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Early Birds and Second Mice in the Stock Market

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Abstract

This paper studies learning in the stock market. Our contribution is to propose a model to illustrate the endogenous timing decision on trading, taking into account the incentive of learning from others about the fundamental value. The model is similar to Easley and O'Hara (1992), except that we introduce less-informed traders whose private information is inferior to fully-informed traders, but superior to that of random noise traders, and a zero-profit market maker. We also allow both types of informed traders to optimize timing of trading. We show that fully-informed traders act as early birds because it is optimal for them to buy or sell at the earliest possible time; meanwhile, less-informed traders could be better off as second mice by delaying transactions to learn from previous trades. The greater information asymmetry between the less-informed traders and the market maker, the larger profits the former could make even though the latter is learning from all trades.

JEL Codes: D83, G12, G14.

Keywords: Market microstructure, insider trading, learning, strategic timing.

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"The early bird catches the worm and the second mouse gets the cheese."

1 Introduction

In many economies, social learning plays an important and strategic role in agents' decision making. For example, in choosing whether to adopt an innovation of uncertain quality, forward-looking consumers may realize the value of delaying decision and waiting for more information from others' adoption behavior or feedback. This "waiting strategy" has attracted the attention of many economists (Frick and Ishii, 2015; Young, 2009; Ellison and Fudenberg, 1993; Jensen, 1982), however, it has not been much explored in the stock market.

In the market microstructure literature, starting from Kyle (1985) and Glosten and Milgrom (1985), it is common to model three types of agents in the stock market: informed traders who have private information about stocks' fundamental value; uninformed traders who trade because of liquidity needs (also known as, noise traders); and the market maker, who behaves competitively and sets prices efficiently conditional on the quantities traded by others. The literature shares the wisdom that: first, an informed trader have the incentive to buy or sell stock shares because the initial stock price quoted by the market maker is unlikely to reflect its fundamental value; second, informed traders' orders transmit information to the market maker who immediately adjusts the price quotes closer to the true value.

Now, suppose that there are other agents who are partially informed about the state of the asset in the stock market, we ask an important question: could those agents benefit by waiting to learn from those with better private information? Moreover, once we allow for this option of delaying the trading opportunity to the future, what will be the optimal trading timing for the perfectly-informed?

Intuitively, a perfectly-informed trader has no obvious incentive to "wait to learn", and neither does an uninformed trader who learns as much as the market maker. The incentive of waiting to learn comes from agents with imperfect private information. In our model, we introduce two types of informed agents: fully-informed and less-informed traders, in addition to noise traders and a zero-profit market maker. In terms of informational characteristics, suppose that less-informed traders receive a private signal on the fundamental value. If this is the case, the market maker is learning from the fully-informed as fast as the less-informed, and thus it is not easy to illustrate the benefits from waiting. We therefore take the "event uncertainty" information structure from Easley and O'Hara (1992), which assumes that there is uncertainty on whether an information event has occurred or not. The information event usually refers to a shock that reveals the stock's fundamental value to some traders. Though this assumption has often been used in sequential trading market microstructure models, except for Cipriani and Guarino (2014), it never plays the role as a layer of information asymmetry between informed traders and the market maker once we control for the information asymmetry on the stock value. In our model, we assume that less-informed traders have private information about the information event, as they know with certainty whether the market is now in a new state due to the information event or remains in the old state. This constitutes the informational advantage of the less-informed relative to the market maker.

To elaborate, we assume that if the event does not occur, all agents in the market share the same information, thus there is no bid-ask spread; and if the event occurs, some traders become fully informed of the new fundamental value, while some become the less-informed type who privately know state has changed and therefore are aware of the existence of fully-informed traders. Due to this informational advantage, the less-informed can infer more accurate information about the fundamental from previous orders compared to the market maker. Therefore, given an early trading opportunity, the less-informed could possibly profit from waiting and learning from others.

When including endogenous timing, solving for the equilibrium can easily become involved. To make the model tractable, we consider three periods and follow Glosten and Milgrom (1985)'s game structure in which the nature randomly chooses one investor with trading opportunity in each period. We find that in equilibrium, when the information event occurs, the fully-informed will act at the earliest possible time to profit, while under some conditions the less-informed prefer to wait until there are previous orders for them to learn from. In this "waiting equilibrium", both types of informed traders make positive profits. Fully-informed traders are the early bird catching worms as the information arbitrage, and less-informed are second mice getting the cheese thanks to learning from predecessors. Moreover, we show that the greater the information asymmetry between the less-informed and the market maker (i.e., the smaller probability of event taking place), the more cheese the former can get.

The benchmark model is simple and stylized in order to capture the key dynamics. In this model, the "waiting strategy" is not literally strategic as the less-informed make zero expected profits if they trade in the first period. In one extension, we try to address this issue by setting an exogenous reward to any order in the first period. We prove that, under some conditions, less-informed traders still prefer to wait even by deviating they are earning positive profits in the first period.

An outline of this paper is as follows. We review the related literature in Section 2. In Section 3, we introduce a stylized model to illustrate the informational benefits from waiting. Section 4 includes two simple model extensions. Finally, in Section 5 we conclude.

2 Literature Review

This paper contributes to an extensive finance literature on the role of information in price formation[1]. Glosten and Milgrom (1985) and Kyle (1985) are the two canonical models of market microstructure. Kyle (1985) considers the problem of a single strategic informed investor who decides how much to trade at each point in time and a zero-profit market maker who learns from the order flow and behave competitively. Glosten and Milgrom (1985) assume that investors, informed investors and purely liquidity traders, arrive one by one randomly and anonymously, and make unit trade decision. Both Glosten and Milgrom (1985) and Easley and O'Hara (1987) show that a positive bid-ask spread is present in a market with noise investors and non-strategic informed investors.

There have been many models extending Kyle (1985)'s monopolistic trader model to include traders with heterogeneous information. Holden and Subrahmanyam (1992) show that if the private signal is common to all informed traders, they trade very aggressively and this competition among them makes their information revealed almost immediately. Foster et al. (1994) analyze the situation in which one informed investor has a better signal than the others, and they find that this investor acts less aggressively on his private signal. In a model with informed traders who receive heterogeneous private signals, Foster and Viswanathan (1996) predict that the higher the correlation between those private signals, the higher the aggressiveness. Back et al. (2000) analyze competition among informed traders in continuous time as Foster and Viswanathan (1996) do in discrete time.

[1] see Pastor and Veronesi (2009) for a survey.

Some other papers suggest that investors could also be heterogeneous in many other dimensions. Harrison and Kreps (1978) assume the existence of heterogeneous expectations within the pool of potential investors and look for consistent price schemes. Zhang (2008) examines the setting of asymmetrically informed traders with disclosure, analyzes an N-period dynamic market where outsiders observe part of the information about a security prior to trading and update their incomplete information by learning from disclosed insider trades during trading. Liu and Zhang (2011) consider a model with four kinds of traders: one insider with private information and shared information, some risk neutral outsiders with access to the shared information, noise traders, and competitive market makers. They find that public information harms the insider but benefits the outsiders and noise traders. Cipriani and Guarino (2014) focus on the herd behavior in financial markets where non-strategic investors receive heterogeneous signals. To overcome the lack of tractability, Cipriani and Guarino (2014) use a particular distributional assumption on the signal which allows them to characterize the equilibrium in a recursive way. Their empirical finding shows that, through the lens of their model, in most of the trading periods, a small positive measure of informed traders herd.

A key assumption of our model is that investors are strategic in choosing when to trade. Back and Baruch (2004) solve a version of Glosten and Milgrom (1985) in which a strategic informed trader optimizes his times of trading. They show that the model converges to Kyle (1985) model as the trade size gets smaller. However, there is no "strategic learning" in their context, which makes their paper significantly different from ours. Finally, in our model, periods without trades also convey information. The importance of these periods is stressed in Diamond and Verrecchia (1987) in the context of short selling restrictions.

3 The Model

This section introduces the baseline model of this paper. We first present our model settings in section 3.1, and later we solve for an equilibrium which illustrates the informational benefits gained through waiting in section 3.2.

3.1 Setup

In the three-period sequential trade model, we consider an asset whose eventual value can take one of the two values: V_h and V_l. There are potential buyers and sellers and a market maker who quotes prices to buy and sell. All agents are risk-neutral in our model. The market maker acts competitively and thus makes zero profit by setting the price equal to the expected value of this asset.

The market's information structure before trading starts, similar to Easley and O'Hara (1992), is illustrated in Figure I. In the original state (at the first node), everyone in the market shares the same prior belief: with probability δ, the asset takes value V_l; and with probability $1-\delta$, it is of value V_h. Then nature selects whether an information event occurs with the probability of occurrence α. If the information event does not occur, there is no change to the informational characteristics of the market. Then all traders are uninformed and they may choose to buy, sell, or not trade with equal probabilities due to their liquidity needs.

If the information event occurs, the market enters a new state and the value of the asset might change. At the second node in the branch of event occurrence, potential buyers and sellers are divided into three groups. The first type, constitutes a proportion of μ, is fully-informed of the eventual asset value in this new state. Less-informed traders, accounting for a proportion of q, form the second type. Those traders privately know the change of the state, but they do not have any updated information on the eventual asset value in the new state. Both fully-informed and less-informed traders are rational and strategic. They buy if they expect the asset value to be higher than the ask price, and sell if the bid price is higher than their expectation. Moreover, they can also choose not to trade in a certain trading period. To ease the exposition, we denote the buy action as B, the sell action as S, and no trading as ϕ. Finally, the third type is the uninformed trader (noise trader) who is assumed to buy, sell, or not trade, all with a probability $\frac{1}{3}$ for simplicity.[2]

The market maker knows this trade process and the parameter values α, δ, μ and q all of which we assume are strictly between zero and one. What she does not know is whether an information event has occurred, whether the eventual asset value becomes V_h or V_l given that it has occurred, and whether any particular trader is informed. Everyone including the market maker can observe all

[2] In Section 4, we provide a different set-up with a slight modification on the current one. In the new setting, we assume that if the event does not occur, only noise traders, accounting for a proportion of $1-\mu-q$ out of all potential buyers and sellers, stay in the market because of liquidity needs.

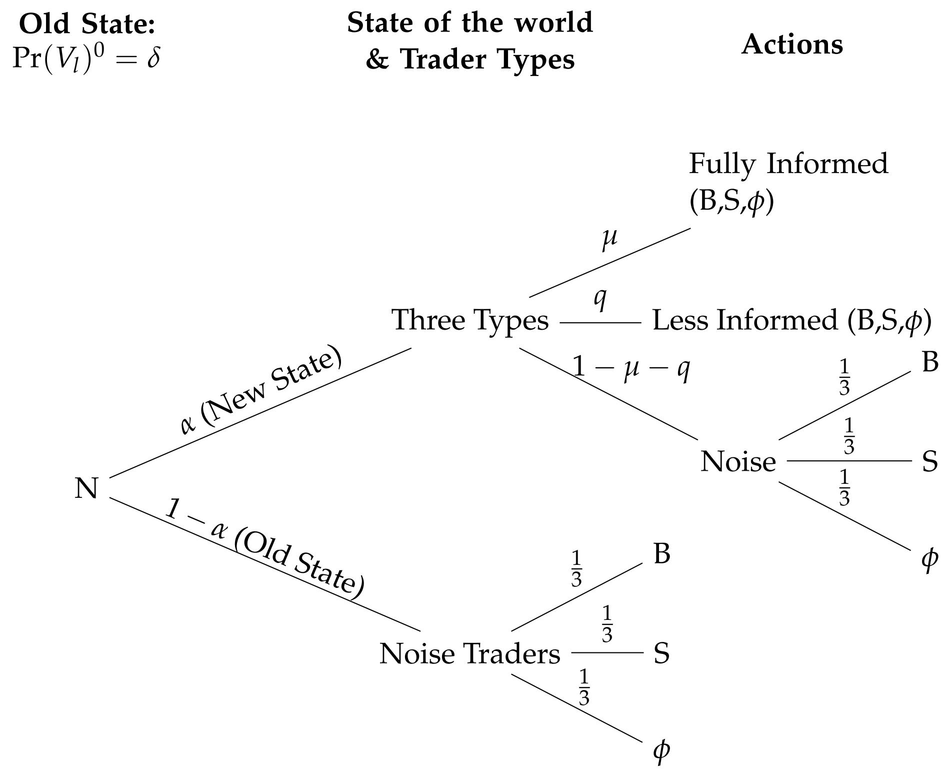


Figure I: Tree diagram of the trading process

trades. Price quotes are revised once the market maker learns new information from the trades, and are always set to her expected fundamental value.

Trading begins after realization of this "event uncertainty". The market operates for three periods ($T = 1, 2, 3$), and in each period, one trader is randomly selected to participate in the trading process. If he buys or sells, he leaves the market after current period, and a new trader of the same type will replace him in the pool of potential buyers and sellers for the next period. If he chooses not to trade, he returns to the pool and will be selected with the same probability in the next period. The potential trader composition is always constant over the three periods: if the market is in the new state, it has a proportion of μ fully-informed, a proportion of q less-informed, and a proportion of $1 - \mu - q$ uninformed; if the market is in the old state, all traders are uninformed.

3.2 The Equilibrium

If information event does not occur, the market outcome is trivial as only noise traders are trading and the price quote set by the market maker is $\delta V_l + (1 - \delta)V_H$. It is not the focus of this paper. Instead, we are mainly interested in the market outcome when an information event occurs, especially in the following candidate equilibrium where FI refers to a fully-informed trader; LI refers to a less-informed trader; MM refers to the market maker; **a** denotes the ask price and **b** denotes the bid price; subscript $\{1,2,3\}$ indicates the period:

$$\begin{aligned}
\{\mathbf{T=1}: &(\text{FI, B if } V_h, \text{ S if } V_l),\ (\text{LI}, \phi),\ (\text{MM}, a_1 = \mathrm{E}_{\mathrm{MM}}[v \mid B_1], b_1 = \mathrm{E}_{\mathrm{MM}}[v \mid S_1]);\\
\mathbf{T=2}: &(\text{FI, B if } V_h, \text{ S if } V_l),\ (\text{LI}, \phi \text{ if } \phi_1), (\text{MM},\ a_2 = \mathrm{E}_{\mathrm{MM}}[v \mid Q_1, B_2],\ b_2 = \mathrm{E}_{\mathrm{MM}}[v \mid Q_1, S_2]);\\
\mathbf{T=3}: &(\text{FI}, \text{B if } V_h, \text{ S if } V_l),\ (\text{LI, B if } B_2 \text{ and } \phi_1, \text{ S if } S_2 \text{ and } \phi_1,\ \phi \text{ if } \phi_2 \text{ and } \phi_1),\\
&(\text{MM},\ a_3 = \mathrm{E}_{\mathrm{MM}}[v \mid Q_1, Q_2, B_3],\ b_3 = \mathrm{E}_{\mathrm{MM}}[v \mid Q_1, Q_2, S_3]).\}
\end{aligned}$$

In this equilibrium, a fully-informed trader chosen to trade in the first period will buy or sell depending on the fundamental, and then he leaves the market. If this trading opportunity is given to a less-informed trader, he will choose to wait and return to the candidate pool for the next period. At $T = 2$, if another fully informed trader is chosen, he buys if the asset value is V_h and sells if it is V_l. If a less-informed trader is chosen, he waits again if there was no previous trade. In the final period, the strategy of a fully-informed trader is the same as in the previous two periods, while a less-informed trader buys (sells) if there was at least a buy (sale) before $T = 3$. The assumptions of risk neutrality and competitive behavior dictate that, in the equilibrium, the market maker sets ask (bid) price equal to the expected value of the asset conditional on trade history and a current buy (sale). The trade outcome in the first (second) period is denoted as Q_1 (Q_2).[3]

While this equilibrium involves many possible realizations, we focus specifically on one of them:

$$\begin{aligned}
\{\mathbf{T=1}: &(\text{LI}, \phi);\\
\mathbf{T=2}: &(\text{FI, B if } V_h, \text{ S if } V_l), (\text{MM},\ a_2 = \mathrm{E}_{\mathrm{MM}}[v \mid \phi_1, B_2],\ b_2 = \mathrm{E}_{\mathrm{MM}}[v \mid \phi_1, S_2]);\\
\mathbf{T=3}: &(\text{LI, B if } B_2, \text{ S if } S_2),\ (\text{MM},\ a_3 = \mathrm{E}_{\mathrm{MM}}[v \mid \phi_1, B_2, B_3],\ b_3 = \mathrm{E}_{\mathrm{MM}}[v \mid \phi_1, S_2, S_3]).\}
\end{aligned}$$

The reason is that this market outcome illustrates "waiting to learn" strategy and its informational

[3] Note that, we have not characterized the best responses of less-informed traders at $T = 2$ and $T = 3$ when there is a buy or sale at $T = 1$. It is not trivial to characterize as the dominant strategy depends on parameter values. Since we only focus on the equilibrium realization where a less-informed is chosen at $T = 1$ and he waits, we skip the analysis of those two responses.

gains. As such, in the rest of this section, we will go through period by period and prove why this realization constitutes an equilibrium. To begin with, we first introduce two propositions that can be straightforwardly proven.

Proposition 1 *Sale (buy) by a fully-informed trader when the fundamental value is V_h (V_l) never constitutes an equilibrium.*

Suppose that in equilibrium, a fully-informed trader sells when the asset value is high. The market maker sets her bid price equal to $\mathrm{E}[v \mid S]$ and it is obvious that $V_h > \mathrm{E}[v \mid S]$. Selling the stock at a price that is lower than the fundamental value yields negative profits for the fully-informed trader, thus it is never optimal. Similar logic also applies to eliminate the case of buying when the asset value is low as an equilibrium.

Proposition 2 *A less-informed trader's optimal strategy is not to trade in the first and any other periods if no trade has occurred before.*

Regardless of the optimal strategy of fully-informed traders and less-informed traders (B, S, or ϕ), upon observing a buy (sale), following Proposition 1, the market maker either updates her belief and forms a higher (lower) expectation of the assets expected value, or her belief remains unchanged. In either case, a less-informed trader expects a non-positive payoff from trading, because he still holds his prior belief, which is the same as the market maker's, without historical trades. Hence, it is optimal to wait for less-informed traders when there is nothing to learn from previous periods.

The First Period (T=1) If a noise trader is selected, he buys, sells, or does not trade with equal probabilities. As stated in Proposition 2, if a less-informed trader is chosen at $T = 1$, he will not trade. For fully-informed traders, at the moment, we take their optimal strategy as buying (selling) if the asset value is V_h (V_l). At the end of this section, we will check for fully-informed traders' profitable deviations and confirm that the equilibrium we are currently characterizing does exist.

We first show how the market maker updates her beliefs upon observing no trade in the first period. Before introducing the algebra, we define the notations for the three different states. **H** represents that the market is in a new state (where an information event occurs) and the asset's fundamental value is V_H; **L** also refers to the new state but with the fundamental being V_l; finally,

0 indicates the state without the occurrence of information event. In the first period, according to Proposition 1 and 2, "no trade" is either because an information event occurs and a less-informed trader is chosen, or, because a noise trader is chosen in any of the three states. The conditional probability of each of the three possible states, are given by Bayes rule:

$$\Pr\{\Psi = 0 \mid \phi_1\} = \frac{(1-\alpha)\frac{1}{3}}{\alpha q + \frac{1}{3}(1-\alpha\mu-\alpha q)} \tag{1}$$

$$\Pr\{\Psi = L \mid \phi_1\} = \frac{\alpha\delta(\frac{1}{3}(1-\mu-q)+q)}{\alpha q + \frac{1}{3}(1-\alpha\mu-\alpha q)} \tag{2}$$

$$\Pr\{\Psi = H \mid \phi_1\} = \frac{\alpha(1-\delta)(\frac{1}{3}(1-\mu-q)+q)}{\alpha q + \frac{1}{3}(1-\alpha\mu-\alpha q)} \tag{3}$$

With the updated probability of three states, we can calculate the conditional probability of the low value V_l given no trade. If the state is 0, the probability remains the prior one: δ. If the state is H, then the probability is zero. In contrast, if the state is L, the probability is one. To summarize, the market-maker's conditional probability of the fundamental being V_L given no trade, denoted by δ_1, is:

$$\begin{aligned}\delta_1 &= \Pr\{v = V_l \mid \phi_1\} \\ &= 1 \cdot \Pr\{\Psi = L \mid \phi_1\} + \delta \Pr\{\Psi = 0 \mid \phi_1\}\end{aligned} \tag{4}$$

Moreover, we also calculate the market market's updated belief on the probability of event occurrence, denoted by α_1:

$$\alpha_1 = \Pr\{\Psi = L \mid \phi_1\} + \Pr\{\Psi = H \mid \phi_1\} \tag{5}$$

We treat (δ_1, α_1) as two state variables of the first period. All these conditional probabilities can be rewritten as functions of the two state variables: $\Pr\{\Psi = L \mid \phi_1\} = \delta_1 - \delta(1-\alpha_1)$, $\Pr\{\Psi = H \mid \phi_1\} = \alpha_1 - (\delta_1 - \delta(1-\alpha_1))$ and $\Pr\{\Psi = 0 \mid \phi_1\} = 1 - \alpha_1$.

Substituting Equations 1, 2 and 3 into Equation 4 and 5 yields:

$$\delta_1 = \delta \tag{6}$$

$$\alpha_1 = \frac{\alpha q + \frac{1}{3}\alpha(1-\mu-q)}{\alpha q + \frac{1}{3}(1-\alpha\mu-\alpha q)} \tag{7}$$

Equation 6 presents an intuitive result: as both of less-informed traders and noise trades do not own more information on the fundamental compared to the market maker, the market maker cannot gain new information and thus her belief on the probability of V_l stays the old level. However, the outcome of no trade is informative regarding the likelihood of an information event taking place. The market maker extracts information from no trade and adjusts her belief on α. Whether the updated α_1 is higher or lower than the prior depends on parameters μ and q.[4]

In the last step, we simplify some conditional probabilities of the state: $\Pr\{\Psi = L \mid \phi_1\} = \delta\alpha_1$, $\Pr\{\Psi = H \mid \phi_1\} = \alpha_1(1-\delta)$ for future needs.

The Second Period (T=2) Continuing on "no trade" outcome, in the second period, we focus on the case that a fully-informed trader is chosen to trade and he buys (sells) if the fundamental value is high (low). Again, we will check later if there exists a profitable deviation to "waiting". Without loss of generality, suppose that the fully-informed trader buys, we show how the market maker sets her ask price.

As in equilibrium, less-informed traders choose to wait if they are chosen, a buy can only come from a fully-informed trader when the state is **H** or a noise one. Following Bayes' rule, the updated conditional probabilities of three possible states, given that a buy occurs, are given by:

$$\begin{aligned}\Pr\{\Psi = 0 \mid \phi_1, B_2\} &= \frac{\Pr\{B_2 \mid \Psi = 0, \phi_1\}\Pr\{\Psi = 0 \mid \phi_1\}\Pr\{\phi_1\}}{\Pr\{B_2 \mid \phi_1\}\Pr\{\phi_1\}} \\ &= \frac{(1-\alpha_1)\frac{1}{3}}{\alpha_1\mu(1-\delta) + \frac{1}{3}(1-\alpha_1\mu-\alpha_1 q)}\end{aligned} \tag{8}$$

where $\Pr\{B_2 \mid \phi_1\} = \Pr\{B_2 \mid \Psi = 0, \phi_1\}\Pr\{\Psi = 0 \mid \phi_1\} + \Pr\{B_2 \mid \Psi = L, \phi_1\}\Pr\{\Psi = L \mid \phi_1\} + \Pr\{B_2 \mid \Psi = H, \phi_1\}\Pr\{\Psi = H \mid \phi_1\}$. Similarly, we have

[4] If $2q > \mu$, we have $\alpha_1 > \alpha$.

$$\Pr\{\Psi = L \mid \phi_1, B_2\} = \frac{\alpha_1\delta \cdot \frac{1}{3}(1-\mu-q)}{\alpha_1\mu(1-\delta) + \frac{1}{3}(1-\alpha_1\mu-\alpha_1 q)} \tag{9}$$

$$\Pr\{\Psi = H \mid \phi_1, B_2\} = \frac{\alpha_1(1-\delta)(\frac{1}{3}(1-\mu-q)+\mu)}{\alpha_1\mu(1-\delta) + \frac{1}{3}(1-\alpha_1\mu-\alpha_1 q)} \tag{10}$$

Knowing these conditional probabilities of states, we are able to calculate two state variables (δ_2, α_2). The market maker sets her ask price equal to $\delta_2 V_l + (1-\delta_2)V_h$.

$$\begin{aligned}
\delta_2 &= \Pr\{v = V_l \mid \phi_1, B_2\} \\
&= 1 \cdot \Pr\{\Psi = L \mid \phi_1, B_2\} + \delta \Pr\{\Psi = 0 \mid \phi_1, B_2\} \\
&= \delta[\frac{\frac{1}{3}(1-\alpha_1\mu-\alpha_1 q)}{\alpha_1\mu(1-\delta) + \frac{1}{3}(1-\alpha_1\mu-\alpha_1 q)}]
\end{aligned} \tag{11}$$

$$\alpha_2 = \Pr\{\Psi = L \mid \phi_1, B_2\} + \Pr\{\Psi = H \mid \phi_1, B_2\} \tag{12}$$

In this period, the market maker extract information from a buy on both the probability of V_l and the probability that there was no information event. It is straightforward to see that $\delta_2 < \delta_1 = \delta$. This result reflects the intuition that a buy serves as a positive signal for the asset value.

We also calculate less-informed traders' updated belief after observing a buy in the second period.

$$\begin{aligned}
\tilde{\delta_2} &= \Pr\{\Psi = L \mid B_2\} \\
&= \frac{\Pr\{B_2 \mid \Psi = L\} \Pr\{\Psi = L\}}{\Pr\{B_2\}} \\
&= \delta[\frac{\frac{1}{3}(1-\mu-q)}{(1-\delta)\mu + \frac{1}{3}(1-\mu-q)}]
\end{aligned} \tag{13}$$

Once again we get $\tilde{\delta_2} < \delta$. This is because the less-informed trader also revises his belief on $V = V_l$ to a lower level after the positive signal. An interesting comparison here is between the market maker's expectation and the less-informed's. It is easy to show that $\tilde{\delta_2} < \delta_2$, and thus $\mathrm{E}_{\mathrm{LI}}(v) > \mathrm{E}_{\mathrm{MM}}(v)$. From here, we can see that less-informed traders extract a stronger message from a trade, because they know with certainty that the fully-informed type is present. It highlights less-informed traders' informational advantage over the market maker, where the incentive of "waiting to learn" comes from.

Finally we simplify some conditional probabilities for the market maker's third-period Bayesian updating: $\Pr\{\Psi = L \mid \phi_1, B_2\} = \delta_2 - \delta(1-\alpha_2)$, $\Pr\{\Psi = H \mid \phi_1, B_2\} = \alpha_2 - (\delta_2 - \delta(1-\alpha_2))$ and $\Pr\{\Psi = 0 \mid \phi_1, B_2\} = 1 - \alpha_2$.

The Third Period (T=3) At $T = 2$, we illustrated less-informed trader's potential informational gains through learning. In the final period, we solve for the conditions under which the less-informed makes positive profits. We begin with the following proposition:

Proposition 3 *Following a no-trade at $T = 1$, and a buy (sale) at $T = 2$, a sale (buy) from a less-informed trader at $T = 3$ cannot constitute an equilibrium.*

It is easy to prove that $\mathrm{E}_{\mathrm{MM}}[v \mid \phi_1, B_2, S_3] < \mathrm{E}_{\mathrm{MM}}[v \mid \phi_1, B_2] < \mathrm{E}_{\mathrm{LI}}[v \mid B_2]$, i.e., the bid price at $T = 3$, equal to $\mathrm{E}_{\mathrm{MM}}[v \mid \phi_1, B_2, S_3]$, is smaller than the less-informed trader's expectation on the fundamental. As such, the less-informed trader will not sell, instead, he either buys or opts out of the market.

We study the equilibrium where the less-informed trader will buy at $T = 3$ conditional on no trade at $T = 1$ and a buy at $T = 2$. The conditional probabilities in the third period can be derived following the Bayes' rule. They are given by:

$$\Pr\{\Psi = L \mid \phi_1, B_2, B_3\} = \frac{(\delta_2 - \delta(1-\alpha_2))(q + \frac{1}{3}(1-\mu-q))}{\alpha_2 q + \frac{1}{3}(1 - \alpha_2\mu - \alpha_2 q) + \mu(\alpha_2(1-\delta) + (\delta - \delta_2))} \tag{14}$$

$$\Pr\{\Psi = 0 \mid \phi_1, B_2, B_3\} = \frac{\frac{1}{3}(1-\alpha_2)}{\alpha_2 q + \frac{1}{3}(1 - \alpha_2\mu - \alpha_2 q) + \mu(\alpha_2(1-\delta) + (\delta - \delta_2))} \tag{15}$$

Finally, we can calculate the ask price $\delta_3 V_l + (1-\delta_3)V_h$, which is a function of all the state variables, $\{\delta_1, \alpha_1, \delta_2, \alpha_2\}$. Those state variables include all relevant information in the trading history.

$$\begin{aligned}\delta_3 &= \Pr\{v = V_l \mid \phi_1, B_2, B_3\} \\ &= 1 \cdot \Pr\{\Psi = L \mid \phi_1, B_2,, B_3\} + \delta \Pr\{\Psi = 0 \mid \phi_1, B_2, B_3\} \\ &= \frac{(\delta_2 - \delta(1-\alpha_2))(q + \frac{1}{3}(1-\mu-q)) + \frac{1}{3}\delta(1-\alpha_2)}{\alpha_2 q + \frac{1}{3}(1 - \alpha_2\mu - \alpha_2 q) + \mu(\alpha_2(1-\delta) + (\delta - \delta_2))}\end{aligned} \tag{16}$$

For a less-informed trader to make positive profits from buying, it requires that $\delta_3 > \tilde{\delta_2}$. This condition is not always satisfied and the inequality sign depends on parameter values. Next, we will present the requirements on parameter values for the equilibrium of our interest, {**T=1**: (LI, ϕ); **T=2**: (FI, B if V_h, S if V_l), $(a_2 = \mathrm{E_{MM}}[v \mid \phi_1, B_2], b_2 = \mathrm{E_{MM}}[v \mid \phi_1, S_2])$; **T=3**: (LI, B if B_2, S if S_2), $(a_3 = \mathrm{E_{MM}}[v \mid \phi_1, B_2, B_3], b_3 = \mathrm{E_{MM}}[v \mid \phi_1, S_2, S_3])$}, to hold.

Equilibrium Conditions We need to show that there is no profitable deviation from the equilibrium to prove its existence. There are two possible deviations that we have not checked yet: a less-informed trader may prefer to opt out than trading at $T = 3$, and a fully-informed trader may prefer to wait than trading at $T = 1$.

To prevent the first deviation, we need parameter values to satisfy the following condition to make sure that a less-informed trader $T = 3$ makes non-negative profits by trading:

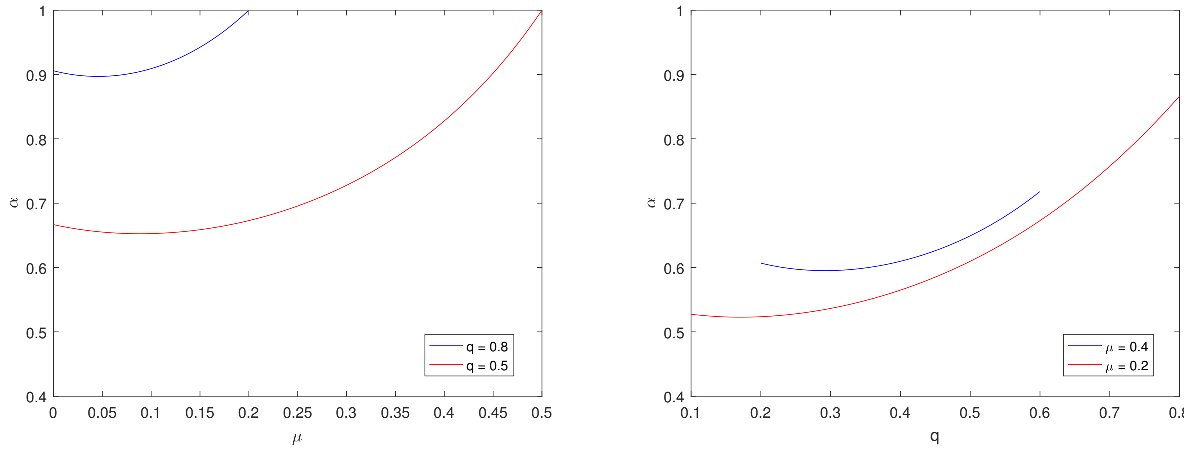
$$\mathrm{E_{MM}}(v \mid \phi, B_2, B_3) < \mathrm{E_{LI}}(v \mid B_2) \Longleftrightarrow \delta_3 > \tilde{\delta_2} \tag{17}$$

After some algebra, we obtain the condition:

$$\alpha(2 + 2q^3 + 3q(1-\mu)\mu - 3\mu^2 + 2\mu^3 - 3q^2(1+\mu)) < 1 \tag{18}$$

This condition is quite intuitive: the smaller the α, the more informational advantage less-informed traders have compared to the market maker, the more likely they beat the market at $T = 3$ by extracting more accurate information from the second-period trade.

Figure II depicts the threshold on α with respect to other two parameters μ and q. Though not universally true, in most cases the threshold α is increasing in μ (for a given q) and q (for a given μ). For a higher μ or q, a less-informed trader and the market maker both learn more from trading outcome, however, the difference between two agents is also bigger. For example, when μ is close to one, a less-informed agent almost learns perfectly about the fundamental after a trade in the second period; when $1 - \mu - q$ is close to one, the market is too noisy to learn, and thus there is no salient difference between the market maker and a less-informed. It implies that, for most part of parameter regions, less-informed traders' informational advantage increases with μ and q, therefore, the constraint on α



(a) Threshold on α w.r.t μ

(b) Threshold on α w.r.t q

Figure II: Conditions for positive benefits from waiting and learning

becomes more relaxed and the threshold on α becomes higher.

Note that in the previous steps we were solving the equilibrium conditional on the asset value being high and a fully-informed trader buying in the second period. The problem of asset value being low and a fully-informed trader selling is symmetric. Conditional on first-period no-trade outcome and a second-period sale, it is optimal for the less-informed trader to sell as long as he makes positive profits[5]: $\mathrm{E}_{\mathrm{MM}}(v \mid \phi, S_2, S_3) > \mathrm{E}_{\mathrm{LI}}(v \mid S_2)$. This inequality requires the same condition on parameter values as stated in Equation 18.

Now, we are able to calculate the expected payoff of waiting for a less-informed trader chosen at $T = 1$. It is given by the following function, and Equation 18 is also the necessary condition for waiting to be strictly beneficial.

$$q\mu[\delta(\mathrm{E}_{\mathrm{MM}}(v \mid \phi, S_2, S_3) - \mathrm{E}_{\mathrm{LI}}(v \mid S_2)) + (1-\delta)(\mathrm{E}_{\mathrm{LI}}(v \mid B_2) - \mathrm{E}_{\mathrm{MM}}(v \mid \phi, B_2, B_3))] \tag{19}$$

The second deviation might be profitable. If the fully-informed trader deviates to waiting, with some values of μ and q, upon observing no trade, the market maker updates her prior α to a lower value. Then in the next period, if the fully-informed trader is selected again and he buys, the market maker thinks it is less likely to be from a fully-informed, so the ask price would be lower compared

[5] Similar to Proposition 3, it is easy to prove that buying cannot be optimal in this scenario.

to the ask price at $T = 1$, and thus the fully-informed trader will make more profits conditional on being selected at T=2. Nevertheless, to avoid tedious algebra, we provide a sufficient condition for this deviation to be non-profitable: $2q > \mu$. This condition is derived from Equation 7 which assures that the updated α is higher, so that the fully-informed trader makes less profits even he is chosen to trade at $T = 2$.

4 Extensions

This section modifies the baseline model in two different ways to show the robustness of previous results.

In the first extension, illustrated in Figure III, we assume that a proportion of $\mu + q$ traders opt out of the market if the state does not change. The rational is as follows: as no change implies no new information about the asset value, rational consumers are aware that, they know as much as the market maker and thus they choose not to trade because trading yields non-positive profits. If the state changes, the game structure is the same as the baseline model, where a proportion of μ traders are fully-informed of the eventual asset value, a proportion of q traders are only informed of the change of state, and the rest traders are noise ones.

Solving for the equilibrium which involves "waiting" follows the same steps proceeded in the previous section. In the Appendix, we skip the algebra and present the expressions of those state variables: $\{\delta_1, \delta_2, \delta_3\}$ and $\{\alpha_1, \alpha_2\}$. We also provide Figure IV to illustrate the threshold on α which is the maximum value for the equilibrium to exist.

By comparing Figure II and Figure IV, we can see that the threshold on α is lower in the extension setting than in the baseline model for the same μ and q. In the new setting, a proportion of $\mu + q$ agents opt out if the state remains unchanged, instead of behaving as noise ones. As a result, there is less noise for the market maker. Hence, a lower α is required to compensate less-informed traders for the reduction in their informational advantage.

Moreover, Figure IV also shows that the threshold on α is decreasing in μ and q for the most part. Though the logic of the threshold increasing in μ and q in Figure II still holds, here a higher μ or q leads to less traders in the market in the case of no information event, therefore, it is easier for the marker maker to figure out the occurrence of an information event. That is why we need a lower α to

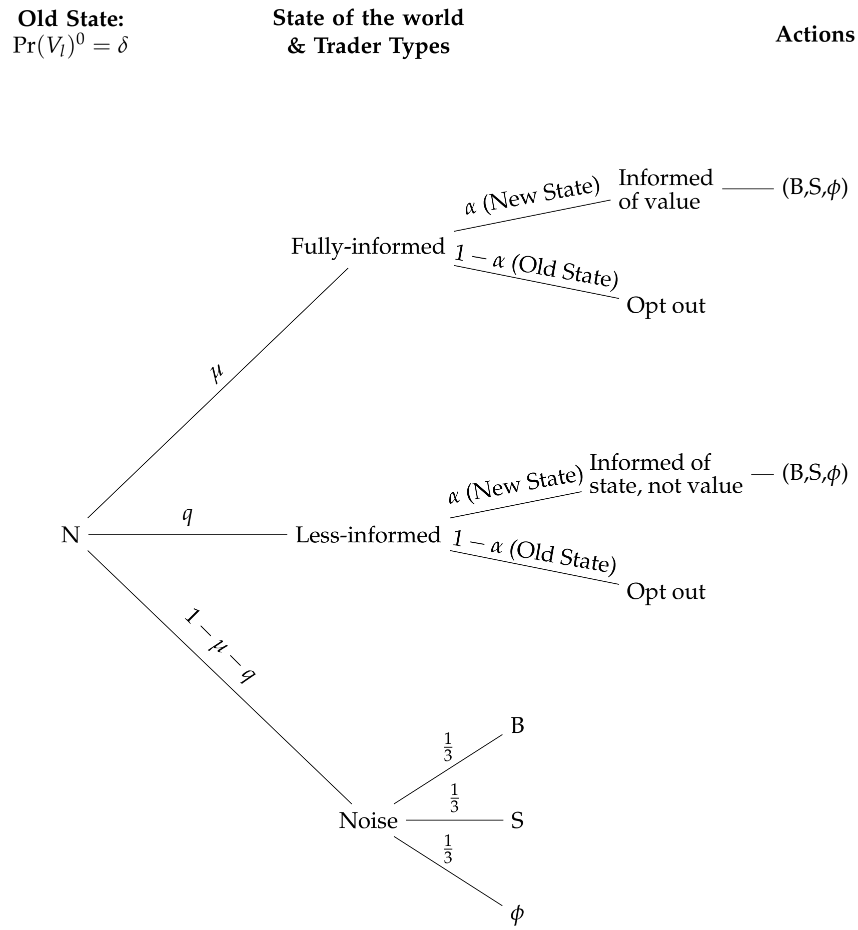
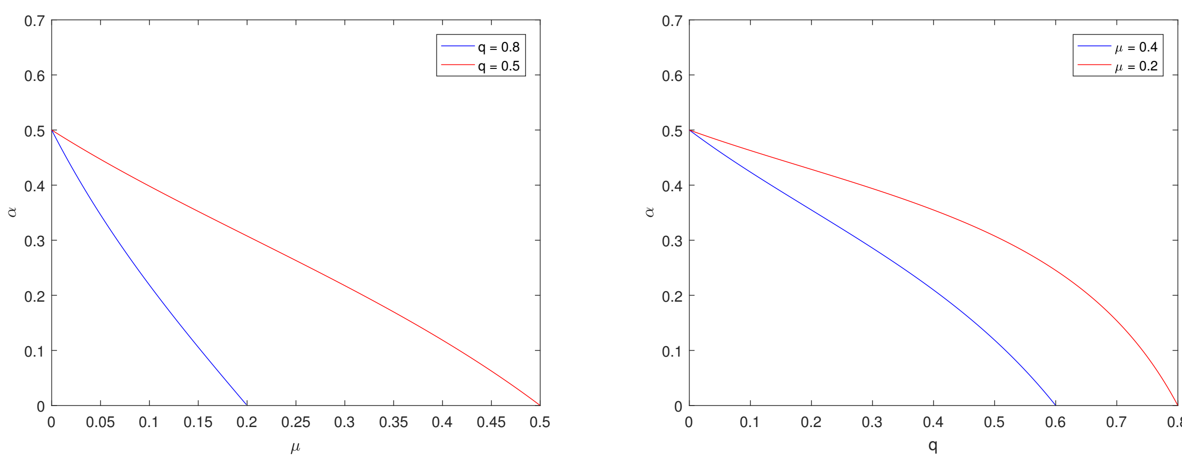


Figure III: Tree diagram of the trading process



(a) Threshold on α w.r.t μ

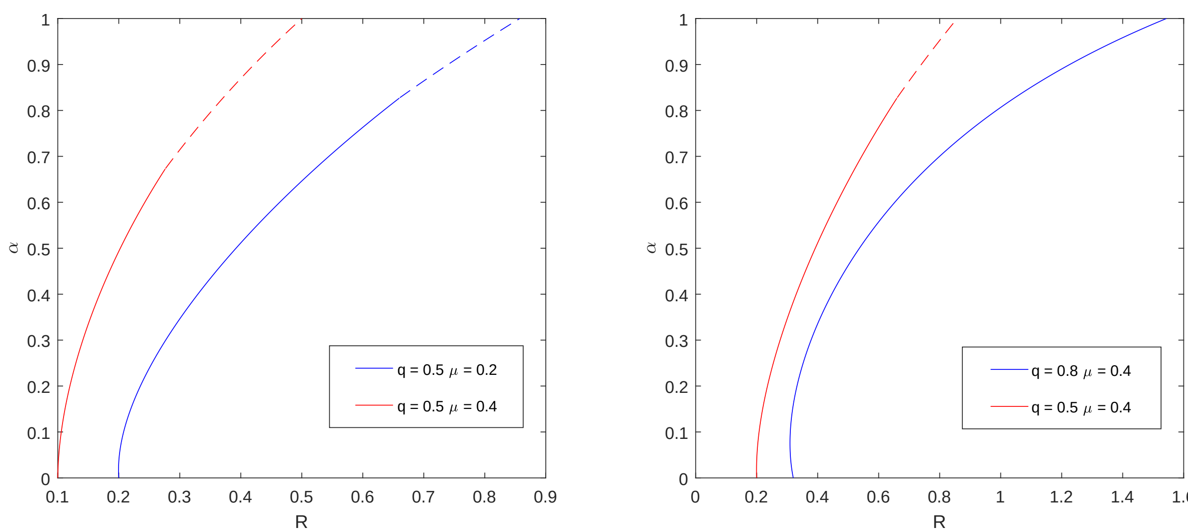
(b) Threshold on α w.r.t q

Figure IV: Conditions for positive benefits from waiting and learning sustain less-informed traders' advantage, and thus their profits from learning.

In the second extension, we try to illustrate "waiting" in a more strategic way. "Waiting" in the baseline model is mandatory as the expected payoff for a less-informed trader to trade at $T = 1$ is non-positive. Suppose that there is an exogenous reward R for providing liquidity in this market. If a less-informed trader deviates to trade in the first period, denoted by $d \in \{1, -1\}$, his expected payoff is given by:

$$\mathrm{E}[v - p(d)] + R|d|$$

where p(1) and p(-1) are the corresponding ask and bid prices set by the market maker in the equilibrium. Under the new assumption, "waiting" might not constitute an equilibrium if the reward parameter R is big enough. To ensure that the deviation is not profitable, i.e., it is optimal for the less-informed trader to wait, we need to impose restrictions on parameter values. A numerical example is shown in Figure V:



(a) Threshold on α w.r.t R (fixed q)

(b) Threshold on α w.r.t R (fixed μ)

Figure V: Conditions for waiting to be optimal

The dashed lines present the set of parameter values to make the less-informed trader break-even from either waiting or trading at $T = 1$. When α is high, as we have analyzed before, it is not profitable for the less-informed to trade even after learning from past trades. If at the same time R is small, corresponding to the left part to the dashed lines, the less-informed trader opts out as they cannot make positive profits either by waiting or trading at $T = 1$. Increasing R leads to profits by trading, corresponding to the right part to the dashed line, where the less-informed chooses to trade.

The solid lines present the combinations of parameter values to make the less-informed trader indifferent between waiting or trading at $T = 1$ (both choices give positive profits). Right to those lines, with a high R and/or a small α, the less-informed trader prefers to trade at $T = 1$. The lines exert an increasing pattern in the figure because the higher the α is, the bigger loss a less-informed trader suffers from trading in the first period ($\mathrm{E}[v - p(d)]$), and thus the higher R it requires to make the less-informed trader indifferent.

5 Concluding Remarks

As the incentive of "waiting to learn" has been explored in various contexts in the economics literature, to the best of our knowledge, there is no paper studying this incentive in the stock market where informational structure is more involved compared to many others. Based on the "event uncertainty" informational structure of Easley and O'Hara (1992), we introduce a new type of traders who are privately informed of the occurrence of an event, but not informed of the asset's fundamental value. This new type of traders, named less-informed traders in our model, have an advantage in learning from the past orders over the market maker, as they know that the perfectly-informed traders are present and thus they are able to draw more precise information from past trading outcomes. Therefore, waiting could benefit those traders. We also show that the "information rent" from learning is higher with greater information asymmetry between less-informed traders and market marker, i.e., an information event is not very likely to occur. Conditional on its occurrence, the less-informed then have a bigger informational advantage.

The key feature of our model is that, the addition of the "event uncertainty" adds another "state" to a standardized game of Glosten and Milgrom (1985) and enables us to create a new tractable layer of information asymmetry between investors and the market maker. This model, however, has a defect: the expected payoff of a selected less-informed trader buying or selling in the first period is negative, and therefore "waiting" seems to be non-strategic. While this feature provides us with an easy solution for market equilibrium and still highlights the positive information benefits from "waiting", we try to relax this restriction by considering the case where less-informed traders can earn an exogenous positive reward by deviating from the waiting equilibrium, whether sell or buy. We can show that the less-informed traders may still prefer the "waiting strategy" under some conditions despite the positive profits from not waiting.

For future work, we plan to endogenize the exogenous trading profits by assuming that less-informed traders can receive a private signal on the asset value. Under this assumption, if a less-informed trader is selected in the first period, similar to a fully-informed trader, he can make a strictly positive amount of profits due to his superior information over the market maker. Our intuition is that, there should be a threshold[6] on the precision of the private signal, above which the incentive to wait exists and below which a less-informed trader optimally chooses to be an early bird instead of a second mouse.

[6] We expect it to be a function of other parameters: e.g., α, the level of information asymmetry on the state of the world between a less-informed and the market maker.

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Appendix A Solutions for the Extension Exercise

The following equations are the expressions of the state variables, $\{\delta_1, \delta_2, \delta_3, \alpha_1, \alpha_2\}$, following the information structure of Figure III in the first extension exercise. For simplicity, we present equations without recursive substitutions. With some algebra, all those variables can be expressed as functions of only parameters μ, q, α, δ.

$$\delta_1 = \delta \tag{20}$$

$$\alpha_1 = \frac{\alpha(q + \frac{1}{3}(1-\mu-q))}{q + \frac{1}{3}(1-\mu-q) + (1-\alpha)\mu} \tag{21}$$

$$\delta_2 = \delta[\frac{\frac{1}{3}(1-\mu-q)}{\alpha_1\mu(1-\delta) + \frac{1}{3}(1-\mu-q)}] \tag{22}$$

$$\alpha_2 = \frac{\alpha_1\frac{1}{3}(1-\mu-q) + \alpha_1(1-\delta)\mu}{\frac{1}{3}(1-\mu-q) + \alpha_1(1-\delta)\mu} \tag{23}$$

$$\delta_3 = \frac{q(\delta_2 - (1-\alpha_2)\delta) + \frac{1}{3}\delta_2(1-\mu-q)}{\frac{1}{3}(1-\mu-q) + \mu(\alpha_2(1-\delta) + (\delta-\delta_2)) + \alpha_2 q} \tag{24}$$

Using those expressions, we will be able to calculate the excepted payoff for a less-informed trader to trade at $T = 3$, and thus find the conditions for positive benefits from waiting to hold.