BANKRUPTCY CODES, LIQUIDATION TIMING, AND DEBT VALUATION

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Abstract

This paper derives closed-form solutions for values of debt and equity in a continuous-time structural model in which the demands of creditors to be repaid cause a firm to be put into bankruptcy. This allows discussing the effect of creditor coordination in recovering money on the values of debt, equity, and the firm. The effects of features of bankruptcy codes that influence creditor coordination such as automatic stays and preference law are also considered. In the model, a lack of creditor coordination reduces the value of debt, but can increase the value of the firm. Automatic stays and preference law increase the value of equity, but can decrease the value of debt and the firm.

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1 Introduction

Leland (1994) argues that the decision to put a firm into bankruptcy depends on the value of its debt and the value of its equity, and that in turn, the value of debt and equity of a firm depend on when that firm will be put into bankruptcy. Therefore, models need to solve for the optimal decision to put a firm into bankruptcy and the values of debt and equity jointly. In Leland's approach, the firm promises a perpetual coupon payment to debtholders. The assumption is that if a coupon payment is missed, the firm is put into bankruptcy immediately. In this kind of framework, bankruptcy happens when the *equity holder* optimally decides to stop injecting funds to ensure that coupons are paid in full.

However, some firms do in practice miss coupon payments without being put into bankruptcy immediately. For example, Varma and Cantor (2005) report that of about slightly more than 1,000 "initial default events" recorded by Moody's for the period between 1983 to 2003, more than 55% consist of missed interest payments and grace period defaults which occur when firms are not in formal bankruptcy (the remaining cases are direct Chapter 11 filings, distressed exchanges, missed principals, direct Chapter 7 filings and Chapter 11 prepacks).

Once payments are missed, the most important provision of a debt contract, the provision to make timely payments, has been breached. In this kind of situation, bankruptcy can happen when *creditors* decide to take legal steps to demand repayment.[1] Furthermore, whether or not creditors will be able to coordinate will matter, and hence features of bankruptcy codes such as automatic stays and preference law that affect the coordination of creditors will matter. This paper asks the following questions: How are the values of debt, equity, and the firm affected if it is the decision of *creditors* to demand repayment that causes bankruptcy? How does the coordination among creditors affect their decision to demand repayment, and hence the values of debt, equity, and the firm? How do features of bankruptcy codes such as automatic stays, preference law, and a policy of equality of distributions to creditors affect the decision to demand repayment, and hence the values of debt, equity and the firm? The paper presents answers to these questions in the context of a continuous-time structural model of debt and equity that produces *closed-form* values of debt, equity, and the firm.

In the model, the equity holder cannot inject funds and default happens when cash flows are insufficient to make the coupon payments. Once the firm is in default, creditors have a right to demand full payment, either collectively or individually. Successful legal action of creditors leads to bankruptcy, which is taken to be synonymous with liquidation.[2]

[1]E.g. LoPucki (1983) provides empirical evidence that this is in fact what happens in practice.

[2]The majority of bankruptcy filings in the U.S are Chapter 7 filings (bankruptcy liquidation) as

The model considers different outcomes that might arise depending on whether or not creditors can coordinate. If creditors can coordinate, the features of bankruptcy codes that affect creditor coordination such as automatic stays, preference law, and equality of distributions are not important. If creditors cannot coordinate, these features become important. To develop an intuition about the effect of such features, I first discuss how uncoordinated creditors would interact in the absence of these features, and then discuss how uncoordinated creditors interact in the presence of such features.

In the model, the equity holder will want the firm to be liquidated at a point that maximizes the value of equity ex post. If the share of the liquidation value that goes to the equity holder is low or zero, the equity holder wants the firm to be liquidated "too late", i.e. the total firm value is higher if the firm is liquidated at an earlier point than the point at which the equity holder desires liquidation.

In the case where creditors can coordinate, they will want the firm to be liquidated at a point that maximizes the value of debt. If the share of the liquidation value that goes to creditors is large or equal to 100%, creditors want the firm to be liquidated "too early", i.e. the total firm value is higher if the firm is liquidated at a later point than the point at which creditors desire liquidation. If the firm is in default when coordinated creditors want the firm to be liquidated, then they can take the firm to court at this point, demand the firm's assets in payment of the debt, and cause liquidation (i.e. they collectively grab assets). This will lead to maximized debt values, but low equity values.

In the case where creditors cannot coordinate, they will compare the costs and benefits of individually grabbing assets to decide when to act, taking into account the possible actions of other creditors.

Consider initially the case in which automatic stays and preference law do not apply, and there is no policy of equality of distributions to creditors. Suppose that whenever the firm is defaulting on at least some of the promised interest payments on its debt, creditors have to decide whether to individually hire a costly lawyer that will attempt to obtain a judgement lien, i.e. attempt to grab assets. In doing so, they consider the probability of being successful in recovering money, which is assumed to depend negatively on the cash available to the firm, and the number of other creditors filing claims; if many creditors file claims and the firm does not have sufficient cash to fight in court, it will be liquidated. Creditors that did file claims will receive a higher liquidation payoff. If few creditors file claims and the firm has sufficient cash to fight in court, the creditors that did file claims

opposed to Chapter 11 (bankruptcy reorganization). For example, the News Release of the Administrative Office of the U.S. Courts of Nov 14, 2003 reports that 21,008 businesses filed for Chapter 7 in the fiscal year 2003, whereas only 9,185 filed for Chapter 11. Chapter 11 (bankruptcy organization) is discussed informally in Section 5. A formal discussion in the given framework is likely to be a fruitful area for future research.

have to pay their lawyers, but are unsuccessful in grabbing assets.

The central feature of this game are strategic complementarities and imperfect information about the actions of other creditors.[3] They produce a critical point at which a sufficient number of creditors will attempt to grab assets such that the firm is liquidated.

For low legal costs, individual creditors rush to grab assets very early and cause even earlier liquidation than coordinated creditors, and hence low debt values, low equity values, and low firm values. For very high legal costs, individual creditors rush to grab assets very late, and cause a liquidation as late as that desired by the equity holder, and hence high equity values, but very low debt values, and low firm values. For intermediate legal costs, it is possible that the asset grab game actually leads to better liquidation timing and hence higher overall firm values.

Now consider the case in which automatic stays and preference law do apply, and there is a policy of equality of distributions to creditors. This changes the cost-benefit calculation of individual creditors. Once a sufficient number of creditors file claims, the firm is put into bankruptcy, an automatic stay applies to most creditors, and preference law ensures that most of the money previously obtained by creditors has to be returned to the trustee, to be shared out equally among creditors. This reduces incentives to grab assets individually, leading to later liquidation. Although this benefits holders of equity, it can hurt creditors. The effect on the value of the firm is ambiguous and depends on the extent to which automatic stays and preference law reduce incentives to grab assets. If the incentives to grab assets are low to start with because the legal cost of grabbing assets is high, discincentivizing asset grabs can worsen liquidation timing. If the incentives to grab assets are high to start with because the legal cost of grabbing assets is low, some disincentivization of asset grabs can improve liquidation timing, but too much disincentivization can worsen liquidation timing. This illustrates how the features of bankruptcy codes that hinder collection efforts by individual creditors will not always have unambiguously beneficial effects.

Finally, adjusting the capital structure of the firm can often reduce inefficiencies produced by incentives that lead to bad liquidation timing, and in some situations, levels of debt that achieve firm-value maximizing liquidation timing exist. If creditors cannot coordinate and asset grabs occur late, be it because of high legal costs of grabbing assets and / or the effect of automatic stays, preference law, and a policy of equality of distributions, raising levels of debt can make individual collection efforts more likely to succeed at an earlier stage, encourage asset grabs and hence achieve optimal liquidation timing.

[3]A technical contribution of the paper is proposing a way of integrating a private information game in a standard continuous-time public-information pricing model with a single filtration via some specific assumptions on the timing of the game.

Related literature In terms of the dynamic pricing literature, the model presented here is related to that of Leland (1994), and other papers that propose extensions of Leland's model. For example, Broadie, Chernov, and Sundaresan (2007) or François and Morellec (2004) augment the Leland (1994) model by allowing for a period prior to liquidation in which the firm defaults on payments, which they label as (Chapter-11-type) bankruptcy; this happens when the asset value falls below a bankruptcy barrier which is endogenously determined and chosen by equity holders. François and Morellec (2004) assume that the firm is liquidated once it has spent enough time below the bankruptcy boundary. In the model of Broadie, Chernov, and Sundaresan (2007), liquidation happens at an ex-post optimal liquidation boundary below the bankruptcy boundary, or if the firm spends a sufficient amount of time under the bankruptcy boundary. In contrast, the model in this paper follows the approach of Naqvi (2008), in which negative dividends or asset sales are not allowed, which implies default occurs when cash flows are insufficient to cover coupon payments. In this approach, default is not synonymous with bankruptcy, and efforts to collect on debt are closely related to the actual incidence of liquidation.

The model here is also related to dynamic pricing models that incorporate strategic debt service games or renegotiation games as in the papers of Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997).[4] In practice, the possibility of renegotiation between equity holders and creditors can be an important additional factor determining actual liquidation outcomes, over and above the decision of creditors to collect on their debt. In order to focus on the decision of creditors to collect on the debt, and how the ability to coordinate influences this decision, and how features of bankruptcy codes influence incentives to collect individually, however, the model presented in this paper considers a situation in which renegotiation or strategic debt service is not possible. This is useful not only for understanding about what might drive liquidation outcomes in situations in which renegotiating costs would be prohibitively high, but also possibly for understanding the bargaining positions of the parties in renegotiation.

The model here is also related to some papers outside the dynamic pricing literature. The asset grab game is a variant of a model which has been used in the context of (static) debt pricing by Morris and Shin (2004), which in turn is closely related to to models used e.g. in the context of currency crises (Morris and Shin, 1998) or bank runs (e.g Goldstein and Pauzner, 2005).[5] Papers that examine creditor coordination include the paper by Gertner and Scharfstein (1991), who look at the aggregate effects of individually optimal decisions to accept an exchange offer. In contrast, here, the focus here is on the decision

[4]Other versions of renegotiation games are considered e.g. by Mella-Barral (1999), Hege and Mella-Barral (2005), or Hackbarth, Hennesy, and Leland (2007).

[5]See also Morris and Shin (2003) for a survey of these models.

to individually collect on debt on which a firm is defaulting. Furthermore, Bolton and Scharfstein (1996) examine a model in which inefficiencies in liquidation or renegotiation that are the result of having a large number of creditors can reduce moral hazard; and Bris and Welch (2005) look at how free riding in debt collection efforts between uncoordinated creditors can be beneficial in reducing socially wasteful expenditures that result from equity holders and debt holders fighting over the liquidation value of the firm. Papers that examine the relationship between capital structure and liquidation decisions include that by Titman (1984). However, he only considers the case that would correspond to the case of coordinated creditors here.

In the next section (Section 2), a basic model is presented, which can accommodate different assumptions about how creditors enforce their rights. Pricing in the context of this model for arbitrary liquidation mechanisms is discussed in Section 3, and liquidation in the context of coordinated and uncoordinated creditors is discussed in Section 4. Section 5 discusses a modified version of the asset grab game that takes into account the effect of automatic stays, preference law, and equality of distributions to creditors. The relation between inefficiencies in liquidation and capital structure is examined in Section 6. Section 7 concludes.

2 The model

Suppose a firm engages in a productive activity that uses a single productive asset and generates a cash flow (net of costs) at each point in time until the firm is liquidated, i.e. the productive asset is sold. The cash flow x evolves according to the following stochastic process under the pricing measure $\mathbb{Q}$ defined by the money market account as the numeraire:

$$dx(t) = \mu x(t)dt + \sigma x(t)d\tilde{W}(t), \tag{1}$$

where $\tilde{W}(t)$ is a $\mathbb{Q}$-Brownian motion. The annualized risk-free interest rate is constant at $r > 0$, μ, σ are constants, and $\mu < r$. I assume existence of a market price of risk process $\nu(t)$, such that the dynamics of the process under the objective measure $\mathbb{P}$ can be written as

$$dx(t) = (\mu + \sigma\nu(t))\, x(t)dt + \sigma x(t)dW(t), \tag{2}$$

where $W(t)$ is a $\mathbb{P}$-Brownian motion. Under some regularity conditions, this process can be thought of as the limit of the discrete time process

$$x_{t+\Delta} = x_t + (\mu + \sigma\nu_t)x_t\Delta + x_t\eta_{t+\Delta}, \quad \eta_{t+\Delta} \sim NID\left(0, \sigma^2\Delta\right), \tag{3}$$

as the size of the discrete time step Δ becomes arbitrarily small.[6]

Here, limits of a discrete-time game based on [3] are taken such that valuation is equivalent to valuation under [1]. This simultaneously allows setting up and solving the game in discrete time as well allowing for closed-form formulas for debt, equity and firm values.

The firm is initially set up with equity and debt; debt is issued with an promised perpetual coupon of c per unit of time. At times at which the cash flow x is sufficient to pay the coupon, the coupon is paid in full, and any remainder is paid out to the equity holder as a dividend. The equity holder pays taxes at rate τ on this dividend; for simplicity, debt is assumed not to be taxed.[7] When the cash flow x is insufficient to pay the coupon, creditors receive all of the cash flow in partial payment of the coupon, and the equity holder receives nothing, i.e. the firm defaults on part of the coupon payment. This setup ignores that defaults have to be cured (i.e. when the debtor is in arrears, interest payments that have been missed have to be paid at a later date if a sufficient amount of money becomes available), and is therefore an approximation.[8]

If the firm is liquidated, the productive asset is sold, i.e. the cash flows are swapped irreversibly for a constant liquidation value $K > 0$, of which a fraction $s > 0$ covers legal costs, such that the net liquidation value is $(1 - s)K$. Note that choosing a constant liquidation value rather than arguing that the liquidation value is a fraction of the pre-liquidation going-concern value of the firm as is often done in the literature is significant in that it means that liquidation (even in the unlevered firm) will be optimal at some positive level of the going-concern value.

I assume that the net liquidation value $(1 - s)K$ is less than or equal to the face value of debt c/r,[9] that in the event of liquidation, absolute priority is respected,[10] and that the net liquidation value therefore goes to creditors.[11] Formally, the payoffs to debt and

[6]E.g. if $\nu(t)$ is a function of time and $x(t)$, then [2] is a stochastic differential equation, and the Euler discretization [3] will converge both weakly (with order 1) and strongly (with order 0.5) to the solution of [2] (see e.g. Kloeden and Platen, 2000).

[7]τ here should therefore be interpreted as a net tax advantage to debt in the sense of Miller (1977). Introducing a tax on debt explicitly would not alter the fundamental trade-off between liquidation inefficiencies and a tax advantage to debt discussed in Section 6.

[8]Although allowing for cures would make the model more realistic, there is a trade-off in terms of complexity versus obtaining closed form solutions, since introducing cures would introduce path dependency. The model of Broadie, Chernov, and Sundaresan (2007) is an example of a similar model that allows for cures but can only be solved numerically.

[9]Since debt is perpetual, there is no payment of principal. Interpreting c/r as the face value of debt (or principal) in this case is common in the literature, see e.g. Mella-Barral and Perraudin (1997).

[10]In practice, absolute priority is typically respected in liquidation, although it is often not respected in reorganization.

[11]One can also consider the less interesting case where $(1 - s)K > c/r$, where the liquidation payoff to creditors is the full face value c/r, and the liquidation payoff to the equity holder is the remaining

equity are as follows:

$$\text{payoff to equity at } t = \begin{cases} (1-\tau)\max(x_t - c, 0) & \text{before liquidation} \\ 0 & \text{at liquidation} \end{cases} \tag{4}$$

and

$$\text{payoff to debt at } t = \begin{cases} \min(c, x_t) & \text{before liquidation} \\ (1-s)K & \text{at liquidation.} \end{cases} \tag{5}$$

In the various versions of the model, the firm will be liquidated when the cash flow reaches a liquidation boundary $\bar{x}$, where the different liquidation mechanisms I consider lead to different liquidation boundaries. In general, the location of the liquidation boundary $\bar{x}$ depends on the actions of the equity holder, and the actions of creditors which depend on whether or not creditors can coordinate.

If creditors can coordinate, they will prefer to liquidate the firm at a liquidation boundary that maximizes the value of debt, but they can only request courts to actually liquidate if the firm is defaulting on coupon payments, i.e. violating its contractual obligations. The equity holder wants liquidation at a liquidation boundary that maximizes the value of equity. The liquidation boundary at which the firm is liquidated depends on who acts first.

If creditors cannot coordinate, they decide to act at a point that maximizes their individual utility given the information they have. Here, the action that they take is to hire or not to hire a lawyer that tries to recover money by obtaining a judgement lien against the firm (grabbing or not grabbing assets). The firm is liquidated if a 'sufficient' fraction of the creditors hire lawyers.

Formally, suppose there exists a continuum of creditors with mass one. Suppose that the court agrees to liquidate immediately before a time $t+\Delta$ only when the fraction l of creditors who decide to hire a lawyer and attempt to grab assets is larger than or equal to $\frac{x_{t+\Delta}}{c}$. This formulation ensures that it will be impossible for the firm to be forcibly liquidated when $x_{t+\Delta} > c$.

All creditors know the cash flow in the previous period t, such that we can interpret it as public information about the cash flow $x_{t+\Delta}$, where the precision of this information is $\alpha = (\sigma^2\Delta)^{-1}$.

At some intermediate time period $t+q$ (where $t < t+q < t+\Delta$), creditors receive a private signal ξ_i (subscript i indexes the different creditors) about the value that the cash flow x will take at $t+\Delta$, given by

$$\xi_i = x_{t+\Delta} + x_t\epsilon_i, \quad \epsilon_i \sim NID\left(0, \frac{1}{\beta}\right), \tag{6}$$

liquidation value after creditors have been paid off $(1-s)K - c/r$. See Appendix B.

where $\text{Cov}(\eta_{t+\Delta}, \epsilon_i) = 0$, i.e. the noise is orthogonal to the innovations in the cash flow. From the signal ξ_i and the public information x_t, creditors form a posterior about the cash flow of the firm in period $t + \Delta$. The differences in posteriors resulting from the differences in the private signal is what creates uncertainty about the actions of other creditors and hence coordination failure. Once creditors have formed their posterior, they act.

t	$t+q$	$t+\Delta$
signals revealed	no trading	signals revealed
x_t revealed	agents receive new signals	$x_{t+\Delta}$ revealed
trading	agents form posterior	trading
prices formed	agents act	prices formed
coupons are paid	firm is possibly liquidated	coupons are paid

Figure 1. Timing assumptions

Markets open at times $t, t+\Delta, t+2\Delta, \ldots$, and the asset grab games are played at some intermediate time periods $t+q, (t+\Delta)+q, \ldots$ etc. in which markets are closed. Assume that whenever trading occurs, the cash flow at that time is public information. As a consequence of these timing assumptions, only public information will be incorporated into prices, which allows valuation by standard martingale techniques. To simplify the pricing argument later, we can also assume that coupons or partial coupons etc. are paid at the times at which markets are open, i.e. $t, t+\Delta, \ldots$ etc.[12]

Attempting to grab assets produces an immediate cost sK. If the firm is pushed into liquidation, creditors that have grabbed assets receive the liquidation value $(1-s)K$, whereas creditors that have not grabbed assets receive 0. If the firm is not pushed into liquidation, creditors that have attempted to grab assets still incur the cost. Figure **??** illustrates the instantaneous payoffs that creditors consider (together with any possible payoffs in the future) when making the decision whether to attempt to grab assets at the intermediate time periods $t+q$.[13]

While these payoffs produce the fundamental feature of the game which is strategic complementarities, potentially more complicated and possibly more realistic payoffs, es-

[12] One way to interpret these assumptions is the following: During the day (t), trading occurs and prices reveal all information. In the evening ($t+q$), markets close. The creditors now receive private information about the financial situation of the firm, e.g. via a tip-off. They have to decide immediately whether or not to call their lawyer. If called, a lawyer starts billing his client immediately, and works throughout the night to prepare the papers to be filed. In the morning, before markets open, all lawyers that have been hired congregate in front of the court house; if the number of lawyers is large, it is clear that the firm will be liquidated. If the firm is not liquidated, when markets open (at $t+\Delta$), trading occurs, and prices reveal all private information of creditors.

[13] Because of the assumption that there exists a continuum of creditors, i.e. that creditors are atomistic, payoffs in the future do not affect the solution of the game. See Section 4.2 and Appendix C for details.

	liquidation	no liquidation
grab assets	$(1-s)K$	$-sK$
do not grab assets	0	0

Figure 2. Payoffs to creditors in the discrete time game.
Payoffs to creditors at the interim stage $t+q$.

pecially those which depend on the actual fraction of creditors attempting to grab assets are conceivable. It would for example be natural to argue that in the event of liquidation, assets will first be shared among those creditors that attempted to grab assets, and all remaining assets will be shared between those that did not attempt to grab assets. In this kind of setup with *one-sided strategic complementarities* (Goldstein and Pauzner, 2005), it is possible to prove existence of equilibria in monotone strategies, and uniqueness for particular structures of noise (Morris and Shin, 2003; Goldstein and Pauzner, 2005), but this greatly complicates solutions and does not change the qualitative flavor of the game. In the limit derived below, the payoff does not depend on the fraction of creditors that attempt to grab assets in any case.

Since creditors maximize expected utility, payoffs will have to be converted into utility before solving the game. I assume that utility is additively separable across time, such that expected utility can be decomposed into an instantaneous (Bernoulli) utility associated with instantaneous payoffs, and a (discounted) continuation utility associated with future payoffs.[14]

3 Pricing

This section describes pricing in the continuous-time limit, taken a critical level $\bar{x}$ of the cash flow at which the firm is liquidated as given. Here, I only discuss the case where the firm is liquidated after default, i.e. $\bar{x} < c$. (This which will turn out to be the relevant case in the subsequent discussion.)[15]

Since claims are perpetual, the usual pricing partial differential equations become ordinary differential equations (ODEs), which can be solved given some appropriate boundary conditions. For the price of debt and equity, requiring that the discounted gains from holding these assets are martingales under $\mathbb{Q}$ produces two pricing ODEs, one for the region where the firm is not defaulting on promised coupon payments ($x(t) \geq c$), and one for the region where the firm is defaulting on part of the promised coupon payments ($x(t) < c$),

[14] A specific choice of utility function will imply a specific form for the market price of risk $\nu(t)$.

[15] Valuation in the simpler case when the firm is liquidated before default, i.e. $\bar{x} \geq c$ is described in Appendix A. This case is only relevant when relaxing the assumption that $(1-s)K < c/r$.

each with two constants of integration. To solve for the price of a claim, four boundary conditions are required.

Let D_1 denote the price of debt in the region where the firm is not defaulting, and let D_2 denote the price of debt in the region where the firm is defaulting. Then the boundary conditions to be imposed for the price of debt can be written as

$$\lim_{x\to\infty} D_1(x) = \frac{c}{r} \tag{7}$$

$$D_1(c) = D_2(c) \tag{8}$$

$$D_1'(c) = D_2'(c) \tag{9}$$

$$D_2(\bar{x}) = (1-s)K. \tag{10}$$

[7] states that as the cash flow becomes very large, debt essentially becomes riskless. [8] is a value-matching condition that states that at the point where the dynamics of the discounted gains process change (when the cash flow is just equal to the coupon), the value of the solution to both differential equations has to be the same. [9] is required to rule out arbitrage as the cash flow falls below the coupon (see e.g. Dixit, 1993), and [10] is another value-matching condition that states that when the firm is liquidated, the price of debt must be equal to the liquidation payoff.

These boundary conditions produce 4 linear equations in 4 unknowns (the constants of integration), which can be easily solved. It is shown in the Appendix that this produces the following equations for the price of debt:

$$D(x,\bar{x}) = \begin{cases} D_1(x,\bar{x}) & \text{when } x \geq c, \text{ i.e. the firm is not defaulting} \\ D_2(x;\bar{x}) & \text{when } x < c, \text{ i.e. the firm is defaulting} \end{cases} \tag{11}$$

where

$$D_1(x,\bar{x}) = \frac{c}{r} + \left[D_2(c,\bar{x}) - \frac{c}{r}\right]\left(\frac{x}{c}\right)^{-\gamma} \tag{12}$$

and

$$D_2(x;\bar{x}) = \left\{\frac{x}{r-\mu} - Z(c)\left(\frac{x}{c}\right)^{\delta}\right\} + \left[(1-s)K - \left\{\frac{\bar{x}}{r-\mu} - Z(c)\left(\frac{\bar{x}}{c}\right)^{\delta}\right\}\right]\left(\frac{x}{\bar{x}}\right)^{-\gamma} \tag{13}$$

where in turn

$$Z(c) = \frac{\gamma}{\delta+\gamma}\left(\frac{1+\gamma}{\gamma}\frac{c}{r-\mu} - \frac{c}{r}\right). \tag{14}$$

and δ and $-\gamma$ are simple functions of the drift μ, volatility σ and the interest rate r.[16]

These formulas can be interpreted. For example, D_1 can be seen to be the sum of a first term c/r, which is the value of receiving the coupon forever, plus a second term. To

[16] More precisely, δ and $-\gamma$ are the positive and negative roots respectively of the characteristic equation of the ODE.

understand the second term, note that $\left(\frac{x}{c}\right)^{-\gamma}$ can be interpreted as the value of a claim that pays one at the time at which the cash flow x hits the coupon c, i.e. when the firm starts to default on coupon payments.[17] The entire second term then is the value of a claim that pays off the amount in the square brackets (which can be shown to be negative) when x hits c, i.e. swaps the value of receiving the coupon forever c/r against D_2.

In interpreting D_2, note that the value of receiving the cash flow forever, which could be labeled as the going-concern value, is easily shown to be

$$\frac{x(t)}{r-\mu} \tag{15}$$

(see Appendix A for details). D_2 is composed of two terms. The first term contains the value of receiving the cash flow forever, minus $Z(c)\left(\frac{x}{c}\right)^{\delta}$. This $Z(c)\left(\frac{x}{c}\right)^{\delta}$ accounts for the fact that if the cash flow x rises above the coupon c, creditors receive at most the coupon c and not the full the cash flow. The second term in D_2 swaps the first term against a liquidation payment of $(1-s)K$ when the cash flow x hits the liquidation boundary $\bar{x}$.

The value of equity is derived in a similar manner. In the appendix, it is shown that for a similar set of boundary conditions, if E_1 and E_2 are defined as

$$E(x,\bar{x}) = \begin{cases} E_1(x,\bar{x}) & \text{when } x \geq c \text{, i.e. the firm is not defaulting} \\ E_2(x;\bar{x}) & \text{when } x < c \text{, i.e. the firm is defaulting,} \end{cases} \tag{16}$$

then

$$E_1(x;\bar{x}) = \left\{(1-\tau)\left(\frac{x}{r-\mu}-\frac{c}{r}\right)\right\} + \left[E_2(c;\bar{x}) - \left\{(1-\tau)\left(\frac{x}{r-\mu}-\frac{c}{r}\right)\right\}\right]\left(\frac{x}{c}\right)^{-\gamma}, \tag{17}$$

and

$$E_2(x;\bar{x}) = \left\{(1-\tau)Z\left(\frac{x}{c}\right)^{\delta}\right\} + \left[0 - \left\{(1-\tau)Z\left(\frac{\bar{x}}{c}\right)^{\delta}\right\}\right]\left(\frac{x}{\bar{x}}\right)^{-\gamma}. \tag{18}$$

E_1 can be seen to consist of two terms. The first term consists of the going concern value minus the value of receiving the coupon forever, which can also be interpreted as the value of receiving the cash flow x net of coupons c, i.e. dividends, forever, all net of taxes. The second term swaps the first term against E_2 when the cash flow x hits the coupon c, i.e. when the firm starts to default on coupon payments.

E_2 again consists of two terms. The first term represents the value to the equity holder associated with the possibility of returning to the region where the firm is not defaulting and pays dividends. The second term swaps the first term against a liquidation payment of 0 when the cash flow x hits the liquidation boundary $\bar{x}$.

[17] It is of course also the Laplace transform of the density of the hitting time of x hitting c.

Finally, the market value of the levered firm V is the sum of the market values of debt (D) and equity (E). For the two regions, we again define

$$V(x, \bar{x}) = \begin{cases} V_1(x, \bar{x}) & \text{when } x \geq c, \text{ i.e. the firm is not defaulting} \\ V_2(x; \bar{x}) & \text{when } x < c, \text{ i.e. the firm is defaulting,} \end{cases} \tag{19}$$

and then have

$$V_1(x; \bar{x}) = E_1(x; \bar{x}) + D_1(x; \bar{x}) = \\ \left\{(1-\tau)\frac{x}{r-\mu} + \tau\frac{c}{r}\right\} + \left[V_2(c; \bar{x}) - \left\{(1-\tau)\frac{c}{r-\mu} + \tau\frac{c}{r}\right\}\right]\left(\frac{x}{c}\right)^{-\gamma}, \tag{20}$$

and

$$V_2(x; \bar{x}) = E_2(x; \bar{x}) + D_2(x; \bar{x}) = \\ \left\{\frac{x}{r-\mu} - \tau Z(c)\left(\frac{x}{c}\right)^{\delta}\right\} + \left[(1-s)K - \left\{\frac{\bar{x}}{r-\mu} - \tau Z(c)\left(\frac{\bar{x}}{c}\right)^{\delta}\right\}\right]\left(\frac{x}{\bar{x}}\right)^{-\gamma}. \tag{21}$$

V_1 consists of two terms. The first terms is the sum of the post-tax going concern value, plus the value of the tax shield. The second term swaps the first term against V_2 when the cash flow x hits the coupon c, i.e. the when the firm starts to default on coupon payments.

V_2 again consists of two terms. The first term is the going concern value (now *not* net of taxes because when the firm is defaulting, no dividends are paid, and hence no taxes on dividends are paid) minus a term that accounts for the fact that if the cash flow x rises above the coupon c, taxes will have to be paid on dividends. The second term swaps the first term against the net liquidation value when the cash flow x hits the liquidation boundary $\bar{x}$.

It can be seen that there are two channels via which the capital structure (determined by the size of c) can influence the market value of the firm. Firstly, raising c implies paying less taxes on the total cash flow paid out, raising the value of the firm. Secondly, changing c might have an effect on the liquidation boundary $\bar{x}$, which can change the market value of the firm.

4 Liquidation

I first describe the critical level of the cash flow at which the firm is liquidated when creditors can coordinate, followed by the critical level of the cash flow at which the firm is liquidated when creditors cannot coordinate. I then compare the liquidation outcomes.

4.1 Coordinated creditors

Given the derived prices of debt and equity, it is now possible to compare the optimal points at which to liquidate the firm; optimal in the sense of maximizing either the value of debt, the value of equity, or the value of the levered firm. At these points, first order conditions (or equivalently smooth pasting conditions) are satisfied for interior solutions. I label these as $\bar{x}_D, \bar{x}_E$ and $\bar{x}_V$:

$$\left.\frac{\partial D}{\partial \bar{x}}\right|_{\bar{x}=\bar{x}_D} \equiv 0, \quad \left.\frac{\partial E}{\partial \bar{x}}\right|_{\bar{x}=\bar{x}_E} \equiv 0, \quad \text{and} \quad \left.\frac{\partial V}{\partial \bar{x}}\right|_{\bar{x}=\bar{x}_V} \equiv 0. \tag{22}$$

The respective values will be maximized at these points given that some second order conditions are satisfied.[18]

Now since $D + E = V$, we have the following relationship between the derivatives,

$$\frac{\partial D}{\partial \bar{x}} + \frac{\partial E}{\partial \bar{x}} = \frac{\partial V}{\partial \bar{x}}, \tag{23}$$

which we can evaluate at $\bar{x}_V$. At this point, the RHS will be zero, which implies that unless this point is also optimal for both classes of claim holders (i.e. at $\bar{x}_V$, $\partial D/\partial \bar{x} = \partial E/\partial \bar{x} = 0$), the terms on the LHS must be of opposite sign. If the derivatives are monotonically decreasing and continuous over the relevant interval, this immediately implies that the points at which the equity holder and creditors respectively would like to liquidate the firm will be on opposite sides of $\bar{x}_V$. In this case, either

$$\bar{x}_D < \bar{x}_V < \bar{x}_E \tag{24}$$

or

$$\bar{x}_E < \bar{x}_V < \bar{x}_D. \tag{25}$$

Intuitively, the firm-value maximizing liquidation boundary must be a compromise (lie between) what the creditors and the equity holder want.

This kind of insight forms the basis of the discussion by Mella-Barral (1999), who looks at the case where the equity holder always injects sufficient cash to ensure that coupons are paid, and the equity holder determines $\bar{x}$, and goes on to produce a version of the argument of Haugen and Senbet (1978), that costless Coasian bargaining between creditors and the equity holder can achieve the firm-value maximizing outcome, with the surplus being divided among creditors and the equity holder. In this kind of setup, however, it is hard to argue for a situation as in [25], since if creditors always receive their coupon, the only way to make them actually want to liquidate the firm is by letting

[18] The sign of the derivatives here does not depend on the cash flow x, and hence characterizing the optimal points $\bar{x}_D, \bar{x}_E$ and $\bar{x}_V$ is straightforward.

the liquidation value be equal to or higher than the value of receiving the coupon forever, which implies that debt in this kind of model has to be worth *at least as much* as equivalent risk-free debt.

In contrast, with the setup presented in Section 2, it is not necessary to require a liquidation payoff that is higher than the value of receiving the coupon forever, as summarized in Proposition 1 below.

Proposition 1. *Under the given assumptions,*

$$0 = \bar{x}_E < \bar{x}_V < \bar{x}_D \leq c. \tag{26}$$

Proof. An intuitive argument for why the proposition holds is as follows (for a formal proof see Appendix B): Since the net liquidation proceeds $(1 - s)K$ are assumed to be less than the face value of debt and absolute priority is assumed to be respected, the liquidation payoff to the equity holder is zero, and the liquidation payoff to creditors is equal to the net liquidation value $(1 - s)K$.[19]

This implies that in liquidation, the equity holder never gains anything, but always loses a positive continuation value — associated with the possibility that the cash flow can always return to the region where it exceeds the promised coupon payment, and that the firm will therefore pay dividends at some point in the future. Hence the equity holder never wants to liquidate. Mathematically, the value of equity is maximized for a choice of liquidation boundary $\bar{x}_E = 0$ which will be hit with probability zero.

Conversely, in liquidation, creditors do gain a positive liquidation payoff, but lose a positive continuation value. The point at which this gain and loss are traded off optimally is given by $\bar{x}_D$. Mathematically, the value of debt is maximized for a choice of liquidation boundary $0 < \bar{x}_D \leq c$. This is above zero because the liquidation payoff is positive, and hence creditors will want to liquidate for very low cash flows / continuation values. It is below c because the liquidation payoff is less than the value of receiving the full coupon c forever (c/r), and hence creditors will not want to liquidate unless the firm is defaulting.[20]

Since the market value of the firm is defined as the sum of the value of debt and equity, the liquidation boundary that maximizes this value $(\bar{x}_V)$ is in between the liquidation boundary that maximizes the value of debt $(\bar{x}_D)$ and the liquidation boundary that maximizes the value of equity $(\bar{x}_E)$, as argued above. □

Compared to the liquidation decision that would maximize the market value of the firm, creditors wants excessive liquidation. This is because the continuation value of

[19] See Appendix B for a discussion of the case when $(1 - s)K > c/r$.

[20] $\bar{x}_D = c$ exactly when $(1 - s)K = c/r$.

creditors is lower than the combined continuation value of all parties.[21]

The actual liquidation outcome under coordinated creditors is as follows. As the cash flow falls, the first liquidation boundary to be hit is that of creditors. Since at that point, the firm is defaulting, creditors are in a position to enforce their rights. Assuming that bargaining costs are prohibitive, creditors will take the firm to court at $\bar{x}_D$, and cause liquidation.

If bargaining costs are not prohibitive such that Coasian bargaining as in Haugen and Senbet (1978) between coordinated creditors and the equity holder is possible, it is clear that liquidation at the firm-value maximizing liquidation point could always be achieved, even when incentives are not aligned. A single bank, for example, might accept a delay in liquidation in exchange for additional cash-flow rights.

However, there is at least anecdotal evidence that this kind of overly aggressive enforcement does seem to occur in the UK, where floating charge holders (typically banks) are a good example of coordinated creditors with strong rights. For instance, it has been argued that in the UK, " . . . the bank may decide against keeping a good company going because it does not see the upside potential" (Hart, 1995, p. 168), or floating charge holders "apply themselves ruthlessly to the realization of assets to satisfy the charge [. . .] in some cases with scant regard for the future of the company" (Woolridge, 1987). This suggests that the concept of creditor-determined premature liquidation is not only of interest as a hypothetical benchmark that is useful to understand bargaining positions, but also of practical interest.

4.2 Uncoordinated creditors

If creditors cannot coordinate, they play an asset grab game in which their actions may fail to collectively maximize the value of debt.

Solving the discrete-time game In the repeated game between uncoordinated creditors described in Section 2, the continuation utility does not depend on the current action of an individual creditor, because creditors are atomistic. This implies that the repeated game can be solved as a series of one-shot games, which greatly simplifies the analysis.[22]

A single-stage asset grab game can be solved using the same procedure as in Morris and Shin (2004), which is described in Appendix C. Solving the game here means finding

[21] This is essentially the point made by Hart (1995, pp. 166); the version presented here is a variation of the version of the argument presented by Naqvi (2008).

[22] For any creditor for whom the utility of grabbing assets is compared to the utility of not grabbing assets, the *same* expression for the continuation utility will appear on both sides of an inequality. The continuation utility therefore cancels out, and only Bernoulli utilities of instantaneous payoffs matter. See Appendix C for details.

the critical level of the cash flow $\bar{x}_{AG}$ (AG for "asset grabs"), such that when the cash flow falls below this critical level, a sufficient number of creditors decide to grab assets and the firm is liquidated. It is shown in the appendix that this critical value of the cash flow $\bar{x}^{AG}_{t+\Delta}$ is given by the implicit function

$$\bar{x}^{AG}_{t+\Delta} = c\Phi\left\{\frac{\alpha}{\sqrt{\beta}}\left(\frac{\bar{x}^{AG}_{t+\Delta}}{x_t} - 1 - (\mu + \sigma\nu_t)\Delta\right) + \frac{\sqrt{\alpha+\beta}}{\sqrt{\beta}}\Phi^{-1}\{\theta\}\right\}, \tag{27}$$

where $\alpha = (\sigma^2\Delta)^{-1}$ is the precision of the public information, and β is the precision of the private information. θ is a ratio that reflects the utilities associated with the instantaneous payoffs in the game:

$$\theta = \frac{u((1-s)K) - u(0)}{u((1-s)K) - u(-sK)}, \tag{28}$$

where $u(\cdot)$ is the (Bernoulli) utility function. θ attains an upper limit at $(1-s)$ when creditors are risk neutral. It is decreasing in s, and in the curvature of the utility function, i.e. risk-aversion.

The critical value of the cash flow $\bar{x}^{AG}_{t+\Delta}$ is unique if

$$c\frac{1}{\sqrt{2\pi}}\frac{\alpha}{\sqrt{\beta}}\frac{1}{x_t} < 1. \tag{29}$$

This type of condition is standard for this type of game, see Appendix C.6 for details. Basically, the quality of private information has to be high enough in relation to the quality of public information in order for creditors to be uncertain about the actions of others and hence coordination failure to be an issue.

The unique equilibrium is the only equilibrium which survives iterated deletion of dominated strategies (see e.g. Morris and Shin (2003)).

Continuous-time limit Now take the continuous time limit. For the cash flow process to tend to a geometric Brownian motion, it is necessary that (loosely speaking)

$$\lim_{\Delta\to dt} \sigma^2\Delta = \sigma^2 dt, \tag{30}$$

i.e. the variance of public information about the innovation in the cash flow to be proportional to time. So the variance of the innovation $\sigma^2\Delta$ is $O(\Delta)$, or the precision $\alpha = (\sigma^2\Delta)^{-1}$ is $O(\frac{1}{\Delta})$.

Now a sufficient condition for the uniqueness of the equilibrium described above to carry over to continuous time, regardless of the cash flow and the coupon c, is that

$$\frac{1}{\beta} = o\left(\Delta^2\right), \tag{31}$$

i.e. that private information becomes more precise at a rate faster than Δ^2, because this ensures that the uniqueness condition [29] is always satisfied. This is just to say that the quality of private information needs to be sufficiently high in relation to the quality of public information in order for creditors to be sufficiently uncertain about the actions of others to obtain coordination failure. As Δ tends to dt, Δ^2 tends to 0, and hence β grows at a faster rate than α. Consequently, $\frac{\alpha}{\sqrt{\beta}}$ tends to zero, so [29] will be satisfied for any permissible $x_{t+\Delta}$. Also, $\frac{\sqrt{\alpha+\beta}}{\sqrt{\beta}}$ tends to 1. The equation for the critical level of the cash flow then reduces to the following:

Proposition 2. *The critical level of the cash flow at which an asset grab occurs is given by*

$$\bar{x}_{AG} = \theta c, \tag{32}$$

where $0 < \theta < 1$ *is given by*

$$\theta = \frac{u((1-s)K) - u(0)}{u\left((1-s)K\right) - u(-sK)}. \tag{33}$$

Proof. Taking limits of [27] as described, we obtain the result. □

This means that in continuous time, the critical level of the cash flow is a fraction θ of the coupon, where this fraction reflects the utility of grabbing assets versus the utility of not grabbing assets in situations in which the firm is liquidated and in situations where it is not liquidated.

θ is decreasing in s; if it is expensive to hire a lawyer to file a claim, creditors will be reluctant to attempt to grab assets, and the cash flow has to be lower before a sufficient number of them act to force liquidation - this is particularly easy to see in the case where creditors are risk neutral, in which case θ attains its upper limit of $(1-s)$, and hence $\bar{x}_{AG}$ is reduced to $(1-s)c$.

θ is also decreasing in the curvature of the utility function or risk aversion. Intuitively, if creditors are more risk averse, they are less willing to incur the certain cost associated with attempting to grab assets in exchange for an uncertain payoff (the grabbed assets if the firm is liquidated).

The solution is constant over time. If we let the intermediate time period $(t+q)$ tend to the period immediately following it, the firm fails at t whenever $x(t)$ hits θc, i.e. when the cash flow is a fraction θ of the coupon.

Conditional on the cash flow in the next period, the probability that a creditor receives a signal which prompts her to grab assets is $\Phi\left\{\frac{1}{x_t}\sqrt{\beta}(\xi_t^* - x_{t+\Delta})\right\}$. As β tends to infinity, this probability tends either to 1 or to 0 for all non-marginal creditors. What this means is that because all creditors essentially receive the same information (as the signal becomes infinitely precise), the creditors will either all grab assets, or will all refrain from doing so. For any non-marginal creditor, the ex-ante probability of grabbing assets when the other creditors do not do so tends to zero. Also, the probability of not grabbing assets if all other creditors are grabbing assets tends to zero. In the limit, creditors receive the same signals, and there is no uncertainty about the cash flow. However, strategic uncertainty remains in the sense that the marginal creditor is completely uncertain about the actions of other creditors.[23]

Since in equilibrium the creditors act in unison, the payoffs are the same for all creditors. By construction, creditors receive the same liquidation payoffs in an asset grab as in the case of coordinated liquidation; this makes comparisons across the different situations easier.[24] Since the equity holder always has incentives to continue, the firm is liquidated when creditors rush to grab assets at $\bar{x}_{AG}$.

4.3 A comparison of liquidation outcomes

I illustrate some general properties of liquidation outcomes produced by the model in the context of a numerical example. The assumed parameters are listed in Table 1. I plot the resulting values of debt, equity, and the firm, and the resulting spreads in Figure 3.

Table 1: Parameters for numerical example

Parameter assumptions for a numerical example, and various model-implied values (equity volatility for liquidation at $\bar{x}_V$). Parameters are the risk-free interest rate r, the drift of cash flows μ, the volatility of cash flows σ, the initial cash flow $x(0)$, the coupon c, the dividend tax (or net tax advantage of debt) τ, the liquidation value of the productive asset K, and the legal cost of grabbing assets s.

r	0.05			
μ	0.03		pre-tax going concern value $\frac{x(0)}{r-\mu}$	250
σ	0.04		value of an all eq. firm $(1-\tau)\frac{x(0)}{r-\mu}$	150
$x(0)$	5	$\Longrightarrow$	face value of debt $\frac{c}{r}$	60
c	3		equity volatility	47%
$\tau^\dagger$	0.4		1-year default probability under $\mathbb{Q}$	24%
K	60			
s	0.2			

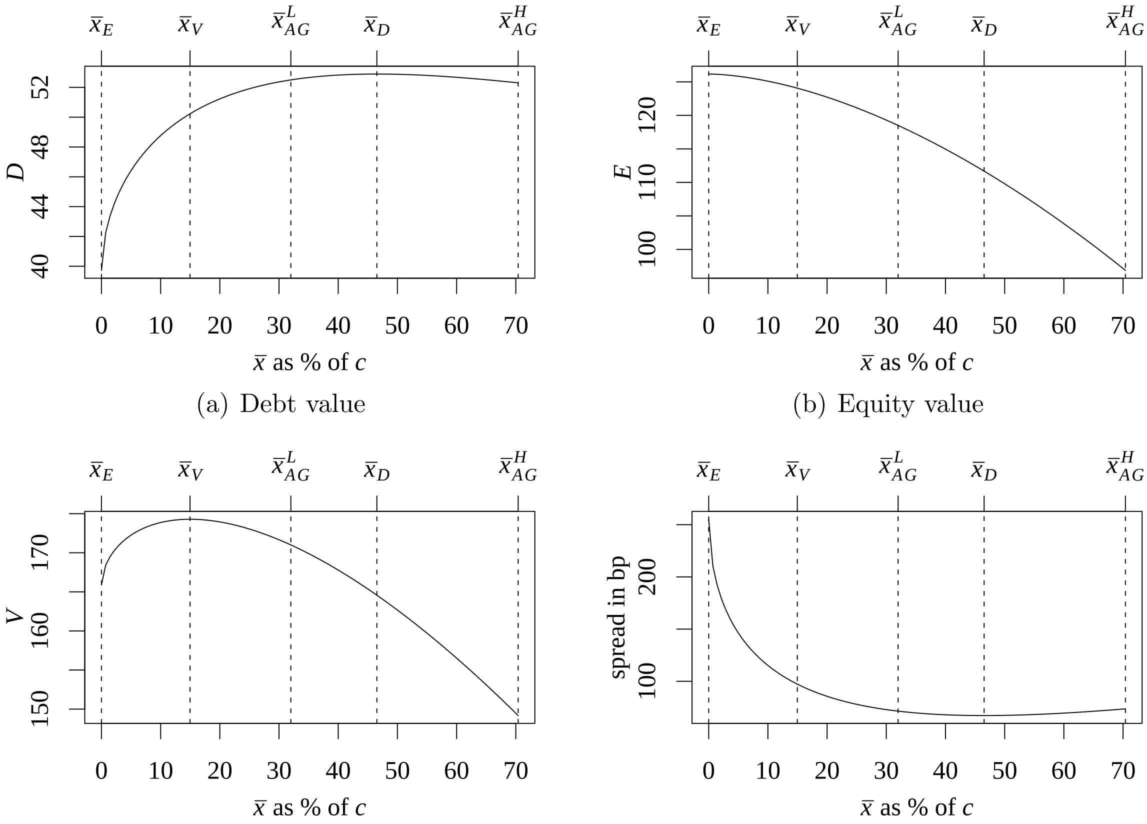
† Kemsley and Nissim (2002) estimate the debt tax shield to be approximately 40% of debt balances.

The parameters are representative of a risky firm. Cash flows ($x(0) = 5$) exceed coupons ($c = 3$), but are very volatile ($\sigma = 0.4$). The (annual) implied probability of default under the pricing measure $\mathbb{Q}$ is therefore very high at 24% (the probabilities of

[23] This is a standard limiting result for this type of game. See Appendix C.7, or the discussion by Morris and Shin (2003).

[24] It might be plausible to argue that legal costs are higher in the asset grab games. A previous version of this paper explored this possibility. Such a version of the model does not produce qualitatively different conclusions.

being liquidated at different boundaries once defaulting is lower).[25] Also, for liquidation at e.g. $\bar{x}_V$ (see below), these parameters would produce an equity volatility of about 47%.



(a) Debt value

(b) Equity value

(c) Firm value

(d) Spread

Figure 3. Debt, equity, firm value and spread as function of $\bar{x}/c$
Debt value, equity value, firm value and spread (in bp, calculated as $c/D-r$), all as function of liquidation boundary $\bar{x}$, measured as % of coupon c. The vertical dashed lines are drawn at $\bar{x}_E$, $\bar{x}_V$, $\bar{x}_D$, and two values of $\bar{x}_{AG}$, respectively, all measured as % of coupon c. Parameters as in Table 1.

In this scenario, the debt-value maximizing liquidation boundary $\bar{x}_D$ is about 46% of the coupon, the equity-value maximizing liquidation boundary $\bar{x}_E$ is 0% of the coupon, and the firm-value maximizing liquidation point $\bar{x}_V$ is about 15% of the coupon, as can be seen in Figures 3(a), 3(b), and 3(c) respectively. Since the firm is very risky, the various option values of waiting to liquidate are high, and hence the various optimal liquidation boundary are low. Figure 3(c) also indicates that liquidation at $\bar{x}_D$ produces lower firm value than liquidation at $\bar{x}_V$. For the given numbers, the difference is about 6% of the maximum attainable firm value for liquidation at $\bar{x}_V$.

Figure 3(d) shows that since the debt value achieves a maximum at $\bar{x}_D$, the spread (calculated as $c/D - r$) achieves a minimum of about 67 bp at this point. The spread

[25] These can be calculated via standard formulas for hitting times of geometric Brownian motions.

corresponding to firm-value maximizing liquidation at $\bar{x}_V$ is about 97 bp. The spreads are relatively low even though the company has a high default probability because the liquidation payoff as a fraction of face value (which we could think of as a measure of recovery) is relatively high at $\frac{(1-s)K}{c/r} = 80\%$.[26]

We can now consider assumptions on parameters that lead to different asset grab liquidation boundaries $\bar{x}_{AG}$. Suppose, for example, a constant relative risk aversion-type utility function with a risk aversion coefficient of 2, but suppose that in a first case the coupon paid by the firm represents 3% of a (high) regular income that creditors receive from other sources, and in the second case it represents 15% of a (low) regular income. In the case where the coupon represents 3% of a high regular income, creditors will be less risk averse vis-a-vis a loss of a given size. It was argued above that this leads to higher asset grab liquidation boundaries, as creditors are less reluctant to grab assets, i.e. less reluctant to pay a lawyer that might not recover anything. In the first case, the asset grab boundary $\bar{x}_{AG}^H$ is consequently at a higher 70% of the coupon, and in the second case, the asset grab boundary $\bar{x}_{AG}^L$ is consequently at a lower 32% of the coupon (where the superscripts H and L distinguish the two cases). It can be seen in Figure 3(c) that the low asset grab liquidation boundary that results from higher risk aversion actually generates *higher* firm values than liquidation by coordinated creditors at $\bar{x}_D$. (Other parameters, in particular s, could be varied to generate different scenarios for $\bar{x}_{AG}$, but they all change $\bar{x}_D$ and the value of debt also, so the numerical comparison is more convoluted and omitted here.)

In general, in cases in which $\bar{x}_{AG} < \bar{x}_D$, firm value can be higher under asset grab liquidation than under liquidation by coordinated creditors, as illustrated by Figure 3(c). In cases in which $\bar{x}_{AG} > \bar{x}_D$, firm value is always lower under asset grab liquidation than under liquidation by coordinated creditors.[27]

When is $\bar{x}_{AG} > \bar{x}_D$? For a sufficiently low legal cost of grabbing assets s, this is always the case. To see this, consider for example the case where $s = 0$. From Proposition 2, it can be seen that in this case, $\bar{x}_{AG} = c$. Intuitively, if attempting to grab assets is costless, creditors will do so as soon as possible. But $\bar{x}_D \leq c = \bar{x}_{AG}$ by Proposition 1. Hence if the legal cost of grabbing assets s is zero (or in general, low enough), the firm value is lower (or at most equal) under asset grab liquidation than under liquidation by coordinated

[26]To see that this can impose limits on spreads, consider the simpler case of perpetual debt that promises coupon c that is paid until liquidation, at which point a fraction $\mathcal{R}$ of the face value of debt (c/r) is paid. The value of debt at liquidation is $\mathcal{R}(c/r)$, and the spread at that point would be $c/(\mathcal{R}(c/r))-r = (1-\mathcal{R})/\mathcal{R}r$. For $\mathcal{R} = 0.8$, this implies that the spread attains a maximum of one quarter of r at this point.

[27]This can be shown formally by an argument similar to the proof of Proposition 1, but is also obvious when considering the ordering of optimal liquidation boundaries in the proposition.

creditors.

For intermediate values of the legal cost of grabbing assets s, and some degree of risk aversion, it can be the case that $\bar{x}_{AG} < \bar{x}_D$, as in the numerical example above. It is in these cases that the firm value can be higher under asset grab liquidation than under liquidation by coordinated creditors.

5 Automatic stays, preference law, and equality of distributions

Bankruptcy codes attempt to prevent asset grabs via automatic stays, preference law, and a policy of equality of distributions to creditors, thus protecting debtors. For example, part of the stated purpose of preference law in the US is the following:

> [...] by permitting the trustee to avoid prebankruptcy transfers that occur within a short period before bankruptcy, creditors are discouraged from racing to the courthouse to dismember the debtor during his slide into bankruptcy.[28]

In practice, firms do apply for protection from their creditors exactly when these attempt to individually grab assets (LoPucki, 1983). This suggests modifying the game by allowing the equity holder to explicitly put the firm into formal (Chapter-7 type) bankruptcy in which case an automatic stay, preference law, and a policy of equality of distributions to creditors applies.

A modeling issue then is whether the equity holder and creditors should move simultaneously, or whether either creditors or the equity holder should move first. In the US, preference law (United States Code, Title 11, Ch. 5.III § 547) specifies that any preferential transfer that a creditor manages to obtain in the period up to 90 days before formal bankruptcy can be avoided; i.e. if a creditor manages to grab assets within 90 days before formal bankruptcy, the grabbed amount has to be returned to the trustee, to be shared equally across all creditors. This suggests that the equity holder can put the firm into formal bankruptcy *after* observing the actions of creditors, but that this move would still affect the payoffs to creditors.

Suppose therefore that at each interim stage $t+q$ we now have the following two-stage sequential move game: First creditors decide whether or not to grab assets. Then the equity holder decides whether or not to file for bankruptcy, having observed the actions of all creditors (see Figure 4).

[28] H.R. Rep. No. 595, 95th Cong., 1st Sess. p. 177 (1977), describing the Bankruptcy Reform Act of 1978. Note that the verb "to avoid" is used here in its legal sense of "to repudiate, nullify or render void".

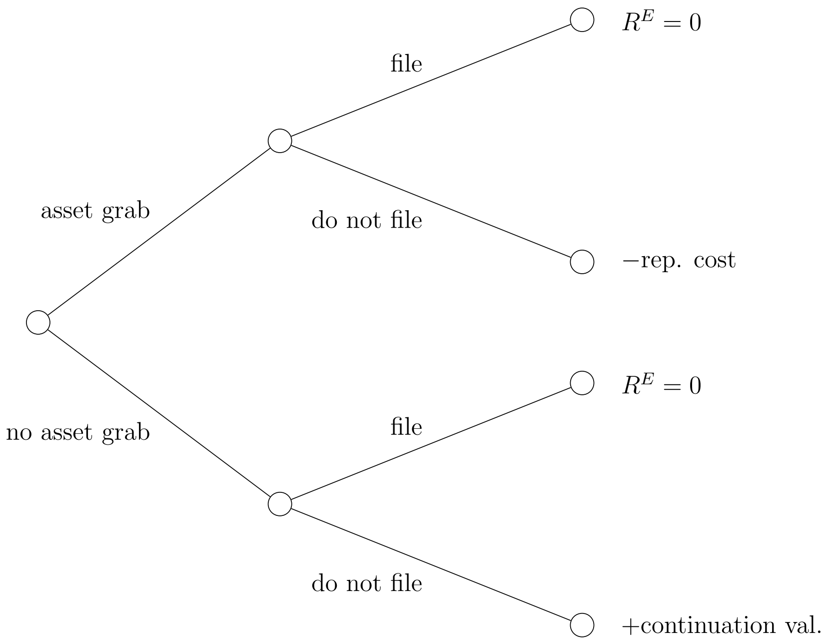


Figure 4. Payoffs to equity in bankruptcy game at $t + q$
First creditors decide whether or not to grab assets. The equity holder observes whether a sufficient number of creditors have grabbed assets such that the firm will be liquidated piecemeal ("asset grab"), or not ("no asset grab"), and will decide to put the firm into bankruptcy or not depending on the given payoffs. As long as the reputational cost is not zero, and the continuation value is above zero, the equity holder puts the firm into bankruptcy only when an asset grab has happened.

A second modeling issue is how to describe the payoffs to the equity holder and creditors in bankruptcy. If absolute priority is respected in Chapter 7, the payoff to the equity holder is zero. In practice, it is likely that an equity holder would have at least a weak preference for orderly Chapter 7 liquidation over a disorderly asset grab liquidation, e.g. because of reputational costs associated with the latter. Suppose that such reputational costs exist, but that they are arbitrarily small. When a large number of creditors grab assets such that an asset grab liquidation would result, the equity holder would then file for bankruptcy, since this is weakly preferred. When a small number of creditors grab assets such that no asset grab liquidation would result, the equity holder does not file for bankruptcy, since in this situation the continuation value to equity (which is always positive) is always higher than the liquidation value (which is zero).

Bankruptcy codes strive to achieve equality of distributions to creditors, but do not necessarily achieve full equality of distributions. For example, preference law does not prevent all types of pre-bankruptcy transfers and requests of the trustee to return grabbed funds are open to a legal challenge. To model this, suppose we modify the payoffs that result from a successful asset grab (that now provokes bankruptcy). Assume that creditors that grabbed assets obtain $(1 - s)K$ as before, but assume that creditors that did not grab assets now obtain $(1-\varepsilon)(1-s)K$, where $0 \le \varepsilon \le 1$. If $\varepsilon = 1$, the payoff to creditors that did not grab assets is 0. In this case, the payoffs are the same payoffs as in the benchmark asset grab game without bankruptcy codes. If $\varepsilon < 1$, then creditors that did not grab assets now receive more than in the benchmark asset grab game without bankruptcy codes. As long as $\varepsilon > 0$, however, they still receive less than creditors that grabbed assets, meaning that bankruptcy codes are not achieving full equality of distributions.[29] If $\varepsilon = 0$, creditors that grabbed assets and creditors that did not grab assets receive the same amount, implying that bankruptcy codes are achieving full equality of distributions. Lowering ε therefore makes payoffs of creditors in bankruptcy more similar. The new payoff matrix for creditors is then given in Figure 5.

	bankruptcy	no bankruptcy
grab assets	$(1-s)K$	$-sK$
do not grab assets	$(1-\varepsilon)(1-s)K$	0

Figure 5. Payoffs to creditors in bankruptcy game at $t+q$
The optimal choice of of equity holders in the second stage (put the firm into bankruptcy when there is a successful asset grab) produces the given payoff matrix for creditors in the first stage.

Following the steps of the argument in the previous section, this game can be solved in the same way, which produces a critical point at which creditors would attempt to grab assets (and provoke the equity holder to put the firm into bankruptcy).

Proposition 3. *The critical level of the cash flow at which an asset grab occurs in the case where bankruptcy codes disincentivize asset grabs is*

$$\bar{x}_{BC} = \zeta c, \tag{34}$$

where $0 < \zeta < \theta$ is given by

$$\zeta = \frac{u((1-s)K) - u((1-\varepsilon)(1-s)K)}{u((1-s)K) - u(-sK)}. \tag{35}$$

Proof. Analogous to the proof of Proposition 2 □

(Here, the subscript "BC" is meant to indicate "bankruptcy codes"). The critical level of the cash flow is a fraction ζ of the coupon, where this fraction again reflects the utility of grabbing assets versus the utility of not grabbing assets. It is easiest to see the influence of the key parameters s and ε in the case where creditors are risk neutral; in this case ζ attains an upper limit of $\varepsilon(1-s)$.

[29] In practice, not all pre-bankruptcy transfers can be avoided by the bankruptcy trustee. ε could e.g. be related to the probability that the bankruptcy trustee successfully challenges a pre-bankruptcy transfer.

Again, if it is expensive to hire a lawyer to file a claim (high s), creditors will be reluctant to attempt to grab assets, and the cash flow has to be lower before a sufficient number of them act to force bankruptcy. Lowering ε makes payoffs in bankruptcy for creditors that grabbed and creditors that did not grab more similar, and therefore reduces incentives to grab, and consequently reduces the critical level of the cash flow at which asset grabs occur. As ε tends to zero, not grabbing assets becomes a (weakly) dominant strategy, illustrating how bankruptcy codes can prevent asset grabs. If $\varepsilon > 0$, then creditors grab assets at some positive level of the cash flow, after which the equity holder puts the firm into bankruptcy, as has been observed to happen in practice (LoPucki, 1983). As long as $\varepsilon < 1$, however, creditors grab assets at a point that is below the point at which they would grab if there were no bankruptcy codes, i.e. $\bar{x}_{BC} < \bar{x}_{AG}$ (since $\zeta < \theta$). The higher ε, the closer $\bar{x}_{BC}$ is to $\bar{x}_{AG}$. Since payoffs to the equity holder and creditors are the same as before, pricing is straightforward.

To extend this to Chapter 11, assumptions about the Chapter 11 payoff to the equity holder would have to be made. In Chapter 7, absolute priority is likely to be respected, whereas there in Chapter 11, there are likely to be deviations from absolute priority in favour of equity holders (Bris, Welch, and Zhu, 2006; Franks and Torous, 1989). This makes modelling Chapter 11 more complicated.

One could assume as a starting point that the bargaining in Chapter 11 is over the difference between the going-concern value and liquidation value of the firm, and that the equity holder has some power in this bargaining process (this would explain the observed deviations from absolute priority). The payoff to the equity holder in Chapter 11 would then in general be positive, and would be positively related to the going-concern value of the firm. Note that in this case, the equity holder should always strictly prefer Chapter 11 to Chapter 7, since the payoff in Chapter 7 is zero.

With these assumptions on payoffs in Chapter 11, it is easy to see that the equity holder would still file (now for Chapter 11) if an asset grab occurred. However, the equity holder would then also have an incentive to file for Chapter 11 in situations in which an asset grab had not occurred - she would file voluntarily when the Chapter 11 payoff is just equal to the continuation value (there would be a boundary at which the equity holder files voluntarily). Whether the equity holder would file for Chapter 11 voluntarily or after an asset grab would then depend on whether the boundary at which the equity holder files voluntarily is above the boundary at which creditors grab assets. If the payoff to the equity holder in Chapter 11 is sufficiently small, and asset grabs occur sufficiently early, the liquidation outcomes would look similar to the liquidation outcome described for the Chapter 7 case above. Conversely, in cases in which the payoff to the equity holder in

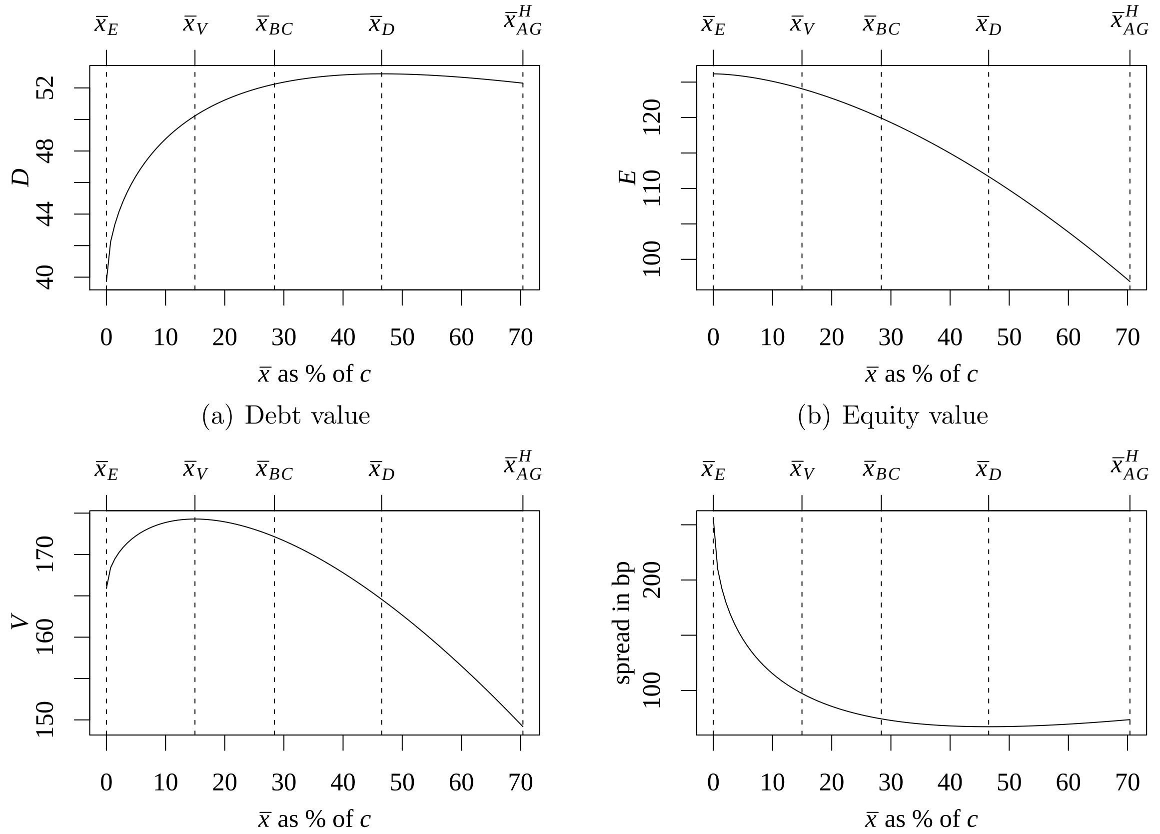
Chapter 11 is large and asset grab occur sufficiently late, one would observe voluntary bankruptcy. A more precise examination of Chapter 11 in the framework proposed here is likely to be a fruitful area for future research.

A comparison of liquidation outcomes In the context of the numerical example of the previous section, the liquidation boundary under bankruptcy codes with features that affect creditor coordination, $\bar{x}_{BC}$, can also be calculated. Assume as before that the coefficient of relative risk aversion is 2, and that the coupon paid by the firm represents 3% of a (high) regular income from other sources. In the absence of features of bankruptcy codes that affect creditor coordination, this produced a asset grab boundary $\bar{x}_{AG} = \bar{x}_{AG}^{H}$ of around 70% of the coupon. Now, assume for example that $\varepsilon = 0.5$, meaning that payoffs in bankruptcy are more similar than before, but still not equal. This produces a $\bar{x}_{BC}$ of around 28% of the coupon. This puts the actual liquidation boundary close to the firm-value maximizing liquidation boundary $\bar{x}_V$ of around 15% of the coupon, as illustrated in Figure 6. The value of the firm if liquidated at $\bar{x}_{BC}$ is about 99% of the maximum attained if liquidated at $\bar{x}_V$.

In the given numerical example, firm value is higher under liquidation at $\bar{x}_{BC}$ than under liquidation under the assumed value of the asset grab liquidation boundary $\bar{x}_{AG} = \bar{x}_{AG}^{H}$. This does not necessarily have to be the case. Consider, for example, the choice of parameters that was considered in the previous section that produced an asset grab liquidation boundary of $\bar{x}_{AG} = \bar{x}_{AG}^{L}$ of around 32% of the coupon, and suppose that bankruptcy codes can achieve full equality of distributions, i.e. achieve $\varepsilon = 0$. From Proposition 3, it can be seen that in this case, $\bar{x}_{BC} = 0$. Intuitively, if in bankruptcy, all creditors receive the same payoff, it becomes a dominant strategy not to grab assets, and hence the firm is never put into bankruptcy (note that $x(t) > 0$ a.s.).

From Figure 3(c), it can then be seen that the firm value is higher for liquidation at $\bar{x}_{AG}^{L}$ than for liquidation at 0. Here, since asset grabs in the absence of features of bankruptcy codes that affect creditor coordination already happen relatively late, delaying asset grabs even further can have a negative effect on firm value.

In general, the firm value can be raised by delaying the incidence of asset grabs when such a delay means that they occur closer to the firm-value maximizing level $\bar{x}_V$. Disincentivizing asset grabs by making distributions to creditors more equal produces just such a delay. However, if asset grabs already occur at a point relatively close to $\bar{x}_V$, e.g. because the legal costs of individually grabbing assets are high, then disincentivizing asset grabs even more can produce excessive continuation ($\bar{x}_{BC} < \bar{x}_V$), which can reduce firm value. It is in this sense that bankruptcy codes that provide moderate protection



(a) Debt value (b) Equity value

(c) Firm value (d) Spread

Figure 6. Debt, equity, firm value and spread as function of $\bar{x}$
Debt value, equity value, firm value and spread (in bp, calculated as $c/D-r$), all as function of liquidation boundary $\bar{x}$, measured as % of coupon c. The vertical dashed lines are drawn at $\bar{x}_E$, $\bar{x}_V$, $\bar{x}_D$, $\bar{x}_{AG}^H$, and at $\bar{x}_{BC}$ respectively, all measured as % of coupon c. Parameters as in Table 1.

can improve liquidation timing when the legal cost of grabbing assets is low, but might worsen liquidation timing when the legal cost of grabbing assets is medium or high.

6 Optimal capital structure

The previous discussion of the effect of various liquidation mechanisms on debt, equity and firm value takes the choice of capital structure as given. The choice of capital structure in this context is synonymous with a choice of the level of coupon c (or equivalently, the face value of debt $\frac{c}{r}$). In this section, I discuss how for a given liquidation mechanism, adjusting the capital structure can achieve an optimum firm value. The capital structure here is optimal in the ex-ante sense of Leland (1994); i.e. I consider the owner of an all-equity firm that wants to exit by selling debt and equity in proportions that maximize the overall proceeds of the sale, taken a liquidation mechanism as given.

We can then consider firm value as a function of the coupon c (or face value of debt

c/r). In general, increasing the level of debt c has two effects on firm value in the model. Firstly, it increases the size of the tax shield, which increases the value of the firm, ceteris paribus. Secondly, it can change incentives to liquidate since it affects the liquidation boundaries; this can increase or decrease the value of the firm, ceteris paribus, depending on whether the effect is to move the liquidation boundary closer towards the firm-value maximizing boundary or further away. If increasing the level of debt produces a distortion of liquidation decisions, there can exist a trade-off between the size of the tax shield, and the distortions introduced, leading to an interior optimum.

To consider possible tradeoffs, it is necessary to understand how the liquidation boundaries change as the coupon c or face value of debt c/r changes.

The firm-value maximizing liquidation boundary $\bar{x}_V$ is decreasing in the level of debt. Intuitively,[30] as the level of debt increases, the size of the debt tax shield increases, which forms part of the continuation value. Since the continuation value increases, the firm-value maximizing liquidation boundary decreases.

The debt-value maximizing liquidation boundary $\bar{x}_D$ is always above the firm-value maximizing boundary (cf. Proposition 1). Intuitively, because the continuation value of creditors is lower than the overall continuation value of all claimants, they have incentives to liquidate prematurely. $\bar{x}_D$ is also a decreasing function of c, that tends to $\bar{x}_V$ as $c \to \infty$. As c increases, creditors obtain a larger part of the overall continuation value, lessening their incentives to liquidate prematurely. In the limit, as $c \to \infty$, the creditors essentially own rights to all future cash flows and therefore are the owners of the firm. Hence they act to maximize the value of the firm.

If creditors are uncoordinated, the liquidation boundaries $\bar{x}_{AG} = \theta c$ and $\bar{x}_{BC} = \zeta c$ are simple linear increasing functions of c, with slope θ or ζ (where $\theta > \zeta$, cf. Propositions 2 and 3). In both cases, increasing the amount of debt makes asset grabs more likely to succeed at a given level of the cash flow, therefore encouraging earlier asset grabs.

Given this behaviour of liquidation boundaries, we can now consider the tradeoff between the tax advantage of debt, and possible distortions of liquidation decisions. If the firm is liquidated at the firm-value maximizing liquidation boundary $\bar{x}_V$, there are no distortions of liquidation decisions. Increasing the level of debt c increases the size of the tax shield, but does not affect liquidation outcomes. The value of the firm, if liquidated at the firm-value maximizing boundary, is therefore always increasing in c.

If the firm is liquidated at the debt-value maximizing liquidation boundary $\bar{x}_D$, i.e. if creditors are coordinated, the distortion in liquidation decisions is reduced by increasing c, in the sense that increasing c moves $\bar{x}_D$ closer to the firm-value optimizing $\bar{x}_V$. Increasing

[30] For a formal discussion of this and other boundaries, see Appendix D.

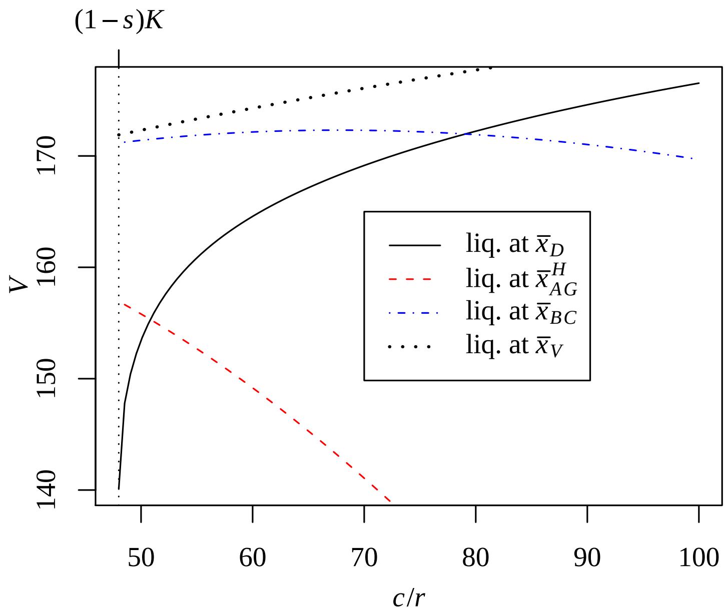


Figure 7. Firm values as function of face value of debt
Firm values as a function of face value of debt c/r. Parameter values as Table 1, except coupons c, which vary. Here, $\bar{x}_{AG}^H$ and $\bar{x}_{BC}$ calculated for the same parameters as in Figures 6(a) to 6(d), except coupons c, which vary.

the level of debt c therefore increases the size of the tax shield, and improves liquidation outcomes. The value of the firm, if liquidated at the debt-value maximizing boundary is therefore also always increasing in c.

In the case where creditors are uncoordinated, and liquidation occurs either at $\bar{x}_{AG}$ or at $\bar{x}_{BC}$, an interior optimum can exist. Here, increasing c will cause earlier and earlier liquidation, which might initially be desirable, but will eventually have a negative effect (ceteris paribus) on the value of the firm. Increasing the level of debt c therefore increases the size of the tax shield, but eventually worsens liquidation outcomes. There can therefore exist a trade-off as in standard trade-off theory between an advantage of debt (the tax shield of debt effect), and a disadvantage associated with debt (the distortion of liquidation decisions).

This is illustrated in the context of the extended numerical example in Figure 7, which plots firm values against coupons. In the case of uncoordinated creditors and bankruptcy codes with features that affect creditor coordination, i.e. liquidation at $\bar{x}_{BC}$, a maximum firm value of about 170 is achieved for a face value of $c/r \approx 67$. The optimal leverage (calculated as market value of debt/ market value of equity) for the relatively risk firm considered in the example is about 56%.

7 Conclusion

This paper presents a continuous-time structural model in which defaulting firms are liquidated when creditors attempt to enforce claims against these firms. It considers how the value of debt, equity, and the firm are affected by the actions of coordinated creditors, and uncoordinated creditors. In the case of uncoordinated creditors, the effect of features of (Chapter-7 type) bankruptcy codes such as automatic stays, preference law, and policies of equality of distributions that affect creditor coordination are also discussed. *Closed-form* solutions are derived for the value of debt, equity, and the firm.

In the model, coordinated creditors can have incentives to liquidate prematurely, in the sense that firm value would be higher if the firm was liquidated later. Uncoordinated creditors care about payoffs in an asset grab game, and can have incentives to try to collect individually (grab assets) either too early or too late, depending on the payoffs in the game. Legal costs of individually grabbing assets can give incentives to uncoordinated creditors to delay grabbing assets. Features of Chapter-7 type bankruptcy codes that affect creditor coordination (automatic stays, preference law, policies of equality of distribution) change the payoffs in the asset grab game such that grabbing assets becomes less attractive, protecting debtors. This leads to later liquidation. Although this debtor protection can improve liquidation timing when the legal costs of grabbing assets is low, it can worsen liquidation timing in situations where the legal costs of grabbing assets is medium or high, as it is likely to lead to excessive continuation in this case. Bankruptcy codes that prevent asset grabs and protect debtors therefore do not necessarily improve liquidation timing.

Inefficiencies arising from non-optimal liquidation can be mitigated to some extent by a judicious choice of capital structure, and in some situations, levels of debt that achieve firm-value maximizing liquidation timing exist. In terms of firm-value maximizing capital structure, a trade-off between a tax advantage of debt versus a disadvantage of debt related to inefficient liquidation exists in various scenarios, and can lead to certain levels of debt being optimal.

The model presented here raises some questions that warrant attention in future research; notably the question of how introducing the option of Chapter-11 type bankruptcy reorganization in addition to Chapter-7 type bankruptcy liquidation in the given framework might affect liquidation timing, and hence the values of debt, equity and the firm.

Also, one issue that has not been explored here is how different liquidation outcomes have an effect on the shape and in particular curvature of the values of debt, equity, and the firm, seen as functions of the cash flow. This curvature in turn has implications e.g. for how the values of the claims are related to cash flow volatility. It is likely that the different liquidation mechanisms have pricing implications across firms (e.g. with different cash flow volatilities), which could be explored in future research.

APPENDIX

A Valuation

Solutions to the valuation problem at hand are well known (see e.g. Dixit, 1993). Let $F(x)$ denote the value of a perpetual claim to flow payoffs of the type $a + bx$, where x is an Itō process, and a, b are constants. Then requiring that the discounted gains process associated with the claim is a martingale under $\mathbb{Q}$ produces a pricing ordinary differential equation (ODE). If x is geometric Brownian motion as in [1], then the ODE will have the following form:

$$\frac{1}{2}\sigma^2 x^2 F''(x) + \mu x F'(x) + a + bx = rF(x). \tag{36}$$

Solutions to this ODE have the form

$$F = Ax^{-\gamma} + Bx^{\delta} + \frac{a}{r} + \frac{bx}{r-\mu}, \tag{37}$$

where A and B are constants of integration to be determined via boundary conditions, and $\delta > 1$ and $-\gamma < 0$ are the positive and negative roots respectively of the characteristic equation of the ODE, given by

$$\{\delta, -\gamma\} = \frac{-\mu + \frac{1}{2}\sigma^2}{\sigma^2} \pm \frac{\sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2 r}}{\sigma^2}. \tag{38}$$

A.1 The going-concern value

The pre-tax going-concern value of the firm (the value of receiving the pre-tax cash flows $x(t)$ forever) can be derived by applying the appropriate boundary conditions to the general solution above, or by direct integration, yielding $\frac{x(t)}{r-\mu}$.

A.2 The value of debt

In the case where $\bar{x} < c$, we need to consider two regions; one in which the coupon is paid in full, and one in which the coupon is only paid in part.

In the region where creditors receive the coupon ($x \geq c$), the price of debt D_1 is given by [37] with $a = c$ and $b = 0$. The formula for D_1 contains two constants of integration. In the region where creditors receive the cash flow ($x \leq c$), the price of debt D_2 is given by [37] with $a = 0$ and $b = 1$. The formula for D_2 also contains two constants of integration.

To produce pricing formulas used for a very general version of the proof of Proposition 1 in Appendix B, I slightly generalize the boundary conditions in the main text by substituting [10] with the condition $D_2(\bar{x}) = R^D$, where R^D represents a general liquidation payoff to creditors. Applying the four boundary conditions to obtain the values of the four constants of integration, D_1 and D_2 are found to be

$$D_1(x,\bar{x}) = \frac{c}{r} + \left[D_2(c,\bar{x}) - \frac{c}{r}\right]\left(\frac{x}{c}\right)^{-\gamma} \tag{39}$$

$$D_2(x;\bar{x}) = \left\{\frac{x}{r-\mu} - Z(c)\left(\frac{x}{c}\right)^{\delta}\right\} + \left[R^D - \left\{\frac{\bar{x}}{r-\mu} - Z(c)\left(\frac{\bar{x}}{c}\right)^{\delta}\right\}\right]\left(\frac{x}{\bar{x}}\right)^{-\gamma}. \tag{40}$$

Setting $R^D = (1-s)K$ produces the formula stated in the main text.

If $\bar{x} \geq c$, we do not need to consider the region in which the coupon is only paid in part. In this case, it can be shown that for the boundary conditions $\lim_{\bar{x}\to\infty} D_1(x) = \frac{c}{r}$ and $D_1(\bar{x}) = R^D$ one finds that

$$D_1(x;\bar{x}) = \frac{c}{r} + \left[R^D - \frac{c}{r}\right]\left(\frac{x}{\bar{x}}\right)^{-\gamma}. \tag{41}$$

A.3 The value of equity

In the case where $\bar{x} < c$, we need to consider two regions; one in which the coupon is paid in full and the equity holder receives dividends, and one in which the coupon is only paid in part, and hence the equity holder receives no dividends.

In the region where the equity holder receives dividends ($x \geq c$), the price of equity E_1 is given by [37], with $a = -(1-\tau)c$ and $b = (1-\tau)$. The formula for E_1 contains two constants of integration. In the region where the equity holder does not receive dividends ($x \leq c$), the price of equity E_2 is given by [37] with $a = 0$ and $b = 0$. The formula for E_2 also contains two constants of integration.

We can impose the following boundary conditions:

$$\lim_{x\to\infty} E_1(x) - (1-\tau)\left(\frac{x}{r-\mu} - \frac{c}{r}\right) = 0 \tag{42}$$

$$E_1(c) = E_2(c) \tag{43}$$

$$E_1'(c) = E_2'(c) \tag{44}$$

$$E_2(\bar{x}) = R^E, \tag{45}$$

To produce pricing formulas used for a very general version of the proof of Proposition 1 in Appendix B, I slightly generalize the boundary conditions used for producing the formulas in the main text by by letting R^E represent a general liquidation payoff to the equity holder (in the main text, R^E is assumed to be zero). Applying the four boundary conditions to obtain the values of the four constants of integration, E_1 and E_2 are found to be

$$E_1(x;\bar{x}) = \left\{(1-\tau)\left(\frac{x}{r-\mu} - \frac{c}{r}\right)\right\} + \left[E_2(c;\bar{x}) - \left\{(1-\tau)\left(\frac{c}{r-\mu} - \frac{c}{r}\right)\right\}\right]\left(\frac{x}{c}\right)^{-\gamma}, \tag{46}$$

$$E_2(x;\bar{x}) = \left\{(1-\tau)Z\left(\frac{x}{c}\right)^{\delta}\right\} + \left[R^E - \left\{(1-\tau)Z\left(\frac{\bar{x}}{c}\right)^{\delta}\right\}\right]\left(\frac{x}{\bar{x}}\right)^{-\gamma}. \tag{47}$$

Setting $R^E = 0$ produces the formula stated in the main text.

If $\bar{x} \geq c$, we do not need to consider the region in which the coupon is only paid in part and the equity holder receives no dividends. In this case, it can be shown that for the boundary conditions $\lim_{\bar{x}\to\infty} E_1(x) - (1-\tau)\left(\frac{x}{r-\mu} - \frac{c}{r}\right) = 0$ and $E_1(\bar{x}) = R^E$ one finds that

$$E_1(x;\bar{x}) = \left\{(1-\tau)\left(\frac{x}{r-\mu} - \frac{c}{r}\right)\right\} + \left[R^E - \left\{(1-\tau)\left(\frac{\bar{x}}{r-\mu} - \frac{c}{r}\right\}\right)\right]\left(\frac{x}{\bar{x}}\right)^{-\gamma}. \tag{48}$$

A.4 The value of the levered firm

Assume that $R^D + R^E = (1-s)K$, i.e. the sum of the liquidation payoffs to creditors and to the equity holder has to equal the net liquidation proceeds (this assumption is satisfied by the liquidation payoffs assumed in the main text).

We can now add the value of equity and the value of debt to obtain the value of the levered firm. For $\bar{x} < c$, one obtains the formula stated in the main text.

For the case where $\bar{x} \geq c$, one obtains

$$V(x;\bar{x}) = \left\{(1-\tau)\frac{x}{r-\mu} + \tau\frac{c}{r}\right\} + \left[(1-s)K - \left\{(1-\tau)\frac{\bar{x}}{r-\mu} + \tau\frac{c}{r}\right\}\right]\left(\frac{x}{\bar{x}}\right)^{-\gamma}. \tag{49}$$

Note that if $\tau = 0$, this simplifies considerably, and the size of the coupon c has no direct impact on the value of the firm (c might still have an indirect effect on V via its effect on the liquidation point $\bar{x}$).

B Optimal liquidation boundaries

Here, I prove a more general version of Proposition 1 that also covers the case when $(1-s)K > c/r$.[31] Assume that the liquidation payoffs to debt and equity are given by $R_D = \min((1-s)K, c/r)$ and $R_E = \max((1-s)K - c/r, 0)$ respectively. This maintains the assumption that absolute priority is respected in liquidation; creditors will be paid off

[31]Note that when $(1-s)K > c/r$, there is a sufficient amount of net liquidation proceeds to ensure that all creditors will be repaid. Asset grabs should therefore not arise. Extending the asset grab game to this case would not be meaningful.

up to the face value of debt in liquidation, any remaining liquidation proceeds go to the equity holder.

The more general general version of Proposition 1 can then be stated in terms of two new propositions as follows:

Proposition A1. *Iff* $\bar{x}_V \geq c$, *then*

$$\bar{x}_E = \bar{x}_V (= \bar{x}_D) \tag{50}$$

Conversely, iff $\bar{x}_V < c$, *then*

$$\bar{x}_E < \bar{x}_V < \bar{x}_D \leq c. \tag{51}$$

Proposition A2. *If*

$$(1-s)K \leq \frac{c}{r}, \tag{52}$$

then $\bar{x}_V < c$.

It can be seen that $(1-s)K \leq c/r$ as assumed in the main text implies $\bar{x}_V < c$ by Proposition A2, and hence the ordering $\bar{x}_E < \bar{x}_V < \bar{x}_D \leq c$ by Proposition A1. Furthermore, using the closed form for $\bar{x}_E$ derived below, we have $\bar{x}_E = 0$ in this case. Taken together, this implies that if $(1-s)K < c/r$, Proposition 1 holds.

Note that $(1-s)K < c/r$ is a *sufficient*, but not *necessary* condition for the ordering $\bar{x}_E < \bar{x}_V < \bar{x}_D \leq c$ to hold (the *necessary and sufficient* condition is $\bar{x}_V < c$, as in Proposition A2).

The proofs of Propositions A1 and A2 are in subsection B.4, and build on the results derived in subsections B.1-B.3.

B.1 The debt-value maximizing liquidation boundary

Consider the case $\bar{x} < c$ initially. Note that D_1 depends on $\bar{x}$ only through the boundary payoff of D_2 at c, and that this dependence is positive. Hence maximizing D w.r.t. $\bar{x}$ is equivalent to maximizing D_2. Note furthermore that D_2 only depends on $\bar{x}$ via A_2. Since $x^{-\gamma} > 0$, choosing $\bar{x}$ to maximize D is equivalent to choosing $\bar{x}$ to maximize the term in A_2. This will yield the optimal boundary from the point of creditors for any $\bar{x} \leq c$.

Since

$$\frac{\partial (A_2 x^{-\gamma})}{\partial \bar{x}} = \bar{x}^{\gamma-1} \left\{ \gamma R^D - (1+\gamma)\frac{\bar{x}}{r-\mu} + (\delta+\gamma)\left(\frac{\bar{x}}{c}\right)^{\delta} Z(c) \right\} x^{-\gamma}, \tag{53}$$

we can define an implicit function

$$f^D(\bar{x}) = \gamma R^D + (1+\gamma)\frac{\bar{x}}{r-\mu} + (\delta+\gamma)\left(\frac{\bar{x}}{c}\right)^{\delta} Z(c) \qquad \text{if } \bar{x} < c. \tag{54}$$

For $\bar{x} \geq c$, the pricing formula is simplified as above. Taking derivatives in this case w.r.t. $\bar{x}$, we obtain

$$\bar{x}^{\gamma-1}\left\{\gamma R^D - \gamma\frac{c}{r}\right\} x^{-\gamma}, \tag{55}$$

suggesting that we can extend the definition of $f^D(\bar{x})$ as follows:

$$f^D(\bar{x}) = \begin{cases} \gamma R^D - (1+\gamma)\frac{\bar{x}}{r-\mu} + (\delta+\gamma)\left(\frac{\bar{x}}{c}\right)^\delta Z(c) & \text{if } \bar{x} < c \\ \gamma R^D - \gamma\frac{c}{r} & \text{if } \bar{x} \geq c. \end{cases} \tag{56}$$

For values of $\bar{x}$ that satisfy $f^D(\bar{x}) = 0$, the value of debt will be maximized, i.e. we could define $\bar{x}_D$ via $f^D(\bar{x}_D) \equiv 0$. It will therefore be useful to discuss the properties of $f^D(\bar{x})$.

With some algebra it can be seen that $(f^D)'(c) = 0$, and $(f^D)''(c) > 0$, so that $f^D(\bar{x})$ attains a local minimum at $\bar{x} = c$. Since $f^D(\bar{x})$ is continuous at c and constant for $\bar{x} > c$, this is also a global minimum value attained for any $\bar{x} \geq c$. If $(1-s)K < \frac{c}{r}$, then $R^D = \min\left((1-s)K, \frac{c}{r}\right) = (1-s)K$. It is then easily seen that this minimum value is

$$f^D(c) = \gamma\left((1-s)K - \frac{c}{r}\right) < 0 \qquad \text{for } \bar{x} \geq c. \tag{57}$$

When $(1-s)K \geq \frac{c}{r}$, $R^D = \frac{c}{r}$, and hence this minimum value is $f^D(c) = 0$.

We can also decompose $f^D(\bar{x})$ in the region where $\bar{x} < c$ as

$$f^D(\bar{x}) = g(\bar{x}) + h(\bar{x}) \qquad \text{if } \bar{x} < c, \tag{58}$$

where $g(\bar{x}) = R^D - (1+\gamma)\frac{\bar{x}}{r-\mu}$, i.e. the linear part of $f^D(\bar{x})$, and $h(\bar{x}) = (\delta+\gamma)\left(\frac{\bar{x}}{c}\right)^\delta Z(c)$, i.e. the non-linear part. In interpreting $h(\bar{x})$, it will be useful to proceed via an intermediate step.

Lemma 1.

$$r > -\gamma\mu. \tag{59}$$

Proof. This follows from the quadratic characteristic equation that $-\gamma$ is supposed to solve:

$$-\mu\gamma - r + \frac{1}{2}\sigma^2\gamma(\gamma+1) = 0 \tag{60}$$

Since $\sigma^2, \gamma > 0$, we have that $r - \frac{1}{2}\sigma^2\gamma(\gamma+1) < r$ and hence $r > -\mu\gamma$. □

Note that we can rearrange $r > -\gamma\mu$ (which is the case according to Lemma 1) to produce $Z(c) > 0$. It follows from $\delta > 1$ and the fact that $Z(c) > 0$ that for $\bar{x} > 0$, $h(\bar{x}) > 0, h'(\bar{x}) > 0$, and $h''(\bar{x}) > 0$, and hence that $f^D(\bar{x})$ is convex for $0 \leq \bar{x} < c$.

To summarize, the function $f^D(\bar{x})$ is continuous (also at c), it is positive at 0, convex and strictly decreasing on $[0, c)$, and flat and non-positive on $[c, \infty)$.

Although there is no closed form for the solution to $f^D(\bar{x}) = 0$, we can determine the existence of a solution and characterize it to some extent. If $(1-s)K < \frac{c}{r}$, $f^D(\bar{x})$ is positive at 0 and monotonic decreasing until it reaches a minimum at c below 0, implying that there exists an $0 < \bar{x}_D < c$, and that the value of debt must be maximized at this point. If $(1-s)K \geq \frac{c}{r}$, then $f^D(\bar{x})$ reaches its minimum of 0 at $\bar{x} = c$ and remains at this level for higher values of $\bar{x}$. This means that in this case, all values of $\bar{x} \in [c, \infty)$ maximize the value of debt. To reduce notational clutter, define

$$\bar{x}_D = \underset{\bar{x} \text{ s.t. } f^D(\bar{x})=0}{\arg\min} \|\bar{x}_V - \bar{x}\|, \tag{61}$$

i.e. in the case where there are many points $\bar{x}$ that satisfy $f^D(\bar{x}) = 0$, let $\bar{x}_D$ denote the point in that set closest to $\bar{x}_V$. For example, if $\bar{x}_V < c$ and the set of points satisfying $f^D(\bar{x}) = 0$ is $[c, \infty)$, we define $\bar{x}_D = c$. If $\bar{x}_V \geq c$, and the set of points satisfying $f^D(\bar{x}) = 0$ is $[c, \infty)$, we define $\bar{x}_D = \bar{x}_V$.

B.2 The equity-value maximizing liquidation boundary

Consider the case $\bar{x} < c$ initially. Note that that E_1 only depends on $\bar{x}$ through E_2, and that this dependence is positive. Maximizing E_2 therefore maximizes E. E_2 is maximized when the term in A_2^E is maximized. Note that

$$\frac{\partial(A_2^E x^{-\gamma})}{\partial \bar{x}} = \bar{x}^{\gamma-1} \left(\gamma R^E - (\delta + \gamma) \left(\frac{\bar{x}}{c}\right)^{\delta} (1-\tau) Z(c) \right) x^{-\gamma}. \tag{62}$$

We can then define

$$f^E(\bar{x}) = \gamma R^E - (1-\tau)(\delta+\gamma) \left(\frac{\bar{x}}{c}\right)^{\delta} Z(c) \qquad \text{if } \bar{x} < c. \tag{63}$$

Given the definition of $h(\bar{x})$, we could also write

$$f^E(\bar{x}) = \gamma R^E - (1-\tau) h(\bar{x}) \qquad \text{if } \bar{x} < c. \tag{64}$$

We note that due to the properties of $h(\bar{x})$, this function is concave and decreasing on $(0, c]$.

Again, for $\bar{x} \geq c$ the pricing formula is simplified as above. Taking derivatives in this case w.r.t. $\bar{x}$, we obtain

$$\bar{x}^{\gamma-1} \left\{ \gamma R^E - (1-\tau) \left((1+\gamma) \frac{\bar{x}}{r-\mu} - \gamma \frac{c}{r} \right) \right\} x^{-\gamma}, \tag{65}$$

suggesting that we can extend the definition of $f^E(\bar{x})$ as follows:

$$f^E(\bar{x}) = \begin{cases} \gamma R^E - (1-\tau)(\delta+\gamma) \left(\frac{\bar{x}}{c}\right)^{\delta} Z(c) & \text{if } \bar{x} < c \\ \gamma R^E - (1-\tau) \left((1+\gamma) \frac{\bar{x}}{r-\mu} - \gamma \frac{c}{r} \right) & \text{if } \bar{x} \geq c. \end{cases} \tag{66}$$

The equity-value maximizing liquidation boundary is defined by $f^E(\bar{x}_E) \equiv 0$. Note that $f^E(\bar{x})$ is continuous (also at c), it is non-negative at 0, concave and strictly decreasing on $(0, c]$, and linear and strictly decreasing on (c, ∞).

A closed form for the solution to $f^E(\bar{x}) = 0$ exists. It is given by

$$\bar{x}_E = \begin{cases} \left(\frac{\gamma}{\delta+\gamma} \frac{R^E}{(1-\tau)Z(c)}\right)^{1/\delta} c & \text{if } \bar{x}_E < c \\ \frac{(r-\mu)}{1-\tau} \frac{\gamma}{1+\gamma} \left((1-s)K - \tau\frac{c}{r}\right) & \text{if } \bar{x}_E \geq c \end{cases} \tag{67}$$

as can be easily seen by solving $f^E(\bar{x}_E) \equiv 0$ for the two regions.

B.3 The firm-value maximizing liquidation boundary

Consider the case $\bar{x} < c$ initially. Note that V_1 only depends on $\bar{x}$ via V_2, and that this dependence is positive. Hence maximizing V w.r.t. $\bar{x}$ is equivalent to maximizing V_2. Taking derivatives of V_2 w.r.t. $\bar{x}$ in this region, we obtain

$$\bar{x}^{\gamma-1} \left\{ \gamma(1-s)K - (1+\gamma)\frac{\bar{x}}{r-\mu} + \tau(\delta+\gamma)\left(\frac{\bar{x}}{c}\right)^{\delta} Z(c) \right\} x^{-\gamma}. \tag{68}$$

We can then define

$$f^V(\bar{x}) = \gamma(1-s)K - (1+\gamma)\frac{\bar{x}}{r-\mu} + \tau(\delta+\gamma)\left(\frac{\bar{x}}{c}\right)^{\delta} Z(c) \qquad \text{if } \bar{x} < c. \tag{69}$$

Given the definition of $h(\bar{x})$, we could also write

$$f^V(\bar{x}) = \gamma(1-s)K - (1+\gamma)\frac{\bar{x}}{r-\mu} + \tau h(\bar{x}) \qquad \text{if } \bar{x} < c. \tag{70}$$

Note that we are adding a convex function ($\tau h(\bar{x})$) to a linear function, such that $f^V(\bar{x})$ will be convex on $[0, c]$.

Again, for the case that $\bar{x} \geq c$, the pricing formula is simplified as above. Taking derivatives in this case w.r.t. $\bar{x}$, we obtain

$$\bar{x}^{\gamma-1} \left\{ \gamma(1-s)K - \left((1-\tau)(1+\gamma)\frac{\bar{x}}{r-\mu} + \tau\gamma\frac{c}{r}\right) \right\} x^{-\gamma}, \tag{71}$$

suggesting that we can extend the definition of $f^V(\bar{x})$ as follows:

$$f^V(\bar{x}) = \begin{cases} \gamma(1-s)K - (1+\gamma)\frac{\bar{x}}{r-\mu} + \tau(\delta+\gamma)\left(\frac{\bar{x}}{c}\right)^{\delta} Z(c) & \text{if } \bar{x} < c \\ \gamma(1-s)K - \left((1-\tau)(1+\gamma)\frac{\bar{x}}{r-\mu} + \tau\gamma\frac{c}{r}\right) & \text{if } \bar{x} \geq c. \end{cases} \tag{72}$$

The firm-value maximizing liquidation point is defined by $f^V(\bar{x}_V) \equiv 0$. Note that $f^V(\bar{x})$ is continuous (also at c). It is positive at 0. Also, it can be seen that $f^D(\bar{x}) + f^E(\bar{x}) = f^V(\bar{x}$. Since $f^D(\bar{x})$ is decreasing on $[0, c)$ and flat on $[c, \infty)$, and $f^E(\bar{x})$ is decreasing on $(0, c]$

and decreasing on (c, ∞), $f^V(\bar{x})$ is strictly decreasing. It is convex on $[0, c]$, and linear on (c, ∞).

The fact that $f^V(\bar{x})$ is positive at 0, and linear and decreasing for high $\bar{x}$ implies that a solution to $f^V(\bar{x}) = 0$ will always exist. The fact that it is strictly decreasing throughout implies that a single solution will exist. There is no closed form for $\bar{x}_V$ when $\bar{x}_V < c$. For $\bar{x}_V > c$, we can see that

$$\bar{x}_V = \frac{r - \mu}{1 - \tau} \frac{\gamma}{1 + \gamma} \left((1 - s)K - \tau \frac{c}{r} \right), \tag{73}$$

which coincides with $\bar{x}_E$ if $\bar{x}_E > c$. We can interpret this equation as saying that the firm should be liquidated when the post-tax going concern value $(1 - \tau)\frac{x}{r-\mu}$ reaches a fraction $\frac{\gamma}{1+\gamma} < 1$ of the liquidation value, net of the tax shield.

If $\tau = 0$, a closed form for $\bar{x}_V$ exists regardless of whether $\bar{x}_V$ is larger or smaller than c:

$$\bar{x}_V = (r - \mu) \frac{\gamma}{1 + \gamma} (1 - s)K. \tag{74}$$

B.4 Proof of Propositions A1 and A2

The proofs of Propositions A1 and A2 are based on some properties of the functions $f^D(\bar{x}), f^E(\bar{x})$, and $f^V(\bar{x})$, which I summarize here for convenience.

1. $f^D(\bar{x})$ is positive at 0, strictly decreasing on the interval $[0, c)$, and constant on $[c, \infty)$. This constant level is either negative iff $(1-s)K < \frac{c}{r}$, or zero iff $(1-s)K \geq \frac{c}{r}$.
2. $f^E(\bar{x})$ is positive at 0 iff $R^E > 0$, and 0 at this point otherwise. It is also strictly decreasing and convex on $(0, \infty)$.
3. $f^V(\bar{x})$ is positive at 0 and strictly decreasing, and

$$f^V(\bar{x}) = f^E(\bar{x}) + f^D(\bar{x}).$$

Proof of Proposition 2 If $(1 - s)K < \frac{c}{r}$, $R^E = 0$ and $R^D = (1 - s)K < \frac{c}{r}$. By (2), $f^E(\bar{x}) < 0$ for all $\bar{x} > 0$. Also, by (1), for $\bar{x} \geq c$, $f^D(\bar{x}) \leq 0$. This implies by (3) that for $\bar{x} \geq c$, $f^V(\bar{x}) < 0$. Since by (3), $f^V(0) = 0$, this implies that $\bar{x}_V < c$.

Proof of Proposition 1 I proceed by proving the following sub-statements in turn:

1. $\bar{x}_V < c \implies \bar{x}_E < \bar{x}_V < \bar{x}_D \leq c$
2. $\bar{x}_E < \bar{x}_V < \bar{x}_D \leq c \implies \bar{x}_V < c$
3. $\bar{x}_V \geq c \implies \bar{x}_V = \bar{x}_E (= \bar{x}_D)$
4. $\bar{x}_V = \bar{x}_E (= \bar{x}_D) \implies \bar{x}_V > c.$

Proof of sub-statement 1 ($\bar{x}_V < c \implies \bar{x}_E < \bar{x}_V < \bar{x}_D \leq c$)
If $(1-s)K < \frac{c}{r}$, $R^E = 0$. In this case, $f^E(\bar{x}) < 0$ for all $\bar{x} > 0$, and zero only at $\bar{x} = 0$ by (2). This implies that $\bar{x}_E = 0$. Also, $f^D(\bar{x})$ achieves a minimum for all $\bar{x} \geq c$ of $f^D(c) < 0$ by (1). Since $f^D(0) > 0$, we have that $\bar{x}_D < c$. Also, for $\bar{x} > 0$, $f^E(\bar{x}) < 0$, and hence by (3), $f^E(\bar{x}) < f^V(\bar{x}) < f^D(\bar{x})$. This in turn implies that $\bar{x}_E < \bar{x}_V < \bar{x}_D < c$.

If $(1-s)K = \frac{c}{r}$, $R^E = 0$. In this case, $f^E(\bar{x}) < 0$ for $\bar{x} > 0$, and zero only at $\bar{x} = 0$ by (2). This implies that $\bar{x}_E = 0$. Also, $f^D(\bar{x})$ achieves a minimum for all $\bar{x} \geq c$ of $f^D(c) = 0$ by (1). Since we define $\bar{x}_D$ to be the point that solves $f^D(\bar{x}) = 0$ closest to $\bar{x}_V$ (see [61]), and $\bar{x}_V < c$, we find that $\bar{x}_D = c$. We have that $\bar{x}_V < \bar{x}_D = c$. Also, for $\bar{x} > 0$, $f^E(\bar{x}) < 0$, and hence by (3) , $f^E(\bar{x}) < f^V(\bar{x})$. This in turn implies that $\bar{x}_E < \bar{x}_V$. Taken together, we have $\bar{x}_E < \bar{x}_V < \bar{x}_D = c$.

If $(1-s)K > \frac{c}{r}$, $R^E > 0$. This implies that $f^E(0) > 0$, and hence $\bar{x}_E > 0$ by (2). Also, $R^D = \frac{c}{r}$. This implies that $f^D(\bar{x})$ achieves its minimum of 0 at any $\bar{x} \geq c$ by (1). Since we define $\bar{x}_D$ to be the point that solves $f^D(\bar{x}) = 0$ closest to $\bar{x}_V$ (see [61]), and $\bar{x}_V < c$, we find that $\bar{x}_D = c$. Also, for $\bar{x} < c$, $f^D(\bar{x}) > 0$ and hence by (3), $f^E(\bar{x}) < f^V(\bar{x})$. This in turn implies that $\bar{x}_E < \bar{x}_V$. Taken together, we have $\bar{x}_E < \bar{x}_V < \bar{x}_D = c$.

Proof of sub-statement 2 ($\bar{x}_E < \bar{x}_V < \bar{x}_D \leq c \implies \bar{x}_V < c$)
If $\bar{x}_V < \bar{x}_D \leq c$, then obviously $\bar{x}_V < c$.

Proof of sub-statement 3 ($\bar{x}_V \geq c \implies \bar{x}_V = \bar{x}_E (= \bar{x}_D)$)
By Proposition A2, we know that if $(1-s)K < \frac{c}{r}$, then $\bar{x}_V < c$, implying that if $\bar{x}_V \geq c$, then $(1-s)K \geq \frac{c}{r}$. So here, we must have $(1-s)K \geq \frac{c}{r}$. This implies that $R^E > 0$, and $R^D = \frac{c}{r}$. This implies that $f^D(\bar{x}) = 0$ for all $\bar{x} \geq c$ by (1). This in turn implies that $f^V(\bar{x}) = f^E(\bar{x})$ for all $\bar{x} \geq c$ by (3). Since $\bar{x}_V > c$, this implies that $\bar{x}_V = \bar{x}_E$. Since we define $\bar{x}_D$ to be the point that solves $f^D(\bar{x}) = 0$ closest to $\bar{x}_V$, and in fact $f^D(\bar{x}) = 0$ for any $\bar{x}_D > c$, we see that $\bar{x}_D = \bar{x}_V$. Taken together, we have that $\bar{x}_V = \bar{x}_E (= \bar{x}_D)$.
Proof of sub-statement 4 ($\bar{x}_V = \bar{x}_E (= \bar{x}_D) \implies \bar{x}_V \geq c$)
By (3), $f^E(\bar{x})$ only coincides with $f^V(\bar{x})$ (such that $\bar{x}_E$ and $\bar{x}_V$ can coincide) when $f^D(\bar{x}) = 0$. By (1), $f^D(\bar{x}) = 0$ for a $\bar{x} < c$ only if $R^D = (1-s)K < \frac{c}{r}$. But this already implies that $R^E = 0$, which by (2) implies that $\bar{x}_E = 0$, so the points cannot coincide. This implies that we need to look for liquidation points $\bar{x} \geq c$ such that $f^D(\bar{x}) = 0$. (By (1), this only happens when $(1-s)K \geq \frac{c}{r}$.) This implies that we need $\bar{x}_V \geq c$.

C Solution of the asset grab game

C.1 Basic procedure

The solution procedure for a single stage of the discrete-time repeated game is the same as that of Morris and Shin (2004). Suppose that creditors follow a switching strategy around a certain posterior belief. Given the posterior belief around which creditors switch, it is possible to derive the critical next-period cash flow for which the firm will be liquidated. Given a critical cash flow for which the firm will be liquidated, it is possible to derive the belief around which creditors switch. This produces two equations in two unknowns, which can then be solved for the critical cash flow for which the firm is liquidated.

C.2 Posteriors

Given the assumptions on the information structure in the main text, posteriors can be worked out as follows. From the signal ξ_i and the public information x_t, creditors form a posterior about the cash flow in period $t+\Delta$, $x_{t+\Delta}$ which is normal with mean and variance given by

$$\rho_i = E[x_{t+\Delta}|\xi_i] = \frac{\alpha(1+(\mu+\sigma\nu_t)\Delta)x_t + \beta\xi_i}{\alpha+\beta}, \text{ and } \mathrm{Var}(x_{t+\Delta}|\xi_i) = \frac{(x_t)^2}{\alpha+\beta}. \tag{75}$$

C.3 Critical value of $x_{t+\Delta}$ for which the firm is liquidated

Suppose creditors follow a switching strategy around ρ^*, i.e. creditors grab assets when their posterior mean is below ρ^*. Then an creditor will not grab assets if and only if the private signal is bigger than

$$\xi^* = \frac{\alpha+\beta}{\beta}\rho^* - \frac{\alpha}{\beta}(1+(\mu+\sigma\nu_t)\Delta)x_t. \tag{76}$$

Conditional on state $x_{t+\Delta}$, the distribution of ξ_i is normal with mean $x_{t+\Delta}$ and precision $\frac{\beta}{x_t^2}$. So the ex-ante probability for any creditor of grabbing assets is equal to $\Phi\left\{\frac{1}{x_t}\sqrt{\beta}(\xi^* - x_{t+\Delta})\right\}$, where $\Phi(\cdot)$ is the cumulative standard normal density function. The fraction of creditors that grab assets will be equal to this ex ante probability for any individual creditor by some version of the law of large numbers.

Since the firm fails if the fraction that grabs assets is $l \geq \frac{x_{t+\Delta}}{c}$, the critical value of $x_{t+\Delta}$ (denoted by $\bar{x}^{AG}_{t+\Delta}$) for which the firm fails at t is given by $\bar{x}^{AG}_{t+\Delta} = c\Phi\left\{\frac{1}{x_t}\sqrt{\beta}\left(\xi^* - \bar{x}^{AG}_{t+\Delta}\right)\right\}$ or

$$\bar{x}^{AG}_{t+\Delta} = c\Phi\left\{\frac{1}{x_t}\left(\frac{\alpha}{\sqrt{\beta}}\left(\rho^* - (1+(\mu+\sigma\nu_t)\Delta)x_t\right) + \sqrt{\beta}\left(\rho^* - \bar{x}^{AG}_{t+\Delta}\right)\right)\right\}. \tag{77}$$

C.4 Critical value of ρ around which creditors switch

Payoffs in any intermediate period $t+q$ are as described in the payoff matrix in the main text.

Creditors will switch if they believe that they will obtain a higher utility from doing so. We find the critical ρ^* around which creditors switch by considering the marginal creditor, for whom the expected utility of not grabbing assets should just equal the expected utility of grabbing assets.

Let F denote the posterior cumulative distribution (given the belief) over the cash flow $x_{t+\Delta}$, let $u(\cdot)$ denote the (Bernoulli) utility function that maps instantaneous payoffs into instantaneous utility, let λ be the subjective discount factor, and let U_L and U_{NL} denote the future utility associated with the firm being liquidated and not liquidated in the current period respectively. Then for the marginal creditor,

$$\begin{aligned} &\int_{-\infty}^{\bar{x}_{t+\Delta}^{AG}} u((1-s)K)dF + \lambda \int_{-\infty}^{\bar{x}_{t+\Delta}^{AG}} U_L dF \quad + \quad \int_{\bar{x}_{t+\Delta}^{AG}}^{\infty} u(-sK)dF + \lambda \int_{\bar{x}_{t+\Delta}^{AG}}^{\infty} U_{NL} dF \\ = \quad &\int_{-\infty}^{\bar{x}_{t+\Delta}^{AG}} u(0)dF + \lambda \int_{-\infty}^{\bar{x}_{t+\Delta}^{AG}} U_L dF \quad + \quad \int_{\bar{x}_{t+\Delta}^{AG}}^{\infty} u(0)dF + \lambda \int_{\bar{x}_{t+\Delta}^{AG}}^{\infty} U_{NL} dF. \end{aligned}$$

The terms in U_L and U_{NL} cancel, since future payoffs do not depend on the actions of an individual creditor. Also, since the Bernoulli utilities do not depend on x directly, we can move them out of the integrals to obtain

$$\begin{aligned} &u((1-s)K)\Pr\left(x_{t+\Delta} \le \bar{x}_{t+\Delta}^{AG}\right) + u(-sK)\Pr\left(x_{t+\Delta} > \bar{x}_{t+\Delta}^{AG}\right) \\ &\qquad\qquad = u(0)\Pr\left(x_{t+\Delta} \le \bar{x}_{t+\Delta}^{AG}\right) + u(0)\Pr\left(x_{t+\Delta} > \bar{x}_{t+\Delta}^{AG}\right). \end{aligned}$$

Noting that $\Pr\left(x_{t+\Delta} \le \bar{x}_{t+\Delta}^{AG}\right) = 1 - \Pr\left(x_{t+\Delta} > \bar{x}_{t+\Delta}^{AG}\right)$, we can rewrite this as

$$\Pr\left(x_{t+\Delta} > \bar{x}_{t+\Delta}^{AG}\right) = \frac{u((1-s)K) - u(0)}{u((1-s)K) - u(-sK)} \equiv \theta, \tag{78}$$

where θ is defined by this expression as in the main text. Note that $0 \le \theta \le (1-s)K$ where the upper limit is attained when creditors are risk neutral. Also note that $\partial\theta/\partial s < 0$ for standard (concave and increasing) utility functions.

For the marginal creditor, the probability $\Pr(x_{t+\Delta} > \bar{x}_{t+\Delta}^{AG})$ is given by

$$\Pr\left(x_{t+\Delta} > \bar{x}_{t+\Delta}^{AG}\right) = \Phi\left\{\frac{\sqrt{\alpha+\beta}}{x_t}\left(\rho^* - \bar{x}_{t+\Delta}^{AG}\right)\right\} \tag{79}$$

where Φ denotes the cumulative normal density function.

We can equate [79] and [78] to obtain

$$\bar{x}_{t+\Delta}^{AG} - \rho^* = \frac{x_t}{\sqrt{\alpha+\beta}}\Phi^{-1}(\theta). \tag{80}$$

C.5 Equilibrium forced reorganisation

Combining equations (80) and (77) we can solve for critical level of the cash flow in the next period causes failure in preceding intermediate period:

$$\bar{x}^{AG}_{t+\Delta} = c\Phi\left\{\frac{\alpha}{\sqrt{\beta}}\left(\frac{\bar{x}^{AG}_{t+\Delta}}{x_t} - 1 - (\mu + \sigma\nu_t)\Delta\right) + \frac{\sqrt{\alpha+\beta}}{\sqrt{\beta}}\Phi^{-1}(\theta)\right\} \tag{81}$$

Reorganisation at time $t+q$ will occur when x hits $\bar{x}^{AG}_{t+\Delta}$ at $t+\Delta$.

C.6 Uniqueness

Proposition A3. *The critical value of the cash flow* $\bar{x}^{AG}_{t+\Delta}$ *is unique if*

$$c\frac{1}{\sqrt{2\pi}}\frac{\alpha}{\sqrt{\beta}}\frac{1}{x_t} < 1. \tag{82}$$

Proof. This is a version of the proof in Morris and Shin (2004). A sufficient condition for a unique solution is that the slope of the RHS of [81], seen as a function of $\bar{x}^{AG}_{t+\Delta}$, is less than one everywhere. This slope is equal to

$$c\varphi\left\{\frac{\alpha}{\sqrt{\beta}}\left(\frac{\bar{x}^{AG}_{t+\Delta}}{x_t} - 1 - (\mu + \sigma\nu_t)\Delta\right) + \frac{\sqrt{\alpha+\beta}}{\sqrt{\beta}}\Phi^{-1}(\theta)\right\}\frac{\alpha}{\sqrt{\beta}}\frac{1}{x_t}. \tag{83}$$

The standard normal density reaches a maximum of $\frac{1}{\sqrt{2\pi}}$ at 0, hence a sufficient condition for a unique solution is that $c\frac{1}{\sqrt{2\pi}}\frac{\alpha}{\sqrt{\beta}}\frac{1}{x_t} < 1$ as stated above. □

C.7 Uncertainty in the limit

In the continuous-time limit, the marginal creditor views the fraction of creditors that attempt to grab assets as a random variable that is uniformly distributed, i.e. the marginal creditor is completely uncertain about this fraction. It is in this sense that strategic uncertainty remains present in the limit.[32]

Proposition A4. *The distribution of the fraction that attempt to grab assets* l *given the belief* ρ^* *of the marginal creditor is uniform in the limit.*

Proof. The proportion of creditors who receive a signal lower than ξ^* is

$$l = \Phi\left\{\frac{\sqrt{\beta}}{x_t}\left(\xi^* - x_{t+\Delta}\right)\right\}. \tag{84}$$

[32] This is a special case of a general result in coordination failure games in which the precision of private information goes to infinity. The general result is discussed in more detail e.g. in Morris and Shin (2003).

What is the probability that a fraction less than y of the other bondholders receive a signal higher than that of the marginal creditor, conditional on the marginal creditor's belief, or what is $\Pr((1-l) < y \mid \rho^*)$?

The event $1 - l < y$ is equivalent to the event $1 - \Phi\left\{\frac{\sqrt{\beta}}{x_t}(\xi^* - x_{t+\Delta})\right\} < y$ or (rearranging) $x_{t+\Delta} < \xi^* + \frac{x_t}{\sqrt{\beta}}\Phi^{-1}\{y\}$. So the probability in question can be expressed as

$$\Pr\left(x_{t+\Delta} < \xi^* + \frac{x_t}{\sqrt{\beta}}\Phi^{-1}\{1-y\} \middle| \rho^*\right). \tag{85}$$

The posterior of the marginal creditor over $x_{t+\Delta}$ has mean ρ^* and variance $\frac{x_t^2}{\alpha+\beta}$, hence this probability is

$$\Pr((1-l) < y \mid \rho^*) = \Phi\left\{\frac{\sqrt{\alpha+\beta}}{x_t}\left(\xi^* + \frac{x_t}{\sqrt{\beta}}\Phi^{-1}\{y\} - \rho^*\right)\right\}. \tag{86}$$

Now as we take limits, $\rho^* \to \xi^*$, since private information becomes infinitely more precise than public information (the creditor attaches all weight to the signal and none to the mean of the prior), and $\frac{\sqrt{\alpha+\beta}}{\sqrt{\beta}} \to 1$. It follows that

$$\Pr((1-l) < y \mid \rho^*) = y, \tag{87}$$

so the cumulative distribution of $1 - l$ is the identity function, which implies that the density of $1 - l$, and hence also l, will be uniform. $\square$

D Liquidation boundaries as a function of c

The firm-value maximizing liquidation boundary I write $\bar{x}_V(c)$ to emphasize that $\bar{x}_V$ can be viewed as a function of c. $\bar{x}_V(c)$ is defined via $f^V(\bar{x}_V(c)) \equiv 0$, where $f^V(\bar{x}; c)$ is given by [72]. Note that in [72], the function is defined separately for $\bar{x} < c$ and $\bar{x} \geq c$. To avoid circularity, we can express the inequality $\bar{x}_V(c) < c$ in terms of a critical level of c, $\tilde{c}$, which is defined via $\bar{x}_V(\tilde{c}) \equiv \tilde{c}$.[33] The inequality $\bar{x}_V(c) < c$ is then equivalent to $c > \tilde{c}$. [72] can be written as

$$f^V(\bar{x}) = \begin{cases} \gamma(1-s)K - \left((1-\tau)(1+\gamma)\frac{\bar{x}}{r-\mu} + \tau\gamma\frac{c}{r}\right) & \text{if } c \leq \tilde{c} \\ \gamma(1-s)K - (1+\gamma)\frac{\bar{x}}{r-\mu} + \tau(\delta+\gamma)\left(\frac{\bar{x}}{c}\right)^{\delta} Z(c) & \text{if } c > \tilde{c}. \end{cases} \tag{88}$$

When $c \leq \tilde{c}$, the amount of debt is "sufficiently low" in relation to collateral for interests to be aligned and for $\bar{x}_V = \bar{x}_D = \bar{x}_E$ in the sense of Proposition A1.

[33]This can be solved in closed form to obtain

$$\tilde{c} = \frac{\gamma r(r-\mu)}{r(1-\tau)(1+\gamma) + \tau\gamma(r-\mu)}(1-s)K.$$

We can now work out the properties of the function $\bar{x}_V(c)$. Note that setting $c = 0$ necessarily implies that $c \leq \tilde{c}$, which implies that

$$f^V(\bar{x}; c) = \gamma(1-s)K - (1-\tau)(1+\gamma)\frac{\bar{x}}{r-\mu} \tag{89}$$

We can solve to obtain

$$\bar{x}_V(0) = \frac{r-\mu}{1-\tau}\frac{\gamma}{1+\gamma}(1-s)K > 0.^{34} \tag{90}$$

Letting $c \to \infty$ necessarily implies that $c > \tilde{c}$. Noting that $\lim_{c\to\infty}\left(\frac{\bar{x}}{c}\right)^\delta Z(c) = 0$ since $\delta > 1$, we see that

$$\lim_{c\to\infty} f^V(\bar{x}; c) = \gamma(1-s)K - (1+\gamma)\frac{\bar{x}}{r-\mu} \tag{91}$$

Solving explicitly, we obtain that

$$\lim_{c\to\infty} \bar{x}_V(c) = (r-\mu)\frac{\gamma}{1+\gamma}(1-s)K.^{35} \tag{92}$$

The derivative $\frac{\partial \bar{x}_V(c)}{\partial c}$ could be worked out from $f^V(\bar{x}; c)$ via the implicit function rule. Here, I just work out the sign of this derivative. Note that due to the properties of $f^V(\bar{x}; c)$ discussed in Appendix B, $\frac{\partial f^V(\bar{x};c)}{\partial \bar{x}} < 0$ everywhere. This implies that $\text{sign}\left(\frac{\partial \bar{x}_V(c)}{\partial c}\right) = \text{sign}\left(\frac{\partial f^V(\bar{x};c)}{\partial c}\right)$. Using $\delta > 1$, it can be verified that $\frac{\partial f^V(\bar{x};c)}{\partial c} < 0$ in both regions, such that $\frac{\partial \bar{x}_V(c)}{\partial c} < 0$ everywhere.

To summarize, $\bar{x}_V(0)$ is positive at $\frac{r-\mu}{1-\tau}\frac{\gamma}{1+\gamma}(1-s)K$, and strictly decreases to $(r-\mu)\frac{\gamma}{1+\gamma}(1-s)K$ as $c \to \infty$.

The debt-value maximizing liquidation boundary Note that for $\bar{x}_V \geq c$ or $c \leq \tilde{c}$, $\bar{x}_D(c) = \bar{x}_V(c)$ since interests are aligned. For $c > \tilde{c}$, as long as $(1-s)K \geq \frac{c}{r}$, $\bar{x}_D(c) = c$. This suggests defining another critical level $\hat{c} = r(1-s)K$. By Proposition A2, $(1-s)K = \frac{c}{r}$ or $c = \hat{c}$ implies that $\bar{x}_V(\hat{c}) < \hat{c}$, and hence $\hat{c} > \tilde{c}$. We can therefore say that for $\tilde{c} < c \leq \hat{c}$, we have $\bar{x}_D(c) = c$.

For $c > \hat{c}$, we have $(1-s)K < \frac{c}{r}$, which implies that $\bar{x}_D(c) < c$. In this region, $\frac{\partial f^D(\bar{x};c)}{\bar{x}} < 0$. By the implicit function rule, the sign of the derivative $\frac{\partial \bar{x}_D(c)}{\partial c}$ is therefore equal to the sign of the derivative $\frac{\partial f^D(\bar{x};c)}{\partial c}$. It can be shown that since $\delta > 1$, this derivative is negative. We also see that

$$\lim_{c\to\infty} f^D(\bar{x}; c) = \gamma(1-s)K - (1+\gamma)\frac{\bar{x}}{r-\mu} \tag{93}$$

[34] This could also be obtained by solving the optimal stopping problem for an all equity firm which has dividends taxed at rate τ.

[35] This could also be obtained by solving the optimal stopping problem for an all equity firm which does not have dividends taxed. This is because if $c \to \infty$, all cash flows go to creditors, and (partial) coupon payments to debt holders are not taxed in the model.

and hence

$$\lim_{c\to\infty} \bar{x}_D(c) = (r-\mu)\frac{\gamma}{1+\gamma}(1-s)K, \tag{94}$$

i.e. the liquidation point preferred by creditors will equal the socially optimal point. Obviously, if the entire firm value consists of the value of debt, maximizing the value of debt is synonymous with maximizing the value of the firm.

The liquidation boundaries in the case of non-coordinated creditors We have that $\bar{x}_{AG}(c) = \theta c$, and $\bar{x}_{BC}(c) = \zeta c$, where neither θ or ζ are functions of c. Both $\bar{x}_{AG}(c)$ and $\bar{x}_{BC}(c)$ are simple linear increasing functions of c. Since the game is set up only for situations in which debt is not fully collateralized, i.e. $(1-s)K < \frac{c}{r}$, these functions are only defined for $c > \hat{c}$.

References

Anderson, R., and S. Sundaresan, 1996, "Design and Valuation of Debt Contracts," *Review of Financial Studies*, 9, 37–68.

Bolton, P., and D. S. Scharfstein, 1996, "Optimal Debt Structure and the Number of Creditors," *Journal of Political Economy*, 104, 1–25.

Bris, A., and I. Welch, 2005, "The Optimal Concentration of Creditors," *Journal of Finance*, 60, 2193–2212.

Bris, A., I. Welch, and N. Zhu, 2006, "The Costs of Bankruptcy: Chapter 7 Liquidation versus Chapter 11 Reorganization," *Journal of Finance*, 61, 1253–1303.

Broadie, M., M. Chernov, and S. Sundaresan, 2007, "Optimal Debt and Equity Values in the Presence of Chapter 7 and Chapter 11," *Journal of Finance*, 62, 1341–1377.

Dixit, A. K., 1993, "The Art of Smooth Pasting," , vol. 55 of *Fundamentals of Pure and Applied Economics*, Harwood Academic Publishers, Reading, UK.

François, P., and E. Morellec, 2004, "Capital Structure and Asset Prices: Some Effects of Bankruptcy Procedures," *Journal of Business*, 77, 387–411.

Franks, J. R., and W. N. Torous, 1989, "An Empirical Investigation of U.S. Firms in Reorganization," *Journal of Finance*, 44, 747–769.

Gertner, R., and D. S. Scharfstein, 1991, "A Theory of Workouts and the Effects of Reorganization Law," *Journal of Finance*, 46, 1189–1222.

Goldstein, I., and A. Pauzner, 2005, "Demand-Deposit Contracts and the Probability of Bank Runs," *Journal of Finance*, 60, 1293–1327.

Hackbarth, D., C. A. Hennesy, and H. E. Leland, 2007, "Can the Trade-off Theory Explain Debt Structure?," *Review of Financial Studies*, 20, 1389–1428.

Hart, O. D., 1995, *Firms, Contracts, and Financial Structure* . Clarendon Lectures in Economics, Oxford University Press, Oxford, UK.

Haugen, R. A., and L. W. Senbet, 1978, "The Insignificance of Bankruptcy Costs to the Theory of Optimal Capital Structure," *Journal of Finance*, 33, 383–393.

Hege, U., and P. Mella-Barral, 2005, "Repeated Dilution of Diffusely Held Debt," *Journal of Business*, 78, 737–786.

Kemsley, D., and D. Nissim, 2002, "Valuation of the Debt Tax Shield," *Journal of Finance*, 57, 2045–2073.

Kloeden, P. E., and E. Platen, 2000, *Numerical Solution of Stochastic Differential Equations*, Springer-Verlag, Heidelberg, Germany.

Leland, H. E., 1994, "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure," *Journal of Finance*, 49, 1213–1252.

LoPucki, L. M., 1983, "The Debtor in Full Control - Systems Failure Under Chapter 11 of the Bankruptcy Code?," *American Bankruptcy Law Journal*, 57, 99–126 and 247–273.

Mella-Barral, P., 1999, "The Dynamics of Default and Debt Reorganization," *Review of Financial Studies*, 12, 535–578.

Mella-Barral, P., and W. Perraudin, 1997, "Strategic Debt Service," *Journal of Finance*, 52, 531–556.

Miller, M. H., 1977, "Debt and Taxes," *Journal of Finance*, 32, 261–275.

Morris, S., and H. S. Shin, 1998, "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks," *American Economic Review*, 88, 587–597.

Morris, S., and H. S. Shin, 2003, "Global Games: Theory and Applications," in Mathias Dewatripont, Lars Peter Hansen, and Stephen J. Turnovsky (ed.), *Advances in Economics and Econometrics*, Cambridge University Press.

Morris, S., and H. S. Shin, 2004, "Co-ordination Risk and the Price of of Debt," *European Economic Review*, 48, 113–153.

Naqvi, H. M., 2008, "The Valuation of Corporate Debt with Default Risk," RMI Working Paper No 08/03.

Titman, S., 1984, "The Effect of Capital Structure on a Firm's Liquidation Decision," *Journal of Financial Economics*, 13, 137–151.

Varma, P., and R. Cantor, 2005, "Determinants of Recovery Rates on Defaulted Bonds and Loans for North American Corporate Issuers: 1983-2003," *Journal of Fixed Income*, 14, 29–44.

Woolridge, F. H., 1987, *Administration Procedure*, Jordans, Bristol, UK.

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