PATENT LICENSING REVISITED: HETEROGENEOUS FIRMS AND PRODUCT DIFFERENTIATION

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Abstract

We analyze the optimal licensing contract that a patentee provides for a cost reducing innovation to a set of firms competing in a downstream market. We study two alternative licensing regimes: (i) a combination of royalties on sales and flat fees and (ii) fixed fees only. The first contract depends on the degree of competition in the final good market: when competition is stronger, the patentee demands a higher royalty. We also show that, contrary to the literature, using fixed fees only is not optimal for the patentee when firms are heterogeneous or when there is product differentiation. The reason is that royalties allow the patent holder to soften competition in the final good market. Finally we show that even though a combination of royalties and fees allows for improved access to the technology, social welfare is lower compared with using fixed fees only.

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1 Introduction

Obtaining a patent for an innovation is a crucial step in many development processes. A patent allows its owner to exclude other firms from producing a similar good or using a related process. Moreover, a patent allows its owner to license the new technology to other firms. Without the protection of a patent, a competing firm could expropriate the knowledge embedded in the innovation.

Moreover, with the protection of a patent, licensing represents an important share of the innovator's return on the investment. Licensing arises in several circumstances. Often, in markets where inventors are not active competitors, the new technologies have, as side-products, uses that may differ from the original application. In other cases, inventors might be financially constrained and unable to undertake the necessary investment to market the results of their research. In these two situations, licensing the right to use the technology to other firms seems to be the natural alternative. Finally, litigation processes often result in the licensing of the patents under dispute.[1]

In this paper we study the optimal licensing agreement between a patent holder and firms that have potential marketable uses for that patent. We consider the case in which several of these firms are competitors in a downstream market, where they sell differentiated products and the patent holder does not compete in the final good market. We also assume that potential licensees have private information about the usefulness of the new technology, which is represented as a cost-reducing innovation.

A feature of this setup is that for a particular firm, the willingness to pay for the new technology depends on how many licenses are allocated. In particular, if firms are paying a flat fee, more licenses imply that more firms have lower costs; therefore, competition becomes fiercer, reducing profits for all firms. As a result, firms are willing to pay less for the license. This mechanism results in a downward sloping

[1] This is the focus of Llobet (2001). In that paper, the patent holder decides to license the innovation to a potential infringer to avoid expensive litigation. One of the most interesting results is that the patent holder does not necessarily benefit from having more protection against future infringement. The reason is that more protection deters the arrival of future improvements on the invention and potential licensees.

demand for licenses.

In this setup with heterogeneous firms, we show that the patent holder can, in general, do better than contracting with only a flat fee or only a royalty on sales. In particular, the patent holder will offer a menu of a combination of royalties and fees to all firms, and the firms will self-select in such a way that those that benefit more from the patent will choose a lower royalty and a higher flat fee. We also show that this menu of royalties and fees serves two purposes. First, it allows the patent holder to discriminate among different firms and extract a larger surplus from them, along the lines of the price discrimination literature. Second, and more important, the use of this royalty is a way of inducing higher prices in the final good market. That is, a royalty on sales decreases the marginal revenue that firms obtain, and for this reason they decide to sell fewer units at a higher price. Higher profits imply a higher willingness to pay for the license, allowing larger license payments. These two forces that induce higher prices in the final good market are limited by the effect denoted in the vertical relationships literature as *double-marginalization*. When the patent holder offers a higher royalty, the increase in profits that the license generates is lower, and so is the willingness to pay to obtain it.

The framework we propose allows us to examine the optimal contract for markets with different degrees of competition in the production of the final good. Of the two effects described before, we show that the price discrimination effect is independent of the differentiation in the final good that firms produce. The second effect, which we interpret as the collusion that the patent holder induces among licensees, is more important as the market becomes more competitive. That is, the patent holder has the ability to increase royalties as a way to soften competition, even though the double-marginalization distortion becomes more important.

By the same token, the number of licenses granted raises as the degree of competition increases. The reason is that, in more homogeneous industries, the profits of a licensee are influenced to a greater extent by the pricing of the competitors. More licenses allow the patent holder to reduce the output of those firms and so increase profits to the rest of the licensees. As a result, the willingness to pay for the patent rises for all licensees.

Our model therefore predicts that royalties will be more important for the patent holder in markets where firms are more heterogeneous (because they allow discrimination of licensees) or markets where competition among potential licensees is stronger (because the patent holder can use royalties to induce collusion). In markets where competition among firms is weaker, royalties should be lower and be accompanied by a higher flat fee, causing fewer distortions.

Despite the relevance of patent licensing there are limited data on these contracts. A few empirical studies reach the conclusion that no general pattern seems to arise; depending on the environment, royalties, flat fees, or combinations of both are used.[2] Because of this disparity in contracts, there is an extensive literature that provides conditions under which each type of licensing contract is optimal for the patent holder.[3] The paper closest to ours is Kamien and Tauman (1986). They assume that there is one patent holder and a number of potential licensees, ex ante identical. These licensees produce a homogeneous good, and, as in our paper, they compete in the final good market. They consider two kinds of competition—Bertrand and Cournot—and two contracts—fees and royalties. They conclude that fees are better than royalties both from the point of view of the patent holder and for society as a whole. In the case of Bertrand competition, the optimal royalty is offered to all firms and it is set to compensate for the decrease in marginal cost, making firms indifferent between obtaining the license or not.[4] Therefore, the equilibrium price remains unchanged, in spite of all firms having the new technology. When using flat fees the patent holder offers a limited number of licenses. Because flat fees do not distort the incentives of firms, the equilibrium price is lower in the case of both Bertrand and Cournot competition. As a result, consumer surplus increases.

Our paper departs from previous studies in several dimensions. We consider firms to be heterogeneous in their uses for the patent, which implies that not all the active

[2] Kamien(1992) cites Rostoker (1984) where he finds that royalties plus fees are used in 46% of the cases, royalties alone in 36%, and fixed fees in another 13%.

[3] Kamien (1992) presents a survey of these papers.

[4] These results apply when the innovation is not *drastic*—that is, when the monopoly price with the new technology is above the original marginal cost, arguably the most usual situation.

firms in the market will obtain a license. Moreover, the decrease in marginal cost that the use of the patent can achieve is private information, which provides some scope for the inventor to offer combinations of royalties and fees to discriminate among different uses of the patent. For this reason, a combination of royalties and fees makes the improvement available to more firms than would be achieved by using a single flat fee. Nevertheless, with royalties, the associated distortions in the final good market resulting from double marginalization often reduce social welfare in cases where only a flat fee is used. This result is the opposite of that obtained in the price discrimination literature, where a monopolist who distinguishes among different consumers with a menu of tariffs can improve social welfare beyond that using a uniform price.

The structure of the paper is as follows. In section 2 we introduce the model and we describe its equilibrium as a function of the number of licenses and the royalties charged. In section 3 we describe the contracts that are implementable and the one that maximizes profits for the patent holder. In section 4 we introduce the optimal flat fee in our context, and section 5 compares the features of both contracts. Section 6 discusses some empirical implications of the model, and section 7 concludes.

2 The Model

The model considers a market with three types of agents: consumers, producers of a variety of final goods, and a patent holder of a cost-reducing innovation. The structure of the final good market corresponds to the *monopolistic competition* framework introduced by Dixit and Stiglitz (1977). There is a continuum of goods along the unit line, indexed by a parameter x, and a representative consumer that chooses how much of each product to consume, according to the utility function

$$u(y) = \left(\int_0^1 y(x)^{\lambda} \, dx \right)^{\frac{1}{\lambda}},$$

where $y(x)$ is the quantity purchased of product x and $\lambda \in [0,1]$ measures how differentiated are the final products. That is, if $\lambda = 1$ products are perfect substitutes, and if $\lambda = 0$ the utility function becomes Cobb-Douglas.[5] The consumer has total income normalized to 1, so that if $p(x)$ is the price of good x, the budget constraint can be written as

$$\int_0^1 p(x)\, y(x)\, dx \leq 1.$$

The utility maximization problem is solved by equating the marginal rate of substitution between any two goods to the ratio of prices; this describes the demand of each good x, $y(x)$, as a function of the demand of any other good x':

$$\frac{p(x)}{p(x')} = \left(\frac{y(x)}{y(x')}\right)^{\lambda-1}. \tag{1}$$

This means that in any equilibrium and for all $x \in [0,1]$ it must be that $p(x) = \kappa y(x)^{\lambda-1}$, where κ is a quantity determined in equilibrium. Substituting $y(x) = \left[\frac{p(x)}{p(x')}\right]^{\frac{1}{\lambda-1}} y(x')$ into the budget constraint and letting

$$P = \left(\int_0^1 p(x)^{-\frac{\lambda}{1-\lambda}} dx\right)^{-\frac{1-\lambda}{\lambda}}$$

denote the *price index*, the consumer's demand for product x is given by

$$y(x) = \left(\frac{P^{\lambda}}{p(x)}\right)^{\frac{1}{1-\lambda}}.$$

Therefore, κ is obtained as

$$\kappa = \frac{p(x)}{y(x)^{\lambda-1}} = P^{\lambda}. \tag{2}$$

Similarly, we denote the *quantity index* as Y, and it is defined by

$$Y = \left(\int_0^1 y(x)^{\lambda}\, dx\right)^{\frac{1}{\lambda}},$$

which implies that for any x,

$$\frac{y(x)^{\lambda-1}}{p(x)} = \frac{Y^{\lambda-1}}{P},$$

so that the demand of each good depends only on its own price and on aggregate prices and quantities.

[5] The model can accomodate a more general specification that includes an outside good, with similar results and a substantially higher level of complexity.

2.1 The Patent Holder and the Final Good Producers

The patent holder has an invention that is protected under a patent. This invention can be used by the continuum of firms in a variety of ways. For simplicity, we assume the patent holder does not compete in the downstream market.[6] The reason is that we focus on the case in which the patent holder lacks the resources to market the final good and licensing is used instead.

A potential licensee producing a downstream good uses an initial technology with a constant marginal cost of c. The patented process allows firms to reduce their marginal cost. However, the magnitude of the reduction is heterogeneous and is private information of the firms. In particular, the parameter x, uniformly distributed in the $[0, 1]$ interval, measures how much the firm is attached to the new technology. We assume that x is private information, so that when negotiating a license the patent holder does not observe the valuation of each prospective patent holder. If the quality of the patented improvement is denoted by θ, a firm x will obtain a reduction in costs of $(1 - x)\theta$:

$$c(x) = \begin{cases} c - (1 - x)\theta & \text{if the patent is licensed} \\ c & \text{otherwise.} \end{cases}$$

Obviously, it has to be that $\theta \leq c$ so that firms operate with non-negative marginal costs. Notice that the firms that benefit the most from the patent are those with the lowest x, while firms with an x close to 1 get only a small decrease in cost.

The demand for the product of a downstream firm is given by equation (2) as $p(x) = \kappa y(x)^{\lambda - 1}$. This demand has the usual properties for $0 < \lambda < 1$. Firms are atomistic and hence, cannot affect the variable κ which depends on the aggregate price index and the size of the market, given by total consumer's income, which has been normalized to 1.

We consider contracts in which the patent holder keeps a percentage α of gross revenues, as a royalty, and charges a flat fee T. In the negotiation the patent holder has all the bargaining power. This seems a reasonable assumption, since firms are

[6] This topic is addressed by Arora and Fosfuri (1998). They provide conditions under which the patent holder sells licenses to some of the competitors in the same market.

atomistic and compete with each other. The profit function of the firm, if a licensing agreement (α, T) is achieved, corresponds to

$$\Pi(x) = \max_{y(x)} y(x)\left((1-\alpha)\,p(x) - c(x)\right) - T,$$

where $p(x) = \kappa y(x)^{\lambda-1}$. In other words, the firm maximizes profits $y(x)\left(p(x) - c(x)\right)$ net of licensing costs, $\alpha p(x)\,y(x) + T$. The optimal choice of $y(x)$ satisfies

$$y(x) = \left(\frac{c(x)}{(1-\alpha)\,\lambda\kappa}\right)^{\frac{1}{\lambda-1}}, \tag{3}$$

and thus the corresponding price of good x is

$$p(x) = \frac{c(x)}{(1-\alpha)\,\lambda}. \tag{4}$$

This expression illustrates the *double-marginalization* that the use of royalties adds to the typical mark-up over marginal cost (since $\alpha > 0$). A higher royalty rate, α, reduces the marginal revenue of the producer, cutting down on quantity and raising prices. Notice that when $\alpha = 0$ and $\lambda = 1$, so that products are perfect substitutes, the price equals marginal cost.

Replacing $y(x)$ and $p(x)$ in the profit function, we obtain

$$\Pi(x) = \left(\frac{1-\lambda}{\lambda}\right) c(x)^{-\frac{\lambda}{1-\lambda}}\left((1-\alpha)\,\lambda\kappa\right)^{\frac{1}{1-\lambda}} - T. \tag{5}$$

It is easy to verify that the profit function, $\Pi(x)$, has the usual properties. That is, profits are decreasing in c and α. Moreover, for a given contract (α, T) and because $c(x)$ is increasing in x, profits decrease with x.

On the other hand, if no license is purchased the price corresponds to $p(x) = \frac{c}{\lambda}$ and profits are

$$\Pi = \left(\frac{1-\lambda}{\lambda}\right) c^{-\frac{\lambda}{1-\lambda}}\left(\lambda\kappa\right)^{\frac{1}{1-\lambda}}.$$

These profits are independent of x. As a result we obtain that if a firm with $x = \widetilde{x}$ is indifferent between buying a license or not, firms with $x < \widetilde{x}$ will benefit from the license, while firms with $x > \widetilde{x}$ will opt out and use the old technology. Hence, the number of licenses sold will correspond to $\widetilde{x}$, defined as

$$\Pi(\widetilde{x}) = \Pi$$

if $\widetilde{x} < 1$. When $\Pi(1) > \Pi$, $\widetilde{x} = 1$. Therefore, using the expression for $\Pi(x)$, we obtain that

$$\widetilde{x} = \min\left\{1 - \frac{1}{\theta}\left[c - \left(\frac{\lambda}{1-\lambda}T + c^{-\frac{\lambda}{1-\lambda}}(\lambda\kappa)^{\frac{1}{\lambda}}\right)\right], 1\right\}. \tag{6}$$

In the next section we study the kind of contracts that allow self-selection of these firms and, among them, characterize the optimal contract for the patent holder.

3 The Optimal Contract

The patent holder decides on the royalty and the fee that potential licensees will pay. Because intermediate firms have different uses for the patent, it is natural to think that the patent holder might be interested in discriminating among them, by providing menus of royalties and fees that induce self-selection of firms.

This option represents an important difference with the literature surveyed by Kamien (1992) that assumes that all firms are identical. By allowing heterogeneity on the uses of the patent we provide additional reasons for the existence of royalties beyond the typical risk-sharing attributes between the creator of a technology and the firms finally using it.[7] Moreover, because we assume that each firm has private information about their valuation for the new technology, the patent holder cannot perfectly discriminate among them. However, even in this case, the patent holder can in general do better than offering a single royalty and fee, (α, T), by offering a profile of contracts, where each contract is geared toward a particular firm. The constraint on these contracts is that they have to induce firms to choose the specific contract designed for them. Using the revelation principle, we only need to focus on *direct revelation* mechanisms $\{\alpha(x), T(x)\}$, where each option of the menu is intended for a particular firm x. For a menu to be incentive compatible—or to induce self-selection—declaring their own type has to be a dominant strategy for each firm. In other words, a menu of contracts $\{\alpha(x), T(x)\}$ is incentive-compatible if for all firms x,

[7] See Bousquet et al. (1998) for a study of the effect of risk-sharing on the optimal license.

$$x \in \arg\max_{\widehat{x}} \Upsilon\left(x, \widehat{x}\right) - T\left(\widehat{x}\right), \tag{7}$$

where

$$\Upsilon\left(x, \widehat{x}\right) = \left(\frac{1-\lambda}{\lambda}\right)\left(c - \left(1-x\right)\theta\right)^{-\frac{\lambda}{1-\lambda}}\left(\left(1-\alpha(\widehat{x})\right)\lambda\kappa\right)^{\frac{1}{1-\lambda}},$$

so that

$$\Pi(x) \equiv \Upsilon\left(x, x\right) - T\left(x\right).$$

The next lemma characterizes these contracts.

Lemma 1 *For a menu of contracts* $\{\alpha\left(x\right), T\left(x\right)\}_{x\in[0,1]}$ *to be incentive-compatible, the following must be true:*

1. $\alpha\left(x\right)$ *is increasing in* x*,*
2. $T\left(x\right)$ *corresponds to*

$$T\left(x\right) = \left(\frac{1-\lambda}{\lambda}\right)\left(c - \left(1-x\right)\theta\right)^{-\frac{\lambda}{1-\lambda}}\left(\left(1-\alpha\left(x\right)\right)\lambda\kappa\right)^{\frac{1}{1-\lambda}} + \int_{x}^{\widetilde{x}} \theta y\left(s\right) ds - \Pi - K, \tag{8}$$

where $K = 0$ *if* $\widetilde{x} < 1$ *and* $K \geq 0$ *if* $\widetilde{x} = 1$*.*

The previous lemma describes the incentive compatible choices as a function of $\widetilde{x}$. In the typical mechanism design jargon, $\widetilde{x}$ is defined using the participation constraint, as obtained in (6). Only firms with $x < \widetilde{x}$ will participate in the mechanism. In particular, if $\widetilde{x} = 1$, all firms decide to obtain a license, and any contract that provides profits of at least Π will be enough to satisfy this constraint. This is the sense of the parameter K introduced in the expression (8). If $\widetilde{x} = 1$ it can be easily shown that $\Pi\left(1\right) = \Pi + K$.

It is easy to see that $K > 0$ will never be profit maximizing, since the patent holder would be able to increase profits by choosing $K = 0$ and raising the fee T to firm x. Therefore, regardless of $\widetilde{x}$ the relevant participation constraint is equivalent to

$$\Pi\left(\widetilde{x}\right) = \Pi. \tag{9}$$

The lemma also shows that a necessary condition for incentive compatibility is that the royalty schedule, $\{\alpha(x)\}_{x=0}^{\widetilde{x}}$, is increasing in x. The intuition is that firms with lower x benefit more from the patent, reducing more their marginal cost of production. For this reason, firms with lower x will produce a higher output and hence they will be more likely to trade a lower per unit royalty for a higher flat fee. This result is a parallel of the *two-part tariff* arguments, where a monopolist selling to heterogeneous consumers decides to charge a higher flat fee and a lower per unit price to consumers that demand more units of the good sold.

The patent holder obtains from each licensee x a revenue of $\alpha(x)\,p(x)\,y(x)+T(x)$, and for this reason, the objective function will be

$$\begin{array}{ll} V = \max_{\alpha(x),\widetilde{x}} & \int_0^{\widetilde{x}} \left(\alpha(x)\,p(x)\,y(x) + T(x)\right) dx, \\ \text{s.t.} & \begin{array}{l} \alpha(x) \geq \alpha(x') \text{ if } x \geq x', \\ \text{(8) and (6).} \end{array} \end{array}$$

That is, from all firms that buy a license—those with $x \leq \widetilde{x}$—the patent holder obtains a royalty $a(x)$ on total sales $p(x)\,y(x)$ plus the associated flat fee. The constraints are a translation of *Lemma 1* to ensure that the contract is incentive-compatible.

The participation constraint (6) restricts the kind of contracts that the patent holder can offer in another important way. The contract chosen by a firm has to provide higher profits than those the firm would obtain by not buying the patent and using the existing technology instead. Notice that, in contrast with the usual mechanism design model, this outside option is *endogenous*; it depends, through the price P, on the number and characteristics of the licenses granted.

We can replace in the objective function $p(x) = \frac{c-(1-x)\theta}{(1-\alpha)\lambda}$ and T from (8). Integrating by parts, the problem can be rewritten as

$$V = \max_{\alpha(x),\widetilde{x}} \int_0^{\widetilde{x}} \left\{ y(x) \left((c-(1-x)\theta) \left(\frac{1}{(1-\alpha(x))\lambda} - 1 \right) - \theta x \right) - \Pi(\widetilde{x}) \right\} dx,$$
$$\text{s.t. } \alpha(x) \geq \alpha(x') \text{ if } x \geq x' \text{and (9).} \tag{10}$$

In this problem the patent holder chooses the optimal royalty profile, $\{\alpha(x)\}_{x=0}^{\widetilde{x}}$, and the marginal firm, $\widetilde{x}$. These two variables are enough to obtain the profile of

$T(x)$. Although this expression is similar to the typical mechanism design problem, one of its features does not allow us to use the standard technique: solving for each of the agents—licensees in this case—separately and then verifying that the optimal contract satisfies the incentive compatibility constraint. Instead, in our model, each particular $\alpha(x)$ affects the choice of the other α's through the price index. That is, a higher $\alpha(x)$ increases the price $p(x)$ that firm x charges, and therefore, when considering a subinterval $[0, \tilde{x}] \subset [0, 1]$, raises the price index P. This in turn affects the valuation for the license of all the other firms.

Nevertheless, the next proposition shows that in fact the optimal mechanism implies that firms that make a better use of the patent will choose contracts with a lower royalty and a higher flat fee, and this royalty will increase as x increases.

Proposition 2 *The optimal royalty corresponds to*

$$\alpha(x) = \gamma(x)(1 - s) + s,$$

where $\gamma(x) = \frac{\theta x}{c-(1-2x)\theta}$ *and*

1. *s is a function of the entire royalty profile $\{\alpha(x)\}_{x=0}^{\tilde{x}}$ and is independent of the particular x's,*
2. $0 \leq s \leq 1$,
3. $V = s$,
4. *$\alpha(x)$ is increasing in x.*

Therefore, the optimal royalty that a patent holder will offer to a set of firms is increasing in x, satisfying the condition for incentive compatibility. Moreover the previous proposition can be interpreted as a convex combination of two effects.

To interpret the first effect, in the next proposition we study a similar problem where the patent holder is facing a single potential licensee, who is a monopolist in the final good market, with a cost function $c(x) = c - (1 - x)\theta$, where x is private information.[8] The royalty in that case would correspond to $\gamma(x)$. In other words,

[8] In our model we cannot make the standard analogy between having a single agent with many potential types or having multiple agents with a given type each.

Proposition 3 *Suppose that a patent holder is willing to license a patent to a single firm with cost function* $c(x) = c - (1-x)\theta$ *when using the innovation. The downstream firm can be of type* x*, which is uniformly distributed between* 0 *and* 1*, and is private information. This firm is a monopolist facing an inverse demand function* $p = \kappa y^{\lambda - 1}$*. The optimal royalty and fee schedule correspond to*

$$\alpha(x) = \frac{\theta x}{c - (1-2x)\theta} = \gamma(x),$$

$$T(x) = \left(\frac{1-\lambda}{\lambda}\right)\left(\frac{c-(1-x)\theta}{(c-(1-2x)\theta)^{\frac{1}{\lambda-1}}} - c^{-\frac{\lambda}{1-\lambda}}\right)(\lambda\kappa)^{\frac{1}{1-\lambda}} + \int_x^{\widetilde{x}} \theta y(s)\, ds.$$

Hence, in the model with multiple downstream firms, the first effect, with weight $1-s$, can be interpreted as the optimal royalty if the patent holder did not take into account the effect of royalty α on the aggregate price index P and therefore on the profits of the other licensees. In other words, $\gamma(x)$ is the optimal royalty that the patent holder would set if he were facing a set of firms that were monopolists in independent markets. In this case, a higher royalty allows for a better screening of the different licensees. The limit of this effect is what the literature on vertical restrictions has denoted as double-marginalization. By charging a royalty on sales, the profit of the licensee decreases, reducing the willingness to pay for the license. The royalty rate $\gamma(x)$ takes into account the trade-off between rent-extraction from the heterogeneous licensees and the problem of double-marginalization that the royalties generate. The importance of rent extraction increases as x raises, since $\gamma'(x) > 0$. It is important to notice that these two effects are independent of λ. Only the flat fee $T(x)$ takes λ into effect, as it computes the degree of monopoly power that the downstream firm has.

The second effect, with weight s, corresponds to the interest of the patent holder in inducing collusion among licensees. The profit that each licensee obtains increases with the price that all the other firms charge through P. Therefore, the optimal policy fully internalizes this effect, and for a given number of licensees $\widetilde{x}$ it corresponds to a royalty of $\alpha(x) = 1$.

The variable, s, that determines the weights of these two effects is a complicated function of the parameters of the model and the entire royalties profile. Given our normalization of the consumer's income, s can range between 0 and 1. This variable counterbalances the double-marginalization cost with the other two effects: collusion and price discrimination. Numerical results show that s is 0 when $\lambda = 0$ and it is increasing in λ. When competition is low, the price discrimination effect is predominant, and the patent holder sets up the royalties as if firms were not competing with each other. As competition among licensees becomes stronger (and λ becomes closer to 1), the firms' price approaches c and as a result the payoff from inducing some collusion increases with respect to the cost of the double-marginalization distortion.

Notice that s can also be interpreted as the optimal royalty for the firm with $x = 0$. That is, $\alpha(0) = s$.

Another surprising interpretation of s is that it corresponds to the profits that the patent holder achieves when choosing the optimal profile of α and T. That is, $s = V$. If consumer income were not normalized to 1, that is $W \neq 1$, then $s = VW$. Moreover, because $s = 0$ when $\lambda = 0$, profits for the patent holder are 0 when final goods are not substitutes of each other and each firm faces a completely inelastic demand. The reason is that the invention does not lead to a competitive advantage nor an increase in the production of any firm. Therefore, firms are willing to pay 0 for it. Profits increase with λ as a result of the competitive advantages that it generates to licensees, and the the willingness to pay increases accordingly.

The number of licenses that the patent holder will offer can be obtained from expression (10), from the first-order condition in terms of $\widetilde{x}$ given by

$$y(\widetilde{x})\left[(c-(1-\widetilde{x})\theta)\left(\frac{1}{(1-\alpha(\widetilde{x}))\lambda}-1\right)-\theta\widetilde{x}\right]-\Pi+\frac{\partial\kappa}{\partial\widetilde{x}}\frac{sP^{-\lambda}}{(1-\lambda)}=0.$$

This condition defines the optimal number of licenses to be offered, given the participation constraint. The first term evaluates the marginal increase in licensing revenue that selling to the firm $\widetilde{x}$ generates, while the second is the minimum amount of profits that the licensee can guarantee for himself by not buying the license. These two terms are equivalent to the typical mechanism design condition. However, one additional term appears: the effect of selling one more license on the equilibrium prices that firms charge. For instance, as the literature advocates, if selling one more license had the effect of lowering P, thus enhancing competition, the effect would be as follows: the willingness to pay of prospective licensees would decrease and the optimal fee would be lower.[9] But as we can see, the sign of this effect is given by the sign of the derivative $\frac{\partial \kappa}{\partial \widetilde{x}}$, which can be positive or negative.

$$\frac{\partial \kappa}{\partial \widetilde{x}} = -\left(1-\lambda\right) P^{\frac{\lambda(2-\lambda)}{1-\lambda}} \left(p\left(\widetilde{x}\right)^{-\frac{\lambda}{1-\lambda}} - \left(\frac{c}{\lambda}\right)^{-\frac{\lambda}{1-\lambda}} \right) \lesseqgtr 0.$$

Therefore, the existent literature is consistent with $\frac{\partial \kappa}{\partial \widetilde{x}} < 0$, which would occur if $p\left(\widetilde{x}\right) < \frac{c}{\lambda}$ (that is, if obtaining a license implies a decrease in the price that the firm will charge). In this case, competition among downstream firms is increased. Notice, however, that the expression for $p\left(x\right)$ depends crucially on the per-unit royalty that the patent holder demands for the license:

$$p\left(x\right) = \frac{c - \left(1-x\right)\theta}{\left(1-\alpha\left(x\right)\right)\lambda}.$$

As it is obvious from this expression, the price must be increasing in x. In particular, if the patent holder sells enough licenses and x approaches 1, the price will be above the original price and therefore $\frac{\partial \kappa}{\partial \widetilde{x}} > 0$. As a result, and especially for high values of λ, the patent holder might be interested in selling to all firms as a way to artificially raise the price that the firms are charging. A consequence of this policy is that for some licensees (those with an x close to 1) the patent holder will have to impose a negative fee—which can be considered a *bribe*—that the firm will accept in exchange for the royalty that will have to be paid.

Next, we explore the case in which $\lambda = 1$, for which there is an explicit solution for the optimal royalty that illustrates the previous point.

3.1 A limiting case, $\lambda = 1$

The monopolistic competition model introduced in the last two sections allows us to study the Bertrand case as the limiting case when λ goes to 1. This case embeds interesting features of the model. Only the most efficient firm is producing, and this result can be achieved in several ways.

[9] Notice that changes in $\widetilde{x}$ also affect the outside option of firms, Π, since it is a function of κ.

When a firm does not obtain the license, profits correspond to

$$\Pi(p) = \begin{cases} \frac{1}{\zeta(p)}\left(1 - \frac{c}{p}\right) & \text{if } p \leq \min\left(\min_x \left(p\left(x\right)\right), 1\right), \\ 0 & \text{otherwise,} \end{cases}$$

where $\zeta(p) \in [0,1]$ is the measure of firms with a price equal to p. That is, the firms that have the lowest price share the consumer income equally, producing $\frac{1}{\zeta(p)p}$ units of the good, at a cost of c.

On the other hand, licensees have the following profit function:

$$\Pi(x) = \begin{cases} \frac{1}{\zeta(p(x))}\left(1 - \alpha(x) - \frac{c-(1-x)\theta}{p(x)}\right) - T(x) & \text{if } p(x) \leq \min\left(\min_{x'}\left(p\left(x'\right)\right), 1\right), \\ 0 & \text{otherwise.} \end{cases}$$

As before, the patent holder obtains two types of rents, the royalty on sales $\alpha(x)$ and the flat fee $T(x)$.

The first thing to notice is that, from the point of view of the consumer, all goods are perfect substitutes and therefore the socially optimal outcome involves the most efficient firm, with $x = 0$, being the only producer. Moreover, the patent holder can control who is producing the good by giving access to the technology to some firms. For example, if the patent holder only sells a license to firm $x = 0$, Bertrand competition will give rise to an equilibrium price of $p(0) = c$, which is the marginal cost of all remaining firms. The patent holder could also increase the equilibrium price above c using the argument outlined in the previous section. That is, by selling a license to all competitors and requiring a sufficiently high royalty payment, the patent holder would cause all firms to raise their price, increasing profits for the firm that sells.

It turns out that in this case both equilibria are equivalent. In the first, the patent holder will sell licenses with a flat fee $T = \frac{\theta}{c}$ and a royalty equal to $\alpha(x) = 0$. In this case, the equilibrium price will be $p(x) = c$ and only the firm located at $x = 0$ will sell.

Given $T = \frac{\theta}{c}$ and a price $p(x) = c$ for $x > 0$, notice that the profits that the firm with $x = 0$ obtains are 0. To the extent that all other firms benefit less from the invention, they will obtain negative profits if they produce (and undercut firm $x = 0$). Hence, profits for the patent holder will be $\frac{\theta}{c}$.

The alternative would be to sell a license to all firms and, according to the comments in the previous section, bribe them by imposing a negative fee T. By doing that, the patent holder can try to raise the price that other firms charge and ease competition in the market. As a result, the firm with $x = 0$ would obtain higher profits, which the patent holder could in turn appropriate.

Given a price p that the patent holder targets as the equilibrium price, the minimum fee to be paid makes any firm indifferent between obtaining the license and not producing and undercutting firm $x = 0$ to be the monopolist. That is,

$$T = 1 - \frac{c}{p}.$$

On the other hand, the firm with $x = 0$ will buy the license (and produce) if

$$1 - \alpha(0) - \frac{c - \theta}{p} \geq 0.$$

Hence, the optimal price that the patent holder will induce corresponds to

$$\begin{aligned} \max_p \; -\left(1 - \frac{c}{p}\right) + \alpha(0) &= \max_p \; -\left(1 - \frac{c}{p}\right) + \left(1 - \frac{c - \theta}{p}\right), \\ s.t.\; T &\leq 0 \end{aligned}$$

where the first term takes into account that the fee has to be paid to all firms in the market. The corner solution of this problem implies that $T = 0$ and, therefore, $p = c$. Moreover, $\alpha(0) = \frac{\theta}{c}$, which corresponds to the previous case.

Obviously, both equilibria are equivalent in terms of allocations and profits. Notice, however, that only the second one turns out to be the limit of the general case when λ goes to 1.

The results of this limiting case are equivalent to those in the literature where firms are assumed to be identical and behave as Bertrand competitors. That is, suppose that all firms obtain a decrease in marginal cost of θ, so that after purchasing the license the marginal cost of all firms is $c - \theta$. Obviously, all firms are willing to pay the same amount to be the only licensee of the technology. In particular, they will obtain profits of $\frac{\theta}{c}$ and this is the fee that the patent holder will request. This section shows that such a result does not generalize to the existence of product differentiation.

4 Flat Fees

The literature has emphasized two kinds of contracts as mechanisms to allocate licenses: flat fees and a combination of flat fees and royalties. In this section we characterize the optimal flat fee and in the next we compare numerically the properties of flat fees with respect to the optimal combination of royalties and fees.

The patent holder will choose a fee T that firms will have to pay in order to obtain a license. With only one instrument, there is no scope for discrimination among different innovators and, therefore, the fee will be unique. An obvious consequence is that the patent holder will always be worse off with respect to the case where royalties are available.[10]

For a given contract, patentees with a lower x obtain higher profits; therefore, if a firm located at x decides to purchase the license, all others with $x' < x$ will obtain it as well. Then, if not all firms buy a license, there will be a threshold denoted as $\overline{x}$ so that all firms with an $x > \overline{x}$ decide to use the old technology.[11]

The corresponding price index in this case is

$$P\left(\overline{x}\right) = \left[\int_0^{\overline{x}} \left(\frac{c-(1-x)\,\theta}{\lambda}\right)^{-\frac{\lambda}{1-\lambda}} dx + \int_{\overline{x}}^1 \left(\frac{c}{\lambda}\right)^{-\frac{\lambda}{1-\lambda}} dx\right]^{-\frac{1-\lambda}{\lambda}},$$

where the individual prices are replaced by their expressions given by $p\left(x\right) = \frac{c(x)}{\lambda}$. In particular, firms with $x > \overline{x}$ have a marginal cost of c. The expression for κ is now

$$\kappa\left(\overline{x}\right) = P\left(\overline{x}\right)^{\lambda},$$

while profits when a license is purchased are

$$\overline{\Pi}\left(x\right) = y\left(x\right)\left(p\left(x\right) - c\left(x\right)\right) = \left(c-(1-x)\,\theta\right)^{-\frac{\lambda}{1-\lambda}} \lambda^{\frac{\lambda}{1-\lambda}} P\left(\overline{x}\right)^{\frac{\lambda}{1-\lambda}} \left(1-\lambda\right).$$

If the firm does not obtain the license, profits are given by

$$\overline{\Pi} = c^{-\frac{\lambda}{1-\lambda}} \lambda^{\frac{\lambda}{1-\lambda}} P\left(\overline{x}\right)^{\frac{\lambda}{1-\lambda}} \left(1-\lambda\right).$$

[10] Except in the case where $\lambda = 1$, since in that case one of the optimal royalties is 0, with a flat fee $T = \frac{\theta}{c}$.

[11] We will denote $\overline{x}$ and $\overline{\Pi}\left(x\right)$ as the number of licenses sold and the profits of each licensee when flat fees are considered.

It is easy to see that the increase in profits from obtaining a license,

$$\Delta\pi(x) = \overline{\Pi}(x) - \overline{\Pi},$$

is decreasing in x. Because a firm x is willing to pay for the license up to the total increase in profits that it generates, the threshold will be $\overline{x} = 0$ if $\Delta\pi(0) \leq T$ and $\overline{x} = 1$ if $\Delta\pi(1) \geq T$; the threshold be defined from $\Delta\pi(\overline{x}) = T$ otherwise. In the interior solution choosing $\overline{x}$ is equivalent to choosing T.

The problem that the patent holder solves is

$$\overline{V} = \max_{\overline{x}} \ \Delta\pi(\overline{x})\,\overline{x}.$$

The optimal number of licenses will therefore be

$$\overline{x} = \underset{\hat{x}}{\operatorname{argmax}} \ \left(c(\hat{x})^{-\frac{\lambda}{1-\lambda}} - c^{-\frac{\lambda}{1-\lambda}}\right) P(\hat{x})\,\hat{x}.$$

The first-order condition of this problem can be written as

$$\begin{aligned} -\overline{x}\Bigg[&\left(c(\overline{x})^{-\frac{\lambda}{1-\lambda}} - c^{-\frac{\lambda}{1-\lambda}}\right) P(\overline{x})^{\frac{2\lambda}{1-\lambda}} \left(p(\overline{x})^{\frac{-\lambda}{1-\lambda}} - \left(\frac{c}{\lambda}\right)^{\frac{-\lambda}{1-\lambda}}\right) + \\ &+\frac{\lambda\theta}{1-\lambda} c(\overline{x})^{\frac{-1}{1-\lambda}} P(\overline{x})^{\frac{\lambda}{1-\lambda}}\Bigg] + \left(c(\overline{x})^{-\frac{\lambda}{1-\lambda}} - c^{-\frac{\lambda}{1-\lambda}}\right) P(\overline{x})^{\frac{\lambda}{1-\lambda}} = 0, \end{aligned} \tag{11}$$

where $p(\overline{x}) = \frac{c(\overline{x})}{\lambda}$. Moreover, notice that $p(\overline{x})^{\frac{-\lambda}{1-\lambda}} - \left(\frac{c}{\lambda}\right)^{\frac{-\lambda}{1-\lambda}} = \lambda^{\frac{\lambda}{1-\lambda}}\left(c(\overline{x})^{-\frac{\lambda}{1-\lambda}} - c^{-\frac{\lambda}{1-\lambda}}\right)$. Since $P(\overline{x})^{\frac{\lambda}{1-\lambda}} > 0$, this condition can be further simplified into

$$-\overline{x}\left[\frac{\lambda\theta}{1-\lambda} c(\overline{x})^{\frac{-1}{1-\lambda}} + \lambda^{\frac{\lambda}{1-\lambda}} \left(c(\overline{x})^{-\frac{\lambda}{1-\lambda}} - c^{-\frac{\lambda}{1-\lambda}}\right)^2 P(\overline{x})^{\frac{\lambda}{1-\lambda}}\right] + \left(c(\overline{x})^{-\frac{\lambda}{1-\lambda}} - c^{-\frac{\lambda}{1-\lambda}}\right) = 0.$$

It is difficult in this case to obtain analytical results for the optimal number of licensees (and therefore the optimal fee). For this reason we rely on numerical simulations that we show next.

5 Comparing Contracts

In this section we use numerical methods to establish the characteristics of both types of contracts, their effect on welfare, and private profits for patent holders. The numerical characterizations that follow correspond to the parametrization $c = 1$ and $\theta = \frac{4}{5}$.

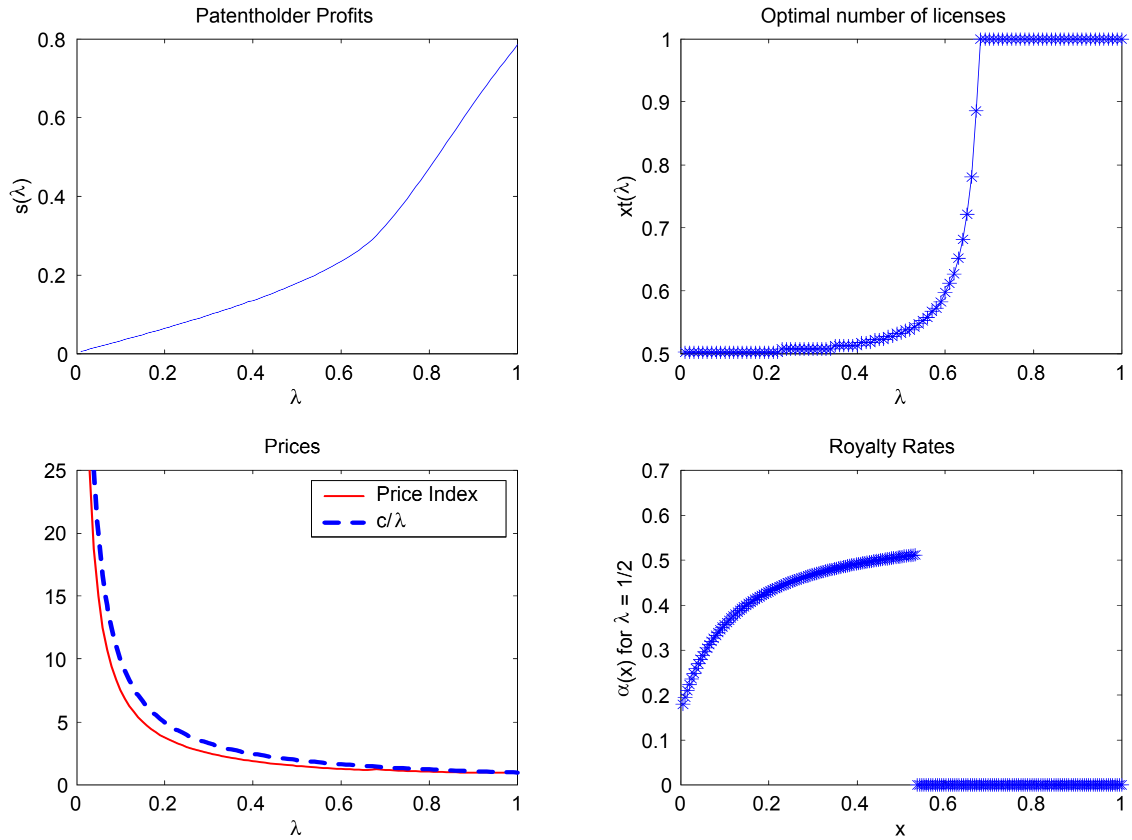

Figure 1: Royalty Rates. Parametrization: $c = 1, \theta = \frac{4}{5}$

5.1 Royalty Rates

Figure 1 illustrates the characteristics of the royalty contract. In particular, it shows that the equilibrium variable $s(\lambda)$ is increasing with the degree of competition λ and belongs to the interval $[0, 1]$ in the upper left panel. The number of licenses, $\tilde{x}$, in the upper right panel increases with the degree of competition; and the royalty rate, $a(x)$, in the lower right panel is increasing in x. Although not shown, the entire royalty profile $\{a(x)\}$ is increasing in λ.

5.2 Number of Licenses

The number of licensees granted with royalties is consistently higher than with flat fees. Figure 2 illustrates this property. This implies that more firms gain ac-

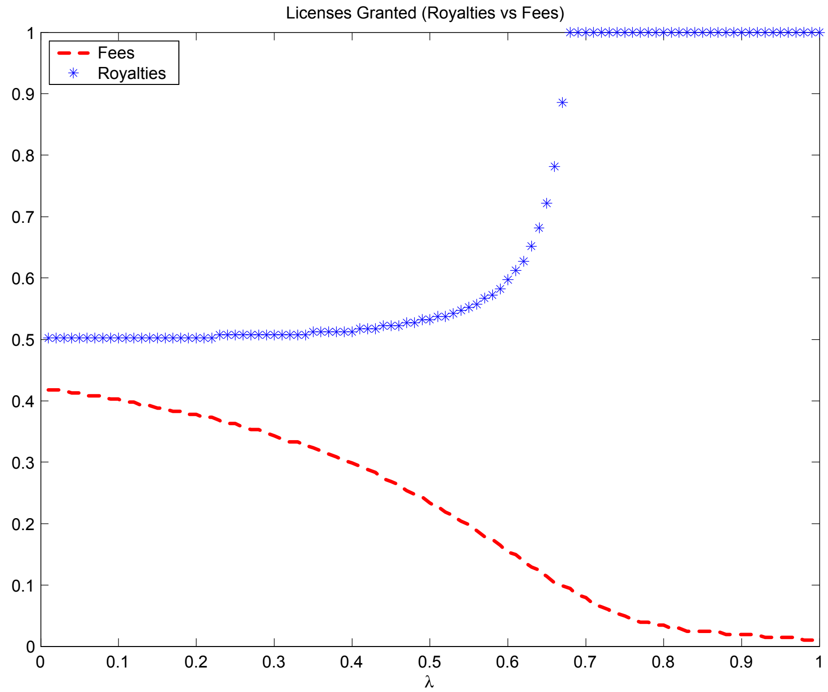

Figure 2: Number of Licenses

cess to the new technology when royalties are used. Furthermore, as the degree of competition increases (the degree of product differentiation declines), the number of licenses granted with royalties increases, while the number of licenses with flat fees declines. This is so because more competition lowers the profits of the producers when more firms have the technology. In this particular parametrization, eventually all firms gain access to the new technology with royalties, but the number declines toward zero when flat fees are used.

5.3 Equilibrium Prices

We now analyze the equilibrium prices that firms set under each type of contract. When royalties are assigned, replacing $\alpha(x)$, we obtain,

$$p(x) = \begin{cases} \frac{c-(1-2x)\theta}{\lambda(1-s)} & \text{if } x \leq \widetilde{x}, \\ \frac{c}{\lambda} & \text{if } x > \widetilde{x}, \end{cases}$$

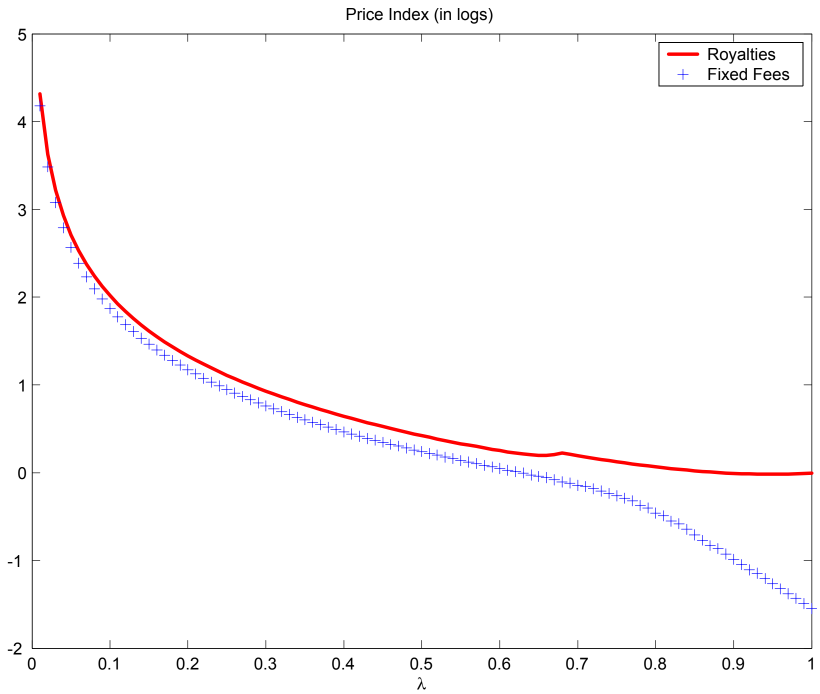

Figure 3: Price Indexes

while in the case of flat fees,

$$p(x) = \begin{cases} \frac{c-(1-x)\theta}{\lambda} & \text{if } x \leq \overline{x}, \\ \frac{c}{\lambda} & \text{if } x > \overline{x}. \end{cases}$$

The results for both contracts are quite different. For low values of x, royalties result in higher prices, due to the wedge that is generated with respect to marginal costs. In particular, since $s(\lambda)$ is increasing in λ the wedge increases as the degree of competition rises. However, for higher values of x this price differential decreases. Figure 3 illustrates this behavior. When royalties are used, the patent holder uses them to artificially raise the price of the final good, softening competition, and raising the resulting prices so that the price index is always higher when royalties are used.

5.4 Welfare

We can also estimate the effect on social welfare of each contract. As usual, the consumer surplus corresponds to the utility of the consumer net of the cost of buying

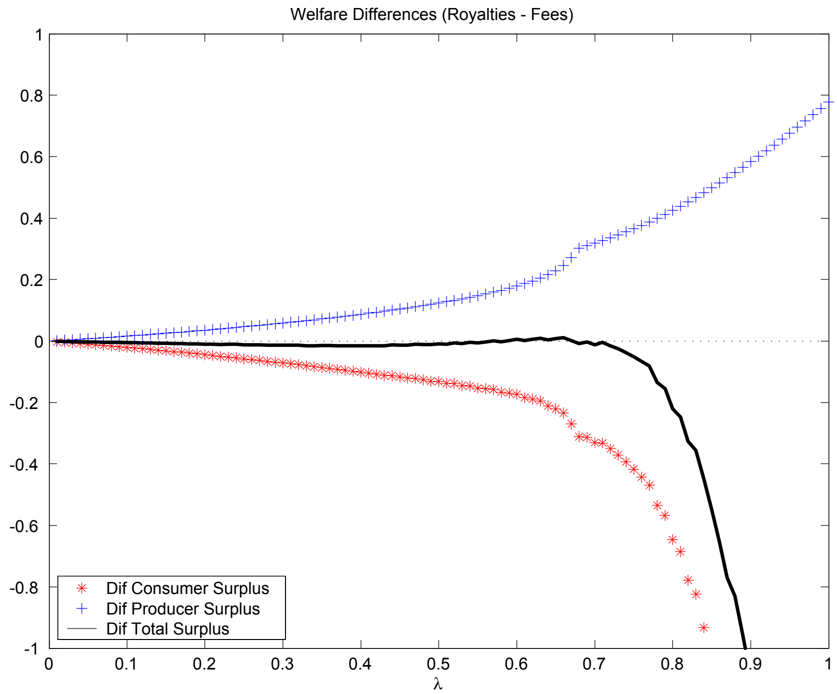

Figure 4: Welfare Comparison

the goods,

$$CS = u(y) = \left(\int_0^1 y(s)^{\lambda} ds\right)^{\frac{1}{\lambda}} - \int_0^1 p(s)\, y(s)\, ds,$$

while the producer surplus corresponds to the sum of the profits of the final good producers and the patent holder. In the case of royalties, they are, respectively,

$$\begin{aligned} FS &= \int_0^{\widetilde{x}} \left[\left(\left(1-\alpha(x)\right)p(x) - c(x)\right)y(x) - T\right] + \\ &+ \int_{\widetilde{x}}^1 y(x)\left[p(x) - c(x)\right] dx, \\ PatS &= \int_0^{\widetilde{x}} \left[\alpha(x)\, p(x)\, y(x) + T(x)\right] dx. \end{aligned}$$

With the use of flat fees the result is

$$\begin{aligned} FS &= \int_0^1 \left[\left(p(x) - c(x)\right)y(x)\right] dx - T\overline{x}, \\ PatS &= T\overline{x}. \end{aligned}$$

Therefore, the total producer surplus can be computed as

$$PS = PatS + FS = \int_0^1 \left(p(x) - c(x)\right) y(x)\, dx.$$

The results of our simulations in Figure 4 show the following: from the producer standpoint, royalties are better bacause, by inducing a higher price for the final good (and therefore a higher price index) and by making the technology available to more firms, royalties decrease the competition among final good producers. Because firms approach the monopolistic price, the distortion on the consumer side is larger, leading to a lower consumer surplus. Total welfare is higher with fees for low and high values of λ while the effect is ambiguous for intermediate values. An opposing effect is that flat fees generate a higher consumer surplus, since they do not distort the price of the final good.

6 Empirical Implications

This paper presents implications about the kind of licensing contracts that we should observe in the marketplace depending, among other things, on the heterogeneity of the uses for the final invention and the degree of market segmentation and product differentiation.

One of the most important implications of the model is that for the same degree of heterogeneity in the use of the innovation, in sectors where products are more homogeneous, royalties should be higher; that is, the royalty schedule $\{\alpha(x)\}$ is increasing in λ. At the same time, the differences in the uses of the technology make the licensees increase their production. As a result, concentration in the sector should be higher. In Figure 5 we present two alternative measures of market concentration and examine their behavior as the degree of product differentiation changes. The first measure is the Herfindahl-Hirschman index. We compute the index as the sum

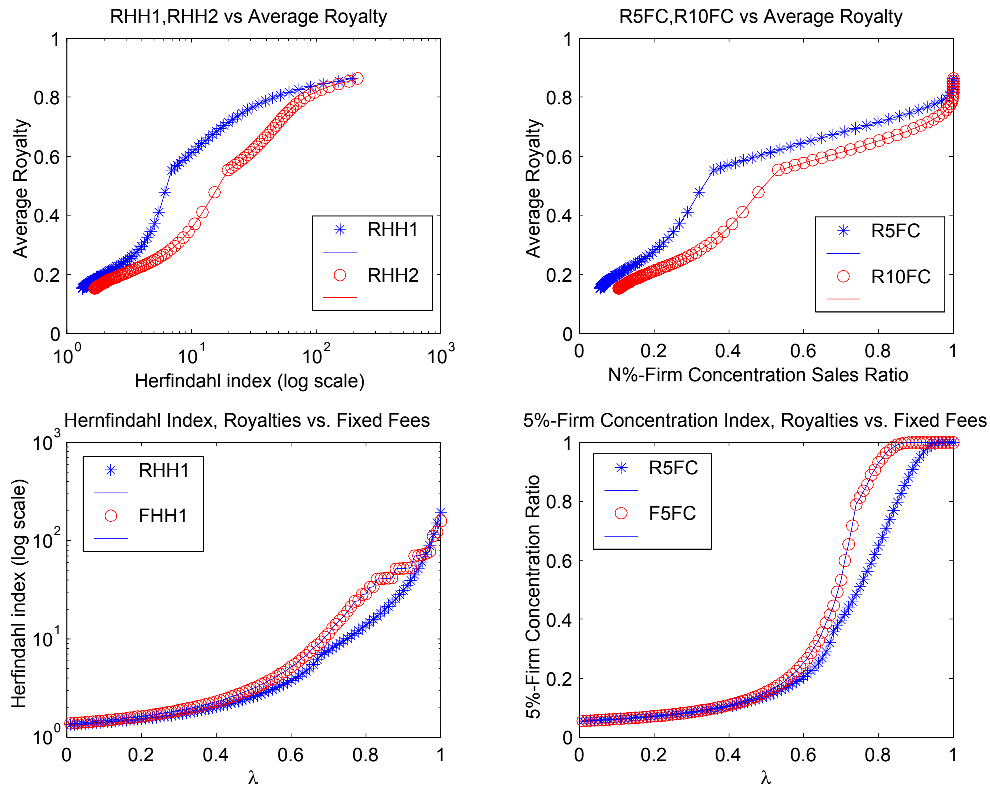

Figure 5: Market Concentration

of squared market shares in terms of output.[12]

$$HH_1 = \frac{\int_0^1 y(x)^2 dx}{\left[\int_0^1 y(x) dx\right]^2}.$$

The second measure is $N\%$-Firm Concentration Ratio, computed as the proportion of sales undertaken by the top N-percent of firms.

$$N\%FC = \frac{\int_0^{\frac{1}{n}} p(x)y(x)dx}{\int_0^1 p(x)y(x)dx}.$$

We can see that there is a positive relationship between the average royalty rate and market concentration. If we compare the degree of market concentration under

[12]An alternative calculation uses the quantity index defined in section 2,

$$HH_2 = \frac{\int_0^1 y(x)^2 dx}{\left[\int_0^1 y(x)^\lambda dx\right]^{\frac{2}{\lambda}}}.$$

the two licensing regimes, we observe that concentration is higher when the patent holder charges a flat fee to license the new technology. The intuition is that with flat fees access to the new technology is limited, as the number of licenses granted is always smaller, and therefore the dispersion among existing firms is higher. Licensed firms produce a larger percentage of total output, and, with a royalty license structure, access to the new technology is more homogeneous.

7 Concluding Remarks

This paper presents a model of patent licensing that integrates two important features largely unexplored in the literature: private information and the degree of competition in the final market. Both of them turn out to have a systematic effect in the optimal contract that the patent holder chooses. Private information gives the patent holder the opportunity to offer a menu of royalties and fees and provide a lower royalty to firms that make a better use of the technology. This scenario increases profits for the patent holder with respect to the usual case of a single flat fee studied in the literature.

The varying degree of market competition also has important implications. In particular, when competition is fiercer and firms enjoy small markups over marginal cost in their prices, the profit-maximizing contract calls for higher royalties and lower flat fees. The reason is that by charging higher royalties the marginal revenue of each firm is reduced, causing a decrease in the quantity they sell and easing the competition. In the same direction, a second remarkable effect arises. The number of licenses is higher when there is more competition, as a way of reducing the total quantity that firms produce, which benefits the patent holder. We have interpreted both effects as the ability of inducing collusion among licensees. Numerical results show, however, that welfare might be lower when a combination of royalties and fees is used rather than using flat fees because of this collusion, in spite of more firms having access to the new technology.

The setup we introduce allows us to ask more general questions. For example, it is easy to accommodate for a second patent holder producing an alternative technology that could benefit more firms with an x closer to 1.

Among the empirical implications of our model, we found that the average royalty rate declines with the degree of product differentiation and that there is a positive relation between the average royalty rate and market concentration among the downstream firms. We also found that market concentration is higher if only flat fees are used to license the new technology. The reason is that, in both cases, licensed firms produce a larger percentage of market output, whereas with royalty contracts more firms gain access to the new technology than with flat fee contracts; therefore, firms are more homogeneous in the first case.

The availability of data on royalty rates and licensing contracts in general is very limited. There are few instances where there is more availability of information on the details of licensing contracts—for example, in the case of franchise contracts and in the case of licensing contracts for international transfer of technology via foreign direct investment. We plan to address the issue of testing the empirical implications of our model in future work.

A Proofs

Proof of Lemma 1

Using the envelope theorem, we observe that profits are decreasing in x,

$$\frac{\partial \Upsilon(x,x)}{\partial x} = -\theta \left[c-(1-x)\theta\right]^{-\frac{1}{1-\lambda}} \left[(1-\alpha(x))\lambda\kappa\right]^{\frac{1}{1-\lambda}} = -\theta y(x) < 0, \tag{12}$$

and the typical sorting condition can be computed as

$$\frac{\partial^2 \Upsilon(x,x)}{\partial x \partial \alpha} = -\theta \frac{\partial y}{\partial \alpha} = \theta \frac{y(x)}{(1-\lambda)(1-\alpha)},$$

which is positive.[13]

[13]In fact, if firms were not atomistic the cross-derivative would be

$$\frac{\partial^2 \Upsilon}{\partial x \partial \alpha} = -\theta \frac{\partial y}{\partial \alpha} = \theta \frac{y(x)}{(1-\lambda)(1-\alpha)} \left[1-\lambda\left(\frac{P}{p(x)}\right)^{\frac{\lambda}{1-\lambda}}\right].$$

This last effect originates from the competition among final good producers. That is, each firm could take into account that an increase in the $\alpha(x)$ not only affects the quantity $y(x)$ and the

Integrating equation (12) into the profit function we have

$$\Pi(\widetilde{x}) - \Pi(x) = \int_{x}^{\widetilde{x}} \frac{\partial \Upsilon(x,x)}{\partial x} dx.$$

When $\widetilde{x} < 1$, $\Pi(\widetilde{x}) = \Pi$. Solving for $T(x)$ we obtain the desired result. When $\widetilde{x} = 1$ we know that $\Pi(1) \geq \Pi$ and it follows that $K \geq 0$.

Proof of Proposition 2

As usual in mechanism design, we assume that the incentive compatibility constraint is satisfied and we later verify. From the problem in (10) we can obtain the first-order condition with respect to $\alpha(x)$ as

$$\int_0^{\widetilde{x}} \left\{ \frac{\partial y(x')}{\partial \kappa} \frac{\partial \kappa}{\partial \alpha(x)} \left[(c - (1-x')\theta) \left(\frac{1}{(1-\alpha(x'))\lambda} - 1 \right) - \theta x' \right] \right.$$
$$\left. \frac{\Pi}{(1-\lambda) k} \frac{\partial \kappa}{\partial \alpha(x)} \right\} dx - \frac{y(x)}{(1-\alpha)(1-\lambda)} \left[(c - (1-x)\theta) \frac{\alpha}{1-\alpha} - \theta x \right]$$
$$= 0$$

If we call s the following function of $\widetilde{x}$,

$$s = \left(\int_0^{\widetilde{x}} \frac{y(x')}{\kappa} \left[(c - (1-x')\theta) \left(\frac{1}{(1-\alpha(x'))\lambda} - 1 \right) - \theta x' \right] dx' - \Pi \widetilde{x} \right) P^{\lambda}.$$

Given the fact that $\frac{\partial y(x)}{\partial \kappa} = \frac{y(x)}{(1-\lambda)\kappa}$ we can rewrite the first-order condition as

$$\frac{\partial \kappa}{\partial \alpha(x)} \frac{s P^{-\lambda}}{1-\lambda} - \frac{y(x)}{(1-\alpha)(1-\lambda)} \left[(c - (1-x)\theta) \frac{\alpha}{1-\alpha} - \theta x \right] = 0$$

and we can obtain the expression for $\frac{\partial \kappa}{\partial \alpha(x)}$ as

$$\frac{\partial \kappa}{\partial \alpha(x)} = P^{\lambda} \lambda \left[\int_0^1 p(x)^{-\frac{\lambda}{1-\lambda}} dx \right]^{-1} \frac{p(x)^{\frac{-\lambda}{1-\lambda}}}{1-\alpha(x)} = \kappa \lambda y(x)^{\lambda} \frac{1}{1-\alpha} P^{\lambda}.$$

Replacing in the previous equation

$$\lambda y(x)^{\lambda} \frac{1}{1-\alpha(x)} \frac{s}{1-\lambda} - \frac{y(x)}{(1-\alpha(x))(1-\lambda)} \left[(c - (1-x)\theta) \frac{\alpha(x)}{1-\alpha(x)} - \theta x \right] = 0$$

corresponding price $p(x)$ but also the aggregate price index, P. Since firms are atomistic, we follow the standard simplification in the literature (see Dixit and Stiglitz, 1993) and we do not include this effect.

and using the expression $y(x) = \left(\frac{P^{\lambda}}{p(x)}\right)^{\frac{1}{1-\lambda}}$, we can solve for $\alpha(x)$ as

$$\alpha(x) = \frac{s(c-(1-x)\theta)+\theta x}{c-(1-2x)\theta}.$$

That $s = V$ can be verified by inspection. The next lemma proves part (i) of the proposition.

Lemma 4 $0 \leq s \leq 1$

Proof. *First we show that $s > 1$ is a contradiction. Assume that this is the case. Then it has to be that*

$$\int_0^{\widetilde{x}} y(x') \left[(c-(1-x')\theta) \left(\frac{1}{(1-\alpha(x'))\lambda} - 1 \right) - \theta x' \right] dx' > 0.$$

Replacing $\alpha(x)$ by its expression we obtain

$$\int_0^{\widetilde{x}} y(x') \left[(c-(1-2x')\theta) \left(\frac{1}{(1-s)\lambda} - 1 \right)' \right] dx' =$$
$$\int_0^{\frac{1}{2}} y(x') \left[(c-(1-2x')\theta) \left(\frac{1}{(1-s)\lambda} - 1 \right)' \right] dx' +$$
$$\int_{\frac{1}{2}}^{\widetilde{x}} y(x') \left[(c-(1-2x')\theta) \left(\frac{1}{(1-s)\lambda} - 1 \right)' \right] dx' < 0$$

since $\widetilde{x} \leq 1$ and $y(x)$ is non-decreasing in x. Therefore $s \leq 1$.

Moreover, notice that because the expression of s is equivalent to V and the patent holder can make positive profits by choosing a flat fee, $s \geq 0$. ■

Proof of Proposition 3

The problem that the patent holder solves corresponds to

$$\max_{\alpha(x),\widetilde{x}} \int_0^{\widetilde{x}} \left\{ y(x) \left[(c-(1-x)\theta) \left(\frac{1}{(1-\alpha(x))\lambda} - 1 \right) - \theta x \right] - \Pi \right\} dx,$$

under the same incentive compatibility constraint described in Lemma 1. The first-order condition becomes in this case

$$\frac{y(x)}{(1-\alpha)(1-\lambda)} \left[(c-(1-x)\theta) \frac{\alpha}{1-\alpha} - \theta x \right] = 0.$$

After solving for α we obtain the desired result.

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