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Productivity and the Welfare of Nations

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Abstract

We show that the welfare of a country's infinitely-lived representative consumer is summarized, to a first order, by total factor productivity (TFP), appropriately defined, and by the capital stock per capita. The result holds for both closed and open economies, regardless of the type of production technology and the degree of product market competition. Welfare-relevant TFP needs to be constructed with prices and quantities as perceived by consumers, not firms. Thus, factor shares need to be calculated using after-tax wages and rental rates. We use these results to calculate welfare gaps and growth rates in a sample of advanced countries with high-quality data on output, hours worked, and capital. We also present evidence for a broader sample that includes both advanced and developing countries.

JEL Codes: D24, D90, E20, O47.

Keywords: Productivity, welfare, TFP, Solow residual.

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1 Introduction

Standard models in many fields of economics posit the existence of a representative household in either a static or a dynamic setting, and then seek to relate the household's welfare to observable aggregate data. A separate large literature examines the productivity residual defined by Solow (1957), and interprets it as a measure of technical change or policy effectiveness. Yet a third literature, often termed "development accounting," studies productivity differences across countries, and interprets them as measures of technology gaps or institutional quality. We show that these three literatures are intimately related. Starting from the standard framework of a representative household that maximizes intertemporal welfare over an infinite horizon, we derive methods for comparing economic well-being over time and across countries. Our methods minimize the need to make parametric assumptions about preferences, technology or market structure. We show that under a wide range of assumptions, welfare can be measured using just two variables, productivity and capital accumulation. We then take our theory to the data, and measure welfare differences over time and across countries.

More precisely, we show that to a first-order approximation the welfare change of a representative household can be fully characterized by three objects: the expected present discounted value of total factor productivity (TFP) growth, the change in expectations of the level of TFP, and the growth in the stock of capital per person, all of which can be calculated using standard national income accounting and labor input data. Welfare-relevant TFP needs to be constructed with prices and quantities as perceived by consumers, not firms, for example, using after-tax wages and rental rates. Our result applies to both closed and open economies, provided TFP is calculated using domestic absorption instead of GDP as the measure of output.

The result sounds similar to one that is proven in the context of a competitive optimal growth model with technology as the only source of stochastic shocks, which might lead one to ask what assumptions on technology and product market competition are required. The answer is, None, because in our setting TFP growth need not measure technical change in an aggregate production function, as it does in a neoclassical growth model. Our result holds for all types of technology, market structure, or frictions affecting firms as long as consumers take prices as given and are not constrained in the amount they can buy or sell at those prices. Thus, for example, the same result holds whether the TFP growth is generated by exogenous technological change, as in the Ramsey model, or is the result of spillovers or profit-maximizing investment in R&D as in endogenous or semi-endogenous growth models.

Our objective is to address a pressing and long-standing issue, the measurement of relative welfare across countries based on national income accounting and labor input data, employing a method firmly grounded in economic theory. We restrict ourselves to comparisons using aggregate data, because these data are all that is available for many countries. Of course this choice comes with a cost: using aggregate data we are unable to address the welfare consequences of inequality within countries. Jones and Klenow (2016) do address the role of inequality in cross countries welfare comparisons and show that the necessary data severely limit the sample of countries that can be considered. They also include life expectancy as a determinant of welfare (see also Becker, Philipson and Soares (2005)), but we will stick with what we can learn using aggregate national income accounting and labor data, well aware that we are putting aside other potential sources of welfare differences.

To derive welfare results, some assumptions regarding preferences are needed. In keeping with our desire to use aggregate data, we assume the existence of a representative household endowed with an additively time-separable intertemporal utility function, with period utility depending upon consumption and leisure. The only restrictions we put on the period utility function are the ones shown by King, Plosser and Rebelo (1988) to be necessary for the existence of a steady state growth path with growing consumption and real wages yielding constant leisure (see also Boppart and Krusell (2020)). We show that data on cross-country differences in TFP and capital intensity, long the staples of discussion in the development and growth literatures, suffice to measure cross-country welfare differences in this setting.

The logic of our argument is based on a dynamic application of the envelope theorem, which implies that the welfare of a representative agent depends to a first order on the expected time paths of the variables that the agent takes as exogenous. In a dynamic growth context, these variables are the prices for factors the household supplies (labor and capital), the prices for the goods it purchases (consumption and investment), and beginning-of-period household assets, which are predetermined state variables. Thus, the TFP that is directly relevant for household welfare is the price-based *dual* Solow residual, which we transform into the familiar primal Solow residual using the national income identity.

Our cross-country welfare result comes from using the link between welfare and exogenous prices implied by economic theory to ask how much a household's welfare would differ if it faced the sequence of prices, not of its own country, but of some other country. More precisely, we perform the thought experiment of having a US household optimizing while facing the expected time paths of all goods and factor prices in, say, France, and owning the initial stock of French assets rather than US assets. The difference between the resulting level of welfare and the welfare of remaining in the US measures the gain or loss to a US household of being moved to France.

Note that our thought experiment is a counterfactual one—the US household in France will choose different time paths for consumption, leisure and saving than the French household, because the two generally have different preferences. Yet we show that the welfare comparisons can be based on the productivity residuals constructed using just the observed, equilibrium data of both countries, without the need to construct any counterfactual quantities. By contrast, the method of comparing welfare by inserting the consumption and leisure of different countries into a fixed utility function requires one to assume that preferences are the same across countries. Otherwise, the observed French levels of consumption and leisure will not be those chosen by a US consumer facing French prices and initial conditions. Furthermore, our thought experiment takes dynamics into account, and allows the consumer to face time-varying paths of prices and assets.

Most importantly, our revealed-preference approach allows one to perform cross-country welfare comparisons without the need to know individuals' preference parameters, other than the discount rate, and without having to assume them equal across countries. Note that our welfare comparisons are from a definite point of view—in the above example, that of a US household. In principle, the result could be different if the USA-France comparison is made by a French household, with different preferences over consumption and leisure. Luckily, our empirical results are not greatly affected by the choice of the "reference country" used for these welfare comparisons. We calculate the one necessary preference parameter, the discount rate, from the Euler equation of the representative household in the reference country, using data on the real rate of return on capital from national income accounting, the growth rate of per-capita consumption, and the population growth rate.

Feenstra, Inklaar and Timmer (2015a) discuss the new data that have been introduced into the 2015 Penn World Tables (PWT) to allow users to compute the welfare measures that we derive in this paper. In particular, they use our paper to motivate the addition of a measure of real domestic absorption (termed CDA in PWT8.1), as well as a TFP measure using absorption as the definition of output (CWTFP in PWT8.1). They note that a strength of our welfare measure is that it requires no assumptions about technology or market structure.[1]

Our approach is based on first-order approximations. Thus, it is best suited for welfare comparisons among economies not too far from a common steady state, which suggests that its empirical implementation should be taken with extra caution in settings not meeting that requirement. Nevertheless, we show through numerical simulation that our approximations remain fairly accurate even in settings where countries have different steady-state levels, or even growth rates, of income.

We illustrate our methods using data for several industrialized countries for which high-quality data are available (Canada, France, Italy, Japan, Spain, the United Kingdom and the United States) and show how their relative welfare levels evolve over time. We also provide welfare comparisons for a larger set of countries that includes both advanced and developing countries. However, we continue to focus on the smaller sample because it has better data: for example, information on hours worked is not available for most of the countries the larger sample. In our benchmark case of optimal government spending and distortionary taxation, the US is the welfare leader among industrialized countries throughout our sample period. In our smaller sample with high-quality data, the US pulls away from all other advanced countries in terms of welfare. The only exception is the UK, which converges steadily to US levels of welfare over time. In both data sets, the welfare differences among countries are driven to a much greater extent by TFP gaps than by differences in capital intensity. This finding echoes the conclusion of the "development accounting" literature, but for welfare differences rather than GDP gaps. Our results are robust to the choice of reference country and are not fundamentally altered by the addition of human capital to the analysis. We also show how TFP dynamics and the treatment of leisure affect the cross-country welfare comparisons.

The paper is structured as follows. The next section presents our analytical framework, and derives the results on the measurement of welfare within single economies and on welfare comparisons across countries. Section 3 extends the basic framework to allow for distortionary taxes and government expenditure, and summarizes our results in their more general form. We take our framework to the data in Section 4. Section 5 relates our work to several distinct literatures. Finally, in the conclusions we summarize our findings and suggest avenues for future

[1] See Feenstra, Inklaar and Timmer (2015b) for a more extensive discussion and a comparison of our methods with other approaches to cross-country welfare measurement.

research.

2 The Productivity Residual and Welfare

Both intuition and formal empirical work link TFP growth to increases in the standard of living, at least as measured by GDP per capita.[2] The usual justification for studying the Solow productivity residual is that, under perfect competition and constant returns to scale, it measures technological change, which contributes to GDP growth, one major determinant of welfare. Indeed, in the basic version of the neoclassical growth model with fixed or variable labor supply, the time path of technology and the initial capital stock are sufficient statistics for consumer welfare. But the intuition based on this result suggests that we should not necessarily care about the Solow residual in an economy with non-competitive output markets, non-constant returns to scale, and possibly other distortions where the residual as defined by Solow (1957) no longer measures technology correctly. It also suggests that the Solow residual and the capital stock would not be sufficient statistics for welfare if there are shocks to the model other than technology.

We show that the link between Solow's residual and welfare is immediate and robust, even when the residual does not measure technical change. Here we build on the intuition of Basu and Fernald (2002) and derive rigorously the relationship between a modified version of the productivity residual and the intertemporal utility of the representative household. The fundamental result we obtain is that, to a first-order approximation, lifetime utility reflects the present discounted value of productivity residuals (plus the initial stock of assets), regardless of the production technology and the degree of product market competition. Our result also establishes that these two variables comprise a sufficient statistic for consumer welfare (to a first order) even if technology is not the only shock. For example, our result holds even with shocks to tax rates, government expenditure, tariff rates, changes in price-cost markups, to name only a few of the cases we cover.

Our results are complementary to those in Solow's classic (1957) paper. Solow established that if there was an aggregate production function with constant returns to scale and if all markets were competitive, then his index measured its rate of change. We now show that under a very different set of assumptions, which are disjoint from Solow's, the familiar TFP index is also the key component of an intertemporal welfare measure. The results are parallel to one another. Solow did not need to assume anything about the consumer side of the economy to give a technical interpretation to his index, but he had to make assumptions about technology and firm market structure. We do not need to assume anything about the firm side (which includes technology and market structure) in order to give a welfare interpretation, but we do need to assume the existence of a representative consumer with a utility function that is consistent with steady state growth (King, Plosser and Rebelo (1988)).[3] Moreover, as we shall show, our approach does not require knowledge of the parameters of the utility function, other than

[2] For a review of the literature linking TFP to GDP per worker, in both levels and growth rates, see Weil (2008).

[3] At a technical level, both results assume the existence of a potential function (Hulten, 1973), and show that TFP growth is the rate of change of that function. In Solow's case, the potential function is the aggregate production function. In our case, it is the representative household's intertemporal utility function.

the discount rate. Essentially, the key parameters of consumer preferences are revealed by the expenditure and distributional shares used to construct the productivity residual. Even though the discount rate depends upon preference parameters, such as the rate of time preference and the intertemporal elasticity of substitution for consumption, we can measure it from the Euler equation of the representative household in the steady state, using data on the real rate of return on capital from national income accounting, the growth rate of per-capita consumption, and the population growth rate.

An obvious alternative for calculating consumer welfare would be to use calibrated or estimated dynamic stochastic general equilibrium (DSGE) models. They routinely assume a representative consumer as well. The advantage of our approach is that imposes less structure on the problem, and therefore it is much less dependent on parametric assumptions. Furthermore, understanding the actual welfare changes in a given historical period requires the modeler to incorporate all of the possible shocks that were relevant: changes in technology, trade policy, market structure, and so on. Our measure allows for multiple shocks that we do not have to specify—basically, shocks to anything other than the utility function itself.[4] However, only a fully-specified DSGE model allows going beyond historical comparisons to conduct counterfactual policy experiments. Thus, there are trade-offs between the two approaches. They are complementary rather than competing, and the choice of which to use depends upon the purpose of the analysis.

2.1 Measuring welfare changes over time

We begin by assuming the familiar objective function for a representative household that maximizes intertemporal utility. In a growth context one often neglects the dependence of welfare on leisure, but Nordhaus and Tobin (1972) suggest that this omission is not innocuous (see the discussion in Section 5). Thus, we assume the household derives utility from both consumption and leisure:

$$W_t = E_t \sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s} \frac{N_{t+s}}{H} \frac{1}{1-\sigma} {C_{t+s}}^{1-\sigma} \nu(\overline{L} - L_{t+s}) \tag{1}$$

where W_t denotes the total welfare of the household, C_t is the per-capita consumption, L_t are per-capita hours of work and $\overline{L}$ is the per-capita time endowment. N_t is population and H is the number of households, assumed to be fixed and normalized to one from now on. Initially, we assume there is no government. Population grows at a constant rate n. To ensure the existence of a well-defined steady-state in which hours of work are constant while consumption and the real wage share a common trend, we assume that the utility function has the King, Plosser and Rebelo (1988) form with $\sigma > 0$ and $\nu(.) > 0$.[5] In a setting with no government, the budget constraint facing the representative consumer and the capital accumulation equation are

[4] In any environment, DSGE or otherwise, welfare comparisons with time-varying preferences raise problematic philosophical issues.

[5] If $\sigma = 1$, then the utility function must be $U(C, \overline{L} - L) = \log(C) + \nu(\overline{L} - L)$. See King, Plosser and Rebelo (1988).

respectively:

$$P_t^I K_t N_t + B_t N_t = (1-\delta) P_t^I K_{t-1} N_{t-1} + \left(1+i_t^B\right) B_{t-1} N_{t-1} + P_t^L L_t N_t + P_t^K K_{t-1} N_{t-1} + \Pi_t N_t - P_t^C C_t N_t \tag{2}$$

$$K_t N_t = (1-\delta) K_{t-1} N_{t-1} + I_t N_t \tag{3}$$

K_t, B_t and I_t denote per-capita capital, net asset holdings and investment; P_t^K, P_t^L, P_t^C and P_t^I denote, respectively, the user cost of capital, the hourly wage, the price of consumption goods and of new capital goods; i_t^B is the nominal rate of returns on assets and Π_t denotes per-capita profits, which are paid lump-sum from firms to consumers. So far we have implicitly assumed, for ease of notation, that there is only one type of consumption and one type of investment good. However, we show in the (2012) working paper version that we can allow for multiple consumption and investment goods. In this case, C_t and I_t should be interpreted as homothetic aggregators of individual consumption and investment goods. P_t^C and P_t^I are the corresponding price indexes.

In a closed economy B_t would denote domestic inside debt. In equilibrium, it would equal zero. It does not matter whether B_t represents a single asset or a full menu of state-contingent assets. In turn, in an open-economy setting, the set of available assets and whether there is full consumption risk-sharing matters for many questions. However, our results hold, even in the open economy, regardless of the structure of asset markets and the degree of risk sharing, as one can verify by inspecting the proofs we present later.[6] For ease of notation, we continue to treat B_t and i_t^B as scalars, but it is important to keep in mind that they may be vectors, and the set of assets in B_t may or may not span the full state space.

Define "equivalent consumption" per person, denoted by C_t^*, as the level of consumption per-capita at time t that, if growing at the steady-state rate g from t onward, with leisure set at its steady-state level, delivers the same per-capita intertemporal utility as the actual stream of consumption and leisure. More precisely, C_t^* satisfies:

$$\begin{aligned} \frac{W_t}{N_t} &= V_t = \sum_{s=0}^{\infty} \frac{(1+n)^s}{(1+\rho)^s} \frac{\left(C_t^*(1+g)^s\right)^{1-\sigma}}{(1-\sigma)} \nu(\overline{L}-L)) \\ &= \frac{1}{(1-\sigma)(1-\beta)} C_t^{*1-\sigma} \nu(\overline{L}-L) \end{aligned} \tag{4}$$

where V_t denotes per-capita utility and $\beta = \frac{(1+n)(1+g)^{1-\sigma}}{(1+\rho)}$ is the discount rate. We will measure welfare changes over time in terms of equivalent consumption per-capita and relate them to observable economic variables.

First we define a few of the key variables used in our analysis. Consider a modified definition of the Solow productivity residual:

$$\Delta \log PR_{t+s} = \Delta \log Y_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log K_{t+s-1} \tag{5}$$

where $\Delta \log Y_t = s_C \Delta \log C_t + s_I \Delta \log I_t$. $\Delta \log Y_t$ is a Divisia index of domestic absorption

[6] The intuition is that the observed time paths of consumption and leisure summarize all of the relevant information about the degree of risk sharing allowed by asset markets.

growth (in real per-capita terms), where demand components are aggregated using constant steady-state shares. s_C and s_I denote the steady-state values of $s_{C,t} = \frac{P_t^C C_t}{P_t^Y Y_t}$ and $s_{I,t} = \frac{P_t^I I_t}{P_t^Y Y_t}$ respectively, and $P_t^Y Y_t$ represents per-capita absorption in current prices. Distributional shares are also defined as the steady-state values, s_L and s_K, of $s_{L,t} \equiv \frac{P_t^L L_t}{P_t^Y Y_t}$ and $s_{K,t} \equiv \frac{P_t^K K_{t-1} N_{t-1}}{P_t^Y Y_t N_t}$ (note that the household receives remuneration on the capital stock held at the end of the last period). We use the word "modified" in describing the productivity residual for several reasons. First, we use real absorption rather than GDP as the measure of output. Second, all shares are calculated at their steady-state values, and hence are not time varying, which is sometimes assumed when calculating the residual.[7] Third, the residual is stated in terms of per-capita rather than aggregate variables, although it should be noted that Solow himself defined the residual on a per-capita basis (1957, equation 2a). Correspondingly, define the log level productivity residual as:

$$\log PR_{t+s} = s_C \log C_{t+s} + s_I \log I_{t+s} - s_L \log L_{t+s} - s_K \log K_{t+s-1} \tag{6}$$

The prices in the budget constraint, equation (2), are defined in nominal terms. It will often be easier to work with relative prices. Taking the purchase price of new capital goods, P_t^I, as numeraire, define the following relative prices: $p_t^K = \frac{P_t^K}{P_t^I}$, $p_t^L = \frac{P_t^L}{P_t^I}$ and $p_t^C = \frac{P_t^C}{P_t^I}$. Real per-capita profits are defined as $\pi_t = \frac{\Pi_t}{P_t^I}$. Our approximations are taken around a steady-state path where the first three relative prices are constant and the wage p^L grows at rate g, as in standard one-sector models of economic growth. We also assume that all per-capita quantity variables other than labor hours (for example Y_t, C_t, I_t, etc.) grow at a common rate g in the steady-state. Note these assumptions imply that all of the shares we have defined above are constant in the steady-state and so is the capital-output ratio, whose nominal steady-state value will be denoted by $\frac{P^I K}{P^Y Y}$.[8] We do not see the assumption of balanced, steady state growth as overly restrictive, as it is consistent with a large range of exogenous, endogenous, and semi-endogenous growth models.

Under these assumptions we can show that welfare changes, as measured by equivalent consumption, C_t^*, are, to a first-order approximation, a linear function of the expectation of present and future total factor productivity growth (and its revision), and of the initial capital stock. This first key result is summarized in:

Proposition 1 *Assume that the representative household in a closed or open economy with no government maximizes (1) subject to (2), taking prices, profits and interest rates as exogenously given. Assume also that population grows at a constant rate n, and the wage and all per-capita quantities other than hours worked grow at rate g in the steady-state. To a first-*

[7] Rotemberg and Woodford (1991) argue that in a consistent first-order log-linearization of the production function the shares of capital and labor should be taken to be constant.

[8] We conjecture, that all our results could be proved in the household environment corresponding to a two-sector growth model as laid out, for example, in Whelan (2003)—assuming that the steady-state shares are also constant, as in Whelan's setup.

order approximation, the growth rate of equivalent consumption can be written as:

$$\Delta \log C_t^* = \frac{(1-\beta)}{s_c}\left[E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s-1} + \frac{1}{\beta}\left(\frac{P^I K}{P^Y Y}\right) \Delta \log K_{t-1}\right] \tag{7}$$

Proof. Proofs of all propositions and extensions are in the (Online) Appendix, Sections A.1 through A.3. The proof for Proposition 1 is in Section A.1 ■

Proposition 1 implies that the expected present value of current and future Solow productivity residuals (or their revision), together with the change in the initial stock of capital per-capita, is a sufficient statistic for the welfare of a representative consumer (where we measure welfare as the log change in equivalent consumption). Note that the term $\Delta E_t \log PR_{t+s} = E_t \log PR_{t+s} - E_{t-1} \log PR_{t+s}$ represents the revision in expectations of the log level of the productivity residual, based on the new information received between $t-1$ and t and it will reduce to a linear combination of the innovations in the stochastic shocks affecting the economy at time t.[9]

Proposition 1 is a statement about the value function. A similar result has been proven in the context of the Ramsey or RBC models with exogenous growth, where productivity measures technology. In that setting, one can show that the maximized value depends on current and expected future technology and current capital. Under a Markov assumption about the evolution of technology, all the expected technology terms can be summarized by technology today. Thus, in this simple context our result nests one that is already well known. But our result is much more general. Since we have not made any assumptions about production technology and market structure, the productivity terms may or may not measure technical change. For example, Solow's residual does not measure technical change in economies where firms have market power, or produce with increasing returns to scale, or where there are Marshallian externalities. Even in these cases, Proposition 1 shows that productivity and the capital stock are jointly a sufficient statistic for welfare.

While the proof of the proposition requires somewhat complex notation and algebra, contained in Appendix A.1 through A.3 , in the remainder of this sub-section we shall try to convey the economic reasoning for the result by considering the simpler case of a closed economy with a zero steady-state growth rate ($g = 0$). (Of course, the formal proof of Proposition 1 allows for $g > 0$ and for the economy to be either open or closed.) We begin by taking a first-order approximation to the level of utility of the household (normalized by population) around the steady state.[10] We then use the household's first-order conditions for optimality and the transversality condition to obtain:

$$\frac{(V_t - V)}{\lambda p^Y Y} = E_t \sum_{s=0}^{\infty} \beta^s \left[s_L \widehat{p_{t+s}^L} + s_K \widehat{p_{t+s}^K} + s_\pi \widehat{\pi}_t - s_C \widehat{p_{t+s}^C}\right] + \frac{1}{\beta}\left(\frac{P^I K}{P^Y Y}\right) \widehat{K_{t-1}} \tag{8}$$

Hatted variables denote log deviations from the steady-state ($\widehat{x}_t = \log x_t - \log x$). Variables

[9] If we assume that the modified log level productivity residual follows a univariate time-series process, then only the innovation of such a process matters for the expectation revision, and the first summation is simply a function of current and past values of productivity.

[10] We approximate the level of V rather than its log because $V < 0$ if $\sigma > 1$.

without time subscripts denote steady-state values. Since $g = 0$, $\beta = \frac{1+n}{1+\rho}$. λ is the Lagrange multiplier associated with the budget constraint expressed, like utility, in per-capita terms. Equation (8) follows almost directly from the Envelope Theorem. An atomistic household maximizes taking as given the sequences of current and expected future prices, lump-sum transfers (Π_t), and predetermined variables (in our environment, just K_{t-1}). Thus, only fluctuations in these objects affect welfare to a first order. The Envelope Theorem plus a bit of algebra shows that each change in exogenous prices or profits needs to be multiplied by its corresponding share to derive its effect on welfare—for example, the larger is s_C the more the consumer suffers from a rise in the relative price of consumption goods. (It may appear that the investment price is missing, but since we normalized the relative price of investment goods to 1 it never changes.) The terms within the summation can be thought of as the dual version of the productivity residual, as we will show shortly.

Two comments are in order. First, the left hand side of the equation has an interesting interpretation: it is the money value of the deviation of per-person utility from its steady-state level, expressed as a fraction of steady-state absorption per person. To understand this interpretation, consider the units. The numerator is in "utils," which we divide by λ, which has units of utils per investment good (since investment goods are our numeraire). The division gives us the deviation of utility from its steady-state value measured in units of investment goods in the numerator. We then scale the result by the real value of per-capita absorption, also stated in terms of investment goods (recall that p^Y is a relative price).

Second, note that we can express welfare change without knowing the parameters of the utility function, other than the discount factor β. The right-hand side contains only expectations of the Solow residual (and its revision) and the initial capital stock per capita. Essentially, the parameters of the utility function are embedded in observed choices for consumption, labor and capital, in the expenditure shares, in the distributional shares and in the capital-output ratio. Our basic idea is to use a revealed-preference approach, akin to the logic behind the economic approach to index-number theory. This approach uses observed choices to infer welfare parameters (as, for example, expenditure shares reveal the relative importance of different prices to the household).

However, we find it more convenient and intuitive to express the left hand side of (8) in terms of equivalent consumption. Using the definition in (4) and taking a first order approximation of $V_t - V$ in terms of $\log C_t^*$ we obtain:

$$\frac{(V_t - V)}{\lambda p^Y Y} = \frac{s_C}{(1-\beta)}(\log C_t^* - \log C) \tag{9}$$

where we have used the fact that in the steady state $C^* = C$ and $U_C = \lambda p^c$.

The right hand side of (8) is written as a function of the log deviation from the steady-state of prices, profits and the initial capital stock. Our results can also be presented using the familiar primal productivity residual rather than stating them in terms of prices and transfers, as in (8). However, if one uses a consistent data set, there is literally no difference between the two. Using the per-capita version of the household budget constraint (2) and the capital accumulation equation (3), one can show that the following relationship must hold at all points in time:

$$s_L\widehat{p^L_{t+s}} + s_K\widehat{p^K_{t+s}} + s_\pi\widehat{\pi}_t - s_C\widehat{p^C_{t+s}} = s_C\widehat{C}_{t+s} + s_I\widehat{I_{t+s}} - s_L\widehat{L}_{t+s} - s_K\widehat{K_{t+s-1}}. \tag{10}$$

Equation (10) says that in any data set where national income accounting conventions are enforced, the primal productivity residual identically equals the dual productivity residual.[11] Thus we can express our results in either form, but using the dual result would require us to provide an empirical measure of lump-sum transfers, which is not needed for results based on the primal residual. For this reason, we work with the primal.

Using (9) and (10) in (8), we can write:

$$(\log C_t^* - \log C) = \frac{(1-\beta)}{s_C}\left[E_t\sum_{s=0}^{\infty}\beta^s\widehat{PR}_{t+s} + \frac{1}{\beta}\left(\frac{P^I K}{P^Y Y}\right)\widehat{K_{t-1}}\right] \tag{11}$$

where now the log deviation of consumption is expressed as a function of the log deviation from steady-state of the productivity residual, defined in equation (6). Taking differences of equation (11) and using the definition of the Solow residual in (5) gives the key equation of Proposition 1, whose proof we have just sketched for the case of $g = 0$.

So far we have used absorption as a measure of output. How would we have to modify our proposition, if one were to use, instead, a standard measure of output, real GDP, defined as consumption, plus investment, plus net exports? Then, we show at the end of Appendix A.1 that the welfare-relevant residual can be written as the sum of a conventionally-defined productivity residual plus additional components that capture terms of trade and capital gains effects. Moreover, in addition to the lagged capital stock, the initial conditions should include the lagged value of the stock of net foreign assets.

However, measuring these extra terms empirically poses major challenges. In contrast, all these problems disappear if the measurement of welfare is based on real absorption rather than GDP as the measure of output. This is why we have put the absorption-based productivity residual at the center of our theoretical analysis and empirical application.

2.2 Implications for Cross-Country Analysis

Proposition 1 pertains to the evolution of welfare in individual economies over time. The indexes we obtain are not comparable across countries. However, in this sub-section we show that similar methods can be used to do a rigorous welfare comparison across countries. More precisely, we show that productivity and the capital stock suffice to calculate differences in welfare across countries, with both variables computed as log level deviations from a reference country.

A comparison of welfare across two countries, i and j, requires either assuming that their respective representative agents possess the same utility function, or making the comparison from the perspective of the representative agent in a reference country, j, who optimizes facing the prices, per capita returns, profits, and initial assets of country i. The former assumption is clearly unappealing, but the latter approach seems difficult since it appears that we need to construct counterfactual choices of consumption and leisure for the household from the refer-

[11] See, for example, Barro and Sala-i-Martin (2004, section 10.2).

ence country, moving to country i. However, we show that our approach does not require the construction of such counterfactual quantities, and can be implemented using only observed cross-country differences in productivity.

We can then study the cross-country difference in the utility of a representative consumer. As in the within-country case, we conduct the comparison using the concept of equivalent consumption. In this context for the representative agent of the reference country j living in country i, equivalent per-capita consumption, $\widetilde{C}_t^{*,i}$ satisfies:

$$\widetilde{V}_t^i = \frac{1}{(1-\sigma^j)\left(1-\beta^j\right)} \left(\widetilde{C}_t^{*,i}\right)^{1-\sigma^j} \nu(\overline{L} - L^j) \tag{12}$$

where $\widetilde{V}_t^i$ denotes per-capita utility of the individual from country j, facing country i's relative prices, per-capita profits and per-capita initial capital stock (we use $\sim$ to denote these counterfactual quantities).[12] Note that $\widetilde{C}_t^{*,i}$ is defined for a constant level of leisure fixed at country j's steady-state level. We will use V_t^j and $C_t^{*,j}$ to denote per-capita utility and equivalent consumption of the individual of country j living in country j. We take first-order approximations of $\widetilde{V}_t^i$, V_t^j, and the budget constraints around the steady state of country j. This gives us:

Proposition 2 *Assume that in a reference country, country j, the representative household maximizes (1) subject to (2), under the assumptions of Proposition 1. Assume now that the household of country j is confronted with the sequence of prices, per-capita returns, profits, and initial capital stock of country i. In a closed or open economy with no government, to a first order approximation, the difference in equivalent consumption between living in country i versus country j is:*

$$\begin{aligned} \log \widetilde{C}_t^{*,i} - \log C_t^{*,j} &= \frac{(1-\beta^j)}{s_c^j} E_t \sum_{s=0}^{\infty} (\beta^j)^s \left(\log \overline{PR}_{t+s}^i - \log PR_{t+s}^j\right) \\ &\quad + \frac{(1-\beta^j)}{\beta^j s_c^j} \left(\frac{P^{I,j} K^j}{P^{Y,j} Y^j}\right) \left(\log K_{t-1}^i - \log K_{t-1}^j\right) \end{aligned} \tag{13}$$

where the productivity terms are constructed in the following fashion:

$$\log \overline{PR}_{t+s}^i = \left(s_C^j \log C_{t+s}^i + s_I^j \log I_{t+s}^i\right) - s_L^j \log L_{t+s}^i - s_K^j \log K_{t+s-1}^i \tag{14}$$

$$\log PR_{t+s}^j = \left(s_C^j \log C_{t+s}^j + s_I^j \log I_{t+s}^j\right) - s_L^j \log L_{t+s}^j - s_K^j \log K_{t+s-1}^j \tag{15}$$

Proof. Proof for Proposition 2 is in Section A.2 of the (Online) Appendix. ■

Welfare differences across countries are therefore summarized by two components. The first component is related to the well-known log difference between TFP levels, which accounts empirically for most of the difference in per-capita income across countries (Hall and Jones (1999)), although here it is the present value of the gap that matters for welfare. In the development accounting literature, this gap is interpreted as a measure of technological or institutional differences between countries. This interpretation, however, is valid only under restrictive assumptions on market structure and technology (perfect competition, constant returns to scale,

[12] In particular, the individuals from country j should face the same rates of return on capital and bonds faced by the individuals from country i. See section A.2 of the Appendix, especially equations (A.32) and (A.33).

no externalities, etc.). We provide a welfare interpretation of cross-country differences in TFP that applies even when these assumptions do not hold. The second component of the welfare difference reflects the difference in capital intensity between the two countries; *ceteris paribus*, a country with more capital per person can afford more consumption or higher leisure. The development accounting literature also uses capital intensity as the second variable explaining cross-country differences in per-capita income.

Our result holds for any kind of technology and market structure, as long as a representative consumer exists, takes prices as given and is not constrained in the amount he can buy and sell at those prices. (We discuss quantity rationing and violations of the law of one price in section A.3.2 in the Appendix and in our (2012) working paper). Notice however, that our measure of per-capita TFP is modified with respect to the traditional growth accounting measure in three ways. First, measuring welfare differences requires comparing not only current log differences in TFP but the present discounted value of future ones as well. Second, the distributional and expenditure shares used to compute the log differences in TFP between countries need to be calculated at their steady-state values in the reference country.[13] Third, we use real absorption rather than GDP as the measure of output.

As in the case of Proposition 1, we shall try to convey the economic reasoning for the result by considering the simple case of a closed economy with a zero steady-state growth rate ($g = 0$). Assume we confront the household from country j with the prices, profits and the initial per-capita capital stock of country i. If we expand the utility of a representative member of the household, denoted by $\widetilde{V}_t^i$, around the steady state of his own country, we obtain:

$$\begin{aligned}\frac{(\widetilde{V}_t^i - V^j)}{\lambda^j p^{Y,j} Y^j} \quad = \quad & E_t \sum_{s=0}^{\infty} (\beta^j)^s [s_L^j (\log p_{t+s}^{L,i} - \log p^{L,j}) + s_K^j (\log\ p_{t+s}^{K,i} - \log\ p^{K,j}) \\ & + s_{\Pi}^j (\log \pi_{t+s}^i - \log \pi^j) - s_C^j (\log p_{t+s}^{C,i} - \log p^{C,j})] \\ & + \frac{1}{\beta^j} \left(\frac{P^{I,j} K^j}{P^{Y,j} Y^j} \right) \left(\log K_{t-1}^i - \log K^j \right)\end{aligned} \tag{16}$$

Now expand per-capita utility for country j's household around its own steady-state and subtract from (16). This yields:

$$\begin{aligned}\frac{(\widetilde{V}_t^i - V_t^j)}{\lambda^j p^{Y,j} Y^j} \quad = \quad & E_t \sum_{s=0}^{\infty} (\beta^j)^s [s_L^j (\log p_{t+s}^{L,i} - \log p_{t+s}^{L,j}) + s_K^j (\log\ p_{t+s}^{K,i} - \log\ p_{t+s}^{K,j}) \\ & + s_{\Pi}^j (\log \pi_{t+s}^i - \log \pi_{t+s}^j) - s_C^j (\log p_{t+s}^{C,i} - \log p_{t+s}^{C,j})] \\ & + \frac{1}{\beta^j} \left(\frac{P^{I,j} K^j}{P^{Y,j} Y^j} \right) \left(\log K_{t-1}^i - \log K_{t-1}^j \right)\end{aligned} \tag{17}$$

Differences in welfare across countries are, therefore, due to differences in their relative prices, per-capita profits and capital intensities. Use (12) to express differences in utility across countries in term of log differences in equivalent consumption on the left hand side of (17). Now normalize by country country j population and productivity and linearize two budget constraints around country j's steady state: first, the budget constraint for the household from

[13] It is standard in the development accounting literature to assume that all countries have the same capital and labor shares in income (often one-third and two-thirds), but to use country-specific shares in expenditure.

country j if moved to country i and, second, its budget constraint when living in its own country. Subtracting one from the other allows us to write the right hand side of (17) in terms of productivity differences and differences in the initial capital stock. This yields equations (13), (14), and (15) in Proposition 2.[14]

Notice that in stating Proposition 2, we have not needed to assume that either the population growth rate n or the per-capita growth rate g is common across countries. Most importantly, we do not need to know the parameters of the utility function (other than β) to perform cross country welfare comparisons, nor do we need to assume that any of those parameters (including β) is common across countries. This is because we are making the comparisons from the point of view of the representative individual in the reference country, who is faced with the exogenous (to the household) prices, lump-sum transfers and initial conditions of country i.

It is also important to emphasize that this thought experiment does not simply assign to the household from the reference country the consumption and leisure choices made by the household from country i. Rather, our approach allows the reference-country household to re-optimize when facing the conditions of country i. Even if faced with the same exogenous variables, the choices of the two households will generally differ, unless their preferences are identical. However, to a first order approximation, the algebraic sum of the terms in prices, profits, and initial conditions for the household from country j facing country i's prices, profits, and endowments equals the algebraic sum of the terms in consumption, labor supply and capital chosen by the individual from country i. We exploit this fact to obviate the need to know counterfactual quantities in calculating the welfare change for a household of country j moving to country i.

If we based our cross-country comparison on consumption and leisure, rather than productivity and the capital stock, we would be left with the insurmountable problem of providing an empirical counterpart for the (unobservable) quantities of the former variables chosen by the household of country j moving to country i. Only if preferences were assumed identical across countries – a rather restrictive assumption – could the issue be resolved by using the actual quantities of country i. Our derivation shows that we can, instead, base the comparison on the actual observed productivity levels and capital stocks of the two countries, even if preferences differ across countries.[15]

We note that in deriving our result we have used the population and productivity levels of the reference country to normalize the budget constraint of the household of the comparison country, and we have expanded around the steady state of the reference country. This naturally raises the issue of the quality of the approximation. We address this issue at the end of Section 3.

[14] We can show that, in the special case in which all countries are in the steady state, share a common growth rate, and consumers do not derive utility from leisure, cross-country welfare comparisons reduce to comparing Net National Product. This result is in the spirit of Weitzman (1976, 2003). We thank Chad Jones for this observation.

[15] See the equations leading up to (A.68) in Appendix A, Section A.2.

3 Extensions

We now show that our method of using TFP to measure welfare can be extended to allow for the presence of distortionary taxes and government expenditure. The first extension is relatively simple and is discussed in the text. The second one is a bit more complex and additional details are presented in Section A.3.1 of the Appendix. At the end of this section we also summarize another extension that allows for human capital accumulation, which is developed in more detail in section A.4 of the Appendix. We also discuss briefly whether and under which conditions we can allow for departures from the assumption of price-taking households, in particular in the form of rationing in the labor market (Section A.3.2 of the Appendix). These extensions require modifications in the formulas for welfare comparisons over time and across countries, and we state the changes to the basic framework that are needed in each case. These results prove that the basic idea of using TFP to measure welfare holds in a variety of economic environments, but they also demonstrate the advantage of deriving the welfare measure from an explicit dynamic model of the household. For brevity, we only discuss the generalization of our measure of welfare changes over time (Proposition 1). Similar results apply to the measure of cross-country welfare differences (Proposition 2).

3.1 Taxes

Consider first an environment with distortionary and/or lump-sum taxes. Since the prices in the budget constraint (2) are those faced by the consumer, in the presence of taxes all prices should be interpreted as after-tax prices. At the same time, the variable that we have been calling "profits," Π_t, can be viewed as comprising any transfer of income that the consumer takes as exogenous. Thus, it can be interpreted to include lump-sum taxes or rebates. Finally one should think of B_t as including both government and private bonds (assumed to be perfect substitutes, for ease of notation).

More precisely, in order to modify (2) to allow for taxes, let τ_t^K be the tax rate on capital income, τ_t^R be the tax rate on revenues from bonds, τ_t^L be the tax rate on labor income, τ_t^C be the *ad valorem* tax on consumption goods, and τ_t^I be the corresponding tax on investment goods.[16] Also, let $P_t^{C'}$ and $P_t^{I'}$ respectively denote the pre-tax prices of consumption and capital goods, so that the tax-inclusive prices faced by the consumer are $P_t^{C'}\left(1+\tau_t^C\right)$ and $P_t^{I'}\left(1+\tau_t^I\right)$. We assume for the time being that the revenue so raised is distributed back to individuals using lump-sum transfers; we consider government expenditures in the next subsection. The representative household's budget constraint now is:

$$\begin{aligned} 0 \;=\; & -P_t^{I'}\left(1+\tau_t^I\right)K_tN_t - B_tN_t + (1-\delta)P_t^{I'}\left(1+\tau_t^I\right)K_{t-1}N_{t-1} + \left(1+i_t^B\left(1-\tau_t^R\right)\right)B_{t-1}N_{t-1} \\ & +P_t^L\left(1-\tau_t^L\right)L_tN_t + P_t^K\left(1-\tau_t^K\right)K_{t-1}N_{t-1} + \Pi_tN_t - P_t^{C'}\left(1+\tau_t^C\right)C_tN_t \qquad (18) \end{aligned}$$

Thus, extending our benchmark case, the exogenous variables in the household's maximization are not only the prices and the initial stocks of capital and bonds, but also the tax rates on labor and capital income, consumption and investment. However, it can be easily shown that

[16] For simplicity, we are assuming no capital-gains taxes and no expensing for depreciation. These could obviously be added at the cost of extra notation.

the basic results (7) and (13) continue to hold. The only modification is that in defining the Solow productivity residual we need to take account of the fact that the national accounts measure factor payments as perceived by firms – that is, before income taxes – while nominal expenditure is measured using prices as perceived from the demand side, thus inclusive of indirect taxes (subsidies) on consumption and investment. Hence, letting $P_t^C = P_t^{C'}\left(1+\tau_t^C\right)$ and $P_t^I = P_t^{I'}\left(1+\tau_t^I\right)$ denote the tax-inclusive prices of consumption and investment goods, the expenditure shares s_C and s_I defined earlier are fully consistent with those obtained from national accounts data, but the factor shares s_L and s_K defined above use the gross income of labor and capital rather than their after-tax income. Thus, to be data consistent, with taxes the welfare residual needs to be redefined in terms of the shares of after-tax returns on labor and capital. Specifically, equation (5) should be re-written as:

$$\Delta \log PR_{t+s} = \Delta \log Y_{t+s} - \left(1-\tau^L\right) s_L \Delta \log L_{t+s} - \left(1-\tau^K\right) s_K \Delta \log K_{t+s-1} \quad (19)$$

and an analogous modification applies to (14) and (15). τ^L and τ^K are the steady-state values of τ_t^L and τ_t^K With these modifications, our results generalize to a setting with distortionary, time-varying taxes on consumption and investment goods and on the household's income coming from labor, capital or financial assets.

3.2 Government Expenditure

With some minor modification, our framework can be likewise extended to allow for the public provision of goods and services (see section A.3.1 in the appendix for the full details). We illustrate this under the assumption that government activity is financed with lump-sum taxes. Using the results from the previous subsection, it is straightforward to extend the argument to the case of distortionary taxes.

Assume that government spending takes the form of public consumption valued by consumers. We rewrite instantaneous utility as:

$$U(C_{t+s}, C_{G,t+s}, L_{t+s}) = \frac{1}{1-\sigma}\Omega(C_{t+s}; C_{G,t+s})^{1-\sigma}\nu(\overline{L} - L_{t+s}) \quad (20)$$

where C_G denotes per-capita public consumption and $\Omega(.)$ is homogenous of degree one in its arguments. Total absorption now includes public consumption: that is, $P_t^Y Y_t = P_t^C C_t + P_t^G C_{G_t} + P_t^I I_t$, where P^G is the public consumption deflator.

In this setting, our earlier results need to be modified to take account that public consumption may not be set by the government at the level that consumers would choose. Intuitively, in such circumstances the value that consumers attach to public consumption may not coincide with its observed value as included in domestic absorption, and therefore in the productivity residual as conventionally defined. Formally, let $s_{c_{Gt}} = \frac{P_t^G C_{G_t}}{P_t^Y Y_t}$ denote the share of public consumption out of domestic absorption, and let $s^*_{c_{G_t}}$ be the share that would obtain if public consumption were valued according to its marginal contribution to the utility of the representative household, rather than using its deflator P_t^G.[17] The welfare-relevant modified Solow

[17] It is easy to verify that $s^*_{c_{G_t}} = \frac{U_{C_{Gt}} P_t^C C_{G_t}}{U_{C_t} P_t^Y Y_t}$.

residual (5) now is:

$$\Delta \log PR_{t+s} = \Delta \log Y_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log K_{t+s-1} + \left(s^*_{c_G} - s_{c_G}\right) \Delta \log C_{G,t+s} \quad (21)$$

and an analogous modification applies to (14) and (15). Hence in the presence of public consumption the Solow residual needs to be adjusted up or down depending on whether public consumption is under- or over-provided (i.e., $s^*_{c_G} > s_{c_G}$ or $s^*_{c_G} < s_{c_G}$ respectively). If the government sets public consumption exactly at the level the utility-maximizing household would have chosen if confronted with the price P^G_t, then $s^*_{c_G} = s_{c_G}$ and no correction is necessary. For want of a better term, we shall refer to this case as 'optimal government consumption.' In turn, in the case in which public consumption is pure waste $s^*_{c_G} = 0$, the welfare residual should be computed on the basis of private absorption, consumption and investment. With the residual redefined in this way, the growth rate of equivalent consumption now is:[18]

$$\Delta \log (C_t)^* = \frac{(1-\beta)}{(s_C + s^*_{C_G})} \left[E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s-1} + \frac{1}{\beta} \left(\frac{P^I K}{P^Y Y} \right) \Delta \log K_{t-1} \right] \quad (22)$$

3.3 Summing up

We can now go back to the two propositions stated earlier. They were formulated for the special case of an economy with no government. In light of the discussion in this section, we can now restate them in a generalized form for an economy with a government sector, which is more appropriate for empirical implementation.

Proposition 1' *Assume a closed or open economy, with public consumption, taxes on labor and capital income at rates τ^L_{t+s} and τ^K_{t+s}, and taxes on consumption and investment expenditure at rates τ^C_{t+s} and τ^I_{t+s}. Assume also that the representative household maximizes intertemporal utility, taking prices, profits, interest rates, tax rates and public consumption as exogenously given. Lastly, assume that population grows at a constant rate n, and the wage and all per-capita quantities other than hours worked grow at rate g in the steady state. To a first-order approximation, the growth rate of equivalent consumption can be written as:*

$$\Delta \log (C_t)^* = \frac{(1-\beta)}{(s_C + s^*_{C_G})} \left[E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s-1} + \frac{1}{\beta} \left(\frac{P^I K}{P^Y Y} \right) \Delta \log K_{t-1} \right] \quad (23)$$

where productivity growth is defined as:

$$\begin{aligned} \Delta \log PR_{t+s} &= s_C \Delta \log C_{t+s} + s_I \Delta \log I_{t+s} + s^*_{C_G} \Delta \log C_{G,t+s} \\ &\quad - \left(1-\tau^L\right) s_L \Delta \log L_{t+s} - \left(1-\tau^K\right) s_K \Delta \log K_{t+s-1} \end{aligned} \quad (24)$$

[18] Government purchases might also yield productive services to private agents. For example, the government could provide education or health services, or public infrastructure, which – aside from being directly valued by consumers – may raise private-sector productivity. In such case, the results in the text remain valid, but it is important to note that the contribution of public expenditure to welfare would not be fully captured by the last term in the modified Solow residual as written in the text. To this term we would need to add a measure of the productivity of public services, which is implicitly included in the other terms in the expression.

Shares and tax rates are evaluated at their steady-state values, and $s^*_{C_G}$ denotes the steady-state share of public consumption in total absorption that would obtain if public consumption were valued according to its marginal contribution to the utility of the representative household.

As explained earlier, the value of $s^*_{C_G}$ depends on the assumptions made about government consumption. This implies that the proposition can encompass a variety of cases with respect to taxation and government spending: 1) wasteful government spending with lump sum taxes (in which case distortionary taxes are set to zero in the productivity equation); 2) optimal government spending with lump sum taxes; 3) wasteful government spending with distortionary taxes; 4) optimal government spending with distortionary taxes.

Our main result regarding welfare differences across countries can be restated in a similar way:

Proposition 2' *Assume that in a reference country j the representative household maximizes intertemporal utility under the assumptions of Proposition 1'. Assume now that the household of country j is confronted with the sequence of prices, tax rates, per-capita profits, other lump sum transfers, public consumption, and endowment of country i. In a closed or open economy with distortionary taxation and government spending, the difference in equivalent consumption between living in country i versus country j can be written, to a first order approximation, as:*

$$\begin{aligned} \ln \widetilde{C}^{*,i}_t - \ln C^{*,j}_t &= \frac{(1-\beta^j)}{(s^j_C + s^{*j}_{C_G})} E_t \sum_{s=0}^{\infty} (\beta^j)^s \left(\log \overline{PR}^i_{t+s} - \log PR^j_{t+s} \right) \\ &+ \frac{(1-\beta^j)}{\beta^j (s^j_C + s^{*j}_{C_G})} \left(\frac{P^{I,j} K^j}{P^{Y,j} Y^j} \right) \left(\log K^i_{t-1} - \log K^j_{t-1} \right) \end{aligned} \tag{25}$$

where $s^{*j}_{C_G}$ the steady-state share of public consumption in total absorption that would obtain if public consumption were valued according to its marginal contribution to the utility of the representative household. The two productivity terms are constructed with all shares and tax rates evaluated at the reference country's steady-state values:

$$\log PR^j_t = s^j_C \log C^j_t + s^j_I \log I^j_t + s^{*j}_{C_G} \log C^j_{G,t} - \left(1 - \tau^{L,j}\right) s^j_L \log L^j_t - \left(1 - \tau^{K,j}\right) s^j_K \log K^j_{t-1} \tag{26}$$

$$\log \overline{PR}^i_t = s^j_C \log C^i_t + s^j_I \log I^i_t + s^{*j}_{C_G} \log C^i_{G,t} - \left(1 - \tau^{L,j}\right) s^j_L \log L^i_t - \left(1 - \tau^{K,j}\right) s^j_K \log K^i_{t-1} \tag{27}$$

Proposition 2' will be the basis for our cross-country welfare comparisons. The derivation shows a result that would be hard to intuit ex ante, which is that to a first-order approximation only the tax rates of the reference country enter the welfare comparison.[19] This asymmetry implies that welfare rankings may depend on the choice of reference country. In our empirical application in Section 4.3 below we take the US as our reference country, but check the robustness of the results by using France instead.

Our setting so far features a single consumption and investment good. However, the exten-

[19] Of course, the tax rates of the comparison country will generally change output and input levels in that country through general-equilibrium effects, which will influence the welfare gap between the two countries. However, the tax rates of the comparison country do not enter the formula directly.

sion to the case of multiple types of consumption and investment goods is straightforward, and is developed in detail in our (2012) working paper.

Another concern, particularly when analyzing a large set of diverse countries, is accounting for differences in human capital. In section A.4 in the appendix we show how to extend our framework in order to incorporate human capital in the spirit of Lucas (1988). As in Lucas, we assume that non-leisure time can be used either to work or to accumulate human capital, and that the accumulation of human capital is linear in its stock. We show that the definition of productivity must account for the effect on total labor input of both hours and human capital changes. Human capital investment does not show up as part of domestic absorption because in the Lucas formulation it is only a subtraction from leisure and does not require any other physical input. However, human capital must now be included among the initial conditions, alongside physical capital.

We have assumed throughout that the household is a price-taker in goods and factor markets, and that it faces no constraints other than the intertemporal budget constraint. In section A.3.2 of the Appendix we show that under some conditions our approach extends to environments where the household does not behave as a price taker, faces a distribution of prices rather than a single price, or faces quantity constraints. We focus on the labor market, which is the market in which the price-taking assumption may seem most questionable.

A potential concern with our main results, as stated in Proposition 1' and 2', is that they are proved using first-order approximations. This approach may seem especially problematic for cross-country comparisons, where gaps in living standards are often large and growth trajectories may differ even in the long run. In (the Online) Appendix B, we consider a set of workhorse models that are standard in the macroeconomic literature. We start from the Ramsey model, with inelastic labor supply, and then move on to the neoclassical growth model with variable labor supply, augmented, in turn, with distortionary taxes, imperfect competition (which is formally the same as the case of distortionary taxation), government expenditure, and production externalities (which introduce increasing returns to scale in production).

We consider first the case of transitional dynamics for within-country analysis, that is cases where a country is converging to its own steady state. We solve for the time path of welfare in these models numerically using third- and fourth-order approximations, and in some cases with exact numerical solutions using global solutions. We then compare the numerical results to the ones would get using our Propositions based on first-order approximations. The error from using the first-order approximations is typically half a percent or smaller, which seems quite acceptable.

We then proceed to examine the accuracy of our cross country results. We allow countries to have different levels or even *trend growth rates* of per-capita income and consumption, even in the steady state. We assume that countries at their own steady states (or balanced growth paths), which allows us to compute the welfare gaps between countries analytically in our simulations. We then compare these exact solutions to our first-order approximations. (Cases where countries are converging dynamically to their own steady states, but the steady states are different across countries, amount to a combination of the within- and cross-country simulations, and the approximation errors would be the sum of the two.) To our surprise, we found that differences in trend growth rates, which we view as a stringent test of our method, are not likely to be a significant source of error. Using our first-order approximation to compute the increase in welfare of going from a country with a 1 percent per-capita growth rate in the steady state to a country with a 2 percent growth rate has an approximation error of only 6 percent relative to the exact calculation. Plausible differences in the capital share or the steady-state *level* of productivity create smaller approximation errors. Larger approximation errors come if there are substantial differences in tax rates between the two countries.

Overall, we are reassured by our findings. We conclude from the numerical experiments that our approximation is very accurate for assessing welfare differences, both over time and across countries.

Finally, empirical implementation of Propositions 1' and 2' requires forecasts of future TFP. For simplicity, below we use univariate models to generate the forecasts. Here we remark on the similarities and differences between our method and traditional index-number theory. Our method is similar to the economic analysis of index numbers, in that we use a non-parametric method based on aggregating data that are exogenous to the household yet relevant for its welfare, where the weights used for aggregation are shares that reveal the importance of each input to the household's utility. The difference is that we are more ambitious, in that we try to compute not just the increment to welfare in each period—a flow measure—but the intertemporal sum of these flows, a stock or present-discounted-value measure. If we wanted to measure only a period flow, we would not need to forecast future TFP, or to take a stand on the size of the discount rate, β. We need to do so only because we are trying to construct a more comprehensive lifetime welfare measure.

4 Empirical Results

4.1 Data and Measurement

We illustrate the potential of our methodology by computing welfare indexes over the period 1985-2005 for a set of large, developed countries for which high-quality time-series data are available: the US, UK, Japan, Canada, France, Italy and Spain. We use two different data sets to compare welfare within a country and across countries. Given the interest in welfare comparisons for a larger group of countries, we also use a third data set for cross-country comparisons comprising 57 developed and developing countries. However, we do not use this sample for our baseline results because consistent time-series data on hours of work (as opposed to employment) are not available for most countries.[20]

To analyze welfare changes over time for our sample of advanced countries, we combine data coming from the OECD Statistical Database with the EU-KLEMS dataset.[21] We construct the log of absorption from the OECD dataset, using data on household final consumption,

[20] Bick, Fuchs-Schundeln and Lagakos (2018) construct hours of work for a large cross section of countries, but their data only cover the year 2005. In section C.3 in the appendix we offer some additional evidence for the smaller or new OECD countries for which hours data are available.

[21] The EU-KLEMS data are extensively documented by O'Mahony and Timmer (2009). We are unable to include Germany in the sample, since official data for unified Germany are available only since 1995 in EU-KLEMS.

gross capital formation and government consumption (where appropriate) at constant national prices, and their respective nominal shares of absorption as weights. Since our theory requires steady-state shares, we use the averages of the observed shares across the twenty years in our sample.

The growth rate of our modified productivity residual is constructed as the log-change in real absorption minus the log changes in capital and labor, each weighted by its income share out of absorption. Data on aggregate production inputs are provided by EU-KLEMS. The capital stock is constructed by applying the perpetual inventory method to investment data. Labor input is the total amount of hours worked by employed persons. To obtain per-capita quantities, we divide absorption, capital, and labor by total population. We assume that economic profits are zero in the steady-state so that we can recover the gross (tax unadjusted) share of capital as one minus the labor share.

In order to compare welfare across countries, we combine data from the Penn World Tables with hours data from EU-KLEMS dataset. Specifically, our basic measure of real absorption is constructed from the Penn World Tables as the weighted average of PPP-converted log private consumption, log gross investment and log government consumption, using as weights their respective shares of absorption in the reference country; as in the within-country case, we use shares that are averaged across the twenty years in our sample.

To construct the modified log productivity residual for each country, we subtract share-weighted log capital and labor from log real absorption. The shares are the compensation of each input out of absorption in the reference country, also in this case kept constant at their average values. The stock of capital in the economy is constructed using the perpetual-inventory method on the PPP-converted investment time series from the Penn World Tables. labor input is total hours worked, from EU-KLEMS.

The broader 57-country sample comprises all countries with population of 2 million or more in 1985 for which sufficient data were available to construct TFP for at least 20 years (1985-2005). For the comparative welfare calculations, absorption, capital, and the factor shares are constructed exactly as before. Since consistent data on hours of work are not available for most countries in the sample, we use, as an imperfect proxy for total labor input, aggregate employment from the ILO's Key Indicators database.

For the empirical exercises including human capital (reported in Section C.5 of the Appendix), we construct per-capita human capital stocks as in Caselli (2005, pp. 685-686), using the Barro and Lee (2010) data on average years of schooling of the population over 25 years of age. The source reports data at 5-year intervals; we use log-linear interpolation to obtain annual data.

Finally, when accounting for distortionary taxation, we use data on average tax rates on capital and labor provided by Boscá et al. (2005).

Since our welfare calculations depend on the expected present discounted value of TFP growth, we need to construct forecasts of future TFP. To keep our empirical illustration simple and uniform across countries, we estimate separate univariate time-series models for each country, using annual data over 1985-2005. We leave for future work the extension to a multivariate forecasting framework.

For each country, we estimate a simple AR process for log TFP, including a linear time trend. Both the autoregressive coefficients and the coefficient of the trend term are country-specific. The trend captures the long-run TFP growth rate, while the AR process allows for stationary fluctuations around that trend. This specification is consistent with the fact that the cross-country welfare comparisons below do not require a common long-run TFP growth rate across countries. We focus on three measures of aggregate TFP suggested by our theory. In all cases, our output concept is absorption rather than GDP, so our TFP indexes are appropriate for measuring welfare in open economies. The first two measures correspond to the case in which government purchases are wasteful, while the third assumes that government purchases are optimal. In the former case, we construct absorption by aggregating consumption and investment only, using shares that sum to $(1 - s_{c_G})$. In the latter, output is the sum (in logs) of consumption, investment and government purchases, aggregated with shares that sum to one. In the first case we assume that taxes are lump-sum, so we do not need to adjust the factor income shares. In the second and third cases, government spending is assumed to be financed with distortionary taxes, and the capital and labor shares employed in the calculation of TFP are corrected for both indirect and income taxes. Since taxes are distortionary in the real world, and it is likely that a major portion of public spending is valued by consumers, we take the latter scenario of distortionary taxes and optimal spending as our benchmark case, but retain in this section the other two scenarios for comparison purposes.

Estimation results (reported in section C1 in the appendix) show that, for all countries and TFP measures in our smaller sample of countries with high-quality data, the log level of TFP is well described by either an AR(1) or AR(2) stationary process around a linear trend. In all cases, once we allow for a time trend, the null of a unit root in the log TFP process can be rejected.

We follow the same procedures for our larger 57-country sample described above, with qualitatively similar results.

4.2 Within-Country Results

We construct country-specific indexes of welfare change over time for our seven benchmark countries using the estimated AR processes described above to form expectations of future levels or differences of TFP. We use equation (23) to express the average welfare change per year in each country in terms of changes in equivalent consumption. Given a value of β and the estimated time-series processes for TFP in each country, we can readily construct the first two terms in equation (24), the present value of expected TFP growth and the change in expectations of that quantity. The third term, involving the change in the capital stock, is observable from our data, given a value of β.

We construct our measure of β using the Euler equation for capital in the steady state so that $\beta = \frac{(1+n)(1+g)}{1+p^K-\delta}$, where g is the steady-state growth rate of per-capita consumption, n the constant population growth rate, and $p^K - \delta$ is the net return to capital (averaged across debt and equity). We calculate $p^K - \delta$ as total payments to capital, net of economic depreciation, divided by the value of the capital stock at the end of the previous period, using national accounts data compiled by Eurostat. To represent steady-state values we have used the averages over

Table 1: Annual Average Log Change in Per-Capita Equivalent Consumption and its Components

	Wasteful Spending Lump-Sum Taxes			Wasteful Spending Distortionary Taxes			Optimal Spending Distortionary Taxes		
	Total	Fraction due to:		**Total**	Fraction due to:		**Total**	Fraction due to:	
		TFP	*Capital*		*TFP*	*Capital*		*TFP*	*Capital*
Canada	**0.014**	0.40	0.60	**0.022**	0.61	0.39	**0.018**	0.65	0.35
France	**0.025**	0.86	0.14	**0.025**	0.86	0.14	**0.021**	0.88	0.12
Italy	**0.024**	0.51	0.49	**0.027**	0.57	0.43	**0.022**	0.60	0.40
Japan	**0.018**	0.42	0.58	**0.023**	0.56	0.44	**0.024**	0.66	0.34
Spain	**0.025**	0.43	0.57	**0.034**	0.58	0.42	**0.034**	0.67	0.33
UK	**0.035**	0.76	0.24	**0.038**	0.78	0.22	**0.032**	0.80	0.20
USA	**0.025**	0.83	0.17	**0.029**	0.85	0.15	**0.024**	0.86	0.14

Note: Sample period: 1985-2005.

our sample period for g, n, and the net return to capital. We calculate β to be 0.946 for the US, which we round to 0.95. We use the same method to construct β for all other countries as well, and compute their welfare changes using country-specific values of β. These range from 0.91 for Italy, to 0.96 for France. In the Appendix (section C.2) we report the results obtained imposing a common value of $\beta = 0.95$ (the US figure) for all countries, and show that they do not change significantly.

The resulting average welfare growth rates for the period 1985-2005 are shown in Table 1. for several fiscal scenarios.[22] Consider first the case of the US, given in the last row. With wasteful government expenditure and lump-sum taxes, as assumed in the first column, the average annual growth rate of welfare in the US is equivalent to a permanent annual increase in consumption of about 2.5 percent. With distortionary taxes, it rises to 2.9 percent. Welfare growth rises because the after-tax shares of capital and labor sum to less than one, so tax-adjusted TFP growth is higher. Maintaining the assumption of distortionary taxes but with optimal government spending, US welfare growth actually declines, to 2.4 percent. This result does *not* mean that the US consumer prefers wasteful to optimal government spending! The level of welfare is surely much lower when the government wastes 20 percent of GDP. However, our results imply that the difference in welfare between the two cases is entirely a *level* difference rather than a *growth rate* difference. In fact, the growth rate is lower with optimal government spending, for the US and several other countries, because real per-capita government expenditures have risen more slowly than the average growth rate of real per-capita consumption and investment.

As in the US, welfare growth in most countries only shows modest variation across the scenarios. The biggest differences arise in countries with high rates of growth of factor inputs and government purchases per capita, such as Spain and, to a lesser extent, Japan. For Spain the differences are large, with growth rates rising nearly a full percentage point from the first column to the third, which shows that assumptions about fiscal policy can matter significantly for welfare. With wasteful government spending, the UK leads our sample of countries in terms

[22] We omit the scenario of optimal spending and lump sum taxes for reasons of space. See our (2012) working paper for those results.

of welfare growth; with optimal spending, it is basically tied with Spain. In both cases, Canada lags behind the other countries, followed by Japan and Italy in the case of wasteful spending, with either lump sum or distortionary taxes.

Table 1 also shows the relative contribution of the two components of welfare–TFP growth and capital accumulation–to the growth rate of welfare in each country. For this decomposition, we treat the expectation-revision term as part of the contribution of TFP. In our benchmark case of optimal spending and distortionary taxes, 60 percent or more of the welfare gains are attributable to TFP in all countries. For the US, the UK and France, the fraction of growth due to TFP is 80 percent or higher. However, for Canada, Japan and Spain, this fraction depends significantly on assumptions about fiscal policy, and can be as low as 40 percent.

In section C.5 in the appendix we include a version of Table 1 where we add human capital as a state variable, and include the joint contribution on human and physical capital to welfare growth. Taking this broad view, we find that the contribution of capital accumulation to welfare growth is typically substantially larger: as high as 40 to 60 percent for several countries in our benchmark case. The US is the only country where the contribution of TFP to welfare growth remains in excess of 80 percent when we allow for human capital accumulation.

4.3 Cross-Country Results

We now turn to measuring welfare differences across the countries in our sample. For each country and time period, we calculate the welfare gap between that country and the US, as defined in equation (25). Recall that this gap is the loss in welfare of a representative US household that is moved permanently to country i starting at time t, expressed as the log gap between equivalent consumption at home and what it would be for the same household abroad. In this hypothetical move, the household loses the per-capita capital stock of the US, but gains the corresponding capital stock of country i. From time t on, the household optimizes using this new level of capital and while facing the same product prices, factor prices, per capita returns, and tax rates, and receives the same lump-sum transfers and government expenditure benefits, as all the other households in country i. We refer to the incremental equivalent consumption that the household would receive from this hypothetical move as "the welfare difference" or "the welfare gap." Note that these gaps are all from the point of view of a US household. Hence, all the shares in (25), even those used to construct output and TFP growth in country i, are the *US* shares. Similarly, the discount rate β used to discount the future by the the Us household in France is, naturally, the US β. This raises the obvious question: would our results be quite different if we took a different country as our baseline?

We present numerical results for our benchmark case of distortionary taxation and optimal government spending, using the US as the reference country (our baseline), as well as those using France as the reference country in panels A and B of Table 2. In the Table we report the average value of the welfare gap over our sample period. Moreover, we continue to break down each gap into the fraction due to the TFP terms versus the part due to the initial capital stock per worker. As the he magnitudes of the gap vary over time, we plot it in Figure 1 for all the countries and time periods in our sample (in panel A for US preferences and shares, and in panel B for French preferences and shares). Note that by definition the gap is zero for the

Table 2: Welfare Gap Relative to the US and its Components; Optimal Government Spending and Distortionary Taxes

	PANEL A US preferences			*PANEL B* French preferences		
	Total	Fraction due to: *TFP*	*Capital*	**Total**	Fraction due to: *TFP*	*Capital*
Canada	**-0.29**	0.92	0.08	**-0.31**	0.86	0.14
France	**-0.19**	0.90	0.10	**-0.25**	0.86	0.14
Italy	**-0.49**	1.00	0.00	**-0.50**	1.00	0.00
Japan	**-0.46**	1.11	-0.11	**-0.39**	1.25	-0.25
Spain	**-0.37**	0.90	0.10	**-0.40**	0.83	0.17
UK	**-0.12**	0.49	0.51	**-0.19**	0.39	0.61
USA	**0.00**	-	-	**0.00**	-	-

Note: estimates refer to the years 1985-2005.

US, since the US household neither gains nor loses by moving to the US at any point in time. The vertical axis shows, therefore, the gain to the US household of moving to any of the other countries at any point in the sample period, expressed in log points of equivalent consumption.

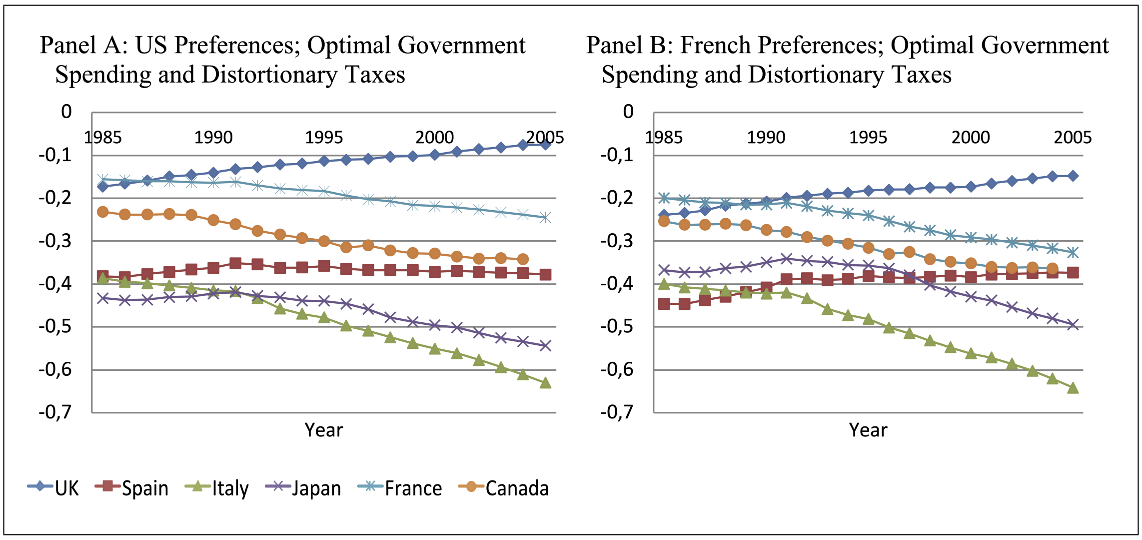

Figure 1: Cross-Country Welfare Comparisons (log Equivalent Consumption relative to the US)

It is instructive to begin by focusing on the beginning and end of the sample for our baseline case (see Panel A of Figure 1). At the beginning of the sample, expected lifetime welfare in both France and the UK was a bit less than 20 percent lower than in the US (gaps of 16 and 17 percent, respectively). This relatively small gap reflects both the long-run European advantage in leisure and the fact that in the mid-1980s the US was still struggling with its productivity slowdown, while TFP in the leading European economies was growing faster than in the US. Capital accumulation was also proceeding briskly in those countries. By the end of the sample, the continental European economies, Canada and Japan are generally falling behind the US, because they had not matched the pickup in TFP growth and investment experienced in the US after 1995 (the exception is Spain, which maintains a similar gap). Italy experiences the greatest relative “reversal of fortune,” ending up with a welfare gap of around 70 percent relative to the US. The results for France are qualitatively similar, but less extreme. Its initial welfare gap of 16 percent relative to the US gradually widens to 24 percent at the end of the sample period.

The only economy in our sample that exhibits convergence to the US throughout the period of analysis is the UK. Although the welfare level of the UK is always below that of the US, the UK shows strong convergence, slicing off half the welfare gap in two decades. This result is interesting, because the UK experienced much the same lack of TFP growth in the late 1990s and early 2000s as the major continental European economies. However, the UK had very rapid productivity growth from 1985 to 1995. The other “Anglo-Saxon” country in our sample, Canada, had a welfare level 23 percent below that of the US in 1985, but the welfare gap had grown to 34 percent by the end of the sample. This result is due primarily to the differential productivity performance of the two countries: TFP in Canada actually fell during the 1990s, and rose only slowly in the early 2000s.

One of the most striking comparisons is between the US and Japan. Even in 1985, when its economic performance was the envy of much of the world, Japan would have been the least attractive country in our sample for a US household contemplating emigration; such a household would give up more than 40 percent of consumption permanently in order to stay in the US instead of moving to Japan. However, like the UK, Japan was closing the gap with the US until the start of its ‘lost decade’ in 1991. The relative performance of the three countries changes dramatically from that point: unlike the UK, which continues to catch up, Japan begins to fall behind the US, first slowly and then more rapidly. Even more striking is the continued divergence of Italy from the US since the early 1990s, reflecting Italy’s dismal TFP performance.

In Table 2, we also investigate whether the cross-country welfare gaps are driven mostly by the TFP gap or by differences in capital per worker. Looking at the 1985-2005 averages in Table 2, Panel A, for our baseline case, we find that for five of the six countries, TFP is responsible for the vast majority of the welfare gap relative to the US. Indeed, for Japan TFP accounts for more than 100 percent of the gap (meaning that Japan has generally had a higher level of capital per person than the US) and for 100 percent of the gap for Italy. The exception to this finding is the UK. The average welfare gap between the US and the UK is driven about equally by TFP and by capital. By the end of the sample, the UK has almost converged to the US in "welfare-relevant TFP," and the difference in per-capita capital between the two economies accounted for most of the (small) welfare gap between the two countries.

We now check the robustness of these results along several dimensions. First, we check whether our welfare ranking is sensitive to the choice of the reference country. We redo the preceding exercises taking France as the baseline country. France is the largest and most successful continental European economy in our sample, and by revealed preference French households place much higher weight on leisure than do US ones. We summarize the results for our baseline case of optimal spending with distortionary taxes in Panel B of Figure 1 and Table 2 . For ease of comparison with the preceding cross-country figures, we still normalize the US welfare level to zero throughout, even though the comparison is done from the perspective of the French household and is based on French shares. Reassuringly, the qualitative results are unchanged. France and the UK start closest to the US in 1985, but even they are well behind the

Table 3: Per-Capita GDP, Consumption and Equivalent Consumption relative to the US

	Equiv. Cons. Opt Gov, Dist Tax	Equiv. Cons. Opt Gov, Dist Tax (US dynamics)	Equiv. Cons. Opt Gov, Dist Tax (No Leisure)	GDP	Consumption
PANEL A: 2005					
Canada	-0.34	-0.27	-0.29	-0.18	-0.32
France	-0.24	-0.21	-0.43	-0.32	-0.40
Italy	-0.63	-0.42	-0.73	-0.37	-0.50
Japan	-0.54	-0.45	-0.64	-0.26	-0.46
Spain	-0.38	-0.39	-0.37	-0.42	-0.53
UK	-0.08	-0.19	-0.14	-0.22	-0.19
USA	0.00	0.00	0.00	0.00	0.00
PANEL B: average 1985-2005					
Canada	-0.29	-0.22	-0.27	-0.16	-0.25
France	-0.19	-0.15	-0.38	-0.26	-0.34
Italy	-0.49	-0.27	-0.59	-0.28	-0.40
Japan	-0.46	-0.37	-0.50	-0.18	-0.39
Spain	-0.37	-0.37	-0.44	-0.45	-0.54
UK	-0.12	-0.22	-0.18	-0.25	-0.22
USA	0.00	0.00	0.00	0.00	0.00

US level of welfare The UK converges towards the US welfare level and so, from a much lower starting point, does Spain. All the other economies, including France, fall steadily farther behind the US over time, and Spain converges much more slowly from around 1990. Interestingly, from the French point of view almost all the other countries (Japan is an exception) are shifted down *vis-a-vis* the US relative to the ranking from the US point of view. Productivity continues to be the most important component of the welfare gap, again with the only exception being the UK at the end of the period.

We report the results of our next two robustness checks in Table 3. Relative to the traditional indexes of country performance, two new factors we highlight are dynamics (the expectation of future productivity change) and the treatment of leisure. To show their importance for cross-country differences in welfare, we have recalculated our basic index assuming, first, that future levels of productivity in all countries are expected to grow at the US rate, and second by assuming that leisure does not enter the utility function. The results appear in the second and third columns of Table 3, while the first column repeats our benchmark result for ease of comparison and includes the 2005 welfare gap, in addition to its average value over the period.

In our first robustness check, we take as given the observed gaps in TFP (and capital) among countries at the beginning of the sample, but assume that TFP in all countries will follow the US dynamics henceforth, both in terms of the long-run trend and the dynamics around that growth path (see column 2). When we assume that future productivity in each country is expected to have the same long run and short run behavior as in the US, significant changes occur for most countries in our sample. More specifically Canada, France, Italy, and

Japan display a smaller gap by the end of the period. There are no significant changes for Spain, and less convergence to the US for the UK. These changes basically reflect differences in the long run behavior of productivity across countries. This exercise shows the importance in projecting forward expected TFP growth in the wealth-type welfare measure we consider, which is inherently an uncertain exercise. This uncertainty is not shared by a flow welfare measure, which in our case would be simply a comparison of welfare-relevant TFP across countries at a point in time. Yet the flow measure also has the disadvantage that it does not take account of past savings. Two countries may have the same current TFP but very different levels of capital per capita. The country with greater capital is able to have greater consumption and leisure on average at all dates, which is not clear from a flow measure. Of course, a country with the same level but a faster growth rate of TFP also has greater welfare possibilities over time. We thus find the wealth-type measure more appealing conceptually, but need to be clear that it is more difficult to implement empirically.

Next, we check the importance of taking account of leisure in our welfare calculation (see column 3). When differences in leisure do not figure in the index, Italy, France, and Japan with lower average hours of work per capita than the US, display a worse performance in terms of welfare relative to the US; the difference is particularly striking for France, whose welfare gap increases by 80 percent.

In our fourth robustness check, whose results are reported in section C.5 of the Appendix, we bring human capital into the analysis. From the derivations in section A.4 of the appendix, this may change our welfare results for two reasons. First, countries may differ in their initial human capital stocks. Second, the series for labor input is now adjusted for human capital. While interesting quantitative differences emerge, the basic patterns in our benchmark results do not change, except for the fact that for all countries the difference in initial capital stocks (comprising both human and physical capital) now accounts for a larger share of the welfare gap vis-a-vis the US.

In Table 3 we also compare our welfare results to those based on traditional measures, namely PPP-adjusted GDP and consumption per capita (see the last two columns). Focusing on Panel A, for the final year of our sample, we see that the three measures sometimes give identical results. For example, the US is atop the world rankings by all three measures, although the gap between the US and the second-ranked country is much smaller in percentage terms for welfare (8 percent) than it is for the other two variables (19 or 22 percent). On the other hand, the differences can be striking. For example, Canada, which leads Spain by 20 percent or more in terms of consumption and GDP per capita, is overtaken by France and nearly tied with Spain in our welfare comparison. Indeed, Spain is last within our group of countries in terms of the conventional metrics of consumption and GDP, but ranks fifth in welfare terms, ahead of Italy and Japan, and close to Canada. For the other countries, the welfare measure is not so kind. Japan trails the US by only 26 percent in GDP per capita, but double that—54 percent—in terms of welfare. Similarly, Italy has more than 60 percent of the per-capita GDP of the US, but only about 40 percent of the welfare level. On the other hand, France trails the US by 40 percent in consumption per-capita, but by almost half that amount in terms of welfare. Thus, our measure clearly provides new information on welfare differences among countries.

4.4 A Broader Cross-Country Sample

The empirical exercises so far are limited to a small set of large advanced countries for which the requisite data were available. However, it may be interesting to assess welfare gaps across countries in a broader sample, including both advanced and developing countries. Thus, as a final empirical exercise, we extend our cross-country results to a large set of countries for the year 2005. We take all countries with available data with populations greater than two million people in 1985. Recall that, aside from absorption, labor and capital aggregates for the countries involved, numerical comparisons of welfare across countries only require information on factor shares, tax rates, and the discount factor for the reference country (which continues to be the US for this exercise). Thus, unlike other measures that have been proposed, our welfare measure is relatively easy to construct for a large sample of countries.[23]

As with the smaller sample of developed countries, we calculated the welfare gaps (always from the perspective of the US household) under our benchmark assumptions regarding public spending (optimal) and taxation (distortionary). Results appear in Table 4. The table presents results without human capital; a similar table that takes account of human capital differences is provided in section C.5 in the appendix. In each case, we estimated country-specific autoregressive models and used them to project the future path of that country's modified productivity residual. For the sake of space, we only report the results of our baseline specification, with optimal government expenditure and distortionary taxes. Except for the different measurement of the labor input, the exercise is therefore the same as that reported in Table 2. However, it has to be taken with caution not only because the measure of the labor input is noisier here, but also because the countries involved are more heterogeneous in terms of the stage of convergence to their (possibly heterogeneous) steady-state. [24]

The first column of Table 4 reports the log differences in per capita welfare relative to the US for the year 2005. No country ranks ahead of the US, but several, such as Australia, Norway, and Singapore, are close to US levels of welfare. All the industrialized countries in the sample rank above the median, with Italy and Portugal bringing up the rear in 23rd and 22nd place for that group. Among the six advanced countries in our earlier exercises, relative ranks are much the same as those shown in Table 2, with the only exception being that France slips slightly behind Canada. The reason is that the French enjoy high levels of leisure, but these come in the form of lower hours per worker rather than a lower employment rate.[25]

In turn, most developing countries exhibit fairly large welfare gaps relative to the US. For

[23] As noted, however, consistent time-series information on hours worked is not available for most countries. Hence, as already mentioned, we measure aggregate labor input using total employment rather than total hours. For the sake of comparability, we use employment even for those countries for which hours data are available. The immediate consequence is that cross-country differences in work hours per person are ignored in the calculation of cross-country differences in the productivity residual, and thus also in the calculation of differences in welfare. We also provide in section C.3 in the appendix some additional evidence using hours of work for a wider sample of smaller or newer OECD countries for which such data are available.

[24] Indeed, the fact that many of the countries in the broader sample are quite far from the US steady state could raise concerns about the accuracy of the first-order approximation underlying the calculations. However, the results in Appendix B, where we allow for level and growth differences in productivity in a calibrated RBC model, suggest that the approximation should remain quite reliable even in this case.

[25] Bick et al. (2018) document that workers in rich countries work fewer hours than those in developing countries. In our sample, for the countries with hours data, the correlation between total employment and total hours of work (both in logs) is 0.53, which confirms that the former is a fairly noisy proxy for the latter.

Table 4: Welfare Gap Relative to the US and its Components in 2005. Benchmark Case (Optimal Government Spending and Distortionary Taxes)

	Welfare Gap	Fraction due to TFP	Fraction due to Capital		Welfare Gap	Fraction due to: TFP	Fraction due to: Capital
USA	0.000	—	—				
Norway	-0.06	1.73	-0.73	Uruguay	-1.47	0.82	0.18
Australia	-0.07	0.95	0.05	Peru	-1.54	0.80	0.20
Singapore	-0.07	1.69	-0.69	Argentina	-1.56	0.85	0.15
United Kingdom	-0.17	0.61	0.39	Malaysia	-1.58	0.88	0.12
Ireland	-0.31	0.88	0.12	Costa Rica	-1.68	0.85	0.15
Netherlands	-0.35	0.94	0.06	Tunisia	-1.84	0.86	0.14
Canada	-0.35	0.92	0.08	Panama	-1.90	0.85	0.15
France	-0.36	0.89	0.11	Guatemala	-1.97	0.82	0.18
Sweden	-0.40	0.90	0.10	Brazil	-1.99	0.87	0.13
Finland	-0.46	0.95	0.05	Egypt	-2.03	0.79	0.21
Denmark	-0.46	0.98	0.02	El Salvador	-2.06	0.82	0.18
Belgium	-0.52	1.02	-0.02	China	-2.08	0.85	0.15
New Zealand	-0.52	0.85	0.15	Thailand	-2.13	0.90	0.10
Austria	-0.55	1.01	-0.01	Venezuela	-2.15	0.88	0.12
Switzerland	-0.58	1.09	-0.09	Sri Lanka	-2.33	0.83	0.17
Spain	-0.59	0.94	0.06	India	-2.38	0.80	0.20
Japan	-0.62	1.05	-0.05	Colombia	-2.44	0.87	0.13
Israel	-0.66	0.89	0.11	Syria	-2.60	0.83	0.17
South Korea	-0.70	0.96	0.04	Ecuador	-2.61	0.89	0.11
Hong Kong	-0.72	1.03	-0.03	Indonesia	-2.63	0.86	0.14
Portugal	-0.91	0.93	0.07	Honduras	-2.80	0.86	0.14
Italy	-0.95	0.99	0.01	Philippines	-2.97	0.85	0.15
Romania	-1.00	0.61	0.39	Morocco	-3.26	0.89	0.11
Iran	-1.08	0.80	0.20	Pakistan	-3.29	0.85	0.15
Chile	-1.17	0.81	0.19	Bolivia	-3.35	0.86	0.14
Turkey	-1.22	0.77	0.23	Uganda	-3.45	0.80	0.20
Jamaica	-1.36	0.86	0.14	Paraguay	-3.86	0.90	0.10
Mexico	-1.47	0.86	0.14	Togo	-4.11	0.85	0.15

example, both China and India trail the US by substantial amounts. (One should remember that these data are for 2005, and China in particular has made substantial relative progress since.) In 2005, countries like Turkey and Malaysia have higher expected welfare than China.

As before, we may ask how these welfare-based country comparisons would relate to those obtained on the basis of per capita consumption or GDP. The answer is that the resulting country ranking would show visible differences – for example, Norway had higher PPP GDP per capita than the US in 2005, but, according to the results in Table 4, lower welfare. On the whole, however, there is broad agreement among the three measures. Indeed, the correlation coefficients between the log differences in welfare shown in Table 4, and the log differences in per capita PPP GDP or consumption (both *vis-a-vis* the US) observed in 2005, equal 0.94 and 0.93, respectively. However, the majority (40) of the 57 countries shown lag further behind the US in terms of welfare than in terms of per-capita GDP. Indeed, the median difference between the two gaps (in percentage terms) equals 10 percent.

Next, we examine the extent to which the 2005 cross-country welfare gaps shown in the first column of Table 4 are driven by differences in TFP and by differences in initial capital per worker. The relevant decomposition is shown in the last two columns of the Table 4. On the whole, TFP accounts for the bulk of the welfare differences. Across countries, its median contribution to the welfare gap equals 87 percent. In a few countries, notably Norway and Singapore, the welfare gaps are due entirely to TFP, with part of the gap offset by an initial capital stock above that of the US.

Given the importance of TFP, we again perform the exercise of assuming that all countries have the TFP dynamics and long-run growth rate of the US, with the results reported in section C.4 of the appendix. As we would predict based on the prior results, this exercise shows some developed countries with less favorable long run dynamics than the US (such as Italy, Japan, and Belgium) converging closer to US welfare levels. China and India, not surprisingly given their growth performance over the sample period, fall further behind US welfare levels.

Lastly, we repeat the exercise in Table 4, but taking into account differences in human capital. The results, in Appendix C.5, do not show substantial differences from those shown here, except for the fact that the fraction of the welfare gap attributable to the initial capital stock tends to become larger for most countries, at the expense of the fraction due to TFP. However, the latter still accounts for the lion's share of the welfare gap.

5 Relationship to the Literature

Measuring welfare change over time and differences across countries using observable national income accounts data has been a long-standing challenge for economists. We note here the similarities and differences between our approach and others found in previous literature. We also suggest ways in which our results might be useful in other fields of economics, where the same question arises in different contexts.

Nordhaus and Tobin (1972) originated one approach, which is to take a snapshot of the economy's flow output at a point in time and then go "beyond GDP," by adjusting GDP in various ways to make it a better flow measure of welfare. Nordhaus and Tobin found that the largest gap between flow output and flow welfare comes from the value that consumers put on leisure. Their result motivated us to add leisure to the utility function in our model, which is standard in business-cycle analysis but not in growth theory. Nordhaus and Tobin's approach has been followed and extended recently by Jones and Klenow (2016) who add other corrections, notably for life expectancy and inequality. Their basic approach is to assume that a consumer with US preferences is assigned the consumption and leisure of another country. The US household is not allowed to optimize given the new conditions (prices, endowments, etc.) found in the other country. By contrast, we allow the household of a reference country to re-optimize dynamically when faced with the paths of exogenous variables of another country, and then compare the difference between the optimized values of utility at home and abroad. Our thought experiment gives the "irreducible" difference in welfare between the two countries, while the approach of assigning choices that are sub-optimal for the assumed preferences and new conditions will produce an upward-biased measure of welfare gaps, unless preferences are identical across countries.

Our approach echoes the methods used in the literature started by Weitzman (1976) and analyzed in depth by Weitzman (2003), with notable contributions from many other authors.[26] This literature also relates the welfare of an infinitely lived representative agent to observables; for example, Weitzman (1976) linked intertemporal welfare to net domestic product. The results in these papers are derived using a number of strong restrictions on the nature of technology, product market competition, and the nature of exogenous shocks to the economy. Most of the analysis in the literature applies to a closed economy where growth is optimal. By contrast, we derive all our results based only on first-order conditions from household optimization, which allows for imperfect competition in product markets of an arbitrary type and for a vast range of production possibilities and shocks, and for the economy to be closed or open.

Our findings shed light on a variety of issues that bedevil the measurement of productivity and allocative efficiency. For example, Baker and Rosnick (2007), reasoning that the ultimate object of growth is consumption, make the reasonable conjecture that one should deflate nominal productivity gains by a consumption price index to create a measure they call "usable productivity." We begin from the assumption that consumption (and leisure) at different dates are the only inputs to economic well-being, but nevertheless show that output should be calculated in the conventional way, rather than being deflated by consumer prices.

Our work clarifies and unifies several results in other literatures, especially international economics. Kohli (2004) shows in a static setting that terms-of-trade changes can improve welfare in open economies even when technology is constant. Kehoe and Ruhl (2008) prove a related result in a dynamic model with balanced trade: opening to trade may increase welfare, even if it does not change TFP. In these models, which assume competition and constant returns, technology is equivalent to TFP. We generalize and extend these results, and show that in a dynamic environment with unbalanced trade welfare can also change with the quantity of net foreign assets and their rates of return.[27] In general, we show that there is a link between

[26] A far from exhaustive list includes Asheim (1994), Arronson and Löfgren (1995), Mino (2004), Sefton and Weale (2006), Basu, Pascali, Schiantarelli and Serven (2009), and Hulten and Schreyer (2010). Reis (2005) analyses the related problem of computing a dynamic measure of inflation for a long-lived representative consumer.

[27] The result that openness does not change TFP may be fragile in models with increasing returns. If opening

observable aggregate data and welfare in an open economy, which is the objective of Bajona, Gibson, Kehoe and Ruhl (2010).

While we agree with the conclusion of these authors that GDP is not a sufficient statistic for uncovering the effect of trade policy on welfare, we show that one can construct such a sufficient statistic by considering a relatively small number of other variables. Our results also shed light on the work of Arkolakis, Costinot and Rodriguez-Clare (2012). These authors show that in a class of modern trade models, which includes models with imperfect competition and micro-level productivity heterogeneity, one can construct measures of the welfare gain from trade without reference to micro data.[28] Our results imply that this conclusion actually holds in a much larger class of models, although the exact functional form of the result in Arkolakis et al. (2012) may not. Finally, since changes in net foreign asset positions and their rates of return are extremely hard to measure, we show that one can measure welfare using data only on TFP and the capital stock, even in an open economy, provided that TFP is calculated using absorption rather than GDP as the output concept.

Our work provides a different view of a large and burgeoning literature that investigates the reasons for output differences across countries. If we specialize our cross-country result to the lump sum taxes-optimal spending case, we obtain something closely related to the results produced by the "development accounting" literature. We show that in that case, (the present value of) the log differences in TFP levels emphasized by the developing accounting literature need to be supplemented with only one additional variable, namely log level gaps in capital per person (also a focus of that literature), in order to serve as a measure of welfare differences among countries.

A number of recent papers suggest that countries can increase output and TFP substantially by allocating resources more efficiently across firms. Our work implies that the literature is correct to focus on the connection between reallocation and aggregate TFP. An increase in aggregate TFP due to reallocation is as much of a welfare gain for the representative consumer as a change in technology with the same magnitude and persistence. This result implies immediately that estimates of TFP losses due to allocative inefficiency (e.g., Hsieh and Klenow, 2009) can be translated to estimates of welfare losses.

6 Conclusions

We show that aggregate TFP, appropriately defined, and the capital stock can be used to construct sufficient statistics for the welfare of a representative consumer. To a first-order approximation, welfare is measured by the expected present value of aggregate TFP and by the initial capital stock. This result holds regardless of the type of production technology and the degree of product market competition, and applies to closed or open economies with or without distortionary taxation. Crucially, TFP has to be calculated using prices faced by households rather than prices facing firms. In modern economies with high rates of income and indirect taxation, the gap between household and firm TFP can be considerable. Finally, in an open

to trade changes factor inputs, either on impact or over time, then TFP as measured by Solow's residual will change as well, which we show has an effect on welfare even holding constant the terms of trade.

[28] Atkeson and Burstein (2010) come to similar conclusions in a related model.

economy, the change in welfare will also reflect present and future changes in the returns on net foreign assets and in the terms of trade. However, these latter terms disappear if absorption rather than GDP is used as the output concept for constructing TFP, and TFP and the initial capital stock are again sufficient statistics for measuring welfare in open economies. Most importantly, these variables also suffice to measure welfare level differences across countries, with both variables computed as log level deviations from a reference country. Notably, our approach allows us to perform these welfare comparisons without having to assume identical preferences across countries and without having to make assumptions about the value of preference parameters. The only parameter we need to know is the discount factor for the country taken as reference in the comparison, which we calculate from the Euler equation for the capital stock, evaluated at the steady state.

We extend the existing literature on intertemporal welfare measurement by deriving all our results from household first-order conditions alone. The generality of our derivation allows us to propose a new interpretation of TFP that sheds new light on several distinct areas of study. For instance, we show that measures of cross-country TFP differences akin to those produced by the "development accounting" literature are crucial for calculating welfare differences among countries. In general, our results imply that all changes in the time path of the Solow residual, whatever their source (for example, technology, increasing returns, or reallocation) are equally important for welfare. While our approach allows us to make historical comparisons, counterfactual policy experiments would require a full general equilibrium model. Thus, our method allows researchers to measure actual welfare using standard data under very general assumptions, while DSGE models can answer "what if" questions, at the cost of making many more assumptions. We view the two approaches as complementary.

We illustrate our results by using national accounts data and standard labor market data to measure welfare growth rates and gaps across countries. Our evidence suggests that expectations about future productivity and the presence of leisure in the utility function are important determinants of welfare rankings. Importantly, in the vast majority of cases, the bulk of the welfare gap relative to the US, our welfare leader among large countries, is due to the productivity gap, rather than the gap in the physical or human capital stock.

Our analysis has been confined to the case of a representative agent, which automatically rules out distributional issues and, more generally, household heterogeneity. Our theoretical framework could be extended in principle to the case of heterogeneous households, since mathematically the case of heterogeneous households within a country is similar to the case that we have already analyzed, of heterogeneous countries within the world. The main barrier to pursing the study of heterogeneity is data availability. For instance, it would require data on the productivity residual at a sufficient number of points of the income distribution and Markov transition matrices for income mobility between those points. While the challenges are great, the payoff would be compelling, and thus this major extension is the subject of our ongoing research. We view the present paper as the first step along this road.

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A Derivations

A.1 Proposition 1

Assume that in the steady state aggregate per capita variables grow at a constant rate g. Thus, they are proportional to some variable $X_t = X_0(1+g)^t$. Rewrite the utility function, the budget constraint and the capital accumulation, equations (1), (2) and (3), in normalized form by dividing by X_t :

$$v_t = \sum_{s=0}^{\infty} \beta^s U(c_{t+s}; \overline{L} - L_{t+s}) \tag{A.1}$$

$$k_t + b_t = \frac{(1-\delta) + p_t^K}{(1+g)(1+n)} k_{t-1} + \frac{(1+r_t)}{(1+g)(1+n)} b_{t-1} + p_t^L L_t + \pi_t - p_t^C c_t \tag{A.2}$$

$$k_t = \frac{(1-\delta)}{(1+g)(1+n)} k_{t-1} + i_t \tag{A.3}$$

where: $v_t = \frac{V_t}{X_t^{(1-\sigma)}}$, $c_t = \frac{C_t}{X_t}$ $k_t = \frac{K_t}{X_t}$, $b_t = \frac{B_t}{P_t^I X_t}$, $p_t^K = \frac{P_t^K}{P_t^I}$, $p_t^L = \frac{P_t^L}{P_t^I X_t}$, $p_t^C = \frac{P_t^C}{P_t^I}$, $(1+r_t) = \left(1+i_t^B\right) \frac{P_{t-1}^I}{P_t^I}$, $\pi_t = \frac{\Pi_t}{P_t^I X_t}$ and $\beta = \frac{(1+n)(1+g)^{1-\sigma}}{(1+\rho)}$.

The first order conditions for this problem are:

$$U_{c_t} - \lambda_t p_t^C = 0 \tag{A.4}$$

$$U_{L_t} + \lambda_t p_t^L = 0 \tag{A.5}$$

$$-\lambda_t + \beta E_t \frac{(1-\delta) + p_{t+1}^K}{(1+g)(1+n)} \lambda_{t+1} = 0 \tag{A.6}$$

$$-\lambda_t + \beta \frac{1}{(1+g)(1+n)} E_t (1+r_{t+1}) \lambda_{t+1} = 0 \tag{A.7}$$

where λ_t is the Lagrange multiplier associated with the budget constraint. Define with $\widehat{x} = \frac{x_t - x}{x}$ the percent deviation from the steady-state of a variable (x is the steady-state value of x_t). Taking a first order approximation of the Lagrangean (which equals the value function along the optimal path), one obtains:

$$
\begin{aligned}
&v_t - v = \\
&E_t[\sum_{s=0}^{\infty} \beta^s (U_c c\widehat{c}_{t+s} + U_L L\widehat{L}_{t+s} \\
&+\lambda p^L L\widehat{L}_{t+s} - \lambda p^C c\widehat{c}_{t+s} - \lambda k\widehat{k}_{t+s} - \lambda b\widehat{b}_{t+s}) \\
&+\sum_{s=0}^{\infty} \beta^{s+1} (\lambda \frac{(1-\delta)+p^K}{(1+g)(1+n)} k\widehat{k}_{t+s} + \lambda \frac{(1+r)}{(1+g)(1+n)} b\widehat{b}_{t+s}) \\
&+\sum_{s=0}^{\infty} \beta^s \widehat{\lambda}_{t+s}\ \lambda(-k - b + \frac{(1-\delta)+p^K}{(1+g)(1+n)} k + \frac{(1+r)}{(1+g)(1+n)} b \\
&+p^L L + \pi - p^C c)] \\
&+\sum_{s=0}^{\infty} \beta^s \lambda (p^L L\widehat{p}^L_{t+s} + \frac{p^K k}{(1+g)(1+n)}\ \widehat{p}^K_{t+s} - p^C_t c\widehat{p}^C_{t+s} + \pi\widehat{\pi}_{t+s} + \frac{rb}{(1+g)(1+n)} \widehat{r}_{t+s}) \\
&+\lambda \frac{(1-\delta)+p^K}{(1+g)(1+n)} k\widehat{k}_{t-1} + \lambda \frac{(1+r)}{(1+g)(1+n)} b\widehat{b}_{t-1} \qquad (A.8)
\end{aligned}
$$

Using the first-order conditions, the first five lines equal zero:

$$
\begin{aligned}
v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^L L\widehat{p}^L_{t+s} + \frac{p^K k}{(1+g)(1+n)}\ \widehat{p}^K_{t+s} - p^C c\widehat{p}^C_{t+s} + \pi\widehat{\pi}_{t+s} + \frac{rb}{(1+g)(1+n)} \widehat{r}_{t+s}) \right] \\
&+\lambda \frac{(1-\delta)+p^K}{(1+g)(1+n)} k\widehat{k}_{t-1} + \lambda \frac{(1+r)}{(1+g)(1+n)} b\widehat{b}_{t-1} \qquad (A.9)
\end{aligned}
$$

Linearize the budget constraint and the law of motion for capital:

$$
\begin{aligned}
&k\ \widehat{k}_t + b\widehat{b}_t - \frac{(1-\delta)+p^K}{(1+g)(1+n)} k\widehat{k}_{t-1} - \frac{(1+r)}{(1+g)(1+n)} b\widehat{b}_{t-1} - p^L L\widehat{L}_t - p^L L\widehat{p}^L_t - \frac{p^K k}{(1+g)(1+n)} \widehat{p}^K_t \\
&- \frac{rb}{(1+g)(1+n)} \widehat{r}_t - \pi\widehat{\pi}_t + p^C c\widehat{c}_t + p^C c\widehat{p}_t = 0 \qquad (A.10)
\end{aligned}
$$

$$
k\widehat{k}_t = \frac{(1-\delta)}{(1+g)(1+n)} k\widehat{k}_{t-1} + i\widehat{i}_t \qquad (A.11)
$$

Using these two equations and the steady-state version of the FOC for capital in (A.9) gives us:

$$
\begin{aligned}
v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^C c\widehat{c}_{t+s} + i\widehat{i}_{t+s} - \frac{p^K}{(1+g)(1+n)} k\widehat{k}_{t+s-1} - p^L L\widehat{L}_{t+s} \right] + \lambda \frac{1}{\beta} k\widehat{k}_{t-1} \\
&+\lambda \sum_{s=0}^{\infty} \beta^s \left[b\widehat{b}_{t+s} - \beta \frac{(1+r)}{(1+g)(1+n)} b\widehat{b}_{t+s} \right] \qquad (A.12)
\end{aligned}
$$

Using the FOC and the transversality condition for bonds, the last line in the equation above equals zero. Hence we get:

$$
\begin{aligned}
v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^C c \widehat{c}_{t+s} + i \widehat{i}_{t+s} - \frac{p^K k}{(1+g)(1+n)} \widehat{k}_{t+s-1} - p^L L \widehat{L}_{t+s} \right] \\
&\quad + \lambda \frac{1}{\beta} k \widehat{k}_{t-1}
\end{aligned} \tag{A.13}
$$

Take the difference between the expected level of intertemporal utility v_t as in (A.13) and v_{t-1} and use the fact that $\frac{x_t - x}{x} \simeq \log x_t - \log x$ for positive x_t. After adding and subtracting $E_t x_{t+s}$ for each variable x_{t+s}, we obtain:

$$
\begin{aligned}
\Delta v_t &= E_t \sum_{s=0}^{\infty} \beta^s \lambda [p^C c \Delta \log c_{t+s} + i \Delta \log i_t - p^L L \Delta \log L_{t+s} - \frac{p^K k}{(1+g)(1+n)} \Delta \log \ k_{t+s-1}] \\
&\quad + \sum_{s=0}^{\infty} \beta^s \lambda [p^C c \left(E_t \log c_{t+s-1} - E_{t-1} \log c_{t+s-1} \right) + i \left(E_t \log i_{t+s-1} - E_{t-1} \log i_{t+s-1} \right) \\
&\quad - p^L L \left(E_t \log L_{t+s-1} - E_{t-1} \log L_{t+s-1} \right) - \frac{p^K k}{(1+g)(1+n)} \left(E_t \log k_{t+s-2} - E_{t-1} \log k_{t+s-2} \right)] \\
&\quad + \lambda \frac{1}{\beta} k \Delta \log k_{t-1}
\end{aligned} \tag{A.14}
$$

Define domestic absorption growth (in normalized form) as:

$$
\Delta \log y_t = s_C \Delta \log c_t + s_I \Delta \log i_t \tag{A.15}
$$

Inserting (A.15) into (A.14) and dividing both terms by $\lambda p^Y y$ one obtains:

$$
\begin{aligned}
\frac{v}{\lambda p^Y y} \frac{\Delta v_t}{v} &= E_t \sum_{s=0}^{\infty} \beta^s [\Delta \log y_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log \ k_{t+s-1}] \qquad \text{(A.16)} \\
&\quad + \sum_{s=0}^{\infty} \beta^s \lambda [(E_t \log y_{t+s-1} - E_{t-1} \log y_{t+s-1}) \\
&\quad - s_L \left(E_t \log L_{t+s-1} - E_{t-1} \log L_{t+s-1} \right) - s_K \left(E_t \log k_{t+s-2} - E_{t-1} \log k_{t+s-2} \right)] \\
&\quad + \frac{1}{\beta} \frac{k}{p^Y y} \Delta \log k_{t-1}
\end{aligned}
$$

The definition of equivalent consumption in terms of normalized variables is:

$$
v_t = \frac{1}{(1-\sigma)(1-\beta)} c_t^{*1-\sigma} \nu(\overline{L} - L) \tag{A.17}
$$

Expanding the right hand side of (A.17) in terms of $\log c_t^*$ and using the FOC for consumption, it follows that:

$$
\frac{v}{\lambda p^Y y} \frac{\Delta v_t}{v} = \frac{s_C}{1-\beta} \Delta \log c_t^* \tag{A.18}
$$

Using the result above in equation (A.16) one obtains, to a first-order approximation:

$$
\begin{aligned}
\frac{s_C}{1-\beta}\Delta\log c_t^* \quad &= \quad E_t\sum_{s=0}^{\infty}\beta^s[\Delta\log y_{t+s} - s_L\Delta\log L_{t+s} - s_K\Delta\log\ k_{t+s-1}] \qquad \text{(A.19)}\\
&+\sum_{s=0}^{\infty}\beta^s\lambda[(E_t\log y_{t+s-1} - E_{t-1}\log y_{t+s-1})\\
&-s_L\left(E_t\log L_{t+s-1} - E_{t-1}\log L_{t+s-1}\right) - s_K\left(E_t\log k_{t+s-2} - E_{t-1}\log k_{t+s-2}\right)]\\
&+\frac{1}{\beta}\frac{k}{p^Y y}\Delta\log k_{t-1}
\end{aligned}
$$

Using the fact that $\Delta\log y_t = \Delta\log Y_t - g$, $\Delta\log k_t = \Delta\log K_t - g, \Delta\log c_t^* = \Delta\log C_t^* - g$, equation (A.19) can be rewritten as:

$$
\begin{aligned}
\frac{s_C}{1-\beta}\Delta\log C_t^* \quad &= \quad E_t\sum_{s=0}^{\infty}\beta^s\Delta\log PR_{t+s}\\
&+\sum_{s=0}^{\infty}\beta^s\left[E_t\log PR_{t+s-1} - E_{t-1}\log PR_{t+s-1}\right]\\
&+\frac{1}{\beta}\frac{k}{p^Y y}\Delta\log K_{t-1} \qquad \text{(A.20)}\\
&+\frac{s_C}{1-\beta}g - \frac{1}{(1-\beta)}g(1-s_K) - \frac{1}{\beta}\frac{k}{p^Y y}g
\end{aligned}
$$

where $\log PR_{t+s}$ is defined in (6). Using equations (A.6) and (A.3) evaluated at the steady-state, one can easily show that $\frac{s_C}{(1-\beta)} - \frac{1}{(1-\beta)}(1-s_K) - \frac{1}{\beta}\frac{k}{p^Y y} = 0$, so that the last line in equation (A.20) equals zero. Multiplying both sides of (A.20) by $\frac{1-\beta}{s_C}$ yields equation (7) in Proposition 1 in the main text.

If we were to use real GDP rather than absorption as a measure of output, then the welfare-relevant residual can be written as the sum of a conventionally-defined productivity residual plus additional components that capture terms of trade and capital gains effects. Moreover, the initial conditions then should include the initial value of the net foreign asset stock. To show this, assume that the domestic economy buys imports $IM_{t+s}N_{t+s}$ at a price P_{t+s}^{IM} and sells domestic goods abroad $EX_{t+s}N_{t+s}$ at a price P_{t+s}^{X}. The current account balance can be written:

$$
B_tN_t - B_{t-1}N_{t-1} = i_tB_{t-1}N_{t-1} + P_t^{EX}EX_tN_t - P_t^{IM}IM_tN_t \qquad \text{(A.21)}
$$

which can be re-written in a normalized form as:

$$
b_t = \frac{(1+r_t)}{(1+g)(1+n)}b_{t-1} + p_t^{EX}ex_t - p_t^{IM}im_t \qquad \text{(A.22)}
$$

Linearizing the equation above, we get:

$$
b\widehat{b}_t = \frac{(1+r)}{(1+g)(1+n)}b\widehat{b}_{t-1} + \frac{rb}{(1+g)(1+n)}\widehat{r}_t + p^{EX}ex\ \widehat{ex}_t + p^{EX}ex\ \widehat{p^{EX}}_t - p^{IM}im\ \widehat{im}_t - p^{IM}im\ \widehat{p^{IM}}_t \qquad \text{(A.23)}
$$

Using equations (A.10), (A.11) and (A.23), and the steady-state version of the FOC for capital and bonds in (A.9) gives us:

$$
\begin{aligned}
v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda [p^C c\widehat{c}_{t+s} + i\widehat{i}_{t+s} + p^{EX} ex\ \widehat{ex}_t - p^{IM} im\ \widehat{im}_t - \frac{p^K}{(1+g)(1+n)} k\widehat{k}_{t+s-1} - p^L L\widehat{L}_{t+s} \\
&\quad + p^{EX} ex\ \widehat{p^{EX}}_t - p^{IM} im\ \widehat{p^{IM}}_t] + \lambda \frac{1}{\beta} k\widehat{k}_{t-1} + \lambda \frac{1}{\beta} b\widehat{b}_{t-1} \qquad \text{(A.24)}
\end{aligned}
$$

Define GDP growth (in normalized form) as:

$$
\Delta \log gdp_t = s'_C \Delta \log c_t + s'_I \Delta \log i_t + s'_X \Delta \log ex_t - s'_M \Delta \log im_t \qquad \text{(A.25)}
$$

where s'_C, s'_I, s'_X and s'_M are respectively the steady-state shares of consumption, investment, exports and imports out of GDP. Using the definition above and some algebra, equation (A.24) can be re-written as:

$$
\begin{aligned}
\Delta \log C_t^* &= \frac{(1-\beta)}{s_c} \left[E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PRTT_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PRTT_{t+s-1} \right. \\
&\quad \left. + \frac{1}{\beta} \left(\frac{P^I K}{P^Y Y} \right) \Delta \log K_{t-1} + \frac{1}{\beta} \left(\frac{P^I}{P^Y Y} \right) \Delta \frac{B_{t-1}}{P^I_{t-1}} \right] \qquad \text{(A.26)}
\end{aligned}
$$

where $\Delta \log PRTT_{t+s}$ is defined as

$$
\begin{aligned}
\Delta \log PRTT_{t+s} &= \Delta \log Y_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log K_{t+s-1} \\
&\quad + s_X \Delta \log p^{EX}_{t+s} - s_M \Delta \log p^{IM}_{t+s} + \left(\frac{Br}{P^Y Y} \right) \Delta \log r_{t+s}
\end{aligned}
$$

A.2 Proposition 2

Consider now a representative household of country j moving to country i. Specifically, consider a fictional household characterized by the same utility function as the representative household of country j that is confronted with the sequence of prices, per-capita profits, per-capita remuneration on capital and bonds, and per-capita initial endowments (the same stock of capital and bonds) of country i. This fictional household maximizes the following intertemporal utility function:

$$
\widetilde{W}_t = E_t \sum_{s=0} N^j_{t+s} \frac{1}{(1+\rho_j)^s (1-\sigma_j)} \left(\widetilde{C}^i_{t+s} \right)^{1-\sigma_j} v \left(\overline{L} - \widetilde{L}^i_{t+s} \right) \qquad \text{(A.27)}
$$

under the following budget constraint:

$$
\begin{aligned}
& -P^I_{t+s}\widetilde{K}^i_{t+s}N^j_{t+s} - \widetilde{B}^i_{t+s}N^j_{t+s} + \left[(1-\delta)P^I_{t+s} + \widetilde{P}^{K,i}_{t+s}\right]N^j_{t+s-1}\widetilde{K}^i_{t+s-1} \\
& +\widetilde{R}^i_{t+s}N^j_{t+s-1}\widetilde{B}^i_{t+s-1} + P^{L,i}_{t+s}\widetilde{L}^i_{t+s}N^j_{t+s} - p^{C,i}_{t+s}\widetilde{C}^i_{t+s}N^j_{t+s} + \Pi^i_{t+s}N^j_{t+s} \\
= \;& 0
\end{aligned} \tag{A.28}
$$

the following capital accumulation equation:

$$N^j_{t+s}\widetilde{K}^i_{t+s} = \widetilde{I}^i_{t+s}N^j_{t+s} + (1-\delta)N^j_{t+s-1}\widetilde{K}^i_{t+s-1} \tag{A.29}$$

and the following initial conditions:

$$\widetilde{K}^i_{t-1} = K^i_{t-1} \tag{A.30}$$

$$\widetilde{B}^i_{t-1} = B^i_{t-1} \tag{A.31}$$

A tilde denotes the (unobservable) quantities that this fictional household choose; $\widetilde{R}^i_{t+s} \geq 1$. Notice that the budget constraint of this fictional household differs from the budget constraint of the representative household of country i for two reasons. First the size of this fictional household is N^j_{t+s} and not N^i_{t+s}. Second, the remuneration to bonds and capital are different. Although the per-capita remunerations to the individuals in this fictional household are the same as the per-capita remuneration to the individuals of the representative household in country i, the remunerations to the entire households are different as the household in country j is growing at a different rate than the household in country i. Specifically, $\widetilde{R}^i_{t+s}$ and $\widetilde{P}^{K,i}_{t+s}$ are defined such that:

$$\widetilde{R}^i_{t+s} = R^i_{t+s}\frac{(1+n_j)}{(1+n_i)} \tag{A.32}$$

and

$$\widetilde{P}^{K,i}_{t+s} = P^{K,i}_{t+s}\frac{(1+n_j)}{(1+n_i)} \tag{A.33}$$

To give an intuition, consider equation (A.32). Assume that the representative individual in country i invests 1$ in bonds in t+s-1. After one year, he will get $\frac{R^i_{t+s}}{(1+n_i)}$ as the household has grown at the growth rate n_i and an increase number of individuals will share the benefits of the bond investment in t+s. Now, assume that the representative individual in the fictional household invest 1$ in bonds in t+s-1. This fictional household is growing at the rate n_j, not n_i. To get the same return as the representative individual in country i, the return on 1$ invested in bonds will have to be $\frac{R^i_{t+s}}{(1+n_j)}$.

In normalized form, the fictional household maximizes:

$$\widetilde{v}^i_t = E_t\sum_{s=0}\beta^{j^s}\frac{1}{1-\sigma_j}\left(\widetilde{c}^i_{t+s}\right)^{1-\sigma_j}v\left(\overline{L} - \widetilde{L}^i_{t+s}\right) \tag{A.34}$$

subject to:

$$-\widetilde{k}^i_{t+s}-\widetilde{b}^i_{t+s}+\frac{\left[(1-\delta)+\widetilde{p}^{K,i}_{t+s}\right]}{(1+n_j)(1+g_j)}\widetilde{k}^i_{t+s-1}+\frac{\widetilde{r}^i_{t+s}}{(1+n_j)(1+g_j)}\widetilde{b}^i_{t+s-1}+p^{L,i}_{t+s}\widetilde{L}^{i,PC}_{t+s}-p^{C,i}_{t+s}\widetilde{c}^i_{t+s}+\pi^i_{t+s}=0 \tag{A.35}$$

$$\widetilde{k}^i_{t+s}=\widetilde{i}^i_{t+s}+\frac{(1-\delta)}{(1+n_j)(1+g_j)}\widetilde{k}^i_{t+s-1} \tag{A.36}$$

$$\widetilde{k}^i_{t-1}=k^i_{t-1} \tag{A.37}$$

$$\widetilde{b}^i_{t-1}=b^i_{t-1} \tag{A.38}$$

where: $\widetilde{v}^i_t \equiv \frac{\widetilde{W}_t}{\left(X^j_t\right)^{1-\sigma_j}N^j_t}$, $\widetilde{c}^i_{t+s}\equiv\frac{\widetilde{C}^i_{t+s}}{X^j_{t+s}}$, $\widetilde{k}^i_{t+s}\equiv\frac{\widetilde{K}^i_{t+s}}{X^j_{t+s}}$, $\widetilde{b}^i_{t+s}\equiv\frac{\widetilde{B}^i_{t+s}}{P^{iI}_{t+s}X^j_{t+s}}$, $p^{iK}_{t+s}\equiv\frac{P^{iK}_{t+s}}{P^{iI}_{t+s}}$, $p^{iL}_{t+s}\equiv\frac{P^{iL}_{t+s}}{P^{iI}_{t+s}X^j_{t+s}}$, $p^{iC}_{t+s}\equiv\frac{P^{iC}_{t+s}}{P^{iI}_{t+s}}$, $\widetilde{r}^i_{t+s}\equiv\widetilde{R}^i_{t+s}\frac{P^{iI}_{t+s-1}}{P^{iI}_{t+s}}$, $\pi^i_{t+s}\equiv\frac{\Pi^i_{t+s}}{P^{iI}_{t+s}X^j_{t+s}}$ and $\beta^j\equiv\frac{(1+n_j)(1+g_j)^{1-\sigma_j}}{(1+\rho_j)}$.

Linearizing $\widetilde{v}^i_t$ around country j's steady state and using the envelope theorem, one obtains:

$$\begin{aligned}\widetilde{v}^i_t \;=\;& v+E_t\sum_{s=0}^{\infty}\beta^{j^s}\lambda \\ &\times\left[\frac{k\left(\widetilde{p}^{K,i}_{t+s}-p^K\right)}{(1+n_j)(1+g_j)}+\frac{b\left(\widetilde{r}^i_{t+s}-r\right)}{(1+n_j)(1+g_j)}+L\left(p^{L,i}_{t+s}-p^L\right)-c\left(p^{C,i}_{t+s}-p^C\right)+\pi(\pi^i_{t+s}-\pi)\right] \\ &+\lambda\frac{\left[(1-\delta)+p^K\right]}{(1+n_j)(1+g_j)}\left(k^i_{t-1}-k\right)+\lambda\frac{r}{(1+n_j)(1+g_j)}\left(b^i_{t-1}-b\right)\end{aligned} \tag{A.39}$$

where v, λ, k, b, c, p^C, p^K, p^L, r and π are the steady state values of the normalized utility, quantities, and prices in country j.

Now take the budget constraint and capital accumulation equation of the representative household in country i:

$$\begin{aligned}&-P^{I,i}_{t+s}K^i_{t+s}N^i_{t+s}-B^i_{t+s}N^i_{t+s}+\left[(1-\delta)P^{I,i}_{t+s}+P^{K,i}_{t+s}\right]N^i_{t+s-1}K^i_{t+s-1} \\ &+R^i_{t+s}N^i_{t+s-1}B^i_{t+s-1}+P^{L,i}_{t+s}L^i_{t+s}N^i_{t+s}-p^{C,i}_{t+s}C^i_{t+s}N^i_{t+s}+\Pi^i_{t+s}N^i_{t+s} \\ =\;& 0\end{aligned} \tag{A.40}$$

and:

$$N^i_{t+s}K^i_{t+s}=I^i_{t+s}N^i_{t+s}+(1-\delta)N^i_{t+s-1}K^i_{t+s-1} \tag{A.41}$$

which in normalized terms become:

$$-k_{t+s}^{i}-b_{t+s}^{i}+\frac{\left[(1-\delta)+p_{t+s}^{K,i}\right]}{(1+n_{i})(1+g_{j})}k_{t+s-1}^{i}+\frac{r_{t+s}^{i}b_{t+s-1}^{i}}{(1+n_{i})(1+g_{j})}+p_{t+s}^{L,i}L_{t+s}^{i}-p_{t+s}^{C,i}c_{t+s}^{i}+\pi_{t+s}^{i}=0$$

and:

$$k_{t+s}^{i}=i_{t+s}^{i}+\frac{(1-\delta)}{(1+n_{i})(1+g_{j})}k_{t+s-1}^{i} \tag{A.42}$$

where $c_{t+s}^{i}\equiv\frac{C_{t+s}^{i}}{X_{t+s}^{j}}$, $k_{t+s}^{i}\equiv\frac{K_{t+s}^{i}}{X_{t+s}^{j}}$, $b_{t+s}^{i}\equiv\frac{B_{t+s}^{i}}{P_{t+s}^{iI}X_{t+s}^{j}}$.
Using (A.32) and (A.33), (A.40) can be re-written as:

$$-k_{t+s}^{i}-b_{t+s}^{i}+\frac{(1-\delta)k_{t+s-1}^{i}}{(1+n_{i})(1+g_{j})}+\frac{\widetilde{p}_{t+s}^{K,i}k_{t+s-1}^{i}}{(1+n_{j})(1+g_{j})}+\frac{\widetilde{r}_{t+s}^{i}}{(1+n_{j})(1+g_{j})}b_{t+s-1}^{i}+p_{t+s}^{L,i}L_{t+s}^{i}-p_{t+s}^{C,i}c_{t+s}^{i}+\pi_{t+s}^{i}=0 \tag{A.43}$$

Now linearize this budget constraint around country's j steady state:

$$\begin{aligned}
&\frac{k\left(\widetilde{p}_{t+s}^{K,i}-p^{K}\right)}{(1+n_{j})(1+g_{j})}+\frac{b\left(\widetilde{r}_{t+s}^{i}-r\right)}{(1+n_{j})(1+g_{j})}+L\left(p_{t+s}^{L,i}-p^{L}\right)-c\left(p_{t+s}^{C,i}-p^{C}\right)+\pi(\pi_{t+s}^{i}-\pi)\\
=\;&(k_{t+s}^{i}-k)-\frac{(1-\delta)}{(1+n_{i})(1+g_{j})}(k_{t+s-1}^{i}-\overline{k})-\frac{p^{K}}{(1+n_{j})(1+g_{j})}(k_{t+s-1}^{i}-\overline{k})\\
&+(b_{t+s}^{i}-b)-\frac{r}{(1+n_{j})(1+g_{j})}(b_{t+s-1}^{i}-b)-p^{L}(L_{t+s}^{i}-L_{t+s})+p^{C}(c_{t+s}^{i}-c)
\end{aligned} \tag{A.44}$$

Replacing into (A.39) we get:

$$\begin{aligned}
\widetilde{v}_{t}^{i}\;=\;&v_{t}+\lambda E_{t}\sum_{s=0}^{\infty}\beta^{j^{s}}\\
&\times\{\overline{p}^{C}(c_{t+s}^{i}-c)+(k_{t+s}^{i}-k)-\frac{(1-\delta)}{(1+n_{i})(1+g_{j})}(k_{t+s-1}^{i}-\overline{k})\\
&-\frac{p^{K}}{(1+n_{j})(1+g_{j})}(k_{t+s-1}^{i}-\overline{k})-p^{L}(L_{t+s}^{i}-L_{t+s})\\
&+(b_{t+s}^{i}-b)-\frac{r}{(1+n_{j})(1+g_{j})}(b_{t+s-1}^{i}-b)\}\\
&+\lambda\frac{\left[(1-\delta)+p^{K}\right]}{(1+n_{j})(1+g_{j})}\left(k_{t-1}^{i}-k\right)+\lambda\frac{r}{(1+n_{j})(1+g_{j})}\left(b_{t-1}^{i}-b\right)
\end{aligned} \tag{A.45}$$

Using the capital accumulation equation (A.42) and rearranging:

$$\begin{aligned}
\widetilde{v}_t^i &= v_t + \lambda E_t \sum_{s=0}^{\infty} \beta^{j^s} [p^C (c_{t+s}^i - c) + (i_{t+s}^i - i) - \frac{p^K}{(1+n_j)(1+g_j)} (k_{t+s-1}^i - k) - p^L (L_{t+s}^i - \overline{L}_{t+s})] \\
&+ \lambda E_t \sum_{s=0}^{\infty} \beta^{j^s} [(b_{t+s}^i - b)(1 - \frac{\beta^j r}{(1+n_j)(1+g_j)})] \qquad \text{(A.46)} \\
&+ \lim_{T\to\infty} \beta^{jT} (b_T^i - b) \\
&+ \lambda \frac{\left[(1-\delta) + p^K\right]}{(1+n_j)(1+g_j)} \left(k_{t-1}^i - k\right)
\end{aligned}$$

Notice that the FOCs for the representative household of country j are:

$$\left(c_t^j\right)^{-\sigma_j} v\left(\overline{L} - L_t^j\right) - \lambda_t p_t^{C,j} = 0 \qquad \text{(A.47)}$$

$$-\frac{1}{1-\sigma_j} \left(c_t^j\right)^{1-\sigma_j} v'\left(\overline{L} - L_t^j\right) + \lambda_t p_t^{L,i} = 0 \qquad \text{(A.48)}$$

$$-\lambda_{t+s} + \beta^j E_t \lambda_{t+s+1} \frac{r_{t+s}^j}{(1+n_j)(1+g_j)} = 0 \qquad \text{(A.49)}$$

$$-\lambda_{t+s} + \beta^j E_t \lambda_{t+s+1} \frac{\left[(1-\delta) + p_{t+s}^{K,j}\right]}{(1+n_j)(1+g_j)} = 0 \qquad \text{(A.50)}$$

which, taken in steady state, imply:

$$(c)^{-\sigma_j} v\left(\overline{L} - L\right) - \lambda p^C = 0 \qquad \text{(A.51)}$$

$$-\frac{1}{1-\sigma_j} (c)^{1-\sigma_j} v'\left(\overline{L} - L\right) + \lambda p^L = 0 \qquad \text{(A.52)}$$

$$1 - \frac{\beta^j r}{(1+n_j)(1+g_j)} = 0 \qquad \text{(A.53)}$$

$$1 - \frac{\beta^j \left[(1-\delta) + p^K\right]}{(1+n_j)(1+g_j)} = 0 \qquad \text{(A.54)}$$

Together with the transversality condition, the two results above imply that equation (A.46) can be re-written as:

$$\begin{aligned}
\widetilde{v}_t^i &= v_t + \lambda E_t \sum_{s=0}^{\infty} \beta^{j^s} [p^C (c_{t+s}^i - c) + (i_{t+s}^i - i) - \frac{p^K}{(1+n_j)(1+g_j)} (k_{t+s-1}^i - k) - p^L (L_{t+s}^i - L_{t+s})] \\
&+ \lambda \frac{1}{\beta^j} \left(k_{t-1}^i - k\right) \qquad \text{(A.55)}
\end{aligned}$$

Next, consider the representative household in country j. Proceeding in a parallel manner as above, it is possible to prove that:

$$
\begin{aligned}
v_t^j &= v_t + \lambda E_t \sum_{s=0}^{\infty} \beta^{j^s} [p^C (c_{t+s}^j - c) + (i_{t+s}^j - i) - \frac{p^K}{(1+n_j)(1+g_j)} (k_{t+s-1}^j - k) - p^L (L_{t+s}^j - L_{t+s})] \\
&\quad + \lambda \frac{1}{\beta^j} \left(k_{t-1}^j - k \right) \qquad \text{(A.56)}
\end{aligned}
$$

Combining the two expressions above, we get:

$$
\begin{aligned}
\frac{\widetilde{v}_t^i - v_t^j}{\lambda} &= E_t \sum_{s=0}^{\infty} \beta^{j^s} \qquad \text{(A.57)} \\
&\quad \times \left[(i_{t+s}^i - i_{t+s}^j) + p^C (c_{t+s}^i - c_{t+s}^j) - \frac{p^K}{(1+n_j)(1+g_j)} (k_{t+s-1}^i - k_{t+s-1}^j) - p^L (L_{t+s}^i - L_{t+s}^j) \right] \\
&\quad + \lambda \frac{1}{\beta^j} \left(\widetilde{k}_{t-1}^i - k_{t-1}^j \right)
\end{aligned}
$$

Now rearranging the terms above and using some basic approximation rules:

$$
\begin{aligned}
\frac{v}{\lambda p^y y} (\log \widetilde{v}_t^i - \log v_t^j) &= E_t \sum_{s=0}^{\infty} \beta^{j^s} \{ s_I (\log i_{t+s}^i - \log i_{t+s}^j) + s_C (\log c_{t+s}^i - \log c_{t+s}^j) \qquad \text{(A.58)} \\
&\quad - s_K (\log k_{t+s-1}^i - \log k_{t+s-1}^j) - s_L (L_{t+s}^i - L_{t+s}^j) \} \\
&\quad + \frac{1}{\beta^j} \frac{k}{p^y y} \left(\log \widetilde{k}_{t-1}^i - \log k_{t-1}^j \right)
\end{aligned}
$$

where s_I, s_C, s_K, and s_L are respectively the steady state shares of investment, consumption, capital and labor in country j.

Now translate the right-hand part in per-capita quantities:

$$
\begin{aligned}
\frac{v}{\lambda p^y y} (\log \widetilde{v}_t^i - \log v_t^j) &= E_t \sum_{s=0}^{\infty} \beta^{j^s} \{ s_I (\log I_{t+s}^i - \log I_{t+s}^j) + s_C (\log C_{t+s}^i - \log C_{t+s}^j) \\
&\quad - s_K (\log K_{t+s-1}^i - \log K_{t+s-1}^j) - s_L (L_{t+s}^i - L_{t+s}^j) \} \qquad \text{(A.59)} \\
&\quad + \frac{1}{\beta^j} \frac{\overline{P^I K}}{P^Y Y} \left(\log K_{t-1}^i - \log K_{t-1}^j \right)
\end{aligned}
$$

Now, define the equivalent consumption of the fictional representative household as:

$$
\widetilde{W}_t^i = \sum_{s=0}^{\infty} \frac{N_t (1+n_j)^s}{(1+\rho_j)^s} \frac{(C_t^* (1+g_j)^s)^{1-\sigma}}{(1-\sigma_j)} \nu(\overline{L} - L)) \qquad \text{(A.60)}
$$

In normalized terms:

$$
\widetilde{v}_t^i = \sum_{s=0}^{\infty} \frac{(1+n_j)^s}{(1+\rho_j)^s} \frac{(c_t^* (1+g_j)^s)^{1-\sigma}}{(1-\sigma_j)} \nu(\overline{L} - L)) \qquad \text{(A.61)}
$$

or:

$$\widetilde{v}_t^i = \frac{1}{(1-\sigma^j)\left(1-\beta^j\right)}\left(\widetilde{c}_t^{*,i}\right)^{1-\sigma^j}\nu(\overline{L}-L^j) \tag{A.62}$$

Linearize this around the steady state of country j, we get:

$$\widetilde{v}_t^i - v = \frac{1}{\left(1-\beta^j\right)}\nu(\overline{L}-L^j)\,(c)^{-\sigma^j}\,(\widetilde{c}_t^{*,i}-c) \tag{A.63}$$

which using equation (A.51) and some basic approximation rules can be re-written as:

$$\frac{v}{\lambda p^y y}(\log \widetilde{v}_t^i - \log v) = \frac{s_C}{\left(1-\beta^j\right)}(\log \widetilde{c}_t^{*,i} - \log c) \tag{A.64}$$

Proceeding in an analogous way as above, we can get the same expression for the representative household of country j:

$$\frac{v}{\lambda p^y y}(\log v_t^j - \log v) = \frac{s_C}{\left(1-\beta^j\right)}(\log c_t^{*,j} - \log c) \tag{A.65}$$

Using the two equations above, we get:

$$\frac{v}{\lambda p^y y}(\log \widetilde{v}_t^i - \log v_t^j) = \frac{s_C}{(1-\beta^j)}(\log \widetilde{c}_t^{*,i} - \log c_t^{*j}) \tag{A.66}$$

In per-capita terms:

$$\frac{v}{\lambda p^y y}(\log \widetilde{v}_t^i - \log v_t^j) = \frac{s_C}{(1-\beta^j)}(\log \widetilde{C}_t^{*i} - \log C_t^{*j}) \tag{A.67}$$

Plugging this into equation (A.59), we finally get:

$$\begin{aligned}(\log \widetilde{C}_t^{*i} - \log C_t^{*j}) \quad = \quad & \frac{(1-\beta^j)}{s_C}E_t\sum_{s=0}^{\infty}\beta^s\{s_I(\log I_{t+s}^i - \log I_{t+s}^j) + s_C(\log C_{t+s}^i - \log C_{t+s}^j) \\ & -s_K(\log K_{t+s-1}^i - \log K_{t+s-1}^j) - s_L(L_{t+s}^i - L_{t+s}^j)\} \\ & +\frac{(1-\beta^j)}{\beta^j s_C}\frac{\overline{P^I K}}{P^Y Y}\left(\log K_{t-1}^i - \log K_{t-1}^j\right)\end{aligned} \tag{A.68}$$

A.3 Extensions

A.3.1 Government Expenditure

Assume that utility depends upon private consumption and government spending on public consumption, as in equation (20). Assume government expenditure is financed through a lump-sum tax. Re-writing the household maximization problem in normalized variables and proceeding in a similar fashion as in the proof of proposition 1, we obtain:

$$
\begin{aligned}
v_t &= v + \lambda E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{U_{c_G} c_G \widehat{c}_{G,t+s}}{\lambda} + p^C c \widehat{c}_{t+s} + i \widehat{i}_{t+s} - \; p^L L \widehat{L}_{t+s} - \frac{p^K k}{(1+g)(1+n)} \widehat{k}_{t+s-1} \right] \\
&\quad + \lambda \frac{1}{\beta} k \widehat{k}_{t-1} \qquad \text{(A.69)}
\end{aligned}
$$

where $c_{G,t} = \frac{C_{G,t}}{X_t}$. The log-change in per-capita domestic absorption, in normalized variables is defined as:

$$
\Delta \log y_t = s_C \Delta \log c_{t+s} + s_{c_G} \Delta \log c_{G,t} + s_I \Delta \log i_t \qquad \text{(A.70)}
$$

where s_{c_G} is the steady state value of $s_{c_G,t} = \frac{P_t^G C_{G,t}}{P_t^Y Y_t}$ and P^G is the public consumption deflator.

Using this result, after some algebra equation (A.69) can be rewritten as:

$$
\begin{aligned}
\frac{v}{\lambda p^Y y} \frac{\Delta v_t}{v} &= E_t \sum_{s=0}^{\infty} \beta^s [\Delta \log y_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log \; k_{t+s-1} + \left(s^*_{c_G} - s_{c_G} \right) \Delta \log c_{G,t}] \\
&\quad + \sum_{s=0}^{\infty} \beta^s \lambda [(E_t \log y_{t+s-1} - E_{t-1} \log y_{t+s-1}) \\
&\quad + \frac{1}{\beta} \frac{k}{p^Y y} \Delta \log k_{t-1} \qquad \text{(A.71)}
\end{aligned}
$$

where $s^*_{c_G}$ is the steady state value of $s^*_{c_G,t} = \frac{U_{c_G,t} c_{G,t}}{\lambda_t}$. From this point, the algebra is very similar to the benchmark case, and yields (21) and (22) in the main text.

A.3.2 Multiple Wages and Labor Market Rationing

So far we have assumed that the household is a price-taker in goods and factor markets, and that it faces no constraints other than the intertemporal budget constraint. We have exploited the insight that under these conditions relative prices measure the representative consumer's marginal rate of substitution between goods, even when relative prices do not measure the economy's marginal rate of transformation. We now ask whether our conclusions need to be modified in environments where the household does not behave as a price taker. We present two examples, and then draw some tentative conclusions about the robustness of our previous results.

Our examples focus on the labor market. It seems reasonable to assume that consumers are price-takers in capital markets; most individuals take rates of return on assets as exogenously given. The assumption is still tenable when it comes to the purchase of goods, although some transaction prices may be subject to bargaining. The price-taking assumption seems most questionable when it comes to the labor market, and indeed several literatures (on labor search, union wage setting, and efficiency wages, to name three) begin by assuming that households are not price takers in the labor market. Thus, we study two examples. One is in the spirit of the dual labor markets literature, where wages are above their market-clearing level in some sectors but not in other. We do not model why wages are higher in the primary sector, but this can be due to the presence of unions or government mandates in formal but not in informal employment, or efficiency wage considerations in some sectors but not in others. Wages in the secondary market are set competitively. The second example is in the spirit of labor market search, and has households face a whole distribution of wages. In both cases households would prefer to supply all their labor to the sector or firm that pays the highest wage, but are unable to do so. In this sense, both examples feature a type of labor market rationing. (In both cases the different wages are paid to identical workers, and are not due to differences in human capital characteristics.)

First, consider the case in which the household can supply labor in two labor markets. The primary market pays a high wage $\overline{P_t^L}$ and the secondary market pays a lower wage $\underline{P_t^L} < \overline{P_t^L}$. Although the worker prefers to work only in the primary sector, the desirable jobs are rationed; he cannot supply more than $\widetilde{L}$ hours in the high-wage job in each period. The representative household faces the following budget constraint:

$$\begin{aligned} P_t^I K_t N_t + B_t N_t &= (1-\delta) P_t^I K_{t-1} N_{t-1} + \left(1+i_t^B\right) B_{t-1} N_{t-1} \\ &\quad + N_t \overline{P_t^L} \widetilde{L} + N_t \underline{P_t^L} (L_t - \widetilde{L}) + P_t^K K_{t-1} N_{t-1} + \Pi_t N_t - P_t^C N_t \end{aligned} \tag{A.72}$$

Assuming that the labor rationing constraint is binding, the logic of our previous derivations remains valid, but now we need to re-define the labor share in terms of the lower wage paid in the secondary market. For instance the modified productivity growth residual for the closed economy is now:

$$\Delta \log PR_t = \log Y_t - \Delta \underline{s_L} \log L_{h,t} - s_K \Delta \log K_{h,t-1} \tag{A.73}$$

where the distributional share of labor $\underline{s_L} \equiv \frac{\underline{P^L} L}{P^Y Y}$ is computed using the *low* wage, paid in the dual labor market, rather than the average wage. The intuition for this result comes from the fact that the marginal wage for the household is $\underline{P_t^L}$ while $N_t(\overline{P_t^L} - \underline{P_t^L})\widetilde{L}$ can be considered as a lump-sum transfer and can be treated exactly like the profit term in the budget constraint. (Thus, we can also allow for arbitrary variations over time in the primary wage $\overline{P_t^L}$ or the rationed number of hours $\widetilde{L}$ without changing our derivations.)

This example shows that in some cases our methods need to be modified if the household is no longer a price-taker. However, in this instance the modification is not too difficult—one can simply decrease the labor share by the ratio of the average wage to the competitive wage. Furthermore, this example shows that imperfect competition in factor markets can introduce an additional gap between the welfare residual and the standard Solow residual that is like a tax wedge, making our modifications to standard TFP even more important if one wants to use TFP data to capture welfare. As is the case with taxes, welfare rises with increases in output holding inputs constant, even if there is no change in actual technology.

Note that we would get a qualitatively similar result if, instead of labor market rationing, we assumed that the household has monopoly power over the supply of labor, as in many New Keynesian DSGE models. As in the example above, we would need to construct the true labor share by using the household's marginal disutility of work, which would be less than the real wage. In this environment, we would obtain the welfare-relevant labor share by dividing the observed labor share by an assumed value for the average markup of the wage over the household's marginal rate of substitution between consumption and leisure.

The second example shows that there are situations where our previous results in sections 2 and 3 are exactly right and need no modification, even with multiple wages and labor market rationing. Consider a household that comprises a continuum of individuals with mass N_t. Suppose that each individual can either not work, or work and supply a fixed number of hours $\widehat{L}$. In this environment, the household can make all its members better off by introducing lotteries that convexify their choice sets. Suppose that the household can choose the probability q_t for an individual to work. The representative household maximizes intertemporal utility:

$$W_t = \sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s} \frac{N_{t+s}}{H} \left[q_t U(C_t^0; \overline{L} - \widehat{L}) + (1-q_t) U(C_t^1; \overline{L}) \right] \tag{A.74}$$

where $q_t U(C_t^0; \overline{L} - \widehat{L}) + (1-q_t) U(C_t^1; \overline{L})$ denotes expected utility prior to the lottery draw. C_t^0 and C_t^1 denote respectively per-capita consumption of the employed and unemployed individuals, while average per-capita consumption, C_t is given by:

$$C_t = q_t C_t^0 + (1-q_t) C_t^1 \tag{A.75}$$

Assume that the individuals that work face an uncertain wage P_t^L, which is observed only after labor supply decisions have been made. More specifically, individual wages in period t are iid draws from a distribution with mean $E_t P_t^L$. Notice that, by the law of large numbers, the household does not face any uncertainty regarding its total wage income. Thus, the budget constraint for the household becomes:

$$\begin{aligned} P_t^I K_t N_t + B_t N_t \quad &= \quad (1-\delta) P_t^I K_{t-1} c + \left(1 + i_t^B\right) B_{t-1} N_{t-1} \\ &\quad + q_t N_t E_t P_t^L \widehat{L} + P_t^K K_{t-1} N_{t-1} + \Pi_t N_t - P_t^C \left(q_t C_t^0 + (1-q_t) C_t^1\right) N_t \end{aligned} \tag{A.76}$$

Following Rogerson and Wright (1988) and King and Rebelo (1999),[29] we can rewrite the per-period utility function as:

$$U(C_t; L_t) = \frac{1}{1-\sigma} {C_t}^{1-\sigma} \nu^*(L_t) \tag{A.77}$$

where $L_t = q_t \widehat{L}$ denotes the average number of hours worked and:

$$\nu^*(L_t) = \left(\frac{L_t}{\widehat{L}} \nu(\overline{L} - \widehat{L})^{\frac{1}{\sigma}} + (1 - \frac{L_t}{\widehat{L}}) \nu(\overline{L})^{\frac{1}{\sigma}} \right)^{\sigma} \tag{A.78}$$

In summary, the maximization problem faced by the household is exactly the same as the one described in section 2, even if identical workers are paid different wages.[30] All the results we

[29] In obtaining this result we use the fact that the marginal utility of consumption of the individuals in the household needs to be equalized at the optimum. This implies: $c_t^0 = c_t^1 \left(\frac{\nu(\overline{L} - \widehat{L})}{\nu(\overline{L})} \right)^{\frac{1}{\sigma}}$.

[30] King and Rebelo (1999) show that in this framework the representative agent has an infinite Frisch labor supply elasticity. This result follows from the assumption that all agents in the household have the same disutility

have derived previously also apply in this new setting. The second example leads to a different result from the first for two reasons. First, it is an environment with job search rather than job queuing. Second, the number of hours supplied by each worker is fixed. Under these two assumptions, the labor supply decision is made *ex ante* and not *ex post*.

From these two examples, it is clear that dropping the assumption that all consumers face the same price for each good or service can—but need not—change the precise nature of the proxies we develop for welfare. Even in the case where the measure changed, however, our conclusion that welfare can be summarized by a forward-looking TFP measure and capital intensity remained robust. While the exact nature of the proxy will necessarily be model-dependent, we believe that our basic insight applies under fairly general conditions.

A.3.3 Summing up

Using the extensions developed in A.3.1-A.3.2 and the others described in the text (distortionary taxes and multiple investment and consumption goods) we can state Proposition 1'. A parallel argument can be used to derive the generalization of Proposition 2, Proposition 2'.

A.4 Human capital

Next we develop an extension of our model in the spirit of Lucas (1988). As in Lucas, assume that non-leisure time can be used either to work or to accumulate human capital, and that the accumulation of human capital is linear in the stock of human capital.[31] The representative household maximizes intertemporal utility:

$$W_t = \sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s} \frac{N_{t+s}}{H} U(C_{t+s}; \overline{L} - L_{t+s} - E_{t+s}) \tag{A.79}$$

where E_t denotes the amount of time devoted to human capital accumulation, under the following budget constraint:

$$\begin{aligned} P_t^I K_t N_t + B_t N_t &= (1-\delta) P_t^I K_{t-1} N_{t-1} + \left(1 + i_t^B\right) B_{t-1} N_{t-1} + P_t^L L_t H_{t-1} N_t + P_t^K K_{t-1} N_{t-1} \\ &\quad + \Pi_t N_t - P_t^C C_t N_t \end{aligned} \tag{A.80}$$

where labor income $P_t^L H_{t-1} L_t N_t$ now depends on the initial level of human capital H_{t-1}. The human capital accumulation equation is assumed to be:

$$(H_t - H_{t-1}) + \delta_H H_{t-1} = F(E_t) H_{t-1} \tag{A.81}$$

of labor. Mulligan (2001) shows that even when all labor is supplied on the extensive margin, one can obtain any desired Frisch elasticity of labor supply for the representative agent by allowing individual agents to have different disutilities of labor. In a more elaborate example, we could use Mulligan's result to show that the only restrictions on the preferences of the representative agent are those that we assume in Section 2.

[31] Lucas showed that this formulation of the human capital accumulation process generates a positive steady-state growth rate, even if there is no exogenous technological progress. If the production function of goods exhibits constant returns in physical and human capital, then all the relevant quantities grow at the same rate. These results extend to the case in which the leisure choice is endogenized, although in this case there are some parameter configurations for which multiple steady-state balanced growth paths exist (Ladron de Guevara, Ortigueira and Santos (1999)).

with $F'(E_t) > 0$ and $F(0) = 0$. Linearizing the maximization problem as before, we get:

$$\begin{aligned} v_t - v &= E_t \sum_{s=0}^{\infty} \beta^s \lambda \left(\frac{p^L L h}{(1+g)} \widehat{p}^L_{t+s} + \frac{p^K k}{(1+g)(1+n)} \widehat{p}^K_{t+s} - p^C_t c \widehat{p}_{t+s} + \pi \widehat{\pi}_{t+s} + \frac{rb}{(1+g)(1+n)} \widehat{r}_{t+s} \right) \\ &\quad + \lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1} + \left[U_L \frac{1+g}{F'(E)h} + \frac{\lambda p^L L}{(1+g)} \right] h \widehat{h}_{t-1} \end{aligned} \quad \text{(A.82)}$$

where $h_t = \frac{H_t}{X_t}$ and $p^L_t = \frac{P^L_t}{P^I_t}$. The FOCs for human capital and labor, in normalized terms, are:

$$\begin{aligned} 0 &= -U_{L_t} \frac{1+g}{F'(E)h_{t-1}} \\ &\quad + \beta E_t \left[U_{L_{t+1}} \frac{(1+g)h_{t+1}}{F'(E)h_t^2} + \frac{L_{t+1} p^L_{t+1}}{(1+g)} \lambda_{t+1} \right] \end{aligned} \quad \text{(A.83)}$$

and

$$0 = -U_{L_t} + \frac{\lambda_t p^L_t h_{t-1}}{(1+g)} \quad \text{(A.84)}$$

while the FOCs for consumption, physical capital and bonds are still defined by equations (A.4), (A.6) and (A.7).

Using the FOCs for physical capital and human capital evaluated in the steady state in (A.82) and the log-linearized version of the budget constraint, we obtain:

$$\begin{aligned} v_t - v &= \lambda E_t \sum_{s=0}^{\infty} \beta^s \left[\left(p^C c \widehat{c}_{t+s} + i \widehat{\imath}_{t+s} \right) - \left(\frac{p^L L h}{(1+g)} \left(\widehat{L}_{t+s} + \widehat{h}_{t+s-1} \right) + \frac{p^K}{(1+g)(1+n)} k \widehat{k}_{t+s-1} \right) \right] \\ &\quad + \frac{\lambda}{\beta} k \widehat{k}_{t-1} + \left(\frac{1}{1-\beta} \right) \frac{\lambda p^L L h}{(1+g)} h \widehat{h}_{t-1} \end{aligned} \quad \text{(A.85)}$$

Following the same steps as with the basic model, it is possible to show that the change in equivalent consumption now also depends upon the change in the initial level of human capital. Moreover, labor input must be adjusted for human capital growth in the definition of productivity growth. Thus, equations (23) and (24) now become:

$$\begin{aligned} \Delta \log (C_t)^* &= \frac{(1-\beta)}{(s_C + s^*_{C_G})} \left[E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s-1} \right. \\ &\quad \left. + \frac{1}{\beta} \left(\frac{P^I K}{P^A A} \right) \Delta \log K_{t-1} + \left(\frac{1}{1-\beta} \right) \left(1 - \tau^L \right) s_L \Delta \log H_{t-1} \right] \end{aligned} \quad \text{(A.86)}$$

and:

$$\begin{aligned} \Delta \log PR_{t+s} &= s_C \Delta \log C_{t+s} + s_I \Delta \log I_{t+s} + s^*_{C_G} \Delta \log C_{G,t+s} \\ &\quad - \left(1 - \tau^L \right) s_L \Delta \log(L_{t+s} H_{t+s-1}) - \left(1 - \tau^K \right) s_K \Delta \log K_{t+s-1} \end{aligned} \quad \text{(A.87)}$$

Summarizing, in the presence of human capital, our welfare results may change for two reasons. First, the definition of productivity must account for the effect on total labor input of both hours and human capital changes. Note that human capital investment does not show up as part of domestic absorption because in the Lucas formulation it is only a subtraction from leisure and does not require any other physical input. Second, human capital must now be included among the initial conditions, alongside physical capital. The latter factor comes into play in our cross-country welfare comparisons only to the extent that the hypothetical move of the household from the reference country (the US) to country i entails losing her initial human capital stock and acquiring the human capital capital stock of country i. In principle, there is no compelling reason why the thought experiment should be framed in this way, rather than allowing the US household to retain the US human capital stock when moving to country i. However, for comparability with the development accounting literature, which assigns a prominent role to cross-country differences in human capital stocks, we opt for assuming that the US household does not retain its human capital when moving to country i.

B The quality of our approximation: some examples

A potential concern with our main results, as stated in Proposition 1' and 2', is that they are proved using first-order approximations. This approach may seem especially problematic for cross-country comparisons, where gaps in living standards are often large. We now use simple general-equilibrium models to investigate the quantitative error introduced by our use of approximations. We consider a set of workhorse models that are standard in the macroeconomic literature, solve them, and then compare the calculated welfare values to our approximated measures.[32]

Assume that there is a fixed number of identical household with an infinite time horizon. The representative household chooses consumption, leisure and investments in capital and bonds to maximize the following intertemporal utility function:

$$W_t = E_t \sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s} \frac{{C_{t+s}}^{1-\sigma}}{(1-\sigma)} (\overline{L} - L_{t+s})^{\frac{1-\gamma}{\gamma}(1-\sigma)}$$

subject to the following budget constraint:

$$I_t + B_t = \left(1 + i_t^B\right) B_{t-1} + P_t^L (1-\tau_t) L_t + P_t^K (1-\tau_t) K_{t-1} - \Pi_t - C_t$$

In this model public expenditure is pure waste and can be financed through an income tax (with tax-rate τ_t) or a lump-sum tax (Π_t). Thus, total public expenditure is:

$$C_{G,t} = P_t^L \tau_t L_t + P_t^K \tau_t K_{t-1} + \Pi_t$$

The law of motion for capital is:

$$K_t = \ (1-\delta) K_{t-1} + I_t$$

There is a large number of firms which operate under perfect competition and are characterized by the same Cobb-Douglas function:

$$Y_t = E_t A_t L_t^{1-\alpha} K_{t-1}^{\alpha}$$

where A_t is the Harrod-neutral technology parameter while E_t measures an externality that arises from aggregate production:

$$E_t = Y_t^{1-\frac{1}{e}}$$

In equilibrium, the national income account equation holds:

$$Y_t = C_t + C_{G,t} + I_t$$

[32]We compare welfare across countries assuming that they are at their steady states. These calculations are exact solutions of the non-linear models. For the comparisons over time in within-country, dynamic settings, we solve the models using third-order approximations and compare our results based on first-order approximations to the third-order solutions. To check the third-order approximations, we also solved the models using fourth-order approximations and solved the simpler models using global methods. In both cases the results were barely distinguishable numerically from the third-order solutions, so we think these are a good baseline for the purposes of checking the first-order approximations.

There are three potential shocks in this economy: to technology A_t, to the lump-sum tax, Π_t, and to the income tax rate, τ_t. The laws of motion of these variables are the following:

$$\log A_t = \rho_1 \log A_t + \lambda t + \varepsilon_{1t}$$

$$\Pi_t = (1 - \rho_2)\Pi^{SS} + \rho_2 \Pi_{t-1} + \varepsilon_{2t}$$

$$\tau_t = (1 - \rho_3)\tau^{SS} + \rho_3 \tau_{t-1} + \varepsilon_{3t}$$

The following table reports the calibration for the benchmark model:

ρ	0.013
σ	1.1
$\frac{1-\gamma}{\gamma}$	1.78
δ	0.012
α	0.4
e	2
λ	0
$\rho_1 = \rho_2 = \rho_3$	0.95
Π^{SS}	0.16
τ^{SS}	0.29
$sd(\varepsilon_1) = sd(\varepsilon_2) = sd(\varepsilon_3)$	0.0075

We have considered four special cases of this model:

1. Ramsey model ($\gamma = 1; e = 1; \Pi^{SS} = 0; \tau^{SS} = 0$)

2. RBC model ($e = 1; \Pi^{SS} = 0; \tau^{SS} = 0$)

3. RBC model with public expenditure financed by distortionary taxes ($e = 1; \Pi^{SS} = 0$)

4. RBC model with public expenditure financed by lump-sum taxes and externalities from production ($\tau^{SS} = 0$)

Note that a simpler version of case 3 (an RBC model with distortionary taxes rebated lump-sum to households, with no public expenditure) is identical to a model with imperfect competition in the goods market. For the equivalence between distortionary taxation on income and the markups that result from imperfect competition, see for example, Correia, Farhi, Nicolini and Teles (2013). The monopolist rebates profits to consumers in a lump-sum fashion, just as the tax revenue is rebated ($\Pi^{SS} < 0$). Our assumed tax rate of 29 percent maps to a markup of price over marginal cost of 1.41, very close to the value of 1.4 assumed by Rotemberg and Woodford (1995). Thus, our experiments with distorted RBC models can also be interpreted as allowing for imperfect competition and monopoly power.

B.1 Within-country analysis

First, we discuss the quality of the approximation in a within-country analysis. Figure (B.1) reports the impulse response function of our measure of approximated welfare and compares it to welfare measures based on third-order approximations, in four standard macro models subject to different types of shocks. Panel (a) reports the impulse response of both equivalent consumption and our approximated measure of it following a one-standard deviation technology shock in a standard Ramsey growth model. The two lines are practically indistinguishable. On impact, equivalent consumption increases by 19.35% while its approximated value increases by 19.32%.[33] In the following periods, the approximated value converges monotonically to the exact one. The non-linearity of the utility function does not have a large effect on the quality of the approximation. In panel (b), we perform the same experiment as in panel (a) but using an extremely concave utility function: we raise the coefficient of relative risk aversion from a common business-cycle value of 1.1 to 10. Following the technology shock, on impact equivalent consumption increases by 19.19% while its approximated value again increases by 19.32%.[34] As we would expect, the approximation error is larger when the utility function is more concave, but the magnitude of the difference is still quite small and converges to zero quickly.

To ensure that the quality of the approximation is not a peculiarity of the Ramsey model, in the following panels, we perform the same exercise in different theoretical frameworks. Panel (c) considers a technological shock in a Real Business Cycle (RBC) model with standard calibration. Panel (d) considers a tax shock in a RBC model with distortionary income taxes and wasteful public expenditure. Panel (e) considers a public expenditure shock in a RBC model with lump-sum taxes, wasteful public expenditure and production externalities. Note that in these last two cases, our welfare-relevant TFP differs from exogenous technology—in the first case due to taxes or imperfect competition, and in the second case because of the externality. In all three cases, however, the first-order approximation gives results that are close to the calculated value for welfare.

B.2 Cross-country analysis

We next evaluate the approximation in a cross-country setting. We use a standard RBC model with distortionary income taxes to analyze welfare gaps between two countries. In this context we run two distinct exercises. First, we analyze welfare gaps in steady between countries characterized by no long-term technological growth (λ=0). Then we turn to countries characterized by (potentially different) long-term technological growth rates (λ>0 and different across countries) and analyze their welfare gaps along the balanced growth path. Comparing countries in either their steady states or balanced growth paths allows us to solve for the exact values of welfare in the two countries, and compare the exact gap to our approximated result. In our theoretical results, which we take to the data, we allow for both steady-state (or balanced growth path)

[33] Here and in the rest of the paper, we use percent (%) change to refer to differences in natural logs multiplied by 100.

[34] It is not a coincidence that the approximated first-period welfare change is the same in the two models. Since both models are neoclassical, the time path of TFP is just the exogenous shocks to technology, which is the same in the two cases. Moreover, since we shock both models starting at their steady states, the period $t-1$ change in the capital stock is also the same in the two models—zero.

welfare gaps *and* transitory welfare gaps between countries. To evaluate the approximation error for this type of comparison, one can take the approximation error for transitory shocks, which we just discussed in the previous subsection, and add it to the approximation error for the steady-state (or balanced growth path) differences.

Let's start from the case in which technology does not follow a trend (λ=0). We consider two countries in their respective steady states. We compute the change in equivalent consumption of a representative agent in a reference country who moves permanently to a different country characterized by different exogenous technology parameters or tax rates. We then compute the approximated change in equivalent consumption and see how it compares to the exact value. We conduct three different experiments, with the results in the three panels of Figure (B.2). First, in panel (a), we consider an increase in the capital elasticity parameter from 0.28 to 0.39 (thus moving from British to Canadian capital shares, the two extremes in our sample): it produces a steady-state increase in equivalent consumption of 73.77%, while the result from our approximation is 72.78%. Second, in panel (b), we consider an increase in the income tax rate from 30% to 40% (thus moving from the average US tax rate over 1985-2005 to the French average over the same period of time): it produces a reduction in equivalent consumption of 16.58 percent while the approximated change is 13.85 percent. Finally, in panel (c), we consider differences in the technology level. The figure illustrates the exact and the approximated change in equivalent consumption when the level of technology drop to a fraction x of its original level. Moving to a country with a level of productivity that is 50% (10%) that of the reference country implies a reduction in equivalent consumption of 69.31% (230.26%), while the approximated value is 65.90% (218.90%). It is interesting to note that the approximation error is largest for differences in tax rates. However, all the changes we consider are large ones. Relative to the large size of the welfare gaps we are considering, we believe the approximation errors are modest and quite acceptable.

Second, we move to the case in which there is long-term technological growth (λ>0). Both countries are assumed to be in their respective balanced growth path. Although they are captured at the moment in which the level of technology is the same, they are characterized by different long-term technological growth rates. Figure (B.3) illustrates the exact and the approximated change in equivalent consumption of a representative agent who moves permanently from a country characterized by a growth rate of 1% along the balanced growth path to a country characterized by a growth rate of x, with x varying from 0.5% to 5%. For instance, moving to a country characterized by a growth rate of 0.05% (2%) implies a log-change in equivalent consumption of -23% (+47%), while the approximated value is -25%(+50%). Even in these extreme scenarios, approximation errors are rather modest.

Figure B.1: Numerical Experiments: Impulse Responses for log Equivalent Consumption

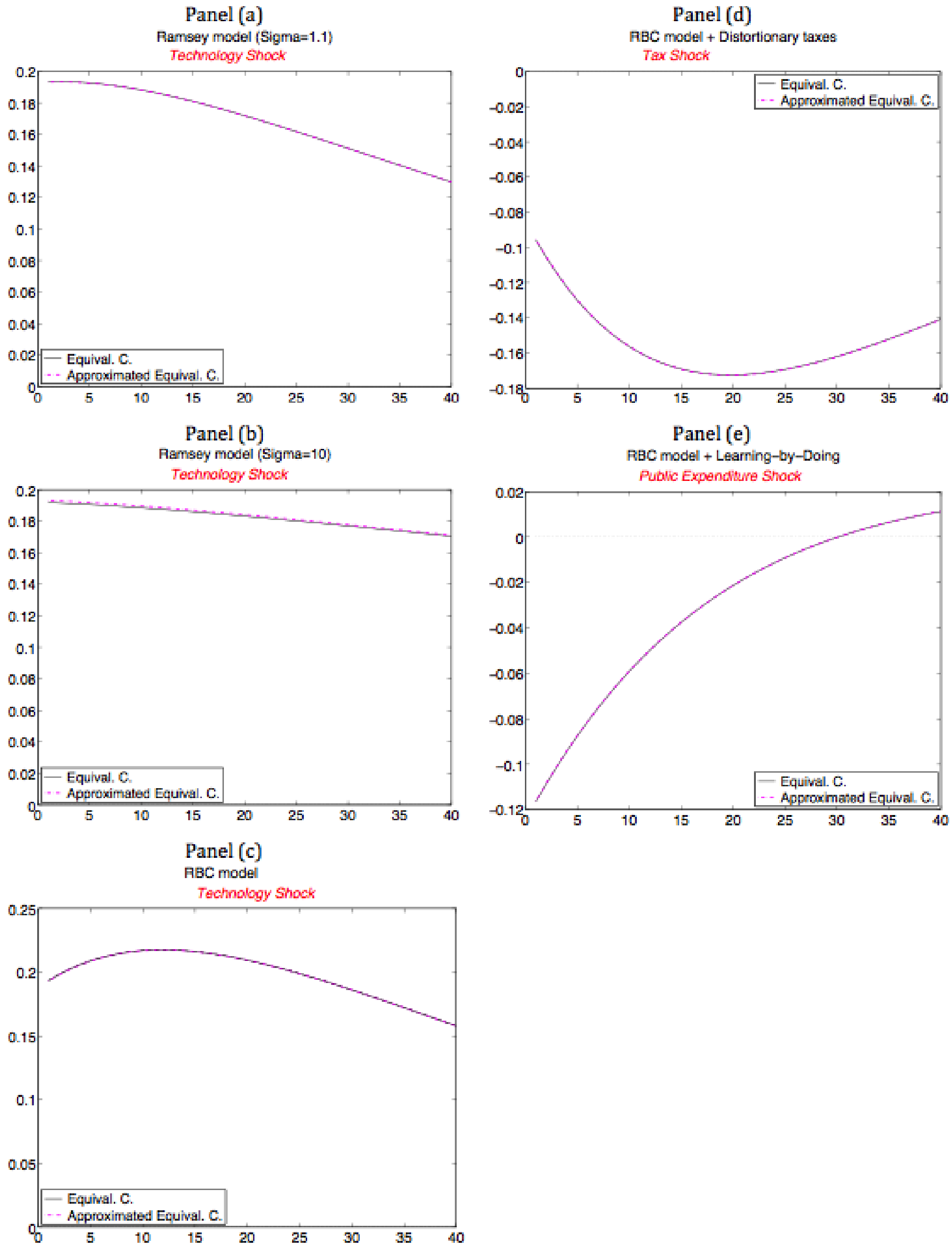


Figure B.2: Numerical Experiments: Cross-Country Gaps for log Equivalent Consumption (1)

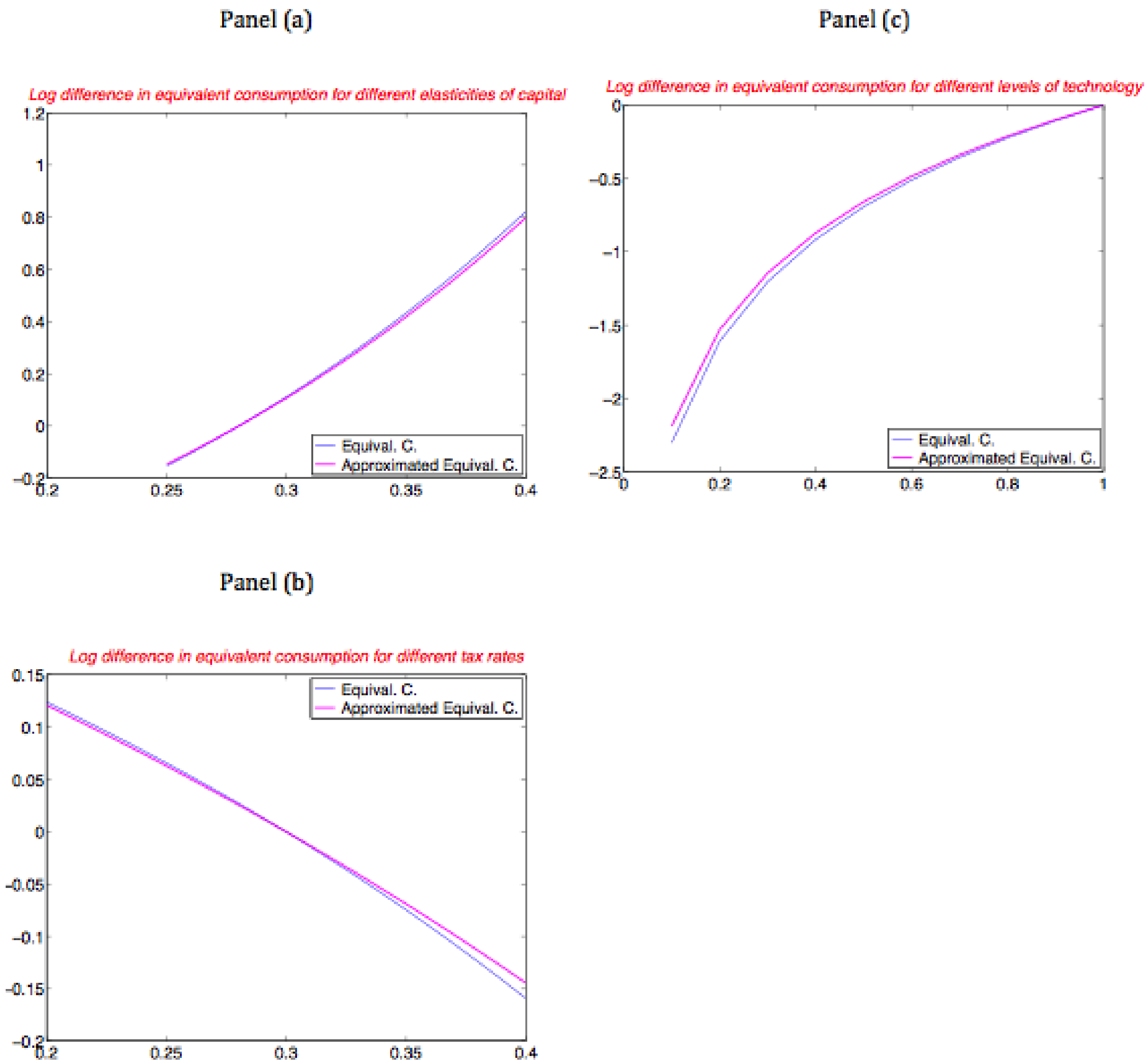
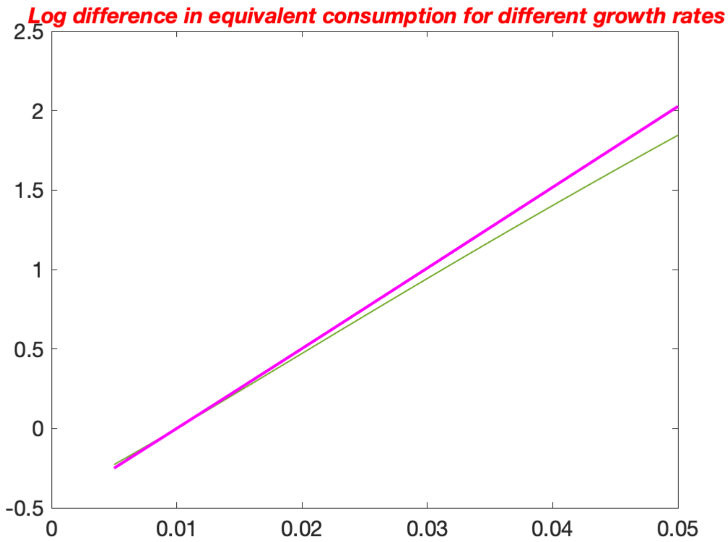


Figure B.3: Numerical Experiments: Cross-Country Gaps for log Equivalent Consumption (2)



C Additional results

C.1 Estimation results

In this subsection, we report the estimates of the AR process of three different welfare-relevant measures of the log-level productivity residual.

For each country, we estimate the following AR(2) stochastic process:

$$\log PR_{t+s} = \alpha + \beta_1 \log PR_{t+s-1} + \beta_2 \log PR_{t+s-2} + \gamma s + \epsilon$$

If the estimated β_2 is not statistically different than zero, we re-estimate an AR(1) process.

The tables C.1-C.3, reported below, illustrate the estimated coefficients for the following cases.

-Table C.1: the welfare-relevant log-productivity is computed for the case optimal spending and distortionary taxes used in the within-country results illustrated in Table 1;

-Table C.2: the welfare-relevant log-productivity is computed for the case optimal spending and distortionary taxes used in the cross-country analysis illustrated in Table 2 (in which aggregate labor input is measured in hours);

-Table C.3: the welfare-relevant log-productivity is computed for the case optimal spending and distortionary taxes used in the cross-country analysis illustrated in Table 4 (in which aggregate labor input is measured using total employment)

Table C.1: Estimated AR process for log-productivity 1

country	α	β_1	β_2	γ
Canada	-5.7855	0.6895	0	0.0029
France	-8.6373	1.1073	-0.3933	0.0044
Italy	-2.0963	0.8744	0	0.0011
Japan	-2.6135	1.4688	-0.5909	0.0013
Spain	-8.1208	1.3878	-0.6231	0.0041
United Kingdom	-15.8368	1.1045	-0.4914	0.0080
United States	-6.8103	0.8003	0	0.0034

Note: Sample period: 1985-2005.

Table C.2: Estimated AR process for log-productivity 2

country	α	β_1	β_2	γ
Canada	-1.7506	0.8858	0	0.0013
France	-5.9577	1.1693	-0.4994	0.0043
Italy	-0.3350	0.7936	0	0.0009
Japan	-2.3427	1.2039	-0.3886	0.0018
Spain	-6.3663	1.1591	-0.4503	0.0043
United Kingdom	-12.4980	1.2018	-0.6021	0.0078
United States	-5.6063	1.1390	-0.3887	0.0038

Note: Sample period: 1985-2005.

Table C.3: Estimated AR process for log-productivity 3

country	α	β_1	β_2	γ
Argentina	3.593727	1.149198	-0.42929	-0.00028
Australia	-5.06739	0.793567	0	0.003721
Austria	0.750041	0.941128	0	0.000035
Belgium	0.761074	0.879179	0	0.000311
Bolivia	8.43918	0.752525	0	-0.003
Brazil	0.118642	1.061114	-0.44284	0.001925
Canada	-1.86154	1.268911	-0.51032	0.002311
Chile	-10.1804	0.776145	0	0.006274
China	-44.7504	1.088401	-0.56033	0.024553
Colombia	12.16802	1.050191	-0.45152	-0.00399
Costa Rica	-0.04953	0.537742	0	0.002481
Denmark	-3.67269	1.056534	-0.49634	0.004326
Ecuador	10.18538	0.40658	0	-0.0021
Egypt	-9.12192	0.717952	0	0.005994
Spain	-0.07686	1.383222	-0.51337	0.000776
Finland	-4.88266	1.404733	-0.73227	0.004283
France	-2.41925	1.139784	-0.50811	0.003318
Guatemala	-10.9873	0.649341	-0.42572	0.009492
Hong Kong	0.674894	0.89909	0	0.000234
Honduras	0.325425	0.702516	0	0.001303
Indonesia	-11.9145	1.09218	-0.43191	0.007579
India	-16.3408	0.793982	0	0.009146
Ireland	-5.49249	1.76736	-1.00499	0.004085
Iran	-7.97859	0.884494	0	0.004599
Israel	-1.0112	1.069697	-0.35724	0.002126
Italy	3.095352	0.939057	0	-0.0012
Jamaica	-16.9635	0.371247	0	0.011792
Japan	0.322815	1.32388	-0.50757	0.000874
South Korea	-7.25026	0.767303	0	0.004898
Sri Lanka	-20.1942	0.525057	0	0.01241
Morocco	9.298045	0.617179	0	-0.00279
Mexico	-4.11902	0.528247	0	0.004576
Malaysia	-15.9967	1.035435	-0.46759	0.010205
Netherlands	-2.80717	1.182371	-0.56978	0.003618
Norway	-8.00474	1.442062	-0.75594	0.005789
New Zealand	-11.35	0.889747	-0.51429	0.009173
Pakistan	1.750647	0.751069	0	0.000321
Panama	-11.5274	0.735962	-0.42443	0.009303
Peru	-4.91871	0.946989	-0.24087	0.004018
Philippines	-14.2813	0.750402	-0.50056	0.010712
Portugal	-1.3192	1.136738	-0.31277	0.00163
Paraguay	17.70428	1.209283	-0.4142	-0.00784
Romania	-9.89026	0.926368	0	0.005338
Singapore	-16.848	0.638489	0	0.010446
El Salvador	-2.18835	0.582368	0	0.003257
Sweden	-4.92617	1.096873	-0.52495	0.004893
Switzerland	1.490719	0.608748	0	0.00147
Syria	-1.9154	0.779799	0	0.002051
Togo	-24.6381	0.322208	0	0.015213
Thailand	-7.53626	1.213338	-0.49704	0.005176
Tunisia	-9.87409	0.757413	0	0.006161
Turkey	-15.9611	0.475331	0	0.01078
Uganda	-44.3501	0.783893	-0.5396	0.025517
United Kingdom	-6.55507	1.618652	-0.94491	0.005145
United States	-2.99646	1.481944	-0.71083	0.002829
Uruguay	-8.38513	1.072941	-0.41764	0.006008
Venezuela	-1.10235	1.062496	-0.42793	0.002415

Note: Sample period: 1985-2005.

C.2 Within-country results with a common discount rate

The results reported in Table 1 are obtained using country-specific discount rates. Here we report the results obtained imposing a common value of $\beta = 0.95$ (the US figure) for all countries. Comparison with Table 1 reveals very little change in the results.

Table C.4: Annual Average Log Change in Per-Capita Equivalent Consumption and its Components (Assume a common discount rate across countries)

	Wasteful Spending Lump-Sum Taxes			Wasteful Spending Distortionary Taxes			Optimal Spending Distortionary Taxes		
	Total	Fraction due to:		**Total**	Fraction due to:		**Total**	Fraction due to:	
		TFP	*Capital*		*TFP*	*Capital*		*TFP*	*Capital*
Canada	**0.013**	0.44	0.56	**0.021**	0.66	0.34	**0.017**	0.69	0.31
France	**0.026**	0.83	0.17	**0.026**	0.83	0.17	**0.022**	0.86	0.14
Italy	**0.018**	0.66	0.34	**0.021**	0.70	0.30	**0.017**	0.72	0.28
Japan	**0.018**	0.42	0.58	**0.023**	0.56	0.44	**0.024**	0.66	0.34
Spain	**0.021**	0.51	0.49	**0.030**	0.66	0.34	**0.031**	0.75	0.25
UK	**0.032**	0.82	0.18	**0.035**	0.83	0.17	**0.030**	0.85	0.15
USA	**0.025**	0.83	0.17	**0.029**	0.85	0.15	**0.024**	0.86	0.14

Note: Sample period: 1985-2005.

C.3 Extended OECD sample

Our baseline results refer to a small group of large OECD economies. In the table below we extend the analysis to smaller OECD economies, and those having joined the organization more recently, for which hours data is available.

Table C.5: Welfare Gap Relative to the US and its Components. Benchmark Case (Optimal Government Spending and Distortionary Taxes) with labor input measured in hours

	Welfare Gap	Fraction due to TFP	Fraction due to Capital
Australia	-0.06	0.98	0.02
Austria	-0.34	1.06	-0.06
Belgium	-0.24	1.11	-0.11
Canada	-0.29	0.92	0.08
Denmark	-0.34	0.99	0.01
Finland	-0.38	1.01	-0.01
France	-0.19	0.90	0.10
Ireland	-0.24	0.77	0.23
Italy	-0.49	1.00	0.00
Japan	-0.46	1.11	-0.11
Netherlands	-0.12	0.99	0.01
Portugal	-0.83	0.91	0.09
South Korea	-0.91	0.91	0.09
Spain	-0.37	0.90	0.10
Sweden	-0.27	0.97	0.03
United Kingdom	-0.12	0.49	0.51
United States	0	-	-

Note: Sample period: 1985-2005.

C.4 US dynamics in the broader sample

The table below reports the results of repeating the exercise in Table 4 assuming that TFP in all countries considered follows the same dynamics as in the US.

Table C.6: Welfare Gap Relative to the US and its Components in 2005. Benchmark Case (Optimal Government Spending and Distortionary Taxes) with US dynamics

	Welfare	Fraction due to	
	Gap	TFP	Capital
USA	0.000	—	—
Norway	-0.21	1.21	-0.21
Australia	-0.22	0.99	0.01
United Kingdom	-0.25	0.74	0.26
Canada	-0.32	0.91	0.09
France	-0.32	0.87	0.13
Austria	-0.34	1.01	-0.01
Netherlands	-0.35	0.93	0.07
Belgium	-0.36	1.03	-0.03
Ireland	-0.38	0.90	0.10
Italy	-0.40	0.96	0.04
Sweden	-0.41	0.91	0.09
Switzerland	-0.42	1.12	-0.12
Denmark	-0.43	0.98	0.02
Spain	-0.46	0.92	0.08
Finland	-0.47	0.95	0.05
Japan	-0.48	1.07	-0.07
Singapore	-0.55	1.09	-0.09
New Zealand	-0.58	0.87	0.13
Israel	-0.61	0.89	0.11
Hong Kong	-0.63	1.04	-0.04
Portugal	-0.87	0.93	0.07
South Korea	-0.95	0.97	0.03
Turkey	-1.37	0.79	0.21
Mexico	-1.40	0.85	0.15
Argentina	-1.43	0.84	0.16
Chile	-1.56	0.86	0.14
Jamaica	-1.59	0.88	0.12
Costa Rica	-1.60	0.85	0.15

	Welfare	Fraction due to:	
	Gap	TFP	Capital
Peru	-1.66	0.82	0.18
Uruguay	-1.68	0.84	0.16
Iran	-1.73	0.88	0.12
Romania	-1.75	0.88	0.12
Malaysia	-1.91	0.90	0.10
Brazil	-1.93	0.86	0.14
Panama	-2.00	0.85	0.15
El Salvador	-2.03	0.82	0.18
Guatemala	-2.05	0.83	0.17
Colombia	-2.06	0.85	0.15
Venezuela	-2.10	0.88	0.12
Tunisia	-2.23	0.89	0.11
Thailand	-2.27	0.90	0.10
Egypt	-2.30	0.81	0.19
Ecuador	-2.32	0.88	0.12
Syria	-2.43	0.82	0.18
Honduras	-2.64	0.85	0.15
Sri Lanka	-2.74	0.86	0.14
Paraguay	-2.81	0.86	0.14
Morocco	-2.90	0.88	0.12
Bolivia	-2.91	0.84	0.16
Indonesia	-2.93	0.87	0.13
China	-3.07	0.90	0.10
Pakistan	-3.08	0.84	0.16
Philippines	-3.11	0.85	0.15
India	-3.18	0.85	0.15
Uganda	-3.97	0.82	0.18
Togo	-4.37	0.86	0.14

C.5 Results with human capital

Table C.7 illustrates the consequences of adding human capital to our within-country analysis. Comparison with Table 1 shows that the resulting changes in welfare growth are very modest under all three scenarios considered. The main difference is that in the configuration when government spending is wasteful and taxes are lump-sum, Italy's welfare growth now outpaces Spain's. On the other hand, inclusion of human capital along with physical capital raises the share of welfare growth attributable to capital accumulation above the levels shown in Table 1 for all countries and under all scenarios. The cases of Spain and Italy are particularly striking in this regard.

Table C.7: Annual Average Log Change in Per-Capita Equivalent Consumption and its Components (Case with human capital)

	Wasteful Spending Lump-Sum Taxes			Wasteful Spending Distortionary Taxes			Optimal Spending Distortionary Taxes		
	Total	Fraction due to:		**Total**	Fraction due to:		**Total**	Fraction due to:	
		TFP	*Capital*		*TFP*	*Capital*		*TFP*	*Capital*
Canada	**0.014**	0.08	0.92	**0.022**	0.47	0.53	**0.018**	0.51	0.49
France	**0.027**	0.27	0.73	**0.026**	0.50	0.50	**0.022**	0.59	0.41
Italy	**0.025**	0.08	0.92	**0.028**	0.34	0.66	**0.022**	0.38	0.62
Japan	**0.018**	0.10	0.90	**0.023**	0.38	0.62	**0.024**	0.52	0.48
Spain	**0.024**	-0.35	1.35	**0.033**	0.20	0.80	**0.034**	0.39	0.61
UK	**0.035**	0.61	0.39	**0.038**	0.68	0.32	**0.032**	0.71	0.29
USA	**0.026**	0.80	0.20	**0.029**	0.83	0.17	**0.024**	0.83	0.17

Note: Sample period: 1985-2005.

In turn, Figure C.1 displays the results of repeating the cross-country exercise shown in the first panel of Figure 1 adding human capital into the model. The welfare gap relative to the US is now wider for all countries considered. However, the magnitude of the change varies across countries. It is especially large for Spain and France, and more modest for the other countries. Further comparison with Figure 1 also shows that the slopes of the lines depicting the welfare gaps over time are fairly similar across the two figures, so the widening of the gaps is roughly the same with or without human capital. At the beginning of the sample period, Spain now shows the largest welfare gap relative to the US, and the UK the smallest one (when ignoring human capital, Japan and France respectively assumed those roles). At the end of the sample period, countries' relative rankings are the same as in the case without human capital, except for the fact that Canada and France switch places.

Lastly, Table C.8 adds human capital into the cross-country welfare calculations for the broader sample considered in Table 4 in the text. Again the main consequence is a widening of the welfare gap relative to the U.S. for virtually all the countries considered. In fact, there are few changes in their relative rankings in terms of welfare: the correlation between the country rankings in Table 4 and those in Table C.8 exceeds .99. In turn, the percentage contribution of TFP to the welfare gaps declines. Its median value falls from 0.87 in Table 4 to 0.77 here, reflecting the fact that for most countries lagging human capital accounts for a significant share

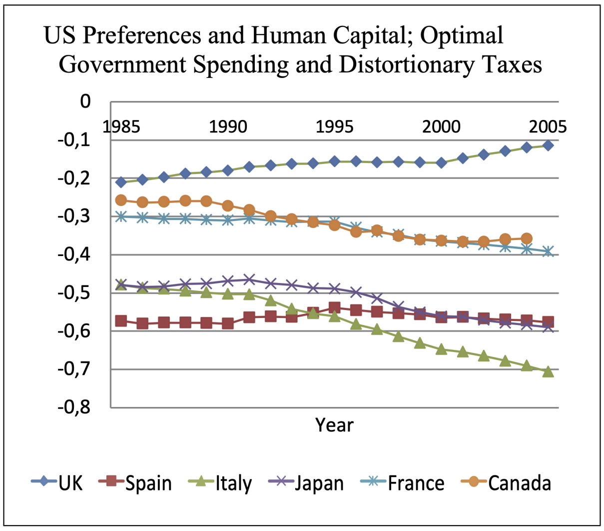


Figure C.1: Cross-Country Welfare Comparisons (log Equivalent Consumption relative to the US)

of the welfare lag vis-a-vis the U.S.

Table C.8: Welfare Gap Relative to the US and its Components in 2005. Benchmark Case (Optimal Government Spending and Distortionary Taxes) with Human Capital

	Welfare	Fraction due to	
	Gap	TFP	Capital
USA	0.000	—	—
Australia	-0.07	0.66	0.34
Norway	-0.12	1.46	-0.46
Singapore	-0.21	0.44	0.56
United Kingdom	-0.22	0.04	0.96
Ireland	-0.34	0.76	0.24
Netherlands	-0.39	0.79	0.21
Canada	-0.39	0.81	0.19
Sweden	-0.45	0.85	0.15
Denmark	-0.47	0.78	0.22
Finland	-0.49	0.77	0.23
France	-0.50	0.71	0.29
Belgium	-0.52	0.88	0.12
New Zealand	-0.54	0.84	0.16
Austria	-0.55	0.80	0.20
Switzerland	-0.59	0.93	0.07
Japan	-0.67	0.97	0.03
Spain	-0.67	0.79	0.21
Israel	-0.68	0.87	0.13
Hong Kong	-0.76	0.88	0.12
South Korea	-0.76	0.90	0.10
Italy	-0.89	0.83	0.17
Portugal	-1.00	0.72	0.28
Chile	-1.21	0.71	0.29
Romania	-1.30	0.46	0.54
Turkey	-1.33	0.57	0.43
Iran	-1.36	0.63	0.37
Jamaica	-1.51	0.78	0.22
Uruguay	-1.53	0.71	0.29

	Welfare	Fraction due to:	
	Gap	TFP	Capital
Argentina	-1.61	0.77	0.23
Mexico	-1.62	0.75	0.25
Peru	-1.62	0.71	0.29
Malaysia	-1.73	0.81	0.19
Costa Rica	-1.76	0.76	0.24
Panama	-2.00	0.79	0.21
Tunisia	-2.01	0.72	0.28
Guatemala	-2.05	0.62	0.38
Brazil	-2.17	0.76	0.24
Egypt	-2.18	0.66	0.34
Venezuela	-2.21	0.75	0.25
China	-2.22	0.75	0.25
Thailand	-2.23	0.77	0.23
El Salvador	-2.26	0.73	0.27
Sri Lanka	-2.40	0.76	0.24
India	-2.48	0.64	0.36
Colombia	-2.55	0.78	0.22
Syria	-2.61	0.69	0.31
Ecuador	-2.65	0.81	0.19
Indonesia	-2.76	0.75	0.25
Honduras	-2.91	0.76	0.24
Philippines	-3.04	0.80	0.20
Morocco	-3.40	0.77	0.23
Pakistan	-3.45	0.75	0.25
Bolivia	-3.47	0.82	0.18
Uganda	-3.57	0.69	0.31
Paraguay	-3.94	0.84	0.16
Togo	-4.28	0.77	0.23