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Twin Default Crises

Caterina Mendicino
Kalin Nikolov
Juan Rubio-Ramirez
Javier Suarez
Dominik Supera

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Abstract

We study the interaction between borrowers' and banks' solvency in a quantitative macroeconomic model with financial frictions in which bank assets are a portfolio of defaultable loans. We show that ex-ante imperfect diversification of bank lending generates bank asset returns with limited upside but significant downside risk. The asymmetric distribution of these returns and their implications for the evolution of bank net worth are important for capturing the frequency and severity of twin default crises - simultaneous rises in firm and bank defaults associated with sizeable negative effects on economic activity. As a result, our model implies higher optimal capital requirements than common specifications of bank asset returns, which neglect or underestimate the impact of borrower default on bank solvency.

JEL Codes: G01, G28, E44.

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Caterina Mendicino
European Central Bank
Caterina.Mendicino@ecb.europa.eu

Kalin Nikolov
European Central Bank
Kalin.Nikolov@ecb.int

Juan Rubio-Ramirez
Emory University
jrubior@emory.edu

Javier Suarez
CEMFI
suarez@cemfi.es

Dominik Supera
Wharton School
superad@wharton.upenn.edu

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1. Introduction

How does the interaction between borrowers' and banks' solvency affect financial and macroeconomic stability? Bank equity declines are the key driver of the economy into banking crises.¹ The main cause of bank equity declines is the realization of abnormally high default rates among bank borrowers.² Yet the quantitative relevance of the channels linking borrower defaults, bank solvency and macroeconomic stability is largely unexplored, mainly because of the difficulties to capture them in a structural manner. Quantifying these channels is essential for a macroprudential calibration of bank capital requirements, which are the main policy tool to protect the economy against bank insolvencies.

In this paper, we build a structural model of bank default risk in which bank assets are a portfolio of defaultable loans. We embed such a model in an otherwise standard quantitative macroeconomic framework with costly state verification frictions. Returns on the banks' loan portfolios are asymmetric and feature limited upside potential but significant downside risk. Importantly, we assume banks to be imperfectly diversified across borrowers, which produces realistic cross-sectional dispersion in their realized loan returns. Aggregate bank equity losses and bank default rates spike when a significant fraction of banks experience large default rates among their borrowers.

Formally, we capture limits to banks' diversification by assuming a segmentation of credit markets into islands. Each bank operates on an island and lends to a continuum of firms that are exposed to both firm- and island-idiosyncratic productivity shocks. Banks diversify away firm-idiosyncratic shocks by lending to all the firms within an island. Island-idiosyncratic shocks instead generate heterogeneity in banks' performance and default outcomes. This is because banks cannot lend across islands. Finally, to generate aggregate fluctuations in macroeconomic and financial variables, we include aggregate total factor productivity (TFP) shocks, as well as firm- and island-risk shocks that affect the dispersion of firm- and island-idiosyncratic shocks, respectively.

The asymmetric distribution of returns on bank loans is a distinctive feature of our macro-banking model. Taking into account the limited upside but significant downside risks associated with bank lending allows our general equilibrium model to replicate key aspects of the relationship between firm and bank defaults and the macroeconomy, including striking non-linearities observed in the data. This turns out to be crucial for the the correct calibration of bank capital requirements.

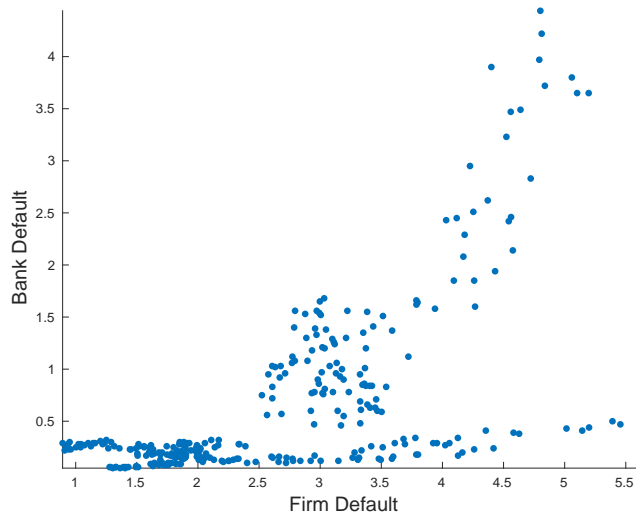
Bank and firm default rates feature positive correlation in the data. However, the overall positive correlation hides substantial shifts in the co-movement between the two default rates over time. Figure 1 displays a scatter plot of the average expected default frequencies (EDFs)

¹In Baron, Verner and Xiong (2019), bank equity losses are found to pre-date other crisis symptoms such as the loss of confidence among investors and to predict subsequent contractions in bank credit and aggregate economic activity.

²This is found when reviewing banking crises that occurred in the 20th century (e.g., BIS, 2004) and also when performing stress tests on European banks (e.g., EBA, 2018; ECB, 2018)

of firms and banks in the euro area (EA) over the period 1992-2016.³ Broadly speaking, one can identify three different regimes in the relationship between firm and bank default. In the most frequent regime, the default rates of both firms and banks are low. In another regime, the firm default rate is high but the bank default rate is modest. The last regime is one in which the default rates of both firms and banks are elevated. The same pattern can be observed in other countries, including the US.

Figure 1: Firm and Bank Default Rates



Scatter plot of Moody's cross-sectional average of expected default frequencies (EDFs) within one year for the 1992:M1 to 2016:M12 (monthly frequency) sample of firms (non-financial corporations) and banks in the EA; series in percent.

Importantly, as documented in Table 1, the growth rates of GDP in the EA, the US and a number of European countries are below normal when firm default is high but much lower when firm and bank defaults are both high.⁴ We will regard observations in this last regime as episodes of *twin default crises*.

These features of the data are at the core of our analysis. We explore the quantitative implications of our model using a high-order perturbation method and estimate its parameters using the generalized method of moments.⁵ We target a large set of unconditional moments in the macro, banking and financial EA data. The asymmetry in bank loan returns allows

³Each dot represents a monthly average of the corresponding probabilities of default over one year. The underlying EDFs are estimates provided by Moody's. See Appendix A for more details on the data.

⁴This is consistent with standard definitions of a systemic financial crisis and the large bank default rates and output losses associated with them (see, e.g., Laeven and Valencia, 2013). Empirically, we deem the EDFs of firms and banks as "high" when they are above their respective 90th percentile in the data.

⁵We rely on a third-order approximation to the underlying policy functions and use pruning to ensure that the model moments exist (see Andreasen, Fernandez-Villaverde and Rubio-Ramirez, 2017).

Table 1: Average Quarterly GDP Growth

	High Firm Def.	Twin Defaults
Euro Area	-0.0466	-0.5842
Germany	-0.2550	-0.6690
France	-0.0718	-0.6605
Italy	-0.0242	-0.5471
Netherlands	-0.5043	-2.1904
Belgium	-0.3645	-0.4051
US	-0.0781	-0.9790

Notes: First column refers to periods of high firm defaults and low bank defaults, whereas the second column uses periods of twin defaults. GDP growth rates (demeaned) are reported in quarterly rates. Sample: EA 1992Q1-2016Q4, US: 1940:Q1-2016:Q4.

our model to reproduce not only the unconditional correlation between firm and bank defaults but also the untargeted frequency and severity of twin default crises (highlighted in Figure 1 and Table 1).⁶ To corroborate our findings, we build a variant of our model that features lognormally distributed bank loan returns and produces bank equity returns and bank failure rates as in the classical Merton model of corporate default (Merton, 1974).⁷ We show that the model with such more symmetric distribution of returns does not reproduce the frequency and severity of twin default crises.

Additionally we show that, both in the data and in the model, the correlation between firm and bank default is larger at higher quantiles of bank default. The model also implies a strong negative link between GDP growth and bank default at lower quantiles of GDP growth, which is consistent with recent findings on the importance of financing conditions as a determinant of the economy’s downside risk (Adrian, Boyarchenko and Giannone, 2019). As before, a Merton-type variation of our model would follow along these dimensions.

After documenting the ability of our model to reproduce these empirical regularities, we investigate the factors that engender twin default crises in the model. The analysis reveals the importance of island-risk shocks: episodes of simultaneously high firm and bank defaults appear as a result of sequences of small negative island-risk shocks that become increasingly amplified as the probability of bank failure grows. Intuitively, the non-linearity in bank asset returns implies that, once banks have a high risk of failure, the marginal impact of additional credit

⁶The focus of our paper on bank lending to firms is consistent with the important role of EA banks in lending to non-financial corporations (NFCs) and the importance of NFC defaults as drivers of credit losses in Europe (EBA, 2018). Our model could be adapted to consider the case in which bank borrowers are households to finance house purchases with mortgages. However, such a setup would be less relevant in the EA since the recourse nature of most European mortgages makes the default rates of these loans very low even in bad times.

⁷The modelling of bank assets as equity claims is a standard feature of macroeconomic models with banks. The symmetric distribution of returns on bank assets (Merton-type formulation) is also common in double-decker models of financial frictions in which, as in our model, banks extend risky loans and feature endogenous bank failure risk (e.g., Clerc et al., 2015; Elenev, Landvoigt and Nieuwerburgh, 2018; Jermann, 2019; Gete and Melkadze, 2020).

losses on banks' solvency and the macroeconomy is much larger than in normal times. When firm and bank default are already high, an island-risk shock of limited size disproportionately increases bank default and leads to a large drop in output.⁸ The importance of the nonlinearities is ratified by the fact that, if solved to a first-order approximation, the model only generates twin default crises if hit by implausibly large realizations of the island-risk shock.

By allowing us to reproduce the frequency and severity of twin default crises, the non-linearity in bank asset returns is also important for the correct calibration of bank capital requirements. In the model, banks finance their lending with insured deposits and their owners' scarce equity, and, thus, bank leverage is determined by the regulatory capital requirement. When this requirement is higher (and hence bank leverage is lower), the effect of island-risk shocks on bank solvency gets significantly dampened since bank equity returns become less sensitive to credit losses. From a normative perspective, however, capital regulation involves a clear trade-off in our model. On the one hand, a higher capital requirement makes banks less vulnerable to credit losses, reducing the probability of twin default crises and the associated deadweight losses. On the other hand, a higher capital requirement also means a higher average cost of funding for banks and, hence, a higher cost of borrowing for firms. Hence preventing or reducing the severity of twin default crises comes along with a reduction in credit and economic activity in normal times.

The net effect of the bank capital requirement on economic activity and welfare depends on which of the two forces dominates. A 15 percent bank capital requirement maximizes the expected utility of the representative household in our model. This ratio is about 5 percentage points higher than the requirement prescribed by a variation of our model that resembles the most standard Merton-type framework used in the literature. This underlines the importance of modelling bank default risk in a structural way and capturing the frequency and severity of twin default crises for the model's policy implications.

Related literature The modelling of bank assets as portfolios of risky loans differentiate our approach from the standard financial accelerator literature, which abstracts from banks (e.g., Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997; Bernanke, Gertler and Gilchrist, 1999) and from subsequent developments where banks directly hold productive assets (e.g. Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011; Brunnermeier and Sannikov, 2016; Rogers, Piazzesi and Schneider, 2019). In such frameworks, abstracting from banks' role as credit intermediaries precludes investigating borrowers' default as a driver of bank equity declines and bank defaults, and thus is an important limitation for the analysis of bank capital requirements.

In other macroeconomic models with banks, the heterogeneity in bank asset returns comes from shocks that affect loan portfolio returns independently of the performance of the individual borrowers (e.g., Clerc et al., 2015; Elenev, Landvoigt and Nieuwerburgh, 2018; Jermann, 2019; Gete and Melkadze, 2020). Such a Merton-type formulation is quantitatively very tractable. However, as pointed out by Gornall and Strebulaev (2018) and Nagel and Purnanandam (2019)

⁸The strong non-linear effects of island-risk shocks is verified by computing generalized impulse response functions as in Andreasen, Fernandez-Villaverde and Rubio-Ramirez (2017).

in partial equilibrium frameworks, it prevents tracing large bank equity losses and bank default back to abnormally high borrower default rates and, thus, fails to capture the downward-skewness of loan portfolio returns. We contribute to this literature by showing that, in a general equilibrium formulation, the imperfect diversification of loan portfolios and the implied asymmetries in their return distributions help to reproduce the state-dependent correlations between firm default, bank default, and GDP growth observed in the data.

Our analysis is related to the growing literature that assesses the determinants of the optimal level of bank capital requirements, where as existing papers emphasize banks' role in liquidity provision (Van Den Heuvel, 2008; Begenau and Landvoigt, 2017; Begenau, 2019), systemic risk taking (Martinez-Miera and Suarez, 2014), imperfect competition (Corbae and D'Erasmus, 2019) and deadweight losses from bank default (Clerc et al., 2015; Mendicino et al., 2018; Elenev, Landvoigt and Nieuwerburgh, 2018; Corbae and D'Erasmus, 2019). We contribute to this literature by highlighting the importance of capturing rare and severe episodes of twin default crises for assessing the extent to which capital requirements can reduce the costs associated with bank failures and generating accurate prescriptions for the optimal level of capital requirements.

Our focus on non-linearities associated with the very nature of credit losses brings a different perspective to the literature that emphasizes the highly non-linear aspects of financial crises through a number of other potential sources: occasionally binding constraints (Mendoza, 2010; Benigno et al., 2013; Bianchi, 2016; Akinci and Queralto, 2016), asset price feedback loops (He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014), sovereign defaults (Bocola, 2016), liquidity problems (Bigio, 2015; De Fiore, Hoerova and Uhlig, 2018) and bank panics (Gertler, Kiyotaki and Prestipino, 2019).

Banking crises are commonly described as systemic events involving investors' loss of confidence in banks, bank failures, and the need for massive government rescue plans (e.g., Schularick and Taylor, 2012; Laeven and Valencia, 2013; Jorda, Schularick and Taylor, 2016). However, Baron, Verner and Xiong (2019) points out that large bank equity declines pre-date the occurrence of other crisis symptoms and predict subsequent contractions in bank credit and aggregate economic activity. Our paper explores the role of credit losses as a cause of large bank equity declines, revealing non-linearities with the potential to replicate the frequency and severity of twin default crises observed in the data.

2. The Model

The model economy is populated by a representative household (the 'household' hereafter) that provides consumption insurance to three types of members: workers, entrepreneurs and bankers. Workers supply labor to the production sector. Entrepreneurs and bankers supply equity to entrepreneurial firms and banks. Entrepreneurial firms pay for the input of production in advance. They use (inside) equity and bank loans to rent labor from workers and purchase capital used to produce the final good. Banks use (inside) equity and deposits from the household to provide loans to firms. Both entrepreneurial firms and banks operate under limited liability and live for one period. Deposits are insured by a deposit guarantee scheme (DGS).

2.1 The Household

In each period some workers become either entrepreneurs or bankers and the same measure of entrepreneurs and bankers retire and become workers again.⁹ At the beginning of each period entrepreneurs receive payments from last period entrepreneurial firms and stay active with probability θ_e or retire otherwise. Upon retirement entrepreneurs transfer any accumulated net worth to the household. At the same time, a mass $(1 - \theta_e)$ of workers become entrepreneurs. The new entrepreneurs receive aggregate endowments $\iota_{e,t}$ from the household. Similarly, at the beginning of each period bankers receive payments from banks that lived last period and stays active with probability θ_b or retire otherwise. Upon retirement they transfer any accumulated net worth to the household. At the same time, a mass $(1 - \theta_b)$ of workers become bankers. The new bankers receive aggregate endowments $\iota_{b,t}$ from the household and pay lump-sum taxes to the DGS so as to cover the losses on the insured deposits in banks that defaulted in the previous period.

The household chooses consumption, C_t , hours worked, H_t , and investments in insured bank deposits, D_t , to maximize the present discounted value of utility

$$\mathbb{E}_t \sum_{s=t}^{\infty} \left(\beta^s \log(C_s) - \frac{\varphi}{1+\eta} H_s^{1+\eta} \right)$$

subject to the budget constraint

$$C_t + D_t = w_t H_t + R_{d,t-1} D_{t-1} + \Upsilon_t + \Xi_t \quad (1)$$

where η is the inverse of the Frisch elasticity of labor supply, φ is the weight of labor supply in the utility of households, w_t is the real hourly wage and $R_{d,t-1}$ is the gross rate of deposits. In addition Υ_t are aggregate net transfers from entrepreneurs and bankers to households, and Ξ_t is profits from the capital producing firms that households own. We are interested in the symmetric equilibrium, hence we assume that the household invests its deposits symmetrically in all the (symmetric) banks in the economy. All the variables in the problem of the household represent aggregate variables.¹⁰

The household's problem yields the following FOCs with respect to consumption,

$$U_{C_t} = \lambda_t, \quad (2)$$

labor supply,

$$-U_{H_t} = w_t \lambda_t, \quad (3)$$

⁹This assumption guarantees that entrepreneurs and bankers do not accumulate enough net worth such that entrepreneurial firms and banks can be entirely financed with internal funds (see Gertler and Kiyotaki, 2010).

¹⁰As see below, there are both aggregated and idiosyncratic shocks. All the conditional expectations are taken with respect to the aggregated shocks.

and demand for the portfolio of insured deposits,

$$1 = \mathbb{E}_t (\Lambda_{t+1}) R_{d,t}, \quad (4)$$

where λ_t is the Lagrange multiplier of the budget constraint and $\Lambda_{t+1} \equiv \beta \frac{\lambda_{t+1}}{\lambda_t}$ is the household's stochastic discount factor.

2.2 Entrepreneurs, Entrepreneurial Firms, Bankers, and Banks

Entrepreneurs and bankers provide equity to entrepreneurial firms and banks, respectively. Both entrepreneurial firms and banks live for one period, operate under limited liability (they can default) and obtain external financing by issuing non-recourse non-contingent debt: bank loans and deposits, respectively. They both operate in a continuum of measure one of islands. In each island there is a continuum of measure one of entrepreneurial firms and a representative bank.

Entrepreneurial firms produce the final good and are subject to both firm- and island-idiosyncratic shocks, whose realizations affect their terminal net worth. Non-defaulted entrepreneurial firms and banks pay their terminal net worth to entrepreneurs and bankers, respectively. Each representative bank diversifies its lending across entrepreneurial firms in its island but not across islands; so its terminal net worth depends on the island-idiosyncratic shock.¹¹ Entrepreneurial firms and banks default when their terminal asset value is lower than their debt repayment obligations. In this case, their lenders take possession of their assets at a cost, equal to a proportion μ_f of assets.

2.2.1 Entrepreneurs

Let $V_{e,t}(n_{e,t}(i))$ be the value of being entrepreneur $i \in (0, 1)$ with net worth $n_{e,t}(i)$ at period t from the perspective of the household to which she belongs. Every period, entrepreneur i decides how much of her net worth, $n_{e,t}(i)$, to invest in a portfolio of equity of the continuum of entrepreneurial firms living in period t , $eq_{e,t}(i)$, and how much to pay back to the household in the form of dividends, $dv_{e,t}(i)$, to maximize

$$V_{e,t}(n_{e,t}(i)) = \max_{eq_{e,t}(i), dv_{e,t}(i)} \{dv_{e,t}(i) + \mathbb{E}_t \Lambda_{t+1} [(1 - \theta_e) n_{e,t+1}(i) + \theta_e V_{e,t+1}(n_{e,t+1}(i))]\} \quad (5)$$

¹¹We assume that banks cannot lend across islands. Imperfect diversification can be interpreted as the result of credit market segmentation and/or specialization. In Europe, banks operate largely within national borders and many specialize in lending to particular industries and sectors (Guiso, Sapienza and Zingales, 2004; De Bonis, Pozzolo and Stacchini, 2011; Behr and Schmidt, 2016; De Jonghe et al., 2019). Geographic and sectoral specialization is also a feature of US small and medium-sized banks (Deyoung et al., 2015; Regehr and Sengupta, 2016) and banks in Peru (Paravisini, Rappoport and Schnabl, 2015).

subject to

$$\begin{aligned} eq_{e,t}(i) + dv_{e,t}(i) &= n_{e,t}(i), \\ n_{e,t+1}(i) &= \theta_e \rho_{e,t+1} eq_{e,t}(i), \text{ and} \\ dv_{e,t}(i) &\geq 0. \end{aligned} \tag{6}$$

The portfolio invested in entrepreneurial equity in period t pays at the beginning of period $t + 1$. From the point of view of the entrepreneur maximizing the value function described above, $\rho_{e,t+1}$ is an exogenous random variable. In equilibrium, $\rho_{e,t+1}$ equals the gross rate of return of the portfolio of entrepreneurial equity. A detailed expression for the equilibrium value of $\rho_{e,t+1}$ is provided below. We are interested in the symmetric equilibrium; hence, we assume that each entrepreneur invests symmetrically in all the (symmetric) entrepreneurial firms of the economy. The constraint $dv_{e,t}(i) \geq 0$ reflects the fact that entrepreneurs can freely pay positive dividends back to the household but the household cannot provide further net worth to the entrepreneurs. All the variables in the problem of the entrepreneur represent per capita variables.¹²

As in Gertler and Kiyotaki (2010), we guess that the value function of being an entrepreneur is linear in her net worth

$$V_{e,t}(n_{e,t}(i)) = \nu_{e,t} n_{e,t}(i),$$

where $\nu_{e,t}$ is the shadow value of one unit of entrepreneurial net worth. Then we can write the Bellman Equation (5) as

$$\nu_{e,t} n_{e,t}(i) = \max_{eq_{e,t}(i), dv_{e,t}(i)} [dv_{e,t}(i) + \mathbb{E}_t \Lambda_{t+1} (1 - \theta_e + \theta_e \nu_{e,t+1}) n_{e,t+1}(i)]. \tag{7}$$

We guess and later verify that, in the proximity of the steady state, $\nu_{e,t} \geq 1$. From the envelope theorem $dv_{e,t}(i) = 0$ whenever $\nu_{e,t} > 1$. Under our parameter values $\nu_{e,t} = 1$ with a probability close to zero. As a result, we impose $dv_{e,t}(i) = 0$ such that the Bellman equation (7) reduces to

$$\nu_{e,t} = \mathbb{E}_t \Lambda_{t+1} (1 - \theta_e + \theta_e \nu_{e,t+1}) \rho_{e,t+1} \tag{8}$$

and the evolution of an entrepreneur's net worth is

$$n_{e,t+1}(i) = \rho_{e,t+1} n_{e,t}(i). \tag{9}$$

Intuitively, we focus on parameterizations for which the inability of the household to transfer as much net worth to entrepreneurs as it wants creates an aggregate shortage of entrepreneurial equity, which keeps the risk-adjusted expected return to entrepreneurial equity greater than

¹²The rate of return $\rho_{e,t+1}$ is a function of time t endogenous aggregate state variables and time $t + 1$ exogenous aggregate state variables (i.e., aggregate shocks). Hence, the value function $V_{e,t}$ is not only a function of the individual state variable, $n_{e,t}(i)$ but also of the aggregate state variables. For this reason the individual entrepreneur needs to guess rules to forecast the aggregate state variables. In equilibrium, those rules need to be coherent with behavior. To simplify notation, we only describe the dependence of the value function with respect to the individual state variable.

the risk-free rate. As a result, the shadow value of funds in the hands of entrepreneurs, $\nu_{e,t}$, is greater than unity (that is, if the household were able to transfer more funds to entrepreneurs, it would do so). The entrepreneur responds to this by retaining all her net worth and paying no dividends until she retires. Equation (8) allows us to define the entrepreneurs' stochastic discount factor for later use as

$$\Lambda_{e,t+1} = \Lambda_{t+1} (1 - \theta_e + \theta_e \nu_{e,t+1}). \quad (10)$$

We need an index i for individual entrepreneurs because each of them has been an entrepreneur for a different length of time and has therefore accumulated a different level of net worth. However, since individual entrepreneur value functions and policy functions are linear in own net worth, the distribution of entrepreneurial wealth is irrelevant for aggregate outcomes. As we will see below, aggregate investment and credit demand depend only on the aggregate wealth of the entrepreneurial sector.

2.2.2 Entrepreneurial firms

Entrepreneurial firms live for a single period and pay out their terminal net worth, if positive, to entrepreneurs. If their terminal net worth is negative, they default. Entrepreneurial firms living at period t produce the final good, y_{t+1} , using labor, h_t , and capital, k_t

$$y_{t+1} = A_{t+1} k_t^\alpha h_t^{1-\alpha}, \quad (11)$$

where A_t is an aggregate TFP shock. At the beginning of the period, entrepreneurial firms buy capital from capital producers at price q_t . In period t , they pay $w_t h_t + q_t k_t$ using equity from entrepreneurs, $EQ_{e,t}$, and loans from their island bank, $B_{f,t}$, with gross loan interest rate $R_{f,t}$. At the beginning of period $t + 1$, the final good is produced and sold to the households and the depreciated capital, $(1 - \delta) k_t$, is sold back to the capital producers at price q_{t+1} . Although entrepreneurial firms can only borrow from island-specific banks, final goods, labor, and capital can move freely across islands.

Since entrepreneurial firms have constant returns, the scale of an individual firm is indeterminate. Without loss of generality, we assume that there is a continuum of measure one of firms and each firm receives an identical amount of equity from entrepreneurs (which is also equal to the aggregate wealth of entrepreneurs). Also, as we will see below, we make the same assumption about individual banks. Given that loans are extended before any shock is realized, borrowers are identical, and hence, the loan amount received by each entrepreneurial firm will be the same and equal to aggregate bank loans.

The idiosyncratic shocks Entrepreneurial firms living at period t face a firm-idiosyncratic shock, ω_i , and an island-idiosyncratic shock, ω_j to the terminal value of their assets (output plus the market value of undepreciated capital). These shocks are independent and they are

realized at the same time. We assume that ω_i and ω_j are log-normally distributed

$$\log(\omega_\vartheta) \sim \mathcal{N}\left(-\frac{\sigma_{\omega_\vartheta,t+1}^2}{2}, \sigma_{\omega_\vartheta,t+1}^2\right),$$

for $\vartheta = i, j$. The standard deviation of the idiosyncratic shocks is time-varying. The law of motion of these two aggregate shocks will be introduced below. There will be persistent shocks, which will change the standard deviations of both the firm and the island-idiosyncratic shocks. These shocks to the risk facing firms and banks will be an important source of aggregate risk in the model and will be vital to generate defaults. We denote the CDFs of ω_i and ω_j by $F_{i,t+1}$ and $F_{j,t+1}$, respectively. The subscript $t+1$ captures the dependence of the CDFs on the aggregate risk shocks.

Terminal net worth of an entrepreneurial firm Entrepreneurial firms default if their terminal assets are insufficient to pay back their loan, $R_{f,t}B_{f,t}$, in full. In other words, the terminal net worth of an entrepreneurial firm experiencing shocks (ω_i, ω_j) at the beginning of period $t+1$ equals

$$\Pi_{i,j,t+1}(\omega_i, \omega_j) = \omega_i \omega_j [q_{t+1}(1-\delta)k_t + y_{t+1}] - R_{f,t}B_{f,t}, \quad (12)$$

and the entrepreneurial firm defaults on its loan when $\Pi_{i,j,t+1}(\omega_i, \omega_j) < 0$.

From Equation (12), it is useful to define the threshold value for the firm-idiosyncratic shock ω_i below which entrepreneurial firms in an island experiencing an island-idiosyncratic shock ω_j default

$$\bar{\omega}_{t+1}(\omega_j) = \frac{R_{f,t}B_{f,t}}{\omega_j(q_{t+1}(1-\delta)k_t + y_{t+1})}. \quad (13)$$

Note that a low realization of the island-idiosyncratic shock increases the threshold at which firms default on their bank. This, in turn, will ensure that the default rate on the portfolio of the bank on island j depends on the realization of its island-idiosyncratic shock. This is going to be a key source of default for the banks in our framework.

The payoff that an entrepreneur would receive from the equity invested across all entrepreneurial firms in the economy is

$$\Pi_{f,t+1} = \int_0^\infty \int_{\bar{\omega}_{t+1}(\omega_j)}^\infty \Pi_{i,j,t+1}(\omega_i, \omega_j) dF_{i,t+1}(\omega_i) dF_{j,t+1}(\omega_j) \quad (14)$$

Entrepreneurial firms choose capital, hours worked, the loan amount, and the gross loan rate to maximize

$$\max_{k_t, h_t, B_{f,t}, R_{f,t}} \mathbb{E}_t \Lambda_{e,t+1} \Pi_{f,t+1}$$

subject to

$$B_{f,t} + EQ_{e,t} = w_t h_t + q_t k_t, \text{ and} \quad (15)$$

$$\mathbb{E}_t \Lambda_{b,t+1} \Pi_{b,t+1} \geq \nu_{b,t} \phi B_{f,t}, \quad (16)$$

where Equation (15) is the entrepreneurial firm's budget constraint and Equation (16) is the bankers' participation constraint, which reflects the competitive pricing of the loans that banks are willing to offer for different leverage and productive choices by entrepreneurial firms. As explained and fully specified in detail below, $\Lambda_{b,t+1}$ is bankers' stochastic discount factor, $\Pi_{b,t+1}$ is the payoff that bankers would receive from equity invested across banks that provide the corresponding bank loans to all the firms operating in their island (that will be defined below), $\nu_{b,t}$ is the shadow value of bankers' net worth, and ϕ is the regulatory capital requirement that determines the fraction of the loan amount $B_{f,t}$ that must be funded with equity at period t .

The entrepreneurial firm's problem yields the following FOCs with respect to capital,

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial k_t} + \zeta_{f,t} q_t - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial k_t} = 0, \quad (17)$$

labor demand,

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial h_t} + \zeta_{f,t} w_t - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial h_t} = 0, \quad (18)$$

loans,

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial B_{f,t}} - \zeta_{f,t} - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial B_{f,t}} + \xi_{f,t} \nu_{b,t} \phi = 0, \quad (19)$$

and the gross loan rate,

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial R_{f,t}} - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial R_{f,t}} = 0, \quad (20)$$

where $\zeta_{f,t}$ is the Lagrange multiplier of the entrepreneurial firm's budget constraint, and $\xi_{f,t}$ is the Lagrange multiplier of the bankers' participation constraint.

Finally, the gross rate of return on the portfolio of equity of an entrepreneur that symmetrically invests in all entrepreneurial firms is

$$\rho_{e,t+1} = \frac{\Pi_{f,t+1}}{EQ_{e,t}}. \quad (21)$$

2.2.3 Bankers

Let $V_{b,t}(n_{b,t}(j))$ be the value of being banker $j \in (0, 1)$ with net worth $n_{b,t}(j)$ at period t from the perspective of the household to which she belongs. Every period, banker j decides how much of her net worth, $n_{b,t}(j)$, to invest in a portfolio of equity of the continuum of banks, $eq_{b,t}(j)$, and how much to pay back to the household in the form of dividends, $dv_{b,t}(j)$, to

maximize

$$V_{b,t}(n_{b,t}(j)) = \max_{eq_{b,t}(j), dv_{b,t}(j)} \{dv_{b,t}(j) + \mathbb{E}_t \Lambda_{t+1} [(1 - \theta_b) n_{b,t+1}(j) + \theta_b V_{b,t+1}(n_{b,t+1}(j))]\} \quad (22)$$

subject to

$$\begin{aligned} eq_{b,t}(j) + dv_{b,t}(j) &= n_{b,t}(j), \\ n_{b,t+1}(j) &= \rho_{b,t+1} eq_{b,t}(j), \text{ and} \\ dv_{b,t}(j) &\geq 0. \end{aligned} \quad (23)$$

The portfolio invested in bank equity in period t pays at the beginning of period $t + 1$. From the point of view of the banker maximizing the value function described above, $\rho_{b,t+1}$ is an exogenous random variable. In equilibrium, $\rho_{b,t+1}$ equals the gross rate of return of the portfolio of entrepreneurial equity. A detailed expression for the equilibrium value of $\rho_{b,t+1}$ is provided below. We are interested in the symmetric equilibrium; hence, we assume that each banker invests symmetrically in all the (symmetric) banks of the economy. The constraint $dv_{b,t}(i) \geq 0$ reflects the fact that bankers can freely pay positive dividends back to the household but the household cannot provide further net worth to the bankers. All the variables in the problem of the banker represent per capita variables.

As in the case of entrepreneurs, we guess that bankers' value function is linear

$$V_{b,t}(n_{b,t}(j)) = \nu_{b,t} n_{b,t}(j),$$

where $\nu_{b,t}$ is the shadow value of a unit of banker net worth. The Bellman equation in (22) becomes

$$\nu_{b,t} n_{b,t}(j) = \max_{eq_{b,t}(j), dv_{b,t}(j)} [dv_{b,t}(j) + \mathbb{E}_t \Lambda_{t+1} (1 - \theta_b + \theta_b \nu_{b,t+1}) n_{b,t+1}(j)]. \quad (24)$$

As before, we guess and later verify that, in the proximity of the steady state, $\nu_{b,t} \geq 1$. From the envelope theorem $dv_{b,t}(j) = 0$ whenever $\nu_{b,t} > 1$. Under our parameter values, $\nu_{b,t} = 1$ with probability close to zero. Hence we impose $dv_{b,t}(j) = 0$ to have

$$\nu_{b,t} = \mathbb{E}_t \Lambda_{t+1} (1 - \theta_b + \theta_b \nu_{b,t+1}) \rho_{b,t+1} \quad (25)$$

and the evolution of a banker's net worth is

$$n_{b,t+1}(j) = \rho_{b,t+1} n_{e,t}(j). \quad (26)$$

Again, just as in the case of entrepreneurs, the inability of the household to transfer as much net worth to bankers as it wants creates an aggregate shortage of bank equity, which keeps the risk-adjusted expected return to bank equity greater than the risk-free rate. As a result, the shadow value of funds in the hands of bankers, $\nu_{b,t}$, is greater than unity. Bankers respond to this by retaining all of their net worth and paying no dividends until they retire.

Equation (24) allows us to define bankers' stochastic discount factor as

$$\Lambda_{b,t+1} = \Lambda_{t+1} (1 - \theta_b + \theta_b \nu_{b,t+1}). \quad (27)$$

As was the case with entrepreneurs, we need an index j for individual bankers because each of them has been a banker for a different length of time and has therefore accumulated a different level of net worth. However, since individual banker value functions and policy functions are linear in own net worth, the distribution of bankers' wealth is irrelevant for aggregate outcomes. As we will see below, aggregate investment and credit demand depend only on the aggregate wealth of the banking sector.

2.2.4 Banks

As entrepreneurial firms, banks live for a single period and pay out their terminal net worth, if positive, to bankers. If their terminal net worth is negative they default. At period t banks use equity $EQ_{b,t}$ from bankers and deposits d_t from households in order to provide loans $b_{f,t}$ to entrepreneurial firms operating in their island. Hence, they face the following balance-sheet constraint

$$b_{f,t} = EQ_{b,t} + d_t. \quad (28)$$

We assume that banks invest symmetrically in all the (symmetric) entrepreneurial firms in their island. Banks also face the following regulatory capital constraint

$$EQ_{b,t} \geq \phi b_{f,t}$$

where ϕ is the capital requirement on loans. As explained below, in equilibrium, the capital constraint is always binding because funding the bank with deposits is always cheaper than funding it with equity, hence

$$EQ_{b,t} = \phi b_{f,t} \quad (29)$$

Gross rate of return on assets of the bank in island j Banks operate under constant returns to scale; hence, their individual loan supply is perfectly elastic as long as the loan rate and the decisions of the borrowing firm satisfy bankers' participation constraint. This constraint plays the role of the zero profit condition in standard production theory and it stipulates that the loan must guarantee the bankers the equilibrium expected rate of return on banker equity. Because the bank is a levered institution with the possibility to default at time $t + 1$, the expected equity return also includes the value of limited liability, which allows shareholders to avoid negative returns. In the remainder of this section, we cover the steps needed to derive a detailed expression for bankers' payoffs.

We start by computing the terminal asset value of the representative bank living in island j . At the beginning of period $t + 1$, firm- and island-idiosyncratic shocks, ω_i and ω_j , hit the entrepreneurial firms living at period t . As derived in Equation (13), conditional on the island-idiosyncratic shock ω_j , an entrepreneurial firm pays back its loan in full when it experiences a firm-idiosyncratic shock no lower than $\bar{\omega}_{t+1}(\omega_j)$. Entrepreneurial firms with firm-idiosyncratic

shocks smaller than $\bar{\omega}_{t+1}(\omega_j)$ default on their loans and the bank only recovers a fraction $1 - \mu_f$ of the terminal value of the entrepreneurial firm's assets. So the total repayment from defaulting entrepreneurial firms in an island experiencing an island-idiosyncratic shock ω_j is

$$(1 - \mu_f) \omega_j [q_{t+1} (1 - \delta) k_t + y_{t+1}] \int_0^{\bar{\omega}_{t+1}(\omega_j)} \omega_i dF_{i,t+1}(\omega_i),$$

In turn, the total repayment from non-defaulting entrepreneurial firms in an island experiencing an island-idiosyncratic shock ω_j is

$$R_{f,t} b_{f,t} \int_{\bar{\omega}_{t+1}(\omega_j)}^{\infty} dF_{i,t+1}(\omega_i).$$

So the gross rate of return on assets of the bank in island j is

$$\tilde{R}_{f,t+1}(\omega_j) = \frac{(1 - \mu_f) \omega_j [q_{t+1} (1 - \delta) k_t + y_{t+1}]}{b_{f,t}} \int_0^{\bar{\omega}_{t+1}(\omega_j)} \omega_i dF_{i,t+1}(\omega_i) + R_{f,t} \int_{\bar{\omega}_{t+1}(\omega_j)}^{\infty} dF_{i,t+1}(\omega_i). \quad (30)$$

Following Bernanke, Gertler and Gilchrist (1999) it is useful to define

$$\Gamma_{i,t+1}(\bar{\omega}_{t+1}(\omega_j)) = \int_0^{\bar{\omega}_{t+1}(\omega_j)} \omega_i dF_{i,t+1}(\omega_i) + \bar{\omega}_{t+1}(\omega_j) \int_{\bar{\omega}_{t+1}(\omega_j)}^{\infty} dF_{i,t+1}(\omega_i)$$

and

$$G_{i,t+1}(\bar{\omega}_{t+1}(\omega_j)) = \int_0^{\bar{\omega}_{t+1}(\omega_j)} \omega_i dF_{i,t+1}(\omega_i).$$

Then Equation (30) can be rewritten more compactly as

$$\tilde{R}_{f,t+1}(\omega_j) = [\Gamma_{i,t+1}(\bar{\omega}_{t+1}(\omega_j)) - \mu_f G_{i,t+1}(\bar{\omega}_{t+1}(\omega_j))] \frac{\omega_j [q_{t+1} (1 - \delta) k_t + y_{t+1}]}{b_{f,t}}. \quad (31)$$

Yet again, it is worth noting that the return of the bank in island j is a function of the island-idiosyncratic shock ω_j . The mechanism works through the impact of ω_j on the default rate of entrepreneurial firms on island j as well as on the recovery value of the assets of defaulted entrepreneurial firms.

Equation (31) shows that the bank's loan portfolio return is a non-linear function of ω_j and hence is not log-normal even if ω_j itself is. Thus our model departs from the standard Merton approach (Merton, 1974) where it is assumed that bank asset returns follow a log-normal process.¹³ Our choice complicates the model solution with respect to the Merton's

¹³Island-idiosyncratic shocks play at bank level the same role as the so-called "single risk factor" in the partial equilibrium model of Vasicek (2002). See also (Gordy, 2003). In contrast to direct implementations of such framework (e.g., Repullo and Suarez, 2013) our continuum of islands (and, hence, risk factors) allows us to capture ex post heterogeneity in bank performance.

approach. More details will be provided in Section 3.1.

We will analyze extensively the implications of this feature of our model in Section 4.1. Most importantly, our approach captures the limited upside of loan payoffs together with the significant downside risks posed by borrower defaults. As we will see our model captures important features of the data, including the conditional and unconditional positive correlation between firm and bank defaults and the frequency and severity of twin default crises. In contrast, the Merton-type version of our model fails to reproduce them. This implies that the Merton-type model generates lower prescriptions for the optimal level of capital requirements than our model.

Terminal net worth of a bank Banks default on their deposits if their revenue at the beginning of period $t + 1$, $\tilde{R}_{f,t+1}(\omega_j) b_{f,t}$, is insufficient to pay the promised repayment, $R_{d,t} d_t$, in full. In other words, the terminal net worth of a bank experiencing shock ω_j at the beginning of period $t + 1$ equals

$$\Pi_{b,t+1}(\omega_j) = \tilde{R}_{f,t+1}(\omega_j) b_{f,t} - R_{d,t} d_t \quad (32)$$

and the bank defaults on its deposits when $\Pi_{b,t+1}(\omega_j) < 0$.

Gross rate of return on the portfolio of bank equity From Equation (32), it is useful to define a threshold value for the island-specific shock ω_j below which the bank in island j defaults. This is implicitly done in the next equation

$$\tilde{R}_{f,t+1}(\bar{\omega}_{j,t+1}) b_{f,t} - R_{d,t} d_t = 0. \quad (33)$$

Equation (33) implies that banks' failure rate at the beginning of period $t + 1$ is $F_{j,t+1}(\bar{\omega}_{j,t+1})$. Thus, the aggregate payoffs of a portfolio containing the equity of all banks are then

$$\Pi_{b,t+1} = \int_{\bar{\omega}_{j,t+1}}^{\infty} \tilde{R}_{f,t+1}(\omega_j) b_{f,t} dF_{j,t+1}(\omega_j) - R_{d,t} d_t (1 - F_{j,t+1}(\bar{\omega}_{j,t+1})) \quad (34)$$

and the gross rate of return on the portfolio of equity of a banker that symmetrically invests in all banks is

$$\rho_{b,t+1} = \frac{\Pi_{b,t+1}}{EQ_{b,t}} = \frac{\Pi_{b,t+1}}{\phi b_{f,t}}. \quad (35)$$

2.3 Capital Production

Capital producers combine the final good, I_t , with the last period capital goods, K_{t-1} , in order to produce new capital goods that competitively sell to entrepreneurial firms at price q_t . The representative capital-producing firm is owned by the household and maximizes the expected

discounted value of profits

$$\max_{I_{t+j}} \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left[q_{t+j} S \left(\frac{I_{t+j}}{K_{t+j-1}} \right) K_{t+j-1} - I_{t+j} \right],$$

where $S \left(\frac{I_{t+j}}{K_{t+j-1}} \right) K_{t+j-1}$ gives the units of new capital produced by investing I_{t+j} and using K_{t+j-1} . The increasing and concave function $S(\cdot)$ captures the existence of adjustment costs, which we specify as in Jermann (1998)

$$S \left(\frac{I_{k,t}}{K_{t-1}} \right) = \frac{a_{k,1}}{1 - \frac{1}{\psi_k}} \left(\frac{I_t}{K_{t-1}} \right)^{1 - \frac{1}{\psi_k}} + a_{k,2},$$

where $a_{k,1}$ and $a_{k,2}$ are chosen to guarantee that in the steady state the investment-to-capital ratio is equal to the depreciation rate and $S'(I_t/K_{t-1})$ equals one.

The FOC of this problem is

$$q_t = \left[S' \left(\frac{I_t}{K_{t-1}} \right) \right]^{-1}. \quad (36)$$

2.4 Deposit Guarantee Scheme

The DGS has to balance its budget period-by-period. Thus, the lump-sum taxes have to cover the cost of guaranteeing deposits every period. Hence, the total lump sum tax T_t imposed on bankers to balance the agency's budget is

$$T_t = F_{j,t}(\bar{\omega}_{j,t}) R_{d,t-1} d_{t-1} - (1 - \mu_j) \int_0^{\bar{\omega}_{j,t}} \tilde{R}_{f,t}(\omega_j) b_{f,t} dF_{j,t}(\omega_j), \quad (37)$$

where the first part is the deposits lost because of bank defaults and the second part is the total repayment from defaulting banks. We can write the above expression as

$$T_t = \Omega_t d_{t-1}, \quad (38)$$

where

$$\Omega_t = F_{j,t}(\bar{\omega}_{j,t}) R_{d,t-1} - \frac{1 - \mu_j}{1 - \phi} \int_0^{\bar{\omega}_{j,t}} \tilde{R}_{f,t}(\omega_j) dF_{j,t}(\omega_j) \quad (39)$$

is the expected loss rate per unit of deposits, which has been written using the fact that $d_t = (1 - \phi) b_{f,t}$. As mentioned, the lump-sum taxes to the DGS are paid by the new bankers.

2.5 Aggregate Shocks

We assume the following AR(1) law of motion for the TFP shock

$$\log(A_{t+1}) = \rho_A \log(A_t) + \sigma_A \epsilon_{A,t+1}, \quad (40)$$

where $\epsilon_{A,t+1}$ is normally distributed with mean zero and variance one.

The standard deviation of the distribution of each idiosyncratic shock is time-varying and evolves as an AR(1) process

$$\log\left(\frac{\sigma_{\omega_{\vartheta},t+1}}{\bar{\sigma}_{\omega_{\vartheta}}}\right) = \rho_{\sigma_i} \log\left(\frac{\sigma_{\omega_{\vartheta},t}}{\bar{\sigma}_{\omega_{\vartheta}}}\right) + \sigma_{\vartheta} \epsilon_{\omega_{\vartheta},t+1} \quad (41)$$

for $\vartheta = i, j$, where $\epsilon_{\omega_{\vartheta},t+1}$ is normally distributed with mean zero and variance one.¹⁴ We will call these shocks firm- and island-risk shocks.

2.6 Aggregation, Market Clearing and Equilibrium Conditions

Aggregation and market clearing conditions are described in Appendix B, whereas the exhaustive list of equilibrium conditions of the model is reported in Appendix C. It is easy to see that the state variables of the model are those included in the following vector

$$\mathbf{w}_t = (D_t, K_t, H_t, N_{e,t}, N_{b,t}, q_t, w_t, R_{f,t}, R_{d,t}, A_{t-1}, \sigma_{\omega_j,t-1}, \sigma_{\omega_i,t-1}). \quad (42)$$

3. Solution, Estimation and Quantitative Results

In this section we present the solution of the model, the estimation of the parameters and the quantitative results.

3.1 Solving the Model

We solve the system of stochastic difference equations implied by the equilibrium conditions using a pruned state-space system for the third-order approximation around the steady state as defined in Andreasen, Fernandez-Villaverde and Rubio-Ramirez (2017). This approach eliminates explosive sample paths and greatly facilitates inference using any method of moments approach. In particular, it ensures the existence of unconditional moments so that we can estimate the parameters of the model by applying the generalized method of moments (GMM).

In order to use perturbation methods to approximate the solution to the model, we need to compute the aggregate loan returns that banks generate conditional on not defaulting, defined

¹⁴This specification is similar to the one adopted in Christiano, Motto and Rostagno (2014).

here as $R_{p,t+1}$. These returns are the first term in Equation (34), namely,

$$R_{p,t+1} \equiv \int_{\bar{\omega}_{j,t+1}}^{\infty} \tilde{R}_{f,t+1}(\omega_j) dF_{j,t+1}(\omega_j). \quad (43)$$

As we mentioned before, the bank's loan return $\tilde{R}_{f,t+1}(\omega_j)$ is not log-normally distributed because ω_j enters non-linearly in its definition. This introduces a complication: the integral in Equation (43) as well as its derivatives cannot be written as an explicit function of the state variables. We overcome this challenge by (i) splitting this integral into the sum of integrals taken over smaller intervals, and (ii) computing a series of quadratic Taylor approximations of $\tilde{R}_{f,t+1}(\omega_j)$ around a mid-point of each interval. Because the powers of log-normally distributed variables are themselves log-normally distributed, the quadratic approximation to the bank profit function is itself approximately log-normally distributed and the expected profits as well as its derivatives can be computed as explicit functions of the state variables. The method is tractable and highly accurate. More details are provided in Appendix E.

3.2 Model Estimation

The estimation of the model is a two-step procedure. First, some parameters are calibrated prior to the estimation procedure, since there is consensus about their value in the literature. Second, we estimate the rest of the parameters using quarterly euro area (EA) data between 1992:Q1 and 2016:Q4.

Table 2: Estimated Parameters

Par.	Description	Value	Par.	Description	Value
χ_b	Bankers' endowment	0.6520	χ_e	Entrepreneurs' endowment	0.5435
$\bar{\sigma}_{\omega_i}$	Mean firm-risk shock	0.3447	$\bar{\sigma}_{\omega_j}$	Mean island-risk shock	0.2625
σ_A	Std TFP shock	0.0044	ρ_A	Persistence TFP shock	0.9804
σ_i	Std firm-risk shock	0.0729	ρ_{σ_i}	Persistence firm-risk shock	0.9141
σ_j	Std island-risk shock	0.0936	ρ_{σ_j}	Persistence island-risk shock	0.7539
ψ_k	Capital adjustment cost	1.9942			

Notes: The reader should note that σ_i is not the standard deviation of firm-risk shock, which is $\frac{\sigma_i}{\sqrt{1-\rho_{\sigma_i}^2}}$. The

same applies for the standard deviation of the island-risk shock.

Since we use quarterly data, the discount factor of the households, β , is set to 0.995. We set the Frisch elasticity of labor supply, η , equal to one, the value of capital depreciation, δ , to 0.025, and the capital-share parameter of the production function, α , to 0.30. The cost parameters μ_f and μ_j are set a common value equal to 0.30.¹⁵ The capital requirement level,

¹⁵Similar values are used, among others, in Carlstrom and Fuerst (1997), who refer to the evidence in Alderson and Betker (1995), where estimated liquidation costs are as high as 36 percent of asset value. Among non-listed bank-dependent firms, these costs can be expected to be larger than those among the highly levered, publicly

ϕ , is set to be 0.08 which was the regulatory minimum in the Basel II regime. We set both θ_e and θ_b to 0.975, implying that bankers and entrepreneurs remain active for ten years on average. Finally, the labor utility parameter, φ , which only affects the scale of the economy, is normalized to one.

In the second step, we estimate the parameters summarized in Table 2 by targeting the unconditional moments described in Table 3.¹⁶ Appendix A describes the data counterpart of these model variables. As evidenced in Table 3, our model matches the targets reasonably well. We target a number of macroeconomic, financial and banking moments. We target the standard deviations of GDP, investment and consumption growth, the mean ratio of corporate loans to GDP (B_t/GDP_t in the model) along with the standard deviation of loan growth, the mean and standard deviation of the loan spread ($R_{f,t} - R_t$ in the model), and the mean and standard deviation of ROE of banks ($\rho_{b,t}$ in the model). Additionally, our moment targets the mean and standard deviation of the conditional expectation of entrepreneurial firm defaults.

$$DF_t = \mathbb{E}_t \left(\int_0^\infty \int_0^{\bar{\omega}_{t+1}(\omega_j)} dF_{i,t+1}(\omega_i) dF_{j,t+1}(\omega_j) \right),$$

and the conditional expectation of the bank default rate

$$DB_t = \mathbb{E}_t \left(\int_0^{\bar{\omega}_{j,t+1}} dF_{j,t+1}(\omega_j) \right).$$

Importantly, our model is able to replicate an important feature of the relationship between firm and bank default. As Table 3 shows, the unconditional correlation between firm and bank default in the data is both large and positive, and our model reproduces it (0.64 in the data versus 0.76 in the model).¹⁷ Matching this correlation turns out to be of first-order importance when drawing conclusions about optimal bank capital requirements.

3.3 Model Validation

The interaction between firm and bank defaults and their implications for GDP growth are crucial aspects of the data our model should be able to capture. As shown in Table 3, the model is able to match the unconditional moments related to defaults and macroeconomic variables targeted in the calibration. In this section we perform model validation by comparing the model's implications for important untargeted moments of firm and bank defaults and GDP growth conditional on high and low default realizations. This ensures that the model is able to characterize the behavior of the economy both in normal and crisis times.

traded US corporations studied in Andrade and Kaplan (1998), where estimated financial distress costs fall in the range of 10 percent to 23 percent. Our choice of 30 percent is consistent with the large foreclosure, reorganization and liquidation costs found in some of the countries analyzed by Djankov et al. (2008).

¹⁶To avoid the impact of the resource costs of default on the measurement of output, we define GDP_t as $GDP_t = C_t + I_t$.

¹⁷A similar degree of correlation can be observed in the US data.

Table 3: Targeted Moments: Baseline Model

Variable	Data	Model	Variable	Data	Model
Std GDP growth	0.6877	0.6217	Std Cons. growth	0.5617	0.4912
Mean Loans/GDP	2.442	2.6386	Std Loan growth	1.1965	0.5936
Mean Loan spread	1.2443	1.0058	Std Loan spread	0.6828	0.7535
Mean Firm default	2.6469	2.2497	Std Firm default	1.0989	2.4384
Mean Bank default	0.6646	0.5860	Std Bank default	0.8438	1.2320
Mean ROE banks	6.4154	6.4652	Std ROE banks	4.1273	3.7971
Corr (FD & BD)	0.6421	0.7648	Std Inv. growth	1.3908	2.0106

Notes: Interest rates, equity returns, default rates, and spreads are reported in annualized percentage points. The standard deviations (Std) of GDP growth, Investment (Inv), and Loan growth are reported in quarterly percentage points.

3.3.1 Defaults and economic performance in the data

As anticipated in the introduction, Figure 1 is a scatter plot of the Moody’s EDFs of firms and banks in the EA over the period 1992:M1 to 2016:M12. The x-axis displays firms’ EDF and the y-axis displays bank’s EDFs.¹⁸

The two default series are positively correlated as successfully matched in the estimation. However, the data highlight three different regimes. In the most frequent regime, the default rates of both firms and banks are low. We think of this as “normal times” or a low default regime. In another regime, the firm default rate is high but the bank default rate is modest. We think of this as a high firm default regime. The last regime is one in which the default rates of both firms and banks are elevated: the twin defaults regime.¹⁹ Following the standard definitions of a systemic banking crisis (see, e.g., Laeven and Valencia, 2013), we think of the twin default s regime as corresponding to crisis times.

Our working definition of high defaults is when the EDF of the corresponding borrower is above the 90th percentile of observations. We therefore define twin defaults s as the simultaneous occurrence of firm and bank EDFs above their respective 90th percentiles. high firm default periods are those where firm EDF is above the 90th percentile and bank EDF is below the 90th percentile. In the low default regime, both EDFs are below the 90th percentile.

Another way of representing the state-contingent relationship between firm and bank default risk is through quantile regressions of the following form:

$$\text{BankDef}_t(\tau) = \zeta_\tau \text{FirmDef}_t, \quad (44)$$

where FirmDef_t is firm EDF and BankDef_t is bank EDF. We use quarterly data from 1992:Q1

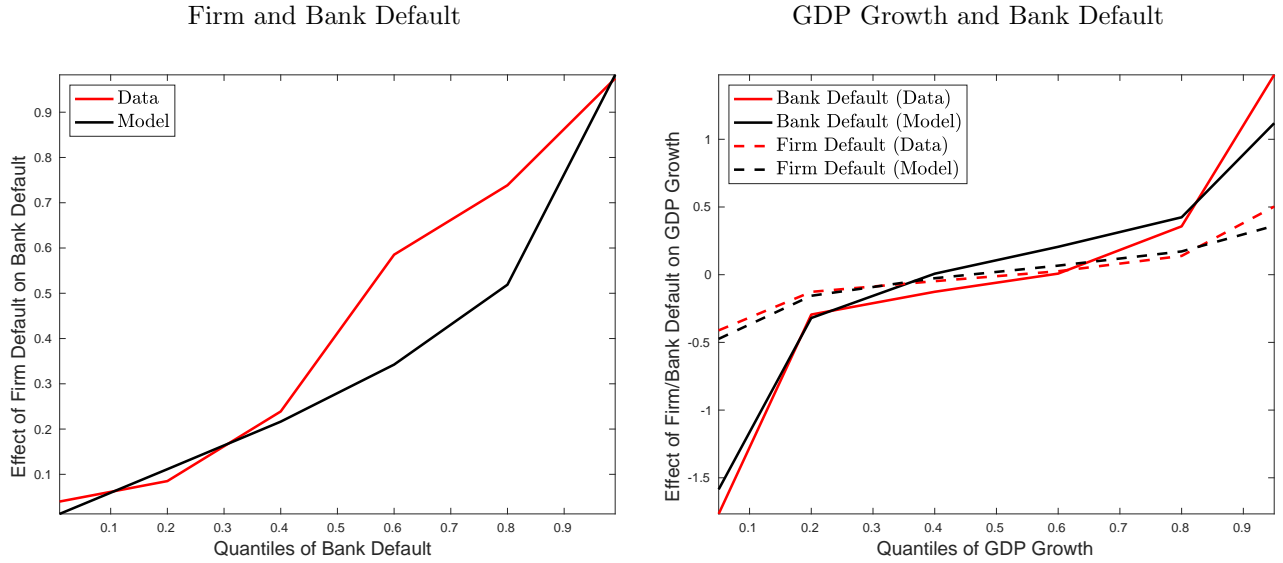
¹⁸Each dot represents a monthly average of the corresponding EDF over one year. The underlying EDFs are provided by Moody’s. See Appendix A for more details on the data.

¹⁹By definition, there is a fourth regime in the data and in the model, where the bank default rate is high but firm default is modest. We do not comment on that regime because the default rate of firms is similar to the one in the twin default s regime for both the data and the model.

to 2016:Q2.

The left panel of Figure 2 (red line) plots the quantile regression coefficients ζ_τ in Equation (44). The non-linearity in the relationship between the two defaults is clearly visible and highly statistically significant. At higher levels of bank default risk, the coefficient obtained by regressing bank on firm defaults is higher. Because the variance of firm and bank defaults is roughly constant across bank default quantiles, the quantile regression coefficients indicate that the correlation between firm and bank default is state dependent and increasing with the bank default rate.

Figure 2: Quantile Regression: Data vs Model



Notes: The left panel of this figure presents coefficients ζ_τ from the quantile regression in Equation (44). The right panel of this figure presents coefficients β_τ from the quantile regression in Equation (45). Both equations are estimated on EA data (1992-2016) and on simulated data from the baseline model.

The other feature of the data we are interested in is the relationship between aggregate economic activity and firm and bank defaults, respectively. A simple way to analyze this relationship is to look at GDP growth during the different firm and bank default regimes discussed above.

Table 1 in the introduction shows for the EA, the US and a number of European countries the average GDP growth rate within the high firm default regime (first column) and twin defaults regime (second column). Average growth rates have been demeaned using the unconditional mean of GDP growth for each country. For all countries, GDP growth is much lower in the twin defaults regime.

Once again, quantile regressions offer another way to represent the non-linear relationship between defaults and macroeconomic outcomes. We investigate the relationship between firm and bank defaults and GDP growth using quantile regressions of the following form

$$\Delta y_t(\tau) = \beta_\tau \text{Def}_{t-1} + \gamma_\tau \Delta y_{t-1}, \quad (45)$$

where Def_{t-1} can either be FirmDef_{t-1} or BankDef_{t-1} and Δy_t represents GDP growth. As before, we use the quarterly data from 1992:Q1 to 2016:Q2.

This quantile regression exercise is similar in spirit to the exercise performed in Adrian, Boyarchenko and Giannone (2019) who run a quantile regression of GDP growth on lagged GDP growth and an index of financial conditions. It is natural for us to use firm and bank defaults as these are the main proxies for financial conditions in our framework. This is why we regress GDP growth on the lagged GDP growth and the lagged level of default (Def_{t-1}) of either firms or banks. The right panel of Figure 2 plots the coefficients for either firm (the dashed red lines) or bank (the solid red lines) default in the corresponding quantile regressions estimated on EA data. The results highlight three key features of the non-linear relationship between defaults and real activity. First, the link between both defaults and economic growth is weak for GDP growth quantiles close to the median. This suggests that defaults (whether bank or firm) have only a weak correlation with GDP growth in normal times.

Second, the negative relationship between bank default and GDP growth becomes quantitatively more negative for the bottom quantiles. Increases in bank defaults have a larger (negative) impact on GDP growth when the economy is already in a recession (i.e. at the bottom quantile for GDP growth).

Third, the above relationship does not hold for firm default. In sharp contrast to the non-linear pattern between bank default and economic activity, the impact of corporate defaults on GDP growth is small and flat across all GDP growth quantiles. Thus, Figure 2 (right panel) clearly shows that it is the risk of bank failures that is driving the deterioration in macroeconomic performance during periods of twin defaults identified in Table 1. This link between bank default and economic performance during twin default crises will explain the importance of capital regulation in mitigating this downside risk to the real economy.

3.3.2 Model implications

Section 3.3.1 established a number of important facts regarding the state-dependent co-movement between default rates and GDP growth. We learned that the marginal impact of corporate failures on bank solvency is stronger when banks are weaker. We saw that twin defaults are associated with deeper recessions. Finally, our results established that the correlation of bank (but not firm) defaults with real activity grows in recessions. These important empirical regularities were not targeted in estimation.

In the previous section, we used a 90th percentile-based criterion to identify the low default, high firm default and twin defaults regimes in the EA data. Here, we use DF_t and DB_t as the model counterparts for firm and bank EDFs, respectively, and we employ the same criterion to split the model-simulated time series into the three regimes.

Table 4 compares the model and data averages for firm default, bank default and GDP growth within each regime for the EA data and the model-simulated data.²⁰ The model does a

²⁰Table 4 also reports the results for the Merton variant of our model. We will comment on those later.

Table 4: Three Default Regimes

	Frequency	GDP growth	Bank default	Firm default
Low Default Regime				
Data	86.0%	0.0923	0.4346	2.3480
Baseline Model	85.9%	0.0588	0.2388	1.5154
Merton-type Model	82.1%	0.0522	0.3351	1.7118
1st Order App.	87.3%	0.0318	0.3130	1.7113
Higher Cap. Req.	91.5%	0.0293	0.0453	1.5406
High Firm Default Regime				
Data	4.0%	-0.0466	0.4033	4.8500
Baseline Model	4.0%	-0.1344	0.9270	6.8463
Merton-type Model	7.5%	-0.1887	0.3980	7.4093
1st Order App.	1.0%	-0.1556	1.4211	5.5756
Higher Cap. Req.	8.3%	-0.3079	0.3387	7.693
Twin Defaults Regime				
Data	7.0%	-0.8189	3.0224	4.6076
Baseline Model	5.9%	-0.5737	4.0225	8.6266
Merton-type Model	1.1%	-0.5399	3.2624	7.7483
1st Order App.	4.1%	-0.2735	2.4816	6.0247
Higher Cap. Req.	0.1%	-0.8990	2.2445	11.833

Notes: This table compares the model and data averages for firm default, bank default and GDP growth within three default regimes for the EA data and the simulated data from different models. The baseline model corresponds to a capital requirement set to 8 percent ($\phi = 0.08$) and solved with third-order perturbation. Merton-type Model corresponds to the model in which the Merton-type specification of bank asset returns is adopted. 1st Order App. corresponds to the model solved with first-order perturbation methods. Higher Cap. Req. corresponds to the model with a capital requirement set to 15 percent ($\phi = 0.15$). Twin Defaults episodes are defined as the simultaneous occurrence of firm and bank default above their respective 90th percentiles. High Firm Default are episodes with firm default above the 90th percentile and bank default below the 90th percentile. In Low Default episodes, both bank and firm default are below the 90th percentile. The default thresholds used to define the three regimes in the Merton-type model and the 1st Order App. model are the ones determined by the baseline model. Model results are based on 1,000,000 simulations. GDP growth is demeaned.

good job of reproducing these untargeted conditional moments. First, it reproduces remarkably well the frequency of the three default regimes. Second, the model reproduces the same ranking observed in the data in terms of the performance of GDP growth in the three regimes. The twin default regime features by far the worst GDP growth realizations, whereas the high firm default regime features a relatively mild recession despite the fact that firms' default rates are very similar.

In the previous section we also used quantile regressions to characterize the non-linear relationships among the two default series and GDP growth. The black lines in Figure 2 show that our model can replicate both quantile regressions remarkably well.²¹ The model is qualitatively and quantitatively consistent with all the key facts identified in our description of the quantile regressions on EA data. The correlation between firm and bank default is higher when banks are more fragile and their probability of default is high. During times of average GDP growth, neither firm nor bank defaults affect economic performance in a significant manner. Bank (but not firm) defaults have a large and negative impact on GDP growth when the economy is already in recession. Despite the fact that these moments were not targeted in estimation, the model delivers a very close match.

4. Understanding the Model

In the previous section we saw that our model was able to replicate the co-movement of firm and bank defaults and GDP growth in the different regimes. Here we dig into the underlying mechanism in order to highlight the role of several important features of our framework in replicating different features of the EA data.

We start in Section 4.1 with the most important innovation of our paper: the modelling of bank assets as debt claims with limited upside. We explain in more detail how our approach works and how it differs from previous studies before showing that it is essential in matching the data. Then, in Section 4.2, we examine the importance of other model ingredients: the firm- and island-idiosyncratic shocks, the two risk shocks (which widen the dispersion of the island- and firm-idiosyncratic shocks) and the approximation order to which we solve the model.

4.1 Bank Asset Returns

As previously noted, a distinguishing feature of our model is the structural approach to loan default risk whereby banks fail when a significant fraction of the borrowers in their imperfectly diversified loan portfolios default. Other macroeconomic models of bank default consider banks with (ex-ante) perfectly diversified loan portfolios. In these models, the (ex-post) heterogeneity in bank asset returns comes from shocks that directly affect the performance of banks' loan portfolios rather than the underlying borrowers, thus making loan returns and their implications

²¹Regression coefficients for the model are obtained using simulations of the model for 100,000 periods. As before, we use DF_t and DB_t as the model counterparts for firm and bank EFDs, respectively.

for bank equity returns and bank failure similar to the classical Merton (1974) approach to corporate default.²²

To create a Merton-type version of our model that is in line with the current literature, we modify Equation (31) in two ways. First, we remove the impact of the island-idiosyncratic shocks by setting them to unity at all times $\omega_j = 1$. This is equivalent to assuming that banks are perfectly diversified across borrowers. Second, in order to introduce ex-post heterogeneity in bank default outcomes, we include a log-normally distributed bank-idiosyncratic shock to bank revenues ω_b . The loan portfolio returns under this specification are determined by

$$\tilde{R}_{f,t+1}^M(\omega_b) = [\Gamma_{i,t+1}(\bar{\omega}_{t+1}(1)) - \mu_f G_{i,t+1}(\bar{\omega}_{t+1}(1))] \frac{\omega_b [q_{t+1}(1 - \delta)k_t + y_{t+1}]}{b_{f,t}}. \quad (46)$$

Identically to the island-idiosyncratic shock, the standard deviation of the distribution of the bank-idiosyncratic shock, ω_b , is also time-varying and evolves as in Equation 41. We keep the rest of the model identical.²³

We are now ready to compare our model with the Merton-type approach. Figure 3 shows the crucial differences between $\tilde{R}_{f,t+1}(\omega_j)$ and $\tilde{R}_{f,t+1}^M(\omega_b)$. In the top left panel of Figure 3 we report $\tilde{R}_{f,t+1}(\omega_j)$ on the y-axis as a function of the island-idiosyncratic shock, ω_j , on the x-axis. In the top right panel, we depict the distribution of $\tilde{R}_{f,t+1}(\omega_j)$. In the bottom left panel of Figure 3 we similarly report, $\tilde{R}_{f,t+1}^M(\omega_b)$ as a function of the idiosyncratic shock to bank loan revenues and in the bottom right plot we depict the distribution of $\tilde{R}_{f,t+1}^M(\omega_b)$.²⁴

The top left panel of Figure 3 clearly shows that bank asset returns are highly non-linear in the island-idiosyncratic shock (ω_j). When ω_j is very high, all borrowers repay and the bank receives the promised repayment, including interest, from all its borrowers. But the upside is limited for the lender as is naturally the case under a standard debt contract. However, the presence of default creates downside for the financial intermediary. As the island idiosyncratic shock takes very low values, some firms start to default and bank asset returns begin to decline in a highly non-linear fashion.

The top right panel of Figure 3 shows the distribution of $\tilde{R}_{f,t+1}(\omega_j)$, which is clearly not log-normal. A large spike occurs when all borrowers repay. Bank asset returns are left-skewed with a fat left tail of low return realizations caused by firm defaults.

In contrast, the bottom panels of Figure 3 show that the Merton approach produces very

²²This is an attractive formulation because it makes the banking problem very tractable. But, as two recent papers (Nagel and Purnanandam, 2019; Gornall and Strebulaev, 2018) have shown, such a modelling strategy is not appropriate for banks even if the banks are financing underlying projects with log-normal returns. This is because financial intermediaries hold portfolios of loans with asymmetrically distributed payoffs. If borrowers repay, they repay a fixed contractual amount. If they default, the recovery value on loans is limited to a fraction of firms' asset values. Such a downwardly skewed payoff structure is not consistent with a log-normal distribution.

²³We estimate the parameters of the Merton-type version of our model to match the set of moments presented in Table 3.

²⁴For these figures, we have fixed $q_{t+1}, k_t, y_{t+1}, b_{f,t}, R_{d,t}, d_t$ to their steady-state values obtained with the parameter values described in Section 3.2. We set $\sigma_{\omega_b, t+1}$ such that the expected bank default equals its targeted value from Table 3. We use 10,000,000 draws of ω_j to plot the histograms.

different bank asset returns as a function of the exogenous bank-idiosyncratic shock to loan revenues. Since $\tilde{R}_{f,t+1}^M(\omega_b)$ is linear in ω_b , it looks like the returns on an equity portfolio in the sense that the bank experiences upside and downside shocks in a fully symmetric fashion. The distribution of returns looks very different too. Since ω_b is log-normal, $\tilde{R}_{f,t+1}^M(\omega_b)$ is log-normal too. Banks in the Merton world are exposed to less risk due to the far smaller left tail. This is important for the model's implications regarding the level of capital requirement needed to make banks safe.

Characterizing bank asset returns in an accurate manner is also essential when studying the relationship between firm and bank defaults. The top right panel of Figure 4 is a scatter plot of firm and bank defaults in the model when we use $\tilde{R}_{f,t+1}^M(\omega_b)$ instead of $\tilde{R}_{f,t+1}(\omega_j)$ and compares it to what our model delivers (top left panel). As one can see the correlation between firm and bank failures is zero when we use $\tilde{R}_{f,t+1}^M(\omega_b)$ instead of $\tilde{R}_{f,t+1}(\omega_j)$.

This should not be a surprise since the shocks used to generate firm and bank default are orthogonal to one another. In contrast, our mechanism treats the two defaults as intimately linked, thus helping to endogenously generate an empirically realistic relationship close to that in the data.

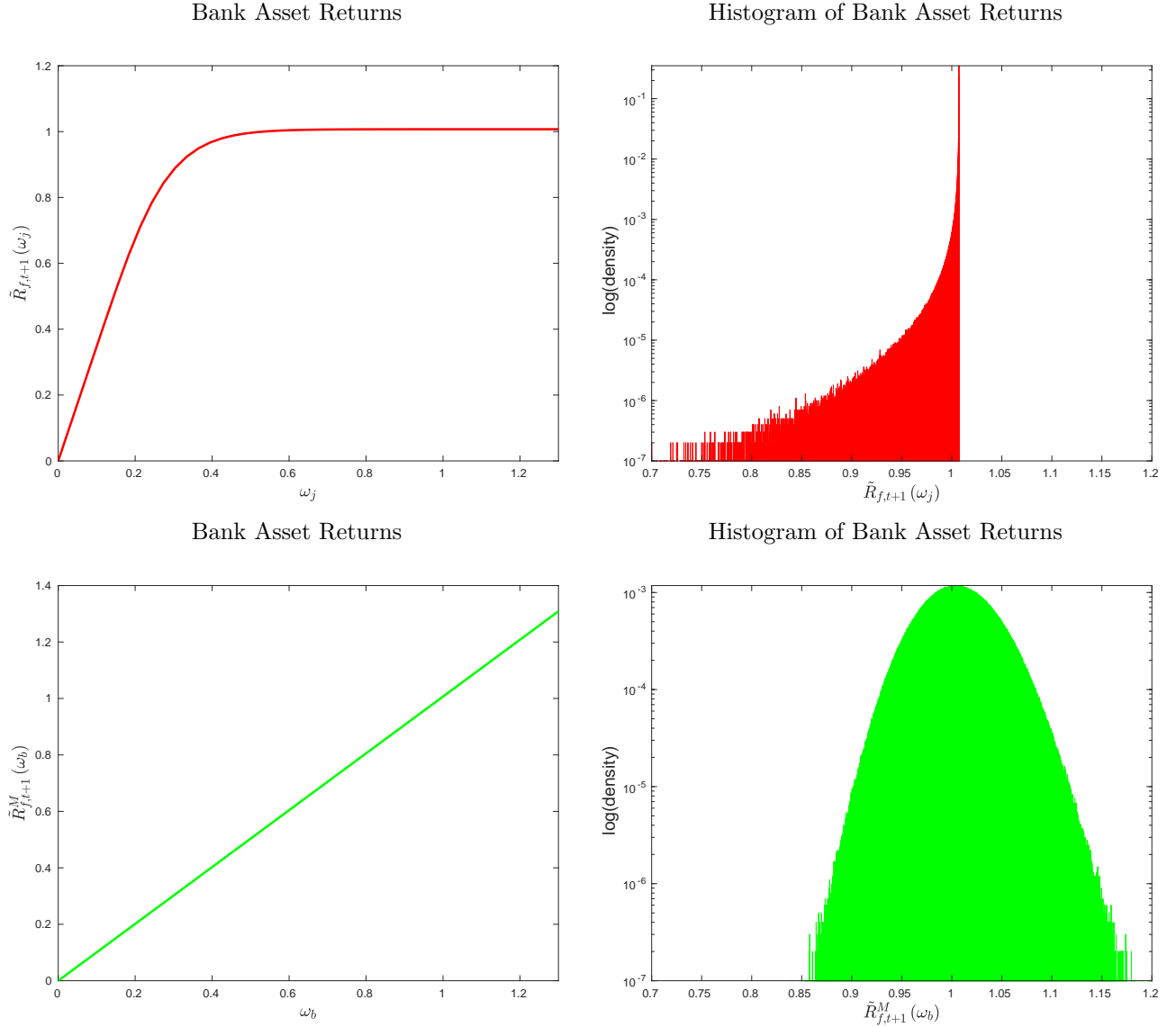
Another way to examine the relationship between firm and bank defaults is through quantile regressions. The bottom panels of Figure 4 compare the quantile regression coefficients for Equations (44) and (45) in our model (black line) to those obtained from a Merton-type version of our model (blue line). For completeness, we also include the relationship in the EA data (red line). Yet again, in the Merton-type approach the relationship between firm and bank defaults is close to zero at all quantiles of bank default. Although the Merton-type version of the model does a better job matching the quantile regression coefficients in Equation (45), it still fails to match the correlation between GDP growth and bank default at both the top and the bottom quantiles of GDP growth.

Failing to capture the non-linear relationship between firm and bank default has important implications for the frequency and severity of the twin defaults regime. In Table 4 we compare the performance of our baseline model with the Merton-type model in terms of the untargated conditional moments in the three default regimes. There are two main data features that the Merton-type approach cannot reproduce. First, the frequency of the twin defaults regime implied by the Merton-type model is significantly lower than the one observed in the data and in our baseline model. Second, the Merton approach underestimates (overestimates) the severity of the twin defaults (high firm default) regime in terms of GDP growth.

4.2 Other Important Features of the Model

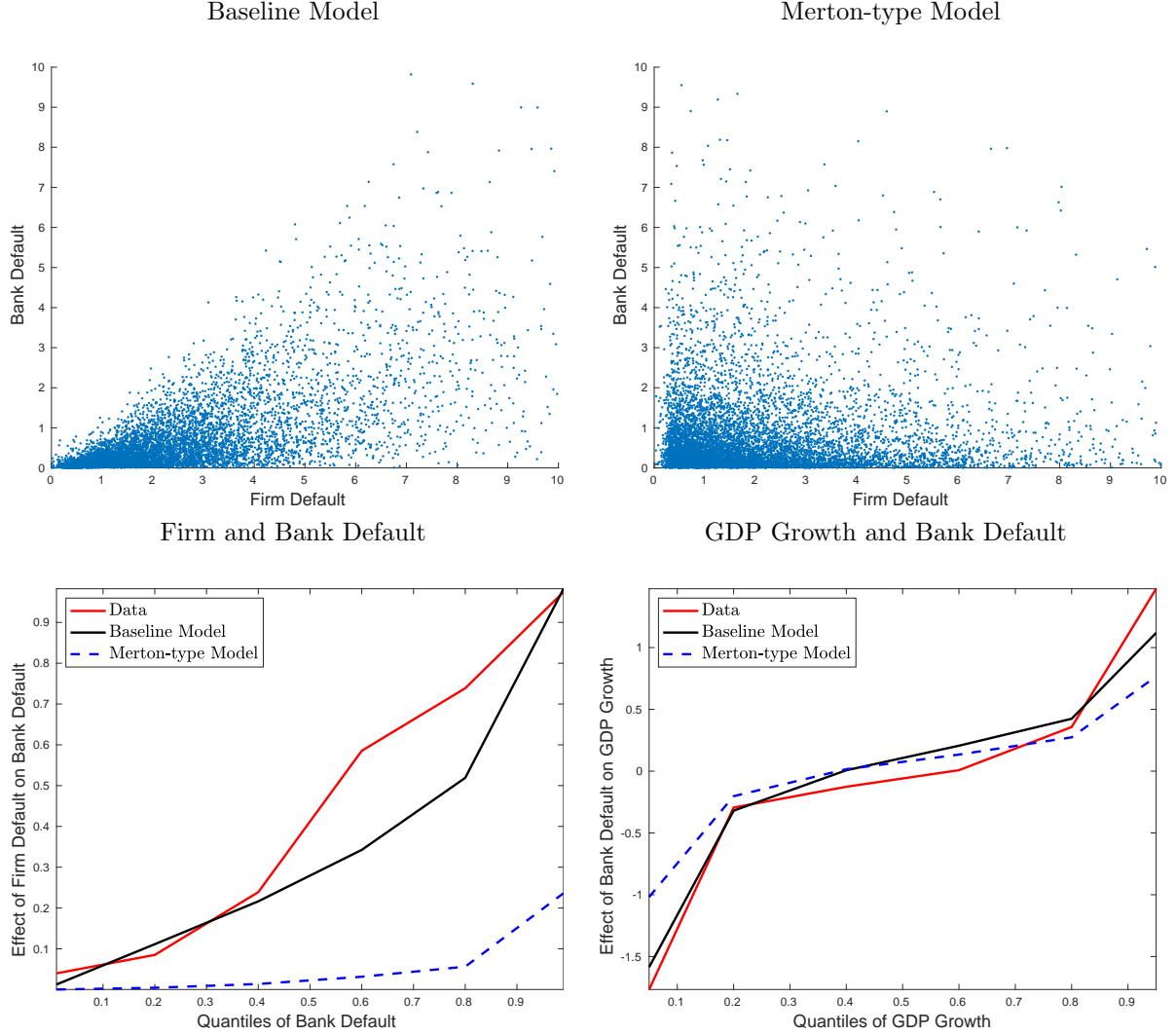
Our model with its endogenous connection between firm and bank solvency features a number of idiosyncratic and aggregate risk shocks that are important for the transmission of firm defaults to bank defaults and to the macroeconomy at large. In this section we investigate the importance of each of these shocks. We do this by removing them on an individual basis and then examining the extent to which this deteriorates the model's performance in replicating the quantile regressions in Section 3.1.

Figure 3: Bank Asset Returns: Baseline vs Merton-type Model



Notes: The top panels of this figure present bank asset returns as a function of the non-diversifiable island shock ω_j (left plot) and the histogram of bank asset returns (right plot) in the baseline model. The bottom panels of this figure present bank asset returns as a function of the bank-idiosyncratic shock ω_b (left plot) and the histogram of bank asset returns (right plot) in the Merton-type version of our model.

Figure 4: Scatter Plots and Quantile Regressions: Baseline vs Merton-type Model



Notes: The top panels display the scatter plot of firm and bank default produced with the baseline model (top left plot) and the Merton-type version of our model (top right). The bottom left panel presents coefficients ζ_τ from the quantile regression in Equation (44), whereas the bottom right panel presents coefficients β_τ from the quantile regression in Equation (45) for the Merton-type model (blue line), the data (red line) and the baseline model (black line).

4.2.1 Island-idiosyncratic and island-risk shocks

We start with the island-idiosyncratic and island-risk shocks. In our framework banks default when they experience abnormally low realizations of the island-idiosyncratic shock. Our model also allows aggregate fluctuations in the non-diversifiable (island) risk by means of island-risk shocks, i.e., shocks to the dispersion of the island-idiosyncratic risk. These shocks increase the probability of very low realizations of the island-idiosyncratic shocks, making banks more vulnerable.

The results of eliminating island-idiosyncratic and island-risk shocks are shown in the top panels of Figure 5. The figure presents the quantile regression coefficients for Equations (44) and (45) for the model without island-idiosyncratic shocks (green line), i.e., when the island-idiosyncratic shock is set to one, and without the island-risk shocks (blue line). The red and black lines correspond to our baseline model and the data, respectively.²⁵

The figure shows that both island-idiosyncratic and island-risk shocks are vital in generating a realistic sensitivity of bank default to firm default and of GDP growth to bank defaults. In the model without island-idiosyncratic shocks, the quantile regression coefficients go to zero because banks are perfectly diversified and their loan portfolio returns are very stable. Firms continue to default because of the firm-idiosyncratic shocks but banks are diversified against these shocks. And while aggregate shocks induce some fluctuations in firm default, these are too small to make banks fail, since our banks' solvency is protected by their equity buffers. Thus, if the bank is fully diversified, bank defaults do not happen and cannot possibly affect GDP growth. The model without island-risk shocks shows that, although eliminating this shock does not lead to fully diversified banks, keeping the non-diversifiable risk (and hence the probability of bank default) low and relatively constant reduces the model's capability of the model to match the sensitivity of bank default to firm default and of GDP growth to bank default. Clearly, the model without island-risk shocks, although it does a better job than the model without island-idiosyncratic shocks, fails to generate the state-dependent relationship between firm and bank defaults and economic activity that we see in the data.

This experiment clearly indicates the importance of both island-idiosyncratic and island-risk shocks in generating realistic conditional and unconditional correlation patterns between firm and bank defaults and economic activity. When the non-diversifiable risk is constant (no island-risk shocks), bank defaults are rare, they are mostly unaffected by firm defaults, and they do not affect real economic activity. When non-diversifiable risk is absent (no island-idiosyncratic shocks), banks do not default.

4.2.2 Firm-idiosyncratic and firm-risk shocks

The other source of risk to firms in our model comes from firm-idiosyncratic and firm-risk shocks, i.e. shocks to the dispersion of the firm-idiosyncratic risk. These shocks capture risks to individual firms that are diversifiable at the individual bank level. The firm-risk shocks increase firm defaults but they affect different banks evenly rather than concentrating the bulk

²⁵Note that when we eliminate the island-idiosyncratic shocks, the island-risk shocks became irrelevant.

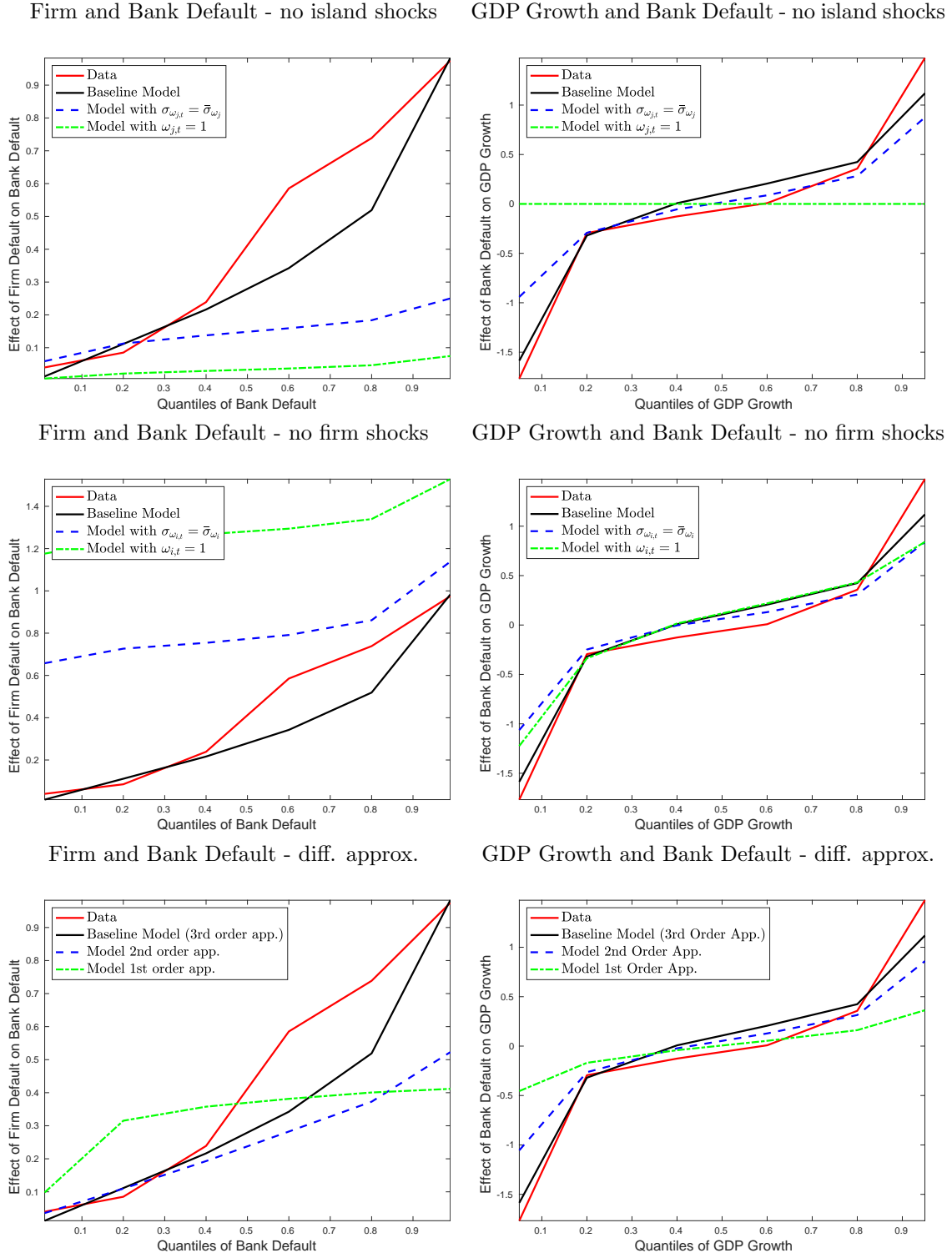
of losses on a few unlucky banks, as is the case for the island-risk shocks.

In this section we investigate how the model's ability to replicate the quantile regression coefficients for Equations (44) and (45) changes when we eliminate the firm-idiosyncratic and -risk shocks. The middle panels of Figure 5 show the results. This time the green line displays the quantile regression coefficients in the model where we set the firm-idiosyncratic shock equal to unity for all firms, while the blue line presents the results from the model where firm-risk shocks are shut down.

Both the green and blue lines display a relationship between firm and bank defaults. Intuitively, the green lines in the middle panels of Figure 5 correspond to an economy with fully non-diversified banks in which the defaults of banks and firms are almost perfectly correlated. This makes the sensitivity of bank default to firm default very large and rather constant over states. The impact of shutting down the firm-risk shocks is qualitatively similar to the elimination of the firm-idiosyncratic shocks but not as quantitatively large with respect to the quantile regression coefficients for Equation (44). The right middle panel shows that eliminating either firm-idiosyncratic or firm-risk shocks generates a state-dependent relationship between bank defaults and economic activity that is too weak compared both with the data and the implications of our baseline model.

This experiment clearly indicates the importance of both firm and island shocks in generating realistic conditional and unconditional correlation patterns between firm and bank defaults and economic activity. When we eliminate non-diversifiable risk (no island shocks), the conditional and unconditional correlation between firm and bank default is too small. Instead, when we eliminate diversifiable risk (no firm shocks), the conditional and unconditional correlation between firm and bank default is too large. In both instances the conditional and unconditional correlation between bank default and economic activity is too low.

Figure 5: Quantile Regressions: Key Model Features



Notes: The figure explores the importance of non-diversifiable risk (top panels), diversifiable risk (middle panels) and approximation order (bottom panels). The left column presents coefficients ζ_τ from the quantile regression in Equation (44), while the right column presents coefficients β_τ from the quantile regression in Equation (45).

4.2.3 Approximation order

Finally, we investigate the role of our solution method by comparing the quantile regressions implied by our baseline model (which is solved using third-order approximation) with the quantile regressions implied by first-order (green lines) or second-order (blue lines) approximate solutions.²⁶ The bottom panels of Figure 5 shows the results.

Both the linear and the second-order model clearly fail to match the non-linearities found in the data. They generate flat quantile regression coefficients in both panels. Intuitively, a model solved to first or second order works well in normal times but fails to generate the sharp and non-linear deterioration of economic and financial conditions in crises or recessions. In contrast, a third-order approximation captures the non-linearity in the co-movements of firm and bank defaults and economic activity.

We have already discussed in Section 4.1 that modelling bank portfolios as consisting of defaultable loans introduces an important non-linearity into bank asset returns and hence into bank default realizations. It is therefore natural that a non-linear solution method is needed to capture such non-linearities in an accurate manner. Our results show that a third-order solution is sufficient for this purpose.

Table 4 also reports the performance of the linear approximated version of the model in terms of untargeted conditional moments in the three default regimes. As we pointed out before, capturing the non-linear relationship between firm and bank default has important implications for the frequency and severity of the twin defaults regime. Indeed, the frequency of the twin defaults regime reproduced by the first-order model is somewhat lower than the one observed in the data and in our baseline model. In addition, it underestimates (overestimates) the severity of the twin defaults (high firm default) regime in terms of GDP growth.

5. The Anatomy of twin default crises

In what follows we investigate how the model generates episodes of twin default crises. The results confirm the importance of island-risk shocks and reveal the ability of the model to endogenously propagate them. In fact, under high leverage and a solution method that captures non-linearities, the model does not require large shocks to generate severe but rare episodes of twin default crises. This analysis will lead us to the policy section, where we examine the role of the capital requirement in helping prevent them.

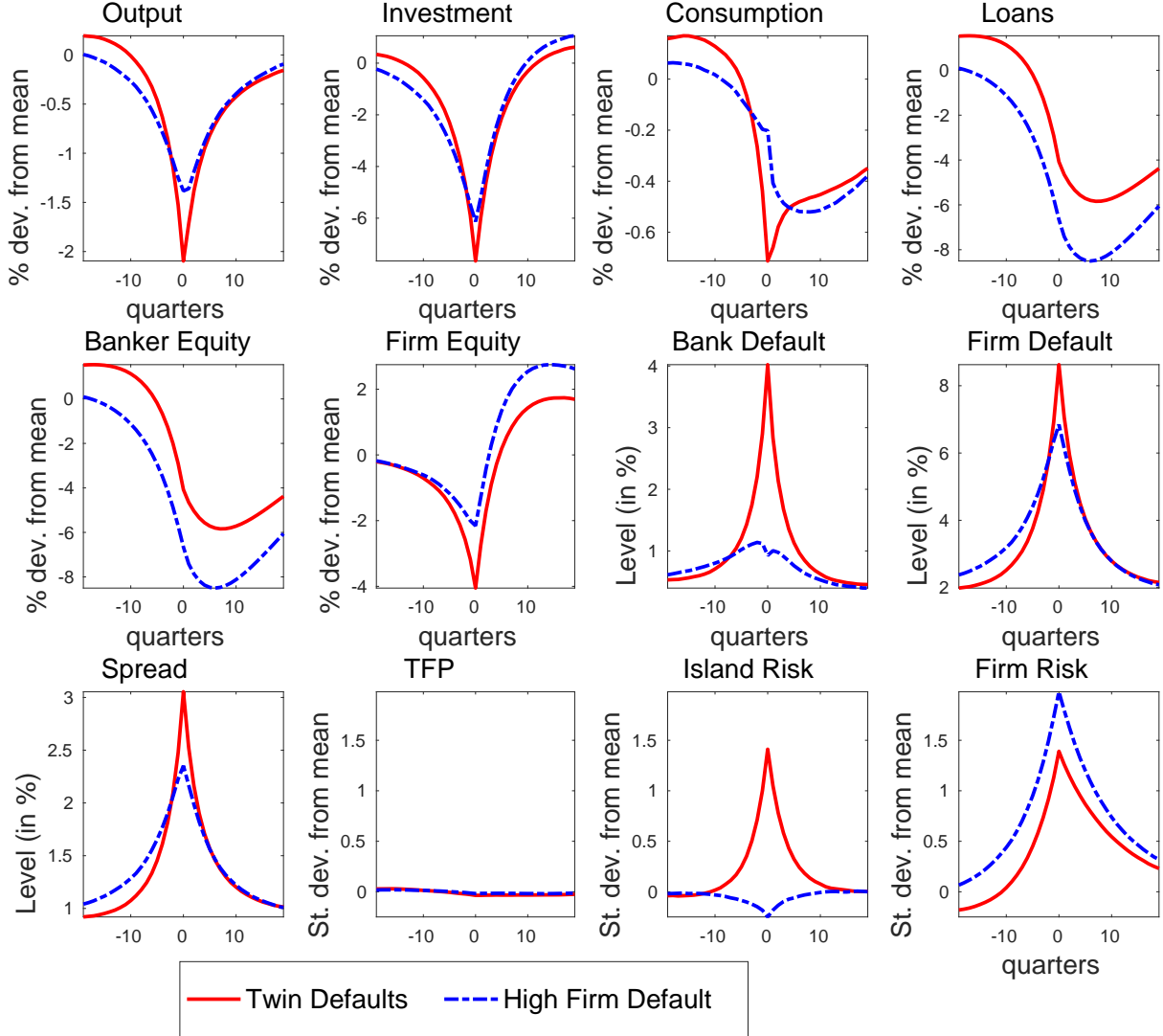
5.1 Average Path to Crisis

Figure 6 shows the average path leading to high firm default (blue line) and twin defaults (red line) regimes. The figure is generated by simulating the model for 1,000,000 periods, identifying periods in which defaults are above the 90th percentile and then computing the

²⁶We estimate the parameters of the first- and second-order approximation versions of our model to match the set of moments presented in Table 3.

average realizations of shocks and endogenous variables for twenty periods before and after the crisis periods.

Figure 6: Paths to Crises



Notes: This figure shows the average path leading to high firm default (blue dashed line) and twin defaults (red solid line) regimes. The figure is generated by simulating the model for 1,000,000 periods, identifying periods in which defaults are above the 90th percentile and then computing the average realizations of shocks and endogenous variables for twenty periods before and after the crisis periods. We define twin defaults as the simultaneous occurrence of firm and bank default above their respective 90th percentiles. High firm default periods are those where firm default is above the 90th percentile and bank default is below the 90th percentile. TFP, Island Risk and Firm Risk represent the level of A_t , $\frac{\sigma_{\omega_j, t+1}}{\sigma_{\omega_j}}$ and $\frac{\sigma_{\omega_i, t+1}}{\sigma_{\omega_i}}$ in their respective standard deviation units.

Several messages stand out from the figure. First of all, the model implies that twin default episodes generate output falls that are twice as large compared to high firm default events. Second, the model captures remarkably well the evolution of bank defaults for both regimes. Bank defaults rise above 4 percent during twin defaults, which is very close to what we observe in the EA data during the recent financial crisis. In contrast, bank failures remain below 1 percent in the episodes of high firm defaults. Both cases are very close to the evidence reported in Table 4. The declines in output, investment and lending are more pronounced in the case of twin defaults when compared with high firm defaults.

The model is also consistent with the empirical finding (Schularick and Taylor, 2012; Jorda, Schularick and Taylor, 2016) that financial crises tend to be preceded by above average economic activity and lending. In the twin defaults regime, output, bank lending, bank capital and consumption reach a cyclical peak around 8-10 quarters before the crisis event and fall sharply as it approaches. In our model, the pre-crisis boom is mainly driven by good (below average) realizations of the firm-risk shock. As a result, corporate leverage is elevated, making firms (and hence banks) more vulnerable to subsequent adverse shocks.

The figure also shows that, on average, TFP remains broadly unchanged in both types of episodes. The two risk shocks instead play an important role. The rise in the firm-risk shocks plays a role in both high firm defaults and twin defaults, while the rise in the island-risk shocks plays a key role in generating twin default crises.

Finally, the model does not need very large risk shocks in order to generate a twin default Crisis. These episodes occur following a sequence of small and positive risk shocks that accumulate into a 1.5 standard deviation increase. As the figures show, the island-risk shocks are crucial to generate bank defaults and, therefore, twin default crises. Firm-risk shocks by themselves can only create high firm default events. The fact that our baseline model does not need very large risk shocks explains why our model can match the frequency of the twin defaults regime in the data.

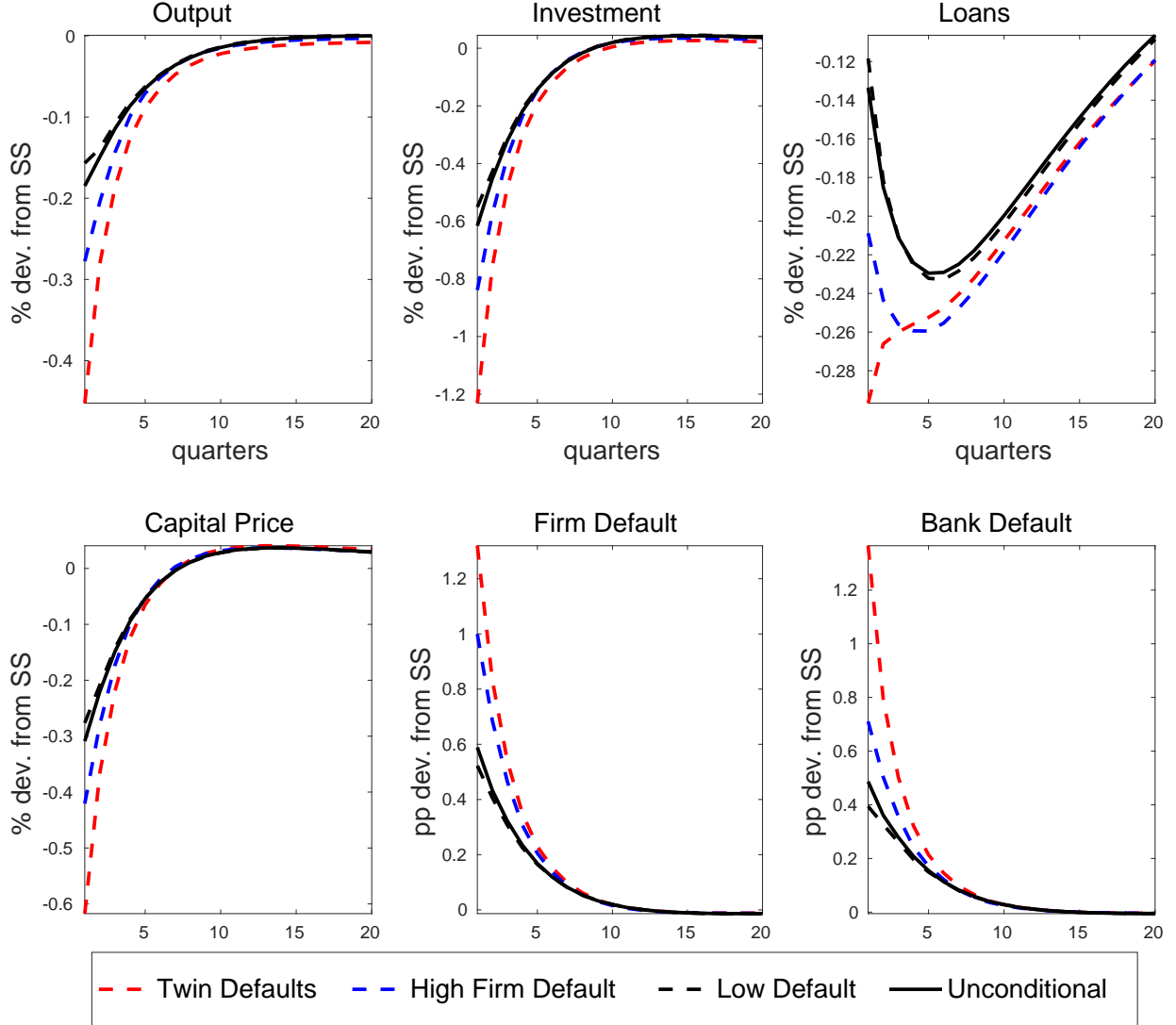
5.2 Generalized Impulse Response Functions to an Island-risk Shock

We now use the Generalized Impulse Response Functions (GIRFs) to show that the economy “accelerates” into a twin default event as the impact of additional island-risk shocks grows. This internal propagation helps the model generate twin default crises without the need for huge shocks. Figure 7 reports three sets of GIRFs to a one standard deviation island-risk shock. The solid line shows the unconditional GIRF, the blue dashed line shows the GIRF conditional on the economy being at a high firm default episode, and the red dashed line shows the GIRF conditional on the economy being in a twin default episode. Details on how to compute both conditional and unconditional GIRFs can be found in Appendix F.

Clearly, the shock has a much larger impact when conditioning on either a twin defaults or a high firm default episode.²⁷ The GDP drop is much larger than the effect in the unconditional GIRF. The same is true for the drop in investment and the price of capital and for the impact on firm and bank defaults. This shows how the model solved with a third-order approximation

²⁷It is important to note that the traditional linear IRFs are independent of the state of the economy.

Figure 7: Conditional Impulse Response Functions: Island Risk Shock



Notes: This figure reports three sets of generalized impulse response functions (GIRFs) to a one standard deviation island-risk shock. The solid black line shows the unconditional GIRF, the black dashed line shows the GIRF conditional on the economy being in a low default episode, the blue dashed line shows the GIRF conditional on the economy being at a high firm default episode, and the red dashed line shows the GIRF conditional on the economy being in a twin defaults episode. We define a twin default s episode as the simultaneous occurrence of firm and bank default above their respective 90th percentiles. High firm default episodes are those where firm default is above the 90th percentile and bank default is below the 90th percentile. In the low default regime, both bank and firm default are below the 90th percentile. Details on how to compute both conditional and unconditional GIRFs can be found in Appendix F.

is able to amplify island-risk shocks during crisis times differently than during normal times. In our model, once the economy finds itself in a situation of high firm default, it becomes very vulnerable to island-risk shocks. In any case the GIRFs conditional only on high firm default shows much less amplification than when we condition on a twin defaults episode.

5.3 Non-linear Effects

What are the key aspects of the model and its solution that help us generate a path to crisis as shown in Figure 6? In this section, we focus our attention on a key element of our setup: the non-linear solution method. The importance of non-linearities is confirmed by the comparison, in Figure 8, of the baseline path to crisis (solid lines) with that obtained with a first-order approximation (dashed lines).²⁸

The figure shows that the first-order approximation needs very large (and therefore highly unlikely) island-risk shocks that need to increase by three standard deviations in the first-order approximation rather than only 1.5 standard deviations in the baseline. It is also the case that, despite the large shocks, the first-order approximation cannot generate a realistic increase in the probability of bank default. These large shocks explain why the linear model can only produce twin defaults with 4 percent probability, while the frequency implied by the baseline model is very closed to the 7 percent observed in the data.

5.4 The Role of Bank Leverage

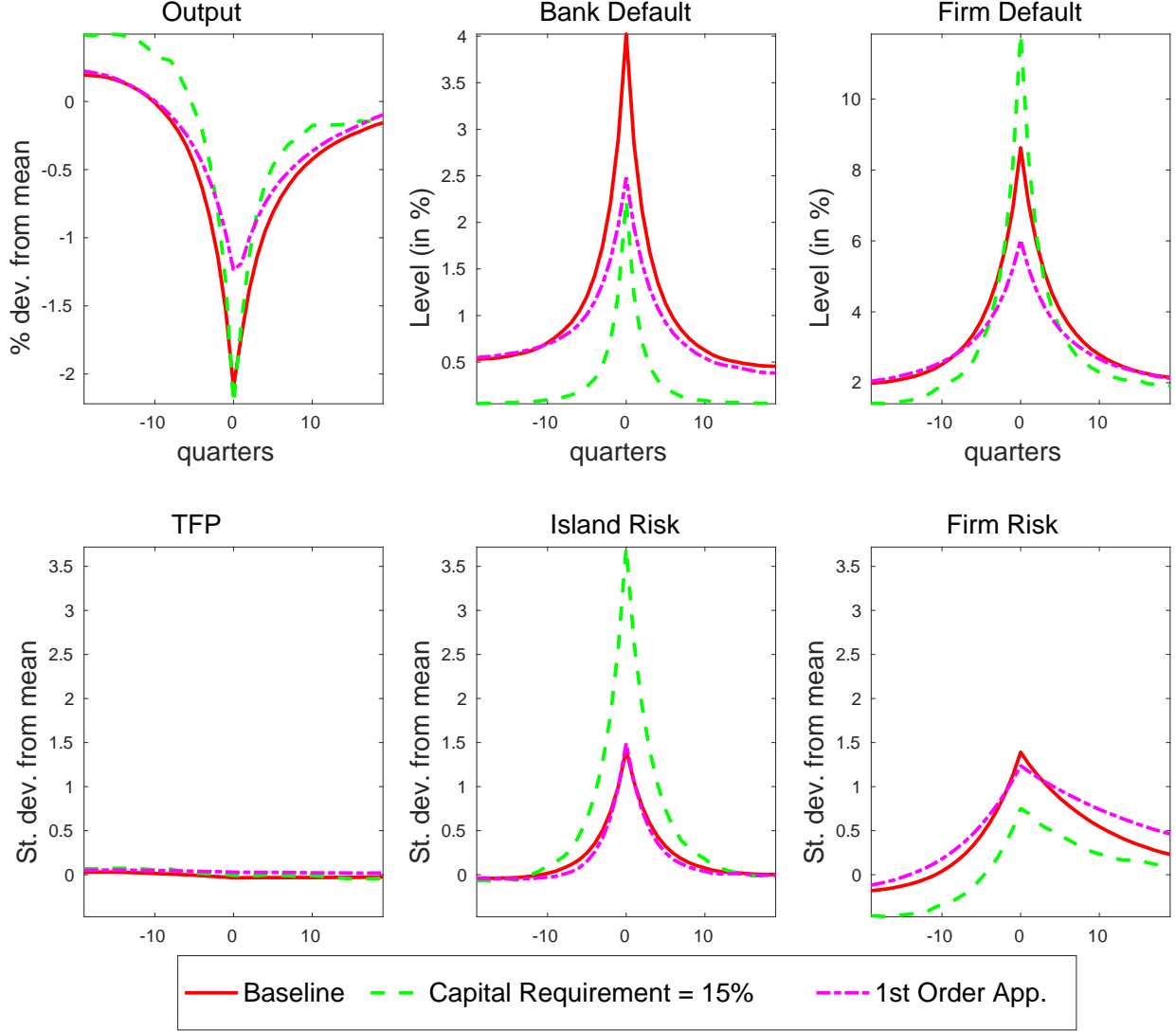
Bank leverage is the second key aspect of the model we want to focus on. Figure 8 also compares the baseline path to crisis with a bank capital requirements of 8 percent (solid lines) with a path implied by a version of the model with a higher bank capital requirement of 15 percent (dashed line).²⁹

The figure clearly shows that the model with a a bank capital requirement of 15 percent needs a 3.5 standard deviation increase in the island-risk shock in order to generate a twin default crisis. Table 4 shows the performance of the model with higher capital requirements in terms of untargeted conditional moments in the three default regimes. The fact that the model requires such a large shock explains why the frequency of the twin defaults (high firm default) regime is dramatically lower (higher) under the 15 percent capital requirement. In the following section, we will see that imposing such a capital requirement is welfare improving because of the high costs associated with twin default crises.

²⁸As it was the case in Table 4, the thresholds used to define the three regimes are always the ones determined by the baseline model.

²⁹As was the case in Table 4, the thresholds used to define the three regimes are always the ones determined by the baseline model.

Figure 8: Path to Twin Defaults in Different Scenarios

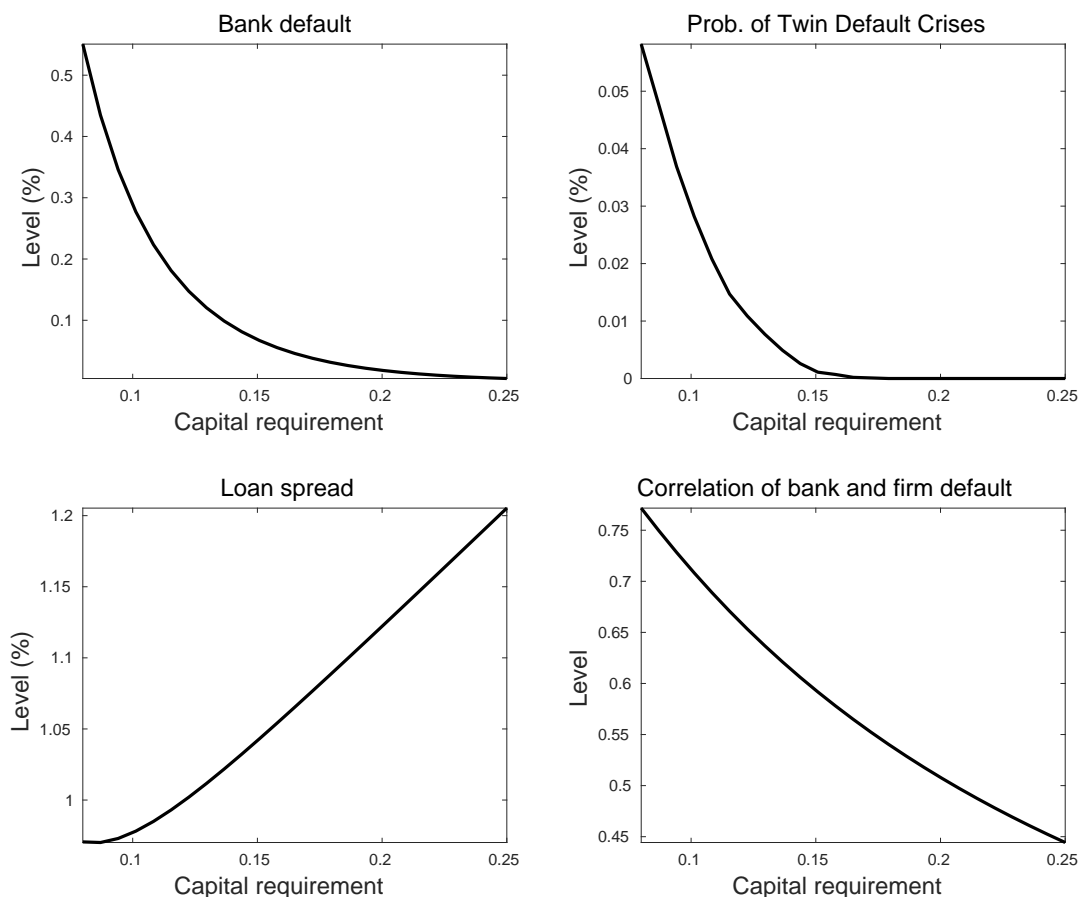


Notes: This figure shows the average path leading to a twin default episode under different model assumptions. Baseline (red solid line) corresponds to our baseline model with the capital requirement sets to 8 percent ($\phi = 0.08$) and solved with third order perturbation methods. Capital Requirement = 15% (green dashed line) corresponds to the model with capital requirement sets to 15 percent ($\phi = 0.15$). 1st Order App. (pink dashed-dotted line) corresponds to the model solved with a first-order perturbation method. The figure is generated by simulating the model for 1,000,000 periods, identifying periods of twin defaults and then computing the average realizations of shocks and endogenous variables for twenty periods before and after the crisis periods. We define a twin default episode as the simultaneous occurrence of firm and bank default above their respective 90th percentiles. The 90th percentile default thresholds used to define the three regimes in the four models are always the ones determined by the baseline model. TFP, Island Risk and Firm Risk represent the level of A_t , $\frac{\sigma_{\omega_j, t+1}}{\bar{\sigma}_{\omega_j}}$ and $\frac{\sigma_{\omega_i, t+1}}{\bar{\sigma}_{\omega_i}}$ in their respective standard deviation units.

6. Implications for Capital Requirements

We conclude our analysis by examining the model's implications for the socially optimal level of bank capital requirements. Figure 9 shows the implications of different values of the capital requirement on the mean of the ergodic distribution of selected variables for our baseline model.³⁰

Figure 9: Comparative Statics with Respect to Capital Requirement Level



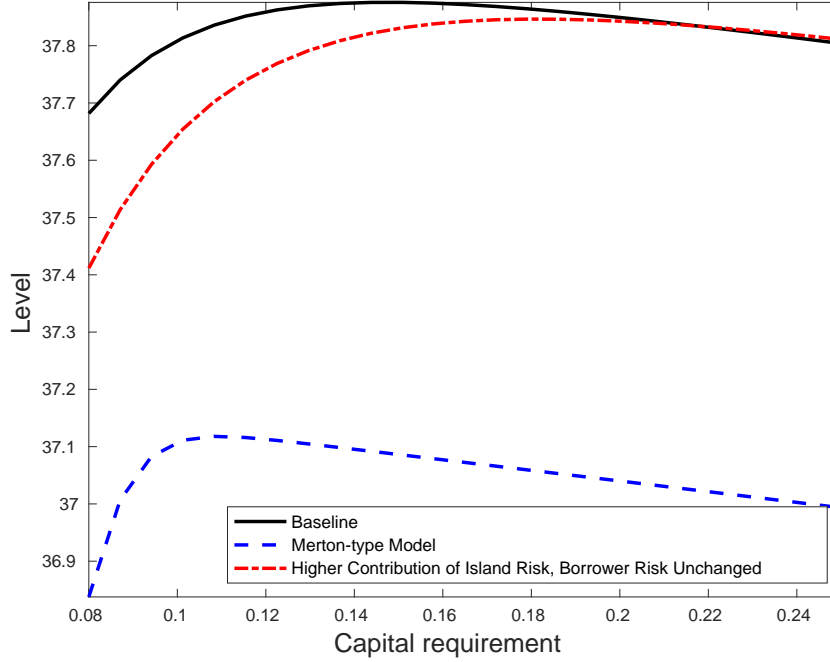
Notes: This figure shows the implications of different values of the capital requirement ϕ on the mean of the ergodic distribution of selected variables for our baseline model.

The figure shows that the imposition of higher capital requirements faces a trade-off between reducing the probability of twin default crises, and maintaining the supply of bank credit in normal times. Higher capital requirements make bank equity returns better protected against the island-risk shocks. When banks are less levered, non-diversifiable risk has a lower impact on

³⁰We consider changes in the capital requirement ϕ while keeping all other parameters unchanged.

banks' equity and this reduces banks' default, as well as, the correlation between firm and bank default. As a result, twin default crises become less frequent and deadweight losses associated with the costs of asset repossession decline. However, higher capital requirements are also costly for the economy. They increase the relative scarcity of bank net worth and the average cost of bank funding. This implies higher borrowing costs, reduced bank credit, and lower investment.

Figure 10: Welfare Effects of the Capital Requirement in Different Scenarios



Notes: This figure reports the ergodic mean of household welfare as a function of the level of bank capital requirements in different scenarios. Baseline (black solid line) corresponds to our baseline model. Merton-type model (blue dashed line) corresponds to the model in which the Merton-type specification of bank asset returns is adopted. Higher Contribution of Island Risk, Borrower Risk Unchanged (red dashed-dotted line) corresponds to the model in which we increase the average standard deviation of the island-idiosyncratic shock by 10 percent and reduce the average standard deviation of the firm-idiosyncratic shock by 6.3 percent while keeping the probability of firm default unchanged. While the average probability of firm default remains equal to 2.25 percent, the probability of bank default increases from 0.59 percent to 1.03 percent and the probability of a twin default crisis increases from 5.8 percent to 8.8 percent.

The trade-off is reflected in the overall effects of higher capital requirements on welfare. The solid black line in Figure 10 reports the ergodic mean of household welfare as a function of the level of bank capital requirements. The optimal bank capital requirement is around 15 percent which is associated with welfare gains of approximately 0.1 percent in certainty equivalent consumption terms relative to the baseline model, where the bank capital requirement was calibrated to be 8 percent.

Starting from the 8 percent capital requirement, welfare first increases because the gains from the reduction in the probability of bank default outweigh the losses from imposing higher

funding costs on banks. At the optimum, the probability of bank default is below 0.1 percent and further reductions in bank failures have a limited impact on welfare. For a capital requirement above 15 percent, the negative effect of elevated borrowing costs for firms dominates and welfare declines.

In what follows we explore how the prescriptions regarding the capital requirement depend on the importance of island-idiosyncratic shocks and on the adoption of the island approach instead of a Merton-type framework. As shown above, these are crucial elements for the occurrence of twin default crises.

To highlight the importance of non-diversifiable island-idiosyncratic shocks for our results, we increase the average standard deviation of the island-idiosyncratic shock and reduce the average standard deviation of the firm-idiosyncratic shock while keeping the probability of firm default unchanged. In this way, we reduce the extent to which banks can diversify away the firm default risk.³¹ The red dashed line in Figure 10 shows household welfare when the island-idiosyncratic shock is more important in the determination of firm default and bank failures are, thus, more frequent. The optimal capital requirement is substantially higher; close to 20 percent.

Figure 10 also reports welfare as a function of the capital requirement under a Merton-type specification of bank asset returns (blue dashed line). This version of the model underestimates the gains from having higher capital requirements with respect to our baseline model. The optimal capital requirement is significantly lower under the Merton-type formulation (around 11 percent). We calibrate the Merton-type version of our model so as to ensure that the mean and standard deviation of bank default are the same as in our baseline model. This implies that the mean deadweight loss due to bank default is the same in both models. However, as shown in Section 4., the correlation between firm and bank default in this version of the model is substantially lower. This means that high bank defaults are not necessarily accompanied by firm defaults and the overall macroeconomic losses (and consumption declines) associated with bank default are lower. As a result, the Merton-type formulation implies a lower optimal level of the bank capital requirement.

7. Conclusions

We build a quantitative macro model with banks that micro-founds the link between borrower and bank defaults. The model reproduces key Euro Area data moments and in particular the unconditional and conditional correlation of firm and bank defaults and their relationship with output. Moreover, the model is able to match the frequency and severity of twin default crises characterized by very high firm and bank defaults as well as deep recessions. Aggregate shocks to non-diversifiable sources of borrowers' default risk for banks play an important role in driving the economy into such crises.

³¹The average standard deviation of the island-idiosyncratic shock is increased by 10 percent, whereas the average standard deviation of the firm-idiosyncratic shock is reduced by 6.3 percent. While the average probability of firm default remains equal to 2.25 percent, the probability of bank default increases from 0.59 percent to 1.03 percent. The probability of twin default crises increases from 5.8 percent to 8.8 percent.

We make an important contribution to the modelling of banks in general equilibrium by properly characterizing the non-linear nature of loan portfolio returns featuring a limited upside but downside risks due to borrower default risk. These non-linearities (captured by solving the model using a third order approximation) turn out to be key to generating twin default crises of realistic frequency and severity.

Turning to the model's policy implications, we show that bank leverage is crucial for the frequency and severity of financial crises. Crisis frequency can therefore be reduced by imposing a higher capital requirement on banks at the cost of reducing credit and output in normal times. We find that a capital requirement of fifteen per cent optimally trades off lower crisis frequency (benefits of higher capital) against lower credit in normal times (costs of higher capital).

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Appendix A Data

- Investment: Gross Fixed Capital Formation, Millions of euros, Chain linked volume, Calendar and seasonally adjusted data, Reference year 1995, Source: the Area Wide Model (AWM) dataset.
- Gross Domestic Product (GDP): we define the GDP as the sum of Consumption and Investment.
- Loans: Outstanding amounts of loans at the end of quarter (stock) extended to non-financial corporations by Monetary and Financial Institution (MFIs) in EA, Source: MFI Balance Sheet Items Statistics (BSI Statistics), Monetary and Financial Statistics (S/MFS), European Central Bank.
- Loan Spread: Spread between the composite interest rate on loans and the composite risk free rate. We compute this spread in two steps.
 1. Firstly, we compute the composite loan interest rate as the weighted average of interest rates at each maturity range (up to 1 year, 1-5 years, over 5 years).
 2. Secondly, we compute corresponding composite risk free rates that take into account the maturity breakdown of loans. The maturity-adjusted risk-free rate is the weighted average (with the same weights as in case of composite loan interest rate) of the following risk-free rates chosen for maturity ranges:
 - 3 month EURIBOR (up to 1 year).
 - German Bund 3 year yield (1-5 years).
 - German Bund 10 year yield (over 5 years for commercial loans).
 - German Bund 7 year yield (5-10 years for housing loans).
 - German Bund 20 year yield (over 10 years for housing loans).

Source: MFI Interest Rate Statistics of the European Central Bank, Bloomberg.

- Return on equity of banks: Bank Equity Return (after tax), EA. Source: Global Financial Development, World Bank.
- Expected default of Banks: Asset weighted average of EDF within one year for the sample of banks in EA. The data comes on the monthly basis. We aggregate it to quarterly series by averaging the monthly series within a quarter. ³² Source: Moody's KMV.
- Expected default of non-financial firms: we compute it using Moody's EDF series for a sample of non-financial corporations in the EA. Since in the Moody's dataset we have an over-representation of large firms and under-representation of small and medium-sized enterprises (SMEs) compared to the loan portfolio of bank in the EA, we proceed in two

³²See detailed EDF description on the Moody's webpage.

steps.³³ Firstly, we construct two separate EDF indices: i) for SMEs, ii) for large firms.³⁴ Secondly, we build an aggregate default series for non-financial firms as a weighted average of EDF indices for SMEs and large firms. As weights we use the share of loans extended by banks in EA to SMEs and large firms respectively.³⁵ The data comes on the monthly basis. We aggregate it to quarterly series by averaging the monthly series within a quarter.

Appendix B Model Aggregation and Market Clearing

In this subsection we describe model aggregation and market clearing conditions.

Final good The clearing of the market for final good requires

$$Y_t = y_t, \quad (47)$$

where aggregate output Y_t equals household consumption, C_t , plus the investment in the production of new capital, I_t , plus the resources absorbed by the costs of repossessing assets from defaulting entrepreneurial firms and banks

$$Y_t = C_t + I_t + \Sigma_{b,t} + \Sigma_{e,t}, \quad (48)$$

where

$$\begin{aligned} \Sigma_{b,t} &= \mu_j \int_0^{\bar{\omega}_{j,t}} \tilde{R}_{f,t}(\omega_j) B_{f,t-1} dF_{j,t}(\omega_j) \text{ and} \\ \Sigma_{e,t} &= \mu_f \int_0^\infty \int_0^{\bar{\omega}_t(\omega_j)} \omega_i \omega_j [q_t(1 - \delta) K_{t-1} + Y_t] dF_{i,t}(\omega_i) dF_{j,t}(\omega_j). \end{aligned}$$

Labor The clearing of the labor market requires

$$H_t = h_t. \quad (49)$$

Physical capital The clearing of the market for physical capital requires

$$K_t = k_t. \quad (50)$$

³³We define SMEs as firms with average total assets below €43 m within the sample period in the database (as in the definition of the European Commission)

³⁴EDF indices are constructed as asset weighted average of EDF within one year for the sample of non-financial firms within the size category

³⁵We obtain the data on the share of SMEs loans in total loans from Financing SMEs and Entrepreneurs database of OECD.

Equity The clearing of the market for equity requires

$$EQ_{e,t} = \int_0^\infty eq_{e,t}(i)di \text{ and} \quad (51)$$

$$EQ_{b,t} = \int_0^\infty eq_{b,t}(j)dj. \quad (52)$$

Loans The clearing of the market for loans, requires

$$B_{f,t} = b_{f,t}. \quad (53)$$

Bank deposits The clearing of the market for bank deposits, requires

$$D_t = d_{f,t}. \quad (54)$$

Law of motion of capital Finally, the law of motion of capital is given by

$$K_t = (1 - \delta) K_{t-1} + S \left(\frac{I_t}{K_{t-1}} \right) K_{t-1}. \quad (55)$$

Law of motion of entrepreneurs' aggregate net worth Let $N_{e,t}$ be the aggregate net wealth of entrepreneurs at period t . Then

$$N_{e,t} = \theta_e \rho_{e,t} N_{e,t-1} + \iota_{e,t}, \quad (56)$$

which reflects the retention of net worth by the fraction θ_e of non-retiring entrepreneurs, whom individual net worth evolves as described by Equation (9), the aggregate endowment $\iota_{e,t}$ added by the entering entrepreneurs, and the fact that aggregate net wealth equals individual net wealth

$$N_{e,t} = \int_0^\infty n_{e,t}(i)di. \quad (57)$$

Law of motion of bankers aggregate net worth Let $N_{b,t}$ be the aggregate net wealth of bankers at period t . Then

$$N_{b,t} = \theta_b \rho_{b,t} N_{b,t-1} + \iota_{b,t} - T_t, \quad (58)$$

which reflects the retention of the aggregate net worth by the fraction θ_b of non-retiring bankers, whom individual net worth evolves as described by Equation (26), the aggregate endowment $\iota_{b,t+1}$ received by the entering bankers, and the fact that aggregate net wealth equals individual net wealth

$$\int_0^\infty N_{b,t} = n_{b,t}(j)dj. \quad (59)$$

Endowments of entering entrepreneurs and bankers We model the aggregate endowment of entering entrepreneurs as a proportion χ_e of the aggregate net worth of the retiring entrepreneurs

$$\iota_{e,t} = \chi_e(1 - \theta_e)\rho_{e,t}N_{e,t-1}. \quad (60)$$

Akin to the case of entrepreneurs, we model the aggregate endowment of entering bankers as a proportion χ_b of the aggregate net worth of the retiring bankers

$$\iota_{b,t} = \chi_b(1 - \theta_b)\rho_{b,t}N_{b,t-1}. \quad (61)$$

Net transfers from entrepreneurs and bankers to the household Let the net transfers received by the household from entrepreneurs and bankers at period t be $\Upsilon_{e,t}$ and $\Upsilon_{b,t}$ respectively. Then, we have

$$\Upsilon_{e,t} = (1 - \theta_e)\rho_{e,t}N_{e,t-1} - \iota_{e,t}, \quad (62)$$

which reflects the the aggregate worth of the fraction $1 - \theta_e$ of retiring entrepreneurs minus the aggregate endowment $\iota_{e,t}$ added by the entering entrepreneurs. Equivalently, we also have that

$$\Upsilon_{b,t} = (1 - \theta_b)\rho_{b,t}N_{b,t-1} - \iota_{b,t}, \quad (63)$$

which reflects the the aggregate worth of the fraction $1 - \theta_e$ of retiring bankers minus the aggregate endowment $\iota_{b,t}$ added by the entering bankers. Thus, we have that the sum of net transfers from entrepreneurs and bankers to the household is

$$\Upsilon_t = (1 - \theta_e)\rho_{e,t}N_{e,t-1} - \iota_{e,t} + (1 - \theta_b)\rho_{b,t}N_{b,t-1} - \iota_{b,t}. \quad (64)$$

Profits from capital production Profits received by households from capital producing firms are

$$\Xi_t = q_t S \left(\frac{I_t}{K_{t-1}} \right) K_{t-1} - I_t. \quad (65)$$

Deposit insurance costs Using $D_t = d_{f,t}$, Equation (38) can be written as

$$T_t = \Omega_t D_{t-1}. \quad (66)$$

Appendix C Model Equilibrium Conditions

We provide the equilibrium conditions for our model. We begin with the equilibrium conditions related to the households, then entrepreneurs and entrepreneurial firms, then bankers and banks, then the capital production sector, and finally the market clearing conditions.

The household Using Equations (2) and (3) we obtain

$$-\frac{U_{H_t}}{U_{C_t}} = w_t, \quad (67)$$

Equation (4) is part of the equilibrium conditions. Hence, we have

$$1 = \mathbb{E}_t \Lambda_{t+1} R_{d,t}. \quad (68)$$

Entrepreneurs Equations (8) and (10) gives us

$$\nu_{e,t} = \mathbb{E}_t \Lambda_{e,t+1} \rho_{e,t+1}. \quad (69)$$

The elements of the law of motion of entrepreneurs' net worth reflected in Equations (56) and (60) are also part of the equilibrium. Hence, we have

$$N_{e,t+1} = \theta_e \rho_{e,t+1} N_{e,t} + \iota_{e,t+1} \text{ and} \quad (70)$$

$$\iota_{e,t+1} = \chi_e (1 - \theta_e) \rho_{e,t+1} N_{e,t}. \quad (71)$$

Entrepreneurial Firm Equations (11)-(21) from the entrepreneurial firms' problem are also part of the equilibrium conditions. Hence, we have

$$Y_{t+1} = A_{t+1} K_t^\alpha (H_t)^{(1-\alpha)}, \quad (72)$$

$$\Pi_{i,j,t+1} (\omega_i, \omega_j) = \omega_i \omega_j (q_{t+1} (1 - \delta) K_t + Y_{t+1}) - R_{f,t} B_{f,t}, \quad (73)$$

$$\bar{\omega}_{t+1} (\omega_j) = \frac{R_{f,t} B_{f,t}}{\omega_j (q_{t+1} (1 - \delta) K_t + Y_{t+1})}, \quad (74)$$

$$\Pi_{f,t+1} = \int_0^\infty \int_{\bar{\omega}_{t+1}(\omega_j)}^\infty \Pi_{i,j,t+1} (\omega_i, \omega_j) dF_{i,t+1} (\omega_i) dF_{j,t+1} (\omega_j), \text{ and} \quad (75)$$

$$B_{f,t} + N_{e,t} = w_t H_t + q_t K_t, \quad (76)$$

$$\mathbb{E}_t \Lambda_{b,t+1} \Pi_{b,t+1} = \nu_{b,t} \phi B_{f,t}, \quad (77)$$

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial K_t} + \zeta_{f,t} q_t - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial K_t} = 0, \quad (78)$$

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial H_t} + \zeta_{f,t} w_t - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial H_t} = 0, \quad (79)$$

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial B_{f,t}} - \zeta_{f,t} - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial B_{f,t}} + \xi_{f,t} \nu_{b,t} \phi = 0, \quad (80)$$

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial R_{f,t}} - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial R_{f,t}} = 0, \quad (81)$$

$$\rho_{e,t+1} = \frac{\Pi_{f,t+1}}{N_{e,t}}, \quad (82)$$

where we have also used the clearing of the market for the final good, labor, physical capital, and the entrepreneurial firms' equity and the fact that aggregate net wealth equals individual net wealth, i.e. Equations (47), (49), (50), (51), and (57) and the balance sheet of the entrepreneurs, i.e. Equation (6), together with $dv_{e,t} = 0$.³⁶

Bankers Equations (25) and (27) gives us

$$\nu_{b,t} = \mathbb{E}_t \Lambda_{b,t+1} \rho_{b,t+1}. \quad (83)$$

The laws of motion of bankers net worth reflected in Equations (58) and (61) also part of the equilibrium. Hence, we have

$$N_{b,t+1} = \theta_b \rho_{b,t+1} N_{b,t} + \iota_{b,t+1} - T_t \text{ and} \quad (84)$$

$$\iota_{b,t+1} = \chi_b (1 - \theta_b) \rho_{b,t+1} N_{b,t}. \quad (85)$$

Banks Equations (28), (29), (31), (33), (34), and (35) from the banks' problem are also part of the equilibrium conditions. Hence, we have

$$B_{f,t} = N_{b,t} + D_t, \quad (86)$$

$$N_{b,t} = \phi B_{f,t}, \quad (87)$$

$$\tilde{R}_{f,t+1}(\omega_j) = [\Gamma_{i,t+1}(\bar{\omega}_{t+1}(\omega_j)) - \mu_f G_{i,t+1}(\bar{\omega}_{t+1}(\omega_j))] \frac{\omega_j [q_{t+1}(1 - \delta) K_t + Y_{t+1}]}{B_{f,t}}, \quad (88)$$

$$\tilde{R}_{f,t+1}(\bar{\omega}_{j,t+1}) B_{f,t} - R_{d,t} D_t = 0, \quad (89)$$

$$\Pi_{b,t+1} = \int_{\bar{\omega}_{j,t+1}}^{\infty} \tilde{R}_{f,t+1}(\omega_j) B_{f,t} dF_{j,t+1}(\omega_j) - R_{d,t} D_t (1 - F_{j,t+1}(\bar{\omega}_{j,t+1})), \text{ and} \quad (90)$$

$$\rho_{b,t+1} = \frac{\Pi_{b,t+1}}{\phi B_{f,t}}, \quad (91)$$

where we have also used the clearing of the market for the final good, physical capital, and banks' equity and the fact that aggregate net wealth equals individual net wealth, i.e. Equations (47), (50), (52), and (59) and the balance sheet of the bankers, i.e. Equation (23), together with $dv_{b,t} = 0$.

³⁶To simplify notation, we say that the derivative $\frac{\partial f}{\partial X}$ is evaluated at X , while the derivative $\frac{\partial f}{\partial x}$ is evaluated at x .

Capital production The evolution of capital is controlled by the FOC of the capital producer and the law of motion of capital, i.e. Equations (36) and (55)

$$q_t = \left[S' \left(\frac{I_t}{K_{t-1}} \right) \right]^{-1} \text{ and} \quad (92)$$

$$K_t = (1 - \delta) K_{t-1} + S \left(\frac{I_t}{K_{t-1}} \right) K_{t-1}. \quad (93)$$

Deposit insurance costs The deposit insurance cost Equation (66)

$$T_t = \Omega_t D_{t-1}. \quad (94)$$

Market clearing The aggregate resource constraint Equation (48) can be written as,

$$Y_t = C_t + I_t + \Sigma_{b,t} + \Sigma_{e,t}, \quad (95)$$

where we have also used the clearing of the market for the final good, labor, and physical capital, i.e. Equations (47), (49), and (50).³⁷

Appendix D Walras Law

This section shows that given the equilibrium conditions (67)-(93), the consumer budget constraint in Equation (1) is equivalent to the resource constraint in Equation (95).

Using Equations (21), (56), (62) and several clearing market conditions is easy to see that profits of entrepreneurial firms, $\Pi_{f,t}$, equals to the aggregate net worth of entrepreneurs after net transfers from households, $N_{e,t}$, plus the net transfers from the entrepreneurs to households, $\Upsilon_{e,t}$. Thus,

$$N_{e,t} + \Upsilon_{e,t} = \Pi_{f,t} = \int_0^\infty \int_{\bar{\omega}_{t+1}(\omega_j)}^\infty \{ \omega_i \omega_j [q_{t+1} (1 - \delta) k_t + y_{t+1}] - R_{f,t} B_{f,t} \} dF_{i,t}(\omega_i) dF_{j,t}(\omega_j). \quad (96)$$

Using Equations (35), (58), (63) and several clearing market conditions is easy that profits of banks, $\Pi_{b,t}$, equals to the aggregate net worth of bankers after net transfers from households, $N_{b,t}$, plus the net transfers from the bankers to households, $\Upsilon_{b,t}$, plus lump-sum taxes T_t . Thus,

$$N_{b,t} + \Upsilon_{b,t} = \Pi_{b,t} - T_t = \int_{\bar{\omega}_{j,t}}^\infty \tilde{R}_{f,t}(\omega_j) b_{f,t-1} dF_{j,t}(\omega_j) - R_{d,t-1} D_{t-1} (1 - F_{j,t}(\bar{\omega}_{j,t})) - T_t, \quad (97)$$

³⁷Appendix D shows that the aggregate resource constraint in Equation (95) implies the household budget constraint in Equation (1).

where we have used several clearing market conditions. From Equation (37) and several clearing market conditions, we also have that

$$T_t = F_{j,t}(\bar{\omega}_{j,t}) R_{d,t-1} D_{t-1} + (1 - \mu_j) \int_0^{\bar{\omega}_{j,t}} \tilde{R}_{f,t}(\omega_j) B_{f,t-1} dF_{j,t}(\omega_j), \quad (98)$$

and therefore

$$\Pi_{b,t} + R_{d,t-1} D_{t-1} - T_t = \int_0^\infty \tilde{R}_{f,t}(\omega_j) B_{f,t-1} dF_{j,t}(\omega_j) - \Sigma_{b,t}. \quad (99)$$

The loan return on of bank in island j is given by

$$\begin{aligned} \tilde{R}_{f,t}(\omega_j) B_{f,t-1} &= \int_0^\infty \min \{ \omega_i \omega_j [q_t (1 - \delta) K_{t-1} + Y_t], R_{f,t-1} B_{f,t-1} \} dF_{i,t}(\omega_i) \\ &\quad - \mu_f \int_0^{\bar{\omega}_t(\omega_j)} \omega_i \omega_j [q_t (1 - \delta) K_{t-1} + Y_t] dF_{i,t}(\omega_i). \end{aligned} \quad (100)$$

If we substitute Equation (100) into Equation (99), then we get

$$\begin{aligned} &\Pi_{b,t} + R_{d,t-1} D_{t-1} - T_t \\ &= \int_0^\infty \int_0^\infty \min \{ \omega_i \omega_j [q_t (1 - \delta) K_{t-1} + Y_t], R_{f,t-1} B_{f,t-1} \} dF_{i,t}(\omega_i) dF_{j,t}(\omega_j) \\ &\quad - \mu_f \int_0^\infty \int_0^{\bar{\omega}_t(\omega_j)} \omega_i \omega_j [q_t (1 - \delta) K_{t-1} + Y_t] dF_{i,t}(\omega_i) dF_{j,t}(\omega_j) - \Sigma_{b,t} \\ &= \int_0^\infty \int_0^\infty \min \{ \omega_i \omega_j [q_t (1 - \delta) K_{t-1} + Y_t], R_{f,t-1} B_{f,t-1} \} dF_{i,t}(\omega_i) dF_{j,t}(\omega_j) - \Sigma_{e,t} - \Sigma_{b,t}. \end{aligned} \quad (101)$$

Clearly, it is the case that

$$\Pi_{f,t} = \int_0^\infty \int_0^\infty \max \{ \omega_i \omega_j [q_t (1 - \delta) K_{t-1} + Y_t], R_{f,t-1} B_{f,t-1} \} dF_{i,t}(\omega_i) dF_{j,t}(\omega_j). \quad (102)$$

Adding (101) and (102), we get

$$\begin{aligned} &\Pi_{f,t} + \Pi_{b,t} + R_{d,t-1} D_{t-1} - T_t \\ &= \int_0^\infty \int_0^\infty \omega_i \omega_j [q_t (1 - \delta) K_{t-1} + Y_t] dF_{i,t}(\omega_i) dF_{j,t}(\omega_j) - \Sigma_{e,t} - \Sigma_{b,t} \\ &= q_t (1 - \delta) K_{t-1} + Y_t - \Sigma_{e,t} - \Sigma_{b,t}. \end{aligned} \quad (103)$$

Using the fact that $N_{e,t} + \Upsilon_{e,t} = \Pi_{f,t}$, $N_{b,t} + \Upsilon_{b,t} = \Pi_{b,t} - T_t$, and $\Upsilon_t = \Upsilon_{b,t} + \Upsilon_{e,t}$ in Equation (103), we get

$$N_{e,t} + N_{b,t} + \Upsilon_t = q_t (1 - \delta) K_{t-1} + Y_t - R_{d,t-1} D_{t-1} - \Sigma_{e,t} - \Sigma_{b,t}. \quad (104)$$

Equation (104) can be written as

$$N_{b,t} + N_{e,t} = Y_t + q_t (1 - \delta) K_{t-1} - R_{d,t-1} D_{t-1} - \Sigma_{e,t} - \Sigma_{b,t} - \Upsilon_t. \quad (105)$$

Using Equations (76) and (86) we obtain

$$N_{b,t} + N_{e,t} - q_t K_t = w_t H_t - D_t. \quad (106)$$

Substituting Equation (106) into Equation (1), we get

$$C_t + q_t K_t = N_{b,t} + N_{e,t} + R_{d,t-1} D_{t-1} + \Upsilon_t + \Xi_t, \quad (107)$$

but since $\Upsilon_t = \Upsilon_{e,t} + \Upsilon_{b,t}$, if we substitute Equation (105) into Equation (107), we get

$$C_t + q_t K_t = Y_t + q_t (1 - \delta) K_{t-1} - \Sigma_{e,t} - \Sigma_{b,t} + \Xi_t. \quad (108)$$

But we have that

$$\Xi_t = q_t S \left(\frac{I_t}{K_{t-1}} \right) K_{t-1} - I_t, \quad (109)$$

but using Equation (55) we obtain that

$$\Xi_t = q_t (K_t - (1 - \delta) K_{t-1}) - I_t. \quad (110)$$

If we substitute Equation (110) into Equation (108) we obtain

$$C_t + I_t = Y_t - \Sigma_{e,t} - \Sigma_{b,t} \quad (111)$$

that it is equal to Equation (95) by the definitions of $\Sigma_{e,t}$ and $\Sigma_{b,t}$.

Appendix E Approximating Banks' Expected Profits

As mentioned, in order to use perturbation methods to approximate the solution to the model we need to compute bank's expected return on the loan portfolio (conditional on not defaulting), defined here as $R_{p,t+1}$, which is part of Equation (34) and is given by the integral defined in Equation (43).

In the subsequent analysis, we take $q_{t+1}, k_t, y_{t+1}, b_{f,t}, R_{d,t}, d_t$ as given and use the notation of $\tilde{R}_{f,t+1}$ to be the function of island shock, ω_j , only. From the analysis in Section 4.1, it should be clear that the bank's loan return $\tilde{R}_{f,t+1}(\omega_j)$ is not log-normally distributed. Mathematically, this is due to the fact that $\Gamma_{i,t+1}(\bar{\omega}_{t+1}(\omega_j))$ and $G_{i,t+1}(\bar{\omega}_{t+1}(\omega_j))$ which enter into $\tilde{R}_{f,t+1}(\omega_j)$ are both non-linear functions of ω_j . As a result of highly non-linear shape of $\tilde{R}_{f,t+1}(\omega_j)$, the integral in Equation (43) cannot be computed as explicit function of the state variables and perturbation methods cannot be applied. We overcome this challenge by (i) splitting this integral into the sum of integrals taken over smaller intervals, (ii) computing a series of

quadratic Taylor approximations of $\tilde{R}_{f,t+1}(\omega_j)$ around a mid-point of each interval.

Formally, we split the domain of ω_j into N intervals of equal length defined on $N + 1$ points x_k ranging from $x_1 = \bar{\omega}_{j,t+1}$ to $x_{N+1} = \omega_j^{\max}$ where the highest point ω_j^{\max} is chosen such that $\tilde{R}_{f,t+1}(\omega_j^{\max}) = R_{f,t}$ almost surely. Given those assumptions, $R_{p,t+1}$ is approximately given by:

$$R_{p,t+1} \approx \sum_{k=1}^N \left(\int_{x_k}^{x_{k+1}} \Theta^k(\omega_j) dF_{j,t+1}(\omega_j) \right) + [1 - F_{j,t+1}(x_{N+1})] R_{f,t} \quad (112)$$

where $\Theta^k(\omega_j)$ is a Taylor approximation of $\tilde{R}_{f,t+1}(\omega_j)$ around a point $\omega_j = \bar{x}_k \equiv \frac{x_{k+1} + x_k}{2}$ and is given by

$$\Theta^k(\omega_j) = \tilde{R}_{f,t+1}(\bar{x}_k) + \tilde{R}'_{f,t+1}(\bar{x}_k)(\omega_j - \bar{x}_k) + \frac{1}{2} \tilde{R}''_{f,t+1}(\bar{x}_k)(\omega_j - \bar{x}_k)^2 \quad (113)$$

All the derivatives of $\tilde{R}_{f,t+1}$ are with respect to ω_j and can be computed as an explicit functions of the state variables. Using the simplified expression for $\Theta^k(\omega_j)$ we can rewrite $\int_{x_k}^{x_{k+1}} \Theta^k(\omega_j) dF_{j,t+1}$ as follows:

$$\int_{x_k}^{x_{k+1}} \Theta^k(\omega_j) dF_{j,t+1} = Q_0(\bar{x}_k) + Q_1(\bar{x}_k) \int_{x_k}^{x_{k+1}} \omega_j dF_{j,t+1} + Q_2(\bar{x}_k) \int_{x_k}^{x_{k+1}} \omega_j^2 dF_{j,t+1} \quad (114)$$

where: $Q_i(\bar{x}_k)$ are just constants given by:

$$\begin{aligned} Q_0(\bar{x}_k) &= [F_{j,t+1}(x_{k+1}) - F_{j,t+1}(x_k)] \left[\tilde{R}_{f,t+1}(\bar{x}_k) - \bar{x}_k \tilde{R}'_{f,t+1}(\bar{x}_k) + \frac{1}{2} \bar{x}_k^2 \tilde{R}''_{f,t+1}(\bar{x}_k) \right], \\ Q_1(\bar{x}_k) &= [F_{j,t+1}(x_{k+1}) - F_{j,t+1}(x_k)] \left[\tilde{R}'_{f,t+1}(\bar{x}_k) - \frac{1}{2} \bar{x}_k \tilde{R}''_{f,t+1}(\bar{x}_k) \right], \\ Q_2(\bar{x}_k) &= [F_{j,t+1}(x_{k+1}) - F_{j,t+1}(x_k)] \left[\frac{1}{2} \tilde{R}''_{f,t+1}(\bar{x}_k) \right], \end{aligned}$$

Given our assumption of log-normally distributed island shock, ω_j , we have expressions for $\int_{x_k}^{x_{k+1}} \omega_j dF_{j,t+1}$ and $\int_{x_k}^{x_{k+1}} \omega_j^2 dF_{j,t+1}$ as explicit functions of the state variables. Consequently, we can easily derive very accurate, the approximation of $R_{p,t+1}$ in Equation (112) as an explicit function of the state variables.

Appendix F IRFs

We consider the GIRF proposed by Koop, Pesaran, and Potter (1996). The GIRF for any variable in the model var in period $t + l$ following a disturbance to the n^{th} shock of size ν_n in period $t + 1$ is defined as

$$GIRF_{var}(l, \epsilon_{n,t+1} = \nu, \mathbf{w}_t) = \mathbb{E}[var_{t+l} | \mathbf{w}_t, \epsilon_{n,t+1} = \nu] - \mathbb{E}[var_{t+l} | \mathbf{w}_t], \quad (115)$$

where $\tilde{\mathbf{z}}_t$ are the value of the state variables of the model at time t (see Equation (42)) and $n \in \{A, \delta, i, j\}$. Hence, the GIRF depend on the value of the state variables when the shocks hits. For example,

$$GIRF_{\Delta \log Y_t}(4, \epsilon_{A,t+1} = -3, (1.1D_{ss}, 0.9K_{ss}, \dots, 1.01\bar{\sigma}_{\omega_i}))$$

is the GIRF of GDP growth, $\Delta \log Y_t$, at period $t + 4$, after a TFP shock of value -3 in period $t + 1$, when D_t was 10 percent above the steady state, K_t was 10 percent below the steady state, \dots , and $\sigma_{\omega_i,t}$ was one percent above steady state.

But GIRF defined in Equation (115) are conditioned on the value of the state variables when the shocks hits. In what follows, instead we want to compute GIRFs that are conditioned on the values of observables when the shocks hits. For example, we would condition on the expected default rate of firms, $ED_{f,t}$, to be above one percent at the time of the shock. In this case, we want to compute the following GIRF

$$GIRF_{var}(l, \epsilon_{n,t+1} = \nu, ED_{f,t} > 0.01) = \int \mathbf{1}_{\{ED_{f,t} > 0.01\}}(\mathbf{w}_t) GIRF_{var}(l, \epsilon_{n,t+1} = \nu, \mathbf{w}_t) f(\mathbf{w}_t) d\mathbf{w}_t, \quad (116)$$

where $\mathbf{1}_{\{ED_{f,t} > 0.01\}}(\mathbf{w}_t)$ takes a value equal to one if the state variables at time t are such that he expected default rate of firms is above one percent at time t and zero otherwise and where $f(\mathbf{w}_t)$ is the unconditional density of the state variables. Of course Equation (116) needs to be computed by simulation.