Capital Requirements, Market Power and Risk-Taking in Banking

Rafael Repullo

CEMFI and CEPR

CEMFI, Casado del Alisal 5, 28014 Madrid, Spain

www.cemfi.es

Abstract

This paper presents a dynamic model of imperfect competition in banking where the banks can invest in a prudent or a gambling asset. We show that if intermediation margins are small, the banks' franchise values will be small, and in the absence of regulation only a gambling equilibrium will exist. In this case, either flat-rate capital requirements or binding deposit rate ceilings can ensure the existence of a prudent equilibrium, although both have a negative impact on deposit rates. Such impact does not obtain with either risk-based capital requirements or nonbinding deposit rate ceilings, but only the former are always effective in controlling risk-shifting incentives.

Keywords: bank regulation, capital requirements, deposit rate ceilings, moral-hazard, risk-shifting, imperfect competition, franchise values.
JEL Classification: G21, G28, D43

1 Introduction

It is well-known that an increase in bank competition that erodes the present value of the banks' future rents (their franchise or charter value) reduces their incentives to behave prudently. The standard regulatory response has been to tighten capital requirements: higher capital implies higher losses for the banks' shareholders in case of default, and hence lower incentives for risk-taking. However, in a recent paper, Hellmann, Murdock and Stiglitz (2000), henceforth HMS, observe that in addition to this *capital at risk effect*, there is a *franchise value effect* that goes in the opposite direction. In particular, they claim that higher capital requirements reduce the banks' franchise values, and hence the payoffs associated with prudent investment, so their overall effect is ambiguous.

The purpose of this paper is to reexamine the relationship between capital requirements, market power, and risk-taking in banking in the context of a dynamic model of imperfect competition in the deposit market where, following HMS, the banks can invest in either a prudent or a gambling asset. The gambling asset is dominated in terms of expected return by the prudent asset, but yields a higher payoff if the gamble succeeds. Imperfect competition is introduced by borrowing from the industrial organization literature on spatial competition, in particular Salop's (1979) circular road model with uniformly distributed consumers (depositors in our case). Banks are located symmetrically around the circle, and compete by offering deposit rates. Travelling to banks is costly for depositors, which is the source of the banks' market power. Like HMS, we assume that deposits are fully insured by a government agency that can be funded by either deposit insurance premia or lump sum taxation.

In the absence of capital requirements, the characterization of the equilibrium of this model is very simple. There are two possible types of (symmetric) equilibrium: a *prudent equilibrium* in which the banks invest in the prudent asset, and a *gambling equilibrium* in which the banks invest in the gambling asset. In both equilibria, the intermediation margin is equal to the ratio between the depositors' unit transport cost and the number of banks.

We show that for low intermediation margins (i.e., very competitive markets) only the gambling equilibrium exists, for high margins (i.e., very monopolistic markets) only the gambling equilibrium exists, and for intermediate margins both types of

equilibria exist. We also show that if the cost of capital exceeds the return of the prudent asset, capital requirements are always effective in ensuring the existence of a prudent equilibrium. The reason for this result is that an increase in capital requirements reduces equilibrium deposit rates in such a way that the banks' franchise value does not change. Hence only the capital at risk effect operates, so higher capital reduces the banks' incentives to invest in the gambling asset.

As an extension of this result we examine the case where capital requirements can discriminate in favor of investment in the prudent asset. We show that risk-based capital requirements are more efficient regulatory tools, because they can ensure the existence of a prudent equilibrium at no cost in terms of bank capital.

Finally, we also analyze the effect of introducing deposit interest rate ceilings. Such regulation has been advocated by HMS as a way to boost the banks' franchise values and hence reduce their risk-taking incentives. We show that deposit rate ceilings are also effective in ensuring the existence of a prudent equilibrium, although they may require very low (even negative) interest rates. Interestingly, we show that the same problem arises with flat-rate capital requirements, but not with risk-based requirements.

As suggested by the title of the paper, our model has three main ingredients: (i) *bank regulation* (in the form of capital requirements, deposit rate ceilings, and deposit insurance), (ii) *imperfect competition* in the deposit market, and (iii) *moral hazard* in the choice of investment. Most of the literature has looked at combinations of either (i) and (ii), or (i) and (iii). The first class of papers includes Chiappori, Perez-Castrillo and Verdier (1995), who study the regulation of deposit rates in the context of a circular road model of banking competition in both the deposit and the loan market, and Matutes and Vives (1996), who discuss the effect of deposit insurance in a Hotelling model of competition in the deposit market. In the second class of papers, Furlong and Keeley (1989) show that higher capital requirements reduce risk-taking incentives in a state preference model of a bank that chooses the level of asset risk, Genotte and Pyle (1991) note that this result may not obtain in a model where the bank endogenously decides the size of its portfolio, Rochet (1992) shows that the effect of capital requirements on risk-taking is ambiguous when the bank's investment decision is taken by a risk averse owner-manager, and Besanko and Kanatas (1996)

show that if in addition to the moral hazard problem in the choice of investment there is a second moral hazard problem in the choice of monitoring effort, higher capital requirements may worsen the second problem and lead to higher risk.

The three ingredients have been considered in a static context by Keeley (1990), who introduces market power by assuming that banks can make positive net present value loans, showing that increased competition may lead to higher risk-taking. Matutes and Vives (2000) get a similar result in the context of a fully fledged model of imperfect competition in the deposit market, and support the use of deposit rate ceilings and direct asset restrictions as regulatory tools. In a dynamic context, there is the paper by Suarez (1994), who constructs a model of a single monopolistic bank that chooses in each period the volatility of its lognormally distributed asset portfolio. Using dynamic programming techniques, he endogenizes the franchise value of the bank and shows that the model has a bang-bang solution: when market power falls below a critical level, the solution jumps from minimal to maximal risk.

Our paper differs from Suarez (1994) in that we introduce a model of monopolistic competition in the deposit market, and we simplify the bank's asset choice by using the simple discrete returns setup of HMS. In addition, we assume that bank capital is costly, but otherwise the two models are very similar. On the other hand, our paper differs from HMS in the explicit modeling of competition in the deposit market, and in the way in which the cost of capital enters the value function of the banks: in our setup bank capital is inside capital provided by the existing shareholders, while they assume that it is outside capital raised in the stock market.

It is worth stressing that unlike in the current policy discussions on capital requirements, the role of capital in our model is not to cover unexpected losses but to provide incentives for prudent bank behavior.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 characterizes the equilibrium of the model when there is a minimum capital requirement. Section 4 analyzes the effects of introducing risk-based capital requirements and deposit interest rate ceilings, and Section 5 concludes. The proofs of all the results are contained in the Appendix.

2 The Model

Consider a discrete time, infinite horizon model of an economy with $n > 2$ *risk neutral banks*. Each bank $j = 1, ..., n$ receives from a *regulator* a license to operate at an initial date $t = 0$. This license is withdrawn at any date when the bank is revealed to be insolvent, that is when the value of its assets is smaller than the value of its deposit liabilities. In this case a new bank is allowed to enter the market, so the total number of banks is always n.[1]

The banks operate in a market with a continuum of measure 1 of *overlapping generations of depositors* distributed uniformly on a circumference of unit length. The n banks are located symmetrically on this circumference. Depositors live for two dates, have a unit endowment in the first date of their life, and only want to consume in the second date of their life. So they will invest their initial endowment in the only asset that is available to them, namely bank deposits. We assume that travelling to banks around the circumference has a cost of μ times the distance between the depositor and the bank.

At each date t the banks compete in this market by offering deposit rates. We will focus on symmetric equilibria in which all the banks choose the same deposit rate. Since depositors have a unit endowment and total measure 1, in equilibrium each bank will get $1/n$ *deposits* at each date. The banks can also raise *equity capital*, which has a perfectly elastic supply at an expected rate of return ρ. This can be rationalized by postulating that bank shareholders are infinitely lived agents with preferences linear in consumption with a discount rate ρ.

The funds raised by the banks can be invested in either of two assets: a *prudent asset*, yielding a return α, and a *gambling asset*, yielding a high return γ with probability $1 - \pi$, and a low return β with probability π.[2] As HMS, we assume that $\alpha > 0$ and $1 + \beta \geq 0$, and that

$$\gamma > \alpha > (1 - \pi)\gamma + \pi\beta. \tag{1}$$

This means the gambling asset is dominated in terms of expected return by the prudent asset, but yields a higher return if the gamble succeeds. We also follow HMS

---

[1] See Perotti and Suarez (2002) for an interesting model in which the number of banks goes down after a bank failure. They show that the associated future increase in the rents of the surviving banks acts as an incentive to current prudent behavior.

[2] All these returns are in net terms per unit of investment.

in assuming that

$$\rho > \alpha, \tag{2}$$

so bank capital is costly in the sense that it requires an expected return higher than the return of the prudent asset.[3]

The regulator requires the banks to hold a *minimum capital k* per unit of deposits, and fully insures their deposits.[4] For simplicity, we assume that deposit insurance premia are zero, and that deposit insurance payouts are financed by lump sum taxes on the old depositors.

The asset choice of any bank is not observed by the depositors or the regulator. However, the regulator can observe if the value of the bank's assets is smaller than the value of its deposit liabilities, in which case the bank is closed,[5] its depositors are compensated, and a new bank enters the market.

3 Characterization of Equilibrium

To analyze the equilibrium of the model it is convenient to proceed in three stages. First, we consider the simple case where the banks can only invest in the prudent asset. Second, we discuss the case where the banks can only invest in the gambling asset. Finally, we look at the general case where the banks can invest in either of the two assets. In all these cases we restrict attention to Markov strategies in which the past influences current play only through its effect on state variables.[6]

3.1 The model with the prudent asset

At each date t each bank j chooses the amount of capital k_{jt} to hold per unit of deposits and the deposit rate r_{jt} to offer, and invests all the funds raised in an asset

---

[3]There is a huge literature in corporate finance starting with Myers and Majluf (1984) that justifies this assumption in terms of asymmetric information costs.

[4]HMS (2000, p.151) argue that "the assumption of deposit insurance best reflects reality," but it also considerably simplifies the analysis. In the absence of deposit insurance, the expected return of the deposits of a bank would depend on its investment decision, so the modelling of competition in the deposit market would be more complicated. See Matutes and Vives (1996) for a model without deposit insurance where the depositors form beliefs about the probability of failure of banks.

[5]See Repullo (2000) for a model in which the regulator's incentives for closing the bank are explicitly analyzed.

[6]See Fudenberg and Tirole (1991), Chapter 13.

that yields the safe return α. To simplify the notation we will omit the subindex t and simply write the bank's decision variables as k_j and r_j. Given the existence of a capital requirement, the bank's choice of capital must satisfy the constraint $k_j \geq k$.

To obtain the symmetric Nash equilibrium of the model of bank competition we first compute the demand for deposits of bank j when it offers the deposit rate r_j while the remaining $n-1$ banks offer the rate r. In this situation bank j only has two effective competitors, namely banks $j-1$ and $j+1$. A depositor located at distance z from bank j and distance $1/n - z$ from bank $j+1$ will be indifferent between going to j or to $j+1$ if the return net of transport costs is the same, that is if

$$r_j - \mu z = r - \mu \left(\frac{1}{n} - z \right).$$

Solving for z in this equation yields

$$z(r_j, r) = \frac{1}{2n} + \frac{r_j - r}{2\mu},$$

so taking into account the symmetric market area between bank j and bank $j-1$ gives the following *demand for deposits* of bank j:

$$D(r_j, r) = 2z(r_j, r) = \frac{1}{n} + \frac{r_j - r}{\mu}. \tag{3}$$

Notice that for $r_j = r$ we have $z(r, r) = 1/2n$, i.e. the mid point between two adjacent banks, and $D(r, r) = 1/n$.

The problem of the bank's shareholders at date t is

$$\max_{k_j \geq k, r_j} \left[-k_j D(r_j, r) + \frac{1}{1+\rho} \left(\alpha - r_j + (1+\alpha)k_j \right) D(r_j, r) + \frac{1}{1+\rho} V_P \right]. \tag{4}$$

The first term in this expression is, with negative sign, the equity contribution of the bank's shareholders at date t (recall that k_j is the amount of capital per unit of deposits). The second term is the discounted value of the bank equity capital at date $t+1$, which equals the value of its assets, $(1+\alpha)(1+k_j)D(r_j, r)$, minus the value of its deposit liabilities, $(1+r_j)D(r_j, r)$. Notice that

$$(1+\alpha)(1+k_j) - (1+r_j) = \alpha - r_j + (1+\alpha)k_j,$$

which is the expression that appears in the objective function. Thus the shareholders get (per unit of deposits) the intermediation margin $\alpha - r_j$ plus the gross return $1+\alpha$

6

of investing capital k_j in the prudent asset. The third term in (4) is the discounted value of remaining open at date $t+1$ and hence obtaining a stream of profits at future dates $t + 2$, $t + 3$, etc. The discount rate used in the last two terms is the cost of capital ρ.[7]

Differentiating the objective function (4) with respect to k_j, and using Assumption (2), gives

$$\left(-1 + \frac{1 + \alpha}{1 + \rho}\right) D(r_j, r) = \frac{\alpha - \rho}{1 + \rho} D(r_j, r) < 0,$$

so we have a corner solution $k_j = k$. Obviously, since the cost of capital ρ is greater than the return α of the prudent asset, it makes no sense for the bank to hold excess capital.

Substituting this result into the objective function (4), differentiating it with respect to r_j, and using the demand function (3), gives the following first order condition:

$$-\frac{k}{\mu} + \frac{1}{1 + \rho}\left[\frac{\alpha - r_j + (1 + \alpha)k}{\mu} - \left(\frac{1}{n} + \frac{r_j - r}{\mu}\right)\right] = 0.$$

The (unique) symmetric Nash equilibrium is then obtained by setting $r_j = r$ in this condition and solving for r, which gives the *equilibrium deposit rate* when the banks can only invest in the prudent asset:

$$r_P(k) = \alpha - \frac{\mu}{n} - \delta_P k, \tag{5}$$

where

$$\delta_P = \rho - \alpha. \tag{6}$$

Since $\delta_P > 0$ by Assumption (2), the equilibrium deposit rate is decreasing in the capital requirement k.

The equilibrium intermediation margin, defined as the difference between the asset return α and the equilibrium deposit rate $r_P(k)$, is

$$\alpha - r_P(k) = \frac{\mu}{n} + \delta_P k.$$

---

[7]In HMS the discount rate of the banks' shareholders does not coincide with the cost of (outside equity) capital. Assuming as in the pecking order theory of Myers and Majluf (1984) that the former is smaller than the latter, the shareholders would want to reinvest all their profits in order to reduce the need to raise outside equity, eventually funding all the capital requirement with inside equity. For this reason, we simply assume that there is no outside equity capital.

Hence the margin is increasing in the ratio between the unit transport cost μ and the number of banks n, in the differential δ_P between the cost of capital ρ and the return of the prudent asset α, and in the level of the capital requirement k. For $k = 0$ the margin equals the ratio μ/n, and for $k \geq 0$ the margin is such that

$$-k + \frac{1}{1+\rho}\left(\alpha - r_P(k) + (1+\alpha)k\right) = -k + \frac{1}{1+\rho}\left(\frac{\mu}{n} + (1+\rho)k\right) = \frac{1}{1+\rho}\frac{\mu}{n}.$$

This means that the outcome of the competition for deposits implies that the banks' shareholders get (per unit of deposits) the margin μ/n plus the required rate of return on their capital. Hence, as in Salop's (1979) model, the ratio μ/n is the appropriate measure of the banks' market power.

Substituting $k_j = k$ and $r_j = r = r_P(k)$ into the objective function (4), and taking into account the fact that by dynamic programming the maximized value is also V_P, yields the equation

$$V_P = \frac{1}{1+\rho}\left(\frac{\mu}{n^2} + V_P\right),$$

so the *banks' franchise value* is

$$V_P = \frac{\mu}{\rho n^2}. \tag{7}$$

This expression is easy to understand. Each bank raises $1/n$ deposits at each date $t = 0, 1, 2, ...$, and gets profits (net of the cost of capital) equal to μ/n^2 at each date $t = 1, 2, 3, ...$, with present value at $t = 0$ equal to

$$\left[\frac{1}{1+\rho} + \frac{1}{(1+\rho)^2} + \frac{1}{(1+\rho)^3} + ...\right]\frac{\mu}{n^2} = \frac{\mu}{\rho n^2}.$$

The franchise value V_P is increasing in the transport cost μ and decreasing in the number of banks n and in the cost of capital ρ. Interestingly, V_P does not depend on the return α of the prudent asset, since this return (net of the intermediation margin) is entirely paid to the depositors. Also, V_P does not depend on the capital requirement k, because the negative effect of the capital requirement is exactly compensated by a reduction in the equilibrium deposit rate $r_P(k)$. Hence the cost of the capital requirement is entirely pass onto the depositors, which are correspondingly made worse off.

3.2 The model with the gambling asset

Suppose next that at each date t each bank j chooses the amount of capital $k_j \geq k$ to hold per unit of deposits and the deposit rate r_j to offer, and invests all the funds raised in an asset that yields the high return γ with probability $1 - \pi$, and the low return β with probability π.

When the gamble fails the value of the bank's assets is $(1 + \beta)(1 + k_j)D(r_j, r)$. We will assume that this is smaller than the value $(1 + r_j)D(r_j, r)$ of its deposit liabilities,[8] so in this case the bank is closed by the regulator, and by limited liability its shareholders get zero. Hence the problem of the bank's shareholders at date t is

$$\max_{k_j \geq k, r_j} \left[-k_j D(r_j, r) + \frac{1 - \pi}{1 + \rho} \left(\gamma - r_j + (1 + \gamma)k_j \right) D(r_j, r) + \frac{1 - \pi}{1 + \rho} V_G \right], \tag{8}$$

where V_G is the bank's franchise value in the model with the gambling asset. Thus the bank's objective function is similar to that of the model with a prudent asset, except that now the asset return is γ instead of α, and the second and third terms are multiplied by the probability $1 - \pi$ that the gamble succeeds.

Differentiating the objective function (8) with respect to k_j, and using Assumptions (1) and (2), gives

$$\left(-1 + \frac{(1 - \pi)(1 + \gamma)}{1 + \rho} \right) D(r_j, r) < -\frac{\pi(1 + \beta)}{1 + \rho} D(r_j, r) \leq 0,$$

so we also have a corner solution $k_j = k$.

Substituting this result into the objective function (8), differentiating it with respect to r_j, and using the demand function (3), gives the following first order condition:

$$-\frac{k}{\mu} + \frac{1 - \pi}{1 + \rho} \left[\frac{\gamma - r_j + (1 + \gamma)k}{\mu} - \left(\frac{1}{n} + \frac{r_j - r}{\mu} \right) \right] = 0.$$

The (unique) symmetric Nash equilibrium is then obtained by setting $r_j = r$ in this condition and solving for r, which gives the *equilibrium deposit rate* when the banks can only invest in the gambling asset:

$$r_G(k) = \gamma - \frac{\mu}{n} - \delta_G k, \tag{9}$$

---

[8]Since $r_j \geq 0$, a sufficient condition is that $(1 + \beta)(1 + k_j) < 0$. In particular, this condition would hold if $1 + \beta = 0$, that is if the gross return when the gamble fails is zero.

where
$$\delta_G = \frac{1+\rho}{1-\pi} - (1+\gamma). \tag{10}$$
Since by Assumptions (2) and (1) we have
$$(1-\pi)\delta_G = (1+\rho) - (1-\pi)(1+\gamma) > (1+\alpha) - (1-\pi)(1+\gamma) > \pi(1+\beta) \geq 0,$$
the equilibrium deposit rate is decreasing in the capital requirement k.

The equilibrium intermediation margin, defined as the difference between the success return γ and the equilibrium deposit rate $r_G(k)$, is
$$\gamma - r_G(k) = \frac{\mu}{n} + \delta_G k.$$
Hence the margin is increasing in the ratio between the unit transport cost μ and the number of banks n, in the cost of capital ρ, in the probability of failure π, and in the level of the capital requirement k, and is decreasing in the success return of the gambling asset γ. For $k=0$ the margin equals the ratio μ/n, and for $k \geq 0$ the margin is such that
$$-k + \frac{1-\pi}{1+\rho}(\gamma - r_G(k) + (1+\gamma)k) = -k + \frac{1-\pi}{1+\rho}\left(\frac{\mu}{n} + \frac{1+\rho}{1-\pi}k\right) = \frac{1-\pi}{1+\rho}\frac{\mu}{n}.$$
Hence the outcome of the competition for deposits implies that the banks' shareholders get (per unit of deposits) the margin μ/n with probability $1-\pi$ plus the required rate of return on their capital.

Substituting $k_j = k$ and $r_j = r = r_G(k)$ into the objective function (8), and taking into account the fact that by dynamic programming the maximized value is also V_G, yields the equation
$$V_G = \frac{1-\pi}{1+\rho}\left(\frac{\mu}{n^2} + V_G\right),$$
so the *bank's franchise value* is
$$V_G = \frac{(1-\pi)\mu}{(\rho+\pi)n^2}. \tag{11}$$
As before, this expression is easy to understand. Each bank raises $1/n$ deposits at each date $t = 0, 1, 2, ...$, and gets profits (net of the cost of capital) equal to μ/n^2 at each date $t = 1, 2, 3, ...$ with probability $(1-\pi)^t$, a stream that has present value at $t=0$ equal to
$$\left[\frac{1-\pi}{1+\rho} + \left(\frac{1-\pi}{1+\rho}\right)^2 + \left(\frac{1-\pi}{1+\rho}\right)^3 + ...\right]\frac{\mu}{n^2} = \frac{(1-\pi)\mu}{(\rho+\pi)n^2}.$$

10

Hence the one-period net expected return, $(1 - \pi)\mu/n^2$, is discounted at a rate that is the sum $\rho + \pi$ of the opportunity cost of bank capital and the probability that the gamble fails and the bank is closed by the regulator.[9]

The franchise value V_G is increasing in the transport cost μ and decreasing in the number of banks n, the cost of capital ρ, and the probability of failure π. As in the case of the model with the prudent asset, the franchise value V_G does not depend on the success return γ of the prudent asset, since this return (net of the intermediation margin) is entirely paid to the depositors. Also, V_G does not depend on the capital requirement k, because the negative effect of the capital requirement is exactly compensated by a reduction in the equilibrium deposit rate $r_G(k)$.

3.3   The general model

If the banks can invest in either the prudent or the gambling asset, there are two possible types of symmetric equilibria: one in which all the banks invest in the prudent asset, and another one in which all the banks invest in the gambling asset. By the arguments in the previous subsections, it is clear that in no case the banks will want to hold any excess capital, so we can set $k_j = k$ and focus on the choice of deposit rates and type of investment.

A *prudent equilibrium* exists if no bank j has an incentive to deviate from a situation in which all the banks offer the deposit rate $r_P(k)$ and invest in the prudent asset, that is if the following condition holds:

$$
\max_{r_j} \left[-kD(r_j, r_P(k)) + \frac{1 - \pi}{1 + \rho} \left(\gamma - r_j + (1 + \gamma)k \right) D(r_j, r_P(k)) + \frac{1 - \pi}{1 + \rho} V_P \right] \leq V_P.
$$
(12)

The left hand side of this expression is the present value of the deviation to the gambling strategy at any date t,[10] while the right hand side is the value of the bank in the prudent equilibrium.

A *gambling equilibrium* exists if no bank j has an incentive to deviate from a situation in which all the banks offer the deposit rate $r_G(k)$ and invest in the gambling

---

[9]Notice the similarity with the discount rate in models where consumers face in each period a constant probability of death; see Blanchard (1985).

[10]See Fudenberg and Tirole (1991), Chapter 4, for a proof that one-stage-deviations are sufficient to characterize subgame perfect equilibria.

asset, that is if the following condition holds:

$$\max_{r_j} \left[-kD(r_j, r_G(k)) + \frac{1}{1+\rho} \left(\alpha - r_j + (1+\alpha)k \right) D(r_j, r_G(k)) + \frac{1}{1+\rho} V_G \right] \leq V_G.$$

(13)

The left hand side of this expression is the present value of the deviation to the prudent strategy at any date t, while the right hand side is the value of the bank in the gambling equilibrium.

We can now state the main result of this section.

Proposition 1 *There are two critical values*

$$m_P(k) = \frac{\gamma - \alpha - (\delta_G - \delta_P)k}{2(h-1)} \qquad and \qquad m_G(k) = hm_P(k),$$

(14)

where

$$h = \sqrt{\frac{\rho + \pi}{(1-\pi)\rho}} > 1,$$

(15)

such that a prudent equilibrium exists if $\mu/n \geq m_P(k)$, and a gambling equilibrium exists if $\mu/n \leq m_G(k)$.

By the definitions (10) and (6) of δ_G and δ_P and Assumption (1) we have

$$\delta_G - \delta_P > (1-\pi)\delta_G - \delta_P = (1+\alpha) - (1-\pi)(1+\gamma) > \pi(1+\beta) \geq 0.$$

Hence the critical values $m_P(k)$ and $m_G(k)$ defined in (14) are linearly decreasing functions of the capital requirement k. Moreover, since $h > 1$ their intercepts, m_P and m_G, satisfy

$$m_P = m_P(0) < m_G(0) = m_G,$$

and we also have

$$m_P(\widehat{k}) = m_G(\widehat{k}) = 0,$$

for

$$\widehat{k} = \frac{\gamma - \alpha}{\delta_G - \delta_P}.$$

(16)

Since $\delta_G - \delta_P > 0$ and $\gamma > \alpha$ by Assumption (1), we have $\widehat{k} > 0$. Hence we have the situation depicted in Figure 1.[11] In region P the intermediation margin μ/n is above the line $m_G(k)$ and only the prudent equilibrium exists. In region G the margin μ/n
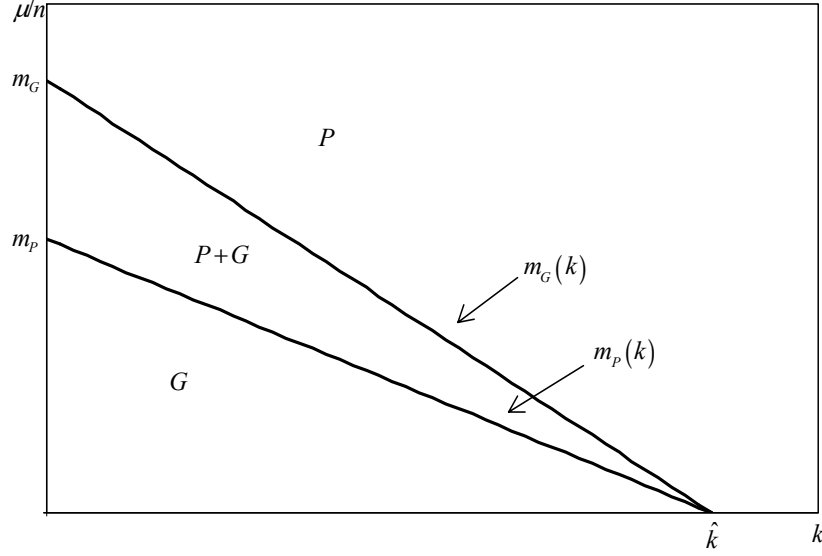
Figure 1: Characterization of equilibrium with capital requirements

is below the line $m_P(k)$ and only the gambling equilibrium exists. And in region P+G where the margin μ/n is between the two lines both types of equilibria exist.

Therefore high values of the measure μ/n of the banks' market power are conducive to the prudent equilibrium. The intuition for this result is fairly obvious. If the banks obtain large rents when open they have an incentive to choose the prudent strategy in order to preserve these rents. Conversely, if the banks obtain little rents when open they have an incentive to gamble. Finally, for intermediate values of the margin μ/n, the strategic interaction among the banks generates multiple equilibria.

To explain this multiplicity observe that, since $m_P(k) > 0$ implies $r_G(k) > r_P(k)$, in region P+G where both types of equilibria exist the gambling equilibrium is characterized by higher deposit rates. Now suppose for simplicity that $k = 0$, and let $r_G = r_G(0)$ and $r_P = r_P(0)$. If all the banks set the high deposit rate r_G and choose the gambling strategy, then by the proof of Proposition 1 a deviating bank choosing the prudent strategy will offer the lower rate $(r_G + r_P)/2$, so if μ/n is not sufficiently large its margin, $\alpha - (r_G + r_P)/2 = (\alpha - \gamma)/2 + \mu/n$, will be small (possibly nega-

---

[11]Figures 1-3 are drawn for the following parameter values: $\alpha = 0.1$, $\beta = -1$, and $\gamma = \pi = \rho = 0.2$.

tive), and the deviation will not be profitable. On the other hand, if all the banks set the low deposit rate r_P and choose the prudent strategy, then by the proof of Proposition 1 a deviating bank choosing the gambling strategy will offer the higher rate $(r_G + r_P)/2$ and it will get more deposits,[12] but if the margin μ/n is sufficiently large the gain when the gamble succeeds will not compensate the loss of future rents when the gamble fails, so the deviation will not be profitable.

The effect of the parameters of the model on the characterization of the regions in the $k - \mu/n$ space is easy to derive by simply looking at the effect of changes in these parameters on the critical values m_P, m_G and $\widehat{k}$ that determine the intersections of the linear functions $m_P(k)$ and $m_G(k)$ with the two axis.

In particular, it is immediate to check that m_P, m_G and $\widehat{k}$ are all increasing in the spread $\gamma - \alpha$ between the success return of the gambling asset and the return of the prudent asset. Hence when the gambling asset becomes relatively more attractive, the gambling equilibrium region becomes larger and the prudent equilibrium region becomes smaller.

On the other hand, m_P, m_G and $\widehat{k}$ are all decreasing in the probability π of failure of the gambling asset,[13] so an increase in the probability $1 - \pi$ of obtaining the future rents associated with the gambling strategy has the same qualitative effect as an increase in the spread $\gamma - \alpha$.

Finally, the effect of the cost of capital ρ is more complicated since it increases the critical values m_P and m_G, and it reduces the critical value $\widehat{k}$.[14] The effect on the intercepts comes from the fact that an increase in ρ reduces the present value of the higher future rents associated with the prudent strategy, making it relatively less attractive than the gambling strategy. But at the same time, the capital requirement has more bite for the gambling strategy, so for large values of k the prudent equilibrium region becomes larger. Hence an increase in the cost of capital has a negative effect on the banks' incentives for prudent investment behavior, unless they operate in an

---

[12]This clearly illustrates the destabilizing role of deposit insurance: When a bank deviates to the gambling strategy the depositors assume that they will always get the high rate $(r_G + r_P)/2$, without taking into account that the bank will fail with probability π.

[13]Since m_P and m_G are decreasing in h and $\partial h/\partial \pi > 0$, and $\widehat{k}$ is decreasing in $\delta_G - \delta_P$ and $\partial(\delta_G - \delta_P)/\partial \pi > 0$.

[14]Since m_P and m_G are decreasing in h and $\partial h/\partial \rho < 0$, and $\widehat{k}$ is decreasing in $\delta_G - \delta_P$ and $\partial(\delta_G - \delta_P)/\partial \rho > 0$.

environment with high capital requirements.

The implications of our results for capital regulation are immediate. If an increase in bank competition reduces the intermediation margin μ/n and pushes the banks to the gambling equilibrium region, then an increase in the capital requirement k can always shift them back to the region where a prudent equilibrium exists.

To see whether this is efficient one needs to compute the welfare gain for both the depositors and the banks' shareholders. In a prudent equilibrium the depositors get

$$1 + r_P(k) = 1 + \alpha - \frac{\mu}{n} - \delta_P k$$

at each date, while the banks' shareholders get the margin μ/n.[15] In a gambling equilibrium the depositors get $1 + r_G(k)$ with probability $1 - \pi$, and (since they are taxed to finance deposit insurance payouts) the liquidation value of the banks' assets $1 + \beta$ with probability π,[16] so in expected terms they receive

$$(1 - \pi)(1 + r_G(k)) + \pi(1 + \beta) = 1 + (1 - \pi)\left(\gamma - \frac{\mu}{n}\right) + \pi\beta - (1 - \pi)\delta_G k.$$

On the other hand, the banks' shareholders obtain at each date the margin μ/n with probability $1 - \pi$, so in expected terms they get $(1 - \pi)\mu/n$.[17] Table 1 summarizes the welfare properties of the two types of equilibrium.

Table 1   Agents' payoffs at each date for the two equilibria

	Depositors	Banks' shareholders
Prudent equilibrium $(\mu/n \geq m_P(k))$	$1 + \alpha - \dfrac{\mu}{n} - \delta_P k$	$\dfrac{\mu}{n}$
Gambling equilibrium $(\mu/n \leq m_G(k))$	$1 + (1 - \pi)\left(\gamma - \dfrac{\mu}{n}\right) + \pi\beta$ $-(1 - \pi)\delta_G k$	$(1 - \pi)\dfrac{\mu}{n}$

---

[15] As noted in Section 3.1, the banks' shareholders obtain $\alpha - r_P(k) + (1 + \alpha)k = \mu/n + (1 + \rho)k$ at each date, but $(1 + \rho)k$ is just the compensation for their investment in bank capital.

[16] We are assuming, without loss of generality, that the returns of the gambling asset are perfectly correlated across banks.

[17] As noted in Section 3.2, the banks' shareholders obtain $\gamma - r_G(k) + (1 + \gamma)k = \mu/n + (1 + \rho)k/(1 - \pi)$ at each date with probability $1 - \pi$, which in expected terms equals $(1 - \pi)\mu/n + (1 + \rho)k$. But $(1 + \rho)k$ is just the compensation for their investment in bank capital.

Hence the payoff of the depositors is decreasing in the level of the capital requirement k for both the prudent and the gambling equilibrium, while that of the banks' shareholders is independent of k. This simply follows from the fact that the cost of the capital requirement is entirely pass onto the depositors.

Table 1 shows that, for any level of the capital requirement k, the banks' shareholders are better off in the prudent than in the gambling equilibrium. The comparison is more complicated for the depositors. For $k = 0$ they prefer the prudent equilibrium if and only if

$$\alpha > (1 - \pi)\gamma + \pi \left(\beta + \frac{\mu}{n} \right). \tag{17}$$

This is stronger than the requirement in Assumption (1) that the return of the prudent asset be greater than the expected return of the gambling asset. The reason why the term $\pi\mu/n$ appears here is that in the prudent equilibrium the margin μ/n is always transferred to the banks, while in the gambling equilibrium this happens only when the gamble succeeds. Since we have shown that the coefficients of k in the depositors' payoffs in Table 1 satisfy $(1 - \pi)\delta_G > \delta_P$, it follows that Assumption (17) is all we need to ensure that, for any level of k, the depositors are better off in the prudent than in the gambling equilibrium.

Now suppose that (17) holds, and that for $k = 0$ only the gambling equilibrium exists. In this case, the minimum capital requirement k^* required to avoid gambling is defined by the condition $m_P(k^*) = \mu/n$,[18] which by (14) gives

$$k^* = \frac{\gamma - \alpha - 2(h - 1)\mu/n}{\delta_G - \delta_P}. \tag{18}$$

By our previous discussion it is clear that

$$\frac{\partial k^*}{\partial(\mu/n)} < 0, \qquad \frac{\partial k^*}{\partial(\gamma - \alpha)} > 0, \qquad \text{and} \qquad \frac{\partial k^*}{\partial\pi} < 0,$$

so the minimum requirement k^* is decreasing in the intermediation margin μ/n and in the probability π of failure of the gambling asset, and is increasing in the spread $\gamma - \alpha$ between the success return of the gambling asset and the return of the prudent asset. On the other hand, the effect of the cost of capital ρ on the minimum requirement k^* is ambiguous.

---

[18]We are implicitly assuming that in the case of multiple equilibria the banks play the equilibrium that yields the highest payoff to them. Since by (7) and (11) we have $V_P > V_G$, this criterion selects the prudent equilibrium.

It is important to note while the banks' shareholders will be better off in the prudent equilibrium with capital requirement k^* than in the gambling equilibrium with no capital requirement, this will not be necessarily so for the depositors. In order to ensure this, Assumption (17) would need to be strengthen to

$$\alpha > (1 - \pi)\gamma + \pi \left(\beta + \frac{\mu}{n} \right) + \delta_P k^*.$$

Alternatively, we could assume that bank failures entail significant administrative and/or bankruptcy costs incurred by the regulator and pass onto the depositors via taxation.

Hence we conclude that capital requirements are good for fostering prudent bank behavior, although they have a negative impact on the depositors who bear the burden of the requirement in the form of lower deposit rates. In addition, the minimum required capital should be higher in environments with relatively attractive gambling assets (in terms of either upside returns or success probabilities), and/or with low intermediation margins.

4 Extensions

In the previous section we have shown that flat-rate capital requirements are an effective policy instrument for addressing the banks' incentives for risk-taking. We now examine two alternative ways of inducing the banks to choose prudent investment strategies. The first one is to have risk-based capital requirements, that is capital requirements that discriminate in favor of investment in the prudent asset. The second one is to introduce deposit interest rate ceilings, that is an upper bound on the rates that banks are allowed to pay to the depositors.

4.1 Risk-based capital requirements

Suppose that the banks are subject to a capital requirement k_P if they invest in the prudent asset and a capital requirement k_G if they invest in the gambling asset, with $k_P < k_G$. Moreover, assume that $k_P = 0$ and $k_G = k > 0$, and let

$$r_P = r_P(0) = \alpha - \frac{\mu}{n} \tag{19}$$

denote the deposit rate in the prudent equilibrium.

A *prudent equilibrium* exists if no bank j has an incentive to deviate from a situation in which all the banks offer the deposit rate r_P and invest in the prudent asset, that is if the following condition holds:

$$\max_{r_j} \left[-kD(r_j, r_P) + \frac{1-\pi}{1+\rho} (\gamma - r_j + k(1+\gamma)) D(r_j, r_P) + \frac{1-\pi}{1+\rho} V_P \right] \leq V_P. \quad (20)$$

The left hand side of this expression is the present value of the deviation to the gambling strategy at any date t, which involves a capital charge k per unit of deposits, while the right hand side is the value of the bank in the prudent equilibrium.

A *gambling equilibrium* exists if no bank j has an incentive to deviate from a situation in which all the banks offer the deposit rate $r_G(k)$ and invest in the gambling asset, that is if the following condition holds:

$$\max_{r_j} \left[\frac{1}{1+\rho} (\alpha - r_j) D(r_j, r_G(k)) + \frac{1}{1+\rho} V_G \right] \leq V_G. \quad (21)$$

The left hand side of this expression is the present value of the deviation to the prudent strategy at any date t, which involves a zero capital charge, while the right hand side is the value of the bank in the gambling equilibrium.

The following result characterizes equilibrium with risk-based capital requirements.

Proposition 2 *If the regulator imposes a capital requirement k for investment in the gambling asset and no requirement for investment in the prudent asset, there are two critical values*

$$m_P'(k) = \frac{\gamma - \alpha - \delta_G k}{2(h-1)} \qquad and \qquad m_G'(k) = hm_P'(k), \quad (22)$$

where h is given by (15), such that a prudent equilibrium exists if $\mu/n \geq m_P'(k)$, and a gambling equilibrium exists if $\mu/n \leq m_G'(k)$.

Since $\delta_G > \delta_G - \delta_P$, the functions $m_P'(k)$ and $m_G'(k)$ in (22) that characterize the equilibria with risk-based capital requirements are steeper than the functions $m_P(k)$ and $m_G(k)$ in (14) that characterize the equilibria with flat-rate capital requirements, so the former intersect the horizontal axis at a point $\widehat{k}' < \widehat{k}$. This implies that the
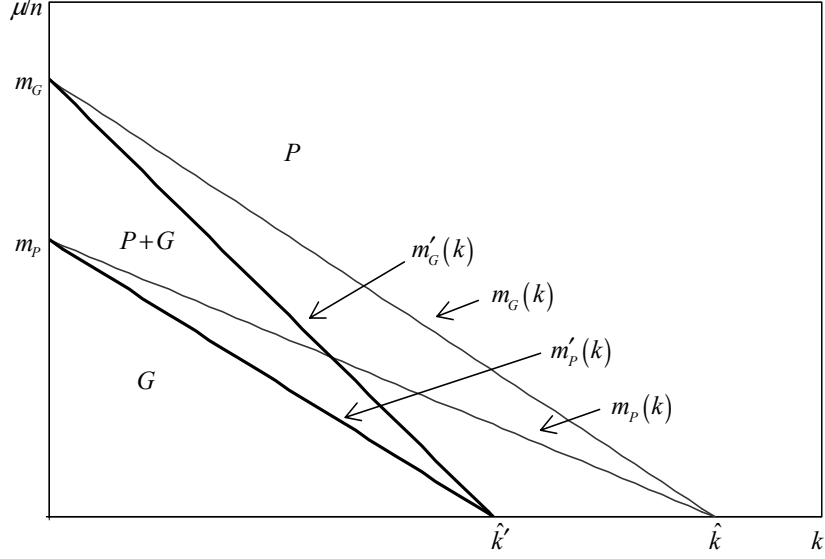
Figure 2: Characterization of equilibrium with risk-based capital requirements

region above the line $m'_G(k)$ where only a prudent equilibrium exists becomes larger, and the regions below the line $m'_P(k)$ and between the two lines where, respectively, only a gambling equilibrium and both types of equilibria exist become smaller. This is illustrated in Figure 2.

The welfare properties of equilibrium with risk-based capital requirements are summarized in Table 2. The only difference with Table 1 is that the now the capital requirement only applies to investments in the gambling asset, so banks do not hold any capital in the prudent equilibrium, and consequently the payoff of the depositors is in this case independent of k. Hence, if Assumption (17) holds, the minimum requirement that ensures that banks will play the prudent equilibrium is such that the depositors are better off in this equilibrium than in the gambling equilibrium with no capital requirement.

Table 2 Agents' payoffs at each date for the two equilibria
with risk-based capital requirements

	Depositors	Banks' shareholders
Prudent equilibrium $(\mu/n \geq m_P'(k))$	$1 + \alpha - \dfrac{\mu}{n}$	$\dfrac{\mu}{n}$
Gambling equilibrium $(\mu/n \leq m_G'(k))$	$1 + (1-\pi)\left(\gamma - \dfrac{\mu}{n}\right) + \pi\beta$ $-(1-\pi)\delta_G k$	$(1-\pi)\dfrac{\mu}{n}$

The conclusion is then that risk-based capital requirements are more efficient tools for the regulator, because they can ensure the existence of a prudent equilibrium at no cost in terms of bank capital. However, we have started our analysis with the assumption that the asset choices of the banks are not observed by the regulator, for otherwise he could directly prevent the banks from investing in the gambling asset. So how could the regulator enforce a risk-based capital requirement if he does not observe the characteristics of the banks' portfolios?

A possible answer to this question would be to set up a regulatory structure that makes it incentive compatible for the banks to reveal their asset choices to the regulator, for example by offering them the possibility to use their own risk management systems in order to compute the capital that they are required to hold. In fact, the Basel Committee on Banking Supervision (2001, pp.8-9) is putting forward revised proposals for a standardized approach for credit risk capital charges, and specific proposals for a new internal ratings-based (IRB) approach that "...incorporates in the capital calculation the bank's own internal estimates of the probability of default...,"[19] and "...provides capital incentives relative to the standardized approach." Thus one can interpret the proposed reform of the Basel Capital Accord as a way to induce the banks to reveal private information about the risks in their loan portfolios by effectively reducing their capital requirements. However, an analysis of the incentive compatibility of these proposals is beyond the scope of this paper.

---

[19]Obviously, this requires that the banks' risk management systems be validated by the regulator, which is the subject of the so-called second pillar of the New Capital Accord.

4.2 Deposit rate ceilings

We now examine the effects a regulation that prevents the banks from offering deposit rates above a ceiling $\bar{r}$. Such a regulation has been advocated by HMS as an efficient way to control risk-shifting incentives. We will show deposit rate ceilings expand the prudent equilibrium region, but that there is a large set of parameter values for which no ceiling greater than or equal to zero can ensure the existence of a prudent equilibrium.

In what follows we assume that there are no capital requirements ($k = 0$), and we let $r_P = r_P(0) = \alpha - \mu/n$ and $r_G = r_G(0) = \gamma - \mu/n$ denote, respectively, the prudent and gambling equilibrium deposit rates in the absence of a ceiling.

If a prudent equilibrium exists when the regulator introduces a deposit rate ceiling $\bar{r}$, the banks' franchise value will be

$$V_P(\bar{r}) = \max\left\{\frac{\mu}{\rho n^2}, \frac{\alpha - \bar{r}}{\rho n}\right\}. \tag{23}$$

To explain this expression notice that if $\bar{r} \geq r_P$, the ceiling is not binding and we have the same franchise value as in (7), namely $\mu/\rho n^2$. On the other hand if $\bar{r} < r_P$, the ceiling is binding, the banks will get profits $(\alpha - \bar{r})/n$ at each date, and the present value of this flow will be $(\alpha - \bar{r})/\rho n$. Since $\mu/\rho n^2 < (\alpha - \bar{r})/\rho n$ if and only if $\bar{r} < \alpha - \mu/n = r_P$, (23) then follows.

Similarly, if a gambling equilibrium exists with ceiling $\bar{r}$, the banks' franchise value will be

$$V_G(\bar{r}) = \max\left\{\frac{(1-\pi)\mu}{(\rho+\pi)n^2}, \frac{(1-\pi)(\gamma - \bar{r})}{(\rho+\pi)n}\right\}. \tag{24}$$

To explain this expression notice that if $\bar{r} \geq r_G$, the ceiling is not binding and we have the same franchise value as in (11), namely $(1-\pi)\mu/(\rho+\pi)n^2$. On the other hand if $\bar{r} < r_G$, the ceiling is binding, the banks will get profits $(\gamma - \bar{r})/n$ at each date t with probability $(1-\pi)^t$, and the present value of this flow will be $(1-\pi)(\gamma - \bar{r})/(\rho+\pi)n$. Since $(1-\pi)\mu/(\rho+\pi)n^2 < (1-\pi)(\gamma - \bar{r})/(\rho+\pi)n$ if and only if $\bar{r} < \gamma - \mu/n = r_G$, (24) then follows.

A *prudent equilibrium* with ceiling $\bar{r}$ exists if no bank j has an incentive to deviate from a situation in which all the banks offer the deposit rate $\min\{\bar{r}, r_P\}$ and invest

in the prudent asset, that is if the following condition holds:

$$\max_{r_j \leq \overline{r}} \left[\frac{1-\pi}{1+\rho}(\gamma - r_j)D(r_j, \min\{\overline{r}, r_P\}) + \frac{1-\pi}{1+\rho}V_P(\overline{r}) \right] \leq V_P(\overline{r}). \qquad (25)$$

The left hand side of this expression is the present value of the deviation to the gambling strategy at any date t, which incorporates the constraint that the deposit rate offered by bank j cannot exceed the ceiling, the while the right hand side is the value of the bank in the prudent equilibrium.

A *gambling equilibrium* with ceiling $\overline{r}$ exists if no bank j has an incentive to deviate from a situation in which all the banks offer the deposit rate $\min\{\overline{r}, r_G\}$ and invest in the gambling asset, that is if the following condition holds:

$$\max_{r_j \leq \overline{r}} \left[\frac{1}{1+\rho}(\alpha - r_j)D(r_j, \min\{\overline{r}, r_G\}) + \frac{1}{1+\rho}V_G(\overline{r}) \right] \leq V_G(\overline{r}). \qquad (26)$$

The left hand side of this expression is the present value of the deviation to the prudent strategy at any date t, which incorporates the constraint that the deposit rate offered by bank j cannot exceed the ceiling, the while the right hand side is the value of the bank in the gambling equilibrium.

The following result characterizes equilibrium with deposit rate ceilings.

Proposition 3 *If $\overline{r} \geq \overline{r}_P$, where*

$$\overline{r}_P = \frac{\alpha h^2 - \gamma}{h^2 - 1} \qquad (27)$$

and h is given by (15), there are two critical values $m_P(\overline{r})$ and $m_G(\overline{r})$, with

$$m_P(\overline{r}_P) = m_G(\overline{r}_P) = \frac{\gamma - \alpha}{h^2 - 1}$$

and $m_P(\overline{r}) < m_G(\overline{r})$ for $\overline{r} > \overline{r}_P$, such that a prudent equilibrium with ceiling $\overline{r}$ exists if $\mu/n \geq m_P(\overline{r})$, and a gambling equilibrium with ceiling $\overline{r}$ exists if $\mu/n \leq m_G(\overline{r}_P)$. On the other hand, if $\overline{r} \leq \overline{r}_P$, a prudent equilibrium with ceiling $\overline{r}$ exists for all μ/n.

The critical values $m_P(\overline{r})$ and $m_G(\overline{r})$, which are defined by (29) and (30) in the Appendix, are increasing in the ceiling $\overline{r}$ for $\overline{r}_P \leq \overline{r} \leq (\alpha+\gamma)/2 - m_P = \gamma - m_G$, and satisfy $m_P(\overline{r}) = m_P$ for $\overline{r} \geq (\alpha+\gamma)/2 - m_P$, and $m_G(\overline{r}) = m_G$ for $\overline{r} \geq \gamma - m_G$. Hence we have the situation depicted in Figure 3. In region P the intermediation margin
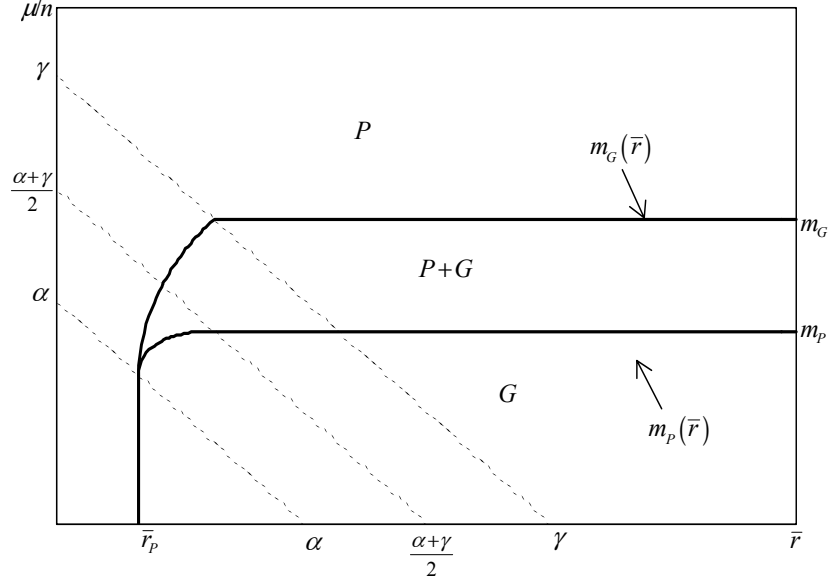
Figure 3: Characterization of equilibrium with deposit rate ceilings

μ/n is above the line $m_G(\overline{r})$ and only the prudent equilibrium exists. In region G the margin μ/n is below the line $m_P(\overline{r})$ and only the gambling equilibrium exists. And in region P+G where the margin μ/n is between the two lines both types of equilibria exist.

Figure 3 shows that, as in the case of capital requirements, if an increase in bank competition reduces the intermediation margin μ/n and pushes the banks to the gambling equilibrium region, there is a deposit rate ceiling $\overline{r}$ that can ensure the existence of a prudent equilibrium. Clearly, for $0 < \mu/n < \alpha - \overline{r}_P$ only a binding ceiling (that is, a ceiling with $\overline{r} < \alpha - \mu/n = r_P$) will work, while for $\alpha - \overline{r}_P \leq \mu/n < m_P$ a nonbinding ceiling will suffice. As noted by HMS, the latter case is especially attractive, since nonbinding ceilings do not distort the level of deposit rates.

The welfare properties of equilibrium with deposit rate ceilings are summarized in Table 3. Comparing it with Table 2, we conclude that in those cases where a prudent equilibrium exists with a nonbinding ceiling, the depositors and the banks' shareholders are indifferent between ceilings and risk-based capital requirements, while in those cases where a binding ceiling is needed to sustain prudent bank behavior, the deposi-

23

tors are worse off and the shareholders are better off than in the prudent equilibrium with risk-based requirements.

<p style="text-align:center">Table 3 Agents' payoffs at each date for the two equilibria
with deposit rate ceilings</p>

	Depositors	Banks' shareholders
Prudent equilibrium $(\overline{r} \leq \overline{r}_P \text{ or } \overline{r} > \overline{r}_P$ and $\mu/n \geq m_P(\overline{r}))$	$1 + \min\left\{\overline{r}, \alpha - \dfrac{\mu}{n}\right\}$	$\max\left\{\alpha - \overline{r}, \dfrac{\mu}{n}\right\}$
Gambling equilibrium $(\overline{r} \geq \overline{r}_P$ and $\mu/n \leq m_G(\overline{r}))$	$1 + (1-\pi)\min\left\{\overline{r}, \gamma - \dfrac{\mu}{n}\right\} + \pi\beta$	$(1-\pi)\max\left\{\gamma - \overline{r}, \dfrac{\mu}{n}\right\}$

However, there is a potential problem with both deposit rate ceilings and flat-rate capital requirements which we have not discussed so far, namely the fact that in order to guarantee the existence of a prudent equilibrium the deposit rates implied by either regulation may be negative. In the case of deposit rate ceilings, this would happen when the critical value $\overline{r}_P$ in (27) is negative, that is when

$$\frac{\gamma}{\alpha} > h^2, \tag{28}$$

because then by Proposition 3 for any $0 < \mu/n < \alpha$ the required ceiling would satisfy $\overline{r} \leq \overline{r}_P < 0$. In the case of flat-rate capital requirements, this would happen when $r_P(k^*) < 0$. Using the definitions of $r_P(k)$ and k^* in (5) and (18), one can show after some tedious manipulations that

$$r_P(k^*) = \frac{1}{\delta_G - \delta_P}\left(\rho(\alpha h^2 - \gamma) + [\gamma - \alpha(2h-1) - \rho(h-1)^2]\frac{\mu}{n}\right).$$

From here it is easy to show that if (28) holds, then $r_P(k^*) < 0$ for any $0 < \mu/n < \alpha$. In other words, flat-rate capital requirements are effective in controlling risk-shifting incentives only if deposit rate ceilings are also effective.[20] On the other hand, risk-based

---

[20] The converse is not true. In particular, one can show that for $2h - 1 < \gamma/\alpha < h^2$ we have $\overline{r}_P > 0$ and $r_P(k^*) < 0$ for μ/n sufficiently close to α.

capital requirements are not subject to this problem, because the capital requirement does not have any effect on deposit rates in the prudent equilibrium.

Hence we conclude that deposit rate ceilings can ensure the existence of prudent equilibria that otherwise would not exist, unless the success return of the gambling asset γ is too large relative to the return of the prudent asset α. The intuition for this result is the following. If investment in the gambling asset becomes very attractive for the banks' shareholders, they will not refrain from investing in this asset even when the future rents associated with investment in the prudent asset reach the upper bound that obtains for $\overline{r} = 0$. In this case, only risk-based capital requirements would be effective, since they directly penalize investment in the gambling asset without distorting prudent equilibrium rates.

5   Conclusion

This paper has reexamined the role of capital requirements and deposit rate ceilings as a regulatory tools to reduce risk-shifting incentives in situations of increased competition in banking. Building on Hellmann, Murdock, and Stiglitz (2000), we have shown that for a particular model of imperfect competition in the deposit market, both instruments are in general effective in preventing the banks from taking excessive risks.

The reason for our different results lies in the fact that in our model the costs of an increase in the capital requirement are fully translated to the depositors. Since the equilibrium intermediation margins remain constant, the banks' franchise values do not change, and so the only effect of a higher requirement is to increase the capital loss to the bank's shareholders in case of default. Deposit rate ceilings, on the other hand, work through a different channel: they provide higher rents and increase the banks' franchise values (if they are binding), and reduce the profits from a deviation to riskier investments (if they are not).

We have also shown that both flat-rate capital requirements and deposit rate ceilings may imply very low (even negative) interest rates, but that this problem does not arise with risk-based capital requirements, that is requirements that penalize investment in riskier assets. Thus if the regulator wants to ensure prudent bank

behavior without distorting the level of deposit rates, she may use either nonbinding deposit rate ceilings or risk-based capital requirements. However, while nonbinding ceilings are only effective when the banks retain significant market power, risk-based requirements, if they can be implemented, work for any degree of market power.

It is important to bear in mind that these results are derived from a model with a number of special features: depositors cannot invest in assets other than bank deposits and their aggregate demand is fixed, banks face an perfectly elastic supply of both the prudent and the gambling asset and a perfectly elastic supply of equity capital, etc. This means that some results may not be robust, and perhaps more importantly, that some issues like the welfare cost of the banks' market power and hence the trade-off between competition and stability of the banking system cannot be addressed. On the other hand, a fully worked out model like the one in this paper may turn out to be a very useful benchmark for further work in this area.

Appendix

Proof of Proposition 1 Differentiating the left hand side of condition (12) with respect to r_j, and using the demand function (3), gives the following first order condition:

$$-\frac{k}{\mu} + \frac{1-\pi}{1+\rho}\left[\frac{\gamma - r_j + k(1+\gamma)}{\mu} - \left(\frac{1}{n} + \frac{r_j - r_P(k)}{\mu}\right)\right] = 0.$$

Solving for r_j in this expression and using the definition (9) of $r_G(k)$ we conclude that the deviating bank j will offer the deposit rate

$$r_j = \frac{r_G(k) + r_P(k)}{2}.$$

Substituting this result back into (12) and rearranging then gives

$$\frac{1-\pi}{1+\rho}\left(\frac{(r_G(k) - r_P(k))^2}{4\mu} + \frac{r_G(k) - r_P(k)}{n} + \frac{\mu}{n^2}\right) + \frac{1-\pi}{1+\rho}V_P \le V_P.$$

Substituting V_P from (7) into this expression, and using the definition (15) of h and the fact that by (5) and (9) we have $r_G(k) - r_P(k) = \gamma - \alpha - (\delta_G - \delta_P)k$, the condition for the existence of a prudent equilibrium simplifies to

$$\frac{1}{4}[\gamma - \alpha - (\delta_G - \delta_P)k]^2 + [\gamma - \alpha - (\delta_G - \delta_P)k]\frac{\mu}{n} - (h^2 - 1)\left(\frac{\mu}{n}\right)^2 \le 0.$$

Since $h^2 > 1$ if and only if $\rho + \pi > (1 - \pi)\rho$, that is if and only if $\pi(1 + \rho) > 0$, it is immediate to check that for $\gamma - \alpha - (\delta_G - \delta_P)k \ge 0$ this inequality will be satisfied if either

$$\frac{\mu}{n} \ge \frac{\gamma - \alpha - (\delta_G - \delta_P)k}{2(h-1)} = m_P(k),$$

or

$$\frac{\mu}{n} \le -\frac{\gamma - \alpha - (\delta_G - \delta_P)k}{2(h+1)}.$$

But this latter case can be disregarded since it violates the assumption $\mu/n > 0$.

Differentiating the left hand side of condition (13) with respect to r_j, and using the demand function (3), gives the following first order condition:

$$-\frac{k}{\mu} + \frac{1}{1+\rho}\left[\frac{\alpha - r_j + k(1+\alpha)}{\mu} - \left(\frac{1}{n} + \frac{r_j - r_G(k)}{\mu}\right)\right] = 0.$$

Solving for r_j in this expression and using the definition (5) of $r_P(k)$ we conclude that the deviating bank j will offer the deposit rate

$$r_j = \frac{r_G(k) + r_P(k)}{2}.$$

Substituting this result back into (13) and rearranging then gives

$$\frac{1}{1+\rho}\left(\frac{(r_G(k) - r_P(k))^2}{4\mu} - \frac{r_G(k) - r_P(k)}{n} + \frac{\mu}{n^2}\right) + \frac{1}{1+\rho}V_G \leq V_G.$$

Substituting V_G from (11) into this expression, and using the definition (15) of h and the fact that $r_G(k) - r_P(k) = \gamma - \alpha - (\delta_G - \delta_P)k$, the condition for the existence of a gambling equilibrium simplifies to

$$\frac{1}{4}[\gamma - \alpha - (\delta_G - \delta_P)k]^2 - [\gamma - \alpha - (\delta_G - \delta_P)k]\frac{\mu}{n} + \left(1 - \frac{1}{h^2}\right)\left(\frac{\mu}{n}\right)^2 \leq 0.$$

Since $h^2 > 1$, it is immediate to check that for $\gamma - \alpha - \delta k \geq 0$ this inequality will be satisfied if

$$\frac{h[\gamma - \alpha - (\delta_G - \delta_P)k]}{2(h+1)} \leq \frac{\mu}{n} \leq \frac{h[\gamma - \alpha - (\delta_G - \delta_P)k]}{2(h-1)} = hm_P(k) = m_G(k).$$

But the first inequality can be disregarded since one can show that

$$D\left(\frac{r_G(k) + r_P(k)}{2}, r_G(k)\right) \geq 0$$

if and only if

$$\frac{\mu}{n} \geq \frac{\gamma - \alpha - (\delta_G - \delta_P)k}{2} > \frac{h[\gamma - \alpha - (\delta_G - \delta_P)k]}{2(h+1)}. \quad\blacksquare$$

Proof of Proposition 2 Differentiating the left hand side of condition (20) with respect to r_j, and using the demand function (3), gives the following first order condition:

$$-\frac{k}{\mu} + \frac{1-\pi}{1+\rho}\left[\frac{\gamma - r_j + k(1+\gamma)}{\mu} - \left(\frac{1}{n} + \frac{r_j - r_P}{\mu}\right)\right] = 0.$$

Solving for r_j in this expression and using the definition (9) of $r_G(k)$ we conclude that the deviating bank j will offer the deposit rate

$$r_j = \frac{r_G(k) + r_P}{2}.$$

28

Substituting this result back into (20) and rearranging then gives

$$\frac{1-\pi}{1+\rho}\left(\frac{(r_G(k)-r_P)^2}{4\mu}+\frac{r_G(k)-r_P}{n}+\frac{\mu}{n^2}\right)+\frac{1-\pi}{1+\rho}V_P\leq V_P.$$

Substituting V_P from (7) into this expression, and using the definition (15) of h and the fact that by (9) and (19) we have $r_G(k)-r_P=\gamma-\alpha-\delta_G k$, the condition for the existence of a prudent equilibrium simplifies to

$$\frac{1}{4}(\gamma-\alpha-\delta_G k)^2+(\gamma-\alpha-\delta_G k)\frac{\mu}{n}-(h^2-1)\left(\frac{\mu}{n}\right)^2\leq 0.$$

Since $h^2>1$, it is immediate to check that for $\gamma-\alpha-\delta_G k\geq 0$ this inequality will be satisfied if either

$$\frac{\mu}{n}\geq\frac{\gamma-\alpha-\delta_G k}{2(h-1)}=m'_P(k),$$

or

$$\frac{\mu}{n}\leq-\frac{\gamma-\alpha-\delta_G k}{2(h+1)}.$$

But this latter case can be disregarded since it violates the assumption $\mu/n>0$.

Differentiating the left hand side of condition (21) with respect to r_j, and using the demand function (3), gives the following first order condition:

$$\frac{\alpha-r_j}{\mu}-\left(\frac{1}{n}+\frac{r_j-r_G(k)}{\mu}\right)=0.$$

Solving for r_j in this expression and using the definition (19) of r_P we conclude that the deviating bank j will offer the deposit rate

$$r_j=\frac{r_G(k)+r_P}{2}.$$

Substituting this result back into (21) and rearranging then gives

$$\frac{1}{1+\rho}\left(\frac{(r_G(k)-r_P)^2}{4\mu}-\frac{r_G(k)-r_P}{n}+\frac{\mu}{n^2}\right)+\frac{1}{1+\rho}V_G\leq V_G.$$

Substituting V_G from (11) into this expression, and using the definition (15) of h and the fact that $r_G(k)-r_P=\gamma-\alpha-\delta_G k$, the condition for the existence of a gambling equilibrium simplifies to

$$\frac{1}{4}(\gamma-\alpha-\delta_G k)^2-(\gamma-\alpha-\delta_G k)\frac{\mu}{n}+\left(1-\frac{1}{h^2}\right)\left(\frac{\mu}{n}\right)^2\leq 0.$$

Since $h^2 > 1$, it is immediate to check that for $\gamma - \alpha - \delta_G k \geq 0$ this inequality will be satisfied if

$$\frac{h(\gamma - \alpha - \delta_G k)}{2(h+1)} \leq \frac{\mu}{n} \leq \frac{h(\gamma - \alpha - \delta_G k)}{2(h-1)} = h m_P'(k) = m_G'(k).$$

But the first inequality can be disregarded since one can show that

$$D\left(\frac{r_G(k) + r_P}{2}, r_G(k)\right) \geq 0$$

if and only if

$$\frac{\mu}{n} \geq \frac{\gamma - \alpha - \delta_G k}{2} > \frac{h(\gamma - \alpha - \delta_G k)}{2(h+1)}. \quad \blacksquare$$

Proof of Proposition 3   To characterize the prudent equilibrium region there are three cases to consider. First, if $\overline{r} \leq r_P = \alpha - \mu/n$, it is immediate to check that $r_j = \overline{r}$ maximizes the left hand side of condition (25), which then becomes

$$\frac{1-\pi}{1+\rho}(\gamma - \overline{r})\frac{1}{n} + \frac{1-\pi}{1+\rho}V_P(\overline{r}) \leq V_P(\overline{r}).$$

Substituting $V_P(\overline{r}) = (\alpha - \overline{r})/\rho n$ into this expression and solving for $\overline{r}$ gives $\overline{r} \leq \overline{r}_P$, where $\overline{r}_P$ is given by (27). Second, if $r_P < \overline{r} < (r_P + r_G)/2$, it is also immediate to check that $r_j = \overline{r}$ maximizes the left hand side of condition (25), which then becomes

$$\frac{1-\pi}{1+\rho}(\gamma - \overline{r})\left(\frac{1}{n} + \frac{\overline{r} - r_P}{\mu}\right) + \frac{1-\pi}{1+\rho}V_P(\overline{r}) \leq V_P(\overline{r}).$$

Substituting $V_P(\overline{r}) = \mu/\rho n^2$ into this expression and rearranging gives

$$(\gamma - \overline{r})(\overline{r} - \alpha) + 2(\gamma - \overline{r})\frac{\mu}{n} - h^2\left(\frac{\mu}{n}\right)^2 \leq 0.$$

It is immediate to check that this inequality will be satisfied if either

$$\frac{\mu}{n} \geq \frac{1}{h^2}\left(\gamma - \overline{r} + \sqrt{(h^2 - 1)(\gamma - \overline{r})(\overline{r} - \overline{r}_P)}\right) = m_P(\overline{r}), \quad (29)$$

or

$$\frac{\mu}{n} \leq \frac{1}{h^2}\left(\gamma - \overline{r} - \sqrt{(h^2 - 1)(\gamma - \overline{r})(\overline{r} - \overline{r}_P)}\right).$$

But this latter case can be disregarded since one can show that for $\overline{r} \leq \alpha$ it violates the assumption $r_P = \alpha - \mu/n < \overline{r}$, and for $\overline{r} > \alpha$ it violates the assumption $\mu/n > 0$.

Finally, if $\overline{r} \geq (r_P + r_G)/2$, by the proof of Proposition 1 we know that a prudent equilibrium exists if $\mu/n \geq m_P$.

To characterize the gambling equilibrium region there are also three cases to consider. First, if $\overline{r} \leq r_P = \alpha - \mu/n$, it is immediate to check that $r_j = \overline{r}$ maximizes the left hand side of condition (26), which then becomes

$$\frac{1}{1+\rho}(\alpha - \overline{r})\frac{1}{n} + \frac{1}{1+\rho}V_G(\overline{r}) \leq V_G(\overline{r}).$$

Substituting $V_G(\overline{r}) = (1 - \pi)(\gamma - \overline{r})/(\rho + \pi)n$ into this expression and solving for $\overline{r}$ gives $\overline{r} \geq \overline{r}_P$, where $\overline{r}_P$ is given by (27). Second, if $r_P < \overline{r} < r_G$, differentiating the left hand side of condition (26) with respect to r_j and solving the corresponding first order condition gives $r_j = (\overline{r} + r_P)/2$. Substituting this result back into (26) we get

$$\frac{1}{1+\rho}\left(\alpha + \frac{\overline{r} + r_P}{2}\right)\left(\frac{1}{n} + \frac{r_P - \overline{r}}{\mu}\right) + \frac{1}{1+\rho}V_G(\overline{r}) \leq V_G(\overline{r}).$$

Substituting $V_P(\overline{r}) = (1 - \pi)(\gamma - \overline{r})/(\rho + \pi)n$ into this expression and rearranging gives

$$h^2(\overline{r} - \alpha)^2 - 2[h^2(\overline{r} - \alpha) + 2(\gamma - \overline{r})]\frac{\mu}{n} + h^2\left(\frac{\mu}{n}\right)^2 \leq 0.$$

It is immediate to check that this inequality will be satisfied if

$$\frac{1}{h^2}\left(h^2(\overline{r} - \alpha) + 2(\gamma - \overline{r}) - 2\sqrt{(h^2 - 1)(\gamma - \overline{r})(\overline{r} - \overline{r}_P)}\right) \leq \frac{\mu}{n}$$

and

$$\frac{\mu}{n} \leq \frac{1}{h^2}\left(h^2(\overline{r} - \alpha) + 2(\gamma - \overline{r}) + 2\sqrt{(h^2 - 1)(\gamma - \overline{r})(\overline{r} - \overline{r}_P)}\right) = m_G(\overline{r}). \qquad (30)$$

But the first inequality can be disregarded since one can show that for $\overline{r} \leq \alpha$ it is implied by the assumption $r_P = \alpha - \mu/n < \overline{r}$, and for $\overline{r} > \alpha$ one can show that

$$D\left(\frac{\overline{r} + r_P}{2}, \overline{r}\right) \geq 0$$

if and only if

$$\frac{\mu}{n} \geq \overline{r} - \alpha > \frac{1}{h^2}\left(h^2(\overline{r} - \alpha) + 2(\gamma - \overline{r}) - 2\sqrt{(h^2 - 1)(\gamma - \overline{r})(\overline{r} - \overline{r}_P)}\right).$$

Finally, if $\overline{r} \geq r_G$, by the proof of Proposition 1 we know that a gambling equilibrium exists if $\mu/n \leq m_G$.

To conclude the proof it is immediate to check that the definitions of $m_P(\overline{r})$ and $m_G(\overline{r})$ in (29) and (30) imply $m_P(\overline{r}_P) = m_G(\overline{r}_P)$ and $m_P(\overline{r}) < m_G(\overline{r})$ for $\overline{r} > \overline{r}_P$. ∎

References

[1] Basel Committee on Banking Supervision (2001): *Overview of The New Basel Capital Accord.*

[2] Besanko, D., and G. Kanatas (1996): "The Regulation of Bank Capital: Do Capital Standards Promote Bank Safety?," *Journal of Financial Intermediation,* 5, 160-183.

[3] Blanchard, O. (1985), "Debt, Deficits, and Finite Horizons," *Journal of Political Economy*, 93, 223-247.

[4] Chiappori, P.-A., D. Pérez-Castrillo, and T. Verdier (1995): "Spacial Competition in the Banking System: Localization, Cross Subsidies, and the Regulation of Deposit Rates," *European Economic Review*, 39, 889-918.

[5] Fudenberg, D., and J. Tirole (1991): *Game Theory*, MIT Press.

[6] Furlong, F. T., and M. C. Keeley (1989): "Capital Regulation and Bank Risk-Taking: A Note," *Journal of Banking and Finance,* 13, 883-891.

[7] Genotte, G., and D. Pyle (1991): "Capital Controls and Bank Risk-Taking," *Journal of Banking and Finance,* 15, 805-824.

[8] Hellmann, T. F., K. C. Murdock, and J. E. Stiglitz (2000): "Liberalization, Moral Hazard in Banking, and Prudential Regulation: Are Capital Requirements Enough?" *American Economic Review,* 90, 147-165.

[9] Keeley, M. C. (1990): "Deposit Insurance, Risk, and Market Power in Banking," *American Economic Review,* 80, 1183-1200.

[10] Matutes, C., and X. Vives (1996): "Competition for Deposits, Fragility, and Insurance," *Journal of Financial Intermediation*, 5, 184-216.

[11] Matutes, C., and X. Vives (2000): "Imperfect Competition, Risk-Taking, and Regulation in Banking," *European Economic Review*, 44, 1-34.

[12] Myers, S. C., and N. S. Majluf (1984): "Corporate Financing and Investment Decisions when Firms Have Information that Investors Do Not Have," *Journal of Financial Economics*, 13, 187-221.

[13] Perotti, E., and J. Suarez (2002): "Last Bank Standing: What Do I Gain If You Fail?," *European Economic Review,* 46, 1599-1622.

[14] Repullo, R. (2000): "Who Should Act as Lender of Last Resort? An Incomplete Contracts Model," *Journal of Money, Credit and Banking*, 32, 580-605.

[15] Rochet, J.-C. (1992): "Capital Requirements and the Behaviour of Commercial Banks," *European Economic Review,* 36, 1137-1170.

[16] Salop, S. (1979): "Monopolistic Competition with Outside Goods," *Bell Journal of Economics,* 10, 141-156.

[17] Suarez, J. (1994): "Closure Rules, Market Power and Risk-Taking in a Dynamic Model of Bank Behaviour," LSE Financial Markets Group Discussion Paper No. 196.