THE PROCYCLICAL EFFECTS OF BANK CAPITAL REGULATION

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Abstract

We develop and calibrate a dynamic equilibrium model of relationship lending in which banks are unable to access the equity markets every period and the business cycle is a Markov process that determines loans’ probabilities of default. Banks anticipate that shocks to their earnings and the possible variation of capital requirements over the cycle can impair their future lending capacity and, as a precaution, hold capital buffers. We compare the relative performance of several capital regulation regimes, including one that maximizes a measure of social welfare. We show that Basel II is significantly more procyclical than Basel I, but makes banks safer. For this reason, it dominates Basel I in terms of welfare except for small social costs of bank failure. We also show that for high values of this cost, Basel III points in the right direction, with higher but less cyclically-varying capital requirements.

JEL Codes: G21, G28, E44.
Keywords: Banking regulation, Basel capital requirements, Capital market frictions, Credit rationing, Loan defaults, Relationship banking, Social cost of bank failure.

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1 Introduction

Discussions on the procyclical effects of bank capital requirements went to the top of the agenda for regulatory reform following the financial crisis that started in 2007. The argument whereby these effects may occur is well-known. In recessions, losses erode banks’ capital, while risk-based capital requirements such as those in Basel II (see BCBS, 2004) become higher. If banks cannot quickly raise sufficient new capital, their lending capacity falls and a credit crunch may follow. Yet, correcting the potential contractionary effect on credit supply by relaxing capital requirements in bad times may increase bank failure probabilities precisely when, due to high loan defaults, they are largest. The conflicting goals at stake explain why some observers (e.g., regulators with an essentially microprudential perspective) think that procyclicality is a necessary evil, while others with a more macroprudential perspective think that it should be explicitly corrected. Basel III (BCBS, 2010) seems a compromise between these two views. It reinforces the quality and quantity of the minimum capital required to banks, but also establishes that part of the increased requirements be in terms of mandatory buffers—a capital preservation buffer and a countercyclical buffer—that are intended to be built up in good times and released in bad times.

This paper constructs a model that captures the key trade-offs in the debate. The model is simple enough to allow us to trace back the effects to a few basic mechanisms. Yet, for the comparison between regulatory regimes (and the characterization of the capital requirements that maximize social welfare) we rely on numerical methods. In our calibration we use evidence from US banks in the period preceding the current financial crisis.

We find that, in spite of inducing banks to hold voluntary capital buffers that are larger in expansions than in recessions, banks’ supply of credit is significantly more procyclical under the risk-based requirements of Basel II than under the flat requirements of Basel I.\(^1\) \(^2\)

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1. The declaration of the G20 Washington Summit of November 14-15, 2008, called for the development of “recommendations to mitigate procyclicality, including the review of how valuation and leverage, bank capital, executive compensation, and provisioning practices may exacerbate cyclical trends.” See also Brunnermeier et al. (2009), FSF (2009), and Kashyap, Rajan, and Stein (2008).

2. Basel I (BCBS, 1988) established a requirement in terms of capital to risk-weighted assets and classified assets in four broad categories. All corporate loans (as well as consumer loans) were in the top risk category.
However, Basel II reduces banks’ probabilities of failure, especially in recessions. For this reason, it dominates Basel I in terms of welfare except for small values of the social cost of bank failure—a parameter with which we capture the externalities behind regulators’ concerns about bank solvency. Moreover, when the social cost of bank failure is around 25% of the initial assets of the failed banks, Basel II implies a cyclical variation in the capital requirements very similar to that of the socially optimal ones. For larger values of the social cost of bank failure, optimal capital charges should be higher than those in Basel II, but their cyclical variation should be comparatively lower. This suggests that, from the lens of our model and for sufficiently large values of the social cost of bank failure, the reforms introduced by Basel III constitute a move in the right direction.

**Modeling strategy**  
Our model is constructed to highlight the primary microprudential role of capital requirements (containing banks’ risk of failure and, thus, deposit insurance payouts and other social costs due to bank failures) as well as their potential procyclical effect on the supply of bank credit. A number of features of the model respond to the desire to keep it transparent about the basic trade-offs. We model the business cycle as a Markov process with two states (expansion and recession), and we abstract from demand-side fluctuations and feedback effects, that could be captured in a fuller macroeconomic model that might embed ours as a building block.

Bank borrowers are overlapping generations of entrepreneurs who demand loans for two consecutive periods. Banks are managed in the interest of their risk-neutral shareholders (providers of their equity capital). Consistent with the view that relationship banking makes banks privately informed about their borrowers, we assume that (i) borrowers become dependent on the banks with whom they first start a lending relationship, and (ii) banks with ongoing relationships have no access to the equity market. The first assumption captures the lock-in effects caused by the potential lemons problem faced by banks when a borrower is switching from another bank. The second assumption captures the implications of these

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3See Boot (2000) for a survey of the relationship banking literature. Several papers explicitly analyze the costs of switching lenders under asymmetric information (e.g., Sharpe, 1990) as well as the trade-offs behind the possible use of multiple lenders as a remedy to the resulting lock-in effects (e.g., Detragiache, Garella,
informational asymmetries for the market for seasoned equity offerings, which can make the
dilution costs of urgent recapitalizations prohibitively costly.\footnote{4}

The combination of relationship lending and the inability of banks with ongoing rela-
tionships to access the equity market establishes a natural connection between the capital
shortages of some banks and the credit rationing of some borrowers at a given date. It also
ensures that two necessary conditions for capital requirements to have aggregate procyclical
effects on credit supply are satisfied: some banks must find it difficult to respond to their
capital needs by issuing new equity, and some borrowers must be unable to avoid credit
rationing by switching to other sources of finance.\footnote{5}

For simplicity, the market for loans to newly born entrepreneurs is assumed to be perfectly
competitive and free from capital constraints. Each cohort of new borrowers is funded by
banks that renew their lending relationships, have access to the equity market, and hence
face no binding limits to their lending capacity.

An important feature of our analysis, distinct from many papers in the literature, is that
we allow banks in their first lending period to raise more capital than needed to just satisfy the
capital requirement. The existence of voluntary capital buffers has been frequently mentioned
as an argument against the prediction of most static models that capital requirements will
be binding and as a factor mitigating their procyclical effects. We find, however, that the
equilibrium buffers (of up to 3.8% in the recession state under Basel II) are not sufficient to
neutralize the effects of the arrival of a recession on the supply of credit to bank-dependent
borrowers (which falls by 12.6% on average in the baseline Basel II scenario).

\textbf{Related literature} Other papers where endogenous capital buffers emerge as a result
of an explicit dynamic optimization problem are Estrella (2004), Peura and Keppo (2006),

\footnote{4}This argument is in line with the logic of Myers and Majluf (1984) and is also subscribed by Bolton
and Freixas (2006). An alternative explanation for banks’ reluctance to raise new equity when their capital
position is impaired is the debt overhang problem (see Myers, 1977, and Hanson, Kashyap, and Stein, 2011).

\footnote{5}These conditions have been noted by Blum and Hellwig (1995) and parallel the conditions in Kashyap,
Stein, and Wilcox (1993) for the existence of a bank lending channel in the transmission of monetary policy.
whose dividend policy and equity raising processes are subject to quadratic adjustment costs in a context where loan losses follow a second-order autoregressive process and bank failure is costly. He shows that the optimal capital decisions of the bank change significantly with the introduction of a value-at-risk capital constraint. Peura and Keppo (2006) consider a continuous-time model in which raising bank equity takes time. A supervisor checks at random times whether the bank complies with a minimum capital requirement and the bank may hold capital buffers in order to reduce the risk of being closed for holding insufficient capital when audited. Similarly, the banks in Elizalde and Repullo (2007) may hold economic capital in excess of their regulatory capital in order to reduce the risk of losing their valuable charter in case of failure. Zhu (2008) adapts the model of Cooley and Quadrini (2001) to the analysis of banks with decreasing returns to scale, minimum capital requirements, and linear equity-issuance costs. Assuming ex-ante heterogeneity in banks’ capital positions, the paper finds that for poorly-capitalized banks, risk-based capital requirements increase safety without causing a major increase in procyclicality, whereas for well-capitalized banks, the converse is true.

Our analysis is simpler along the dynamic dimension than most of the papers mentioned above. However, differently from them, we construct an equilibrium model of relationship banking with endogenous loan rates and a focus on the implications of capital requirements for aggregate bank lending, bank failure probabilities, and social welfare. In this sense, our paper is also related to recent attempts to incorporate bank capital frictions and capital requirements into macroeconomic models. Van den Heuvel (2008) assesses the aggregate steady-state welfare cost of capital requirements in a setup where deposit funding (as opposed to equity funding) provides unique liquidity services to consumers. Meh and Moran (2010), Gertler and Kiyotaki (2010), Martinez-Miera and Suarez (2012) and many others consider models where aggregate bank capital is a state variable whose dynamics is constrained by the evolution of the limited wealth of the population of bankers. In most of these papers bank capital requirements are binding at all times, although some papers like Gerali et al. (2010) induce the existence of buffers by postulating that the deviation from some ad hoc target capital ratio involves a quadratic cost. The procyclical effects of capital requirements
is the focus of attention in Angelini et al. (2010), where there are no loan defaults or bank failure, making their model silent on an important aspect of the relevant welfare trade-offs, and in Brunnermeier and Sannikov (2011), where requirements take the form of value-at-risk constraints on a trading book and risk comes from the evolution of asset prices.

Our paper is complementary to contributions focused on the qualitative trade-offs involved in the design of regulation under the new macroprudential perspective. The early contributions of Daníelsson et al. (2001), Kashyap and Stein (2004), Gordy and Howells (2006), and Saurina and Trucharte (2007), and the more recent of Brunnermeier et al. (2009), and Hanson, Kashyap, and Stein (2011) note the potential importance of the procyclical effects of capital requirements and elaborate on the pros and cons of the various policy options for their correction.

The list of options is long and includes (i) smoothing the inputs of the regulatory formulas by promoting the use of through-the-cycle (rather than point-in-time) estimates of the probabilities of default (PDs) and losses-given-default (LGDs) that feed them (see Caterineu-Rabell, Jackson, and Tsomocos, 2005), (ii) smoothing or cyclically-adjusting the output of the regulatory formulas (see Repullo, Saurina, and Trucharte, 2010), (iii) forcing the building up of buffers based on cyclically sensitive variables such as bank profits and credit growth (see CEBS, 2009, and BCBS, 2010), (iv) adopting countercyclical provisioning (see Burroni et al., 2009), (v) exercising regulatory discretion with countercyclical goals in mind, and (vi) relying on contingent convertibles and other forms of capital insurance (see Kashyap, Rajan, and Stein, 2008). As most other papers in the literature, our model is too stylized to formally capture the differences between these proposals, and hence to inform the comparison between them (which is largely driven by legal, accounting, and political economy issues potentially affecting their effectiveness, predictability, manipulability, risk of capture, and cost of implementation). Our analysis is more informative on the level and degree of cyclical adjustment of the capital requirements that regulators should target to impose in one way or another.

Empirical studies focused on the impact of bank regulation on bank capital decisions and the supply of credit are abundant but often little conclusive as they are plagued with
problems of endogeneity and poor identification. Due to the Lucas’ critique, the results from reduced-form analyses of the dynamics of bank capital buffers under specific regulatory regimes cannot be extrapolated for the assessment of new regulatory regimes. Yet the relevance of banks’ capital constraints for determining the supply of credit is documented, among others, by Bernanke and Lown (1991), who examine credit supply in the years after the introduction of Basel I, Ivashina and Scharfstein (2010), who show that after the demise of Lehman Brothers poorly capitalized banks contracted their credit disproportionately more than better capitalized banks, and by Aiyar, Calomiris, and Wieladek (2012), who document sizable loan supply effects following discretionary shifts in the level of capital requirements in the UK from 1998 to 2007.

Outline of the paper The rest of the paper is organized as follows. Section 2 presents the model. In Section 3 we analyze the capital decision of a representative bank, define the equilibrium, and provide the comparative statics of equilibrium loan rates and capital buffers. In Section 5 we discuss our calibration of the model. Section 5 reports the quantitative results concerning the loan rates, capital buffers, credit rationing, and probabilities of bank failure under the various regulatory regimes. In Section 6 we compare these regimes in terms of social welfare and characterize the optimal capital requirements. Section 7 discusses the robustness of our results to changes in some of the key assumptions of the model. Section 8 contains our concluding remarks. The Appendix gathers the proofs of the analytical results and shows the relationship between the single common risk factor model used in the calibration and the Basel II formula for capital requirements.

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2 The Model

Consider a discrete-time infinite-horizon economy with three classes of risk-neutral agents: entrepreneurs, investors, and banks. Entrepreneurs finance their investments by borrowing from banks. Investors provide funds to the banks in the form of deposits and equity capital. Banks channel funds from investors to entrepreneurs. There is also a government that insures bank deposits and imposes minimum capital requirements on banks.

2.1 Entrepreneurs

Entrepreneurs belong to overlapping generations whose members remain active for up to two periods (three dates). Each generation is made up of a measure-one continuum of ex-ante identical and penniless individuals. Entrepreneurs born at a date \( t \) have the opportunity to undertake a sequence of two independent one-period investment projects at dates \( t \) and \( t + 1 \). Each project requires a unit investment and yields a pledgeable return \( 1 + a \) if it is successful, and \( 1 - \lambda \) if it fails, where \( a > 0 \) and \( 0 < \lambda < 1 \).

All projects operating from date \( t \) to date \( t + 1 \) have an identical probability of failure \( p_t \). The outcomes of these projects exhibit positive but imperfect correlation, so their aggregate failure rate \( x_t \) is a continuous random variable with support \([0, 1]\) and cumulative distribution function (cdf) \( F_t(x_t) \) such that the probability of project failure satisfies

\[
p_t = E_t (x_t) = \int_0^1 x_t dF_t(x_t).
\]

For simplicity, we consider the case in which the history of the economy up to date \( t \) only affects \( F_t(x_t) \) (and thus \( p_t \)) through an observable state variable \( s_t \) that can take two values, \( l \) and \( h \), and follows a Markov chain with transition probabilities

\[
q_{ss'} = \Pr (s_{t+1} = s' \mid s_t = s), \text{ for } s, s' = l, h.
\]

Moreover, we assume that the cdfs corresponding to the two states, \( F_l(\cdot) \) and \( F_h(\cdot) \), are ranked in the sense of first-order stochastic dominance, so that \( p_l < p_h \). Thus states \( l \) and \( h \) may be interpreted as states of expansion (low business failure) and recession (high business failure), respectively.
2.2 Investors

At each date $t$, there is a large number of investors willing to supply banks with deposits and equity capital in a perfectly elastic fashion at some required rate of return. The required interest rate on bank deposits (which are assumed to be insured by the government) is normalized to zero. In contrast, the required expected return on bank equity is $\delta \geq 0$. This excess cost of bank capital $\delta$ is intended to capture in a reduced-form manner distortions (such as agency costs of equity) that imply a comparative disadvantage of equity financing relative to deposit financing (and in addition to deposit insurance).\footnote{Further to the reasons for the extra cost of equity financing offered by the corporate finance literature, Holmström and Tirole (1997) and Diamond and Rajan (2000) provide agency-based explanations specifically related to banks’ monitoring role. For the positive results of the paper, $\delta$ may also be interpreted as the result of debt tax shields, but in this case it should not constitute a deadweight loss in the normative analysis (see Admati et al., 2011).}

2.3 Banks

Banks are infinitely lived competitive intermediaries specialized in channeling funds from investors to entrepreneurs. Following the literature on relationship banking, we assume that each entrepreneur relies on a sequence of one-period loans granted by the single bank from which the first loan is obtained. Setting up the relationship with the entrepreneur involves a setup cost $\mu$ which is subtracted from the bank’s first period revenues.\footnote{This cost might include personnel, equipment, and other operating costs associated with the screening and monitoring functions emphasized in the literature on relationship banking.} Finally, for simplicity, we abstract from the possibility that part of the second period investment be internally financed by the entrepreneur.\footnote{This simplification is standard in relationship banking models; see, for example, Sharpe (1990) or Von Thadden (2004). Moreover, if entrepreneurs’ first-period profits are small relative to the required second-period investment, the quantitative effects of relaxing this assumption would be negligible.}

Banks are funded with insured deposits and equity capital, but access to the latter is affected by an important imperfection: while banks renewing their portfolio of lending relationships can unrestrictedly raise new equity, recapitalization is impossible for banks with ongoing lending relationships. Our goal here is to capture in a simple way the long delays or high dilution costs that a bank with opaque assets in place may face when arranging
an equity injection.\footnote{These costs are typically attributed to asymmetric information. Specifically, if banks learn about their borrowers after starting a lending relationship (like in Sharpe, 1990) and borrower quality is asymmetrically distributed across banks, the market for seasoned equity offerings (SEOs) is likely to be affected by a lemons problem (like in Myers and Majluf, 1984). Specifically, after a negative shock, banks with lending relationships of poorer quality will be more interested in issuing equity at any given price, which would explain why the prices at which new equity can be raised may be unattractive to banks with higher-quality relationships and why, in sufficiently adverse circumstances, the market for those SEOs may collapse.}

Banks are managed in the interest of their shareholders, who are protected by limited liability. A capital requirement obliges them to keep a capital-to-loans ratio of at least $\gamma_s$ when the state of the economy is $s$. This formulation encompasses several regulatory scenarios that will be compared below: a laissez-faire regime with no capital requirements ($\gamma_l = \gamma_h = 0$), a regime with flat capital requirements such as those of Basel I (which for corporate loans sets a requirement of Tier 1 capital of $\gamma_l = \gamma_h = 4\%$), and a regime with risk-sensitive capital requirements such as those of Basel II or Basel III (where the cyclical variation in the risk-based inputs of the regulatory formula implies $\gamma_l < \gamma_h$).\footnote{The precise Basel formula that makes $\gamma_s$ an increasing function of the probability of default of the loans (the probability of project failure $p_s$) is described in Section 4.}

### 2.4 Government policies and social welfare

The government performs two tasks in this economy. First, it insures bank deposits (raising lump-sum taxes in order to cover the cost of repaying depositors in case of bank failure). Second, it imposes minimum capital requirements on banks.

In the normative analysis below, we will assess the welfare implications of the various regulatory scenarios taking into account possible negative externalities associated with bank failures, which will be assumed to imply a social cost equal to a proportion $c$ of the initial assets of the failed banks.\footnote{The externalities commonly identified in the literature include the disruption of the payment system, the erosion of confidence on similar banks and the rest of the financial system, the deterioration of public finances derived from the cost of resolving or supporting banks in trouble, the fall in economic activity associated with a potential credit crunch, and the damage to the general economic climate. For an empirical assessment of these costs, see Laeven and Valencia (2008, 2010).} Specifically, given that investors (depositors and bank shareholders) in equilibrium will break even in expected net present value terms over their relevant investment horizons, we will measure social welfare as the sum of the expected residual in-
come flows obtained by entrepreneurs from their investment projects minus the expected
cost of deposit insurance payouts and the expected social cost of bank failures.

3 Equilibrium Analysis

In this section we characterize banks’ equilibrium capital and lending decisions and derive
some comparative statics results on equilibrium loan rates and capital buffers.

3.1 Banks’ optimization problem

We assume that entrepreneurs born at date $t$ obtain their first period loans from banks that
can unrestrictedly raise capital at this date. This is consistent with the assumption that
banks with ongoing lending relationships face capital constraints, and allows us to analyze
the banking industry as if it were made of overlapping generations of banks that operate for
two periods, specialize in loans to their contemporaneous entrepreneurs, and can only issue
equity when they start operating.

Consider a representative bank that lends a first unit-size loan to the measure one con-
tinuum of entrepreneurs born at date $t$, possibly refinances them at date $t + 1$, and ends its
activity at date $t + 2$. Denote the states of the economy at dates $t$ and $t + 1$ by $s$ and $s'$,
respectively. At date $t$ the bank raises $1 - k_s$ deposits and $k_s$ capital, with $k_s \geq \gamma_s$ to satisfy
the capital requirement, and invests these funds in a unit portfolio of first period loans.\footnote{Notice that the bank may start up with a buffer of capital $k_s - \gamma_s > 0$ in order to better accommodate shocks that impair its capacity to lend in the second period.}
The interest rate on these loans, $r_s$, will be determined endogenously, but is taken as given
by the perfectly competitive bank.\footnote{This corresponds to the idea that entrepreneurs can shop around for their first period loans before becoming locked in for their second period loans.}

At date $t + 1$ the bank obtains revenue $1 + r_s$ from the fraction $1 - x_t$ of performing loans
(those extended to entrepreneurs with successful projects) and $1 - \lambda$ from the fraction $x_t$ of
defaulted loans, and incurs the setup cost $\mu$. So its assets are worth $1 + r_s - x_t(\lambda + r_s) - \mu$,
while its deposit liabilities are $1 - k_s$ (since the deposit rate has been normalized to zero).
Thus, the net worth (or available capital) of the bank at date \( t + 1 \) is

\[
k'_s(x_t) = k_s + r_s - x_t(\lambda + r_s) - \mu, \tag{2}
\]

where \( x_t \) is a random variable whose conditional cdf is \( F_s(x_t) \).

The entrepreneurs that started up at date \( t \) demand a second unit-size loan at date \( t + 1 \).\(^{15}\) Since they are dependent on the bank at this stage, their demand is inelastic. Thus, the second period loan rate will be \( a \), assigning all the pledgeable return from the investment in the period to the bank.

To comply with capital regulation, funding all second period projects at date \( t + 1 \) would require the bank to have an amount of capital equal to \( s_0 \), where \( s_0 \) is the state of the economy at that date. There are three cases to consider. First, if \( k'_s(x_t) < 0 \) the bank fails, the deposit insurer takes over the bank and repays the depositors, and the entrepreneurs dependent on the bank cannot invest. Second, if \( 0 \leq k'_s(x_t) < \gamma_{s'} \) the bank’s available capital cannot support funding all the second period projects, so some entrepreneurs are credit rationed. Third, if \( k'_s(x_t) \geq \gamma_{s'} \) the bank can fund all the second period projects and, on top of that, pay a dividend \( k'_s(x_t) - \gamma_{s'} \) to its shareholders at date \( t + 1 \).\(^{16}\)

Which case obtains depends on the realization of the default rate \( x_t \). Using the definition (2) of \( k'_s(x_t) \), it is immediate to show that the bank fails when \( x_t > \tilde{x}_s \), where

\[
\tilde{x}_s = \frac{k_s + r_s - \mu}{\lambda + r_s}. \tag{3}
\]

The bank has insufficient lending capacity (and rations credit to some of the second period projects) when \( \tilde{x}_{ss'} < x_t \leq \tilde{x}_s \), where

\[
\tilde{x}_{ss'} = \frac{k_s + r_s - \mu - \gamma_{s'}}{\lambda + r_s}. \tag{4}
\]

And the bank has excess lending capacity (and pays a dividend to its shareholders) when \( x_t \leq \tilde{x}_{ss'} \).

\(^{15}\)Note that this includes entrepreneurs that defaulted on their initial loans. This is because under our assumptions such default does not reveal any information about their second period projects.

\(^{16}\)Since entrepreneurs born at date \( t + 1 \) borrow from banks that can raise equity at that date, the bank lending to entrepreneurs born at date \( t \) can use the excess capital either to pay a dividend to its shareholders or to reduce the deposits to be raised at this date. However, with deposit insurance and an excess cost of bank capital \( \delta \geq 0 \), the second alternative is strictly suboptimal.
The following proposition provides an expression of the net present value for the shareholders of a bank that can raise capital at date $t$. Since the result follows quite directly from the sequence of definitions that it contains, we will omit its proof, replacing it with the brief explanation given below.

**Proposition 1** The net present value for the shareholders of a representative bank that in state $s$ has capital $k_s$ and faces an interest rate $r_s$ on its unit of initial loans is

$$v_s(k_s, r_s) = \frac{1}{1 + \delta} \mathbb{E}_t[v_{ss'}(x_t)] - k_s,$$  

where

$$v_{ss'}(x_t) = \begin{cases} 
\pi_{s'} + k_s'(x_t) - \gamma_{s'}, & \text{if } x_t \leq \widehat{x}_{ss'}, \\
\pi_{s'} - \frac{k_s'(x_t)}{\gamma_{s'}}, & \text{if } \widehat{x}_{ss'} < x_t \leq \widehat{x}_s, \\
0, & \text{if } x_t > \widehat{x}_s,
\end{cases}$$

is the conditional equity value at date $t + 1$, inclusive of dividends, and

$$\pi_{s'} = \frac{1}{1 + \delta} \int_0^1 \max \{\gamma_{s'} + a - x_{t+1}(\lambda + a), 0\} \, dF_{s'}(x_{t+1})$$

is the discounted gross return that equity earns on each unit of loans made at date $t + 1$.

The operator $\mathbb{E}_t(\cdot)$ in (5) takes into account the uncertainty at date $t$ about both the state of the economy at date $t + 1$ (which affects $\gamma_{s'}$ and $\pi_{s'}$) and the default rate $x_t$ of initial loans (which determines the capital $k'_s(x_t)$ available at $t + 1$). Expected future payoffs in (5) and (7) are discounted at the shareholders’ required expected return $\delta$. The three expressions in the right-hand-side of (6) correspond to three cases mentioned above. With excess lending capacity, the bank funds all the second period projects, which yields a discounted gross return $\pi_{s'}$, and pays a dividend $k'_s(x_t) - \gamma_{s'}$. With insufficient lending capacity, the bank funds a fraction $k'_s(x_t)/\gamma_{s'}$ of the second period projects, which yields a discounted gross return $\pi_{s'} k'_s(x_t)/\gamma_{s'}$. Finally, in case of bank failure, the shareholders get a zero payoff.\(^{17}\)

\(^{17}\)As specified in (7), $\pi_{s'}$ is obtained by integrating with respect to the probability distribution of the default rate $x_{t+1}$ the net worth that the bank generates at date $t + 2$ out of each unit of lending at date $t + 1$. The expression in the integrand of (7) is identical to (2) except for the fact that the bank’s capital is $\gamma_{s'}$, the loan rate is $a$, the setup cost $\mu$ has already been incurred, and shareholders’ limited liability is taken into account using the max operator.
The representative bank that first lends to a generation of entrepreneurs in state \( s \) takes the initial loan rate \( r_s \) as given and chooses its capital \( k_s \) so as to maximize \( v_s(k_s, r_s) \) subject to the requirement \( k_s \geq \gamma_s \) insofar as the resulting value is not negative. If it were negative, shareholders would prefer not to operate the bank. To guarantee that operating the bank is profitable, we henceforth assume that the following sufficient condition holds.

**Assumption 1** \( v_s(\gamma_s, a) \geq 0 \) and \( \pi_s - \gamma_s \geq 0 \) for \( s = l, h \).

This assumption states that making loans at a rate equal to the project’s net success return \( a \) while satisfying the capital requirement with equality constitutes a non-negative net present value investment for the bank’s shareholders in the two lending periods.

The following result characterizes the initial capital decision of the bank.

**Proposition 2** The capital decision \( k_s \) of a representative bank that in state \( s \) faces an interest rate \( r_s \) on its unit of initial loans always has a solution, which may be interior or at the corner \( k_s = \gamma_s \). When the solution is interior, the probability that in the next period the bank ends up with excess lending capacity in the low default state \( s' = l \) and rations credit in the high default state \( s' = h \) is strictly positive.

The existence of a solution follows directly from the fact that \( v_s(k_s, r_s) \) is continuous in \( k_s \), for any given interest rate \( r_s \). We show in the Appendix that the function \( v_s(k_s, r_s) \) is neither concave nor convex in \( k_s \), and its maximization with respect to \( k_s \) may have interior solutions or corner solutions with \( k_s = \gamma_s \).\(^{18}\) The intuition for the positive probability that (in an interior solution) the bank ends up with excess lending capacity in state \( s' = l \) and rations credit in state \( s' = h \) is the following. If in the two possible states at date \( t + 1 \) the bank had a probability one of finding itself with excess lending capacity, then it would have an incentive to reduce its capital at date \( t \) in order to lower its funding costs. Conversely, if in the two possible states at date \( t + 1 \) the bank had a probability one of finding itself with insufficient lending capacity, then it would have an incentive to increase its capital at date \( t \) in order to relax its capital constraint at date \( t + 1 \).

\(^{18}\)Note that since the function \( v_s(k_s, r_s) \) is not concave in \( k_s \), there may be multiple optimal values of \( k_s \) corresponding to any \( r_s \).
3.2 Equilibrium

In order to define an equilibrium, it only remains to describe how the loan rate \( r_s \) applicable to lending relationships starting in state \( s \) is determined. Under perfect competition, the pricing of initial loans must be such that the net present value of the representative bank for its shareholders is zero under its optimal capital decision. Were it negative, no bank would extend these loans. Were it positive, banks would have an incentive to expand the scale of their activities. Hence in each state of the economy \( s = l, h \) we must have

\[
v_s(k_s^*, r_s^*) = 0,
\]

for

\[
k_s^* = \arg \max_{k_s \geq \gamma_s} v_s(k_s, r_s^*).
\]

An equilibrium is a sequence of pairs \( \{(k_t, r_t)\} \) describing the capital-to-loans ratio \( k_t \) of the banks that can issue equity at date \( t \) and the interest rate \( r_t \) on their initial loans, such that each pair \( (k_t, r_t) \) satisfies (8) and (9) for \( s = s_t \), where \( s_t \) is the state of the economy at date \( t \). The following result proves the existence of an equilibrium.

**Proposition 3** There exists a unique \( r_s^* \) that satisfies equilibrium conditions (8) and (9).

The uniqueness of \( r_s^* \) follows from the fact that, for each initial state \( s \), the net present value of the bank is an overall continuous and increasing function of \( r_s \) (after taking into account how the capital decision \( k_s \) varies with \( r_s \)). Moreover, such function is negative for sufficiently low values of \( r_s \) and, by Assumption 1, non-negative when \( r_s \) equals \( a \), which guarantees the existence of a unique solution.

3.3 Comparative statics

Table 1 summarizes the comparative statics of the equilibrium initial loan rate \( r_s^* \), which are derived in the Appendix. The table shows the sign of the derivative \( dr_s^*/dz \) obtained by differentiating (8) with respect to a parameter denoted generically by \( z \).
Table 1. Comparative statics of the initial loan rate $r_s^*$

<table>
<thead>
<tr>
<th>$z = a \lambda \mu \delta \gamma_l \gamma_h q_{sh}$</th>
<th>$\frac{dr_s^*}{dz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$+$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$+$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$+$</td>
</tr>
<tr>
<td>$\gamma_l$</td>
<td>$+$</td>
</tr>
<tr>
<td>$\gamma_h$</td>
<td>$+$</td>
</tr>
<tr>
<td>$q_{sh}$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

The effects of the various parameters on $r_s^*$ are inversely related to their impact on bank profitability. Other things equal, the success return $a$ impacts positively on the profitability of continuation lending; the loss given default $\lambda$ affects negatively the profitability of both initial lending (directly) and continuation lending (directly and by reducing the availability of capital in the second period); the setup cost $\mu$ has a similar negative effect, with no direct effect on the profitability of continuation loans; the cost of bank capital $\delta$ increases the cost of making loans in both periods; the capital requirements $\gamma_l$ and $\gamma_h$ increase the burden of capital regulation in the corresponding initial or continuation state; finally, in any regulatory regime with $\gamma_l \leq \gamma_h$, the probability of ending up in the high default state $q_{sh}$ decreases the profitability of continuation lending because in state $h$ loan losses are higher and the capital requirement is not lower than in state $l$.\textsuperscript{19}

Table 2 summarizes the comparative statics of the equilibrium initial capital $k_s^*$ chosen by the representative bank in an interior solution. As further explained in the Appendix, we decompose the total effect of the change in any parameter $z$ in a direct effect, for constant $r_s^*$, and a loan rate effect, due to the change in $r_s^*$. Since $k_s$ and $r_s$ are substitutes in providing the bank with sufficient capital for its continuation lending (see the expression for $k'_s(x_t)$ in (2)), it turns out that $\frac{dk_s^*}{dr_s}$ is negative, implying that the signs of the loan rate effects are the opposite to those in Table 1.

\textsuperscript{19}Obviously, the probability of ending up in the low default state $q_{sl} = 1 - q_{sh}$ has the opposite effect.
Table 2. Comparative statics of the initial capital $k_s^*$
(in an interior equilibrium)

<table>
<thead>
<tr>
<th>$z$</th>
<th>$a$</th>
<th>$\lambda$</th>
<th>$\mu$</th>
<th>$\delta$</th>
<th>$\gamma_l$</th>
<th>$\gamma_h$</th>
<th>$q_{sh}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\partial k_s^*}{\partial z}$ (direct effect)</td>
<td>+</td>
<td>?</td>
<td>+</td>
<td>−</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>$\frac{\partial k_s^<em>}{\partial k_s} \frac{dr_s^</em>}{dz}$ (loan rate effect)</td>
<td>+</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>$\frac{dk_s^*}{dz}$ (total effect)</td>
<td>+</td>
<td>?</td>
<td>?</td>
<td>−</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

For the parameters $a$ and $\delta$, the direct and the loan rate effects point in the same direction, so the total effect can be signed: higher profitability of continuation lending and lower costs of bank capital encourage banks to increase self-insurance against default shocks that threaten their continuation lending. For the setup cost $\mu$, the direct and the loan rate effects have unambiguous but opposite signs, so the total effect is ambiguous. The positive direct effect comes from the fact that $\mu$ subtracts to the bank’s continuation lending capacity exactly like $k_s$ adds to it (see again (2)).

The direct effects on $k_s$ of parameters $\lambda$, $\gamma_l$, $\gamma_h$, and $q_{sh}$ have ambiguous signs. Increasing any of these parameters reduces the profitability of continuation lending (and the value of holding excess capital in the initial lending period) but impairs the expected capital position of the bank when such lending has to be made (so the prospects of ending up with insufficient capital increase). This means that the profitability of continuation lending and the need for self-insurance move in opposite directions. This ambiguity extends to the total effects.

The details of the relevant analytical expressions suggest that the shape of the distributions of default rates matter for the determination of the unsigned effects, which could only be assessed either empirically or by numerically solving the model under some realistic parameterization. In the rest of the paper, we resort to the second alternative.
4 Calibration

This section presents the parameterization under which we derive our quantitative results. We start by specifying the distributions of the default rate in each state, $F_l(x_t)$ and $F_h(x_t)$, as well as the capital regulation regimes, determining $\gamma_l$ and $\gamma_h$, that will be compared. Finally, we discuss the values given to the parameters of the model: the projects’ success return $a$ and loss given default $\lambda$, the cost of setting up a lending relationship $\mu$, the excess cost of bank capital $\delta$, the transition probabilities $q_{ss'}$ for $s, s' = l, h$, and the parameters in the distributions specified for the default rate. In the calibration, one period is one year.

4.1 Default rate distributions

We assume that the probability distributions of the loan default rate $x$ are those implied by the single common risk factor model of Vasicek (2002), which was the model used to provide a value-at-risk foundation to the capital requirement formulas of Basel II (see Gordy, 2003). As shown in the Appendix, this model implies

$$F_s(x) = \Phi \left( \sqrt{1 - \rho_s} \frac{\Phi^{-1}(x) - \Phi^{-1}(p_s)}{\sqrt{\rho_s}} \right),$$

for $s = l, h$, where $\Phi(\cdot)$ is the cdf of a standard normal random variable and $\rho_s \in (0, 1)$ is a parameter that measures the dependence of individual defaults on the common risk factor (and thus determines the degree of correlation between loan defaults). With this formulation, the distribution of the default rate in state $s$ is fully parameterized by the probability of default $p_s$ and the correlation parameter $\rho_s$.\footnote{It is easy to show that increases in $p_s$ produce a first-order stochastic dominance shift in the distribution of $x$, and increases in $\rho_s$ produce a mean-preserving spread in the distribution of $x$.}

4.2 Regulatory regimes

The quantitative analysis in the paper is based on the assumption that the empirical counterpart of the equity capital that appears in the model (and to which the capital requirements $\gamma_l$ and $\gamma_h$ refer to) is what Basel regulations define as Tier 1 capital (essentially, common equity). Both Basel I and Basel II established (i) an overall requirement in terms of the
sum of Tier 1 and Tier 2 capital (where the latter included substitutes of common equity with lower loss-absorbing capacity such as convertible and subordinated debt), and (ii) the additional requirement that at least half of the required capital had to take the (presumably more expensive) form of Tier 1 capital. However, the regulatory response to the financial crisis that started in 2007, known as Basel III, has upgraded the role of the second requirement after assessing that only (the core of) Tier 1 capital is truly capable of protecting banks against insolvency (see BCBS, 2010). Consistent with this view, we will focus on Tier 1 capital requirements but we will incorporate an adjustment to capture the incidence of the overall Tier 1 + Tier 2 requirement on banks’ cost of funding.

The positive part of our quantitative analysis considers three capital regulation regimes. In the laissez-faire regime, a purely theoretical benchmark, we set $\gamma_h = \gamma_l = 0$. In the Basel I regime we set $\gamma_h = \gamma_l = 0.04$, which corresponds to the minimum Tier 1 capital requirement on all non-mortgage credit to the private sector set by the Basel Accord of 1988 (i.e. one half of the overall 8% requirement of Tier 1 + Tier 2 capital).

The Tier 1 capital requirements of the Basel II regime are obtained by dividing by two the overall requirement of Tier 1 + Tier 2 capital given by the Basel II formula. For corporate exposures of a one-year maturity, this implies:

$$\gamma_s = \frac{\lambda}{2} \Phi \left( \frac{\Phi^{-1}(p_s) + \sqrt{\rho(p_s) \Phi^{-1}(0.999)}}{\sqrt{1 - \rho(p_s)}} \right), \quad (11)$$

where

$$\rho(p_s) = 0.12 \left( 2 - \frac{1 - e^{-50p_s}}{1 - e^{-50}} \right). \quad (12)$$

The term $\rho(p_s)$ reflects the way in which Basel regulators calibrated the correlation parameter $\rho_s$ in (10) as a decreasing function of the probability of default $p_s$. The rationale for this

\[21\text{The formula has an explicit value-at-risk interpretation: given the distribution of the default rate in (10), it requires Tier 1 + Tier 2 capital sufficient to cover loan losses with a confidence level of 99.9\%.}\]

\[22\text{See BCBS (2004, paragraph 272). The full Basel II formula incorporates an adjustment factor that is increasing in the maturity of the loan, and equals one for a maturity of one year. Also, Basel II distinguishes between expected losses, equal to $\lambda p_s$, which should be covered with general loan loss provisions, and the remaining part of the charge, $\lambda (\gamma_s - p_s)$, which should be covered with capital. However, from the perspective of our analysis, provisions are just another form of equity capital, so the distinction between these components is immaterial to our calculations.}\]
assumption is that, in the cross-section, riskier firms are typically smaller firms for which idiosyncratic risk factors are more important than the common risk factor, so their defaults are less correlated with each other. Since this argument does not apply to the time-series dimension on which we focus, we will parameterize $\rho_s$ as a constant $\rho$ equal to the weighted average of $\rho(p_s)$ for $s = l, h$, where the weights are the unconditional probabilities of each state $s$.$^{23}$

Additionally to the three regimes compared in the positive part of our analysis, in the normative part we will characterize an optimal minimum capital regime in which the capital requirements $\gamma_l$ and $\gamma_h$ are set to maximize our measure of social welfare.

### 4.3 Parameter values

Table 3 describes our baseline parameterization of the model. The value of the success return $a$ determines the interest rate of second period loans (measured as a spread over the risk-free deposit rate, which has been normalized to zero). Standard statistical sources do not provide banks’ marginal lending and borrowing interest rates. A common approach is to proxy them with implicit average rates obtained from accounting figures. According to the FDIC Statistics on Banking, Total interest income of all US commercial banks was, on average, 5.74% of Earning assets in the pre-crisis years 2004-2007, while Total interest expense was 2.32% of Total liabilities. This implies an average net interest margin of 3.42%.$^{24}$ Adding Service charges on deposit accounts, which were 0.55% of Total deposits, produces an average intermediation margin of 3.97% on deposit-funded activities during the referred period. This justifies our choice of $a = 0.04$.

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$\lambda$</th>
<th>$\mu$</th>
<th>$\delta$</th>
<th>$p_l$</th>
<th>$p_h$</th>
<th>$q_{ll}$</th>
<th>$q_{hh}$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.04</td>
<td>0.45</td>
<td>0.03</td>
<td>0.08</td>
<td>0.010</td>
<td>0.036</td>
<td>0.80</td>
<td>0.64</td>
<td>0.174</td>
</tr>
</tbody>
</table>

$^{23}$These probabilities are $\phi_l = (1 - q_{hh})/(2 - q_{ll} - q_{hh})$ and $\phi_h = (1 - q_{ll})/(2 - q_{ll} - q_{hh})$, respectively.

$^{24}$The data is available at http://www2.fdic.gov/SDI/SOB/
Parameter $\lambda$ determines the loss given default (LGD) of the loans to projects that fail. We take the value $\lambda = 0.45$ from the Basel II “foundation Internal Ratings-Based (IRB) approach” for unsecured corporate exposures, which was calibrated in line with industry estimates of this parameter.$^{25}$

The value of the setup cost $\mu$ is hard to establish directly from the data, since its empirical counterpart is included in the broader category of non-interest expense in banks’ accounts. In the FDIC Statistics on Banking the ratio of Total non-interest expense of all US commercial banks to Total assets for years 2004-2007 has an average of 3.97%. The role of $\mu$ in the model is to reduce the profitability of bank lending in order to have realistic initial loan rates. Taking $\mu = 0.03$ we obtain first period loan spreads (over the risk-free deposit rate) of about 100 basis points in the low default state.

For the calibration of the excess cost of bank capital $\delta$ we take into account that the regulatory regimes that we compare are described in terms of minimum requirements of Tier 1 capital. However, Basel I and Basel II also required the total amount of Tier 1 + Tier 2 capital to be at least twice as much as the minimum requirement of Tier 1 capital. Instead of considering this second requirement and explicitly modeling the two classes of capital and the frictions possibly affecting each of them, we take a shortcut and make $\delta$ equal to two times the reference estimate of banks’ excess cost of equity financing.$^{26}$

To set a reference estimate for $\delta$, one may follow the literature on entrepreneurial financing, which commonly assumes a spread between the rates of return required by entrepreneurs and those required by their lenders.$^{27}$ Carlstrom and Fuerst (1997) and Gomes, Yaron, and Zhang (2003), among others, set the spread at 5.6%, while Iacoviello (2005) opts for a more conservative 4%.$^{28}$ An alternative approach, proposed by Van den Heuvel (2008), is to at-

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$^{25}$The implications of allowing for cyclical variation in $\lambda$ will be discussed in Section 7.

$^{26}$Since the excess cost $\delta$ also applies to the capital buffers held on top of the regulatory requirements, our strategy implicitly assumes that Tier 1 capital buffers are matched with buffers of Tier 2 capital of the same size.

$^{27}$Most papers in the capital structure tradition (e.g. Hennessy and Whited, 2007) focus on the net tax disadvantages of equity financing (vis-à-vis debt financing), an aspect of the differential cost of equity funding that does not constitute a deadweight loss from a social welfare perspective (see Admati et al., 2011) and from which we wish to abstract in order to facilitate the normative analysis in Section 6 below.

$^{28}$The spreads found in the entrepreneurial financing literature may be interpreted as a reduced-form discount for the lack of diversification or liquidity associated with entrepreneurs’ equity stakes. If extended
tribute the spread between the costs of banks’ equity and deposit funding to the unique liquidity services associated with deposits. He compares the average return on subordinated bank debt (which counts as Tier 2 capital for regulatory purposes, but has the same tax advantages as standard debt) with the average net return of deposits. He finds a spread of 3.16% that can be considered a lower bound estimate of the cost of Tier 1 capital since its main component, common equity, presumably involves larger informational and agency costs than subordinated debt. Given that the various candidate estimates fluctuate around a mid value of 4%, we set $\delta = 2 \times 0.04 = 0.08$.

Under the default rate distributions in (10) and with a state-invariant correlation parameter $\rho$, the only parameters of the model subject to Markov chain dynamics are the probabilities of default $p_l$ and $p_h$. To set them we look at the Special Report “Commercial Banks in 1999” of the Federal Reserve Bank of Philadelphia, that offers data on the experience of US commercial banks during the 1990s.\(^{29}\) In years around the 1990-1991 recession the aggregate ratio of Non-performing loans to Total loans was slightly above 3%, declined to slightly above 2% in 1993, and remained below 1.5% (with a downward trend) for the rest of the decade. Against this background, the choices in Table 3 ($p_l = 0.01$ and $p_h = 0.036$) are fine-tuned so as to imply that the unconditional mean of the Tier 1 capital requirements of Basel II (i.e. the weighted average of the values $\gamma_l = 3.2\%$ and $\gamma_h = 5.5\%$ obtained from (11) and (12), where the weights are the unconditional probabilities of each state) equals 4%, exactly as in the Basel I regime. This will allow us to attribute the differences in results across these regulatory regimes to a cyclical rather than a level effect.

We set the transition probabilities of the Markov process, $q_{ll}$ and $q_{hh}$, so as to produce expected durations of $(1 - q_{ll})^{-1} = 5$ years for the low default state and $(1 - q_{hh})^{-1} = 2.8$ years for the high default state.\(^{30}\) These durations are derived from the analysis of the annual ratio

\(^{29}\)See http://www.philadelphiafed.org/files/bb/bbspecial.pdf. Similar reports for years after 1999 confirm the overall picture, but offer the information with a breakdown (large banks vs. small banks) that does not make the numbers directly comparable.

\(^{30}\)The expected duration of state $s$ is $(1 - q_{ss}) + 2q_{ss}(1 - q_{ss}) + 3q_{ss}^2(1 - q_{ss}) + \ldots = (1 - q_{ss})^{-1}$.\)
of Net loan and lease charge-offs to Gross loans and leases for the FDIC-insured commercial banks over the period 1969-2004.\textsuperscript{31} After detrending the series using the standard HP-filter for annual data, we find 20 below-average yearly observations in 4 complete low default phases (implying an average duration of $20/4 = 5$ years) and 14 above-average observations in 5 complete high default phases (implying an average duration of $14/5 \simeq 2.8$ years).\textsuperscript{32}

Finally, as explained above, we set the value of the correlation parameter, $\rho = 0.174$, equal to the weighted average of the values of $\rho(p_s)$ obtained from (12), where the weights are the unconditional probabilities of each state $s$ ($\phi_l = 0.643$ and $\phi_h = 0.357$).

5 Quantitative Results

This section describes the equilibrium loan rates, capital buffers, credit rationing, and bank solvency that obtain when solving the model using the parameterization described in the previous section. The outcomes presented in the first three panels of Table 4 come from directly solving the equilibrium equations (8) and (9) in each state. Credit rationing is defined as the proportion of second period projects that cannot be undertaken because of banks’ insufficient lending capacity or failure. In the fourth panel of Table 4 we report the expected credit rationing in state $s'$ for each possible sequence of states $(s, s')$, which using the notation in Section 3.1 can be formally written as:\textsuperscript{33}

\[
CR_{ss'} = \int_{\tilde{x}_{ss'}}^{\tilde{x}_{s'}} \left[ 1 - \frac{K_s'(x)}{\gamma_{s'}} \right] dF_s(x) + [1 - F(\tilde{x}_{ss'})],
\]

(13)

where the first term reflects the rationing due to banks’ insufficient lending capacity and the second the rationing due to bank failure. Table 4 also reports the unconditional mean of this variable across all possible trajectories of the economy. The fifth panel shows the probabilities of bank failure of first and second period banks in each state, as well as their average values across states and the overall average across banks.

\textsuperscript{31}The FDIC Historical Statistics on Banking are available at http://www2.fdic.gov/hsob/index.asp.
\textsuperscript{32}The observations of 1969 and 2004 belong to censored below-average phases and are not taken into account. The matched durations are consistent with the results in Koopman, Lucas, and Klaassen (2005), who identify a stochastic cycle in US business failure rates with a period of between 8 and 11 years.
\textsuperscript{33}The need to take expectations comes from the fact that credit rationing in a period in which the prevailing state is $s'$ varies with the realization of the default rate in the previous period.
Table 4. Equilibrium loan rates, capital buffers, credit rationing, and bank solvency under different regulatory regimes
(all variables in %)

<table>
<thead>
<tr>
<th></th>
<th>Laissez-faire</th>
<th>Basel I</th>
<th>Basel II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loan rate in state s</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_l^*$</td>
<td>0.8</td>
<td>1.3</td>
<td>1.3</td>
</tr>
<tr>
<td>$r_h^*$</td>
<td>2.5</td>
<td>3.2</td>
<td>3.3</td>
</tr>
<tr>
<td>Capital in state s</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_l^*$</td>
<td>4.2</td>
<td>6.7</td>
<td>6.9</td>
</tr>
<tr>
<td>$k_h^*$</td>
<td>3.4</td>
<td>6.3</td>
<td>6.7</td>
</tr>
<tr>
<td>Capital buffer in state s</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta_l = k_l^* - \gamma_l$</td>
<td>4.2</td>
<td>2.7</td>
<td>3.8</td>
</tr>
<tr>
<td>$\Delta_h = k_h^* - \gamma_h$</td>
<td>3.4</td>
<td>2.3</td>
<td>1.2</td>
</tr>
<tr>
<td>Expected credit rationing in state $s'$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(s, s') = (l, l)$</td>
<td>3.2</td>
<td>2.4</td>
<td>0.9</td>
</tr>
<tr>
<td>$(s, s') = (l, h)$</td>
<td>3.2</td>
<td>2.4</td>
<td>12.6</td>
</tr>
<tr>
<td>$(s, s') = (h, l)$</td>
<td>17.2</td>
<td>9.3</td>
<td>5.3</td>
</tr>
<tr>
<td>$(s, s') = (h, h)$</td>
<td>17.2</td>
<td>9.3</td>
<td>12.4</td>
</tr>
<tr>
<td>Unconditional</td>
<td>8.2</td>
<td>4.9</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Probability of bank failure

<table>
<thead>
<tr>
<th></th>
<th>Laissez-faire</th>
<th>Basel I</th>
<th>Basel II</th>
</tr>
</thead>
<tbody>
<tr>
<td>First period banks, $s = l$</td>
<td>3.17</td>
<td>0.20</td>
<td>0.16</td>
</tr>
<tr>
<td>First period banks, $s = h$</td>
<td>17.15</td>
<td>2.87</td>
<td>2.25</td>
</tr>
<tr>
<td>Unconditional</td>
<td>8.16</td>
<td>1.15</td>
<td>0.90</td>
</tr>
<tr>
<td>Second period banks, $s = l$</td>
<td>0.55</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>Second period banks, $s = h$</td>
<td>10.21</td>
<td>1.50</td>
<td>0.76</td>
</tr>
<tr>
<td>Unconditional</td>
<td>4.02</td>
<td>0.56</td>
<td>0.31</td>
</tr>
<tr>
<td>Unconditional, all banks</td>
<td>6.09</td>
<td>0.86</td>
<td>0.61</td>
</tr>
</tbody>
</table>

This table reports the results from numerically solving for the equilibrium of the model under the parameterization described in Table 3. Rows labeled ‘unconditional’ show weighted averages based on the unconditional probabilities of each state. Expected credit rationing in state $s'$ is the expected proportion of second period projects that cannot be undertaken because of banks’ insufficient lending capacity or failure. We report its unconditional mean as well as values conditional on the various combinations of the state of the economy in the reference period $s'$ and in the previous period $s$. When reporting the probabilities of bank failure, ‘first period banks’ and ‘second period banks’ refer to banks funding first and second period projects, respectively.

5.1 Loan rates

Initial loan rates are always higher in the high default state $h$, reflecting the need to compensate banks for both a higher probability of default and a lower prospective profitability.
of continuation lending (since $q_{hh} = 0.64 > 0.20 = q_{lh}$, so the high default state $h$ is more likely to occur after state $h$ than after state $l$). The loan rates obtained under Basel I and Basel II are virtually identical (but clearly higher than those emerging in the laissez-faire regime) because the average capital effectively used by the representative bank in its two lending periods ends up being very similar in both regimes.\footnote{Specifically, as further commented below, first period capital decisions are very similar, while second period capital coincides with the regulatory minimum, whose average across states in Basel II has been set in the calibration equal to the 4\% requirement of Basel I. This is in line with previous results obtained in a static framework (Repullo and Suarez, 2004).}

### 5.2 Capital buffers

As shown in the second and third panel of Table 4, the model produces positive capital buffers even in the laissez-faire regime. The rationale for these buffers is to preserve banks’ future lending capacity. For instance, the unregulated first period bank in the low default state $l$ chooses a capital-to-loans ratio of 4.2\% and does so for the sole purpose of preserving its capacity to make profitable loans in the second period (which, in its case, only requires not failing in the first period). Interestingly, the capital chosen by this bank as a buffer in the high default state $h$ (in spite of the much higher probability of failure) falls to 3.4\% because, given the persistence of each state, second period lending is expected to be less profitable and hence less worthy to preserve.

The buffers are also positive, though not as sizable, in the Basel I and Basel II regimes. The average buffers are very similar in these two regimes but their cyclical pattern is markedly different. Even though Basel II would allow a first period bank in state $l$ to operate with $\gamma_l = 3.2\%$ of Tier 1 capital, the bank chooses a capital of 6.9\%, which is even higher than the 6.7\% that this bank would choose in the Basel I regime under a requirement of 4\%. In addition to the desire to preserve lending capacity if state $l$ still prevails in the next period, this decision reflects the optimal response to anticipating that, if the economy switches to state $h$, the capital requirement will increase to $\gamma_h = 5.5\%$. Symmetrically, the buffer in state $h$ is significantly smaller under Basel II than under Basel I because the bank anticipates that if the economy switches to state $l$ the capital requirement will be lower, while if it remains
in state $h$ its lending will not be so profitable after all.

## 5.3 Credit rationing

The results regarding credit rationing allow us to visualize the magnitude of the concern that leads banks to hold capital buffers. In spite of the equilibrium buffers, credit rationing is significant, especially when the economy comes from or ends up in a high default state. In the laissez-faire and Basel I regimes, credit rationing does not depend on the arrival state $s'$ (since the capital requirement does not vary across states) but just on bank profits in the previous period, whose distribution depends on the departure state $s$. Realizations of the default rate that leave banks with insufficient lending capacity are more likely when $s = h$. Consequently, expected credit rationing in the Basel I (laissez-faire) regime is $2.4\%$ ($3.2\%$) after a low default state period and $9.3\%$ ($17.2\%$) after a high default state period.

In the Basel II regime, the impact of loan defaults on banks’ lending capacity is also present, but the overall effects are dominated by the cross-state variation in capital requirements: the two sequences ending with $s' = h$ exhibit the largest credit rationing (slightly above $12\%$). In contrast, credit rationing in the sequence $(s, s') = (l, l)$ is only $0.9\%$. Unconditionally, the laissez-faire regime produces the largest credit rationing ($8.2\%$), followed by Basel II ($5.6\%$) and Basel I ($4.9\%$).

Thus, the main difference between Basel I and Basel II lies in the distribution of credit rationing across state sequences: Basel II produces a larger average supply of credit during long expansion periods ($(l, l)$ sequences) as well as a much lower average supply of credit when the economy enters recession ($(l, h)$ sequences) and while the recession lasts ($(h, h)$ sequences). In other words, Basel II amplifies the impact of the business cycle on credit supply.\(^{35}\)

\(^{35}\)The good news is that Basel II may help sustain a larger supply of credit during the process of recovery (i.e. along $(h, l)$ sequences).
5.4 Bank failure probabilities

In spite of their capital buffers, banks in our economy are more likely to fail in their first lending period than in their second lending period. This is due to the incidence of the setup cost $\mu$ as well as the fact that first period loan rates are competitive, while second period rates are monopoly rates (because of the hold-up problem). Conditional bank failure probabilities are, realistically, closely related to the loan default cycle: in the Basel regimes, banks are between 15 and 50 times more likely to fail in the high than in the low default state. The laissez-faire regime involves an average probability of failure (6% per year) much higher than Basel I (0.86%) or Basel II (0.61%). Basel II implies greater bank solvency than Basel I because it concentrates the protection coming from bank capital in the high default state. In combination with the results on credit rationing, these results point to a non-trivial welfare comparison between the two Basel regimes that we will investigate in Section 6.

5.5 Understanding the forces behind the results

To further understand the forces driving banks’ equilibrium capital decisions in the first lending period, which are key to the overall results, this section discusses the effects of changing two parameters that play an important role in the underlying optimization. In the interest of space, we focus on the Basel II regime. The first column of Table 5 reproduces the equilibrium outcomes obtained under our baseline parameterization.

The second column shows the results for the scenario in which the excess cost of bank capital is raised from 8% to 9%. This change reduces the profitability of second period lending as well as the direct cost of holding a capital buffer in the first period. Banks react by reducing the capital buffer in both states, which produces a strong impact on credit rationing, especially when ending up in the high default state ($CR_{lh}$ rises from 12.6% to 20.2% and $CR_{hh}$ from 12.4% to 14.3%). The reduction in capital buffers increases the probability of failure of first period banks. And the unconditional probability of bank failure rises from 0.61% to 0.68%.
Table 5. Effect of the excess cost of bank capital and the persistence of the low default state on equilibrium outcomes under Basel II
(all variables in %)

<table>
<thead>
<tr>
<th></th>
<th>Baseline parameterization</th>
<th>Higher cost of bank capital</th>
<th>Higher duration of state l</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loan rate in state (s)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(r^*_l)</td>
<td>1.3</td>
<td>1.4</td>
<td>1.3</td>
</tr>
<tr>
<td>(r^*_h)</td>
<td>3.3</td>
<td>3.4</td>
<td>3.3</td>
</tr>
<tr>
<td>Capital in state (s)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(k^*_l)</td>
<td>6.9</td>
<td>6.4</td>
<td>6.7</td>
</tr>
<tr>
<td>(k^*_h)</td>
<td>6.7</td>
<td>6.4</td>
<td>6.7</td>
</tr>
<tr>
<td>Capital buffer in state (s)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Delta_l = k^*_l - \gamma_l)</td>
<td>3.8</td>
<td>3.3</td>
<td>3.5</td>
</tr>
<tr>
<td>(\Delta_h = k^*_h - \gamma_h)</td>
<td>1.2</td>
<td>0.1</td>
<td>1.2</td>
</tr>
<tr>
<td>Expected credit rationing in state (s')</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>((s, s') = (l, l))</td>
<td>0.9</td>
<td>1.3</td>
<td>1.2</td>
</tr>
<tr>
<td>((s, s') = (l, h))</td>
<td>12.6</td>
<td>20.2</td>
<td>18.2</td>
</tr>
<tr>
<td>((s, s') = (h, l))</td>
<td>5.3</td>
<td>6.1</td>
<td>5.3</td>
</tr>
<tr>
<td>((s, s') = (h, h))</td>
<td>12.4</td>
<td>14.3</td>
<td>12.4</td>
</tr>
<tr>
<td>Unconditional</td>
<td>5.6</td>
<td>7.3</td>
<td>5.9</td>
</tr>
</tbody>
</table>

Probability of bank failure

<table>
<thead>
<tr>
<th></th>
<th>Baseline parameterization</th>
<th>Higher cost of bank capital</th>
<th>Higher duration of state l</th>
</tr>
</thead>
<tbody>
<tr>
<td>First period banks, (s = l)</td>
<td>0.16</td>
<td>0.22</td>
<td>0.20</td>
</tr>
<tr>
<td>First period banks, (s = h)</td>
<td>2.25</td>
<td>2.55</td>
<td>2.25</td>
</tr>
<tr>
<td>Unconditional</td>
<td>0.90</td>
<td>1.05</td>
<td>0.85</td>
</tr>
<tr>
<td>Second period banks, (s = l)</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Second period banks, (s = h)</td>
<td>0.76</td>
<td>0.76</td>
<td>0.76</td>
</tr>
<tr>
<td>Unconditional</td>
<td>0.31</td>
<td>0.31</td>
<td>0.28</td>
</tr>
<tr>
<td>Unconditional, all banks</td>
<td>0.61</td>
<td>0.68</td>
<td>0.56</td>
</tr>
</tbody>
</table>

This table has the same structure as Table 4 and reproduces in its first column the equilibrium outcomes obtained in the Basel II regime under the baseline parameterization of the model. The second column reports the Basel II outcomes when the excess cost of bank capital is raised from its baseline value of 8 percent to 9 percent. The third column reports the results obtained when changing the baseline value of the transition probability \(q_{ll}\) so as to lengthen the expected duration of the low default state from 5 to 6 years (i.e. we set \(q_{ll} = 0.833\) rather than \(q_{ll} = 0.8\)).

The third column in Table 5 reports the results obtained when the transition probability \(q_{ll}\) is raised so as to lengthen the expected duration of the low default state from 5 to 6 years. This shift decreases the risk that banks raising capital in the low default state face higher capital requirements (and a reduction in the profitability of lending) during their second
lending period. The change in $q_l$ only affects the outcomes obtained over state sequences initiated in the low default state. Banks’ equilibrium buffer in such state falls from 3.8% to 3.5%, which implies that expected credit rationing rises from 12.6% to 18.2% if the economy switches to the high default state (and from 0.9% to 1.2% if the economy remains in the low default state). The probability of failure of first period banks in the low default state increases slightly (from 0.16% to 0.20%) but unconditionally banks’ average solvency rises since the economy is less likely to visit the state in which the risk of bank failure is the highest.

6 Welfare analysis

In our risk-neutral economy, social welfare can be measured by the sum of the expected net present value of the income flows that the various agents extract, directly or indirectly through bank claims, from the funding or ownership of entrepreneurs’ investment projects. These income flows have been already presented in prior sections, with two exceptions which play a key role in the normative results. First, we are going to consider that bank failures cause negative externalities that amount to a loss of social welfare equal to a proportion $c$ of the initial assets of the failed banks. We will present the welfare comparison of the various regulatory regimes and find the welfare maximizing values of the minimum capital requirements $\gamma_l$ and $\gamma_h$ for values of $c$ ranging from 0% to 60%.\footnote{Laeven and Valencia (2008, 2010) provide a discussion and an empirical assessment of the social costs of bank crises. They differentiate between the direct costs of bank resolution, the overall deterioration of public finances (measured by the amount of government debt), and the output losses in the recessions normally following a bank crisis. They report the costs (which vary widely across various crisis episodes around the world) as a percentage of GDP. Translating their numbers into our setup is not direct, since the ratio of bank assets to GDP varies significantly over time and across countries. Skepticism about the appropriate choice of $c$ leads us to present the results for the range $c \in [0, 0.60]$.}

We are also going to consider that entrepreneurs extract more from their investment projects than just the residual part of the pledgeable success return left after repaying the bank loans. This is consistent with corporate finance and control theories that emphasize the role of control rents (Hart, 1995) and show that incentive problems give rise to (endogenous) fractions of corporate value that cannot be pledged to outside investors (Tirole,
2005). We will assume that entrepreneurs appropriate a non-pledgeable income \( b \) per period whenever their investment projects get developed and succeed. The practical implication of this parameter is to introduce an extra cost associated with credit rationing. Absent direct empirical estimates of this parameter, we set \( b = a = 0.04 \), which implies that the overall net present value generated by the investment projects is roughly twice as large as if only pledgeable returns were taken into account.\(^{37}\)

Depositors are fully protected by deposit insurance and receive their required rate of return (normalized to zero) with probability one, so their stake in social welfare is just zero and we can ignore their payoffs in the welfare calculations. Similarly, by the zero net present value condition (8) that characterizes equilibrium, bank shareholders break even on expectation over any whole two-period interval following a recapitalization of their banks, so we can also ignore their payoffs. This leaves us with entrepreneurs (as the projects’ residual claimants) and the government (as insurer of bank deposits and internalizer of the social cost of bank failures) as the only two relevant classes of agents with a non-trivial stake in social welfare.

Assuming the entrepreneurs and the government discount their payoffs at the risk-free deposit rate (that we have normalized to zero), it is convenient to think of the social welfare criterion as the expected net present value of the payoffs that accrue to entrepreneurs and the government in connection with the undertaking and funding of the projects of a given cohort of entrepreneurs. In parallel to the expressions used in Section 3, we will provide expressions for welfare and its various components conditional on the states \( s \) and \( s’ \) faced by the reference cohort of entrepreneurs in their first and second investment periods.

Therefore, social welfare over the investment sequence \((s, s')\) can be written as:

\[
W_{ss'} = U_{ss'} + DI_{ss'} + BF_{ss'},
\]  \( (14) \)

\(^{37}\)Our results for \( b = 0 \) (available from the authors upon request) suggest that in the absence of a significant non-pledgeable component in projects’ returns, the social costs of capital requirements due to credit rationing are overwhelmed by the social cost of bank failures, tilting the welfare balance strongly in favor of a risk-based regime such as Basel II, but with higher capital requirements in each state.
where
\[ U_{ss'} = (1 - p_s)(a - r_s^* + b) + (1 - CR_{ss'})(1 - p_s')b \] (15)
are the expected payoffs of the entrepreneurs over their two investment periods, inclusive of non-pledgeable returns,
\[ DI_{ss'} = \int_{\tilde{x}_s}^{1} k'_s(x)dF_s(x) + (1 - CR_{ss'})\int_{\tilde{x}_s'}^{1} [\gamma_{s'} + a - x(\lambda + a)]dF_{s'}(x). \] (16)
are the (negative) payoffs to the government stemming from its role as insurer of bank deposits,
\[ BF_{ss'} = -c\{1 - F(\tilde{x}_s) + (1 - CR_{ss'})[1 - F(\tilde{x}_{s'})]\} \] (17)
are the (negative) payoffs due to the social cost of bank failures, and \( \tilde{x}_{s'} = (\gamma_{s'} + a)/(\lambda + a) \) is the threshold default rate above which second period banks fail.

To explain (15), notice that the first term accounts for the payoff of entrepreneurs’ first period projects, which comprise a pledgeable return \( (1 + a) - (1 + r_s^*) = a - r_s^* \) as well as a non-pledgeable return \( b \) if their projects succeed. The second term accounts for the non-pledgeable return \( b \) obtained from second period projects insofar as they are undertaken and succeed. In (16) the two terms account for expected deposit insurance payouts associated with banks involved in first and second period projects, respectively (which are obtained by integrating banks’ net worth at the end of the corresponding period over the realizations of the default rate for which the banks fail). Finally, (17) is the expected social cost of bank failure obtained by multiplying the proportional cost \( c \) by banks’ average assets and their probabilities of failure in each of the two lending periods.

Our measure of social welfare \( W \) is the expected value of \( W_{ss'} \) over the four possible sequences \((s, s')\) weighted by their ergodic probabilities. Our optimal capital requirements, \( \gamma^*_l \) and \( \gamma^*_h \), are the values of \( \gamma_l \) and \( \gamma_h \) that maximize \( W \).\(^{38}\)

\(^{38}\)To avoid computational problems associated with the possible existence of multiple local maxima, we find \((\gamma^*_l, \gamma^*_h)\) by evaluating \( W \) over a fine and wide grid of possible values of \( \gamma_l \) and \( \gamma_h \).
Figure 1. Social welfare vs. the social cost of bank failure

This figure depicts our measure of social welfare $W$ as a function of the social cost of bank failure $c$ for each of the four regulatory regimes that we compare: the laissez-faire regime, Basel I, Basel II, and the regime with optimal capital requirements. The underlying parameterization is described in Table 3.

Figure 1 depicts $W$ as a function of the social cost of bank failure and for each of the regulatory regimes examined in Section 5 as well as under the optimal capital requirements. The comparison with the laissez-faire regime shows that capital regulation adds to social welfare even in the polar scenario where the social costs of bank failure represented by $c$ are zero, which means that the loss of entrepreneurial surplus associated with credit rationing provides a rationale for imposing banks a requirement of capital above the amounts that they would voluntarily choose in the absence of regulation. The social welfare associated with the laissez-faire regime falls quite dramatically with $c$ due to the high probability of failure that characterizes such a regime. The linear fall of social welfare with $c$ in the Basel regimes is a direct result of the probabilities of bank failure associated with each of them.
(lower in Basel II than in Basel I). In the regime with optimal capital requirements social welfare falls less than proportionally with \( c \) because \( \gamma_l^* \) and \( \gamma_h^* \) increase as \( c \) falls, offering greater protection against bank failure.

The impact of the excess cost of bank capital and the cost of credit rationing on the welfare comparisons is visible at various levels. The cost of credit rationing explains why Basel I (which, as previously discussed implies essentially the same average levels of capital and, hence, the same overall cost of bank capital as Basel II) slightly dominates Basel II for very low values of \( c \) (less than 5%): the small discrepancy is the net result of the better performance of Basel I in terms of credit rationing and its worse performance in terms of bank failure risk. Opposite to our priors when initiating this research project, the welfare losses due to credit rationing affect very little the comparison between Basel I and Basel II, which is mainly driven by the value of the social cost of bank failure.

Figure 2 depicts the optimal capital requirements \( \gamma_l^* \) and \( \gamma_h^* \) over the same range of values of \( c \) as in Figure 1, together with the Basel II requirements \( \gamma_l = 3.2\% \) and \( \gamma_h = 5.5\% \). The cost of bank capital (relative to the low incidence of credit rationing at the implied equilibrium levels of capital) explains why for low values of \( c \) (less than 25%), the socially optimal capital requirements would be comparatively much more cyclical and lower in level than the requirements of Basel II. For instance, with \( c = 0 \), the socially optimal \( \gamma_l^* \) is zero, while \( \gamma_h^* \) is close to 4%. The picture changes as \( c \) increases: the goal of reducing banks’ probability of failure in state \( l \) as well as in preserving their capacity to lend when the economy switches to state \( h \) makes the optimal capital requirements higher in level and less cyclically varying. Curiously, under our calibration, the optimal level and degree of cyclical dependence of the optimal requirements virtually coincide with those associated with Basel II when the social cost of bank failure amounts to about 25% of bank assets. For larger values of \( c \), \( \gamma_l^* \) and \( \gamma_h^* \) are higher, but less cyclically varying, than their Basel II counterparts.
This figure depicts the optimal capital requirements for different values of the social cost of bank failure $c$. The horizontal lines correspond to the (Tier 1) capital requirements of Basel II, which are included to facilitate the comparison. The underlying parameterization is described in Table 3.

Figure 2 provides an interesting preliminary assessment of the direction taken in Basel III. As described in BCBS (2010), this new international agreement on regulatory standards reinforces capital regulation by means of higher requirements of core Tier 1 capital and by complementing them with a capital preservation buffer and a countercyclical buffer. The idea behind these mandatory buffers is to force banks to build them up in good times (state $l$) but allowing their use in bad times (state $h$). Under the lens of our model, this new regulation would be consistent with a social welfare maximizing choice under the assessment that the social cost of bank failure is at or above the highest levels of $c$ depicted in Figure 2. It should be noted that for these levels of the social cost of bank failure the optimal capital requirements have much less cyclical variability than the Basel II requirements, which
supports the introduction of the new mandatory buffers.\(^{39}\)

7 Discussion

In this section we discuss some simplifying features of our model, including the distribution of banks’ market power in the first and the second lending periods, the use of short-term loans, and the assumption that banks with ongoing relationships have no access to the equity market. We consider the possible effects of relaxing these assumptions, placing the emphasis on implications regarding capital buffers and credit rationing, which are the most distinctive features of our model.

7.1 Competition and market power

In our model banks are perfectly competitive in the market for first period loans and act as monopolists in the market for second period loans. These assumptions guarantee tractability and internal consistency. First, we avoid the complications associated with modeling imperfect competition in the market for initial loans. In such a scenario, loans from different banks would not be perfect substitutes for the initial borrowers, and banks could extract positive surplus from starting lending relationships with specific borrowers, in which case the assumption that all new relationships go to banks that can unrestrictedly raise capital would be less justified. On the one hand, some initial borrowers might find that their most preferred banks suffer capital constraints. On the other, banks would have to take the value of these prospective relationships into account when deciding their capital buffers. Ignoring these complications does not seem to obviously bias our results in one direction or another, while it is clear that adding them would break the simple OLG structure of the model.

As for second period loans, we have assumed that borrowers are fully locked in to their initial lender. This is consistent with the existence of an (unmodeled) asymmetric information problem (e.g., one that makes borrowers searching for a new bank after one period look like lemons in the refinancing market). If the market for continuation loans were more

\(^{39}\) See Repullo and Saurina (2011) for a critique of the use of the credit-to-GDP gap in the design of the countercyclical capital buffer of Basel III.
competitive, the degree of effective competition and the resulting loan rates would vary with
the lending capacity of the banks involved. Even in the polar case of perfect competition,
banks would be able to appropriate scarcity rents from their (non-rationed) borrowers when
rationing emerges in equilibrium. Relative to the current modeling, a more competitive
market for second period loans would entail lower continuation rents for banks, and hence
lower incentives to keep capital buffers. So our assumption yields a conservative estimate of
the procyclicality induced by capital regulation.

7.2 Short-term loan contracts

We have described the relationship between entrepreneurs and banks as instrumented by a
sequence of one-period loans. One might wonder whether this builds in the “imperfection”
that drives our main results. The answer is yes and no. With long-term contracts there
might be room for improving over the credit allocation outcomes obtained in our analysis.
For example, for given capital buffers, setting higher loan rates in the first period and lower
loan rates in the second would reduce the incidence of credit rationing, since banks would
have more capital to support their second period lending. But in the context of our model,
long-term contracts pose important commitment problems. In particular, they would have to
specify the loan rates in the first and the second period, as well as the rationing scheme to be
used in those cases where the bank ends up with insufficient lending capacity (since otherwise
the bank might try to renegotiate the terms of the second period loans by threatening the
entrepreneurs with rationing). This means that default rates in the first period would have
to be verifiable, and banks would have to be restricted in their ability to pay dividends, since
lack of capital in the second period might otherwise be strategically used by banks to extract
a larger surplus from their locked-in borrowers.

\footnote{40Under our assumptions, either all banks or no bank are capital constrained in the market for second
period loans, so the second period loan rate would be either the monopoly rate $a$ or some break-even rate
that makes continuation lending a zero net present value investment for the banks.}


7.3 Imperfect access to the equity market

The assumption that banks with ongoing relationships have no access to the equity market is obviously crucial for our results. With perfect, frictionless access to the equity market in the interim period, there would be no credit rationing among second period borrowers, except in the rare event that their bank fails (and it is not recapitalized by its shareholders). Banks in such a context would most likely hold no buffers, at least under the high capital requirements of Basel I or Basel II. Given the ample evidence on capital market imperfections, the key question is whether the specificities of our approach—that ties these imperfections to the informational asymmetries associated with relationship lending—drive the results.

A more general way of capturing capital market imperfections would be to assume that access can occur with some (exogenous) probability \( v < 1 \). Changes in \( v \) could then be used to evaluate the marginal effects of the friction on capital buffers and credit rationing. One could also explore situations in which \( v \) is contingent on the state \( s' \) of the economy at the interim date. This extension would probably reinforce our conclusions about the procyclical effects of risk-based capital requirements such as those of Basel II. If with some probability banks can access the equity market in the interim period, they would have lower incentives to keep capital buffers, so depending on parameter values the incidence of credit rationing could be higher.\(^41\)

7.4 Other extensions

The framework used in this paper could also be extended in a number of other directions. First, we could consider lending relationships that extend over more than two periods. If relationships last for \( T \) periods and banks cannot raise equity for the whole length of the

\(^{41}\)One really strong assumption that we are making is that banks can frictionless access the equity market when they renew their stock of lending relationships. This assumption is instrumental to achieving a tractable OLG structure but has the unattractive feature of making first period borrowers immune to rationing. A modeling alternative would be to assume a structure similar to the one in the popular Calvo (1983) model of staggered price setting, i.e., that in each period a fraction of the banks can issue new equity. In this context one would have to discuss the allocation of the newly born entrepreneurs to the existing banks. Would they demand loans to the recapitalizing banks only? If not, how would the pricing of the new loans be determined and what would be the effect on banks’ incentives to hold excess capital?
relationship, the qualitative results should be very similar to ours. Such a model would, of course, yield richer dynamics, as the effect of a shock would propagate over several periods. Second, we could incorporate cyclically-varying demand for loans. One easy way of introducing a downward sloping aggregate demand for loans would be to assume that entrepreneurs are heterogeneous in their opportunity cost of becoming active in the first period. The time-series variability in projects' success probabilities would tend to produce a larger demand for loans in the low than in the high default state. Further cyclical variability in this demand might be introduced by replacing the current success return $a$ for some $a_s$. Finally, one could allow for feedback effects from constrained to unconstrained entrepreneurs by letting $a_t = a(I_t)$ instead of $a$, where $a(I_t)$ is an increasing (and possibly concave) function and $I_t$ is the aggregate investment at date $t$. This would capture demand externalities or technological complementarities similar to those studied in endogenous growth theory.

Additional cyclical might be introduced by allowing for cyclical variation in the loss given default (LGD) parameter $\lambda$. Using data on bond defaults, Altman et al. (2005) find that LGDs are positively correlated with default rates, thus suggesting that $\lambda$ might be higher in state $h$ than in state $l$. Moreover, under the so-called “advanced Internal Ratings-Based (IRB) approach” of Basel II, banks must compute their capital requirements taking into account the estimate of $\lambda$ derived from their internal models. This means that in an advanced IRB regulatory environment capital requirements would exhibit even more cyclicality than in the “foundation IRB” environment that we have considered. Conceptually, the implications of a cyclically-varying $\lambda$ are thus very similar (both within and between regulatory regimes) to those of increasing the amplitude of the cyclical variation in the probability of default parameter $p_s$, which would certainly tend to exacerbate the cyclical effects that we find.

8 Conclusion Remarks

In many early views on the implications of Basel II it was standard to first recognize the potential cyclical effects of risk-based capital requirements and then qualify that, given than most banks held capital in excess of the regulatory minima, the practical incidence of this
problem was likely to be small if not negligible. After the experience of the crisis that started in 2007, the temptation is to presume that the buffers voluntarily chosen by the banks will never be sufficient to prevent the procyclicality of bank lending, and to conclude that a radical adjustment of the regulation is needed.

The early discussions on the topic unveiled two related misconceptions. The first misconception is that the holding of capital buffers means that capital requirements are “not binding.” Under a purely static perspective this would be tautologically true. Small changes in a non-binding constraint would typically not alter banks’ capital decisions. In a dynamic problem, however, this needs not be the case: banks may hold capital buffers because they wish to reduce the risk of facing a statically binding requirement in the future. Perhaps these precautions make future requirements “not binding” when the time comes, but clearly their presence alters banks’ capital decisions and the whole development of future events.

A second related misconception is to accept that the cyclical behavior of capital buffers observed under some particular regulatory regime can be extrapolated to other regulatory regime. The results in this paper suggest that the behavior of buffers can radically differ across regimes. Predictions obtained from reduced-form models estimated under a different regulatory regime do not resist the Lucas’ critique.

Our model makes it possible to evaluate the cyclical effects of bank capital regulation without incurring in these misconceptions. One key contribution of this paper is to show that the interaction of relationship lending (which makes some borrowers dependent on the lending capacity of the specific bank with which they establish a relationship) with frictions in banks’ access to equity markets (which makes some banks’ lending capacity a function of their historically determined capital positions and the capital requirements imposed by regulation) has the potential to cause significant cyclical swings in the supply of credit. Under a realistic parameterization, the model shows that the risk-based requirements of Basel II imply a much larger procyclicality in banks’ lending capacity than the flat requirement of Basel I. Specifically, despite Basel II inducing banks to hold larger buffers during expansions (in order to prepare for the rise in capital requirements when entering a recession), the arrival of recessions is likely to produce sizeable credit rationing among borrowers dependent
on capital-constrained banks.

The second key contribution of the paper is to perform an explicit welfare analysis of the various capital regimes under consideration. Subject to all the caveats stemming from the many simplifying assumptions of the model (including abstracting from demand side fluctuations and aggregate feedback effects), we believe that our analysis helps identify (and give a tentative realistic quantification to) some of the key trade-offs relevant for the assessment of how different capital regulation regimes compare in the presence of procyclical effects, and for the design of optimal capital requirements.\footnote{To overcome some of these limitations, one could take our model as a building block for a fuller dynamic general equilibrium model with a production sector partly composed of entrepreneurial firms that rely on relationship bank lending. One could also think about extensions, such as those suggested in Section 7.3, that generalize our modeling of the frictions related to banks’ access to equity financing.}

We present our welfare results for a wide range of values of the social cost of bank failure (a parameter with which we capture the externalities behind regulators’ concerns about bank solvency). Opposite to our initial conjecture, we find that Basel II dominates Basel I in terms of social welfare for all except very low levels of such cost. This result is a call for caution against the simple claim that if regulation induces cyclicality it needs to be radically adjusted: the adjustment is not a free lunch. In fact, when the social cost of bank failure is around 25\% of the initial assets of the failed banks, Basel II implies a cyclical variation in the capital requirements very similar to that of the socially optimal ones. For larger values of the social cost of bank failure, our results suggest that Basel III points in the right direction: imposing higher capital requirements, but making them exhibit a lower degree of cyclical variation that those in Basel II—a challenging goal left to the operation of the new capital conservation and countercyclical buffers.
Appendix

Proof of Proposition 2 The existence of a solution to the bank’s capital decision problem follows, by Weierstrass theorem, from the fact that the net present value $v_s(k_s, r_s)$ is continuous in $k_s$ for any given interest rate $r_s$. The function $v_s(k_s, r_s)$ may be written as

$$v_s(k_s, r_s) = qd v_{sf}(k_s, r_s) + qsh v_{sh}(k_s, r_s), \tag{18}$$

where

$$v_{ss'}(k_s, r_s) = \frac{1}{1 + \delta} \left[ \int_0^\infty \pi_{s'} + k_s'(x) - \gamma_{s'} \right] dF_s(x) + \frac{\pi_{s'}}{\gamma_{s'}} \int_0^\infty k_s'(x) dF_s(x) - k_s. \tag{19}$$

Using the definitions of $k_s'(x)$ in (2), $\tilde{x}_s$ in (3), and $\tilde{x}_{ss'}$ in (4), one can establish the following properties of function $v_{ss'}(k_s, r_s)$:\(^43\)

1. For $k_s \leq \mu - r_s$ we have $\tilde{x}_{ss'} < \tilde{x}_s \leq 0$, so

$$\frac{\partial v_{ss'}}{\partial k_s} = -1 < 0.$$

2. For $\mu - r_s < k_s \leq \mu - r_s + \gamma_{s'}$ we have $\tilde{x}_{ss'} \leq 0 < \tilde{x}_s$, so

$$\frac{\partial v_{ss'}}{\partial k_s} = \frac{\pi_{s'}}{(1 + \delta) \gamma_{s'}} F_s(\tilde{x}_s) - 1 \leq 0, \quad \text{and} \quad \frac{\partial^2 v_{ss'}}{\partial k_s^2} = -\frac{\pi_{s'} F_s'(\tilde{x}_s)}{(1 + \delta) \gamma_{s'} (\lambda + r_s)} > 0.$$

3. For $\mu - r_s + \gamma_{s'} < k_s < \mu + \lambda + \gamma_{s'}$ we have $0 < \tilde{x}_{ss'} < 1$, so

$$\frac{\partial v_{ss'}}{\partial k_s} = \frac{1}{(1 + \delta) \gamma_{s'}} [\pi_{s'} F_s(\tilde{x}_s) - (\pi_{s'} - \gamma_{s'}) F_s(\tilde{x}_{ss'})] - 1 \leq 0,$$

and

$$\frac{\partial^2 v_{ss'}}{\partial k_s^2} = \frac{1}{(1 + \delta) \gamma_{s'} (\lambda + r_s)} [\pi_{s'} F'_s(\tilde{x}_s) - (\pi_{s'} - \gamma_{s'}) F'_s(\tilde{x}_{ss'})] \leq 0.$$

4. For $\mu + \lambda + \gamma_{s'} \leq k_s$ we have $1 \leq \tilde{x}_{ss'} < \tilde{x}_s$, so

$$\frac{\partial v_{ss'}}{\partial k_s} = \frac{1}{1 + \delta} - 1 < 0.$$

Hence the function $v_{ss'}(k_s, r_s)$ is linearly decreasing or strictly convex for $k_s \leq \mu - r_s + \gamma_{s'}$, linearly decreasing for $k_s \geq \mu + \lambda + \gamma_{s'}$, and may be increasing or decreasing, and concave or convex for $\mu - r_s + \gamma_{s'} < k_s < \mu + \lambda + \gamma_{s'}$. Since $\gamma_l \leq \gamma_h$ implies $\mu - r_s + \gamma_l \leq \mu - r_s + \gamma_h$

\(^{43}\)Note that in (19) we do not have to worry about values of $\tilde{x}_s$ and $\tilde{x}_{ss'}$ smaller than 0 or greater than 1, because the distribution $F_s(x)$ has support $[0, 1].$
and $\mu + \lambda + \gamma_l \leq \mu + \lambda + \gamma_h$, it follows that the problem $\max_{\gamma_s \leq k_s \leq 1} v_s(k_s, r_s)$ cannot have an interior solution with $k_s \leq \mu - r_s + \gamma_l$ or with $k_s \geq \mu + \lambda + \gamma_h$. Hence if there is an interior solution it must involve $k_s \in (\mu - r_s + \gamma_l, \mu + \lambda + \gamma_h)$. Since $\hat{x}_{sl} > 0$ for $\mu - r_s + \gamma_l < k_s$, it follows that there is a positive probability $F_s(\hat{x}_{sl})$ that the bank has excess lending capacity in state $l$ (and given that $\hat{x}_{sl} > \hat{x}_{sh}$ possibly also in state $h$). Also, since $\hat{x}_{sh} < 1$ for $k_s < \mu + \lambda + \gamma_h$, it follows that there is a positive probability $1 - F_s(\hat{x}_{sh})$ that the bank has insufficient lending capacity in state $h$ (and given that $\hat{x}_{sl} > \hat{x}_{sh}$ possibly also in state $l$). Finally, assuming that $\mu + \lambda + \gamma_h < 1$ (in our calibration $\mu + \lambda + \gamma_h = 0.535$), the fact that $v_s(k_s, r_s)$ is decreasing in $k_s$ for $k_s \geq \mu + \lambda + \gamma_h$ implies that there cannot be a solution at the corner $k_s = 1$. ■

**Proof of Proposition 3** By the theorem of the maximum, the function $v_s(k_s(r_s), r_s)$, where $k_s(r_s)$ denotes banks’ optimal choice of $k_s$ given $r_s$, is continuous in $r_s$. Moreover, we have

$$\frac{dv_s}{dr_s} = \frac{\partial v_s}{\partial k_s} \frac{dk_s}{dr_s} + \frac{\partial v_s}{\partial r_s}. $$

When $k_s(r_s)$ is interior the first term is zero, by the envelope theorem, and the second is positive, because $r_s$ has a positive impact on $k'_s(x_s)$ and consequently on $v_{s,s'}(x_t)$. When $k_s(r_s)$ is at the corner $\gamma_s$ we have $dk_s/dr_s = 0$, while the second term is positive as in the previous case. Hence we have $dv_s/dr_s > 0$. Moreover, for sufficiently low interest rates we have $v_s(k_s(r_s), r_s) < 0$, while for $r_s = a$ Assumption 1 implies $v_s(k_s(r_s), r_s) > 0$. Hence we conclude that there is a unique $r^*_s$ that satisfies $v_s(k_s(r^*_s), r^*_s) = 0$. ■

**Comparative statics of the equilibrium loan rate** The sign of $dr^*_s/dz$ for $z = \alpha, \lambda, \mu, \delta, \gamma_l, \gamma_h, q_{sh}$ is obtained by total differentiation of (8):

$$\frac{\partial v_s}{\partial k_s} \frac{dk^*_s}{dz} + \frac{\partial v_s}{\partial r_s} \frac{dr^*_s}{dz} + \frac{\partial v_s}{\partial z} = 0. \tag{20}$$

When $k^*_s$ is interior, the first term in (20) is zero, by the envelope theorem. Hence we have

$$\frac{dr^*_s}{dz} = -\left(\frac{\partial v_s}{\partial r_s}\right)^{-1} \frac{\partial v_s}{\partial z}. \tag{21}$$

In the proof of Proposition 3 we have noted that $\partial v_s/\partial r_s > 0$, so the sign of $dr^*_s/dz$ is opposite to the sign of $\partial v_s/\partial z$. Similarly, in a when $k^*_s = \gamma_s$ we have $dk^*_s/dz = 0$ for all $z \neq \gamma_s$, in which case the first term in (20) is also zero and (21) obtains again. Finally, for $z = \gamma_s$, we have $dk^*_s/d\gamma_s = 1$, which implies

$$\frac{dr^*_s}{d\gamma_s} = -\left(\frac{\partial v_s}{\partial r_s}\right)^{-1} \left(\frac{\partial v_s}{\partial \gamma_s} + \frac{\partial v_s}{\partial k_s}\right),$$

41
where $\partial v_s / \partial k_s \leq 0$, since otherwise $k_s^* = \gamma_s$ would not be optimal. With these expressions in mind, the results in Table 1 can be immediately related to the (self-explanatory) signs of the partial derivatives of $v_s(k_s^*, r_s^*)$ that we summarize in Table A1 (and whose detailed expressions we omit, for brevity).

Table A1. Effects on the net present value of the bank

<table>
<thead>
<tr>
<th>$z$</th>
<th>$a$</th>
<th>$\lambda$</th>
<th>$\mu$</th>
<th>$\delta$</th>
<th>$\gamma_l$</th>
<th>$\gamma_h$</th>
<th>$q_{sh}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\partial v_s / \partial z$</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Comparative statics of the equilibrium capital When the equilibrium capital in state $s$ is at the corner $k_s^* = \gamma_s$, with $\partial v_s / \partial k_s < 0$, marginal changes in any parameter other than $\gamma_s$ have no impact on $k^*_s$, while obviously $dk^*_s / d\gamma_s = 1$. Thus, in what follows we focus on the more interesting interior solution case.\(^{44}\)

The sign of $dk^*_s / dz$ for $z = a, \lambda, \mu, \delta, \gamma_l, \gamma_h, q_{sh}$ is obtained by total differentiation of the first-order condition $\partial v_s / \partial k_s = 0$ that characterizes an interior equilibrium:

$$
\frac{\partial^2 v_s}{\partial k^2_s} \frac{dk^*_s}{dz} + \frac{\partial^2 v_s}{\partial k_s \partial r_s} \frac{dr^*_s}{dz} + \frac{\partial^2 v_s}{\partial k_s \partial z} = 0,
$$

(22)

where $\partial^2 v_s / \partial k^2_s < 0$ by the second-order condition, which implies

$$
\frac{dk^*_s}{dz} = - \left( \frac{\partial^2 v_s}{\partial k^2_s} \right)^{-1} \left( \frac{\partial^2 v_s}{\partial k_s \partial z} + \frac{\partial^2 v_s}{\partial k_s \partial r_s} \frac{dr^*_s}{dz} \right).
$$

Hence the sign of $dk^*_s / dz$ coincides with the sign of the first term in brackets, which has two components: the direct effect of $z$ on $k^*_s$ (for constant $r^*_s$) and the loan rate effect (due to the effect of $z$ on $r^*_s$). The signs of the direct effects shown in the first row of Table 2 coincide with the signs of the cross derivatives $\partial^2 v_s / \partial k_s \partial z$ summarized in Table A2 (whose detailed expressions we omit, for brevity).

\(^{44}\)The case with $k_s^* = \gamma_s$ and $\partial v_s / \partial k_s = 0$ is a mixture of both cases since, depending on the sign of the effect of the marginal variation in a parameter, the optimal decision might shift from being at the corner to being interior. A similar complexity may occur if the change in a parameter breaks some underlying indifference between an interior and a corner solution (or between two interior solutions). We will omit the discussion of these cases, for simplicity.
Table A2. Effects on the marginal value of capital

<table>
<thead>
<tr>
<th></th>
<th>( z = \alpha )</th>
<th>( \lambda )</th>
<th>( \mu )</th>
<th>( \delta )</th>
<th>( \gamma_h )</th>
<th>( \gamma_l )</th>
<th>( q_{sh} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\partial^2 v_s}{\partial k_s \partial z} )</td>
<td>+</td>
<td>?</td>
<td>-</td>
<td>-</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

The loan rate effects shown in the second row of Table 2 can be easily determined from the results summarized on Table 1 and the fact that \( \frac{\partial^2 v_s}{\partial k_s \partial r_s} \) is negative. To see the latter, one can verify that the way \( k_s \) and \( r_s \) enter in \( v_s(k_s, r_s) \) implies \( \frac{\partial^2 v_s}{\partial k_s \partial r_s} = \frac{\partial^2 v_s}{\partial k_s^2} < 0 \) (see the proof of Proposition 2 for some relevant intermediate expressions). Hence using (18) we conclude

\[
\frac{\partial^2 v_s}{\partial k_s \partial r_s} = q_{sl} \frac{\partial^2 v_{sl}}{\partial k_s \partial r_s} + q_{sh} \frac{\partial^2 v_{sh}}{\partial k_s \partial r_s} = q_{sl} \frac{\partial^2 v_{sl}}{\partial k_s^2} + q_{sh} \frac{\partial^2 v_{sh}}{\partial k_s^2} = \frac{\partial^2 v_s}{\partial k_s^2} < 0,
\]

where the last inequality follows from the second-order condition for an interior solution in \( k_s \).

The single common risk factor model and the Basel II formula

The derivation of (10) can be summarized as follows. Suppose that the project undertaken by entrepreneur \( i \) in a date in which the state of the economy is \( s \) fails if \( y_i < 0 \), where \( y_i \) is a latent random variable defined by

\[
y_i = \alpha_s + \sqrt{\rho_s} \ u + \sqrt{1 - \rho_s} \ \varepsilon_i,
\]

where \( \alpha_s \) is a state-contingent parameter that determines the mean of the latent variable, \( u \) is the single common risk factor, \( \varepsilon_i \) is an idiosyncratic risk factor, and \( \rho_s \in (0, 1) \) is a (potentially state dependent) parameter that determines the extent of correlation in project failures. Suppose further that \( u \) and \( \varepsilon_i \) are standard normal random variables, independently distributed from each other and over time, as well as, in the case of \( \varepsilon_i \), across projects. The probability of failure of the project of entrepreneur \( i \) in state \( s \) is \( p_s = \Pr (y_i < 0) = \Phi(-\alpha_s) \), since \( y_i \sim N(\alpha_s, 1) \), which implies \( \alpha_s = -\Phi^{-1}(p_s) \).

With a continuum of projects, the failure rate \( x \) (the fraction of projects that fail) will only be a function of the realization of the common factor \( u \). Specifically, by the law of large numbers, the effects of the idiosyncratic factors \( \varepsilon_i \) will be diversified away and \( x \) will coincide with the probability of failure of a (representative) project conditional on the state of the economy \( s \) and the realization of \( u \):

\[
x = g_s(u) = \Pr \left( -\Phi^{-1}(p_s) + \sqrt{\rho_s} \ u + \sqrt{1 - \rho_s} \ \varepsilon_i < 0 \mid u \right) = \Phi \left( \frac{\Phi^{-1}(p_s) - \sqrt{\rho_s} \ u}{\sqrt{1 - \rho_s}} \right).
\]
The cdf of the failure rate is \( F_s(x) = \Pr(g_s(u) \leq x) = \Pr(u \geq g_s^{-1}(x)) \) (since \( g'_s(u) < 0 \)), so using the definition of \( g_s(u) \) and the fact that \( u \sim N(0, 1) \) we get (10).

The idea behind the capital requirements associated with the Basel II formula is to require banks to have enough overall (Tier 1 + Tier 2) capital to cover their one-year ahead loan losses with a probability of 99.9%. In the context of our model, the implied capital requirement in state \( s \) is then \( \lambda F_s^{-1}(0.999) \), where \( \lambda \) is the loss given default and \( F_s^{-1}(0.999) \) is the 99.9% quantile of the distribution of the default rate. To obtain an explicit formula for this quantile, one can invert the cdf of the default rate \( x \) in state \( s \) as given by (10). This yields a formula very similar to that in (11) except for two differences. The first one is that the term in \( \rho_s \) that appears in (10) is replaced in (11) by the function \( \rho(p_s) \). This function was introduced by the Basel regulators on the basis of cross-sectional evidence pointing to default correlation being smaller for exposures with higher probabilities of default. The second difference is that (11) refers to the minimum regulatory requirement of Tier 1 capital only, which is just one half of the minimum overall (Tier 1 + Tier 2) capital requirement established in the Basel II formula.
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