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working paper
2508

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May 2025

CENTRO DE ESTUDIOS
MONETARIOS Y FINANCIEROS
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CEMFI Working Paper No. 2508
May 2025

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Abstract

We examine the interaction between banks and money market funds (MMFs) in a setup where the latter can experience large redemptions following an aggregate liquidity shock (as in March 2020). In the model MMFs and bank deposits are alternatives for firms'management of their cash holdings. MMFs experiencing correlated redemptions get forced to sell assets to banks in narrow markets, producing asset price declines. Ex post the price declines damage firms' capacity to cover their needs with the redeemed shares. Ex ante the prospect of such an effect reduces the attractiveness of MMFs relative to bank deposits. Yet the equilibrium allocation of firms' savings exhibits an excessive reliance on MMFs since firms fail to internalize their effect on the size of the pecuniary externalities caused by future redemptions. This provides a rationale, distinct from first mover advantages, for the macroprudential regulation of the investment in MMFs.

JEL Codes: G01, G21, G23.

Keywords: Liquidity management; liquidity risk; pecuniary externalities; money markets.

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Acknowledgement

We would like to thank Albert Banal-Estanol, Douglas Diamond, Sebastian Fanelli, Douglas Gale, Tommaso Gasparini, David Martinez-Miera, Rafael Repullo, Lin Shen, and other participants at the 4th Banca d'Italia, Bocconi University and CEPR Conference on "Financial Stability and Regulation", 7th Madrid-Barcelona Workshop on Banking and Corporate Finance, 8th Banco de Portugal Conference on Financial Intermediation, 13th NYU-LawFin/SAFE-ESCP BS Law & Finance Conference, 30th Finance Forum, 31st European Summer Symposium in Financial Markets in Gerzensee, 2023 RiskLab/Bank of Finland/ESRB Conference on Systemic Risk Analytics, and seminar presentations at CEMFI, Chicago Booth, and CNMV, for helpful comments and suggestions. Martin Farias acknowledges financial support from the Maria de Maeztu Unit of Excellence CEMFI MDM-2016-0684, funded by MCIN/AEI/10.13039/501100011033, the Santander Research Chair, Fundación Carolina and CEMFI. Javier Suarez acknowledges support from grant PID2022-142998NB-I00 funded by MICIU/AEI /10.13039/501100011033 and by ERDF, EU.

1 Introduction

Money market funds (MMFs) are mutual funds that invest in short-term financial assets including deposits, treasuries, commercial paper, and repurchase agreements (repos). With assets under management exceeding $8 trillions globally, MMFs constitute an important investment vehicle for firms and households and an important source of funding for the issuers of money market securities, including governments, financial institutions, and non-financial corporations. Banks, in particular, place in MMFs a significant fraction of their commercial paper, certificates of deposit, and repos. MMFs also interact with banks in the secondary markets for money market securities. The resulting interconnections with final investors, banks, and other issuers of short-term assets make shocks and frictions potentially affecting MMFs a source of concern and instability for the whole financial system and the broader economy.

The vulnerabilities associated with MMFs became evident during the financial turmoil triggered by the onset of the Covid-19 pandemic in March 2020, when investors in demand for cash redeemed substantial amounts of savings from MMFs.[1] In face of unusually high redemptions, MMFs had to sell relatively illiquid assets—typically hold to maturity—inducing downward pricing spirals in the corresponding secondary and primary markets. Fear of fire sales and contagion across the financial system led central banks to intervene with lending facilities and asset purchase programs specifically devised to directly or indirectly facilitate liquidity to the MMFs sector.[2] These events placed the assessment of the contribution of MMFs to financial instability at the top of regulators' agenda.[3]

This paper aims to provide an analytical insight on the topic. We build a model with a rationale for the coexistence of banks and MMFs and explicit financial and trading links between both sectors (and between them and the real economy). The model is explicit about the trade-off underlying investors' decision to keep their liquid savings in the form of bank deposits or MMFs shares. Deposits promise a fixed conversion value but ther attractiveness is reduced by bank-specific frictions (which we model as a probability that they cannot be converted into cash because of idiosyncratic problems at bank level).[4] MMFs shares can

[1] See Financial Stability Board (2020, 2021) for an account of the performance of MMFs around March 2020. Redemptions by non-financial corporations wishing to guarantee their capacity to face unexpected liquidity shortfalls played a major role. According to Aramonte, Schrimpf and Shin (2022) another driver of the "dash for cash" was the rise in margins associated with derivative positions following the generalized rise in uncertainty and market volatitity.

[2] Haddad, Moreira, and Muir (2021) provide evidence of the connection between fire sales and price declines in the US, and of the effectiveness of the interventions put in place by the Fed.

[3] See, for instance, Financial Stability Board (2021) or International Monetary Fund (2021).

[4] The risk of deposit illiquidty might stem from purely operational reasons or deeper liquidity or solvency

always be converted into cash but their redemption value fluctuates in response to aggregate liquidity shocks.

In our model, investors anticipate the risk of declines in the redemption value of MMFs shares and optimize their initial liquidity allocation accordingly. However, a pecuniary externality leads to channelling an excessive fraction of initial savings to MMFs. We show that a tax on savings allocated to MMFs (or any macroprudential policy with equivalent impact) can restore the constrained efficiency of the competitive equilibrium, inducing outcomes characterized by lower (but not null) price declines and less negative real effects (fall in investment) in response to aggregate liquidity shocks.

Our three-date model features firms, banks and MMFs. All agents are risk neutral. The model focuses on the liquidity provision role of banks and MMFs and, thus, abstracts from other important roles of these intermediaries, including the provision of funding to firms, households or the government. Firms have savings at the initial date which they might need at either the interim or the final date, and banks and MMFs provide alternative liquid ways in which to carry those savings into the future. Thus, at the initial date, firms split their savings into bank deposits and MMFs shares, banks collect deposits and issue commercial paper to invest in bank assets, and MMFs invest in a portfolio of bank commercial paper, thus indirectly supporting also to the investment in bank assets.

At the interim date, firms face both aggregate and idiosyncratic liquidity shocks. The aggregate shock takes the form of a "dash for cash" (like the one observed in March 2020): it calls investors to keep up to the final date a minimal fraction of their savings in the form of liquid bank deposits.[5] We model firms' idiosyncratic liquidity shocks as coming from the emergence of investment opportunities (projects) whose positive NPV would be wasted if not undertaken. Firms deal with these shocks by using and/or rebalancing their holdings of bank deposits and MMFs shares. MMFs accommodate their redemptions by selling commercial paper back to the banks in a frictional secondary market in which selling pressures translate into price declines.[6] Banks offset flutuations in deposits and the funding needs implied by their trade in the secondary commercial paper market with variations in their holdings of bank assets.[7]

problems. Even in the presence of deposit insurance, the deposits at a failing bank can turn illiquid due to administrative delays as well as in the balances exceeding the amounts under insurance coverage.

[5] This shock can be interpreted as a version of a liquidity shock *a la* Holmström and Tirole (1998); under this interpretation the liquid deposits would be subsequently used by their original holders to pay for some (potential) abnormal expense instead of being held until the final date.

[6] The causal impact of fire sales by mutual funds on asset price declines is empirically documented, among others, by Haddad, Moreira, and Muir (2021), Jiang, Li, Sun, and Wang (2022), and Giannetti and Chotibhak (2022).

[7] The baseline formulation describes bank assets as safe short-term assets. Results would be similar if

We model the low market liquidity of commercial paper as arising from costs that banks face when purchasing commercial paper in the secondary market. Introducing these costs is a simple way to capture (in reduced form) a variety of imperfections that impair the functioning of secondary markets for short-term debt securities in the real world. One specific imperfection that we plan to explore in a fuller structural manner in a future version of the paper is the existence of capital requirements, which introduce a regulatory capital cost to the acquisition of commercial paper in secondary markets.[8] Other reasons for secondary market prices to respond to selling pressure include the informational frictions highlighted in the market microstructure literature, costs implied by due diligence, and costs implied by search frictions and congestion in predominantly over-the-counter markets which feature very few transactions in normal times.[9]

We are able to characterize analytically the properties of the competitive equilibrium of the model. At the interim date, when firms are not hit by the aggregate liquidity shock, the redemptions faced by MMFs are motivated by purely idiosyncratic (and thus diversifiable) liquidity needs, secondary market prices remain close to fundamental value, and the investment of firms receiving investment projects is high. In contrast, when the aggregate liquidity shock realizes, the reallocation of savings from MMFs to bank deposits implies large redemptions, declines in the secondary market price of commercial paper, and a reduction in the investment that firms with investment projects can undertake.

The equilibrium of the model at the initial date is characterized by an indifference condition. At the margin, firms are indifferent between placing their savings in the safer but seldom idiosyncratically illiquid deposits of their banks and MMFs with prices that fluctuate across aggregate states. The equilibrating mechanism is as follows: if the proportion of aggregate liquid savings placed in MMFs were smaller, the size of redemptions in the interim aggregate illiquid state would be smaller and secondary market prices would remain higher, making MMFs dominate bank deposits at the initial date. Symmetrically, if the savings placed in MMFs were larger, the price of MMFs shares would experience larger declines in

banks invest in long-term assets but can borrow frictionlessly against them (e.g. using a central bank facility) at the interim date.

[8] If banks' equity capital is fixed at the interim date and the capital requirement becomes binding when the economy is hit by the aggregate liquidity shock, the secondary market for commercial paper will exhibit "capital-in-the-market pricing." Akin to the well-known cash-in-the-market pricing of Allen and Gale (1994), such pricing can make secondary market prices a decreasing function of the selling pressure produced by MMFs redemptions.

[9] Indeed, in normal circumstances, the illiquidity of commercial paper does not pose a problem for its holders who will typically hold these instruments to maturity (and arrange their portfolio in such a manner that they continuously recover cash, if needed, from maturing assets). This also explains the absence of active market making in these securities.

the aggregate illiquid state and deposits would dominate ex ante as a savings mean. The ex ante indifference condition pins down a unique aggregate allocation of savings between deposits and MMFs shares at the initial date (and consistent secondary asset prices at the interim states).

The competitive equilibrium of the model is not constrained efficient. The variation of secondary market prices with the volume of MMF redemptions interacts with firms' liquidity needs in a manner that generates a negative pecuniary externality. The redemptions of MMFs shares in the illiquid state exacerbate price declines and damage firms' capacity to invest ex post. A constrained social planner would reduce the savings placed in MMFs at the initial date. This would contribute to keeping secondary market prices higher in the illiquid aggregate state and support firms' profitable investment in that state.

As in other models with pecuniary externalities in a financial frictions setup (e.g. Lorenzoni, 2008, Dávila and Korinek, 2017), the root cause of inefficiency is the impact of prices on the underlying financial constraints. The existing assets provide firms with imperfect insurance against the shocks that they may experience at the interim date. The degree of insurance associated with MMFs shares depends on the aggregate redemptions experienced at that date which in turn are related to the size of the initial allocation of savings to MMFs. Individual firms choosing between deposits and MMFs at the initial date do not internalize their contribution to the fall of secondary market and share redemption prices in the aggregate illiquid state.

The identified externality provides a rationale for macroprudential policy aimed at enhancing the resilience of the MMFs sector. In the context of our model, restoring constrained efficiency requires attaining the proper allocation of savings across deposits and MMFs at the initial date and that can be achieved by, e.g., imposing a (Pigouvian) tax on MMFs savings. Other regulatory options, such as imposing liquidity requirements on MMFs (e.g. some minimum holding of bank deposits at the initial date so as to facilitate the accommodation of redemptions without a price impact at the interim date) might similarly reduce the inefficiency caused by the pecuniary externality. Importantly, however, policy interventions affecting the ex ante allocation of liquid savings across banks and MMFs are superior to imposing penalties or limitations to the agents redeeming their MMFs shares in the illiquid state since that would harm the (imperfect) liquidity insurance function that MMFs provide (and that a welfare maximizing intervention should aim to preserve or enhance).[10]

[10] In our formulation, the pricing and timing of MMFs share redemptions in the interim date prevents the dilution of non-redeeming investors, thus avoiding the inefficiency caused by first mover advantages, which has been the main focus of prior literature.

Related literature. We provide a first model of the interactions between banks and MMFs in a market equilibrium setup. Our contribution thus fits into the growing literature on the coexistence of bank and non-bank financial intermediaries (Plantin, 2015; Gertler, Kiyotaki and Prestipino, 2016; Moreira and Savov, 2017; Begenau and Landvoigt, 2018; Bengui and Bianchi, 2018; Ordoñez,2018; Martinez-Miera and Repullo, 2019; Jeanne and Korinek, 2020). In particular, our framework is related to other models where non-bank financial intermediaries add to banks in the provision of safe assets to investors (e.g. via the pooling and tranching of risky assets as in Gennaioli, Shleifer and Vishny, 2013; Ferrante, 2018; and Segura and Villacorta, 2020). However, in contrast to many papers in this tradition, we do not resort to infinite risk-aversion to motivate the preference of investors for assets delivering stable payoffs. Instead, we model this preference as driven by a classical precautionary motive: the explicit value of preserving the capability to undertake investment projects or satisfy liquidity needs if they arise.[11] Additionally to rationalizing the coexistence of banks and MMFs as liquidity providers, we are explicit about their interactions in the primary and secondary markets for money market securities.

Our model fits also into the literature studying financial fragility in the mutual fund sector (Chen, Goldstein and Jiang, 2010; Cipriani, Martin, McCabe, and Parigi 2014; Goldstein, Jiang and Ng, 2017; Cipriani and La Spada, 2020; Voellmy, 2021). This literature typically studies the fragility stemming from the existence of first mover advantages and potential runs in partial equilibrium frameworks, providing insights on how MMFs' pricing of redemptions (as well as the presence redemption fees or gates) affect, avoid or contribute to trigger investors' runs. This literature shows that the importance of first mover advantages can be largely reduced by the removal of market practices such as the promise of "stable net asset value" and by the introduction of anti-dilution pricing schemes such as swing pricing (see, Jin, Kacperczyk, Kahraman, and Suntheim, 2022, for recent empirical evidence on the effectiveness of the latter). In our model, the timing and pricing of redemptions allows us to abstract from first mover advantages and runs, and thus to focus on the pecuniary externalities caused at the market level which cannot be avoided by improvements on the private contracting side.[12]

Our paper is also connected to the literature studying the effects of bank regulation on liquidity provision. Our regulatory justification for the frictions that produce asset price

[11] See Cipriani and La Spada (2021) for evidence that MMFs investors value the stability of the redemption value associated with their shares.

[12] In this sense, we are closer to Cucic (2021), who studies issues of contract design and liquidity provision to MMFs in a model in which asset liquidations following high redemptions are subject to cash-in-the-market pricing.

declines when MMFs sell off their assets in secondary markets is related to the market microstructure models of Cimon and Garriott (2019) and Saar, Sun, Yang and Zhu (2020), where capital regulations induce banks to reduce the inventories used in market making activities and thereby increase the cost of immediacy. Regulation also plays a role in d'Avernas and Vandeweyer (2020) who model the trade of liquidity between leveraged shadow banks and banks using repos; they show that banks' intraday liquidity requirements can limit the supply of repo funding and explain spikes in its cost.

Finally, we contribute to the policy debate on the macroprudential regulation of mutual funds and, in particular, MMFs and debt funds. The turmoil triggered by the onset of the Covid-19 pandemic reinforced the interest of regulators in addressing the contribution of mutual funds to financial stability (e.g., see Bailey, 2021, Capotă, Grill, Molestina Vivar, Schmitz, and Weistroffer 2021, Clarida, Duygan-Bump and Scotti, 2021, Eren, Schrimpf, and Sushko, 2020). Our general equilibrium analysis highlights that the rationale for the macroprudential regulation of MMFs (or mutual funds more generally) does not only depend on vulnerabilities intrinsic to the institutional architecture of these funds, but also on features of the markets where they operate and the other intermediaries with which they trade. It also highlights the importance of focusing the normative discussion not just on signals of instability such as redemptions and the price declines caused by them (which might be unavoidable elements of a second best solution) but on the economic function played by the funds within the financial system and their contribution to an efficient provision of liquidity to investors.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 states the conditions that characterize the competitive equilibrium. Section 4 analyzes the main properties of the competitive equilibrium. Section 5 contains the efficiency analysis. Section 6 concludes.

2 Model

The economy lasts for three dates indexed by $t = 0, 1, 2$. The model is composed by measure-one continua of firms, banks, and MMFs. At $t = 0$ agents make their initial portfolio and fund raising decisions. At $t = 1$ assets yield their short-term returns; agents reoptimize following the realization of idiosyncratic and aggregate shocks; banks and MMFs accommodate the potential outflows and inflows of deposits and MMFs shares; trade in secondary asset markets occurs. At $t = 2$ assets yield their final returns and agents obtain their final payoffs.

Aggregate uncertainty is fully resolved at $t = 1$ where the economy may be in a normal state $\omega = 0$ or an illiquid state $\omega = 1$ with probabilities $1-\gamma$ and γ, respectively. In the next subsections, we describe each of the agent classes, introducing the idiosyncratic shocks that some of them experience at $t = 1$ and the decisions faced at each date.

2.1 Firms

Firms are indexed by i. They are owned and managed by risk neutral shareholders who aim to maximize the expected value of the terminal payoffs received from each firm at $t = 2$.

At $t = 0$ all firms are identical. They are endowed with net worth e_0^f and can invest in bank deposits d_0^f and MMFs shares m_0^f.[13] The price and return characteristics of each of these cash-like assets will be explained below when describing banks and MMFs, respectively. For notational simplicity, we assume each firm holds a well-diverfied portfolio of bank deposits across all banks.[14]

At $t = 1$ firms experience an aggregate liquidity shock if $\omega = 1$ as well as idiosyncratic shocks $\psi_i \in \{0, 1\}$ to their investment opportunities. The aggregate "dash for cash" liquidity shock ($\omega = 1$) means that all firms must hold liquid deposits at least equal to θe_0^f between $t = 1$ and $t = 2$.[15] This shock can be thought as capturing in reduced form the necessity to hold some minimal fully liquid savings for precautionary reasons (because of potential falls in sales, rises in input prices or margin calls related to financial exposures). The idiosyncratic investment shock ($\psi_i = 1$) occurs with probability π and provides the firm with an investment project that yields a gross return A at $t = 2$ per each unit of funds k invested at $t = 1$. On top of this, a fraction ϵ of the bank deposits brought from $t = 0$ become illiquid at $t = 1$ and are automatically rolled over until $t = 2$. These illiquid deposits do not qualify from the perspective of satisfying the liquidity needs θe_0^f emerging under $\omega = 1$.

To accommodate the investment and liquidity shocks at $t = 1$ firms can resort to their outstanding liquid bank deposits, to the redemption of MMFs shares at the prevailing price $q_1(\omega)$ or to any combination of these alternatives. Let $s_i^f = (\omega, \psi_i) \in S^f$ represent the state of firm i at $t = 1$. Then, for each state $s_i^f, \in S^f$, firms decide the bank deposits $d_1^f(s_i^f)$, MMFs shares $m_1^f(s_i^f)$, and real investment $k_1^f(s_i^f)$ with which to proceed to $t = 2$.

[13] As it will become clear, what we label as "firms" could be any agent, including households, that allocate their savings across bank deposits and MMFs shares at $t = 0$. For households, the investment projects introduced below might be reintrepreted as consumption opportunities from which they derive utility.

[14] This allows firms to diversify away the bank-idiosycratic risk affecting bank deposits at the interim date and allows us to analyze the model without keeping track of the distribution of bank shocks across firms. For an ex ante perspective, however, this simplification is inconsequential since firms are risk neutral, technologies are linear, and the diversified risk is orthogonal to other risks.

[15] We make the liquidity shock proportional to e_0^f to obtain scale free results regarding the allocation of firms' endowment across deposits and MMFs shares.

At $t = 2$, firms simply receive the final payoffs associated with the financial assets and real investment brought from $t = 1$.

2.2 Banks

Banks are indexed by j. They are owned and managed by risk neutral shareholders who aim to maximize the expected value of the terminal equity payoffs received from each bank at $t = 2$.

At $t = 0$ all banks are identical. They collect deposits d_0^b and issue commercial paper cp_0^b to invest an amount $a_0^b = d_0^b + cp_0^b$ in a safe, short-term bank asset with a gross return $1 + r_0$ at $t = 1$.[16]

At $t = 1$ a fraction ϵ of randomly selected banks become illiquid. So it is convenient to use the binary variable δ_j to indicate whether bank j remains liquid ($\delta_j = 0$) or not ($\delta_j = 1$) and $s_j^b = (\omega, \delta_j) \in S^b$ to represent the state of bank j at $t = 1$. Liquid banks can freely collect new deposits $d_1^b(s_j^b)$, buy commercial paper with face value $t_1^b(s_j^b)$ in the secondary market, and invest $a_1^b(s_j^b)$ in a safe, short-term bank asset with a return $1 + r_1$ at $t = 2$.[17] In contrast, the balance sheet of illiquid banks freezes, meaning that prior deposits become non-convertible and are automatically rolled over until $t = 2$, they cannot take new deposits or participate in the secondary market for commercial paper, and prior asset investments are rolled over too.[18] So for illiquid banks we have $d_1^b(s_j^b) = d_0^b/p_1^D(\omega)$, $t_1^b(s_j^b) = 0$, and $a_1^b(s_j^b) = (1 + r_0)a_0^b$.

Bank deposits are one-period zero-coupon debt issued at unit prices p_0^D at $t = 0$ and $p_1^D(\omega)$ at $t = 1$.[19] Bank commercial paper is two-period zero-coupon debt issued at price p_0^{CP} at $t = 0$ and maturing at $t = 2$. Commercial paper trades in the secondary market at a price $p_1^{CP}(\omega)$ at $t = 1$.

Importantly, we assume that banks have to incur a cost $\lambda(\omega)$ per unit of commercial paper bought in the secondary market at $t = 1$. Capturing frictions that grow with the

[16]The analysis could be extended to allow banks to initially invest in long-term assets which at $t = 1$ might be convertible in central bank reserves through their sale or their use as collateral in, e.g., repo transactions or a borrowing facility of the central bank. In this case, the rate of conversion of bank assets into liquidity at $t = 1$ might depend on applicable prices and haircuts but the key insights from the analysis of the current setup would continue to apply.

[17]Formally, for liquid banks, we will not impose any sign constraint on either $t_1^b(s_j^b)$ or $a_1^b(s_j^b)$, which means that these banks can both buy and sell commercial paper and the safe short-term asset at $t = 1$.

[18]Under this formulation, banks turning illiquid at $t = 1$ do not fail and pay the roll-over value of the initial deposits at $t = 2$. The illiquidity shock might then be interpreted as the result of operational risk (e.g. a cyber-attack or the collapse of the bank's IT system) or some more fundamental shock to the liquidity of bank assets.

[19]Deposits turning illiquid at $t = 1$, however, see their maturity effectively extended until $t = 2$.

volumes of commercial paper that banks have to absorb (or with the selling pressure in the secondary market) at $t = 1$, we assume $\lambda(\omega)$ is (linearly) increasing in the aggregate amount of commercial paper bought by banks at that date:

$$\lambda(\omega) = \frac{v}{e_0^f} \int t_1^b(s_j^b)\, dj, \tag{1}$$

where v is a constant.[20] The presence of e_0^f in (1) is explained to avoid scale effects, so that the importance of the secondary market frictions grows with the volume of secondary trade *relative to* outstanding amounts (rather than the absolute volume of trade).

At $t = 2$, banks receive the final payoffs from their assets, repay their deposits and commercial paper, and distribute the residual net worth to their shareholders.

2.3 MMFs

MMFs act as investment vehicles for the owners of their shares. All MMFs are initially identical and remain identical throughout the analysis so we will refer to the variables describing each fund and the sector as a whole with the same notation. At $t = 0$, MMFs receive an inflow m_0^f of savings from firms and invest it in a portfolio of commercial paper issued by banks. Thus they demand commercial paper with total face value $cp_0^m = m_0^f / p_0^{CP}$.[21] The unit value of MMFs shares at $t = 0$ is normalized to one, $q_0^m = 1$, so that the number of initial MMFs shares is just m_0^f.

At $t = 1$ MMFs face potential net redemptions of their shares from firms, $m_0^f - \int m_1^f(s_i^f)di$. Each MMFs share can be redeemed at its marked-to-market valuation, $q_1(\omega)$, which is determined as the result of dividing the market value of MMFs assets (the commercial paper that they hold) by the total number of MMFs shares. MMFs accommodate their redemptions with net sales of commercial paper with total face value $t_1^m(\omega)$ so that

$$p_1^{CP}(\omega)\, t_1^m(\omega) = q_1(\omega) \left(m_0^f - \int m_1^f(s_i^f)\, di \right), \tag{2}$$

where $p_1^{CP}(\omega)$ is the secondary market price of commercial paper at $t = 1$ in state ω.[22]

[20] While we do not need to impose a sign restriction on $t_1^b(s_j^b)$, it will become clear that in equilibrium banks never face a strictly positive net demand for commercial paper at $t = 1$ since MMFs experience no strictly positive net inflows from firms at $t = 1$. So there is no loss of generality in treating $t_1^b(s_j^b)$ as non-negative.

[21] We could extend the model to allow MMFs to also invest in other securities (e.g. government bonds). The extended model would work in a qualitatively similar way provided that the sale of securities by MMFs to banks in the aggregate illiquid state is subject to the same type of frictions (at the margin) as the sale of commercial paper under the current formulation.

[22] While in equilibrium, under our assumptions, MMFs will never be net buyers of commercial paper in the secondary market, we do not impose any sign constraint on $t_1^m(\omega)$, or the net redemptions $m_0^f - \int m_1^f(s_i^f)di$.

At $t = 2$ MMFs distribute the residual value of their commercial paper holdings among the outstanding shares on a pro-rata basis, determining the corresponding terminal payoff per share, $q_2(\omega)$.

2.4 Overview and interpretation of the model

As an overview of the timing and key ingredients of the model, the various panels of Figure 1 display the balance sheets (or uses and sources of funds, when so indicated) of firms, banks, and MMFs at each date. The balance sheets at $t = 2$ represent financial statements right before banks and MMFs settle their obligations vis-à-vis their liability holders.

The model contains a number of financial frictions and shocks that rationalize the coexistence of banks and MMFs or, equivalently, of bank deposits and bank commercial paper (since MMFs in this economy are essentially the vehicle through which final investors invest in the latter). Investment opportunities in the model include the safe short-term assets in which banks can invest at $t = 0, 1$ and the investment projects that (some of the) firms obtain at $t = 1$. To make the model interesting, we assume $A > 1 + r_1$ so that the firms receiving an investment project at $t = 1$ can obtain greater returns than the banks investing in their own assets between $t = 1$ and $t = 2$.

Importantly, firms lack access to external financing at $t = 1$. They can only undertake their projects or satisfy their liquidity needs with the liquid deposits and MMFs shares brought from $t = 0$. And there are no insurance markets in which they can buy specific protection for the investment shocks or the liquidity shocks. So they imperfectly self-insure through the allocation of their initial endowments across deposits and MMFs shares—or, equivalently as said above, across deposits and banks' commercial paper.

Firms' choice between deposits and commercial paper at $t = 0$ is not trivial because of different liquidity properties of each of these instruments. Deposits are non-tradable. Those made at $t = 0$ mature, in principle, at $t = 1$, but if the issuing bank turns illiquid, they are frozen (or forcely rolled over) until $t = 2$. In this case deposits fail to serve their role in insuring against investment and liquidity shocks. Commercial paper issued at $t = 0$ matures at $t = 2$ but is tradable at $t = 1$. In the absence of frictions to the trading of commercial paper at $t = 1$, commercial paper would have the advantage vis-a-vis deposits of remaining liquid (via tradability) even when the issuing bank turns illiquid. However, the presence of trading costs introduces a countervailing effect: it depresses the resale value of commercial paper (or the redemption value of MMFs shares) when there are net sales from the initial holders (and, more specifically, as we will see, when the economy is hit by the aggregate liquidity shock).

While it would be possible to capture additional differences between deposits and commercial paper funding from the perspective of the issuers (e.g., in terms of banks' management of their refinancing needs at $t = 1$), those affecting their holders are enough to produce trade-offs leading to interior equilibrium solutions to firms' saving allocation problem at $t = 0$ and, thus, to the coexistence of the two instruments.

Balance sheets at $t = 0$:

Firms

Deposits $p_0^D d_0^f$	Net worth e_0^f
MMFs shares m_0^f	

Banks

Assets a_0^b	Deposits $p_0^D d_0^b$
	CP $p_0^{CP} cp_0^b$

MMFs

Commercial paper (CP) $p_0^{CP} cp_0^m$	Shares m_0^f

Balance sheets (or uses and sources of funds, when indicated) at $t = 1$:

Firm i (uses and sources of funds)

Illiquid deposits ϵd_0^f	Illiquid deposits ϵd_0^f
Deposits $p_1^D(\omega) d_1^f(s_i^f)$	Past liquid deposits $(1-\epsilon)\, d_0^f$
MMFs shares $q_1(\omega) m_1^f(s_i^f)$	Past MMFs shares $q_1(\omega) m_0^f$
Investment in project $k_1^f(s_i^f)$	

Bank j (uses and sources of funds)

Assets $a_1^b(s_j^b)$	Past assets $(1+r_0)\, a_0^b$
CP $p_1^{CP}(\omega)\,(1+\lambda(\omega))\, t_1^b(s_j^b)$	Net deposit funding $p_1^D(\omega) d_1^b(s_j^b) - d_0^b$

MMFs

CP $p_1^{CP}(\omega)\,(cp_0^m - t_1^m(\omega))$	Shares $q_1(\omega) \int m_1^f(s_i^f) di$

Balance sheets at $t = 2$:

Firm i

Illiquid deposits $\epsilon d_0^f / p_1^D(\omega)$	Net worth
Liquid deposits $d_1^f(s_i^f)$	
MMFs shares $q_2(\omega) m_1^f(s_i^f)$	
Real assets $A k_1^f(s_i^f)$	

Bank j

Assets $(1+r_1)\, a_1^b(s_j^b)$	Deposits $d_1^b(s_j^b)$
CP $t_1^b(s_j^b)$	CP cp_0^b
	Net worth

MMFs

CP $cp_0^m - t_1^m(\omega)$	Shares $q_2(\omega) \int m_1^f(s_i^f) di$

Figure 1 Balance sheets of firms, banks, and MMFs.

As the normative analysis of the model will reveal, the resale value of commercial paper (or redemption value of MMFs shares) will interact with firms' investment and liquidity needs in a way that generates a negative pecuniary externality associated with firms' initial investment in commercial paper (or MMFs shares), making the competitive equilibrium of the model constrained inefficient. Importantly, restoring constrained efficiency will not imply prohibiting the investment in commercial paper (or in MMFs) but rebalancing the initial allocation of firms' savings taking the marginal externality into account.

In the following sections of the paper we first characterize the competitive equilibrium of the model and then study its efficiency properties, showing specific policy interventions through which a social planner might restore its constrained efficiency.

3 Equilibrium conditions

A competitive equilibrium in this economy consists of an allocation $\left\{d_0^f, m_0^f, d_0^b, cp_0^b, a_0^b, cp_0^m, \{d_1^f(s^f), m_1^f(s^f), k_1^f(s^f)\}_{s^f\in S^f}, \{d_1^b(s^b), t_1^b(s^b), a_1^b(s^b)\}_{s^b\in S^b}, \{t_1^m(\omega)\}_{\omega=0,1}\right\}$ and prices $\{p_0^D, p_0^{CP}, \{p_1^D(\omega), p_1^{CP}(\omega)\}_{\omega=0,1}\}$ such that firms, banks and MMFs solve their optimization problems and markets clear. To obtain the equations that characterize the equilibrium of the model, we proceed by backwards induction, studying optimization and market clearing conditions at $t = 0, 1, 2$ in reverse order.

3.1 Equilibrium at the final date ($t = 2$)

At the final date there are no decisions to be made. The owners of the firms receive their final net worth, which consists of bank deposits, MMFs shares and, if applicable, the return of the investment project undertaken at $t = 1$. Thus the value of firms to their owners can be written as

$$V_2^f(s_i^f) = \frac{\epsilon d_0^f}{p_1^D(\omega)} + d_1^f(s_i^f) + q_2(\omega)\, m_1^f(s_i^f) + \psi_i A k_1^f(s_i^f), \tag{3}$$

where the first term accounts for the repayment of deposits invested at $t = 0$ in a bank turning illiquid at $t = 1$ and the other terms account for the gross returns of asset holdings decided at $t = 1$ (liquid deposits, MMF shares and real investment).

Banks simply obtain the terminal value of their assets and commercial paper holdings, and repay the face value of their claims to depositors and commercial paper holders. Any residual net worth would be returned to bank shareholders, whose value would then be

$$V_2^b(s_j^b) = (1 + r_1)\, a_1^b(s_j^b) + t_1^b(s_j^b) - d_1^b(s_j^b) - cp_0^b. \tag{4}$$

MMFs fully distribute the value of their remaining commercial paper among the firms holding their (not previously redeemed) shares. The liquidation value of each share, $q_2(\omega)$, can be obtained from the balance sheet of the representative MMF:

$$q_2(\omega)\int m_1^f(s_i)di = cp_0^m - t_1^m(\omega). \tag{5}$$

3.2 Equilibrium at the interim date ($t = 1$)

Agents enter $t = 1$ with assets and liabilities carried from the previous period (see Figure 1). Additionally, the aggregate shock ω realizes and each firm i and bank j learn the realization of their idiosyncratic shocks, ψ_i and δ_j, respectively.

3.2.1 Firms at $t = 1$

Firms arrive at $t = 1$ with bank deposits d_0^f and MMFs shares m_0^f. They choose the amounts of deposits $d_1^f(s_i^f)$ and MMFs shares $m_1^f(s_i^f)$ to carry to $t = 2$, as well as their real investment $k_1^f(s_i^f)$, which will be zero in the absence of an investment project ($\psi_i = 0$). Firms' continuation value is determined by the the maximization of the expected value of their final worth, $V_2^f(s_i^f)$, subject to the relevant constraints:

$$V_1^f(d_0^f, m_0^f; s_i^f) = \max_{\left\{d_1^f(s_i^f), m_1^f(s_i^f), k_1^f(s_i^f)\right\}} \left\{ \frac{\epsilon d_0^f}{p_1^D(\omega)} + d_1^f(s_i^f) + q_2(\omega)\, m_1^f(s_i^f) + \psi_i A k_1^f(s_i^f) \right\} \tag{6}$$

$$\text{s.t.:} \quad p_1^D(\omega)\, d_1^f(s_i^f) + \psi_i k_1^f(s_i^f) = (1-\epsilon)\, d_0^f + q_1(\omega)\left(m_0^f - m_1^f(s_i^f)\right), \tag{7}$$

$$d_1^f(s_i^f) \geq \omega\theta e_0^f, \text{ and} \tag{8}$$

$$m_1^f(s_i^f), k_1^f(s_i^f) \geq 0. \tag{9}$$

The budget constraint in (7) imposes that liquid bank deposits and real investment at $t = 1$ must be financed with the outstanding liquid deposits or the redemption of MMFs shares brought from $t = 0$. The requirement of holding liquid deposits of at least θe_0^f in the illiquid state appears in (8). Condition (9) adds non-negativity constraints for MMFs shares and real investment at $t = 1$.

The solution of this problem generally depends on the holdings (d_0^f, m_0^f) brought from $t = 0$ and the state vector s_i^f that summarizes the shocks experienced by each firm. To streamline the presentation, we focus our analysis on parameterizations of the model for which firms' equilibrium decisions are as in the *conjectured solution* that we describe next, along with the conditions required for the optimality of such solution:

1. Firms with an investment project aim to undertake it at maximum scale, liquidating all their financial assets except illiquid deposits and the minimal liquid deposits θe_0^f required in state $\omega = 1$. This means that in states $s_i^f = (\omega, 1)$, for any ω, firms choose:

$$d_1^f(s_i^f) = \omega\theta e_0^f, \tag{10}$$

$$m_1^f(s_i^f) = 0, \text{ and} \tag{11}$$

$$k_1^f(s_i^f) = (1-\epsilon)\, d_0^f + q(\omega)m_0^f - p_1^D(\omega)\,\omega\theta e_0^f \geq 0. \tag{12}$$

Using (7) to substitute for $k_1^f(s_i^f)$ in (6), we can observe that, for firms with $\psi_i = 1$, the objective function in (6) is (weakly) decreasing in both $d_1^f(s_i^f)$ and $m_1^f(s_i^f)$ if and only if

$$-Ap_1^D(\omega) + 1 \leq 0, \text{ and} \tag{13}$$

$$-Aq_1(\omega) + q_2(\omega) \leq 0, \tag{14}$$

for all ω. So the optimality of the conjectured solution requires:

$$A \geq \max\left\{\frac{1}{p_1^D(0)}, \frac{1}{p_1^D(1)}, \frac{q_2(0)}{q_1(0)}, \frac{q_2(1)}{q_1(1)}\right\}, \tag{15}$$

that is, the return on the investment project must be, in all aggregate states, at least as large as the returns of any of the financial alternatives (deposits and MMFs shares).

2. Firms without an investment project when the aggregate liquidity shock realizes ($\omega = 1$) hold the minimal amount of liquid deposits θe_0^f required in that state, thus minimizing their (strictly positive) redemption of MMFs shares.[23] So in state $s_i^f = (1, 0)$, firms choose:

$$d_1^f(s_i^f) = \theta e_0^f, \tag{16}$$

$$m_1^f(s_i^f) = \frac{q_1(1)\, m_0^f + (1-\epsilon)\, d_0^f - p_1^D(1)\,\theta e_0^f}{q_1(1)} \geq 0, \text{ and} \tag{17}$$

$$k_1^f(s_i^f) = 0. \tag{18}$$

From (6) and (7), for this solution to be optimal, the return to investing in MMFs shares at $t = 1$ must be at least as large as that of saving in bank deposits, which requires

$$\frac{q_2(1)}{q_1(1)} \geq \frac{1}{p_1^D(1)}. \tag{19}$$

[23] Intuitively, this behavior implies the lowest possible fire-sale effects on commercial paper prices (and hence MMFs redemption values) in the aggregate illiquid state.

3. Firms without an investment project when the aggregate liquidity shock does not realize ($\omega = 0$) remain indifferent between investing their liquid resources in deposits or in MMFs shares. This means that in state $s_i^f = (0,0)$, firms find it optimal to choose any combination of $d_1^f(s_i^f)$ and $m_1^f(s_i^f)$ satisfying their budget constraint, which in this state simplifies to

$$p_1^D(0)\, d_1^f(s_i^f) = (1-\epsilon)\, d_0^f + q_1(0)\left(m_0^f - m_1^f(s_i^f)\right). \tag{20}$$

Considering (6) and (7), these firms' indifference between MMFs shares and deposits at $t = 1$ arises if and only if

$$\frac{q_2(0)}{q_1(0)} = \frac{1}{p_1^D(0)}. \tag{21}$$

Intuitively, firms' behavior in the conjectured solution is compatible with an equilibrium in which (strictly positive net) redemptions of MMFs shares in the aggregate illiquid state reduce the price of commercial paper (and hence the redemption value of those shares). The decline in the price of MMFs shares is large enough for firms without investment projects to (at least weakly) prefer investing in MMFs shares to investing in deposits at $t = 1$, but not large enough to induce firms with investment projects to pass them up. Yet, the fall in the redemption value of the MMFs shares bought at $t = 0$ causes these firms to invest less. Thus, for parameterizations confirming the existence of this equilibrium, a marginal worsening of the effects of commercial paper sales on MMFs redemption prices in the illiquid state will cause further declines in firms' real investment, being the channel for the transmission of a welfare-relevant pecuniary externality.[24]

3.2.2 Banks at $t = 1$

Banks arrive at the interim date with assets a_0^b, deposits d_0^b, and outstanding commercial paper cp_0^b. Illiquid banks ($\delta_j = 1$) get their balance sheet frozen, which means continuing with rolled-over assets $a_1^b(s_j^b) = (1 + r_0)a_0^b$, rolled-over deposits $d_1^b(s_j^b) = d_0^b/p_1^D(\omega)$, and zero purchases of commercial paper, $t_1^b(s_j^b) = 0$, as previously described. In contrast, liquid banks ($\delta_j = 0$) choose $a_1^b(s_j^b)$, $d_1^b(s_j^b)$ and $t_1^b(s_j^b)$ to maximize the expected value of their final net worth, $V_2^b(s_j^b)$, subject to the relevant constraints. So their continuation value can be

[24] Alternative parameterizations may lead to equilibria where the pecuniary externality is also present in the liquid state, although at a lower intensity. They might also lead to situations in which the aggregate liquidity shock is so large under $\omega = 1$ that even firms with $\psi = 1$ prefer investing in MMF shares at $t = 1$. In such a case, however, real investment would not vary at the margin with the fall in MMFs redemption values at $t = 1$, so the normative analysis would be less interesting.

expressed as:

$$V_1^b\left(d_0^b, cp_0^b; s_j^b\right) = \max_{\left\{a_1^b(s_j^b), d_1^b(s_j^b), t_1^b(s_j^b)\right\}} \left\{(1+r_1)\, a_1^b(s_j^b) + t_1^b(s_j^b) - d_1^b(s_j^b) - cp_0^b\right\} \tag{22}$$

$$\text{s.t.:} \quad a_1^b(s_j^b) + p_1^{CP}(\omega)\,(1+\lambda(\omega))\, t_1^b(s_j^b) = (1+r_0)\,(p_0^D d_0^b + p_0^{CP} cp_0^b) + (p_1^D(\omega)\, d_1^b(s_j^b) - d_0^b) \tag{23}$$

$$\text{and } d_1^b(s_j^b) \geq 0, \tag{24}$$

where we have used (4) to write the objective function, and the balance sheet constraint of the initial date to substitute $p_0^D d_0^b + p_0^{CP} cp_0^b$ for a_0^b in the budget constraint (23).

Using (23) to further replace $a_1^b(s_j^b)$ in (22), the problem of the bank simplifies to maximizing the resulting objective function with respect to $d_1^b(s_j^b)$ and $t_1^b(s_j^b)$ subject only to the non-negativity of $d_1^b(s_j^b)$. Since the reparameterized problem is linear in its decision variables, the first order conditions characterizing an interior solution deliver the following pricing conditions for deposits and commercial paper at $t = 1$:[25]

$$p_1^D(\omega) = \frac{1}{1+r_1} \text{ and} \tag{25}$$

$$p_1^{CP}(\omega) = \frac{1}{(1+r_1)\,(1+\lambda(\omega))}. \tag{26}$$

Condition (25) implies that the supply of bank deposits by liquid banks at $t = 1$ is perfectly elastic at an implicit yield equal to the rate of return r_1 that the bank can earn on its assets. Similarly, condition (26) makes the bank willing to purchase (or issue if $t_1^b(s_j^b) < 0$) any amount of commercial paper insofar as its price implies a net yield (after accounting for the trading cost $\lambda(\omega) > 0$ if $t_1^b(s_j^b) > 0$) also equal to the return on its assets.

3.2.3 MMFs at $t = 1$

At $t = 1$ MMFs sell commercial paper in the amount $t_1^m(\omega)$ required to accommodate firms' net redemptions, as determined by (2). The marked-to-market pricing of MMFs shares at $t = 1$ implies:

$$q_1(\omega) = \frac{p_1^{CP}(\omega)\, cp_0^m}{m_0^f}. \tag{27}$$

Putting together (2) and (27) implies

$$t_1^m(\omega) = \left(1 - \frac{\int m_1^f(s_i^f)\, di}{m_0^f}\right) cp_0^m. \tag{28}$$

[25] Notice that a potential corner solution to liquid banks' problem involving $d_1^b(s_j^b) = 0$ would not be compatible with firms' decisions in the equilibrium on which we focus, so we can safely ignore it.

Thus, MMFs sell a proportion of their initial commercial paper holdings exactly equal to the redeemed proportion of their shares. So, by construction, the price at which MMFs shares are redeemed prevents the dilution of the non-redeeming investors and rules out first-mover advantages in our model.[26]

3.2.4 Market clearing at $t = 1$

The clearing of the markets for deposits at liquid banks and commercial paper at $t = 1$ requires

$$\int d_1^b(s_j^b)\,dj - \frac{\epsilon d_0^f}{p_1^D(\omega)} = \int d_1^f(s_i^f)\,di \quad \text{and} \tag{29}$$

$$\int t_1^b(s_j^b)\,dj = t_1^m(\omega), \tag{30}$$

respectively.[27]

3.3 Equilibrium at the initial date ($t = 0$)

At the initial date agents solve their optimization problems taking prices as given and anticipating the equilibrium prices and returns emerging at $t = 1$ and $t = 2$ under each possible realization of the aggregate shock ω at $t = 1$.

3.3.1 Firms at $t = 0$

Firms decide the allocation of their net worth endowment e_0^f across deposits d_0^f and MMFs shares m_0^f in order to maximize their expected net worth at $t = 2$. Their optimization problem can be written as

$$\max_{\{d_0^f, m_0^f\}} \quad \mathbb{E}_0\left[V_1^f\left(d_0^f, m_0^f; s_i^f\right)\right] \tag{31}$$

$$\text{s.t.:} \quad p_0^D d_0^f + m_0^f = e_0^f \text{ and} \tag{32}$$

$$d_0^f, m_0^f \geq 0, \tag{33}$$

where $V_1^f(d_0^f, m_0^f; s_i^f)$ is their continuation value at $t = 1$, which is determined in (6)-(9).

[26] First mover advantages can be relevant in practice due to dynamics within periods in which investors (sequentially) approach the MMFs to redeem their shares. By abstracting from them, we keep the model parsimonious and highlight the existence of frictions that lead to inefficiency in the allocation of savings to MMFs even in the absence of first mover advantages.

[27] To explain the left hand side of (29), notice that bank deposits $d_1^b(s_j^b)$ include the rolled-over deposits of illiquid banks, while the deposits $d_1^f(s_i^f)$ only include firms' deposits at liquid banks.

The conjectured solution to firms' problem at $t = 1$ is linear in d_0^f and m_0^f, while (6) is linear in firms' decisions at $t = 1$, so the objective function in (31) is linear in d_0^f and m_0^f. The constraints in (32) and (33) are also linear in both decision variables. Using (32) to substitute for d_0^f in the objective function, the problem can be re-written as one of choosing just $m_0^f \in [0, e_0^f]$ to maximize the resulting linear objective function. Having an interior solution would then require that the derivative of that objective function with respect to m_0^f is zero, that is:

$$\frac{d\mathbb{E}_0\left[V_1^f\left((e_0^f - m_0^f)/p_0^D, m_0^f; s_i^f\right)\right]}{dm_0^f} = 0. \tag{34}$$

Computing this derivative leads us to the next result. All proofs appear in the Appendix.

Lemma 1 *Under the conjectured solution to firms' problem at $t = 1$, firms will be indifferent between investing in deposits or in MMFs shares at $t = 0$ if and only if equilibrium prices satisfy the following condition:*

$$\begin{aligned} \frac{1}{p_0^D}\left\{(1-\epsilon)\left[\pi A + (1-\pi)\left(\frac{1-\gamma}{p_1^D(0)} + \frac{\gamma q_2(1)}{q_1(1)}\right)\right] + \epsilon\left(\frac{1-\gamma}{p_1^D(0)} + \frac{\gamma}{p_1^D(1)}\right)\right\} = \\ \pi A\left[(1-\gamma)\, q_1(0) + \gamma q_1(1)\right] + (1-\pi)\left[(1-\gamma) q_2(0) + \gamma q_2(1)\right] \end{aligned} \tag{35}$$

Condition (35) is essentially a non-arbitrage condition that equates the expected return from investing in bank deposits (the term in the left hand side of the expression) to that of investing in MMFs shares (the two positive terms in the right hand side). In the expression for the expected returns to investing in bank deposits at $t = 0$, the first term within the curly brackets reflects that the fraction of deposits remaining liquid allow to optimally undertake the investment project with return A if the firm receives it and, otherwise, either the rollover of the investment in bank deposits (when $\omega = 0$) or the investment in (relatively cheap) MMFs shares (when $\omega = 1$). The second term accounts for the returns at $t = 2$ of the fraction of deposits turning illiquid at $t = 1$.

The expected return from investing in MMFs is that of redeeming the shares at $t = 1$ to undertake real investment with gross return A if the firm obtains an investment project at that date (which happens with probability π) plus that of just keeping the MMFs shares until $t = 2$ if the investment project does not realize (that is, with probability $1 - \pi$). Elements in $q_t(\omega)$ reflect the role in the calculations of the redemption or terminal values of the MMFs shares at each state or date under firms' conjectured optimal behavior at $t = 1$.

3.3.2 Banks at $t = 0$

Banks decide the supply of deposits d_0^b and the issuance of commercial paper cp_0^b with which to finance their assets a_0^b in order to maximize their expected net worth at $t = 2$. Building on the expression for the continuation value of the bank at $t = 1$ in (22)-(24) (where we had already used the balance sheet constraint $a_0^b = p_0^D d_0^b + p_0^{CP} cp_0^b$ to substitute for a_0^b), the problem of the representative bank at the initial date can be written as:

$$\max_{\{d_0^b, cp_0^b\}} \quad \mathbb{E}_0 \left[V_1^b \left(d_0^b, cp_0^b; s_j^b \right) \right] \tag{36}$$

$$\text{s.t.:} \qquad d_0^b, cp_0^b \geq 0. \tag{37}$$

Since this problem is linear in d_0^b and cp_0^b, sustaining interior solutions requires banks to be indifferent between any possible choices of both variables. Thus, the derivatives of $\mathbb{E}_0[V_1^b(d_0^b, cp_0^b; s_j^b)]$ with respect to d_0^b and cp_0^b must both equal zero, which implies

$$p_0^D = \frac{1}{1 + r_0} \text{ and} \tag{38}$$

$$p_0^{CP} = \frac{1}{(1 + r_0)(1 + r_1)}. \tag{39}$$

Condition (38) shows that deposits taken at $t = 0$ facilitate the investment in bank assets with return r_0 for one period so, if deposits paid a yield different from r_0, banks would strictly prefer to supply either no deposits at all or an infinite amount of them. In a similar vein, issuing commercial paper allows the bank to invest in bank assets for two consecutive periods so (39) is the condition for this activity not to generate either strictly negative or strictly positive net worth returns.

Not surprisingly, given the constant returns to scale technology (and no initial net worth) with which the competitive banks operate in our economy, the conditions compatible with reaching interior solutions to banks' decision problems at $t = 0$ and $t = 1$ imply that banks' terminal net worth will be zero under both realizations of the aggregate shock ω.

3.3.3 MMFs at $t = 0$

MMFs use the funds that firms invest in their shares, m_0^f, to buy bank commercial paper cp_0^m according to their balance sheet constraint:

$$m_0^f = p_0^{CP} cp_0^m. \tag{40}$$

3.3.4 Market clearing at $t = 0$

The clearing of the markets for deposits and commercial paper at the initial date requires

$$d_0^b = d_0^f \text{ and} \tag{41}$$
$$cp_0^b = cp_0^m, \tag{42}$$

respectively.

4 Equilibrium analysis

In the previous section, after defining a competitive equilibrium in the context of our model, we obtained the conditions that characterize it under a "conjectured" configuration of the solution to firms' problem at $t = 1$. In this section we develop some formal results that describe the implications of these conditions for equilibrium prices and quantities and provide explicit restrictions on parameters under which a (unique) equilibrium with the conjectured configuration exists. We also provide comparative statics results showing the impact of parameters on equilibrium prices and quantities.

4.1 Equilibrium prices

The first result in this section refers to the pricing of deposits and commercial paper and the valuation of MMFs shares in the conjectured equilibrium.

Lemma 2 *Conditions for the existence of the conjectured equilibrium imply the following asset prices and valuations of MMFs shares:*

$$p_0^D = \tfrac{1}{1+r_0}, \qquad p_0^{CP} = \tfrac{1}{(1+r_0)(1+r_1)}; \tag{43}$$

$$p_1^D(0) = \tfrac{1}{1+r_1}, \qquad p_1^{CP}(0) = \tfrac{1}{1+r_1}, \qquad q_1(0) = 1 + r_0; \tag{44}$$

$$p_1^D(1) = \tfrac{1}{1+r_1}, \qquad p_1^{CP}(1) = \tfrac{1}{(1+r_1)(1+\lambda(1))}, \qquad q_1(1) = \tfrac{1+r_0}{1+\lambda(1)}; \tag{45}$$

$$q_2(0) = q_2(1) = (1 + r_0)(1 + r_1). \tag{46}$$

Under the conjectured configuration of equilibrium, most asset prices are trivially connected to the rates of return r_t on banks' safe short-term assets at $t = 0$ and $t = 1$. The only exceptions are the prices of commercial paper and MMFs shares at $t = 1$ in the state of aggregate illiquidity, which suffer the extra discount $\lambda(1)$ due to the frictions in the secondary market for commercial paper (the costs that banks face in absorbing such commercial paper at $t = 1$). The following result shows that combining the prices in Lemma 2 with the indifference condition provided in Lemma 1 leads to a unique candidate equilibrium value for $\lambda(1)$.

Lemma 3 *Conditions for the existence of the conjectured equilibrium determine a unique candidate equilibrium value* $\lambda^* > 0$, *implicitly defined by*

$$\pi\epsilon[A - (1 + r_1)] = \gamma\lambda^* \left[\frac{\pi A}{1 + \lambda^*} + (1 - \epsilon)(1 - \pi)(1 + r_1) \right], \tag{47}$$

for the price discount $\lambda(1)$ *suffered by commercial paper in the state of aggregate illiquidity* $(\omega = 1)$.

Equation (47) restates the condition for firms to be indifferent between investing in bank deposits or in MMFs shares at $t = 0$. After taking relevant equilibrium prices into account, (47) is a condition that only involves parameters and the endogenous secondary-market discount λ^* suffered by commercial paper in state $\omega = 1$. To understand the expression that determines this discount, it is better to rearrange the terms in (47) in the following manner

$$\epsilon \left\{ \pi[A - (1 + r_1)] + (1 - \pi)\gamma\lambda^*(1 + r_1) \right\} = \gamma \left\{ \pi \frac{\lambda^* A}{1 + \lambda^*} + (1 - \pi)\lambda^*(1 + r_1) \right\}, \tag{48}$$

and to think of each side as collecting expected terminal net worth losses associated with each saving alternative relative to having arrived at $t = 1$ with a full unit of liquid deposits. The left hand side of (48) collects the terms multiplied by ϵ, thus showing all the costs implied by having a fraction of illiquid bank deposits. Frozen deposits at $t = 1$ impede the firm with an investment project (so the probability π) to profit from the differential return between the real investment, A, and the investment in bank deposits, $1 + r_1$. Instead, frozen deposits when the firm does not have a real investment opportunity only imply missing the option, in the aggregate illiquid state (so the probability γ), of obtaining an extra return $\lambda^*(1 + r_1)$ by investing in relatively cheap MMFs shares rather than deposits between $t = 1$ and $t = 2$.[28]

The right hand side of (48) collects the costs implied by bringing savings from $t = 0$ in the form of MMFs shares whose redemption at $t = 1$ is potentially subject to the discount λ^* in the aggregate illiquid state (so the probability γ affecting the whole expression). The first term in curly brackets is the value loss that occurs from having to undertake the investment

[28] Notice that the return from investing in underpriced MMFs shares is $q_2(1)/q_1(1) = (1 + r_1)(1 + \lambda^*)$, while deposits only pay $1 + r_1$.

project with funds obtained through the redemption of underpriced MMFs shares.[29] The second term is, as in the left hand side, the loss due to missing the option, in the absence of an investment project, of investing in underpriced MMFs shares at $t = 1$.

The comparative statics of λ^* as defined by (47) illustrates the influence of the various elements of our theory of the coexistence of deposits and MMFs shares.

Proposition 1 *Parameters affect the equilibrium price discount λ^* featured by the secondary market in the state of aggregate illiquidity as indicated by the signs below each parameter in the following representation of equation (47):*

$$\lambda^* = L(\underset{+}{\pi},\ \underset{+}{A},\ \underset{+}{\epsilon},\ \underset{-}{\gamma},\ \underset{0}{\theta},\ \underset{0}{r_0},\ \underset{-}{r_1},\ \underset{0}{\upsilon},\ \underset{0}{e_0^f}). \tag{49}$$

Accordingly, the equilibrium price discount in the state of aggregate illiquidity increases with those parameters that make deposits comparatively less attractive (including ϵ but also π and A) and decreases with those that make deposits more attractive (γ and r_1). The logic for all these results is to offset variations that would otherwise break firms' indifference condition at the initial date. Perhaps the most surprising results are those referred to the probability and attractiveness of firms' investment projects (π and A). They imply that, for a fixed λ^*, the prospect of finding better uses for liquidity at the interim date, makes MMFs shares relatively more attractive than deposits. This "procyclical" attractiveness of MMFs is related to the fact that a deposit turning illiquid would imply a full loss of the net present value of the investment that could have been undertaken with the corresponding funds, while the realization of the illiquid state when holding MMFs shares only implies a partial loss (due the price discount λ^*) in the value of the funds available to undertake the investment.

Before turning to the analysis of the implications for equilibrium quantities, the following lemma revisits the condition in (15) that makes firms with investment projects at the interim date to prefer their undertaking to the holding of MMFs shares or bank deposits up to the final date.

Lemma 4 *Under the pricing conditions in Lemmas 2 and 3, the condition*

$$\gamma \geq \frac{\pi\epsilon}{\pi\epsilon + (1-\epsilon)} \tag{50}$$

is necessary and sufficient for the optimality of firms' conjectured equilibrium behavior.

[29] Notice that the redemption of MMFs shares under $\omega = 1$ yields $1/(1 + \lambda^*)$ while a liquid deposit would have yielded 1, so the return lost on the investment project is $A - A/(1 + \lambda^*) = \lambda^* A/(1 + \lambda^*)$.

Thus, under the prices that sustain the conjectured equilibrium, the probability of the illiquid state γ must be large enough relative to (the product of) the probability of receiving an investment project π and the probability of deposits turning illiquid ϵ. This is a rather technical condition which can be easily satisfied. If γ were too low, the discount λ^* required for firms' indifference at $t = 0$ would be so high that in the interim date under $\omega = 1$ even firms with investment projects would prefer to invest in MMFs rather than in their own projects.[30]

4.2 Equilibrium quantities

The last step in the characterization of the conjectured equilibrium (and the conditions for its existence) involves considering firms' portfolio decisions at $t = 0$ and their compatibility with market clearing at $t = 1$ under the prices implied by Lemmas 2 and 3. By construction, those prices make firms indifferent between bank deposits and MMFs shares at $t = 0$, so all that we need to check is the existence of a compatible initial portfolio allocation (d_0^f, m_0^f), with $p_0^D d_0^f + m_0^f = e_0^f$, $m_0^f \geq 0$ and $d_0^f \geq 0$. Since in the conjectured equilibrium $p_0^D = 1/(1+r_0)$, we can describe the portfolio allocation decision as the choice of a portfolio weight $x_0^f \equiv m_0^f/e_0^f \in [0,1]$ for the MMFs shares and use $d_0^f = (1+r_0)(1-x_0^f)e_0^f$ and $m_0^f = x_0^f e_0^f$ to substitute for d_0^f and m_0^f, respectively.

Now, the clearing of the market for commercial paper at $t = 1$ in state $\omega = 0$ under $\lambda(0) = 0$ requires, by (1) and (30), a zero net sale of commercial paper, $t_1^m(0) = 0$. This in turn requires, by (28), that the redemptions of MMFs shares by firms with investment projects can be accommodated with a rise in the holdings of the firms without them. As the next lemma formally establishes, this is possible provided that the initial portfolio weight of MMFs shares is not too high:

Lemma 5 *Sustaining an equilibrium with $\lambda(0) = 0$ requires the initial portfolio weight of MMFs shares to be lower than a certain threshold:*

$$x_0^f \leq \bar{x}_0^f \equiv \frac{(1-\pi)(1-\epsilon)}{\pi + (1-\pi)(1-\epsilon)} \in (0,1). \tag{51}$$

Intuitively, the threshold $\bar{x}_0^f$ is decreasing in π and ϵ because the initial investment in bank deposits must be large enough to allow firms without investment projects whose deposits remain liquid at $t = 1$ to buy all the shares sold by the redeeming firms in the liquid state $\omega = 0$.

[30] In this case, the configuration of equilibrium would be different from the postulated one. For low enough γ, a regime might arise in which firms invest all their savings in MMFs at $t = 0$ and only partially switch to deposits at $t = 1$ when experiencing the liquidty shock.

Of course, equilibrium also requires the clearing of the secondary market for commercial paper in state $\omega = 1$. In the proof of the following proposition first we use (1) together with other equilibrium conditions to establish an intuitive increasing relationship

$$\Lambda(x_0^f) = \frac{v\{[\pi + (1-\pi)(1-\epsilon)]\,(1+r_0)(1+r_1)x_0^f - (1-\pi)\,[(1-\epsilon)(1+r_0)(1+r_1) - \theta]\}}{1 + v(1-\pi)[(1-\epsilon)(1+r_0)(1+r_1)(1-x_0^f) - \theta]} \tag{52}$$

between the weight of MMFs shares in firms' initial portfolio, x_0^f, and the discount $\lambda(1)$ that would clear the interim-date secondary market for commercial paper in state $\omega = 1$. The proposition further identifies conditions on parameters for having a (unique) initial portfolio weight $x^* \in (0, \bar{x}_0^f]$ under which $\Lambda(x^*) = \lambda^*$ and all other conditions for the existence of the conjectured equilibrium are satisfied.

Proposition 2 *Under (50), if the size of the liquidity need in state $\omega = 1$ satisfies*

$$\frac{\lambda^*}{1+\lambda^*}\frac{[\pi + (1-\pi)\,(1-\epsilon)] + v(1-\pi)(1-\epsilon)(1+r_0)(1+r_1)\pi}{v(1-\pi)[\pi + (1-\pi)\,(1-\epsilon)]} \leq \theta < (1-\epsilon)(1+r_0)(1+r_1), \tag{53}$$

the conjectured equilibrium exists and features an initial portfolio weight for MMFs shares $x^* \in (0, \bar{x}_0^f]$ recursively determined by $x^* = \Lambda^{-1}(\lambda^*)$.

The conditions guaranteeing the existence of the conjectured equilibrium are intuitive and hold over a positive-measure region of the parameter space. The range of values of θ delimited in (53) is of positive length for sufficiently large values of v, the parameter that graduates the sensitivity of secondary market prices for commercial paper to the selling pressure observed at $t = 1$ (and which, as established in Proposition 1, does not affect λ^*). Satisfying (50) requires γ to be sufficiently large, and increasing γ only affects (53) through λ^*, which is decreasing in γ. So the range of values of θ for which both (50) and (53) hold expands with γ.

Figure 2 provides a graphical representation of Proposition 2. It depicts the discount $\lambda(1)$ at which commercial paper trades in the secondary market in state $\omega = 1$ in the vertical axis and the weight x_0^f of firms' initial portfolio in the horizontal axis. The horizontal line at $\lambda(1) = \lambda^*$ describes the only discount for which, according to Lemma 3 firms are indifferent between deposits and MMFs shares at $t = 0$. The upward slopping curve describes $\Lambda(x_0^f)$ as defined in (52), that is, the locus of pairs $(x_0^f, \lambda(1))$ compatible with the clearing of the secondary market for commercial paper in state $\omega = 1$. Under the conditions in (53), this line has a negative intercept at $x_0^f = 0$ and satisfies $\Lambda(\bar{x}_0^f) \geq \lambda^*$, which guarantees the existence of a (unique) intersection with the horizontal line $\lambda(1) = \lambda^*$ at some $x_0^f = x^* \in (0, \bar{x}_0^f]$ that identifies the equilibrium weight of MMFs shares in firms' initial portfolio.

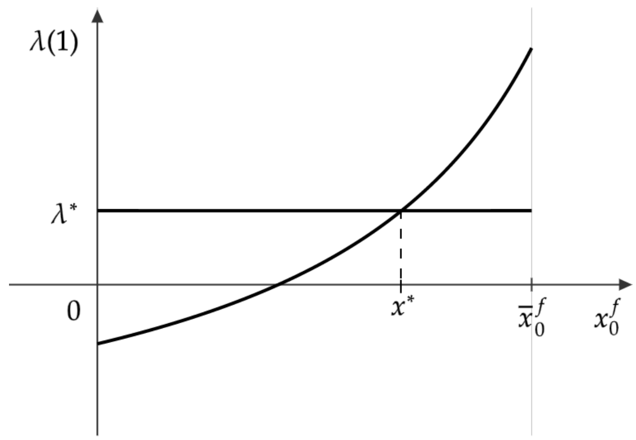

Figure 2 Determination of the unique equilibrium weight of MMFs shares in firms' initial portfolio.

The equilibrium weight of MMFs shares in firms' portfolio, $x^* = \Lambda^{-1}(\lambda^*)$, depends on parameters directly, by the form of $\Lambda(x_0^f)$ in (52), and indirectly, through λ^*. Any parameter that increases (decreases) λ^* without directly entering in (52) also increases (decreases) x^*. Likewise, parameters that shift $\Lambda(x_0^f)$ downwards (upwards) without affecting λ^* will also have a positive (negative) effect on x^*. When parameters shift both λ^* and $\Lambda(x_0^f)$ in the same direction, the final effect on x^* is ambiguous.[31] This explains the results summarized in our next proposition.

Proposition 3 *Parameters affect the equilibrium weight of MMFs shares in firms' portfolio, x^*, directly through $\Lambda(x_0^f)$ and indirectly through λ^*. The signs of each effect and the overall effect are indicated in the following table (interrogation signs denote ambiguity):*

	π	A	ϵ	γ	θ	r_0	r_1	v	e_0^f
Direct effect on x^*	–	*0*	–	*0*	–	+	+	–	*0*
Indirect effect via λ^*	+	+	+	–	*0*	*0*	–	*0*	*0*
Overall effect on x^*	*?*	+	*?*	–	–	+	*?*	–	*0*

(54)

A few clear-cut predictions on parameters affecting x^* emerge. The positive effect of A on x^* is driven by the motives explained after Proposition 1, whereby, at the margin, the investment in MMFs shares allows a better preservation of the value of potential investment

[31] For the parameters with ambiguous effects, numerical explorations of the equilibrium show that the final effect on x^* can go in either direction depending on the dominance of the direct or the indirect effect, which is in turn governed, among other factors, by the price sensitivity parameter v. Direct effects tend to dominate when v is large, while indirect effects tend to dominate when v is small.

projects than bank deposits (since the liquidity problems affecting the former translate into reductions in the scale of the investment, while the potential illiquidity of bank deposits fully impedes the investment). The negative effects on x^* of the probability γ and size θ of the aggregate liquidity shock, as well as the parameter v which measures the importance of the frictions in the secondary market for commercial paper at the interim date, are all explained by the negative effects of aggregate liquidity frictions on the attractiveness of MMFs. Altogether, these results suggest procyclicality in x^*, in the sense that firms' propensity to invest in MMFs is larger when they have a prospect of obtaining more valuable investment projects and of facing more benign aggregate liquidity conditions at the interim date.

5 Efficiency analysis

Our general equilibrium setup features frictions on multiple dimensions. Specifically, markets incompleteness makes firms unable to borrow to undertake their investment project or accommodate liquidity needs at $t = 1$, and there are no markets at $t = 0$ in which to buy explicit insurance against investment and liquidity shocks. Firms self-insure against those shocks by investing their endowments at $t = 0$ in bank deposits and MMFs shares.

The potential illiquidity of deposits and MMFs shares at $t = 1$ (arising from the possibility of banks turning idiosyncratically illiquid in the case of deposits, and from the presence of secondary market frictions that affect the pricing of MMFs shares in the aggregate illiquid state) makes the choice between these alternatives not trivial for firms, potentially leading to a competitive equilibrium in which firms invest in both deposits and MMFs shares. This section studies the efficiency properties of such an equilibrium. We show first that, due to the underlying pecuniary externalities, the equilibrium is not constrained efficient and then that a social planner could induce a constrained efficient allocation of firms' savings across bank deposits and MMFs by setting a suitable Pigouvian tax on the investments in the latter.

5.1 Constrained inefficiency of the competitive equilibrium

Following Davila and Korinek (2017) (who nicely summarize prior contributions to the analysis of economies with financial frictions), the efficiency analysis in this section starts with a relatively narrow notion of constrained efficiency. We examine whether the allocation that firms make of their savings across bank deposits and MMFs shares at $t = 0$ is the same that a benevolent social planner would make under the same financial structure, constraints, and otherwise laissez-faire functioning of the economy at $t = 0$, $t = 1$ and $t = 2$. Under this narrow concept of constrained efficiency, the planner does not directly interfere with the decision problems of banks and valuation rules of MMFs at any date or with firms' decisions after $t = 0$.

So we consider a social planner who aims to maximize firms' value at $t = 0$ (which is the expected value of their terminal net worth) by directly controlling the weight of MMFs shares in firms' initial portfolio x_0^f. Beyond this, the intervention firms' decision problems at $t = 1$ and $t = 2$. Since banks continue to obtain zero terminal net worth in all states, maximizing firms' initial value can be regarded equivalent to maximizing aggregate social surplus or welfare.

Assuming that under the intervention firms' optimal decisions at $t = 1$ are as in the "conjectured solution" described in subsection 3.2.1 (an assumption whose validity can be checked ex post), all the pricing and valuation conditions obtained in Lemma 2 remain valid in the *intervened equilibrium*. In this case, the intervention only affects the equilibrium value of $\lambda(1)$ which does no longer need to be determined as in (47) since the social planner does not need to satisfy firms' *private* indifference condition (35). Instead, the planner makes its (centralized) choice of x_0^f taking into account that in the intervened equilibrium the price discount will be determined, by market clearing, as $\lambda(1) = \Lambda(x_0^f)$.

Formally, the planner solves

$$\begin{array}{ll} \max\limits_{x_0^f \in [0,1]} & \mathbb{E}_0 \left[V_1^f \left((1 - x_0^f) e_0^f / p_0^D, x_0^f e_0^f; s_i^f \right) \right] \\ \text{s.t.:} & \lambda(1) = \Lambda(x_0^f), \\ & \text{and the pricing conditions in Lemma 2.} \end{array} \tag{55}$$

The objective function in (55) is the same that firms maximize in (31), but we have used $d_0^f = (1 - x_0^f) e_0^f / p_0^D$ and $m_0^f = x_0^f e_0^f$ to write the optimization in terms of the portfolio weight $x_0^f \equiv m_0^f / e_0^f \in [0, 1]$. Plugging all the relevant pricing conditions in the objective function, the first order condition for an interior solution to the planner's problem is

$$\frac{\partial \mathbb{E}_0 \left[V_1^f \left((1 - x_0^f) e_0^f / p_0^D, x_0^f e_0^f; s_i^f \right) \right]}{\partial x_0^f} + \frac{\partial \mathbb{E}_0 \left[V_1^f \left((1 - x_0^f) e_0^f / p_0^D, x_0^f e_0^f; s_i^f \right) \right]}{\partial \lambda(1)} \Lambda'(x_0^f) = 0, \tag{56}$$

where the first term is the private marginal value of investing in MMFs (exactly as taken into account by individual firms in the competitive equilibrium) and the second term is the uninternalized effect of such investment on firm value via $\lambda(1)$.

The proof of the following proposition shows that when evaluated at the competitive equilibrium, the first term in the left hand side of (56) is zero, while the second term is strictly negative. This implies that the social planner might increase welfare by choosing a weight of MMFs shares in firms' initial portfolio x_0^f strictly lower than x^*.

Proposition 4 *The competitive equilibrium with* $\lambda(1) = \lambda^*$ *and* $x_0^f = x^*$ *characterized in Section 4 is not constrained efficient in that welfare can be increased by reducing* x_0^f *relative to* x^*.

Proposition 4 implies that the social planner can increase firms' initial value by reducing their investment in MMFs at $t = 0$. This reallocation would produce positive welfare effects by reducing the price discount experienced by commercial paper prices and thus the prices of MMFs shares in the state of aggregate illiquidity. The lower discount allows firms with investment opportunities to undertake them at a larger scale. If the size of the liquidity need θe_0^f leads firms without investment opportunities to also make net redemptions of MMFs shares in the illiquid state, then the reduction in $\lambda(1)$ is also ex post beneficial to them. But even if the need θe_0^f is not so large and these firms can profit from the discount $\lambda(1)$ by increasing their holdings of MMFs shares, the excess profitability of the real investment opportunities of the other group of firms would make the ex post redistribution ex ante welfare improving.

Numerical examples confirm that, under the conditions in which the competitive equilibrium is as characterized in prior sections, the social planner's problem has a unique solution featuring $x_0^f = x^{SP}$ with $x_0^{SP} < x^*$. In the examples, x^{SP} is interior and given by the unique value of x_0^f that solves (56).

5.2 Inducing constrained efficiency with a tax

Assuming that the solution to the constrained social planner's problem involves an initial weight of MMFs shares in firms' initial portfolio x^{SP} given by the unique solution to (56), we now analyze the possibility of implementing the corresponding intervened equilibrium as a competitive equilibrium with taxes. Specifically, we consider the case in which the social planner, instead of directly controlling x_0^f, establishes a tax τ per unit of funds invested in MMFs shares at $t = 0$ and rebates the tax revenue to the firms also at $t = 0$ through a lump sum transfer $L = \tau m_0^f$.

Proposition 5 *The constrained efficient allocation associated with* $x^{SP} < x^*$ *can be implemented as a competitive equilibrium with a tax* $\tau^{SP} > 0$ *on savings in MMFs shares at* $t = 0$.

Intuitively, the social planner can set the tax τ^{SP} in a Pigouvian manner: to make firms internalize at the initial date the relevant marginal costs of investing in MMFs shares. The tax reduces the investment in MMFs shares and, thus, the redemptions faced by MMFs in the aggregate illiquid state. This allows to sustain a lower price discount for the commercial paper traded in the secondary market and, thus, for the redemption value of MMFs shares in that state.

5.3 Penalizing investment versus penalizing redemptions

Under the notion of constrained efficiency considered in this section, in the competitive equilibrium firms allocate excessive savings in MMFs shares at the initial date. They fail to internalize the impact of their decisions on social surplus. Specifically, the losses implied by the price discounts experienced by commercial paper and MMFs shares when, in the state of aggregate illiquidity, MMFs face large redemptions. We have shown that social surplus can be increased (and constrained efficiency restored) by imposing a Pigouvian tax on firms' investment in MMFs shares at $t = 0$.

Are there other interventions that might restore constrained efficiency? Are there interventions that might increase social surplus more generally? A preliminary answer to both questions is "possibly yes," since we are considering a setup with severe market incompleteness and richer forms of policy intervention (with taxes and subsidies not only at $t = 0$ but also at $t = 1$ and $t = 2$ and possibly contingent on the aggregate state that realizes at $t = 1$) might help bring the economy closer to the first best. However, characterizing the first best and the interventions that could bring outcomes closer to the first best is beyond the scope of the current paper.

Having said that, our analysis may help assess the merits of some policy proposals put forward in light of the problems experienced by MMFs in March 2020. Some of those proposals suggest penalizing redemptions (e.g. with redemption fees) after diagnosing the problem as one in which excessive redemptions produce excessive declines in asset prices and the redemption value of MMFs shares. We conjecture that, from perspective of our analysis, such a policy would be inferior to the Pigouvian tax solution in Proposition 5. The reason for this is that, although the anticipated redemption fees would have the effect of ex ante discouraging the investment in MMFs (very much like the Pigouvian tax), they would also worsen the "liquidity insurance" function of MMFs (relative to bank deposits), as they would penalize firms precisely in the contingencies in which they redeem their shares, which is when they value liquidity the most. The detailed analytical exploration of this conjecture is left for a future version of this paper.

6 Conclusions

We have examined the interaction between banks and money market funds (MMFs) in a setup where the latter can experience large redemptions following an aggregate liquidity shock (as in March 2020). In the model MMFs and bank deposits are alternatives for firms' management of their cash holdings. MMFs experiencing correlated redemptions get forced to sell assets to banks in a narrow secondary market, producing asset price declines. Ex post the price declines damage firms' capacity to cover their needs with the redeemed shares. Ex ante the prospect of such effect reduces the attractiveness of MMFs relative to bank deposits. Yet the equilibrium allocation of firms' savings across bank deposits and MMFs shares exhibits an excessive reliance on the latter since firms fail to internalize their effect on the size of the pecuniary externalities caused by future redemptions. This provides a rationale, distinct from first mover advantages, for the macroprudential regulation of the investment in MMFs.

The model sustaining our main conclusions so far is based on some simplifying assumptions (e.g. regarding the assets in which banks invest and the root cause of the frictions that make secondary market prices respond to selling pressure when confronting an aggregate liquidity shock). We plan to relax some of these assumptions in a future version of the paper. The future version will also include a more exhaustive discussion of the empirical and policy implications of the analysis.

References

Allen, F., and D. Gale (1994) "Liquidity preference, market participation and asset price volatility," *American Economic Review* 84, 933-955.

Aramonte, S., A. Schrimpf, and H. S. Shin (2022) "Non-bank financial intermediaries and financial stability," CEPR DP 16962.

Bailey, A. (2021) "Taking our second chance to make MMFs more resilient," Speech at the ISDA 35th Annual General Meeting.

Begenau, J., and T. Landvoigt (2018) "Financial regulation in a quantitative model of the modern banking system," Available at SSRN 2748206.

Bengui, J., and J. Bianchi (2018) "Macroprudential policy with leakages," National Bureau of Economic Research WP 25048.

Capotă, L.-D., M. Grill, L. Molestina Vivar, N. Schmitz, and C. Weistroffer (2021) "How effective is the EU money market fund regulation? Lessons from the Covid-19 turmoil," ECB Macroprudential Bulletin 12.

Chen, Q., I. Goldstein, and W. Jiang (2010) "Payoff complementarities and financial fragility: Evidence from mutual fund outflows," *Journal of Financial Economics* 97, 239-262.

Cimon, D., and C. Garriott (2019) "Banking regulation and market making," *Journal of Banking & Finance* 109, 105653.

Cipriani, M., and G. La Spada (2020) "Sophisticated and unsophisticated runs," FRB of New York Staff Report.

Cipriani, M., and G. La Spada (2021) "Investors' appetite for money-like assets: The MMF industry after the 2014 regulatory reform," *Journal of Financial Economics* 140, 250-269.

Cipriani, M., A. Martin, P. E. McCabe, and B. M. Parigi (2014) "Gates, fees, and preemptive runs," FRB of New York Staff Report.

Clarida, R., B. Duygan-Bump, and C. Scotti (2021) "The Covid-19 crisis and the Federal Reserve's policy response," FRB Finance and Economics Discussion Series 2021-135.

Cucic, D. (2020) "Regulating liquidity risk in mutual funds," mimeo.

dAvernas, A., and Q. Vandeweyer (2020) "Intraday liquidity and money market dislocation," mimeo.

Davila, E., and A. Korinek (2017) "Pecuniary externalities in economies with financial frictions," *Review of Economic Studies* 85, 352-395.

Eren, E., A. Schrimpf, and V. Sushko (2020) "US dollar funding markets during the Covid-19 crisis—The money market fund turmoil," BIS Technical Report.

Ferrante, F. (2018) "A model of endogenous loan quality and the collapse of the shadow banking system," *American Economic Journal: Macroeconomics* 10, 152-201.

Financial Stability Board (2020) Holistic Review of the March Market Turmoil, Reports to the G20, 17 November

Financial Stability Board (2021) Policy Proposals to Enhance Money Market Fund Resilience, Final Report, 11 October.

Gennaioli, N., A. Shleifer, and R. Vishny (2013) "A model of shadow banking," *Journal of Finance* 68, 1331-1363.

Gertler, M., N. Kiyotaki, and A. Prestipino (2016) "Wholesale banking and bank runs in macroeconomic modeling of financial crises," in *Handbook of Macroeconomics*, Vol. 2, Elsevier, 1345-1425.

Giannetti, M., and J. Chotibhak (2022) "Bond price fragility and the structure of the mutual fund industry," CEPR DP 17050.

Goldstein, I., H. Jiang, and D. T. Ng (2017) "Investor flows and fragility in corporate bond funds," *Journal of Financial Economics* 126, 592-613.

Haddad, V., A. Moreira, and T. Muir (2021) "When selling becomes viral: Disruptions in debt markets in the COVID-19 Crisis and the Fed's response," *Review of Financial Studies* 34, 5309-5351.

Holmström, B., and J. Tirole (1998) "Private and public supply of liquidity," *Journal of Political Economy* 106, 1-40.

International Monetary Fund (2021) "Investment funds and financial stability. Policy considerations," Monetary and Capital Markets Department DP/2021/018.

Jeanne, O., and A. Korinek (2020) "Macroprudential regulation versus mopping up after the crash," *Review of Economic Studies* 87, 1470-1497.

Jiang, H., Y. Li, Z. Sun, and A. Wang (2022) "Does mutual fund illiquidity introduce fragility into asset prices? Evidence from the corporate bond market," *Journal of Financial Economics* 143, 277-302.

Jin, D., M. Kacperczyk, B. Kahraman, F. Suntheim (2022) "Swing pricing and fragility in open-end mutual funds," *Review of Financial Studies* 35, 1-50.

Lorenzoni, G. (2008) "Inefficient credit booms," *Review of Economic Studies* 75, 809-833.

Martinez-Miera, D., and R. Repullo (2019) "Markets, banks, and shadow banks," ECB Working Paper 2234.

Moreira, A., and A. Savov (2017) "The macroeconomics of shadow banking," *Journal of Finance* 72, 2381-2432.

Ordoñez, G. (2018) "Sustainable shadow banking," A*merican Economic Journal: Macroeconomics* 10, 33-56.

Plantin, G. (2015) "Shadow banking and bank capital regulation," *Review of Financial Studies* 28, 146-175.

Saar, G., J. Sun, R. Yang, and H. Zhu (2020) "From market making to matchmaking: Does bank regulation harm market liquidity?," available at SSRN 3399063.

Segura, A., and A. Villacorta (2020) "Demand for safety, risky loans: A model of securitization," Bank of Italy Temi di Discussione No. 1260.

Voellmy, L. (2021) "Preventing runs with fees and gates," *Journal of Banking and Finance* 125, 106065.

A Appendix: Proofs

Proof of Lemma 1. We start by stating the final net worth of a firm in each of the idiosyncratic states $s_i^f = (\omega, \psi_i)$ that it can experience at $t = 1$. The following expressions are the result of replacing the guessed solution to its optimization problem in the objective function (6) in each state:

$$V_1^f\left(d_0^f, m_0^f; (1,\omega)\right) = \frac{\epsilon d_0^f}{p_1^D(\omega)} + \omega\theta e_0^f + A\left[(1-\epsilon)d_0^f + q_1(\omega)m_0^f - \omega p_1^D(\omega)\theta e_0^f\right], \tag{A.1}$$

$$V_1^f\left(m_0^f, d_0^f; (0,0)\right) = \frac{\epsilon d_0^f}{p_1^D(0)} + \frac{(1-\epsilon)d_0^f}{p_1^D(0)} + q_2(0)m_0^f, \tag{A.2}$$

$$V_1^f\left(d_0^f, m_0^f; (1,0)\right) = \frac{\epsilon d_0^f}{p_1^D(1)} + \theta e_0^f + q_2(1)\left[m_0^f + \frac{(1-\epsilon)d_0^f - p_1^D(1)\theta e_0^f}{q_1(1)}\right]. \tag{A.3}$$

Next, we compute $\mathbb{E}[V_1^f(d_0^f, m_0^f; s_i^f)]$ as a weighted average of the three expressions above, with weights corresponding to the probabilities of the corresponding combinations of idiosyncratic and aggregate shocks. Grouping together the resulting terms in θ, d_0^f, and m_0^f, the expression becomes

$$\begin{aligned}\mathbb{E}_0\left[V_1^f\left(d_0^f, m_0^f; s_i^f\right)\right] &= \gamma\left[1 - \pi A p_1^D(1) - (1-\pi)\frac{q_2(1)}{q_1(1)}p_1^D(1)\right]\theta e_0^f \\ &+ \left\{(1-\epsilon)\left[\pi A + (1-\pi)\left(\frac{1-\gamma}{p_1^D(0)} + \frac{\gamma q_2(1)}{q_1(1)}\right)\right] + \epsilon\left(\frac{1-\gamma}{p_1^D(0)} + \frac{\gamma}{p_1^D(1)}\right)\right\} d_0^f \\ &+ \left\{\pi A\left[(1-\gamma)\, q_1(0) + \gamma q_1(1)\right] + (1-\pi)\left[(1-\gamma)q_2(0) + \gamma q_2(1)\right]\right\} m_0^f \end{aligned} \tag{A.4}$$

The problem of the firm at $t = 0$ is to maximize (A.4) with respect to d_0^f and m_0^f subject to the budget constraint (32) and the non-negativity constraints (33). Using (32), to replace d_0^f with $(e_0^f - m_0^f)/p_0^D$, the resulting expression is linear in m_0^f, so obtaining an interior solution requires having $d\mathbb{E}_0[V_1^f((e_0^f - m_0^f)/p_0^D, m_0^f; s_i^f)]/dm_0^f = 0$, which is equivalent to (35).■

Proof of Lemma 2. The values of p_0^D and p_0^{CP} come directly from firms' indifference conditions stated in (38) and (39), respectively. The values of $p_1^D(0)$ and $p_1^D(1)$ come directly from (25).

To explain the value of the remaining prices, notice that MMFs' initial balance sheet in (40) implies

$$\frac{cp_0^m}{m_0^f} = \frac{1}{p_0^{CP}}, \tag{A.5}$$

that is, an investment in $1/p_0^{CP}$ units of commercial paper per each unit of MMFs shares at $t = 0$. Together with the marked-to-market pricing of MMFs shares at $t = 1$ in (27), this implies

$$q_1(\omega) = \frac{p_1^{CP}(\omega)cp_0^m}{m_0^f} = \frac{p_1^{CP}(\omega)}{p_0^{CP}}, \tag{A.6}$$

which intuitively means that redeeming shareholders simply obtain at $t = 1$ the same returns as the would have obtained by directly investing in commercial paper between dates $t = 0$ and $t = 1$.

Consistently with this, if we combine the equation determining the terminal value of MMFs shares in (5) with the expression for the secondary market sales of commercial paper $t_1^m(\omega)$ in (28) we obtain

$$q_2(\omega)\int m_1^f(s_i)di = cp_0^m - t_1^m(\omega) = cp_0^m - \left(1 - \frac{\int m_1^f(s_i^f)di}{m_0^f}\right)cp_0^m = \frac{cp_0^m}{m_0^f}\int m_1^f(s_i)di. \tag{A.7}$$

Thus, we have, using (A.5),

$$q_2(\omega) = \frac{cp_0^m}{m_0^f} = \frac{1}{p_0^{CP}}, \tag{A.8}$$

which means that non-redeeming shareholders obtain at $t = 2$ the same returns as if they had directly invested in commercial paper at $t = 0$. Then the result $q_2(0) = q_2(0) = (1+r_0)(1+r_1)$ in the lemma comes from just plugging the value of $p_0^{CP} = 1/[(1+r_0)(1+r_1)]$ into the equation above.

Now, banks' indifference condition in (26) directly explains the expression for $p_1^{CP}(1)$ in the lemma, as well as that for $p_1^{CP}(0)$ provided that $\lambda(0) = 0$, which is, in turn, implied by firms' indifference condition in state $\omega = 0$ in (21) and the values of $q_2(0)$ and $p_1^D(0)$.

Finally, plugging the expressions for $p_1^{CP}(\omega)$ and p_0^{CP} in (A.6) leads to the expressions for $q_1(0)$ and $q_1(1)$ stated in the lemma and completes the proof.■

Proof of Lemma 3 Plugging the prices in Lemma 2 into the indifference condition provided in (35) leads, after some algebra, to an equation in $\lambda(1)$ equivalent to (47):

$$\pi\epsilon[A - (1 + r_1)] = \gamma\lambda(1)\left[\frac{\pi A}{1 + \lambda(1)} + (1 - \epsilon)(1 - \pi)(1 + r_1)\right]. \tag{A.9}$$

In this equation, the left hand side is a positive constant, while the right hand side is zero for $\lambda(1) = 0$ and grows monotonically and unboundedly with $\lambda(1)$ for $\lambda(1) > 0$. So the equation implicitly defines a unique value of the price discount $\lambda(1) = \lambda^* > 0$ compatible with the conjectured equilibrium.■

Proof of Proposition 1 The results follow from the full differentiation of (47) with respect to λ^* and each of the parameters. Parameters with 0 are those not entering in (47). In several cases, signing the involved derivatives requires taking into account that (47) holds. Details are omitted for brevity and can be provided upon request.■

Proof of Lemma 4 Under the candidate equilibrium prices in Lemma 2, the condition in (15) reduces to

$$A \geq (1+r_1)(1+\lambda(1)), \tag{A.10}$$

since all other terms in the max operator equal $1+r_1$. Now, solving for A in (A.9), we obtain

$$\pi\epsilon[A-(1+r_1)] = \gamma\lambda(1)\left[\frac{\pi A}{1+\lambda(1)} + (1-\epsilon)(1-\pi)(1+r_1)\right] \tag{A.11}$$

$$A = \frac{\pi\epsilon(1+r_1) + (1-\epsilon)(1-\pi)(1+r_1)\gamma\lambda(1)}{\left(\pi\epsilon - \frac{\pi\gamma\lambda(1)}{1+\lambda(1)}\right)}, \tag{A.12}$$

which substituted into (A.10) reduces the condition to

$$\frac{\pi\epsilon + (1-\epsilon)(1-\pi)\gamma\lambda(1)}{\pi\epsilon(1+\lambda(1)) - \pi\gamma\lambda(1)} \geq 1, \tag{A.13}$$

whose denominator must be positive because of (A.12) and $A > 0$. Rearranging terms, after some algebra, $\lambda(1)$ drops out and the inequality in (50) arises.■

Proof of Lemma 5 The analysis of firms' problem at $t = 1$ implies having $m_1^f(s_i^f) = 0$ when $s_i^f = (1,0)$, while when $s_i^f = (0,0)$ firms are indifferent between deposits and MMFs shares. However, for such firms $m_1^f(s_i^f)$ is bounded above by the value that solves (20) when $d_1^f(s_i^f) = 0$, that is,

$$\bar{m}_1^f(0,\delta_i,0) \equiv \frac{(1-\delta_i)\, d_0^f}{1+r_0} + m_0^f, \tag{A.14}$$

where we have used the fact that $q_1(0) = 1 + r_0$ in the conjectured equilibrium. To have $t_1^m(0) = 0$, we need, by (28), $\int m_1^f(s_i^f)di = m_0^f$ under $\omega = 0$, that is, the maximum feasible MMFs share holding's by firms with $s_i^f = (0,0)$ to be no lower than m_0^f:

$$(1-\pi)\left[\frac{(1-\epsilon)d_0^f}{1+r_0} + m_0^f\right] \geq m_0^f \Leftrightarrow \frac{(1-\pi)(1-\epsilon)d_0^f}{1+r_0} \geq \pi m_0^f. \tag{A.15}$$

But with $d_0^f = (1+r_0)(1-x_0^f)e_0^f$ and $m_0^f = x_0^f e_0^f$, this means having

$$(1-\pi)(1-\epsilon)(1-x_0^f) \geq \pi x_0^f \Leftrightarrow x_0^f \leq \frac{(1-\pi)(1-\epsilon)}{\pi + (1-\pi)(1-\epsilon)} \in (0,1).■ \tag{A.16}$$

Proof of Proposition 2 This proof has two steps:

Step 1. We first show that (1) together with other equilibrium conditions imply the relationship between values of x_0^f and $\lambda(1)$ provided in (52). Plugging (30) in (1) and using (28) to substitute for $t_1^m(1)$ yields:

$$\lambda(\omega) = \frac{v}{e_0^f}\left(1 - \frac{\int m_1^f(s_i^f)\,di}{m_0^f}\right) cp_0^m. \tag{A.17}$$

But MMFs' initial balance sheet in (40) implies $cp_0^m = m_0^f/p_0^{CP}$, while $p_0^{CP} = 1/[(1+r_0)(1+r_1)]$ by Lemma 2. So (A.17) can be rewritten as

$$\lambda(\omega) = \frac{v(1+r_0)(1+r_1)}{e_0^f}\left(m_0^f - \int m_1^f(s_i^f)\,di\right). \tag{A.18}$$

Now, firms' optimal decisions in state $\omega = 1$ imply

$$\left(\int m_1^f(s_i^f)di\right)_{|\omega=1} = (1-\pi)\left[\frac{(1-\epsilon)d_0^f - p_1^D(1)\theta e_0^f}{q(1)} + m_0^f\right], \tag{A.19}$$

where $p_1^D(1) = 1/(1+r_1)$ and $q(1) = (1+r_0)/(1+\lambda(1))$ by Lemma 2. Plugging (A.19) in (A.18) yields

$$\lambda(1) = \frac{v(1+r_0)(1+r_1)}{e_0^f}\left[\pi m_0^f - (1-\pi)\frac{(1-\epsilon)d_0^f - \theta e_0^f/(1+r_1)}{(1+r_0)/(1+\lambda(1))}\right], \tag{A.20}$$

which, after some algebra, simplifies to

$$\lambda(1) = v\left[\pi(1+r_0)(1+r_1)\frac{m_0^f}{e_0^f} - (1-\pi)(1+\lambda(1))\left[(1-\epsilon)(1+r_1)\frac{d_0^f}{e_0^f} - \theta\right]\right]. \tag{A.21}$$

Finally, replacing m_0^f/e_0^f by x_0^f and d_0^f/e_0^f by $(1+r_0)(1-x_0^f)$, and solving for $\lambda(1)$, we obtain:

$$\lambda(1) = \frac{v\{[\pi + (1-\pi)(1-\epsilon)](1+r_0)(1+r_1)x_0^f - (1-\pi)[(1-\epsilon)(1+r_0)(1+r_1) - \theta]\}}{1 + v(1-\pi)[(1-\epsilon)(1+r_0)(1+r_1)(1-x_0^f) - \theta]}, \tag{A.22}$$

which justifies the form of $\Lambda(x_0^f)$ in (52).

Step 2. Next we show the results in the statement of the proposition. Assumption (50) is needed for the existence of the conjectured equilibrium by Lemma 4. The second inequality in (53) (which means that if firms initially invest all their savings in bank deposits, $x_0^f = 0$, they are able to accommodate their liquidity needs in state $\omega = 1$) is sufficient for having $\Lambda(0) < 0$. But if $\Lambda(0) < 0$ then, by Lemma 5, having $\Lambda(\bar{x}_0^f) \geq \lambda^*$ becomes a necessary and sufficient condition for the existence of a (unique) initial portfolio weight $x^* \in (0, \bar{x}_0^f]$ under which $\Lambda(x^*) = \lambda^*$. Plugging $\bar{x}_0^f$ from (51) in (52) yields

$$\Lambda(\bar{x}_0^f) = \frac{v(1-\pi)[\pi + (1-\pi)(1-\epsilon)]\theta}{[\pi + (1-\pi)(1-\epsilon)] + v(1-\pi)\{(1-\epsilon)(1+r_0)(1+r_1)\pi - [\pi + (1-\pi)(1-\epsilon)]\theta\}}, \tag{A.23}$$

thus reducing the condition $\Lambda(\bar{x}_0^f) \geq \lambda^*$ to the first inequality in (53).

Solving for $x^* = \Lambda^{-1}(\lambda^*)$ using (52), we obtain:

$$x^* = \frac{\lambda^* + (1+\lambda^*)v(1-\pi)[(1-\epsilon)(1+r_0)(1+r_1) - \theta]}{v(1+r_0)(1+r_1)\{[\pi + (1-\pi)(1-\epsilon)] + \lambda^*(1-\pi)(1-\epsilon)\}}. \blacksquare \tag{A.24}$$

Proof of Proposition 3 The results follow from the arguments provided prior to the proposition in the main text, the results in Proposition 1, and the full differentiation of (52) with respect to x_0^f and each of the parameters. Signing the direct effects requires, in some cases, using the fact that $\Lambda(x^*) = \lambda^*$ and the assumptions under which the conjectured equilibrium exists. Details are omitted for brevity and can be provided upon request.$\blacksquare$

Proof of Proposition 4 In this proof we want to show that in the competitive equilibrium firms' initial investments in MMFs shares generate pecuniary externalities that are detrimental to firms' value. For that, we start from the expression for $\mathbb{E}_0[V_1^f(d_0^f, m_0^f; s_i^f)]$ in (A.4) (proof of Lemma 1), which under the pricing conditions in Lemma 2 becomes

$$\begin{aligned}\mathbb{E}_0\left[V_1^f\left((1-x_0^f)e_0^f/p_0^D, x_0^f e_0^f; s_i^f\right)\right] &= e_0^f\Bigg(\gamma\left[1 - \frac{\pi A}{1+r_1} - (1-\pi)(1+\lambda(1))\right]\theta \\ &+ (1+r_0)\left\{(1-\epsilon)\left[\pi A + (1-\pi)(1+r_1)(1+\gamma\lambda(1))\right] + \epsilon(1+r_1)\right\}(1-x_0^f) \\ &+ (1+r_0)\left\{\pi A\left[(1-\gamma) + \frac{\gamma}{1+\lambda(1)}\right] + (1-\pi)(1+r_1)\right\}x_0^f\Bigg).\end{aligned} \tag{A.25}$$

The partial derivative of this expression with respect to x_0^f yields

$$\begin{aligned}\frac{\partial\mathbb{E}_0\left[V_1^f\left((1-x_0^f)e_0^f/p_0^D, x_0^f e_0^f; s_i^f\right)\right]}{\partial x_0^f} &= -e_0^f(1+r_0)\Bigg\{(1-\epsilon)\left[\pi A + (1-\pi)(1+r_1)(1+\gamma\lambda(1))\right] \\ &-\epsilon(1+r_1)\pi A\left[(1-\gamma) + \frac{\gamma}{1+\lambda(1)}\right] - (1-\pi)(1+r_1)\Bigg\},\end{aligned} \tag{A.26}$$

which does not directly depend on x_0^f and equals zero when evaluated at $\lambda(1) = \lambda^*$, since it is precisely the indifference condition that characterizes firms' portfolio choice in the competitive equilibrium (Lemma 3). Thus we have that, when evaluated at the competitive equilibrium, the total derivative of $\mathbb{E}_0[V_1^f((1\text{–}x_0^f)e_0^f/p_0^D, x_0^f e_0^f; s_i^f)]$ with respect to x_0^f reduces to the second term in the left hand side of (56):

$$\frac{d\mathbb{E}_0\left[V_1^f\left((1\text{–}x_0^f)e_0^f/p_0^D, x_0^f e_0^f; s_i^f\right)\right]}{dx_0^f} = \frac{\partial\mathbb{E}_0\left[V_1^f\left((1\text{–}x_0^f)e_0^f/p_0^D, x_0^f e_0^f; s_i^f\right)\right]}{\partial\lambda(1)}\Lambda'(x_0^f). \tag{A.27}$$

From (52) we know that $\Lambda'(x_0^f) > 0$, while

$$\begin{aligned}\frac{\partial\mathbb{E}_0\left[V_1^f\left((1\text{–}x_0^f)e_0^f/p_0^D, x_0^f e_0^f; s_i^f\right)\right]}{\partial\lambda(1)} &= -e_0^f\Bigg[\gamma(1-\pi)\theta \\ -(1-\epsilon)(1-\pi)(1+r_0)(1+r_1)\gamma(1\text{–}x_0^f) &+ \frac{\pi A\gamma(1+r_0)}{(1+\lambda(1))^2}x_0^f\Bigg].\end{aligned} \tag{A.28}$$

Thus, if we show that (A.28) is negative when evaluated at $\lambda(1) = \lambda^*$ and $x_0^f = x^*$, we will confirm that the competitive equilibrium is inefficient and the social planner might increase firms' value at $t = 0$ by choosing some $x_0^f < x^*$. The sign of (A.28) is the opposite of that of the expression contained in square brackets. So we want to show that

$$\frac{\pi A(1+r_0)}{(1+\lambda(1))^2}x_0^f - (1-\epsilon)(1-\pi)(1+r_0)(1+r_1)(1\text{–}x_0^f) + (1-\pi)\theta > 0, \tag{A.29}$$

or, equivalently, after some re-arrangement,

$$\frac{A}{(1+\lambda(1))(1+r_1)}\pi\frac{1+r_0}{1+\lambda(1)}x_0^f - (1-\pi)\left[(1-\epsilon)(1+r_0)(1\text{–}x_0^f) - \frac{\theta}{1+r_1}\right] > 0. \tag{A.30}$$

To prove this, we proceed in two steps.

Step 1. In the competitive equilibrium, $A/[(1+\lambda(1))(1+r_1)] \geq 1$ by (15) and the equilibrium prices in Lemma 2. But, then, a sufficient condition for (A.30) is having

$$\pi\frac{1+r_0}{1+\lambda(1)}x_0^f - (1-\pi)\left[(1-\epsilon)(1+r_0)(1\text{–}x_0^f) - \frac{\theta}{1+r_1}\right] > 0. \tag{A.31}$$

Step 2. Condition (A.31) is equivalent to having $t_1^m(1) > 0$, which in turn is a necessary condition for having $\lambda(1) > 0$ by (1).To see this, notice that the definition of x_0^f and the equilibrium prices in Lemma 2 make (A.31) equivalent to

$$\begin{aligned}&\pi q_1(1)m_0^f - (1-\pi)\left[(1-\epsilon)d_0^f - p_1^D(1)\theta e_0^f\right] \\ =\ & q_1(1)\left\{m_0^f - (1-\pi)\left[m_0^f + \frac{(1-\epsilon)d_0^f - p_1^D(1)\theta e_0^f}{q_1(1)}\right]\right\} \\ =\ & q_1(1)\left[m_0^f - \int m_1^f(s_i^f)di\right] = p_1^{CP}(1)t_1^m(1) > 0,\end{aligned} \tag{A.32}$$

where the second equality uses firms' equilibrium demand for MMFs shares in state $\omega = 1$ which, in equilibrium, comes exclusively from the firms without investment projects in that state and is determined by (17). The last equality in (A.32) uses the fact that MMFs accommodate their net redemptions at $t = 1$ with net sales of commercial paper in the secondary market, as specified in (2).■

Proof of Proposition 5 Expressing the lump sum transfer as a proportion $l = L/e_0^f$ of firms' initial net worth, we can equivalently write

$$l = \tau x_0^f. \tag{A.33}$$

The problem of the representative firm at $t = 0$ can then stated as follows

$$\max_{x_0^f \in [0,1]} \mathbb{E}_0 \left[V_1^f \left(\left[1 + l - (1+\tau)\, x_0^f \right] e_0^f / p_0^D, x_0^f e_0^f; s_i^f \right) \right] \tag{A.34}$$

where the objective function in (A.34) is the same that firms maximize in (31) but we have used the new budget constraint

$$p_0^D d_0^f + (1+\tau)\, m_0^f = e_0^f + L \tag{A.35}$$

and the definition $x_0^f = m_0^f / e_0^f$ to express d_0^f as

$$\frac{e_0^f + L - (1+\tau)\, m_0^f}{p_0^D} = \left[1 + l - (1+\tau)\, x_0^f \right] \frac{e_0^f}{p_0^D} \tag{A.36}$$

and m_0^f as $x_0^f e_0^f$.

The problem in (A.34) is linear x_0^f, so the existence of an interior solution requires, similarly to the case of the competitive equilibrium without taxes (Lemma 1), that an indifference condition holds:

$$\begin{aligned} &\frac{1}{p_0^D} \left\{ (1-\epsilon) \left[\pi A + (1-\pi) \left(\frac{1-\gamma}{p_1^D(0)} + \frac{\gamma q_2(1)}{q_1(1)} \right) \right] + \epsilon \left(\frac{1-\gamma}{p_1^D(0)} + \frac{\gamma}{p_1^D(1)} \right) \right\} \\ &= \frac{1}{1+\tau} \left\{ \pi A \left[(1-\gamma)\, q_1(0) + \gamma q_1(1) \right] + (1-\pi) \left[(1-\gamma) q_2(0) + \gamma q_2(1) \right] \right\}. \end{aligned} \tag{A.37}$$

This condition is equivalent to (35) except for the fact that the right hand side (that accounts for the expected return of investing one unit of the initial net worth in MMFs shares) is divided by $1+\tau$.

Conditional on the choice of x_0^f by firms at $t = 0$, the optimization and market clearing conditions of all agents at $t = 1$ and $t = 2$ are exactly as in the equilibrium without taxes. If under the relevant x_0^f firms behave at $t = 1$ as in the conjectured solution described in subsection 3.2.1, all the pricing and valuation conditions obtained in Lemma 2 remain valid. Plugging these conditions in (A.37) leads, after some reordering, to

$$[\epsilon - (1-\epsilon)\tau]\,\pi[A - (1+r_1)] = \gamma\lambda(1)\left[\frac{\pi A}{1+\lambda(1)} + [\tau + (1+\tau)(1-\epsilon)(1-\pi)](1+r_1)\right], \tag{A.38}$$

which is the counterpart of (47) in Lemma 3. This equation uniquely determines a candidate equilibrium value of $\lambda(1)$, decreasing in τ, for each relevant value of τ. Of course, for $\tau = 0$, (A.38) yields $\lambda(1) = \lambda^*$ as in the competitive equilibrium without taxes.

To implement the constrained efficient portfolio weight x^{SP} that solves (56), the social planner can identify $\lambda^{SP} = \Lambda(x^{SP})$ using (52) and then find the unique tax rate τ^{SP} for which (A.38) yields $\lambda(1) = \lambda^{SP}$.■