STATE-CONTINGENT BANK REGULATION WITH UNOBSERVED ACTIONS AND UNOBSERVED CHARACTERISTICS

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Abstract

This paper studies bank regulation in the presence of deposit insurance, where banks have private information on their own ability and their investment strategy. Banks choose the mean and variance of their portfolio return. Regulators wish to control banks' risk choice, even though all agents are risk neutral and there are no deadweight costs of bank failure, because high risk adversely affects banks' ex ante incentives along other dimensions. Regulatory tools studied are capital requirements and return-contingent fines. Regulators can seek to separate bank types by offering a menu of contracts. We use numerical methods to study the properties of the model with two different bank types. Without fines, capital requirements only have limited ability to separate bank types. When fines are added, separation is much easier. Fine schedules and capital requirements are tailored to bank type. Low quality banks are fined when they produce high returns in order to control risk-taking behavior. High quality banks face fines on lower returns to prevent low-type banks from pretending they are high quality. Combining state-contingent fines with capital regulation significantly improves upon pure capital regulation.

JEL Codes: G28, G21
Keywords: Bank regulation, moral hazard and hidden information

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1 Introduction

This paper studies bank regulation in a model where deposit insurance induces a wedge between private decisions and the social optimum. In this model, each bank has private information on its ability and chooses both the mean and variance of its investment portfolio. The regulator can set capital requirements and impose state-contingent fines. Furthermore, the regulator may offer banks a menu of regulatory contracts. A key feature of this model is that banks that choose a lower portfolio variance also choose a portfolio with a higher mean. Thus, in contrast to our usual finance intuition, bank portfolio returns endogenously exhibit a “reverse mean-variance trade-off”. This feature can be exploited by the regulator to improve social welfare.

Often, the goal of regulation is described as “ensuring the safety and soundness of the banking system”. That is, the regulator seeks to reduce the overall risk of the bank sector. This goal is usually motivated by a desire to protect taxpayer liability, reduce failure resolution costs, or prevent systemic risk. We develop an alternative rationale for reducing bank risk that is complementary to, but distinct from, these standard rationales. In our paper, the cost to society of high failure risk is due to the way high risk distorts the ex-ante incentives of banks. For example, White (1991) argues that the cost of the United States Savings and Loans crisis was not primarily the deadweight societal cost of resolving the failed thrift institutions ex post. Rather, it was the cost of poor investment decisions made by thrifts before the wave of thrift failures started in the mid-1980’s.

It is well known that many savings and loan institutions were technically insolvent in the early 1980’s because they held a large amount of low-interest mortgages made before the inflation of the late 1970’s but had to pay the much higher market rate of interest for short-term liabilities that existed in the early 1980’s. Mistaken attempts at deregulating the S&L’s without proper supervisory safeguards gave these insolvent thrifts the opportunity to increase their portfolio risk, in essence, to gamble for resurrection. White (1991) provides evidence that failed thrifts were more likely to have engaged in real estate lending and other new activities that were not in the traditional purview of thrifts.

In this paper, we argue that this sort of fall in the diligence with which banks construct their asset portfolio is associated with an increase in bank risk. More precisely, if banks were required (or induced) to reduce the variance of their portfolio, they would also tend to expend more effort increasing
the mean of their portfolio return. Thus, the welfare-maximizing regulator ought to be concerned about reducing bank risk, but not necessarily because risk per se is costly, but because reduced bank risk leads banks to make better investments, thereby increasing the mean output of the economy and enhancing aggregate welfare.

In our model, the regulator cannot control bank risk directly, because the distribution of banks’ portfolio returns is private information. This difficulty in determining banks’ ex ante return distributions, especially the return variance, is a fundamental practical problem for bank regulators. The Basel Committee on Bank Regulation continues to struggle with a practical way of measuring bank portfolio risk in its efforts to implement risk-based capital requirements. We capture this difficulty in an extreme way by assuming that risk is completely unobservable to the regulator. Therefore, risk must be controlled indirectly. The main regulatory tools available to do so in our model are state-contingent fines. These fines differ from most current regulatory practice, which relies primarily on non-state-contingent regulatory tools such as ex ante capital requirements. The Basel Accord of 1988 bases capital minimums on a crude risk-weighting of total assets held by a bank. Similarly, the Federal Deposit Insurance Corporation Improvement Act of 1991 (FDICIA) imposes capital requirements for U.S. banks. But recently, some state-contingent devices have become part of the regulatory tool kit. The prompt corrective action provisions of FDICIA can be viewed as state-contingent: if a bank’s capital falls below a minimal value, sanctions can be imposed, including closure. State-contingent tools play an even more prominent role in the “internal models approach,” a 1995 modification to the Basel accord that applies to the trading books of large money center banks. This approach allows the bank to set its regulatory capital level using the Value at Risk (VaR) estimate produced by the bank’s own risk model. Regulators backtest these models to determine if the bank’s model is adequate or if it is accurately reporting its results. If a bank’s model performs poorly, sanctions can be imposed. As argued by Rochet (1999), these checks introduce some state-contingency into the regulatory mechanism.1 Our model explic-

1The “pre-commitment approach” is another state-contingent mechanism. Under pre-commitment, banks would be allowed to choose their own level of capital but would be subject to a fine if this capital did not cover ex post losses. In the proposal, fines were to be used as the penalty but other penalties, such as increased capital or increased regulatory scrutiny, could be used as well (Kupiec and O’Brien (1995a,b)). This approach was actually put forth by regulators for public comment (Office of the Federal Register (1995)) but has
itly studies state-contingent regulation by allowing fines to depend on the return produced by the bank.

This paper builds on Marshall and Prescott (2001), who study state-contingent fines in a two-dimensional moral hazard model where both the mean and variance of the bank’s portfolio return is private information. They find that for lognormal distributions of returns it is optimal to impose fines on banks that produce extremely high returns. This seemingly perverse result is driven by the need to control risk taking. It is desirable to impose fines on return realizations with the highest deterrence effect per dollar of fines assessed in equilibrium. In the absence of limited liability, the optimal fine would be placed on the extreme left-hand tail of the distributions. However, with limited liability fines cannot be assessed on the left-hand tail so they are assessed on the right-hand tail instead.

An obvious issue raised by this result is that if some banks are higher quality than others, and if bank quality is private information, fines on high returns may simply punish the high quality banks rather than deterring risk-taking by low quality banks, and may even deter innovation. (See Boyd (2001)). By incorporating unobservable heterogeneity in bank types, our model can address this trade-off. The model contains both low and high quality banks. Low type banks have little incentive to expend effort to increase their portfolio quality. In contrast, high-type banks make the socially optimal effort choice even in the absence of regulation. We find that, as in Marshall and Prescott (2001), the optimal contract still imposes fines on high returns for the low-type banks. However, these fines are not imposed on the high-type banks because the regulator can separate types by offering a menu of contracts. Low-type banks choose the contract with high return fines, while high-type banks choose an alternative contract. However, because the regulator cannot observe bank type, the contracts on the menu must induce self selection. Consequently, some punitive measures need to be not been adopted.

We also build other papers in the literature on bank regulation in the presence of private information. Antecedent papers include Giammarino, Lewis, and Sappington (1993), Campbell, Chan, and Marino (1992), Boyd, Chang and Smith (1998, 2002), Nagarajan and Sealy (1998), Besanko and Kanatas (1996), and Matutes and Vives (2000). Also relevant is the small principal-agent literature on when the agent controls risk. See, for example, Palomino and Prat (2003).

The result is similar to that of Green (1984) who studied risk control in a non-banking environment.
included in the high-type contract, even though, in the absence of unobserved heterogeneity the high-type would be self-regulating. This must be done to convince the low-type to truthfully report their type. Furthermore, we find that the costs of private information about bank type are borne entirely by the higher quality bank. The lower-quality bank always receives at least as much utility as it would have under type-observability.

The problem is very difficult to analyze. There are two dimensions to the moral-hazard problem and there is private information on bank type. Furthermore, the capital requirement, while observable, affects incentives. Because of these complications, we explore the model by using numerical methods to solve and analyze specific parametric examples. We transform the problem into a linear program as in Myerson (1982), Prescott and Townsend (1984), and others. Even solving this linear program is not straightforward because of the large number of off-equilibrium strategies that need to be checked to preserve incentive compatibility. We use a method developed in Prescott (2003) that efficiently checks these strategies. The numerical methods are described in Appendix B.

The remainder of the paper is organized as follows: Section 2 develops the model. Section 3 derives some comparative static results on the connection between bank behavior and capital requirements. It also provides some conditions under which a decreased return variance induces a bank to increase its portfolio mean. Section 4 reports in detail the optimal contracts for a variety of parametric examples. The final section offers some concluding comments. Technical details can be found in the two appendices.

2 The Model

2.1 Households

There are two periods and a single consumption good. There is a continuum of risk-neutral households of measure one who consume in the second period.
only. The households own all the assets in the economy, consume all the output, and operate all the banks. Each household includes one “banker”, who is one of two types, low and high. Low-type bankers are bad at operating a bank while high-type bankers are good at bank operation. (We discuss the consequences of bank type more formally below.) Let $h_i$ denote the fraction of households with a type $i$ banker, for $i \in \{low, high\}$.

In the first period, a household receives an endowment of one unit of the consumption good. Each type of household must split this endowment between capital to use in its own bank and funds to deposit in other banks. Demand deposits pay off one unit of the consumption good in the second period for each unit invested in the first period. In addition to its pecuniary payoff, a unit of bank deposits provides liquidity services with utility value $\rho > 0$. All demand deposits are government insured, so the household is indifferent about which bank holds its deposits.

For simplicity, we assume that each bank can only be of size one.\footnote{This extreme span of control assumption is often made in the bank regulation literature. For example, see Boyd, Chang, and Smith (2002).} A type-$i$ bank funds its investments with deposits from outsiders $D_i \in [0, 1]$. The remaining portion of the investment, $1 - D_i$, is funded by the banker’s own funds. These own funds will be called capital. Since every bank must be the same size, the deposits made by the household must equal $D_i$, the deposits its bank takes from other households. We make the assumption that a household may not place deposits in its own bank. This assumption captures the idea that depositors do not monitor their bank because of deposit insurance.

We do not model the individual assets of a bank’s investment portfolio. Instead, we assume that the bank chooses the distribution of its portfolio return. Let $r$ denote the gross return accruing to a bank. This portfolio return has a cdf $F(\cdot|\mu, \sigma)$, where $F$ is a two-parameter family of probability distributions completely characterized by its mean $\mu$ and variance $\sigma^2$. For simplicity of exposition, we assume that there exists a pdf corresponding to $F(\cdot|\mu, \sigma)$. This pdf is denoted $f(\cdot|\mu, \sigma)$. For most commonly used two-parameter distributions, the value of $f(r|\mu, \sigma)$ is decreasing in $\mu$ for $r$ sufficiently small. Accordingly, we assume that there exists an $r^*(\mu, \sigma)$ such that

$$\frac{\partial f(r|\mu, \sigma)}{\partial \mu} < 0, \forall r < r^*(\mu, \sigma).$$

For example, equation (1) holds for the normal distribution with $r^*(\mu, \sigma) = \phi$.
it holds for the log normal distribution for an $r^* (\mu, \sigma) > \mu$.

The bank chooses two characteristics of the portfolio. The first is the portfolio standard deviation $\sigma$, which measures the bank’s risk choice. The second is the bank’s level of screening effort, $s \geq 0$. We think of screening effort as the amount of diligence applied in evaluating loans and other assets. Screening positively affects the mean of the distribution, denoted $\mu_i(s)$, where

$$\mu'_i (s) > 0 \quad (2)$$
$$\mu''_i (s) < 0. \quad (3)$$

The only difference between the two bank types is that a given amount of screening effort results in a higher mean return for the high types than for the low types. That is,

$$\mu_{\text{high}}(s) > \mu_{\text{low}}(s), \forall s.$$

The cdf of the return of bank of type $i$ that chooses screening $s$ and risk $\sigma$ will be denoted $F_i(\cdot | s, \sigma)$ where

$$F_i (r | s, \sigma) \equiv F (r | \mu_i(s), \sigma). \quad (4)$$

The pdf corresponding to this cdf will be denoted $f_i(\cdot | s, \sigma)$. Screening $s$ also has a utility cost $\gamma s$, with $\gamma > 0$.

In addition to the banks and households, there is a regulator who seeks to maximize social welfare. (We characterize explicitly the regulator’s objective in Section 2.2, below.) The regulator may impose fines, $g_i(r)$, as a function of the bank’s type and ex post return. These fines are non-negative, and are constrained by limited liability of the bank. Consequently,

$$0 \leq g_i(r) \leq \max\{0, r - D_i\}. \quad (5)$$

The expected payoff of the $i^{th}$ bank, denoted $v_i$, is

$$v_i \equiv \int_{D_i}^{\infty} [r - D_i - g_i(r)] f_i (r | s_i, \sigma_i) dr. \quad (6)$$

The household purchases consumption in the second period using its bank deposits (which are distinct from but equal in amount to the deposits received by its banker), plus the profits from its banker’s activities, less a lump sum tax (common across types) with expected value $T$ that is used by the deposit
insurer to pay off the depositors of failed banks. Therefore, the expected consumption for a household with bank type \(i\), denoted \(C_i\), is subject to the constraint

\[ C_i \leq D_i + v_i - T. \] (7)

Household utility is a linear function of consumption, liquidity services provided by deposits and the disutility of screening effort:

\[ C_i + \rho D_i - \gamma s_i. \] (8)

Using equations (6) and (7) we can write the bank’s utility function (8) in an alternative form:

\[
\int_0^{D_i} (D_i - r) f_i(r|s_i, \sigma_i) \, dr - \int_0^{D_i} g(r_i) f_i(r|s_i, \sigma_i) \, dr + \rho D_i + \mu_i(s_i) - \gamma s_i - T.
\] (9)

The first term in equation (9) has the form of the payoff to a put option with strike price \(D_i\). This term captures what is commonly referred to as the deposit insurance put option. In effect, deposit insurance gives the bank an option to put the bank to the deposit insurer in exchange for the insurer taking over the liability \(D_i\) owed to the depositors. The second term is the income lost from fines. The third term is the value of liquidity services received from deposits. The fourth term is the mean return to the bank’s portfolio. The fifth term is the lost utility due to screening. Notice that the household/banker takes taxes as given.6

2.2 Regulator

To cover the cost of bank failure, the regulator can use lump sum taxes and the fines it collects. We assume that fines are costly to collect, due to their

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6Static models of bank regulation often include a franchise value or charter value term in the bank’s objective function. (See, e.g., Keeley (1990), Marshall and Venkataraman (1999).) This term is a stand-in for the present value of the bank’s future operations, which is lost to the bank owners in the event of bankruptcy. Concern for lost franchise value acts as a disincentive to risk taking, and thus can offset the risk-encouraging effects of the deposit insurance put option. Franchise value could easily be incorporated into this model. We refrain from doing so in order to focus attention specifically on the way the deposit insurance put option distorts bank incentives.
punitive nature. In particular, there is a deadweight cost of $\tau \geq 0$ per unit fine collected. Therefore, taxes must satisfy

$$T = \sum_i h_i \left[ \int_0^{D_i} (D_i - r) f_i(r|s_i, \sigma_i) dr - \int_0^\infty (1 - \tau) g_i(r) f_i(r|s_i, \sigma_i) dr \right].$$

(10)

The regulator maximizes the share-weighted average of the ex ante utilities of the two types of households, as given in equation (9), subject to equations (5), (6), (7), and (10).\textsuperscript{7} However, the regulator takes into account the effect on utility of taxes $T$ in equation (9) while the bank takes them as exogenous. These taxes lay at the heart of the distortion caused by deposit insurance.

Suppose the regulator could observe banks’ types and control their choices of $\{D_i, s_i, \sigma_i\}$. Substituting equations (6), (7), and (10) into the utility function of each bank (equation (8)) and then weighing each type by their fraction of the population, one obtains the following expression for the regulator’s objective function

$$\sum_i h_i \left[ \rho D_i + \mu_i(s_i) - \gamma s_i - \int_0^\infty \tau g_i(r) f_i(r|s_i, \sigma_i) dr \right].$$

(11)

The object in square brackets in equation (11) is the utility of the type-$i$ household from the perspective of the regulator. Ignoring fines and taxes (the latter of which does not affect the bank’s decisions), the only difference between this expression and equation (9) is that equation (9) includes the payoff to the deposit insurance put option. Because of this difference in objective functions, an unregulated banks’ decisions would generally be socially suboptimal. As we shall see, banks have an incentive to take on too much leverage, too much risk, and apply insufficient screening effort.

2.3 Formal Statement of the Regulator’s Problem

In the most general specification of the model, the regulator observes deposits $D$ and the bank’s ex post return $r$, but the bank’s type $i$ and action pair $(s, \sigma)$ is private information. Thus, the regulator’s problem is to use instruments that condition on $D$ and $r$ to elicit information about bank type and to influence the bank’s action choices.

\textsuperscript{7}We do not address potential incentive problems with the regulator’s behavior.
Formally, the problem takes the following steps: First, banks send reports to the regulator on their type $i$. The content of these reports cannot be verified by the regulator so the bank can say anything. However, we know by the Revelation Principle that as long as we impose the right incentive constraints, we can restrict ourselves to a direct mechanism where a bank directly reports its type. Second, based on this report of $i$, the regulator sets a deposit level $D_i$, recommends a screening-risk pair $\{s_i, \sigma_i\}$, and sets a schedule of fines $g_i(r)$ that depends on the portfolio return $r$. The deposit level is interpreted as a capital requirement, since capital equals $1 - D_i$. We refer to a triplet of the deposit level, screening, and risk, $(D, s, \sigma)$, as an assignment. Third, in response to the assignment and fine schedule, the bank chooses its screening and risk levels (which need not equal the $(s, \sigma)$ pair recommended in the regulator’s assignment). Fourth and finally, the return is realized, fines and taxes are assessed, the depositors are paid off (either by the bank or the deposit insurer), and each household consumes.

Using the Revelation Principle, the regulator’s problem can be summarized as follows:

**Regulator’s Problem**

\[
\max_{D_i, s_i, \sigma_i, g_i(r)} \sum_i h_i \left( \rho D_i + \mu_i(s_i) - \gamma s_i - \int_0^\infty \tau g_i(r) f_i(r|s_i, \sigma_i) \, dr \right)
\]

subject to equation (5), the following truth-telling incentive constraints

\[
\int_0^{D_i} (D_i - r) f_i(r|s_i, \sigma_i) \, dr + \rho D_i + \mu_i(s_i) \\
-\gamma s_i - \int_0^\infty g_i(r) f_i(r|s_i, \sigma_i) \, dr - T \\
\geq \max_{s, \sigma} \int_0^{D_j} (D_j - r) f_i(r|s, \sigma) \, dr + \rho D_j + \mu_i(s) \\
-\gamma s - \int_0^\infty g_j(r) f_i(r|s, \sigma) \, dr - T, \, \forall i \in \{\text{low, high}\}, \, j \neq i,
\]
the following moral-hazard incentive constraints that for all $i$

$$\begin{align*}
(s_i, \sigma_i) &= \arg \max_{s, \sigma} \left[ \int_0^{D_i} (D_i - r) f_i(r|s, \sigma) dr + \rho D_i + \mu_i(s) \right] \\
&\quad - \gamma s - \int_0^\infty g_i(r)f_i(r|s, \sigma) dr - T ,
\end{align*}$$

(14)

and the constraint in equation (10) that lump-sum taxes cover the costs of bank failure resolution net of fines collected.

The moral hazard incentive constraints (14) require that, for the deposit level and fine schedule specified for each bank type, the recommended values of screening effort and risk are those that would be chosen by that type. The truth telling constraints (13) guarantee that banks truthfully report their type. The max operator on the right-hand side of equation (13) is needed because the optimal contract does not specify the off-equilibrium strategy to be used by a bank that lies. The utility from this strategy needs to be calculated to properly assess the value to a bank of misrepresenting its type. Finally, note that while $T$ is in both sets of incentive constraints it enters as a constant and has no effect on sets of feasible allocations that satisfy either constraint.

### 3 Some Useful Comparative Static Results

Before analyzing the complete Regulator’s Problem it is useful to look at the bank’s incentives in the absence of fines. These comparative static results indicate the direction in which the bank would change its actions in response to possible regulatory policies.

#### 3.1 Unregulated banks choose suboptimally low screening

Let us consider first the incentives of unregulated banks. In particular, we set $g_i(r) = 0, \forall r$. According to equation (11), the choice of $\sigma$ does not directly affect the value of the regulator’s objective in the absence of fines. Thus, the key concern of the regulator is to move the banks’ screening effort toward the social optimum. According to equation (11), the socially optimal screening level for bank $i$ is characterized by

$$\mu'_i(s_i) = \gamma ,$$

(15)
that is, the marginal increase in the mean return must equal $\gamma$, the marginal cost of screening. However, according to equation (9), the privately optimal screening level is characterized by

$$
\mu'_i(s_i) = \gamma - \int_0^{D_i} (D_i - r) \frac{\partial f_i(r|s_i, \sigma_i)}{\partial s} dr.
$$

We now show that if $D$ is not too big relative to the mean of the distribution then the bank’s choice of screening is strictly lower than the social optimum. Equations (15) and (16) differ by a term that captures the way the deposit insurance put option varies with $s$. Using equations (1), (2), and (4), we can sign this term, as follows:

$$
\int_0^{D_i} (D_i - r) \frac{\partial f_i(r|s_i, \sigma_i)}{\partial s} dr = \mu'_i(s_i) \int_0^{D_i} (D_i - r) \frac{\partial f_i(r|\mu_i(s_i), \sigma_i)}{\partial \mu} dr < 0 \text{ if } D < r^*(\mu, \sigma).
$$

According to equation (17), the left-hand side of (16) exceeds the left-hand side of (15) as long as the deposit level $D$ is not too big.\(^8\) If this condition holds then equation (3) implies that the value of $s_i$ ensuring equation (16) is strictly lower than the value of $s_i$ implied by equation (15). In other words, the unregulated bank’s screening choice is strictly less than the socially optimal screening level. The condition that $D$ is not too big is not too restrictive. For example, in the case of the normal or log normal family of distributions, inequality (1) holds for all $r < \mu$. Since $D \leq 1$ and the mean of the gross return exceeds unity (assuming that the bank expects a positive net return to its investments), then the condition that $D$ not be too big holds automatically.

If bank screening effort were observable, the regulator could presumably mandate the optimal screening level directly. However, throughout this paper we assume that screening effort is unobservable to the regulator. So how might the regulator induce the bank to increase its screening level? The conventional regulatory tool is increased capital. It turns out that mandating higher capital does indeed induce higher screening effort. A second, less obvious tool, which will be an important focus later in this paper, is to induce the bank to reduce its risk choice. In the following, we explore each of these approaches in turn.

\(^8\)In fact, the condition $D < r^*(\mu, \sigma)$ is sufficient, but not necessary for the left-hand side of equation (17) to be negative. All that is needed is for $D$ to be sufficiently small that the set of $r < r^*(\mu, \sigma)$ dominates the sign of the weighted integral in equation (17).
3.2 Inducing higher screening effort via capital regulation

Suppose fines are excluded from the available regulatory instruments, so the only regulatory tool is capital requirements. We show here that higher capital tends to induce higher screening. Recall that capital simply equals $1 - D$, so mandating increased capital is equivalent to mandating a lower value for $D$. Suppose the regulator sets $D$ and the bank then chooses $s$. The following comparative static result holds:

**Proposition 1:** Suppose $s > 0$, $D < r^*(\mu, \sigma)$. Then, holding $\sigma$ constant, the bank’s choice of $s$ is decreasing in $D$.

(Proof See Appendix A.)

According to Proposition 1, the regulator can induce banks to increase $s$ by requiring them to reduce $D$, in other words, by increasing capital.

3.3 How does the risk level affect screening effort?

For most of this paper we assume that the bank’s choice of risk, $\sigma$, is private information. But suppose for a moment that $\sigma$ were observable to the regulator, and that the regulator could mandate a $\sigma$ level for the bank. How would the regulator’s choice of $\sigma$ affect the bank’s choice of $s$? In this section we show that, for a wide range of specifications, $s$ would be decreasing in $\sigma$. That is, the regulator could induce higher screening (thereby offsetting the distortions induced by the deposit insurance put option) by reducing the bank’s portfolio risk.

To demonstrate this assertion, let us totally differentiate the bank’s first order condition (16) with respect to $\sigma$ and rearrange to get

$$\frac{\partial s}{\partial \sigma} = - \left[ \int_0^D (D - r) f_{s\sigma}(r|\sigma) dr \right] / \left[ \int_0^D (D - r) f_{ss}(r|\sigma) dr + \mu''(s) \right]$$

According to equation (4),

$$f_{s\sigma}(r|\sigma) = f_{\mu\sigma}(r|\mu, \sigma) \mu'(s)$$
so equation (18) can be written
\[
\frac{\partial s}{\partial \sigma} = \frac{-\int_0^D (D-r) f_{\mu \sigma} (r|s, \sigma) \, dr}{\int_0^D (D-r) f_{ss} (r|s, \sigma) \, dr + \mu'' (s)} \tag{19}
\]

According to second-order condition (25) in the Appendix A, the denominator of (19) is negative. So to sign \( \frac{\partial s}{\partial \sigma} \) we must determine the sign of the numerator of equation (19). This object cannot be signed unambiguously.

\( \mu' (s) > 0 \) by construction, so
\[
\frac{\partial s}{\partial \sigma} < 0 \text{ if and only if } \int_0^D (D-r) f_{\mu \sigma} (r|\mu, \sigma) \, dr < 0. \tag{20}
\]

The integral in equation (21) is the second (cross) derivative of the deposit insurance put option with respect to the mean and standard deviation of the return distribution. So equations (20) - (21) say that increased portfolio risk induces a reduction in screening effort if and only if the sign of this cross derivative is negative.

The sign of this cross derivative depends on the sign of \( f_{\mu \sigma} (r) \), which in turn depends on where \( r \) is located relative to the mean and standard deviation of the distribution, as well as on the shape of the distribution itself. It is a property of most commonly used two-parameter distributions that \( f_{\mu \sigma} (r) < 0 \) for all \( r \) sufficiently small. In particular, if \( f \) is normal, then
\[
f_{\mu \sigma} (r|\mu, \sigma) < 0 \text{ if } r < \mu - \sigma \sqrt{3}. \tag{22}
\]

We were unable to obtain a similar analytic result for the log normal distribution, but a grid search over a wide range of \( \mu \)'s and \( \sigma \)'s reveals that, for \( f \) lognormal, a sufficient condition for \( f_{\mu \sigma} (r|\mu, \sigma) < 0 \) is
\[
r < e^{E \log (r) - \sigma (\log (r)) \sqrt{3}}. \tag{23}
\]

More precisely, there exists an \( r^* (\mu, \sigma) \) satisfying
\[
\log [r^* (\mu, \sigma)] > E \log (r) - \sigma (\log (r)) \sqrt{3}
\]
s.t.
\[
f_{\mu \sigma} (r|\mu, \sigma) < 0 \text{ if } r < r^* (\mu, \sigma).
\]

In practice, we find that for most values of \( \{\mu, \sigma\} \), \( \log [r^* (\mu, \sigma)] \approx E \log (r) - \sigma (\log (r)) \sqrt{3} \).
where $E \log (r)$ and $\sigma (\log(r))$ denote the mean and standard deviation, respectively, of $\log (r)$. The analogy between equations (22) and (23) is obvious.

These results suggest that, unless $\mu$ is very small or $\sigma$ is very big, inequality (21) should hold. To check this conjecture, we numerically evaluate the left-hand side of inequality (21) on a grid of $\{\mu, \sigma, D\}$ combinations with support

\[
\begin{aligned}
\mu & \in [0.5, 2.5] \\
\sigma & \in [0.05, 1.0] \\
D & \in [0.1, 1.0]
\end{aligned}
\]

If we think of the return horizon as one year, these grids encompass the realistic cases. For the log normal distribution, inequality (21) holds for all values of $\{\sigma, D\}$ as long as $\mu \geq 1.2$ (that is, a mean net return of at least 20%). If $\mu \geq 1.0$ (positive mean net return), inequality (21) holds except for very high risk levels ($\sigma \geq 0.7$), and is there only when $D = 1$ (i.e., zero capital). We perform the same experiment with the beta distribution with support $(0, 3)$ that is used in Section 4.2, below.\(^{10}\) With this distribution, inequality (21) holds for all $\{\sigma, D\}$ in this grid as long as $1.0 \leq \mu \leq 2.0$ (that is, the mean net return is between 0 and 100%). Thus, $\frac{\partial s}{\partial \sigma} < 0$ for these two distributions when the mean and variance are in the empirically plausible region.

This section showed that, for a wide variety of plausible distributions, reducing the risk of a bank’s portfolio tends to increase its level of screening effort. But, for most of this paper we assume that the regulator cannot observe the bank’s portfolio risk. How then are the results of this section useful? While the regulator cannot mandate a risk level, the regulator may be able to *convince* the bank to choose lower risk by indirect means, such as ex post return-contingent fines. According to the results of this section, doing so will result in the bank choosing a higher screening level. This is what we find in our numerical simulations, described in Sections 4.3 and 4.4.2, below.

\(^{10}\)The standard beta distribution has support $(0, 1)$. As in our baseline numerical example of section 4.2, we assume in these experiments that the support on $r$ is $(0, 3)$, so the relevant density function $f (r|\mu, \sigma)$ is the standard beta evaluated at $\{r/3, \mu/3, \sigma/3\}$. 

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4 An Example

It is very difficult to characterize the Regulator’s Problem analytically. First, it contains a moral hazard problem with two dimensions (screening and risk) on which the bank chooses its hidden action. Second, the moral hazard is preceded by hidden information on bank quality. Third, the regulator also chooses a deposit level, which, while observable, complicates the problem because it directly affects incentive constraints.

Because of these difficulties, we adopt the strategy of solving numerical examples to learn about the properties of this model. In this section we first describe our approach to solving the model. We then use this approach to solve a set of examples that illustrate how fines are an effective way to control risk, and by extension, control screening. In particular, our examples illustrate that high-return fines can be used to deter risk-taking by the low-type banks whether or not bank type is public information.

4.1 Solving the model numerically

We solve the Regulator’s Problem numerically by formulating it as a linear program. Linear programs are an effective tool for computing solutions to mechanism design problems. They can be used to solve problems with arbitrary specifications of preferences and technologies. Furthermore, linear programs can be efficiently solved using widely available, high quality software.

To formulate the problem as a linear program, one first must discretize all underlying variables. In our model, we discretize returns \( r \), and fines \( g \), screening levels \( s \), risk choices \( \sigma \), and deposit levels \( D \). The regulator then chooses a joint probability distribution over the possible combinations of these variables. Embedded in this joint probability distribution is the terms of the regulatory contract. Thus, this solution method allows for the possibility of randomized contracts. Indeed, allowing for randomization is the key step in transforming the problem into a linear program. However, we will focus primarily on cases where the contracts are deterministic (that is, where the probability distribution associated with the optimal contract is

\[11\text{Myerson (1982) and Prescott and Townsend (1984) are early papers that set these problems up as linear programs. For a survey on the use of linear programming methods to solve mechanism design problems see Prescott (1999).} \]
The major complication with this approach is checking the truth-telling incentive constraints. As mentioned above, the truth-telling constraints guarantee that the utility from correctly reporting type dominates the utility from lying and then taking some feasible off-equilibrium strategy. One way to guarantee truth-telling is to check each possible off-equilibrium strategy. This is the standard method, as in Myerson (1982). Unfortunately, as we describe in Appendix B, our problem involves an astronomical number of such constraints. Instead, we use a method developed in Prescott (2003) that takes advantage of the max operator in equation (13) to more efficiently check the truth-telling constraints. A detailed, self-contained, description of the numerical solution procedure can be found in Appendix B.

4.2 Parameterization

We assume that there are four possible levels of screening effort,
\[ s \in \{0.2, 0.3, 0.4, 0.5\} \]
and two possible standard deviations for the bank portfolios: \[ \sigma \in \{0.6, 1.0\} \].

We assume that the increasing, concave function \( \mu_i \) mapping bank type \( i \)'s screening effort into its portfolio mean takes the negative exponential form:
\[ \mu_i(s) = \mu_{i, \text{max}} - (\mu_{i, \text{max}} - \mu_{i, \text{min}}) e^{-a_i s} \]
where, for \( i = \text{low, high} \), \( \{\mu_{i, \text{max}}, \mu_{i, \text{min}}, a_i\} \) are non-negative type-specific parameters with \( \mu_{i, \text{min}} < \mu_{i, \text{max}} \). Note that \( \mu_i(0) = \mu_{i, \text{min}} \) and \( \mu_i(\infty) = \mu_{i, \text{max}} \). In the parametric examples we present below, we assume that
\[ \mu_{i, \text{min}}^{\text{low}} = 0; \quad \mu_{i, \text{min}}^{\text{high}} = 0.6; \]

12 In the case where bank type is observable, the optimal contract can always be achieved with deterministic contracts. Intuitively, the only way randomization could be useful is if the bank makes a decision prior to the realization of the random assignment. When type is observable, the only choices that the banks make are their screening and risk choices. These are made after the assignment is made, so the regulator need never randomize the assignment.

13 The use of only two risk levels can be motivated by the result in Marshall and Prescott (2001), that if banks’ portfolio returns are log normal, and if banks can choose from a closed interval of risk levels, their optimal choice would always be one of the two endpoints. That is, for any give combination \( \{D, s\} \), an interior solution for \( \sigma \) is never chosen by the bank.
\[\mu_{\text{low}}^\text{max} = 1.7; \quad \mu_{\text{high}}^\text{max} = 2.3; \quad a_{\text{low}} = a_{\text{high}} = 5.\]

Note that the \(\mu_{\text{high}}(\cdot)\) schedule represents a parallel upward shift of the \(\mu_{\text{low}}(\cdot)\) schedule.

Our solution method requires discretization of the distributions. Since discretization effectively imposes upper and lower bounds on the distribution, it is natural to start with distributions that have bounded support. In particular, we assume that all return distributions are from the beta family of distributions. Since the beta is a two-parameter distribution, it is completely described by its mean and variance (along with its upper and lower bounds). We assume the support of all distributions is the open interval \((0, 3.0)\). We thus must construct a beta distribution with this support for each combination of \(\{\text{type}, s, \sigma\}\). We then discretize these distributions on the following seven-point grid:\textsuperscript{14}

\[r \in \{0.04, 0.5, 1.0, 1.5, 2.0, 2.5, 2.96\}.\]

Finally, we set the cost of capital \(\rho = 0.05\), the social cost of fines \(\tau = 0.01\), and the cost of screening effort \(\gamma = 1.0\).

Before turning to the optimal regulation in this example, let us look at the optimal bank choices in the unregulated case where no fines are imposed and \(D = 1\) (zero capital). Table 1 gives the value of the bank’s objective function (equation (9)) and the regulator’s objective function (equation (11)) for each type and each choice of \(\{s, \sigma\}\). (The value of the bank’s objective excludes the lump sum tax, which does not affect bank incentives.) Note first that the regulator wants both banks to choose \(s = 0.4\). Lower values of screening effort are insufficiently productive, but the highest value \(s = 0.5\) incurs a suboptimally high disutility of effort. Note also that, with no deadweight cost of bankruptcy, the regulator is indifferent between high and low risk.

\textsuperscript{14}For each combination of screening effort and risk, there is a unique beta distribution on support \((0, 3)\). We discretize each distribution by evaluating the beta density at each of the seven grid points, and then adding \(\varepsilon_i\) to the \(i\)th grid point’s probability, where \(\{\varepsilon_i\}_{i=1}^7\) are chosen to minimize \(\sum_{i=1}^7 \varepsilon_i^2\) subject to the constraints that the resulting probabilities sum to unity and that the mean and variance of the discrete distribution exactly match the mean and variance of the original beta distribution. Since very small probabilities often introduce numerical instability in our solution algorithm, we then do a second round of ad hoc adjustments to ensure that no single probability is less than 0.001, that the means hold exactly, and that the variances are close to the target variances.
Turning to the values of the banks’ objectives, the high type’s preferred action is $s = 0.4$ and $\sigma = 1.0$. Since this maximizes the regulator’s objective, the high-type bank is *self-regulating* in this example. In contrast, the low-type bank prefers the highest risk level with $s = 0.3$. So the task facing the regulator is to induce the low-type bank to increase its screening to $s = 0.4$. Note also that if the low type were *forced* to choose the lower risk level, the optimal choice of screening would coincide with the regulatory optimum $s = 0.4$. However, if the high type were forced to choose low risk, its objective would be maximized at the suboptimally high level $s = 0.5$. These effects of risk reduction are examples of the result in Section 3.3 that for a large set of plausible portfolio distributions, screening is decreasing in risk. This will be an important consideration in the optimal regulatory design. Inducing the low type directly to choose the socially optimal screening level is difficult. However, if the regulator can induce the low type to choose low risk, it will be optimal for the bank to then select the optimal screening level, which is the prime concern of the regulator.

At first glance, the results in Table 1 suggest that the screening level $s = 0.2$ is extraneous since it appears to be neither privately nor socially optimal: No unregulated bank would ever choose this screening level, nor would the regulator ever mandate it. However, as we shall discuss below, once fines are imposed screening may be so unappealing to the bank that a $s = 0.2$ deviation is a possibility that needs to be prevented.

### 4.3 Optimal Regulation when bank type is observed

Marshall and Prescott (2001) studied optimal regulation in this model when bank type is observable. It is useful to revisit this simpler case in the context of our baseline parameterization to provide a benchmark against which the results with unobserved heterogeneity can be compared. Several of the forces operational in that model are operational in the heterogenous agent model as well. Furthermore, the implications of the model for this case raise some interesting issues that the analysis with unobserved heterogeneity may be able to clarify.

We determine the optimal contract for each of the two types, both for pure capital requirements (no fines permitted) and for the case where both capital requirements and ex post fines are available. The results in the case of pure capital regulation for the two bank types are displayed in the first column of Table 2. The optimal contracts induce high screening and high
variance for both types. The high type is fully leveraged, while the low type has a capital level of 14%. The low type needs this capital requirement to induce socially optimal screening. This is an example of Proposition 1 in Section 3.2, above. In contrast, as we discussed in Section 4.2, above, the high type chooses this optimal screening level even in the absence of a capital requirement.

We now introduce fines as a second regulatory instrument in addition to capital. The results for this case are displayed in the first column of Table 3. Note that in this case fines completely displace capital, since capital is a more costly way of influencing bank incentives than state-contingent fines. In particular, the optimal contract assigns the low type 0% capital ($D = 1$), with the low risk and $s = 0.4$. The low type is induced to take this strategy by the use of a fine of 1.8910 that is assessed if the highest return is produced. There are no fines imposed on the other returns. Fining a bank for doing well may seem counterintuitive but it is actually a simple application of the likelihood ratio principle, which is relevant for optimal incentive contracts. This principle can be understood as follows: Fines have two effects on the moral-hazard incentive constraints: They hurt banks who take the recommended strategy, and they also hurt those that deviate. The relative size of these effects at a particular return level $r$, is determined by the likelihood ratio, $LR(r)$, defined as

$$LR(r) \equiv \frac{\text{prob}(r \mid \text{deviating strategy followed})}{\text{prob}(r \mid \text{recommended strategy followed})}.$$  

This ratio can be interpreted as the prevention per unit of fine assessed in equilibrium. It is generally desirable to impose the fine on the return with the highest likelihood ratio.

For this example, the investment strategy of most concern to the regulator is where $s = 0.2$ and $\sigma = 1.0$. The likelihood ratio for this deviation is highest at $r = 0.04$, the return in the extreme left-hand tail of the distribution. The regulator would like to impose the fine on this return but cannot because of limited liability. The return with the next highest likelihood ratio for this deviation is $r = 2.96$, the return in the extreme right-hand tail. Fines can be assessed for this return without violating limited liability, so this is the return level that receives the fine.

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15 For more on likelihood ratios in moral hazard models see Hart and Holmstrom (1987).
16 This is the deviating strategy with respect to which the incentive constraint binds.
Inspection of this contract reveals why capital is not used. The fine on the highest return is strictly below the maximum feasible fine, given limited liability, of 1.96. If this fine were insufficient to induce optimal screening, it could be increased at the margin, thereby strengthening incentives without using costly capital. In our numerical simulations, we find that as long as fines have a low regulatory cost compared to capital, capital and fines coexist in the same contract only if at least one fine is at the bound imposed by the limited liability constraint.

Another important point that is illustrated by this example is that the binding incentive constraint need not be near the equilibrium choice. That is, it need not be a local incentive constraint.

In particular, while the lowest screening level \( s = 0.2 \) is never chosen by an unregulated bank, the binding incentive constraint in this example is to prevent deviation from \( \{ s = 0.4, \sigma = 0.6 \} \) to \( \{ s = 0.2, \sigma = 1.0 \} \). The reason is that the high fine on the right-hand tail of the distribution induces the bank to reduce the probability of the highest return. But there are two ways it can do so: by reducing \( \sigma \) or by reducing \( \mu \) via a reduction in screening effort all the way to \( s = 0.2 \). The former is the effect desired by the regulator; the latter needs to be avoided as represented by the binding incentive constraint.

This example illustrates why the first-order approach to incentive constraints does not work in this model. In the first-order approach (see Hart and Holmstrom (1987)) the incentive constraints are replaced with the first-order conditions to the agents subproblem; but these are necessary rather than sufficient conditions. Without strong assumptions, there is no guarantee that a solution to the program with the first-order conditions is the same as the solution to the global program. In particular, the program utilizing first-order conditions would treat certain globally infeasible allocations as if they were feasible. In our example, since local incentive constraints do not bind, it would actually select one of these (infeasible) allocations as the solution to the regulatory problem. Our linear programming method is a global method so we do not have to worry about this possibility.

### 4.4 Optimal regulation with unobserved heterogeneity

According to the results in Section 4.3, the optimal structure of ex post fines for the low type is to impose large fines on the highest return level. In other words, banks are penalized for doing extremely well. This seemingly perverse result is actually quite intuitive, since an extremely high return
to a low-type bank is a reasonably good signal of excessive risk-taking. The penalty structure could be duplicated by requiring the bank to issue warrants or convertible debt with a high strike price. However, one important concern with such a regulatory fine schedule is that some banks may have a high return not because they took excessive risks, but simply because they are good banks. They may have better management or they may have a more favorable investment opportunity set. In this section, we address this concern by solving the Regulator’s Problem defined earlier, in which bank type is unobservable. With unobserved bank quality, we can investigate the trade-offs between the good and bad incentive effects of high-return fines.

4.4.1 Optimal capital requirements without fines in the presence of unobserved heterogeneity

When fines are not used, the optimal capital requirements when bank type is unobserved in the baseline case are displayed in the last three columns of Table 2. The table gives the optimal regulatory contracts for the two types as the fraction of high types, $h_{\text{high}}$, varies from .01 to .995.

When $h_{\text{high}}$ is not too big (less than 86%), the optimal regulatory strategy is to assign both types 14% capital, which is the optimal type-observable contract for the low type. That is, no effort is made to separate types. The reason is that this contract is the least expensive contract (from the regulator’s standpoint) that induces the low-type bank to choose the highest screening level. While this contract is decidedly inferior from the perspective of the high-type bank, the only alternative from the regulator’s perspective is to let the low-type bank choose a lower screening level. The social cost to this alternative exceeds the social cost of the capital requirements imposed on the high-type bank.

When $h_{\text{high}}$ is very high (87% or higher), the regulator again makes no effort to separate type. For these values of $h_{\text{high}}$, the regulator simply assigns both types of banks zero capital, which of course is the optimal type-observable contract for the high type. The low type bank responds by setting $s = 0.3$, a socially suboptimal screening level. There are so few low-type banks that the regulator is willing to tolerate this suboptimal screening by low-type banks in order to save the capital costs incurred by the high-type banks.

Finally, for $h_{\text{high}}$ in a narrow range near 0.86, the optimal regulation is to separate types by offering different contracts to each type. While this
would be impossible with non-random contracts (since both bank types would choose the lower-capital alternative), it is possible in theory to separate types by offering the low-type bank a random contract while offering the high-type bank a non-random contract. The reason this is possible is that the bank’s objective function is nonlinear in $D$ because it incorporates the payoff to the deposit insurance put option. The expected payoff of a put option is strictly convex in its strike price. In our example, the put option term has strictly greater curvature for the low type bank than for the high type bank, so, as an implication of Jensen’s inequality, the low-type bank has a greater preference for randomness. To separate types, one simply offers the high type bank a nonrandom capital level while offering the low type bank a random capital level with a somewhat lower mean. We see this in the second-to-last column of Table 2 for $h_{\text{high}} = 86\%$. The optimal contract exploits this possibility by giving the high type a deterministic contract with 1% capital, while giving the low type a mixed contract that imposes 14% capital with 7% probability and zero capital with 93% probability. We find it interesting that in the numerical simulations we have studied, this type of randomized contract is optimal only for a very narrow set of parameters. For the vast majority of examples, separation of types with pure capital regulation is suboptimal.

4.4.2 Optimal regulation using both capital and fines in the presence of unobserved heterogeneity

The optimal contracts for the baseline parameterization when both fines and capital are used are displayed in Table 3. The first two rows of Table 3 give details of the contract for the low type. The second row is only used when the optimal contract is a mixture of two contract elements. The third row gives the optimal contract for the high type. (In the baseline parameterization mixed contracts are never used for the high-type bank.) As described in Section 4.3, the first column gives the optimal type-observable contracts. The remaining three columns give the contracts as the fraction of high types $h_{\text{high}}$ ranges from 1% to 99.5%.

When bank type is unobservable, the optimal type-observable contracts for the two bank types cannot be implemented simultaneously. The reason is that the contract for the high type (zero capital, zero fines) is more attractive to the low type than its own contract (zero capital, high fine on the highest output level). So, the low type would invariably misrepresent its type, receive the high-type contract, and then use a suboptimally low screening level.
To remedy this problem the regulator can choose one of three alternatives:

1. Assign the low type its optimal observable-type contract, and impose sufficient penalties on the high type (either in the form of fines or capital requirements) so that the low type has no incentive to misrepresent type.

2. Assign the high type its optimal observable-type contract, and give the low type a (socially suboptimal) contract that is sufficiently attractive so that low-type banks have no incentive to misrepresent type.

3. Assign neither type its optimal observable-type contract; and craft a set of contracts such that neither bank type has an incentive to misrepresent type.

Not surprisingly, we find that the optimal choice among these three alternatives depends on the fraction $h_{\text{high}}$ of high-type banks. In particular, alternative (1) is used when $h_{\text{high}}$ is relatively low, alternative (2) is used when $h_{\text{high}}$ is extremely high, and alternative (3) is used for intermediate values of $h_{\text{high}}$.

In the baseline case, the low-type contract is the same as the optimal contract under type-observability as long as the fraction of high types is not too high ($h_{\text{high}}$ below 88%). This is clearly evident in Table 3. For these values of $h_{\text{high}}$, however, the contract for the high type imposes fines at return levels 2.0 and 2.5. These fines are substantial. At return 2.0, over 89% of the bank’s profit (after paying off depositors) is fined away; at return 2.5 approximately 29% of the profit is fined. So, while the low type receives the same contract as she would if type could be observed, the high type must pay heavy fines in order to dissuade the low types from masquerading as high types.

Neither fine in the high-type contract is constrained by limited liability, so one might think that these two fines play distinct roles. This is partially correct. Two constraints bind in this contract. The first binding constraint is the truth-telling constraint preventing low types from profiting by misrepresenting their type and choosing $\{s, \sigma\} = \{0.3, 1.0\}$. The second binding constraint is the incentive constraint preventing the high type from choosing the suboptimally high screening effort of $s = 0.5$. Consider Table 4, which gives the values of bank and regulatory objectives for $\{s, \sigma\}$ pair under both the optimal low-type contract and the optimal high-type contract. Note that
the highest value of the low type’s objective from truth-telling is 1.7924, attained at the socially optimal action pair \( \{ s, \sigma \} = \{ 0.4, 0.6 \} \). But the low type can attain this precise value by misrepresenting herself as a high type and choosing the socially suboptimal action pair \( \{ s, \sigma \} = \{ 0.3, 1.0 \} \). Thus, the truth-telling constraint binds with respect to this deviating action. If the fine on \( eitChe \) return 2.0 or 2.5 were reduced, this constraint would be violated and the low-type bank would profit from lying and misbehaving.

Note also that the high-type’s objective attains its maximum of 1.69761 at either the socially optimal action pair \( \{ s, \sigma \} = \{ 0.4, 0.1 \} \) or at the socially suboptimal pair \( \{ s, \sigma \} = \{ 0.5, 0.1 \} \). Thus, the high-type’s incentive constraint binds with respect to this latter action. If the fine on \( r = 2.5 \) were reduced (without a concomitant reduction in the fine on \( r = 2.0 \)), this constraint would be violated.\(^{17}\) In other words, if only a fine at \( r = 2.0 \) were used to dissuade low-type banks from lying, the fine itself would induce high-type banks to screen excessively, an action that bank would never take in the unregulated case. An additional fine at \( r = 2.5 \) is needed to correct this perverse incentive. This is an example of a possibility that can occur in optimal mechanism design when both moral hazard and adverse selection are present: A contractual provision designed to alleviate the adverse selection problem can itself exacerbate the moral hazard problem, requiring additional regulatory correction.

This contract on the high types is inferior to their optimal type-observable contract, both from the perspective of social utility and private bank utility. Table 5 reports the social value of each contract, the private value of each contract before taxes (which is what matters for incentives) and the private value of the contract after taxes. For purposes of utility comparison, we treat the type-observable case as if the tax levies on the two types acted as actuarially fair deposit insurance premiums. That is, the costs of failure resolution for a given bank type are assessed only on the banks of that type.\(^{18}\)

Table 5 shows that the social utility of the optimal observable-type contract for high banks is 1.71993. In contrast, the social utility of the equilibrium high-type contract when bank type is unobservable is lower. For \( h_{high} \) below 0.88, this value is 1.71867. This small difference is the social cost of private information about bank characteristics. The reason this cost is so

\(^{17}\)If both fines are reduced at an appropriate rate, the incentive constraint need not be violated.

\(^{18}\)Note that this sort of type-dependent tax levy is infeasible when type is unobservable.
small is that in both contracts the high bank makes the optimal screening choice \( s = 0.4 \), so the only difference from a societal standpoint is the dead-weight cost of the fines. When \( \tau \) is only 1%, this deadweight cost is small; the regulator is willing to pay this cost when there are not too many high banks in the economy. However, the *private* cost to the high banks of type-unobservability is much larger. The private after-tax value to the high bank of its optimal observable-type contract equals the social value of 1.71993,\(^{19}\) while the corresponding private value of the equilibrium contract under type-unobservability varies from 1.63905 to 1.70902 as \( h_{\text{high}} \) varies from 0.01 to 0.88. All these values are *less* than the private value accruing to the high type when bank type is observable. In contrast, Table 5 shows that, when bank type is unobservable, the low-type banks receive a private after-tax value that in all cases *exceeds* what they receive under type-observability. In particular, this value when type is observed is 1.11990, whereas when type is private information these values range between 1.12067 and 1.19065 (for \( h_{\text{high}} \) between 0.01 to 0.88).

Thus, the optimal contracts under type-unobservability in effect transfer value from high to low types. In this sense, the more productive banks bear the full costs of type-unobservability, even though these high-type banks are completely self-regulating when type is public knowledge. This is relevant for regulatory practice. It is often argued that the vast majority of banks have strong incentives to behave in a prudent and value-maximizing manner. These banks are essentially self-regulating, since their private incentives are well aligned with social imperatives. However, if there are enough poor quality banks, and if the regulators have imperfect information about bank quality, then it may be necessary to impose a heavy superstructure of regulation on the high banks just in order to affect the incentives of the low banks.

Contrast the optimal contracts in the case described above to the case where the fraction of high types reaches 99.5% or higher. This case is displayed in the last column of Table 3. At this point the regulator simply assigns the high type its optimal contract under type-observability. With our parameterization, this contract is completely unregulated (zero capital, zero fines). Since we require both fines and capital to be non-negative, there is

\(^{19}\)That these two values are identical follows from our assumption that, when type is observable, the costs of bank failure resolution for type \( i \) banks are assessed only on banks of that type, along with the fact that the value of the deposit insurance put option exactly equals the cost of paying off the depositors in failed banks.
no way to dissuade the low-type bank from misrepresenting type. (No other contract can dominate the zero capital, zero fine contract in the low type’s private valuation.) As a result, the two types must be assigned the same contract and the truth-telling constraints hold trivially. The low-type bank then chooses a suboptimally low screening level of 0.3. However, the fraction of low-type banks is so small that the regulator simply does not care about the suboptimally low mean output from these banks. (For higher values of $\tau$, this threshold is reached at lower values of $h_{\text{high}}$.)

Finally, let us briefly consider the intermediate case where $h_{\text{high}}$ is between 0.89 and 0.99. When the fraction of high types is in this range, the social cost of fines on the high-type banks is sufficiently onerous that the regulator wishes to reduce these fines. But to maintain truth-telling, the regulator must simultaneously increase the value of the low-type contracts. If it did not do so, the low type banks would misrepresent themselves as high types. In all the examples we have computed, the regulator gives additional value to the low-type banks by assigning them a mixed contract that randomizes between a high-fine, low (or zero) capital contract and a low-(or zero-) fine, high capital contract. As an example, consider the second-to-last column in Table 3, which exhibits the optimal contract in our baseline model with $h_{\text{high}} = 0.89$. As compared to the contract for $h_{\text{high}} = 0.88$, the regulator reduces the fines on the high type, imposing a zero fine on $r = 2.5$ and a lower fine on $r = 2.0$. In order to increase the utility of the low type, the regulator randomizes between the optimal type-observable contract (zero capital, fine of 1.8910 on the highest return) and the optimal type-observable contract with capital only (14% capital, zero fines). This random contract gives the low-type bank higher private value, first because it imposes a lower expected fine, and second because (as discussed in Section 4.4.1) it exploits the differences in the curvature of the two types’ utility functions. As shown in the second-to-last column of Table 5, the social value of this mixed contract is lower than the non-random contract, but the regulator is willing to forego this value in order to reduce the fine on the high-type banks. Note that both the private and social values of the high-type contract are higher in the case than when $h_{\text{high}} = 0.88$. 

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5 Conclusion

This paper studies a bank capital regulation model in which deposit insurance causes a potentially lower level of expected output because it creates a taste for risk that reduces marginal incentives to exert screening effort. Capital regulation of the sort commonly seen in regulatory practice is fairly effective at offsetting this distortion. However, state-contingent tools are shown to be more powerful. Fines can induce optimal screening effort while economizing on (or eliminating entirely) the use of costly capital.

We learned a number of lessons from this exercise. First, a powerful reason the regulator might seek to deter risk is to induce banks to choose a higher mean portfolio. Second, unobserved heterogeneity does not eliminate the usefulness of state-contingent fines as a regulatory tool. On the contrary, fines are still useful, both to deter misrepresentation of type and to deter suboptimal choices after type has been truthfully revealed. Third, the likelihood ratio principle guides the choice of return levels on which to impose the fines. In particular, fines at the highest return level tend to be used whenever the objective is to deter high risk-taking. Fourth, for most distribution of types, the low type receives its optimal type-observable contract. The utility given to the low type by these contracts is at least as great as that under type-observability. In contrast, the high type only receives its type-observable contract when there are so few low types that the regulator is unconcerned with separating types. Otherwise, the high type is given a contract that provides a strictly lower utility level than he would receive under type-observability. Finally, fines are potentially a less costly way of separating types than the pure capital requirements that are the focus of much regulatory practice. When fines are precluded, the regulator generally gives up any attempt to separate types, even though this is feasible in principle via randomization. However, when fines are included in the regulatory tool kit (with a relatively low cost $\tau$ of 0.01), the regulator almost always chooses to separate types.

The model justifies the regulatory focus on capital adequacy and safety-and-soundness (interpreted as risk reduction), since both of these approaches can potentially offset the distortions induced by the deposit insurance put option. However, a reservation one might raise with the results of this paper is that state-contingent fines per se are not typically observed in regulatory practice. Furthermore, the equilibrium contracts in this paper often require fines on high returns, an approach that could encounter political and even
legal obstacles. In future research, we are considering other regulatory instruments, such as costly risk audits, that have the potential of delivering similar results as those found in this paper while conforming more closely to observed practice.
Appendix A: Proof of Proposition 1

When fines are set to zero, the bank’s objective in equation (9) becomes
\[
\int_0^D (D - r) f(r|s) dr + \rho D + \mu(s) - \gamma s - T.
\]
If \( s > 0 \), then the first- and second-order necessary conditions for optimal choice of \( s \) are:
\[
\int_0^D (D - r) f_s(r|s) dr + \mu'(s) - \gamma = 0 \tag{24}
\]
and
\[
\int_0^D (D - r) f_{ss}(r|s) dr + \mu''(s) < 0 \tag{25}
\]
To determine \( s'(D) \), the response of \( s \) to a change in \( D \), totally differentiate equation (24) with respect to \( D \):
\[
F_s(D|s) + s'(D) \left[ \int_0^D (D - r) f_{ss}(r|s) dr + \mu''(s) \right] = 0 \tag{26}
\]
which implies
\[
s'(D) = \frac{-F_s(D|s)}{\int_0^D (D - r) f_{ss}(r|s) dr + \mu''(s)}. \tag{27}
\]
Second order condition (25) implies that
\[
\text{sign} [s'(D)] = \text{sign} [F_s(D|s)] \tag{28}
\]
\[
= \text{sign} (\mu'(s) [F_{\mu}(D|\mu, \sigma)]) \tag{29}
\]
According to equations, (1), (2), and (29), \( s'(D) < 0, \forall D < r^*(\mu, \sigma) \) as in the statement of the proposition.

Appendix B: Solving the Regulator’s Problem with Heterogeneous Agents

We solved our numerical examples by formulating the Regulator’s Problem as a linear program and then solving the problem using standard linear programming code. There are two steps to the linearization. The first step is to allow
randomization in the contractual terms. This means that the regulator may randomly recommend \((D, s, \sigma)\) combinations to each bank type. We write this probability distribution as \(\omega_i(s, \sigma, D)\). Fines now need to depend on the realization of \(\omega_i(s, \sigma, D)\). Fines could also be random but because of the linear preferences and objective function we can write them as \(g_i(r, s, \sigma, D)\).

The second step of the linearization is to discretize the sets of variables, that is, the \(g, r, s, \sigma,\) and \(D\). These grids are straightforward except for the fine grid. The upper bound on fines depends on \(D\) and \(r\) because of limited liability. For this reason we use as our fine grid \(\{0, \max\{0, r - D\}\}\). Because of the linear preferences, a two point grid is all we need to capture all relevant fines. (Lotteries over the two points capture everything in between.)

To formulate the problem as a linear program we solve for the joint distribution over the grid of variables for each type. Let \(\pi_i(g, r, s, \sigma, D)\) denote the conditional joint probability of a type-\(i\) bank receiving assignment \((D, s, \sigma)\), realizing return \(r\) (if the recommended \((s, \sigma)\) are taken) and being assessed fine \(g\). (To keep the notation simple, we do not write out the explicit dependence of \(g\) on the realization of \(r\) and \(D\).) Embedded in this object are the two choice variables of the regulator, the mixing probabilities \(\omega_i(s, \sigma, D)\) and the fine schedule \(g_i(r, s, \sigma, D)\). They are related to the joint distribution as follows:

\[
\omega_i(s, \sigma, D) = \sum_{g,r} \pi_i(g, r, s, \sigma, D) \tag{30}
\]

and

\[
g_i(r, s, \sigma, D) = \sum_g \pi_i(g|r, s, \sigma, D)g. \tag{31}
\]

Equation (31) gives the expected level of the fine given the return, assignment, and reported type, which is all we need for utility and welfare purposes.

Our strategy is to let the regulator directly choose the joint probability distribution \(\pi_i(g, r, s, \sigma, D)\). To guarantee that this object is a probability distribution, we restrict its elements to be non-negative and we require that

\[
\forall i, \sum_{g,s,\sigma,D} \pi_i(g, r, s, \sigma, D) = 1. \tag{32}
\]

In choosing the joint distribution, the regulator is implicitly choosing the probability of assignments, (30), and the fine schedule, (31). Still, there is a technological relationship between the return and the investment strategy...
that must not be violated. In particular, the identity
\[ \pi_i(g, r, s, \sigma, D) = \pi_i(g| r, s, \sigma, D) f_i(r| s, \sigma) \omega_i(s, \sigma, D) \]
must hold. This identity can be guaranteed to hold by the system of linear equations
\[ \forall i, D, \bar{r}, \bar{s}, \bar{\sigma}, \sum_g \pi_i(g, \bar{r}, \bar{s}, \bar{\sigma}, D) = f_i(\bar{r}| \bar{s}, \bar{\sigma}) \sum_{g,r} \pi_i(g, r, s, \sigma, D). \quad (33) \]

The next set of constraints are the moral-hazard constraints. These constraints guarantee that the bank takes the recommended investment strategy conditional on truthfully reporting its type, as follows:
\[ \forall i, D, s, \sigma, b_s, b_\sigma, \sum_{g,r} \pi_i(g, r, s, \sigma, D)(r - D - g) + \sum_{g,r} \pi_i(g, r, s, \sigma, D)(-\gamma s) \geq \sum_{g,r} \pi_i(g, r, s, \sigma, D)(r - D - g) + \sum_{g,r} \pi_i(g, r, s, \sigma, D)(-\gamma \bar{s}) \quad (34) \]
The left-hand side of (34) is the utility from taking the recommended action while the right-hand side is the utility from taking deviating strategy \((\bar{s}, \bar{\sigma})\). Both sides are weighed by the marginal distribution \(\omega_i(s, \sigma, D)\). Notice that the \(\rho D\) and \(T\) terms have been dropped as they cancel out on both sides of the constraint.

The last set of constraints are the truth-telling constraints. These constraints are the most problematic ones for computational purposes. First, we write these constraints in the same form as used in Myerson (1982). Next, we write them in a form that is more useful for computational purposes.

Let \(\delta\) denote a function mapping the set of possible action pairs \((s, \sigma)\) into itself. Intuitively, if the regulator recommends action pair \((s, \sigma)\), a possible deviating action pair would be \(\delta(s, \sigma)\) (so the deviating screening level would be \(\delta_1(s, \sigma)\), and the deviating risk level would be \(\delta_2(s, \sigma)\)). Using this notation, one can write the truth-telling constraints in the following way, as

\[ \text{\footnotesize While equations (34) - (37) are written differently than the corresponding constraints in the Regulator’s problem, they are equivalent. They just have not been algebraically manipulated to break out the deposit insurance put option as a separate term.} \]
in Myerson (1982):

\[
\sum_{g,s,\sigma,D,r \geq D} \pi_i(g,r,s,\sigma,D)(r - D - g) + \sum_{g,s,\sigma,D,r} \pi_i(g,r,s,\sigma,D)((1 + \rho)D - \gamma s)
\]

\[
\geq \sum_{g,s,\sigma,D,r \geq D} \pi_j(g,r,s,\sigma,D) \frac{f_i(r | \delta_1(s,\sigma), \delta_2(s,\sigma))}{f_j(r | s,\sigma)}(r - D - g)
\]

\[
+ \sum_{g,s,\sigma,D,r} \pi_j(g,r,s,\sigma,D) \frac{f_i(r | \delta_1(s,\sigma), \delta_2(s,\sigma))}{f_j(r | s,\sigma)}((1 + \rho)D - \gamma s), \forall \delta, j \neq i
\]

In words, the left-hand side of (35) gives the expected value to a type-\(i\) bank of truthfully reporting its type and selecting the recommended action pair \((s, \sigma)\). The right hand side gives the expected value if that bank misrepresents itself as a type-\(j\) bank, and then, if it receives a recommended action \((s, \sigma)\), the bank actually chooses the deviating actions \(\delta(s, \sigma)\). The key point to note about this constraint is that it must hold for all possible functions \(\delta\). These functions must specify all of the possible off-equilibrium strategies that a bank can take in response to a recommended \((s, \sigma)\) pair. There are a huge number of these functions. Since these recommendations may be random, each possible off-equilibrium strategy needs to include a response to each possible recommendation. In particular, if there are \(n_s\) possible screening levels and \(n_\sigma\) possible risk levels, then there are \((n_s n_\sigma)\) constraints of the form (35) per \((i, j)\) pair, deposit combination.\(^{21}\)

Fortunately, this serious curse of dimensionality can be dealt with by reformulating the truth-telling constraint. In this reformulation, an additional choice variable \(w(s, \sigma, D, i, j)\) is introduced that keeps track of the maximum off-equilibrium utility a type-\(i\) agent can receive if he reports his type as \(j\) and is recommended \((D, s, \sigma)\).\(^{22}\) This solution strategy was anticipated in the way we wrote equation (13) in the Regulator’s Problem, where the utility from off-equilibrium strategies was dealt with by using the max operator rather than enumerating all the possible off-equilibrium strategies. The

\[^{21}\]In the examples of section 4 there are four possible screening levels and two possible risk levels, so the total number of constraints (35) per \((i, j)\) pair, deposit combination would equal 16,777,216.

\[^{22}\]This strategy is based on the one used by Prescott (2003) to deal with a similar model where the shock was to an agent’s marginal disutility of effort.
off-equilibrium utility constraints are \( \forall s, \sigma, \hat{\sigma}, D, i, j \neq i : \)

\[
    w(s, \sigma, D, i, j) \geq \sum_{g} \sum_{r \geq D} \pi_j(g, r, s, \sigma, D) \frac{f_i(r \mid \hat{\sigma})}{f_j(r \mid r, s, \sigma)} (r - D - g) + \quad (36)
    \sum_{g, r} \pi_j(g, r, s, \sigma, D) \frac{f_i(r \mid \hat{\sigma})}{f_j(r \mid r, s, \sigma)} ((1 + \rho)D - \gamma s).
\]

These constraints give the most utility a type-\( i \) bank can receive if it reports that it is a type-\( j \) bank and is assigned \((D, s, \sigma)\). This utility is weighed by \( \omega_j(s, \sigma, D) \). The off-equilibrium utility can now be used to guarantee truth-telling. The truth-telling constraints are

\[
\forall i, j \neq i, \sum_{g, s, \sigma, D, r \geq D} \pi_i(g, r, s, \sigma, D)(r - D - g)
+ \sum_{g, s, \sigma, D, r} \pi_i(g, r, s, \sigma, D)((1 + \rho)D - \gamma s) \geq \sum_{s, \sigma, D} w(s, \sigma, D, i, j).
\]

The left-hand side is the utility from telling the truth and taking the recommended action while the right-hand side is the utility the agent would receive from lying and then taking the best off-equilibrium strategy possible.

The result of this reformulation is that for the example in the paper with eight different investment strategies we only need \((n_s n_\sigma)^2 + 1\) constraints per \((i, j)\) pair, deposit combination to satisfy the above truth-telling condition. This substantial reduction in the size of the linear program made it feasible for us to study the problem in this paper.

The program is

\[
    \max_{\pi_i(\cdot) \geq 0, w_i(\cdot)} \sum_i h_i \left[ \sum_{g, r, s, \sigma, D} \pi_i(g, r, s, \sigma, D)(r - \gamma g + \rho D - s) \right]
\]

subject to probability measure constraints (32), technology constraints (33), moral-hazard constraints (34), off-equilibrium incentive constraints (36), and truth-telling constraints (37).

The program is a linear program. There is a finite number of constraints and a finite number of choice variables: \( \pi_i(g, r, s, \sigma, D) \) and \( w(s, \sigma, D, i, j) \) for each type \( i, j \neq i \) and each point in the \((g, r, s, \sigma, D)\) grid. We wrote our code for creating the linear programming coefficients in Matlab. The linear
program was then solved by calling Minos, a Fortran program solver developed at the Stanford Systems Optimization Laboratory. Minos was called using the TOMLAB optimization library. To check the accuracy of the code we also independently programmed the problem in the GAMS programming language, and then called Minos from GAMS.
References


<table>
<thead>
<tr>
<th>Type = low</th>
<th>Private Value</th>
<th>Social Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s, σ) = (0.20, 0.60)</td>
<td>1.1219</td>
<td>0.9246</td>
</tr>
<tr>
<td>(s, σ) = (0.30, 0.60)</td>
<td>1.1725</td>
<td>1.0707</td>
</tr>
<tr>
<td>(s, σ) = (0.40, 0.60)</td>
<td>1.1853</td>
<td>1.1199</td>
</tr>
<tr>
<td>(s, σ) = (0.50, 0.60)</td>
<td>1.1600</td>
<td>1.1105</td>
</tr>
<tr>
<td>(s, σ) = (0.20, 1.00)</td>
<td>1.3220</td>
<td>0.9246</td>
</tr>
<tr>
<td>(s, σ) = (0.30, 1.00)</td>
<td>1.3520</td>
<td>1.0707</td>
</tr>
<tr>
<td>(s, σ) = (0.40, 1.00)</td>
<td>1.3441</td>
<td>1.1199</td>
</tr>
<tr>
<td>(s, σ) = (0.50, 1.00)</td>
<td>1.3050</td>
<td>1.1105</td>
</tr>
<tr>
<td>Type = high</td>
<td>Private Value</td>
<td>Social Value</td>
</tr>
<tr>
<td>(s, σ) = (0.20, 0.60)</td>
<td>1.5596</td>
<td>1.5246</td>
</tr>
<tr>
<td>(s, σ) = (0.30, 0.60)</td>
<td>1.6864</td>
<td>1.6707</td>
</tr>
<tr>
<td>(s, σ) = (0.40, 0.60)</td>
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<td>1.7199</td>
</tr>
<tr>
<td>(s, σ) = (0.50, 0.60)</td>
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<td>1.7105</td>
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<tr>
<td>(s, σ) = (0.20, 1.00)</td>
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<tr>
<td>(s, σ) = (0.30, 1.00)</td>
<td>1.7865</td>
<td>1.6707</td>
</tr>
<tr>
<td>(s, σ) = (0.40, 1.00)</td>
<td>1.8231</td>
<td>1.7199</td>
</tr>
<tr>
<td>(s, σ) = (0.50, 1.00)</td>
<td>1.8126</td>
<td>1.7105</td>
</tr>
</tbody>
</table>

Notes: Table 1 displays the value of the bank’s objective (“Private Value”) and the regulator’s objective (“Social Value”) as a function of type (low or high), risk level σ and screening level s, when capital and fines are both set to zero. The private value is exclusive of the lump sum tax T. The parameters correspond to the baseline parameterization in Section 4.2: μmin = {0, 0.6}, μmax = {1.7, 2.3}, a = {5, 5}, ρ = 0.05, τ = 0.01, γ=1.0.
### Table 2: Optimal Capital Regulation in the Baseline Case

<table>
<thead>
<tr>
<th>Bank Type: Private or Public Information?</th>
<th>Public</th>
<th>Private</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>High type %:</strong></td>
<td><strong>NA</strong></td>
<td><strong>1% - 85%</strong></td>
</tr>
<tr>
<td><strong>Low Type</strong></td>
<td><strong>Probability of Assignment:</strong></td>
<td><strong>Screening Level (s):</strong></td>
</tr>
<tr>
<td>Contract Assignment 1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>14%</td>
<td>14%</td>
</tr>
<tr>
<td><strong>Contract Assignment 2 (if applicable)</strong></td>
<td><strong>Probability of Assignment:</strong></td>
<td><strong>Screening Level (s):</strong></td>
</tr>
<tr>
<td>Low Type</td>
<td>0.93</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>High Type</strong></td>
<td><strong>Probability of Assignment:</strong></td>
<td><strong>Screening Level (s):</strong></td>
</tr>
<tr>
<td>Contract Assignment</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>0%</td>
<td>14%</td>
</tr>
</tbody>
</table>

**Notes for Table 2:** This table gives the details of the optimal contracts for the low type bank and the high type bank in the baseline case when fines are prohibited, so the regulator only uses capital requirements. The parameterization used is: $\rho = 0.05$, $\gamma = 1.0$. The first two panels, labeled “Low Type,” give the details of the contract for the low type bank. (The second panel is only used if the optimal contract for the low type randomizes between two contract assignments.) The third panel gives the contract for the high type bank. Each panel gives the probability of the assignment (1.0 unless a random contract is used), and the assigned screening level, risk level, and capital level. The first column, labeled “Public”, gives the optimal contract when type is public information. The remaining three columns give the optimal contract when type is private information and the percentage of high type banks takes three different ranges. The parameters correspond to the baseline parameterization in Section 4.2: $\mu_{\min} = \{0, 0.6\}$, $\mu_{\max} = \{1.7, 2.3\}$, $a = \{5, 5\}$, $\rho = 0.05$, $\tau = 0.01$, $\gamma = 1.0$. 
Table 3: Optimal Contracts in the Baseline Case with Capital and Fines

<table>
<thead>
<tr>
<th>Bank Type: Private or Public Information?</th>
<th>Public</th>
<th>Private</th>
</tr>
</thead>
<tbody>
<tr>
<td>High type %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of Assignment:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Screening Level (s):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk Level (σ):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital Level (1-D):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fines</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Return = 0.04:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Return = 0.5:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Return = 1:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Return = 1.5:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Return = 2:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Return = 2.5:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Return = 2.96:</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Low Type

Contract Assignment 1

Probability of Assignment: 1 1 62 1
Screening Level (s): 0.4 0.4 0.4 0.3
Risk Level (σ): 0.6 0.6 0.6 1
Capital Level (1-D): 0% 0% 0% 0%
Fines
Return = 0.04: 0 0 0 0
Return = 0.5: 0 0 0 0
Return = 1: 0 0 0 0
Return = 1.5: 0 0 0 0
Return = 2: 0 0 0 0
Return = 2.5: 0 0 0 0
Return = 2.96: 1.8910 1.8910 1.8910 0

Contract Assignment 2 (if applicable)

Probability of Assignment: .38 0 0 0
Screening Level (s): 0.4 0 0 0
Risk Level (σ): 1.0 0 0 0
Capital Level (1-D): 14% 0 0 0
Fines
Return = 0.04: 0
Return = 0.5: 0
Return = 1: 0
Return = 1.5: 0
Return = 2: 0
Return = 2.5: 0
Return = 2.96: 0

High Type

Contract Assignment

Probability of Assignment: 1 1 1 1
Screening Level (s): 0.4 0.4 0.4 0.4
Risk Level (σ): 1 1 1 1
Capital Level (1-D): 0% 0% 0% 0%
Fines
Return = 0.04: 0 0 0 0
Return = 0.5: 0 0 0 0
Return = 1: 0 0 0 0
Return = 1.5: 0 0 0 0
Return = 2: 0 0.8917 0.8812 0
Return = 2.5: 0 0.4321 0 0
Return = 2.96: 0 0 0 0

Notes for Table 3: This table gives the details of the optimal contracts for the low type bank and the high type bank under the baseline parameterization: τ = 0.01, τ = 0.05, γ = 1.0. The first two panels, labeled “Low Type,” give the details of the contract for the low type bank. The second panel is only used if the optimal contract randomizes between two contract assignments. The third panel gives the contract for the high type bank. (In the baseline parameterization, the high type bank is never assigned a randomized contract.) Each panel gives the probability of the assignment (1.0 unless a randomized contract is used), the assigned screening level, risk level, and capital level, and the amount of fines on each of the seven return levels. The first column gives the optimal contract when type is public information. The remaining three columns give the optimal contract when type is unobservable and the percentage of high type banks takes three different values or ranges. The parameters correspond to the baseline parameterization in Section 4.2: μ mê = {0, 0.6}, μ mê = {1.7, 2.3}, a = {5, 5}, ρ = 0.05, τ = 0.01, γ = 1.0.
### Table 4: Private and Social Value of Regulated Banks

<table>
<thead>
<tr>
<th>Type = low</th>
<th>Optimal Contract of Low-type Banks</th>
<th>Optimal Contract of High-type Banks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Private Value</td>
<td>Social Value</td>
</tr>
<tr>
<td>(s, σ) = ( 0.20, 0.60 )</td>
<td>1.11999</td>
<td>0.92458</td>
</tr>
<tr>
<td>(s, σ) = ( 0.30, 0.60 )</td>
<td>1.16939</td>
<td>1.07064</td>
</tr>
<tr>
<td>(s, σ) = ( 0.40, 0.60 )</td>
<td>1.17924</td>
<td>1.11986</td>
</tr>
<tr>
<td>(s, σ) = ( 0.50, 0.60 )</td>
<td>1.14933</td>
<td>1.11034</td>
</tr>
<tr>
<td>(s, σ) = ( 0.20, 1.00 )</td>
<td>1.17924</td>
<td>0.92317</td>
</tr>
<tr>
<td>(s, σ) = ( 0.30, 1.00 )</td>
<td>1.13388</td>
<td>1.06850</td>
</tr>
<tr>
<td>(s, σ) = ( 0.40, 1.00 )</td>
<td>1.04524</td>
<td>1.11694</td>
</tr>
<tr>
<td>(s, σ) = ( 0.50, 1.00 )</td>
<td>0.94440</td>
<td>1.10684</td>
</tr>
<tr>
<td>Type = high</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(s, σ) = ( 0.20, 0.60 )</td>
<td>1.53659</td>
<td>1.52437</td>
</tr>
<tr>
<td>(s, σ) = ( 0.30, 0.60 )</td>
<td>1.57752</td>
<td>1.66958</td>
</tr>
<tr>
<td>(s, σ) = ( 0.40, 0.60 )</td>
<td>1.47614</td>
<td>1.71743</td>
</tr>
<tr>
<td>(s, σ) = ( 0.50, 0.60 )</td>
<td>1.31935</td>
<td>1.70637</td>
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<td>(s, σ) = ( 0.20, 1.00 )</td>
<td>1.23535</td>
<td>1.52008</td>
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<tr>
<td>(s, σ) = ( 0.30, 1.00 )</td>
<td>1.09230</td>
<td>1.66373</td>
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<tr>
<td>(s, σ) = ( 0.40, 1.00 )</td>
<td>0.96035</td>
<td>1.71130</td>
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<td>(s, σ) = ( 0.50, 1.00 )</td>
<td>0.84346</td>
<td>1.70076</td>
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</tbody>
</table>

**Notes:** Table 4 displays the value of the bank’s objective (“Private Value”) and the regulator’s objective (“Social Value”) for the optimal low-type contract and the optimal high-type contract as a function of type (low or high), risk level σ and screening level s. The optimal contract for a low-type bank has zero capital and a fine of 1.8910 on the highest return. The optimal contract for a high-type bank has zero capital and a fine of 0.8917 on the return equal to 2.0 and a fine of 0.4321 on the return equal to 2.5. The parameters correspond to the baseline parameterization in Section 4.2: \( \mu^{\text{min}} = \{0, 0.6\}, \mu^{\text{max}} = \{1.7, 2.3\}, a = \{5, 5\}, \rho = 0.05, \tau = 0.01, \gamma = 1.0. \)
<table>
<thead>
<tr>
<th>Bank Type: Private or Public Information?</th>
<th>Public</th>
<th>Private</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>High type %:</strong></td>
<td>NA</td>
<td>1%</td>
</tr>
<tr>
<td>Social value of contract:</td>
<td>1.11987</td>
<td>1.11987</td>
</tr>
<tr>
<td>Private value of contract before taxes:</td>
<td>1.17924</td>
<td>1.17924</td>
</tr>
<tr>
<td>Private value of contract after taxes:</td>
<td>1.11987</td>
<td>1.12067</td>
</tr>
<tr>
<td><strong>Low Type</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Social value of contract:</td>
<td>1.71993</td>
<td>1.71867</td>
</tr>
<tr>
<td>Private value of contract before taxes:</td>
<td>1.82314</td>
<td>1.69761</td>
</tr>
<tr>
<td>Private value of contract after taxes:</td>
<td>1.71993</td>
<td>1.63904</td>
</tr>
<tr>
<td><strong>High Type</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Social value of contract:</td>
<td>1.71993</td>
<td>1.71867</td>
</tr>
<tr>
<td>Private value of contract before taxes:</td>
<td>1.82314</td>
<td>1.69761</td>
</tr>
<tr>
<td>Private value of contract after taxes:</td>
<td>1.71993</td>
<td>1.63904</td>
</tr>
</tbody>
</table>

**Notes to Table 5:** This table gives the social value (the value in the regulator’s objective function) and the private value (the value in the bank’s objective function) of optimal contracts both before and after taxes. The parameterization for the baseline case is: $\tau = 0.01$, $\rho = 0.05$, $\gamma = 1.0$. The first column, labeled “Public” gives these valuations for the optimal contract when bank type is observable. The remaining four columns give the values for the optimal contracts when type is unobservable, and the percentage of high types takes on four values. The parameters correspond to the baseline parameterization in Section 4.2: $\mu_{\text{min}} = \{0, 0.6\}$, $\mu_{\text{max}} = \{1.7, 2.3\}$, $\alpha = \{5, 5\}$, $\rho = 0.05$, $\tau = 0.01$, $\gamma = 1.0$.
0101 Manuel Arellano: “Discrete choices with panel data”.
0102 Gerard Llobet: “Patent litigation when innovation is cumulative”.
0103 Andres Almazán and Javier Suárez: “Managerial compensation and the market reaction to bank loans”.
0104 Juan Ayuso and Rafael Repullo: “Why did the banks overbid? An empirical model of the fixed rate tenders of the European Central Bank”.
0105 Enrique Sentana: “Mean-Variance portfolio allocation with a Value at Risk constraint”.
0106 José Antonio García Martín: “Spot market competition with stranded costs in the Spanish electricity industry”.
0107 José Antonio García Martín: “Cournot competition with stranded costs”.
0108 José Antonio García Martín: “Stranded costs: An overview”.
0109 Enrico C. Perotti and Javier Suárez: “Last bank standing: What do I gain if you fail?”.
0110 Manuel Arellano: “Sargan's instrumental variable estimation and GMM”.
0201 Claudio Michelacci: “Low returns in R&D due to the lack of entrepreneurial skills”.
0202 Jesús Carro and Pedro Mira: “A dynamic model of contraceptive choice of Spanish couples”.
0203 Claudio Michelacci and Javier Suarez: “Incomplete wage posting”.
0204 Gabriele Fiorentini, Enrique Sentana and Neil Shephard: “Likelihood-based estimation of latent generalised ARCH structures”.
0205 Guillermo Caruana and Marco Celentani: “Career concerns and contingent compensation”.
0206 Guillermo Caruana and Liran Einav: “A theory of endogenous commitment”.
0207 Antonia Díaz, Josep Pijoan-Mas and José-Víctor Rios-Rull: “Precautionary savings and wealth distribution under habit formation preferences”.
0208 Rafael Repullo: “Capital requirements, market power and risk-taking in banking”.
0302 Cristina Barceló: “Housing tenure and labour mobility: A comparison across European countries”.
0303 Victor López Pérez: “Wage indexation and inflation persistence”.
0304 Jesús M. Carro: “Estimating dynamic panel data discrete choice models with fixed effects”.
0305 Josep Pijoan-Mas: “Pricing risk in economies with heterogenous agents and incomplete markets”.
0306 Gabriele Fiorentini, Enrique Sentana and Giorgio Calzolari: “On the validity of the Jarque-Bera normality test in conditionally heteroskedastic dynamic regression models”.
0307 Samuel Bentolilla and Juan F. Jimeno: “Spanish unemployment: The end of the wild ride?”.
0308 Rafael Repullo and Javier Suarez: “Loan pricing under Basel capital requirements”.

0309 Matt Klaeffling and Victor Lopez Perez: “Inflation targets and the liquidity trap”.

0310 Manuel Arellano: “Modelling optimal instrumental variables for dynamic panel data models”.

0311 Josep Pijoan-Mas: “Precautionary savings or working longer hours?”. 

0312 Meritxell Albertí, Ángel León and Gerard Llobet: “Evaluation of a taxi sector reform: A real options approach”.

0401 Andres Almazan, Javier Suarez and Sheridan Titman: “Stakeholders, transparency and capital structure”.

0402 Antonio Diez de los Ríos: “Exchange rate regimes, globalisation and the cost of capital in emerging markets”. 

0403 Juan J. Dolado and Vanessa Llorens: “Gender wage gaps by education in Spain: Glass floors vs. glass ceilings”.

0404 Sascha O. Becker, Samuel Bentolila, Ana Fernandes and Andrea Ichino: “Job insecurity and children’s emancipation”.

0405 Claudio Michelacci and David Lopez-Salido: “Technology shocks and job flows”. 

0406 Samuel Bentolila, Claudio Michelacci and Javier Suarez: “Social contacts and occupational choice”.

0407 David A. Marshall and Edward Simpson Prescott: “State-contingent bank regulation with unobserved actions and unobserved characteristics”.