Career Concerns and Contingent Compensation

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Abstract

This paper considers a two-period model in which managers have superior information about their ability to forecast the realization of given investment projects. Firms compete for managers by offering short-run contracts. As future salaries depend on current play through its impact on managerial reputation, managers' investment decisions are affected by their concern for their future careers. We analyze the interaction between these implicit incentives, created by managers' career concerns, and the explicit incentives made possible by contingent compensation. We show that managers' career concerns create perverse incentives that are robust to the introduction of contingent contracting. We also find that while managerial compensation is monotonically increasing in profit at date 2, it is not at date 1. Two numerical exercises relate the implications of our results to the literature on the link between pay and performance. In line with empirical findings, we find that: i) the pay-performance sensitivity is highest in the final period of managers' employment; ii) higher pay-performance sensitivities are associated with a lower variance of profits.

1 Introduction

The literature on managerial compensation has dedicated a close attention to the trade-off in the provision of incentives and insurance arising because of managers' risk and effort aversions. In this paper we depart from this view and focus on the impact of managers' talent on managerial compensation and incentives. In particular we consider a market for managerial services and analyze the interplay of implicit (reputational) and explicit (compensation) incentives of managers. A manager is assumed to have private information about his ability to distinguish a profitable investment project from an unprofitable one. Firms have imperfect knowledge about this ability, and therefore the investment decision of a manager and its final outcome are used to update the estimates of his skills. Because of this dependance on the past, a manager's choices are influenced by his preoccupation with his future salaries, a component which we will refer to as career or reputational concern.

This paper studies how, when, and to what extent short-run contingent contracts can offset the distortions created by career concerns and analyzes the implications of this on investment decisions, contracts, and the link between pay and performance measures. To address these issues we construct a two-period model of managerial compensation with career concerns and characterize equilibrium contracts and investment decisions.

We find that managers' reputational concerns create perverse incentives that can be only partially offset by contingent compensation. In particular we show that (i) investment decisions in period 2 are efficient, but investment decisions in period 1 depend on managers' initial reputation: A manager will overinvest when his reputation is low, invest efficiently when it is high and underinvest when it is intermediate; (ii) managerial compensation is monotonically increasing in performance in period 2 but not in period 1: Generous compensation in the face of poor results may be required to induce managers to take appropriate investment actions that may compromise their future careers.[1]

We explore the implications of our results by performing two numerical exercises on the joint distribution of equilibrium salaries and performance measures with the goal to verify whether the predictions of the model are consistent with existing empirical studies.

Our first numerical exercise finds that the pay-performance sensitivity is higher in the

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[1]This result is a possible explanation for frequently observed contractual provisions, as, for instance, "golden parachutes," through which top managers enjoy very generous compensation in the face of such negative events as dismissals and hostile takeovers.

final period of managers' employment than in the initial one, in line with Gibbons and Murphy's (1992) empirical finding that the pay-performance elasticities of CEO's of large US companies increase as they approach retirement. Our second numerical exercise demonstrates that the pay-performance sensitivity is decreasing in the firm's variance of profits, in line with the empirical findings of Aggarwal and Samwick (1999).

The theoretical models that motivated the previous empirical research focused their attention on the optimal design of incentives for homogenous risk averse managers to exert appropriate levels of effort.[2] In contrast, our model proposes a different perspective on the agency relationship between managers and shareholders–one that is centered on competing firms that try to provide incentives to make appropriate decisions for risk neutral managers who may have private information about their forecasting ability. Although we share the general view that managerial risk aversion is a fundamental component of the agency relationship between management and ownership, in this paper we abstract from it to focus on the impact of career concerns and asymmetric information on managerial contracting. Our numerical exercises show that our predictions are not invalidated by documented empirical regularities, and we therefore consider our model as a possibly complementary view of the nature of the agency relationship between management and ownership.

Our work is related to the literature on managerial career concerns initiated by Fama (1980) and Holmström (1982). Fama (1980) argued that a manager's career concern provides incentives to make optimal decisions over and above the predictions of a static model. Holmström (1982) showed that career concerns are not necessarily sufficient to align managers' interests with firms' objectives. This work started a research agenda that has mainly concentrated on how reputational concerns may provide incentives to (partially) solve static inefficiencies.[3]

Holmström and Ricart i Costa (1986) were among the first to analyze the possibly perverse incentives of reputational concerns: "[...] reputation is the source rather than the resolution of incentive problems."[4] Since in our model, without career concerns efficiency

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[2]Several authors, including, Holmström and Ricart i Costa (1986) and Murphy (1999), have in fact questioned the importance of managerial effort aversion in the agency relationship between shareholders and top executives.

[3]Among others, this line of research has been explored by Diamond (1989), Gibbons and Murphy (1992), Meyer and Vickers (1997), and Dewatripont, Jewitt, and Tirole (1999a, 1999b).

[4]Holmström and Ricart i Costa (1986), page 837, fn. 2. Similar arguments have been defended by several other authors and notably by Ricart i Costa, (1988, 1989), Kanodia, Bushman, Dickhaut (1989), Sharfstein

would be attained in equilibrium, in spirit our work is more closely related to research in this latter direction.

Our work also relates to research on the impact of career concerns on managerial attitudes towards investment. Hirshleifer and Thakor (1992), Kanodia, Bushman and Dickhaut (1989) and Zwiebel (1995), among others, have argued, that career concerns result in managerial conservatism and therefore, in systematic underinvestment. Other authors have maintained that career concerns lead to excessive risk taking or overinvestment (see, e.g., Ricart i Costa (1988), or Holmström and Ricart i Costa (1986), for sufficiently low risk aversion). Prendergast and Stole (1996) have argued that managers tend to be prone to exaggeration in the beginning of their careers and to conservatism at the end. Our result is that in the beginning of a manager's career his attitude to investment depends on his initial reputation: when his initial reputation is bad, overinvestment arises, for intermediate values underinvestment results, while a manager with a sufficiently good initial reputation takes efficient investment decisions.

This paper shares the view of another strand of literature, including Jeon (1998) and Kanodia, Bushman and Dickhaut (1989), that focuses on forecasting as the main input provided by managers and the desire to build a reputation for accuracy as the source of the agency problem in an asymmetric information setting. Our work differs from theirs, and in fact from virtually all of the literature on career concerns, in that it takes into account the possibility of contingent compensation to provide explicit incentives to either reinforce or counterbalance the implicit incentives provided by career concerns.[5]

Our results also provide implications on the relationship between managerial pay and measures of firm performance and our contributions are best understood in relationship to existing work by Gibbons and Murphy (1992) and Aggarwal and Samwick (1999).

Gibbons and Murphy (1992) see career concerns as providing managers with incentives to work hard in the early stages of their careers. Given managers are risk averse, making

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and Stein (1990), Hirshleifer and Thakor (1992), Prendergast (1993), Zwiebel (1995), Prendergast and Stole (1996), Jeon (1998), and Morris (2001).

[5] Some of the previously mentioned papers do allow for the possibility of current compensation being contingent on current performance but restrict the way in which this can happen, by linking current compensation linearly to current profits as in Gibbons and Murphy (1992), Prendergast and Stole (1996) and Kanodia, Bushman and Dickhaut (1989). Prendergast (1993) is the only notable exception in considering the possibility of making compensation contingent on current performance to create incentives to exert forecasting effort. Since this is seen to create incentives to misreport the findings, however, mechanism design in Prendergast (1993) is the solution to the shirking problem but also the source of a problem of dishonest reporting.

compensation responsive to random performance is costly and an extensive use of contingent compensation is made only when necessary, i.e., in the late stages of a manager's career, when the incentives provided by career concerns are fading. The implication of this is that the pay-performance sensitivity increases as retirement approaches.

Our paper is an attempt to show that, while career concerns may be responsible for the increase in pay-performance sensitivity, the reasons behind this may be altogether different. We show that competition ensures that pay is monotonic in performance in the second period, and we view career concerns as creating perverse incentives that require nonmonotonic pay schedules in the early stages of a manager's career.

Aggarwal and Samwick (1999) study the relationship between pay of US top executives and their firms' performance and in particular focus on how the pay-performance sensitivity varies with the variance of firms' profits. Their work is motivated by a classical principal-agent model in which a risk averse manager is given incentives to exert high effort through a contract that makes his pay linearly dependent on his firm's profit. Managerial risk aversion implies that the slope of the optimal contract is decreasing in the variance of firm's profit, a prediction validated by their empirical analysis.

Our second numerical exercise shows that the joint distribution of salaries and profits generated by our model is in line with Aggarwal and Samwick's (1999) empirical findings in that the pay-performance sensitivity is decreasing in the variance of firms' profits, but our model proposes a different explanation for this regularity. Competition ensures that managers with a high reputation at the end of their careers are offered contracts with salary payments that are very sensitive to performance. Because these managers also generate low variances of profits, a negative association between firms' profit variances and pay-performance sensitivity arises.[6] In this sense our results question the direction of causality maintained in the empirical studies (from variance to pay-performance sensitivity) and propose a new standpoint on the problem.

The main contributions of this paper derive from its focus on contingent compensation. On one side we verify that the perverse incentives of career concerns are robust to the introduction of contingent contracting. On the other we are able to characterize the impact of

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[6]Unlike the classical literature that sees firms' profits variance as exogenous and independent of the manager, we assume that all firms are ex-ante identical but the distribution of net profits and, therefore, their variance depends on the type of the manager–both directly, through his forecasting ability, and indirectly, through his equilibrium investment decisions.

contingent contracting on equilibrium outcomes, and in particular, on investment decisions and on the link between managerial pay and firms' performance.

The paper is organized as follows. Section 2 presents the model. Section 3 introduces the equilibrium concept used in the paper. Section 4 offers a characterization of the equilibrium. Section 5 studies the implications of our analysis on the link between managerial pay and firms' performance measures. Section 6 concludes.

2   The Model

All firms are risk neutral and each has an investment project available which has a revenue of $z > 0$ with probability p, and 0 with probability $1 - p$. We normalize the cost of the investment project to 1 and assume that the investment project is ex-ante profitable, $pz - 1 > 0$.[7] Firms have available ex-ante identical investment projects.

Managers are risk neutral[8] and live for two periods, $t = 1, 2$. Each manager has an innate ability to forecast the realization of a given investment project. If employed, a manager only has to decide whether to invest (I) or not (N) in the project available to the firm.[9] In focusing on investment decisions only, we subscribe to the widely held view that "[i]n a managerial context [...] effort is only part of the overall incentive problem" and that it is more important to "worry about how effective [...] managers are at making decisions".[10]

For simplicity, we assume that only two types of managers exist, good and bad, $\tau \in \{G, B\}$. A good manager is always able to forecast the realization of the investment project, whereas a bad manager is never able to do so. Thus, before making the decision to invest or not a manager receives a signal $\sigma \in \{V, L, H\}$, where V is interpreted as a void signal and L and H as the low and high signal, respectively. The probability of the project having the high return ($z > 0$) conditional on the received signal will be

$$\Pr(z \mid \sigma) = \begin{cases} p & \text{if } \sigma = V \\ 0 & \text{if } \sigma = L \\ 1 & \text{if } \sigma = H \end{cases}.$$

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[7] Qualitatively similar results are obtained in the case in which $pz - 1 < 0$.

[8] Since risk neutrality makes the manager indifferent among contracts with the same expected value, many different equilibrium contracts will exist. For this reason, in section 4.2.2 we focus on the minimum variance contract that is arbitrarily close to the contract that would be preferred by a manager with an arbitrarily low degree of risk aversion.

[9] A more realistic description would be one in which the manager has to decide whether to invest in a safe or risky project. We have chosen not to depart from the typical description in the literature. As is clear, this alternative interpretation only requires mechanical rewording.

[10] Holmström and Ricart-i-Costa (1986), page 835.

In other words, while signals H and L ensure, respectively, the success or the failure of the investment project, the void signal, V, provides no additional information and the conditional probability of success is, therefore, equal to the prior, p.[11] A good manager receives signal H with probability p, and L with probability $1-p$, whereas a bad manager gets signal V with probability 1. At the beginning of the first period there is no asymmetric information about a manager's type so that a given manager is commonly believed to be good with probability μ and bad with probability $1-\mu$. Once a manager is employed, however, he either gets signal H or L or he gets signal V and in this way he learns his type with probability 1.

We denote the observable final outcome of the investment process at date $t = 1, 2$, by $P^t \in \{N, F, S\}$, where N indicates that no investment took place and F and S indicate that the investment was carried out and it was, respectively, a failure or a success. An investment action profile for a manager at date t is a vector $i^t = (i_V^t, i_L^t, i_H^t) \in \{N, I\}^3$ where i_V^t, i_L^t, and i_H^t denote the decision to invest (I) or not (N) at date $t = 1, 2$ when the signal received by the manager is respectively V, L, or H.

Firms offer one-period contracts to managers. Given one of the main goals of this paper is to show that reputational concerns generate efficiency losses, we consider contracts that are contingent on the realization of the investment project, $P^t \in \{N, F, S\}$. This seems important, as it guarantees that these losses are robust to the introduction of optimal contracting. A contract is therefore a triple $w^t = \left(w_N^t, w_F^t, w_S^t\right)$, with the constraint that no salary can be negative, $w_{P^t} \geq 0$, $P^t \in \{N, F, S\}$, $t = 1, 2$.[12] In other words, while we only allow short run contracts, we allow salary payments to be made after the realization of the investment process and we therefore introduce the possibility for compensation to depend also on contemporaneous measures of performance.

Given contracts are short run and the investment projects are serially independent, firms' time horizons are irrelevant and they can be treated as short-run profit maximizers. Managers live for two periods and they maximize their expected discounted lifetime salary

$$U = E\left[w^1(\mu) + \delta w^2(\mu^2)\right],$$

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[11] Managers with high ability are often described in the literature as being able to generate high expected return investment projects, i.e., as being able to come up with good ideas. In contrast to this, we refer to managerial ability as the ability to forecast the realization of a given project: all investment projects are drawn from the same distribution, regardless of the manager's ability, but different managers may have different abilities to forecast their realizations.

[12] We use superscripts for time indices and subscripts for realizations of random variables.

where $\mu^2 = E\left[\mu \mid P^1\right]$ is the probability of the manager being good in period 2 conditional upon the realization of the investment process at time 1, P^1, and where $\delta \in \mathbb{R}_+$ is the discount factor.[13]

We finally make the assumption that a firm can function without a manager, but that in this case it will get signal V with probability 1. Given investment projects are assumed to be ex-ante profitable, investing is the optimal decision for a firm with no manager (and therefore, without a signal on the profitability of the investment project). This implies that a firm with no manager gets an expected profit of $pz - 1$.[14] For managers to care about their reputation, it is necessary to assume that they appropriate at least part of their reputational rents. For this to happen we need to assume that they are scarce. To simplify notation and wording in the paper we will consider the case in which two firms compete for every single manager. In particular, we assume that every manager is offered at most a countable set of contracts of the form $\left(w_N^t, w_F^t, w_S^t\right) \in \mathbb{R}_+^3$ by each firm and chooses one contract (if any) out of them. All our results generalize to the case in which the measure of the set of managers is lower than the measure of the set of firms and firms are allowed to make offers to all managers.[15]

In the following we summarize the extensive form of the game.

1. Period 1.

   N *Nature* chooses the type of manager, G with probability μ and B with probability $1 - \mu$, and (independently) the realization of the investment project, S with probability p and F with probability $1 - p$.

   F Without observing nature's choices, each of the two *firms* competing for a given manager (commonly known to be good with probability μ) offers him at most a countable set of contracts, each of them of the form $\left(w_N^1, w_F^1, w_S^1\right) \in \mathbb{R}_+^3$.

   M Without observing nature's choices, the *manager* either accepts an offer or rejects them all.

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[13]While this is not important for our results, we also allow cases in which $\delta > 1$, as they can represent situations in which the factor of growth of the scale of the second period investment project with respect to the first period one more than compensates the manager's time preference factor.

[14]The qualitative nature of the results of the paper would not change if we made the assumption that the reservation level for the firm is 0 rather than $pz - 1$, i.e., if we assumed that a manager is an essential input in the production process.

[15]In this case firms are allowed to offer different countable sets of contracts to managers with different beginning of period probabilities of being good.

R If the manager *rejects* all offers, play restarts at the beginning of period 2.

A If the manager *accepts* an offer, he is hired.

  σ The manager receives a signal, $\sigma^1 \in \{V, L, H\}$.

  i The manager decides whether to invest or not in the given project, $i \in \{I, N\}$.

  P The manager's play and the realization of the investment project in case the manager decided to invest are observed, $P^1 \in \{N, F, S\}$. The firm pays the manager a salary according to the contract accepted by the manager.

2. Period 2 has the same structure as period 1 with one exception. Given that the type of the manager is the same in both periods, having observed σ^1 in period 1, the manager knows his type when he considers whether to accept any second period offer, while firms do not have access to this information. Second period realization of the investment project is independent of the first period one.

We now discuss our modeling choices and the consequences of making different assumptions.

The assumption that no long term contracts are available is necessary for our results. If a long term binding contract were feasible at the beginning of a manager's working life, when no asymmetric information exists, it would be possible to design it in such a way that managers have no incentives to distort their investment decisions away from the optimum. We rule out such binding contracts, as the information revealed after the first period creates incentives for at least one of the parties to break the contract.

In this paper we consider a screening model, i.e., a situation in which the uninformed party (firms) make offers. Our results are not sensitive to this choice. In a previous version of this paper, Caruana and Celentani (1999), we studied the same problem in a signaling setting, i.e., in the case in which the informed party (the manager) makes contract offers. In this case the sets of accepted contracts and equilibrium path investment actions are the same as in the present paper as long as the second period continuation equilibria (i) satisfy full extraction of the surplus by the manager, (ii) are Pareto optimal in terms of the investment action profile played on the equilibrium path, and (iii) survive the *intuitive criterion*. An alternative formulation of a signaling environment is one in which, in the spirit of Maskin

and Tirole (1992), the manager offers a menu of contracts, the firm accepts or rejects the menu, and finally the manager chooses one contract out of the previously proposed (and accepted) set. With this formulation in all equilibria the set of accepted contracts and the equilibrium path investment decisions would be the same as in the present paper.

This paper takes the view that it is important to study the implications of career concerns when asymmetric information on managerial ability exists. We assume that information is initially symmetric and that an asymmetry arises only in the course of the first period. We consider this assumption realistic, but it is also important to remark that the equilibrium outcomes of our model also arise when information is asymmetric from the beginning, although in this latter case the set of equilibria is larger.

3 Strategies and Equilibrium Concept

In each of the two periods, two firms compete for a manager believed to be good with probability $\mu \in [0, 1]$ by offering each at most a countable set of short-run (one period) contracts, $w(\mu) \in \mathbb{R}^3_+$, conditioning on the public history of the game.

In each of the two periods, the manager accepts a contract or none out of the sets of contracts offered to him by the two firms. If the manager accepts, he receives a private signal on the profitability of the project, $\sigma^t \in \{V, L, H\}$. Conditioning on the public and the private history of the game, the manager then decides whether to invest or not in the given project.

Consider a manager who is believed to be good with probability μ at the beginning of period 1. His first period investment strategy, after having accepted offer w^1, can be denoted as

$$i^1\left(\mu, w^1\right) = \left(i^1_V\left(\mu, w^1\right), i^1_L\left(\mu, w^1\right), i^1_H\left(\mu, w^1\right)\right) \in \{I, N\}^3$$

where $i^1_{\sigma^1}\left(\mu, w^1\right) \in \{I, N\}$ denotes the manager's investment decision conditional on private signal $\sigma^1 \in \{V, L, H\}$. Similarly, the manager's second period investment strategy, after having accepted contract w^1 (in period 1), after receiving signal σ^1 (in period 1), given P^1, the outcome of the investment decision process in period 1, and after having accepted the second period offer w^2 can be written as

$$i^2\left(\mu, w^1, \sigma^1, P^1, w^2\right) = \left(i^2_V\left(\ldots\right), i^2_L\left(\ldots\right), i^2_H\left(\ldots\right)\right) \in \{I, N\}^3$$

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where $i_{\sigma^2}^2 (\dots) \in \{I, N\}$ denotes the manager's investment decision conditional on private signal $\sigma^2 \in \{V, L, H\}$.

Given that no additional use of notation will be made, we choose not to provide a full description of strategies and strategy spaces. Also, for notational convenience we will occasionally omit arguments whenever this cannot cause any confusion.

The equilibrium concept we use is perfect Bayesian equilibrium. To ensure the existence of such equilibria we will assume the following standard tie-breaking rules: (i) whenever indifferent between the two investment actions (I and N), the manager will play the one with the higher expected profit; (ii) whenever indifferent between accepting a contract or rejecting all, the manager will accept a contract.

As will become clear later on, it is possible that in equilibrium certain realizations of the investment process in period 1 are observed with probability 0. Given Bayesian updating is not defined after such events, we construct beliefs after such zero-probability realizations making the assumption that each type of manager is equally likely to make a mistake in playing his investment strategy.

4 Equilibrium Characterization

The following Lemma presents two simple results that are useful for the subsequent analysis. Let $E[\pi \mid i, \mu]$ denote the expected profit gross of salary payments as a function of the investment action profile $i \in \{I, N\}^3$ and of the beginning of period probability of the manager being good, $\mu \in [0, 1]$.

LEMMA 1   1. On the equilibrium path $i^t \in \{(I, I, I), (N, N, I), (I, N, I)\}$, $t = 1, 2$.

    2. $E[\pi \mid (I, N, I), \mu] = (pz - 1) + \mu (1 - p) > E[\pi \mid i, \mu]$, for $i \in \{(I, I, I), (N, N, I)\}$.

    3. $E[\pi \mid (N, N, I), \mu] = \mu p (z - 1) \geq (pz - 1) = E[\pi \mid (I, I, I), \mu]$ if and only if

$$z \leq \frac{1 - \mu p}{p (1 - \mu)}. \tag{1}$$

PROOF. For part 1 suppose that in equilibrium a first period offer w^1 is made such that

$$i^1 \left(\mu, w^1\right) \notin \{(I, I, I), (N, N, I), (I, N, I)\}.$$

Because $w^1 \in \mathbb{R}_+^3$ and given the expected gross profit from $i \notin \{(I, I, I), (N, N, I), (I, N, I)\}$ is strictly less than $pz - 1 > 0$, the net expected payoff to the firm offering this contract is

strictly less than $pz - 1 > 0$. Given a firm with no manager earns $pz - 1 > 0$, withdrawing all offers is a profitable deviation for the firm and a contradiction arises. The same arguments applies for second period offers. Parts 2 and 3 are established through straightforward computations. ∎

Part 1 of Lemma 1 establishes that the investment action profile played on the equilibrium path in any of the two periods has to be one of (I, I, I), (N, N, I), or (I, N, I). Part 2 states that, among the three previous investment action profiles, (I, N, I) always generates the highest expected profit gross of salary payments. Part 3 finally establishes (1) as a necessary and sufficient condition for the expected profit gross of salary payments generated by (N, N, I) to be no lower than the one generated by (I, I, I).

With Lemma 1 we proceed to subsection 4.1 which solves for second period continuation equilibria. In subsection 4.2, we then move back to the first period and characterize the equilibrium path investment decisions and managerial contracts.

4.1 Second Period

At the beginning of the second period the manager has already privately learned his type. Let $\mu_2 \in [0, 1]$ denote the firms' (posterior) probability assessment that he is the good manager.[16] The following Proposition characterizes second period continuation equilibria.

PROPOSITION 1 *Assume that at the beginning of period 2 firms believe that the manager is good with probability $\mu_2 \in [0, 1]$. Then, in any continuation equilibrium:*

1. *The unique contract which is accepted by both types of managers is:*

$$w^2 (\mu_2) = \left(w_N^2 (\mu_2), w_F^2 (\mu_2), w_S^2 (\mu_2) \right) = \left(\frac{\mu_2 (1-p)}{1 + (1-p)\mu_2}, 0, \frac{\mu_2 (1-p)}{p \left(1 + (1-p)\mu_2 \right)} \right).$$

2. *The investment action profile played by the manager on the equilibrium path is:*

$$i^2 = (I, N, I)$$

PROOF. Appendix.

Proposition 1 analyzes the impact of asymmetric information on second period continuation equilibria and shows that they are

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[16] In equilibrium this probability is computed from the prior probability, first period equilibrium strategies, and the realization of the first period investment process. Note, that although the manager learns privately his type, firms may in fact also learn his type from first period play, as shown by Lemma 2, below. In other words, there are cases in which in equilibrium μ_2 is either 0 or 1.

1. *Pooling*: A manager that is believed to be good with probability μ_2 at the beginning of period 2 will accept offer $w^2(\mu_2)$ *regardless of whether he is in fact good or bad.*

2. *Efficient*: On the equilibrium path the manager plays the efficient investment action profile.

To see why second period continuation equilibria are efficient, note that competition for managers tends to lead to surplus maximization (efficiency). In the manager's second period of life, his last, nothing counters this tendency to efficiency as the manager's only incentives are the explicit ones provided by the contract he accepts.

To see that second period continuation equilibria are pooling it is necessary to show that offers that are more attractive to a good manager are not profitable deviations for the firms. The following argument will show that this is the case because, once the incentive compatibility constraints are kept into account, the indifference curves of the two types of managers do not intersect, and offers that are preferred by a good manager are also preferred by a bad manager.

Without loss of generality assume first that $w_F^2(\mu_2) = 0$[17] and consider pairs $\left(w_N^2, w_S^2\right) \in \mathbb{R}_+^2$. Figure 1 depicts the indifference curves for the good and the bad manager in that space keeping into account their incentive compatibility constraints. Given that in the second period the incentives are simply determined by explicit compensation, it is easy to see that so long as $w_S \geq w_N$ (i.e., below the 45 degree line) the typical indifference curve for the good manager is the negatively sloped line represented as U^G as a good manager's expected utility is $pw_S + (1-p)w_N$ (if $w_S < w_N$ the manager would always refrain from investing and his utility would be w_N). The typical indifference curve for the bad manager instead is like the kinked line represented as U^B: Above line (IC) (i.e., whenever $pw_S^2(\mu_2) < w_N^2(\mu_2)$) a bad manager chooses not to invest and gets w_N, whereas below line (IC) (i.e., when $pw_S^2(\mu_2) \geq w_N^2(\mu_2)$) he chooses to invest and gets pw_S.

In equilibrium managers fully extract their expected value and thus firms only make $pz - 1$ in expected terms. In Figure 1 we represent the condition ensuring this as the broken double line (FE). Given this, it is easy to verify that any contract on (FE) but different from the intersection of lines (IC) and the rightmost part of (FE) is such that a profitable deviation for a firm exists (attracting only good managers), whereas no such deviation exists

[17] By this we mean that (i) in equilibrium $w_F^2(\mu_2) = 0$ has to hold and (ii) if no profitable deviation with $w_F^2(\mu_2) = 0$ from a proposed equilibrium exists, then no profitable deviation exists at all.

for the contract at that intersection. It is easy to check that this last contract is the one mentioned in Proposition 1,[18] and the result follows.

An interesting implication of Proposition 1 is that equilibrium wages are increasing in gross profit as they are highest when the outcome is success, intermediate when the outcome is no investment, and lowest when the outcome is failure. For expositional purposes, Figure 2 depicts $w_S^2(\mu_2)$ and $w_N^2(\mu_2)$ for the case in which $p = .6$, a case we will also consider in subsection 4.2.2.

4.2 First Period

Consider the continuation game starting after a manager believed to be good with probability μ accepts first period contract offer w^1. From Proposition 1, and given the tie-breaking rule assumed in section 3, all perfect Bayesian equilibria of this continuation game are identical in first period investment strategies and in the distribution over second period accepted contracts.

Let $i(w^1, \mu)$ denote such a first period investment strategy and let $E[w^2(\mu^2) \mid \tau]$ be the expected value of the accepted second period contract for a manager of type $\tau \in \{G, B\}$ who is believed to be good with probability μ^2 in the beginning of period 2.

For a fixed $w = (w_N, w_F, w_S) \in \mathbb{R}_+^3$ let $E[w \mid i, \mu]$ be the expected salary payment when the manager is commonly believed to be good with probability μ (at the beginning of the period) and when he plays the investment strategy i.

Since there is no asymmetry of information at the beginning of period 1, and because the two firms are competing for one manager, by a standard Bertrand pricing argument, offers that will be accepted with positive probability by the manager have to be such that the proposing firm's expected profits are exactly $pz - 1$.[19] Given a manager accepts the offer that guarantees him the highest lifetime utility, in period 1 the offers that are accepted

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[18]Notice that the contract in Lemma 1 is such that $pw_S^2(\mu_2) = w_N^2(\mu_2)$ and gives the firm expected profit $pz - 1$.

[19]More precisely, if this were not the case, there is an $\varepsilon > 0$ such that a firm that has an equilibrium payoff no higher than the minimum of the two would have an incentive to offer ε more for all realizations of the first period investment process. This is the case as offering ε more for all realizations of the first period investment process implies that the continuation strategies after acceptance by the manager are unchanged.

in equilibrium with positive probability are such that

$$w^1 \in \underset{w^1}{\arg\max} \quad E\left[w^1 + \delta E\left[w^2(\mu^2) \mid \tau\right] \mid i\left(w^1, \mu\right), \mu\right]$$

$$\text{s.t.} \quad w_N^1, w_F^1, w_S^1 \geq 0$$

$$E\left[\pi - w^1 \mid i^1\left(w^1, \mu\right), \mu\right] = pz - 1.$$

The objective function is a manager's lifetime expected discounted salary given the fact that both he and the firm correctly anticipate continuation equilibrium play. In other words, it already incorporates the ex-post incentives that a manager will face after observing his private signal, that is, the reputational consequences of his play. This is taken into account by having the first and second period expected salaries conditioned on the prior probability of the manager being good and on the continuation equilibrium investment strategy $i^1\left(w^1, \mu\right)$.

The following Lemma provides the probability of the manager being good, conditional on the first period investment strategy and the public realization of the first period investment process.

LEMMA 2 *Let $w^1 \in \mathbb{R}_+^3$ be given.*

1. *If $i^1\left(w^1, \mu\right) = (I, I, I)$, then $\mu^2(S) = \mu^2(F) = \mu^2(N) = \mu$.*

2. *If $i^1\left(w^1, \mu\right) = (I, N, I)$, then $\mu^2(S) = \mu$, $\mu^2(F) = 0$, $\mu^2(N) = 1$.*

3. *If $i^1\left(w^1, \mu\right) = (N, N, I)$, then $\mu^2(S) = 1$, $\mu^2(F) = \mu$, $\mu^2(N) = \frac{\mu(1-p)}{1-\mu p}$.*

PROOF. In the second case $\mu^2(.)$ is pinned down by Bayes's rule. In the first case (for $\mu^2(N)$) and the third case (for $\mu^2(F)$) this cannot be done. As was remarked in subsection 3 we make the assumption that the two types of players are equally likely to make a mistake in their investment decision. This assumption implies that if $i^1\left(w^1, \mu\right) = (I, I, I)$, then $\mu^2(N) = \mu$, and if $i^1\left(w^1, \mu\right) = (N, N, I)$, then $\mu^2(F) = \mu$. ∎

Lemma 3 uses Proposition 1 and Lemma 2 to compute continuation equilibria payoffs, a result that simplifies the computation of the first period equilibrium.

LEMMA 3 *For all $w^1 \in \mathbb{R}_+^3$, $E\left[E\left[w^2\left(\mu^2\right) \mid \tau\right] \mid i^1\left(w^1, \mu\right), \mu\right] = \mu(1-p)$.*

PROOF. Since we know from Lemma 1 that in any continuation equilibrium

$$i^1 \left(w^1, \mu \right) \in \{(I, I, I), (N, N, I), (I, N, I)\},$$

we need to consider three different cases, depending on which of the previous investment strategies is played after a given $w^1 \in \mathbb{R}_+^3$. This can be checked through straightforward calculations using $w^2 \left(\mu^2 \right)$ from Proposition 1 and the posterior probabilities of the manager being good under $i^1 \left(w^1, \mu \right) \in \{(I, I, I), (N, N, I), (I, N, I)\}$ provided in Lemma 2. ∎

From Lemma 3, the maximization problem above can be simplified as follows

$$\max_{w^1} \quad E \left[w^1 \mid i \left(w^1, \mu \right), \mu \right] + \delta \mu \left(1 - p \right)$$
$$\text{s.t.} \quad w_N^1, w_F^1, w_S^1 \geq 0$$
$$E \left[\pi - w^1 \mid i^1 \left(w^1, \mu \right), \mu \right] = pz - 1$$

Thus, the first period offers that are accepted are simply those that maximize the manager's first period payoff, because the expected second period payoffs do not depend on the first period accepted offer. This is not to say that career concerns do not have an ex-ante-impact, as $i^1 \left(w^1, \mu \right)$ does depend on career concerns for any given $w^1 \in \mathbb{R}_+^3$ and $\mu \in (0, 1)$. In other words, career concerns are important to the extent that they induce the manager ex-post, i.e., once he has observed his private signal and, consequently, learned his type, to make certain investment decisions rather than others. Since different investment strategies imply different expected net profits, career concerns do play a role in the manager's first period equilibrium accepted offer.

4.2.1 First Period Investment Strategies

Let

$$\widetilde{z} \left(\mu \right) = \left(1 - \mu p \right) \left(\frac{1}{p(1 - \mu)} - \delta \frac{\left(1 - p \right) \left(\mu p^2 - 4\mu p + 3\mu + 1 \right)}{\left(2 - p \right) \left(1 + \mu \left(1 - p \right) \right) \left(1 - 3\mu p + \mu + \mu p^2 \right)} \right)$$
$$\mu^* = \frac{\delta}{2 + \delta - p}.$$

The following proposition provides a complete characterization of the investment action profiles played on the first period equilibrium path.

PROPOSITION 2 *Let* $(\mu, z, \delta) \in [0, 1] \times \left(\frac{1}{p}, \infty \right) \times \mathbb{R}_+$ *be given and let* i^1 *and* w^1 *be part of an equilibrium. Then,*

1. $i^1\left(w^1,\mu\right) = (I,N,I)$, *if and only if* $\mu \in [\mu^*, 1]$.

2. $i^1\left(w^1,\mu\right) = (N,N,I)$ *if and only if* $\mu \in [0,\mu^*)$ *and* $z \in \left[\frac{1}{p}, \widetilde{z}\left(\mu\right)\right]$.

3. $i^1\left(w^1,\mu\right) = (I,I,I)$ *if and only if* $\mu \in [0,\mu^*)$ *and* $z > \widetilde{z}\left(\mu\right)$.

PROOF. Appendix.

The results of Proposition 2 are depicted in Figure 3, where, for a given value of p, we plot in the (μ, z) space the investment action profile played in the first period equilibrium path. Note that Lemma 10 in the Appendix shows that $\widetilde{z}\left(\mu\right)$ is increasing.

The result of Proposition 2 can be summarized in the following terms.

By an equilibrium argument, a manager will always prefer to accept an offer that induces him to play the investment action profile with the highest surplus. If a nonempty set of offers such that, after accepting any element of the set, the manager will play according to the first best profile (I,N,I), in equilibrium the manager will accept an offer from this set and will then play according to (I,N,I).

Suppose now the previous set is empty and assume that the expected surplus of (N,N,I) is larger than that of (I,I,I). By the same argument as above if a nonempty set of offers exists such that after accepting any element of the set the manager will play according to (N,N,I), in equilibrium the manager will accept an offer from this set and will then play according to (N,N,I) (underinvestment). If the previous set is empty, or if the expected surplus of (N,N,I) is smaller than that of (I,I,I), in equilibrium the manager will invest regardless of the signal he gets, (I,I,I) (overinvestment).

Proposition 2 uses the previous arguments to characterize the sets of parameters for which each of the three investment action profiles described above is played in the first period equilibrium path:

1. Suppose a manager plays according to (I,N,I) on the first period equilibrium path. The implicit reputational incentives for the bad manager to deviate are decreasing in μ and the surplus that can be used to offset these reputational incentives through explicit compensation is increasing in μ (and is independent of z). As a consequence (I,N,I) can be played in the first period in equilibrium only when μ is above a given threshold. Proposition 2 provides an explicit calculation of this threshold, μ^*.

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2. Suppose a manager plays according to (N, N, I) on the first period equilibrium path. The implicit reputational incentives for the bad manager to deviate are decreasing in μ and the surplus that can be used to offset these reputational incentives through explicit compensation is increasing in μ and decreasing in z. This implies that these reputational incentives can be countered when μ is high and z is low, or, in other words, in a region lying below an increasing function on the (μ, z) space, $\widetilde{z}(\mu)$. Once again, Proposition 2 provides an explicit calculation of such function, $\widetilde{z}(\mu)$.[20]

By part 1 of Proposition 2 if the initial reputation of the manager is sufficiently good, then in the first period the manager will play the efficient investment action profile. If his initial reputation is not sufficiently good, then, by parts 2 and 3, over- or underinvestment will occur in equilibrium depending on whether the initial reputation of the manager is bad or intermediate, with the region of underinvestment getting smaller the more profitable the investment project is in expected terms (the higher z) and eventually disappearing (when $z \geq \widetilde{z}(\mu^*)$).

The rest of this section completes the analysis of equilibrium by turning attention to first period equilibrium accepted contracts.

4.2.2 Characterization of First Period Contracts

The set of first period contracts that can be accepted in an equilibrium can simply be derived as the set of contracts $w^1 \in \mathbb{R}^3_+$ that are such that the continuation equilibrium first period investment strategy, $i^1(w^1, \mu)$, satisfies the conditions of Proposition 2 and such that in the continuation equilibrium firms break even,

$$E\left[\pi - w^1 \mid i^1\left(w^1, \mu\right), \mu\right] = pz - 1.$$

The assumption of managerial risk neutrality implies that many different first period contracts (with the same expected value) can be accepted by the manager in different equilibria, and that all equilibria are identical in the distribution over investment outcomes and in the expected first period payoff to the manager and the firm. Although the objective of the paper is to focus on career concerns alone, the multiplicity of equilibria in first period accepted contracts is an artificial product of the extreme assumption of risk neutrality. Any

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[20]Since it can be verified that $\widetilde{z}(\mu) < \frac{1-\mu p}{p(1-\mu)}$, there are cases in which the first period investment strategy played in equilibrium is (I, I, I), despite of the fact that it is dominated, not only by (I, N, I), but also by (N, N, I).

arbitrarily small amount of risk aversion would break the tie among all first period contracts with the same expected value. Although we do not reproduce the proof here, it is easy to verify that for any sequence of strictly concave utility functions that tend to a linear utility function, the limit optimal contract in a set of contracts with the same expected value is the one that minimizes variance. For this reason, whenever multiple equilibrium accepted contracts exist, in the rest of the paper we focus on the one with the minimum variance.

To provide an intuition about the results we will consider a numerical example in which we will assume that $\delta = 1$, $p = .6$, and $z = 1.7$. The qualitative nature of the results for this example can be verified to be independent of the parameter values.

First and second period equilibrium contracts are plotted against the beginning of period probability of the manager being good in Figures 4 and 2, respectively.[21] Notice that in this case $z < \widetilde{z}(\mu^*)$, so that there are managers of ex ante types μ such that case 2 of Proposition 2 (underinvestment) arises.

As Figure 4 shows, there are three different regions of first period contracts, each corresponding to one of the three regions mentioned in Proposition 2.

Consider first $\mu \in [0, \widetilde{\mu})$, with $\widetilde{\mu} = 0.34145$. In this region, the prior probability of a manager being good is too low for a contract to exist, in which firms break even and the equilibrium investment decisions are different from $i^1(w^1, \mu) = (I, I, I)$. The equilibrium contract is therefore $w = (0, 0, 0)$, and managerial compensation is constant in firm performance.

In the second region, corresponding to $\mu \in [\widetilde{\mu}, \mu^*)$, $\mu^* = 0.41667$, while the manager's prior probability of being good is not sufficient to have a contract that has firms break even, and such that the efficient investment strategy is played, there is a set of contracts that ensure $i^1(w^1, \mu) = (N, N, I)$. Although by Proposition 2, all such contracts can arise in equilibrium, in Figure 4 we single out the contract that minimizes the variance of compensation.[22] From Figure 4 it is easy to see that for all values of μ in this region $w_N > w_S > w_F$, so that managerial compensation is not monotonically increasing in firms'

[21]While the contracts for the second period can be easily plotted using the results of Lemma 1, the equilibrium contracts for period 1 have been computed by a Matlab program, developed by Ramón Xifré. We use the same program in section 5 below to analyze the aggregate relationship between managerial pay and firm performance. The program is available upon request from the authors.

[22]Since in this region failure never occurs in equilibrium, the value of w_F is in fact irrelevant for the variance of compensation. In Figure 4 we depict the highest w_F that satisfies the incentive compatibility constraints, given w_S and w_N. In other words all values of w_F on or below the levels shown in Figure 4 are equivalent both from the point of view of incentives and from the point of view of the variance of managerial compensation.

profits. Note that $w_N > w_S$ to guarantee that the bad manager (who gets the void signal) prefers not to invest despite his unambiguous reputational incentives to invest (from Lemma 2 in this region $\mu^2(N) < \mu^2(F)$ and $\mu^2(N) < \mu^2(S)$).

In the third region, corresponding to $\mu \in [\mu^*, 1]$, the manager's initial reputation is sufficient for a set of contracts to exist, such that in the continuation equilibrium the manager will play the efficient investment strategy, $i^1(w^1, \mu) = (I, N, I)$. From Figure 4 it is easy to check that in this area $w_F > w_S > w_N$, so that, again, managerial compensation is nonmonotonic in firms' performance. It is easy to verify from Lemma 2 that in this area the expected posterior for a manager who gets the void signal is μp when investing, while it is equal to 1 when not investing. This implies that the bad manager has implicit (reputational) incentives not to invest. To counterbalance these implicit incentives, the contract has to satisfy $w_F > w_N$, as to ensure that the (bad) manager who gets the void signal makes the efficient decision to invest.

5   The Link Between Pay and Performance

The goal of this section is to investigate some implications of our results on the link between managerial compensation and firm performance.

For this purpose we first compute the joint probability distribution over salaries and profit generated by equilibrium play. As was done in the previous section, we cope with the multiplicity of first period equilibrium contracts, by singling out the equilibrium contract with the lowest variance for each value of μ.

We then turn to the link between pay and performance by performing regression analyses on the equilibrium distribution over salaries and profits. Because we our interested in finding out whether the equilibrium prediction of our models are consistent with available evidence, we will try to reproduce the econometric analyses of two works on the link between pay and performance, Gibbons and Murphy (1992) and Aggarwal and Samwick (1999).

In this section we take into account managerial heterogeneity, in the sense that we consider a population of managers with different prior probabilities of being good in the beginning of their careers. We also take the view that, while a manager and the firms competing for him know the manager's idiosyncratic μ, i.e., his idiosyncratic prior probability of being good, the econometrician has no such direct knowledge.

5.1 Career Concerns and the Link Between Pay and Performance

In this subsection we analyze the link between pay and performance by considering separately the joint distribution of compensation and profit for each of the two periods.[23] We consider profits both net and gross of salary payments. We use each of these distributions to regress salaries on profits and we interpret the resulting OLS coefficients as measures of the pay-performance sensitivity.

To check the robustness of our results we have repeated the above computations for many different initial distributions of μ given by different Beta distributions, $Be(\alpha, \beta)$, with different values for the parameters α and β. Given the qualitative results are the same for all distributions, in this subsection, as well as in the following one, we will report our results on only four different distributions representing a uniform distribution ($\alpha = \beta = 1$), two skewed distributions ($\alpha = 3$, $\beta = 1.5$, right skewed, and $\alpha = 1.5$, $\beta = 3$, left skewed) and a symmetric distribution with a lower variance than the uniform distribution ($\alpha = \beta = 3$). Figure 5 provides the plots of densities for each of these four cases.

The computations to be presented below are performed for the same parameter specification used in the example in subsection 4.2.2 above. The qualitative nature of the results have been verified to be the same for all the other parameter specifications that we have considered.

The first and second column of Table 1 report the OLS coefficients for, respectively, first and second period when salaries are regressed on profits gross of salary payments and show that the magnitude of the pay-performance sensitivity is always higher in the second period than in the first.[24]

The third and fourth column of Table 1 report the OLS coefficients for first and second period when salaries are regressed on profits net of salary payments. The results on the relative magnitude of the pay-performance sensitivity in the first and the second period are preserved although the pay-performance sensitivity in the first period is now negative.

To perform comparisons with empirical studies that use changes in firms' stock market valuations the appropriate measure would seem to be profits net of salary payments because

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[23] Notice that the second period aggregate distribution is computed using the posterior probability of each type of manager's being good in the beginning of period 2 given equilibrium play and equilibrium learning about his type.

[24] We also computed pay-performance elasticities and pay–performance correlations and found that the values of the coefficients are always larger in the second period than in the first.

firms' market valuations should discount the cost of managerial compensation. Despite this observation, we regard the results for the case of profits gross of salary payments as more interesting for the following reason. As was mentioned in section 2, for managers to care about their reputation it is necessary that they appropriate at least part of their reputational rents. For the sake of simplicity, our model makes the extreme assumption that competition ensures that a manager appropriates *all* his reputational rents. This implies that in our model, unlike in reality, a manager's compensation package may be sizable with respect to the firm's profit,[25] so that managerial compensation may even reverse the relative orderings of profit realizations. In other words, we believe that the negative coefficients in the regressions of first period salaries and net profits are an artificial consequence of the disproportionate relative size of managerial compensation to firms' profits and we therefore focus our attention on the relationship between salaries and profits gross of managerial compensation.[26]

The results reported in Table 1 are in line with Gibbons and Murphy's (1992) empirical finding that the sensitivity of managerial pay to firm performance increases as retirement nears. Gibbons and Murphy's (1992) theoretical explanation for this result is that as retirement nears stronger explicit (compensation) incentives are needed to substitute the fading implicit (reputational) incentives. The reason behind the same result in our model is, however, completely different and will be discussed in the following.

The difference between the joint distributions of salaries and profits in the first and the second period depends on two main factors: (a) Managerial compensation in period 2 is increasing in profit but is nonmonotonic in period 1 (see Figures 2 and 4); (b) As time goes by, firms get additional information about managers and update their estimates of their abilities. This suggests that the difference between pay-performance sensitivity in the first and the second period could be decomposed into two effects : the *career concerns effect* arising because of (a) and the *learning effect* due to (b). Because learning in our model is sometimes rather extreme–at the end of the first period it is learned that some managers are good and some bad with probability 1–it is important to assess the contribution of each

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[25] Many empirical studies have documented the fact that CEO's compensation packages are very small in comparison to firms' changes of value.

[26] This idea can be formalized as follows. Suppose that the regression coefficient with profits gross of salary payments is positive and the one with profits net of salary payments is negative. If managerial salaries w are multiplied by a constant κ, then it is easy to show that for a sufficiently low value of κ the regression coefficient with profits net of salary payments is also positive.

of these effects to the overall result to ensure it is not driven by these somewhat extreme assumptions. To do so, we performed the same exercise in the second period but, instead of using the posterior distribution over μ generated by equilibrium play and learning, we used the prior distributions (as in Figure 5)–thus shutting down the learning effect. The resulting slopes (not reported here) are very close to the slopes reported in Table 1, but in fact between 4.5% and 8.3% higher than them. The implication of this is that in these cases the learning effect is not very sizable in absolute terms, but in fact it is also negative: Keeping equilibrium learning into account does not increase the slope for the second period, it reduces it. Given this, the difference in compensation schedules, the career concern effect, emerges as the driving force behind the increasing pay-performance sensitivity.[27]

5.2   Risk and Executive Compensation

The relationship between executive pay and firm performance has been the object of extensive research. Holmström and Milgrom (1987) studied the optimal compensation schedule for a repeated agency problem and found that, under appropriate conditions, it is linear in firm performance and that its slope is decreasing in the variance of profits, which is equal to the (exogenous) variance of a firm specific error term. To test this prediction Aggarwal and Samwick (1999) propose the following specification as an approximation of the optimal contract:[28]

$$w_{ijt} = \gamma_0 + \gamma_1 \pi_{jt} + \gamma_2 F\left(\sigma_{jt}^2\right)\pi_{jt} + \gamma_3 F\left(\sigma_{jt}^2\right) + \lambda_t + \varepsilon_{it}. \tag{2}$$

Subindices i, j, and t refer, respectively, to the executive, the firm and the period, w_{ijt} is the executive's compensation, π_{jt} is the return to shareholders, $F\left(\sigma_{jt}^2\right)$ is the (cumulative) distribution function of the variance of returns of the firm, λ_t is a year effect, and ε_{it} is the error term. The pay-performance sensitivity for a manager working for a firm with variance σ_{jt}^2 is $\gamma_1 + \gamma_2 F\left(\sigma_{jt}^2\right)$, and this specification makes it is easy to compute the pay-performance sensitivity at any percentile of the distribution of variances. For example, the pay-performance sensitivities of the managers working for the firm with the lowest, median, and highest variances are, respectively, γ_1, $\gamma_1 + 0.5\gamma_2$, and $\gamma_1 + \gamma_2$. The prediction of the standard principal-agent model is that $\gamma_1 > 0$ and $\gamma_2 < 0$ ($\gamma_1 + \gamma_2 > 0$). In other words, while higher performance leads to higher compensation, the effect of returns on

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[27]Other simulations we performed show that the learning effect can be both positive and negative.

[28]See equation (2) in Aggarwal and Samwick (1999), page 77.

compensation will be smaller at firms with more variable returns. The classical principal-agent model makes no clear prediction about the relationship between variance of returns and the *level* of compensation (as opposed to the slope of the pay-performance schedule), but Aggarwal and Samwick (1999) also introduce $\gamma_3 F\left(\sigma_{jt}^2\right)$ to make sure that their estimates of γ_2 "are not affected by any relationship between the variance and the level of compensation that may happen to exist in the cross section."[29]

Aggarwal and Samwick (1999) estimate the above equation using US data for 1993-96. Wages are defined to be yearly dollar compensations to CEO's (in thousands of dollars) and returns as yearly dollar returns to firms (in millions of dollars). To compute the variances of returns for each individual firm they use monthly data observations of stock returns in the previous 60 months. Their main results on the relationship between pay-performance sensitivity and variance of firm returns are presented in column 1 of their Table 3 where they provide the median regression estimates of the coefficient in the above specification: $\gamma_1 = 27.596$, $\gamma_2 = -26.147$.[30] Both coefficients are significantly different from 0 and are consistent with the predictions of the standard principal-agent model: the estimated pay-performance sensitivities of the managers working for the firm with the lowest, median, and highest variances turn out to be, respectively,

$$
\begin{aligned}
\gamma_1 &= 27.596 \\
\gamma_1 + 0.5\gamma_2 &= 14.5225 \\
\gamma_1 + \gamma_2 &= 1.449.
\end{aligned}
$$

To verify whether our model is consistent with Aggarwal and Samwick's (1999) results we estimate the coefficients of the same specification as in (2) using the joint distribution of salaries, profits and theoretical profit variances generated by our model. Because Aggarwal and Samwick (1999) do not take into account the numbers of years before retirement, we use the average of the joint distributions for the first and the second period, thereby implicitly assuming that half of the managers are in the early stage of their career (period 1) and the other half in their final stage (period 2) with the idea that two overlapping generations of managers live at the same date.[31] Since we only consider one date, we ignore the year

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[29] Aggarwal and Samwick (1999), page 78.

[30] Given the right skewness of compensation distributions, due to the fact that some CEO's are also the founders and main shareholders of some companies, Aggarwal and Samwick (1999) advocate the use of median regression rather than OLS to get estimates that depend less on these outliers.

[31] If we define the second period of a CEO as the last three years in office, and the previous years as the

effect, λ_t. Finally, given our compensation realizations do not exhibit the same outliers as the data we compute OLS estimates of the coefficients.

As in the previous subsection, to check the robustness of our results we have repeated the above computations for many different initial distributions of μ given by different Beta distributions, $Be(\alpha, \beta)$, with different values for the parameters α and β. Given the results are qualitatively the same for all distributions, we report our results only for the same four different distributions considered in the previous subsection (see Figure 5). Our results for these cases are summarized in Table 2: As in Aggarwal and Samwick (1999), γ_1 is positive and γ_2 negative;[32] Table 2 also provides the pay-performance sensitivities for the managers who are employed by the firm with the lowest, median, and largest variance in the population.

Since the pay-performance sensitivity is in fact decreasing in the (theoretical) variance of firm returns, our model turns out to be observationally equivalent to the standard principal-agent model and the documented relationship between risk and pay-performance sensitivity may be due to reasons different from the ones put forward by it.

Figures 6 and 7 show the variance of profits of a firm as a function of its manager's age and beginning of period probability of being good. When a manager is in his first period and has a low initial value of μ the variance of gross profit is maximal (Figure 6) and his pay does not depend on realized profit at all ($w_N = w_F = w_S = 0$, Figure 4). For both intermediate and higher values of μ, the variance of profit will be lower (Figure 6) and his pay will depend on profits but in a nonmonotonic way (Figure 4). While the first observation seems to contribute to a negative association of pay-performance sensitivity and variance, the second has an unclear effect.

Consider now a manager in his second period. From Figure 2 it is easy to see that both $w_S - w_N$ and $w_N - w_F$ are increasing in μ.[33] Given profit realizations (before salaries) are

first period (as in Gibbons and Murphy, 1992), assuming that half of the population of CEO is in its second period is probably an overestimate. We checked that our results were valid also for lower fractions of old managers in the population, but given we do not regard our analysis as a quantitative calibration exercise, we present the results for equal fractions of young and old managers.

[32] We should mention that while Aggarwal and Samwick (1999) find a positive value for γ_3, our numerical experiments provide negative values. As will be clear below this is a consequence of the fact that better managers generate lower variances *and* are better paid so that a negative relationship arises between variance and the *level* of compensation. We believe that if we allowed firms of different sizes and allowed larger firms to hire better managers, the size effect might counterbalance the effect mentioned above and reverse the sign of this coefficient.

[33] In Figure 2 we do not depict w_N which is identically equal to 0.

independent of μ, the previous observation clarifies that in the second period a manager's pay-performance sensitivity is increasing in μ, his beginning of period probability of being good. Figure 7 shows, on the other hand, that the variance of profits is decreasing in the same probability so that a negative association between pay-performance sensitivity and profit variance arises.

The previous arguments suggest that the negative association between pay-performance sensitivity and variance of profits is mainly due to the compensation patterns of heterogeneous managers in the late stages of their careers. This intuition is confirmed by the fact that running similar regressions for managers in their first and second period separately, the results we have presented are confirmed (and in fact magnified in absolute terms) for managers in their second period, but are unclear for managers in their first period.

Aggarwal and Samwick (1999) also show that omitting variances (i.e., imposing $\gamma_2 = \gamma_3 = 0$) leads to an estimate of the pay-performance sensitivity of 3.47 (a result in line with Jensen and Murphy's (1990) findings) as opposed to a pay-performance sensitivity at the median variance of 14.52. We performed a similar exercise and also found that in this case the pay-performance sensitivities are lower than the corresponding estimates of pay-performance sensitivities at the median variance reported in Table 2.

The previous discussion shows that the data generated by our model is consistent with the empirical evidence. Our results therefore suggest that the correlation between variance of profits and pay-performance sensitivity might be spurious in the sense that it could be the consequence of (unobserved) heterogeneity of managerial skills. At a more general level, while we are convinced about the merits of the interpretation proposed by the standard principal-agent theory, we believe that our results provide a possibly complementary explanation of the relationship between risk and executive compensation.

6   Conclusion

In this paper we study a situation in which a manager's preoccupation for his career creates a discrepancy between his interests and those of the firm. We show that the existence of asymmetric information on managers' abilities to forecast the realization of investment projects is sufficient to create this divergence. In the context of a two-period model we study the way and the extent to which this divergence can be solved using explicit compensations schemes. Allowing salaries to depend on contemporaneous performance is not

always sufficient to induce managers to make efficient investment decisions in the early stages of their careers. When the initial reputation of the manager is bad, exaggeration (overinvestment) arises, for intermediate values of initial reputation, conservatism (under-investment) results, while the efficient investment decisions are taken in equilibrium only when the initial reputation of the manager is sufficiently good.

We characterize equilibrium contracts and show that the asymmetry of information on managers' abilities has very different impacts at different stages of a manager's career. While second period compensation is monotonically increasing in performance, the asymmetry of information creates perverse incentives in the first period (career concerns) that can be (partially) amended only through nonmonotonic pay schemes.

To investigate some of the implications of our results, we have carried out two numerical exercises that study the link between pay and performance generated by our model.

In our first exercise we find that the pay-performance sensitivity in the second period of a manager's career is larger than in the first. We argue that this increase is due to the *career concern effect* that arises because of the divergence between first and second period compensation schemes and implies a more direct link between pay and performance in the second period, when pay is monotonic in performance, than in the first, when it is not. The result is supported by the increase in pay-performance elasticity of CEOs of large US companies in their final years of employment documented by Gibbons and Murphy (1992). Our model therefore provides an alternative explanation for this empirical finding.

Our second exercise analyzes the relationship between risk and executive compensation. Standard principal-agent models with managerial risk aversion imply that a lower pay-performance sensitivity is an optimal response to an increase in the exogenous variance of profits. Aggarwal and Samwick's (1999) empirical findings have given support to this view.

Our results also imply that the pay-performance sensitivity is positive but decreasing in the variance of firm's profits. Our work is, therefore, also in line with Aggarwal and Samwick's (1999) findings but it suggests that the negative association between risk and pay-performance sensitivity could also arise because the managers with the highest pay-performance sensitivities are the ones who *generate* the lowest variance of profits. Since the two models are observationally equivalent in this dimension, we see our result as an alternative or possibly complementary explanation for this empirical regularity.

Our model can produce a number of additional testable predictions that can shed light

on the intratemporal and intertemporal relationships among managerial compensation and measures of firm performance, such as investment or profit. We believe that our results provide an encouraging first step in this direction.

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A   Appendix

A.1   Proof of Proposition 1

We will prove a sequence of claims.

1. No separating equilibrium exists, that is to say no equilibrium exists in which different types of manager accept different offers.

   Suppose such an equilibrium exists, and let $\left(w_N^B, w_F^B, w_S^B\right)$ denote the offer accepted by a bad manager. For any firm offering this contract not to have an incentive to withdraw it, it is necessary that $w_N^B = w_F^B = w_S^B = 0$. Let $\left(w_N^G, w_F^G, w_S^G\right) \neq \left(w_N^B, w_F^B, w_S^B\right)$ denote the offer the good manager accepts. Since $\left(w_N^G, w_F^G, w_S^G\right) \in \mathbb{R}_+^3$, the salary in at least one state has to be strictly positive. Given this, the bad manager would prefer $\left(w_N^G, w_F^G, w_S^G\right)$ to $\left(w_N^B, w_F^B, w_S^B\right) = (0, 0, 0)$ and a contradiction is obtained.

   Consider now pooling equilibria.

2. If a pooling equilibrium exists it has to be such that firms's expected profits are exactly $pz - 1$.

   If firms's expected profits were less than $pz - 1$ a profitable deviation for them would be to withdraw all offers and invest on their own, which gives an expected profit of $pz - 1$. Let (π_1, π_2) denote the expected equilibrium payoffs to firms 1 and 2, suppose $\max(\pi_1, \pi_2) > pz - 1$ and suppose, without loss of generality, that $\pi_1 = \max(\pi_1, \pi_2)$. Let $\widetilde{w} = (\widetilde{w}_N, \widetilde{w}_F, \widetilde{w}_S)$ denote the offer made by firm 1 and that is accepted with positive probability by the manager. Then, it is straightforward to recognize that there is an $\varepsilon > 0$, such that $\widehat{w} = (\widetilde{w}_N + \varepsilon, \widetilde{w}_F + \varepsilon, \widetilde{w}_S + \varepsilon)$ is a profitable deviation for firm 2 as $\widehat{w}$ is strictly preferred for all $\varepsilon > 0$ and there is an $\varepsilon > 0$ such that the expected profit to firm 2 is strictly larger than π_2.

3. If a pooling equilibrium exists it has to be such that the efficient investment strategy is played.

   Suppose this is not the case and let (w_N, w_F, w_S) denote the offer which is accepted by both types of manager. Suppose that after having accepted (w_N, w_F, w_S) the manager does not play according to (I, N, I). From Lemma 1 he will play either (I, I, I) or (N, N, I):

(a) Suppose the manager plays (I, I, I). This implies that $(w_N, w_F, w_S) = (0, 0, 0)$. Consider (w'_N, w'_F, w'_S) such that $w'_N = w'_F = w'_S = \mu_2(1-p) - \varepsilon$ with $\varepsilon \in (0, \mu_2(1-p))$. This offer is strictly preferred by both types of managers, and since after accepting this offer the manager would play (I, N, I), the expected profit to the firm would be $pz - 1 + \varepsilon$, a contradiction

(b) Suppose the manager plays (N, N, I). This implies that $w_N \geqslant pw_S + (1-p)w_F$. Consider now an alternative offer $(w'_N, w'_F, w'_S) = (w_N - \alpha, 0, w_S + \varepsilon)$ with $\varepsilon \in \left(0, \frac{w_N}{p} - w_S\right)$ and $\alpha \in \left(0, \frac{p}{(1-p)}\varepsilon\right)$. It is easy to check that such an offer is strictly preferred only by the good manager and it would yield expected gross profits of $p(z - 1 - w'_S) - (1-p)w'_N$ which, for ε sufficiently small, can be shown to be larger than $\mu p(z - 1 - w_S) - (1 - \mu p)w_N$. Therefore, (w'_N, w'_F, w'_S) would be a profitable deviation and a contradiction arises.

4. If a pooling equilibrium exists, the offer accepted by both types of managers is such that $w_F = 0$.

Recall from 3 above that, if a pooling equilibrium exists it has to be such that the efficient investment action profile is played. Suppose, contrary to the claim that the offer accepted in equilibrium by both types of manager, $w = (w_N, w_F, w_S)$ is such that $w_F > 0$. Consider another offer $\widehat{w} = (\widehat{w}_N, \widehat{w}_F, \widehat{w}_S) = (w_N, 0, w_S + \varepsilon)$. It is easy to recognize that there is an $\varepsilon > 0$ such that $\widehat{w}$ is strictly preferred only by the good manager and that gives an expected payoff to the firm strictly larger than $pz - 1$, a contradiction.

5. No equilibrium can exist in which both managers accept an offer different from

$$\left(w_N^2, w_F^2, w_S^2\right) = \left(\frac{\mu_2(1-p)}{1 + (1-p)\mu_2}, 0, \frac{\mu_2(1-p)}{p(1 + (1-p)\mu_2)}\right).$$

From 2-4 above we know that if a pooling equilibrium exists, it is such that the manager plays the efficient investment action profile, such that firms' expected profits are $pz - 1$ and such that the accepted offer is such that $w_F = 0$. Given this if a pooling equilibrium exists the accepted offer $(w_N, 0, w_S) \in \mathbb{R}^3_+$ has to be such that

$$pw_S \geq w_N \tag{3}$$

$$pw_S + (1-p)\mu_2 w_N = \mu_2(1-p) \tag{4}$$

The thick segment on Figure 8 depicts the contracts that satisfy all previous conditions. The lines (IC) and (FE) correspond to the conditions (3) and (4). Notice that their intersection lies on $\left(w_S^2, w_N^2\right)$. Since expected salaries for the good and the bad manager are, respectively, $p w_S + (1-p) w_N$ and $p w_S$, U_G and U_B represent the indifference curves of the good and the bad manager respectively. Consider any contract on the thick segment of (FE) different from $\left(w_S^2, w_N^2\right)$ such as contract w depicted in Figure 8. All contracts in the interior of the triangle marked with a circle are strictly preferred by the good manager and give an expected payoff strictly larger than $pz - 1$, thus constituting a profitable deviation.

6. There exists an equilibrium in which both types of manager accept offer

$$\left(w_N^2, w_F^2, w_S^2\right) = \left(\frac{\mu_2(1-p)}{1 + (1-p)\mu_2}, 0, \frac{\mu_2(1-p)}{p\left(1 + (1-p)\mu_2\right)}\right).$$

First notice that if $w_S \geq w_N$ a good manager invests efficiently and gets utility $p w_S + (1-p) w_N$, so that for all contracts below the 45 degree line his indifference curve is like the negatively sloped line Figure 1. Consider now the bad manager. Since the bad manager invests if and only if $p w_S \geq w_N$, i.e., if the contract he accepts is below the (IC) line, his utility below the (IC) line will be w_S and above it will be w_N so that his indifference curve will be like the kinked line in Figure 1.

Consider now the indifference curves of the two types of manager passing through the contract mentioned in the claim (at the intersection of the rightmost part of (FE) and (IC) as in Figure 1). Suppose that a profitable deviation for a firm exists that attracts only the good manager. This means that there is a contract below the 45 degree line, above the indifference curve for the good manager and below the indifference curve for the bad manager. From Figure 1 it is easy to see that if the good manager prefers a contract to

$$\left(\frac{\mu_2(1-p)}{1 + (1-p)\mu_2}, 0, \frac{\mu_2(1-p)}{p\left(1 + (1-p)\mu_2\right)}\right),$$

the bad manager will too and a contradiction is obtained. Suppose now that a profitable deviation exists that attracts both types of manager. It is easy to see that any contract preferred by both types of manager will be above the full extraction condition (FE) and will therefore imply a profit less than $pz - 1$ for the firm, a contradiction.

It is easy to see that no profitable deviation can exist that attracts only the bad manager, and the claim follows.

From 1-6 the claim of Proposition 1 follows. ∎

A.2 Proof of Proposition 2

From Proposition 1 the offer accepted by the manager in all second period continuation equilibria is:

$$\left(w_N^2 \left(\mu^2 \right), w_F^2 \left(\mu^2 \right), w_S^2 \left(\mu^2 \right) \right) = \left(\frac{\mu^2(1-p)}{1+(1-p)\mu^2}, 0, \frac{\mu^2(1-p)}{p\left(1+(1-p)\mu^2\right)} \right).$$

This implies that expected second period payoff for all second period continuation equilibria for a manager of type $\tau = G, B$, given that the perceived probability that he is good conditional on the history at the end of the first period is μ^2, will be:

$$E\left[w^2 \mid i^2 \left(w^2 \right), \mu^2, G \right] = \frac{\mu^2(1-p)(2-p)}{1+\mu^2(1-p)}$$

$$E\left[w^2 \mid i^2 \left(w^2 \right), \mu^2, B \right] = \frac{\mu^2(1-p)}{1+\mu^2(1-p)}$$

Given these preliminaries we will now prove Proposition 2 through a series of Lemmas. Given all second period continuation equilibria are payoff equivalent, all the statements to be proved are valid for all continuation equilibria and therefore no explicit reference to second period continuation equilibria will be made.

A.2.1 Part 1

LEMMA 4 *Let $(\mu, z, \delta) \in [0,1] \times \left(\frac{1}{p}, \infty \right) \times \mathbb{R}_+$ be given. Suppose that there exists a $w^1 \in \mathbb{R}_+^3$ such that $i^1 \left(w^1, \mu \right) = (I, N, I)$. Then $\forall (\mu', z') : \mu' \geq \mu$ and $z' \geq \frac{1}{p}$, there exists a $w^{1\prime} \in \mathbb{R}_+^3$ such that $i^1 \left(w^{1\prime}, \mu \right) = (I, N, I).$*

PROOF. Suppose that there exists a $w^1 \in \mathbb{R}_+^3$ such that $i^1 \left(w^1, \mu \right) = (I, N, I)$. This means that

$$
\begin{align}
w_N^1 + \delta \frac{1-p}{2-p} &\leq (1-p) w_F^1 + p \left(w_S^1 + \delta \frac{\mu(1-p)}{1+\mu(1-p)} \right) \tag{5} \\
w_N^1 + \delta (1-p) &\geq w_F^1 \tag{6} \\
w_N^1 + \delta (1-p) &\leq w_S^1 + \delta \frac{\mu(1-p)(2-p)}{1+\mu(1-p)} \tag{7} \\
\mu(1-p) &= \mu(1-p) w_N^1 + (1-\mu)(1-p) w_F^1 + p w_S^1 \tag{8}
\end{align}
$$

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It is easy to see that (5)-(8) hold for all $z \geq \frac{1}{p}$ as they are independent of z.

Now consider $\mu' > \mu$ and note that μ appears only in (5), (7), and (8). Totally differentiating (8) with respect to w_S^1 and μ and rearranging we get

$$\frac{dw_S^1}{d\mu} = \frac{1-p}{p}\left(1 - w_N^1 + w_F^1\right) \geq 0 \tag{9}$$

with the inequality following from the fact that, by (8) and non-negativity of salaries, $w_N^1 \leq 1$.

Consider now $\mu' > \mu$. By (9) one can choose $w_S^{1\prime} > w_S^1$ such that given $\left(w_N^1, w_F^1\right)$ (8) holds. Now it is easy to check that (5)-(7) are also satisfied, due to the fact that $w_S^{1\prime} > w_S^1$ and $\mu' > \mu$ and therefore

$$\frac{\mu'(1-p)}{1+\mu'(1-p)} > \frac{\mu(1-p)}{1+\mu(1-p)}.$$

∎

LEMMA 5 *Let* $z \in \left(\frac{1}{p}, \infty\right)$ *and* $\mu = \mu^* = \frac{\delta}{2+\delta-p}$ *be given. Then* $i^1\left(w^1, \mu\right) = (I, N, I)$ *if and only if*

$$\left(w_N^1, w_F^1, w_S^1\right) = \left(0, \frac{\delta(1-p)(1+\delta-p)}{(1+\delta)(2-p)}, (1-p)\frac{\delta}{1+\delta}\right).$$

PROOF. Let $\mu = \mu^* = \frac{\delta}{2+\delta-p}$. From (5) and (7), substituting the full extraction constraint (8), we have

$$w_N^1 \leq (1-p)\delta\frac{w_F^1(1+\delta)(2-p) - \delta(1-p)(1+\delta-p)}{(2-p)^2(1+\delta)^2}$$

$$w_N^1 \leq (1-p)\frac{w_F^1(1+\delta)(2-p) - \delta(1-p)(1+\delta-p)}{(1+\delta)(-2p+p^2-\delta)}$$

whose only nonnegative solution is $w_N^1 = 0$, $w_F^1 = \frac{\delta(1-p)(1+\delta-p)}{(1+\delta)(2-p)}$. It is easy to verify that this solution satisfies (6). From the full extraction condition we get $w_S^1 = (1-p)\frac{\delta}{1+\delta}$ which concludes the proof. ∎

LEMMA 6 *Let* $z \in \left(\frac{1}{p}, \infty\right)$ *and* $\mu < \mu^* = \frac{\delta}{2+\delta-p}$ *be given. Then, there exists no* $w^1 \in \mathbb{R}_+^3$ *such that* $i^1\left(w^1, \mu\right) = (I, N, I)$.

PROOF. Let $(\mu, z) \in [0, 1] \times \left(\frac{1}{p}, \infty\right)$ be given and consider any $w^1 \in \mathbb{R}_+^3$ such that $i^1\left(w^1, \mu\right) = (I, N, I)$. From (5) and (7), substituting the full extraction constraint (8), we have

$$w_N^1 \leq \frac{\mu(1-p)}{1+\mu(1-p)}w_F^1 - (1-p)\frac{\left(\mu(p-1)^2 + (1-\mu p)\right)\delta - \mu(1+\mu(1-p))(2-p)}{(1+\mu(1-p))^2(2-p)} \tag{10}$$

$$w_N^1 \leq -\frac{(1-\mu)(1-p)}{(p+\mu(1-p))}w_F^1 + (1-p)\frac{\mu(1+\mu(1-p)) - p(1-\mu)\delta}{(p+\mu(1-p))(1+\mu(1-p))} \tag{11}$$

A necessary condition for existence of $\left(w_N^1, w_F^1\right) \in \mathbb{R}_+^2$ satisfying (10) and (11) is that $w_F^1 \in \mathbb{R}_+$ exists that satisfies (10) and (11) for $w_N^1 = 0$. We therefore have

$$
\begin{aligned}
w_F^1 &\geq \frac{\left(\mu(p-1)^2 + (1-\mu p)\right)\delta - \mu(1+\mu(1-p))(2-p)}{\mu(1+\mu(1-p))(2-p)} \\
w_F^1 &\leq \frac{\mu(1+\mu(1-p)) - p(1-\mu)\delta}{(1-\mu)(1+\mu(1-p))}
\end{aligned}
$$

and a necessary condition for existence of $w_F^1 \in \mathbb{R}_+$ satisfying the above inequalities is[34]

$$
\frac{\left(\mu(p-1)^2 + (1-\mu p)\right)\delta - \mu(1+\mu(1-p))(2-p)}{\mu(2-p)} \leq \frac{\mu(1+\mu(1-p)) - p(1-\mu)\delta}{(1-\mu)}
$$

which can be shown to be equivalent to $\mu \geq \mu^*$. ∎

Since, as it was argued in subsection 4.2, in equilibrium a manager always prefers a first period offer w^1 such that $i^1\left(w^1, \mu\right) = (I, N, I)$ rather than any offer $w^{1\prime}$ such that $i^1\left(w^{1\prime}, \mu\right) \neq (I, N, I)$, part 1 of Proposition 2 follows easily from Lemmas 4 and 6 that show that a $w^1 \in \mathbb{R}_+^3$ such that $i^1\left(w^1, \mu\right) = (I, N, I)$ and satisfying full extraction exists if and only if $\mu \geq \mu^* = \frac{\delta}{2+\delta-p}$.

A.2.2 Part 2

A necessary condition that has to be satisfied for $i^1\left(w^1, \mu\right) = (N, N, I)$ is that profits are higher than with (I, I, I), that is

$$
\mu p(z-1) \geq pz - 1 \tag{12}
$$

Suppose that there exists $w^1 \in \mathbb{R}_+^3$ such that $i^1\left(w^1, \mu\right) = (N, N, I)$. Recall from Lemma 2 that

$$
\begin{aligned}
\mu^2(N) &= \frac{\mu(1-p)}{1-\mu p} \tag{13} \\
\mu^2(S) &= 1 \tag{14} \\
\mu^2(F) &= \mu. \tag{15}
\end{aligned}
$$

To ease notation in the following we will make use of (14) and (15) but will not substitute (13) until it will become necessary. Under the assumption of part 2 of Proposition 2 we

---

[34]Notice that $\mu(1+\mu(1-p)) - p(1-\mu)\delta > 0$.

have

$$w_N^1 + \delta \frac{\mu^2 (N) (1-p)}{1 + \mu^2 (N) (1-p)} \geq (1-p)\left[w_F^1 + \delta \frac{\mu (1-p)}{1 + \mu (1-p)}\right] + p\left[w_S^1 + \delta \frac{1-p}{2-p}\right] \quad (16)$$

$$w_N^1 + \delta \frac{\mu^2 (N) (1-p) (2-p)}{1 + \mu^2 (N) (1-p)} \geq w_F^1 + \delta \frac{\mu (1-p) (2-p)}{1 + \mu (1-p)} \quad (17)$$

$$w_N^1 + \delta \frac{\mu^2 (N) (1-p) (2-p)}{1 + \mu^2 (N) (1-p)} \leq w_S^1 + \delta (1-p) \quad (18)$$

$$(1-\mu) w_N^1 + \mu (1-p) w_N^1 + \mu p w_S^1 = \mu p (z-1) - (pz-1) \quad (19)$$

Lemmas 7-9 prepare for the proof of part 2 of Proposition 2.

LEMMA 7 *Suppose that* $\left(w_N^1, w_F^1, w_S^1\right) \in \mathbb{R}_+^3$ *satisfies (16). Then* $\left(w_N^1, 0, w_S^1\right) \in \mathbb{R}_+^3$ *(i) also satisfies (16) and (ii) satisfies (17) with strict inequality.*

PROOF. Part (i) of the Lemma is trivial. Given this, to prove part (ii), set $w_F^1 = 0$. Adding $\delta \frac{\mu^2 (N)(1-p)(1-p)}{1+\mu^2 (N)(1-p)}$ to both sides of (16) and simplifying we get

$$w_N^1 + \delta \frac{\mu^2 (N) (1-p) (2-p)}{1 + \mu^2 (N) (1-p)} \geq p w_S^1 + \delta \left[\frac{\mu (1-p) (1-p)}{1 + \mu (1-p)} + p\frac{1-p}{2-p} + \frac{\mu^2 (N) (1-p) (1-p)}{1 + \mu^2 (N) (1-p)}\right] \quad (20)$$

Given (20) for (17) to hold it is sufficient that

$$p w_S^1 + \delta \left[\frac{\mu (1-p) (1-p)}{1 + \mu (1-p)} + p\frac{1-p}{2-p} + \frac{\mu^2 (N) (1-p) (1-p)}{1 + \mu^2 (N) (1-p)}\right] \geq \delta \frac{\mu (1-p) (2-p)}{1 + \mu (1-p)}$$

or

$$p w_S^1 + \delta \left[p\frac{1-p}{2-p} + \frac{\mu^2 (N) (1-p) (1-p)}{1 + \mu^2 (N) (1-p)} - \frac{\mu (1-p)}{1 + \mu (1-p)}\right] \geq 0 \quad (21)$$

Since $w_S^1 \geq 0$ and the term in bracket can be shown to be nonnegative for $(\mu, p) \in (0,1)^2$, (21) holds and (17) holds strictly. ∎

LEMMA 8 *Suppose that there exist a* $w^1 \in \mathbb{R}_+^3$ *such that* $i^1\left(w^1, \mu\right) = (N, N, I)$ *and such that (16) holds with strict inequality. Then there exists a* $w^{1\prime} \in \mathbb{R}_+^3$ *such that* $i^1\left(w^{1\prime}, \mu\right) = (N, N, I)$ *and (16) holds with equality.*

PROOF. Suppose that the assumption of the Lemma is true. By Lemma 7 set $w_F^1 = 0$ without loss of generality. From (19) totally differentiating with respect to w_N^1 and w_S^1 we get

$$(1 - \mu p) dw_N^1 + \mu p dw_S^1 = 0$$
$$\frac{dw_S^1}{dw_N^1} = -\frac{1 - \mu p}{\mu p} < 0.$$

This implies that we can increase w_S^1 and decrease w_N^1 without violating (18) until (16) is satiated. By Lemma 7, if (16) holds, so does (17). ∎

LEMMA 9 *Suppose that for a given $(\mu, z, \delta) \in [0,1] \times \left(\frac{1}{p}, \infty\right) \times \mathbb{R}_+$ a $w^1 \in \mathbb{R}_+^3$ exists such that $i^1\left(w^1, \mu\right) = (N, N, I)$. Then there exist a $w^{1\prime} \in \mathbb{R}_+^3$ such that (16) and (19) hold with equality and (17) and (18) hold with strict inequality.*

PROOF. By Lemma 7 set $w_F^1 = 0$ without loss of generality. The fact that (17) holds with strict inequality when (16) holds (strictly or not) has already been shown in Lemma 7. By Lemma 8 we can now concentrate on the case in which (16) holds with equality. If $w_S^1 > w_N^1$, then (18) trivially holds strictly. Consider now the case in which $w_N^1 \geq w_S^1$. By (16) holding with equality and by

$$\delta \frac{\mu(1-p)}{1+\mu(1-p)} < w_S^1 + \delta \frac{1-p}{2-p}$$

we have that

$$w_N^1 + \delta \frac{\mu^2(N)(1-p)}{1+\mu^2(N)(1-p)} < w_S^1 + \delta \frac{1-p}{2-p}.$$

Multiplying through by $(2-p)$ we get

$$w_N^1(2-p) + \delta \frac{\mu^2(N)(1-p)(2-p)}{1+\mu^2(N)(1-p)} < w_S^1(2-p) + \delta(1-p).$$

Subtracting $w_N^1(1-p)$ from both sides we get

$$w_N^1 + \delta \frac{\mu^2(N)(1-p)(2-p)}{1+\mu^2(N)(1-p)} < w_S^1(2-p) - w_N^1(1-p) + \delta(1-p). \qquad (22)$$

Recalling that $w_N^1 \geq w_S^1$ and adding $(1-p)\left(w_N^1 - w_S^1\right) \geq 0$ to the right hand side of (22) we get

$$
\begin{aligned}
w_N^1 + \delta \frac{\mu^2(N)(1-p)(2-p)}{1+\mu^2(N)(1-p)} &< w_S^1(2-p) - w_N^1(1-p) + (1-p)\left(w_N^1 - w_S^1\right) + \delta(1-p) \\
&= w_S^1(2-p) - w_N^1(1-p) + (1-p)w_N^1 - (1-p)w_S^1 + \delta(1-p) \\
&= w_S^1 + \delta(1-p)
\end{aligned}
$$

which concludes the proof. ∎

LEMMA 10 *Let $(\mu, z, \delta) \in [0,1] \times \left(\frac{1}{p}, \infty\right) \times \mathbb{R}_+$ be given. Suppose that there exists a $w^1 \in \mathbb{R}_+^3$ such that $i^1\left(w^1, \mu\right) = (N, N, I)$ and such that first period expected profit is higher than under (I, I, I). Then, for all (μ', z') such that $\mu' \geq \mu$ and $z' \in \left[\frac{1}{p}, z\right]$, there exists $w^{1\prime} \in \mathbb{R}_+^3$ such that $i^1\left(w^{1\prime}, \mu\right) = (N, N, I)$.*

PROOF. By Lemma 7 set $w_F^1 = 0$ without loss of generality. Rewrite (16) as

$$w_N^1 + \delta (1-p) \left[\frac{\mu^2 (N)}{1 + \mu^2 (N)(1-p)} - \frac{\mu (1-p)}{1 + \mu (1-p)} \right] \geq p \left[w_S^1 + \delta \frac{1-p}{2-p} \right]$$

or

$$w_N^1 + \delta (1-p) f(\mu, p) \geq p \left[w_S^1 + \delta \frac{1-p}{2-p} \right] \tag{23}$$

where

$$\begin{aligned}
f(\mu, p) &= \frac{\mu^2 (N)}{1 + \mu^2 (N)(1-p)} - \frac{\mu (1-p)}{1 + \mu (1-p)} \\
&= \frac{\frac{\mu(1-p)}{1-\mu p}}{1 + \frac{\mu(1-p)}{1-\mu p}(1-p)} - \frac{\mu (1-p)}{1 + \mu (1-p)}
\end{aligned}$$

and notice that

$$\frac{\partial f(\mu, p)}{\partial \mu} > 0$$

for all $(\mu, p) \in (0,1)^2$.

From Lemmas 7, 8 and 9 we know that if for a given $(\mu, z, \delta) \in [0,1] \times \left(\frac{1}{p}, \infty \right) \times \mathbb{R}_+$ there exists a $w^1 \in \mathbb{R}_+^3$ such that $i^1 (w^1, \mu) = (N, N, I)$, then there also exists a $w^{1\prime} \in \mathbb{R}_+^3$ such that (16) and (19) hold with equality and (17) and (18) hold as strict inequalities. Rewrite (23) (equivalent to (16)) with equality

$$w_N^1 + \delta (1-p) f(\mu, p) = p \left[w_S^1 + \delta \frac{1-p}{2-p} \right].$$

Since $\frac{\partial f(\mu,p)}{\partial \mu} > 0$ for all $(\mu, p) \in (0,1)^2$, when μ increases to $\mu' \in (\mu, \mu^*)$, if we increase w_S^1 and w_N^1 by the same (absolute) amount so as to satisfy (19) for μ', (23) will still be satisfied and so will (17) and (18).

To conclude the proof we only need to show that the difference between the expected profit under (N, N, I) and under (I, I, I) is increasing with μ and decreasing with z. Let

$$h(\mu, z) = \mu p (z-1) - (pz - 1).$$

Then it is easy to see that

$$\begin{aligned}
\frac{\partial h(\mu, z)}{\partial \mu} &= p(z-1) > 0 \\
\frac{\partial h(\mu, z)}{\partial z} &= \mu p - p = -p(1-\mu) < 0.
\end{aligned}$$

∎

So far we have proved that, if there exists a contract such that in the continuation equilibrium $i^1\left(w^1, \mu\right) = (N, N, I)$ for a given (μ, z), there will also exist a contract such that in the continuation equilibrium $i^1\left(w^1, \mu'\right) = (N, N, I)$ for all $\mu' \in [\mu, \mu^*)$ and $z' \in \left[\frac{1}{p}, z\right]$. The following Lemma provides an explicit expression for the frontier $\widetilde{z}(\mu)$ such that there exists a $w^1 \in \mathbb{R}^3_+$ such that $i^1\left(w^1, \mu\right) = (N, N, I)$ if and only if $(\mu, z) \in (0, 1) \times \left[\frac{1}{p}, \widetilde{z}(\mu)\right]$ and shows that the set $(0, 1) \times \left[\frac{1}{p}, \widetilde{z}(\mu)\right]$ has positive measure on the space (μ, z).]

LEMMA 11 *Let $(p, \delta) \in (0, 1) \times \mathbb{R}_+$ be fixed. Let $\widetilde{z}(\mu)$ be such that there exists a $w^1 \in \mathbb{R}^3_+$ such that $i^1\left(w^1, \mu\right) = (N, N, I)$ if and only if $(\mu, z) \in (0, 1) \times \left[\frac{1}{p}, \widetilde{z}(\mu)\right]$. Then*

1. *$\widetilde{z}(\mu) = (1 - \mu p)\left(\frac{1}{p(1-\mu)} - \delta\frac{(1-p)\left(\mu p^2 - 4\mu p + 3\mu + 1\right)}{(2-p)(1+\mu-\mu p)(1-3\mu p + \mu + \mu p^2)}\right)$;*

2. *There exists a $\mu \in (0, \mu^*)$ such that $\widetilde{z}(\mu) = \frac{1}{p}$;*

3. *$\widetilde{z}(\mu^*) > \frac{1}{p}$;*

4. *$(\mu', z') \in (0, 1) \times \left[\frac{1}{p}, \widetilde{z}(\mu)\right]$ implies (12).*

PROOF. By Lemma 7 set $w^1_F = 0$ without loss of generality. By Lemma 9 we can restrict without loss of generality to $w^1 \in \mathbb{R}^3_+$ such that (16) and (19) hold with equality. We then have

$$w^1_N = p w^1_S + \delta\left[\frac{\mu(1-p)^2}{1 + \mu(1-p)} - \frac{\mu^2(N)(1-p)}{1 + \mu^2(N)(1-p)} + p\frac{1-p}{2-p}\right] \tag{24}$$

$$w^1_N = -\frac{\mu p}{1 - \mu p} w^1_S + \frac{\mu p(z-1) - (pz-1)}{1 - \mu p} \tag{25}$$

A necessary and sufficient condition for a solution $\left(w^1_N, w^1_S\right) \in \mathbb{R}^2_+$ to exist is that

$$\frac{\mu p(z-1) - (pz-1)}{1 - \mu p} \geq \delta\left[\frac{\mu(1-p)^2}{1 + \mu(1-p)} - \frac{\mu^2(N)(1-p)}{1 + \mu^2(N)(1-p)} + p\frac{1-p}{2-p}\right] \tag{26}$$

Since (26) implies (12) part 4 follows. From (26) we get

$$\widetilde{z}(\mu) = (1 - \mu p)\left(\frac{1}{p(1-\mu)} - \delta\frac{(1-p)\left(\mu p^2 - 4\mu p + 3\mu + 1\right)}{(2-p)(1+\mu-\mu p)(1-3\mu p + \mu + \mu p^2)}\right)$$

which proves part 1. Since by Lemma 10 $\widetilde{z}(\mu)$ is increasing, to prove parts 2 and 3 it is sufficient to show that $\widetilde{z}(0) < \frac{1}{p}$ and $\widetilde{z}(\mu^*) > \frac{1}{p}$: which can be verified through straightforward calculations. ∎

Recall from (1) that if μ and z are such that $z \leq \frac{1-\mu p}{p(1-\mu)}$ the expected gross profit of (N, N, I) is larger than that of (I, I, I). Given this and noticing that $\widetilde{z}(\mu^*) \leq \frac{1-\mu p}{p(1-\mu)}$, it

is easy to see that if $(\mu, z) \in (0,1) \times \left[\frac{1}{p}, \tilde{z}(\mu)\right]$ in equilibrium a manager always prefers a first period offer w^1 such that $i^1(w^1, \mu) = (N, N, I)$ rather than any offer $w^{1\prime}$ such that $i^1(w^{1\prime}, \mu) = (I, I, I)$. Given this part 2 of Proposition 2 follows directly from Lemmas 10 and 11.

A.2.3 Part 3

To prove part 3 of Proposition 2, consider the following Lemma.

LEMMA 12 *Let $(\mu, z, \delta) \in [0,1] \times \left(\frac{1}{p}, \infty\right) \times \mathbb{R}_+$ be given. Then, $i^1(w^1, \mu) = (I, I, I)$ if and only if $w^1 = (0,0,0)$.*

PROOF. Suppose that there exists a $w^1 \in \mathbb{R}_+^3$ such that $i^1(w^1, \mu) = (I, I, I)$. Using Lemma 2 the incentive compatibility constraints and the full extraction condition are:

$$w_N^1 + \delta\frac{\mu(1-p)}{1+\mu(1-p)} \leq (1-p)\,w_F^1 + pw_S^1 + \delta\frac{\mu(1-p)}{1+\mu(1-p)}$$

$$w_N^1 + \delta\frac{\mu(1-p)(2-p)}{1+\mu(1-p)} \leq w_F^1 + \delta\frac{\mu(1-p)(2-p)}{1+\mu(1-p)}$$

$$w_N^1 + \delta\frac{\mu(1-p)(2-p)}{1+\mu(1-p)} \leq w_S^1 + \delta\frac{\mu(1-p)(2-p)}{1+\mu(1-p)}$$

$$0 = (1-p)\,w_F^1 + pw_S^1$$

whose only solution is

$$w_N^1 = w_S^1 = w_F^1 = 0.$$

Noticing that this is the only solution for all $(\mu, z) \in [0,1] \times \left[\frac{1}{p}, \infty\right)$ concludes the proof. ∎

By parts 1 and 2 of Proposition 2 for $\mu \in [0, \mu^*)$ and $z > \tilde{z}(\mu)$ there exist no $w^1 \in \mathbb{R}_+^3$ such that $i^1(w^1, \mu) \in \{(N, N, I), (I, N, I)\}$ and by Lemma 12 we can now conclude that $\mu \in [0, \mu^*)$ and $z > \tilde{z}(\mu)$ the manager's first period offer w^1 will be such that $i^1(w^1, \mu) \in (I, I, I)$ thus proving part 3 of Proposition 2.
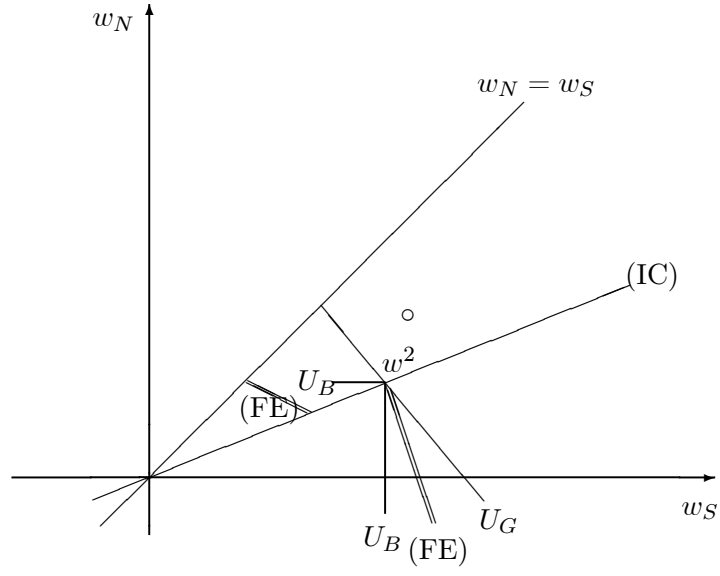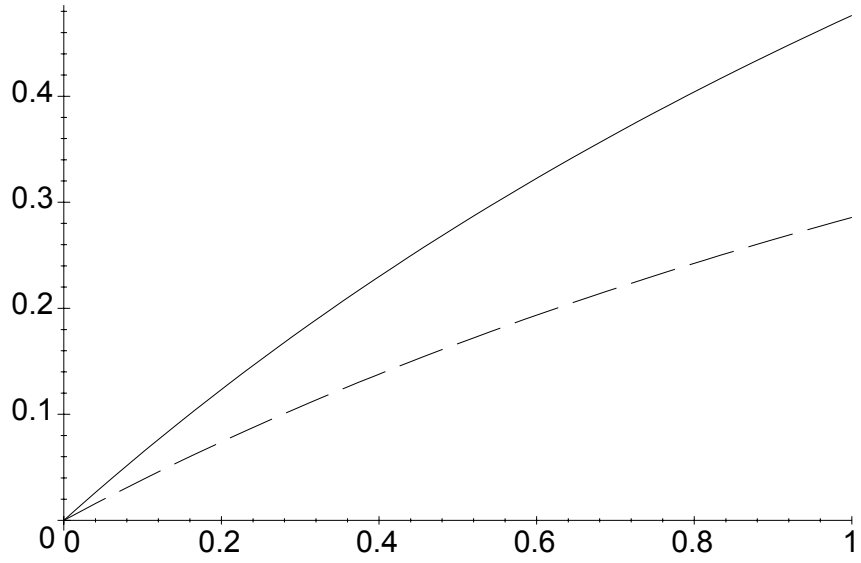
Figure 1

Figure 2: Second period equilibrium contracts. $w_N^2(\mu)$, $w_S^2(\mu)$ are represented, respectively, by the dashed and the solid curves. $w_F^2(\mu)$ is identically equal to 0.
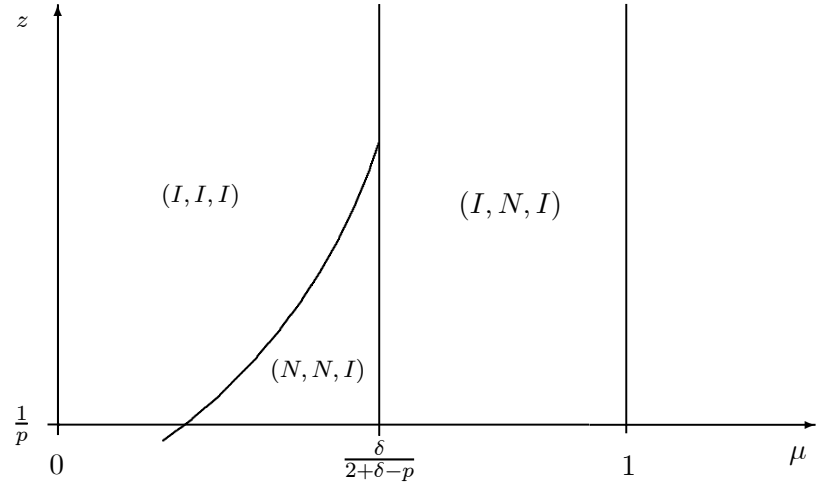
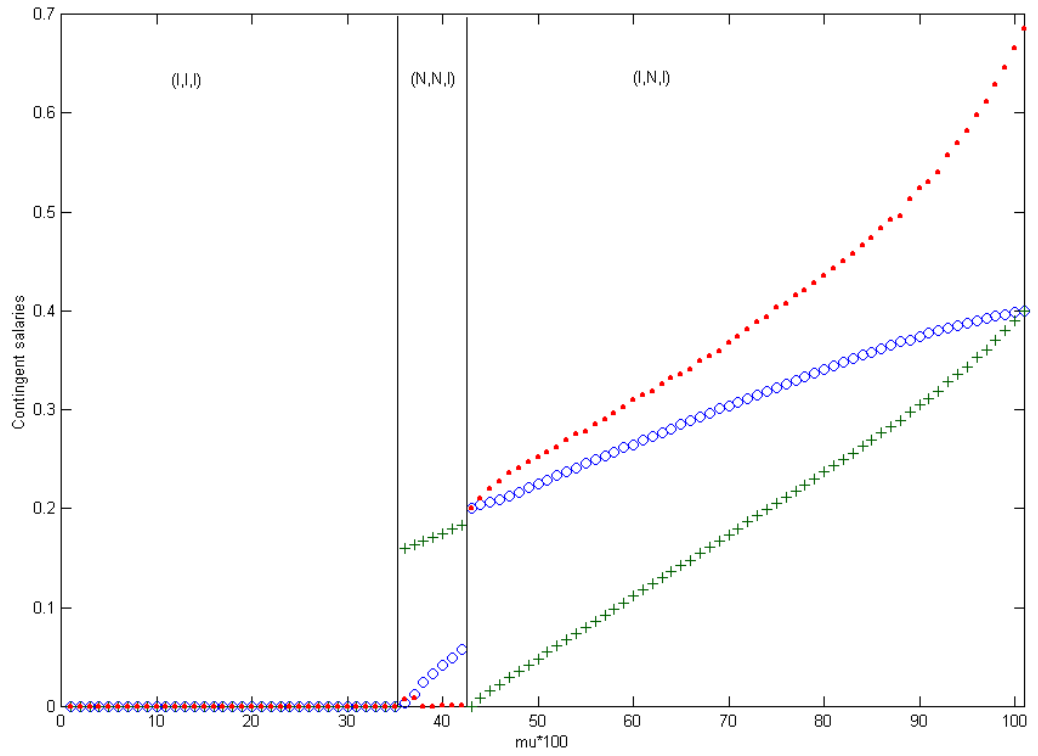**Figure 3**: Equilibrium investment action profiles

Figure 4: First period equilibrium contracts.

$w_N^1(\mu)$, $w_F^1(\mu)$, $w_S^1(\mu)$ are represented, respectively, by plus signs, dots and circles.
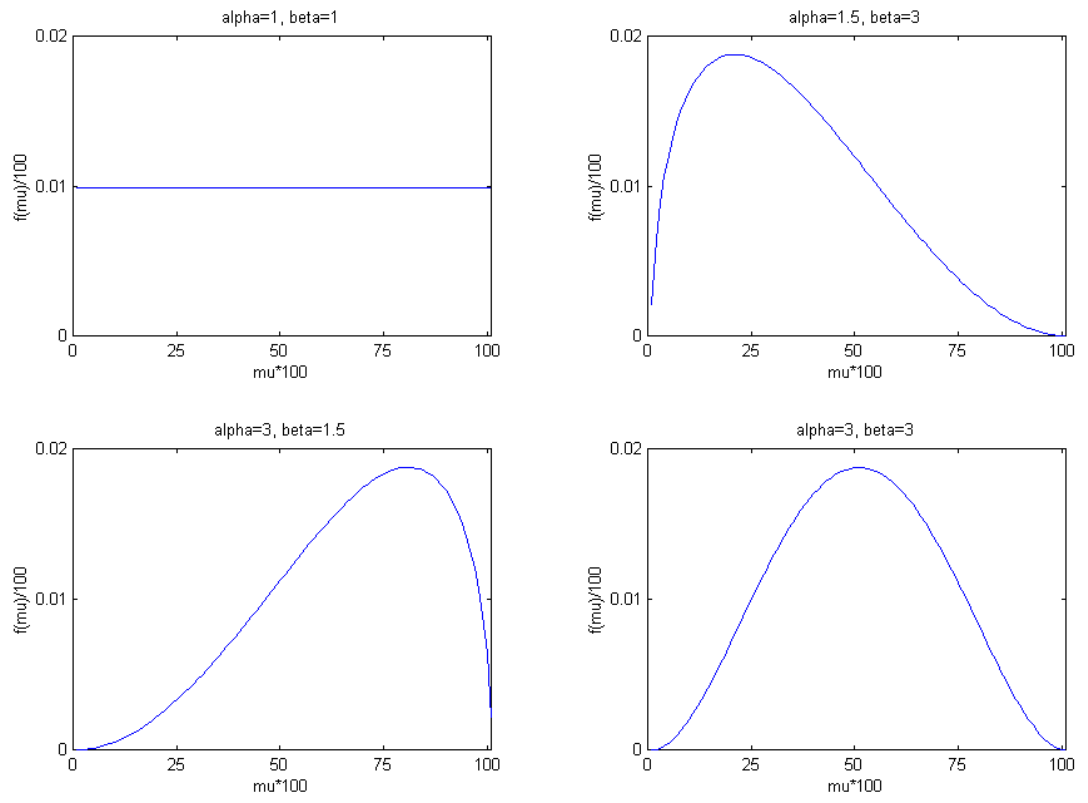
Figure 5: Density functions of Beta distributions

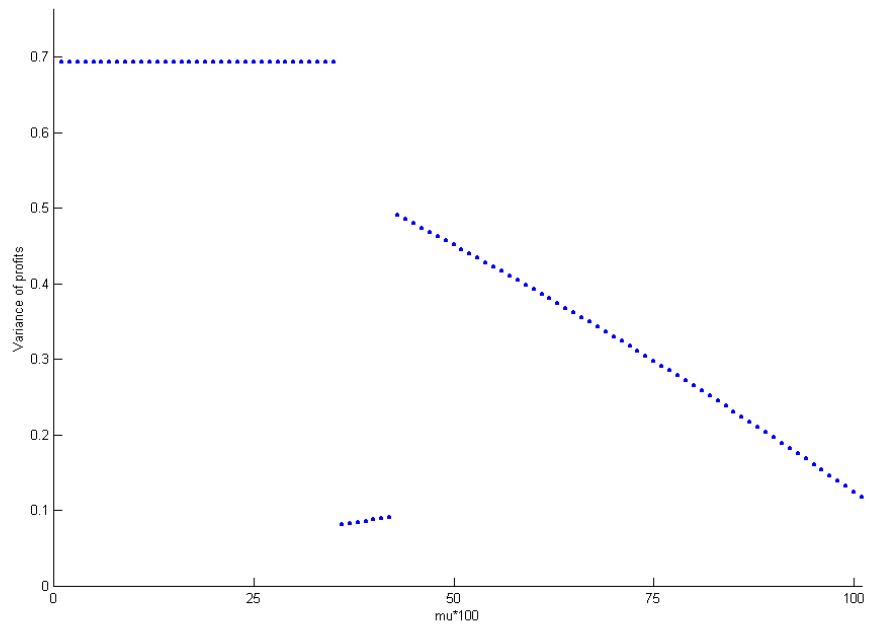Horizontal axis: 100μ; Vertical axis: $f\left(\mu\right)/100$

Figure 6: Variance of profits for a firm with a manager in his first period and beginning of period probability of being good equal to μ.
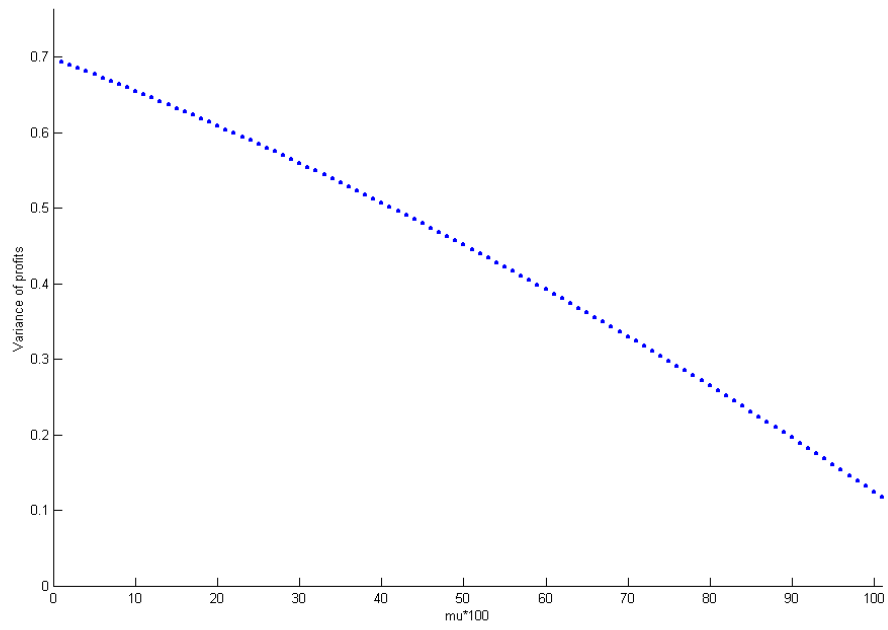
Figure 7: Variance of profits for a firm with a manager in his second period and beginning of period probability of being good equal to μ.
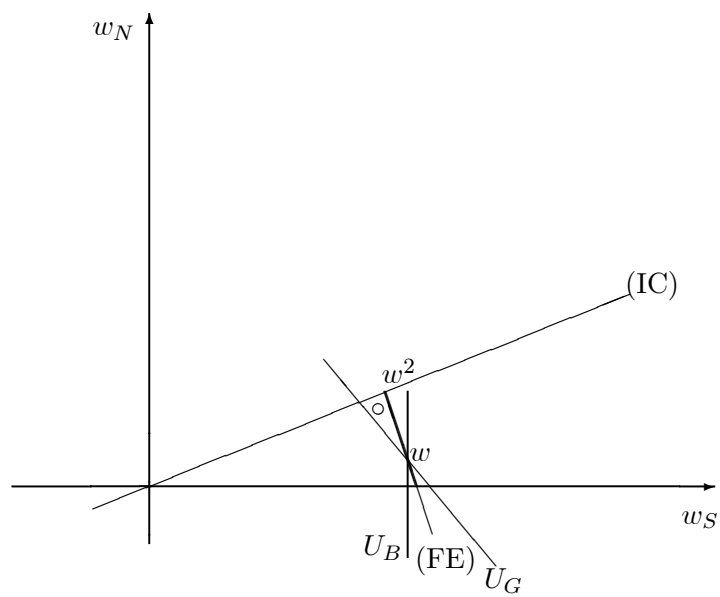
Figure 8

Table 1
OLS coefficients for periods 1 and 2

α	β	Profits gross of salary payments		Profits net of salary payments	
		Period 1	Period 2	Period 1	Period 2
1	1	0.0439	0.1438	-0.0072	0.1021
3	1.5	0.0284	0.1863	-0.0118	0.1531
1.5	3	0.0191	0.1043	-0.0064	0.0841
3	3	0.0245	0.1475	-0.0117	0.1191

Table 2
OLS coefficients for specification (2)
(salaries vs. profits gross of salary
payments; average of two periods)

α	β	γ_1	γ_2	$\gamma_1 + 0.5\gamma_2$	$\gamma_1 + \gamma_2$
1	1	0.1767	-0.1732	0.0901	0.0035
3	1.5	0.1642	-0.1219	0.1032	0.0423
1.5	3	0.1399	-0.1376	0.0711	0.0023
3	3	0.1620	-0.1484	0.0878	0.0136