

# working paper

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### Structural Estimation of a Model of School Choices: the Boston Mechanism vs. its Alternatives

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## Abstract

We develop a model of school choices by households under the popular Boston mechanism (BM) and a new method to fully solve household problem that is infeasible to solve via traditional method. We estimate the joint distribution of household preferences and sophistication types using administrative data from Barcelona. Our counterfactual policy analyses show that a change from BM to the student deferred acceptance mechanism would create more losers than winners and decrease the average welfare by 1,020 euros, while a change from BM to the top trading cycles mechanism has the opposite effect and increases the average welfare by 460 euros.

JEL Codes: C78, D63, I24.

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# 1 Introduction

In many countries, every child is guaranteed free access to education in some public school. However, not all public schools are of the same quality, nor are higher-quality schools distributed evenly across residential areas. Designed to broaden households' access to schools beyond their neighborhoods, public school choice systems have been increasingly adopted in many countries, including the U.S.<sup>1</sup> On the one hand, the quality of schools to which students are assigned can have significant long-term effects for individual families as well as important implications on the efficiency and equity for a society.<sup>2</sup> On the other hand, schools are endowed with certain capacities and not all choices can be satisfied. As a result, how to operationalize school choice, i.e., what procedure should be used to assign students to schools, becomes a non-trivial question that remains heatedly debated on among policy makers and researchers.

One important debate centers around a procedure known as the Boston mechanism (BM), which was used by Boston Public Schools (BPS) between 1999 and 2005 to assign K-12 pupils to city schools, and still is one of the most popular school choice systems in the world. In BM, a household submits its applications in the form of an ordered list of schools. All applicants are assigned to their first choices if there are enough seats in those schools. If a given school is over-demanded, applicants are accepted in the order of their priorities for that school.<sup>3</sup> Those rejected from their first choices face a dramatically decreased chance of being accepted to any other desirable schools since they can only opt for the seats that remain free after everyone's first choice has been considered. As a result, some parents may refrain from ranking schools truthfully, which makes BM vulnerable to manipulation (Abdulkadiroğlu and Sönmez (2003)). In 2005, the BPS replaced BM with the Gale-Shapley student deferred acceptance mechanism (GS), originally proposed by Gale and Shapley (1962), which

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<sup>1</sup>Some studies have explored exogenous changes in families' school choice sets to study the impacts of school choice on students' achievement, e.g., Abdulkadiroğlu, Angrist, Dynarsky, Kane and Pathak (2010), Deming, Hastings, Kane and Staiger (2014), Hastings, Kane and Staiger (2009), Lavy (2010), Mehta (2013) and Walter (2013). Other studies focus on how the competition induced by student's school choices affects school performance, e.g., Hoxby (2003) and Rothstein (2006).

<sup>2</sup>See Heckman and Mosso (2014) for a comprehensive review of the literature on human development and social mobility.

<sup>3</sup>Priorities for a given school are often determined by whether or not one lives in the zone that contains that school, whether or not one has siblings enrolled in the same school, and some other socioeconomic characteristics, with a random lottery to break the tie.

provides incentives for households to reveal their true preferences.<sup>4</sup>

Although the vulnerability of BM to manipulation is widely agreed upon, it remains unclear whether or not it should be replaced in other cities as well.<sup>5</sup> In practice, the switch decision by the BPS was resisted by some parents.<sup>6</sup> In theory, the efficiency and equity comparison between BM and its alternatives remains controversial.<sup>7</sup> The welfare implications of various mechanisms thus become an empirical question, one that needs to be answered before a switch from BM to GS or some other mechanisms is recommended more widely.

To answer this question, one needs to quantify two essential but unobservable factors underlying households' choices, which is what we do in this paper. The first factor is household preferences, without which one could not compare welfare across mechanisms even if household choices were observed under each alternative mechanism. Moreover, as choices are often not observed under counterfactual scenarios, one needs to predict which households would change their behaviors and how their behaviors would change, were the current mechanism switched to a different one. The knowledge of household preferences alone is not enough for this purpose. Although BM gives incentives for households to act strategically, there may exist non-strategic households that simply rank schools according to their true preferences.<sup>8</sup> A switch from BM to GS, for example, will induce behavioral changes only among strategic households who hide their true preferences under BM. Therefore, the knowledge about the distribution of household types (strategic or non-strategic) becomes a second essential factor for one to assess the impacts of potential reforms in the school choice system.

We develop a model of school choices under BM by households who differ in both their preferences for schools and their strategic types. Non-strategic households fill out their application forms according to their true preferences. Strategic households take admissions risks into account to maximize their expected payoffs, who may hide

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<sup>4</sup>See Abdulkadiroğlu, Pathak, Roth and Sönmez (2005) for a description of the Boston reform.

<sup>5</sup>Pathak and Sönmez (2013) document switches in Chicago and England from certain forms of the Boston mechanism to less-manipulable mechanisms, and argue that these switch decisions revealed government preferences against mechanisms that are (excessively) manipulable.

<sup>6</sup>See Abdulkadiroğlu, Che, and Yasuda (2011) for examples of the concerns parents had.

<sup>7</sup>See the literature review below.

<sup>8</sup>There is direct evidence that both strategic and non-strategic households exist. For example, Abdulkadiroğlu, Pathak, Roth and Sönmez (2006) show that some households in Boston obviously failed to strategize. Calsamiglia and Güell (2014) prove that some households obviously behave strategically. Estimation results in our paper are such that both types exist.

their true preferences. A household’s expected payoff depends on how it selects and ranks schools on its application list. The traditional way to solve this problem involves selecting the best permutation from all possible ones out of the set of schools. When the number of schools is relatively big, such a solution soon becomes infeasible because the dimensionality of this problem grows exponentially with the number of schools. We discover two properties of the student-school allocation mechanism that have not been utilized in this literature, and show that the problem can be *fully* solved via backward induction even when the household faces a large choice set that is infeasible to handle using the traditional method.

We apply our model to a rich administrative data set from Barcelona, Spain, where a BM system has been used to allocate students across the over 300 public schools. The data contains information on applications, admissions and enrollment for all Barcelona families who applied for schools in the public school system in the years 2006 and 2007. In particular, we observe the entire application list submitted by each applicant, who can rank-list up to 10 out of the over 300 schools. We also observe applicants’ family addresses, hence home-school distances, and other family characteristics that allow us to better understand their decisions. Between 2006 and 2007, there was a drastic change in the official definition of school zones that significantly altered the set of schools a family had priorities for in the school assignment procedure. We estimate our model via simulated maximum likelihood using the 2006 pre-reform data. We conduct an out-of-sample validation of our estimated model using the 2007 post-reform data. The estimated model matches the data in both years well.

The results of the out-of-sample validation provide enough confidence in the model to use it to perform counterfactual policy experiments, where we assess the performance of two popular and truth-revealing alternatives to BM: GS and the top trading cycles mechanism (TTC) (Abdulkadiroğlu and Sönmez (2003)).<sup>9</sup> We find that a change from BM to GS benefits fewer than 12% of the households while hurting 33% of households. An average household loses by an amount equivalent to 1,020 euros. In contrast, a change from BM to TTC benefits 25% of households and hurts 21% of them. An average household benefits by an amount equivalent to 460 euros. Compared to TTC, BM and GS inefficiently assign households to closer-by but lower-

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<sup>9</sup>TTC was inspired by top trading cycles introduced by Shapley and Scarf (1974) and adapted by Abdulkadiroğlu and Sönmez (2003). To our knowledge, only New Orleans has implemented TTC.

quality schools. On the equity side, a switch from BM to GS is more likely to benefit those who live in higher-school-quality zones than those who live in lower-school-quality zones, hence enlarging the cross-zone inequality. In contrast, the quality of the school zone a household lives in does not impact its chance to win or to lose in a switch from BM to TTC. We also find that while TTC enables 59% of households whose favorite schools are out of their zones to attend their favorite schools, this fraction is only 47% under BM and 42% under GS.

Our paper contributes to the literature on school choices, in particular, the literature on the design of centralized choice systems initiated by Balinski and Sönmez (1999) for college admissions, and Abdulkadiroğlu and Sönmez (2003) for public school choice procedures.<sup>10</sup> Abdulkadiroğlu and Sönmez (2003) formulate the school choice problem as a mechanism design problem, and point out the flaws of BM, including manipulability. They also investigate the theoretical properties of two alternatives of BM: GS and TTC. Since then, researchers have been debating on the properties of BM. Some studies suggest that the fact that strategic ranking may be beneficial under BM creates a potential issue of equity since parents who act honestly (non-strategic parents) may be disadvantaged by those who are strategically sophisticated (e.g., Pathak and Sönmez (2008)). Using the pre-2005 data provided by the BPS, Abdulkadiroğlu, Pathak, Roth and Sönmez (2006) find that households that obviously failed to strategize were disproportionately unassigned. Calsamiglia and Miralles (2014) show that under certain conditions, the only equilibrium under BM is the one in which families apply for and are assigned to schools in their own school zones. **CC: On the other hand they show that weaker conditions lead to GS assigning children to their neighborhood school, independently of their truthfully revealed preferences. These arguments cause concern about inequality in the assignment across zones.** Besides equity, BM has also been criticized on the basis of efficiency. Experimental evidence from Chen and Sönmez (2006) and theoretical results from Ergin and Sönmez (2006) show that GS is more efficient than BM in complete information environments. *However*, in a series of studies, Abdulkadiroğlu, Che, and Yasuda (2011); Featherstone and Niederle (2011); and Miralles (2008) all provide examples of specific environments where BM is more efficient than GS.<sup>11</sup> The theoretical debates are still ongoing. For example, some

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<sup>10</sup>While student priorities for a certain college depends on the college’s own “preferences” over students; student priorities for public schools are defined by the central administration.

<sup>11</sup>Abdulkadiroğlu, Che, and Yasuda (2011) also point out that some non-strategic parents may

recent studies challenge the robustness of the results from Abdulkadiroğlu, Che, and Yasuda (2011) to their assumptions about preferences and the priority structure, e.g., Troyan (2012), Akyol (2014) and Lu (2015).

Although there have been extensive theoretical discussions about the strength and weakness of alternative school choice mechanisms, empirical studies designed to quantify the differences between these alternatives have been sparse.<sup>12</sup> Hwang (2015) set-identifies household preferences under the assumption that a household would rank a popular school on its report only if it prefers this school to less popular ones. He (2012) estimates an equilibrium model of school choice under BM using data from one neighborhood in Beijing that contains four schools, for which households have equal priorities to attend. Under certain assumptions, he estimates household preference parameters by grouping household choices, without having to model the distribution of household sophistication types. On the one hand, the approach in He (2012) allows one to be agnostic about the distribution of household strategic types during the estimation, hence imposing fewer presumptions on the data. On the other, it restricts his model’s ability to conduct cross-mechanism comparisons. Assuming all households are strategic, Agarwal and Somaini (2015) interpret a household’s submitted report as a choice of a probability distribution over assignments. Similar to our approach, they exploit the observed assignment outcomes and estimate household preferences without having to solve for the equilibrium. They introduce a class of mechanisms for which consistent estimation is feasible and establish conditions under which preferences are non-parametrically identified. In an extension, they allow for the existence of both strategic and non-strategic households. They apply their method to the Controlled Choice Plan in Cambridge, MA, in which each household can rank up to 3 out of 25 possible school programs. Although these households face a much smaller choice set than those in Barcelona, they also find that the average household welfare would be lower under GS than under the status quo.

Our paper well complements the three papers mentioned above. We estimate both household preferences and the distribution of strategic types. We show how one can solve and estimate a model where households face a large choice set that is infeasible to handle using the traditional method. We apply our model to the administrative

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actually be better off under BM than under GS.

<sup>12</sup>With a different focus, Abdulkadiroğlu, Agarwal and Pathak (2014) show the benefits of centralizing school choice procedures, using data from New York city where high school choices used to be decentralized.



data that contain the application, assignment and enrollment outcomes for the entire city of Barcelona, where households are given priorities to schools in their own school zones. With access to this richer data set, we are able to form a more comprehensive view of the alternative mechanisms in terms of the overall household welfare and the cross-neighborhood inequality.

Also related to our paper are studies that use out-of-sample fits to validate the estimated model.<sup>13</sup> Some studies do so by exploiting random social experiments, e.g., Wise (1985), Lise, Seitz and Smith (2005) and Todd and Wolpin (2006), or lab experiments, e.g., Bajari and Hortacsu (2005). Other studies do so using major regime shifts. McFadden and Talvitie (1977), for example, estimate a model of travel demand before the introduction of the BART system, forecast the level of patronage and then compare the forecast to actual usage after BART's introduction. Pathak and Shi (2014) aim at conducting a similar validation exercise on the data of school choices before and after a major change in households' choice sets of public schools, introduced in Boston in 2013.<sup>14</sup> Some studies, including our paper, deliberately hold out data to use for validation purposes. Lumsdaine, Stock and Wise (1992) estimate a model of worker retirement behavior of workers using data before the introduction of a temporary one-year pension window and compare the forecast of the impact of the pension window to the actual impact. Keane and Moffitt (1998) estimate a model of labor supply and welfare program participation using data after federal legislation that changed the program rules. They used the model to predict behavior prior to that policy change. Keane and Wolpin (2007) estimate a model of welfare participation, schooling, labor supply, marriage and fertility on a sample of women from five US states and validate the model based on a forecast of those behaviors on a sixth state.

The rest of the paper is organized as follows. The next section gives some background information about the public school system in Barcelona. Section 3 describes the model. Section 4 explains our estimation and identification strategy. Section 5 describes the data. Section 6 presents the estimation results. Section 7 conducts counterfactual experiments. The last section concludes. The appendix contains further details and additional tables.

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<sup>13</sup>See Keane, Todd and Wolpin (2011) for a comprehensive review.

<sup>14</sup>The authors are waiting for the post-reform data to finish their project.

## 2 Background

### 2.1 The Public School System in Spain

The public school system in Spain consists of over 300 schools that are of two types: public and semi-public.<sup>15</sup> Public schools are fully financed by the autonomous community government and are free to attend.<sup>16</sup> The operation of public schools follows rules that are defined both at the national and at the autonomous community level. Depending on the administrative level at which it is defined, a rule applies uniformly to all public schools nationally or autonomous-community-wise. This implies that all public schools in the same autonomous community are largely homogenous in terms of the assignment of teachers, school infrastructure, class size, curricula, and the level of (full) financial support per pupil.

Semi-public schools are run privately and funded via both public and private sources.<sup>17</sup> The level of public support per pupil for semi-public schools is defined at the autonomous community level, which is about 60% of that for public schools. Semi-public schools are allowed to charge enrollee families for complementary services. In Barcelona, the service fee per year charged by semi-public schools is 1,280 euros on average with a standard deviation of 570 euros.<sup>18</sup> On average, of the total financial resources for semi-public schools, government funding accounts for 63%, service fees account for 34%, and private funding accounts for 3%. Semi-public schools have much higher level of autonomy than public schools. They can freely choose their infrastructure facilities, pedagogical preferences and procedures. Subject to the government-imposed teacher credential requirement, semi-public schools have controls over teacher recruiting and dismissal. However, there are some important regulations semi-public schools are subject to. In particular, all schools in the public school system, public or semi-public, have to unconditionally accept all the students that are

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<sup>15</sup>Semi-public schools were added into the system under a 1990 national educational reform in Spain (LOGSE). In our sample period, there were 158 public schools and 159 semi-public schools.

<sup>16</sup>Spain is divided into 17 autonomous communities, which are further divided into provinces and municipalities. A large fraction of educational policies are run at the autonomous community (*Comunidades Autonomas*) and municipality levels (*municipios*) following policies determined both at the national and at the local levels. In particular, the Organic Laws (*Leyes Orgánicas*) establish basic rules to be applied nationally; while autonomous communities further develop these rules through what are called *Decretos*.

<sup>17</sup>See <http://www.idescat.cat/cat/idescat/publicacions/catalog/pdfdocs/dossier13.pdf> for details.

<sup>18</sup>The median annual household income is 25,094 euros in Spain and 26,418 euros in Catalunya.

assigned to them via the centralized school choice procedure that we describe in the next subsection; and no student can be admitted to the public school system without going through the centralized procedure. In addition, all schools have the same national limit on class sizes.

Outside of the public school system, there are a small number of private schools, accounting for only 4% of all schools in Barcelona. Private schools receive no public funding and charge very high tuition, ranging from 5,000 to 16,000 euros per year in Barcelona. Private schools are subject to very few restrictions on their operation; and they do not participate in the centralized school choice program.<sup>19</sup>

## 2.2 School Choice within the Public School System

The Organic Law 8/1985 establishes the right for families to choose schools in the public school system for their children. The national reform in 1990 (LOGSE) extended families' right to guarantee the universal access for a child 3 years or older to a seat in the public school system, by requiring that preschool education (ages 3-5) be offered in the same facilities that offer primary education (ages 6-12). Although a child is guaranteed a seat in the public school system, individual schools can be over-demanded. The Organic Law from 2006 (LOE) specifies broad criteria that autonomous communities shall use to resolve the overdemand for schools. Catalunya, the autonomous community for the city of Barcelona, has its own *Decretos* in which it specifies, under the guideline of LOE, how overdemand for given schools shall be resolved. In particular, it describes broad categories over which applicants may be ranked and prioritized, known as the priority rules.

Families get access to schools in the public school system via a centralized school choice procedure run at the city or municipality level, in which almost all families participate.<sup>20</sup> Every April, participating families with a child who turns three in that calendar year are asked to submit a ranked list of up to 10 schools. Households who submit their applications after the deadline (typically between April 10th and April 20th) can only be considered after all on-time applicants have been assigned.<sup>21</sup> All

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<sup>19</sup>For this reason, information on private schools is very limited. Given the lack of information on private schools and the small fraction of schools they account for, we treat private schools as part of the (exogenous) outside option in the model.

<sup>20</sup>For example, in 2007, over 95% of families with a 3-year old child in Barcelona participated in the application procedure.

<sup>21</sup>See Calsamiglia and Güell (2014) for more details on the application forms and the laws under-

applications are typed into a centralized system, which assigns students to schools via a Boston mechanism.<sup>22</sup> The final assignment is made public and finalized between April and May, and enrollment happens at the beginning of September, when school starts. In the assignment procedure, all applicants are assigned to their first choice if there are enough seats. If there is overdemand for a school, applicants are prioritized according to the government-specified priority rules. In Catalunya, a student's priority score is a sum of various priority points: the presence of a sibling in the same school (40 points), living in the zone that contains that school (30 points), and some other characteristics of the family or the child (e.g., disability (10 points)). Ties in total priority scores are broken through a fair lottery. The assignment in every round of the procedure is final, which implies that an applicant rejected from her first-ranked school can get into her second-ranked school only if this school still has a free seat after the first round. The same rule holds for all later rounds.

In principle, a family can change schools within the public school system after the assignment. This is feasible only if the receiving school has a free seat, which is a near-zero-probability event in popular schools. The same difficulty of transferring schools persists onto the preschool-to-primary-school transition because a student has the priority to continue her primary-school education in the same school she enrolled for preschool education, and because school capacities remain the same in preschools and primary schools (which are offered in the same facilities). A family's initial school choice continues to affect the path into secondary schools as students are given priorities to attend specific secondary schools depending on the schools they enrolled for primary-school education. On the one hand, besides the direct effect of quality of the preschool on their children's development, families' school choice for their 3-year-old children have long-term effects on their children's educational path due to institutional constraints. On the other hand, the highly centralized management of public schools in Barcelona reduces the stakes families take by narrowing the differences across schools.

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lying this procedure.

<sup>22</sup>We will describe the exact procedure in the model section.

## 2.3 Changes in the Definition of Zones (2007)

Before 2007, the city of Barcelona was divided into fixed zones; families living in a given zone had priorities for *all* the schools in that zone.<sup>23</sup> Depending on their specific locations within a zone, families could have priorities for in-zone schools that were far away from their residence while no priority for schools that were close-by but belonged to a different zone. This is particularly true for families living around the corner of different zones. In 2007, a family’s school zone was redefined as the smallest area around its residence that covered the closest 3 public and the closest 3 semi-public schools, for which the family was given residence-based priorities.<sup>24</sup> The 2007 reform was announced abruptly on March 27th, 2007, before which there had been no public discussions about it. Families were informed via mail by March 30th, who had to submit their lists by April 20th.

## 3 Model

### 3.1 Primitives

There are  $J$  public schools distributed across various school zones in the city. In the following, schools refer to non-private (public, semi-public) schools unless specified otherwise. There is a continuum of households of measure 1 (we use the words household, applicant, student and parent interchangeably). Each household submits an ordered list of schools before the official deadline, after which a centralized procedure is used to assign students according to their applications, the available capacity of each school and a priority structure.<sup>25</sup> A student can either choose the school she is assigned to or the outside option.

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<sup>23</sup>Before 2007, zones were defined differently for public and semi-public schools. A family living at a given location had priorities for a set of public schools defined by its public-school zone, and a set of semi-public schools defined by its semi-public-school zone. Throughout the paper, in-zone schools refer to the union of these two sets of schools; and two families are said to live in the same zone if they have the same set of in-zone schools.

<sup>24</sup>There were over 5,300 zones under this new definition. See Calsamiglia and Güell (2014) for a detailed description of the 2007 reform.

<sup>25</sup>As mentioned in the background section, almost all families participate in the application procedure. For this reason, we assume that the cost of application is zero and that all families participate. This is in contrast with the case of college application, which can involve significant monetary and non-monetary application costs, e.g., Fu (2014).

### 3.1.1 Schools

Each school  $j$  has a location  $l_j$ , a vector  $w_j$  of observable characteristics, and a characteristic **CC: take s out**  $\zeta_j$  that is observable to households but not the researcher. All school characteristics are public information.<sup>26</sup> No school can accommodate all students, but the total capacity of all schools is at least 1, hence each student is guaranteed a seat in the public school system.

### 3.1.2 Households

A household  $i$  has characteristics  $x_i$ , a home location  $l_i$ , idiosyncratic tastes for schools  $\epsilon_i = \{\epsilon_{ij}\}_j$ , and a type  $T \in \{0, 1\}$  (non-strategic or strategic).<sup>27</sup> Household tastes and types, known to households themselves, are unobservable to the researcher. We assume the vector  $\epsilon_i$  is independent of  $(x_i, l_i)$  and follows an i.i.d. distribution  $F_\epsilon(\epsilon)$ .<sup>28</sup> The fraction of strategic households varies with household characteristics and home locations, given by  $\lambda(x_i, l_i)$ . Conditional on observables, the two types differ only in their behaviors (as we specify below), but share the same distribution of preferences.

**Remark 1** *It is worth noting that we do not take a stand on why some households are strategic while some are not. This is an important research question, especially if a policy change may affect the fraction of strategic households. This is less of a concern in our case, because the major goal of this paper is to investigate the impact of switching BM to some other mechanisms that are truth-revealing. Under a truth-revealing mechanism, all households, strategic or not, will rank schools according to their true preferences, i.e., types no longer matter. Once we recover household preferences and the (current) distribution of strategic versus non-strategic types in the data, we can compare the current regime with its alternatives without knowing how household types will change in the new environment.*

<sup>26</sup>We assume that households have full information about school characteristics. Our data do not allow us to separate preferences from information frictions. Some studies have taken a natural or field experiment approach to shed light on how information affects schooling choices, e.g., Hastings and Weinstein (2008) and Jensen (2010).

<sup>27</sup>Our model is flexible enough to accommodate but **CC: take out “does”** not impose any restriction on the existence of either strategic and non-strategic types. The distribution of the two types is an empirical question. With a parsimonious two-point distribution of sophistication types, the model fits the data well. We leave, as a future extension, more general specifications of the type distribution with more than two levels of sophistication.

<sup>28</sup>In particular, we assume each component of  $\epsilon_i$  follows  $N(0, \sigma_\epsilon^2)$ .

As is common in discrete choice models, the absolute level of utility is not identified, we normalize the ex-ante value of the outside option to zero for all households. That is, a household's evaluation of each school is relative to its evaluation of the outside option, which may differ across households. Let  $d_{ij} = d(l_i, l_j)$  be the distance between household  $i$  and school  $j$ , and  $d_i = \{d_{ij}\}_j$  be the vector of distances to all schools for  $i$ . Household  $i$ 's utility from attending school  $j$ , regardless of its type, is given by

$$u_{ij} = U(w_j, x_i, d_{ij}, \zeta_j) + \epsilon_{ij},$$

where  $U(w_j, x_i, d_{ij}, \zeta_j)$  is a function of the school and household characteristics and the home-school distance.<sup>29</sup>

Between application and enrollment (about 6 months), the value of the outside option is subject to a shock  $\eta_i \sim i.i.d. N(0, \sigma_\eta^2)$ . A household knows the distribution of  $\eta_i$  before submitting the application, and observes it afterwards. For example, a parent may experience a wage shock that changes her ability to pay for the private school. This post-application shock rationalizes the fact that some households in the data chose the outside option even after being assigned to the schools of their first choice. Due to the potential shocks to the outside option, applying for schools in the public school system provides an option value for households.

**Remark 2** *Following the literature on school choice mechanisms, our model abstracts from peer effects and social interactions.<sup>30</sup> The major complication is the potential multiple equilibria problem arising from peer effects and social interactions, even under mechanisms such as GS and TTC. Comparing BM with its alternatives would become infeasible because, on the one hand, theory does not provide guidance about the nature of the set of equilibria; on the other, searching for all possible equilibria numerically is infeasible.*

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<sup>29</sup>Our initial estimation allows a function of zone characteristics to also enter household utility function in order to capture some common preference factors that exist among households living in the same zone. In a likelihood ratio test, we cannot reject that the simpler specification presented here explains the data just as well as the more complicated version.

<sup>30</sup>See Epple and Romano (2011) and Blume, Brock, Durlauf and Ioannides (2011) for comprehensive reviews on peer effects in education and on social interactions, respectively.

## 3.2 Priority and Assignment

In this subsection, we describe the official rules on priority scores and the assignment procedure.

### 3.2.1 The Priority Structure

A household  $i$  is given a priority score  $s_{ij}$  for each of the schools  $j = 1, \dots, J$ . Let  $S_i = \{s_{ij}\}_j$  be the vector of  $i$ 's priority scores. These priority scores follow a simple and transparent government rule and are known before application starts. In particular, the priority score  $s_{ij}$  is determined by household characteristics, its home location and the location of the school. Locations matter only up to whether or not the household locates within the school zone a school belongs to. Let  $z_l$  be the school zone that contains location  $l$ ;  $I(l_i \in z_{l_j})$  indicates whether or not household  $i$  lives in school  $j$ 's zone. Household characteristics  $x_i$  consists of two parts: demographics  $x_i^0$  and the vector  $\{\text{sib}_{ij}\}_{j=0}^J$ .  $\text{Sib}_{ij} = 1$  ( $\text{sib}_{ij} = 0$ ) if student  $i$  has some (no) sibling enrolled in school  $j$ ; if  $\text{sib}_{i0} = 1$  then the student has some sibling enrolled in the outside option. Priority score  $s_{ij}$  is given by

$$s_{ij} = x_i^0 a + b_1 I(l_i \in z_{l_j}) + b_2 \text{sib}_{ij}, \quad (1)$$

where  $a$  is the vector of official bonus points that applies to household demographics,  $b_1 > 0$  is the bonus point for schools within one's zone, and  $b_2$  is the bonus point for the school one's sibling is enrolled in.<sup>31</sup> To reduce its own computational burden, the administration stipulates that a student's priority score of her first choice carries over for all schools on her application list, which is common across the entire country of Spain.<sup>32</sup> We take this feature into account in our application. From (1), one can notice that different households may have the same priority score to school  $j$ . If a school is over-demanded, households are ranked first of all by their scores, and in case of a tie, the tied households are ranked by a **CC: unique random lottery number** drawn after households have submitted their applications.

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<sup>31</sup>It follows from the formula that a student can have 2, 3 or 4 levels of priority scores, depending on whether or not the school is in-zone or out-of-zone, whether or not one has sibling(s) in some in-zone and/or out-of-zone schools. See the appendix for details.

<sup>32</sup>For example, if a student lists an in-zone sibling school as her first choice, she carries  $x_i^0 a + b_1 + b_2$  for all the other schools she listed regardless of whether or not they are within her zone and whether or not she has a sibling in those schools.



### 3.2.2 The Assignment Procedure: BM

Schools are gradually filled up over rounds. There are  $R < J$  rounds, where  $R$  is also the official limit on the length of an application list.

Round 1: Only the first choices of the students are considered. For each school, consider the students who have listed it as their first choice and assign seats of the school to these students one at a time following their priority scores from high to low (with random numbers as tie-breakers) until either there are no seats left or there is no student left who has listed it as her first choice.

Round  $r \in \{2, 3, \dots, R\}$ : Only the  $r^{th}$  choices of the students *not previously assigned* are considered. For each school with still available seats, assign the remaining seats to these students one at a time following their priority scores from high to low (with random lotteries as tie-breakers) until either there are no seats left or there is no student left who has listed it as her  $r^{th}$  choice.

The procedure terminates after any step  $r \leq R$  when every student is assigned a seat at a school, or if the only students who remain unassigned listed no more than  $r$  choices. A student who remains unassigned after the procedure ends can propose a school that still has empty seats and be assigned to it.

One can use a triplet  $(\bar{r}_j, \bar{s}_j, cut_j)$  to characterize the admissions probabilities to each school  $j$ , where  $\bar{r}_j$  is the round at which  $j$  fills up its slot ( $\bar{r}_j > R$  if  $j$  is a leftover school),  $\bar{s}_j$  is the priority score for which lottery numbers are used to break ties for  $j$ 's slots,  $cut_j$  is the cutoff of the random lottery number for admission to  $j$ . School  $j$  will admit any  $r^{th}$ -round applicant before  $\bar{r}_j$ , any  $\bar{r}_j^{th}$ -round applicant with score higher than  $\bar{s}_j$ , and any  $\bar{r}_j^{th}$ -round applicant with score  $\bar{s}_j$  and random lottery higher than  $cut_j$ ; and it will reject any other applicant. Notice that, once the random lottery numbers are drawn, admissions are fully determined. As mentioned earlier, when making its application decision, a household knows its priority scores but not its random numbers **CC: take “s” out**, which introduces uncertain admissions, i.e., probabilities strictly between 0 and 1, for a household in many school-round cases. The assignment procedure implies that the admissions probability is (weakly) decreasing in priority scores within each round, and is (weakly) decreasing over rounds for all priority scores. In particular, the probability of admission to a school in Round  $r + 1$  for the highest priority score is (weakly) lower than that for the lowest score in Round  $r$ .

### 3.3 Household Problem

We start with a household's enrollment problem. After seeing the post-application shock  $\eta_i$  to its outside option and the assignment result, a household chooses between the school it is assigned to and the outside option. Let the expected value of being assigned to school  $j$  be  $v_{ij}$ , such that

$$v_{ij} = E_{\eta_i} \max \{u_{ij}, \eta_i\}. \quad (2)$$

As seen from the assignment procedure, if rejected by all schools on its list, a household can opt for a school that it prefers the most within the set of schools that still have empty seats after everyone's applications have been considered. Label these schools as "leftovers," and  $i$ 's favorite "leftover" school as  $i$ 's backup. The value ( $v_{i0}$ ) of being assigned to its backup school for household  $i$  is given by

$$v_{i0} = \max \{v_{ij}\}_{j \in \text{leftovers}}. \quad (3)$$

In the following, we describe a household's application problem, in which it chooses an ordered list of up to  $R$  schools. We do this separately for non-strategic and strategic households.

#### 3.3.1 Non-Strategic Households

A non-strategic household lists schools on its application form according to its true preferences  $\{v_{ij}\}_j$ . Without further assumptions, any list of length  $n$  ( $1 \leq n \leq R$ ) that consists of the ordered top  $n$  schools according to  $\{v_{ij}\}_j$  is consistent with non-strategic behavior, which makes the prediction of allocation outcomes ambiguous. To avoid such a situation, we impose the following extra structure: suppose household  $i$  ranks its backup school as its  $n_i^*$ -th favorite, then the length of  $i$ 's application list  $n_i$  is such that

$$n_i \geq \min \{n_i^*, R\}. \quad (4)$$

That is, when there are still slots left on its application form, a non-strategic household will list at least up to its backup school. We provide further discussion about this assumption at the end of this subsection.

Let  $A_i^0 = \{a_1^0, a_2^0, \dots, a_{n_i}^0\}$  be an application list for non-strategic ( $T = 0$ ) household

$i$ , where  $a_r^0$  is the ID of the  $r^{th}$ -listed school and  $n_i$  satisfies (4). The elements in  $A_i^0$  are given by

$$\begin{aligned} a_1^0 &= \arg \max_j \{v_{ij}\}_j \\ a_r^0 &= \arg \max_j \{v_{ij} | j \neq a_{r' < r}\}_j, \text{ for } 1 < r \leq \min \{n_i^*, n_i\}. \end{aligned} \tag{5}$$

That is, the list reveals a household's true preferences such that the  $r^{th}$ -listed school is its  $r^{th}$  favorite school. This has to hold for all  $n_i$  elements if the backup school is ranked below  $n_i^*$ , and for the first  $n_i^*$  schools if the backup school is ranked above  $n_i$ .<sup>33</sup> Define  $\mathbf{A}^0(x_i, \epsilon_i, l_i)$  as the set of lists that satisfy (4) and (5) for a non-strategic household with characteristics  $x_i$ , location  $l_i$  and tastes  $\epsilon_i$ . If  $n_i^* \geq R$ , the set  $\mathbf{A}^0(\cdot)$  is a singleton, and the length of the application list  $n_i = R$ . If  $n_i^* < R$ , all lists in the set  $\mathbf{A}^0(\cdot)$  are identical up to the first  $n_i^*$  elements, and they all imply the same allocation outcome for household  $i$ .

**More about Condition (4)** In order to predict allocation outcomes and to calculate welfare, we need to predict the content of a household's application list at least up to the point beyond which listing any additional schools will not affect the allocation outcome. Consider an application list of full length  $A_i^0 = \{a_1^0, a_2^0, \dots, a_R^0\}$ , if none of the  $R$  schools listed admits the household for sure in the round it is listed, then the entire list is outcome-relevant. If some elements in  $A_i^0$  are such that  $i$ 's admissions probability to  $a_r^0$  is 1 in Round  $r$ , then the list is outcome-relevant only up to its  $r^*$ -th element  $a_{r^*}^0$ , where  $a_{r^*}^0$  is the foremost-listed school that admits the household for sure. To predict the outcome, we could impose a different condition labeled Condition S (S for strong) that, when the list is incomplete, a non-strategic household list at least up to  $a_{r^*}^0$ . However, Condition S implicitly requires that a non-strategic household know that its admissions probability to School  $a_{r^*}^0$  in Round  $r^*$  is 1, which involves a substantial amount of sophistication. In comparison, Condition (4) is a much weaker requirement that a non-strategic household know which schools will be leftovers and list at least up to its backup school. It requires far less sophistication than to know the admissions probabilities by school and by round.<sup>34</sup> One reason is

<sup>33</sup>Consistent with our assumption that a non-strategic household know that it has sure access to its backup school, we do not require that schools listed after one's backup school be ranked.

<sup>34</sup>In this model, we have assumed that it is free to fill in the application and, if failing to be assigned within  $R$  rounds, to propose a leftover school. Given the knowledge of the set of leftover

the high persistence in whether or not a school was left over, which was true even between the two years before and after the drastic re-definition of priority zones: 265 out of the 317 schools were either left over twice or never left over in the years of 2006 and 2007. Such high persistence makes it easy to predict the set of leftover schools. Therefore, it may be reasonable to believe that even the non-strategic households may have this (minimal) level of sophistication. With this weak requirement, Condition (4) achieves the same goal as Condition S.<sup>35</sup>

### 3.3.2 Strategic Households

Strategic households are fully aware of the admissions probabilities in all rounds and take them into account when applying for schools.<sup>36</sup> A household's expected payoffs depend not only on which schools it includes on its application list, but also on how these schools are ordered.<sup>37</sup> Therefore, the direct solution to this problem involves choosing the best permutation from all possible ones out of the set of schools. Formally, let  $\mathbf{P}(J; R)$  be the set of all possible permutations of size  $1, 2, \dots, R$  out of elements in  $J$ , and  $|\mathbf{P}(J; R)|$  be its size. An optimal list for a strategic household  $i$  solves the following problem

$$\max_{A \in \mathbf{P}(J; R)} \pi(A, S_i, x_i, l_i, \epsilon_i), \quad (6)$$

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schools and that leftover schools have 100% admissions probabilities, a non-strategic household would be indifferent between adding or not adding its backup school to an incomplete list. Condition (4) specifies that, if indifferent, a non-strategic household will add its backup school. It is also consistent with a situation where the cost of proposing a leftover school after being rejected in all rounds is higher than listing one more school to one's list.

<sup>35</sup>To see why Condition (4) achieves the same goal as its much stronger counterpart, consider the following exhaustive cases. Case 1: None of its  $R$  favorite schools admits the household for sure. Both Condition S and Condition (4) require the same full list of length  $R$ . Case 2: At least one of its  $R$  favorite schools admits the household for sure. By definition, a backup school admits the student for sure and therefore  $r^* \leq n_i^*$ . If  $r^* < n_i^*$ , i.e., the first sure-to-get school is preferable to the backup: lengthening the list to  $n_i^*$  will not change the outcome, because only the first  $r^*$  elements are outcome-relevant. If  $r^* = n_i^*$ , then both conditions lead to the same list.

<sup>36</sup>We assume that strategic households have fully rational expectation of the admissions probabilities because it is a clear baseline. As a justification, the BM mechanism has been practiced in Barcelona for over 20 years, which presumably has given households a lot of background information. Nevertheless, a more flexible model would allow for some other groups, who are strategic but only partially informed of the admissions probabilities. Such a model is a straight-forward extension to our framework but will impose great challenges for identification. We leave it for future work.

<sup>37</sup>Because admissions probabilities are score-school-round-specific, for a given household, the admissions probabilities to a given school vary with where the household puts it on the application list.

where  $\pi(A, \cdot)$  is the expected value yielded by list  $A$ . To the best of our knowledge, choosing the best permutation out of all possible ones has been the method used in all empirical studies on BM and other manipulable mechanisms, which has been feasible because households under those studies face a small number of schools to choose from. However, when the total number of schools  $J$  is relatively big and the length limit  $R$  on the application list goes beyond 1,  $\mathbf{P}(J; R)$  soon becomes unmanageably large. In the case of Barcelona, with  $J = 317$  and  $R = 10$ ,  $|\mathbf{P}(J; R)|$  is around  $9 \times 10^{24}$ .

Further examination of the allocation mechanism reveals two properties that have not been explored in the literature. We utilize these two properties to develop a solution method that fully solves the problem but in a much simpler way.<sup>38</sup> The two properties are:

- (1) **Sequentiality:** Once the applications are submitted, students are allocated sequentially round by round.
- (2) **Limited Dependence:** If rejected from all previous rounds, one's probability of being allocated in the current round depends on one's previous choices in a very limited way. As a result, the *continuation* value of going to Round  $r$  onwards depends on what are listed in the first  $r - 1$  slots, but only in a **CC: easily trackable** way.

Let  $\xi_i^r \sim \text{uniform}[0, 1]$  be  $i$ 's lottery number in Round  $r$ . In the following, we first explain our solution method for the simplest BM case, where a **CC: a different lottery number is drawn for each school, not each round**. random lottery numbers are re-drawn for each round, i.e.,  $\xi_i^r$  are i.i.d. In this case, Property (2) takes the extreme form of no dependence. Then, we describe the method to solve for the BM case used in many cities, where each household only has one random number drawn before Round 1, i.e., **CC:  $\xi_{i,j}^r = \xi_{i,j}^1$  for all  $j$  and  $r$** . Finally, we solve our case where both the lottery number and the priority score for the first round carry over all future rounds.

### Case 1) BM with No Dependence: multiple tie-breakers.

Let  $p_j^r(S_i)$  be the probability of being admitted to school  $j$  in Round  $r$  for a student with priority score  $S_i$ , who listed  $j$  as the  $r^{th}$  application.<sup>39</sup> Each household

<sup>38</sup>To the best of our knowledge, our paper is first to develop and use this solution method in this literature.

<sup>39</sup>Notice that  $p_j^r(S_i)$  only depends on the  $j^{th}$  element of  $S_i$ , i.e., one's probability of getting into school  $j$  in a particular round only depends on  $s_{ij}$ . However, we use this notation to avoid complicated expressions (e.g.,  $p_{a_{r'}}^r(s_{ia_{r'}})$ ) later on.

takes the vector of admissions probabilities  $\{p_j^r(S_i)\}_{j,r}$  as given, which depend only on  $S_i \equiv \{s_{ij}\}_{j=1}^J$ , not on one's choices.<sup>40</sup> As a result, given a vector of priority scores  $S_i$ , the *continuation* value of going to Round  $r$  onwards is independent of what **CC:"are" out and put is** listed in the first  $r-1$  slots. Because of sequentiality, even though the entire application list has to be submitted at once, the solution can be feasibly derived by backward induction. In particular, if unassigned in all  $R$  rounds, the continuation value is given by

$$V^{R+1}(S_i, x_i, l_i, \epsilon_i) = v_{i0}.$$

The value of being in Round  $r \leq R$  for household  $i$  with some priority score  $S_i$  is given by

$$\begin{aligned} V^r(S_i, x_i, l_i, \epsilon_i) &= \max_{j \in J} \{p_j^r(S_i) v_{ij} + (1 - p_j^r(S_i)) V^{r+1}(S_i, x_i, l_i, \epsilon_i)\}, \\ \text{s.t. } p_j^r(S_i) &= \begin{cases} 1 & \text{if } r < \bar{r}_j \text{ or } (r = \bar{r}_j \text{ and } s_{ij} > \bar{s}_j), \\ 1 - \text{cut}_j & \text{if } r = \bar{r}_j \text{ and } s_{ij} = \bar{s}_j, \\ 0 & \text{otherwise,} \end{cases} \end{aligned} \quad (7)$$

where the second last line follows from the uniform distribution with support  $[0, 1]$ . An optimal list, denoted as  $A_i^1 = \{a_{i1}^1, \dots, a_{iR}^1\}$ , can be derived starting from  $a_{iR}^1$  to  $a_{i1}^1$ , where each  $a_{ir}^1$  is the argmax of (7).

Compared to that of  $\mathbf{P}(J; R)$ , the dimensionality of this backward induction problem is drastically lower at  $J \times R$ , i.e., choosing the best school out of  $J$  for  $R$  times. Intuitively, the complexity involved in searching for a best permutation out of  $\mathbf{P}(J; R)$  is the same as that in a backward induction where the value function  $V^r(\cdot)$  depends on how all previous  $r-1$  choices are listed. However, as discovered earlier, the nature of Case 1) is such that  $V^r(\{a_{r'}\}_{r'=1}^{r-1}, S_i, x_i, l_i, \epsilon_i) = V^r(S_i, x_i, l_i, \epsilon_i)$ . Failing to utilize sequentiality and limited dependence, the traditional solution of searching for the best permutation makes the problem unnecessarily complicated.

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<sup>40</sup>It is a widely used assumption in the literature that households are smaller players who take admissions probabilities as given, e.g., Abdulkadiroğlu, Che, and Yasuda (2011), Hatfield, Kojima and Narita (2014), Azevedo and Hatfield (2015), Azevedo and Leshno (2015) and Agarwal and Somaini (2015). See Kojima (2015) for a survey.

**Case 2) BM with Constant Lottery Number** When a household only has one lottery number that is constant over rounds, **CC: correlation is introduced** between admissions probabilities across rounds. In particular, the fact that one is rejected by its  $r^{th}$  choice  $a_r$  when it belongs to the tied group reveals that its lottery number is below the threshold  $cut_{a_r}$ . Therefore, the probability of being admitted in Round  $r+1$  to other schools conditional on being rejected by  $a_r$  is lower than the unconditional probability. **CC: Hence, a rational household should consider the updated beliefs when choosing each of the schools down the list, although the full list is submitted all at once.** Formally, let  $\bar{\xi}_i^r \in [0, 1]$  be the upper bound of one's random number conditional on its rejection history ( $\bar{\xi}_i^1 = 1$ ). This upper bound becomes an additional state variable in the value function and  $p_j^r(S_i)$  should be adjusted to  $p_j^r(S_i|\bar{\xi}_i^r)$ , such that

$$V^r(S_i, x_i, l_i, \epsilon_i, \bar{\xi}_i^r) =$$

$$\max_{j \in J} \left\{ p_j^r(S_i|\bar{\xi}_i^r) v_{ij} + \left(1 - p_j^r(S_i|\bar{\xi}_i^r)\right) V^{r+1}(S_i, x_i, l_i, \epsilon_i, \bar{\xi}_i^{r+1}) \right\} \quad (8)$$

$$s.t. \quad \bar{\xi}_i^{r+1} = \begin{cases} \min \{ cut_j, \bar{\xi}_i^r \} & \text{if } s_{ij} = \bar{s}_j \text{ and } r = \bar{r}_j, \\ \bar{\xi}_i^r & \text{otherwise,} \end{cases} \quad (9)$$

$$p_j^r(S_i|\bar{\xi}_i^r) = \begin{cases} 1 & \text{if } r < \bar{r}_j \text{ or } (r = \bar{r}_j \text{ and } s_{ij} > \bar{s}_j) \\ \max \left\{ 0, \frac{\bar{\xi}_i^r - cut_j}{\bar{\xi}_i^r} \right\} & \text{if } r = \bar{r}_j \text{ and } s_{ij} = \bar{s}_j, \\ 0 & \text{otherwise.} \end{cases} \quad (10)$$

Condition (9) is the updating rule for the state variable  $\bar{\xi}_i^{r+1}$ . Upon rejection, which occurs with probability  $1 - p_j^r(S_i|\bar{\xi}_i^r)$ ,  $\bar{\xi}_i^{r+1}$  (weakly) decreases to  $\min \{ cut_j, \bar{\xi}_i^r \}$  if  $i$  belongs to the tied priority group in Round  $r$  for school  $j$ , and it remains unchanged otherwise. Notice that the choice of  $j$  conditional on  $\bar{\xi}_i^r$  will fully determine  $\bar{\xi}_i^{r+1}$ . The second equality in the admissions probability rule (10) follows the truncated uniform distribution with support  $[0, \bar{\xi}_i^r]$ .

To describe an optimal list  $A_i^1 = \{a_{i1}^1, \dots, a_{iR}^1\}$ , let  $\mathbf{a}_{ir}(\bar{\xi}_i^r)$  be the argmax of (8) given state variable  $\bar{\xi}_i^r$  and **CC: let**  $\Xi_i^r(\bar{\xi}_i^r)$  be the  $\bar{\xi}_i^{r+1}$  associated with  $\mathbf{a}_{ir}(\bar{\xi}_i^r)$ . Given that  $\bar{\xi}_i^1 \in \{1\}$ , the optimal choice for Round 1 ( $a_{i1}^1$ ) is the argmax of (8) with  $\bar{\xi}_i^1 = 1$ . For Round  $r > 1$ ,  $a_{ir}^1$  is the argmax of (8) with  $\bar{\xi}_i^r = \Xi_i^{r-1}(\bar{\xi}_i^{r-1})$ .

Let  $O^r \subseteq J$  be the subset of schools that have been filled up by the beginning

of Round  $r$  ( $O$  for “out”), the size of which is given by  $|O^r| \equiv \sum_{j=1}^J I(r > \bar{r}_j)$ . As shown in Appendix A3, the dimension of the problem in Case 2) is lower than  $J \left(1 + \sum_{r=2}^R |O^r|\right)$ , which is in turn much smaller than  $|\mathbf{P}(J; R)|$ .

### Case 3) BM with Constant Lottery Number and Constant Priority Score

When the priority score of one’s top-listed school carries over to future rounds, the continuation values for  $r > 1$  will depend on the school listed for Round 1 ( $a_1$ ) via the first element in the value function, i.e., the vector of priority scores now becomes a vector of identical elements with  $s_{ia_1} \mathbf{1} = [s_{ia_1}, \dots, s_{ia_1}]$ , where  $\mathbf{1}$  is a  $J$ -dimensional vector of 1’s. With  $s_{ia_1} \mathbf{1}$  being the priority score vector, the problem for  $r > 1$  remains the same as described in (8) to (10). For Round 1, one solves the following problem  $V^1 \left( S_i, x_i, l_i, \epsilon_i, \bar{\xi}_i^1 = 1 \right) =$

$$\max_{j \in J} \left\{ p_j^1(s_{ij} \mathbf{1} | 1) v_{ij} + (1 - p_j^1(s_{ij} \mathbf{1} | 1)) V^2 \left( s_{ij} \mathbf{1}, x_i, l_i, \epsilon_i, \bar{\xi}_i^2 \right) \right\}, \quad (11)$$

*s.t.* Conditions (9), (10).

That is, the choice in Round 1 governs the vector of priority scores. An optimal list can be described similar to that in Case 2). As shown in Appendix A3, the dimension of the problem in Case 3) is smaller than  $J \left(1 + (R - 1) |\Omega_i| + \sum_{r=2}^R |O^r|\right)$ , where  $|\Omega_i|$  is the number of different levels of priority scores Household  $i$  can have.<sup>41</sup> Again, the dimensionality is much lower than  $|\mathbf{P}(J; R)|$ .

**Remark 3** *There can be multiple optimal application lists yielding the same value. Let  $\mathbf{A}^1(x_i, l_i, \epsilon_i)$  be the set of optimal lists for a strategic household. All lists in the optimal set, including the one derived by backward induction, are identical up to the payoff-relevant part of the lists and imply the same allocation outcome. For example, consider a list  $A^1 = \{a_1^1, \dots, a_r^1, \dots, a_R^1\}$ , by the specification of  $\{u_{ij}\}$ , each  $a_r^1$  is generically unique if no school listed before it has a 100% admissions rate for the household. However, if for some  $r < R$ , the admissions rate for the  $r^{\text{th}}$  listed school is one, then any list that shares the same first  $r$  ordered elements is also optimal. See the appendix for other cases.*

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<sup>41</sup>Recall that a household can have several levels of priority scores,  $|\Omega_i| \in \{2, 3, 4\}$ , depending on whether or not one has siblings enrolled in the system and whether one’s siblings’ schools are in or out of one’s school zone.



## 4 Estimation

### 4.1 Further Empirical Specification

#### 4.1.1 Utility Function

As described in detail in Appendix B1, the utility function (net of individual tastes) takes the following form

$$U(w_j, x_i, d_{ij}, \zeta_j) = \tau_1 I(\text{single parent}) + \tau_2 (\text{sib}_{ij} - \text{sib}_{i0}) - C(d_{ij}) \quad (12) \\ + \sum_{e=1}^3 (\delta_{0e} + \delta_{1e} \zeta_j + w_j \alpha_e) I(\text{edu}_i = e).$$

The first line of (12) specifies the part of utility that varies systematically across households even conditional on education.  $\tau_1$  captures the possible differential evaluation of the public system by single parents.  $\tau_2$  is the utility from enrolling in the same school as one's older sibling:  $\tau_2$  is added to  $i$ 's evaluation of  $j$  if a sibling is enrolled in  $j$ ,  $\tau_2$  is subtracted from  $i$ 's evaluation of all schools in the system if a sibling is in the outside option and hence making the outside option more attractive.  $C(d_{ij})$  is a non-linear distance cost function. The second line of (12) specifies the part of utility that varies systematically across households with different education levels  $\text{edu}_i \in \{1, 2, 3\}$ . In particular, we allow for households of different education levels to have different views of the public system via  $\delta_{0e}$ , school observable characteristics via  $\alpha_e$  and unobservable characteristics via  $\delta_{1e}$ , with  $\delta_{02} = 0$ ,  $\delta_{12} = 1$  normalized for the middle-education group.<sup>42</sup>

#### 4.1.2 Unobserved School Characteristics

We still need to specify the relationship between the unobserved school characteristics  $\zeta_j$  and the observable ones  $w_j$ . It is more realistic to allow for correlation between  $\zeta_j$  and  $w_j$ , rather than assuming that they are orthogonal. As a result,  $\alpha$  in (12) may not be consistently estimated. However, one can combine the effects of  $(w_j, \zeta_j)$  and

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<sup>42</sup>The vector of school observable characteristics that enters the utility function consists of quality, quality<sup>2</sup>, capacity, capacity<sup>2</sup>, tuition and an indicator of semi-public school.

rewrite the second line of (12) as

$$\sum_e (\delta_{0e} + \delta_{1e}\kappa_j + w_j\rho_e) I(edu_i = e). \quad (13)$$

The reduced-form parameters  $\rho$  and  $\{\kappa_j\}_j$  are, each, combinations of the structural parameters  $\alpha$ ,  $\delta$  and  $\zeta$ .<sup>43</sup> For the goal of this paper, it is sufficient to estimate  $\rho$ ,  $\{\kappa_j\}_j$  and all structural parameters except  $\alpha$  and  $\zeta$ .

**Remark 4** *Given our model specification, households' evaluations for schools, i.e., (12) and (13), are invariant to our counterfactual policy changes, which means it is unnecessary to recover  $\alpha$  and  $\zeta$  in order to conduct counterfactual experiments. Because of this fact and because our data do not contain proper instrumental variables for us to disentangle the effects of  $w_j$  and  $\zeta_j$ , our estimation will recover  $\rho$ ,  $\{\kappa_j\}_j$  and all structural parameters except  $\alpha$  and  $\zeta$ . As a cost, we may not be able to consistently translate the unit of welfare from utils to units of school characteristics, e.g., euros (the tuition unit).*

## 4.2 Likelihood

The model is estimated via the simulated maximum likelihood estimation method. The estimates of the model parameters should maximize the probability of the observed application and enrollment outcomes conditional on household observables  $(x_i, l_i)$ , school observables  $(w_j, l_j)$ , and student-school assignments.<sup>44</sup> Denote the vector of model parameters as  $\Theta \equiv [\Theta_u, \Theta_T]$ , where  $\Theta_u$  is the vector of parameters that govern household preferences, and  $\Theta_T$  is the vector of parameters that govern the distribution of household types. In particular,  $\Theta_u$  is composed of 1) the parameters that enter the first line of (12), 2) the school effect parameters  $\delta$ ,  $\rho$  and  $\{\kappa_j\}_j$ , 3) the dispersion of household tastes for schools  $\sigma_e$ , and 4) the dispersion of post-application shocks to the value of the outside option  $\sigma_\eta$ .

Let  $O_i \equiv [\tilde{A}_i, \tilde{e}_i | \tilde{j}_i]$  be the observed outcomes for household  $i$ , where  $\tilde{A}_i$  is the observed application list,  $\tilde{j}_i$  is the school student  $i$  was assigned to, and  $\tilde{e}_i$  is the observed enrollment decision given  $\tilde{j}_i$  as the school  $i$  was assigned to. Recall that a

<sup>43</sup>In particular,  $\kappa_j \equiv \zeta_j + \alpha_2 w_j$ ,  $\rho_e = \alpha_e - \delta_{1e} \alpha_2$  for  $e = 1, 3$ , and  $\rho_2 = 0$ .

<sup>44</sup>Notice that given applications, student assignment is a mechanical procedure that does not depend on parameters of the model, so it does not contribute to the likelihood per se.

household can either enroll in the assigned school or choose the outside option, hence  $\tilde{e}_i = I(\text{enroll})$ , where  $I(\cdot)$  is the indicator function.

Conditional on being type  $T$ , the probability of observing  $O_i$  is given by

$$L_i^T(\Theta_u) = \int \left\{ I\left(\tilde{A}_i \in \mathbf{A}^T(x_i, l_i, \epsilon_i; \Theta_u)\right) \times \left[ \tilde{e}_i \Phi\left(\frac{\bar{u}_{i\tilde{j}}(\Theta_u) + \epsilon_{i\tilde{j}_i}}{\sigma_\eta}\right) + (1 - \tilde{e}_i) \left(1 - \Phi\left(\frac{\bar{u}_{i\tilde{j}}(\Theta_u) + \epsilon_{i\tilde{j}_i}}{\sigma_\eta}\right)\right) \right] \right\} dF_\epsilon(\epsilon; \sigma_\epsilon),$$

where  $\mathbf{A}^T(x_i, l_i, \epsilon_i; \Theta_u)$  is the set of model-predicted optimal application lists for a type- $T$  household with  $(x_i, l_i, \epsilon_i)$ .  $\bar{u}_{i\tilde{j}}(\Theta_u)$  is the model-predicted utility of attending School  $\tilde{j}_i$ , net of individual taste.  $\Phi\left(\frac{\bar{u}_{i\tilde{j}}(\Theta_u) + \epsilon_{i\tilde{j}_i}}{\sigma_\eta}\right)$  is the model-predicted probability that this household will enroll in  $\tilde{j}_i$ , which happens if only if the post-application shock to the outside option is lower than the utility of attending  $\tilde{j}_i$ .

To obtain household  $i$ 's contribution to the likelihood, we integrate over the type distribution

$$L_i(\Theta) = \lambda(x_i, l_i; \Theta_T) L_i^1(\Theta_u) + (1 - \lambda(x_i, l_i; \Theta_T)) L_i^0(\Theta_u).$$

Finally, the total log likelihood of the whole sample is given by

$$\mathbf{L}(\Theta) = \sum_i \ln(L_i(\Theta)).$$

### 4.3 Identification

We give an overview of the identification in this subsection and leave the formal proof in Appendix B2. Besides the regular conditions such as utility functions be continuous, the identification relies on the following assumptions.

IA1: There does not exist a vector of household observable  $x$  and a school  $j$ , such that all households with  $x$  have probability zero of being admitted to school  $j$ .

IA2: Household tastes  $\epsilon$  are drawn from an i.i.d. unimodal distribution, with mean normalized to zero. Tastes are independent of school characteristics, household observables  $(x, l)$  and household type  $(T)$ .

IA3: At least one continuous variable in the utility function is excluded from the type distribution. Conditional on variables that enter the type distribution function, the excluded variable is independent of household type  $T$ .

To illustrate the identification challenge, consider a situation where each household

only applies to one school, which is a less favorable situation for identification because we would have less information, and suppose there is no post-application shock.<sup>45</sup> If all households are non-strategic, the model boils down to a multinomial discrete choice model with a household choosing the highest  $\bar{u}_{ij}(\Theta_u) + \epsilon_{ij}$ . The identification of such models is well-established under very general conditions (e.g., Matzkin (1993)). If all households are strategic, the model is modified only in that a household considers the admissions probabilities  $\{p_{ij}\}_j$  and chooses the option with the highest expected value. With  $\{p_{ij}\}$  observed from the data, this model is identified with the additional condition IA1, which requires that for any  $x$ , the expected value of applying for school  $j$  is not degenerate.<sup>46</sup> The challenge exists because we allow for a mixture of both types of households. In the following, we first explain IA2-IA3, then give the intuition underlying the identification proof.

### 4.3.1 IA2 and IA3 in Our Framework

We observe application lists with different distance-quality-risk combinations with different frequencies in the data. The model predicts that households of the same type tend to make similar application lists. Given IA2, the distributions of type-related variables will differ around the modes of the observed choices, which informs us of the correlation between type  $T$  and these variables. IA3 guarantees that different behaviors can arise from exogenous variations *within a type*. To satisfy IA3, we need to make some restrictions on how household observables  $(x_i, l_i)$  enter type distribution and utility. Conditional on distance, a non-strategic household may not care too much about living to the left or the right of a school, but a strategic household may be more likely to have chosen a particular side so as to take advantage of the priority zone structure.<sup>47</sup> However, given that households, strategic or not, share the same preferences about school characteristics and distances, there is no particular reason to believe that everything else being equal, the strategic type will live closer to a particular school than the non-strategic type would just for pure distance concerns. In other words, because the only difference between a strategic type and a non-

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<sup>45</sup>The post-application shock is identified from the observed allocation and enrollment outcomes.

<sup>46</sup>Agarwal and Somaini (2015) show conditions under which one can nonparametrically identify household preferences when all of them are strategic.

<sup>47</sup>Without directly modeling households' location choices, we allow household types to be correlated with the characteristics of the school zones they live in. We leave the incorporation of household location choices for future extensions.

strategic type is whether or not one considers the admissions probabilities, which are affected by one's home location only via the zone to which it belongs to, we assume that home location  $l_i$  enters the type distribution only via the school zone  $z_{l_i}$ , i.e.,

$$\lambda(x_i, l_i) = \lambda(x_i, z_{l_i}).$$

In contrast, household utility depends directly on the home-school distance vector  $d_i$ . Conditional on being in the same school zone, households with similar characteristics  $x$  but different home addresses still face different home-school distance vectors  $d$ , as required in IA3.

### 4.3.2 The Intuition for Identification

Conditional on  $(x, z_l)$ , the variation in  $d$  induces different behaviors within the same type; and conditional on  $(x, z_l, d)$ , different types behave differently. In particular, although households share the same preference parameters, different types of households will behave as if they have different sensitivities to distance. For example, consider households with the same  $(x, z_l)$  and a good school  $j$  out of their zone  $z_l$ , as the distance to  $j$  decreases along household addresses, more and more non-strategic households will apply to  $j$  because of the decreasing distance cost. However, the reactions will be much less obvious among the strategic households, because they take into account the risk of being rejected, which remains unchanged no matter how close  $j$  is as long as it is out of  $z_l$ . The different distance-elasticity among households therefore inform us of the type distribution within  $(x, z_l)$ .<sup>48</sup> This identification argument does not depend on specific parametric assumptions. For example, Lewbel (2000) shows that similar models are semiparametrically identified when an IA3-like excluded variable with a large support exists. However, to make the exercise feasible, we have made specific parametric assumptions.<sup>49</sup> Appendix B2 shows a formal proof of identification given IA1-IA3 and these additional specifications.

The argument above focuses on the special excluded variable  $d$ , but one can obtain

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<sup>48</sup>Although our identification does not rely on the following extreme case, one can also take the argument to the case of households along the border of two zones. Were all households non-strategic, the applications should be very similar by households across the border. In contrast, were all households strategic, drastically different applications could occur.

<sup>49</sup>We have to use parametric assumptions because semiparametric estimation is empirically infeasible, and because the support of  $d$  is bounded by the size of the city, which is not large enough relative to the (unbounded) support of household tastes, as required in Lewbel (2000).

further information for identification by comparing the observed households' choices with schools they did not choose, in terms of observable factors such as distance, quality and fees. Due to unobserved school characteristics, some seemingly good schools may in fact be unattractive, making it not as popular as it "should have been" among most households. Controlling for such common factors, a household may still leave out of its list some schools that seem to be better than its chosen ones due to unobserved tastes. IA2 implies that tastes are independent of household-school-specific admissions probabilities. Therefore, idiosyncratic preferences should not lead to a systematic relationship between a household's choice of not listing a school and its chance of getting into that school. However, as will be shown in Section 5, the left-out schools for most households in our data were disproportionately unlikely to be those the households had good chance to get into. Such behavior is highly consistent with strategizing instead of truth-telling.

### 4.3.3 Obviously Non-Strategic Households

The identification of our model is further facilitated by the fact that we can partly observe household type directly from the data: there is one particular type of "mistakes" that a strategic household will never make, which is a sufficient (but not necessary) condition to spot a non-strategic household. Intuitively, if a household's admissions status is still uncertain for all schools listed so far, and there is another school  $j$  it desires, it never pays to waste the current slot listing a zero-probability school instead of  $j$  because the admissions probabilities decrease over rounds.<sup>50</sup> The idea is formalized in the following claim and proved in the appendix.<sup>51</sup>

**Claim 1** *An application list with the following features is sufficient but not necessary evidence that the household must be non-strategic: 1) for some  $r^{th}$  element  $a_r$  on the list, the household faces zero admissions probability at the  $r^{th}$  round, and 2) it faces admissions probabilities lower than 1 for all schools listed in previous rounds, and 3) it faces a positive but lower than 100% admissions probability for the school listed in*

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<sup>50</sup>Abdulkadiroğlu, Pathak, Roth and Sönmez (2006) use a mistake similar to Feature 1) in Claim 1 to spot non-strategic households, which is to list a school over-demanded in the first round as one's second choice.

<sup>51</sup>If the support of household characteristics is full conditional on being obviously non-strategic, household preferences can be identified using this subset of households without IA1, since  $\epsilon$  is independent of  $(x, l)$ . However, our identification is only facilitated by, not dependent on the existence of obviously non-strategic households.

a later slot  $r'' \geq r + 1$  and no school listed between  $a_r$  and  $a_{r''}$  admits the household with probability 1.

## 5 Data

Our analysis focuses on the applications among families with children that turned 3 years old in 2006 or 2007 and lived in Barcelona. For each applicant, we observe the list of schools applied for, the assignment and enrollment outcomes. We also have information on the applicant’s home address, family background, and the ID of the school(s) her siblings were enrolled in the year of her application. For each school in the public school system, we observe its type (public or semi-public), a measure of school quality, school capacity and the level of service fees. The final data set consists of merged data sets from five different administrative units: the *Consorci d’Educacio de Barcelona* (local authority handling the choice procedure in Barcelona), *Department d’Ensenyament de Catalunya* (Department of Education of Catalunya), the *Consell d’Avaluacio de Catalunya* (public agency in charge of evaluating the Catalunya educational system), the *Instituto Nacional de Estadística* (national institute of statistics) and the *Institut Catala d’Estadística* (statistics institute of Catalunya).<sup>52</sup>

### 5.1 Data Sources

From the *Consorci d’Educacio de Barcelona*, we obtain access to every applicant’s application form, as well as the information on the school assignment and enrollment outcomes. An application form contains the entire list of ranked schools a family submitted. In addition, it records family information that was used to determine the priority the family had for various schools (e.g., family address, the existence of a sibling in the first-ranked school and other relevant family and child characteristics). The geocode in this data set allows us to compute a family’s distance to each school in the city.

From the Census and local register data, we obtain information on the applicant’s family background, including parental education and whether or not both parents

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<sup>52</sup>These five different data sources were merged and anonymized by the Institut Catala d’Estadística (IDESCAT).

were registered in the applicant’s household. Since information on siblings who were not enrolled in the school the family ranked first is irrelevant in the school assignment procedure, it is not available from the application data. However, such information is relevant for family’s application decisions. From the Department of Education, we obtained the enrollment data for children aged 3 to 18 in Catalunya. This data set is then merged with the local register, which provides us with the ID of the schools enrolled by each of the applicant’s siblings at the time of the application.

To measure the quality of schools, we use the external evaluation of students conducted by *Consell d’Avaluacio de Catalunya*. Since 2009, such external evaluations have been imposed on all schools in Catalunya, in which students enrolled in the last year of primary school are tested on math and language subjects.<sup>53</sup> From the 2009 test results that we obtained, we calculated the average test score across subjects for each student, then use the average across students in each school as a measure of the school’s quality.<sup>54</sup> Finally, to obtain information on the fees charged by semi-public schools (public schools are free to attend), we use the survey data collected by the *Instituto Nacional de Estadística*.<sup>55</sup>

## 5.2 Admissions Thresholds and Sample Selection

It is well-known that BM can give rise to multiple equilibria, which can greatly complicate the estimation of an equilibrium model.<sup>56</sup> However, assuming each household is a small player that takes the admissions thresholds, hence admissions probabilities, as given, we can recover all the model parameters by estimating an individual decision model. This is possible because the assignment procedure is mechanical and because we observe the applications and assignment results for *all* participating families, which we use to simulate the triplet  $(\bar{r}_j, \bar{s}_j, cut_j)$  for each school as follows. Taking the observed applications and the priority rules in Barcelona as given, we

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<sup>53</sup>As mentioned in the background section, a student has the priority to continue her primary-school education in the same school (with the same capacity) she enrolled for preschool education, which makes it very unlikely that one can transfer to a better school between preschool-primary school transition. For example, at least 94% of the 2010 preschool cohort were still enrolled in the same school for primary school education in 2013.

<sup>54</sup>Following the same rule used in Spanish college admissions, we use unweighted average of scores across subjects for each student.

<sup>55</sup>See <http://www.idescat.cat/cat/idescat/publicacions/catalogue/pdfdocs/dossier13.pdf> for a summary of the survey data.

<sup>56</sup>He (2012) did not detect multiple equilibria in his simulations and hence estimated the equilibrium model assuming uniqueness.



make 1,000 copies for each observed application list and assign each copy a random lottery number. We simulate the assignment results in this enlarged market (1,000 times as big as the observed market), which yields  $(\bar{r}_j, \bar{s}_j, cut_j)$  for each school. The simulated admissions thresholds are treated as the ones that the households expected when they applied.

In 2006, 11,871 Barcelona households participated in the application for schools in the Barcelona public school system. After we calculated the admissions probabilities using the entire sample, we select the estimation sample as follows. We drop 3,152 observations whose home location information cannot be consistently matched with the GIS (geographic information system) data, for example, due to typos.<sup>57</sup> We exclude 191 families whose children have special (physical or mental) needs or who submitted applications after the deadline, the latter were ineligible for assignment in the regular procedure. We drop 31 households whose applications, assignment and/or enrollment outcomes are inconsistent with the official rule, e.g., students being assigned to over-demanded schools they did not apply for. Finally, we delete observations missing critical information such as parental education and the enrollment information of the applicant’s older sibling(s).<sup>58</sup> The final sample size for estimation is 6,836.

### 5.3 Summary Statistics

There were 158 public schools and 159 semi-public schools in our sample period. Table 1 summarizes school characteristics separately for the two groups of schools. The first row summarizes school quality as measured by the average test scores of students in each school.<sup>59</sup> The average quality of public schools is 7.4 with a standard deviation of 0.8. Semi-public schools have higher average quality of 8.0 and a smaller dispersion of 0.5. Although public schools are free to attend, semi-public schools

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<sup>57</sup>We know the priority scores these households had for schools on their application list, which enables us to include these households in the calculation of the overall admissions probabilities.

<sup>58</sup>Our model distinguishes between high-school education and college education. Therefore, the observations excluded from the estimation sample include 748 parents who reported their education levels as “high school or above.” In policy simulations, however, we do include this subsample and simulate their application behaviors in order to be able to conduct the city-wise assignment under alternative mechanisms. We interpolate the probability of each of these 748 households as being high school or college educated by comparing them with those who reported exactly high-school education or college education. We estimate the probabilities via a flexible function of all the other observable characteristics, such as gender, residential area, age, number of children etc. The model fit for this subsample is as good as that for the estimation sample, available on request.

<sup>59</sup>We measure test scores on a scale from 0 to 10, distance in 100 meters and tuition in 100 euros.

charge on average 1,280 euros per year with a standard deviation of 570 euros. The average capacity for the incoming 3-year-old students in public schools is 1.4 classes, as compared to 1.8 in semi-public schools.

Table 1 School Characteristics

	Public	Semi-Public	All
Quality	7.4 (0.8)	8.0 (0.5)	7.7 (0.7)
Fees (100 Euros)	0	12.8 (5.7)	6.4 (7.5)
# Classes	1.4 (0.5)	1.8 (1.0)	1.6 (0.8)
Observations	158	159	317

Table 2 summarizes the household characteristics of the 2006 estimation sample. Among all households, about 30% parents had less than high school education and about 40% had college education.<sup>60</sup> For about 15.8% of the sample, only one parent was registered in the applicant’s household. We refer such households as “single parent” households throughout the paper. Over 42% of applicants had at least one older sibling enrolled in some preschool or primary school in 2006, almost all of these older siblings were enrolled in the Barcelona public school system (40.7% out of 42.2%). Depending on their home locations, the numbers of schools for which households had priorities were different, so was the average quality of these schools. On average, a household had priority for 22 schools in 2006 with a standard deviation of almost 8 schools. The average quality of schools within one’s priority zone was 7.8 and the cross zone dispersion was 0.3.

Table 2 Household Characteristics

Parental Edu <sup>a</sup> < HS	29.8%
Parental Edu = HS	30.4%
Parental Edu > HS	39.8%
Single Parent	15.8%
Have school-age older sibling(s)	42.2%
# Schools in Zone	22.3 (7.9)
Average school quality in zone	7.8 (0.3)
Observations	6,836

<sup>a</sup>Parental Edu: mother’s edu if she is present, otherwise father’s edu.

<sup>60</sup>Following the literature on child development, we use mother’s education as the definition of parental education if the mother is present in the household, otherwise, we use the father’s education.

Table 3 shows the number of schools households listed on their application forms. Households were allowed to list up to 10 schools, but most of households listed no more than 3 schools, with 47% of households listing only one school. Across different educational groups, parents with lower-than-high-school education were more likely to have a shorter list, while parents with exact high school education tended to list more schools than the others. Single parents also tended to list more schools compared to both-parent households.

Table 3 Number of Schools Listed (%)

	1	2	3	4 or more
All	46.9	12.4	16.9	23.8
Parental Edu < HS	49.8	15.0	19.5	15.7
Parental Edu = HS	43.4	12.1	18.4	26.2
Parental Edu > HS	47.4	10.6	13.9	20.1
Single-Parent	43.3	14.7	16.8	25.2

Table 4 shows the round at which households were assigned. By definition, a household was assigned to its  $r^{th}$  listed school if it was assigned in round  $r$ ; and remained unassigned if it failed to get in any of its listed schools. Ninety three percent of households were assigned in the first round; 2.8% were assigned in the second round and 2.7% were unassigned. Across educational groups, college-educated parents were most likely to be assigned to their first choices (93.7%), followed by the lowest educational group. Comparing across all education groups, the middle-education group had the lowest fraction of households assigned to their first choices and the lowest fraction assigned within 10 rounds. Single parents were more likely to be assigned to their first choice compared to their counterpart.

Table 4 Assignment Round (%)

	1st	2nd	3rd-10th	Unassigned
All	93.0	2.8	1.5	2.7
Parental Edu < HS	93.2	2.7	1.6	2.5
Parental Edu = HS	92.0	3.5	1.5	3.0
Parental Edu > HS	93.7	2.3	1.3	2.7
Single-Parent	94.0	1.9	1.4	2.7

Although *as an equilibrium outcome*, most households were assigned to their first choices, it **CC:it is *not* the case** that there was little or no risk involved in the application process. Among those assigned to their first choices, the average admissions probability was 94.6%, with the lowest admissions probability being 0.2. Moreover, the entire set of 317 schools was mainly divided into two groups (Table 15). The first group, accounting for 44% of all schools, were filled up in Round 1, of which 84% rejected some applicants in that round. The second group, accounting for 40% of all schools, were leftover schools. These figures point to the high stake households were faced with. A large number of schools were over-demanded; and once rejected in Round 1, most schools one could get into were leftovers. The fact that most households were assigned to their first choices seems to suggest both the prevalence of strategic play and a large amount of coordination in equilibrium.

Given that most households were assigned to their first choices, Table 5 summarizes the characteristics of the top-listed schools. For all students, the average quality of the top-listed schools was 7.9. The home-school distance was about 710 meters. The distance-quality trade-offs seem to differ across educational groups: as parental education goes up, the quality of top-listed schools increases while the distance decreases. Single parents were more likely to top-list a school with higher quality yet longer distance, compared to an average household.

Table 5 Top-Listed Schools

	Quality	Distance (100m)	Fees (100Euros)
All	7.9 (0.6)	7.1 (8.7)	8.1 (7.7)
Parental Edu < HS	7.6 (0.7)	5.2 (6.2)	5.4 (6.6)
Parental Edu = HS	7.9 (0.5)	7.0 (8.6)	8.1 (7.5)
Parental Edu > HS	8.2 (0.4)	8.7 (9.9)	9.9 (8.1)
Single-Parent	8.0 (0.6)	8.1 (9.9)	8.6 (8.4)

Table 6 “Better” Schools than the Top-Listed One

	% Households	# Better Sch	%Better w/ Higher $p$
All Households (6,836)			
Have Sch. Better in 3 Aspects	40.7%	5.2 (9.9)	14.1%
Have Sch. Better in 2 Aspects	99.8%	75.9 (45.1)	10.5%
Sib Sch. not Top-listed (4,025)			
Have Sch. Better in 3 Aspects	39.3%	4.6 (8.7)	24.8%
Have Sch. Better in 2 Aspects	99.8%	77.3 (44.2)	17.8%

% Households: % of households that satisfy the condition specified in each row.

#Better Sch: average (std.dev.) num. of better schools for households with such schools.

%Better w/ higher  $p$ : % of better sch with higher admission prob. than one's top choice.

To show some suggestive evidence of strategic behavior, we compare a household's top-listed schools with other schools in terms of quality, tuition and distance. A school is labeled as “better” than the top-listed school for the specific household in all three aspects if it had higher quality, lower tuition and shorter home-school distance. It is labeled as “better” in two aspects if it failed to meet one of the three conditions. The upper panel of Table 6 shows that, of all 6,836 households in the sample, 41% had at least one school that was better in 3 aspects. Among these households, the average number of such better schools was 5.2 (standard deviation of 9.9). Almost all households had some schools better than their top choices in 2 aspects, with the average number of such schools being 76.<sup>61</sup> Of course, households' unobservable tastes and/or unobserved school characteristics may drive households' top choice over these seemingly better schools. However, these unobservables are far from sufficient. First, comparing these household-specific “better” schools across households, we find very limited, if any, overlapping, which suggests that school unobservables played a very limited role. **CC: I am not sure I understand what you mean here...you want to say that unobserved characteristics outweigh observed ones to justify that not everybody is getting their true first best, right? But I do not see how showing that better than applied for school are different for different people helps in that. Each individuals' first choice is different because its safe school is different. Whether the first choice is the actual first best or not depends on whether unobserved characteristics**

<sup>61</sup>Appendix Table A1 provides more detailed descriptions of schools better in specific pair of aspects.

can justify the chosen school being better than the other seemingly better schools (in terms of observables). If everybody has identical preferences in terms of the observables I can see that negative unobserved on the best schools could explain the behaviors being truthful but only if the choice was the same for everybody. If the chosen school is different... I am not sure that the statement offers evidence that unobserved cannot rationalize everything. I like the second better, where you see the correlation between better-preferred and probability of assignment...<sup>62</sup> Second, these “better” schools were disproportionately unlikely to be schools for which the household had higher admissions probabilities. For example, for each of the 2,783 households that had some schools better than their top choices in 3 aspects, we calculate the fraction of these better schools for which the household had higher admissions probabilities than its top choice. On average, this fraction was only 14% (last column of Table 6). Similar statistics is reported in the second row, where only 10% of schools that were better in 2 aspects also had higher admissions probabilities than one’s top choice. Such patterns are hard **CC: to** rationalize with truthful reporting, unless households’ unobserved tastes vary directly with the household-specific admissions probabilities. To address the concern that sibling schools may be a dominant factor in one’s application decisions, the lower panel of Table 6 restricts attention to the 4,025 households who did not top-list a sibling school (4,011 of them did not have a sibling school). The same pattern persists: there existed “better” schools than the top-listed ones, and they were very unlikely to be the ones with higher admissions probabilities, which is consistent with strategic behavior rather than truth telling. Together with the rest of schools listed on one’s application list, the fact that households systematically avoided “better” schools with low admissions probabilities as their first choices provides information for us to identify the distribution of household strategic types and that of their preferences.

Table 7 lists the fraction of all students, assigned or unassigned, who were enrolled in the public school system (recall that a household can propose a school with an available seat and be assigned to it after the regular admissions if the household remains unassigned to any of its listed schools). Overall, 97% of applicants were enrolled in the public school system. Applicants with college-educated parents and/or single parents were less likely to enroll. The last row shows that 2.2% of households

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<sup>62</sup>Detailed report available upon request.

chose not to enroll even though they had been assigned to their first choice. The ex-post shocks introduced in the model are meant to rationalize such behaviors. Finally, Table A2 in the appendix shows that the probabilities of being assigned in Round 1 were lower for applicants who did not enroll in the public system than for those who did, suggesting that households who took higher risks might have better outside options.

Table 7 Enrollment in Public System (%)

All	96.7
Parental Edu < HS	97.0
Parental Edu = HS	97.1
Parental Edu > HS	96.3
Single-Parent	96.1
Assigned in Round 1	97.8

## 6 Results

### 6.1 Parameter Estimates

Table 8 presents the estimated parameters governing household preferences. The left panel reports the structural parameters (standard errors in parentheses) that govern the parts of utility function that vary within the same education group, the dispersion of tastes and that of post-application shocks. The linear preference parameter on distance is normalized to -1.<sup>63</sup> The cost of distance is convex with the square term being 0.05, although not precisely estimated. In addition, we allow for two jumps in the cost of distance. The first jump is set at 500 meters, which is meant to capture an easy-to-walk distance even for the 3-year old. Another jump is at the 1 kilometer threshold, which is a long yet perhaps still manageable walking distance. As households may have to rely on some other transportation methods when a school is beyond walking distances, it is not surprising to see that the cost of distance jumps significantly at the thresholds, by about 5.5 kilometers at the first threshold and by another 4.6 kilometers at the second. The next row shows that it is especially attractive for a household to send the child to the same school where her older sibling

<sup>63</sup>As in other discrete choice models, we need to normalize one coefficient in the utility function in order to identify the  $\sigma_\epsilon$  and  $\sigma_\eta$ .

was enrolled in. This parameter adjusts such that most (97%) but not all households with sibling schools top-listed such schools. Compared to both-parent households, single parent households value schools in the public system less. The last two rows on the left panel show, respectively, the dispersion of household preferences across schools and that of post-application shocks.<sup>64</sup> The ex-ante value of the outside option is normalized to zero, while the average value of schools ( $\kappa_j$ ) is estimated to be 3,391. Therefore, a relatively large shock is necessary for a household to give up the assigned public school, especially if it is the household's first choice. The variance of the idiosyncratic tastes is relatively small, which is consistent with households' low willingness to take risks (Table 6).

Table 8 Preference Parameters

Structural Parameter Estimates <sup>a</sup>			Summarize School FE <sup>b</sup>		
			Edu < HS	Edu = HS	Edu > HS
Distance <sup>2</sup>	-0.05 (0.04)	Constant	2766.3	2783.3	2423.0
Distance>5 (100m)	-55.3 (7.1)	Quality	152.0	176.4	187.8
Distance>10 (100m)	-46.5 (7.9)	Fee	-1.0	-0.6	-0.3
Sibling School	1339.0 (86.5)	Semi-Public	-0.6	6.5	0.8
Single Parent	-404.3 (12.2)	Capacity		0.7	
$\sigma_\epsilon$ (taste dispersion)	66.3 (6.2)	Capacity <sup>2</sup>		-0.001	
$\sigma_\eta$ (post-app shock)	1937.8 (18.7)	Quality <sup>2</sup>		-9.9	

<sup>a</sup>Structural preference parameter estimates.

<sup>b</sup>OLS regression of the estimated school value parameters on observables.

It will be overwhelming yet non-informative to directly report the over 300 parameter estimates ( $\kappa_j$ ) related to school values.<sup>65</sup> Instead, we run an OLS regression of school value parameter estimates on observable characteristics as a summary. The form of the OLS regression follows the structural model specification in Section 4.1.2 and Appendix B1. The right panel of Table 8 reports the results. These OLS estimates will be unbiased *only if* the unobserved school characteristics are uncorrelated with the observable ones.<sup>66</sup> As mentioned earlier, this potential correlation

<sup>64</sup>Households take expectations over these ex-post shocks when applying, thus application provides another benefit, i.e., an option value.

<sup>65</sup>The estimates and standard errors of these parameters are available upon request.

<sup>66</sup>Because these OLS estimates are used only to summarize the school values and because they are subject to biases, their standard errors are not reported.



does not affect our analyses and counterfactual experiment results, which use the consistently-estimated school values. However, caveats should be taken when relating school values and/or welfare to school characteristics (fees, quality and capacity). With these caveats, the right panel reveals the following messages, all of which line up with our intuition. 1) Across the three education groups (less than high school, high school, above high school), the middle group values schools within the public school system more than the other two groups, especially the college-educated group. One explanation is that college-educated parents are more likely to be able to afford a costly outside option. These numbers are consistent with the observed behaviors. For example, Table 5 and Table 7 show that although college-educated parents and single parents were more likely to be assigned to their top choices, they were less likely to enroll their children in the public school system. 2) Higher educated parents value school quality more, and are less sensitive to fees.<sup>67</sup> Our findings that parents of different education levels differ in their views of the trade-offs among quality, fees and distance are consistent with those in the literature.<sup>68</sup> 3) Households prefer schools with larger capacity, which is consistent with the fact that larger schools tend to have more resources and lower close-down risks. 4) Everything else being equal, semi-public schools are more preferable except for the low-educated group, presumably because such schools have more flexibility than government-run public schools.

Table 9 presents the estimated parameters governing the probability that a household is strategic, which takes a logistic functional form. Single parents and parents with higher education levels are more likely to be strategic. We do not find that strategic households are more likely to live in zones with more schools, and in fact, the coefficient is slightly negative. However, we do find that strategic households are more likely to live in zones with higher average school quality.<sup>69</sup> Households with older children, which have already gone through the admissions process before, are more likely to be strategic.<sup>70</sup>

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<sup>67</sup>Preferences for quality peak beyond the maximum quality in the data for the college-educated group, at 99th percentile for the middle-educated group, and around 60th percentile for the lowest-educated group.

<sup>68</sup>For example, Burgess et al (2009), Hastings, Kane and Staiger (2008), He (2012) and Abdulkadiroğlu, Agarwal and Pathak (2014).

<sup>69</sup>We allow for the *correlation* between zone characteristics and types. The finding that strategic households are more likely to live in better zones is consistent with our intuition that strategic households may choose home locations to utilize the residence-based priority structure. However, without modeling residential choices, we cannot interpret this correlation as causation in any direction.

<sup>70</sup>One interesting extension of our model is to incorporate the dynamic considerations households

Table 9 Type Distribution

Constant	-18.9 (2.3)
Single Parent	0.3 (0.5)
Education < HS	-0.1 (0.2)
Education > HS	0.7 (0.3)
No. schools in zone	-0.1 (0.2)
Average school quality in zone	3.1 (1.1)
Have an older sibling	49.0 (24.7)

Based on the estimates in Table 9, Table 10 shows the simulated type distribution in our sample. The left panel shows that 96% of all households were strategic, i.e., very few households applied without considering the odds of being admitted.<sup>71</sup> As education level goes up, the fraction of strategic households grows from 95% to 98%. Households with single parents and those with older children were both more likely to be strategic. The upper-right panel of Table 10 shows the average characteristics of the zones in which different types of households lived. On average, strategic (non-strategic) households lived in zones with 22.3 (21.8) schools and the average quality of these schools was 7.9 (7.7).

Table 10 Strategic vs. Non-Strategic Type: Simulation

	Strategic (%)		Strategic	Non-Strategic
All	96.3	Schools in zone		
Parental Edu < HS	94.7	No. Schools	22.3	21.8
Parental Edu = HS	95.8	Ave. quality	7.9	7.7
Parental Edu > HS	97.8			
Single-Parent	96.6			
Have an older sibling	97.1			

## 6.2 Model Fits and Out-of-Sample Validation

The 2007 re-definition of priority zones abruptly changed the school-household-specific priorities. For example, the number of schools to which a household had priority be-  
with multiple children may have.

<sup>71</sup>We find a much smaller fraction of non-strategic households than Abdulkadiroğlu, Pathak, Roth and Sönmez (2006) did. The main reason is our incorporation of the outside option and the leftover schools into the framework, which rationalizes the choices by a substantial fraction of households that might be categorized as non-strategic otherwise. Another reason is the long (over 20 years) practice of BM in Barcelona, where parents have become very familiar with the mechanism.

came 7.0 on average with a standard deviation of 1.5 in 2007, as compared to the 22.3 (7.9) figure in 2006. More importantly, the priority schools became those that surrounded each home location, which also changed the risk-quality-distance trade-offs faced by households. In this section, we show the model fits for both the 2006 and the 2007 samples.<sup>72</sup> To simulate the 2007 outcomes, we first calculate the admissions probabilities in 2007 via the same procedure as we do for the year 2006, using the entire 2007 sample. Then we use the selected 2007 sample of 7,437 households to conduct an out-of-sample validation, where the sample selection rule is the one used for the 2006 sample.<sup>73</sup> To the extent that the change came as a surprise to households, it is reasonable to believe households had been unable to reallocate before submitting their applications in 2007. As such, we simulate the distribution of 2007 household types using the characteristics of their residential zones according to the 2006 definition. We also assume that strategic households had rational expectation about admissions probabilities in 2007.<sup>74</sup>

Considered as the most informative test of the model, the first two rows of Table 11 explore the changes in the definition of priority zones. The 2007 reform led to situations where some schools were in the priority zone for a household in one year but not in the other, which would affect the behavior of a strategic household. As shown in the first row of Table 11, in 2006, 24% of the households in our sample top-listed a school that was in their priority zone by the 2006 definition but not by the 2007 definition. In 2007, the fraction of households that top listed these schools dropped to 12%. On the other hand, the second row of Table 11 shows that the fraction of households that top-listed schools in their priority zone only by the 2007 definition but not by the 2006 definition increased from 3% to 12% over the two years. The model is able to replicate such behaviors and predicts the changes as being from 24% to 14.6% for the first case, and from 4.5% to 11% for the second case. The next 3 rows of Table 11 show that the model fits the data well in terms of the observable characteristics of the top-listed schools. In particular, the model replicates the fact that top-listed schools in 2007 were of similar quality, shorter distance and lower

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<sup>72</sup>Appendix Table A3-A7 show the fits for subgroups of households conditional on demographics.

<sup>73</sup>In 2007, 12,335 Barcelona households participated. We follow the same sample selection rule as that for the 2006 sample. In particular, the 7,437 households in 2007 do not include the 998 parents who reported “high school or above” as their education levels. We interpolate the probability of being college-educated for these parents and include them in the counterfactual policy experiments.

<sup>74</sup>As shown below, we can fit the data in both years, suggesting that our assumptions about the type distribution and the rational expectation for the 2007 case are not unreasonable.

tuition, relative to those in 2006.

Table 11 Top-Listed Schools

	2006		2007	
	Data	Model	Data	Model
In Zone 06 Only (%)	24.1	24.1	12.0	14.6
In Zone 07 Only (%)	3.0	4.5	12.0	10.9
Quality	7.9	7.9	7.9	7.9
Distance (100m)	7.1	7.2	6.6	7.0
Fee (100 Euros)	8.1	8.1	7.9	7.8

As mentioned in the model section, there can be multiple lists that are payoff-equivalent and imply the same allocation results. All these lists have identical ordered elements that are allocation-relevant, which is what our model can explain. For example, consider a list of length 4, the third element of which was a leftover school. Our model is designed to replicate the first three elements of that list, not how many schools would be listed beyond that point. Table 12 presents the model fit for the length of the allocation-relevant part of household application lists. In both years, about 86% of households' lists contained only one allocation-relevant school and fewer than 3% of households had more than 2 relevant schools on their lists, which is not surprising given that most households were assigned in the first round. The model-predicted distribution of the list length lies slightly to the right of the data distribution.

Table 12 Relevant List Length (%)

	2006		2007	
	Data	Model	Data	Model
1	85.8	83.1	86.1	83.2
2	11.5	14.5	11.7	10.3
$\geq 3$	2.7	3.4	2.2	6.5

Table 13 shows the rounds at which households were assigned, given the observed admissions probabilities. The model slightly under-predicts the fraction of households assigned in Round 1 for 2006. Table 14 shows that the model closely replicates the enrollment rate within the public school system. In particular, with the ex-post

shocks, the model replicates the non-enrollment behavior by households who were assigned to their first choices.<sup>75</sup>

Table 13 Assignment Round (%)

	2006		2007	
	Data	Model	Data	Model
1	93.0	91.3	92.0	92.6
2	2.8	4.0	3.1	3.6
$\geq 3$	1.5	0.9	1.8	0.6
Unassigned	2.7	3.8	3.1	3.2

Table 14 Enrollment in the Public System (%)

	2006		2007	
	Data	Model	Data	Model
All	96.7	96.5	97.6	96.6
Assigned in Round 1	97.8	97.1	98.3	97.1

We estimate our model by solving individual households' problem, taking the data admissions probabilities as given. This estimation method avoids having to solve for the equilibrium under BM, which is known for its multiple equilibria. However, it is still worth checking that the model prediction lines up with the observed equilibrium. To do so, we contrast the model-predicted admissions, generated by the BM allocation mechanism and the model-predicted household application profiles, with the data.<sup>76</sup> Table 15 reports the fractions of schools filled in each round. In both years, the model underpredicts (overpredicts) the number of schools filled in the first (second) round, but the overall fit is good.

<sup>75</sup>Appendix D conducts a comparative statics analysis and contrasts our model predictions with those from a counterpart model with fewer strategic households.

<sup>76</sup>In this case, we run the BM algorithm given the simulated applications to allocate households, instead of allocating them according to the observed admissions probabilities as in Table 13. To simulate the allocation results using the BM mechanism, we need to use all households in the sample. Therefore, we include households whose education levels are interpolated (Footnotes 58 and 73).

Table 15 School Filled Round\*(%)

	2006		2007	
Obs: 317	Data	Model	Data	Model
1	44.2	42.0	46.4	44.8
2	7.6	10.6	11.4	12.4
$\geq 3$	8.2	9.0	6.0	8.6
Leftover	40.0	38.4	36.3	34.2

\*A school is filled in Round  $r$  if it has open seats in rounds 1 to  $r$ , but not in later rounds.

## 7 BM vs. GS vs. TTC

Using the estimated model, we are ready to answer the question we posed at the beginning of the paper. How does the current Boston mechanism (BM) compare with two of its alternatives, the Gale-Shapley student deferred acceptance mechanism (GS) and the top trading cycles mechanism (TTC)? In a different experiment, presented in the appendix, we assess the impacts of the 2007 reform, which changed the student-school priority structure by redefining priority zones. In both experiments, households' welfare refers to their evaluations of their assignment outcomes relative to their outside options, i.e.,  $v_{ij}$ .<sup>77</sup>

### 7.1 Theoretical Background

This subsection briefly discusses the properties of the three alternative mechanisms; Appendix E contains detailed descriptions.<sup>78</sup> The GS procedure is similar to BM with the key differences that students are only temporarily assigned to schools in each round and that one's chance of being finally admitted to a school does not depend on the ranking of the school on her application list. The TTC algorithm has a very different structure. Intuitively, in each round TTC creates cycles of trade between individuals. Each individual in a cycle trades off a seat in her highest-priority school for a seat in her announced most preferred school among those that still have

<sup>77</sup>All simulations include the interpolated sample, as mentioned in footnotes 58 and 73. All simulations use the school-household-specific priority scores given by (1), as they were defined by the official rules in the relevant year.

<sup>78</sup>See Abdulkadiroğlu and Sönmez (2003) for further theoretical discussions.

seats. Whenever such a cycle is formed the allocation is final.

Three properties are considered as desirable for a mechanism, i.e., Pareto efficiency, truth revealing and the elimination of justified envy (also known as stability).<sup>79</sup> Unfortunately, all three properties may not hold simultaneously. BM satisfies none of the properties. GS and TTC are both truth revealing.<sup>80</sup> Between the other two conflicting properties, GS eliminates justified envy at the cost of Pareto efficiency, while TTC achieves Pareto efficiency at the cost of stability.

Despite the fact that BM does not satisfy any of the three desirable properties, the welfare comparison between BM and its alternatives is ambiguous.<sup>81</sup> The ambiguity arises from the coexistence of two competing forces. On the one hand, BM can lead to potential misallocations because households hide their true preferences. This source of misallocation is absent in the truth-revealing GS and TTC. On the other hand, BM may better “respect” households’ cardinal preferences than GS and TTC (Abdulkadiroğlu, Che and Yasuda (2011)). BM-induced household behaviors increase the chance that the “right match” is formed, where a school being matched to households that value it more.<sup>82</sup> Under a truth-revealing mechanism, households who share the same ordinal preferences will rank schools the same way in their applications and be given the same chance of being allocated to various schools, regardless of who will gain the most from each school. Given that it is theoretically inconclusive, the welfare comparison between BM and GS or TTC becomes an empirical question, one that we answer below.

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<sup>79</sup>Stability requires that there be no unmatched student-school pair  $(i, j)$  where student  $i$  prefers school  $j$  to her assignment and she has higher priority at  $j$  than some other student who is assigned a seat at school  $j$ .

<sup>80</sup>This is true when one’s priority scores do not depend on one’s choices. As such, throughout our experiments, we consider GS and TTC with the standard priority structure, e.g., one’s priority score in Round 1 does not carry over to future rounds.

<sup>81</sup>The ambiguity has been reflected in the conflicting findings in the theoretical and lab experimental studies that compare BM with GS. Pareto efficiency is an ordinal concept, which does not necessarily imply the highest level of total household welfare.

<sup>82</sup>The intuition can be explained by the following simple example with equal priorities. Consider three schools and a set of households who share the same ordinal but different cardinal preferences for these schools, where the schools are ranked from high to low as Schools 1, 2 and 3. Under BM, the strategic decision is whether to take the high risk and top-list School 1 or to play it safe and top-list School 2. Given the same evaluation for School 1, a household whose evaluations for Schools 2 and 3 are similar is more likely to choose the risky strategy because it has less to lose from the gamble. Given the same evaluation for School 3, a household that values School 1 much higher than School 2 is more likely to choose the risky strategy because it has more to gain from the gamble.

## 7.2 Results

Under both GS and TTC with the standard priority structure, all households, strategic or not, will list schools according to their true preferences. As such, we simulate each household’s application list according to their true preferences and assign them using GS and then using TTC.<sup>83</sup> We compare the results from these two counterfactual mechanisms with those from the baseline.<sup>84</sup> We present our results under the more recent, i.e., the 2007, priority zone structure.<sup>85</sup>

**Remark 5** *In the following, we will present consequences of the reforms from BM to GS and TTC on the total household welfare, the distribution of winners and losers among different subgroups of households, as well as the assignment outcomes. Notice that the level of total household welfare is not necessarily the criterion for social welfare, which may involve different weights across households. Given that we can calculate the welfare changes at the household level, our results can be used to calculate any weighted social welfare. Moreover, household welfare may not be the only factor that policy makers consider. For example, policy makers may put high value on truth-revealing and the elimination of justified envy, which will make GS particularly attractive. Therefore, we do not necessarily recommend one mechanism over another in this paper. However, given a social objective, our results can be easily used for policy-making purposes.*

### 7.2.1 Household Welfare Comparison

To form the basis for comparison, the first column of Table 16 shows the average and the dispersion (standard deviations in parentheses) of welfare among the population under BM. The second column of Table 14 shows changes in welfare ( $\Delta$ utils) when BM is replaced by GS. The average household welfare decreases by 5.4 from the BM level of 3,811. As many other policy changes, a change from BM to GS has different impacts on households qualitatively and quantitatively, which leads to a wide

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<sup>83</sup>Notice that all allocation mechanisms we consider use random lotteries to rank students with the same priority score. As such, for each experiment we simulate the overall allocation procedure and obtain the outcomes for all students for a given set of random lotteries. We repeat this process many times to obtain the expected (average) outcomes for each simulated student.

<sup>84</sup>Notice that to simulate GS and TTC, it is sufficient to know household preferences. However, to compare GS or TTC with the baseline (Boston) mechanism, one needs to know the distribution of household strategic types.

<sup>85</sup>The 2006 results are similar, available on request.



dispersion of welfare changes across households, with an overall standard deviation of 31. Comparing across different groups of households, the average welfare decreases more for non-strategic than for strategic households, and more for lower-educated households.<sup>86</sup>

Table 16 Household Welfare: BM vs. GS vs. TTC

%	BM <sup>a</sup>	GS-BM <sup>b</sup>		TTC-BM <sup>c</sup>	
		$\Delta$ utils	$\Delta$ 100 euros	$\Delta$ utils	$\Delta$ 100 euros
All	3,811 (633)	-5.4 (30.9)	-10.2 (71.8)	1.9 (40.5)	4.6 (96.3)
Strategic	3,810 (633)	-5.3 (30.3)	-10.2 (69.9)	1.6 (39.9)	3.9 (94.3)
Non-strategic	3,818 (629)	-5.7 (41.0)	-19.7 (131.8)	8.3 (51.5)	19.7 (131.8)
Edu < HS	3,690 (600)	-9.9 (28.3)	-10.0 (28.6)	0.1 (31.4)	0.1 (31.7)
Edu = HS	3,934 (619)	-5.7 (33.0)	-18.3 (106.3)	2.5 (44.1)	8.0 (141.9)
Edu > HS	3,792 (647)	-2.1 (30.3)	-3.7 (54.6)	2.7 (42.8)	4.8 (77.2)

<sup>a</sup>welfare under BM, <sup>b</sup>change from BM to GS, <sup>c</sup>change from BM to TTC

$\Delta$ utils: welfare change in utils.  $\Delta$ 100 euros: welfare change in 100 euros.

To translate the welfare from utils to more intuitive measures, we use the education-specific coefficients for fees as reported in Table 8.<sup>87</sup> Column 3 of Table 14 shows that, on average, the change from BM to GS causes a welfare loss of 1,020 euros. Although the welfare loss in utils decreases with education, the decreasing price sensitivity across education groups yields a different ranking of lost euros. For example, the fee-equivalent measure for the lowest-educated group is only 1,000 euros, as compared to 1,830 euros for the middle education group. Clearly, one should not compare the euro losses directly across education groups because they view the same euro amount differently.<sup>88</sup>

The last two columns of Table 16 compares BM with TTC. For an average household, the change from BM into TTC increases the welfare by 460 euros. As such, TTC leads to the highest total household welfare among all three alternatives. The

<sup>86</sup>Our finding that GS decreases welfare for both strategic and non-strategic households is consistent with some recent theoretical work, e.g., Abdulkadiroğlu, Che, and Yasuda (2011).

<sup>87</sup>As mentioned earlier, these coefficients may be biased if the unobserved and the observed school characteristics are correlated. Welfare changes measured in euros, therefore, may be biased. However, welfare changes measured in utils are free of this problem.

<sup>88</sup>Although the households we study face a much larger choice set and a more complicated problem under BM as a result of the special priority rule in Barcelona, our findings are not peculiar. For example, Hwang (2015) and Agarwal and Somaini (2015), who study BM with standard priority structure, also find that BM generates higher welfare than GS.

gains are especially large for the non-strategic households, measured at 1,970 euros. The average welfare increases for all education groups, with the lowest group gaining the least amount.

**Result 1:** In terms of the level of total household welfare, the three mechanisms are ranked as  $TTC > BM > GS$ .

Table 17 Winners and Losers (%)

	BM to GS		BM to TTC	
	Winner	Loser	Winner	Loser
All	11.7	33.0	25.0	21.5
Strategic	11.6	32.8	24.6	21.7
Non-strategic	15.3	36.7	32.3	17.5
Edu < HS	7.8	37.0	23.1	22.1
Edu = HS	13.0	35.5	27.4	23.0
Edu > HS	13.3	28.2	24.3	20.0

Table 17 reports fractions of winners and losers under each change. The first two columns show the case for a change from BM to GS, which benefits 12% of households while hurting 33% of them. Moreover, the fact that there are more losers than winners holds for all subgroups of households. In contrast, as shown in Columns 3 and 4, a change from BM to TTC generates more winners (25%) than losers (22%); and this pattern persists for all subgroups of households.

**Result 2:** There are more losers than winners from a change of BM into GS, and more winners than losers from a change of BM into TTC.

### 7.2.2 Cross-Zone Inequality

A household's welfare can be significantly affected by the school quality within its zone not only because of the quality-distance trade-off, but also because of the quality-risk trade-off created by the priority structure. For equity concerns, a replacement of BM will be more desirable if it is more likely to benefit those living in poor-quality zones. Table 18 tests whether or not each of the counterfactual reforms meets this goal by showing the zone quality among winners and losers from each reform. The first two columns show the case for the change from BM to GS. The winners are

those who live in better zones than the losers: the average zone quality among all winners is 7.83 while that among losers is 7.78. This difference is almost 20% of a standard deviation of quality across zones. As shown in the next three rows, this pattern persists across all educational groups. Therefore, a change from BM to GS increases the dependence of welfare on zone quality, which is against the goal of equity across zones. The next two columns show that changing from BM to TTC, the average zone quality is similar across winners and losers, which implies that such a reform is unlikely to reduce or to enlarge the cross-zone inequality as compared to BM.

**Result 3:** Welfare dependence on zone quality increases with a change from BM to GS, and remains unaffected by a change from BM to TTC.

Table 18 Zone Quality: Winners vs. Losers

	BM to GS		BM to TTC	
	Winner	Loser	Winner	Loser
All	7.83	7.78	7.80	7.80
Edu < HS	7.72	7.67	7.69	7.68
Edu = HS	7.79	7.77	7.79	7.79
Edu > HS	7.90	7.88	7.89	7.91

**The Cost of the Elimination of Justified Envy** Underlying the results in Table 18 is the residence-based priority and the high respect GS has for priorities, the latter enabling GS to eliminate justified envy but not without cost. **CC: As shown by Calsamiglia and Miralles (2014) GS shapley will lead to an overassignment to applicants to neighborhood schools if there is common agreement on what the set of worse schools is in the system.**<sup>89</sup> The first three columns of Table 19 show the fractions of households assigned to schools in their own school zones

<sup>89</sup>Under BM, a risk-taking poor-zone household only needs to compete with other households who top-listed the same school, since the assignment is final at each round. Under GS, the same poor-zone household has to compete not only with those who have the same favorite school but also with those who are unable to get their favorite schools, because the assignment in each round is only temporary. This can make it harder for a poor-zone household to get into a better school out of its zone under GS than under BM, which in turn can make such a household worse off under GS. Under TTC, having high priority to a better school increases one's chance to form a trading cycle. However, conditional on forming a cycle, the assignment does not depend one's priority for the receiving school. Therefore, who wins and who loses from the change of BM into TTC depends much less on the quality of one's own zone. See the appendix for the details of the GS and the TTC algorithms.

under alternative mechanisms. The first row shows that about 70% of all households are assigned to schools in zone under GS, followed by the case of BM (65%) and finally TTC (58%). The second row shows this fraction among households whose favorite schools are in their school zones. While over 90% of these households are assigned to in-zone schools under BM and GS, this figure is only 85% under TTC. Most illustrative of the point, the third row shows this fraction among households whose favorite schools are out of their school zones. Even though a household would like to attend a school out of its zone, due to its high respect for priorities, GS assigns 38% of them within their zone, as compared to 28% under BM and 19% under TTC.

Table 19 The Cost of the Elimination of Justified Envy

	Assigned in Zone (%)			Assigned to Favorite (%)		
	BM	GS	TTC	BM	GS	TTC
All Households	65.1	70.1	58.4	68.3	64.1	67.8
Favorite is in Zone	90.2	91.7	85.5	82.8	79.3	74.0
Favorite is out of Zone	28.4	38.4	18.8	47.2	41.8	58.9

The last three columns of Table 19 shows the extent to which the respect for priorities hampers households' chances of being assigned to their favorite schools. Row 1 shows that over all households, the chance of being assigned to their favorite schools is the about 68% under both BM and TTC, and 64% under GS. Row 2 shows that BM is the best at accommodating households preferences if their favorite schools are in their school zones, followed by GS and then TTC. Finally, while TTC enables 59% of households whose favorite schools are out of their zones to attend their favorite schools, this fraction is only 47% under BM and 42% under GS.

**Result 4:** GS assigns the largest fraction of households to in-zone schools, followed by BM and then TTC. In terms of enabling households to get out of their zones to attend their desired schools, the three mechanisms are ranked as  $TTC > BM > GS$ .

**Remark 6** *Like most studies on school choice mechanisms, our cross-mechanism comparisons takes student-school priority structures as given.<sup>90</sup> These structures differ across cities; and they play an essential role in the allocation of students. We leave*

<sup>90</sup>See Kominers and Sönmez (2012) and Dur, Kominers, Pathak, and Sönmez (2013) for examples of theoretical studies on priority structures.

it for future research, with data from multiple cities, to understand the trade-offs and social objectives underlying these different priority structures.

### 7.2.3 School Assignment

Table 20 compares the assignment outcomes across mechanisms. The first three columns show the changes in the characteristics of schools households are assigned to when BM is replaced by GS.<sup>91</sup> For an average household, school quality increases by 0.04, school-home distance reduces by 40 meters, and fee increases by 8 euros.<sup>92</sup> The low-education group sees the smallest increase in quality, the smallest deduction in distance, while the largest increase in fees, which explains why the average welfare (utils) decreases the most for this group (Table 16).

Table 20 School Assignment

	GS-BM			TTC-BM		
	Quality	Distance(100m)	Fees(euro)	Quality	Distance	Fees
All	0.04 (0.4)	-0.4 (4.7)	7.8 (333.2)	0.05 (0.5)	0.6 (5.5)	12.4 (427.0)
Edu < HS	0.02 (0.3)	-0.1 (4.2)	8.8 (284.6)	0.03 (0.4)	0.5 (4.7)	2.4 (360.4)
Edu = HS	0.05 (0.5)	-0.5 (5.1)	6.3 (356.0)	0.06 (0.5)	0.7 (5.9)	17.1 (461.1)
Edu > HS	0.05 (0.5)	-0.5 (4.7)	8.3 (343.9)	0.06 (0.5)	0.7 (5.6)	15.3 (438.8)

The last three columns of Table 20 show the changes when BM is replaced by TTC. School quality increases by 0.05 on average, with the lowest-educated group experiencing the smallest increase of 0.03. School-home distance increases by about 60 meters for an average household. All households are assigned to slightly more expensive schools on average. These figures illustrate the fact that the current BM leads to misallocation as people hide their true preferences, and that it makes households inefficiently apply for close-by schools that they have higher priority for, while giving up higher-quality schools with longer distance that they have lower priority for.

**Result 5:** Compared to TTC, both BM and GS inefficiently assign students to schools that are of shorter distance but lower quality.

<sup>91</sup>The baseline case is presented in the appendix Table A9.

<sup>92</sup>A non-zero average change in quality is possible because there are more school seats than students city-wise.

## 8 Conclusion

We have developed and estimated a model of school choices by households under the Boston mechanism. We have recovered the joint distribution of household preferences and their strategic vs. non-strategic types, using administrative data from Barcelona before a drastic change in the definition of households' priority school zones. The estimated model has been validated using data after this drastic change.

We contribute to the on-going debates on school choice mechanism designs by quantifying the welfare impacts of replacing the Boston mechanism with its two alternatives, GS and TTC. A change from the Boston mechanism to GS creates more losers than winners. This change also increases the dependency of a household's welfare on the quality of its school zones, leading to further inequality concerns across residential zones. In contrast, a change from the Boston mechanism to TTC creates more winners than losers. However, the change of BM to TTC is unlikely to reduce or to enlarge the cross-zone inequality.

The methods developed in this paper and the main empirical findings are promising for future research. One particularly interesting extension is to incorporate household's residential choices into the framework of this paper. Individual households may relocate in order to take advantage of changes in school choice mechanisms and/or in residence-based priority structures. Such individual incentives will in turn affect the housing market. There is a large literature on the capitalization of school quality for housing prices, as reviewed by Black and Machin (2010) and Gibbons and Machin (2008).<sup>93</sup> An important yet challenging research project involves combining this literature and the framework proposed in our paper, in order to form a more comprehensive view of the equilibrium impacts of school choice mechanisms on households' choices of schools and residential areas, and on the housing market.

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<sup>93</sup>Ries and Somerville (2010) exploit changes in the catchment areas of public schools in Vancouver and find significant effects of school performance on housing prices. Epple and Romano (2003) conjecture that school choice systems can eliminate the capitalization of school quality on the housing market. Machin and Salvanes (2010) exploit policy reforms in Oslo that allowed students to attend schools without having to live in the school's catchment area, and find a significant decrease in the correlation between a school's quality and housing prices.

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## Appendix

### A1. Properties of the optimal list for a strategic household.

Consider an optimal list  $A_i^1 = \{a_1^1, \dots, a_r^1, \dots, a_{R'}^1\}$  derived by the backward induction as in Section 3.3.2, if the student does not face 100% admissions rate for any of the first  $r - 1$  listed schools, and she does for the  $r^{th}$  listed school ( $p_{a_r^1}^r(s\mathbf{1}|\bar{\xi}_i^r) = 1$ ), then the following lists all generate the same value for the household as  $A_i^1$  does, and hence are all optimal:

- 1) a list that shares the same first  $r$  elements of  $A_i^1$ .<sup>94</sup>
- 2) a list of length  $n$  ( $r < n \leq R$ ), which shares the same first  $r - 1$  elements of  $A_i$  **CC: and the  $n^{th}$  element is  $a_r^1$ ,  $p_{a_r^1}^n(s\mathbf{1}|\bar{\xi}_i^r) = 1$  and for all elements  $r' \in \{r, \dots, n - 1\}$ , the household faces 0 admissions probability.**
- 3) Furthermore, if this  $r^{th}$  listed school is one's backup school with  $p_{a_r}(\cdot) = 1$ , then any list of length  $n$  ( $r - 1 \leq n \leq R$ ) is also optimal if it has the same first  $r - 1$  elements of  $A_i^1$  and the admissions probabilities to the other elements are all 0. **CC: we can eliminate this last paragraph now, right?**

**“For components on a list that do not affect the value of the list, we do not impose that they be in the set  $J_i^*$  chosen in the first part of a strategic household's decision procedure”.**

### A2. Proof for Claim 1

An application list with the following features reveals that the household must be non-strategic: 1) for some  $r^{th}$  ( $r > 1$ ) element  $a_r$  on the list  $p_{a_r}^r(\cdot) = 0$ , and 2)

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<sup>94</sup>In particular, one optimal list may have the same school  $j$  listed in two different rounds  $r < r'$ , with  $p_j^r(s\mathbf{1}|\bar{\xi}_i^r) = 1$ .

$p_{a_{r'}}^{r'}(\cdot) < 1$  for all  $r' < r$ , and 3) for some  $r'' \geq r + 1$ ,  $0 < p_{a_{r''}}^{r''}(\cdot) < 1$  and  $p_{a_{r''''}}^{r''''}(\cdot) < 1$  for any  $r < r''' < r''$ .

Without Feature 2), the list can still be strategically optimal due to Remark 3. Without Feature 3) a household may still be strategic if it prefers some sure-to-get-in school listed later over any of the schools listed after  $a_r$ , including  $a_r$ . All three features guarantee that the household is non-strategic.

**Proof.** Take a given list that satisfies all three features in Claim 1:  $A = \{a_1, \dots, a_r, \dots, a_{r''}, \dots\}$ , where  $a_{r''}$  is the *first* school that satisfies Feature 3). This implies that  $p_{a_{r''''}}^{r''''}(s\mathbf{1}|\bar{\xi}_i^{r''''}) = 0$  for all  $r < r''' < r''$ , since Feature 3) ensures that they be smaller than 1 and  $r''$  is the first to be strictly positive. Let  $W_i^r(A)$  be the residual value of this list starting from the  $r^{th}$  element.

$$\begin{aligned}
W_i^r(A) &= p_{a_r}^r \left( s\mathbf{1}|\bar{\xi}_i^r \right) v_{ia_r} + (1 - p_{a_r}^r \left( s\mathbf{1}|\bar{\xi}_i^r \right)) W_i^{r+1}(A) \\
&= W_i^{r+1}(A) \\
&= p_{a_{r+1}}^{r+1} \left( s\mathbf{1}|\bar{\xi}_i^r \right) v_{ia_{r+1}} + (1 - p_{a_{r+1}}^{r+1} \left( s\mathbf{1}|\bar{\xi}_i^r \right)) W_i^{r+2}(A) \\
&= W_i^{r+2}(A) \\
&= \dots \\
&= p_{a_{r''-1}}^{r''-1} \left( s\mathbf{1}|\bar{\xi}_i^r \right) v_{ia_{r''-1}} + (1 - p_{a_{r''-1}}^{r''-1} \left( s\mathbf{1}|\bar{\xi}_i^r \right)) W_i^{r''}(A) \\
&= W_i^{r''}(A).
\end{aligned}$$

Consider an alternative (not necessarily optimal) application list  $B = \{a_1, \dots, a_{r''}, \dots, a_{r''}, \dots\}$ , which differs from  $A$  only in that  $a_r$  is replaced by  $a_{r''}$ . Note that  $p_{a_{r''}}^r(\cdot) = 1$  since the school is filled up in round  $r'' > r$ .<sup>95</sup> The residual value of this list at its  $r^{th}$  element

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<sup>95</sup>Note that for both list  $A$  and list  $B$ , the value of  $\bar{\xi}_i^r$  in every round is the same because there is no updating for rounds when  $p_{a_{r''''}}^{r''''}(s\mathbf{1}|\bar{\xi}_i^{r''''}) = 0$  for all  $r < r''' < r''$ . As a result,  $W_i^{r+1}(B) = W_i^{r+1}(A)$ .

(now  $a_{r''}$ ) is given by

$$\begin{aligned}
W_i^r(B) &= p_{a_{r''}}^r \left( s\mathbf{1}|\bar{\xi}_i^r \right) v_{ia_{r''}} + (1 - p_{a_{r''}}^r \left( s\mathbf{1}|\bar{\xi}_i^r \right)) W_i^{r+1}(B) \\
&= p_{a_{r''}}^r \left( s\mathbf{1}|\bar{\xi}_i^r \right) v_{ia_{r''}} + (1 - p_{a_{r''}}^r \left( s\mathbf{1}|\bar{\xi}_i^r \right)) W_i^{r+1}(A) \\
&= p_{a_{r''}}^r \left( s\mathbf{1}|\bar{\xi}_i^r \right) v_{ia_{r''}} + (1 - p_{a_{r''}}^r \left( s\mathbf{1}|\bar{\xi}_i^r \right)) W_i^{r''}(A) \\
&> p_{a_{r''}}^{r''} \left( s\mathbf{1}|\bar{\xi}_i^{r''} \right) v_{ia_{r''}} + (1 - p_{a_{r''}}^{r''} \left( s\mathbf{1}|\bar{\xi}_i^{r''} \right)) W_i^{r''}(A) \\
&= W_i^{r''}(A)
\end{aligned}$$

The inequality holds because  $p_{a_{r''}}^r \left( s\mathbf{1}|\bar{\xi}_i^r \right) > p_{a_{r''}}^{r''} \left( s\mathbf{1}|\bar{\xi}_i^{r''} \right)$  (admissions probabilities decrease over rounds) and

$$v_{ia_{r''}} = E \max \left\{ u_{ia_{r''}}, \eta \right\} > E(\eta) = 0.$$

Given that the first  $r-1$  elements are also unchanged, it is immediate that the value of the whole list  $W_i^1(B) > W_i^1(A)$ . ■

### A3. Dimensionality of Strategic Household's Problem

#### Case 2) Constant Lottery Number

Let  $O^r \subseteq J$  be the subset of schools that have been filled up by the beginning of Round  $r$ , the size of which is given by  $|O^r| \equiv \sum_{j=1}^J I(r > \bar{r}_j)$ . Let  $J^r(S_i) \subseteq O^{r+1}$  be the subset of schools for which a household with priority score vector  $S_i$  can possibly be subject to lotteries in round  $r$ , the size of which is given by

$$|J^r(S_i)| \equiv \sum_{j=1}^J I(s_{ij} = \bar{s}_j, r = \bar{r}_j). \quad (14)$$

Let  $NA^r(S_i) \subseteq O^{r+1}$  be the subset of schools that would reject  $i$  for sure in Round  $r$  ( $NA$  for not available).

In Round 1,  $\bar{\xi}_i^1 \in \{1\}$ . For Round  $r > 1$ , including the unconditional upper bound of 1, the maximum number of different values the state variable  $\bar{\xi}_i^r$  can take is  $1 + \sum_{r'=1}^{r-1} |J^{r'}(S_i)|$ , which happens when the  $cut_j$ 's are all different and those occur in Round  $r$  are uniformly higher than those occurring in Round  $r+1$ .<sup>96</sup> Notice 1)

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<sup>96</sup>For example, suppose there is one  $cut_1$  in Round 1 and one  $cut_2$  in Round 2, so that  $\bar{\xi}_i^1 \in$

$\bar{\xi}_i^r = 1$  for  $r > 1$  is possible only if the school listed  $a_{r'} \in NA^{r'}(S_i)$  for all  $r' < r$ , and 2)  $\{\cup_{r' < r} J^{r'}(S_i), \cup_{r' < r} NA^{r'}(S_i)\} \subseteq O^r$  hence  $1 + \sum_{r'=1}^{r-1} |J^{r'}(S_i)| \leq |O^r|$ ; and the inequality is strict if  $|\cup_{r' < r} NA^{r'}(S_i)| > 1$ . Therefore, given there are  $J$  schools to choose from in each round (including those with zero admissions probabilities), the dimension of the problem in Case 2) cannot be larger than  $J \left(1 + \sum_{r=2}^R (1 + \sum_{r'=1}^{r-1} |J^{r'}(S_i)|)\right) \leq J \left(1 + \sum_{r=2}^R |O^r|\right)$ , which is much smaller than  $|\mathbf{P}(J; R)|$ .

### Case 3) Constant Lottery Number and Constant Priority Score

In Round 1, a household's state variable is again  $\bar{\xi}_i^1 \in \{1\}$ . In Round  $r > 1$ , besides the state variable  $\bar{\xi}_i^r$ , there is also an additional state variable, the priority score of one's top-listed school. Let  $\Omega_i$  be the support of Household  $i$ 's priority scores and  $|\Omega_i|$  be the size of  $\Omega_i$ . For a given  $s$  in  $\Omega_i$ , the number of schools for which Household  $i$  can possibly be subject to lotteries in round  $r > 1$  is given by  $|J^r(s\mathbf{1})| \equiv \sum_{j=1}^J I(s = \bar{s}_j, r = \bar{r}_j)$ . Therefore given  $s$ , the maximum number of different values the state variable  $\bar{\xi}_i^r$  can take is at most  $1 + \sum_{r'=1}^{r-1} |J^{r'}(s\mathbf{1})|$ . Notice that  $\cup_{r' < r, s \in \Omega_i} J^{r'}(s\mathbf{1}) \subseteq O^r$ , hence  $\sum_{s \in \Omega_i} (1 + \sum_{r'=1}^{r-1} |J^{r'}(s\mathbf{1})|) \leq |\Omega_i| + |O^r|$ . Therefore, the dimension of the problem in Case 3) cannot be larger than  $J \left(1 + \sum_{s \in \Omega_i} \sum_{r=2}^R (1 + \sum_{r'=1}^{r-1} |J^{r'}(s\mathbf{1})|)\right) = J \left(1 + \sum_{r=2}^R \sum_{s \in \Omega_i} (1 + \sum_{r'=1}^{r-1} |J^{r'}(s\mathbf{1})|)\right) \leq J \left(1 + \sum_{r=2}^R (|\Omega_i| + |O^r|)\right) = J \left(1 + (R-1)|\Omega_i| + \sum_{r=2}^R |O^r|\right)$ , which is in turn smaller than  $|\mathbf{P}(J; R)|$ .

### B1. Detailed Functional Forms

Household Characteristics:  $x_i = [x_{i1}, \dots, x_{i5}]$ , where  $x_{i1} = I(\text{edu}_i < \text{high school})$ ,  $x_{i2} = I(\text{edu}_i = \text{high school})$ ,  $x_{i3} = I(\text{edu}_i \geq \text{College})$ ,  $x_{i4} = I(\text{single parent}_i = 1)$ ,  $x_{i5} = \text{sibling's school}$  ( $x_{i5} = 0$  if outside school,  $\in \{1, \dots, J\}$  if non-private school,  $-9$  if no sibling).

School Characteristics:  $w_j = [w_{j1}, w_{j2}, w_{j3}, w_{j4}]$ , where  $w_{j1}$  is school quality,  $w_{j2}$  is tuition level,  $w_{j3}$  is capacity, and  $w_{j4} = 1$  if the school is semi-public, 0 otherwise.

Home-school distance:  $d_{ij}$ , measured in 100 meters.

Zone Characteristics: Let  $N_z$  be the number of schools in zone  $z$ , and  $\bar{q}_z$  be the average school quality in zone  $z$ .

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$\{1, \text{cut}_1\}$ . Given the rule that  $\bar{\xi}_i^2 = \min\{\text{cut}_2, \bar{\xi}_i^1\}$ , if  $\text{cut}_1 \leq \text{cut}_2$  then the  $\bar{\xi}_i^2 \in \{1, \text{cut}_1\}$ , while if  $\text{cut}_1 > \text{cut}_2$ ,  $\bar{\xi}_i^2 \in \{1, \text{cut}_1, \text{cut}_2\}$ .

### B1.1 Utility functions

Household preference heterogeneity is captured via three channels: 1) households of different characteristics may have different overall evaluation of public schools relative to the outside option; 2) households with different parental education may have different trade-offs among distance, quality, tuition costs and unobserved school characteristics; 3) each household has its idiosyncratic vector of tastes for schools. Formally, Household  $i$ 's utility from attending school  $j$  is given by  $u_{ij} = U(w_j, x_i, d_{ij}, \zeta_j) + \epsilon_{ij}$ . Define  $g^*(\cdot)$  and  $C(\cdot)$  such that  $U(w_j, x_i, d_{ij}, \zeta_j) = g^*(w_j, x_i, \zeta_j) - C(d_{ij})$ .  $g^*(w_j, x_i, \zeta_j) =$

$$\begin{aligned} & \tau_1 x_{i4} + \tau_2 [I(x_{i5} = j) - I(x_{i5} = 0)] \\ & + \sum_{m=1}^3 x_{im} (\delta_{0m} + \delta_{1m} \zeta_j) + w_{j1} \left( \sum_{m=1}^3 \alpha_m x_{im} \right) + w_{j2} \left( \sum_{m=1}^3 \alpha_{3+m} x_{im} \right) \\ & + \alpha_7 w_{j3} + \alpha_8 w_{j3}^2 + \alpha_9 w_{j1}^2 + w_{j4} \left( \sum_{m=1}^3 \alpha_{9+m} x_{im} \right), \end{aligned}$$

where the last two rows describe the part of education-specific utility that depends on  $w_j$  and  $\zeta_j$ , with the form of

$$\sum_e (\delta_{0e} + \delta_{1e} \zeta_j + w_j \alpha_e) I(edu_i = e).$$

The cost for distance is given by

$$C(d_{ij}) = [d_{ij} + c_1 d_{ij}^2 + c_2 I(d_{ij} > 5) + c_3 I(d_{ij} > 10)].$$

### B1.2 Type distribution

$$\lambda(x_i, l_i) = \lambda(x_i, z_{l_i}) = \frac{\exp(\beta_0 + \sum_{m=1}^4 \beta_m x_{im} + \beta_5 I(x_{i5} \geq 0) + \beta_6 N_{z_{l_i}} + \beta_7 \bar{q}_{z_{l_i}})}{1 + \exp(\beta_0 + \sum_{m=1}^4 \beta_m x_{im} + \beta_5 I(x_{i5} \geq 0) + \beta_6 N_{z_{l_i}} + \beta_7 \bar{q}_{z_{l_i}})}.$$

### B2. Identification

Since the dispersion of post-application shocks is mainly identified from the enrollment decisions, to ease the illustration, we show the identification of the model



without post-application shocks. A household has observables  $(x_i, l_i)$  and can be one of two types  $T = 0, 1$ . Home-school distance is given by  $d_{ji} = d(l_i, l_j)$  and  $z_{l_i}$  is the zone that  $l_i$  belongs. Let the taste for school be  $\epsilon_{ij} \sim i.i.d. N(0, 1)$ .<sup>97</sup> In line with (IA2) and (IA3) in the paper, assume that  $d$  is independent of  $T$  conditional on  $(x, z_l)$  and  $\epsilon$  is independent of  $(x, l, T)$ . To give the idea, consider the case where a household can apply only to one school from the choice set of schools 1 and 2, and where all households face the same admissions probabilities. Household-specific admissions probabilities provide more variations, which will provide more identification power.

Let  $\bar{u}_{ij}$  be the utility net of individual taste,  $\bar{u}_{ij} = g(\kappa_j, w_j, x_i) - C(d_{ij})$ , where  $g(\cdot)$  is the reduced form function given by

$$g(\kappa_j, w_j, x_i) = \tau_1 x_{i4} + \tau_2 [I(x_{i5} = j) - I(x_{i5} = 0)] + \sum_e (\delta_{0e} + \delta_{1e} \kappa_j + \rho_e w_j) I(edu_i = e).$$

Let  $p_j > 0$  be the probability of admission to school  $j$  and  $p_1 \neq p_2$  (IA1). Let  $y$  be the decision to list School 1.  $y$  is related to the latent variable  $y^*$  in the following way

$$y(x_i, l_i, \epsilon_i, T) = 1 \text{ if only if}$$

$$y^*(x_i, l_i, \epsilon_i, T) = T(p_{i1}u_{i1} - p_{i2}u_{i2}) + (1 - T)(u_{i1} - u_{i2}) > 0.$$

Hence the probability of observing the decision to list 1 by someone with  $(x_i, l_i)$  is

$$H(x_i, l_i) = \lambda(x_i, z_{l_i}) \Phi\left(\frac{p_1 \bar{u}_{i1} - p_2 \bar{u}_{i2}}{\sqrt{p_1^2 + p_2^2}}\right) + (1 - \lambda(x_i, z_{l_i})) \Phi\left(\frac{\bar{u}_{i1} - \bar{u}_{i2}}{\sqrt{2}}\right).$$

Fix  $(x, z_l)$ ,  $H(\cdot)$  only varies with  $d$ , so we can suppress the dependence on  $(x, z_l)$  and let  $g(\kappa_j, w_j, x_i) = g_j$  such that

$$\begin{aligned} H(d) &= \lambda \Phi\left(\frac{(p_1 g_1 - p_2 g_2) - (p_1 C(d_1) - p_2 C(d_2))}{\sqrt{p_1^2 + p_2^2}}\right) \\ &\quad + (1 - \lambda) \Phi\left(\frac{(g_1 - g_2) - (C(d_1) - C(d_2))}{\sqrt{2}}\right). \end{aligned} \tag{15}$$

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<sup>97</sup>Given that the linear distance enters the utility function with coefficient of minus one, the standard deviation of  $\epsilon$  is identified from the variation in distance within  $(x, z_l)$  group. To simplify the notation, we will present the case where  $\sigma_\epsilon$  is normalized to 1.

### B2.1 Identification of $g(\cdot)$ and $\lambda(\cdot)$

The following theorem shows that fix any  $(x, z_l)$ ,  $g(\kappa_j, w_j, x)$  and  $\lambda(x, z_l)$  are identified.

**Theorem 1** *Assume that 1)  $\lambda \in (0, 1)$ , 2) there exists an open set  $D^* \subseteq D$  such that for  $d_{ij} \in D^*$ ,  $C'(d_{ij}) \neq 0$ . Then the parameters  $\theta = [g_1, g_2, \lambda]'$  in (15) are locally identified from the observed application decisions.*

**Proof.** The proof draws on the well-known equivalence of local identification with positive definiteness of the information matrix. In the following, I will show the positive definiteness of the information matrix for model (15).

Step 1. Claim: *The information matrix  $I(\theta)$  is positive definite if and only if there exist no  $\omega \neq 0$ , such that  $\omega' \frac{\partial H(d)}{\partial \theta} = 0$  for all  $d$ .*

The log likelihood of an observation  $(y, d)$  is

$$L(\theta) = y \ln(H(d)) + (1 - y) \ln(1 - H(d)).$$

The score function is given by

$$\frac{\partial L}{\partial \theta} = \frac{y - H(d)}{H(d)(1 - H(d))} \frac{\partial H(d)}{\partial \theta}.$$

Hence, the information matrix is

$$I(\theta|d) = E \left[ \frac{\partial L}{\partial \theta} \frac{\partial L}{\partial \theta'} | d \right] = \frac{1}{H(d)(1 - H(d))} \frac{\partial H(d)}{\partial \theta} \frac{\partial H(d)}{\partial \theta'}.$$

Given  $H(d) \in (0, 1)$ , it is easy to show that the claim holds.

Step 2. Show  $\omega' \frac{\partial H(d)}{\partial \theta} = 0$  for all  $d \implies \omega = 0$ .

Define  $p_j^* = \frac{p_j}{\sqrt{p_1^2 + p_2^2}}$ ,  $B_1(d) = (p_1^* g_1 - p_2^* g_2) - (p_1^* C(d_1) - p_2^* C(d_2))$ , and  $B_0(d) = \left( \frac{(g_1 - g_2) - (C(d_1) - C(d_2))}{\sqrt{2}} \right)$ ,  $\frac{\partial H(d)}{\partial \theta}$  is given by:

$$\begin{aligned} \frac{\partial H(d)}{\partial \lambda} &= \Phi(B_1(d)) - \Phi(B_0(d)) \\ \frac{\partial H(d)}{\partial g_1} &= \lambda \phi(B_1(d)) p_1^* + (1 - \lambda) \phi(B_0(d)) \frac{1}{\sqrt{2}} \\ \frac{\partial H(d)}{\partial g_2} &= -\lambda \phi(B_1(d)) p_2^* - (1 - \lambda) \phi(B_0(d)) \frac{1}{\sqrt{2}}. \end{aligned}$$

Suppose for some  $\omega$ ,  $\omega' \frac{\partial H(d)}{\partial \theta} = 0$  for all  $d$  :

$$\begin{aligned} & \omega_1 [\Phi(B_1) - \Phi(B_0)] + \omega_2 \left( \lambda \phi(B_1) p_1^* + (1 - \lambda) \phi(B_0) \frac{1}{\sqrt{2}} \right) \\ & - \omega_3 \left( \lambda \phi(B_1) p_2^* + (1 - \lambda) \phi(B_0) \frac{1}{\sqrt{2}} \right) = 0 \end{aligned}$$

Take derivative with respect to  $d_2$  evaluated at some  $d_2 \in D^*$

$$\begin{aligned} & \omega_1 \left[ \phi(B_1) p_2^* - \frac{\phi(B_0)}{\sqrt{2}} \right] C'(d_2) + \omega_2 \left( \lambda \phi'(B_1) p_1^* p_2^* + (1 - \lambda) \phi'(B_0) \frac{1}{2} \right) C'(d_2) \\ & - \omega_3 \left( \lambda \phi'(B_1) (p_2^*)^2 + (1 - \lambda) \phi'(B_0) \frac{1}{2} \right) C'(d_2) = 0. \end{aligned} \quad (16)$$

Define  $\gamma(d) = \frac{\phi(B_1)}{\phi(B_0)}$ , divide (16) by  $\phi(B_0)$  :

$$\begin{aligned} & \omega_1 \left[ \gamma(d) p_2^* - \frac{1}{\sqrt{2}} \right] - \omega_2 \left( \lambda B_1 \gamma(d) p_1^* p_2^* + (1 - \lambda) B_0 \frac{1}{2} \right) \\ & + \omega_3 \left( \lambda B_1 \gamma(d) (p_2^*)^2 + (1 - \lambda) B_0 \frac{1}{2} \right) = 0 \end{aligned}$$

$$\gamma(d) [\omega_1 p_2^* - \lambda B_1 p_2^* (\omega_2 p_1^* - \omega_3 p_2^*)] - \left[ \frac{\omega_1}{\sqrt{2}} + (\omega_2 - \omega_3) (1 - \lambda) B_0 \frac{1}{2} \right] = 0 \quad (17)$$

Since  $\gamma(d)$  is a nontrivial exponential function of  $d$ , (17) hold for all  $d \in D^*$  only if both terms in brackets are zero for each  $d \in D^*$ , i.e.

$$\begin{aligned} & \omega_1 p_2^* - \lambda B_1(d) p_2^* (\omega_2 p_1^* - \omega_3 p_2^*) = 0 \\ & \frac{\omega_1}{\sqrt{2}} + (\omega_2 - \omega_3) (1 - \lambda) B_0(d) \frac{1}{2} = 0. \end{aligned} \quad (18)$$

Take derivative of (18) again with respect to  $d_2$ , evaluated at  $d_2 \in D^*$  :

$$\begin{aligned} & -\lambda C'(d_2) (p_2^*)^2 (\omega_2 p_1^* - \omega_3 p_2^*) = 0 \\ & (\omega_2 - \omega_3) (1 - \lambda) C'(d_2) \frac{1}{2\sqrt{2}} = 0. \end{aligned}$$

Since  $\lambda \in (0, 1)$ ,  $p_j > 0$  (hence  $p_2^{*2} > 0$ ) and  $C'(d_2) \neq 0$  for some  $d$ , we have

$$\begin{aligned}\omega_2 p_1^* - \omega_3 p_2^* &= 0 \\ \omega_2 - \omega_3 &= 0.\end{aligned}$$

Given  $p_1 \neq p_2$  (hence  $p_1^* \neq p_2^*$ ), follows that  $\omega = 0$ . ■

### B2.2 Identification of $C(d_{ij})$ and $(\tau, \delta, \kappa, \rho)$

1) Given the identification result from B2.1, and given that  $C(d_{ij})$  is common across  $(x, z_l)$ 's, the parameters in  $C(d_{ij})$  solves for the system of equations (15), where one equation corresponds to one  $(x, z_l)$ .

2) Once the value of  $g(\kappa_j, w_j, x_i)$  is identified for each  $(j, x_i)$ , the parameters governing  $g(\cdot)$  are identified using the variations in  $(w_j, x_i)$ . In particular, for the middle-education group  $\delta_{02} = 0$ ,  $\delta_{12} = 1$ ,  $\alpha_2 = 0$  are normalized, which means  $g(\kappa_j, w_j, x_i) = \tau_1 x_{i4} + \tau_2 [I(x_{i5} = j) - I(x_{i5} = 0)] + \kappa_j$ . As a result,  $(\tau, \{\kappa_j\}_j)$  are identified using the variation of  $(x_{i4}, x_{i5})$  within the middle-education group, so the value  $(\delta_{0e} + \delta_{1e}\zeta_j + w_j\alpha_e)$  is known for  $e = 1, 3$ . The variation of  $w_j$  thus identifies  $(\delta_e, \alpha_e)$ .

### C. Priority Score Structure

Case 1: Those who do not have a sibling in school have two levels:  $x_i a$  ( $x_i a + b_1$ ) for out-of-zone (in-zone) schools.

Case 2: Those whose sibling(s) is (are) in in-zone schools have 3 levels:  $x_i a$  ( $x_i a + b_1$ ) for out-of-zone (in-zone) non-sibling schools, and  $x_i a + b_1 + b_2$  for sibling schools.

Case 3: Those whose sibling(s) is (are) in out-of-zone schools have 3 levels:  $x_i a$  ( $x_i a + b_1$ ) for out-of-zone (in-zone) non-sibling schools, and  $x_i a + b_2$  for sibling schools.

Case 4: Those with sibling(s) in some in-zone school and sibling(s) in some out-of-zone school have 4 levels:  $x_i a$  ( $x_i a + b_1$ ) for out-of-zone (in-zone) non-sibling schools, and  $x_i a + b_2$  ( $x_i a + b_1 + b_2$ ) for out-of-zone (in-zone) sibling schools.

### D. Comparative Statics Analysis

Appendix Table A8 contrasts our model predictions with those from a counterpart model with only 80% strategic households, which is achieved by adjusting the constant in the type distribution function. With fewer strategic household, the model predicts that the top-listed schools are of higher quality, longer distance and lower cost. More importantly, only 88% of households will be assigned to their first choices, as compared

to the 91% prediction from the baseline model. The counterpart model also predicts a lower fraction of households enrolled in the public system.

## E. The GS and TTC Algorithms

**E1. The GS algorithm** assigns students as follows.

Round 1: Each school  $j$  tentatively assigns its seats to students who top-listed it, one at a time following their priority order. If school  $j$  is over-demanded, lower-ranked applicants are rejected.

In general, at Round  $r$ : Each school  $j$  considers the students it has been holding, *together with* students who were rejected in the previous round but listed  $j$  as their  $r^{th}$  choice. Seats in school  $j$  are tentatively assigned to these students, one at a time following their priority order. If school  $j$  is over-demanded, lower-ranked applicants are rejected.

The algorithm terminates when no student is rejected and each student is assigned her final tentative assignment.

The key differences between GS and BM are 1) in each round, students are only temporarily assigned to a school until the whole procedure ends; and 2) temporarily held students are considered based only on priorities along with students who were rejected from their choices in previous rounds and added into a school’s student pool in the current round. As such, a previously held student can be crowded out by a newly-added student who has higher priority. That is, top-listing a school does not improve one’s chance of being finally admitted to this school, which makes truth-telling a (weakly) dominant strategy for households under GS. Moreover, GS eliminates justified envy. The appealing properties of GS, however, may conflict with Pareto efficiency, as shown by Abdulkadiroğlu and Sönmez (2003).

**E2. The TTC algorithm** assigns students as follows.

Round 1: Assign a counter for each school which keeps track of how many seats are still available at the school, initially set to equal the school capacity. Each school points to the student who has the highest priority for the school. Each student points to her favorite school *under her announced preferences*.<sup>98</sup> This will create ordered lists of distinct schools ( $j$ ) and distinct students ( $i$ ) :  $(j_1, i_1, j_2, i_2, \dots)$ , where  $j_1$  points at  $i_1$ ,  $i_1$  points at  $j_2$ , and  $j_2$  points at  $i_2$ , etc. Because there are finite number of

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<sup>98</sup>A student announces her entire list of schools before the assignment starts. As such, the “pointing” by a student is mechanically following her announced list.

schools, at least one cycle will be formed, where  $i_k$  ( $k \geq 1$ ) points at  $j_1$ . Although there may be multiple cycles formed in a round, each school can be part of at most one cycle and each student can be part of at most one cycle. Every student in a cycle is assigned a seat at the school she points to and is removed. The counter of each school in a cycle is reduced by one and if it reduces to zero, the school is also removed. Counters of all schools that are not in any cycle stay put.

In general, at Round  $r$ : Each remaining school points to the student with highest priority among the remaining students and each remaining student points to her favorite school among the remaining schools. Every student in a cycle is assigned a seat at the school that she points to and is removed. The counter of each school in a cycle is reduced by one and if it reduces to zero the school is also removed. Counters of all other schools stay put.

The algorithm terminates when all students are assigned a seat.

Intuitively, in each round TTC creates cycles of trade between individuals. Each individual in a cycle trades off a seat in her highest-priority school for a seat in her most preferred school among those that still have seats. Whenever such a cycle is formed the allocation is final. Hence, the only way for an individual to improve her allocation is through “stealing” another individual’s school assignment, which will in turn make this other individual worse off. As such, TTC is Pareto efficient as shown by Abdulkadiroğlu and Sönmez (2003), who also prove that TTC is truth-revealing. However, TTC does not eliminate justified envy because student-school priorities are ignored in the TTC trade between individuals.

## **F. Policy Evaluation: The 2007 Reform**

The 2007 reform gives priorities for households to access schools that are closest to their home locations. Depending on households’ home locations and strategic types, the reform may have affected them differently. In order to assess these impacts, we simulate the counterfactual outcomes for the 2007 applicants had they lived under the 2006 regime, taking as given the 2006 admissions probabilities. The results generated from this experiment can be interpreted in two ways: 1) the results are at the individual level, i.e., “what would have happened to a 2007 applicant had she applied in 2006?” 2) assuming that the 2006 and 2007 cohorts are two i.i.d. random samples drawn from the same distribution, the results tell us “what would have happened to all 2007 households if the reform had not happened and if they had played the same equilibrium as the 2006 cohort?”

The first two columns of Table A10 present the fractions of winning and losing households due to the 2007 reform. About 17% of households gained and 7% of households lost from the reform. More non-strategic households were affected, with 21% winners and 12% losers. Across educational groups, the high-school educated group was the most likely to win (18%) and also the most likely to lose (8%) from the reform. The last two columns of Table A10 show the changes in welfare. Overall, the gain from the 2007 reform was equivalent to 1,430 euros. The average welfare impacts were smaller for strategic households than for non-strategic households. Across educational groups, households with higher education gained more.

## G. Additional Tables

### G1. Data

Table A1 “Better” Schools Than the Top-Listed One

	% Households	# Better Sch	%Better w/ Higher $p$
All Households (6836)			
Have Sch. Better in Quality, Fees	97.5%	66.6 (44.3)	9.1%
Have Sch. Better in Quality, Dist	41.1%	4.7 (10.6)	14.5%
Have Sch. Better in Fees, Dist	62.2%	14.3 (30.6)	24.3%
Sib School not Top-listed (4025)			
Have Sch. Better in Quality, Fees	98.2%	69.4 (44.2)	15.4%
Have Sch. Better in Quality, Dist	39.6%	4.3 (9.1)	25.7%
Have Sch. Better in Fees, Dist	60.2%	12.4 (27.0)	42.7%

% Households: % of households that satisfy the condition specified in each row.

#Better Sch: average (std.dev.) num. of better schools for households with such schools.

%Better w/ higher  $p$ : % of better sch with higher admission prob. than one’s top choice.

Table A2 Prob of Admission to one’s First Choice  $p_{ia_1}^1(S_i)$

	Enrolled	Opted out
All Households	91.8%	75.3%
Assigned within 10 Rounds	92.7%	86.8%
Unassigned within 10 Rounds	48.5%	44.8%

Admission prob in Round 1, averaged for each group of households.

### G2. Model Fit

Table A3 Model Fit: Assignment Round 2006 (%)

	Edu < HS		Edu = HS		Edu > HS		Single Parents	
	Data	Model	Data	Model	Data	Model	Data	Model
1	93.2	91.8	92.0	91.0	93.7	91.2	94.0	93.4
2	2.7	2.8	3.5	4.0	2.3	4.8	1.9	2.8
$\geq 3$	1.6	1.3	1.5	0.9	1.3	0.4	1.4	0.2
Unassigned	2.5	4.2	3.0	3.9	2.7	3.3	2.7	3.6

Table A4 Model Fit: Top-Listed Schools 2006

	Quality		Distance (100m)		Tuition (100 Euros)	
	Data	Model	Data	Model	Data	Model
Parental Edu < HS	7.6	7.6	5.2	6.0	5.4	5.7
Parental Edu = HS	7.9	7.9	7.0	7.1	8.1	8.3
Parental Edu > HS	8.2	8.2	8.7	8.2	9.9	9.6
Single-Parent	8.0	8.0	8.1	8.0	8.6	8.6

Table A5 Model Fit: Enrollment in Public System

	2006		2007	
	Data	Model	Data	Model
Parental Edu < HS	96.9	96.2	96.4	96.2
Parental Edu = HS	97.1	97.0	98.4	97.0
Parental Edu > HS	96.3	96.4	97.8	96.7
Single-Parent	96.1	95.5	97.0	95.8

Table A6 Model Fit: Assignment Round 2007 (%)

	Edu < HS		Edu = HS		Edu > HS		Single Parents	
	Data	Model	Data	Model	Data	Model	Data	Model
1	91.0	92.0	90.6	91.8	93.7	93.7	91.3	92.4
2	3.7	3.8	3.2	3.8	2.2	3.1	2.9	3.1
$\geq 3$	1.6	1.2	2.2	0.4	1.6	0.4	1.4	0.4
Unassigned	3.5	3.0	4.0	4.0	2.5	2.8	3.3	3.1



Table A7 Model Fit: Top-Listed Schools 2007

	Quality		Distance (100m)		Tuition (100 Euros)	
	Data	Model	Data	Model	Data	Model
Parental Edu < HS	7.5	7.6	5.2	5.9	5.3	5.4
Parental Edu = HS	8.0	7.9	6.3	6.8	8.2	8.0
Parental Edu > HS	8.2	8.2	7.8	8.2	9.7	9.5
Single-Parent	8.0	8.0	6.8	7.2	8.2	8.0

### G3. Baseline Model vs. Model with Fewer (80%) Strategic Households

Table A8 Model vs. 80% Strategic (2006)

	Data	Model	80% Strategic*
Top-Listed School			
Quality	7.9	7.9	8.0
Distance (100m)	7.1	7.2	7.4
Fee (100 Euros)	8.1	8.1	8.0
Assignment Round (%)			
1	93.0	91.3	88.0
2	2.8	4.0	3.8
$\geq 3$	1.5	0.9	1.4
Unassigned	2.7	3.8	6.8
Enrollment			
Enrolled Public System (%)	96.7	96.5	95.0

\*Adjust the fraction of strategic household to 80% by changing the constant term in the type distribution function, while keeping all other parameters as they are in the baseline model.

### G4. Counterfactual Experiments

Table A9 School Assignment: BM

	Quality	Distance (100m)	Fees (100Euros)
All	7.8 (0.7)	7.3 (7.9)	7.7 (7.4)
Edu < HS	7.4 (0.7)	6.6 (6.9)	5.2 (6.2)
Edu = HS	7.8 (0.6)	7.1 (7.2)	7.7 (7.1)
Edu > HS	8.0 (0.7)	8.1 (8.3)	9.3 (8.0)

Table A10 Impact of the 2007 Reform

	Winner(%)	Loser(%)	$\Delta$ utils	$\Delta$ 100 Euros
All	16.7	6.7	6.7 (29.5)	14.3 (68.4)
Strategic	16.6	6.5	6.6 (28.7)	14.1 (65.9)
Non-strategic	20.8	11.9	8.2 (43.8)	18.9 (108.4)
Edu < HS	14.7	7.0	4.9 (25.7)	5.0 (26.0)
Edu = HS	18.1	7.7	7.1 (31.0)	22.9 (99.8)
Edu > HS	17.1	5.9	7.6 (30.6)	13.7 (55.2)

\*Compare the welfare of a 2007 household under the 2007 regime with its would-be welfare under the 2006 regime. Winners have higher welfare under the 2007 regime.