Pricing Risk in Economies with Heterogenous Agents and Incomplete Markets *

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Abstract

Habit formation has been proposed as a possible solution to the equity premium puzzle. This paper extends the class of models that support the habits explanation in order to account for heterogeneity in earnings, wealth, habits and consumption. I find that habit formation does indeed increase the equity premium. However, contrary to earlier results, the habit hypothesis does not imply a price for risk as big as the one measured in the data. There are three reasons for this. First, households in a habits economy modify their consumption/savings decision. Second, they modify their portfolio choice. These two changes in behavior diminish the consumption fluctuations faced by households. And third, the composition of the set of agents pricing risk in the economy changes so that relatively better self-insured households end up pricing risk.

Keywords: Equity Premium; Habit Formation; Incomplete Markets

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1 Introduction

The equity premium puzzle, as stated by Mehra and Prescott (1985), describes the inability of the standard macroeconomic models to generate a differential return between risky and risk free assets as large as found in the data. In other words, quantitative macroeconomic models produce a compensation for risk that is too small compared to its empirical counterpart.

Habit formation has been proposed as an explanation for the equity premium puzzle because it increases the utility losses from consumption fluctuations and therefore it increases the compensation required to hold risky assets. Habit formation is an interesting hypothesis because it distinguishes between preferences over consumption in different states of the world and preferences over consumption in different points in time. This feature allows to tackle separately the equity premium and the risk free rate.[1] Several authors, as Constantinides (1990), Abel (1990), Heaton (1995), Campbell and Cochrane (1999) and Boldrin, Christiano, and Fisher (1997), show how the extra parameters introduced by the habit formation hypothesis can be used to match the equity premium and other related statistics.

Nevertheless, the models used to defend the habits hypothesis as a solution to the equity premium puzzle may have been too stylized. In particular, the introduction of habits has no effect on two important household decisions. First, households cannot change their consumption/savings decision. However, as shown for instance by Jermann (1998) and Lettau and Uhlig (2000), in general equilibrium models with endogenous consumption the path of consumption generated in equilibrium once we add habit formation turns out to be too smooth. Second, households cannot change their portfolio decision. The reason is that these earlier models were built around the hypothesis of a representative agent, who holds the market portfolio in equilibrium. Recent research on heterogeneous agents economies with incomplete markets has shown the quantitative importance of changing portfolio decisions. For instance, Heaton and Lucas (1996) propose a two-agent economy where the fluctuations in individual incomes are large enough to generate big equity premia. However, in order to diminish their exposure to risk households respond to shocks by changing their portfolio choices and thus the economy displays a very small equity premium. Only when households face costly portfolio reallocations does the equity premium grow. There is a third element overlooked by the earlier models with habit formation. In a heterogeneous agents economy not all households solve their portfolio choice problem with an interior solution. Indeed, many of them end up with a corner solution (i.e. they would

[1] Another way of disentangling preferences over time and preferences over states is the recursive utility function pioneered by Epstein and Zin (1989). See Weil (1989) for a discussion on the risk free rate puzzle.

like shorter positions in one of the assets). But the market price of risk depends on the marginal rate of substitution of the agents holding their desired portfolio. One important element not emphasized so far in the asset pricing literature is that the composition of the set of pricing agents changes as we modify the model economy. In this sense, the addition of habit formation into a model economy with heterogeneous agents changes who has an interior solution to the portfolio choice problem and therefore whose consumption fluctuations matter for pricing risk.

This paper takes seriously the habit formation hypothesis and measures the increase in the price of risk that it generates in economies where consumption and portfolio choices come from household decisions. The paper also quantifies the relative importance of each decision margin and the relative importance of the change in the equilibrium composition of the set of pricing agents. To this end, this paper introduces the habit formation hypothesis in a standard general equilibrium model with incomplete markets and heterogenous agents who face fluctuations in labor income as big as measured in the data. In the model economy, agents differ in habit stocks as well as in earnings and in wealth holdings. The source of these differences is the absence of markets to insure against idiosyncratic shocks to labor income. As shown by Hansen and Jagannathan (1991), the equity premium can be decomposed into two terms: the market price of risk or Sharpe ratio and the amount of risk, which is the standard deviation of the returns of the risky asset. This paper focuses on the price of risk. The reason is that in order to model consumer behavior in a detailed manner the production side is purposely kept simple. For this reason, within the model a single parameter (the volatility of the aggregate shock) drives two important statistics: the amount of risk and the volatility of aggregate series. I calibrate this parameter to the volatility of the aggregate series (i.e. aggregate consumption) and I do not attempt to match the amount of risk of the economy.

The main result is that the habit formation hypothesis increases the price of risk by as much as 37% compared to an analogous economy without habit formation. This conforms to the intuition in the earlier literature that higher costs of consumption fluctuations should result in a higher compensation for holding risky assets. However, contrary to earlier work, the price of risk found is still one order of magnitude below its empirical estimates. As hinted in the previous paragraphs, there are three reasons for this. First, precautionary savings are increased, second the composition of the portfolio is changed, and third the composition of the set of pricing agents changes. I look at each element in turn. I find that each of these mechanisms account for 79.3%, 20.2% and 0.5% of the difference between the model's price of risk and the one generated by a habits economy where consumption and portfolio decisions are kept as in a non habits economy.

The intuition of the results is as follows. When adding habit formation to the model economy, instead of demanding a bigger compensation to hold a risky asset, households modify their behavior in order to face smaller consumption fluctuations. The result is that the degree of consumption smoothness achieved is high enough to prevent the habit formation preferences from generating large fluctuations in marginal utilities. To be precise, when adding habits to the standard model, the average of individual level consumption fluctuations falls by more than 50%. In addition, the changes in the portfolio decisions have some general equilibrium effects. In this type of economy not everybody solves their portfolio choice problem with interior solution. Typically, households with little wealth need a hedge against labor income risk. Since the variance of labor earnings is countercyclical, by borrowing as much as possible in the risky asset and investing in bonds, asset-poor households create a portfolio whose returns are negatively correlated with their labor market risk. On the contrary, asset-rich households are very well self-insured and go as short as possible in bonds and invest in risky assets to obtain higher expected returns. Between these two opposite cases, a fraction of the population in the model have an interior solution to the portfolio choice problem. These agents constitute what I will call the set of pricing agents. If we increase the utility costs of risk, the equilibrium composition of the set of pricing agents changes. It turns out that the set of agents with interior solution to their portfolio choice problem will contain better self-insured households. Therefore, not only average consumption fluctuations fall but also the composition of the set of agents that matter for pricing risk changes towards agents that face relatively lower consumption fluctuations.

The remainder of the paper is organized as follows. Section 2 describes the model economy and its implications for asset pricing. Section 3 shows how I calibrate the different model economies to US data. Then, the results are presented in section 4. Finally, section 5 concludes. The computational method, sketched in section 2, is detailed in the appendix together with some standard accuracy measures for the numerical solutions.

2 The model economies

The basic structure of the economies in this paper is the standard growth model with aggregate uncertainty and heterogeneous agents. The economy is populated by a representative firm and by a continuum of infinitely-lived households. Households are ex-ante identical and differ in equilibrium because of the different realization of their labor market shocks and the different decisions they take as a response. Markets are incomplete in the sense that households cannot write contracts contingent on the realization of their idiosyncratic shock to labor earnings. This model structure was pioneered by Huggett (1993) and Aiyagari (1994) and it was extended to include aggregate uncertainty by Krusell and Smith (1998). In what follows I detail the specifics of the studied model economies and refer to these authors for a more rigorous treatment.

2.1 Production

Each period t, the representative firm uses aggregate capital $K_t \in \mathbb{R}_+$ and aggregate labor $L_t \in \mathbb{R}_+$ to produce $Y_t \in \mathbb{R}_+$ units of a single homogenous good with an aggregate technology $Y_t = F(z_t, K_t, L_t)$. I assume the standard Cobb-Douglas function, $F(z, K, L) = zK^{1-\theta}L^{\theta}$ with $0 < \theta < 1$, where $z_t \in Z \equiv \{z_g, z_b\}$ is the date t realization of the aggregate productivity shock. The aggregate productivity shock follows a stationary Markov process with transition function $\Gamma_z(z, z') = \Pr(z_{t+1} = z' \mid z_t = z)$, with z_0 given. Productive capital depreciates at an exogenous rate $\delta \in (0, 1)$. Since households own the productive capital, the firm's problem is static and the representative firm chooses capital and labor by equating their marginal products to their prices: $w = F_L(z, K, L)$ and $R^s = F_K(z, K, L) + (1 - \delta)$, where w is the wage rate and R^s is the gross return on capital holdings.

2.2 Households

Households own the capital of this economy. Individual capital holdings $s \in S \equiv [\underline{s}, \infty)$ are rented in competitive markets to the representative firm and may also be traded among households themselves. Since production is stochastic, the return on capital holdings is also stochastic and will depend on the realization of the aggregate shock next period. In this economy, households can also trade a risk free bond $b \in B \equiv [\underline{b}, \infty)$. One unit of the risk free bond entitles a known payment of R^b units of the consumption good next period. Notice that both assets are restricted by a lower bound. If these lower bounds are below 0, households can go short on the given asset. In addition, I set up a lower bound on total asset holdings $b + s \geq \underline{\omega}$.

For convenience, I decompose the idiosyncratic risk of labor income into two parts. First, there is an employment opportunity shock. Employment possibilities $e_t \in E \equiv \{0, 1\}$ come stochastically and depend on the aggregate technology level z. At every period of time, households may ($e_t = 1$) or may not ($e_t = 0$) be given an employment opportunity. Since agents do not value leisure the employment opportunity will be taken. Conditional on z_t and z_{t+1} the process is independently distributed across agents and Markovian with transition matrix $\Gamma_e(z, z', e, e') \equiv \Pr(e_{t+1} = e' \mid e_t = e, z_t = z, z_{t+1} = z')$. Second, when given an employment opportunity, agents also get an endowment of efficiency units of labor.

Efficiency units of labor, represented by $\xi_t \in \Xi \equiv \{\xi_1, \xi_2, ..., \xi_{n_\xi}\}$, are independent across households with Markov transition matrix $\Gamma_\xi(\xi, \xi') \equiv \Pr(\xi_{t+1} = \xi' \mid \xi_t = \xi)$. Notice that this process is independent of the aggregate shock z. When not given an employment opportunity households are assumed to operate a home technology that provides them with d units of the consumption good. I split the idiosyncratic shock into these two components purely for quantitative reasons. The employment shock will be used to generate individual uncertainty correlated to aggregate uncertainty whereas the efficiency units shock will be used to introduce an amount of uncertainty in the labor market compatible with observed data. As pointed out by Mankiw (1986) and Constantinides and Duffie (1996), the interaction between idiosyncratic uncertainty and aggregate uncertainty is crucial for the asset pricing implications of this type of models. In particular, if the variance of the process for individual earnings is higher in downturns than in peaks, then the addition of idiosyncratic uncertainty raises the equity premium. The reason is that in this situation equity turns out to be an asset that pays well when idiosyncratic uncertainty is low and pays poorly when idiosyncratic uncertainty is high. Using data from the PSID, Storesletten, Telmer, and Yaron (2001) show that idiosyncratic uncertainty in the labor market seems to be larger in downturns than in peaks. The model in this paper captures this feature by calibrating an employment process that generates higher and longer unemployment rates in downturns. Since there is a continuum of households, a law of large number applies and all individual uncertainty is washed out in the aggregate. The law of large numbers has two implications in this economy. First, the proportion of households in every efficiency level is constant and state independent. Therefore so is the average efficiency of the labor force. For this reason, with no loss of generality, I normalize the unconditional expectation of efficiency units of labor $E[\xi]$ to one. Second, by imposing certain conditions on the transition matrix for employment, the share of employed individuals at a given time t and hence the aggregate labor of the economy are a function of only the aggregate technology level z_t. Therefore, aggregate labor does not need to be a state variable of the economy. This structure is imposed for simplicity and follows Krusell and Smith (1998).[2] I will call the unemployment rates of the economy in good and bad times u_g and u_b.

Households derive utility from both current consumption and their own history of past consumption. Present consumption will be denoted by $c_t \in \mathbb{R}_+$ and past consumption will accumulate in a stock of habits denoted by $h_t \in \mathbb{R}_+$. The habit stock evolves according to the law of motion $h_{t+1} = \psi(c_t, h_t)$ with partial derivatives $\psi_c \in (0, 1]$ and $\psi_h \in [0, 1)$. Per period utility will be denoted by $u(c_t, h_t)$. I will use a standard CES class of functions. Habit formation is modelled as in Abel (1990), Fuhrer (2000) and in Díaz, Pijoan-Mas, and

[2] See the conditions required in section 3.

Ríos-Rull (2003). They characterize it by using the following utility function:

$$u(c,h) = \frac{\left(ch^{-\gamma}\right)^{1-\sigma} - 1}{1-\sigma} \quad \text{with} \quad \gamma \in (0,1)$$

and the following law of motion for habits:

$$\psi(c,h) = (1-\lambda)\,h + \lambda c \quad \text{with} \quad \lambda \in (0,1]$$

Notice that the non-habits case has a representation under this formulation by setting γ equal to zero.[3] The habit formation hypothesis requires $u_{ch} > 0$ and $u_h < 0$. The former property, known as adjacent complementarity, means that the history of past consumption increases the marginal value of current consumption whereas the latter property distinguishes habit formation from durable consumption. These two properties impose $\gamma > 0$ and $\sigma > 1$. Since agents are infinitely-lived, their total utility at time t will be the infinite discounted sum of expected period utilities: $\sum_{j=0}^{\infty} \beta^j E_t \left[u\left(c_{t+j}, h_{t+j}\right)\right]$, where $\beta \in (0,1)$ is the exogenous discount factor. Notice therefore that the consumer understands that by choosing the current consumption level c_t she will modify her own preferences for consumption in the future through the changes in h_{t+j} for $j > 0$.

I formulate the household problem recursively. I drop the time subscript and denote by prime those variables dated in the next period. Each individual state is given by the vector j formed by the agents's wealth ω, stock of habits h, employment opportunity e and efficiency units endowment ξ, plus the distribution of agents μ over this vector and the aggregate shock z. We define household wealth $\omega \in \Omega = [\underline{\omega}, \infty)$ as the sum of bonds, capital, the income generated by them and labor earnings. μ is a probability measure over a σ-algebra generated by the set $J \equiv \Omega \times R_+ \times E \times \Xi$. The transition function for the measure μ is given by $\mu' = Q\left(z, \mu, z'\right)$. Agents maximize the discounted sum of expected utilities by choosing consumption c, risk free bonds b and individual capital holdings s subject to the feasibility constraints, the budget constraint, the law of motion for habits, the transition matrices for the exogenous shocks and the transition function for the aggregate state. The gross return on bonds depends on today's aggregate state (so it is known at the time of

[3]There is an alternative way of modeling habit formation in which the habit stock enters the utility function as a *survival consumption* level

$$u(c,h) = \frac{\left(c - \gamma h\right)^{1-\sigma} - 1}{1-\sigma}$$

With this utility function the Arrow-Pratt coefficient of risk aversion changes with the cycle and it is the only way to obtain a counter-cyclical equity premium in a representative agent economy. However, in a heterogeneous agents framework this is not necessary since the negative correlation between the equity premium and the cycle can be obtained with power utility as it is shown in section 4. The choice of the *survival consumption* formulation presents serious problems when a model is calibrated to individual data. See Díaz, Pijoan-Mas, and Ríos-Rull (2003) for details.

taking decisions) and the gross return on capital depends on today's aggregate state and also on next period's realization of the aggregate shock. The individual problem can be written as:

$$v(j,z,\mu) = \max_{c,b,s} \left\{ u(c,h) + \beta E_{e',\xi',z'|e,\xi,z} \left[v(j',z',\mu') \right] \right\} \tag{1}$$

subject to,

$$c = \omega - b - s \tag{2}$$

$$h' = \psi(c,h) \tag{3}$$

$$\omega' = bR^b(z,\mu) + sR^s(z,\mu,z') + \mathbf{I}_{e'=1} w(z,\mu,z')\,\xi' + \mathbf{I}_{e'=0} d \tag{4}$$

$$\mu' = Q(z,\mu,z') \tag{5}$$

$$c \geq 0; \quad b \geq \underline{b}; \quad s \geq \underline{s}; \quad b+s \geq \underline{\omega} \tag{6}$$

where $\mathbf{I}_l$ is an indicator function that takes value 1 when the statement l is true and 0 otherwise. The expression $E_{l'|l}[m]$ is the operator for the mathematical expectation of m with respect the distribution of l' conditional on l. The laws of motion for e', ξ' and z' are implicit in the expectation operator. We are looking for the policy functions $c = g^c(j,z,\mu)$, $b = g^b(j,z,\mu)$ and $s = g^s(j,z,\mu)$.

2.3 Equilibrium

We will only look at allocations in equilibrium. The following definition establishes what is an equilibrium for this economy.

Definition 1 *An equilibrium for this economy is a set of functions* $\left\{v, g^c, g^b, g^s, R^b, R^s, w\right\}$ *and a transition function* Q *for the aggregate state such that: (i) factor prices satisfy the firms' optimality conditions,* $w(z,\mu,z') = F_L(z',K',L')$ *and* $R^s(z,\mu,z') = F_K(z',K',L') + (1-\delta)$*; (ii) given pricing functions* $\left\{R^b, R^s, w\right\}$*, a law of motion* Q *and the exogenous transition matrices* $\{\Gamma_z, \Gamma_e, \Gamma_\xi\}$*, functions* $\left\{v, g^c, g^b, g^s\right\}$ *solve the household problem; (iii) labor market clears,* $L = \int e\xi d\mu = 1 - u_z$*; (iv) capital market clears,* $\int g^s(j,z,\mu)\,d\mu = K'$*; (v) bonds market clears,* $\int g^b(j,z,\mu)\,d\mu = 0$*; and (vi) the transition function* $Q(z,\mu,z')$ *is generated by the optimal decisions* $\left\{g^c, g^b, g^s\right\}$*, the law of motion for habits* ψ *and the transition matrices* $\{\Gamma_e, \Gamma_\xi\}$*.*

2.4 Solution of the model

Computation of this class of models is very demanding. In order to predict next period's prices agents need to keep track of μ, the distribution of households over shocks, asset holdings and habit stocks. Therefore, the state space contains an object of infinite dimensionality that cannot be stored by a computer. To get around this problem, I follow the *partial information* approach used by Krusell and Smith (1998). The approach is based on assuming that by only using part of the information contained in μ agents can predict next period's aggregate state (and hence prices) almost as well as by using the whole distribution. Krusell and Smith (1998) show that typically the first moment of the marginal distribution of wealth suffices to predict prices. Also Young (2002) shows that for a wider variety of related models first moments are almost sufficient statistics. Two important findings of this paper are that with habit formation preferences, (a) it is also true that only the first moments of μ matter and (b) the first moment of the marginal distribution of agents over habits does not bring any additional information in predicting tomorrow's state once we are considering the marginal distribution of assets (or its first moment). Therefore, the approximation result by Krusell and Smith (1998) holds once we consider non time-separable preferences. The computational appendix at the end of the paper gives details on this. For non-technical readers it suffices to say that hereafter I will use K instead of μ as the endogenous aggregate state of the economy.

2.5 The equity premium

In order to understand the determinants of the equity premium, let's first look at the optimality conditions of the household. Combining the two Euler equations for the household problem we can write

$$E_{z'|z}\left[E_{e',\xi'|e,\xi,z,z'}\left[v_{\omega}\left(j',z',K'\right)\right]\left(R^{s}\left(z,K,z'\right)-R^{b}\right)\right]=0 \tag{7}$$

which is generally known as the pricing equation. It tells us that households choose bonds and capital such that at the margin the value of investing in each asset is the same. Or more technically, the optimal portfolio decision implies that the expected return of each asset, weighted by the expected marginal value of wealth in each state, must be equal to each other. This is the condition that non constrained optimizing agents will satisfy. Obviously, some agents will be in a corner solution by setting $b=\underline{b}$ or $s=\underline{s}$ and will not satisfy equation (7) with equality.

The equity premium is the difference between the expected return of the risky asset and the return of the risk free asset. The optimal portfolio choice condition given by equation (7) lets us express the equity premium as follows,

$$E_{z'|z}\left[R^{s\prime}\right]-R^{b}=SD_{z'|z}\left[R^{s\prime}\right]\ CV_{z'|e,\xi,z}\left[E_{e',\xi'|e,\xi,z,z'}\left[v_{\omega}'\right]\right] \tag{8}$$

where SD denotes standard deviation and CV denotes coefficient of variation. Equation (8) tells us that the equity premium can be decomposed into two terms. The first term is the conditional standard deviation of the return of the risky asset, and it is generally known as the amount of risk. The second term is the coefficient of variation, with respect to the aggregate shock, of the expected marginal value of wealth. It measures the volatility of marginal utilities with respect to changes in the aggregate shock. This term is the price of risk and corresponds to the Sharpe ratio. As argued in the introduction, this second term depends on household decisions and it will be the focus of the quantitative work.

3 Calibration

The model period is imposed to be a quarter. The simulated economies are targeted to reproduce statistics from the US macroeconomic data and from the US cross-sectional distribution of labor earnings. The calibration strategy is as follows. Most parameters are either predetermined, that is to say, selected from other studies, or targeted to statistics whose computation does not require solving for the whole model. Then, two important parameters are calibrated in equilibrium to ensure that the model economy displays certain properties we may regard as important for asset pricing. These properties are the following: (a) the amount of wealth of the economy and (b) the volatility of aggregate consumption growth. The former is important because borrowing and saving are the only (self) insurance mechanisms available to workers. The latter is a crucial ingredient used in the early literature on the equity premium puzzle and habit formation. Therefore, I calibrate the discount factor β and the amplitude of the aggregate shock z such that, given the rest of the parameters, the model economies reproduce the capital to labor ratio and the standard deviation of aggregate consumption growth. I detail the calibration process in the subsequent subsections.

3.1 Earnings process

First of all, I pick values for the employment and efficiency units shocks in order to ensure that the households in the model face the same volatility in labor earnings as it is measured in the data.

The employment shock is characterized by four two by two transition matrices plus the employment levels in good times and in bad times, which gives 10 independent parameters. This means that we need 10 calibration targets. First, following Krusell and Smith (1997) I set average duration of unemployment spells in good and bad times equal to 1.5 and 2.5 quarters respectively.[4] Second, to avoid aggregate labor being a state variable four extra

[4] İmrohoroğlu (1989) calibrates the employment shock similarly and picks average durations equal to 1.66 and 2.33 respectively.

conditions are imposed: (1) employment when $z = z_g$ must be always the same regardless of the previous period realization of z

$$\begin{aligned} 1 - u_g &= (1 - u_g)\,\Gamma_e(z_g, z_g, 1, 1) + u_g\,\Gamma_e(z_g, z_g, 0, 1) \\ 1 - u_g &= (1 - u_b)\,\Gamma_e(z_b, z_g, 1, 1) + u_b\,\Gamma_e(z_b, z_g, 0, 1) \end{aligned}$$

and likewise (2) employment when $z = z_b$ must also be the same regardless of the previous period realization of z

$$\begin{aligned} 1 - u_b &= (1 - u_g)\,\Gamma_e(z_g, z_b, 1, 1) + u_g\,\Gamma_e(z_g, z_b, 0, 1) \\ 1 - u_b &= (1 - u_b)\,\Gamma_e(z_b, z_b, 1, 1) + u_b\,\Gamma_e(z_b, z_b, 0, 1) \end{aligned}$$

Third, the probability for the unemployed of finding a job when moving from good to bad times is set to zero and the probability for the employed to enter unemployment when moving from bad to good times is also set to zero: $\Gamma_e(z_g, z_b, 0, 1) = 0$ and $\Gamma_e(z_b, z_g, 1, 0) = 0$. Finally, I choose the unemployment levels for good and bad times. I target the average and standard deviation of the Bureau of Labor Statistics unemployment rate for the period 1948-2001, which are 5.63% and 1.61% respectively. Under the constraint that u_g and u_b are equidistant from the average, this gives values for the unemployment rates in good and bad times: $u_g = 0.0417$ and $u_b = 0.0719$.

Table 1: **The distribution of earnings**

	data	model
share of earnings of top 20%	60.2%	61.7%
share of earnings of bott 40%	3.8%	4.5%
gini index of earnings	0.61	0.55
persistence top 20%	68%	46%

Note: Cross section statistics in the first column are from the Survey of Consumer Finances, 1998. Persistence is the probability that those households in the top 20% of the income distribution (PSID, 1989) are still in the top 20% five years later (PSID, 1994). Data quoted from Budría, Díaz-Giménez, Quadrini, and Ríos-Rull (2002).

Regarding the efficiency units shock ξ, I establish three points and try to replicate some cross-section and time series statistics of the US earnings distribution. Table 1 shows these statistics (column 1) together with the ones produced by the chosen parameterization of the efficiency units shock (column 2). Table 2 presents the parameters for the efficiency units shock that generate the statistics in column 2 of table 1.

Finally, the home production parameter d is set equal to 5% of the average quarterly earnings.

Table 2: **Stochastic process for efficiency units**

	ξ_1	ξ_2	ξ_3
	30.0	8.0	1.0
	$\Gamma_\xi(\xi_1,.)$	$\Gamma_\xi(\xi_2,.)$	$\Gamma_\xi(\xi_3,.)$
$\Gamma_\xi(.,\xi_1)$	0.9850	0.0025	0.0050
$\Gamma_\xi(.,\xi_2)$	0.0100	0.9850	0.0100
$\Gamma_\xi(.,\xi_3)$	0.0050	0.0125	0.9850

3.2 Technology parameters

To pick values for the technology parameters, I will mainly focus on macroeconomic data. We need to define measurements from the US economy consistent with the model economies. I follow Cooley and Prescott (1995) to construct aggregate series for consumption, investment, output and capital consistent with the model. For the period 1946 to 2001 these series deliver a capital to output ratio of 12.56 and an investment to output ratio of 0.35. The depreciation rate δ is chosen such that in the non-stochastic steady state the model investment to output ratio matches the equivalent statistic in the data. The labor share θ is set equal to 0.60 as in Cooley and Prescott (1995).

Following İmrohoroğlu (1989), I calibrate the two independent parameters of the transition matrix for the aggregate shock to match the average duration of good and bad times. I pick the average duration of each state equal to 8 quarters as Krusell and Smith (1997) do. The levels of the shock are model dependent and are set to reproduce the fluctuations of aggregate consumption measured in the data. In particular, I target the standard deviation of the aggregate consumption growth, which is equal to 0.52, and I impose the shock to be symmetric.

Finally, we have to set the lower bounds on total net worth and on each type of asset. I set the lower bound on total net worth $\underline{\omega}$ equal to five times the average (quarterly) income of the economy. The lower bounds $\underline{b}$ and $\underline{s}$ are set such that the borrowing limit on total wealth can be reached by use of either asset. Therefore, $\underline{b} = \underline{\omega}$ and $\underline{s} = \underline{\omega}$.

3.3 Preference parameters

I need to choose values for the parameters β, σ, γ and λ. For the non-habits economy I choose $\sigma = 2$ in order to obtain an intertemporal elasticity of substitution for consumption equal to 0.5, which is in line with many empirical estimates. However, the value of σ consistent with an intertemporal elasticity of substitution equal to 0.5 in the habits economies will be different. In particular, as shown by Díaz, Pijoan-Mas, and Ríos-Rull (2003), the intertemporal elasticity of substitution for the habits economies becomes $(\gamma + (1-\gamma)\,\sigma)^{-1}$. Therefore, I need to pick γ before choosing σ. There are some empirical papers estimating habit parameter values. However, results are very different among them, depending on both the data and model specification.[5] The strategy followed in this paper is to allow for a *strong* habit process and interpret the results as an upper bound. First, I set $\gamma = 0.75$. γ determines the weight of habits on the utility function. The value chosen implies that households care more about relative consumption than about the consumption level itself. Second, I set $\lambda = 0.25$. This generates a highly persistent habits process. A persistent habit process is used by Constantinides (1990) and Heaton (1995) to obtain sizeable equity premia. In addition, Díaz, Pijoan-Mas, and Ríos-Rull (2003) show in a similar model without aggregate risk that the more persistent the process, the higher the effect of habits in the consumption/savings decision. I set $\sigma = 5$ in order to keep the intertemporal elasticity of substitution equal to 0.5. Finally, as discussed above, the discount factor β is calibrated in equilibrium to match the capital to output ratio, which is measured to be 12.56.

4 Results

First of all I solve for the benchmark economy without habits. I calibrate it as stated in section 3 and call it *HA*. Together with *HA* I solve for a similar economy *HAH* where I add habit formation. Table 3 presents selected statistics from these two model economies (see the second and the third columns) as well as the equivalent statistics from data (first column). The first two rows report the capital to labor ratio and the standard deviation of aggregate consumption growth. As explained in section 3, in the data they are measured to be 12.56 and 0.52 respectively. The economy *HA* is calibrated to reproduce these two targets whereas the economy *HAH* is calibrated to reproduce just the former.

A first quantitative result to highlight is the correlation between the price of risk and the business cycle. The simulated economy *HA* produces a correlation between the Sharpe ratio and the rate of growth of output equal to -0.37. The equivalent statistic for the *HAH* economy is -0.33. This is important because Campbell and Cochrane (1999) argue that in order to model habit formation preferences the survival consumption approach is consistent with the fact that the price of risk is counter-cyclical whereas the power utility is not. This is true in a representative agent framework. However, in a heterogeneous agents setting the power utility can also generate this result.

[5]Some empirical studies on habit formation are Fuhrer (2000), Dynan (2000) and Heaton (1995). See Díaz, Pijoan-Mas, and Ríos-Rull (2003) for a survey.

Table 3: **Statistics of simulated economies**

	data	*model economies*			
		HA	HAH	HAC	$HACH$
K/Y	12.56	12.56	12.56	12.56	12.56
$SD\,[g_C]$	0.52	0.52	0.26	0.52	0.26
$corr\,[g_Y, Sharpe]$	–	-0.37	-0.33	0.36	0.38
$Sharpe\ ratio$ (%)	27.0	1.24	1.70	17.9	31.0
$\int CV\,[Ec']\,d\mu$ (%)	–	0.79	0.31	0.74	0.36

Note: The first column refers to US quarterly data from 1946 to 2001, except *Sharpe ratio* that refers to 1948-1996. $SD[g_C]$ is the unconditional standard deviation of the rate of growth of the log of aggregate consumption in quarterly terms. $corr\,[g_Y, Sharpe]$ is the correlation between the rate of growth of aggregate output and the Sharpe ratio. The actual value of the Sharpe ratio is quoted from Lettau and Uhlig (1997). The last row reports the average (over all households) of the conditional coefficient of variation of the expected consumption c'. The coefficient of variation operator refers to the distribution of z' conditional on z. The expectation operator on consumption and earnings refers to the joint distribution of e' and ξ' conditional on e, ξ, z and z'.

Regarding the price of risk Lettau and Uhlig (1997), using US quarterly data from 1948 to 1996, estimate a value of 27% for the Sharpe ratio. We see that the Sharpe ratio obtained in economy *HA* is 1.24%. This figure represents a very small fraction of the observed price of risk. This result illustrates the essence of the equity premium puzzle. For reasonable parameter values, the price of risk that households require to hold risky assets is nowhere around the observed value in the data.[6] Now let's turn our attention to the HAH economy. We observe that habit formation increases the equilibrium price of risk. The Sharpe ratio increases about 37% from 1.24% to 1.70%. However the value obtained is still far below its empirical counterpart. Contrary to earlier results, as Constantinides (1990), Abel (1990) and Campbell and Cochrane (1999), habit formation does not seem to provide a satisfactory answer to the equity premium puzzle. The reason is threefold and I will develop it in the following paragraphs. First, households change their consumption/savings decisions; second, households change their portfolio choice; third, the composition of the set of agents pricing assets changes too. Notice that the last two motives are absent in representative agent economies.

4.1 Consumption fluctuations fall

In this economy forward-looking households have two decision margins whereby adjust to the higher utility losses from consumption fluctuations. First, as shown by Díaz, Pijoan-Mas, and Ríos-Rull (2003), they may rise their precautionary savings in order to have

[6] As discussed in the introduction, the production side of the model is very simple and the volatility of the returns has not been calibrated. Due to this fact the model equity premium equals 0.18%. Calibrating the volatility of aggregate consumption growth also implies not trying to match the volatility of aggregate output. The resulting standard deviation of the aggregate output growth equals 4.6.

a buffer stock against income fluctuations and second they may change the composition of their portfolios such that the covariance between their labor income and their capital income diminishes. In the end, the use of these two decision margins are targeted to lower the fluctuations in consumption. Consistently with this idea, I find that the economy *HAH* produces a standard deviation for aggregate consumption growth of 0.26, half its counterpart in both the non-habits economy and data. Another way to see this result is by looking at the fluctuations faced by individual households. The last row in table 3 reports the average over the whole population of the coefficient of variation (with respect to the conditional distribution of the aggregate shock) of the expected consumption next period. Precisely, $\int_J CV_{z'|z}\left[E_{e',\xi'|e,\xi,z,z'}\left[g^c\left(j',z',K'\right)\right]\right]d\mu$, which I abbreviate in the table by $\int CV\left[Ec'\right]d\mu$. This statistic gives a measure of individual level consumption fluctuations. As it happens with aggregate consumption, the comparison of the values of this statistic in the economies *HA* and *HAH* shows that individual consumption also fluctuates much less when households form habits. The actual value falls from 0.79% in economy *HA* to 0.31% in economy *HAH*.

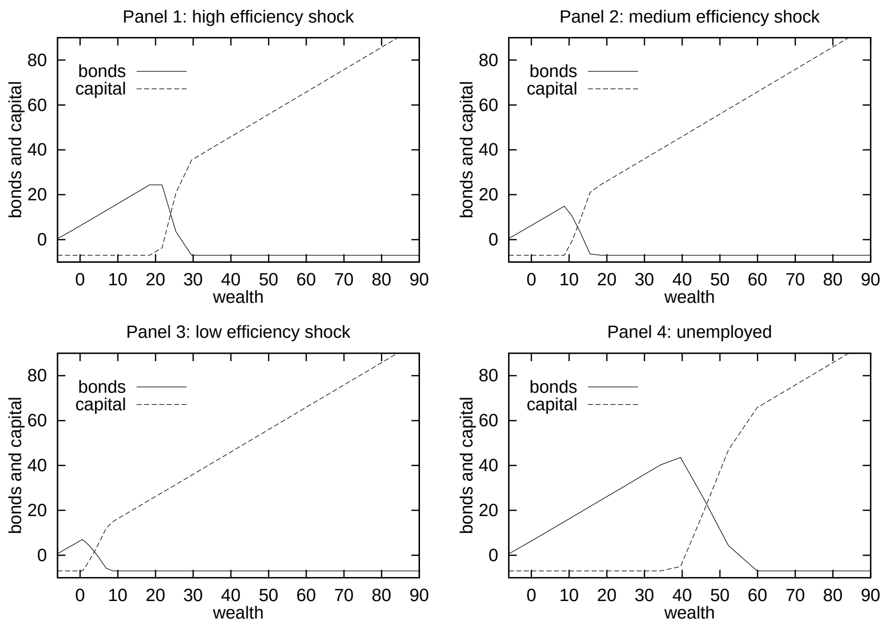
4.2 Policy functions

In order to understand how the trade of assets affects the price of risk it is useful to see the portfolio choices of different types of households. In figures 1 and 2 I plot the policy functions for the economy *HAH*. Each panel in every figure corresponds to each possible earnings state. Figure 1 holds the habit stock constant and displays the portfolio choice for each possible value of wealth ω. Figure 2 holds wealth constant and displays the portfolio choice for each possible value of habits h. We observe three distinct patterns. First, as wealth increases households put relatively more capital in their portfolio. Second, for a given level of wealth, the higher the efficiency level the lower the amount of capital. And third, the amount of capital decreases with the habit stock.

Regarding the first result, higher wealth implies (1) having a lower proportion of labor earnings in next period's expected income and (2) being further away from the borrowing constraints. Therefore, (1) the variability of expected income is lower and (2) it translates to a lesser degree into consumption variability. Therefore, wealth-rich households are more willing to take risk in exchange of higher expected returns. In contrast, wealth-poor agents go as short as possible in capital and invest in bonds. They sacrifice expected returns but in exchange get a portfolio that pays well when their marginal value of wealth is high (downturns) and pays bad when their marginal value of wealth is low (peaks).[7]

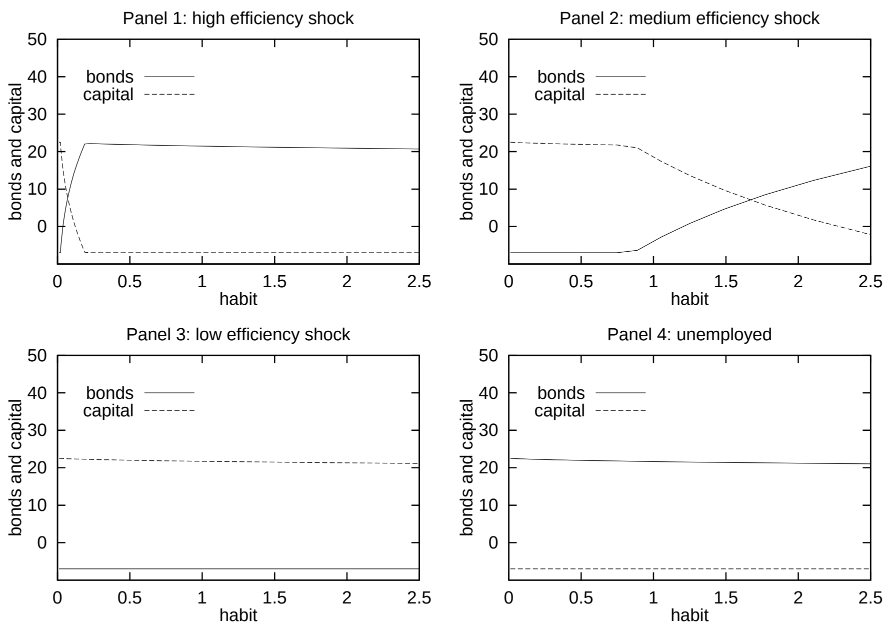
[7]Notice that this result is the opposite of what other authors as Cocco, Gomes, and Maenhout (2005) have found. These authors model labor income as an homoscedastic process with (an estimated) low

Figure 1: **Policy functions (economy *HAH*)**



Note: each panel plots g^b and g^s as a function of wealth ω only. The aggregate shock has been set equal to z_g, the aggregate capital equal to its time series mean and the habit stock equal to its cross-sectional median.

Figure 2: **Policy functions (economy *HAH*)**



Note: each panel plots g^b and g^s as a function of the habit stock h. The aggregate shock has been set equal to z_g, the aggregate capital equal to its time series mean and the individual wealth equal to its cross-sectional median.

The second result implies that low-efficiency households are buying risk from high-efficiency ones in exchange of higher expected payoffs. Note that once we control for wealth, the sole role of the efficiency units shock is to predict next period's efficiency units endowment. The process for ξ is such that $E[\xi' \mid \xi, e' = 1]$ is increasing in ξ. I.e., conditional on being employed next period, the expected amount of efficiency units next period is increasing in the amount of efficiency units in the current period. Therefore, the higher ξ, the larger the income difference between being employed and unemployed and hence the larger the conditional variability of expected labor earnings. This result relies on unemployment risk being unrelated to the earnings position.[8]

Finally, regarding the habits dimension figure 2 shows that the amount of capital decreases with the habit stock. The picture is similar for different levels of wealth (not shown). This is not surprising. Since the habit stock increases the volatility of marginal utilities, for the same level of wealth agents that have enjoyed a history of higher consumption are less willing to take risk than agents that have enjoyed a history of lower consumption. In other words, the new rich are less reluctant to take risk because if things go wrong they do not lose as much as agents that have already got used to a certain status.

Empirical results by Bertaut and Starr-McCluer (2002) are not inconsistent with these findings. Using data from the *Survey of Consumer Finances*, these authors find that, controlling for education and other household observable variables, household wealth increases the share of risky assets in the portfolio whereas household income decreases the share of risky assets.

4.3 The composition effect

Equilibrium asset prices depend on the consumption fluctuations of those individuals that solve the portfolio choice problem with an interior solution. The addition of habit formation changes households decisions and the way they trade assets. This implies that in equilibrium the composition of the set of agents that have an interior solution to their portfolio choice problem also changes. The change in the composition of the set of pricing agents is a force preventing the price of risk from increasing too much. Throughout the paper I refer to this

correlation with stock returns. It turns out that under this stochastic structure labor income becomes a good substitute of the risk free asset and the standard result of Merton (1971) applies. In this paper labor income is heteroscedastic, with variance negatively correlated with the returns of the risky asset. This makes capital holdings very undesirable for asset poor households. Quantitative models by Krusell and Smith (1997) and Storesletten, Telmer, and Yaron (2001) also display this property.

[8]The result does not need to hold when unemployment probability is related to the skill level. High efficiency workers may face a higher differential in earnings between employed and unemployed status but they may also have a lower probability of unemployment. Overall, conditional variance of earnings could be lower for high earners than for low earners.

mechanism as *composition effect*.

Table 4: **Statistics of simulated economies. Composition effect**

	economy HA			*economy HAH*		
	(1)	*(2)*	*(3)*	*(1)*	*(2)*	*(3)*
all households	1.00	0.79	1.31	1.00	0.31	1.71
pricing households	0.27	0.53	1.08	0.32	0.25	1.47
corner choice for bonds	0.44	0.38	0.79	0.39	0.18	1.05
corner choice for capital	0.29	1.63	2.32	0.29	0.56	2.91

Note: for each panel, the first column reports the sum of households over the set defined by each row. The second column reports the average consumption volatility within the given set (in percentage terms). The third column reports the average volatility of marginal utilities within the given set (in percentage terms).

In table 4 I report some statistics related to the risk faced by households according to their portfolio choices. The first column of each panel in table 4 reports the proportion of individuals in every set for a given model period. In the economy without habit formation only 27% of households solve their portfolio choice problem with an interior solution. Adding habits increases this proportion to 32%. The second column reports the coefficient of variation of expected consumption. We observe that in both economies, households with a corner choice for capital are those who face the highest volatility in their consumption. These agents build portfolios that pay well in downturns and bad in peaks. Inverting the correlation between labor income risk and financial returns implies giving up expected value in exchange of insurance against labor earnings volatility. In contrast, the households that go as short as possible in bonds and invest in capital are those that face smaller consumption fluctuations. This type of portfolio maximizes expected value at the cost of higher exposure to risk. The pricing agents are in between these two extremes. The third column in each panel reports the volatility of marginal utilities, $CV_{z'|z}\left[E_{e',\xi'|e,\xi,z,z'}\left[v'_{\omega}\right]\right] d\mu$, which is the price of risk required for a given household in order to willingly hold its current portfolio.[9]

The *composition effect* works as follows. When adding habit preferences to the model, the utility costs of consumption fluctuations increase. Households that were holding both bonds and capital will switch to borrow as much as possible in capital and invest in bonds in order to diminish their exposure to risk. Additionally, households with a corner choice for bonds will start to introduce some bonds above the lower bound while diminishing the amount of capital and will hence become pricing agents. Therefore, the pool of households with interior solution to the portfolio choice problem changes introducing individuals with

[9]Remember that this is the second term of the right hand side of equation (7). Therefore, from table 4 we observe that the Sharpe ratio for this particular model period is 1.08% in the *HA* economy and 1.47% in the *HAH* economy.

small consumption fluctuations that were not pricing risk in the economy *HA* and losing individuals with high consumption fluctuations.

4.4 The importance of each channel

Table 5: **Sharpe ratio under different scenarios.**

	Demanded price of risk at selected percentiles (%)				
	$p25$	$p50$	$p75$	$p95$	$p99$
economy *HA*	1.31	1.31	1.31	1.31	1.31
non habits policy functions	4.51	5.35	5.92	9.72	17.67
non habits consumption, optimal portfolio	3.78	4.58	5.59	9.05	16.75
optimal consumption and portfolio	1.52	1.55	1.59	1.69	1.78
pricing agents in *HAH* economy	1.50	1.53	1.57	1.60	1.66

Note: The entries in the table report, for different households, the actual coefficient of variation of the expected marginal value of wealth under different scenarios. The first row considers the pricing agents in economy *HA* with habits preferences and policy functions as in the *HA* economy. The second row allows households to choose their optimal portfolio. The third row allows households to choose their optimal portfolio and savings decisions. The fourth row computes the average over the pricing agents in the *HAH* economy.

In order to quantify the relative importance of each channel, I look at the price of risk required by different households in different scenarios. In particular, I will evaluate the coefficient of variation of the individual marginal value of wealth for households with non-corner solution to their portfolio choice problem in the following situations. First, I compute the price of risk demanded by the pricing agents of the non-habits economy when we change their preferences to include habit formation but force them to keep the same policy functions. Second, I compute the price of risk demanded by these same agents when I still force them to consume as non habits individuals but I let them optimize their portfolio. Third, I let them choose all their optimal policy functions. And fourth, as before but I compute the price of risk for the pricing agents of the habits economy instead. Notice that we have to choose an equilibrium where to do these measurements, that is to say, we have to choose a set of forecasting rules and a set of market prices. I will report results for the equilibrium of the *HA* economy. Things are very similar when doing this decomposition in the *HAH* equilibrium. These four cases correspond to the second, third, fourth and fifth rows in table 5. For each case, I report the coefficient of variation of the marginal value of wealth at different points of the distribution within the set of pricing agents. These points are the first quartile, the median, the third quartile and the 95 and 99 percentiles. We observe that with the addition of habits the price of risk required by households that behave as in the non habits economy increases very much. The median household requires a price of risk of 5.35%, whereas the top percentile household requires a price of risk of

17.67%. The value for the median is more than four times the equilibrium price of risk of economy *HA* whereas the latter value is within the order of magnitude of the empirically measured price of risk. If we looked at individuals with a corner choice for capital in the original equilibrium, we would find them to demand even larger compensations.

As we let these households modify their behavior according to their habits preferences they will reduce their exposure to risk and therefore they will demand lower compensations for holding risky assets. If we let households readjust their portfolios while keeping their consumption as in the non-habits case, the required price of risk diminishes. For instance, the median falls to 4.58%. If we also let them adjust their consumption the fall is very big, with the median going to 1.55%. Finally, if we compute the price of risk for the set of pricing agents in the *HAH* economy, there is a further although very small fall, with the median going to 1.53%. These successive falls in the price of risk let us decompose the relative importance of each mechanism in preventing the habits formation preferences from solving the equity premium puzzle. As measured at the median, the difference between the price of risk in the economy with habits preferences but no change in behavior and the price of risk in the habits economy is 3.82 percentage points. The change in the portfolio choice accounts for the 20.2% of this difference, the change in the consumption/savings decision accounts for the 79.3% and the *composition effect* for a further 0.5%.

4.5 Matching the Sharpe ratio

The results shown in the previous sections are based on economies where households can hold negative wealth (non collateralized debt) up to 5 times the quarterly average income in the economy (or 1.25 times the annual average income). What these loose borrowing constraints imply is that there are plenty of opportunities to insure against labor income uncertainty by use of the financial markets. Overall, the incomplete markets economy is not so different from a complete markets economy because the financial markets offer good opportunities of insurance. In this section I go to the other extreme. Namely, households are not allowed to hold negative wealth. Furthermore, they cannot go short in any asset by use of the other one as collateral. I solve for a new economy *HAC* with the same calibration targets as economy *HA* but with the borrowing constraints $\underline{b}$ and $\underline{s}$ set equal to zero. I present some selected statistics for this economy in the fourth column of table 3.

Consistently with the results of Heaton and Lucas (1996) and Krusell and Smith (1997), adding constraints to financial markets increases the market price of risk. The Sharpe ratio jumps to 17.9%, a value not far away from its empirical counterpart and almost 15 times as big as in the economy *HA*. The comparison of the Sharpe ratios in economies *HA* and *HAC* gives us a good measure of the quantitative importance of borrowing as a mechanism of insurance. The market price of risk in the economy without borrowing is still below the empirical Sharpe ratio although already quite close to it. This finding is similar to the one found by Krusell and Smith (1997), who report for an economy without borrowing a Sharpe ratio of 21.0%. Krusell and Smith (1997) consider an heterogeneous agents model where the only source of idiosyncratic uncertainty is given by an employment shock. They set a much more volatile employment process with $1 - u_g = 0.96$ and $1 - u_b = 0.90$. This pair of values delivers a standard deviation for the unemployment rate of 3.0%, a figure almost twice as big as the value computed from BLS data. A major drawback of their choice is that it generates counter-cyclical wages.[10]

In this context, the interaction of habit formation and heterogeneity produces a price of risk consistent with the observed data. In the last column of table 3 I provide the statistics corresponding to the habits counterpart of economy *HAC*, which I label *HACH*. In this economy the market price of risk reaches a value of 31.0%. This is a 73% increase with respect to the non habits economy, which dwarfs the 37% increase observed in the borrowing economy.

5 Conclusions

This paper argues that given the observed earnings fluctuations faced by households it is difficult to interpret the equity premium as a risk premium. This is so even if one considers preferences with habit formation.[11] Within a standard equilibrium model economy, the optimal response of households to an increase in the utility costs of risk is to reduce their exposure to income fluctuations. In equilibrium the price of risk increases modestly because households manage to smooth their consumption fluctuations and because the households that end up pricing risk are better self-insured (i.e. they face smaller fluctuations in their marginal rate of substitution). In contrast, the early literature on habits and asset pricing has tended to take consumption fluctuations as given without allowing forward-looking agents to modify their behavior.

I find that, when adding habit formation to a fully specified general equilibrium model with heterogeneous agents, there are many things that change along with the preference hypothesis. First, individuals change their consumption/savings choices. Households save

[10] The marginal product of labor is decreasing in labor and increasing in the aggregate shock z. In peaks the increase in labor due to such a volatile employment process is so big that more than offsets the increase in z.

[11] McGrattan and Prescott (2000) and McGrattan and Prescott (2005) also claim that it may well be that the equity premium is not a risk premium at all. They find that the differential return between the risky and risk free assets is perfectly reasonable once intangible assets, foreign assets and different taxation issues are taken into account.

in advance creating stocks of precautionary savings to smooth out fluctuations in income. Second, individuals change their portfolio choices. The proportions of risky and risk free assets are modified in order to change the covariance between labor income and financial income. As a result, households manage to diminish the fluctuations in consumption. And third, the changes in the portfolio decisions also change the equilibrium composition of the set of households that have an interior solution to their portfolio choice problem. Adding habits makes relatively better self-insured households be the ones pricing the risk.

The amount of asset trading in equilibrium is a crucial element for these results. If we solve for an economy where households are neither allowed to borrow nor allowed to go short in any asset, the addition of habit formation does generate a price of risk as large as in the data. The reason is that the trade of risky and risk free assets available when households can borrow is a very good mechanism to insure against labor income fluctuations. Absent this insurance technology, the extra utility costs of consumption fluctuations introduced by habit formation make households require a very high premium to hold risky assets.

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Appendix

A Computational Procedures

This appendix explains the computer algorithm used to solve the model. The algorithm is based on the *partial information* approach used by Krusell and Smith (1997).[12] The model solved in this paper requires extending the algorithm in order to include an extra individual state variable, the habit stock h, and a new dimension in the aggregate distribution of households. These two issues justify the need to make explicit what is actually done in this paper.

The general strategy is as follows. I replace the endogenous state μ by its first moments K and H. Using only first moments, we have to replace the equilibrium condition *(vi)* in section 2.3 by

$$K' = f^K(z, K, H) \tag{9}$$

and introduce a new equation to predict aggregate habits

$$H' = f^H(z, K, H) \tag{10}$$

We also need to approximate $R^b(z, \mu)$, which is a direct function of the distribution of agents, so I postulate:

$$R^b = f^{R^b}(z, K, H) \tag{11}$$

Under this approximation, the state space of the household problem is reduced. In order to predict prices, instead of z and μ, consumers only need z, K and H. Of course, the forecasting rules $\left\{f^K, f^H, f^{R^b}\right\}$ are unknown and they are part of the solution. Therefore, solving the household problem implies maximizing equation (1) subject to the constrains (2), (3), (4) and (6) and to the forecasting rules (9), (10) and (11). The problem is that the forecasting rules f^K, f^H and f^{R^b} are not known. I start explaining how to solve the household problem for given forecasting rules and then I discuss how to find the forecasting rules consistent with a rational expectations equilibrium.

A.1 Solving the household's problem

For the household problem the state space is given by the individual vector $j = \{\omega, h, e, \xi\}$ plus the aggregate variables z, K and H. I collapse the three exogenous and stochastic state variables e, ξ and z into one variable ϵ that can take $n_\epsilon = n_z(n_\xi + 1) = 8$ different values. We are therefore left with the two endogenous individual state variables ω and h, the exogenous stochastic shock ϵ and the exogenous (at the household level) aggregate variables K and H. Let's define labor earnings in terms of the newly defined exogenous stochastic process ϵ as $\nu(\epsilon, K)$. Households have to solve the following system formed by

[12] They were already extending previous work by Castañeda, Díaz-Giménez, and Ríos-Rull (1998) and Krusell and Smith (1998). Ríos-Rull (1998) explains it in good detail.

the FOC, the constraints and the forecasting rules:

$$\begin{aligned}
&0 = u_c(c,h) + \lambda\beta E_{\epsilon'|\epsilon}\left[v_h(\omega',h',\epsilon',K',H')\right] - \beta E_{\epsilon'|\epsilon}\left[v_\omega(\omega',h',\epsilon',K',H')\,R^s\left(\epsilon',K',R^b\right)\right]\\
&0 = u_c(c,h) + \lambda\beta E_{\epsilon'|\epsilon}\left[v_h(\omega',h',\epsilon',K',H')\right] - \beta E_{\epsilon'|\epsilon}\left[v_\omega(\omega',h',\epsilon',K',H')\,R^b\right]\\
&c = \omega - b - s\\
&h' = \lambda c + (1-\lambda)h\\
&\omega' = bR^b + sR^s(\epsilon',K') + \nu(\epsilon',K')\\
&K' = f^K(\epsilon,K,H)\\
&H' = f^H(\epsilon,K,H)\\
&R^b = f^{R^b}(\epsilon,K,H)\\
&c \geq 0;\quad b \geq \underline{b};\quad s \geq \underline{s};\quad b+s \geq \underline{\omega}
\end{aligned} \tag{12}$$

A standard way to solve this problem would be to use the envelope conditions to substitute out v_ω and v_h from the FOC and then solve for the unknown policy functions $\left\{g^c, g^b, g^s\right\}$. However, doing so with the habits dependence would imply adding variables dated up to the infinity and therefore losing the recursive properties of the problem. What I will do instead is to find a solution for the unknown function $\{v_\omega, v_h\}$ together with the policy functions $\left\{g^c, g^b, g^s\right\}$ as in a value function iteration algorithm.

For a given pair $\{v^0_\omega, v^0_h\}$, the system (12) delivers a set of policy functions $\left\{g^{0,c}, g^{0,b}, g^{0,s}\right\}$. Then, we can substitute them in the right hand side of the envelope conditions to obtain a new pair of derivatives for the value function $\{v^1_\omega, v^1_h\}$,

$$v^1_\omega(\omega,h,\epsilon,K,H) = u_c(c,h) + \lambda\beta E_{\epsilon'|\epsilon}\left[v^0_h(\omega',h',\epsilon',K',H')\right] \tag{13}$$

$$v^1_h(\omega,h,\epsilon,K,H) = u_h(c,h) + (1-\lambda)\beta E_{\epsilon'|\epsilon}\left[v^0_h(\omega',h',\epsilon',K',H')\right] \tag{14}$$

where c, b and s are replaced by $g^{0,c}(\omega,h,\epsilon,K,H)$, $g^{0,b}(\omega,h,\epsilon,K,H)$ and $g^{0,s}(\omega,h,\epsilon,K,H)$. The system of equations (12) together with the envelope conditions (13) and (14) define a mapping T from the cartesian product of the space where v_ω and v_h belong into itself. Solving the household problem amounts to finding a fixed point of this mapping, i.e., a pair such that $\{v^*_\omega, v^*_h\} = T\{v^*_\omega, v^*_h\}$. One problem with this approach is that the space where v_ω and v_h belong to is unknown. I need thus to specify a class of functions that the computer can understand in order to approximate for this space. I will do as follows. For every value of ϵ, I approximate $\{v_\omega, v_h\}$ piece-wise linearly in a four-dimensional grid.[13] Given an initial guess $\{v^0_\omega, v^0_h\}$, I solve the system (12) to get the policy functions $\left\{g^{0,c}, g^{0,b}, g^{0,s}\right\}$. Then, using the envelope conditions (13) and (14), I obtain a new pair $\{v^1_\omega, v^1_h\}$. If the new pair $\{v^1_\omega, v^1_h\}$ is close to $\{v^0_\omega, v^0_h\}$ I have found an approximation to the fixed point of the mapping T and I take $\left\{g^{0,c}, g^{0,b}, g^{0,s}\right\}$ as the solution of the model. If not, I update $\{v^0_\omega, v^0_h\} = \{v^1_\omega, v^1_h\}$ and start again. Notice that there is no contraction theorem for this mapping, which means that there is no guarantee to succeed by using this successive approximations approach. For the iterations to make good progress, it turns out to be very important to select proper initial conditions $\{v^0_\omega, v^0_h\}$.

[13] In the K and H dimension there is not much curvature, so I use fewer points than in the ω and h dimensions. I typically use 6 points for the aggregate variables, 60 for wealth ω and 33 for the habit stock h. This implies solving the system (12) at $570,240$ different points for every pair $\left\{v^0_\omega, v^0_h\right\}$.

A.2 Finding the equilibrium forecasting rules

The nature of the stationary stochastic equilibrium implies keeping track of a distribution μ in order to forecast future prices. The *partial information* approach is based on using just a subset of moments of μ instead. I will use only first moments. We need to find a vector of forecasting functions $f \equiv \left\{f^K, f^H, f^{R^b}\right\} \in \mathcal{F} \equiv \mathcal{F}^K \times \mathcal{F}^H \times \mathcal{F}^{R^b}$ consistent with rational expectations. I.e., given that agents forecast K, H and R^b with certain f, the simulated economy should display this same behavior. Or in other words, the simulated series for K, H and R^b should be *well* predicted by f. The idea is to start with an initial f^0, solve the household's problem defined in section A.1, simulate the economy for a long series of periods and estimate a new f^1 within the same parametric class $\mathcal{F}$. Krusell and Smith (1997) show that one needs to make one correction to this procedure. Precisely, the market for bonds does not clear in every period and state. In order to achieve the bond market clearing in every period and state, I define the following problem:

$$V\left(\omega, h, \epsilon, K, H, R^b\right) = \max_{c,b,s} \left\{u(c,h) + \beta E_{\epsilon'|\epsilon}\left[v\left(\omega', h', \epsilon', K', H'\right)\right]\right\} \tag{15}$$

subject to

$$\begin{aligned}
&c = \omega - b - s \\
&h' = \lambda c + (1-\lambda) h \\
&\omega' = bR^b + sR^s\left(\epsilon', K'\right) + \nu\left(\epsilon', K'\right) \\
&K' = f^K\left(\epsilon, K, H\right) \\
&H' = f^H\left(\epsilon, K, H\right) \\
&R^b = f^{R^b}\left(\epsilon, K, H\right) \\
&c \geq 0; \quad b \geq \underline{b}; \quad s \geq \underline{s}; \quad b + s \geq \underline{\omega}
\end{aligned}$$

This problem differs from the original one in the fact that R^b is a state variable for today, although tomorrow's R^b is perceived to follow the forecasting rule f^{R^b}. I.e., tomorrow's value function is given by problem (12). In this manner one can find an R^b that exactly clears the bond market. The solution to this problem delivers the policy functions $g^c\left(\omega, h, \epsilon, K, H, R^b\right)$, $g^b\left(\omega, h, \epsilon, K, H, R^b\right)$ and $g^s\left(\omega, h, \epsilon, K, H, R^b\right)$. At this stage I can state the algorithm as follows

1. Guess an initial f^0.
2. Solve the household' problem given by the system (12).
3. Simulate the economy,
   (a) set an initial distribution of agents over ω, h and ϵ,
   (b) Look for the R^b that clears the market for bonds. To do so, guess an initial $R^{b,0}$ and solve the problem (15) to find $g^c\left(\omega, h, \epsilon, K, H, R^{b,0}\right)$, $g^b\left(\omega, h, \epsilon, K, H, R^{b,0}\right)$ and $g^s\left(\omega, h, \epsilon, K, H, R^{b,0}\right)$.[14] If there is an excess of lending in the bond market

[14] Solving the problem (15) is quite straightforward since the function v has already been obtained in step 2.

try $R^{b,1} < R^{b,0}$, if there is an excess of borrowing try $R^{b,1} > R^{b,0}$. Go on until finding an $R^{b,*}$ that clears the market,[15,16]

(c) get the next period distribution over ω, h and ϵ by use of $g^c\left(\omega, h, \epsilon, K, H, R^{b,*}\right)$, $g^b\left(\omega, h, \epsilon, K, H, R^{b,*}\right)$ and $g^s\left(\omega, h, \epsilon, K, H, R^{b,*}\right)$ and the law of motion for the shock ϵ,

(d) come back to step (b). Do it for a large number of periods.

4. Drop a number of observations from the beginning such that the remaining time series is clean from the initial conditions. Use the simulated series for K, H and $R^{b,*}$ to estimate new forecasting rules f^1.

5. Compare f^1 and f^0. If they are similar we are done, if not start again by setting $f^0 = f^1$ and going back to point 2.

There is just one last issue to be clarified. Once we have agreed to use only first moments, which is the proper class $\mathcal{F}$ where to define our forecasting rules? In a problem without habit formation Krusell and Smith (1997) show that a log-linear specification on the first moment of the wealth distribution does a good job. I therefore set up the following functional forms for the forecasting rules:

$$\begin{aligned}
\log K' &= \nu_{K0}(z) + \nu_{K1}(z)\log K + \nu_{K2}(z)\log H \\
\log H' &= \nu_{H0}(z) + \nu_{H1}(z)\log K + \nu_{H2}(z)\log H \\
R^b &= \nu_{R0}(z) + \nu_{R1}(z)\log K + \nu_{R2}(z)\log H + \nu_{R3}(z)(\log K)^2 + \nu_{R4}(z)(\log H)^2 + \\
&\quad \nu_{R5}(z)(\log K)(\log H)
\end{aligned}$$

where notice that the coefficients depend on the aggregate shock and therefore estimation of these forecasting rules implies running two different regressions for each. However, as shown in the appendix B.2, my findings are that we do not need so much information. Aggregate habits do not substantially improve the forecasts. This actually means that the aggregate habit stock turns out not to be a state variable of the system. The forecasting rules end up being:

$$\begin{aligned}
\log K' &= \nu_{K0}(z) + \nu_{K1}(z)\log K \\
R^b &= \nu_{R0}(z) + \nu_{R1}(z)\log K + \nu_{R2}(z)(\log K)^2
\end{aligned}$$

and one can drop H from all the formulations of the problem.

B Accuracy of solutions

There are two levels of accuracy in the solutions to the model that we may be worried about. First, for a given vector of forecasting rules f, we may wonder how accurate are

[15] Or until $R^{b,1} \simeq R^{b,0}$

[16] An alternative approach would be to solve the problem generally for a grid of different values of R^b and then interpolate the different guesses $R^{b,0}$, $R^{b,1}$, ... until market clears. The problem with this is its inexactitude. We would need an extremely fine grid on R^b to make the results along different periods of the simulation consistent among them.

the solutions to the household problem. Second, we may want to know how good are the forecasting rules f compared to what the economy actually delivers, that is to say, how well are the households doing in terms of predicting prices. In section B.1 I present accuracy measure for the solutions to the household problem and in section B.2 I discuss the forecasting rules used and their actual predictive power.

B.1 The household problem

The solution method explained in the previous section is based on solving the two Euler equations *exactly* at the given grid points. However, the supports for our state variables (with the exception of the shocks) are continuous. We may be interested in knowing how accurate is the solution outside the grid points; in other words, we may be interested in knowing how far from a zero of the Euler equations we are at any point of the state space.

To convey a meaningful measure of distance from a zero in the Euler equations, I follow Judd (1992) in reporting the relative consumption error. To compute this measure, we need to take the policy functions $\{g^c, g^b, g^s\}$ at any given point of the state space and check which relative change in consumption would yield equality in the Euler equation. If that number is for example 0.01 it means that for every dollar spent in consumption the household makes an error of one cent. I report the base 10 log of this measure. Therefore, if the error measure is -2 we say that the household makes one dollar mistake for every 100 dollars spent or one every 1000 if the error measure is -3.

I report these relative errors for both Euler equations only for those points in the state space where the Euler equations are solved with interior solution. Figures 3 and 4 plot the relative errors for the economy *HA* and figures 5 and 6 for the economy *HAH*. Since we cannot go beyond 3D graphics, I have fixed an arbitrary value for the aggregate state (z, K and H) and plotted the errors for the two equation at all possible points in the wealth, habits and idiosyncratic shock dimensions.

While useful, this graphical information is overwhelming. As a summary of the accuracy of the solutions I have taken the average of these errors over the equilibrium distribution of households in a given period and then taken the base 10 log. For the economy *HA* the average of errors is -3.37 for the bonds equation and -3.38 for the equation for capital (with the maximum errors equal to -2.52 and -2.84 respectively). In the *HAH* economy the equivalent figures are -3.08 and -3.12 (with the maximum errors equal to -2.55 and -2.37 respectively). This means that the error, on average, is less than one dollar for every one thousand dollars spent in consumption.[17]

B.2 The forecasting rules

As discussed in the section A.2, we can try to predict the evolution of the economy by use of a log-linear function of the first moments of the distribution μ. For the non-habits

[17] This errors are computed with the forecasted value of next period aggregate capital. In any case, as shown in section B.2, the actual and predicted values for aggregate capital are almost identical. Indeed, the largest relative discrepancy between the actual and predicted values is 0.067% in the non habits economy and 0.088% in the habits economy.

Figure 3: **Errors in the Euler Equation for bonds. Economy *HA***

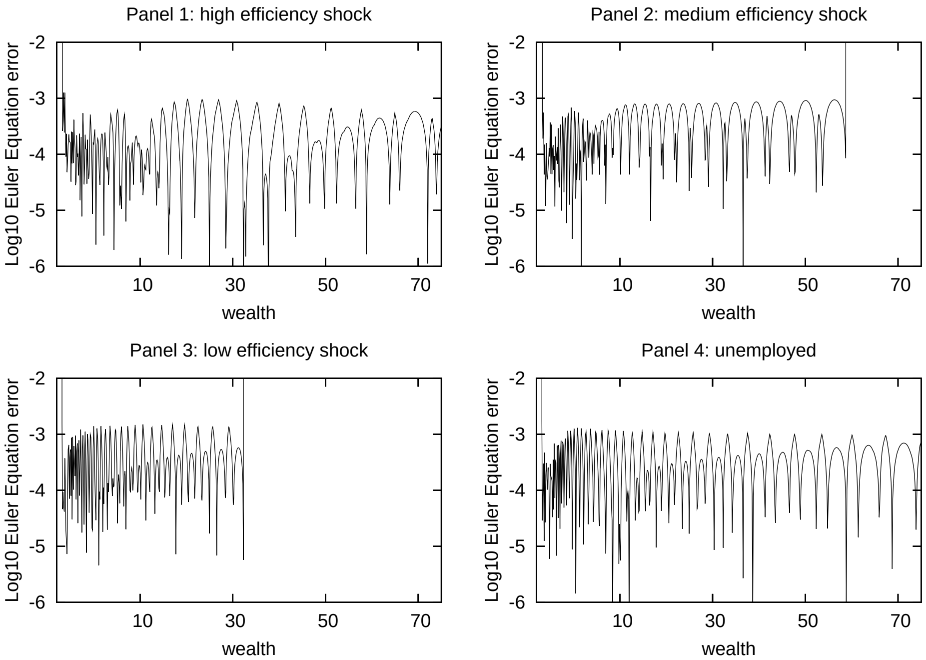


Figure 4: **Errors in the Euler Equation for capital. Economy *HA***

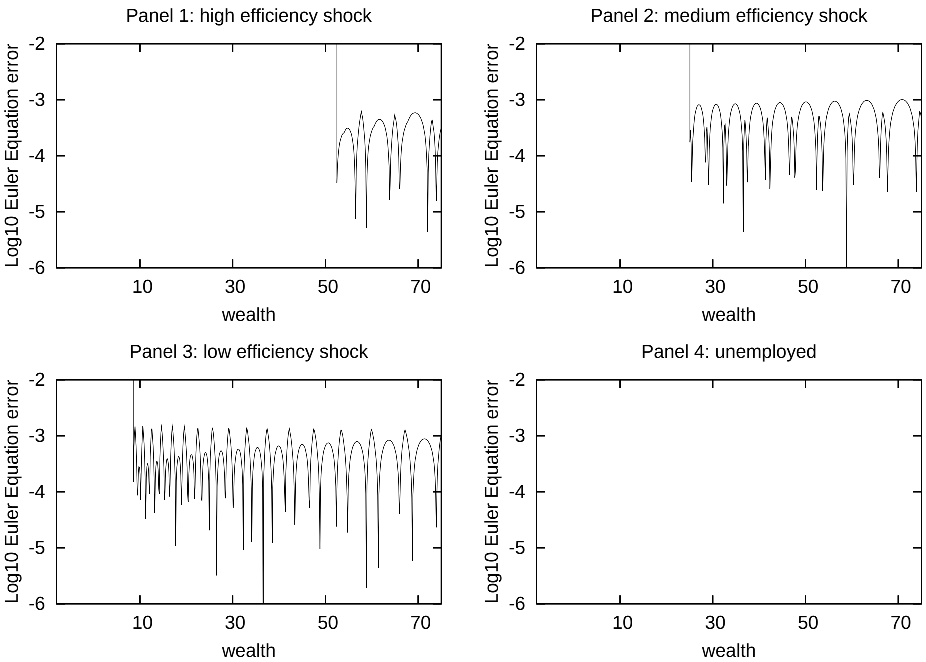


Figure 5: **Errors in the Euler Equation for bonds. Economy *HAH***

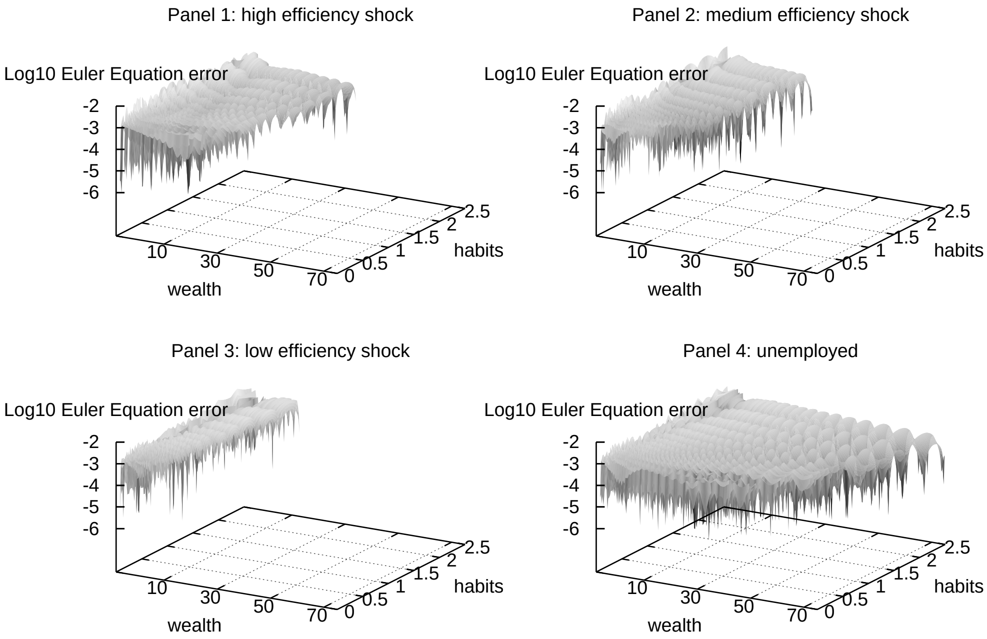
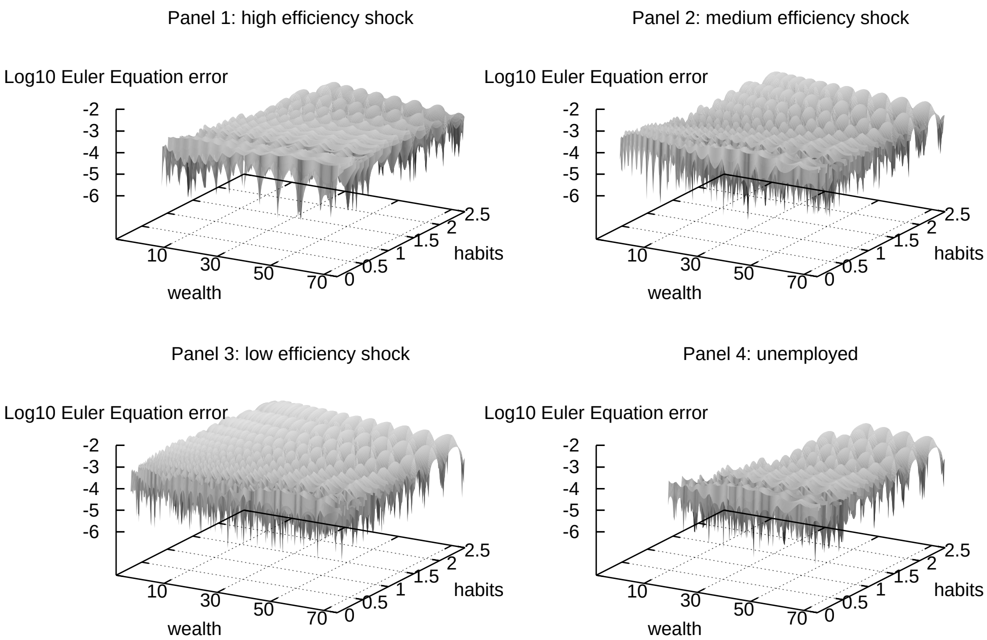


Figure 6: **Errors in the Euler Equation for capital. Economy *HAH***



economy HA the estimated forecasting rules are:

$$\begin{array}{llcll}
\text{if } z = z_g & \log K' & = & 0.083 + 0.973 \log K & \text{with } R^2 = 0.999987 \\
 & R^b & = & 1.105 - 0.052 \log K + 0.006 \left(\log K\right)^2 & \text{with } R^2 = 0.999999
\end{array}$$

$$\begin{array}{llcll}
\text{if } z = z_b & \log K' & = & 0.058 + 0.978 \log K & \text{with } R^2 = 0.999994 \\
 & R^b & = & 1.094 - 0.048 \log K + 0.005 \left(\log K\right)^2 & \text{with } R^2 = 0.999999
\end{array}$$

Notice that the R^2 are very big, larger than 0.9999 in all cases, which tells us that almost all the variation in the time series of $\log K$ and R^b is well predicted by these forecasting rules.

What about the economies with habits? As already anticipated in previous sections, one important finding of this paper is that the first moment of the marginal distribution of agents over habits does not bring any valuable information in predicting tomorrow's state once we are already considering the marginal distribution of assets (or its first moment). For this reason, I have solved the habits economies of this paper with only aggregate capital K as endogenous aggregate state variable. The drop of aggregate habits H does not suppose any change in the results and it dramatically speeds up the computations. Nevertheless, to convince the reader I also present some results for the HAH economy solved with forecasting rules that include the aggregate habit stock.

The estimated forecasting rules for the HAH economy when the aggregate habit stock is included are as follow:

$$\begin{array}{llcll}
\text{if } z = z_g & \log K' & = & +0.035 + 0.988 \log K - 0.028 \log H & \text{with } R^2 = 0.999986 \\
 & \log H' & = & -0.149 + 0.048 \log K + 0.907 \log H & \text{with } R^2 = 0.999992 \\
 & R^b & = & 1.106 - 0.052 \log K + 0.006 \left(\log K\right)^2 & \\
 & & - & 0.001 \log H - 0.001 \left(\log H\right)^2 & \text{with } R^2 = 0.999999
\end{array}$$

$$\begin{array}{llcll}
\text{if } z = z_b & \log K' & = & +0.010 + 0.993 \log K - 0.027 \log H & \text{with } R^2 = 0.999994 \\
 & \log H' & = & -0.161 + 0.051 \log K + 0.903 \log H & \text{with } R^2 = 0.999994 \\
 & R^b & = & 1.095 - 0.048 \log K + 0.005 \left(\log K\right)^2 & \\
 & & + & 0.001 \log H - 0.000 \left(\log H\right)^2 & \text{with } R^2 = 0.999999
\end{array}$$

If we drop the habit stock from the forecasting rules, we obtain a set of forecasting rules very similar to the ones for the HA economy:

$$\begin{array}{llcll}
\text{if } z = z_g & \log K' & = & 0.083 + 0.973 \log K & \text{with } R^2 = 0.999971 \\
 & R^b & = & 1.105 - 0.052 \log K + 0.006 \left(\log K\right)^2 & \text{with } R^2 = 0.999999
\end{array}$$

$$\begin{array}{llcll}
\text{if } z = z_b & \log K' & = & 0.056 + 0.978 \log K & \text{with } R^2 = 0.999977 \\
 & R^b & = & 1.094 - 0.048 \log K + 0.005 \left(\log K\right)^2 & \text{with } R^2 = 0.999999
\end{array}$$

Notice that the loss of predictive power as measured by the R^2 is virtually unnoticeable. Regarding the accuracy measures for the euler equations, the average errors in the euler equations are -3.07 and -3.11 which are almost identical to the ones obtained with the basic HAH economy.

Finally, it needs to be shown that the economy HAH, if solved with the aggregate habit stock as state variable, does not change in any perceptible way from the version without H, which is the one reported in the tables within the main part of the paper. Solving the *HAH* economy with *H* for the same parameter values as the *HAH* in the main part of the paper delivers a capital to output ratio of 12.56, a standard deviation of aggregate consumption growth of 0.26 and a Sharpe ratio of 0.017. All these three statistics are identical to the ones obtained in the *HAH* economy solved without aggregate habit as a state variable.