INFLATION TARGETS AND THE LIQUIDITY TRAP

Matt Klaeffling and Victor Lopez Perez

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CEMFI
Casado del Alisal 5, 28014 Madrid
Tel. (34) 914 290 551. Fax (34) 914 291 056
Internet: www.cemfi.es

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Abstract

The presence of a lower bound of zero on nominal interest rate has important implications for the conduct of optimal monetary policy. Standard rational expectations models can have alternative steady states as well as non-unique laws of motion, i.e. there can be possible sunspot equilibria. Such complications can be ruled out under a number of alternative assumptions. In this paper we analyse the relevance of the zero lower bound for alternative levels of inflation in a standard Neo-Keynesian model, where stability is assured by assuming that fiscal policy turns expansionary at the zero lower bound.

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Matt Klaeffling
Lehman Brothers
MKlaeffl@lehman.com

Victor Lopez
ECB and CEMFI
Victor.Lopez@ecb.int

1 Introduction

The zero bound on the nominal interest rate is an issue that has received increasing attention from economists in recent times. What seemed to be a topic of purely theoretic interest during the high-inflation period of the 1970s and early 1980s has turned into a hot policy debate due to the steady decline of inflation rates and, above all, the Japanese fall into the liquidity trap in the nineties. As a result, there has developed a rapidly growing literature analyzing issues related to the zero bound[1] and trying to learn from the Japanese experience[2].

Central bankers are concerned about the zero bound on nominal interest rates because it may render nominal interest rate policies unable to create the stimulus needed by an economy when output is below trend and inflation expectations are below target. This simply means that policy becomes less effective under certain circumstances. More importantly, theoretical difficulties arise. As was shown in a series of papers by Benhabib, Schmitt-Grohe, and Uribe (1999, 2000a, 2000b, 2001), the presence of a lower bound on nominal interest rates implies the existence of an alternative steady state, in which the inflation rate is negative (and equal in absolute value to the equilibrium real interest rate) and nominal interest rates are zero. Importantly, the standard steady state is unstable: there are an infinite number of trajectories that take the economy from the standard equilibrium to the alternative one. Moreover, this alternative steady state is indeterminate, i.e. random shocks to expectations (*sunspots*) are compatible with rational expectations. These theoretical considerations have important policy implications because the possibility of sunspots

[1] See Benhabib, Schmitt-Grohe and Uribe (1999, 2000a, 2000b, 2001), Bernanke (2000), Buiter and Panigirtzoglou (1999), Clouse, Henderson, Orphanides, Small and Tinsley (2000), Goodfriend (2000), Krugman (1998), McCallum (2000), Reifschenider and Williams (2000), Saunders (2000) or Wolman (1999).

[2] See, for example, Bernanke (1999), Cecchetti (1998), Posen (1998) or Okina (1999).

implies that the economy possesses an uncontrollable risk (cost) for the policy authority. A theoretically satisfactory analysis of the relevance of the issue of the lower bound in theoretical models therefore has to deal with the potential multiplicity of steady states. Relative to these issues this paper is modest in scope in that it ignores the sunspot issue and assumes unicity of the equilibrium.

This paper discusses the question of the level of inflation that maximises the welfare of a representative agent within the framework of a Neo-Keynesian rational expectations model. In the context of this model, optimal policy will depend on the trade-off implicit in choosing a target level of inflation. The level of inflation here is postulated to be positively correlated with the variance of demand shocks. Thus, higher levels of the inflation target induce higher macroeconomic variance, which is considered undesirable[3]. On the other hand, a higher level of inflation serves as a protective barrier from the zero-bound of nominal interest rates, the trap out of which escape is costly. Costly escape out of the trap is modelled by assuming that otherwise neutral fiscal policy turns expansionary[4], thus increasing inflationary pressure, reducing the real interest rate, and ultimately pushing production back towards its equilibrium level. The cost of this intervention is incurred because we assume that government expenditure is essentially wasteful in this model[5]. Finally, monetary policy is designed to maximise welfare of the representative consumer. Welfare essentially consists of two parts, an expected value term, which is decreasing in government expenditure (consumption equalling production minus government expenditure) and the adjustment for the concavity of the utility function, which is decreasing in the

[3] This view is consistent with the distortions related to interactions of inflation with the tax system (Feldstein (1997)) and the empirical finding that inflation level and inflation variance are positively correlated both over time and across countries (Okun (1971), Okun (1975), Taylor (1981) and Ball and Cecchetti (1990)).

[4] In the spirit of Svensson (2001).

[5] This is only a technical assumption. One can conceive alternative forms of public expenditures that have a positive return.

variance of consumption.

The value added by the paper is twofold. On the one hand, it allows for the reaction of the economy to a shock to be state-dependent. This is specially true in the context of the zero bound on nominal interest rates since the degree of effectiveness of the monetary decisions is very limited when interest rates are close to zero[6]. On the other hand, this paper embeds the relationship between the inflation rate and the volatility of the shocks, widely documented in the literature, in an otherwise standard model with nominal rates bounded at zero.

It should be noted that the results of the paper naturally depend on the parameters we use to calibrate the model. In particular, they are sensitive to the assumption on the equilibrium real interest rate. Under the assumption on the equilibrium real interest rate being equal to 2%, we find two main results: First, the probability of hitting the zero lower bound upsurges non-linearly when the inflation target decreases, increasing rapidly as the inflation targets drops below 1 percent and being around 5 percent for an inflation target of zero. And second, the simple economy we propose implies that 2 per cent is the inflation target that maximises the expected utility of a representative consumer.

However, if the equilibrium real interest rate is set equal to 3%, the probability for the non-negativity constraint on nominal interest rates to be binding plummets to negligible figures for all non-negative inflation targets and then the welfare maximising inflation target turns out to be zero. Therefore, given the large degree of uncertainty surrounding the estimates of the equilibrium real interest rate, a welfare-maximising central bank, in the context of this model, should weigh these two different scenarios when choosing the quantitative definition of its policy objective. Indeed, a risk-averse policy-maker would presumably buy insurance by means of attaching a higher weight

[6] See Kimura *et al.* (2002) for empirical evidence on the Japanese economy.

to the “2% real rate” scenario.

Section 2 briefly surveys the literature on optimal inflation and the nominal interest rate zero bound. Section 3 explains the model in greater detail. Section 4 produces the main results, notably, probability estimates of falling into the liquidity trap for several inflation targets and an analysis of the optimal inflation target. Section 5 concludes.

2 Optimal Inflation and the Zero Lower Bound

2.1 Optimal Inflation

When discussing the optimal inflation target it is compulsory to recall Milton Friedman (1969), who proposed that the optimal inflation rate should be negative and equal in absolute value to the real interest rate. According to the Fischer equation the nominal interest rate would then be zero and real balances would be held at a zero marginal cost[7]. This is the famous Friedman rule. The reasoning behind this is that there is a social cost associated with holding currency relative to investing it at a positive interest rate. Since the production of currency is essentially of zero cost there would be, in the words of Robert Lucas, “one of the few legitimate ‘free lunches’ economics has discovered in 200 years of trying”.

Four years later, Phelps (1973) noted that the Friedman argument ignored the fact that inflation allows the government to extract an inflation tax through seigniorage. In the absence of seigniorage, the government will have to rely on alternative (distortionary) means of collecting income. Depending on the welfare cost of these alternative means, which depend on the tax code and the elasticities of factor supplies, the optimal level of inflation will be correspondingly higher. Cogley (1997) notes

[7]Which is equal to the marginal cost of production of currency.

that the inflation tax may in fact have a higher distortionary cost than other forms of taxation, and argues in favor of an inflation rate of 1 percent on the basis of the econometric evidence of distortionary taxation of Mulligan and Sala-i-Martin (1997) and Braun (1994)[8]. Niccolini (1997) argues for a positive inflation tax in the presence of an underground economy that the fiscal authority cannot tax otherwise. Aizeman (1987) and Vegh (1998) argue for an inflation tax based on the collection costs that are associated with other forms of taxation. Whereas these papers focus on calculating the optimal level of inflation others directly focus on the welfare implications of alternative inflation rates. Two such applications are Lucas (2000) and Wolman (1997). While Lucas argues that the reduction from the historic rate of 5 per cent to 0 percent exploits most of the welfare gains relative to the Friedman optimal rule, Wolman shows how this conclusion can be turned upside down by using a different money-demand equation, i.e. he shows how for a different functional form for money demand the bulk of the welfare gain lies in reducing the inflation rate from zero to the Friedman-optimal rate of minus the real interest rate.[9]

Summers (1991) advocated a positive inflation target to deal with several "real-world problems". One of these problems[10] is that nominal interest rates are bounded at zero. Money has a pecuniary rate of return of zero (abstracting from insurance costs, storage costs and taxes) and a non-pecuniary return as a unit of account and a medium of exchange higher than other financial assets. If the nominal interest rate for a close but not perfect substitute for money is negative, an agent can maximise

[8]See also Kimbrough (1986), Guidotti and Vegh (1993), Christiano, Eichenbaum, and Evans (1996), Correira and Teles (1996, 1999), De Fiore and Teles (1999) and De Fiore (2000) on the optimal inflation rate in various theoretic economies.

[9]One important caveat in calculating the welfare effects of reducing inflation to the Friedman optimal rate from estimated (or for that mater calibrated) mony-demand specifications is that one has to extrapolate results into a range of nominal interest rates that have never been observed historically.

[10]Apart from inflation measurement bias and downward nominal wage rigidities.

returns by holding money at a zero interest rate rather than using it to buy a close substitute at a negative interest rate. In such a situation, the economy could find itself in what is called a liquidity trap.

2.2 The Zero Lower Bound

Following Svensson (2000), "In a liquidity trap the economy is satiated with liquidity and the nominal interest rate is zero. (...) If equilibrium real interest rates are positive, equilibrium expected inflation will be negative. (...) Thus, by a liquidity trap, I mean a situation with zero interest rates, persistent deflation and persistent deflation expectations". The liquidity trap so defined is a nominal downward spiral. This is the sense in which there may exist an alternative steady-state in addition to the standard one in which the inflation rate is equal to the policy target and output equals potential output[11].

Policy prescriptions that assure the uniqueness of the standard steady state and the economy's law of motion have been advanced by a number of authors. These policies are usually ordered into three main groups: providing more liquidity to the economy, affecting expectations directly and taxing money holdings.

As to the first group, the basic idea is that the central bank may increase the monetary base by purchasing a variety of assets. For example, the monetary authority may buy Treasury bills via open-market operations (Clouse *et al.* (2000)), government bonds (Clouse *et al.* (2000) and Bernanke (2000)), foreign currencies in exchange markets (Meltzer (1999), Bernanke (2000), Clouse *et al.* (2000), McCallum (2000) and Svensson (2001)) and private sector securities (Bernanke (2000) and Clouse *et al.* (2000)). In addition, the central bank may lend money to the private sector (Clouse

[11] See Reifschneider and Williams (2000) and Benhabib, Schmitt-Grohe, and Uribe (1999, 2000a, 2000b, 2001).

et al. (2000)) or may let money rain (Clouse *et al.* (2000), Bernanke (2000) and Benhabib, Schmitt-Grohe and Uribe (2000a)).

The second set of proposals relies on affecting expectations directly to drive the economy out of the trap. To get to this goal, the monetary authority must have the credibility that it will adhere to what it proposes and, what is more important, the credibility that it can deliver on its proposal. To gain credibility, monetary authority may adhere to a commitment to an explicit inflation target for several years into the future (Krugman (1998) and Bernanke (2000)), to money-growth targets (Hetzel (1999)) or to maintain nominal interest rates at zero level after the liquidity trap has been abandoned (Okina (1999)). Another possibility for the central bank is to write options on the Treasury bond rate that will prevail at some point in the future (Tinsley (1999)). Inside this group, it may be useful to consider agreements with the fiscal authority to a contingency plan to be implemented immediately if a zero bound situation were to occur (Svensson (2001)). Benhabib, Schmitt-Grohe and Uribe (2000) propose an inflation sensitive fiscal policy that calls for lowering taxes when inflation subsides and show that the rule can rule out liquidity traps by making them fiscally unsustainable[12]. The channel through which the liquidity trap is eliminated here is basically that a decline in taxes increases the household's after-tax wealth, which induces an aggregate excess demand for goods[13].

The third group of potential alternatives to the interest rate channel basically consists of various ways of taxing money holdings (Keynes (1923), Gesell (1949), Buiter and Panigirtzoglou (1999) and Goodfriend (2000)). Such taxes are oftentimes also referred to as Gesell taxes. By taxing money holdings, the opportunity cost of

[12]Since sustainability of the fiscal policy is a prerequisite for a rational expectations equilibrium, no equilibrium that would imply an unsustainable fiscal policy can exist.

[13]Note that the magnitude of this effect goes back to the classical Keynes-Pigou debates (Keynes (1936), Pigou (1950)).

holding money is positive in a context of zero (or even slightly negative) nominal interest rates. Hence, the demand for short-term bonds would be positive since they are not taxed by the Gesell tax. The aforementioned elements enable the policy-maker to decrease the short-term nominal interest rate below zero and to avoid the liquidity trap simultaneously. The higher the tax rate on money holdings, the larger the extra room of manoeuvre provided by the tax. Nonetheless, these policy actions may be accompanied by so high administrative costs that they appear uninteresting in practice.

In summary, there are two distinct arguments that are complementary to the analysis of the liquidity trap. First, how can the likelihood of nominal interest rates dropping to zero be minimised and, second, if we do arrive at zero nominal interest rates, how can we escape from the trap. At this point it is important to note that all of the above policy recommendations assure that while we may end up at the zero bound, we do not enter the trap per se. To see this recall that the liquidity trap is defined as an alternative equilibrium - a deflationary spiral - and as such it is a problem to the extent that it would lead the economy to converge to an alternative (suboptimal) steady state. At this other steady state there could theoretically be *sunspot* dynamics of unbounded variance. As a result, all policies that allow the economy to slide inside the trap are intrinsically inefficient. Hence, the only way any policy can protect the economy from the trap is to rule it out as an equilibrium. This argument can be captured in Figure 1, which is borrowed from Benhabib, Schmitt-Grohe and Uribe (2000a), and shows the level of the nominal interest rate as a function of the level of inflation.

In the graph R denotes the equilibrium real interest rate as determined by the steady state rate of time preference of the consumers, and π is the rate of inflation (varying throughout the horizontal axis). $R+\pi$ denotes the nominal interest rate defined as the rate that equilibrates the money market[14](vertical axis) and $I(\pi)$ denotes the supply side "price", the policy rule that determines the nominal interest rate as a function of the nominal interest rate target and the state of the economy to which the policy maker reacts (also throughout the vertical axis). Following Benhabib *et al.* (2000a) we assume that $\frac{\partial I(\pi)}{\partial \pi} \mid_{\pi=\pi^*} > 1$, an assumption that is satisfied, for example, by the Taylor rule as a policy function (see Section 3),

$$I(\pi) = R + \overline{\pi} + \theta^{\pi}(\pi_t - \overline{\pi}) + \theta^{y}(y_t - \overline{y}),$$

where $\overline{\pi}$ and $\overline{y}$ denote the target levels (steady states) of inflation and output, respectively. In this case $\frac{\partial I(\pi)}{\partial \pi} = \theta^{\pi} > 1$ since $\theta^{\pi} = 1.5$ in the Taylor rule[15].

Clearly, there exist two steady states. The first one is the standard case, where $\pi = \pi^* > 0$ and the nominal interest rate $I(\pi^*)$ is positive. This equilibrium is unstable. The second one represents the liquidity trap case, where $\pi = \pi^L = -R$ and the nominal interest rate $I(\pi^L)$ equals zero. Benhabib *et al.* (2000a) demonstrate that the standard equilibrium is unstable whilst the deflationary spiral is stable. We can then uniformly represent all of the policy recommendations regarding escapes from the trap as ways to limit the support of the level of inflation. In fact, one way or another all policy solutions lead to violations of some transversality condition for the equilibrium associated with π_L (see Benhabib *et al.* (2000a)) or increase the lower bound on inflation beyond the level associated with the liquidity trap π_L. In the presence of appropriate policy, the equilibrium is then unique.

[14]By the Fischer equation, the nominal interest rate equals the real interest rate, R, plus the expected future inflation rate. In equilibrium, the expected future inflation rate will be equal to the rate of inflation, π.

[15]The reason why $\frac{\partial I(\pi)}{\partial \pi} > 1$ is a particularly appropriate assumption in the case of Taylor-type policy rules is that this assumption usually is necessary to assure local determinacy of the economy around the standard (target) steady state.

2.3 The Relevance of the Zero Lower Bound

It seems clear that the importance of the zero bound on nominal interest rates as a constraint on monetary policy depends on several factors, such as the frequency, the magnitude and the persistence of the shocks that hit the economy. To analyse the probability of being caught in the liquidity trap, researchers have followed two complementary paths: to use historical data or to rely on simulation analysis.

2.3.1 Historical analysis

The main conclusion that is obtained from the analysis of historical time series is that the probability of hitting the zero bound is essentially zero for an inflation rate of 2 percent. Of course, this is a result that is conditional on an equilibrium real interest rate in line with long-run averages of industrial countries.

Clouse *et al.* (2000) review the history of nominal interest rates in the United States since 1860. They report that for the period between 1860 and 1930 short-term interest rates were well above zero in the United States, despite a series of inflationary and deflationary cycles. Short-term nominal interest rates hit the zero bound by 1932 as a consequence of deflation that began in 1929[16] (with the price level declining 25 percent between 1929 and 1932). From 1932 to 1948 nominal interest rates were under 1 percent, very close to the constraint. The authors construct a proxy for the room available to the monetary authority to diminish nominal rates in response to shocks. The Great Depression is said to stand out "not because of relative little room for easing at the outset of the downturn in 1929 but for ultimately running out of room despite the initial room to ease". Since 1950, nominal interest rates have been well above the zero bound. Also, as noted by Summers (1991), this nominal interest

[16] Actually, a mild deflation in the Consumer Price Index was already under way since the end of 1924.

rate history implies ex-post real interest rates in the United States have actually been lower than zero in about one-third of the years since World War II[17]. Clouse *et al.* also analyse the Japanese experience as well and find that Japan in the 1990s had a delayed decline in long-term yields that was very similar to the experience of the 1930s in the United States. Figure 2 shows the path of interest rates and inflation for Japan during the 90s.

Putting the pieces together we can state that historically the nominal bound has been important in the U.S. during the Great Depression and in Japan during the 90s.[18] In any event, given the shortcomings implied by a purely historical analysis to provide any policy recommendation, many studies supplement the historical analysis with a simulation study.

2.3.2 Simulation analysis

By defining artificial economies and analysing the effects of simulated shocks, researchers have found that the relationship between the inflation target and the probability of the zero bound to be binding is a non-linear one, such that as inflation approaches zero, the likelihood of encountering the zero bound increases at an increasing rate. The prevailing view seems to be that an inflation target of 2 percent would be high enough to sufficiently reduce the effect of the zero bound on the effectiveness of monetary policy.

Cozier and Lavoie (1994) present a calibrated reduced form model with an aggregate demand equation, an expectations augmented Phillips curve, an exchange rate

[17] The real after-tax rate has actually been negative in about 75 per cent of these years

[18] In addition, the US historically has had the privilege of its currency serving as an international financial safe haven, which has resulted in lower real interest rates than in countries of similar macroeconomic performance (see Campbell (1999)), which in turns means that the issue of the zero bound is *ceteris paribus* (i.e. controlling for differences in inflation targets) of accentuated importance for the Federal Reserve Board.

equation, and a forward looking monetary policy rule. They find that the probability of falling into the trap is 3.5 percent at a 1 percent inflation rate and 5 percent at a zero inflation rate. Fuhrer and Madigan (1997) evaluate the zero bound importance by comparing the response of their model (composed by a backward-looking IS curve, a Phillips curve and a monetary policy reaction function) to IS curve shocks at inflation targets of zero and 4 percent. They conclude that, at a zero inflation target, monetary policy is significantly constrained by the zero bound.

Orphanides and Wieland (1998) propose a model quite similar to Fuhrer and Madigan's but disaggregating the IS curve into its components. They use estimated shock processes and compare the variance of output at different inflation targets, finding a significant effect on economic performance if monetary authority sets an inflation target lower than 1 percent. More specifically, they consider two types of rules, attributed to Taylor (1993) and Henderson-McKibbin (1993), and find that the probability of the restriction to be binding is always higher for the latter, increasing from 10% at an inflation rate of 1 percent to 30 percent as the inflation rate drops to zero. However, they find negligible risk for an inflation rate of 2 percent.

Reifschneider and Williams (2000) use the Federal Reserve Board's econometric model of the U.S. economy and reach the conclusion that the zero bound could be a significant constraint on policy in very low inflation environments. As regards the two policy rules mentioned above, they find a probability of hitting the bound of 31 per cent for an inflation target of zero and 7 per cent for an inflation target of 4 per cent when the Henderson-McKibbin rule is used. These probabilities fall to 14 per cent and less than 1 per cent when the Taylor rule is the policy rule. When the zero bound is considered, Reifschneider and Williams, as opposed to Orphanides and Wieland, find that the Henderson-McKibbin rule outperforms the Taylor rule regarding output gap stabilisation[19] but at the cost of higher interest rate volatility.

Last but not least, Coenen and Wieland (2003) utilise a model taken from an earlier piece of research (Coenen and Wieland (2002)), which comprises three economies: the United States, the euro-area and Japan. Then, they conduct a simulation exercise aiming to calculate the frequency of bind of the zero bound on the Japanese nominal interest rates. This frequency turns out to be between 5 and 10 percent when the equilibrium nominal interest rate is 4% and increases rapidly when a lower equilibrium rate is assumed.

3 A Simple Model

3.1 Model specification

The present example builds around the paradigm of the neoclassical synthesis (Goodfried and King (1997)) and is a nutshell version of a class of models that includes Yun (1996), Jeanne (1997), Gali (2001) or Christiano, Eichenbaum, and Evans (2001) among many others. Rather than deriving the model's equilibrium-defining equations here again we refer to the above papers. The linearised economy can be concisely described by four building blocks, all variables being log-deviations with respect to their steady state values. First, an IS curve

$$y_t = \frac{1}{-\gamma}\left[i_t - E_t\left[\pi_{t+1}\right]\right] + E_t\left[y_{t+1}\right] + g_t - E_t[g_{t+1}] + z_t \tag{1}$$

This forward-looking aggregate demand equation can be derived from the representative consumer Euler condition imposing good markets clearing. y_t denotes the output gap at period t, defined as the log-difference between real output and potential

[19] Reifschneider and Williams assume that the equilibrium real interest rate equals 2.5% whilst Orphanides and Wieland chose 1%. This difference is, presumably, the underlying factor behind the discrepancy.

output (the prevalent one in absence of nominal rigidities). π_{t+1} is the inflation rate at period $t+1$, defined as the log-difference between prices at $t+1$ and prices at t. g_t represents public expenditure at time t. i_t is the nominal interest rate. E_t is the rational expectations operator and γ denotes the constant risk aversion coefficient of the representative household. The IS shock z_t in this model could be interpreted as a preference shock,

$$z_t = \phi z_{t-1} + \xi_t \qquad |\phi| < 1 \tag{2}$$

Ball and Cecchetti (1990) found an empirical relationship between long-run inflation and the variance of the shocks that hit the economy. In this spirit, let's assume a linear relationship between the unconditional standard deviation of ξ_t and the absolute value of the inflation target[20] (that is equal to the steady-state inflation rate in this model).

$$\sigma_\xi = \delta_0 + \delta_1 * abs(\overline{\pi}) \qquad \delta_0 > 0,\ \delta_1 > 0 \tag{3}$$

where $\overline{\pi}$ denotes the inflation target or steady-state inflation rate. Thus, we assume that the distortions driven by inflation are minimised when the long-run inflation rate, i.e. the policy target, is zero.[21] The second building block is the New Keynesian Phillips Curve,

$$\pi_t = \beta E_t \left[\pi_{t+1}\right] + \lambda y_t, \tag{4}$$

[20] Although we consider a symmetric effect (around zero) of the inflation rate on the variance of the shock, a theoretical argument may be mentioned for inflation below zero to have a larger effect than inflation above zero. From a theoretical perspective, an inflation rate below zero implies a larger probability for the economy to fall into the deflationary spiral. In such a situation *sunspot* shocks, whose variance is unbounded, may appear. However, *sunspot* shocks are not handled here as the fiscal stimulus is assumed to shield the economy from the deflationary risk. As regards the empirical evidence, the Japanese experience is rather uninformative about this issue. The sample period is very short on one side. On the other side, it is difficult to disentangle the source of the Japanese problem characterised by a situation of deflation amid a financial crisis and a deep process of reallocation of resources.

[21] As Woodford (1999).

where λ is known as the short-run slope of the Phillips curve. This forward-looking aggregate supply equation could be obtained from aggregation of the firms' optimal staggered price setting rules and assuming a linear relationship between real marginal costs and the output gap. The third block is a truncated Taylor rule[22],

$$i_t = \max\{\theta^\pi E_t[\pi_{t+1}] + \theta^c E_t[c_{t+1}], -\overline{r} - \overline{\pi}\} \tag{5}$$

where $\overline{r}$ is the equilibrium real interest rate, c_{t+1} symbolises the consumption gap at time $t+1$ (consumption is linked to output by the good-market clearing condition $Y_t = C_t + G_t$, where G_t denotes the level of public expenditure at time t) and (θ^π, θ^c) are the Taylor rule coefficients.[23] Therefore, the central bank would follow a forward-looking Taylor rule as long as the nominal interest rate that comes from the rule were zero or above. Otherwise, the policy rate would be zero.

In order to exclude a potential alternative deflationary steady state we simplistically could assume that whenever the interest rate is zero the fiscal authority stimulates demand directly via government expenditure. Hence, the fourth and last equation of the model is

$$g_t = \begin{cases} 0 & \text{if} \quad i_t = \theta^\pi E_t[\pi_{t+1}] + \theta^c E_t[c_{t+1}] \\ x > 0 & \text{otherwise} \end{cases} \tag{6}$$

Public expenditure is financed by constant lump-sum taxes. Taxes do not appear in the model as all variables are log-deviations from their steady-state values.

[22] Although we recognise that standard Taylor rules are not formulated in terms of the consumption gap but the output gap, in the context of our model it seems more reasonable to include the consumption gap in the monetary policy rule. The reason is that public expenditure becomes positive only when the economy is in a very bad situation (the zero bound is binding) and, therefore, an interest rate hike induced by positive expected public expenditure is the last thing the economy is claiming for. The results are qualitatively similar when the consumption gap is replaced with the output gap.

[23] Despite the fact that this truncated Taylor rule is not microfounded, it can be seen as a rather good approximation to the behaviour of the central banks in major industrialised countries (see Clarida, Gali and Gertler, 1998).

The fact that the Taylor rule is truncated is the cornerstone of our analysis for the fact that its non-linearity implies that we cannot apply standard solution techniques. To show this we will slightly have to manipulate the IS and the PC curves. The Phillips Curve, (4), can be solved forward to obtain

$$\pi_t = \lambda \sum_{s=0}^{\infty} \beta^s E_t y_{t+s} = E_t \left[\frac{\lambda}{1-\beta L^{-1}} y_t \right] \tag{7}$$

where L denotes the lag operator. Likewise, we solve the IS curve, (1), forward, which yields

$$y_t = E_t \left[\frac{1}{1-L^{-1}} \left[\frac{1}{-\gamma} \left(i_t - \pi_{t+1} \right) + z_t \right] \right] \tag{8}$$

Finally, we substitute for future π_t from (7) into (8) to obtain

$$y_t = E_t \left[\frac{(1-\beta L^{-1})}{(1-\alpha_1 L^{-1} - \alpha_2 L^{-2})} \lambda \left(\frac{1}{-\gamma} i_t + z_t \right) \right]$$

where $\alpha_1 = 1 + \beta + \lambda$ and $\alpha_2 = -\beta$. Which means that output today is a forward-looking ARMA(2,1), a weighted average of expected future demand shocks and interest rates. As said above, we cannot use standard system-reduction techniques to solve this problem for any given point in time since today's actions are weighted averages of two alternative monetary policy regimes, with weights that depend on today's economic state.

In this model we have only one economic shock, ξ_t, the shock to the IS curve. This shock is termed *fundamental*. It is the only shock that can affect the economy when the economy is outside the trap (i.e. when the Taylor rule is active). But there is a second shock, which can be labelled as a *sunspot* shock, that might affect the economy if it is inside the liquidity trap. This shock is not linked to the fundamentals of the economy (even though its dynamics are) and is therefore termed *nonfundamental*. The reason is that while nominal interest rates are positive the monetary policy rule defines a unique rational expectations equilibrium but if the economy is caught in the liquidity trap, the monetary policy rule is passive and the rational expectations equilibrium is not unique. To restore uniqueness, expansionary fiscal policy is assumed to take the economy out of the trap in this model. Therefore, *sunspot* shocks do not play a role anymore.

3.1.1 The effects of a *fundamental* shock if there is no trap in the model

Let's start considering two extreme cases. First, we will assume that there is no liquidity trap in the model (the inflation target is high enough). In this case, a positive demand shock results in an increase of consumption and inflation. Given the Taylor rule this produces an increase of nominal and real interest rates. It is this increase along with the decaying effect of the demand shock that drives the economy back to the steady state. Figure 3 plots the impulse-responses. The local dynamics here are the same as around the standard steady state.

3.1.2 The effects of a negative *fundamental* shock if the economy is always in the trap

The second extreme case is to assume that the economy is in the trap and it will remain there forever. In this case, monetary policy is ineffective and public expenditure does not help to stabilise the economy because $g_t = E_t[g_{t+1}] = x$, for every t.

Further, the passivity of policy means that we have an economic structure that does not yield enough transversality conditions to pin down all economic variables as functions of the demand shock alone. Instead, we have a situation where the state vector is composed out of the economic state variable that is the demand shock, as well as an additional *canonical* state variable that is constructed as a linear combination of economic variables and can be given the interpretation of a variable capturing the expectational state of the economy. Other than in the standard case, where expectations are uniquely defined by the current economic state variable of the system, we here have a case where expectations have marginal causal power. The state vector that determines the dynamics of the economy is then bivariate, composed by z_t and an additional variable. Since the law of motion of the state vector does not have a diagonal transition matrix, a shock to demand will affect this second state variable dynamically and will have a total effect on the economy that is given by the combined effect of these two variables.

However, since this extreme case will be ruled out once the economy is able to escape out of the trap, the *canonical* state variable is assumed to be zero every period and the effects of a *fundamental* (negative) shock can be seen in Figure 4. The responses of macroeconomic variables are much larger than before, since neither monetary policy nor fiscal policy are able to provide the mechanisms needed to smooth the path of the economy.

3.2 Model solution

We here report the main steps that allow us to numerically approximate the model solution. The solution involves three fundamental steps and is essentially a parameterised expectations version of the weighted residual method (see Christiano and Fisher (1997)). First, we have to approximate the mappings from the state of the economy to consumption and inflation respectively. Second, to weigh the loss function over the grid of the state. And third, to find starting values for the procedure, calibrate some parameters and minimise the loss function to compute the others.

a) *Mapping functions*: The model is a forward-looking rational expectations model, which means that today's actions are functions of the entire future path of shocks hitting the system. From the perspective of the representative consumer and firm, this means that their optimal consumption and pricing decisions depend on future shocks as well as the form of monetary policy in action at all points in time in the future. We thus have to calculate the state-depending probabilities of the two possible events: "at the bound at point $t+s$" and "not at the bound at point $t+s$". These probabilities depend on the state of the economy today, t, which in turn allow us to weigh the two macroeconomic regimes that pin down the optimal consumption and pricing decisions. In other words, the model solution should involve the two macroeconomic regimes which could be in place at all points in time from today on. What is known is that optimal consumption and pricing are expectations of functions of these future regimes and they have to be measurable with respect to the time-t state variable z_t and the inflation target $\overline{\pi}$.

We now proceed as follows. First, we define a grid Ω^z on z_t over which to evaluate the model solution. Then, for every $\overline{\pi}$, we approximate the functions that determine the parameters that map the state into consumption and inflation.

$$\begin{aligned} c_t &= f_c(z_t, \overline{\pi}) z_t \\ \pi_t &= f_\pi(z_t, \overline{\pi}) z_t \end{aligned} \tag{9}$$

These approximations are done assuming that the mapping is a combination of the linear mapping functions under the two extreme cases analysed above: "the economy never ends up in the trap" and "the economy is always in the trap",

$$\begin{aligned} f_c(z_t, \overline{\pi}) &= \varpi(z_t, \overline{\pi}) c\,(1) + (1 - \varpi(z_t, \overline{\pi})) c(2) \\ f_\pi(z_t, \overline{\pi}) &= \varpi(z_t, \overline{\pi}) \pi\,(1) + (1 - \varpi(z_t, \overline{\pi})) \pi(2) \end{aligned} \tag{10}$$

where $c(1)$ denotes the constant coefficient of the linear mapping function for the case "never in the trap" and by analogy $c(2)$ is the constant coefficient of the linear mapping function for the case "always in the trap". Hence, the responses of consumption and inflation are going to lay somewhere in between these two extreme cases, depending on the state of the economy and the inflation target chosen.

The weight $\varpi(z_t, \overline{\pi})$ is allowed to vary depending on the values of z_t and $\overline{\pi}$. The lower z_t or $\overline{\pi}$, *ceteris paribus*, the lower $\varpi(z_t, \overline{\pi})$. We made use of tenth-order Chebyshev polynomials[24] to capture the non-linearities stemming from the monetary and fiscal policy rules,

$$\varpi(z_t, \overline{\pi}) = chebyshev(\frac{z_t - z_{\min}}{2z_{\max}}, \overline{\pi})$$

where *chebyshev*$(\bullet)$ symbolises the 10th-order Chebyshev polynomial fitted under the inflation target $\overline{\pi}$, and $z_{\min}$ and $z_{\max}$ are the smallest and largest values for z_t we consider. These bounds are symmetric around zero ($z_{\min} = -z_{\max}$) and their values are chosen to define an interval covering more than 99.80 per cent of the probability mass of the state variable[25]. Therefore, according to the interpretation of $\varpi(z_t, \overline{\pi})$ provided above, this function is expected to be increasing in z_t and to shift upwards when the inflation target $\overline{\pi}$ augments.

b) *Loss function*: Then, we calculate the percentage errors of the aggregate demand and supply equations over the grid Ω^z. Therefore, we define the following loss function L for the grid point ω_z and the set of parameters ψ.

$$\psi = [\overline{\pi}, \gamma, \phi, \delta_0, \delta_1, \beta, \lambda, \theta^\pi, \theta^c, \overline{r}, x, \varpi(\bullet)]$$

$$L(\psi, \omega_z) = norm \begin{bmatrix} \frac{1}{\gamma}[i_t - E_t[\pi_{t+1}(\psi, \omega_z)]] - E_t[\Delta y_{t+1}(\psi, \omega_z)] + E_t[\Delta g_{t+1}] - \omega_z \\ \pi_t(\psi, \omega_z) - \beta E_t[\pi_{t+1}(\psi, \omega_z)] - \lambda y_t(\psi, \omega_z) \end{bmatrix}$$

As the model is not linear, the expectations about future economic variables are not trivially computed. The right way of proceeding is the following: given a grid point ω_z, an additional grid of shocks is defined Ω^ξ. For each grid point ω_z and

[24] The appeal of Chebyshev polynomials consists on compelling the approximation error to be arbitrarly small as the order of the polynomial augments.

[25] If at any point during the simulations the state variable moves below (above) the lower (upper) bound we simply set it equal to the lower (upper) bound.

each point of Ω^{ξ} the trajectories of the relevant variables are computed. Finally, uncertainty is integrated out by means of Gaussian quadrature.

Next, we have to decide how to weigh the loss function at various points of the grid. To this effect the related literature on functional approximation[26] proposes a variety of ways such as collocation, squared residuals, etc. In general there is no natural answer to the question of how to best weigh the losses over the grid. In the minimisation step we compute the euclidean norm of the loss function values over the grid , i.e.

$$L^m = \|L(\psi, \omega_{z_{\min}}), ..., (\psi, \omega_{z_{\max}})\| \tag{11}$$

c) ***Calibration and minimisation routine*:** The final step is to calibrate some parameters and compute the others by loss function minimisation. Table 1 shows the values assigned to calibrated parameters. γ represents the constant relative risk aversion coefficient. We set it equal to 3 to obtain a larger degree of risk-aversion than the one implied by log-preferences[27]. ϕ is the AR(1) coefficient of the stochastic process of the demand shock. We calibrate it equal to 0.85 to obtain the degree of persistence observed in reality for output and inflation. δ_1 is set equal to 0.07, based on the results reported by Ball and Cecchetti (1990) and δ_0 is assigned the value 0.0036 to match the output gap standard deviation after World War II for major industrialised countries with average inflation for the euro area after the ECB was founded. β is the intertemporal discount factor. It is set equal to 0.995, since units of time in the model are quarters. λ is the slope of the Phillips curve. It is assigned a small value (0.05) consistent with the estimated degree of price stickyness

[26] Judd (1996, 1998), McGrattan (1999).

[27] This value is consistent with the findings reported by Hamada (1997) and Guo and Withelaw (2001).

for major industrialised countries[28]. The nominal interest rate rule parameters $(\theta^{\pi}, \theta^{c})$ are calibrated following Taylor (1993) i.e.(1.5,0.5). The equilibrium real interest rate $(\overline{r})$ is set equal to 2%. However, due to the large degree of uncertainty surrounding the value of this parameter, we also report the main results of the paper when the equilibrium real rate is assumed to be 3% (see Appendix A). The public expenditure parameter is assigned the value 0.03, consistent with the constraint on fiscal deficits imposed by the Stability and Growth Pact (3 per cent of GDP per year)[29].

Finally, we compute the parameters of the Chebyshev polynomial by means of minimising the loss function (12) for each inflation target $\overline{\pi}$,

$$\psi^{*} = \min_{\psi} \left\| L(\psi, \omega_{z_{\min}}), ..., (\psi, \omega_{z_{\max}}) \right\|$$

Computationally we do this by using the Levenberg-Marquardt algorithm, for inflation targets changing from -2 per cent (the so-called Friedman rule) to 4 per cent. Since with one state variable the problem is computationally very manageable we use a grid of 25 points for the state space and another grid of 25 points for the Gaussian quadrature procedure to integrate uncertainty out. The next section reports our numerical results.

4 Main results

4.1 Reduced form coefficients

Figure 5 plots the value of the weighting function $\varpi(z_t, \overline{\pi})$ as the state variable z_t varies (for $\overline{\pi}$ equal to -2 percent, the Friedman rule). Not surprisingly, $\varpi(\bullet)$ increases when the state of the economy improves. Moreover, the non-linearity implied by the

[28] See Lansing (2001) or Roberts (2001). This value of λ is consistent with the range reported by McCallum and Nelson (2000) for the impact of the output gap on inflation.

[29] Note that the level of potential output is normalised to 1.

zero lower bound on nominal interest rates turns out evident.

Figure 6 charts the same plot but now $\overline{\pi}$ is set equal to 2 percent. As expected, a higher inflation target shifts the weights upwards since it reduces the likelihood of hitting the zero bound.

These exercises illustrate that our reduced form equations behave in an appropriate manner. Therefore, we can make use of them to analyse the relevance of the zero bound on nominal interest rates under different inflation targets.

4.2 The Probability of Hitting the Zero Bound

Figure 7 shows the probability of hitting the zero bound implied by our model[30]. Like other scholars, we find that the probability increases more than proportionally when the inflation target decreases. When the equilibrium real interest rate is assumed to be 2 percent, this model implies that the probability of hitting the zero bound becomes negligible only at an inflation target above or equal to 1 per cent (the results under the assumption $\overline{r} = 0.03$ are reported in Appendix A).

For comparison, recall that Orphanides and Wieland (1998) report that the probability of the zero bound to be a binding constraint to nominal interest rate rules in the U.S. is negligible for inflation targets around 2 percent in their model. These authors report results for an estimated backward-looking sticky-price model for two alternative policy rules, the Henderson and McKibbin (1993) rule and the Taylor (1993) rule, which is also the one we use. Relative to the latter the former has both a higher coefficient on the output gap (2 instead of 0.5) and on inflation (2 instead of 1.5). They find that the probability of hitting the bound is strictly higher for the

[30]This probability is computed by starting 40 simulations of 2500 periods each from the deterministic steady state. We then drop the first 250 data points to limit dependence on initial conditions and calculate the fraction of times that the zero lower bound was binding. Exact quadrature-based methods were also used, leading to very similar results.

Henderson and McKibbin rule than for the Taylor rule increasing from essentially zero per cent at an inflation target of two per cent to 10 and 30 per cent as the inflation target drops to 1 and 0 per cent (for the Taylor rule they report 0, 3 and 16 per cent respectively). Nonetheless, they rely on linear methods to conduct their simulation exercises. This is not correct since the model is not linear once the zero bound on the nominal interest rate is introduced.

On the contrary, we correctly take into account the non-linear feature of the model when solving it. Interestingly, we do find smaller probabilities: around 5 per cent for the non-negativity constraint to be binding when the inflation target is zero, and decreases to 2 per cent and 1 per cent when the target raises to 1 per cent and 2 per cent respectively.

4.3 The Inflation Targets and Welfare

To run welfare analysis, the first requirement must be a utility function. We consider here a standard constant relative risk aversion functional form consistent with the equations presented above. Thus, the one-period utility function of the representative consumer is given by

$$U(C_t) = \frac{1}{1-\gamma} C_t^{1-\gamma} \tag{12}$$

where C denotes household consumption, defined by the good-market clearing condition $Y = C + G$ (there is neither investment nor foreign sector in the economy). A second order Taylor approximation around the stochastic steady state yields the following expression for expected utility

$$E[U(C_t)] \simeq \frac{1}{1-\gamma} C_{ss}^{1-\gamma} - \frac{\gamma C_{ss}^{-\gamma-1}}{2} E\left[(C_t - C_{ss})^2 \right] \tag{13}$$

where C_{ss} represents the steady state consumption level. Equation (13) may be divided into two parts. One the one hand, a higher level of steady state consumption implies more expected utility. In our economy, public expenditure is useless to the representative household but drains scarce resources that constrain the steady state consumption. Therefore, this term is expected to increase when the inflation target increases as the probability of falling into the trap decreases.

On the other hand, a higher consumption variance decreases expected welfare, since the representative agent is risk-averse ($\gamma > 0$). Regarding this, there are two different effects in our model. A higher inflation target means a higher variance of the exogenous shock and thus a higher consumption variance. However, at the same time, a lower inflation target means that monetary policy may become ineffective for some periods of time, increasing macroeconomic fluctuations. Moreover, if the economy hits the zero bound, public expenditure increases rapidly, reducing consumption and increasing consumption variance. The final effect depends on these trade-offs.

We now compute steady state welfare by means of Gaussian quadrature. Given that the law of motion of the state vector is univariate Markovian, this is a very efficient way to compute the entire distribution of the economy. For a generic function $g(Y)$ of the variables of the economy, Y, we compute the expected value as follows

$$E[g(Y)] = \int g(Y(z))dF(z) \simeq \sum_{\omega_z \epsilon \Omega^z} g(Y(z_{\omega_i}))\vartheta_{\omega_z} \tag{14}$$

where $F(z)$ denotes the cumulative distribution function of the state vector, z, and ϑ_{ω_z} denotes the appropriate weight associated to the grid point ω_z on the state grid Ω^z, which in our case is obtained from Legendre's formula.

When the equilibrium real interest rate is assumed to be 2 percent, the final effect is depicted in Figure 8 (the results under the assumption $\overline{r} = 0.03$ are reported in appendix A). It shows the welfare gains (in output gap standard deviations) with respect to the Friedman rule. Three conclusions may be drawn: First, to target a zero inflation target is far from maximising welfare. Second, it could be seen that the welfare-maximising inflation target implied by the model is around 2 per cent. At this point, the trade-off is exploited optimally. And third, welfare losses of moving between 1 per cent to 3 per cent are relatively small[31].

However, if the equilibrium real interest rate is set equal to 3%, the abovesaid results change and the welfare maximising inflation target turns out to be zero (see Appendix A). As pointed out above, given the large degree of uncertainty surrounding the estimates of the equilibrium real interest rate, a welfare-maximising central bank, in the context of this model, should weigh these two different scenarios when choosing the quantitative definition of its policy objective. Indeed, a risk-averse policy-maker would presumably buy insurance by means of attaching a higher weight to the "2% real rate" scenario.

Naturally, the results obtained in this paper just like the results obtained in the other studies cited above are only of practical importance to the degree that the theoretical economies capture relevant features of real-world economies. While the model of this paper is very simple, it captures most of the features of the latest generation of neo-Keynesian sticky price models. As such, it is an interesting benchmark against which we can compare larger models using the techniques outlined in this paper. More importantly, the central element of our analysis is the mechanism implemented to escape from the trap. In our case fiscal policy assures the existence of a unique equilibrium, which we then approximate. This equilibrium has as central feature deficitary spending that is effective instantaneously and lasts only while the nominal interest rate is zero. This means that, in our simple theoretical world, fiscal

[31]This a common feature to most exercises that use second order Taylor approximations to standard utility functions.

policy helps to re-boost the economy as soon as monetary policy becomes ineffective.

In order to introduce a higher degree of realism into our model we may want to add a number of features that are characteristic for government spending and that are generically cited by opponents of fiscal stimulus packages. First, fiscal policy is subject to well-known decision and implementation lags. As a result the timing of the action is an issue. Second, once the stimulus package is decided upon, the amount chosen may be inadequate, thus failing to achieve the policy's objective, or overshooting the goal. The latter combined with the lagged response might well lead to economic instability in the sense of generating undesirably persistent economic fluctuations. Finally, experience in all of the major economies over the past century has shown that government spending, while relatively easy to increase, has proven to be notoriously difficult to decrease. As a result, the negative effects of government spending (here simply modelled through its wastefulness, but ignoring the distortions introduced through the tax system) might well amplify the overall cost of fiscal policy to a multiple of the cost here calculated. All these considerations may imply that our calculated optimal inflation target might well be biased towards zero, because losses of falling into the liquidity trap could be underestimated. However, we also could consider public expenditure not being completely useless, but entering into the utility function with a coefficient less than one. In this case the bias works on the opposite direction with the final effect depending on the values of the parameters of the model.

4.4 The bias stemming from the linear approach

As mentioned above, previous research on liquidity traps and the zero bound on nominal interest rates[32] is characterised by applying linear techniques to obtain the

[32] See, for example, Orphanides and Wieland (1998) and Reifschneider and Williams (2000).

model solution, i.e. to find the set of equations that drive the behaviour of the key macroeconomic variables under the rational expectations assumption. However, once the non-negativity constraint is taken into account, the model becomes non-linear. As a result, these widely-used linear techniques lead to a bias when calculating the first moments of the unconditional distribution of the variables included into the model.

The intuition may be set out by the following example. In the vicinity of the zero bound on nominal interest rates, the magnitude of the effects produced by a positive shock on the economy are smaller than the magnitude of the effects caused by a negative shock, because in the latter situation the probability for the central bank to have enough room of manoeuvre to implement an appropriate reduction in the policy rate decreases. A linear solution method implies both effects to be quantitatively identical. Nevertheless, this is no longer true in the context of the zero bound. We quantify this bias by calculating the first moments of the linear and non-linear approximations by quadrature.

Table 2 summarises the results for the means of c_t and π_t. Each entry represents the difference between the corresponding moment calculated by solving the model as it were linear minus the one calculated by using the non-linear technique proposed here. For example, if the inflation target were 0%, the (wrong) linear method implies a consumption average 0.15 percentage points larger than the one obtained by applying the non-linear method. As expected a priori, this bias increases as long as the inflation target decreases and, therefore, the probability of hitting the zero bound is higher (e.g. if $\overline{\pi}$ were -1%, the bias would be around +1%). Finally, as could be inferred from Table 2, the bias surrounding the means of the inflation rate is quantitative smaller.

5 Conclusions

This paper studies the question of the quantitative relevance of the zero lower bound within the framework of a standard Neo-Keynesian sticky-price model. In order to assure the global uniqueness of the steady state we assume that otherwise neutral fiscal policy becomes expansionary at the zero lower bound.

The value added by the paper is twofold. On the one hand, it allows for the reaction of the economy to a shock to be state-dependent. This is specially true in the context of the zero bound on nominal interest rates since the degree of effectiveness of the monetary decisions is very limited when interest rates are close to zero. On the other hand, this paper embeds the relationship between the inflation rate and the volatility of the shocks, widely documented in the literature, in an otherwise standard model with nominal rates bounded at zero.

The results of the paper depend on the value we choose to calibrate the parameters of the model. Indeed, these results are sensitive to the assumption on the equilibrium real interest rate. If this parameter were calibrated to be equal to 2% we would draw two main results from the model: first, the likelihood for the non-negativity constraint on nominal interest rates to be binding upsurges non-linearly when the inflation target decreases, increasing rapidly as the inflation targets drops below 1 percent and being around 5 percent for an inflation target of zero. Second, the simple model we have presented here implies that 2 percent would be the inflation target that would maximise the expected utility of the representative consumer.

However, if the equilibrium real interest rate were set equal to 3%, the probability for the zero lower bound falls to the vicinity of zero for inflation targets equal to or larger than zero. Indeed, the welfare maximising inflation target turns out to be zero. Hence, since the degree of uncertainty surrounding the estimates of the equilibrium real interest rate is non-negligible, a benevolent policy-maker which aims to maximise the expected welfare of the representative consumer, ought to attach some weights to these two alternative scenarios (among others) when facing the decision of choosing the quantitative definition of its policy objective. Indeed, a risk-averse policy-maker would presumably buy insurance by attaching a higher weight to the "2% real rate" scenario.

Furthermore, this model does not take into account all possible economic costs that may be connected with an inflation rate different from zero (inflation premia, tax distortions, redistribution effects...) and that could point to the optimality of a lower inflation target. Moreover, if public expenditure were assumed to provide utility to the agents or the fiscal authority were allowed to adopt preemptive measures to avoid the zero bound situation, the welfare maximising inflation target would tend to fall. On the other hand, a more realistic implementation of the fiscal sector, characterised by slow government action and bureaucratic problems, may turn the zero–interest rate situation even less desirable, pointing to the optimality of an even higher inflation target. The final effect remains unknown at this stage.

Further research may point to calculate the welfare-maximising nominal interest rate rule in the vicinity of the zero bound. This rule should be asymmetric (since the rule becomes useless below certain threshold) and state-dependent, i.e. the coefficients of the policy rule ought to be allowed to change as long as nominal interest rate is falling towards zero. Under the optimal monetary policy rule, the welfare maximising inflation objective very likely will be lower than otherwise.

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References

[1] Aizeman, J. (1987), "Inflation, Tariffs, and Tax Enforcement Costs", *Journal of International Economic Integration*, 2

[2] Ball, L. and S. Cecchetti (1990), "Inflation and Uncertainty at Short and Long Horizons", *Brookings Papers of Economic Activity* 1:1990, pp. 215-245.

[3] Benhabib, J., S. Schmitt-Grohe and M. Uribe (1999), "Monetary Policy and Multiple Equilibria", Centre for Economic Policy Research, Discussion Paper No. 2316.

[4] Benhabib, J., S. Schmitt-Grohe and M. Uribe (2000a), "Avoiding Liquidity Traps", Working Paper, Rutgers University.

[5] Benhabib, J., S. Schmitt-Grohe and M. Uribe (2000b), "Liquidity Traps with Global Taylor Rules", Working Paper, Rutgers University.

[6] Benhabib, J., S. Schmitt-Grohe and M. Uribe (2001), "The Perils of Taylor Rules", *Journal of Economic Theory* 96, pp. 40-69.

[7] Bernanke, B.S. (1999), "Japanese Monetary Policy: A Case of Self-Induced Paralysis?", presented at ASSA meetings, 9 January 2000.

[8] Bernanke, B.S. (2000), "Comment on Delong 'America's Historical Experience with Low Inflation"', *Journal of Money, Credit and Banking* 32(4), pp. 994-997.

[9] Braun, R. A. (1994), "How Large Is the Optimal Inflation Tax?", *Journal of Monetary Economics* 34, pp. 201-214.

[10] Buiter, W. and N. Panigirtzoglou (1999), "Liquidity Traps: How to Avoid Them and How to Escape Them", NBER Working Paper No. W7245.

[11] Campbell, J.Y. (1999), "Asset Prices, Consumption, and the Business Cycle", Chapter 19 in John Taylor and Michael Woodford eds., *Handbook of Macroeconomics*, Amsterdam: North-Holland.

[12] Cecchetti, S. (1998), "Understanding the Great Depression: Lessons for Current Policy", in M. Wheeler, ed., *The Economics of the Great Depression*, pp. 171-194.

[13] Christiano, L. J., M. Eichenbaum, and C. Evans (1996), "The Effects of Monetary Policy Shocks: Some Evidence from the Flow of Funds", *Review of Economics and Statistics* 78(1), February, pp. 16-34.

[14] Christiano, L. J., M. Eichenbaum, and C. Evans (2001), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", Federal Reserve Bank of Cleveland Working Paper No. 01-07.

[15] Christiano, L. J. and Fisher, J. D. M. (1997), "Algorithms for Solving Dynamic Models with Occasionally Binding Constraints", NBER Technical Working Paper No. 218.

[16] Clarida, R., J. Gali and M. Gertler (1998), "Monetary Policy Rules in Practice: Some International Evidence", *European Economic Review,* 42, pp. 1033-1067.

[17] Clouse, J., D. Henderson, A. Orphanides, D. Small and P. Tinsley (2000), "Monetary Policy When the Nominal Short-Term Interest Rate is Zero", Finance and Economics Discussion Paper No. 2000-51. Board of Governors of the Federal Reserve System.

[18] Coenen, G., and V. Wieland (2002), "Inflation Dynamics and International Linkages: a Model of the United States, the Euro area and Japan", ECB Working Paper No. 181, September.

[19] Coenen, G., and V. Wieland (2003), "The Zero-Interest-Rate Bound and the Role of the Exchange Rate for Monetary Policy in Japan", ECB Working Paper No. 218, March.

[20] Cogley, T., (1997), "What is the Optimal Rate of Inflation?", *FRBSF Economic Letter* 97-27, September.

[21] Correia, I., and Teles, P. (1996), "Is the Friedman Rule Optimal when Money is an Intermediary Good?", *Journal of Monetary Economics* 38.

[22] Correia, I., and Teles, P. (1999), "The Optimal Inflation Tax", *Review of Economic Dynamics* 2:2, pp. 325-346.

[23] Cozier, B., and C. Lavoie (1994), "Is There a Floor to Nominal Interest Rates? Evidence an Implications for the Conduct of Monetary Policy", presented at the CEA meetings, June.

[24] De Fiore, F., and Teles, P. (1999), "The Optimal Mix of Taxes on Inflation, Money, and Income", Banco de Portugal Working Paper, 2-99.

[25] De Fiore, F. (2000), "The Optimal Inflation Tax when Taxes are Costly to Collect", ECB Working Paper No. 38.

[26] Feldstein, M. (1997), "The Costs and Benefits of Going from Low Inflation to Price Stability", in Romer and Romer eds., *Reducing Inflation: Motivation and Strategy*, Chicago: University of Chicago Press, pp. 123-166.

[27] Friedman, M. (1969), "The Optimum Quantity of Money", in *The Optimum Quantity of Money and Other Essays*, pp. 1-50, University of Chicago Press.

[28] Fuhrer, J. C., and B. Madigan (1997), "Monetary Policy when Interest Rates are Bounded at Zero", *Review of Economics and Statistics* 79, pp. 573-585.

[29] Gali, J. (2001), "New Perspectives on Monetary Policy, Inflation and the Business Cycle", forthcoming in *Advances in Macroeconomic Theory*, edited by M. Dewatripont, L. Hansen and S. Turnovsky. Cambridge University Press.

[30] Gesell, S. (1949), "Die natuerlich Wirtschaftsordnung", Rudolf Zitzmann Verlag: Lauf bei Nuernberg.

[31] Goodfried, M., and King, R. (1997), "The New Neoclassical Synthesis and the Role of Money", NBER Macroannual.

[32] Goodfriend, M. (2000), "Overcoming the Zero Bound on Interest Rate Policy", *Journal of Money, Credit and Banking* 32(4), pp. 1007-1035.

[33] Guidotti, P. E., and C. Vegh (1993), "The Optimal Inflation Tax when Money Reduces Transaction Costs: A Reconsideration", *Journal of Monetary Economics* 31(2), pp. 189-205.

[34] Guo, H., and R. Whitelaw (2001), "Risk and Return: Some New Evidence", Federal Reserve Bank of St. Louis, Working Paper No. 2001-001A.

[35] Hamada, K. (1997), "Inflation Risks, Ageing of the Population, and the Functions of Public and Personal Pensions", IPTP Monthly Report No. 107, August.

[36] Henderson, D., and W. McKibbin (1993), "A Comparison of Some Basic Monetary Policy Regimes for Open Economies: Implications of Different Degrees of Instrument Adjustment and Wage Persistence", *Carnegie-Rochester Series on Public Policy* 39, pp. 221-317.

[37] Hetzel, R. L. (1999), "Japanese Monetary Policy: A Quantity Theory Perspective", *Economic Quarterly* 85.

[38] Jeanne, O. (1997), "Generating Real Persistent Effects of Monetary Shocks: How Much Nominal Rigidity Do We Really Need?", NBER Working Paper No. 6258.

[39] Judd, K. L. (1996), "Approximation, Perturbation, and Projection Solution Methods in Economics", in *Handbook of Computational Economics*, Kendrick, D. A., and Amman, H. (eds.), 1996, Amsterdam: North Holland.

[40] Judd, K. L. (1998), "Numerical Methods in Economics", The MIT Press.

[41] Keynes, J.M. (1923), "A Tract on Monetary Reform", in *The Collected Writings of John Maynard Keynes*, Volume IV, Macmillan Cambridge University Press.

[42] Keynes, J.M. (1936), "The General Theory of Employment, Interest and Money", in *The Collected Writings of John Maynard Keynes*, Volume VII, Macmillan Cambridge University Press.

[43] Kimbrough, K. (1986), "The Optimum Quantity of Money Rule in the Theory of Public Finance", *Journal of Monetary Economics*, 18.

[44] Kimura, T., H. Kobayashi, J. Muranaga and H. Ugai (2002), "The Effect of Quantitative Monetary Easing at Zero Interest Rates", *mimeo.*

[45] Krugman, P.R. (1998), "It's Back: Japan's Slump and the Return of the Liquidity Trap", *Brookings Papers on Economic Activity* 49(2), pp. 137-206.

[46] Lansing, K. J. (2001), "Learning about a Shift in Trend Output: Implications for Monetary Policy and Inflation", Working Paper No. 00-16, Federal Reserve Bank of San Francisco, January.

[47] Lucas, R. E., Jr. (2000), "Welfare and Inflation", *Econometrica* 68.

[48] McCallum, B. T. (2000), "Theoretical Analysis Regarding a Zero Lower Bound on Nominal Interest Rates", *Journal of Money, Credit and Banking* 32(4), pp. 870-904.

[49] McCallum, B. T., and E. Nelson (2000), "Timeless Perspective vs. Discretionary Monetary Policy in Forward-Looking Models", NBER Working Paper No. 7915, September.

[50] McGrattan, E. R. (1999), "Application of Weighted Residual Methods to Dynamic Economic Models", in Marimon and Scott (eds.), *Computational Methods for the Study of Dynamic Economies*, Oxford University Press.

[51] Meltzer, A. H. (1999), "Liquidity Claptrap", *The International Economy* (November-December), pp. 18-23.

[52] Mulligan, C. B., and X. Sala-i-Martin (1997), "The Optimum Quantity of Money: Theory and Evidence", *Journal of Money Credit and Banking*, November.

[53] Niccolini, J. (1997), "Tax Evasion and the Optimal Inflation Tax", *Journal of Development Economics*, 55.

[54] Okina, K. (1999), "Monetary Policy Under Zero Inflation: A Response to Criticisms and Questions Regarding Monetary Policy", Bank of Japan Institute for Monetary and Economic Studies Discussion Paper No. 99-e-20.

[55] Okun, A. (1971), "The Mirage of Steady Inflation", *Brookings Papers on Economic Activity 1971*:2, pp. 485-498.

[56] Okun, A. (1975), "Inflation: Its Mechanics and Welfare Costs", *Brookings Papers on Economic Activity*, 1975:2.

[57] Orphanides, A., and V. Wieland (1998), "Price Stability and Monetary Policy Effectiveness when Nominal Interest Rates are Bounded at Zero", FEDS Working Paper No. 35. Board of Governors of the Federal Reserve.

[58] Phelps, E. (1973), "Inflation in the Theory of Public Finance", *Scandinavian Journal of Economics* 75, pp. 67-82.

[59] Pigou, A.C. (1950), "Keynes General Theory: A Retrospect", in A.C.Pigou: *Collected Economic Writings*, vol. 13.

[60] Posen, A.S. (1998), "Restoring Japan's Economic Growth", Institute for International Economics ed.

[61] Reifschneider, D., and J. C. Williams (2000), "Three Lessons for Monetary Policy in a Low Inflation Era", *Journal of Money, Credit and Banking* 32(4), pp. 936-966.

[62] Roberts, J. M. (2001), "How Well Does the New Keynesian Sticky-Price Model Fits the Data?" Finance and Economics Discussion Paper No. 2001-13, Board of Governors of the Federal Reserve System, February.

[63] Saunders, A. (2000), "Low Inflation: The Behaviour of Financial Markets and Institutions", *Journal of Money, Credit and Banking* 32(4), pp. 1058-1087.

[64] Summers, L. (1991), "Panel Discussion: Price Stability, How Should Long-Term Monetary Policy Be Determined?", *Journal of Money, Credit and Banking* 23(3), pp. 625-631.

[65] Svensson, L. (2000), "How Should Monetary Policy Be Conducted in an Era of Price Stability?", NBER Working Paper No. 7516.

[66] Svensson, L. (2001), "The Zero Bound in an Open Economy: A Foolproof Way of Escaping from a Liquidity Trap", *Monetary and Economic Studies* 19, pp. 277-312.

[67] Taylor, J. B. (1981), "On the Relation Between the Variability of Inflation and the Average Inflation Rate", *Carnegie-Rochester Conference Series on Public Policy* 15, North-Holland.

[68] Taylor, J. B. (1993), "Discretion Versus Policy Rules in Practice", *Carnegie-Rochester Series on Public Policy 39*, pp. 195-214.

[69] Tinsley, P.A. (1999), "Short Rate Expectations, Term Premiums, and Central Bank Use of Derivatives to Reduce Policy Uncertainty", Finance and Economics Discussion Series 1999-14, Board of Governors of the Federal Reserve System.

[70] Vegh, C. (1998), "Government Spending and Inflationary Finance: A Public Finance Approach", IMF Staff Papers 36.

[71] Wolman, A.L. (1997), "Zero Inflation and the Friedman Rule: A Welfare Comparison", *Economic Quarterly* 83(4), pp. 1-21. Federal Reserve Bank of Richmond.

[72] Wolman, A.L. (1999), "Real Implications of the Zero Bound on Nominal Interest Rates", 1152, Society for Computational Economics Working Paper No. 1152.

[73] Woodford, M. (1999), "Optimal Monetary Policy Inertia", Princeton University, June.

[74] Yun, T. (1996), "Nominal Price Rigidity, Money Supply Endogeneity and Business Cycles", *Journal of Monetary Economics* 37(2), pp. 345-370.

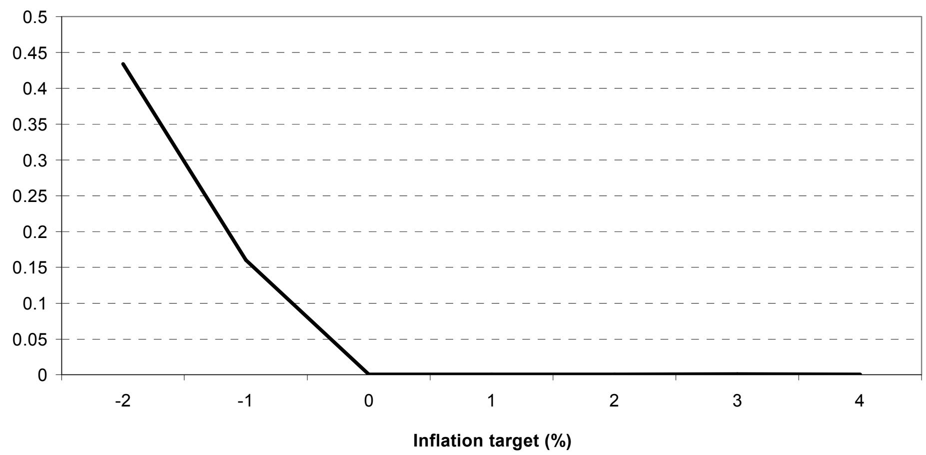
A Main results under a different assumption on the equilibrium real interest rate

This appendix reports the main results of the paper under the assumption of a higher equilibrium real interest rate: 3 percent.

A.1 The Probability of Hitting the Zero Bound

Figure A1 shows the probability of hitting the zero bound under inflation targets between -2 and 4 percent when the equilibrium real interest rate is set equal to 3 percent. As expected, the probabilities are much lower than previously reported, since the buffer the policy-maker enjoys is much larger. In particular, the likelihood for the non-negativity constraint to be binding falls below 0.1% for inflation targets equal to or larger than zero. Note that Figure A1 is not the result of just shifting Figure 7 to the left, since this model takes explicitly into account the link between the long-run inflation rate and the variance of the shocks.

Figure A1: Probability of hitting the zero bound (r=3%)



A.2 The Welfare-Maximising Inflation Target

Figure A2 depicts the welfare-based comparison (in terms of relative gains with respect to $\overline{\pi} = -2\%$) of the non-negative inflation targets when the equilibrium real interest rate is equal to 3 percent. Not surprisingly, the welfare-maximising inflation target implied by the model falls with respect to Figure 8 because the probability of hitting the zero bound and thereby the cost of choosing a low inflation target is smaller now. Indeed, the welfare-maximising inflation target turns out to be zero. In other words, when the equilibrium real interest rate is 3 percent, the trade-off presented in this paper disappears as the zero lower bound is almost never binding under non-negative inflation targets.

Figure A2: Welfare gains relative to a -2% inflation target (r=3%, output gap standard deviations)

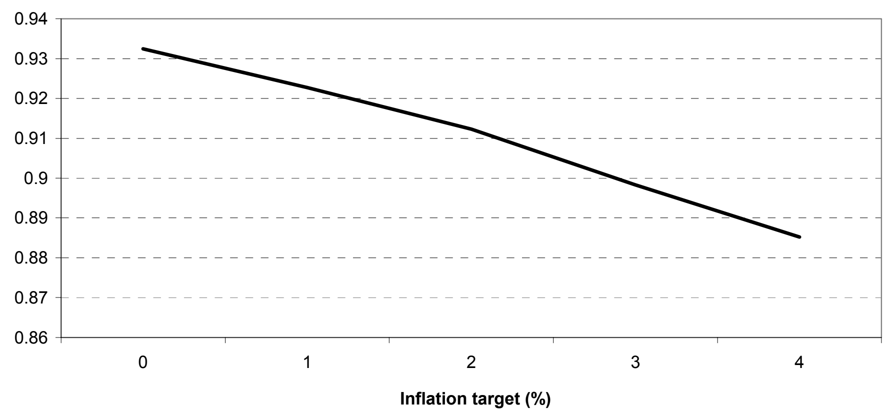


Table 1: Calibration of the model.

Parameter	γ	ϕ	δ_0	δ_1	β	λ	θ^{π}	θ^{c}	$\overline{r}$	x
Value	3	0.85	0.0036	0.07	0.995	0.05	1.5	0.5	0.02	0.03

Table 2: Bias arising from using (wrong) linear model solution methods instead of non-linear methods.

	$\overline{\pi} = 0\%$	$\overline{\pi} = 1\%$	$\overline{\pi} = 2\%$	$\overline{\pi} = 3\%$	$\overline{\pi} = 4\%$
Mean of c_t	+ 0.15 %	+ 0.12 %	+ 0.10 %	+ 0.08 %	+ 0.02 %
Mean of π_t	+ 0.05 %	+ 0.04 %	+ 0.03 %	+ 0.03 %	+ 0.01 %

Figure 1: The liquidity trap

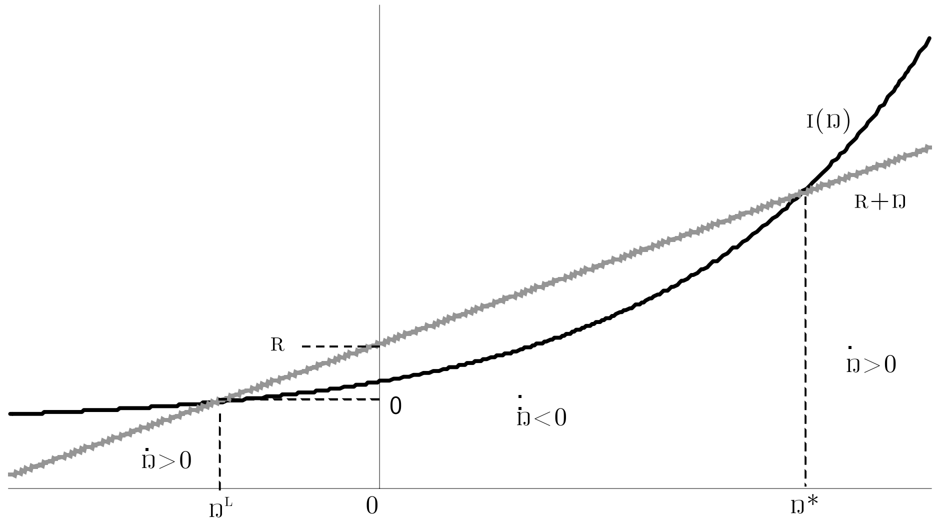


Figure 2: The liquidity trap in Japan

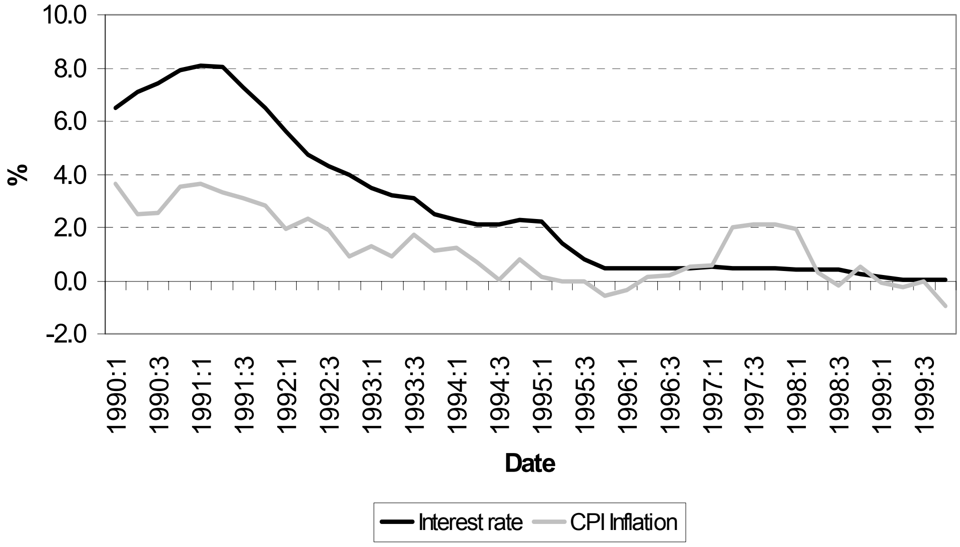


Figure 3: Response to a positive shock (no trap)

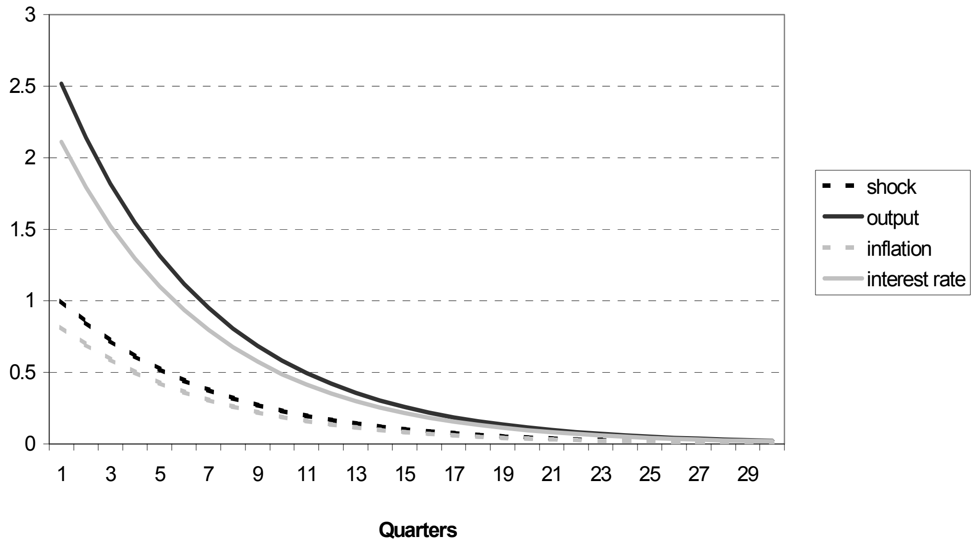


Figure 4: Response to a negative shock (always in the trap)

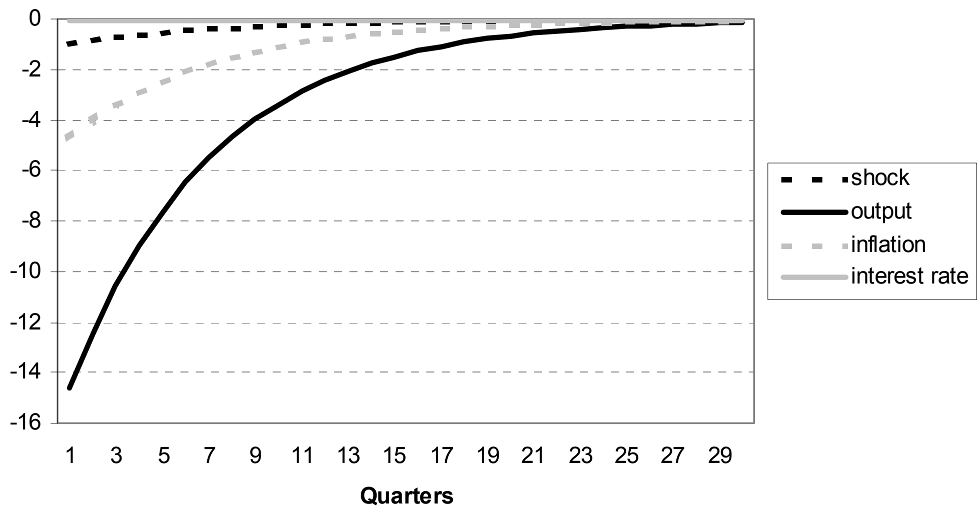


Figure 5: Weighting function (pi=-2%)

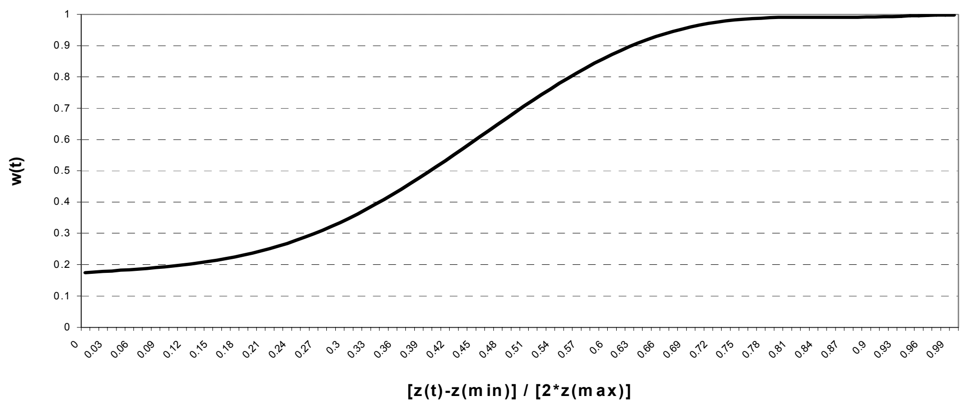
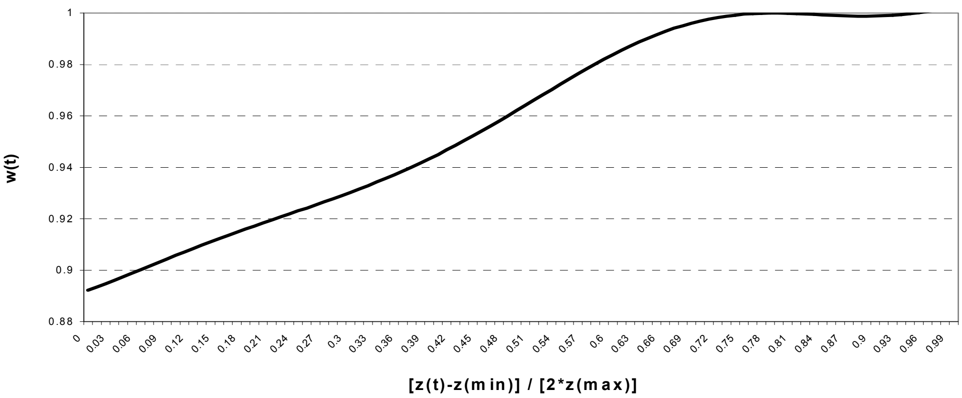


Figure 6: Weighting function (pi=2%)



$z_{\min}$ and $z_{\max}$ were chosen to define a symmetric interval covering ± 4 standard deviations of the state variable. The value of the weighting function for $z_t = 0$ may be found by looking at 0.5 in the horizontal axis. Positive values of z_t are represented to the right of 0.5 whilst the negative ones may be found to the left. A value of 0 in the horizontal axis symbolises a value of z_t equal to -4 standard deviations below the mean.

Figure 7: Probability of hitting the zero bound

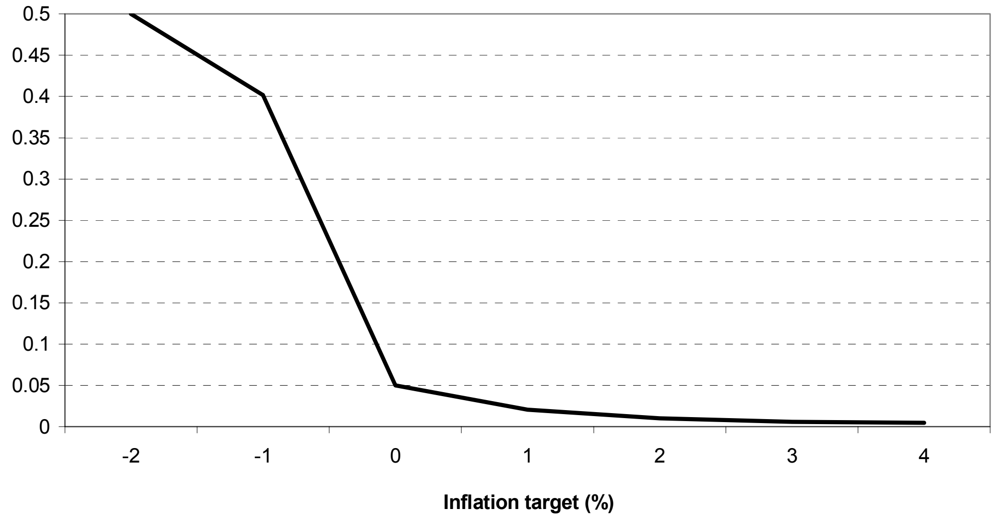
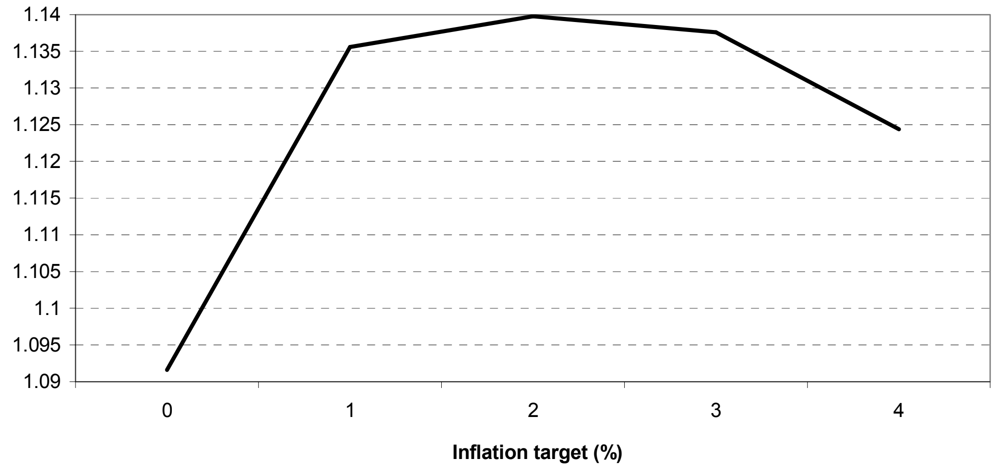


Figure 8: Welfare gains relative to the Friedman rule (output gap standard deviations)



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