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Abstract

We study how the co-existence of fixed-term (FT) and open-ended (OE) contracts shapes firm dynamics, firm selection, worker allocation, aggregate productivity, and output. Using rich Spanish administrative data, we document that the use of fixedterm contracts is very heterogeneous across firms within narrowly defined sectors. Particularly, the relationship between the share of temporary workers and firm size is positive within firm but negative between firms. To explain these facts, we write a model of firm dynamics with technology heterogeneity, search-and-matching frictions, and a two-tier labor market structure. Our model emphasizes a key trade-off between contracts, namely, that while FT contracts give flexibility to firms, they also create more worker turnover, which is costly through the need to hire new workers and through the loss of firm-specific human capital. We find that limiting the use of FT contracts decreases the share of temporary employment and increases aggregate productivity —as better firm selection offsets increased misallocation of workers— but it also increases unemployment, output, and welfare.

JEL Codes: D83, E24, J41, L11.

Keywords: Dual Labor Markets, Temporary Contracts, Firm Dynamics, Unemployment.

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1 Introduction

Many labor markets are characterized by a two-tier system, with the co-existence of open-ended contracts (OECs), protected by large termination costs, and fixed-term contracts (FTCs) of short duration. While extensive research has examined the impact of labor market duality on workers, its effects on firm choices and firm outcomes remain largely unexplored. In this paper we analyze how different firms use each type of contract, and how these choices affect the allocation of workers across firms and the equilibrium distribution of firms. This allows us to assess the macroeconomic effects of different policies aimed at reducing the share of temporary workers in the economy.

We start by documenting new facts about the heterogeneous use of FTCs across firms. To do so, we exploit Spanish administrative firm-level data for the period 2004-2019. The case of Spain is of particular interest because its labor market duality is stark, with a high incidence of FTCs and a strong employment protection for OECs.[1] We uncover three important facts. First, there is a large degree of heterogeneity in the usage of FTCs across firms. For instance, among firms with 11 to 50 employees, the average temporary share is 25%, but the firms at the 10th and 90th percentiles of the distribution employ nearly 0% and 68% of workers under FTCs, respectively. Second, although the use of FTCs varies greatly across narrowly defined industries, provinces, and time, most of the variation occurs due to firm-specific factors within industry, province, and time period, of which time-invariant unobserved firm characteristics play an overwhelmingly relevant role. Third, exploiting the panel dimension of our data, we find that the *within-firm* variation uncovers a positive correlation between firm size and the share of temporary workers, while the *between-firm* variation shows that this relationship is negative.

To understand these facts and the macroeconomic consequences of dual labor markets, we write a model of firm dynamics with search-and-matching frictions and a two-tier labor market structure. Our model emphasizes what we think it is the key trade-off between contracts, namely, that while FTCs give flexibility to firms to adjust employment to changing production opportunities, they also create more worker turnover. The costs of worker turnover may come through the need to hire new workers, but also through the loss of firm-specific human capital.[2] Different from the literature, this makes the firm choice between FTCs and OECs far from trivial.

[1] In 2019, the share of workers under an FTC was 26.3% in Spain, the second highest among OECD countries, just behind the 27.0% in Chile and ahead of the 24.4% in South Korea, 21.8% in Poland, 20.7% in Portugal, or 20.3% in Netherlands (see https://data.oecd.org/emp/temporary-employment.htm).

[2] Instead, we do not focus on demand seasonality —given the relatively small importance of industry fixed effects— or worker screening —given the 6-month probation period contained in OECs in Spain.

We model multi-worker firms operating a decreasing returns-to-scale technology. As in Kaas and Kircher (2015) and Schaal (2017), these firms can direct unemployed workers by posting (and committing to) dynamic long-term contracts. As in Gavazza, Mongey and Violante (2018) and Carrillo-Tudela, Gartner and Kaas (2023), firms may improve their hiring prospects by exerting higher recruiting effort. Our first innovation is that firms may simultaneously post open-ended and fixed-term contracts, which differ in terms of recruiting costs, the rate at which workers separate from the firm, and the ability of the worker to accumulate human capital on the job. Our second innovation is the emphasis on firm heterogeneity: firms differ ex-ante in an unobserved *permanent* technology type and ex-post in persistent and *transitory* productivity, in the number of workers, and in the skill and contract composition of their workforce. The firm's technology type determines the permanent component of firm productivity, as well as the relative productivity of workers of different human capital levels. Human capital is accumulated within the firm and is firm-specific. These two properties provide a value for worker stability (better achieved through OECs) while maintaining a tractable framework in which unemployed workers remain identical. In addition, there is firm exit and entry. Firms exit the economy if they are hit by a destruction shock, or if they lose or fire their last remaining worker, which generates endogenous firm selection. In equilibrium, unemployed workers remain ex-ante indifferent between applying for a job under either contract, as well as between the firms that offer them, because less ex-post attractive offers are posted in tighter markets. Firms wishing to expand quickly can offer higher value to workers (attracting longer queues), exert higher recruiting effort, or both. These choices, when aggregated across firms, imply different matching efficiencies across OE and FT labor markets, which is key to fitting the data on aggregate labor market flows by contract type.

We calibrate the model to match cross-sectional features of our firm-level data as well as to the aggregate flows into and out of unemployment by contract type. The calibration delivers two key results: (i) high-skilled workers are more productive in high-type firms, and (ii) recruiting costs are higher for OECs and hence matching efficiency is larger in the FT market. These two results explain the main empirical facts that we documented in the data. First, because of (i), high-type firms use OECs more extensively to retain a larger share of their workers and enhance their chances of accumulating human capital on the job. This generates the inverse *between-firm* relationship between firm size and the temporary share that we see in the data. Second, because of (ii), the job-filling rate is larger and the cost for firms to attract workers is lower in the FT market, while worker turnover is higher in FT positions. This trade-off is valued differently by different firms: for given productivity, and because of decreasing returns to scale, the larger the firm the lower its opportunity cost of leaving a vacancy unfilled. This implies that firms choose a higher temporary share as they grow toward their optimal size, which generates the positive *within-firm* relationship between firm size and the temporary share.

The calibrated economy exhibits two additional features that will be consequential for our policy counterfactuals. First, labor market frictions imply a significant degree of misallocation of workers between firms, with output losses of 5.3% relative to an economy where the marginal product of each type of worker is equalized across all existing firms. Second, there is substantial firm selection: the share of high permanently productive firms is 13.8% among entrants but 19.1% among incumbents. This selection happens through the higher exit rate of the less productive firm type, which on average operate with lower levels of employment and higher shares of temporary workers.[3]

In the last part of the paper, we use the model to study the effects of dual labor markets on the aggregate economy. We do so by comparing different policy tools aimed at reducing the share of workers employed under FTCs.

Countries with a two-tier labor market system impose a legal (and relatively short) maximum duration that a worker can stay at the firm under an FTC. We start our policy analysis by reducing the duration of FTCs, which is a common policy tool for countries that want to limit the use of FTCs (i.e., Spain in its 2022 labor market reforms). The effects of a reduction on the duration of FTCs are ex-ante ambiguous. On the one hand, reducing the maximum duration of FTCs brings flexibility gains, since firms become less likely to pay firing costs when they get hit by negative productivity shocks. On the other hand, it entails turnover losses, as firms incur hiring costs more frequently to maintain their employment levels and lose valuable firm-specific human capital. Given the short duration of FTCs in Spain, in our calibrated economy, the second effect dominates, making FTCs less attractive to firms when their duration is reduced. Firms react by hiring proportionately more from the OE market (through higher recruiting effort and by offering better-paid contracts) and increasing the promotion rate of their incumbent FT workers, which leads to a decline in the share of temporary employment and a decline in worker turnover. However, the policy comes with a surge in unemployment, coming from a decline in job-finding rates that is stronger than the decline in the job-separation rate.

The policy also leads to a slight increase in aggregate productivity —by 0.17% when FTC duration is decreased from half a year to 1 month. We show, through an analytical decomposition, that this is because the productivity losses that stem from increased misallocation of workers across firms (i.e., from having less flexibility to adjust employment to shocks as

[3]In an extension, we study a version of our economy that generates selection upon entry as well. The results for this version of the model are discussed in Section 5.1.4.

firms use fewer FTCs) are lower than the gains due to (i) better firm selection (i.e. worse firm types being relatively more damaged and exiting more), and (ii) increased human capital accumulation from lower worker turnover and hence longer worker tenure in the firm.[4] All in all, aggregate output and welfare decline (by 9.75% and 2.07%, respectively).

As an alternative policy, we introduce a tax on the use of FTCs, mirroring several policies in place in France, Portugal, and Spain. When calibrated to produce the same reduction in the aggregate share of FTCs as our baseline policy of shortening FTC duration, the effect of the tax on aggregate productivity is quantitatively similar. However, the negative effects on unemployment, output, and welfare are less severe, mostly due to the effects of the tax on job destruction being weaker. Finally, we show that an outright ban on FTCs would be suboptimal: even though FTCs are an impediment to human capital accumulation within the firm, they provide flexibility to firms who need to readjust employment in response to adverse shocks, and they are an effective hiring tool that lessens unemployment. In net, welfare would decrease by 1.22% if FTCs were banned completely.

All in all, these various policy exercises suggest that restricting the use of FTCs may not lead to productivity losses as commonly thought: although the allocation of workers across firms worsens, this is always offset by a better selection of firms in equilibrium. However, restricting the use of FTCs is still a bad idea because it leads to higher unemployment and lower output and welfare.

Related literature There is a large literature studying the effects of employment contract duality on aggregate labor market flows. Blanchard and Landier (2002), Cahuc and Postel-Vinay (2002), Bentolila, Cahuc, Dolado and Le Barbanchon (2012), and Sala, Silva and Toledo (2012) study the effect of dual labor markets in models with search and matching frictions *à la* Mortensen and Pissarides (1994). In these models, search is random and firms do not choose which type of contracts to offer. If they did, they would hire all new workers in FTCs as in Costain, Jimeno and Thomas (2010) because, from the firm's side, the flexibility of FTCs dominates OECs.[5] Hence, these models are not designed to understand the differential choices of FTCs vs OECs across firms. Furthermore, because firms can only hire one worker, they cannot be used to link contract choices to firm dynamics. Our paper contributes to this literature on both of these fronts.

Several papers provide arguments for the coexistence of FTCs and OECs. Cahuc, Charlot

[4]With shorter FTCs, incumbent firms are slightly smaller, which also brings aggregate productivity gains because technology exhibits decreasing returns to scale. However, the contribution of this channel to productivity gains is comparatively small across all of our policy experiments.

[5]Still, the endogenous firing decision in these models allows to make separations contingent on contract type and hence the aggregate share of FTCs is not undetermined in equilibrium.

and Malherbet (2016b) allows for firms to be heterogeneous in their expected job duration so that firms prefer to use FTCs for jobs of short expected duration (in order to save on firing costs) and OECs for jobs of long expected duration (to save on vacancy posting costs). In the context of directed search models, Berton and Garibaldi (2012) argue that an advantage of OECs over FTCs for firms is that the vacancy-filling rate will be higher in equilibrium when posting OECs as more job-seekers will self-select into the OE market, which offers them higher-value jobs ex-post. Our model features a similar equilibrium logic. However, by properly parameterizing the hiring cost function of the OE and FT markets, we can obtain vacancy-filling rates that are larger in the FT than in the OE markets, a feature that is needed to match the large gap in labor market flows between contract types in the data. Other explanations for the coexistence of OECs and FTCs are that duality diminishes on-the-job search and hence allows firms to retain high-quality workers (as in Cao, Shao and Silos (2013)), and that FTCs can be used by firms to overcome financial constraints (as in Caggese and Cuñat (2008)).

We also relate to a macro literature studying the equilibrium dynamics of multi-worker firms in the context of frictional labor markets. We use a directed search framework with dynamic long-term contracts in the spirit of Kaas and Kircher (2015) and Schaal (2017). We adapt this framework to a continuous-time setting with slow-moving state transitions similar to Roldan-Blanco and Gilbukh (2021), and extend it to incorporate segmented labor markets, different ex-ante firm types, and double-sided ex-post heterogeneity. An alternative approach in the literature has been to assume random search and study firm dynamics in the context of decreasing returns and Nash bargaining (as in Elsby and Michaels (2013) or Acemoglu and Hawkins (2014)), or in settings with on-the-job search and a variety of wage-setting protocols (e.g. Moscarini and Postel-Vinay (2013), Coles and Mortensen (2016), Bilal, Engbom, Mongey and Violante (2022), Gouin-Bonenfant (2022), Elsby and Gottfries (2022), Audoly (2023), and Gulyas (2023)). These models have proved very successful for non-dual markets. Our contribution to this literature is to provide a quantitative model for the firm and aggregate labor market dynamics of tiered markets.

On the policy side, several papers have studied the effects of changes in employment protection legislation in dual labor markets contexts. For example, Daruich, Di Addario and Saggio (2023) shows that relaxing constraints on FTCs relative to OECs in Italy failed to increase overall employment, while Cahuc, Carry, Malherbet and Martins (2022) show that a policy intended to restrict the use of FTCs by new establishments of large firms in Portugal did not increase the number of permanent contracts and ended up decreasing employment in large firms. We complement these studies by arguing that changing the duration of FTCs yields significant effects on aggregate productivity and employment that mask various selection and reallocation forces.

Finally, even though dual labor markets are typically associated to European economies due to specific legislation, recent papers have documented a *de facto* duality in the U.S. labor market as well (e.g. Gregory, Menzio and Wiczer (2022) and Ahn, Hobijn and Şahin (2023)). Our model can also speak to the duality of labor market outcomes in economies without firing costs like the US. When facing a negative productivity or demand shock, firms using technologies more reliant on firm-specific human capital will be more conservative in their firing policy, hoarding labor for the next positive shock.

Outline The rest of the paper is organized as follows. Section 2 describes the data and our main empirical findings. Section 3 present the model and Section 4 discusses the estimation of its parameters. Section 5 describes our various policy experiments. Section 6 offers concluding remarks. Proofs and additional results, tables, and figures can be found in the Online Appendix.

2 Empirical Findings

Data We use annual data for Spain from the *Central de Balances Integrada* (CBI) dataset, a comprehensive and unbalanced panel of confidential firm-level balance-sheet data compiled and processed by the *Central de Balances*, a department within Banco de España.[6] This dataset covers the quasi-universe of Spanish firms, including large and small firms as well as privately held and publicly traded firms. Among many other items from the balance sheet of firms, the data provide information on total employment and the type of employment contract. We use data for the period 2004-2019. We restrict our sample to firms observed for at least 5 years and whose average employment over the period is at least one worker. After some cleaning, we keep data for 7,153,669 firm-year observations, corresponding to 705,879 different firms. Remarkably, the average share of FTCs in our sample aggregates very well to the aggregate temporary share from the labor force survey data (see Figure E.1 in the Online Appendix E). Our companion paper, Auciello-Estévez, Pijoan-Mas, Roldan-Blanco and Tagliati (2023), describes the data in more detail and expands on some of the empirical results presented below.

Cross-sectional distribution In our sample, the average share of temporary workers across firms is 18.1%. This average masks an enormous amount of heterogeneity, with the shares of temporary employment at the 25th, 75th, and 90th percentiles of the distribution

[6] See Almunia, López-Rodríguez and Moral-Benito (2018) for details.

Table 1: Temporary share, descriptive statistics

Firm size (employment)	Share of firms (%)	Distribution of firm-level share of temporary workers Mean	p10	p25	p50	p75	p90	p95
Total	**100**	**0.181**	**0.000**	**0.000**	**0.027**	**0.294**	**0.591**	**0.800**
1-10	77.65	0.164	0.000	0.000	0.000	0.250	0.541	0.776
11-50	19.04	0.250	0.000	0.031	0.163	0.391	0.677	0.825
51-100	1.78	0.255	0.000	0.034	0.160	0.393	0.701	0.861
101-200	0.99	0.237	0.000	0.029	0.147	0.361	0.645	0.833
201-500	0.30	0.222	0.000	0.026	0.137	0.329	0.589	0.796

Note: Selected moments of the distribution of the share of workers under temporary contracts. Each row corresponds to one firm size category, each column to a different moment of the distribution. p10 to p95 refer to percentiles 5 to 95 of the distribution. CBI data pooled over all years.

being 0.0%, 29.4%, and 59.1%, respectively, see first row in Table 1. This heterogeneity persists when we look at the temporary share within firm-size bins. For instance, within the subset of firms between 11 and 50 workers, the average share of temporary workers is 25.0%, and the shares of temporary workers at the 25th, 75th and 90th percentiles are 3.1%, 39.1%, and 67.7%, respectively. Finally, in Table 1 we also observe that the use of temporary workers tends to increase with firm size —a feature that we will discuss in more detail later— and that most of Spanish firms (more than 95%) have 50 workers or less.

Aggregate determinants Next, in order to understand the heterogeneity in the use of temporary contracts across firms, we propose a regression of the type:

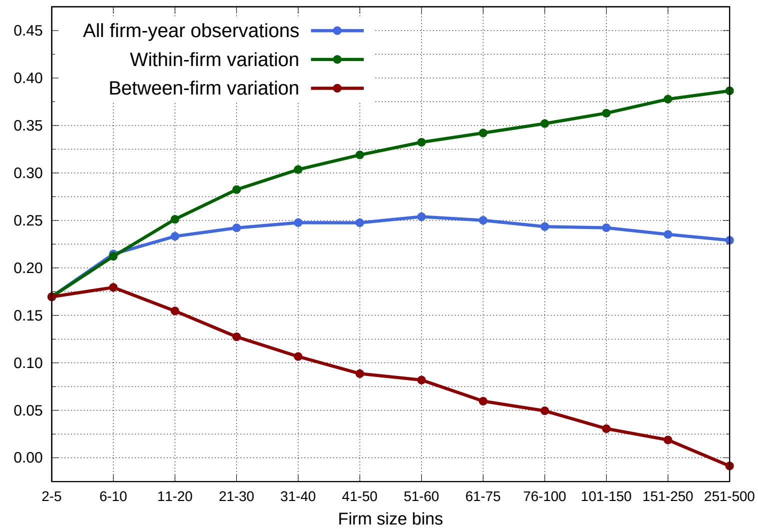
$$TempSh_{ft} = \left(\alpha_i + \alpha_p + \alpha_t\right) + \alpha_f + \mathbf{X}_{ft}\boldsymbol{\beta} + \epsilon_{ft} \tag{1}$$

where $TempSh_{ft}$ is the share of temporary workers of firm f at time t; α_i, α_p, and α_t, are 4-digit industry, province, and year fixed effects; α_f are firm fixed effects; and $\mathbf{X}_{ft}$ is a vector of covariates.[7]

We find that the aggregate fixed effects (α_i, α_p, α_t) are important. The temporary share differs widely across sectors (ranging from 7.7% in "*Real estate activities*" to 43.1% in "*Employment activities*"), across provinces (ranging between 11.5% in Barcelona to 38.8% in Huelva), and over time (the temporary share is strongly procyclical, ranging from 23.9% in 2006 to 15.8% in 2012). However, the R^2 of regression (1) with only the aggregate fixed effects $(\alpha_i, \alpha_p, \alpha_t)$ is 16%, or 17% when interacting industry and year fixed effects. That is, more than 80% of the variation in the usage of FTCs remains within industry, province, and time period. In particular, firm fixed effects explain nearly half of the overall variation:

[7] We index each data point by ft instead of $ipft$ since there is no variation in industry i and (almost) no variation in province p at the firm level.

Figure 1: Temporary share, by firm size: within- and between-firm variation.



Notes: The green line reports the coefficients of the size dummies of a regression of temporary share that controls for aggregate and firm-level fixed effects. The red line reports the firm fixed effects of the same regression against dummies of average firm size. The blue line reports the size dummies of a regression of the temporary share that controls for aggregate but not firm-level fixed effects.

the R^2 of regression (1) with aggregate fixed effects increases from 16% to 61% with the inclusion of α_f into the specification (or to 57% when looking at the adjusted R^2).

Temporary contracts and firm size To uncover the firm-level determinants of temporary employment, we run regression (1) with size-bin dummies (2-5 employees, 6-10, 11-20, 21-30, ...) in the vector $\mathbf{X}_{ft}$ together with the aggregate and firm-level fixed effects. As shown in Figure 1, there is a complex relationship between the use of temporary contracts and firm size. The green line represents the estimated $\boldsymbol{\beta}$ coefficients in the regression with both aggregate and firm fixed effects included, and it captures an increasing within-firm variation between the temporary share and firm size.[8] The red line plots the estimated α_f against the (time-series) average of employment of each firm, and it captures a declining between-firm variation between the temporary share and firm size. Both lines are scaled to deliver the same value for the size bin of 2-5 workers. For comparison, the blue line represents the estimated coefficients in a regression that does not include firm fixed effects, and hence it does not distinguish between within- and between-firm variation. The within-firm variation suggests that firms make use of FTCs to grow or decline in size: at the firm level, FTCs are useful to help employment track productivity or demand changes. The between-firm variation suggests that there are important technology differences across firms, whereby firms that are (permanently) larger prefer a lower share of temporary contracts, while firms

[8]The positive within-firm relationship between the temporary share and firm size is not driven by ageing, as the relationship is robust to adding firm age in the right-hand side.

that are (permanently) smaller prefer a higher share of temporary contracts.

Taking stock Our empirical analysis shows that most variation in the temporary share across firms is explained by firm-specific factors, and not by industry, province, or year effects. Firms use temporary contracts when they grow or decline (within-firm, the temporary share *increases* with employment), but larger firms make less use of temporary contracts (between-firm, the temporary share *declines* with employment). One can abstract from seasonality (possibly captured by industry and province fixed effects) or business cycle fluctuations (captured by the year dummies). With these results in mind, in the next section we write a firm-dynamics model with a dual labor market structure to study the implications of labor market duality for the aggregate economy.

3 Model

3.1 Environment

Time is continuous and infinite. We consider a stationary economy populated by a mass of workers with fixed unit measure and an endogenous measure of firms. Firms and workers are risk-neutral and infinitely-lived, and share a common time discount rate, $\rho > 0$.[9]

Technology A firm has a type $\varphi \in \Phi \equiv \{\varphi_1, \ldots, \varphi_{k_\varphi}\}$ that is kept fixed throughout its life. There are two possible worker skill levels, high (H) and low (L). Let $n_j = 0, 1, 2, \ldots$ denote the number of workers with skill $j \in \mathcal{J} \equiv \{H, L\}$ in a firm. All firms operate the following decreasing returns to scale production function:

$$y(n_H, n_L, z, \varphi) = e^{z+\zeta(\varphi)} \Big(\omega(\varphi) n_H^{\alpha} + \big(1 - \omega(\varphi)\big) n_L^{\alpha} \Big)^{\frac{\nu}{\alpha}}, \tag{2}$$

where $\nu \in (0, 1)$ is the degree of decreasing returns to scale, $\alpha < 1$ controls the elasticity of substitution of workers of different skills,[10] $\omega(\varphi) \in (0, 1)$ measures the relative productivity of high-skilled workers for a firm of permanent type $\varphi \in \Phi$, and $\zeta(\varphi) > 0$ denotes the permanent component of firm productivity for a firm of type $\varphi \in \Phi$. The random variable

[9]Throughout the paper, our convention on notation is the following: for some generic object, we reserve a roman font A, or Greek letters α, for generic variables and deep parameters; the calligraphic font $\mathcal{A}$ is for (countable) sets; the arrow notation $\vec{a}$ is for vectors; the bold font $\mathbf{A}$ is for value functions; and the typeset font $\mathtt{A}$ is for measures of agents.

[10]The elasticity of substitution is $\frac{1}{1-\alpha} > 0$. Skill types are complements in production if $\alpha < \nu$. In this case, the production function is supermodular, i.e. $\partial^2 Y / \partial n_H \partial n_L > 0$, $\forall z, \varphi$.

$z \in \mathcal{Z} \equiv \{z_1 < \cdots < z_{k_z}\}$ stands for the idiosyncratic and transitory productivity of the firm, which follows a continuous-time Markov chain with intensity rates $\lambda(z'|z)$.[11]

Firms hire workers by posting two types of contract, *fixed-term* (FT) and *open-ended* (OE), indexed by $i \in \mathcal{I} \equiv \{FT, OE\}$. A worker is uniquely identified by its type, $(i, j) \in \mathcal{I} \times \mathcal{J}$.

Skill accumulation All starting jobs are in low-skilled positions and, as in Ljungqvist and Sargent (2007), workers can access high-skilled positions through skill upgrades that arrive on the job. The skills are lost upon displacement. These assumptions make human capital firm-specific.[12] We assume that only OE workers can upgrade to high-skilled jobs. In particular, a low-skill OE worker transitions to a high-skill job with intensity $\tau > 0$ (a parameter), with the opposite transition (skill obsolescence) being impossible. This assumption captures the idea that workers under FTCs do not have time to accumulate skills due to their short tenure at the firm.[13] An alternative interpretation is that worker training by the firm is much more likely to be offered to workers under OECs than under FTCs due to the higher turnover of the latter type of worker.[14] This provides an important motive for offering OECs to workers: it is the only way to ensure that a fraction of workers understand the firm's operations and can be entrusted with tasks requiring higher skill.

Worker transitions Firms may promote their FT workers into an OEC, with the opposite transition being illegal. Firms choose a promotion rate p for each one of their FT workers (if any), which carries a promotion cost of $\xi n_{FT} p^{\vartheta}$ units of the firm's output, with $\xi > 0$ and $\vartheta > 1$. When an FT worker is promoted into an OEC, her job description does not change (i.e. FT workers remain low-skill upon promotion). One can think of the promotion costs as administrative or legal costs for contract conversion, as extra screening costs before conversion, or even as training costs (given that all converted FT workers are exposed to a future skill upgrade).

Firms may lose workers for three different reasons: (i) because of an exogenous firm

[11] The transition rates satisfy, $\forall z \in \mathcal{Z}$: $\lambda(z|z) \leq 0$; $\lambda(z'|z) \geq 0$, $\forall z' \neq z$; $\sum_{z' \in \mathcal{Z}} \lambda(z'|z) = 0$; and $\sum_{z' \in \mathcal{Z}} \lambda(z'|z) < +\infty$.

[12] Firm-specific human capital allows to simplify the model because it makes all unemployed individuals identical. An empirical literature highlights the importance of firm-specific human capital for worker wage growth (Topel (1991), Dustmann and Meghir (2005), or Buchinsky, Fougère, Kramarz and Tchernis (2010)).

[13] In our calibrated model, a newly hired FT worker only has an average of half a year before its contract is terminated, which is little time to acquire firm-specific skills. If we allowed workers under FTCs to accumulate human capital, in equilibrium only a tiny fraction of them would do so, and this would come at a high cost in terms of extra state space.

[14] Indeed, several papers show that having a temporary contract diminishes the probability of receiving on-the-job training (e.g. Alba-Ramirez (1994), Dolado, Felgueroso and Jimeno (2000), Bratti, Conti and Sulis (2021), Cabrales, Dolado and Mora (2017)). Additional evidence of this mechanism is that wage returns to experience are much larger for experience years accumulated under OECs than FTCs (see e.g. Garcia-Louzao, Hospido and Ruggieri (2023)).

exit shock, with intensity $s^F \geq 0$, dissolving the firm entirely and sending all of its workers into unemployment; (ii) because the contract expires, at rate $s_i^W \geq 0$ for each contract type $i \in \mathcal{I}$; or (iii) because the firm endogenously decides to fire workers. For the latter case, the firm must choose a per-worker firing rate $\delta_{ij} \geq 0$ for each worker type $(i, j) \in \mathcal{I} \times \mathcal{J}$, which carries a layoff cost equal to $\chi n_{ij} \delta_{ij}^{\psi}$ units of the firm's output, with $\chi > 0$ and $\psi > 1$. This cost is meant to capture expenses associated with laying off workers, such as administrative expenses and legal costs.[15]

Potential entrants In the event that a firm loses all of its workers, it exits the market and becomes a *potential entrant*. To post a contract, potential entrants must incur a flow cost $\kappa > 0$ and, upon entry, draw a permanent type $\varphi \in \Phi$ and an initial idiosyncratic productivity $z \in \mathcal{Z}$ from some π_φ and π_z distributions, respectively.[16] Firms enter with one worker (under an OEC or an FTC), which they attract with the same search-and-matching technology as that of operating firms. We describe this technology next.

Search and matching Search is directed. Every instant of time, a firm (i) opens one vacancy of each type $i \in \{FT, OE\}$, (ii) chooses the terms of the offered contracts, and (iii) chooses the recruiting intensity rates (v_{OE}, v_{FT}) for each vacancy.[17] We assume that a recruiting intensity $v_i > 0$ entails a recruiting cost of $A_i v_i^{\varsigma}$ units of the firm's output, where $A_i > 0$ and $\varsigma > 1$ are parameters. The recruiting cost shifter $A_i > 0$ is potentially contract-specific.[18]

Denote the employment vector of a firm by $\vec{n} \equiv (n_{OEH}, n_{OEL}, n_{FT}) \in \mathcal{N}$, where $\mathcal{N}$ denotes the set of all integer triplets excluding the zero vector. Let $(\vec{n}_t^{t+s}, z_t^{t+s})$ be the full history of possible firm states between dates t and $t+s$. A contract of type $i \in \mathcal{I}$ for skill type $j \in \mathcal{J}$ offered by a firm of permanent type $\varphi \in \Phi$ is a set of complete state-dependent

[15] Mirroring Spanish law, the firing cost parameters do not depend explicitly on contract type. The cost does not include transfers between the employer and the worker (severance payments) because they have no effect on allocations given that the optimal contract maximizes the joint surplus.

[16] Note that this implies that entrants cannot choose their type. See Section 5.1.4 for a model extension where they do.

[17] This implies that firms wishing to grow fast must increase the filling rate of vacancies as they cannot do it by raising the number of vacancies. We make this assumption for tractability, but Gavazza *et al.* (2018) and Carrillo-Tudela *et al.* (2023) show that, in practice, the vacancy yield is much more important for firm growth than the vacancy rate.

[18] This could be due to several reasons. For instance, there might be a tougher screening process for incoming OE workers, as these workers will eventually be assigned to high-skilled tasks, which may require more talent or specific knowledge, making hiring under this contract more costly for the same recruiting intensity. Another interpretation is that the value of properly screening candidates is larger for OE workers, who are expected to stay longer with the firm. Finally, in Spain there are intermediary companies (*empresas de trabajo temporal*) connecting workers seeking and firms offering FTCs, easing the matching process and lowering the costs of recruiting FT workers.

sequences of wages $w_{ij}(\vec{n}_t^{t+s}, z_t^{t+s}; \varphi)$, recruiting intensities $v_i(\vec{n}_t^{t+s}, z_t^{t+s}; \varphi)$, firing rates $\delta_{ij}(\vec{n}_t^{t+s}, z_t^{t+s}; \varphi)$ and, only for workers employed under FTCs, intensities $p(\vec{n}_t^{t+s}, z_t^{t+s}; \varphi)$ of promotion into an OEC, conditional on no worker separation and firm survival.

We assume the following commitment structure. On the worker's side, workers may forfeit their contract and quit the firm, but in that case, they must go back to unemployment (and consume flow utility $b > 0$), from where they can regain employment. On the firm's side, by contrast, there is full commitment to both the contract type as well as to the contractual terms, which cannot be revised or renegotiated for the duration of the match. Therefore, contracts must always comply with the firm's initial promises. Moreover, we assume that the firm cannot discriminate between workers with the same contract type and job description, i.e. all n_{ij} workers of type (i, j) obtain the same contract (though, of course, their individual employment histories may differ).

Given these assumptions, a distinct labor market segment (or "submarket") is indexed by (i) the contract's type $i \in \{FT, OE\}$, and (ii) the long-term value that the worker can expect to obtain from it, denoted W. Each worker can simultaneously search in at most one submarket (i, W), and each firm can simultaneously post only one offer W of each contract type i. We denote by $\mathtt{V}_i(W)$ the intensity-weighted measure of vacancies in submarket (i, W).[19] Likewise, we denote by $\mathtt{U}_i(W)$ the measure of unemployed workers applying to the same submarket (i, W). Then, the measure $\mathtt{M}_i(W)$ of matches in market segment (i, W) is equal to $M(\mathtt{V}_i(W), \mathtt{U}_i(W))$, where $M : \mathbb{R}_+^2 \to \mathbb{R}_+$ is a constant-returns-to-scale matching function whose parameters are common across all submarkets.

As there is a continuum of firms within each submarket and each firm faces the same hiring frictions, by the law of large numbers an individual firm exerting recruiting effort v in submarket (i, W) obtains $v\eta\big(\theta_i(W)\big)$ new hires, where $\theta_i(W) \equiv \mathtt{V}_i(W)/\mathtt{U}_i(W)$ is the effective market tightness and $\eta : \theta \mapsto M(1, \theta^{-1})$ is the aggregate job-filling rate per effective vacancy. Similarly, we denote by $\mu(\theta) = \theta\eta(\theta)$ the aggregate job-finding rate in a submarket with tightness θ.[20]

Recursive contracts Because contracts are large and complex objects, we focus on their recursive formulation. We focus on a Markov Perfect Equilibrium, in which contracts are only functions of the firm's state. A firm's state is (i) its permanent type $\varphi \in \Phi$, (ii) its employment vector and productivity $(\vec{n}, z) \in \mathcal{N} \times \mathcal{Z}$, and (iii) the set

[19]Precisely, the effective measure of vacancies in submarket (i, W) is defined as $\mathtt{V}_i(W) \equiv \int_{\Omega_i(W)} v_{if} \mathrm{d}f$, where v_{if} is the recruiting intensity of firm f for contract $i \in \{FT, OE\}$, and $\Omega_i(W)$ is the set of firms that offer value W for contract i. See further details in Online Appendix A.5.

[20]These rates satisfy standard properties: $\eta(\theta)$ is decreasing and convex, $\mu(\theta)$ is increasing and concave, and $\lim_{\theta \to 0} \mu(\theta) = \lim_{\theta \to +\infty} \eta(\theta) = 0$, and $\lim_{\theta \to +\infty} \mu(\theta) = \lim_{\theta \to 0} \eta(\theta) = +\infty$.

$\vec{W} \equiv (W_{OEH}, W_{OEL}, W_{FT}) \in \mathbb{R}^3_+$ of outstanding values that the firm promised to its incumbent workers. Then, a recursive contract is defined by:[21]

$$\vec{C}_{ij} \equiv \left\{ w_{ij}, v_i, \delta_{ij}, p, W'_{ij}(\vec{n}', z') \right\}.$$

For each worker $(i, j) \in \mathcal{I} \times \mathcal{J}$, a contract includes a wage w_{ij}, a recruiting effort v_i, a per-worker layoff rate δ_{ij}, a promotion rate p for FT workers, and a continuation promise $W'_{ij}(\vec{n}', z')$ for each new possible set of states $(\vec{n}', z')$ of the firm. The new state vector is:

$$(\vec{n}', z') \in \left\{ \begin{array}{l} (n_{OEH},\ n_{OEL} + 1,\ n_{FT},\ z),\ (n_{OEH},\ n_{OEL},\ n_{FT} + 1,\ z), \\ (n_{OEH} + 1,\ n_{OEL} - 1,\ n_{FT},\ z) \\ (n_{OEH} - 1,\ n_{OEL},\ n_{FT},\ z),\ (n_{OEH},\ n_{OEL} - 1,\ n_{FT},\ z),\ (n_{OEH},\ n_{OEL},\ n_{FT} - 1,\ z), \\ (n_{OEH},\ n_{OEL} + 1,\ n_{FT} - 1,\ z), \\ \{(n_{OEH},\ n_{OEL},\ n_{FT},\ z'), : z' \in \mathcal{Z}\} \end{array} \right\},$$

depending on which type of transition the firm has in the next stage (including hiring, promotion, separation, skill upgrades and productivity shocks, respectively). Henceforth, we will make use of the short-hand notation $\vec{n}^+_{ij} \equiv (n_{ij} + 1, \vec{n}_{-(ij)})$, $\vec{n}^-_{ij} \equiv (n_{ij} - 1, \vec{n}_{-(ij)})$, $\vec{n}^p \equiv (n_{OEH}, n_{OEL} + 1, n_{FT} - 1)$, and $\vec{n}^\tau \equiv (n_{OEH} + 1, n_{OEL} - 1, n_{FT})$, to denote the various size transitions, where $\vec{n}_{-(ij)} \equiv \vec{n} \backslash \{n_{ij}\}$, for any $i \in \{FT, OE\}$ and $j \in \{H, L\}$.[22]

3.2 Equilibrium

3.2.1 Unemployed Worker's Problem

Unemployed workers consume a flow utility $b > 0$ while searching in the labor market. Search is directed toward the submarket (i, W) that offers the most profitable expected return for workers. Thus, the value of unemployment is $\mathbf{U} = \max_{(i,W)} \{\mathbf{U}_i(W)\}$, where $\mathbf{U}_i(W)$ solves the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\rho \mathbf{U}_i(W) = b + \mu\big(\theta_i(W)\big) \max \Big\{ W - \mathbf{U}_i(W), 0 \Big\}. \tag{3}$$

As workers prefer the most profitable offers, when unemployed, they must remain indifferent ex-ante between all of those offers to which they decide to apply. Therefore, the following

[21] To alleviate notation, we do not index recursive contracts explicitly by $(\vec{n}, z, \varphi, \vec{W})$.

[22] As $\vec{n}$ may contain more than one instance of the same element, the symbols $\cup$ and $\backslash$ represent *multiset* union and difference operators, respectively, meaning $\{a, b\} \cup \{b\} = \{a, b, b\}$ instead of $\{a, b\} \cup \{b\} = \{a, b\}$, and $\{a, b, b\} \backslash \{b\} = \{a, b\}$ instead of $\{a, b, b\} \backslash \{b\} = \{a\}$.

complementary slackness condition must hold:

$$\forall (i, W) \in \mathcal{I} \times \mathbb{R}_+ : \quad \mathbf{U}_i(W) \leq \mathbf{U}, \text{ with equality if, and only if, } \mu\big(\theta_i(W)\big) > 0.$$

This condition states that a submarket must maximize the value of remaining unemployed, or else it is never visited by workers. Imposing this condition into equation (3) we find:

$$\theta(W) = \mu^{-1}\left(\frac{\rho\mathbf{U} - b}{W - \mathbf{U}}\right). \tag{4}$$

Equation (4) defines the equilibrium function that maps promised values to market tightness, for any given value of unemployment $\mathbf{U}$. Market tightness is decreasing in W (more attractive contracts for workers ex-post attract more workers per job posting ex-ante), and increasing in $\mathbf{U}$ (a better outside option for workers makes jobs relatively less attractive ex-ante).[23] Market tightness, however, does not depend explicitly on the contract type i because of the indifference condition and the fact that the value of a job is summarized only by W. However, in equilibrium, differences in primitives across contracts will lead different firms to offer different values W for different contracts $i = OE, FT$, giving rise to heterogeneity in aggregate job-filling and job-finding rates by contract type.

3.2.2 Joint Surplus Problem

Next, we characterize the optimal contract menu chosen by firms. For each worker of type (i, j) employed by the firm, this implies finding the vector $\vec{C}_{ij} = \{w_{ij}, v_i, \delta_{ij}, p, W'_{ij}(\vec{n}', z')\}$ that maximizes firm value. As the firm has commitments to its pre-existing workers, this problem is subject to a promise-keeping constraint. As workers do not commit, there is also a worker-participation constraint. We write the firm's and employed worker's value functions in Online Appendix A.1.

As it turns out, however, we can find the optimal contract from a simpler and equivalent problem, which involves maximizing the *joint surplus* of the match, that is, the sum of the firm's value and the values of all of its workers. For a match between a firm of type φ in state $(\vec{n}, z, \vec{W})$ and its workers, we define the joint surplus as:

$$\mathbf{\Sigma}(\vec{n}, z, \varphi) \equiv \mathbf{J}\big(\vec{n}, z, \varphi, \vec{W}\big) + \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} n_{ij} W_{ij}, \tag{5}$$

[23]To save on notation, we write $\theta(W)$ instead of $\theta(W, \mathbf{U})$. Note, however, that $\mathbf{U}$ is an endogenous object which both firms and workers take as given when making decisions.

where $\mathbf{J}(\cdot)$ denotes the value of the firm, and recall that $\vec{W} \equiv \{W_{ij}\}$ are the outstanding promised values. As discussed in detail below, the joint surplus is independent of promised values and, anticipating this result, on the left-hand side of equation (5) we have written $\mathbf{\Sigma}(\vec{n}, z, \varphi)$ instead of $\mathbf{\Sigma}(\vec{n}, z, \varphi, \vec{W})$. In Online Appendix A.1, we then show that $\mathbf{\Sigma}(\vec{n}, z, \varphi)$ solves the HJB equation:[24]

$$
\begin{aligned}
(\rho + s^F)\mathbf{\Sigma}(\vec{n}, z, \varphi) = \max_{p,\{v_i,\delta_{ij},W'_{iL}(\vec{n}^+_{iL},z)\}} \Bigg\{ & \underbrace{\mathbf{S}(\vec{n}, z, \varphi)}_{\text{Flow surplus}} + \underbrace{\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} n_{ij}(\delta_{ij} + s^W_i)\Big(\mathbf{\Sigma}(\vec{n}^-_{ij}, z, \varphi) - \mathbf{\Sigma}(\vec{n}, z, \varphi)\Big)}_{\text{Worker separation and firing}} \\
& + \underbrace{\sum_{i\in\mathcal{I}} v_i \eta\Big(W'_{iL}(\vec{n}^+_{iL}, z)\Big)\Big(\mathbf{\Sigma}(\vec{n}^+_{iL}, z, \varphi) - \mathbf{\Sigma}(\vec{n}, z, \varphi)\Big)}_{\text{Hiring an FT or OEL worker}} + \underbrace{n_{FT}\, p\Big(\mathbf{\Sigma}(\vec{n}^p, z, \varphi) - \mathbf{\Sigma}(\vec{n}, z, \varphi)\Big)}_{\text{Promotion of FT workers}} \\
& + \underbrace{n_{OEL}\tau\Big(\mathbf{\Sigma}(\vec{n}^\tau, z, \varphi) - \mathbf{\Sigma}(\vec{n}, z, \varphi)\Big)}_{\text{Skill upgrade of OEL workers}} + \underbrace{\sum_{z'\in\mathcal{Z}} \lambda(z'|z)\Big(\mathbf{\Sigma}(\vec{n}, z', \varphi) - \mathbf{\Sigma}(\vec{n}, z, \varphi)\Big)}_{\text{Productivity shock}} \Bigg\}, \qquad (6)
\end{aligned}
$$

subject to $W'_{iL}(\vec{n}^+_{iL}, z) \geq \mathbf{U}$, $\forall i \in \mathcal{I}$, a worker participation constraint that entices workers to remain matched by promising them more utility that their outside option.[25] Equation (6) states that the joint surplus is composed of the flow surplus, plus the changes in joint surplus value due to worker separation, firing, hiring, promoting, skill upgrading, and productivity shocks, where:[26]

$$
\begin{aligned}
\mathbf{S}(\vec{n}, z, \varphi) \equiv\ & \underbrace{y(\vec{n}, z, \varphi)}_{\text{Firm's output}} + \underbrace{\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} n_{ij}(\delta_{ij} + s^W_i + s^F)\mathbf{U}}_{\text{Workers' outside options}} - \underbrace{\sum_{i\in\mathcal{I}} A_i v_i^{\varsigma}}_{\text{Recruiting costs}} \\
& - \underbrace{\xi n_{FT} p^{\vartheta}}_{\text{Promotion costs}} - \underbrace{\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} \chi n_{ij}\delta_{ij}^{\psi}}_{\text{Firing costs}} - \underbrace{\sum_{i\in\mathcal{I}} v_i \eta\Big(W'_{iL}(\vec{n}^+_{iL}, z)\Big) W'_{iL}(\vec{n}^+_{iL}, z)}_{\text{Expected value delivered to new hires}}. \qquad (7)
\end{aligned}
$$

Equipped with this formulation, we arrive at our main equivalence result (for the proof, see Online Appendix A.1):

Proposition 1 *The firm's and joint surplus problems are equivalent.*

[24]Throughout, we write the job-filling rate as $\eta(W(\cdot)) \equiv \eta(\theta(W(\cdot)))$, with $\theta(W)$ defined in equation (4).

[25]Notice that the promise-keeping constraint does not appear as a constraint in the joint surplus problem because definition (5) imposes that it holds with equality in equilibrium. See Online Appendix A.1 for the formal argument.

[26]In words, the flow surplus is the sum of the firm's flow output and worker's outside options in case of separation (first line of (7)), net of four types of costs: recruiting costs, promotion costs, firing costs, and the costs of having to deliver the promised value in case of successful hiring (in expected terms).

This result is reminiscent of other directed search models with multi-worker firms and long-term contracts (e.g. Schaal (2017)). As in those models, the firm's choices guarantee that the joint surplus is maximized because the contract space is complete, all agents have linear utilities, and these utilities are transferable. Thus, the set of optimal contracts can be found in two stages. In the first stage, we take first-order conditions of problem (6) to find the optimal recruiting intensity, job-filling rates, firing rates, and promotion rates for each job and each contract. Then, in the second stage, we find the set of promised utilities $\vec{W}$ that implement this allocation. We detail these two stages below.

Stage 1 Taking first-order conditions of problem (6) for $W'_{iL}(\vec{n}^+_{iL}, z, \varphi)$ yields:

$$\left.\frac{\partial \eta(W)}{\partial W}\right|_{W'_{iL}(\vec{n}^+_{iL},z,\varphi)} W'_{iL}(\vec{n}^+_{iL}, z, \varphi) + \eta\left(W'_{iL}(\vec{n}^+_{iL}, z, \varphi)\right) = \left(\mathbf{\Sigma}(\vec{n}^+_{iL}, z, \varphi) - \mathbf{\Sigma}(\vec{n}, z, \varphi)\right) \left.\frac{\partial \eta(W)}{\partial W}\right|_{W'_{iL}(\vec{n}^+_{iL},z,\varphi)}, \tag{8}$$

showing that a firm in state $(\vec{n}, z, \varphi)$ equates the expected marginal cost (left-hand side) to the expected marginal benefit (right-hand side) of promising value $W'_{iL}(\vec{n}^+_{iL}, z, \varphi)$ to new hires. The marginal cost is given by the increase in the joint surplus that goes to the new hires and the marginal benefit is given by the joint surplus that the new hires will produce.

The optimal recruiting, firing, and promotion rates for a firm in state $(\vec{n}, z, \varphi)$ satisfy:

$$v_i(\vec{n}, z, \varphi) = \left[\eta\left(W'_{iL}(\vec{n}^+_{iL}, z, \varphi)\right)\left(\frac{\mathbf{\Sigma}\left(\vec{n}^-_{ij}, z, \varphi\right) - \mathbf{\Sigma}(\vec{n}, z, \varphi) - W'_{iL}(\vec{n}^+_{iL}, z, \varphi)}{\varsigma A_i}\right)\right]^{\frac{1}{\varsigma-1}}, \tag{9}$$

$$\delta_{ij}(\vec{n}, z, \varphi) = \left[\frac{\mathbf{\Sigma}\left(\vec{n}^-_{ij}, z, \varphi\right) - \mathbf{\Sigma}(\vec{n}, z, \varphi) + \mathbf{U}}{\psi\chi}\right]^{\frac{1}{\psi-1}}, \tag{10}$$

$$p(\vec{n}, z, \varphi) = \left[\frac{\mathbf{\Sigma}\left(\vec{n}^p, z, \varphi\right) - \mathbf{\Sigma}(\vec{n}, z, \varphi)}{\vartheta\xi}\right]^{\frac{1}{\vartheta-1}}. \tag{11}$$

In all three cases, firms equate marginal costs to marginal gains, given by the corresponding changes in the joint surplus value. We note that, despite not having a fixed cost in production, there is still endogenous firm exit. This will happen whenever the joint surplus $\mathbf{\Sigma}(\vec{n}, z, \varphi)$ of one-worker firms is below the value of unemployment $\mathbf{U}$ of its worker, in which case δ_{ij} will be positive.[27] This situation is more likely to happen for low values of z and for less productive firm types φ.

[27] Note that the joint surplus $\mathbf{\Sigma}\left(\vec{n}^-_{ij}, z, \varphi\right)$ of losing the last worker is zero due to the free entry condition.

Stage 2 The equilibrium promised values that implement these policies can be found by ensuring that the resulting surplus is distributed across agents to maximize firm profits while keeping ex-ante promises at every point in the state space, as explained in Online Appendix A.1. In practice, this allows us to construct the whole sequence of promised values through an iterative procedure: the promised value of a firm with size vector $(n_{ij}, \vec{n}_{-(ij)})$ must coincide with the optimal upsize policy (for a type-ij worker) of a firm with size vector $(n_{ij} - 1, \vec{n}_{-(ij)})$, as well as with the optimal downsize policy of a firm with size vector $(n_{ij} + 1, \vec{n}_{-(ij)})$.

When $(n_{ij}, \vec{n}_{-(ij)}) = (1, \vec{0})$, promised values must be consistent with the optimal choices of potential entrants. These firms have no workers and perceive a value $\mathbf{J}^e$, with:

$$\rho \mathbf{J}^e = -\kappa + \sum_{\varphi \in \Phi} \pi_\varphi(\varphi)\, \widetilde{\mathbf{J}}^e(\varphi), \tag{12}$$

where $\widetilde{\mathbf{J}}^e(\varphi)$ is the expected value of entry for firms of type φ defined by:

$$\widetilde{\mathbf{J}}^e(\varphi) \equiv \sum_{z \in \mathcal{Z}} \pi_z(z) \sum_{i \in \mathcal{I}} \left[\max_{W_i} \left\{ \eta(W_i) \left(\mathbf{J}\left(\vec{n}^e_{iL}, z, \varphi, \{W_i\}\right) - \mathbf{J}^e \right) \text{ s.t. } W_i \geq \mathbf{U} \right\} \right], \tag{13}$$

where we have used the notation $\vec{n}^e_{ij} \equiv (n^e_{ij}, \vec{n}^e_{-(ij)}) = (1, \vec{0})$. Taking first-order conditions pins down the initial promise by entering firms and, by the iterative procedure outlined above, the entire sequence of promised values. We assume free entry into the labor market, i.e. we allow the aggregate measure of firms to freely adjust in equilibrium. Thus, in an equilibrium with positive firm entry, we must have $\mathbf{J}^e = 0$, which pins down the average labor market tightness of the economy.

3.2.3 Closing the Model

To complete the characterization of the equilibrium, we must determine the steady-state distribution of firms and workers. The law of motion for the measure of firms in each state, $\mathtt{f}_t(\vec{n}, z, \varphi)$, is characterized by a set of Kolmogorov forward equations, which we provide in full in Online Appendix A.2. This appendix also shows how to obtain the measure of operating firms, $\mathtt{F}$, as well as the unemployment rate, $\mathtt{U}$, in a stationary equilibrium in which $\frac{\partial}{\partial t}\mathtt{f}_t(\vec{n}, z, \varphi) = 0$.

4 Estimation

This section describes how we bring the model to the data. We start by describing our data sources and several details about the model parameterization (Sections 4.1 and 4.2). Next, we discuss our calibration strategy (Sections 4.3 and 4.4), and validate it with a global identification exercise and a set of non-targeted moments (Section 4.5). Finally, we explore some features of the calibrated economy, with a special emphasis on the sources of firm selection and static employment misallocation (Section 4.6).

4.1 Data Sources

We compute moments from the *Central de Balances* (CBI) firm-level data introduced in Section 2. We focus on a sub-sample of firms with at most 60 workers, representing 97.3% of firms from the full sample.[28] These data come at the yearly frequency. For our variables of interest (firm size, share of temporary workers, and value added per worker), we regress out the aggregate fixed effects (industry, time, province) to ensure that differences across firms do not reflect these other factors, which are not in the model. We also use aggregate data on worker flows into and out of employment by contract type from *Encuesta de Población Activa* (EPA), the Spanish labor force survey. These data come at a quarterly frequency.

4.2 Parameterization

We set the model period to one quarter to match the EPA time frequency, as the CBI data (at a yearly frequency) is used for stock variables. The productivity shock z is represented by the vector of values $\{z_1, \ldots, z_{k_z}\}$ and the k_z-by-k_z matrix of intensity rates $\{\lambda(z'|z)\}$. As this is a potentially large number of parameters, we recover them from the discretization of an Ornstein-Uhlenbeck diffusion process for idiosyncratic productivity (in logs):

$$d \ln(z_t) = -\rho_z \ln(z_t) dt + \sigma_z dB_t, \tag{14}$$

where B_t is a Wiener process, and (ρ_z, σ_z) are positive persistence and dispersion parameters.[29] Further, we choose a standard Cobb-Douglas specification for the matching function:

[28] All of the empirical facts that we established in Section 2 for the full sample also hold for this sub-sample. For instance, we still find that the share of temporary workers increases (respectively, decreases) in firm size when looking at within-firm (respectively, between-firm) variation. See Figure E.2 in the Online Appendix.

[29] Particularly, we recover the $\{\lambda(z'|z)\}$ intensity rates and $\{z_i\}_{i=1}^{k_z}$ productivity realizations from discretizing this process using the Euler-Maruyama and Tauchen (1986) methods (details in Online Appendix D.1). For the entrant firms' productivity distribution π_z we take the ergodic distribution associated with the (calibrated)

$\mathtt{M}(\mathtt{V},\mathtt{U}) = \mathtt{V}^{\gamma}\mathtt{U}^{1-\gamma}$, where $\gamma \in (0,1)$ is the matching elasticity, implying meeting rates $\mu(\theta) = \theta^{\gamma}$ for the worker, and $\eta(\theta) = \theta^{\gamma-1}$ for the firm. This functional form leads to convenient analytical representations for the promised value and the job-filling rate (see Online Appendix A.3). Finally, to avoid a problem of state space dimensionality, we set $(N_{OEH}, N_{OEL}, N_{FT}) = (30, 15, 15)$, which imposes an upper bound of 60 workers per firm, and we assume $k_{\varphi} = 2$ firm types and $k_z = 7$ productivity states.[30]

4.3 Externally Set Parameters

Given the parameterization described above, we have 24 parameters to calibrate. Of these, 9 parameters, namely $\mathbf{p}_{\text{ext}} \equiv \big(\rho, \gamma, \rho_z, \sigma_z, \varsigma, \psi, \vartheta, \tau, \zeta(\varphi_1)\big)$, are set externally (see Table 2). We fix the discount rate to $\rho = 0.0123$, corresponding to an annualized discount rate of $(1+\rho)^4 - 1 \approx 5\%$. For the matching elasticity, we choose $\gamma = 0.5$, a standard value in the literature (e.g. Petrongolo and Pissarides (2001)).[31] The productivity parameters (ρ_z, σ_z) introduced in equation (14) are calibrated to match a yearly autocorrelation of firm-level TFP of 0.81 and a yearly volatility of 0.34. We take these values from Ruiz-García (2021), which estimates an AR(1) process for firm-level TFP using Spanish firm-level balance sheet data from CBI, the same data source that we use in our empirical analysis. As we explain in Online Appendix D.1, these targets imply that $\rho_z = 0.0513$ and $\sigma_z = 0.1833$. We set the cost curvature parameters of the recruiting, firing, and promotion technologies to $\varsigma = \psi = \vartheta = 2$, so that the recruiting intensity, layoff, and promotion rates are linear in the corresponding net surplus changes.[32] We set the skill conversion rate to $\tau = 1/8$, such that the average duration before a skill upgrade is 8 quarters. This follows from Baley, Figueiredo, Mantovani and Sepahsalari (2023), who show that the returns to (occupational) experience are concave and almost exhausted after two years. Finally, we normalize the permanent productivity component of type-φ_1 firms to $\zeta(\varphi_1) = 1$, which comes without loss as our economy exhibits size-neutrality —only the *relative* size $\zeta(\varphi_2)/\zeta(\varphi_1)$ matters, and this ratio will be calibrated internally.

Markov chain implied by equation (14).

[30] Given these choices, the support of the state space has dimension 111,091. This makes the calculation of the equilibrium, particularly of the invariant distribution of firms, computationally challenging. Online Appendix A.2 discusses this issue and how we get around it.

[31] This value is routinely used in models estimated to U.S. data, but it has also been used for European labor markets, specifically in models of dual labor markets (e.g. Thomas (2006), Costain *et al.* (2010) and Bentolila *et al.* (2012)).

[32] We do this for symmetry with the linearity in the hiring policy when $\gamma = 0.5$ (equation (A.27) in the Online Appendix).

Table 2: Externally Set Parameters

Parameter		Value	Target/Source
ρ	Discount rate	0.0123	5% annual discount rate
γ	Matching elasticity	0.5000	Petrongolo and Pissarides (2001)
ρ_z	Mean-reversion in productivity	0.0513	Ruiz-García (2021)
σ_z	Productivity dispersion	0.1833	Ruiz-García (2021)
ς	Recruiting cost curvature	2.0000	Linear marginal cost of recruiting
ψ	Firing cost curvature	2.0000	Linear marginal gain of promoting
ϑ	Promotion cost curvature	2.0000	Linear marginal gain of firing
τ	Rate of skill upgrade	0.1250	Baley *et al.* (2023)
$\zeta(\varphi_1)$	Permanent productivity φ_1 firms	1.0000	Normalization (without loss)

Note: Set of externally calibrated parameters. See Section 4.3 for details.

4.4 Internally Set Parameters

The parameters $\mathbf{p}_{\text{int}} \equiv \left(\kappa, \zeta(\varphi_2), \nu, \omega(\varphi_1), \alpha, \omega(\varphi_2), \chi, A_{OE}, A_{FT}, s_{OE}^W, s_{FT}^W, \xi, \pi_\varphi, s^F, b\right)$, are calibrated internally. As the joint estimation of these 15 parameters and the assignment of firms to types φ is numerically unfeasible, we use the two-step procedure in Bonhomme, Lamadon and Manresa (2022). First, we assign individual firms to types by using some statistics from the data without explicitly solving the model. Then, we estimate the model parameters by the Simulated Method of Moments conditional on this assignment, using the algorithm described in Online Appendix D.2.

4.4.1 Assigning Firms to Permanent Types

The model predicts that firms of different technology types φ differ ex-post in their size, due to $\zeta(\varphi)$, and in their temporary share conditional on firm size, due to $\omega(\varphi)$. In the data, we have documented a between-firm negative correlation between employment and the temporary share (Figures 1 and E.2). We classify firms into two different technology types with the aim of reproducing this relationship. First, we regress the temporary share against dummies of firm size and unobserved firm fixed effects and we keep the estimated firm fixed effects. These can be thought of as capturing the “permanent temporary share” of each firm. Second, we take the time series average of firm size for each firm. This can be thought of as the “permanent size” of each firm. Third, we group all firms into 50 groups (2% of the population each) based on the “permanent temporary share”, from smallest to largest. Fourth, we compute the average “permanent temporary share” and the average “permanent size” in each group. Finally, we run a k-means algorithm with these two variables across all 50 groups to create only two groups, which are our two types (φ_1, φ_2).

Table 3: Assignment of firms in the data to φ types

	Firm size (average, in # workers)	**Temporary share** (average, in %)	**Share of firms** (total, in %)
Firms of type φ_1	9.8	4.3	19.7
Firms of type φ_2	6.8	23.2	80.3
All firms	7.4	19.5	100.0

Note: Results from classifying firms in the data into the two permanent types of the model. See Section 4.4.1 for details.

The results of this procedure are in Table 3. The first type (labeled φ_1), represent a smaller share of firms (19.7%), make a moderate use of temporary contracts (4.3% of the firm's employment, on average), and are larger in size (9.8 employees). The remainder share of firms (80.3%), those of type φ_2, make ample use of temporary contracts (23.2%), and are smaller (6.8 employees).

4.4.2 Targeted Moments and Model Fit

Once we have classified firms in the data into the model's types, we select various moments to identify our parameters. In this section, we discuss intuitively how each parameter is identified by each moment. Table 4 presents the parameter values and model fit. Section 4.4.3 will verify these intuitions with a formal identification exercise.

Average and relative firm size We target the average firm size and the relative size of firms of different types, which help identify κ and $\zeta(\varphi_2)/\zeta(\varphi_1)$, respectively. First, average firm size equals the ratio of the employment rate, $\mathtt{E}$, to the measure of operating firms, $\mathtt{F}$. This is mainly affected by the entry cost parameter, κ.[33] Our target for average firm size is 7.35. Second, the *relative* firm size between φ_2 and φ_1 firm types is driven by the ratio of their permanent productivities, $\zeta(\varphi_2)/\zeta(\varphi_1)$. Because $\zeta(\varphi_1)$ is normalized to 1, this pins down $\zeta(\varphi_2)$. In the data, the average firm size of φ_2 firms is a fraction 0.693 of the average size of φ_1 firms. To match this, the calibration predicts that the permanent productivity component of low-type firms is 5.6% lower than that of high-type firms, $\zeta(\varphi_2) = 0.9439$.

Productivity and temporary share by firm characteristics Next, we want the model to be consistent with the observed relationship between (i) firm productivity and firm

[33]To see this, note that κ increases the firm's expected value of entry through the free entry condition. In turn, the firm's expected value of entry decreases monotonically with the mass of firms relative to the mass of unemployed, $(\mathtt{F} + \mathtt{F}^e)$ / $(1 - \mathtt{E})$ as this ratio determines the average labor market tightness and hence it lowers the probability of succesful entry. With steady state unemployment $1 - \mathtt{E}$ pinned down by the aggregate labor market flows (which are either direct calibration targets or implied by our calibration targets, see details below), κ determines the mass of firms $\mathtt{F} + \mathtt{F}^\mathtt{e}$ and hence the average firm size, $\mathtt{E}/\mathtt{F}$.

characteristics, and (ii) temporary share and firm characteristics. These relationships should help identify the parameters in the production function, $\big(\nu, \alpha, \omega(\varphi_1), \omega(\varphi_2)\big)$.

To see this, let $n = n_H + n_L$ denote the total number of workers in a firm, our observable measure of size in the data. Using equation (2), we can write log output per worker as:

$$\ln\left(\frac{y(n_H, n_L, z, \varphi)}{n}\right) = z + \zeta(\varphi) - (1-\nu)\ln(n) + \frac{\nu}{\alpha}\ln\left(\omega(\varphi)\left(\frac{n_H}{n}\right)^{\alpha} + \big(1-\omega(\varphi)\big)\left(\frac{n_L}{n}\right)^{\alpha}\right). \tag{15}$$

If we could observe all the variables of this equation, a non-linear least squares regression could recover the degree of decreasing returns to scale ν from the partial effect of firm size on firm productivity. Moreover, we could recover the relative productivity of high and low-skilled workers by firm type, $\omega(\varphi)$, and the elasticity of substitution between the two, $\frac{1}{1-\alpha}$, from the partial effect of changes in the skill composition on firm productivity. However, in our CBI data we observe neither firm productivity z nor the share of high-skilled workers, n_H/n.

To circumvent this issue, we adopt an indirect inference strategy and consider a simplified version of the equation above, which can be estimated both in the data and in the model. In particular, we compute a second-order expansion of the last term in equation (15), we measure output per worker as the ratio of value added (VA_{it}) to employment (Emp_{it}), we proxy the skill rate with the temporary share (as they are strongly negatively correlated in the model but the former is unobserved in the data), and we send the firm temporary TFP component z to the error term. The resulting regression is:

$$\ln\left(\frac{VA_{it}}{Emp_{it}}\right) = \text{constant} + \beta_0^A\,\mathbf{1}\big[\varphi_i = \varphi_2\big] + \beta_1^A\ln(Emp_{it}) + \beta_2^A TempSh_{it} + \beta_3^A TempSh_{it}^2 + \epsilon_{it}^A, \tag{16}$$

for firm i at time t, where $\mathbf{1}[\varphi_i = \varphi_2]$ is an indicator variable for permanent firm type. We run this regression by OLS both in the model and the data, and we target the regression coefficients β_1^A (which helps us identify ν) and β_2^A (which contains identification power to jointly pin down $\omega(\varphi_1)$, $\omega(\varphi_2)$ and α). We pool firms of different types together but control for firm type by including the fixed effect $\mathbf{1}[\varphi_i = \varphi_2]$, which eliminates endogeneity concerns due to $\zeta(\varphi)$.

To obtain more identification for these parameters, next we aim to match the effect of firm size and firm type on the temporary share. To do so, we follow again an indirect inference approach and run the following OLS regression both in the model and the data:

$$TempSh_{it} = \beta_0^B\,\mathbf{1}\big[\varphi_i = \varphi_2\big] + \sum_{n=1}^{N_{\text{bins}}}\beta_n^B\,\mathbf{1}\big[Emp_{it} \in \text{SizeBin}_n\big] + \epsilon_{it}^B. \tag{17}$$

The β_0^B coefficient indicates the differential choice of temporary share by firms of different type (and same size), and it is informative about the relative productivity $\omega(\varphi_1)/\omega(\varphi_2)$ of high-skill workers between firm types. The $\{\beta_n^B\}$ coefficients, in turn, indicate the partial effect of firm size on the temporary share within a firm type, capturing the within-firm variation seen in Figure E.2.[34] In particular, we target $\beta_2^B - \beta_1^B$, which captures the intensity by which the share of temporary workers changes within firm type, on average, when firms transition from the first to the second size bin.[35] The moment $(\beta_2^B - \beta_1^B)$, together with β_2^A above, are informative about the *level* productivity of high-skill workers, say $\omega(\varphi_2)$ and the parameter α driving the elasticity of substitution between high- and low-skilled workers.

Panel A of Table E.1 in Online Appendix E shows the empirical coefficients for regression (16). We find that, conditional on type, larger firms are associated with slightly higher productivity ($\beta_1^A = 0.081$). This is the result of the positive correlation between firm size (n) and transitory firm productivity (z) in the cross-section, which dominates the direct negative effect of decreasing returns to scale that we obtain in the calibration. We also find that, conditional on type, a larger temporary share is associated with a lower firm productivity ($\beta_2^A = -0.104$), consistent with the idea that low-skilled workers, which tend to have FTCs, are less productive. Finally, we also find that low types are, on average, less productive ($\beta_0^A = -0.003$), which is not surprising given the results of our classification approach described in Table 3.

Panel B of Table E.1, in turn, shows the empirical coefficients of regression (17). The β_0^B coefficient equals 0.201 in the data, indicating that, conditional on firm size, φ_2-type firms (the less productive ones) use a 20.1 percentage points higher fraction of temporary contracts. On the other hand, the $\{\beta_n^B\}$ coefficients are monotonically increasing in size bins n, confirming that the within-firm positive relationship between firm size and temporary employment that we saw in Figures 1 and E.2 also holds when we condition on firm type instead of on individual firm fixed effects.

In Panel B of Table 4, we report the calibrated model's fit on this regression evidence. In our indirect inference approach, we obtain $\nu = 0.8565$, showing decreasing returns to scale within the typical range of empirical estimates found in the literature. Because temporary contracts are more prevalent among less skilled workers, this recovers $\omega(\varphi_1) = 0.6 > 0.4702 = \omega(\varphi_2)$. Moreover, we find $\alpha = 0.7493$, which delivers a considerably high elasticity of substitution between worker skill types, $\frac{1}{1-\alpha} = 3.99$.

[34] In practice, we build size bins using 5-worker increments. As our sample contains firms with up to 60 employees, this implies $N_{\text{bins}} = 12$ size bins: 1-5, 6-10, 11-15, and so on up to 56-60.

[35] We choose to match the gap between these coefficients because the first two size bins comprise the vast majority of firms in our data.

Worker stocks and flows Next, aiming to identify $(A_{FT}, A_{OE}, s^W_{FT}, s^W_{OE}, \xi)$, we target the employment-to-unemployment (EU) and unemployment-to-employment (UE) quarterly flow rates by contract type, as well as the average (employment-weighted) share of temporary employment, all of which we take from the Spanish labor force survey (EPA). We think of these empirical rates as resulting from the ergodic distribution of a Markov chain with 3 states (unemployed, employed under an FTC, and employed under an OEC), with 5 independent transitions.[36] This system uniquely pins down two steady-state ratios: the temporary share and the unemployment rate. We then use as targeted moments 4 of the 5 transitions (leaving the FT-to-OE rate free), and the average temporary share (leaving the unemployment rate free).

Which parameters are informed by these five targets? First, the two UE rates (for FTCs and OECs) are primarily affected by the recruiting cost parameters (A_{OE}, A_{FT}), as these act as shifters for the number of matches that take place in each market per unit time, given a market tightness. In the data, UE transitions are far more frequent to FTCs (18.77% quarterly) than to OECs (2.79%). When targeting these rates, the calibrated model predicts $A_{FT} = 0.0095$ and $A_{OE} = 2.6332$. That is, the model rationalizes the difference in UE rates by making OE hires more costly: at the same level of spending in recruiting, there are fewer search units in the OE market and hence, at the same promised value, there are fewer matches per unit of time in the OE than in the FT market. These differences in recruiting costs endogenously imply that matching efficiency is much higher in the FT labor market.[37]

Secondly, the EU rates are most directly affected by the worker exogenous separation rates, (s^W_{FT}, s^W_{OE}). In the data, transitions into unemployment are far more common in the FT market (12.85% quarterly) than in the OE market (1.38%). The calibration delivers that the separation rate of FT workers is high (equal to 0.5010, i.e. an average duration of about 6 months on the job), and that OE jobs last for about 6 and a half years on average (conditional on no endogenous separations). The exogenous separations in FTCs may be driven by voluntary quits or by regulations on the maximum duration of these contracts. The latter will be our preferred interpretation in our policy analysis of Section 5.

Finally, the promotion cost parameter ξ determines the flow of workers being promoted from FTCs to OECs. If we were to target this flow, then together with the other four flows described above, we would uniquely pin down the unemployment rate and the temporary share. To identify ξ, we instead target the average temporary share in the

[36] These transitions are: U-to-E on an FTC and vice versa, U-to-E on an OEC and vice versa, and E on an FTC to E on an OEC. Notice that as skill is unobserved in the data, this empirical model abstracts from different flow rates by worker skill type. Moreover, consistent with the model, we set the OE-to-FT flow rates to zero.

[37] Online Appendix A.5 describes how to compute aggregate matching efficiency in market $i = OE, FT$, which we denote Γ_i. In the calibrated economy, we find a large gap: $\Gamma_{FT}/\Gamma_{OE} = 81.9$.

EPA data (21.9%).[38] With these targets on worker flows, our calibrated model implies an unemployment rate of 15.1%.[39]

Other moments Four parameters remain to be calibrated: χ, $\pi_\varphi(\varphi_1)$, s^F and b. First, the firing cost shifter parameter χ is identified by the quarterly dismissal rate in Spain, which we define as the percentage of employed individuals that, from one quarter to the next, lose their job due to being fired or laid off (i.e., excluding contract expiration, voluntary quits, and other types of separations such as retirements). Using EPA data, we find that the average quarterly dismissal rate over the period 2004Q1-2018Q4 equals 0.9%, which we pick as our target.[40] Constructing the equivalent object in the model, we obtain $\chi = 1$.[41]

Second, the probability of entering as a φ_1-type, $\pi_\varphi(\varphi_1)$, is pinned down by the share of incumbent firms classified as high type in the data.[42] This yields $\pi_\varphi(\varphi_1) = 13.84\%$, lower than the 19.1% of φ_1-type incumbent firms in the stationary distribution, indicating higher survival probabilities for the high-type firms.

Third, to pin down s^F we target the entry rate of firms, which is around 7% (per year), see García-Perea, Lacuesta and Roldan-Blanco (2021). In the model, the inflow of firms equals its outflow, so we compute the firm entry rate as the ratio of entrants to the total measure of operating firms.[43] This gives $s^F = 0.0173$.

Finally, we calibrate the opportunity cost of employment, b, to match that the income flow from unemployment represents 70% of the average worker productivity, a customary target in the literature (e.g. Hall and Milgrom (2008)).

4.4.3 Global Identification Results

To validate the identification of the 15 internally estimated parameters, we run the following exercise. For each parameter-moment pair established in the text (see Table 4

[38]Recall that the EPA numbers are always slightly higher than the firm-level ones (see Figure E.1). We choose the EPA number to be consistent with the fact that our targets for the worker flow rates have been computed using these data.

[39]This is in the middle of the unemployment figures observed in Spain during our period of analysis (2004-2019). In the data, the unemployment rate obtains its lowest value in 2007 (at 8.2%) and its highest value in 2013 (at 26.1%).

[40]The EPA survey asks individuals to report the reasons for leaving their last job, giving as one of the options: “dismissal from or withdrawal of post”. We use this variable to construct our dismissal rate.

[41]To put this number in perspective, total firing costs represent 0.18% of aggregate output and give rise to 0.17% welfare losses in the calibrated economy.

[42]In the model, we compute $\mathtt{F}_{\varphi_1}/\mathtt{F}$, where $\mathtt{F}_{\varphi_1} \equiv \sum_i \sum_j \sum_z \mathtt{f}\big(\{n_{ij}\}, z, \varphi_1\big)$, and $\mathtt{f}\big(\{n_{ij}\}, z, \varphi_1\big)$ is the stationary measure of type-φ_1 firms of productivity z with n_{ij} workers of skill $j = L, H$ in contracts of type $i = FT, OE$.

[43]Formally, $\frac{\mathtt{F}^e}{\mathtt{F}} \sum_{\varphi} \pi_\varphi(\varphi) \Big\{ \sum_{z^e} \pi_z(z^e) \sum_i \eta_i \big(W'_{iL}(\vec{n}^e_{iL}, z^e, \varphi)\big) \Big\}$, where $\mathtt{F}^e$ is the measure of potential entrants.

Table 4: Internally Estimated Parameters and Model Fit

Parameter		**Value**	**Moment**	**Model**	**Data/Source**	
κ	Fixed firm entry cost	6619.4	Average firm size	7.391	7.350	CBI
$\zeta(\varphi_2)$	Permanent productivity φ_2 firms	0.9439	Relative size of φ_2 firms	0.696	0.693	CBI
ν	Returns-to-scale parameter	0.8565	β_1^A coefficient, eq. (16)	0.087	0.081	CBI
α	Substitutability b/w worker types	0.7493	β_2^A coefficient, eq. (16)	-0.101	-0.104	CBI
$\omega(\varphi_1)$	Productivity H workers, φ_1 firms	0.6000	β_0^B coefficient, eq. (17)	0.229	0.201	CBI
$\omega(\varphi_2)$	Productivity H workers, φ_2 firms	0.4702	$(\beta_2^B - \beta_1^B)$ gap, eq. (17)	0.036	0.032	CBI
χ	Firing cost shifter	1.0000	Dismissal rate (quarterly)	0.010	0.009	EPA
A_{OE}	Recruiting cost shifter (OECs)	2.6332	UE rate, OE (quarterly)	0.028	0.028	EPA
A_{FT}	Recruiting cost shifter (FTCs)	0.0095	UE rate, FT (quarterly)	0.208	0.188	EPA
s_{OE}^W	OEC destruction rate	0.0392	EU rate, OE (quarterly)	0.019	0.014	EPA
s_{FT}^W	FTC destruction rate	0.5010	EU rate, FT (quarterly)	0.128	0.129	EPA
ξ	Promotion cost shifter	0.7991	Temporary share	0.218	0.219	EPA
$\pi_\varphi(\varphi_1)$	Prob. entering as type φ_1	0.1384	Share of φ_1-type firms	0.197	0.197	CBI
s^F	Firm destruction rate	0.0173	Firm entry rate (annual)	0.072	0.070	INE
b	Employment opportunity cost	1.0076	Leisure value to output	0.728	0.700	.

Note: The model period is one quarter. This table reports the values of the parameters estimated internally. UE and EU rates in the data are averages over HP-filtered quarterly series from EPA over the period 2005Q1-2018Q4 (data before 2005 is unavailable), see Appendix A.4 for details. "Temporary share" is employment weighted. *Data sources:* "CBI" means our sub-sample from the Central de Balances Integrada data; "INE" means data from the Instituto Nacional de Estadística; "EPA" means data from the Encuesta de Población Activa.

for the summary), we allow for quasi-random variation in all remaining parameters and solve the model for each such parameter configuration.[44] As a result, for each level of the identified parameter, we obtain a whole distribution for the targeted moment. In Figure E.3 of Online Appendix E we plot, for each parameter-moment pair, the median of this distribution (black dots) and the inter-quartile range (shaded area), together with the empirical target (dashed line).

We consider that a parameter is well-identified by the moment when (i) the distribution changes across different values of the parameter; (ii) the rate of this change is high; (iii) the inter-quartile range of the moment's distribution is narrow throughout the support for the parameter; (iv) the empirical target falls within the inter-quartile range.[45] Because all the remaining parameters are not fixed but instead vary in a quasi-random fashion within a wide support, this method gives us a global view of identification. Figure E.3 shows

[44]More precisely, we use the following procedure, inspired by Daruich (2023). First, we set wide enough bounds for each parameter from the $\mathbf{p}_{\text{int}}$ parameter vector. Then, we pick quasi-random realizations from the resulting hypercube using a Sobol sequence, which successively forms finer uniform partitions of the parameter space. Finally, for each parameter combination, we solve the model and store the relevant moments. To implement this last step, we use a high-performance computer, which allows us to parallelize the numerical solution and saves us a large amount of computational time.

[45]Criterion (i) implies that the moment is globally sensitive to variation in the parameter; (ii) gives an idea of how strong this sensitivity is; (iii) measures how much other parameters matter to explain variation in the moment; and (iv) implies that the empirical target is not an outlier occurrence.

that the parameters are well-identified by their corresponding moments along most of the aforementioned criteria.

4.5 Non-Targeted Moments

We validate the calibration by confronting the model against a set of non-targeted moments. We focus on moments related to features of the cross-sectional distribution of firms against firm size, temporary employment and employment growth.[46] First, Figure E.4 in Online Appendix E shows that the model aligns well with the empirical distribution of firms and employment by firm size.

Second, we validate the model's predictions on firm employment growth.[47] Figure E.5 compares the distributions of year-on-year employment growth for total, FT and OE employment, in the data versus the model. The calibrated model correctly predicts the dispersion in all three distributions, an indication that the parameters of our idiosyncratic productivity process are well identified. The model, however, slightly underpredicts the frequency of small adjustments (near-zero growth rates) in total employment. This discrepancy is driven solely by OE employment, as the distribution of FT employment growth is well-matched.

Third, Figure E.6 shows the average temporary share by size bins, in the data and the model. The model matches the overall increase in temporary employment across size, except at the very bottom of the firm distribution.

4.6 Selection and Misallocation

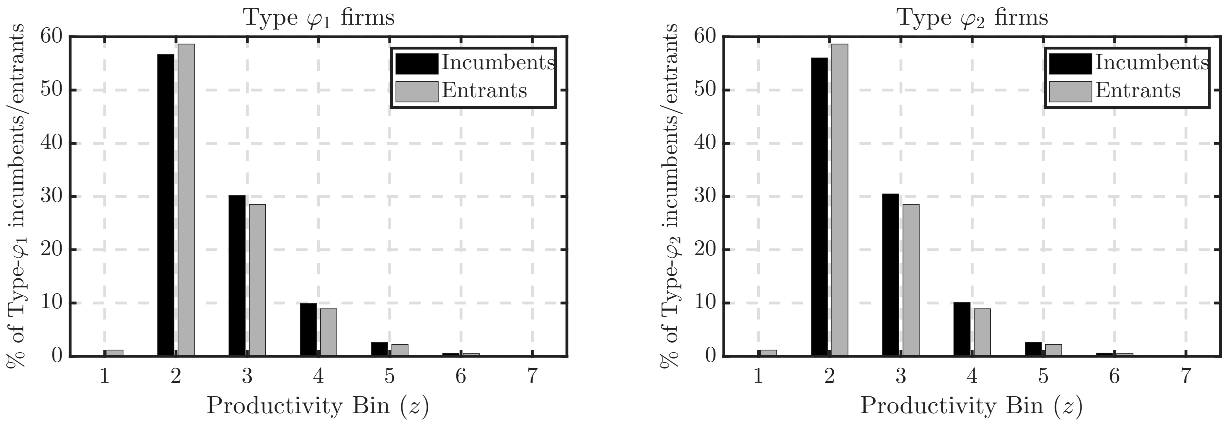
Before proceeding to our policy exercises, we study the extent of firm selection and worker misallocation generated by the search-and-matching frictions and the dual labor market structure in our calibrated model.

Selection First, as we discussed earlier, there is selection of firms *across* types. A larger share of incumbents (19.7%) is of the high type compared to entrants (13.84%). Thus, survival rates are lower among the low types. Second, there is also selection *within* types. The black bars in Figure 2 show the share of incumbent firms of each type that lie within

[46]To compute these moments in the model, we rely on simulated data. We simulate 600,000 firms (about the same number as our sample in the data), drawn from the stationary distribution in the initial period, and simulate them over 400 quarters (i.e. 100 years). We compute results by partitioning each quarter into 90 sub-periods (that is, we record outputs from each firm at a daily frequency).

[47]For any simulated firm i, year-on-year employment growth is always computed as $g_{i,t} = \frac{Emp_{i,t+1} - Emp_{i,t}}{\frac{1}{2}\left(Emp_{i,t+1} + Emp_{i,t}\right)}$, where $Emp_{i,t}$ denotes total, FT, or OE employment. This measure, which we borrow from Davis, Haltiwanger and Schuh (1998), accounts for entry ($g_{i,t} = 2$) and exit ($g_{i,t} = -2$) by treating them symmetrically.

Figure 2: Selection within Permanent Firm Types.



Notes: This figure shows the percentage of type-φ incumbents (black bars) and the percentage of type-φ entrants (gray bars) that are of each productivity level z in the stationary distribution, for φ_1-type firms (left-hand side plot) and φ_2-type firms (right-hand side plot). Within each plot, the black bars add up to 100%, and so do the gray bars.

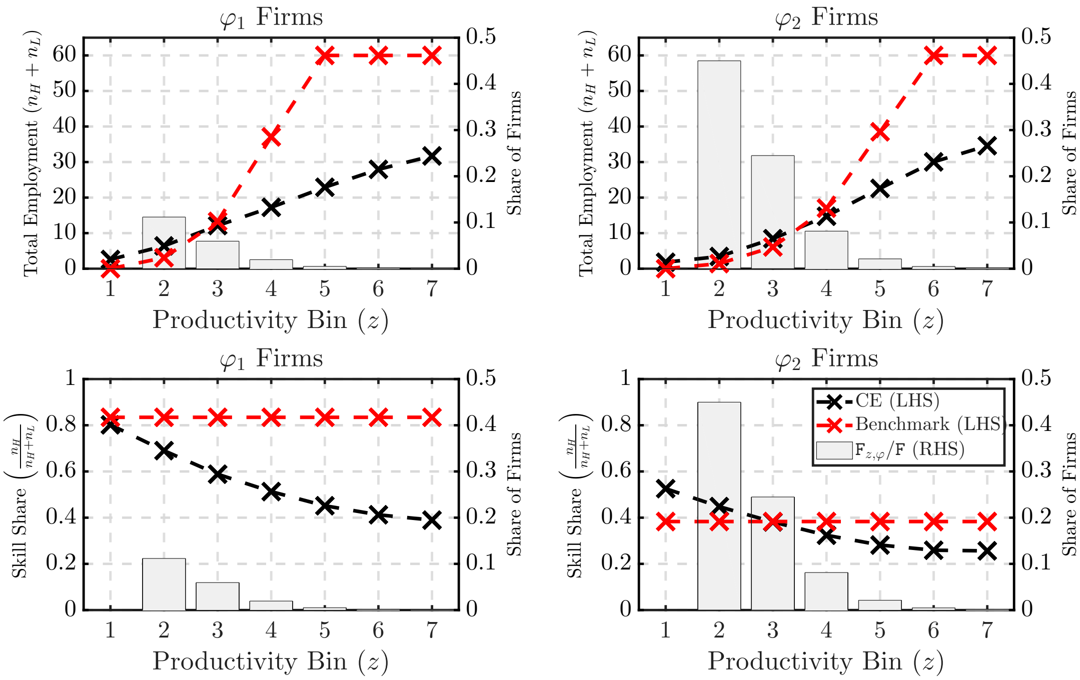
each productivity bin z in the stationary distribution. The gray bars do the same for entrants.[48] The within-type distributions are nearly identical for the two φ types. In both cases, we observe selection once again: incumbents have, on average, higher z productivity levels than entrants, suggesting that lower-z entrants have higher exit probabilities.

Misallocation Next, we describe the extent to which employment is misallocated both *within* and *across* firms. Online Appendix B.1 derives the allocation of workers that would result from (i) maximizing output without being subject to the constraints imposed by the search frictions of the market economy, but (ii) taking the allocation of firms across productivity bins (φ, z) as given. In this (henceforth "Benchmark") allocation, the marginal product of either type of labor is equalized across all firms. This means that (i) the allocation of workers and skills is identical between firms of the same productivity class (φ, z); (ii) more productive firms (higher φ or higher z) employ more workers (due to decreasing returns to scale); and (iii) the allocation of worker skills to firms is increasing in φ and independent of z due to $\omega(\varphi)$ varying with the former but not with the latter.

The competitive allocation differs from this benchmark in two ways. First, within firm productivity classes, firms in the competitive allocation display different amounts of workers and skills. Second, the average amount of workers and skills allocated across firms of the same productivity class (z, φ) in the competitive equilibrium differs from the Benchmark allocation. Both of these translate into output losses. Quantitatively, we find that our calibrated economy produces 5.32% less output per worker than the Benchmark economy with the same level of employment and the same distribution of firms, showing that search

[48]That is, within each plot, the black bars add up to 100%, and so do the gray bars.

Figure 3: Misallocation Across Productivity Classes in the Baseline Calibration.



Notes: This figure compares the allocation of employment (top row) and of the skill share (bottom row) for firms of different z's (horizontal axis), for the Competitive Equilibrium (CE) and the "Benchmark" allocation. Plots on the left-hand side (resp. right-hand side) column are for φ_1 firms (resp. φ_2 firms). For the CE, we provide the average employment within each productivity group (black dashed lines). For the Benchmark, we provide the employment level of the corresponding productivity group (red dashed line). The magnitudes for both can be read from the left-hand axes of each plot. We also plot the stationary distribution of firms across productivity levels in the CE (shaded columns within each plot, right-hand axes). Employment is capped at 60 employees, for illustrative purposes. Full details of the Benchmark solution can be found in Online Appendix B.1.

and matching frictions in Spain are quite damaging for welfare. About two-thirds of this loss is due to the misallocation between productivity groups (z, φ). For instance, while the market allocates 26.1% percent of employment to high type firms, in the Benchmark allocation we find 34.4% percent of employment being employed by these firms. The remaining one-third of the output loss is due to heterogeneity of employment and skill within the productivity groups (z, φ) component.

Behind these numbers, there is substantial heterogeneity, as more (less) productive firms receive too little (too much) employment —see the top two panels in Figure 3— and high-skill employment is too low in high-type firms —see the bottom two panels.

5 Macroeconomic Implications of Dual Labor Markets

We are ready to quantify the macroeconomic effects of dual labor markets and the impact of policies that regulate FTCs. In this section, we study three distinct policies: limiting the maximum duration of FTCs (Section 5.1), taxing the use of FTCs (Section 5.2), and banning the use of FTCs (Section 5.3).

5.1 Reducing the Maximum Duration of FTCs

A key characteristic of dual labor markets is the (relatively short) time limit that workers can spend in a firm under an FTC. When the limit is reached, firms are forced to convert the contract into an OEC or let the worker go. Reducing the maximum duration of FTCs is also a common policy tool for countries that want to limit the use of FTCs, as in the 2022 labor market reforms implemented in Spain.

We explore the consequences of changes in the legal maximum duration of FTCs in the context of our calibrated model. To do so, we solve for a series of economies in which we vary the exogenous separation rate for FT workers, s_{FT}^{W}, such that the average duration for FTCs moves between 1 month and 1 year (in the calibrated economy, the average duration is approximately 6 months). We leave all other parameters unchanged at their calibrated values, and compare across steady-state solutions. The results for this series of exercises are reported in Figure 4. Tables 5 and 6 provide exact numbers for the two policies at the extreme (1 month and 1 year average duration of FTCs).

A policy that reduces the legal duration of FTCs from the baseline duration down to 1 month achieves the intended effect of reducing the overall temporary share of the economy (see Panel (f) in Figure 4). Indeed, the temporary share declines from 21.8% to 1.5%.[49] The policy slightly increases aggregate productivity, by 0.2% (see Table 6), and comes at the expense of a 9.75% reduction in output (Panel (r)) due to the sharp increase in the unemployment rate (Panel (q)), from 15.1% to 23.5%.[50] All in all, limiting FTC duration to 1 month would reduce welfare by 2.07% (Panel (t) and last row of Table 6).[51] In what follows, we discuss these results in detail.

5.1.1 Aggregate Employment and its Composition

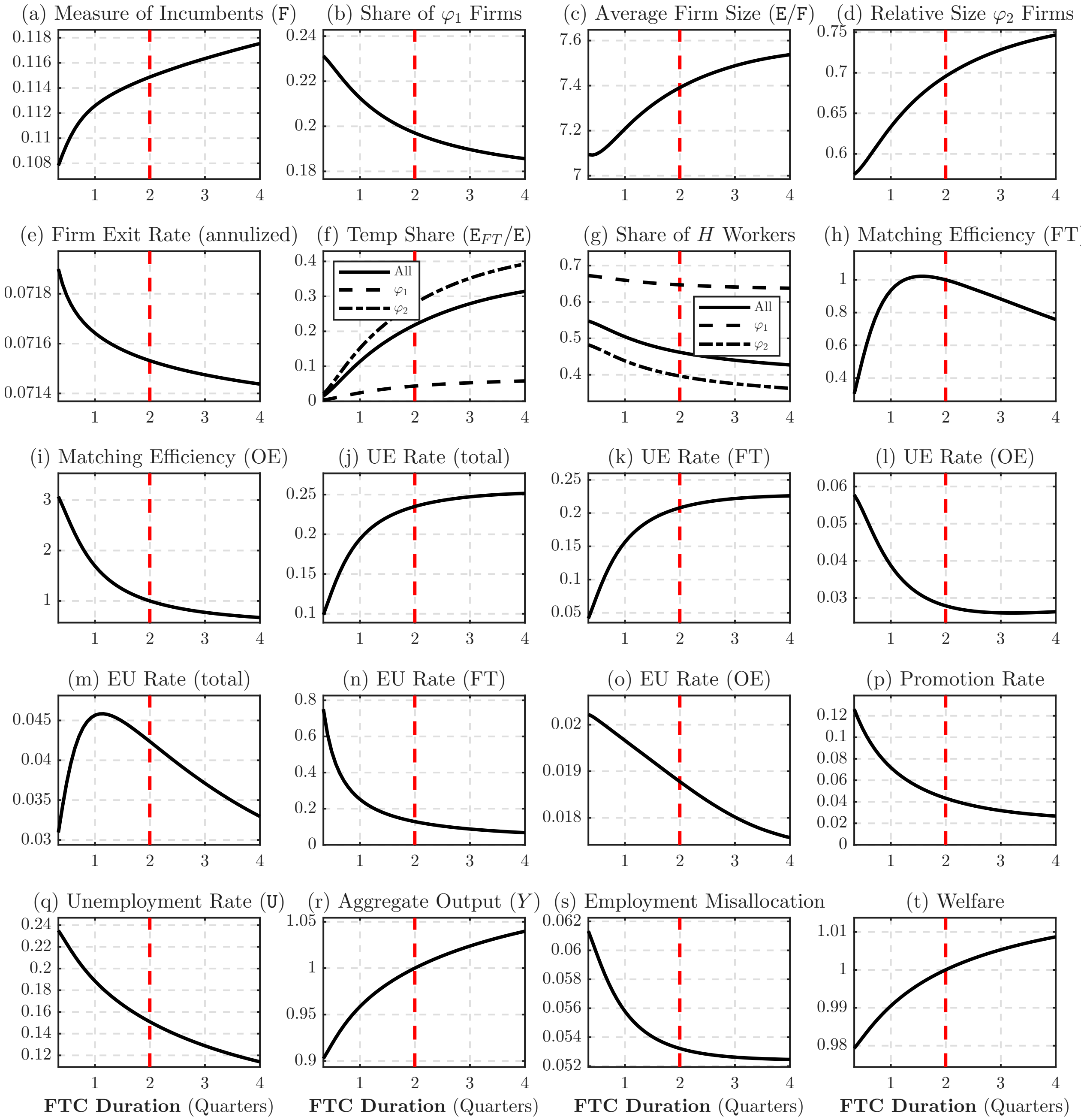
In the model, firms face a trade-off between the higher recruiting costs of workers under OECs and the costs of higher turnover of workers under FTCs, which manifests as more frequent recruitment costs, the opportunity costs of unfilled vacancies, and the loss

[49] These figures are for the employment-weighted average temporary share of the economy. The firm-weighted share declines from 21.0% to 1.7%.

[50] The dynamics of unemployment are written in equations (A.29a) to (A.29d) of the Online Appendix.

[51] To compute welfare, we use equation (A.36) in Online Appendix A.6. In words, welfare equals the present discounted value of production net of firing, recruiting, and promotion costs over all operating firms, plus the value of home production across all unemployed workers, net of the total entry costs paid by all potential entrant firms.

Figure 4: Effects of FTC duration on selected equilibrium variables.



Notes: This figure shows the effects of reducing the maximum duration of FTCs on a number of macroeconomic aggregates of interest. For all panels, the horizontal axis represents $1/s_{FT}^{W}$, and is measured in quarters. The plots show different stationary solutions of the model, keeping all parameters fixed at their baseline calibration values except for s_{FT}^{W}. The red dashed vertical line shows the expected duration in the baseline calibration. Aggregate output, matching efficiency, and welfare are all normalized to one at the baseline calibration. For the computation of EU and UE rates, see Online Appendix A.4.

of human capital. When FTCs expire more quickly, the turnover of workers with FTCs increases, and this trade-off is tilted in favor of OE hiring. As a result, firms' recruiting effort and contract value increase for OE workers —which raises the matching efficiency in this market, see Panel (i)— and decreases them in the FT market —which reduces its matching efficiency, see Panel (h).

This intuition helps explain the policy effects on aggregate worker flows. On the one hand, as firms increasingly prefer to hire through the OEC margin, the aggregate UE flow declines (Panel (j)) due to lower matching efficiency in the OEC market.[52] On the other hand, the changes in the EU flow are milder because the job destruction rate of temporary workers increases mechanically, but the share of FTCs declines. These two counteracting forces shape the aggregate EU flow, which increases for small reductions of the maximum duration of FTCs but declines for larger ones. In net, the declining job-finding rates overwhelm the decrease in job-destruction rates, leading to the increase in unemployment mentioned above (Panel (q)).

Finally, as firms prefer to employ more workers under an OEC, the share of high-skill workers in the economy increases substantially, from 46.2% in the baseline to 54.8% under the policy that reduces FTC duration (Panel (g)).

5.1.2 The Measure of Firms and its Composition

As a byproduct of the effects described above, the policy also carries important implications for the total measure of operating firms and the distribution of firm productivity.

First, with FTCs being less useful for firms, there is a reduction in the value of firm incumbency, which reduces the measure of incumbent firms, $\mathtt{F}$ (Panel (a)). As there is a decrease in $\mathtt{F}$ but the employment rate $\mathtt{E} = 1 - \mathtt{U}$ also declines, the net effect on firm size is ambiguous. Quantitatively, Panel (c) shows that firms are slightly smaller under the policy reform that reduces FTC duration.

Second, there are strong selection effects, with a higher share of firms of the more productive permanent type φ_1 (Panel (b)). This is because firms of the less productive type φ_2 rely more on FT workers, and the worsening of these contracts damages them relatively more, thereby increasing their exit risk.

[52] The decline in the aggregate job-finding rate masks an increase in the job-finding rates of the OE market (Panel (l)) and a decrease in the job-finding rates of the FT market (Panel (k)). Also, the promotion rate increases (Panel (p)).

Table 5: Effects of changes in the average duration of FTCs on macroeconomic aggregates

	(A)	(B)	(C)
	Short duration (1 month)	*Baseline* (6 months)	*Long duration* (1 year)
Measure of incumbent firms	0.108	0.115	0.118
... Share of type-φ_1 firms	23.1 %	19.7 %	18.6 %
Average firm size	7.09	7.39	7.54
... Relative size φ_2 firms	0.575	0.696	0.747
Firm exit rate (annualized)	7.19 %	7.15 %	7.14 %
Average temporary share (Employment weighted)	1.5 %	21.8 %	31.4 %
Average temporary share (Firm weighted)	1.7 %	21.0 %	31.1 %
... within φ_1 firms	0.4 %	4.0 %	5.8 %
... within φ_2 firms	2.1 %	25.2 %	39.2 %
Share of H workers	54.8 %	46.2 %	42.7 %
... within φ_1 firms	67.2 %	64.7 %	63.8 %
... within φ_2 firms	48.2 %	39.6 %	36.3 %
Matching Efficiency (FTCs)	0.73	2.40	1.82
Matching Efficiency (OECs)	0.09	0.03	0.02
UE rate (total)	9.9 %	23.5 %	25.2 %
... UE rate (FT)	4.1 %	20.8 %	22.6 %
... UE rate (OE)	5.8 %	2.8 %	2.6 %
EU rate (total)	3.1 %	4.2 %	3.3 %
... EU rate (FT)	75.1 %	12.8 %	6.7 %
... EU rate (OE)	2.0 %	1.9 %	1.8 %
Promotion rate	12.6 %	4.3 %	2.7 %
Unemployment rate	23.5 %	15.1 %	11.4 %

Notes: This table shows the effects of reducing the maximum duration of FTCs on a number of macroeconomic aggregates of interest. Column (B) corresponds to the baseline calibration; in column (C), we set $s^W_{FT} = 1/4$ so that FTCs expire on average after 1 year; in column (A) we set $s^W_{FT} = 3$, so that FTCs expire on average after 1 month. UE, EU and promotion rates are quarterly figures, whereas the firm entry rate is an annual figure. For the computation of EU and UE rates, see Online Appendix A.4. For the computation of aggregate matching efficiency by contract type, see Online Appendix A.5.

5.1.3 Aggregate Productivity

Next, we turn to the effects of FTCs on aggregate productivity. Although the overall effects of the policy are small in net terms (an increase in aggregate productivity by 0.17%), this masks large changes that roughly offset each other.

The understand these different effects, we provide a novel decomposition.[53] Let us start by defining aggregate output as

$$Y \equiv \sum_{n_H=0}^{+\infty} \sum_{n_L=0}^{+\infty} \sum_{z\in\mathcal{Z}} \sum_{\varphi\in\Phi} \Big[y(n_H, n_L, z, \varphi)\, \mathtt{f}(n_H, n_L, z, \varphi) \Big], \tag{18}$$

[53] In fact, our decomposition formula should be general to models of firm dynamics featuring decreasing returns to scale in production, firm selection, and worker misallocation.

where recall that $\mathtt{f}(n_H, n_L, z, \varphi) \geq 0$ is the measure of operating firms that are of type φ, have productivity z, and employ $n_H \equiv n_{OEH}$ high-skill workers and $n_L \equiv n_{FT} + n_{OEL}$ low-skill workers. The production function $y(\cdot)$ is given in equation (2). Our object of interest is aggregate productivity, defined as output per worker, $Y/\mathtt{E}$.

To decompose aggregate productivity, we need to introduce some new notation. First, let $n \equiv n_H + n_L$ be total firm employment. Second, let $\widehat{n} \equiv n/(\mathtt{E}/\mathtt{F})$ be total firm employment relative to the average firm size, so that if $\widehat{n} > 1$ (respectively, $\widehat{n} < 1$) the firm is larger (respectively, smaller) than the average firm in the economy. And third, let $h \equiv n_H/n$ be the skill share within the firm. Let $\widehat{\mathcal{N}}$ and $\mathcal{H}$ denote the supports of $\widehat{n}$ and h, which, because of our assumptions, are countable sets. Then, in Online Appendix B.3 we show that aggregate output per worker can be written as

$$\frac{Y}{\mathtt{E}} = \underbrace{\left(\frac{\mathtt{F}}{\mathtt{E}}\right)^{1-\nu}}_{\text{Firm size component}} \left[\sum_{z \in \mathcal{Z}} \sum_{\varphi \in \Phi} \underbrace{\frac{\mathtt{F}_{z,\varphi}}{\mathtt{F}}}_{\text{Firm selection component}} \underbrace{\left(\sum_{\widehat{n} \in \widehat{\mathcal{N}}} \sum_{h \in \mathcal{H}} y^h_{z,\varphi}(\widehat{n}, h) \mathrm{g}_{z,\varphi}(\widehat{n}, h) \right)}_{\text{Worker reallocation component}} \right]. \tag{19}$$

In these equations, $\mathtt{F}_{z,\varphi} \geq 0$ is the measure of operating firms of productivity type (z, φ), so that $\sum_z \sum_\varphi \mathtt{F}_{z,\varphi} = \mathtt{F}$; we define $y^h_{z,\varphi}(\widehat{n}, h) \equiv \widehat{n}^\nu y\,(h, 1-h, z, \varphi)$, with $y(\cdot)$ given by equation (2); and $\mathrm{g}_{z,\varphi}(\widehat{n}, h) \in (0,1)$, which is defined precisely in equation (B.4) of the Online Appendix, is the fraction of firms of productivity type (z, φ) that have relative employment $\widehat{n}$ and skill share h.

Equation (19) shows that we can decompose aggregate productivity changes into changes in three distinct components. The first term, the *firm size component*, captures productivity gains from increasing the number of firms per worker, which, through decreasing returns to scale ($\nu < 1$), increases aggregate productivity. Our quantitative results (second row in Table 6) show that the *firm size* component pushes for an increase in productivity when the duration of FTCs decreases, as the number of firms per worker increases (from 13.53 to 14.10 firms per hundred workers). In isolation, this channel would imply an increase in output per worker of 0.59%.

The second term, the *firm selection component*, captures the effects of changes in firm composition on aggregate productivity. Changes in the percentages of firms in each productivity bin lead to productivity gains if more productive firms become more abundant in the economy. This channel works through firm exit: to the extent that the policy change lowers the survival probability of the less productive firms, aggregate productivity will rise.[54] We find that the firm selection effect would yield, in isolation, a 1.44% increase in

[54] See Section 5.1.4 for a model extension where there is also selection upon entry.

Table 6: Effects of changes in average FTC duration on productivity, misallocation and welfare

	Short duration (1 month)	*Long duration* (1 year)
Change in output per worker, of which:	0.17 %	-0.35 %
(a) *Firm size channel*	0.59 %	-0.28 %
(b) *Firm selection channel*	1.44 %	-0.50 %
(c) *Reallocation channel*, of which:	-2.57 %	0.34 %
... between-firm component	-2.62 %	0.36 %
... within-firm component	3.30 %	-1.68 %
Change in output	-9.75 %	3.88 %
Output loss from misallocation (in levels)	6.13 %	5.25 %
Change in welfare	-2.07 %	0.85 %

Notes: This table shows the effect of the policy, expressed in percentage changes with respect to the baseline calibration of 6 months FTC duration (with the exception of the output loss from misallocation, which is expressed in levels). Welfare is computed as in equation (A.36), see Online Appendix A.6.

aggregate productivity in response to the policy change (third row in Table 6). Indeed, as FTC duration decreases, the new distribution of firms features a higher share of φ_1 firms, as seen in Panel (b) of Figure 4.

Cutting FTC duration would increase aggregate productivity if the firm size and selection effects were the only channels through which dual labor markets affected productivity. One final contributor to productivity, however, must be taken into account. This third channel, the *worker reallocation component*, captures the effects of employment reallocation *within* and *across* firms. Through this channel, aggregate productivity improves in response to a policy change when the relative allocation of workers across firms improves, either through a reallocation of total employment (relative to average firm size) or through a reallocation of human capital within firms. The probability mass function $g_{z,\varphi}(\widehat{n}, h)$ measures how firms are distributed in the space of relative sizes and human capital. Changes in this distribution command aggregate productivity gains or losses through employment reallocation dynamics. With the reduction in the duration of FTCs, the allocation of employment across firms worsens: the reallocation channel would, in isolation, lead to a 2.57% decrease in productivity if FTCs expired after 1 month vis-a-vis 6 months, on average (fourth row in Table 6).

As the worker reallocation channel plays a quantitatively important role in explaining the policy's effects on aggregate productivity, it is worth further decomposing it. Precisely, we split the joint probability $g_{z,\varphi}(\widehat{n}, h)$ introduced in equation (19) into the product of conditional and marginal probabilities:

$$g_{z,\varphi}(\widehat{n}, h) = g^{A}_{z,\varphi}(h|\widehat{n})\, g^{B}_{z,\varphi}(\widehat{n}) \tag{20}$$

The first term, $g^A_{z,\varphi}(h|\widehat{n})$, reflects a *within-firm* reallocation component, as it captures how the skill composition of workers changes within firms of the same productivity (z, φ) and same relative size $\widehat{n}$. The second term, $g^B_{z,\varphi}(\widehat{n})$, reflects a *between-firm* reallocation component, as it captures how the relative number of workers $\widehat{n}$ changes across firms of different productivities (z, φ). We find that the worsening in productivity due to the worker reallocation channel is solely due to the between-firm component, which pushes productivity down. Intuitively, when FTCs become of shorter duration and firms increase the share of OECs at the expense of FTCs, they lose flexibility to adjust their total number of employees in response to productivity shocks, a is a standard misallocation effect from higher firing costs (Hopenhayn and Rogerson (1993)). The within-firm component, instead, yields productivity gains from increased human capital per worker, driven by lower turnover.

Finally, we can explore how the effects of the policy are heterogeneous across firms of different productivity classes, (z, φ). We relegate the results and the discussion to Online Appendix B.2, where we emphasize heterogeneous effects on employment misallocation.

5.1.4 Model Extension: Selection Upon Entry

A quantitatively important margin to evaluate the effects of policy on aggregate productivity is the firm selection channel. In the baseline model, firm entry is fully undirected, in the sense that potential entrants know neither their type φ nor their initial productivity shock z when they pay the entry cost κ. This means that changes in the economic environment leading to relative changes in the value of entry for different types of firms do not change the composition of entrants. Thus, all selection occurs on the exit margin.

In this Section, we explore how the effects of shortening the duration of FTCs change when we allow for the selection of entrants as well as exiters. To do so, we assume that potential entrants can choose their type φ. The technology choice φ entails a flow entry cost $\kappa(\varphi) - \varepsilon_\varphi$, where $\kappa(\varphi)$ is common to all entrants and ε_φ is idiosyncratic. Finally, we assume that the common component $\kappa(\varphi)$ is known before entry, but the actual realization of ε_φ is not. This captures uncertainty about the actual costs of entry, which leads to smooth changes in the share of entrants of each type.

Under these assumptions, the ex-ante value of entry $\mathbf{J}^e$, common to all entrants, is

$$\mathbf{J}^e = \mathbb{E}\left[\max_{i\in\{1,2\}}\left\{\mathbf{J}^e(\varphi_i) - \kappa(\varphi_i) + \varepsilon_{\varphi_i}\right\}\right], \tag{21}$$

where the expectation $\mathbb{E}[\cdot]$ is taken over all possible realizations of ε_{φ_1} and ε_{φ_2}. Free entry requires that $\mathbf{J}^e = 0$. The idea is that a more productive technology yields higher value of entry $\mathbf{J}^e(\varphi)$ but it also entails a higher entry cost $\kappa(\varphi)$.

We make this problem tractable with the assumption of Extreme Value distributions for ε_{φ_1} and ε_{φ_2} with dispersion parameter σ_ε. In Online Appendix C, we show that for any chosen elasticity $1/\sigma_\varepsilon$ of entry composition $\pi_\varphi(\varphi_1)/\pi_\varphi(\varphi_2)$ to the differential value of entry $\mathbf{J}^e(\varphi_1) - \mathbf{J}^e(\varphi_2)$, there is a pair of entry costs $\big(\kappa(\varphi_1), \kappa(\varphi_2)\big)$ that delivers the exact same model outcomes as the calibrated economy with undirected entry. This is useful because it implies that, given a chosen elasticity, it is trivial to recalibrate the economy with directed entry. For this robustness exercise, we chose arbitrarily an elasticity of one, obtain the $\kappa(\varphi_1)$ and $\kappa(\varphi_2)$ that deliver the same model outcomes as the calibrated economy with undirected entry, and perform the policy reform of varying the duration of FTCs.

When we shorten the duration of FTCs to 1 month, the share of φ_1 firms increases more than in the case of undirected entry: a 16.3 ppt increase as compared to 3.4 ppt increase with undirected entry (see Table E.2 in the Online Appendix). The larger selection effect is by construction, but the exact amount depends on the chosen elasticity of entry $1/\sigma_\varepsilon$.

The consequences of a larger share of φ_1 firms in the economy are important. First, there is an overall smaller measure of incumbent firms, as φ_1 firms have a higher demand of workers and crowd out other firms by increasing average labor market tightness and lowering the value of entry for all firms. This results in average firm size increasing slightly, from 7.39 to 7.61 workers (as opposed to decreasing to 7.09 when entry is undirected).

Second, although the effect of the policy on the temporary share is not different between the two models, the EU flow declines more when entry is directed (to 2.8% instead of to 3.1%), leading to both higher human capital accumulation (57.6% of workers are high-skill under the policy with directed entry, instead of 54.8% with undirected entry) as well as to a lower increase in the unemployment rate (22.5%, instead of 23.5%).

Third, output per worker increases slightly more than in the model with undirected entry, but there are substantial changes in its different components (see Table E.3). As firms now become larger in response to the policy, the *firm size channel* contributes to a *loss* in productivity (of 0.43%, as opposed to a gain of 0.59% in the model with undirected entry). The *firm selection channel* leads to a much larger productivity gain of 6.79%, above the 1.44% gain obtained in the economy without directed entry. However, the *worker reallocation channel* now leads to productivity losses of 8.92%, much larger than the 2.57% losses in the economy without directed entry. This worsening of worker reallocation is partly due to (i) the fact that φ_1 firms, by using a lower share of FT workers, are less flexible to change size with productivity shocks, and in part also because (ii) φ_2 firms experience productivity losses as their share of skilled workers grows above their optimal size.

Finally, due to the slightly lower increase in unemployment in the economy with directed entry, the output response barely changes: while output dropped by 9.75% in the baseline, here it does so by 8.47%. Similarly, while welfare dropped by 2.07% with undirected entry, here it drops by only 1.27%. This is due to the combination of output not falling as much and savings on the recurrent recruiting costs associated with the FTCs.

5.2 Taxing the Use of FTCs

An alternative policy to limit the temporary share is to directly tax the stock of workers employed under FTCs. This type of policy is currently in place in countries such as France, Portugal, and Spain (see, e.g., Cahuc, Benghalem, Charlot, Limon and Malherbet (2016a)). We study the effect of such a policy by introducing a linear tax on the number of FT workers, so that the flow of firm net revenues becomes $y(\vec{n}, z, \varphi) - \tau_{FT} n_{FT}$. Proceeds are rebated lump sum to ensure that the tax is resource-neutral. Thus, changes in welfare arise only from changes in firm policies and the ensuing changes in the stationary distribution, not from the fact that the economy has more or fewer resources.

We set the tax rate τ_{FT} to reduce the share of FT employment as much as in the economy that limits the maximum duration of FTCs to 1 month.[55] We report the results of these two economies in Online Appendix Tables E.4 and E.5. The effect of this tax on aggregate productivity is of the same magnitude as that of reducing the maximum duration of FTCs. Firm selection is unchanged, and so is the misallocation of workers. However, the effects on unemployment, output, and welfare are slightly different. With a tax on the use of FTCs there is less job destruction and worker turnover compared to the policy that reduces the maximum duration of FTCs: the EU rate goes down to 2.2% in the economy with a tax on FTCs (becoming 13.4% for FT workers), as opposed to going down to 3.1% (and 75.1% for FT workers) in the economy that limits the maximum duration of FTCs. As a consequence, the increase in the unemployment rate is lower, from 15.1% in the baseline to 21.5% in the economy with taxes, compared with 23.5% in the economy that limits the duration of FTCs.

Because productivity hardly varies across the policies, the effects on aggregate output and welfare are dominated by the effects on total employment. Output falls by 7.36% in the economy with taxes, compared to 9.75% in the economy that limits the maximum duration of FTCs. For welfare, these figures are -1.51% and -2.07%, respectively. Thus, taxes on the stock of workers employed under FTCs are preferable over placing stricter limits to the maximum duration of these contracts, as this latter policy generates more job destruction.

[55] This implies a tax rate of $\tau_{FT} = 40\%$.

5.3 Banning FTCs

Finally, we explore the consequences of banning the use of FTCs altogether, see Column (C) of Online Appendix Tables E.4 and E.5. footnote We eliminate labor market duality by making FTCs indistinguishable from OECs. For this, we (i) set $s^W_{FT} = s^W_{OE}$ and $A_{FT} = A_{OE}$, (ii) allow FTCs to acquire skill $j = H$ at the same rate τ as OECs, and (iii) set the promotion cost shifter ξ to infinity. This implies that, as in the benchmark economy, every firm posts two vacancies at each point in time; however, unlike the benchmark economy, the two vacancies are identical and equal to the OEC vacancy of the dual markets economy. We find that output per worker increases slightly (by 0.13%), but so does unemployment (from 15.1% to 20.7%), leading to overall output and welfare losses equal to 6.53% and 1.22%, respectively. To understand the effect on productivity, recall that FTCs are useful because they give firms flexibility to adjust employment to time-varying productivity, as they yield both a lower cost of recruiting effort as well as a shorter fixed duration. At the same time, FTCs are bad for the economy because they help less productive firm types survive. Our results show that when banning FTCs, the productivity losses from the worker reallocation channel are slightly smaller than the productivity gains from the firm selection channel. The increase in unemployment is the result of job-finding rates falling more than the job-separation rates, which underscores the importance of the high matching efficiency in the FT market.

6 Conclusion

Many labor markets are characterized by a dual structure, in which firms can offer both open-ended contracts (OECs) with high termination costs and fixed-term contracts (FTCs) of short duration. Using rich administrative firm-level data for Spain (2004-2019), we document substantial heterogeneity in the use of FTCs across firms, even within narrowly defined industries. Though larger firms exhibit lower shares of temporary employment, this relationship is reversed when exploiting within-firm variation.

We build and calibrate a model of firm dynamics with search-and-matching frictions to explore the macroeconomic implications of dual labor markets. To match the micro-level facts, our calibrated model assigns the permanently larger and more productive firms a stronger return of high-skill workers. Workers accumulate firm-specific human capital on the job, and this rationalizes that these firms rely less on temporary employment in the cross-section of firms, as worker turnover damages human capital accumulation. On the other hand, within-firm variation is driven by the trade-off between higher worker stability and higher hiring costs of workers under OECs. Firms that are far from their optimal size feature a large marginal product of labor and are especially damaged by worker turnover, so they are willing to pay the higher recruiting costs of OECs.

We find that reducing the average duration of FTCs lowers the share of temporary employment, and makes the economy more productive thanks to strong firm and worker selection effects. However, the policy turns out to be ill-advised, as it increases unemployment and decreases overall welfare via the misallocation of employment across firms. Taxing firms for their use of FTCs would lead to similar results, but with lower unemployment increases and hence lower welfare losses.

All in all, our paper emphasizes that the firm side is an important dimension to consider when quantifying the aggregate implications of labor market duality, a phenomenon which is pervasive in both emerging and developed economies.

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Dual Labor Markets and the Equilibrium Distribution of Firms

Josep Pijoan-Mas and Pau Roldan-Blanco

Appendix Materials (for Online Publication Only)

A Additional Theoretical Results

A.1 Firm's and Joint Surplus Problem Equivalence

In order to show the equivalence, first we write the employed worker's and the firm's problem.

Employed Worker's Problem Given a menu of contracts $\vec{C} = \{w_{ij}, v_i, \delta_{ij}, p, W'_{ij}(\vec{n}', z') : (i,j) \in \mathcal{I} \times \mathcal{J}\}$ currently paid by the firm, the value of a worker of type (i,j) satisfies the following HJB equation:

$$
\begin{aligned}
\rho \mathbf{W}_{ij}(\vec{n}, z, \varphi; \vec{C}) \; &= \; w_{ij} + (\delta_{ij} + s_i^W + s^F)\Big(\mathbf{U} - \mathbf{W}_{ij}(\vec{n}, z, \varphi; \vec{C})\Big) \\
\textit{Same type co-worker separates:} \quad & + (n_{ij} - 1)(\delta_{ij} + s_i^W)\Big(W'_{ij}(\vec{n}_{ij}^-, z) - \mathbf{W}_{ij}(\vec{n}, z, \varphi; \vec{C})\Big) \\
\textit{Different type co-worker separates:} \quad & + \sum_{(i',j') \neq (i,j)} n_{i'j'}(\delta_{i'j'} + s_{i'}^W)\Big(W'_{ij}(\vec{n}_{i'j'}^-, z) - \mathbf{W}_{ij}(\vec{n}, z, \varphi; \vec{C})\Big) \\
\textit{Firm hires with either contract:} \quad & + \sum_{i' \in \mathcal{I}} v_{i'} \eta \Big(W'_{i'L}(\vec{n}_{i'L}^+, z)\Big)\Big(W'_{ij}(\vec{n}_{i'L}^+, z) - \mathbf{W}_{ij}(\vec{n}, z, \varphi; \vec{C})\Big) \\
\textit{FT worker is promoted:} \quad & + n_{FT}\, p \Big(W_{ij}^p(\vec{n}^p, z) - \mathbf{W}_{ij}(\vec{n}, z, \varphi; \vec{C})\Big) \\
\textit{Skill upgrade for OE low-types:} \quad & + n_{OEL} \tau \Big(W_{ij}^\tau(\vec{n}^\tau, z) - \mathbf{W}_{ij}(\vec{n}, z, \varphi; \vec{C})\Big) \\
\textit{Productivity shock:} \quad & + \sum_{z' \in \mathcal{Z}} \lambda(z'|z)\Big(W'_{ij}(\vec{n}, z') - \mathbf{W}_{ij}(\vec{n}, z, \varphi; \vec{C})\Big). \qquad \text{(A.1)}
\end{aligned}
$$

where we have defined:

$$
W_{ij}^p(\vec{n}^p, z) \equiv \begin{cases} \dfrac{1}{n_{FT}}\Big(W'_{OEL}(\vec{n}^p, z) + (n_{FT} - 1) W'_{FT}(\vec{n}^p, z)\Big) & \text{if } i = FT, \; \forall j \in \{L, H\} \\ W'_{OE,j}(\vec{n}^p, z) & \text{if } i = OE, \; \forall j \in \{L, H\} \end{cases}
$$

$$
W_{ij}^\tau(\vec{n}^\tau, z) \equiv \begin{cases} W'_{FT}(\vec{n}^\tau, z) & \text{if } i = FT, \; \forall j \in \{L, H\} \\ \dfrac{1}{n_{OEL}}\Big(W'_{OEH}(\vec{n}^\tau, z) + (n_{OEL} - 1) W'_{OEL}(\vec{n}^\tau, z)\Big) & \text{if } (i,j) = (OE, L) \\ W'_{OEH}(\vec{n}^\tau, z) & \text{if } (i,j) = (OE, H) \end{cases}
$$

Firm's Problem An operating firm of type $\varphi \in \Phi$ in state $(\vec{n}, z, \vec{W})$ must choose a menu of contracts $\vec{C}_{ij} = \{w_{ij}, v_i, \delta_{ij}, p, W'_{ij}(\vec{n}', z')\}$ for each $(i,j) \in \mathcal{I} \times \mathcal{J}$, where recall that $\vec{W} = \{W_{ij}\}$ denotes the set outstanding promises to its current workers. Let $\mathbf{J}(\vec{n}, z, \varphi, \vec{W})$ be the value of this firm. The HJB equation is:

$$
\begin{aligned}
\rho \mathbf{J}(\vec{n}, z, \varphi, \vec{W}) \;=\; \max_{\{w_{ij}, v_i, \delta_{ij}, p, W'_{ij}(\vec{n}', z')\}} \Bigg\{ & y(\vec{n}, z, \varphi) - \xi n_{FT} p^{\vartheta} + s^F \left(\mathbf{J}^e - \mathbf{J}(\vec{n}, z, \varphi, \vec{W}) \right) \\
\textit{Wage bill, firing and recruiting costs:} \qquad & + \sum_{i \in \mathcal{I}} \Bigg[\sum_{j \in \mathcal{J}} \Bigg(- w_{ij} n_{ij} - \chi n_{ij} \delta_{ij}^{\psi} - A_i v_i^{\varsigma} \\
\textit{Workers type } (i,j) \textit{ separate:} \qquad & + n_{ij} (\delta_{ij} + s_i^W) \left(\mathbf{J}(\vec{n}_{ij}^-, z, \varphi, \vec{W}'(\vec{n}_{ij}^-, z)) - \mathbf{J}(\vec{n}, z, \varphi, \vec{W}) \right) \Bigg) \\
\textit{Hiring under contract } i\textit{:} \qquad & + v_i \eta \left(W'_{iL}(\vec{n}_{iL}^+, z) \right) \left(\mathbf{J}(\vec{n}_{iL}^+, z, \varphi, \vec{W}'(\vec{n}_{iL}^+, z)) - \mathbf{J}(\vec{n}, z, \varphi, \vec{W}) \right) \Bigg] \\
\textit{FT workers promoted:} \qquad & + n_{FT} p \left(\mathbf{J}(\vec{n}^p, z, \varphi, \vec{W}'(\vec{n}^p, z)) - \mathbf{J}(\vec{n}, z, \varphi, \vec{W}) \right) \\
\textit{Skill upgrade for OE low-types:} \qquad & + n_{OEL} \tau \left(\mathbf{J}(\vec{n}^{\tau}, z, \varphi, \vec{W}'(\vec{n}^{\tau}, z)) - \mathbf{J}(\vec{n}, z, \varphi, \vec{W}) \right) \\
\textit{Productivity shock:} \qquad & + \sum_{z' \in \mathcal{Z}} \lambda(z'|z) \left(\mathbf{J}(\vec{n}, z', \varphi, \vec{W}'(\vec{n}, z')) - \mathbf{J}(\vec{n}, z, \varphi, \vec{W}) \right) \Bigg\},
\end{aligned} \tag{A.2}
$$

where $\boldsymbol{J}^e$ is a firm's value of having no workers (satisfying equation (12)). Problem (A.2) is subject to two constraints:

$$
\mathbf{W}_{ij}(\vec{n}, z, \varphi; \vec{C}) \;\geq W_{ij}, \tag{A.3a}
$$

$$
W'_{ij}(\vec{n}', z') \;\geq \mathbf{U}, \qquad \forall (\vec{n}', z'), \tag{A.3b}
$$

for all $(i,j) \in \mathcal{I} \times \mathcal{J}$. Constraint (A.3a) is a *promise-keeping* constraint: the firm must deliver an expected value to each worker (left-hand side) that is no lower than the outstanding promise (right-hand side). This constraint is in place because of the firm's initial commitment to the posted contracts. Constraint (A.3b) is a *worker-participation* constraint: for every possible future state $(\vec{n}', z')$, the value that each worker obtains cannot be lower than the outside option. This constraint is in place because workers do not commit, and must therefore be enticed to remain matched.

With these equations in place, we are now ready to prove the equivalence result (Proposition 1):

Proposition 1 *The firm's and joint surplus problems are equivalent, in the following sense:*

1. *For any set of contracts $\vec{C}$ that solves problem (A.2)-(A.3a)-(A.3b), the subset*

$$\vec{C}_\Sigma \equiv \left\{ v_i, \delta_{ij}, p, W'_{iL}(\vec{n}^+_{iL}, z) \right\}_{(i,j)\in\mathcal{I}\times\mathcal{J}} \subset \vec{C}$$

solves problem (6).

2. *Conversely, if $\vec{C}_\Sigma$ constitutes a solution to problem (6), then there exists a unique set of wages and continuation promises $\vec{C}_W \equiv \{w_{ij}, W'_{ij}(\vec{n}^-_{ij}, z), W'_{ij}(\vec{n}^p, z), W'_{ij}(\vec{n}^\tau, z), \{W'_{ij}(\vec{n}, z')\}_{z'\in\mathcal{Z}}\}_{(i,j)\in\mathcal{I}\times\mathcal{J}}$ such that $\vec{C}_W \cup \vec{C}_\Sigma$ solves problem (A.2)-(A.3a)-(A.3b).*

Proof of Proposition 1 Let $\vec{C} = \{\vec{C}_{ij}\}$, with $\vec{C}_{ij} = \left\{ \overline{w}_{ij}, \overline{v}_i, \overline{\delta}_{ij}, \overline{p}, \overline{W}'_{ij}(\vec{n}', z') \right\}$, denote a given policy. Then, we can re-write the problem of a type-φ firm in (A.2)-(A.3a)-(A.3b) as follows:

$$\mathbf{J}(\vec{n}, z, \varphi, \vec{W}) = \max_{\vec{C}} \widetilde{\mathbf{J}}\left(\vec{n}, z, \varphi, \vec{W} \;\middle|\; \vec{C}\right), \quad \text{subject to} \begin{cases} \mathbf{W}_{ij}\left(\vec{n}, z, \varphi; \vec{C}\right) \geq W_{ij}, & \forall (i,j) \\ \overline{W}'_{ij}(\vec{n}', z') \geq \mathbf{U}, & \forall (\vec{n}', z'), \ \forall (i,j) \end{cases}$$

where:

$$\begin{aligned} \widetilde{\mathbf{J}}\left(\vec{n}, z, \varphi, \vec{W} \;\middle|\; \vec{C}\right) \equiv \frac{1}{\overline{\rho}\left(\vec{n}, z, \varphi|\vec{C}\right)} \Bigg\{ & y(\vec{n}, z, \varphi) - \xi n_{FT}\overline{p}^{\vartheta} + \sum_{i\in\mathcal{I}} \Bigg[- A_i \overline{v}_i^{\varsigma} + \sum_{j\in\mathcal{J}} \Bigg(- \overline{w}_{ij} n_{ij} - \chi n_{ij} \overline{\delta}_{ij}^{\psi} \\ & + n_{ij}\left(\overline{\delta}_{ij} + s_i^W\right) \mathbf{J}\left(\vec{n}^-_{ij}, z, \varphi, \vec{W}'(\vec{n}^-_{ij}, z)\right) \Bigg) + \overline{v}_i \eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL}, z)\right) \mathbf{J}\left(\vec{n}^+_{iL}, z, \varphi, \vec{W}'(\vec{n}^+_{iL}, z)\right) \Bigg] \\ & + \overline{p} n_{FT} \mathbf{J}\left(\vec{n}^p, z, \varphi, \vec{W}'(\vec{n}^p, z)\right) + n_{OEL}\tau\, \mathbf{J}\left(\vec{n}^\tau, z, \varphi, \vec{W}'(\vec{n}^\tau, z)\right) \\ & + \sum_{z'\in\mathcal{Z}} \lambda(z'|z) \mathbf{J}\left(\vec{n}, z', \varphi, \vec{W}'(\vec{n}, z')\right) \Bigg\} \end{aligned} \tag{A.4}$$

and where we have defined:

$$\overline{\rho}\left(\vec{n}, z, \varphi|\vec{C}\right) \equiv \rho + s^F + n_{FT}\overline{p} + n_{OEL}\tau + \sum_{i\in\mathcal{I}} \left[\overline{v}_i \eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL}, z)\right) + \sum_{j\in\mathcal{J}} n_{ij}\left(\overline{\delta}_{ij} + s_i^W\right) \right]$$

as the effective discount rate of the firm. By monotonicity of preferences, if $\vec{C}$ is optimal, then the promise-keeping constraint (A.3a) must hold with equality: $\mathbf{W}_{ij}\left(\vec{n}, z, \varphi; \vec{C}\right) = W_{ij}$, $\forall (i,j) \in \mathcal{I}\times\mathcal{J}$.[56] Imposing this into equation (A.1) allows us to solve for wages:

[56] Otherwise, the firm could increase its value by offering a combination of flow and continuation payoffs to the workers that would yield lower value to them and still comply with the firm's initial promises, a contradiction with $\vec{C}$ being optimal.

$$
\begin{aligned}
\overline{w}_{ij} = \overline{\rho}\left(\vec{n}, z, \varphi \middle| \vec{C}\right) W_{ij} - \Bigg[& (\overline{\delta}_{ij} + s_i^W + s^F)\mathbf{U} + (n_{ij} - 1)\left(\overline{\delta}_{ij} + s_i^W\right)\overline{W}'_{ij}(\vec{n}^-_{ij}, z) + \sum_{(i',j')\neq(i,j)} n_{i'j'}\left(\overline{\delta}_{i'j'} + s_{i'}^W\right)\overline{W}'_{ij}(\vec{n}^-_{i'j'}, z) \\
& + n_{FT}\overline{p}\left(\mathbf{1}[i = FT]\frac{\overline{W}'_{OEL}(\vec{n}^p, z) + (n_{FT} - 1)\overline{W}'_{FT}(\vec{n}^p, z)}{n_{FT}} + \mathbf{1}[i = OE]\overline{W}'_{OE,j}(\vec{n}^p, z)\right) \\
& + n_{OEL}\tau\left(\mathbf{1}[(i,j) = (OE, L)]\frac{\overline{W}'_{OEH}(\vec{n}^\tau, z) + (n_{OEL} - 1)\overline{W}'_{OEL}(\vec{n}^\tau, z)}{n_{OEL}} + \mathbf{1}[(i,j) \neq (OE, L)]\overline{W}'_{ij}(\vec{n}^\tau, z)\right) \\
& + \sum_{i'\in\mathcal{I}} \overline{v}_{i'}\eta\left(\overline{W}'_{i'L}(\vec{n}^+_{i'L}, z)\right)\overline{W}'_{ij}(\vec{n}^+_{i'L}, z) + \sum_{z'\in\mathcal{Z}} \lambda(z'|z)\overline{W}'_{ij}(\vec{n}, z')\Bigg]
\end{aligned}
\tag{A.5}
$$

where $\mathbf{1}[\cdot]$'s are indicator functions. Define the joint surplus under policy $\vec{C}$ as:

$$
\widetilde{\mathbf{\Sigma}}\left(\vec{n}, z, \varphi, \vec{W}\middle|\vec{C}\right) \equiv \widetilde{\mathbf{J}}\left(\vec{n}, z, \varphi, \vec{W}\middle|\vec{C}\right) + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} n_{ij}W_{ij} \tag{A.6}
$$

Likewise, define the *maximized* joint surplus as:

$$
\mathbf{\Sigma}(\vec{n}, z, \varphi, \vec{W}) \equiv \max_{\vec{C}}\left\{\widetilde{\mathbf{\Sigma}}\left(\vec{n}, z, \varphi, \vec{W}\middle|\vec{C}\right), \text{ such that } \overline{W}'_{ij}(\vec{n}', z') \geq \mathbf{U},\ \forall(\vec{n}', z'),\ \forall(i,j)\right\} \tag{A.7}
$$

Plugging (A.5) inside (A.4) and collecting terms:

$$
\begin{aligned}
\underbrace{\widetilde{\mathbf{J}}\left(\vec{n}, z, \varphi, \vec{W} \;\middle|\; \vec{C}\right) + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} n_{ij}W_{ij}}_{=\,\widetilde{\mathbf{\Sigma}}\left(\vec{n},z,\varphi,\vec{W}|\vec{C}\right)} = \Bigg\{ & y(\vec{n}, z, \varphi) - \xi n_{FT}\overline{p}^{\vartheta} - \sum_{i\in\mathcal{I}} A_i\overline{v}_i\varsigma + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\left(n_{ij}(\overline{\delta}_{ij} + s_i^W + s^F)\mathbf{U} - \chi n_{ij}\overline{\delta}_{ij}^{\psi}\right) \\
& + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} n_{ij}(\overline{\delta}_{ij} + s_i^W)\underbrace{\left(\mathbf{J}(\vec{n}^-_{ij}, z, \varphi, \vec{W}'(\vec{n}^-_{ij}, z)) + (n_{ij} - 1)\overline{W}'_{ij}(\vec{n}^-_{ij}, z) + \sum_{(i',j')\neq(i,j)} n_{i'j'}\overline{W}'_{i'j'}(\vec{n}^-_{i'j'}, z)\right)}_{=\,\mathbf{\Sigma}\left(\vec{n}^-_{ij},z,\varphi,\vec{W}'(\vec{n}^-_{ij},z)\right)} \\
& + \underbrace{\sum_{i\in\mathcal{I}}\left[\sum_{j\in\mathcal{J}} n_{ij}\left(\sum_{i'\in\mathcal{I}}\overline{v}_{i'}\eta\left(\overline{W}'_{i'L}(\vec{n}^+_{i'L}, z)\right)\overline{W}'_{ij}(\vec{n}^+_{i'L}, z)\right) + \overline{v}_i\eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL}, z)\right)\mathbf{J}(\vec{n}^+_{iL}, z, \varphi, \vec{W}'(\vec{n}^+_{iL}, z))\right]}_{[*]} \\
& + \ n_{FT}\overline{p}\ \underbrace{\left(\mathbf{J}(\vec{n}^p, z, \varphi, \vec{W}'(\vec{n}^p, z)) + (n_{FT} - 1)\overline{W}'_{FT}(\vec{n}^p, z) + (n_{OEL} + 1)\overline{W}'_{OEL}(\vec{n}^p, z)\right)}_{=\,\mathbf{\Sigma}\left(\vec{n}^p,z,\varphi,\vec{W}'(\vec{n}^p,z)\right)} \\
& + \ n_{OEL}\tau\ \underbrace{\left(\mathbf{J}(\vec{n}^\tau, z, \varphi, \vec{W}'(\vec{n}^\tau, z)) + (n_{OEL} - 1)\overline{W}'_{OEL}(\vec{n}^\tau, z) + (n_{OEH} + 1)\overline{W}'_{OEH}(\vec{n}^\tau, z)\right)}_{=\,\mathbf{\Sigma}\left(\vec{n}^\tau,z,\varphi,\vec{W}'(\vec{n}^\tau,z)\right)}
\end{aligned}
$$

$$+\sum_{z'\in\mathcal{Z}}\lambda(z'|z)\underbrace{\Big(\mathbf{J}(\vec{n},z',\varphi,\vec{W}'(\vec{n},z'))+\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}n_{ij}\overline{W}'_{ij}(\vec{n},z')\Big)}_{=\mathbf{\Sigma}\left(\vec{n},z',\varphi,\vec{W}'(\vec{n},z')\right)}\Bigg\}\frac{1}{\overline{\rho}\left(\vec{n},z,\varphi|\vec{C}\right)} \tag{A.8}$$

where terms in blue follow from definitions (A.6) and (A.7). The term labeled $[*]$ can be written as:

$$\begin{aligned}
[*] &= \sum_{i\in\mathcal{I}}\left[\sum_{j\in\mathcal{J}}n_{ij}\left(\sum_{i'\in\mathcal{I}}\overline{v}_{i'}\eta\left(\overline{W}'_{i'L}(\vec{n}^+_{i'L},z)\right)\overline{W}'_{ij}(\vec{n}^+_{i'L},z)\right)+\overline{v}_i\eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL},z)\right)\mathbf{J}(\vec{n}^+_{iL},z,\varphi,\vec{W}'(\vec{n}^+_{iL},z))\right]\\
&= \sum_{i\in\mathcal{I}}\overline{v}_i\eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL},z)\right)\left[\mathbf{J}(\vec{n}^+_{iL},z,\varphi,\vec{W}'(\vec{n}^+_{iL},z))+n_{iL}\overline{W}'_{iL}(\vec{n}^+_{iL},z)+\sum_{(i',j')\neq(i,L)}n_{i'j'}\overline{W}'_{i'j'}(\vec{n}^+_{iL},z)\right]\\
&= \sum_{i\in\mathcal{I}}\overline{v}_i\eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL},z)\right)\left[\mathbf{J}(\vec{n}^+_{iL},z,\varphi,\vec{W}'(\vec{n}^+_{iL},z))+(n_{iL}+1)\overline{W}'_{iL}(\vec{n}^+_{iL},z)+\sum_{(i',j')\neq(i,L)}n_{i'j'}\overline{W}'_{i'j'}(\vec{n}^+_{iL},z)-\overline{W}'_{iL}(\vec{n}^+_{iL},z)\right]\\
&= \sum_{i\in\mathcal{I}}\overline{v}_i\eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL},z)\right)\mathbf{\Sigma}(\vec{n}^+_{iL},z,\varphi,\vec{W}'(\vec{n}^+_{iL},z))-\sum_{i\in\mathcal{I}}\overline{v}_i\eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL},z)\right)\overline{W}'_{iL}(\vec{n}^+_{iL},z)
\end{aligned}$$

Putting this back into equation (A.8), we find:

$$\begin{aligned}
\widetilde{\mathbf{\Sigma}}\left(\vec{n},z,\varphi,\vec{W}\middle|\vec{C}\right) \;=\; \frac{1}{\overline{\rho}\left(\vec{n},z,\varphi|\vec{C}\right)}\Bigg\{& y(\vec{n},z,\varphi)-\xi n_{FT}\overline{p}^{\vartheta}-\sum_{i\in\mathcal{I}}A_i\overline{v}_i^{\varsigma}+\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\Bigg(n_{ij}(\overline{\delta}_{ij}+s_i^W+s^F)\mathbf{U}-\chi n_{ij}\overline{\delta}_{ij}^{\psi}\\
&-\overline{v}_i\eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL},z)\right)\overline{W}'_{iL}(\vec{n}^+_{iL},z)+n_{ij}(\overline{\delta}_{ij}+s_i^W)\mathbf{\Sigma}(\vec{n}^-_{ij},z,\varphi,\vec{W}'(\vec{n}^-_{ij},z))\\
&+\overline{v}_i\eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL},z)\right)\mathbf{\Sigma}(\vec{n}^+_{iL},z,\varphi,\vec{W}'(\vec{n}^+_{iL},z))\Bigg)+n_{FT}\overline{p}\mathbf{\Sigma}(\vec{n}^p,z,\varphi,\vec{W}'(\vec{n}^p,z))\\
&+n_{OEL}\tau\mathbf{\Sigma}(\vec{n}^{\tau},z,\varphi,\vec{W}'(\vec{n}^{\tau},z))+\sum_{z'\in\mathcal{Z}}\lambda(z'|z)\mathbf{\Sigma}(\vec{n},z',\varphi,\vec{W}'(\vec{n},z'))\Bigg\}
\end{aligned}\tag{A.9}$$

Note that the right-hand side of (A.9) is independent of $\vec{W}$ and w, so we can omit these as arguments of the $\mathbf{\Sigma}$ function in equation (A.9), and further simplify the equation into:

$$\begin{aligned}
\widetilde{\mathbf{\Sigma}}\left(\vec{n},z,\varphi\middle|\vec{C}_{\Sigma}\right) \;=\; &\frac{1}{\overline{\rho}\left(\vec{n},z,\varphi|\vec{C}\right)}\Bigg\{y(\vec{n},z,\varphi)-\xi n_{FT}\overline{p}^{\vartheta}-\sum_{i\in\mathcal{I}}A_i\overline{v}_i^{\varsigma}+\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\Bigg(n_{ij}(\overline{\delta}_{ij}+s_i^W+s^F)\mathbf{U}-\chi n_{ij}\overline{\delta}_{ij}^{\psi}\\
&-\overline{v}_i\eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL},z)\right)\overline{W}'_{iL}(\vec{n}^+_{iL},z)+n_{ij}(\overline{\delta}_{ij}+s_i^W)\mathbf{\Sigma}(\vec{n}^-_{ij},z,\varphi)+\overline{v}_i\eta\left(\overline{W}'_{iL}(\vec{n}^+_{iL},z)\right)\mathbf{\Sigma}(\vec{n}^+_{iL},z,\varphi)\Bigg)\\
&+n_{FT}\overline{p}\mathbf{\Sigma}(\vec{n}^p,z,\varphi)+n_{OEL}\tau\mathbf{\Sigma}(\vec{n}^{\tau},z,\varphi)+\sum_{z'\in\mathcal{Z}}\lambda(z'|z)\mathbf{\Sigma}(\vec{n},z',\varphi)\Bigg\}
\end{aligned}\tag{A.10}$$

Thus, out of the full set $\vec{C} = \left\{\overline{w}_{ij}, \overline{v}_i, \overline{\delta}_{ij}, \overline{p}, \overline{W}'_{ij}(\vec{n}', z')\right\}_{(i,j)\in\mathcal{I}\times\mathcal{J}}$, only the elements

$$\vec{C}_\Sigma \equiv \left\{\overline{v}_i, \overline{\delta}_{ij}, \overline{p}, \overline{W}'_{iL}(\vec{n}^+_{iL}, z)\right\}_{(i,j)\in\mathcal{I}\times\mathcal{J}} \subset \vec{C}$$

matter for the joint surplus. The optimal contract is then:

$$\vec{C}^*_\Sigma = \arg\max_{\vec{C}_\Sigma} \left\{\widetilde{\mathbf{\Sigma}}\left(\vec{n}, z, \varphi \middle| \vec{C}_\Sigma\right) \text{ s.t. } W'_{iL}(\vec{n}^+_{iL}, z) \geq \mathbf{U},\ \forall i \in \mathcal{I}\right\} \tag{A.11}$$

Wages $\{\overline{w}_{ij}\}_{(i,j)\in\mathcal{I}\times\mathcal{J}}$ are given by equation (A.5), while the remaining continuation values are chosen to be consistent with the solution to problem (A.11). Summing up: by expressing the firm's problem in terms of continuation promises, we have shown that the optimal contract must maximize the joint surplus. Conversely, for any combination of continuation promises that maximizes the joint surplus, there is a unique wage and set of outstanding promises that maximizes the firm's value subject to the promise-keeping constraint. ■

A.2 Equilibrium Distributions of Firms and Workers

A.2.1 Preliminaries

Let $\mathtt{f}_t(\vec{n}, z, \varphi) > 0$ be the measure of type-φ firms with employment vector $\vec{n} = \{n_{ij}\} \in \mathcal{N}$ and idiosyncratic productivity $z \in \mathcal{Z}$ at time $t \in \mathbb{R}_+$. Further, let $\mathtt{F}_t \equiv \sum_{\vec{n}\in\mathcal{N}} \sum_{z\in\mathcal{Z}} \sum_{\varphi\in\Phi} \mathtt{f}_t(\vec{n}, z, \varphi)$ be the total measure of operating firms, and let $\mathtt{F}^e_t > 0$ be the measure of potential entrants. Both $\mathtt{F}_t$ and $\mathtt{F}^e_t$ are endogenous objects, and are determined in equilibrium.

Firms in state $(\vec{n}, z, \varphi)$ seek to hire new workers of type (i, L) in market segment $W'_{iL}(\vec{n}^+_{iL}, z, \varphi)$ for each contract $i \in \mathcal{I}$. Let $\theta_i(\vec{n}^+_{iL}, z, \varphi) \equiv \theta\left(W'_{iL}(\vec{n}^+_{iL}, z, \varphi)\right)$ denote the equilibrium tightness in the submarket where firms of type $(\vec{n}, z, \varphi)$ look to hire an additional (low-skill) worker under contract i. By equation (4), we have

$$\theta_i(\vec{n}^+_{iL}, z, \varphi) = \mu^{-1}\left(\frac{\rho\mathbf{U} - b}{W'_{iL}(\vec{n}^+_{iL}, z, \varphi) - \mathbf{U}}\right) \tag{A.12}$$

By definition, market tightness must adjust to guarantee that:

$$\mathtt{u}_{it}(\vec{n}^+_{iL}, z, \varphi)\theta_i(\vec{n}^+_{iL}, z, \varphi) = v_i(\vec{n}, z, \varphi)\mathtt{f}_t(\vec{n}, z, \varphi) \tag{A.13}$$

for all t, where $\mathtt{u}_{it}(\vec{n}^+_{iL}, z, \varphi)$ is the measure of unemployed workers looking to be hired in a type-i contract by a firm in state $(\vec{n}, z, \varphi)$, and $v_i(\vec{n}, z, \varphi)$ denotes this firm's recruiting effort.[57] Equation (A.13) states that, in equilibrium, the total number of vacancies created by firms in a given state equals

[57] Recall that each firm posts one vacancy of type i at each time t, so v_i can be interpreted both as the recruiting effort of an individual firm, as well as the *effective* measure of vacancies per firm.

the total number of vacancies found by workers in the corresponding submarket, or

$$\underbrace{\mu\Big(\theta_i(\vec{n}^+_{iL}, z, \varphi)\Big)}_{\substack{\text{Job-finding rate}\\ \text{per worker}}} \underbrace{\mathtt{u}_{it}(\vec{n}^+_{iL}, z, \varphi)}_{\text{\# job seekers}} = \underbrace{v_i(\vec{n}, z, \varphi)\eta\Big(\theta_i(\vec{n}^+_{iL}, z, \varphi)\Big)}_{\text{Job-filling rate per firm}} \underbrace{\mathtt{f}_t(\vec{n}, z, \varphi)}_{\text{\# firms}} \tag{A.14}$$

Next, denote by $\mathtt{e}_{ij,t}(\vec{n}, z, \varphi)$ the measure of workers of type (i, j) employed by firms of type $(\vec{n}, z, \varphi)$ at time t. The assignment of workers to firms satisfies:

$$\mathtt{e}_{ij,t}(\vec{n}, z, \varphi) = n_{ij}\mathtt{f}_t(\vec{n}, z, \varphi) \tag{A.15}$$

for every state. The unit measure of workers must be either matched with a firm or unmatched and searching. This gives us a formula for the unemployment rate, $\mathtt{U}_t = 1 - \mathtt{E}_t$, where:

$$\mathtt{E}_t = \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{z\in\mathcal{Z}}\sum_{\varphi\in\Phi} n_{ij}\mathtt{f}_t(\{n_{ij}\}, z, \varphi). \tag{A.16}$$

A.2.2 Kolmogorov Forward Equations

Next, we write down the dynamics of the distribution of firms, which satisfy a set of Kolmogorov forward equations. The law of motion for the measure of firms in state $(\vec{n}, z, \varphi)$ is given by

$$\begin{aligned}\frac{\partial \mathtt{f}_t(\vec{n}, z, \varphi)}{\partial t} + o(\vec{n}, z, \varphi)\mathtt{f}_t(\vec{n}, z, \varphi) = &\sum_{i\in\mathcal{I}} v_i(\vec{n}^-_{iL}, z, \varphi)\eta\Big(\theta_i(\vec{n}, z, \varphi)\Big)\mathtt{f}_t(\vec{n}^-_{iL}, z, \varphi)\\ &+ \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}(n_{ij}+1)\Big(\delta_{ij}(\vec{n}^+_{ij}, z, \varphi) + s^W_i\Big)\mathtt{f}_t(\vec{n}^+_{ij}, z, \varphi)\\ &+ (n_{FT}+1)p(\vec{n}^-_p, z, \varphi)\mathtt{f}_t(\vec{n}^-_p, z, \varphi)\\ &+ (n_{OEL}+1)\tau\mathtt{f}_t(\vec{n}^-_\tau, z, \varphi)\\ &+ \sum_{\hat{z}\neq z}\lambda(z|\hat{z})\mathtt{f}_t(\vec{n}, \hat{z}, \varphi),\end{aligned} \tag{A.17}$$

with $\theta_i(\cdot)$ given by equation (A.12), and

$$o(\vec{n}, z, \varphi) \equiv s^F + \sum_{i\in\mathcal{I}} v_i(\vec{n}, z, \varphi)\eta\Big(\theta_i(\vec{n}^+_{iL}, z, \varphi)\Big) + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} n_{ij}\Big(\delta_{ij}(\vec{n}, z, \varphi) + s^W_i\Big) + n_{FT}p(\vec{n}, z, \varphi) + n_{OEL}\tau + \sum_{\hat{z}\neq z}\lambda(\hat{z}|z)$$

is the outflow rate. The right-hand side of equation (A.17) gives all the inflows into state $(\vec{n}, z, \varphi)$. Inflows come from firms with $\vec{n}^-_{iL} \in \{(n_{OEH}, n_{OEL}, n_{FT}-1), (n_{OEH}, n_{OEL}-1, n_{FT})\}$ that hire a worker with contract i, firms with $\vec{n}^+_{ij} \equiv (n_{ij}+1, \vec{n}_{-(ij)})$ that fire a type-(i, j) worker, firms with $\vec{n}^-_p \equiv (n_{OEH}, n_{OEL}-1, n_{FT}+1)$ that promote an FT worker into an OEC, firms with $\vec{n}^-_\tau \equiv (n_{OEH}-1, n_{OEL}+1, n_{FT})$ for whom a low-skill OE worker has a skill upgrade, and firms at $\vec{n} = (n_{ij}, \vec{n}_{-(ij)})$ that transition into productivity z from some $\widehat{z} \neq z$.[58]

On the other hand, the dynamics of potential entrants are given by

$$\frac{\partial \mathtt{F}_t^e}{\partial t} + \mathtt{F}_t^e \sum_{\varphi\in\Phi} \sum_{z^e\in\mathcal{Z}} \pi_\varphi(\varphi)\pi_z(z^e) \sum_{i\in\mathcal{I}} \eta\Big(\theta_i(\vec{n}_{iL}^e, z^e, \varphi)\Big) = s^F \mathtt{F}_t + \sum_{\varphi\in\Phi} \sum_{z\in\mathcal{Z}} \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} \Big(\delta_{ij}(\vec{n}_{ij}^e, z, \varphi) + s_i^W\Big) \mathtt{f}_t(\vec{n}_{ij}^e, z, \varphi) \tag{A.18}$$

where, on the right-hand side, the first additive term are inflows from exiting firms and the second term are inflows from firms with $\vec{n}_{ij}^e \equiv (n_{ij}, \vec{n}_{-(ij)}) = (1, \vec{0})$ that lose their last worker. On the left-hand side, the second additive term collects outflows from successful entrants.

A.2.3 Finding the Stationary Distribution

To find the stationary distribution, we impose $\frac{\partial \mathtt{f}_t(\vec{n},z,\varphi)}{\partial t} = \frac{\partial \mathtt{F}_t^e}{\partial t} = 0$ into equations (A.17) and (A.18), and solve for the resulting system of equations. In the numerical implementation, we solve for the *share*, not the measure, of firms in each state $s \equiv (n_{OEH}, n_{OEL}, n_{FT}, z, \varphi)$. States take values in $\overline{\mathcal{N}}_{OEH} \times \overline{\mathcal{N}}_{OEL} \times \overline{\mathcal{N}}_{FT} \times \{z_1, \ldots, z_{k_z}\} \times \{\varphi_1, \varphi_2\}$, where we denote $\overline{\mathcal{N}}_{ij} \equiv \{0, 1, 2, \ldots, N_{ij}\}$, for some positive integers $\{N_{ij}\}$. Then, there is one inactive state, where $n_{OEH} = n_{OEL} = n_{FT} = 0$, which we label $s = 0$, and $S \equiv \big[(N_{OEH}+1)\cdot(N_{OEL}+1)\cdot(N_{FT}+1) - 1\big] \cdot k_z \cdot 2$ active states. Denote by $\phi_s \in [0,1]$ the share of firms in state $s = 1, \ldots, S$, such that $\sum_{s=1}^S \phi_s = 1 - \phi_0$, where $\phi_0 > 0$ is the share of potential entrant firms. Stacking all of these shares into the column vector $\vec{\phi} \equiv [\phi_0, \phi_1, \ldots, \phi_s]^\top$, we have a system of $S+1$ non-linear equations, which in matrix form reads:

$$\mathbf{G}^\top \vec{\phi} = \vec{0} \tag{A.19}$$

The object $\mathbf{G}$ is a $(S+1)$-dimensional infinitesimal generator matrix, which registers inflows in the diagonal and outflows in the off-diagonal, such that:

$$\mathbf{G} = \begin{pmatrix} -\sum_{s\neq 0} t_{0,s} & t_{0,1} & t_{0,2} & \cdots & t_{0,S} \\ t_{1,0} & -\sum_{s\neq 1} t_{1,s} & t_{1,2} & \cdots & t_{1,S} \\ t_{2,0} & t_{2,1} & -\sum_{s\neq 2} t_{2,s} & \cdots & t_{2,S} \\ \vdots & \vdots & \ddots & \vdots & \\ t_{S,0} & t_{S,1} & t_{S,2} & \cdots & -\sum_{i\neq S} t_{S,i} \end{pmatrix}$$

The transition rates $t_{s,s'}$ are built using the optimal policies, following the laws of motion stated in equations (A.17) and (A.18). To solve for $\vec{\phi}$, we write system (A.19) as $(\mathbf{G}^\top + \mathbf{I})\vec{\phi} = \vec{\phi}$, where $\mathbf{I}$ is a $(S+1)$-dimensional identity matrix. This means that $\vec{\phi}$ can be computed as the eigenvector of $\mathbf{G}^\top + \mathbf{I}$ that is associated with the unit eigenvalue. We exploit this fact to find our invariant distribution.

[58] Inflows from hiring must be multiplied by $\pi_z(z)\pi_\varphi(\varphi)$ whenever they come from successful entrants, i.e. for $(n_{iL}, n_H) = (1, 0)$, $\forall i$.

In our calibration, we have $S = 111,091$, so $\mathbf{G}$ and $\mathbf{I}$ are very large matrices. However, as the vast majority of transitions are illegal, $\mathbf{G}$ has many zero entries, so in practice we define $(\mathbf{G}, \mathbf{I})$ as sparse matrices to be able to save on computing time and memory.

Computing the whole equilibrium for the baseline calibration once takes 3 minutes and 30 seconds on a Mac Studio equipped with an Apple M3 Ultra processor, a 32-core CPU and 96 GB unified memory.

A.2.4 Aggregate Measures of Agents

Having found the invariant distribution, we make the following normalization:

$$\widetilde{\mathtt{f}}_s \equiv \frac{\phi_s}{1-\phi_0}$$

for each state $s = 1, \ldots, S$ corresponding to a point $(n_{OEH}, n_{OEL}, n_{FT}, z, \varphi)$ in the state space. In words, $\widetilde{\mathtt{f}}_s$ is the share of firms in state s relative to all operating firms, i.e. $\widetilde{\mathtt{f}}(\vec{n}, z, \varphi) \equiv \mathtt{f}(\vec{n}, z, \varphi)/\mathtt{F}$. To proceed, we use identity (A.15) to compute:

$$\widetilde{\mathtt{e}}_{ij}(\vec{n}, z, \varphi) = n_{ij}\widetilde{\mathtt{f}}(\vec{n}, z, \varphi)$$

That is, $\widetilde{\mathtt{e}}_{ij} \equiv \mathtt{e}_{ij}/\mathtt{F}$ is the measure of workers of type (i, j) employed in firms of type $(\vec{n}, z, \varphi)$, as a share of the total measure of operating firms. From this we can find $\widetilde{\mathtt{E}}_{ij} \equiv \mathtt{E}_{ij}/\mathtt{F} = \sum_{\vec{n}\in\mathcal{N}}\sum_z\sum_\varphi \widetilde{\mathtt{e}}_{ij}(\vec{n}, z, \varphi)$, i.e. the total measure of employed individuals of type (i, j) per firm, and $\widetilde{\mathtt{E}} \equiv \mathtt{E}/\mathtt{F} = \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} \widetilde{\mathtt{E}}_{ij}$, i.e. the average firm size.

To save on notation, let $\mathtt{f}^v_i(\vec{n}, z, \varphi) \equiv v_i(\vec{n}, z, \varphi)\mathtt{f}(\vec{n}, z, \varphi)$ denote the aggregate measure of effective vacancies available for contract i in a given state $(\vec{n}, z, \varphi)$. Similarly, let $\widetilde{\mathtt{f}}^v_i(\vec{n}, z, \varphi) \equiv \mathtt{f}^v_i(\vec{n}, z, \varphi)/\mathtt{F}$. Using (A.13), we know:

$$\begin{aligned}
\mathtt{u}_{OE}(n_{OEH}, n_{OEL}+1, n_{FT}, z, \varphi) &= \mathtt{F}\,\frac{\widetilde{\mathtt{f}}^v_{OE}(n_{OEH}, n_{OEL}, n_{FT}, z, \varphi)}{\theta_{OE}(n_{OEH}, n_{OEL}+1, n_{FT}, z, \varphi)} \\
\mathtt{u}_{FT}(n_{OEH}, n_{OEL}, n_{FT}+1, z, \varphi) &= \mathtt{F}\,\frac{\widetilde{\mathtt{f}}^v_{FT}(n_{OEH}, n_{OEL}, n_{FT}, z, \varphi)}{\theta_{FT}(n_{OEH}, n_{OEL}, n_{FT}+1, z, \varphi)}
\end{aligned}$$

To make progress, in the first equation we add up both sides across all $(n_{OEH}, n_{FT}, z, \varphi)$, as well as over $n_{OEL} = 1, \ldots, N_{OE}$ (i.e. omitting $n_{OEL} = 0$). Similarly, in the second equation we add across all $(n_{OEH}, n_{OEL}, z, \varphi)$, as well as over $n_{FT} = 1, \ldots, N_{FT}$ (i.e. omitting $n_{FT} = 0$). That is:

$$\sum_{n_{OEH}}\sum_{n_{OEL}\neq 0}\sum_{n_{FT}}\sum_z\sum_\varphi \mathtt{u}_{OE}(n_{OEH}, n_{OEL}+1, n_{FT}, z, \varphi) = \mathtt{F}\left(\sum_{n_{OEH}}\sum_{n_{OEL}\neq 0}\sum_{n_{FT}}\sum_z\sum_\varphi \frac{\widetilde{\mathtt{f}}^v_{OE}(n_{OEH}, n_{OEL}, n_{FT}, z, \varphi)}{\theta_{OE}(n_{OEH}, n_{OEL}+1, n_{FT}, z, \varphi)}\right) \tag{A.20}$$

$$\sum_{n_{OEH}}\sum_{n_{OEL}}\sum_{n_{FT}\neq 0}\sum_{z}\sum_{\varphi}\mathtt{u}_{FT}(n_{OEH},n_{OEL},n_{FT}+1,z,\varphi)=\mathtt{F}\left(\sum_{n_{OEH}}\sum_{n_{OEL}}\sum_{n_{FT}\neq 0}\sum_{z}\sum_{\varphi}\frac{\widetilde{\mathtt{f}}^{v}_{FT}(n_{OEH},n_{OEL},n_{FT},z,\varphi)}{\theta_{FT}(n_{OEH},n_{OEL},n_{FT}+1,z,\varphi)}\right) \tag{A.21}$$

Developing the left-hand side of (A.20) and (A.21) yields:

$$\begin{aligned}
\sum_{n_{OEH}}\sum_{n_{OEL}\neq 0}\sum_{n_{FT}}\sum_{z}\sum_{\varphi}\mathtt{u}_{OE}(n_{OEH},n_{OEL}+1,n_{FT},z,\varphi) &= \mathtt{U}_{OE}-\sum_{n_{OEH}}\sum_{n_{FT}}\sum_{z}\sum_{\varphi}\mathtt{u}_{OE}(n_{OEH},1,n_{FT},z,\varphi)\\
&=\underbrace{1-\mathtt{E}-\mathtt{U}_{FT}}_{=\mathtt{U}_{OE}}-\sum_{z}\sum_{\varphi}\left(\frac{\mathtt{F}^{e}}{\theta_{OE}(0,1,0,z,\varphi)}+\sum_{(n_{OEH},n_{FT})\neq(0,0)}\frac{\mathtt{f}^{v}_{OE}(n_{OEH},0,n_{FT},z,\varphi)}{\theta_{OE}(n_{OEH},1,n_{FT},z,\varphi)}\right)\\
&=1-\mathtt{U}_{FT}-\mathtt{F}\left[\widetilde{\mathtt{E}}+\sum_{z}\sum_{\varphi}\left(\frac{\widetilde{\mathtt{F}}^{e}}{\theta_{OE}(0,1,0,z,\varphi)}+\sum_{(n_{OEH},n_{FT})\neq(0,0)}\frac{\widetilde{\mathtt{f}}^{v}_{OE}(n_{OEH},0,n_{FT},z,\varphi)}{\theta_{OE}(n_{OEH},1,n_{FT},z,\varphi)}\right)\right]
\end{aligned}$$

and

$$\begin{aligned}
\sum_{n_{OEH}}\sum_{n_{OEL}}\sum_{n_{FT}\neq 0}\sum_{z}\sum_{\varphi}\mathtt{u}_{FT}(n_{OEH},n_{OEL},n_{FT}+1,z,\varphi) &= \mathtt{U}_{FT}-\sum_{n_{OEH}}\sum_{n_{OEL}}\sum_{z}\sum_{\varphi}\mathtt{u}_{FT}(n_{OEH},n_{OEL},1,z,\varphi)\\
&=\mathtt{U}_{FT}-\sum_{z}\sum_{\varphi}\left(\frac{\mathtt{F}^{e}}{\theta_{FT}(0,0,1,z,\varphi)}+\sum_{(n_{OEH},n_{OEL})\neq(0,0)}\frac{\mathtt{f}^{v}_{FT}(n_{OEH},n_{OEL},0,z,\varphi)}{\theta_{FT}(n_{OEH},n_{OEL},1,z,\varphi)}\right)\\
&=\mathtt{U}_{FT}-\mathtt{F}\left[\sum_{z}\sum_{\varphi}\left(\frac{\widetilde{\mathtt{F}}^{e}}{\theta_{FT}(0,0,1,z,\varphi)}+\sum_{(n_{OEH},n_{OEL})\neq(0,0)}\frac{\widetilde{\mathtt{f}}^{v}_{FT}(n_{OEH},n_{OEL},0,z,\varphi)}{\theta_{FT}(n_{OEH},n_{OEL},1,z,\varphi)}\right)\right]
\end{aligned}$$

respectively. Substituting these back into equations (A.20) and (A.21) yields:

$$\begin{aligned}
&1-\mathtt{U}_{FT}-\mathtt{F}\left[\widetilde{\mathtt{E}}+\sum_{z\in\mathcal{Z}}\sum_{\varphi\in\Phi}\left(\frac{\widetilde{\mathtt{F}}^{e}}{\theta_{OE}(0,1,0,z,\varphi)}+\sum_{(n_{OEH},n_{FT})\neq(0,0)}\frac{\widetilde{\mathtt{f}}^{v}_{OE}(n_{OEH},0,n_{FT},z,\varphi)}{\theta_{OE}(n_{OEH},1,n_{FT},z,\varphi)}\right)\right]\\
&\quad=\mathtt{F}\left(\sum_{n_{OEH}}\sum_{n_{OEL}\neq 0}\sum_{n_{FT}}\sum_{z\in\mathcal{Z}}\sum_{\varphi\in\Phi}\frac{\widetilde{\mathtt{f}}^{v}_{OE}(n_{OEH},n_{OEL},n_{FT},z,\varphi)}{\theta_{OE}(n_{OEH},n_{OEL}+1,n_{FT},z,\varphi)}\right)
\end{aligned}$$

and

$$\mathtt{U}_{FT}-\mathtt{F}\left[\sum_{z\in\mathcal{Z}}\sum_{\varphi\in\Phi}\left(\frac{\widetilde{\mathtt{F}}^{e}}{\theta_{FT}(0,0,1,z,\varphi)}+\sum_{(n_{OEH},n_{OEL})\neq(0,0)}\frac{\widetilde{\mathtt{f}}^{v}_{FT}(n_{OEH},n_{OEL},0,z,\varphi)}{\theta_{FT}(n_{OEH},n_{OEL},1,z,\varphi)}\right)\right]$$

$$= \mathtt{F} \left(\sum_{n_{OEH}} \sum_{n_{OEL}} \sum_{n_{FT} \neq 0} \sum_{z \in \mathcal{Z}} \sum_{\varphi \in \Phi} \frac{\widetilde{\mathtt{f}}^{v}_{FT}(n_{OEH}, n_{OEL}, n_{FT}, z, \varphi)}{\theta_{FT}(n_{OEH}, n_{OEL}, n_{FT} + 1, z, \varphi)} \right)$$

Solving for $\mathtt{F}$ on each equation yields:

$$\mathtt{F} = \frac{1 - \mathtt{U}_{FT}}{\widetilde{\mathtt{E}} + \widetilde{\mathtt{U}}_{OE}} \qquad \text{and} \qquad \mathtt{F} = \frac{\mathtt{U}_{FT}}{\widetilde{\mathtt{U}}_{FT}} \tag{A.22}$$

respectively, where we have defined:

$$\widetilde{\mathtt{U}}_{OE} \equiv \sum_{z \in \mathcal{Z}} \sum_{\varphi \in \Phi} \left[\frac{\widetilde{\mathtt{F}}^{e}}{\theta_{OE}(0,1,0,z,\varphi)} + \sum_{(n_{OEH}, n_{FT}) \neq (0,0)} \frac{\widetilde{\mathtt{f}}^{v}_{OE}(n_{OEH}, 0, n_{FT}, z, \varphi)}{\theta_{OE}(n_{OEH}, 1, n_{FT}, z, \varphi)} \right.$$
$$\left. + \sum_{n_{OEH}} \sum_{n_{OEL} \neq 0} \sum_{n_{FT}} \frac{\widetilde{\mathtt{f}}^{v}_{OE}(n_{OEH}, n_{OEL}, n_{FT}, z, \varphi)}{\theta_{OE}(n_{OEH}, n_{OEL} + 1, n_{FT}, z, \varphi)} \right]$$

and

$$\widetilde{\mathtt{U}}_{FT} \equiv \sum_{z \in \mathcal{Z}} \sum_{\varphi \in \Phi} \left[\frac{\widetilde{\mathtt{F}}^{e}}{\theta_{FT}(0,0,1,z,\varphi)} + \sum_{(n_{OEH}, n_{OEL}) \neq (0,0)} \frac{\widetilde{\mathtt{f}}^{v}_{FT}(n_{OEH}, n_{OEL}, 0, z, \varphi)}{\theta_{FT}(n_{OEH}, n_{OEL}, 1, z, \varphi)} \right.$$
$$\left. + \sum_{n_{OEH}} \sum_{n_{OEL}} \sum_{n_{FT} \neq 0} \frac{\widetilde{\mathtt{f}}^{v}_{FT}(n_{OEH}, n_{OEL}, n_{FT}, z, \varphi)}{\theta_{FT}(n_{OEH}, n_{OEL}, n_{FT} + 1, z, \varphi)} \right]$$

Solving for $\mathtt{U}_{FT}$ from equation (A.22) gives

$$\mathtt{U}_{FT} = \frac{\widetilde{\mathtt{U}}_{FT}}{\widetilde{\mathtt{E}} + \widetilde{\mathtt{U}}_{OE} + \widetilde{\mathtt{U}}_{FT}} \tag{A.23}$$

From this, we can finally obtain the aggregate measure of operating firms as

$$\mathtt{F} = \left(\widetilde{\mathtt{E}} + \widetilde{\mathtt{U}}_{OE} + \widetilde{\mathtt{U}}_{FT} \right)^{-1} \tag{A.24}$$

Equation (A.24) reflects a feasibility condition: the sum of the measures of all employed and unemployed workers must equal one, the size of the labor force. Once we have the measure of operating firms $\mathtt{F}$, we can compute all the remaining aggregates. Namely, (i) the total measure of potential entrants is $\mathtt{F}^{e} = \phi_0 \mathtt{F}$; (ii) the employment and unemployment rates are given by $\mathtt{E} = \widetilde{\mathtt{E}}\mathtt{F}$ and $\mathtt{U} = 1 - \mathtt{E}$; (iii) the aggregate temporary share is given by $\mathtt{E}_{FT}/\mathtt{E}$; and (iv) the aggregate share of skilled workers (or "skill share") is $\mathtt{E}_{OEH}/\mathtt{E}$.

A.3 Cobb-Douglas Matching Function

Assume $\mathtt{M}(\mathtt{V},\mathtt{U}) = \mathtt{V}^{\gamma}\mathtt{U}^{1-\gamma}$. Using equation (8), some algebra shows:

$$W'_{iL}(\vec{n}^{+}_{iL}, z, \varphi) = \gamma \mathbf{U} + (1-\gamma)\Big(\mathbf{\Sigma}(\vec{n}^{+}_{iL}, z, \varphi) - \mathbf{\Sigma}(\vec{n}, z, \varphi)\Big) \tag{A.25}$$

This expression is intuitive: the continuation promise $W'_{iL}(\vec{n}^{+}_{iL}, z)$ for a new worker under contract i is a weighted average of the worker's outside option, $\mathbf{U}$, and the marginal net joint surplus gain from the hire, $\mathbf{\Sigma}(\vec{n}^{+}_{iL}, z, \varphi) - \mathbf{\Sigma}(\vec{n}, z, \varphi)$. The object $(1-\gamma)$ gives the share of the overall gains (net of the outside option) that accrue to the new worker. On the other hand, and using the definition of joint surplus (equation (5)), the firm obtains the following change in value from hiring:

$$\begin{aligned} \mathbf{J}\Big(\vec{n}^{+}_{iL}, z, \varphi, \vec{W}'(\vec{n}^{+}_{iL}, z, \varphi)\Big) - \mathbf{J}\big(\vec{n}, z, \varphi, \vec{W}\big) \;&=\; \underbrace{\gamma\Big(\mathbf{\Sigma}(\vec{n}^{+}_{iL}, z, \varphi) - \mathbf{\Sigma}(\vec{n}, z, \varphi) - \mathbf{U}\Big)}_{\text{New surplus, shared with new hire}} \\ &\quad + \underbrace{\sum_{i' \in \mathcal{I}} \sum_{j \in \mathcal{J}} n_{i'j}\Big(W'_{i'j}(\vec{n}, z, \varphi) - W'_{i'j}(\vec{n}^{+}_{iL}, z, \varphi)\Big)}_{\text{Transfer of value between firm and pre-existing workers}} \end{aligned} \tag{A.26}$$

The firm's marginal gain in value is composed of two terms.

1. The firm absorbs the share γ of the net gain in joint surplus that is not absorbed by the new hire.

2. Since we assume that all workers within the firm that are employed under the same contract must earn the same value, there must be a transfer of value between the firm and all its pre-existing workers after hiring takes place.

The aggregate job-filling rate for type-φ firms in state $(\vec{n}, z)$ can then be written as follows:

$$\eta(\vec{n}, z, \varphi) = \left[(1-\gamma)\frac{\mathbf{\Sigma}(\vec{n}^{+}_{iL}, z, \varphi) - \mathbf{\Sigma}(\vec{n}, z, \varphi) - \mathbf{U}}{\rho \mathbf{U} - b}\right]^{\frac{1-\gamma}{\gamma}} \tag{A.27}$$

Finally, with Cobb-Douglas matching the free-entry condition (equations (12)-(13)) reads:

$$\kappa = \gamma \left(\frac{1-\gamma}{\rho \mathbf{U} - b}\right)^{\frac{1-\gamma}{\gamma}} \left[\sum_{\varphi \in \Phi} \pi_{\varphi}(\varphi) \sum_{z^e \in \mathcal{Z}} \pi_z(z^e) \sum_{i \in \mathcal{I}} \Big(\mathbf{\Sigma}(\vec{n}^{e}_{iL}, z^e, \varphi) - \mathbf{U}\Big)^{\frac{1}{\gamma}}\right] \tag{A.28}$$

A.4 Worker Flow Rates

In the data We compute the EU and UE quarterly rates by type of contract using data from the *Encuesta de Población Activa* (EPA), the Labor Force Survey compiled by the *Instituto Nacional de Estadística* (INE), the Spanish national statistical agency. The data come at the quarterly frequency for the period 2006Q1-2019Q4. Denote by $UE^{i,\text{data}}_{t,t+1}$ the U-to-E flow from quarter t to $t+1$ into a contract of type $i = OE, FT$, and similarly for $EU^{i,\text{data}}_{t,t+1}$. E-to-E flows from an FTC into an OEC are denoted by $EE^{FtoO,\text{data}}_{t,t+1}$. Labor market rates are then defined as follows:

$$\widehat{UE}^{\text{data}}_{i,t} \equiv \frac{\sum UE^{i,\text{data}}_{t,t+1}}{\sum U^{\text{data}}_t} \qquad \text{and} \qquad \widehat{EU}^{\text{data}}_{i,t} \equiv \frac{\sum EU^{i,\text{data}}_{t,t+1}}{\sum E^{i,\text{data}}_t}$$

where $\sum$ denotes the sum of sample weights for all observations in that category, $\sum U^{\text{data}}_t$ is the number of unemployed at time t, and $\sum E^{i,\text{data}}_t$ is the number of employed in contract type i at time t. Similarly, the promotion rate in the data is computed as follows:

$$\widehat{EE}^{\text{data}}_{FtoO,t} \equiv \frac{\sum EE^{FtoO,\text{data}}_{t,t+1}}{\sum E^{FT,\text{data}}_t}$$

where $E^{FT,\text{data}}_t$ is the stock of FT workers in quarter t. The values reported in Table 4 are time-series averages over the HP-filtered quarterly series $\left\{\widehat{UE}^{\text{data}}_{i,t}, \widehat{EU}^{\text{data}}_{i,t}, \widehat{EE}^{\text{data}}_{FtoO,t}\right\}^{2019Q4}_{t=2005Q2}$.

In the model As we do not have empirical flows for within-firm skill upgrades, we categorize workers into four status: high-skill and low-skill employed with an OEC, employed with an FTC, and unemployed. The following equations describe flows between these states in the model:

$$\frac{\partial \mathtt{E}_{OEH}}{\partial t} = -\lambda_{EU_{OEH}}\mathtt{E}_{OEH} + \lambda_{EE_{LH}}\mathtt{E}_{OEL} \tag{A.29a}$$

$$\frac{\partial \mathtt{E}_{OEL}}{\partial t} = -\left(\lambda_{EU_{OEL}} + \lambda_{EE_{LH}}\right)\mathtt{E}_{OEL} + \lambda_{EE_{FtoO}}\mathtt{E}_{FT} + \lambda_{UE_{OE}}\mathtt{U} \tag{A.29b}$$

$$\frac{\partial \mathtt{E}_{FT}}{\partial t} = -\left(\lambda_{EU_{FT}} + \lambda_{EE_{FtoO}}\right)\mathtt{E}_{FT} + \lambda_{UE_{FT}}\mathtt{U} \tag{A.29c}$$

$$\frac{\partial \mathtt{U}}{\partial t} = \lambda_{EU_{OEH}}\mathtt{E}_{OEH} + \lambda_{EU_{OEL}}\mathtt{E}_{OEL} + \lambda_{EU_{FT}}\mathtt{E}_{FT} - \left(\lambda_{UE_{OE}} + \lambda_{UE_{FT}}\right)\mathtt{U} \tag{A.29d}$$

where we have defined the following average intensities:

$$\lambda_{EU_{OE,j}} \equiv \frac{EU_{OE,j}}{\mathtt{E}_{OE,j}} \qquad \lambda_{EE_{FtoO}} \equiv \frac{EE_{FtoO}}{\mathtt{E}_{FT}} \qquad \lambda_{UE_{OE}} \equiv \frac{UE_{OE}}{\mathtt{U}}$$

$$\lambda_{EU_{FT}} \equiv \frac{EU_{FT}}{\mathtt{E}_{FT}} \qquad \lambda_{UE_{FT}} \equiv \frac{UE_{FT}}{\mathtt{U}} \qquad \lambda_{EE_{LH}} \equiv \frac{EE_{LH}}{\mathtt{E}_{OEL}}$$

with

$$EU_{OE,j} \equiv \sum_{\vec{n}}\sum_{z}\sum_{\varphi}\left(\delta_{OE,j}(\vec{n},z,\varphi) + s^W_{OE} + s^F\right)\mathtt{e}_{OE,j}(\vec{n},z,\varphi) \qquad EE_{FtoO} \equiv \sum_{\vec{n}}\sum_{z}\sum_{\varphi} p(\vec{n},z,\varphi)\mathtt{e}_{FT}(\vec{n},z,\varphi)$$

$$UE_{OE} \equiv \sum_{\vec{n}}\sum_{z}\sum_{\varphi}\mu\left(\theta_{OE}(\vec{n}^+_{OEL},z,\varphi)\right)\mathtt{u}_{OE}(\vec{n}^+_{OEL},z,\varphi) \qquad EU_{FT} \equiv \sum_{\vec{n}}\sum_{z}\sum_{\varphi}\left(\delta_{FT}(\vec{n},z,\varphi) + s^W_{FT} + s^F\right)\mathtt{e}_{FT}(\vec{n},z,\varphi)$$

$$UE_{FT} \equiv \sum_{\vec{n}}\sum_{z}\sum_{\varphi}\mu\left(\theta_{FT}(\vec{n}^+_{FT},z,\varphi)\right)\mathtt{u}_{FT}(\vec{n}^+_{FT},z,\varphi) \qquad EE_{LH} \equiv \sum_{\vec{n}}\sum_{z}\sum_{\varphi}\tau\mathtt{e}_{OEL}(\vec{n},z,\varphi)$$

for $j = H, L$. Note, in particular, that since all firms face the same rate of skill upgrade, we have that $\lambda_{EE_{LH}} = \tau$. We can write system (A.29a)-(A.29d) in vector-matrix form as follows:

$$\frac{\partial}{\partial t}\begin{bmatrix} \mathtt{E}_{OEH} \\ \mathtt{E}_{OEL} \\ \mathtt{E}_{FT} \\ \mathtt{U} \end{bmatrix} = \begin{pmatrix} -\lambda_{EU_{OEH}} & \lambda_{EE_{LH}} & 0 & 0 \\ 0 & -(\lambda_{EU_{OEL}} + \lambda_{EE_{LH}}) & \lambda_{EE_{FtoO}} & \lambda_{UE_{OE}} \\ 0 & 0 & -(\lambda_{EU_{FT}} + \lambda_{EE_{FtoO}}) & \lambda_{UE_{FT}} \\ \lambda_{EU_{OEH}} & \lambda_{EU_{OEL}} & \lambda_{EU_{FT}} & -(\lambda_{UE_{OE}} + \lambda_{EU_{FT}}) \end{pmatrix}\begin{bmatrix} \mathtt{E}_{OEH} \\ \mathtt{E}_{OEL} \\ \mathtt{E}_{FT} \\ \mathtt{U} \end{bmatrix}$$

Setting the right-hand side to the zero vector and solving the resulting system of linear equations will give us the stationary measures in the reduced-form model. By construction, the resulting stocks coincide exactly with the ones derived from firm-level flows in Online Appendix A.2.4.

Using these results, we can now construct flow rates. As the model is set in continuous time, we must produce discrete-time approximations in order to have numbers that can be compared to the ones from the quarterly data. For this, we compute for each contract type $i = OE, FT$:

$$\widehat{UE}_i^{\text{model}} = \frac{1 - e^{-UE_i dt}}{\mathtt{U}} \qquad \text{and} \qquad \widehat{EU}_i^{\text{model}} = \frac{1 - e^{-\sum_{j=L,H} EU_{ij} dt}}{\sum_{j=L,H} \mathtt{E}_{ij}}$$

In these ratios, in the numerator we have transformed instantaneous Poisson rates into quarterly probabilities by setting $dt = 1/4$.[59] For the overall UE and EU rates, we compute:

$$\widehat{UE}_{\text{total}}^{\text{model}} = \frac{1 - e^{-\sum_i UE_i dt}}{\mathtt{U}} \qquad \text{and} \qquad \widehat{EU}_{\text{total}}^{\text{model}} = \frac{1 - e^{-\sum_i \sum_j EU_{ij} dt}}{\sum_i \sum_j \mathtt{E}_{ij}}$$

Similarly, to obtain the promotion rate at the quarterly frequency in the model, we compute:

$$\widehat{EE}_{FtoO}^{\text{model}} = \frac{1 - e^{-EE_{FtoO} dt}}{\mathtt{E}_{FT}}$$

For estimation purposes, we treat $\widehat{UE}_i^{\text{model}}$, $\widehat{EU}_i^{\text{model}}$ and $\widehat{EE}_{FtoO}^{\text{model}}$ as the direct model counterparts of $\widehat{UE}_i^{\text{data}}$, $\widehat{EU}_i^{\text{data}}$ and $\widehat{EE}_{FtoO}^{\text{data}}$, respectively.

A.5 Aggregate Matching Function and Aggregate Matching Efficiency

This appendix shows that our model with recruiting intensity generates endogenous differences in aggregate matching efficiency between contracts.[60] Recall that a submarket is a pair (i, W), where $i \in \{FT, OE\}$ is the contract type and W is the promised utility. At any given time, a firm f posts

[59] In particular, the numerator is the probability that there is *at least* one transition (i.e. one or more transitions) within a given quarter, which we compute as the complementary probability of no transitions.

[60] The intuitions are similar to those developed in Gavazza *et al.* (2018) and Carrillo-Tudela *et al.* (2023). Relative to the former, we explore aggregate matching intensity in the context of directed search. Relative to the latter, we introduce dual labor markets.

one vacancy of each contract type i and chooses the intensity v_{if} with which to recruit into it. Assuming a Cobb-Douglas matching function, the number of matches in market (i, W) is $\mathtt{M}_i(W) \equiv (\mathtt{V}_i(W))^\gamma (\mathtt{U}_i(W))^{1-\gamma}$, where $\mathtt{U}_i(W)$ is the measure of unemployed workers seeking employment in submarket (i, W), and $\mathtt{V}_i(W) \equiv \int_{\Omega_i(W)} v_{if} \mathrm{d}f$ is the effective measure of vacancies in submarket (i, W), where $\Omega_i(W)$ is the set of firms that offer value W for contract i. The total number of matches for contract i, $\mathtt{M}_i$, can be found by aggregating $\mathtt{M}_i(W)$ across all promised values W:

$$\mathtt{M}_i \equiv \int \mathtt{M}_i(W) \mathrm{d}W = \Gamma_i \mathtt{V}_i^\gamma \mathtt{U}_i^{1-\gamma}, \tag{A.30}$$

where $\mathtt{V}_i = \mathtt{F} + \mathtt{F}^e$ is the number of *actual* vacancies posted for contract i (equal to the sum of operating firms and potential entrants, as each such firm posts exactly one vacancy of each type), $\mathtt{U}_i \equiv \int \mathtt{U}_i(W) dW$ is the total number of job seekers for contract i, and Γ_i is the *aggregate matching efficiency* for contract i, defined by

$$\Gamma_i \equiv \int \left(\frac{\mathtt{V}_i(W)}{\mathtt{V}_i} \right)^\gamma \left(\frac{\mathtt{U}_i(W)}{\mathtt{U}_i} \right)^{1-\gamma} dW. \tag{A.31}$$

In a version of the model without endogenous recruiting intensity (e.g. $v_{if} = v$), differences in aggregate matching efficiency between markets would emerge only from the fact that firms and unemployed workers do not distribute uniformly across markets (as search is directed).[61] With recruiting intensity, however, there is an added component affecting matching efficiency, stemming from underlying differences in recruiting costs across contracts, which operates over and above differences in promised values.

To see how directed search and recruiting costs both generate differences in matching efficiency, note that the *aggregate* job-finding and job-filling rates in contract i can be written as

$$\frac{\mathtt{M}_i}{\mathtt{U}_i} = \Gamma_i \Theta_i^\gamma \qquad \text{and} \qquad \frac{\mathtt{M}_i}{\mathtt{V}_i} = \Gamma_i \Theta_i^{\gamma-1}, \tag{A.32}$$

respectively, where $\Theta_i \equiv \mathtt{V}_i / \mathtt{U}_i$ is the *aggregate market tightness* for contracts of type i. Therefore, we can compute the aggregate UE rate (the fraction of the unemployed that find a job with contract i) by

$$UE_i \equiv \frac{\mathtt{M}_i}{\mathtt{U}} = \Gamma_i \Theta_i^\gamma \frac{\mathtt{U}_i}{\mathtt{U}}, \tag{A.33}$$

where $\mathtt{U} \equiv \sum_i \mathtt{U}_i$ is the total pool of unemployed workers. In the data, $UE_{FT} > UE_{OE}$. Note

$$\frac{UE_{FT}}{UE_{OE}} = \frac{\Gamma_{FT}}{\Gamma_{OE}} \left(\frac{\mathtt{U}_{FT}}{\mathtt{U}_{OE}} \right)^{1-\gamma}. \tag{A.34}$$

As OECs offer better deals ex-post, we have $\mathtt{U}_{FT} < \mathtt{U}_{OE}$. Therefore, in order to obtain $UE_{FT} > UE_{OE}$, it must be that $\Gamma_{FT} > \Gamma_{OE}$. In the calibration, this will be the result of recruiting costs being sufficiently

[61] Indeed, in such an environment, we would have $\Gamma_i = 1$ if search was random.

low for FTCs compared to OECs, for given recruiting intensity (i.e. $A_{FT} < A_{OE}$).

In the stationary equilibrium of the model, all firms in a given state $(\vec{n}, z, \varphi)$ deliver the same promised value and choose identical recruiting effort $v_i(\vec{n}, z, \varphi)$. Using (A.13), then (A.31) reads

$$\Gamma_i = \sum_{\vec{n}} \sum_{z} \sum_{\varphi} \left[\underbrace{\frac{v_i(\vec{n}, z, \varphi)}{\mathtt{V}_i}}_{\text{Relative recruiting intensity}} \left(\underbrace{\frac{\theta_i(\vec{n}, z, \varphi)}{\Theta_i}}_{\text{Relative market tightness}} \right)^{1-\gamma} \mathtt{f}(\vec{n}, z, \varphi) \right]. \tag{A.35}$$

Equation (A.35) says that the aggregate matching efficiency in contract i is determined by two components: (i) the relative recruiting intensity in each market, and (ii) the relative degree of market tightness in each market (which results from the fact that search is directed). How these translate into matching efficiency depends on the distribution of firms $\{\mathtt{f}(\vec{n}, z, \varphi)\}$ and on the matching elasticity, γ.

A.6 Aggregate Welfare

We compute welfare in the stationary equilibrium as the present discounted value of production net of firing, recruiting, and promotion costs over all operating firms, plus the value of home production b across all unemployed workers, net of the total entry costs paid by all potential entrant firms, that is:

$$\begin{aligned} \mathscr{W} = \frac{1}{\rho} \Bigg\{ & - \mathtt{F}^e \kappa + \mathtt{U} b + \sum_{\vec{n} \in \mathcal{N}} \sum_{\vec{z} \in \mathcal{Z}} \sum_{\varphi \in \Phi} \mathtt{f}(\vec{n}, z, \varphi) \times \dots \\ & \dots \times \left[y(\vec{n}, z, \varphi) - \xi n_{FT} \left[p(\vec{n}, z, \varphi) \right]^{\vartheta} - \sum_{i \in \mathcal{I}} \left(A_i \left[v_i(\vec{n}, z, \varphi) \right]^{\varsigma} + \sum_{j \in \mathcal{J}} \chi n_{ij} \left[\delta_{ij}(\vec{n}, z, \varphi) \right]^{\psi} \right) \right] \Bigg\} \end{aligned} \tag{A.36}$$

B Productivity and Misallocation

B.1 Output-Maximizing Benchmark Allocation of Workers to Firms

Consider being able to freely allocate workers to firms in order to maximize output without being constrained by the search frictions present in the competitive equilibrium, but taking as given the distribution of firms across productivity types (z, φ). In this section we show that, in such a benchmark allocation of workers, the marginal product of each type of labor is equalized across firms. Hence, the allocation of high and low skilled workers is identical across firms of the same productivity type, (z, φ).

We explore the case in which the total measure of employed workers $\mathtt{E}$ is fixed, but its split between skill types, $\mathtt{E}_L$ and $\mathtt{E}_H$ is not.[62] Moreover, $\mathtt{F}_{z,\varphi}$, the distribution of firms across productivity types (z, φ),

[62] It is also possible to solve for the allocation in which the measures of employed workers by skill type are fixed but total employment is not. These results are available upon request.

is taken as given. Then, we can obtain the first-best allocation of workers $n_H^*(z,\varphi)$ and $n_L^*(z,\varphi)$ across skill types (z,φ) as the solution to the following problem:

$$\max_{\{n_H(z,\varphi),n_L(z,\varphi)\}} \sum_{z\in\mathcal{Z}}\sum_{\varphi\in\Phi} y(n_H,n_L,z,\varphi)\mathtt{F}_{z,\varphi} \qquad \text{s.t.} \sum_{z\in\mathcal{Z}}\sum_{\varphi\in\Phi} n_H(z,\varphi)\mathtt{F}_{z,\varphi}+\sum_{z\in\mathcal{Z}}\sum_{\varphi\in\Phi} n_L(z,\varphi)\mathtt{F}_{z,\varphi}=\mathtt{E}$$

Let $\lambda \geq 0$ be the Lagrange multiplier associated to the constraint. The first-order conditions are

$$\nu y(n_H,n_L,z,\varphi)^{1-\frac{\alpha}{\nu}}\omega(\varphi)n_H^{\alpha-1}=\lambda=\nu y(n_H,n_L,z,\varphi)^{1-\frac{\alpha}{\nu}}(1-\omega(\varphi))n_L^{\alpha-1}$$

Taking the ratio of the two gives:

$$n_{HL}^*(\varphi)\equiv\frac{n_H^*(z,\varphi)}{n_L^*(z,\varphi)}=\left(\frac{\omega(\varphi)}{1-\omega(\varphi)}\right)^{\frac{1}{1-\alpha}} \tag{B.1}$$

Therefore, a firm's skill share is a function of the firm's permanent type, but not its productivity:

$$h^*(\varphi)\equiv\frac{n_H^*(z,\varphi)}{n_H^*(z,\varphi)+n_L^*(z,\varphi)}=\left(1+\frac{1}{n_{HL}^*(\varphi)}\right)^{-1}$$

Next, using the first-order conditions again, we obtain the total employment demand ($n\equiv n_H+n_L$):

$$n(z,\varphi;\lambda)=\left[\frac{\nu}{\lambda}\omega(\varphi)\Big(h^*(\varphi)\Big)^{\alpha-1}\left(\omega(\varphi)\Big(h^*(\varphi)\Big)^{\alpha}+(1-\omega(\varphi))\Big(1-h^*(\varphi)\Big)^{\alpha}\right)^{\frac{\nu}{\alpha}-1}e^{z+\zeta(\varphi)}\right]^{\frac{1}{1-\nu}} \tag{B.2}$$

To characterize $n^*(z,\varphi)$ we need to get rid of λ. To do so, we aggregate equation (B.2):

$$\mathtt{E}=\left(\frac{\nu}{\lambda}\right)^{\frac{1}{1-\nu}}\sum_{z\in\mathcal{Z}}\sum_{\varphi\in\Phi}\left[\omega(\varphi)\Big(h^*(\varphi)\Big)^{\alpha-1}\left(\omega(\varphi)\Big(h^*(\varphi)\Big)^{\alpha}+(1-\omega(\varphi))\Big(1-h^*(\varphi)\Big)^{\alpha}\right)^{\frac{\nu}{\alpha}-1}e^{z+\zeta(\varphi)}\right]^{\frac{1}{1-\nu}}\mathtt{F}_{z,\varphi}$$

Solving for $\frac{\nu}{\lambda}$ and plugging back into (B.2) gives us the total employment of a firm of type (z,φ):

$$n^*(z,\varphi)=\frac{\mathtt{E}\left[\omega(\varphi)\Big(h^*(\varphi)\Big)^{\alpha-1}\left(\omega(\varphi)\Big(h^*(\varphi)\Big)^{\alpha}+(1-\omega(\varphi))\Big(1-h^*(\varphi)\Big)^{\alpha}\right)^{\frac{\nu}{\alpha}-1}e^{z+\zeta(\varphi)}\right]^{\frac{1}{1-\nu}}}{\sum_{z'\in\mathcal{Z}}\sum_{\varphi'\in\Phi}\left[\omega(\varphi')\Big(h^*(\varphi')\Big)^{\alpha-1}\left(\omega(\varphi')\Big(h^*(\varphi')\Big)^{\alpha}+(1-\omega(\varphi'))\Big(1-h^*(\varphi')\Big)^{\alpha}\right)^{\frac{\nu}{\alpha}-1}e^{z'+\zeta(\varphi')}\right]^{\frac{1}{1-\nu}}\mathtt{F}_{z',\varphi'}}$$

B.2 Misallocation and Heterogeneous Effects in the Policy Counterfactual

Figure E.7 shows the percentage changes in employment, the skill share, and the temporary share for firms of different permanent types and productivities when moving from the baseline to the policy that limits the duration of FTCs to 1 month. When possible (i.e. for total employment and the skill share), we also show the policy response in the output-maximizing Benchmark economy (red lines).

First, we see that all low-type (φ_2) firms reduce employment. However, among them, highly productive (z_3 and above) firms reduce it more than they should, while more unproductive (z_1 and z_2) firms decrease it less than what would minimize misallocation (red lines). In a nutshell, the patterns of between-firm misallocation that we identified in the top panels of Figure 3 get exacerbated. Most high-type (φ_1) firms increase employment, while the benchmark's prescription is to decrease it.

Second, though all firms increase their share of skilled workers, the adjustment is more dramatic among low-type (φ_2) firms. However, as we saw in the bottom panels of Figure 3, it is precisely the high-type firms that are furthest from their optimal allocation of human capital. Therefore, though beneficial for the overall levels of skill in the economy, the policy fails to sufficiently reallocate high-skill workers into the right firms.

Finally, consistent with these results, all firms reduce their share of temporary employment, although the decrease is particularly pronounced among low-type and/or low-productivity firms, whose levels of temporary employment are overall much higher.

These changes in the allocation of workers across and within firms are reflected in changes in aggregate misallocation (see Table 6).

B.3 Productivity Decomposition

In order to decompose aggregate productivity into the different parts shown in equation (19), we first need to transform equation (18) in two steps.

First, note that the measure of firms in state (n_H, n_L, z, φ) can always be written as $\mathtt{f}(n_H, n_L, z, \varphi) = \mathtt{F}_{z,\varphi}\widetilde{\mathtt{f}}_{z,\varphi}(n_H, n_L)$, where $\widetilde{\mathtt{f}}_{z,\varphi}(n_H, n_L)$ denotes the share of operating firms of type (z, φ) with employment vector (n_H, n_L), and $\mathtt{F}_{z,\varphi} \equiv \sum_{n_H} \sum_{n_L} \mathtt{f}(n_H, n_L, z, \varphi)$ is the measure of firms in state (z, φ), so that $\sum_z \sum_\varphi \mathtt{F}_{z,\varphi} = \mathtt{F}$. Then, we can write equation (18) as:

$$\frac{Y}{\mathtt{E}} = \frac{\mathtt{F}}{\mathtt{E}} \left[\sum_{z \in \mathcal{Z}} \sum_{\varphi \in \Phi} \frac{\mathtt{F}_{z,\varphi}}{\mathtt{F}} \left(\sum_{n_H=0}^{+\infty} \sum_{n_L=0}^{+\infty} y(n_H, n_L, z, \varphi) \widetilde{\mathtt{f}}_{z,\varphi}(n_H, n_L) \right) \right]$$

Second, given that the production function $y(n_H, n_L, z, \varphi)$ is homogeneous of degree ν in n_H and n_L, we can rewrite it as a function of total employment, $n \equiv n_H + n_L$, and the skill share in the firm, $h \equiv n_H/n$, so that

$$y(n_H, n_L, z, \varphi) = \underbrace{n^{\nu} y(h, 1-h, z, \varphi)}_{\equiv y^h(n,h,z,\varphi)}$$

Notice that, because of the discreteness of the state space, h takes values in the subset of rational numbers contained in the unit interval: if $n = 1$, then $h \in \mathcal{H}_1 \equiv \{0, 1\}$; if $n = 2$, then $h \in \mathcal{H}_2 \equiv \{0, 1/2, 1\}$; if $n = 3$, then $h \in \mathcal{H}_3 \equiv \{0, 1/3, 2/3, 1\}$; and so on. In general, we have $h \in \mathcal{H} \equiv \bigcup_{n \in \mathcal{N}} \mathcal{H}_n = \mathbb{Q} \cap [0, 1]$, where $\mathbb{Q}$ is the set of all rational numbers.

To identify the size effect, instead of firm employment n we will use employment relative to average firm size, $\widehat{n} \equiv n/(\mathtt{E}/\mathtt{F})$, such that

$$y^h(n, h, z, \varphi) = \left(\frac{\mathtt{E}}{\mathtt{F}}\right)^{\nu} y^h_{z,\varphi}(\widehat{n}, h)$$

Notice that, for given $\mathtt{E}/\mathtt{F}$, normalized firm employment $\widehat{n}$ is a random variable with discrete support, denoted $\widehat{\mathcal{N}}$. Given this, aggregate productivity $Y/\mathtt{E}$ can be written as:

$$\frac{Y}{\mathtt{E}} = \left(\frac{\mathtt{F}}{\mathtt{E}}\right)^{1-\nu} \left[\sum_{z \in \mathcal{Z}} \sum_{\varphi \in \Phi} \frac{\mathtt{F}_{z,\varphi}}{\mathtt{F}} \left(\sum_{\widehat{n} \in \widehat{\mathcal{N}}} \sum_{h \in \mathcal{H}} y^h_{z,\varphi}(\widehat{n}, h) \mathrm{g}_{z,\varphi}(\widehat{n}, h)\right)\right] \tag{B.3}$$

where

$$\mathrm{g}_{z,\varphi}(\widehat{n}, h) \equiv \sum_{n_H=0}^{+\infty} \sum_{n_L=0}^{+\infty} \widetilde{\mathtt{f}}_{z,\varphi}(n_H, n_L)\, \mathbf{1}\left[\left(\frac{n_H + n_L}{\mathtt{E}/\mathtt{F}} = \widehat{n}\right) \wedge \left(\frac{n_H}{n_H + n_L} = h\right)\right] \tag{B.4}$$

is the share of firms of type (z, φ) that have relative size $\widehat{n}$ and skill share h. This is equivalent to the result shown in equation (19). Finally, the joint probability $\mathrm{g}_{z,\varphi}(\widehat{n}, h)$ can be written as:

$$\mathrm{g}_{z,\varphi}(\widehat{n}, h) = \underbrace{\mathrm{g}^A_{z,\varphi}(h|\widehat{n})}_{\substack{\text{Within} \\ \text{component}}} \; \underbrace{\mathrm{g}^B_{z,\varphi}(\widehat{n})}_{\substack{\text{Between} \\ \text{component}}}$$

for some conditional probability mass function $\mathrm{g}^A_{z,\varphi}(h|\widehat{n})$ and some marginal probability mass function $\mathrm{g}^B_{z,\varphi}(\widehat{n})$. On the one hand, $\mathrm{g}^A_{z,\varphi}(h|\widehat{n})$ is the share of (z, φ) that have skill share h conditional on having relative size $\widehat{n}$, so its contribution quantifies the within-firm, across worker component of productivity. On the other hand, $\mathrm{g}^B_{z,\varphi}(\widehat{n})$ is the share of firms of type (z, φ) that are of relative size $\widehat{n}$, regardless of their skill share, so its contribution quantifies the between-firm component of productivity.

C Model Extension with Directed Entry

Using extreme value shocks, the directed entry problem in Section 5.1.4 obtains convenient closed-form solutions. In particular, let the shocks be distributed Gumbel, $\varepsilon_\varphi \sim G(\mu_\varepsilon, \sigma_\varepsilon)$. Then, one can obtain: (i) an expression for the value of entry (equation (21)) as

$$\mathbf{J}^e = \sigma_\varepsilon \ln\left(\sum_{\varphi\in\Phi} \exp\left(\frac{\mathbf{J}^e(\varphi) - \kappa(\varphi)}{\sigma_\varepsilon}\right)\right) + \mu_\varepsilon + \sigma_\varepsilon\gamma_\varepsilon \tag{C.1}$$

where γ_ε is Euler's constant, and (ii) an expression for the fraction of entrants of each type as

$$\pi_\varphi(\varphi) = \frac{\exp\left(\frac{\mathbf{J}^e(\varphi) - \kappa(\varphi)}{\sigma_\varepsilon}\right)}{\sum_{\varphi'\in\Phi} \exp\left(\frac{\mathbf{J}^e(\varphi') - \kappa(\varphi')}{\sigma_\varepsilon}\right)} \tag{C.2}$$

To understand how selection upon entry works, note that the fraction of firms choosing technology φ increases with the value $\mathbf{J}^e(\varphi)$ of entering with technology φ, and that how much selection changes with the change of value is decreasing in the variance parameter σ_ε. In particular, for the case with only two values of φ one can easily write,

$$\frac{d\ln\left(\frac{\pi_\varphi(\varphi_1)}{\pi_\varphi(\varphi_2)}\right)}{d\left(\mathbf{J}^e(\varphi_1) - \mathbf{J}^e(\varphi_2)\right)} = \frac{1}{\sigma_\varepsilon} \tag{C.3}$$

Thus, $1/\sigma_\varepsilon$ gives us the semi-elasticity of the odds ratio of entering as a φ_1 type to the gain in value of entry as a type φ_1. The *actual* entry elasticity is:

$$\mathcal{E}_{\text{entry}} \equiv \frac{d\ln\left(\frac{\pi_\varphi(\varphi_1)}{\pi_\varphi(\varphi_2)}\right)}{d\ln\left(\mathbf{J}^e(\varphi_1) - \mathbf{J}^e(\varphi_2)\right)} = \frac{\mathbf{J}^e(\varphi_1) - \mathbf{J}^e(\varphi_2)}{\sigma_\varepsilon} \tag{C.4}$$

To implement directed entry in practice, first we normalize the location parameter to $\mu_\varepsilon = -\sigma_\varepsilon\gamma_\varepsilon$. This is a way to undo the effect of σ_ε on the ex-ante value of entry, $\mathbf{J}^e$.[63] Second, to recover the $\kappa(\varphi_1)$ and $\kappa(\varphi_2)$ parameters, we use equation (C.1) and the free entry condition $\mathbf{J}^e = 0$ to write:

$$0 = \ln\left(\sum_{\varphi\in\Phi} \exp\left(\frac{\mathbf{J}^e(\varphi) - \kappa(\varphi)}{\sigma_\varepsilon}\right)\right)$$

[63] When σ_ε is large, the probability of getting a high draw of ε_φ increases, and this increases the expected value function. Setting $\mu_\varepsilon = -\sigma_\varepsilon\gamma_\varepsilon$ removes this effect.

Then using equation (C.2) for $\pi_\varphi(\varphi_1)$, we obtain:

$$\kappa(\varphi) = \mathbf{J}^e(\varphi) - \sigma_\varepsilon \ln\left(\pi_\varphi(\varphi)\right),$$

for $\varphi \in \{\varphi_1, \varphi_2\}$. That is, the average cost of entry of each technology is given by the value of entry of that technology (which can be computed with equation (13)), the parameter σ_ε, and the calibrated entry fraction of firms of each type.

While the model with directed entry delivers the same model outcomes as the baseline model with random entry, the counterfactuals are going to differ. In particular, as equation (C.3) shows, counterfactuals that change the value of entry with each technology differently, $\mathbf{J}^e(\varphi_1) - \mathbf{J}^e(\varphi_2)$, will change the number of entrants with each technology in the model with random entry.

D Numerical Appendix

D.1 Idiosyncratic Productivity

In the model, idiosyncratic productivity z is governed by a continuous-time Markov chain (CTMC), with associated infinitesimal generator matrix:

$$\mathbf{\Lambda}_z = \begin{pmatrix} -\sum_{j\neq 1}\lambda_{1j} & \lambda_{12} & \dots & \lambda_{1k} \\ \lambda_{21} & -\sum_{j\neq 2}\lambda_{2j} & \dots & \lambda_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{k1} & \lambda_{k2} & \dots & -\sum_{j\neq k}\lambda_{kj} \end{pmatrix}$$

where $\lambda_{ij} > 0$ is short-hand for $\lambda(z_j|z_i)$, $z_i, z_j \in \mathcal{Z}$. For the numerical implementation, we recover these rates by assuming an Ornstein-Uhlenbeck (OU) process for z_t (in logs):

$$d\ln(z_t) = -\rho_z \ln(z_t)dt + \sigma_z dB_t$$

where B_t is a Wiener process, and $\rho_z, \sigma_z > 0$.[64] This is a continuous-time process defined on a continuous support. To simulate such a process, and draw a one-to-one mapping between the (ρ_z, σ_z) parameters and the $\{\lambda_{ij}\}$ rates, we use the following steps:

1. First, we approximate the process in discrete time. For a given time interval $[0, T] \subset \mathbb{R}_+$, we partition the space into M subintervals of equal length $dt > 0$, i.e. $\mathcal{T} \equiv \{0 = t_0 < t_1 < \cdots < t_M = T\}$ with $t_{m+1} - t_m = dt$ and $dt = T/M$. As the model is calibrated at the quarterly frequency, dt represents a quarter. Then, we approximate the OU process using the Euler-Maruyama method:

[64] In levels, this is a diffusion of the type $dz_t = \mu(z_t)dt + \sigma(z_t)dB_t$, with $\mu(z) = z\left(-\rho_z \ln(z) + \frac{\sigma_z^2}{2}\right)$ and $\sigma(z) = \sigma_z z$.

$$\ln(z_m) = (1-\rho_z dt)\ln(z_{m-1}) + \sigma_z\sqrt{dt}\,\varepsilon^z_m, \qquad \varepsilon^z_m \sim \text{i.i.d. } \mathcal{N}(0,1) \tag{D.1}$$

for each $m = 1, \ldots, M$. From Ruiz-García (2021), we know that the autocorrelation coefficient and the dispersion in firm-level TFP in Spain is 0.81 and 0.34 at an annual frequency. Therefore, for persistence, we set $\rho_z = 1 - (0.81)^{1/4} = 0.0513$ for our quarterly calibration. Moreover, we compute a quarterly figure for dispersion from the yearly dispersion parameter as $\sigma_z = 0.34\left(\sum_{q=1}^{4}(0.81)^{(q-1)/2}\right)^{-1/2} = 0.1833$.

2. To estimate the discrete-time AR(1) process (D.1), we use the Tauchen (1986) method. The outcome of this method is a transition probability matrix $\mathbf{\Pi}_z = (\pi_{ij})$, where π_{ij} denotes the probability of a z_i-to-z_j transition in the $\mathcal{T}$ space.

3. For the mapping from $\{\pi_{ij}\}$ transition probabilities to $\{\lambda_{ij}\}$ intensity rates, we use that any CTMC with generator matrix $\mathbf{\Lambda}_z$ maps into a discrete-time Markov chain with transition matrix $\mathbf{\Pi}_z(t)$ at time t in which holding times between arrivals are independently and exponentially distributed, so that $\mathbf{\Pi}_z(t) = e^{\mathbf{\Lambda}_z t}$. Then, we can solve for $\{\lambda_{ij}\}$ to obtain:

$$\lambda_{ij} = \begin{cases} -\frac{\pi_{ij}}{1-\pi_{ii}}\frac{\ln(\pi_{ii})}{dt} & \text{for } i \neq j \\ \frac{\ln(\pi_{ii})}{dt} & \text{otherwise} \end{cases}$$

D.2 Stationary Solution Algorithm

We solve the model on a grid $\overline{\mathcal{N}}_{OEH} \times \overline{\mathcal{N}}_{OEL} \times \overline{\mathcal{N}}_{FT} \times \mathcal{Z} \times \Phi$, where $\overline{\mathcal{N}}_{ij} \equiv \{0,1,2,\ldots,N_{ij}\}$, $(i,j) \in \mathcal{I} \times \mathcal{J}$, for sufficiently large positive integer N_{ij}. In practice, we use $(N_{OEH}, N_{OEL}, N_{FT}) = (30,15,15)$.

Step 0. Set $\iota = 0$. Choose a guess $\mathbf{U}^{(0)} > b/\rho$.

Step 1. At iteration $\iota \in \{0,1,2,\ldots\}$, given a guess $\mathbf{U}^{(\iota)}$, use Value Function Iteration to solve for the object $\mathbf{\Sigma}^{(\iota)} \in \overline{\mathcal{N}}_{OEH} \times \overline{\mathcal{N}}_{OEL} \times \overline{\mathcal{N}}_{FT} \times \mathcal{Z} \times \Phi$:[65]

$$\begin{aligned}\mathbf{\Sigma}^{(\iota)}(\vec{n},z,\varphi) = \frac{1}{\rho^{(\iota)}(\vec{n},z,\varphi)}\Bigg\{ & y(\vec{n},z,\varphi) - \xi n_{FT}\left[p^{(\iota)}(\vec{n},z,\varphi)\right]^{\vartheta} - \sum_{i\in\mathcal{I}} A_i\left[v_i^{(\iota)}(\vec{n},z,\varphi)\right]^{\varsigma} \\ & + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\left[n_{ij}\left(\delta_{ij}^{(\iota)}(\vec{n},z,\varphi) + s_i^W + s^F\right)\mathbf{U}^{(\iota)} - \chi n_{ij}\left[\delta_{ij}^{(\iota)}(\vec{n},z,\varphi)\right]^{\psi} + n_{ij}\left(\delta_{ij}^{(\iota)}(\vec{n},z,\varphi) + s_i^W\right)\mathbf{\Sigma}^{(\iota)}(\vec{n}_{ij}^{-},z,\varphi)\right] \\ & + \sum_{i\in\mathcal{I}}\left[v_i^{(\iota)}(\vec{n},z,\varphi)\eta^{(\iota)}(\vec{n},z,\varphi)\max\left(\mathbf{\Sigma}^{(\iota)}(\vec{n}_{iL}^{+},z,\varphi) - \mathbf{U}^{(\iota)}, \gamma\left(\mathbf{\Sigma}^{(\iota)}(\vec{n}_{iL}^{+},z,\varphi) - \mathbf{U}^{(\iota)}\right) + (1-\gamma)\mathbf{\Sigma}^{(\iota)}(\vec{n},z,\varphi)\right)\right]\end{aligned}$$

[65] To arrive at this expression, we have used results (8) and (A.27) into equation (A.10).

$$+ n_{FT} p^{(\iota)}(\vec{n}, z, \varphi) \boldsymbol{\Sigma}^{(\iota)}(\vec{n}^{p}, z, \varphi) + n_{OEL} \tau \, \boldsymbol{\Sigma}^{(\iota)}(\vec{n}^{\tau}, z, \varphi) + \sum_{z' \in \mathcal{Z}} \lambda(z'|z) \boldsymbol{\Sigma}^{(\iota)}(\vec{n}, z', \varphi) \Bigg\}$$

for each $\varphi \in \Phi$, where:

$$\delta_{ij}^{(\iota)}(\vec{n}, z, \varphi) \equiv \left[\frac{1}{\psi \chi} \left(\mathbf{U}^{(\iota)} + \boldsymbol{\Sigma}^{(\iota)}(\vec{n}_{ij}^{-}, z, \varphi) - \boldsymbol{\Sigma}^{(\iota)}(\vec{n}, z, \varphi) \right) \right]^{\frac{1}{\psi - 1}}$$

$$p^{(\iota)}(\vec{n}, z, \varphi) \equiv \left[\frac{1}{\vartheta \xi} \left(\boldsymbol{\Sigma}^{(\iota)}(\vec{n}^{p}, z, \varphi) - \boldsymbol{\Sigma}^{(\iota)}(\vec{n}, z, \varphi) \right) \right]^{\frac{1}{\vartheta - 1}}$$

$$v_{i}^{(\iota)}(\vec{n}, z, \varphi) \equiv \left[\left(\frac{\gamma}{\varsigma A_i} \right) \eta^{(\iota)}(\vec{n}, z, \varphi) \left(\boldsymbol{\Sigma}^{(\iota)}(\vec{n}_{iL}^{+}, z, \varphi) - \boldsymbol{\Sigma}^{(\iota)}(\vec{n}, z, \varphi) - \mathbf{U}^{(\iota)} \right) \right]^{\frac{1}{\varsigma - 1}}$$

$$\eta^{(\iota)}(\vec{n}, z, \varphi) \equiv \left[\left(\frac{1 - \gamma}{\rho \mathbf{U}^{(\iota)} - b} \right) \left(\boldsymbol{\Sigma}^{(\iota)}(\vec{n}_{iL}^{+}, z, \varphi) - \boldsymbol{\Sigma}^{(\iota)}(\vec{n}, z, \varphi) - \mathbf{U}^{(\iota)} \right) \right]^{\frac{1-\gamma}{\gamma}}$$

$$\rho^{(\iota)}(\vec{n}, z, \varphi) \equiv \rho + s^{F} + n_{FT} p^{(\iota)}(\vec{n}, z, \varphi) + n_{OEL} \tau + \sum_{i \in \mathcal{I}} \left[v_{i}^{(\iota)}(\vec{n}, z, \varphi) \eta^{(\iota)}(\vec{n}, z, \varphi) + \sum_{j \in \mathcal{J}} n_{ij} \left(\delta_{ij}^{(\iota)}(\vec{n}, z, \varphi) + s_{i}^{W} \right) \right]$$

Step 2. Use a non-linear equation solver to find $\mathbf{U}^{*}$ as the solution to the free-entry condition:

$$-\kappa + \sum_{\varphi \in \Phi} \pi_{\varphi}(\varphi) \cdot \mathbf{J}_{e}^{(\iota)}(\varphi; \mathbf{U}^{*}) = 0$$

where

$$\mathbf{J}_{e}^{(\iota)}(\varphi; \mathbf{U}^{*}) \equiv \gamma \left(\frac{\rho \mathbf{U}^{*} - b}{1 - \gamma} \right)^{\frac{\gamma - 1}{\gamma}} \left[\sum_{z^{e} \in \mathcal{Z}} \pi_{z}(z^{e}) \left(\sum_{i \in \mathcal{I}} \left(\boldsymbol{\Sigma}^{(\iota)}(\vec{n}_{iL}^{e}, z^{e}, \varphi) - \mathbf{U}^{*} \right)^{\frac{1}{\gamma}} \right) \right]$$

is the value of entering with type φ, with $\vec{n}_{i}^{e} = (n_{i}^{e}, \vec{n}_{-(ij)}^{e}) = (1, \vec{0})$.

Step 3. Compute

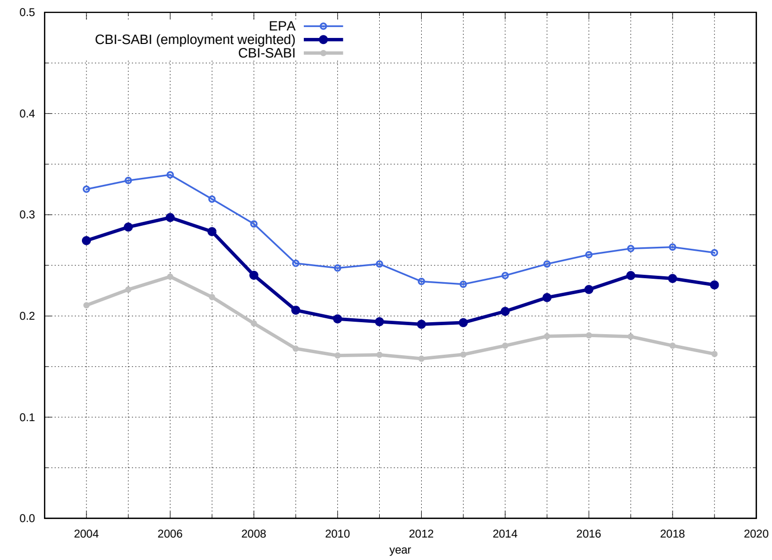
$$\epsilon^{(\iota)} \equiv \left| \frac{\mathbf{U}^{(\iota)} - \mathbf{U}^{*}}{\mathbf{U}^{*}} \right|$$

Proceed to Step 4 if $\epsilon^{(\iota)} < \varepsilon$ for some small tolerance $\varepsilon > 0$. Otherwise, set $\mathbf{U}^{(\iota+1)} = \varrho \cdot \mathbf{U}^{*} + (1 - \varrho) \cdot \mathbf{U}^{(\iota)}$, where $\varrho \in (0, 1]$ is a dampening parameter, and go back to Step 1 with $[\iota] \leftarrow [\iota + 1]$.

Step 4. Find the distribution and aggregate measures following Online Appendix A.2.

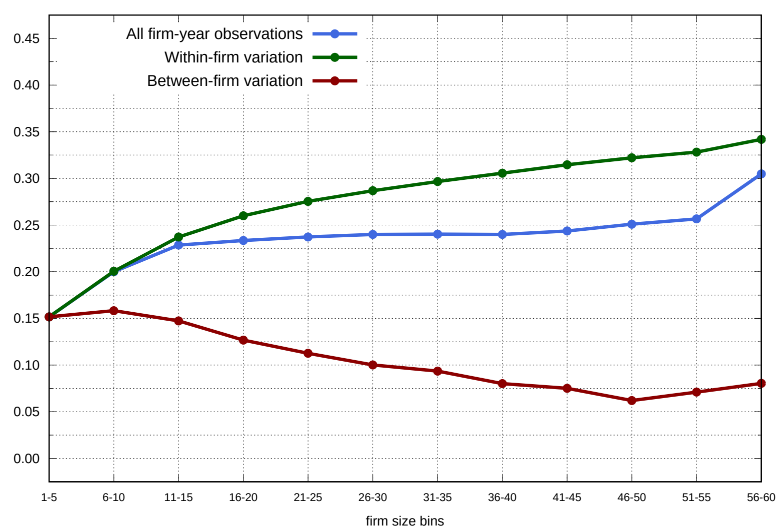
E Additional Figures and Tables

Figure E.1: Aggregate temporary share over time



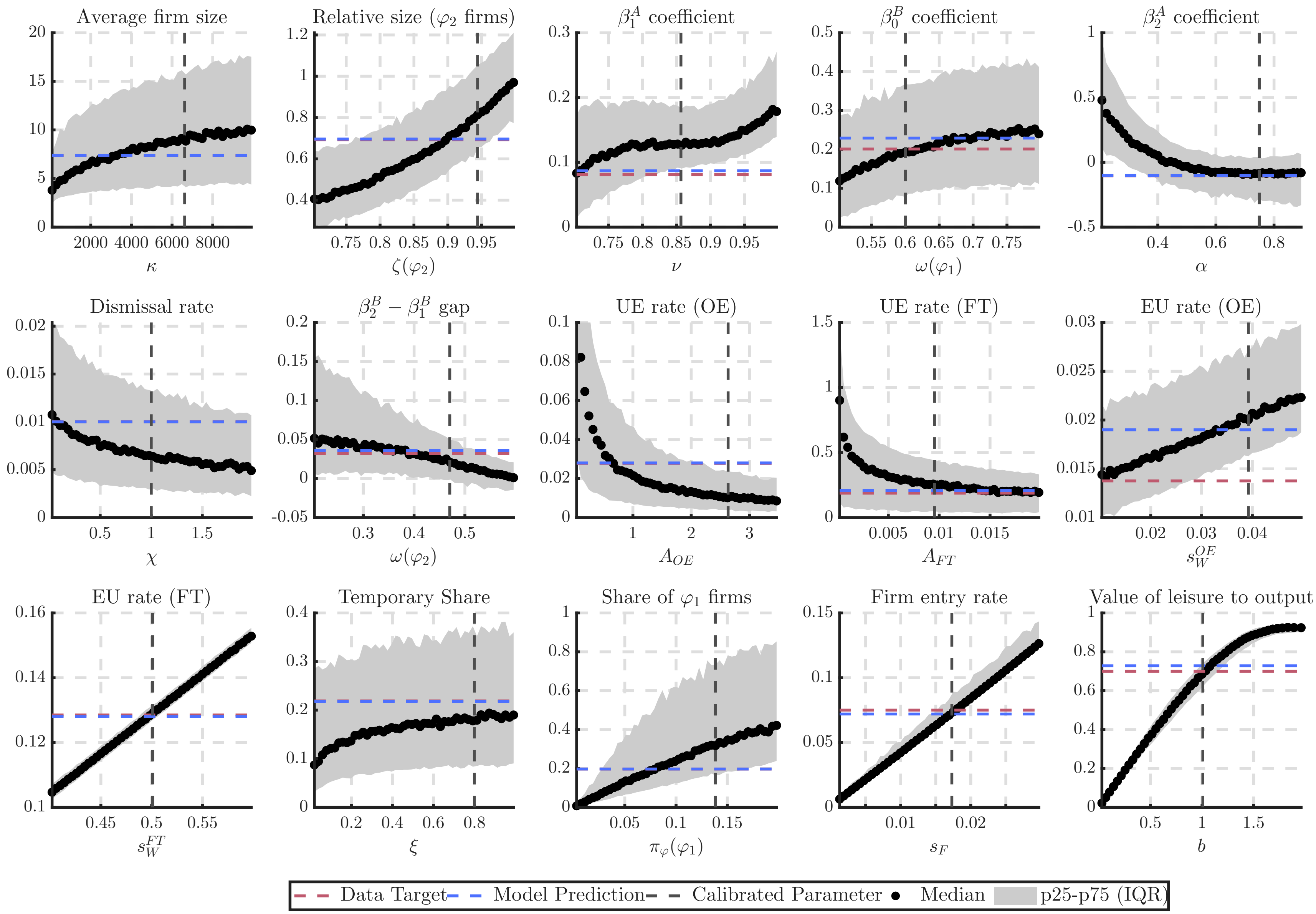
Notes: Light blue line: share of temporary workers in the Encuesta de Población Activa (the Spanish Labor Force Survey); grey line: average share of temporary workers across firms in our firm-level data; dark blue line: employment-weighted average share of temporary workers across firms in our firm-level data, which corresponds to the average share of temporary workers in the economy.

Figure E.2: Temporary share, by firm size, in the sub-sample.



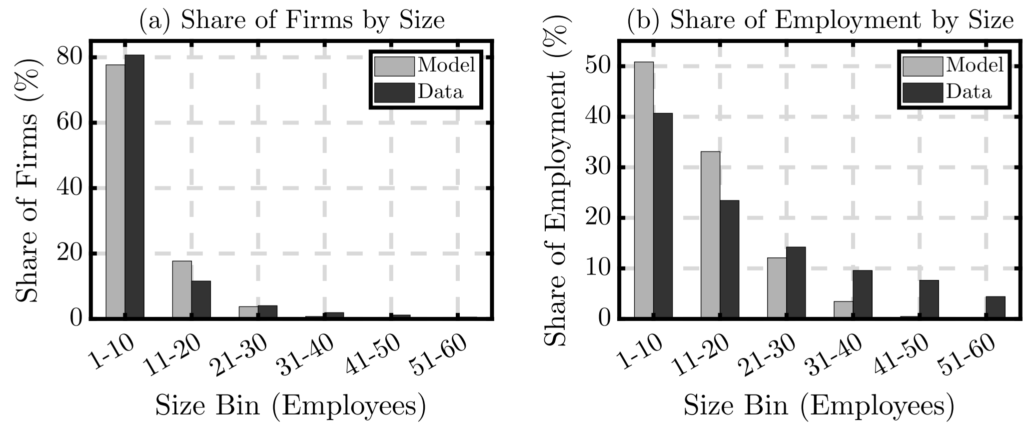
Notes: This figure is the analogue of Figure 1 in the main text, but for the sub-sample used to calibrate the model.

Figure E.3: Global identification results.



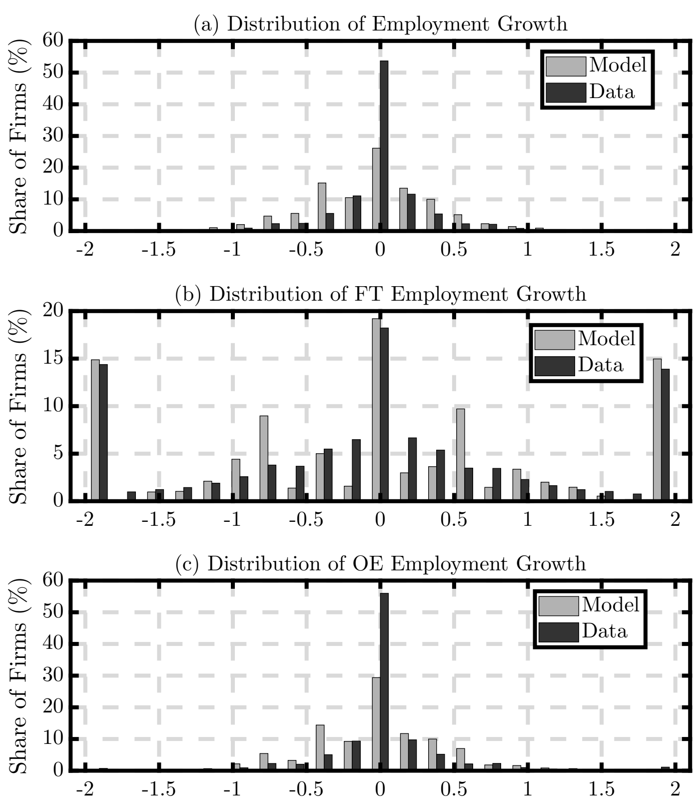
Notes: Global identification results based on approximately 50,000 model solutions. The black dots are the median of the distribution of each targeted moment for given value of the chosen parameter, generated from underlying random variation in all other parameters. The shaded areas are the 25th and 75th percentiles. The red dashed horizontal line is the data target, the blue dashed horizontal line is the model's prediction for the moment, and the black vertical line is the value for the calibrated parameter.

Figure E.4: Non-Targeted Moments: Distribution of Firms and Employment by Size. Model versus Data.



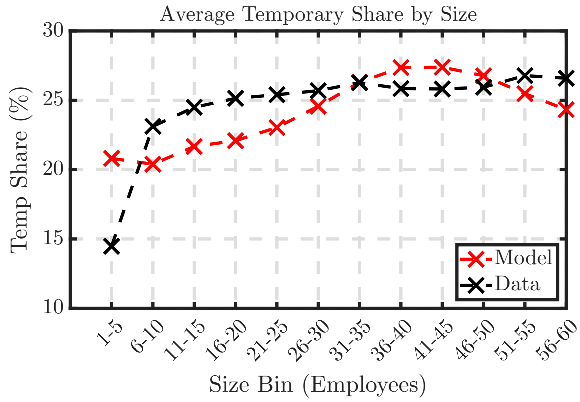
Notes: This figure plots the distribution of firms and employment by firm size (total number of employees), in the data and in the calibrated model.

Figure E.5: Non-Targeted Moments: Distribution of Yearly Employment Growth Rates. Model versus Data.



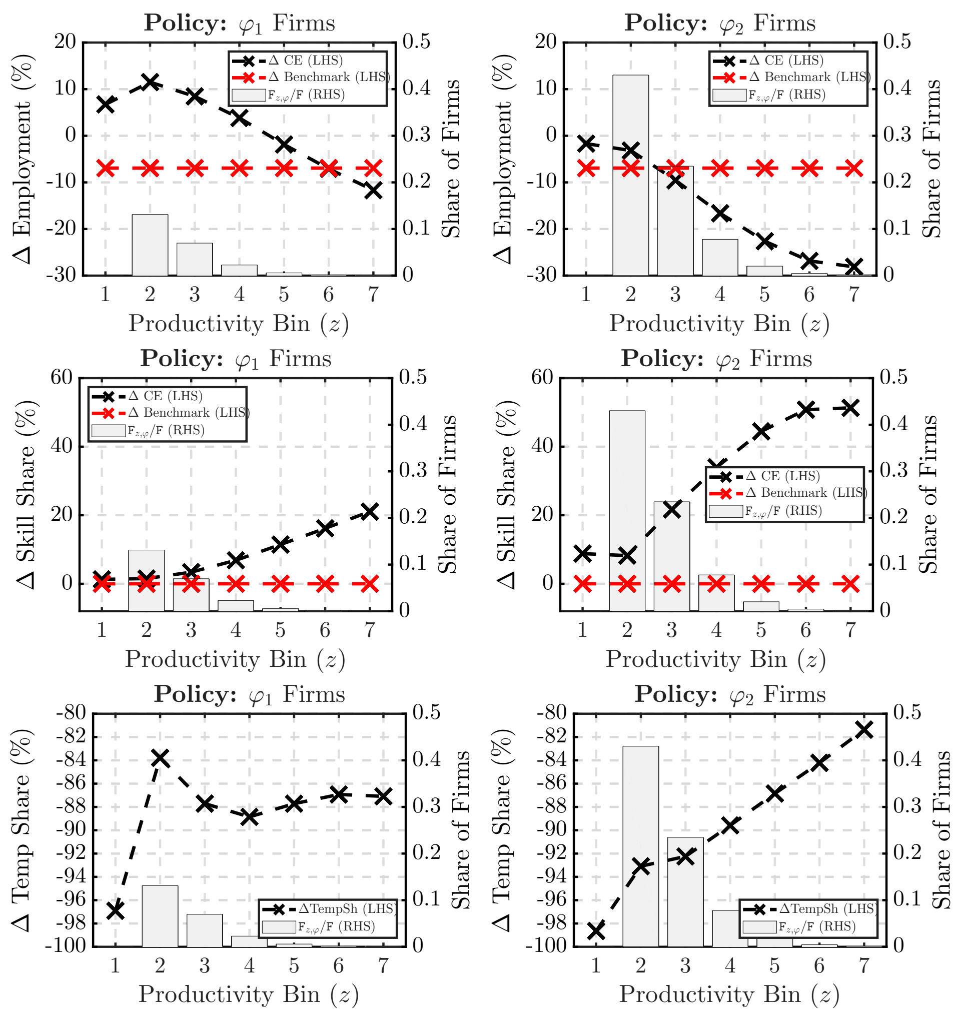
Notes: This figure plots the distribution of employment growth rates for total employment (top), FT employment (middle) and OE employment (bottom), in the data and in the calibrated model.

Figure E.6: Non-Targeted Moments: Average Temporary Share by Firm Size. Model versus Data.



Notes: This figure plots the average temporary share by employment bins, in the data and in the calibrated model.

Figure E.7: Heterogeneous effects by productivity groups of shortening FTC duration.



Notes: These figures show the percentage change in employment (top row), the skill share (middle row) and the temporary share (bottom row) for high-type (left column) and low-type (right column) firms and for each transitory productivity bin (bins on the horizontal axis) when moving from the baseline duration of FTCs to a policy that limits FTCs to 1 month. When available (i.e. for total employment and the skill share), we also show the percentage change for the Benchmark allocation that maximizes total output.

Table E.1: Regression evidence in the data

A. Productivity: $\ln(VA_{it}/Emp_{it})$			**B. Temporary share:** $TempSh_{it}$		
β_0^A	$\mathbf{1}[\varphi_i = \varphi_2]$	-0.003	β_0^B	$\mathbf{1}[\varphi_i = \varphi_2]$	0.201
β_1^A	$\ln(Emp_{it})$	0.081	β_1^B	$Emp_{it} \in [1, 5)$	0.005
β_2^A	$TempSh_{it}$	-0.104	β_2^B	$Emp_{it} \in [5, 10)$	0.037
β_3^A	$TempSh_{it}^2$	-0.092	β_3^B	$Emp_{it} \in [10, 15)$	0.080
			β_4^B	$Emp_{it} \in [15, 20)$	0.099
			β_5^B	$Emp_{it} \in [20, 25)$	0.113
			β_6^B	$Emp_{it} \in [25, 30)$	0.122
			β_7^B	$Emp_{it} \in [30, 35)$	0.129
			β_8^B	$Emp_{it} \in [35, 40)$	0.135
			β_9^B	$Emp_{it} \in [40, 45)$	0.142
			β_{10}^B	$Emp_{it} \in [45, 50)$	0.150
			β_{11}^B	$Emp_{it} \in [50, 55]$	0.156
			β_{12}^B	$Emp_{it} \in [55, 60]$	0.190
# observations		6,316,320	# observations		6,664,229
R^2		0.01	R^2		0.14

Notes: Results, in the firm-level data, from regression (16), in Panel A, and regression (17), in Panel B. Variables are all net of aggregate FE (sector, region, and province). All coefficients are significant at the 1% level.

Table E.2: Effects of changes in average FTC duration on macroeconomic aggregates when entry is directed

	(A) *Short duration* (1 month)	(B) *Baseline* (6 months)	(C) *Long duration* (1 year)
Measure of incumbent firms	0.102	0.115	0.119
... Share of type-φ_1 firms	36.0 %	19.7 %	15.6 %
Average firm size	7.61	7.39	7.47
... Relative size φ_2 firms	0.575	0.696	0.747
Firm exit rate (annualized)	7.18 %	7.15 %	7.14 %
Average temporary share (Employment weighted)	1.2 %	21.8 %	32.6 %
Average temporary share (Firm weighted)	1.5 %	21.0 %	32.0 %
... within φ_1 firms	0.4 %	4.0 %	5.8 %
... within φ_2 firms	2.1 %	25.2 %	39.3 %
Share of H workers	57.6 %	46.2 %	41.7 %
... within φ_1 firms	67.3 %	64.7 %	63.8 %
... within φ_2 firms	48.2 %	39.6 %	36.2 %
Matching Efficiency (FTCs)	0.63	2.40	1.87
Matching Efficiency (OECs)	0.09	0.03	0.02
UE rate (total)	9.4 %	23.5 %	25.7 %
... UE rate (FT)	3.7 %	20.8 %	23.3 %
... UE rate (OE)	5.8 %	2.8 %	2.5 %
EU rate (total)	2.8 %	4.2 %	3.4 %
... EU rate (FT)	75.2 %	12.8 %	6.7 %
... EU rate (OE)	1.9 %	1.9 %	1.8 %
Promotion rate	12.9 %	4.3 %	2.6 %
Unemployment rate	22.5 %	15.1 %	11.4 %

Notes: This table is the same as Table 5 in the main text, but for the economy with directed entry and an entry elasticity of one. For the description of the extension of the model with directed entry, see Online Appendix C.

Table E.3: Effects of changes in average FTC duration on productivity, misallocation and welfare when entry is directed

	(A) *Short duration* (1 month)	(B) *Long duration* (1 year)
Change in output per worker, of which:	0.22 %	-0.42 %
(a) *Firm size channel*	-0.43 %	-0.15 %
(b) *Firm selection channel*	6.79 %	-1.74 %
(c) *Reallocation channel*, of which:	-8.92 %	1.17 %
... between-firm component	-8.99 %	1.20 %
... within-firm component	-2.75 %	-0.92 %
Change in output	-8.47 %	3.89 %
Output loss from misallocation (in levels)	6.61 %	5.05 %
Change in welfare	-1.27 %	0.73 %

Notes: This table is the same as Table 6 in the main text, but for the economy with directed entry and an entry elasticity of one. For the description of the extension of the model with directed entry, see Online Appendix C.

Table E.4: Effects on macroeconomic aggregates of alternative policies

		(A)	(B)	(C)
	Baseline calibration	Short duration	Linear Tax on FT Employment	Ban on FTCs
Measure of operating firms	0.115	0.108	0.110	0.110
... Share of type-φ_1 firms	19.7 %	23.1 %	23.1 %	22.9 %
Average firm size	7.39	7.09	7.11	7.22
... Relative size φ_2 firms	0.696	0.575	0.577	0.576
Firm entry rate (annualized)	7.15 %	7.19 %	7.17 %	7.19 %
Average temporary share (Emp. weighted)	21.8 %	1.5 %	1.5 %	.
Average temporary share (Firm weighted)	21.0 %	1.7 %	1.8 %	.
... within φ_1 firms	4.0 %	0.4 %	0.2 %	.
... within φ_2 firms	25.2 %	2.1 %	2.2 %	.
Share of H workers	46.2 %	54.8 %	54.7 %	54.2 %
... within φ_1 firms	64.7 %	67.2 %	67.3 %	67.1 %
... within φ_2 firms	39.6 %	48.2 %	48.0 %	47.6 %
Matching efficiency (FTCs)	2.40	0.73	0.09	.
Matching efficiency (OECs)	0.03	0.09	0.09	0.05
UE rate (total)	23.5 %	9.9 %	7.8 %	7.8 %
... UE rate (FT)	20.8 %	4.1 %	1.4 %	.
... UE rate (OE)	2.8 %	5.8 %	6.4 %	.
EU rate (total)	4.2 %	3.1 %	2.2 %	2.1 %
... EU rate (FT)	12.8 %	75.1 %	13.4 %	.
... EU rate (OE)	1.9 %	2.0 %	2.0 %	.
Promotion rate	4.3 %	12.6 %	14.3 %	.
Unemployment rate	15.1 %	23.5 %	21.5 %	20.7 %

Notes: This table shows the effects of various policies on a number of macroeconomic aggregates of interest. The linear tax rate in column (B) has been chosen to ensure that the effect of the policy on the (employment-weighted) average temporary share is the same as for the baseline policy discussed in the paper (i.e., a decrease in FTC duration to 1 month).

Table E.5: Effects on productivity, misallocation and welfare of alternative policies

	(A)	(B)	(C)
	Short duration	Linear Tax on FT Employment	Ban on FTCs
Change in output per worker, of which:	0.17 %	0.17 %	0.13 %
(a) *Firm size channel*	0.59 %	0.57 %	0.34 %
(b) *Firm selection channel*	1.44 %	1.45 %	1.31 %
(c) *Reallocation channel*, of which:	-2.57 %	-2.54 %	-2.20 %
... between-firm component	-2.62 %	-2.57 %	-2.17 %
... within-firm component	3.30 %	3.17 %	1.80 %
Change in output	-9.75 %	-7.36 %	-6.53 %
Output loss from misallocation (in levels), of which:	6.13 %	6.11 %	5.90 %
Change in welfare	-2.07 %	-1.51 %	-1.22 %

Notes: This table shows the effect of various policies, expressed in percentage changes with respect to the baseline calibration (with the exception of the output loss from misallocation, which is expressed in levels). Welfare is computed as in equation (A.36), see Online Appendix A.6. The linear tax rate in column (B) has been chosen to ensure that the effect of the policy on the (employment-weighted) average temporary share is the same as for the baseline policy discussed in the paper (i.e., a decrease in FTC duration to 1 month).