quarterly US real GDP and the unemployment rate over the period 1948:Q2–1987:Q4.

We have obtained the data used by Herwartz (2019) from the *Journal of Applied Econometrics* repository and estimated an exactly identified VAR with eight lags, imposing the restriction that the upper-right element of the long run multiplier matrix is zero. Our estimation results are virtually indistinguishable from those reported in equation (5) of Herwartz (2019).

We then apply our independence tests using 9,999 resampling replications. Once again, the evidence is mixed but does not point to strong model misspecification. Tests based on integer moments do not reject independence when focusing on cokurtosis components, with a p-value of 0.738. In contrast, the test based on the two coskewness moments produces a very small p-

value of 0.004. This rejection is entirely driven by the moment combining the level of the supply shock with the square of the demand shock. More specifically, the negative sign of the associated influence function suggests a negative linear relationship between the level of the supply shock and the volatility of the demand shock.

The discrete grid tests do not formally reject the null of independence at the 5% level, although the p-values are relatively close to conventional thresholds. Using partitions into three and four segments yields p-values of 0.102 and 0.069, respectively. We do not consider finer partitions, as the sample size is limited and the expected number of observations per cell would be too small to ensure reliable inference.

Finally, the consistent continuous grid test do not reject independence for any conventional value of the regularisation parameter. The resulting p-values range from 0.191 ($\alpha = 10^{-6}$) to 0.204 ($\alpha = 10^{-8}$). These findings are broadly consistent with those in Herwartz (2019), who reports 0.147 as the p-value of the independence test of Bakirov, Rizzo and Székely (2006).

6 Conclusions

This paper proposes a battery of specification tests designed to assess a fundamental assumption underlying Gaussian SVAR analysis: the stochastic independence of the structural shocks. Although Gaussian PML estimation remains the dominant approach in those models due to its simplicity and robustness to distributional misspecification, it offers no guarantee that the shocks extracted from imposed identification restrictions truly behave as independent driving forces. Our contribution consists in translating the logic of independence-based identification in the non-Gaussian literature into a practical set of diagnostic tools applicable to conventional Gaussian SVARs, in particular those identified with short-run and long-run restrictions.

We propose three complementary testing approaches. First, test statistics exploiting coskewness and cokurtosis offer a simple and computationally fast diagnostic, particularly well-suited for detecting possibly asymmetric tail-dependence in estimated shocks. Second, discrete-grid contingency-type tests compare the empirical joint distribution of the shocks against the product of their marginals, allowing the researcher to trace which specific regions of the joint distribution violate independence. Third, the continuous-grid test provides a consistent inference procedure based on the entire support of the joint distribution using continuum-of-moments methods and regularisation techniques to ensure stability. Together, these tools provide a flexible diagnostic device adaptable to diverse empirical settings and sample sizes.

Our simulation results show that the proposed tests behave well in practice. Across a range of empirically relevant DGPs, identification schemes and distributional specifications, size distor-

tions remain small - particularly when coupled with resampling-based critical values - and they have non-trivial power against economically relevant alternative models that involve some forms of dependence between the shocks. Importantly, the different approaches exhibit complementary comparative advantages: integer-moment statistics are powerful against non-Gaussian dependence, discrete-grid methods yield interpretable rejection patterns even in moderately small samples, while continuous-grid tests provide a consistent procedure regardless of the nature of the joint dependence. Taken together, our findings suggest that empirical practice need no longer rely on untested identification assumptions because their plausibility can be empirically assessed rather than taken on faith.

Our two empirical applications illustrate this point. The Kilian (2009) oil-market SVAR, a cornerstone of applied macro-energy analysis, delivers mixed results: integer moment tests detect correlation between powers of the identified shocks, which is in line with interpretations that precautionary demand is partly absorbing inventory or speculative behaviour. In contrast, discrete and continuous grid tests do not reject independence at conventional significance levels. The Blanchard and Quah (1989) decomposition, on the other hand, appears largely consistent with independence, though some interesting traces of dependence emerge in specific cross-moments. In summary, these applications highlight the value of specification testing as a lens through which researchers can evaluate the credibility and interpretation of shocks, and, if necessary, refine model design by relaxing timing assumptions or modifying identifying restrictions.

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Appendices

A Asymptotic distribution of the Gaussian PMLEs with independent shocks

Supplemental Appendix C of AFS24 derives analytical expressions for the conditional variance of the score and the expected value of the Hessian of SVAR models with cross-sectional independent non-Gaussian shocks when the distributions assumed for estimation purposes may well be misspecified but all the parameters that characterise the conditional mean and covariance functions are consistently estimated, as in the case of finite normal mixtures (see Fiorentini and Sentana (2023) for the general case). Given that a Gaussian distribution is a trivial example of such mixtures, we can specialise their expressions for the Gaussian PMLEs. To do so, we only need to realise two things. First, there are no shape parameters, so all terms involving $\boldsymbol{\varrho}$ disappear. Second, the fact that

$$\frac{\partial \ln f(\varepsilon_{it}^*)}{\partial \varepsilon_i^*} = -\varepsilon_{it}^* \quad \text{and} \quad \frac{\partial^2 \ln f(\varepsilon_{it}^*)}{(\partial \varepsilon_i^*)^2} = -1 \tag{A1}$$

simplifies considerably the expressions for the elements of the covariance matrix of the score and the (minus) expected value of the Hessian. Specifically, if we denote by $\boldsymbol{\upsilon}_{i0}$ the true values of the shape parameters of the distribution of the i^{th} shock, which is potentially of unbounded dimension, the necessary ingredients for the variance of the score will be

$$\begin{aligned}
\mathrm{O}_{ll}(\boldsymbol{\upsilon}_{i0}) &= V\left[\frac{\partial \ln f(\varepsilon_{it}^*)}{\partial \varepsilon_i^*}\middle|\boldsymbol{\upsilon}_{i0}\right] = V(\varepsilon_{it}^*|\boldsymbol{\upsilon}_{i0}) = 1,\\
\mathrm{O}_{ls}(\boldsymbol{\upsilon}_{i0}) &= cov\left[\frac{\partial \ln f(\varepsilon_{it}^*)}{\partial \varepsilon_i^*}, \frac{\partial \ln f(\varepsilon_{it}^*)}{\partial \varepsilon_i^*}\varepsilon_{it}^*\middle|\boldsymbol{\upsilon}_{i0}\right] = E(\varepsilon_{it}^{*3}|\boldsymbol{\upsilon}_{i0}) = \phi(\boldsymbol{\upsilon}_{i0}),\\
\mathrm{O}_{ss}(\boldsymbol{\upsilon}_{i0}) &= V\left[\frac{\partial \ln f(\varepsilon_{it}^*)}{\partial \varepsilon_i^*}\varepsilon_{it}^*\middle|\boldsymbol{\upsilon}_{i0}\right] = V(\varepsilon_{it}^{*2}|\boldsymbol{\upsilon}_{i0}) = \kappa(\boldsymbol{\upsilon}_{i0}) - 1,
\end{aligned}$$

which can be consistently estimated from the third and fourth sample moments of the estimated shocks. As for the expected Hessian, we will have

$$\begin{aligned}
\mathrm{H}_{ll}(\boldsymbol{\upsilon}_{i0}) &= -E\left[\frac{\partial^2 \ln f(\varepsilon_{it}^*)}{(\partial \varepsilon_i^*)^2}\middle|\boldsymbol{\upsilon}_{i0}\right] = 1,\\
\mathrm{H}_{ls}(\boldsymbol{\upsilon}_{i0}) &= -E\left[\frac{\partial^2 \ln f(\varepsilon_{it}^*)}{(\partial \varepsilon_i^*)^2}\cdot\varepsilon_{it}^*\middle|\boldsymbol{\upsilon}_{i0}\right] = 0,\\
\mathrm{H}_{ss}(\boldsymbol{\upsilon}_{i0}) &= -E\left\{\frac{\partial^2 \ln f(\varepsilon_{it}^*;\boldsymbol{\varrho}_\infty)}{(\partial \varepsilon_i^2)}(\varepsilon_{it}^*)^2\middle|\boldsymbol{\upsilon}_{i0}\right\} = E(\varepsilon_{it}^{*2}|\boldsymbol{\upsilon}_{i0}) = 1,
\end{aligned}$$

which do not require any estimation.

Analogous simplifications apply to the covariances between the influence functions underlying

our different tests and the Gaussian scores in view of (A1).

On the other hand, we must explicitly take into account that, unlike in the case of models estimated with non-Gaussian independent shocks, not all elements of the matrix $\mathbf{C}$ will be identified without further restrictions, so that both the variance of the score and the expected Hessian that they obtain for a model expressed in terms of $\mathbf{c} = vec(\mathbf{C})$ will be singular. The next appendix discusses how to deal with this issue.

B The relationship between restricted and unrestricted elements of C

The purpose of this appendix is to establish the relationship between $\mathbf{c}$ and $\mathring{\mathbf{c}}$ in the case of short- and long-run restrictions.

In the first case, the relationship is trivial given that $\mathbf{c} = vec(\mathbf{C})$ and $\mathring{\mathbf{c}} = vech(\mathbf{C})$. Specifically, let $\mathbf{L}_N$ denote the usual elimination matrix, which is the unique $N(N+1)/2 \times N^2$ matrix such that $\mathbf{L}_N' vech(\mathbf{P}) = vec(\mathbf{P})$ for any lower triangular matrix $\mathbf{P}$ (see chapter 5 and of Magnus (1988)). The elimination matrix is such that $\mathbf{L}_N vec(\mathbf{G}) = vech(\mathbf{G})$ for any square matrix $\mathbf{G}$ regardless of its nature. Consequently, $\mathring{\mathbf{c}} = \mathbf{L}_N \mathbf{c}$. In fact, in the case of short-run restrictions, we can easily obtain the direct relationship between $\mathring{\mathbf{c}} = vech(\mathbf{\Sigma}_L)$ and $\boldsymbol{\sigma} = vech(\mathbf{\Sigma})$ as follows. The differential of $\mathbf{\Sigma} = \mathbf{\Sigma}_L \mathbf{\Sigma}_L'$ is

$$d\mathbf{\Sigma} = d\mathbf{\Sigma}_L \cdot \mathbf{\Sigma}_L' + \mathbf{\Sigma}_L \cdot d\mathbf{\Sigma}_L',$$

whence

$$dvec(\mathbf{\Sigma}) = (\mathbf{\Sigma}_L \otimes \mathbf{I}_N) dvec(\mathbf{\Sigma}_L) + (\mathbf{I}_N \otimes \mathbf{\Sigma}_L) dvec(\mathbf{\Sigma}_L') = [(\mathbf{\Sigma}_L \otimes \mathbf{I}_N) + (\mathbf{I}_N \otimes \mathbf{\Sigma}_L)\mathbf{K}_{NN}] dvec(\mathbf{\Sigma}_L),$$

where $\mathbf{K}_{NN}$ is the usual symmetric commutation matrix that transforms $vec(\mathbf{G})$ into $vec(\mathbf{G}')$ and vice versa for any square matrix $\mathbf{G}$ (see Magnus and Neudecker (2019)). Unfortunately, this transformation is singular, which means that we must find an analogous transformation between the corresponding $dvech's$. In this context, we can make use of the elimination to write the previous expression as

$$dvech(\mathbf{\Sigma}) = [\mathbf{L}_N(\mathbf{\Sigma}_L \otimes \mathbf{I}_N)\mathbf{L}_N' + \mathbf{L}_N(\mathbf{I}_N \otimes \mathbf{\Sigma}_L)\mathbf{K}_{NN}\mathbf{L}_N'] dvech(\mathbf{\Sigma}_L). \tag{B2}$$

We can then use the results in chapter 5 of Magnus (1988) to show that the above mapping will be lower triangular of full rank as long as $\mathbf{\Sigma}_L$ has full rank, which means that we can readily obtain the Jacobian matrix of $\boldsymbol{\sigma}$ with respect to $\mathring{\mathbf{c}}$. In addition, the full rank nature of this Jacobian implies that we can also obtain the Jacobian of $\mathring{\mathbf{c}}$ with respect to $\boldsymbol{\sigma}$ by using the

inverse mapping theorem.

From a numerical point of view, the calculation of both $\mathbf{L}_N(\mathbf{\Sigma}_L \otimes \mathbf{I}_N)\mathbf{L}_N'$ and $\mathbf{L}_N(\mathbf{I}_N \otimes \mathbf{\Sigma}_L)\mathbf{K}_{NN}\mathbf{L}_N'$ is straightforward. Specifically, the effect of premultiplying by the $\frac{1}{2}N(N+1)\times N^2$ matrix $\mathbf{L}_N$ is to eliminate rows $N+1$, $2N+1$ and $2N+2$, $3N+1$, $3N+2$ and $3N+3$, etc. Similarly, given that $\mathbf{L}_N\mathbf{K}_{NN}vec(\mathbf{A}) = vech(\mathbf{A}')$, the effect of postmultiplying by $\mathbf{K}_{NN}\mathbf{L}_N'$ is to delete all columns but those in positions 1, $N+1$, $2N+1$,...,$N+2$, $2N+2$,..., $N+3$, $2N+3$,..., N^2 (see Supplemental Appendix D of Fiorentini and Sentana (2021) for further details).

The Jacobian linking $\mathbf{c}$ and $\mathring{\mathbf{c}}$ in the case of long-run restrictions is more involved because their relationship is less direct. Nevertheless, we can obtain it by applying the implicit mapping theorem to the identification condition, which says that $vecl[\mathbf{C}'(\mathbf{I}_N - \sum_{j=1}^p \mathbf{A}_j')^{-1}] = \mathbf{0}$, where $vecl(.)$ is the operator that maps by columns the strict lower triangle of a square matrix. In this context, the formal analysis can be considerably simplified if we make use of the the strict elimination matrix $\mathbf{S}_N$, which is the unique $N(N-1)/2 \times N^2$ matrix such that $\mathbf{S}_N'vecl(\mathbf{O}) = vec(\mathbf{O})$ for any strictly lower triangular matrix $\mathbf{O}$. This matrix is such that $\mathbf{S}_N vec(\mathbf{G}) = vecl(\mathbf{G})$ for any square matrix $\mathbf{G}$ regardless of its nature (see chapter 6 of Magnus (1988) for further details). Naturally, $vech(\mathbf{G})$ and $vecl(\mathbf{G}')$ together represent a mere reordering of the elements of $vec(\mathbf{G})$, as formally shown in Theorem 6.10.i in Magnus (1988), which states that $(\mathbf{L}_N', \mathbf{K}_{NN}\mathbf{S}_N')$ is an orthogonal matrix.

Let us define the matrix

$$\mathbf{B} = \mathbf{I}_N - \textstyle\sum_{j=1}^p \mathbf{A}_j \tag{B3}$$

so that we can write

$$\mathbf{F} = (\mathbf{I}_N - \textstyle\sum_{j=1}^p \mathbf{A}_j')^{-1}\mathbf{C} = \mathbf{B}^{-1}\mathbf{C}.$$

The product rule for differentials immediately implies that

$$d\mathbf{F} = d(\mathbf{B}^{-1}) \cdot \mathbf{C} + \mathbf{B}^{-1} \cdot d\mathbf{C} = -\mathbf{B}^{-1} \cdot d\mathbf{B} \cdot \mathbf{B}^{-1}\mathbf{C} + \mathbf{B}^{-1} \cdot d\mathbf{C}.$$

If we write this in vector form, we obtain

$$dvec(\mathbf{F}) = -(\mathbf{C}'\mathbf{B}^{-1\prime} \otimes \mathbf{B}^{-1})dvec(\mathbf{B}) + (I \otimes \mathbf{B}^{-1})dvec(\mathbf{C}).$$

Therefore,

$$\begin{aligned} \frac{\partial vec(\mathbf{F})}{\partial vec'(\mathbf{B})} &= -(\mathbf{C}'\mathbf{B}^{-1\prime} \otimes \mathbf{B}^{-1}), \\ \frac{\partial vec(\mathbf{F})}{\partial vec'(\mathbf{C})} &= (\mathbf{I}_N \otimes \mathbf{B}^{-1}). \end{aligned}$$

If we do the same with (B3), we obtain

$$\begin{aligned} d\mathbf{B} &= -\sum_{j=1}^{p} d\mathbf{A}_j, \\ dvec(\mathbf{B}) &= -\sum_{j=1}^{p} dvec(\mathbf{A}_j) \end{aligned}$$

and

$$\frac{\partial vec(\mathbf{B})}{\partial vec'(\mathbf{A}_j)} = -\mathbf{I}_N \quad \forall j.$$

Consequently, the chain rule for first derivatives leads to

$$\frac{\partial vec(\mathbf{F})}{\partial vec'(\mathbf{A}_j)} = \frac{\partial vec(\mathbf{F})}{\partial vec'(\mathbf{B})} \frac{\partial vec(\mathbf{B})}{\partial vec'(\mathbf{A}_j)} = -(\mathbf{C}'\mathbf{B}^{-1\prime} \otimes \mathbf{B}^{-1}) \quad \forall j.$$

If we then transform $vec(\mathbf{F})$ into $vecl(\mathbf{F}')$ by premultiplying by the matrix $\mathbf{S}_N\mathbf{K}_{NN}$ defined above, we can show that

$$\begin{aligned} \frac{\partial vecl(\mathbf{F}')}{\partial vec'(\mathbf{A}_j)} &= -\mathbf{SK}(\mathbf{C}'\mathbf{B}^{-1\prime} \otimes \mathbf{B}^{-1}) \quad \forall j, \\ \frac{\partial vecl(\mathbf{F}')}{\partial vec'(\mathbf{C})} &= \mathbf{SK}(\mathbf{I} \otimes \mathbf{B}^{-1}). \end{aligned}$$

On this basis, the implicit function theorem allows us to obtain the Jacobian of $\mathbf{c}$ with respect to $\mathring{\mathbf{c}}$ and $\mathbf{a}$.

C Standardised multivariate discrete mixtures of normals with identical margins

Consider the following mixture of two N-variate normals

$$\mathbf{y} \sim \begin{cases} N(\boldsymbol{\nu}^a, \boldsymbol{\Gamma}^a) & \text{with probability } \lambda, \\ N(\boldsymbol{\nu}^b, \boldsymbol{\Gamma}^b) & \text{with probability } 1-\lambda, \end{cases} \tag{C4}$$

which trivially implies that all N marginal distributions are mixtures of univariate normals. Specifically,

$$y_n \sim \begin{cases} N(\nu_n^a, \gamma_{nn}^a) & \text{with probability } \lambda, \\ N(\nu_n^b, \gamma_{nn}^b) & \text{with probability } 1-\lambda, \end{cases} \tag{C5}$$

for $n = 1, \ldots, N$. Therefore, to guarantee that not only the unconditional means and variances are identical across series, but also that their entire marginal distributions are identical, we must impose that both

$$\boldsymbol{\nu}^a = \nu^a \boldsymbol{\ell}_N, \;\; \boldsymbol{\nu}^b = \nu^b \boldsymbol{\ell}_N, \tag{C6}$$

and

$$vecd(\boldsymbol{\Gamma}^a) = \varpi^a \boldsymbol{\ell}_N, \;\; vecd(\boldsymbol{\Gamma}^b) = \varpi^b \boldsymbol{\ell}_N, \tag{C7}$$

where $\boldsymbol{\ell}_N$ is a vector of N ones and $vecd(.)$ is the operator that stacks the diagonal elements of a square matrix in vector form (see chapter 7 of Magnus (1988) for further details). If we also want to ensure that all the unconditional means are 0, we need to further impose

$$\nu^a = \delta(1-\lambda) \quad \text{and} \quad \nu^a = -\delta\lambda,$$

so that $\delta = \nu^1 - \nu^2$ is a scalar parameter that controls the differences between the means of the components. Under these circumstances, the unconditional covariance matrix of $\mathbf{y}$ will be

$$[\lambda\boldsymbol{\Gamma}^a + (1-\lambda)\boldsymbol{\Gamma}^b] + \lambda(1-\lambda)\delta^2\boldsymbol{\ell}_N\boldsymbol{\ell}_N', \tag{C8}$$

where the first term in square brackets is the average of the covariance matrices of the components and the second one the covariance matrix of their means. For this matrix to be the identity matrix of order N, we need that both

$$vecd[\lambda\boldsymbol{\Gamma}^a + (1-\lambda)\boldsymbol{\Gamma}^b] + \lambda(1-\lambda)\delta^2 vecd(\boldsymbol{\ell}_N\boldsymbol{\ell}_N') = vecd(\mathbf{I}_N) \tag{C9}$$

and

$$veco[\lambda\boldsymbol{\Gamma}^a + (1-\lambda)\boldsymbol{\Gamma}^b] + \lambda(1-\lambda)\delta^2 veco(\boldsymbol{\ell}_N\boldsymbol{\ell}_N') = veco(\mathbf{I}_N), \tag{C10}$$

where $veco(.)$ is the operator that stacks by columns the off-diagonal elements of a square matrix (see Magnus and Sentana (2020) for further details). Restriction (C9) reduces to

$$\lambda\varpi^a + (1-\lambda)\varpi^b + \lambda(1-\lambda)\delta^2 = 1,$$

with $\varpi^a, \varpi^b > 0$, in view of (C7) and the fact that $vecd(\boldsymbol{\ell}_N\boldsymbol{\ell}_N') = vecd(\mathbf{I}_N) = \boldsymbol{\ell}_N$. If we define $\kappa = \varpi^a/\varpi^b$ as the ratio of the common variances of the components, this is equivalent to

$$\varpi^a = \frac{1-\lambda(1-\lambda)\delta^2}{\lambda + (1-\lambda)\kappa}, \tag{C11}$$

whose positivity requires that

$$\delta^2 < \frac{1}{\lambda(1-\lambda)}. \tag{C12}$$

In turn, restriction (C10) requires that

$$\lambda\gamma^a_{nm} + (1-\lambda)\gamma^b_{nm} + \lambda(1-\lambda)\delta^2 = 0 \quad \forall n \neq m$$

because $veco(\boldsymbol{\ell}_N\boldsymbol{\ell}_N') = \boldsymbol{\ell}_{N(N-1)}$ while $veco(\mathbf{I}_N) = \mathbf{0}_{N(N-1)}$. In other words, we need

$$\frac{\gamma^a_{nm}}{1-\lambda} + \frac{\gamma^b_{nm}}{\lambda} = -\delta^2 \quad \forall n \neq m. \tag{C13}$$

In addition, we need to guarantee that $\boldsymbol{\Gamma}^a$ and $\boldsymbol{\Gamma}^b$ are positive (semi-)definite. One particular way

of achieving this requirement is to assume that both $\mathbf{\Gamma}^a$ and $\mathbf{\Gamma}^b$ have equicorrelated structures. Specifically,

$$\mathbf{\Gamma}^a = \varpi^a \left[(1-\rho^a)\mathbf{I}_N + \rho^a \boldsymbol{\ell}_M \boldsymbol{\ell}'_M\right] \quad \text{and } \mathbf{\Gamma}^b = \varpi^a \kappa \left[(1-\rho^b)\mathbf{I}_N + \rho^b \boldsymbol{\ell}_M \boldsymbol{\ell}'_M\right], \tag{C14}$$

with

$$-1/(N-1) \leq \rho^a, \rho^b \leq 1 \tag{C15}$$

to guarantee that their eigenvalues, which are $\varpi^a(1-\rho^a)$ and $\varpi^a[(1-\rho^a)+\rho^a N]$, and $\varpi^b\kappa(1-\rho^b)$ and $\varpi^a\kappa[(1-\rho^b)+\rho^b N]$, respectively, are non-negative.

In this equicorrelated case, $\gamma^a_{nm} = \varpi^a\rho^a$ and $\gamma^b_{nm} = \varpi^a\kappa\rho^b \; \forall n \neq m$, so that (C13) simplifies to

$$\varpi^a[\lambda\rho^a + (1-\lambda)\kappa\rho^b] + \lambda(1-\lambda)\delta^2 = 0.$$

If we then combine this restriction with (C11), we end up with the relationship

$$\frac{[1-\lambda(1-\lambda)\delta^2][\lambda\rho^a + (1-\lambda)\kappa\rho^b]}{\lambda + (1-\lambda)\kappa} = -\lambda(1-\lambda)\delta^2. \tag{C16}$$

An even more restricted case arises by assuming that $\rho^a = \rho^b = \rho$ so that $\mathbf{\Gamma}^b = \kappa\mathbf{\Gamma}^a$, when (C16) simplifies to

$$\rho = -\frac{\lambda(1-\lambda)\delta^2}{1-\lambda(1-\lambda)\delta^2} \tag{C17}$$

Notice, though, that the equicorrelated structure for each component becomes increasingly difficult to make compatible with the unconditional orthogonality of the series as N grows in view of (C15).

Table 1: Monte Carlo size: Short-run restrictions, $T = 250$.

	DGP 0						DGP 1						DGP 2					
	Asymptotic critical values			Resampling critical values			Asymptotic critical values			Resampling critical values			Asymptotic critical values			Resampling critical values		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
Panel A: Integer moment tests																		
Skewness	10.8	5.6	1.2	10.5	5.6	1.1	10.0	5.6	1.7	9.8	5.1	1.0	9.9	5.3	1.6	9.8	4.8	1.1
Kurtosis	9.3	4.9	1.5	9.8	5.0	1.1	8.8	5.8	2.9	9.6	4.8	1.1	8.9	5.6	2.8	9.4	4.9	1.1
Joint	10.1	5.6	1.7	10.3	5.4	1.1	9.9	6.6	3.4	9.5	5.2	1.1	9.8	6.8	3.2	9.6	5.1	1.2
Panel B: Discrete grid tests																		
$H = 2$	10.4	5.1	0.9	10.2	5.3	1.0	10.3	5.3	1.0	10.0	5.3	0.8	10.0	4.7	0.9	9.6	4.6	1.1
$H = 3$	10.1	5.0	1.0	10.1	5.2	0.9	9.9	4.9	1.0	9.9	4.9	1.0	10.1	5.0	1.0	10.0	5.0	1.0
$H = 4$	9.8	4.8	1.0	9.9	4.9	1.1	10.2	5.1	0.9	10.1	4.8	0.8	10.1	5.1	1.0	10.3	4.9	0.9
$H = 5$	10.0	4.9	0.9	10.0	4.9	0.9	10.1	4.9	0.8	10.3	5.1	0.9	9.9	5.0	0.8	10.1	5.0	0.9
Panel C: Continuous grid tests																		
$\alpha = 10^{-5}$				10.2	5.3	1.0				10.5	5.4	1.2				10.6	5.2	1.0
$\alpha = 10^{-6}$				10.0	5.1	1.0				10.6	5.3	1.2				10.4	5.3	1.1
$\alpha = 10^{-7}$				10.0	5.1	1.0				10.5	5.3	1.2				10.3	5.3	1.1
$\alpha = 10^{-8}$				10.1	5.1	1.0				10.5	5.3	1.2				10.3	5.2	1.1

Notes: Monte Carlo empirical rejection rates of Gaussian SVAR specification tests based on 10,000 replications with $\mathbf{C}_I$. Details on the DGPs: DGP 0: $\boldsymbol{\varepsilon}_t^*$ follows a standardised bivariate Gaussian; DGP 1: ε_{1t}^* and ε_{2t}^* follow two independent univariate scale mixture of normals in which the higher variance component has probability $\lambda = \frac{1}{5}$ and the ratio of the two variances is $\varkappa = \frac{1}{6}$; DGP 2: ε_{1t}^* and ε_{2t}^* follow two independent univariate location-scale mixture of normals in which the higher variance component has probability $\lambda = \frac{1}{5}$, $\delta = -0.67$ and $\varkappa = 0.18$. See sections 3.1, 3.2.1 and 3.2.2 for a detailed description of the integer moments-based, discrete and continuous grid test statistics, respectively.

Table 2: Monte Carlo size: Short-run restrictions, $T = 1,000$.

	DGP 0						DGP 1						DGP 2					
	Asymptotic critical values			Resampling critical values			Asymptotic critical values			Resampling critical values			Asymptotic critical values			Resampling critical values		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
Panel A: Integer moment tests																		
Skewness	10.2	5.0	0.9	10.0	5.0	0.9	10.3	5.7	1.5	10.6	5.3	1.1	10.2	5.3	1.4	10.3	5.4	1.1
Kurtosis	9.4	4.9	1.2	9.8	4.9	1.0	9.5	5.8	2.4	10.1	5.1	1.0	9.8	6.1	2.4	10.4	5.2	1.2
Joint	9.6	4.9	1.3	9.8	4.9	1.0	10.7	6.7	2.8	10.4	5.0	1.1	10.5	6.5	2.6	10.3	5.2	1.1
Panel B: Discrete grid tests																		
$H = 2$	10.6	5.3	1.3	10.4	5.5	1.2	10.0	5.1	1.0	9.9	4.6	1.1	10.2	5.1	0.9	10.3	5.1	0.9
$H = 3$	10.4	5.2	1.1	10.1	5.0	1.0	9.8	5.0	1.0	9.7	5.0	1.0	9.9	4.7	0.9	9.7	4.4	0.9
$H = 4$	10.0	5.1	1.0	10.0	4.9	1.2	10.1	4.9	1.0	10.1	5.0	1.0	10.6	5.1	1.1	10.6	5.1	1.1
$H = 5$	10.0	5.2	1.2	10.0	4.9	1.2	9.9	4.7	0.8	9.8	4.9	1.0	10.1	5.3	1.0	10.2	5.0	0.9
Panel C: Continuous grid tests																		
$\alpha = 10^{-5}$				9.9	4.8	0.9				10.0	4.9	1.0				9.8	4.9	0.9
$\alpha = 10^{-6}$				9.8	4.8	1.0				10.3	5.0	1.0				10.0	5.2	0.9
$\alpha = 10^{-7}$				9.7	4.8	1.1				10.3	5.1	1.0				10.1	5.3	1.0
$\alpha = 10^{-8}$				9.9	4.9	1.0				10.4	5.2	1.0				10.1	5.2	0.9

Notes: Monte Carlo empirical rejection rates of Gaussian SVAR specification tests based on 10,000 replications with $\mathbf{C}_I$. Details on the DGPs: DGP 0: $\boldsymbol{\varepsilon}_t^*$ follows a standardised bivariate Gaussian; DGP 1: ε_{1t}^* and ε_{2t}^* follow two independent univariate scale mixture of normals in which the higher variance component has probability $\lambda = \frac{1}{5}$ and the ratio of the two variances is $\varkappa = \frac{1}{6}$; DGP 2: ε_{1t}^* and ε_{2t}^* follow two independent univariate location-scale mixture of normals in which the higher variance component has probability $\lambda = \frac{1}{5}$, $\delta = -0.67$ and $\varkappa = 0.18$. See sections 3.1, 3.2.1 and 3.2.2 for a detailed description of the integer moments-based, discrete and continuous grid test statistics, respectively.

Table 3: Monte Carlo size: Long-run restrictions, $T = 250$.

	DGP 0						DGP 1						DGP 2					
	Asymptotic critical values			Resampling critical values			Asymptotic critical values			Resampling critical values			Asymptotic critical values			Resampling critical values		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
Panel A: Integer moment tests																		
Skewness	10.1	5.2	1.0	10.5	5.3	1.0	9.8	5.3	1.6	9.4	4.8	1.1	10.6	5.9	1.2	9.9	5.1	1.1
Kurtosis	8.8	4.7	1.5	10.1	5.2	1.2	11.0	7.7	4.1	9.7	5.4	1.2	11.4	7.9	4.1	9.8	5.3	1.4
Joint	9.3	4.9	1.5	10.1	5.0	1.1	12.4	8.6	4.3	10.1	5.3	1.3	12.3	8.6	4.4	10.1	5.6	1.5
Panel B: Discrete grid tests																		
$H = 2$	9.6	4.7	0.9	10.0	5.0	1.0	10.0	5.0	0.9	10.3	5.2	0.9	9.9	5.0	0.8	9.8	4.9	0.8
$H = 3$	9.7	4.6	0.8	10.3	5.1	0.8	10.2	4.9	1.0	10.1	5.2	1.0	10.0	5.2	0.8	9.9	5.1	1.0
$H = 4$	9.6	4.8	0.9	10.4	5.1	1.1	10.3	5.0	0.8	10.3	5.2	1.0	10.8	5.4	1.0	10.7	5.2	1.1
$H = 5$	9.4	4.7	0.9	9.9	4.9	1.0	10.1	5.1	1.0	10.0	5.1	1.1	10.2	5.1	0.9	10.1	5.1	0.9
Panel C: Continuous grid tests																		
$\alpha = 10^{-5}$				10.2	4.9	0.9				10.7	5.4	1.1				10.8	5.8	1.2
$\alpha = 10^{-6}$				10.3	5.0	0.9				10.5	5.3	1.1				10.7	5.6	1.3
$\alpha = 10^{-7}$				10.4	5.0	0.9				10.5	5.2	1.1				10.6	5.6	1.3
$\alpha = 10^{-8}$				10.4	5.1	0.9				10.5	5.2	1.1				10.5	5.6	1.3

Notes: Monte Carlo empirical rejection rates of Gaussian SVAR specification tests based on 10,000 replications with $\mathbf{C}_{II}$. Details on the DGPs: DGP 0: $\boldsymbol{\varepsilon}_t^*$ follows a standardised bivariate Gaussian; DGP 1: ε_{1t}^* and ε_{2t}^* follow two independent univariate scale mixture of normals in which the higher variance component has probability $\lambda = \frac{1}{5}$ and the ratio of the two variances is $\varkappa = \frac{1}{6}$; DGP 2: ε_{1t}^* and ε_{2t}^* follow two independent univariate location-scale mixture of normals in which the higher variance component has probability $\lambda = \frac{1}{5}$, $\delta = -0.67$ and $\varkappa = 0.18$. See sections 3.1, 3.2.1 and 3.2.2 for a detailed description of the integer moments-based, discrete and continuous grid test statistics, respectively.

Table 4: Monte Carlo size: Long-run restrictions, $T = 1,000$.

	DGP 0						DGP 1						DGP 2					
	Asymptotic critical values			Resampling critical values			Asymptotic critical values			Resampling critical values			Asymptotic critical values			Resampling critical values		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
Panel A: Integer moment tests																		
Skewness	10.0	5.0	0.9	10.3	4.9	0.8	10.3	5.5	1.5	10.4	5.3	1.1	10.0	5.5	1.4	10.0	5.1	1.1
Kurtosis	9.2	4.9	1.2	9.7	5.0	1.0	10.1	6.3	2.7	10.0	5.0	0.9	10.3	6.7	2.7	9.8	5.2	1.1
Joint	9.3	4.8	1.2	9.9	4.9	1.1	11.4	7.2	3.2	10.2	5.0	1.0	11.2	7.1	2.9	10.1	5.1	1.1
Panel B: Discrete grid tests																		
$H = 2$	10.1	5.3	1.1	10.3	5.5	1.1	9.7	4.9	0.9	9.9	4.9	1.1	9.9	5.1	0.9	10.1	5.1	0.9
$H = 3$	10.1	4.9	1.0	10.7	5.1	1.0	9.9	5.1	0.9	10.0	5.3	0.9	10.1	5.0	0.9	9.8	4.9	1.0
$H = 4$	9.9	4.9	1.0	10.2	5.2	1.1	10.2	5.0	1.0	10.0	5.2	1.0	10.6	5.5	1.2	10.5	5.4	1.2
$H = 5$	9.9	5.0	1.1	10.0	5.3	1.1	10.0	4.9	0.9	10.0	5.0	0.9	10.4	5.2	0.9	10.0	4.8	0.9
Panel C: Continuous grid tests																		
$\alpha = 10^{-5}$				10.5	5.3	1.0				10.0	5.1	1.1				10.3	5.0	1.2
$\alpha = 10^{-6}$				10.3	5.3	1.1				10.4	5.2	1.1				10.2	5.1	1.1
$\alpha = 10^{-7}$				10.0	5.5	1.1				10.2	5.1	1.0				10.1	5.1	1.0
$\alpha = 10^{-8}$				10.2	5.4	1.1				10.3	5.0	1.1				10.3	5.1	1.0

Notes: Monte Carlo empirical rejection rates of Gaussian SVAR specification tests based on 10,000 replications with $\mathbf{C}_{II}$. Details on the DGPs: DGP 0: $\boldsymbol{\varepsilon}_t^*$ follows a standardised bivariate Gaussian; DGP 1: ε_{1t}^* and ε_{2t}^* follow two independent univariate scale mixture of normals in which the higher variance component has probability $\lambda = \frac{1}{5}$ and the ratio of the two variances is $\varkappa = \frac{1}{6}$; DGP 2: ε_{1t}^* and ε_{2t}^* follow two independent univariate location-scale mixture of normals in which the higher variance component has probability $\lambda = \frac{1}{5}$, $\delta = -0.67$ and $\varkappa = 0.18$. See sections 3.1, 3.2.1 and 3.2.2 for a detailed description of the integer moments-based, discrete and continuous grid test statistics, respectively.

Table 5: Monte Carlo power: Short-run restrictions, $T = 250$.

	$\mathbf{C}_I$						$\mathbf{C}_{II}$														
	DGP 3			DGP 4			DGP 0			DGP 1			DGP 2			DGP 3			DGP 4		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
Panel A: Integer moment tests																					
Skewness	39.6	28.5	11.4	71.2	60.4	34.8	10.2	4.8	1.0	40.8	30.4	13.3	86.7	79.6	59.9	41.6	29.8	11.7	73.6	63.5	38.5
Kurtosis	75.3	62.5	31.0	75.9	63.0	30.9	9.4	4.9	1.4	96.6	92.0	64.5	96.6	92.4	65.4	77.2	64.1	31.6	77.4	64.2	33.2
Joint	76.2	63.2	31.0	82.9	71.0	38.0	9.6	5.1	1.1	95.8	89.4	61.1	97.3	93.4	68.9	77.1	64.6	30.8	83.6	72.2	38.0
Panel B: Discrete grid tests																					
$H = 2$	17.2	10.2	2.5	24.2	16.0	5.5	9.6	4.6	1.1	18.0	10.2	2.6	25.7	16.1	4.8	16.0	9.2	2.2	20.2	12.0	4.0
$H = 3$	18.3	10.7	3.3	28.0	17.8	6.0	10.2	5.4	1.0	23.4	14.8	4.8	36.2	24.6	8.6	19.1	11.2	2.7	25.7	15.3	5.1
$H = 4$	21.0	12.0	3.1	30.9	19.4	5.8	9.3	4.1	0.9	27.7	16.6	5.3	41.6	28.7	10.6	21.3	13.0	3.0	29.0	18.5	5.3
$H = 5$	22.4	13.2	3.8	34.4	22.9	7.5	9.0	4.3	0.8	30.8	20.0	6.0	46.8	33.0	12.7	21.5	13.0	3.0	32.3	20.2	6.7
Panel C: Continuous grid tests																					
$\alpha = 10^{-5}$	29.2	18.6	5.7	56.4	43.8	21.2	9.5	4.8	1.2	45.8	33.2	13.5	75.2	63.8	38.7	29.8	20.0	6.6	53.3	40.1	19.0
$\alpha = 10^{-6}$	29.4	18.8	5.7	55.7	42.6	20.5	9.9	4.9	1.1	45.9	33.3	13.7	73.7	62.4	37.4	29.2	18.9	5.8	52.6	40.2	18.1
$\alpha = 10^{-7}$	29.5	19.0	6.0	55.8	42.7	20.7	9.8	4.9	1.0	46.3	33.9	13.9	73.2	62.1	36.8	29.6	18.6	5.8	52.1	40.4	18.1
$\alpha = 10^{-8}$	30.1	19.5	6.0	55.8	42.6	20.5	9.6	5.0	1.0	47.2	34.6	14.4	73.2	62.4	37.1	30.0	19.4	5.9	52.2	40.6	18.1

Notes: Monte Carlo empirical rejection rates of Gaussian SVAR specification tests based on 10,000 replications. Details on the DGPs: DGP 0: $\boldsymbol{\varepsilon}_t^*$ follows a standardised bivariate Gaussian; DGP 1: ε_{1t}^* and ε_{2t}^* follow two independent univariate scale mixture of normals in which the higher variance component has probability $\lambda = \frac{1}{5}$ and the ratio of the two variances is $\varkappa = \frac{1}{6}$; DGP 2: ε_{1t}^* and ε_{2t}^* follow two independent univariate location-scale mixture of normals in which the higher variance component has probability $\lambda = \frac{1}{5}$, $\delta = -0.67$ and $\varkappa = 0.18$; DGP 3: $(\varepsilon_{1t}^*, \varepsilon_{2t}^*)$ follows a standardised bivariate scale mixture of two zero mean normals with scalar covariance matrices in which the higher variance component has probability $\lambda = \frac{1}{5}$ and the ratio of the two variances is $\varkappa = \frac{1}{6}$; DGP 4: $\varepsilon_{1t}^*, \varepsilon_{2t}^*)$ follows a standardised bivariate discrete mixture of two normals with parameters chosen in such a way that the margins are identical with $\delta = -0.67$ and $\varkappa = 0.18$. See sections 3.1, 3.2.1 and 3.2.2 for a detailed description of the integer moments-based, discrete and continuous grid test statistics, respectively.

Table 6: Monte Carlo power: Short-run restrictions, $T = 1,000$.

	$\mathbf{C}_I$						$\mathbf{C}_{II}$														
	DGP 3			DGP 4			DGP 0			DGP 1			DGP 2			DGP 3			DGP 4		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
Panel A: Integer moment tests																					
Skewness	40.7	30.8	14.3	98.6	97.0	89.2	9.5	4.6	1.0	43.6	32.8	15.8	99.9	99.9	99.2	41.6	30.5	14.2	98.5	97.5	90.6
Kurtosis	99.8	99.3	86.3	99.8	99.0	87.9	10.0	5.2	1.0	100.0	100.0	99.4	100.0	100.0	99.8	99.7	98.8	86.7	99.8	99.0	88.6
Joint	99.7	98.8	85.7	100.0	99.8	93.7	9.7	4.9	0.7	100.0	100.0	99.5	100.0	100.0	99.9	99.6	98.4	84.7	100.0	99.6	92.6
Panel B: Discrete grid tests																					
$H = 2$	23.9	15.0	4.6	53.4	40.2	18.8	9.8	5.0	0.8	29.5	19.6	6.5	62.8	48.0	24.7	25.28	15.4	4.3	44.7	32.8	14.0
$H = 3$	37.9	26.0	9.1	71.6	59.8	34.3	9.8	5.0	1.1	54.7	41.2	20.1	87.0	79.1	55.2	36.48	24.6	8.6	67.7	54.8	29.1
$H = 4$	47.6	35.0	15.6	82.3	73.0	48.6	9.9	4.8	1.0	70.7	59.4	30.6	95.7	91.4	74.0	48.4	36.4	14.6	78.8	68.7	41.3
$H = 5$	56.8	42.7	20.8	87.9	80.5	56.5	9.7	4.8	1.2	81.4	71.4	43.9	97.9	96.2	83.6	56.1	42.2	19.5	86.6	78.3	51.8
Panel C: Continuous grid tests																					
$\alpha = 10^{-5}$	77.1	65.4	40.2	96.1	93.2	77.9	9.2	5.1	1.0	98.7	96.7	84.6	100.0	99.9	98.6	77.3	66.3	39.9	96.1	92.4	76.2
$\alpha = 10^{-6}$	74.0	62.6	37.6	94.3	89.2	71.6	9.1	5.4	1.0	98.2	95.1	82.8	100.0	99.6	97.7	76.0	63.3	37.1	94.1	88.9	70.1
$\alpha = 10^{-7}$	72.6	60.7	35.4	93.1	87.7	68.4	9.6	5.0	1.2	97.6	94.4	80.8	100.0	99.4	96.8	73.7	61.6	34.6	92.7	86.9	66.8
$\alpha = 10^{-8}$	73.0	60.4	35.1	92.4	87.4	67.5	9.4	4.9	1.1	97.6	94.5	80.8	100.0	99.4	96.6	74.1	61.2	34.4	92.3	86.4	65.8

Notes: Monte Carlo empirical rejection rates of Gaussian SVAR specification tests based on 10,000 replications. Details on the DGPs: DGP 0: $\boldsymbol{\varepsilon}_t^*$ follows a standardised bivariate Gaussian; DGP 1: ε_{1t}^* and ε_{2t}^* follow two independent univariate scale mixture of normals in which the higher variance component has probability $\lambda = \frac{1}{5}$ and the ratio of the two variances is $\varkappa = \frac{1}{6}$; DGP 2: ε_{1t}^* and ε_{2t}^* follow two independent univariate location-scale mixture of normals in which the higher variance component has probability $\lambda = \frac{1}{5}$, $\delta = -0.67$ and $\varkappa = 0.18$; DGP 3: $(\varepsilon_{1t}^*, \varepsilon_{2t}^*)$ follows a standardised bivariate scale mixture of two zero mean normals with scalar covariance matrices in which the higher variance component has probability $\lambda = \frac{1}{5}$ and the ratio of the two variances is $\varkappa = \frac{1}{6}$; DGP 4: $(\varepsilon_{1t}^*, \varepsilon_{2t}^*)$ follows a standardised bivariate discrete mixture of two normals with parameters chosen in such a way that the margins are identical with $\delta = -0.67$ and $\varkappa = 0.18$. See sections 3.1, 3.2.1 and 3.2.2 for a detailed description of the integer moments-based, discrete and continuous grid test statistics, respectively.

Table 7: Monte Carlo power: Long-run restrictions, $T = 250$.

	$\mathbf{C}_{II}$						$\mathbf{C}_{III}$														
	DGP 3			DGP 4			DGP 0			DGP 1			DGP 2			DGP 3			DGP 4		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
Panel A: Integer moment tests																					
Skewness	36.8	25.6	10.2	66.1	54.6	28.8	9.9	4.3	1.0	12.6	7.0	1.4	18.8	11.2	3.5	37.3	26.6	10.0	65.9	53.7	29.5
Kurtosis	69.1	54.9	26.2	68.8	54.4	26.0	10.3	5.5	1.0	20.6	12.2	3.8	21.2	12.4	4.2	68.9	54.1	25.3	68.5	55.4	24.6
Joint	70.0	55.6	27.1	77.3	62.8	30.4	10.2	4.8	0.9	19.2	11.3	3.3	20.4	12.3	4.0	69.6	56.2	25.7	76.9	64.1	29.9
Panel B: Discrete grid tests																					
$H = 2$	16.6	9.8	2.5	21.6	13.0	4.5	10.7	5.2	1.1	12.0	5.8	1.4	11.4	6.4	1.5	16.4	9.8	2.6	22.2	13.8	4.1
$H = 3$	16.4	9.0	2.6	27.4	16.2	5.6	9.7	4.3	0.8	10.8	5.8	1.4	13.6	6.9	1.6	17.6	10.1	2.4	25.5	15.8	5.1
$H = 4$	19.2	10.2	2.7	28.4	18.4	5.3	9.5	5.0	1.4	11.1	6.0	1.3	13.2	7.0	1.9	19.7	11.4	3.0	29.4	18.1	5.7
$H = 5$	19.4	10.7	2.9	30.6	19.4	7.0	8.8	4.7	0.7	12.1	6.6	1.3	12.9	8.2	2.2	20.1	12.2	3.4	31.2	19.9	7.1
Panel C: Continuous grid tests																					
$\alpha = 10^{-5}$	25.9	15.2	4.4	48.7	36.3	14.0	9.7	5.1	1.2	14.7	9.0	2.3	19.4	11.6	4.0	24.9	15.6	4.7	49.3	35.3	13.8
$\alpha = 10^{-6}$	25.8	15.5	4.6	48.1	35.5	13.8	9.7	5.1	1.0	15.0	8.9	2.4	19.3	11.2	4.0	25.5	15.2	4.8	48.6	34.8	13.6
$\alpha = 10^{-7}$	26.2	15.3	4.6	48.2	34.9	14.1	9.9	5.4	1.1	15.1	9.0	2.5	19.3	11.2	3.8	25.6	15.0	5.0	48.16	34.6	13.5
$\alpha = 10^{-8}$	26.6	15.5	4.7	48.3	35.0	13.8	10.0	5.3	1.1	14.9	9.2	2.6	19.3	11.0	3.8	25.7	15.2	4.9	48.00	34.5	13.6

Notes: Monte Carlo empirical rejection rates of Gaussian SVAR specification tests based on 10,000 replications. Details on the DGPs: DGP 0: $\boldsymbol{\varepsilon}_t^*$ follows a standardised bivariate Gaussian; DGP 1: ε_{1t}^* and ε_{2t}^* follow two independent univariate scale mixture of normals in which the higher variance component has probability $\lambda = \frac{1}{5}$ and the ratio of the two variances is $\varkappa = \frac{1}{6}$; DGP 2: ε_{1t}^* and ε_{2t}^* follow two independent univariate location-scale mixture of normals in which the higher variance component has probability $\lambda = \frac{1}{5}$, $\delta = -0.67$ and $\varkappa = 0.18$; DGP 3: $(\varepsilon_{1t}^*, \varepsilon_{2t}^*)$ follows a standardised bivariate scale mixture of two zero mean normals with scalar covariance matrices in which the higher variance component has probability $\lambda = \frac{1}{5}$ and the ratio of the two variances is $\varkappa = \frac{1}{6}$; DGP 4: $(\varepsilon_{1t}^*, \varepsilon_{2t}^*)$ follows a standardised bivariate discrete mixture of two normals with parameters chosen in such a way that the margins are identical with $\delta = -0.67$ and $\varkappa = 0.18$. See sections 3.1, 3.2.1 and 3.2.2 for a detailed description of the integer moments-based, discrete and continuous grid test statistics, respectively.

Table 8: Monte Carlo power: Long-run restrictions, $T = 1,000$.

	$\mathbf{C}_{II}$						$\mathbf{C}_{III}$														
	DGP 3			DGP 4			DGP 0			DGP 1			DGP 2			DGP 3			DGP 4		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
Panel A: Integer moment tests																					
Skewness	39.8	30.2	13.7	98.2	96.8	87.1	10.3	5.04	0.7	13.5	7.2	1.8	41.4	29.0	10.7	40.9	30.4	13.6	98.4	96.5	86.9
Kurtosis	99.6	98.8	85.0	99.8	98.5	86.1	10.0	4.96	1.0	46.3	31.2	9.6	46.5	31.6	10.4	99.6	98.7	85.1	99.7	98.8	86.4
Joint	99.3	98.1	84.4	100.0	99.8	92.0	10.4	5.48	0.9	39.2	26.4	7.1	40.8	27.6	8.9	99.4	98.1	83.4	100.0	99.6	92.1
Panel B: Discrete grid tests																					
$H = 2$	24.5	15.3	4.3	53.1	39.9	17.7	10.6	5.4	1.1	10.2	5.3	1.2	15.6	8.8	1.7	24.0	15.2	4.2	52.4	39.4	17.7
$H = 3$	37.2	25.5	9.4	70.3	58.9	32.7	10.5	4.8	0.6	12.4	6.4	1.6	17.7	10.9	2.5	36.8	25.8	9.3	70.0	57.4	31.6
$H = 4$	46.6	34.0	13.8	81.9	72.7	46.6	11.1	5.4	1.1	14.7	7.3	1.8	20.5	11.4	2.7	48.4	36.0	13.8	81.6	70.8	45.2
$H = 5$	56.8	43.7	20.3	87.4	78.6	53.4	11.4	6.1	1.4	13.8	7.3	2.0	21.5	13.0	3.4	57.0	43.1	20.7	87.8	79.2	53.3
Panel C: Continuous grid tests																					
$\alpha = 10^{-5}$	74.6	62.5	35.5	95.5	91.7	71.8	10.3	5.6	1.2	27.2	17.6	6.2	36.0	25.8	11.1	74.0	62.3	36.0	94.3	90.2	70.3
$\alpha = 10^{-6}$	72.2	59.4	32.4	93.8	88.2	64.9	10.4	5.3	1.0	25.9	16.4	5.4	34.0	24.4	10.3	72.3	60.2	33.0	92.0	86.1	63.4
$\alpha = 10^{-7}$	70.0	57.7	30.2	92.4	85.4	61.4	9.8	5.3	1.0	25.5	15.7	5.5	33.4	22.6	9.6	70.7	58.0	31.8	90.5	84.3	60.4
$\alpha = 10^{-8}$	70.1	57.8	30.0	91.9	85.0	60.4	9.8	5.4	1.1	25.5	15.7	5.2	32.9	22.6	9.8	70.6	58.0	31.7	90.2	83.5	59.0

Notes: Monte Carlo empirical rejection rates of Gaussian SVAR specification tests based on 10,000 replications. Details on the DGPs: DGP 0: $\boldsymbol{\varepsilon}_t^*$ follows a standardised bivariate Gaussian; DGP 1: ε_{1t}^* and ε_{2t}^* follow two independent univariate scale mixture of normals in which the higher variance component has probability $\lambda = \frac{1}{5}$ and the ratio of the two variances is $\varkappa = \frac{1}{6}$; DGP 2: ε_{1t}^* and ε_{2t}^* follow two independent univariate location-scale mixture of normals in which the higher variance component has probability $\lambda = \frac{1}{5}$, $\delta = -0.67$ and $\varkappa = 0.18$; DGP 3: $(\varepsilon_{1t}^*, \varepsilon_{2t}^*)$ follows a standardised bivariate scale mixture of two zero mean normals with scalar covariance matrices in which the higher variance component has probability $\lambda = \frac{1}{5}$ and the ratio of the two variances is $\varkappa = \frac{1}{6}$; DGP 4: $(\varepsilon_{1t}^*, \varepsilon_{2t}^*)$ follows a standardised bivariate discrete mixture of two normals with parameters chosen in such a way that the margins are identical with $\delta = -0.67$ and $\varkappa = 0.18$. See sections 3.1, 3.2.1 and 3.2.2 for a detailed description of the integer moments-based, discrete and continuous grid test statistics, respectively.