

Cooperation and Retaliation in Legislative Bargaining*

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Abstract

We study a legislative-bargaining divide-the-pie game in which some legislators have the ability to affect the amount of resources to be distributed (positively or negatively). If included in the winning coalition, these legislators cooperate and increase the size of the pie. If excluded, they retaliate and decrease it. Cooperation and retaliation produce significant changes in the equilibrium allocation relative to Baron and Ferejohn (1989). In particular, we find that, i) cooperating and retaliating districts are more likely to be included in the winning coalition, ii) the equilibrium might feature larger-than-minimum winning coalitions, and iii) there exist equilibria with inefficient output losses.

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1 Introduction

In democracy, members of parliament regularly bargain over the budget allocation across different areas of government and regions based on estimates of economic conditions. Legislators who are satisfied with the bargaining outcome may be willing to cooperate and contribute to increase central government's resources. Those who are less satisfied would not cooperate and could even choose to retaliate, slowing down economic activity, or reducing the central government's tax base. Taking these spillovers into account, in the form of cooperation or retaliation, the size of the pie to be distributed is endogenous to the coalition approving the budget. Thus, the presence of these bargaining spillovers might modify an agenda setter's proposal on how to distribute aggregate resources.

In this paper, we study how the presence of cooperation or retaliation, by opening an informal platform to influence formal policy-making, affects the outcome of legislative bargaining, potentially leading to inefficient policy choices. In a first stage, some districts decide whether to become active, i.e. acquire the ability to engage in cooperation or retaliation. In a second stage, the legislature adjourns knowing districts' types and a "divide-the-pie" legislative bargaining game follows.¹ Importantly, active districts not included in the winning coalition cause a loss in output (direct from retaliation, or due to not cooperating to increase resources). This implies that the resources to be divided are endogenous, and depend on the number of active districts that are excluded from the winning coalition.

Whether an active district is included in the winning coalition depends on how costly it is for an agenda setter to gain the support of its legislator to vote in favor of the proposed distribution of rents, relative to the potential loss of resources and the cost of including a passive district. Broadly speaking, if the difference in the continuation values of an active district and a passive one is smaller than the potential output loss, then the legislator from the active district will be offered his continuation value and will be part of the winning coalition.

The agenda setter may propose to form minimum winning coalitions, i.e., coalitions in which the total number of members who approve the proposal is necessary and sufficient. When there are not enough active districts to muster a majority, she will always call the necessary and sufficient passive members to reach the needed majority. An increase in the number of active districts leads agenda-setters to consider larger-than-minimal winning coalitions to increase the size of rents. Whether all active districts are included in the winning coalition, or only a subset of them, depends on the voting rule, the number of

¹Throughout, we denote the resources to be divided in legislative bargaining as pie, rents or output.

active districts, and their bargaining power. These results are in line with the empirical literature, in which larger-than-minimal winning coalitions are the norm.²

When there is a relatively large discounting of the future, all active districts are included in the coalition, with no output losses. Inefficiencies may arise in equilibrium when the cost of including an active district in the winning coalition is so large that some are left out. Intuitively, if an active district is included in the winning coalition with certainty, this would give the district effective veto power, and with low discounting of the future such district could appropriate most of the pie. Not including active districts with certainty reduces their continuation values. This is done up to the point that the agenda setter is indifferent between calling an extra active district and the output damage from leaving it outside the winning coalition. In equilibrium, the expected value from legislative bargaining is higher for active districts (this is due to active legislators being more likely to be called into the winning coalition). We also show that all legislators that have the option to become active in the first stage decide to do so.³

Note that supermajorities do not necessarily benefit active districts. Although marginal increases of the needed majority may increase active districts' continuation values, the effect is non-monotonic. For instance, in the extreme case of unanimity rule, all active districts must be included in the coalition. This rule makes all legislators, from active and passive districts, equally needed and ex-ante payoffs must be identical for all legislators.

In legislatures, the districts' representatives are agents of their constituencies. While we model the decision to become active as made by the legislator, in some circumstances this is the result of grassroots movements. For example, the Great Recession, and the slow recovery from it, produced an outburst of protests in established democracies around the world. Occupy Wall Street in the United States, indignados in Spain, the anti-austerity movement in Greece are examples of demonstrations that can have an impact on economic activity, and may have affected legislators' actions.⁴

Our results are not only present in bargaining in formal legislatures. A polluting country which does not support the outcome of an international environmental agreement may threaten to sustain pollution (generating a negative externality over all other countries) unless it obtains a better deal. Conversely, countries may allocate more effort in reducing contamination if they perceive a benefit from cooperation. An example of how cooperation and retaliation forces might shape international agreements is the clean development

²See Knight (2008) and references therein.

³This decision is not trivial since in some equilibria there are output losses.

⁴Petitions from citizens at large, or from experts, can be seen as another example of grassroots activities that can affect legislators' actions.

mechanism set up in the aftermath of the 1997 Kyoto Protocol.⁵

Our work is linked to the theory of political failure by which an inefficient allocation of resources can be caused by politically determined policy choices.⁶ In this line, whether the outcome of bargaining in legislatures is economically efficient has been studied through the lenses of the formal rules of bargaining, following up on Rubinstein (1982) and Baron and Ferejohn (1989)'s application to procedural rules of legislative bargaining. The latter is a model of a non-cooperative zero-sum game that shows how the final distribution of resources (a dollar) is affected by the majority rule, the choice of the agenda setter (recognition probabilities), and the exact details of the bargaining rules (for instance, the presence of amendments).

The early papers on multilateral bargaining in legislatures (Austen-Smith and Banks, 1988; Baron and Ferejohn, 1989; Baron, 1991; Romer and Rosenthal, 1979) initiated a large body of literature on this topic. Some of these papers focus on the static setting, mostly showing the effect of institutional changes, e.g. different procedural rules, on policy outcomes.⁷ Snyder et al. (2005) look at voting power and recognition probabilities when legislators voting weights depend on the party shares in the election. Duggan and Banks (2000); Banks (2006) generalize the work-horse models by looking at multidimensional policies and general status quo. Most of these models account for inefficiencies when a proposal is passed with delay. Other papers have considered a dynamic setting: Riboni (2010) models endogenous status quo, in which yesterday's policy is today's policy if an agreement is not reached. Similarly, Diermeier and Fong (2011), look at an endogenous status quo with persistent agenda-setting power. Macroeconomic models also incorporate a streamlined model of the legislative bargaining allowing for intertemporal linkages in fiscal policy, decided in the legislature (Battaglini and Coate, 2007, 2008; Leblanc et al., 2000; Piguillem and Riboni, 2015, 2018)

A notable feature of most of the models above is that a proposal is passed with the minimum amount of votes required, i.e., with minimum winning coalitions.⁸ Banks (2000) and Groseclose and Snyder (2000) study larger than minimal winning coalitions in a setup

⁵The clean development mechanism allows countries to implement part of their committed emission abatement targets through projects in countries that have ratified the Kyoto protocol but are not subject to such targets. This gives incentives to ratify the protocol both to countries that have to reduce emissions, as they can do so at a lower cost, and to countries that do not have to reduce emissions, as they will be recipients of foreign investment.

⁶For a general treatment, see Acemoglu (2003).

⁷Lya Eraslan (2002); Eraslan and McLennan (2013) provide a general model with heterogeneous recognition probabilities and discounting.

⁸In his classical work, Riker (1962) poses that minimum winning coalitions go hand in hand with zero sum games. This is disputed since Shepsle (1974).

with sequential voting, which allows for buying cheaper coalitions than minimum winning coalitions. This issue is also studied in a model of “pivotal bribing” in committees in Dal Bó (2007), and in a model of lobbying in Hummel (2009).

To our knowledge, ours is the first paper to study anonymous (up to the district’s type) legislative bargaining in which the size of the pie depends on the composition of the winning coalition.⁹ The paper closer to ours is Baranski (2016). Similarly to our modeling assumptions, the size of rents is endogenous to the agenda setters’ equilibrium proposals. The difference is that, in Baranski (2016) players are ex-ante identical, they all receive an endowment, and they can participate in joint production. In equilibrium, the identity of who produces is non-strategic and depends on the agenda setter’s proposal. While some results are qualitatively similar to ours, we delve deeper in two directions: we allow for non-minimal winning coalitions and for agents that can *decrease* the pie. Similarly, Calvert and Dietz (2006) and Cardona and Rubí-Barceló (2014) consider consumption externalities in the bargaining stage. The latter shows that these externalities affect ex-ante investment, leading to inefficient outcomes. On the same lines, in Harstad (2005), ex-ante investment and, therefore, the size of the pie diminish with the majority rule.¹⁰

In most papers, policy making takes place exclusively within formal institutions, disregarding informal channels of influence. An exception is Scartascini and Tommasi (2012), where political actors can choose to play in the legislative arena, or outside of it.¹¹ If they stay outside parliament, they become active in the “informal arena” and they channel their demands through mobilizations, riots, strikes, etc. Protests are placated with transfers from the formal institution. The authors focus on the long run determinants of institutionalization of policy making, understood as the fraction of actors choosing the formal arena. Contrary to Scartascini and Tommasi (2012), in our paper all demands are channeled inside the parliament, the size of the pie depends on the winning coalition, and the legislative game is repeated until there is an agreement. Also, we allow for positive and negative actions, which can take place simultaneously.

Other studies on political actions outside the parliament focus on the causes of protests, broadly defined. Ray and Esteban (2017) discuss how excluded factions (ethnic groups in their papers) can cause conflict and retaliation. Moreover, they link conflict with inequality, lower economic activity and development. In terms of our setup, the exclu-

⁹Eraslan and Merlo (2017) consider a model in which players are heterogeneous with respect to the potential surplus they bring to the bargaining table, and thus the size of the pie depends on the (random) identity of the agenda setter.

¹⁰Furthermore, Harstad (2005) follows Riker (1962) to model legislative bargaining, and restricts attention to minimum winning coalitions.

¹¹The legislative bargaining game has one round, equivalent to $\delta = 0$ in our setup.

sion of an ethnic group from the winning coalition can backlash into conflict. Edmond (2013) is a recent example of theoretical work on the coordination aspects of protesting, emphasizing (weak) institutional quality as a catalyst for protesting. Battaglini (2017) focuses on whether protests (or petitions) have the power of the wisdom of the commons in influencing policy makers through aggregation of preferences.

The rest of the paper is organized as follows. Section 2 describes the environment, and defines the equilibrium concept. Section 3 presents the main results, and section 3.1 provides some comparative static results, and solves for the decision to become active. Section 5 concludes.

2 Model

We consider an economy with n districts represented by n legislators who have to decide how to divide aggregate resources, \tilde{Y} . Legislators bargain over the distribution of resources using procedural rules as in Baron and Ferejohn (1989) with equal probabilities of recognition: from the set of legislators N with $|N| = n$, one is randomly chosen to make a proposal $x \in X \subset R^n$, where X is the set of all proposals that satisfy the budget constraint. That is, a proposal assigns $x_j \geq 0$ to each district $j \in N$, represented by legislator which we also index with j , such that $\sum_j x_j \leq \tilde{Y}$. Let the voting rule q be such that if a (super) majority of $n/2 < q \leq n$ votes to approve the proposal, resources are distributed and the game is over. If the proposal is not approved, the game begins again. There can be infinite sessions, and amendments are not possible. We assume $u_j(x) = x_j$ for all j , and all players discount the future with $\delta \leq 1$. Last, we focus on stationary equilibria, therefore our equilibrium concept is stationary subgame-perfect Nash.

When the legislature convenes, some districts are “active”, if they can take a costless action that affects the amount of aggregate resources, or “passive”, if they cannot. There can be two types of active districts, “productive”, which means the action they can take allows them to increase aggregate resources by η , or “destructive”, meaning the action they can take allows them to reduce aggregate resources by η . Let r^+ be the number of productive districts, r^- the number of destructive districts, and $n - r^+ - r^-$ the number of passive districts. In section 4 we endogeneize districts’ types by giving them a choice on whether or not to become active before the legislature convenes.

If a legislator from a productive district does not receive resources, i.e., is not included in the winning coalition, then it will choose not to increase aggregate output.¹² Since the

¹²This is a loose use of the term coalitions. We mean that a legislator or district is included in the

district will receive no resources if excluded from the winning coalition, this amounts to specifying the legislator's action when indifferent. If a legislator from a destructive district is not included in the winning coalition, then it will choose to destroy resources (feasibility requires that $n\eta < Y$). Since destruction is costless, this means we are assuming that if indifferent a destructive district would actually destroy resources. Let $m^+ \leq r^+$ ($m^- \leq r^-$) be the number of productive (destructive) districts in the winning coalition. Hence, the pie to be distributed is the following:

$$\tilde{Y}(m^+, m^-) = Y + [m^+ + (m^- - r^-)]\eta.$$

The presence of costless actions to increase or reduce output introduces two innovations with respect to Baron and Ferejohn (1989): first, the resources to be distributed, \tilde{Y} , are endogenous and depend on the winning coalition. Second, ex ante payoffs are not necessarily the same across districts, even if they have the same probability of being agenda setters.

2.1 Strategies and Equilibria

Let $i \in \{0, +, -\}$ denote the legislators' type, that is, whether they come from productive ($i = +$), destructive ($i = -$), or passive districts ($i = 0$). Let \mathcal{C}^i be the set that contains all possible winning coalitions. A coalition $C^i \in \mathcal{C}^i$ contains (at least) $q - 1$ elements, anonymous up to districts' type, with a legislator of type i as the agenda setter.

Let $\theta = (q, r^+, r^-, n, \delta, \eta, Y)$ be the vector of primitives of the game that determine the information set in the collection of information sets Θ , with $\theta \in \Theta$ common knowledge. A pure strategy, s_j^i , for an agenda setter of type i , is an action that offers certain distribution of rents to all members of C^i and 0 to all districts excluded from the coalition.

$$s_j^i : \Theta \rightarrow \mathcal{C}^i \times X.$$

The distribution of the pie must be feasible: $\sum_j x_j \leq \tilde{Y}$. While for non-agenda setters, for all i , a pure strategy is defined by:

$$a_j^i : \Theta \rightarrow \{\text{yes}, \text{no}\}.$$

Following Baron and Ferejohn (1989), we assume that legislators will vote to accept the proposal when indifferent, i.e. we prevent them from mixing between **yes** and **no** winning coalition if he was offered enough to vote yes for the proposal.

strategies. Let S_j^i be the set of feasible proposals in pure strategies, hence a proposal in mixed strategies π_j^i is the following

$$\pi_j^i : S_j^i \rightarrow [0, 1],$$

such that $\sum_{s_j^i \in S_j^i} \pi_j^i(s_j^i) = 1$ for all i and j . More generally, the pair $\sigma_j^i = (\pi_j^i, a_j^i)$ is a mixed (stationary) strategy. A subgame-perfect Nash equilibrium specifies the equilibrium strategies for any player, at any node. Stationarity implies that, conditional on being an agenda setter or not, the equilibrium strategies are the same in every node up to the player's type. Following our notation, since a pure strategy is a degenerate mixed strategy, we define our equilibrium only in terms of the latter.

Definition 1 (Stationary Subgame-Perfect Nash Equilibria). Let the n-tuple $(\sigma_j^{i*})^n$ be a stationary subgame perfect Nash Equilibria, for all $i \in \{0, +, -\}$ and $j \in N$, if

$$u_j^i(\sigma_j^{i*}, \sigma_{-j}^{i*}) \geq u_j^i(\sigma_j^i, \sigma_{-j}^{i*}),$$

for all σ_j^i and for all σ_{-j}^{i*} .

Due to anonymity, an agenda setter of type i cannot discriminate within types when making the proposal. Hence, in any stationary equilibrium, the continuation values of all the legislators of type i are identical, and have the same set of possible strategies. In terms of notation, anonymity allows us to drop the j indexes and look for symmetric stationary equilibria.

At any point in time, a proposal is accepted whenever at least q legislators obtain at least as much benefit by voting **yes** now than by voting **no** and waiting for the next round, i.e., they must receive at least their continuation value.

In what follows we restrict the analysis to q -supermajorities that exclude the unanimity rule. Since all legislators must receive their continuation value to approve a proposal, with $q = n$ they all have to be included in a winning coalition. Hence, they all have the same continuation value and the asymmetry between types disappears. In that case, equilibrium is the same as in Baron and Ferejohn (1989) with unanimity rule.

Remark 1 (Unanimity rule). With $q = n$, equilibrium in this game is identical to Baron and Ferejohn (1989), and the expected payoff of all legislators is $\frac{1}{n}(Y + r^+\eta)$. Similarly, when $\delta = 0$.

A minimum winning coalition is one in which exactly $q - 1$ members plus the agenda setter vote **yes**. Larger-than-minimal winning coalitions, i.e., when more than $q - 1$

members plus the agenda setter vote **yes**, might arise in equilibrium if the benefit of adding a district to the coalition outweighs its cost. Note that it can never be the case that “additional legislators” (those beyond q) come from passive districts when they have positive continuation values, as they would suppose a cost for the agenda setter without any gain. Thus, we are led to the following property of equilibria.

Remark 2. In larger-than-minimal winning coalitions, all members, except perhaps the agenda setter, come from active districts.

Remark 2 simplifies the characterization of equilibria. Let $m^+(i) = E(m^+ - \mathbb{1}_{i=+} | \sigma^i)$ ($m^-(i) = E(m^- - \mathbb{1}_{i=-} | \sigma^i)$) be the expected number of productive (destructive) legislators who would vote yes following a proposal from an agenda setter of type i . Let $C_{m^+, m^-}^i \in \mathcal{C}^i$ be the coalition composed of the agenda setter of type i , $m^+(i)$ ($m^-(i)$) legislators from productive (destructive) districts, and $\max\{0, q-1-m^+(i)-m^-(i)\}$ from passive ones. Let x_+^i , x_-^i , and x_0^i be the amounts offered to them and δv^+ , δv^- , and δv^0 their continuation values. An agenda setter’s strategy can be summarized by $m^+(i)$, $m^-(i)$, x_0^i , x_+^i , and x_-^i , such that it maximizes her payoff, subject to the acceptance of the proposal and feasibility constraints:

$$\begin{aligned}
& \max_{m^+(i), m^-(i), x_0^i, x_+^i, x_-^i} && \tilde{Y}(m^+(i) + m^-(i) + \mathbb{1}_{i \neq 0}) - m^+(i)x_+^i - m^-(i)x_-^i \\
& && - \max\{0, q-1-m^+(i)-m^-(i)\}x_0^i \\
& \text{s.t.} && x_k^i \geq \delta v^k, \quad \forall i, k = 0, +, - \\
& && \tilde{Y}(m^+(i) + m^-(i) + \mathbb{1}_{i \neq 0}) = Y + [m^+(i) + (m^-(i) - r^-) + \mathbb{1}_{i \neq 0}]\eta, \quad \forall i \\
& && m^k(i) \leq r^k - \mathbb{1}_{i=k}, \quad \forall i, k = +, - \\
& && m^k(i) \geq \max\{0, q + r^k - n - \mathbb{1}_{i=k}\}, \quad \forall i, k = +, -.
\end{aligned}$$

Given that the agenda setter’s utility is decreasing in x_k^i , constraints for x_k^i are always binding, and $x_k^0 = x_k^+ = x_k^- = \delta v^k$, for $k = 0, +, -$. Since the agenda setter takes as given continuation values, her strategy is then reduced to choosing $m^+(i)$, and $m^-(i)$, i.e. the composition of her coalition. Characterization of equilibria is simplified due to the following lemma.

Lemma 1. For all $1 \leq r^+ \leq n$, $1 \leq r^- \leq n - r^+$, and for all $0 \leq \delta \leq 1$, it is always the case that $v^+ = v^-$.

Proof. All proofs are in the appendix. □

Thus, the agenda setter is indifferent about the composition of her coalition as long as $m^+(i) + m^-(i)$ is constant.¹³ Her problem can be characterized as choosing $m(i) \equiv m^+(i) + m^-(i)$. Since productive and destructive districts have the same values, we distinguish districts only whether they are active or passive, and denote $r \equiv r^+ + r^-$. We will denote active districts as $i = 1$, and passive districts we keep denoting as $i = 0$. The agenda setter's problem is now given by

$$\begin{aligned} \max_{m(i)} \quad & \tilde{Y}(m(i) + i) - m(i)\delta v^1 - \max\{0, q - 1 - m(i)\}\delta v^0 & (1) \\ \text{s.t.} \quad & \tilde{Y}(m(i) + i) = [Y - r^-\eta] + (m(i) + i)\eta, \forall i \\ & m(i) \leq r - i, \forall i \\ & m(i) \geq \max\{0, q + r - n - i\}, \forall i. \end{aligned}$$

The first order condition of problem (1) is given by

$$\eta - \delta v^1 + \mathbb{1}_{m(i) < q-1} \delta v^0 - \bar{\lambda} + \underline{\lambda} = 0, \quad (2)$$

where $\bar{\lambda}$ and $\underline{\lambda}$ are, respectively, the multipliers on the upper and lower bounds of $m(i)$. The indicator function tells us that for minimum winning coalitions the agenda setter contemplates replacing a member from a passive district with one from a active district, while for larger-than-minimal coalitions, the decision is on enlarging the coalition with new members from active districts.¹⁴

At any point in time, any agenda setter of type i chooses a proposal that maximizes her utility. And taking into account the stationarity of equilibria, (i) she will not offer more than the continuation value to any legislator, and (ii) she will make a proposal that is accepted. Hence the game will end in the first period. Furthermore, by construction, these strategies are subgame perfect at any continuation subgame. Since individual deviations from equilibrium strategies do not affect the players' continuation values, the agenda setter takes the continuation values as given, and chooses $m(i)$ such that equation (2) holds. Moreover, the second order conditions trivially hold due to the linearity of the objective function and the constraints. For the same reason, uniqueness is not warranted.

To solve equation (2), we need to compute continuation values. For any v^0 and v^1 , let the cost of forming a coalition be: $e(C_m^i) = m(i)\delta v^1 + \max\{0, q - 1 - m(i)\}\delta v^0$. Let ρ^i

¹³Note that $\tilde{Y}(m^+(i) + m^-(i) + \mathbb{1}_{i \neq 0}) = Y - [m^+ + (r^- - m^-)]\eta = [Y - r^-\eta] + (m^+(i) + m^-(i) + \mathbb{1}_{i \neq 0})\eta$. Furthermore, if r^+ or r^- is zero then the problem can also trivially be cast in terms of $m^+(i) + m^-(i)$.

¹⁴To be more precise, when $m(i) = q - 1$, the left derivative of (1) is $\eta - \delta v^1 + \delta v^0$, while its right derivative is $\eta - \delta v^1$.

be the probability of a type i legislator being called into a coalition by the agenda setter. Then, taking into account that the probability of recognition as an agenda setter is the same for all types, stationarity implies that, for all i , we can write valuations as follows:

$$v^i = \frac{1}{n}(\tilde{Y} - e(C_m^i)) + \frac{n-1}{n}\rho^i\delta v^i. \quad (3)$$

From the equations above we can solve for v^i as a function of ρ^i , which depend on the coalitions proposed by legislators of type i , summarized in $m(i)$. That is, we need to calculate $\rho^i(m(0), m(1))$.

Suppose we take a passive legislator and we want to find out the probability that he is called into a coalition, ρ^0 . With probability $r/(n-1)$, the agenda setter comes from an active district, hence, the probability that a passive legislator is called in the coalition depends on how many passive districts the active agenda setter needs to call, $\max\{0, q - m(1) - 1\}$, divided by the total number of available passive districts ($n - r$). With probability $\frac{n-r-1}{n-1}$ the agenda setter is from a passive district, and the probability that a passive legislator is called in the coalition depends on how many passive districts the passive agenda setter needs to call, $\max\{0, q - m(0) - 1\}$, divided the total number of available passive districts ($n - r - 1$). Similarly for the case of a legislator from an active district that is not the agenda setter. Hence,

$$\rho^0(m(0), m(1)) = \left[\frac{r}{n-1} \frac{q - m(1) - 1}{n-r} + \frac{n-r-1}{n-1} \frac{q - m(0) - 1}{n-r-1} \right], \quad (4)$$

$$\rho^1(m(0), m(1)) = \left[\frac{r-1}{n-1} \frac{m(1)}{r-1} + \frac{n-r}{n-1} \frac{m(0)}{r} \right]. \quad (5)$$

Any stationary subgame-perfect Nash equilibrium must solve the system of equations (2), and (3) for all i . Before studying the general case we show that $v^1 \geq v^0$ and gain intuition by considering a simple example.

Lemma 2. For all $1 \leq r \leq n$, and for all $0 \leq \delta \leq 1$, it is always the case that $v^1 \geq v^0$.

Example: $\delta = 1$, $r = r^- = 1$. With a very large valuation of the future and a single destructive district, a passive agenda setter must decide whether or not to include the active district in the winning coalition. Suppose that in equilibrium the latter is always included, i.e. $\rho^1 = 1$. Then it is straightforward to find that

$$\begin{aligned} v^1 &= Y, \\ v^0 &= 0. \end{aligned}$$

Since $n\eta < Y$, $\delta(v^1 - v^0) > \eta$, and it is not optimal to have $\rho^1 = 1$. Thus, $\rho^1 < 1$ and there are expected output losses in equilibrium.

3 Analysis

We denote “corner-equilibria” those equilibria in which either $\underline{\lambda} > 0$, $\bar{\lambda} > 0$, or $m(i) = q-1$, and “interior equilibria” those equilibria in which at least one type of agenda setter’s choice is unconstrained, i.e., one for which $\underline{\lambda} = \bar{\lambda} = 0$, and $m(i) \neq q-1$. Since in interior equilibria $m(i)$ generically will not be an integer, we are led to the following characterization of equilibria.¹⁵

Remark 3. Interior equilibria are mixed-strategy equilibria, and corner equilibria are pure strategy equilibria.

Lemma 3 provides our first result. It establishes that when districts can take costless actions to change the size of the pie, larger-than-minimum winning coalitions are possible in equilibrium.

Lemma 3. Winning coalitions are minimal if and only if $\delta \geq \delta_q$. If $r \leq q-1+i$, $\delta_q = 0$.

Relative to the voting rule, q , the number of active districts, r , and the potential change in output, η , the discount factor determines how costly it is to get a legislator’s support. When the discount factor is large enough, only minimum winning coalitions can be sustained in equilibrium. Indeed, for high δ , $\delta \geq \delta_q$, since legislators give a relatively large weight to the future, their continuation values are large. Thus, the cost of adding a non-necessary legislator into the winning coalition is large as well. In this case, the agenda setter does not want to form a larger-than-minimal winning coalition. Conversely, for low δ , $\delta < \delta_q$, the legislators’ continuation values are small, and the cost of including an extra active district in the coalition might be smaller than the output loss if excluded.

Hence, when the number of active districts is small, all coalitions are minimal. While when $r > q-1+i$, an agenda setter of type i might be willing to form larger-than-minimal winning coalitions. In this case, even though larger-than-minimum coalitions are more expensive to build, they maximize the agenda setters’ utility in equilibrium because she acts as a residual claimant. She adds a non-necessary active district even though the cost

¹⁵Our distinction between pure and mixed-strategy equilibria relates to whether strategies call for an integer number of legislator of each type, or if there is randomization between two different integers. In legislative bargaining, due to anonymity, strategies are usually mixing in the sense that there is randomization between legislators of a given type.

of the coalition increases by δv^1 because rents increase by η , and so her utility increases by $\eta - \delta v^1$.

Since Riker (1962), the literature on bargaining has tried to conciliate the theoretical prediction of minimum winning coalitions with the evidence, which does not support it.¹⁶ Lemma 3 provides a rationale for larger than-minimum winning coalitions: they are an equilibrium if and only if the cost of additional legislators is lower than the increase in output from including them in the coalition.

The following proposition characterizes equilibria.

- Proposition 1.** (i) For all r , and for $\delta \in [\delta_q, \bar{\delta}]$, there are only corner equilibria including $m(i) = \min\{q - 1, r - i\}$ active districts.
(ii) For all r , and for $\delta > \bar{\delta}$, there are only interior equilibria.
(iii) For $r > q - 1 + i$, and for $\delta \in [0, \underline{\delta}]$, there are only corner equilibria with $m(i) = r - i$.
(iv) For $r > q - 1 + i$, and for $\delta \in (\underline{\delta}, \delta_q)$, there are only interior equilibria with $q - i < m(i) < r - i$.

Proposition 1 presents our second result. It shows that it is possible that output be inefficient in equilibrium, which happens whenever $m(i) + i < r$. This result reflects the fact that in models of legislative bargaining with linear utility, the agenda setter's actions can be interpreted as if she only cared about the welfare of the winning coalition. Thus, if the cost of adding an active district, or replacing a passive by an active one, is higher than the output gain, not all active districts will be called into the coalition. In contrast, a social planner that cared for aggregate social welfare would never exclude active districts, as this implies an inefficient loss of output.

In (i) and (ii), since $\delta \geq \delta_q$, and independently of the number of active districts, the agenda setter only proposes minimum winning coalitions. Equilibria for (i) and (ii) are depicted in figure 1. In this case, the legislators in active districts included in the coalition are so at the expense of passive ones. In other words, since $\mathbb{1}_{m(i) < q-1} = 1$, the interior equilibrium condition is

$$\delta(v^1 - v^0) = \eta.$$

When $\delta \in [\delta_q, \bar{\delta}]$ all the available active districts to complete a minimum winning coalition are called into it. That is, with $r \leq q - 1 + i$, $m(i) = r - i$ and there are minimum winning coalitions with no output loss. With $r > q - 1 + i$, and $m(i) = q - 1$, some active districts are left out of the winning coalition and the equilibrium is inefficient.

¹⁶See Knight (2008) and references therein.

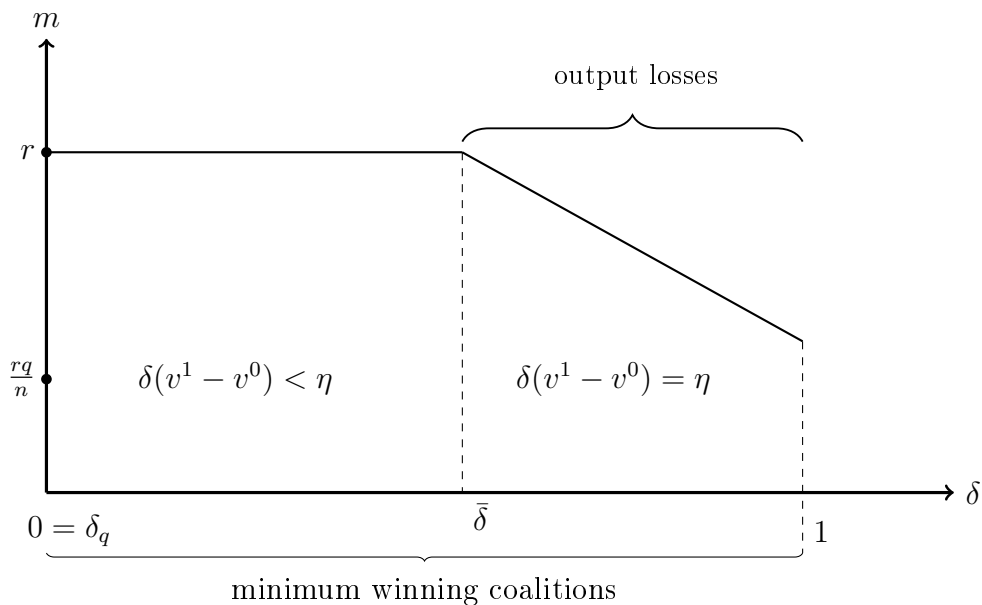


Figure 1: Equilibria when $r \leq q - 1 + i$

For the equilibria described in (iii) and (iv), $\mathbb{1}_{m(i) < q-1} = 0$, the interior equilibrium condition is

$$\delta v^1 = \eta.$$

Similarly to lemma 3, for δ large enough, the benefits to include active districts beyond the minimum-winning coalition must be in balance with the costs. Therefore, for $\underline{\delta} < \delta < \delta_q$, there are mixed strategy equilibria with larger-than-minimum winning coalitions. If δ becomes so small that the benefits of including active districts beyond $q - 1$ is always larger than its cost, then there is a unique pure strategy equilibrium in which all active districts are called into the coalition.¹⁷ Figure 2 describes equilibria for (iii) and (iv).

Corollary 1. Legislators from active districts have a higher probability of being in the winning coalition.

Corollary 1 presents our third, and final, result. It shows that active districts are more likely to be called into a winning coalition. In fact, as we show in the proof, it is precisely their higher probability of being in the winning coalition that leads them to have higher ex ante payoffs.

¹⁷Whenever equilibrium is characterized by $m(i) = r - i$, i.e. when all active districts are included in the coalition, the game has zero-sum properties.

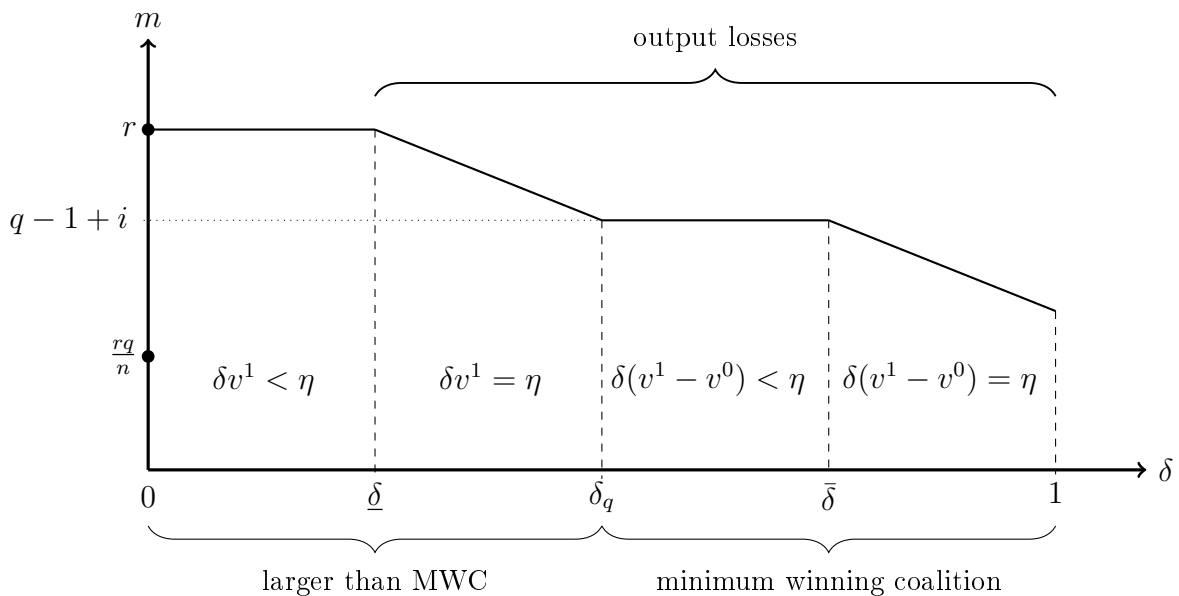


Figure 2: Equilibria when $r > q - 1 + i$

3.1 Comparative Statics

The next proposition summarizes some comparative static results. In particular, we are interested in the effect of parameter changes on the likelihood of having minimum winning coalitions and on output losses, defined as $(r - E(m))\eta$, where $E(m) \equiv \frac{n-r}{n}m(0) + \frac{r}{n}(m(1) + 1)$. As expected, these results depend on the effect of parameter changes on the continuation values of active and passive districts.

Proposition 2. Consider the effects of an increase in the following parameters,

- i) q : makes minimum winning coalitions more likely, and reduces output losses. Active districts' payoffs increase, except when $r \leq q - 1 + i$ and $\delta < \bar{\delta}$.
- ii) η : makes minimum winning coalitions less likely. The effect on output losses is ambiguous.
- iii) r^+ , r^- : makes minimum winning coalitions less likely. The effect on output losses is ambiguous.
- iv) δ : reduces the expected number of active districts in interior equilibria, thus increasing output losses.

An increase in the needed supermajority (weakly) raises the number of both types of legislators in the winning coalition. The increase in the expected number of active districts reduces output losses in equilibrium. The mechanism by which active legislators are (weakly) more likely to be part of the winning coalition depends on whether the

number of active legislators is higher or lower than q .

First, consider the case of a large number of active legislators ($r > q - 1 + i$), depicted in figure 3. We find that q has no effect on $\underline{\delta}$, nor on $m(i)$ for $\delta \in (\underline{\delta}, \delta_q)$. Thus, the supermajority does not affect the equilibrium, in particular output losses, for $\delta \in [0, \delta_q)$. According to remark 2, in the region of minimum winning coalitions, $\delta \in [\delta_q, \bar{\delta}]$, additional legislators needed to achieve the new supermajority come from active districts. Thus, in this region, an increase in q reduces output losses. Finally, for $\delta > \bar{\delta}$, $E(m)$ increases with q (since otherwise v^0 would increase more than v^1), thus reducing output losses.

If there is a small number of legislators ($r \leq q - 1 + i$), the agenda setter's initial response to an increase in q is to call more passive districts into the minimum winning coalition (if $\delta \in [0, \bar{\delta})$, there is no other course of action as all active districts are already in the coalition). This increases passive districts' continuation values, giving the agenda setter incentives to increase the probability of calling active districts when using a mixing strategy. As a result, $\bar{\delta}$ increases with q , as does $E(m)$ for $\delta > \bar{\delta}$. Thus, an increase in q reduces output losses.

The different mechanisms that explain the decrease in output losses with greater supermajorities are then consistent with a non-monotonic effect of q on the continuation values of active districts. For $r > q - 1 + i$ some active districts are left out from the minimum winning coalition, thus increasing the needed supermajority increases the probability that they are called into it, rising their continuation value. For $r \leq q - 1 + i$, when $\delta < \bar{\delta}$, the effect of an increase in the supermajority reverses, as this now increases the probability that passive districts are called into the coalition. The increase in passive districts continuation values must be met, due to feasibility, by a decrease in active players' continuation values. Since when $r \leq q - 1 + i$, $\bar{\delta}$ is increasing in q , there is always a supermajority above which the continuation values of active districts is decreasing in q .¹⁸

Next we consider an increase in η . For this, note that a proportional increase in Y and η leads to proportional increases in v^i , and no effect on thresholds or optimal strategies. Thus, changes in η can be interpreted as comparing different economies in a cross-section, or the same economy over the business cycle (for the latter an increase in η reflects a fall in Y). An increase in η increases the agenda setter's incentives to include active districts in the winning coalition (in particular, the thresholds for larger than minimum winning coalitions including all active districts, $\underline{\delta}$, and for minimum winning coalitions, δ_q , increase). This increases the expected number of active districts in the coalition ($E(m)$), but increases the cost of active districts left out from it. Hence, the effect on

¹⁸Formally, this threshold supermajority corresponds to q such that $\bar{\delta} = 1$. See (11) in the appendix.

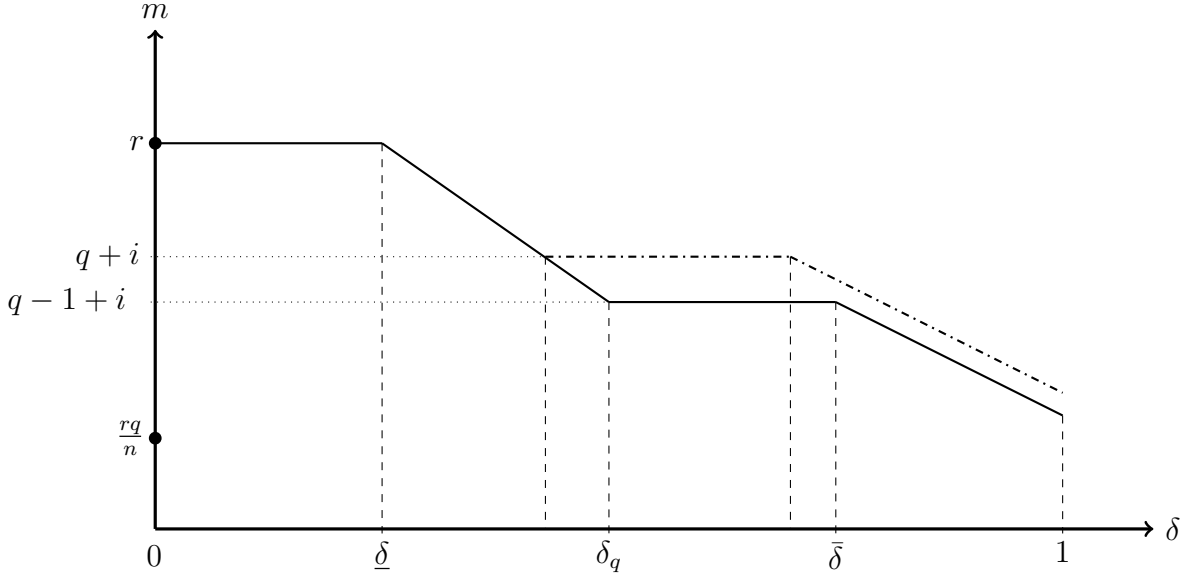


Figure 3: Comparative statics in q : $r > q + i$

output losses, $(r - E(m))\eta$, is generally ambiguous. For example, with a large number of active districts ($r > q - 1 + i$), when mixed strategies are an equilibrium with larger than minimum winning coalitions, an increase in η reduces output losses. With minimum winning coalitions that do not include all active districts, an increase in η increases output losses.¹⁹

An increase in the number of active districts has two effects. First, it gives the agenda setter incentives to increase the number of districts called into the winning coalition. Second, it reduces the probability of a given active district to be called into the winning coalition. These two effects have opposite effects on the continuation value of active districts. When $r > q - 1 + i$, $\underline{\delta}$, $E(m)$ for $\delta \in (\underline{\delta}, \delta_q)$, and δ_q increase with r^- or r^+ . From the latter, minimum winning coalitions are part of the equilibrium for a smaller set of δ . While an increase in r^+ reduces output losses for $\delta \in (\underline{\delta}, \delta_q)$, the effect of r^- is ambiguous (the higher is r^- the more likely output losses increase). For $\delta \in [\delta_q, \bar{\delta}]$, an increase in r^- or r^+ increases output losses, and for $\delta > \bar{\delta}$ the effect is ambiguous (it can be shown that output losses increase with r^- or r^+ when $\delta \approx 1$).

Finally, increases in the discount factor lead to increased output losses, as this increases the continuation value of active districts inducing the agenda setter to call them less often into the winning coalition.

¹⁹When $\delta > \bar{\delta}$, the effect is ambiguous. It can be shown that when $\delta \approx 1$, output losses increase with η .

4 Choice of Becoming Active

Having characterized the equilibria for a given number of active districts we now endogenize districts' choice on whether to become active or not. We assume there is no cost to becoming active. Before the legislature convenes, each district observes a signal, w_j , that is i.i.d. across districts. With probability β^+ , the signal takes the value 1, with probability β^- it takes the value -1 , and with probability $1 - \beta^+ - \beta^-$ it takes the value zero. If $w_j = 1$, district j has the option of becoming "productive". If $w_j = -1$, then district j has the option of becoming "destructive". Finally, if $w_j = 0$ district j can take no action.²⁰ We will now show that all districts for which $w_j \neq 0$ will choose to exercise their options.

With a bit of an abuse in notation, let's assume that r districts have the option to either become productive or destructive, and denote by $v^i(\cdot)$ ex ante payoffs as a function of the number of active districts. Without loss of generality, we consider the decision problem in one of these districts, that takes as given that the other $r - 1$ districts will become active. Thus, this district is in effect comparing payoffs $v^1(r)$ and $v^0(r - 1)$. Given that becoming active is assumed to be costless, it will be in the districts interest to do so whenever $v^1(r) \geq v^0(r - 1)$.²¹ Note that the presence of output losses for some equilibria renders this condition non trivial.

Proposition 3. For all $1 \leq r \leq n$, and for all $0 \leq \delta \leq 1$, it is always the case that $v^1(r) \geq v^0(r - 1)$.

We thus verify that all districts that have an option to become active will do so. The assumption that becoming active is costless allows to characterize this decision without having to find explicit expressions for $v^1(r)$ and $v^0(r)$. If instead we assume that the action is costly, then each district, upon observing the realization of signals, would have to compare the expected gain from becoming active with the cost. Furthermore, if information is imperfect, such that each district only observes their signal, the expected gain, $E[v^1(r) - v^0(r - 1)]$, depends on the distribution of r (which depends on parameters β^+ and β^-). Thus, the decision on becoming active requires knowing $v^i(r)$ for all i and r .²²

²⁰Thus, parameters β^- and β^+ can be seen as measures of institutional quality, or as measures of the degree of discretion that districts have to shield regional output from national taxation, or to promote growth opportunities with spillovers.

²¹We make the assumption, standard in the legislative bargaining literature, of selecting legislators choices when indifferent. Furthermore, from the proof it can be seen that, if $\delta > 0$, $v^1(r) > v^0(r - 1)$.

²²A microfoundation for actions with imperfect information is to have citizens (or a subgroup of them, such as public servants or scientists) in district i observe a noisy signal of the realization of a variable θ_i that summarizes institutional quality or growth opportunities in their district and decide non cooperatively whether to engage in destructive/productive action or not. If the mass of citizens choosing to act

Denote by z the cost of becoming active. For small z , e.g. $z < \min_r[v^1(r) - v^0(r - 1)]$, and provided $\delta > 0$, proposition 3 continues to hold, and all districts that have the option to become active will do so. Lemmas 1, 2, and 3, propositions 1 and 2, and corollary 1 would hold as well.²³

5 Conclusions

We introduce a simple, and arguably natural, assumption in the canonical model of legislative bargaining: some legislators have the ability to either “grease” or “sand” the wheels of policy-making. These legislators, if satisfied with the outcome of the bargaining, cooperate to increase output, rents, or resources available for taxation. Conversely, if unsatisfied, they may retaliate reducing output, rents, or the tax base. With this assumption, the pie to be distributed in the legislative bargaining game becomes endogenous, and determined by the composition of the winning coalition.

Given their ability to affect the level of aggregate resources, active districts are more likely to be called into a winning coalition than passive districts. How much more likely depends on parameters that determine the type of equilibrium. When legislators are sufficiently impatient, all active districts will be called into the winning coalition. As patience increases, so does the continuation value of active districts. When the agenda setter is choosing the composition of her winning coalition, she trades off the higher cost of active districts against the increase in output they produce. Therefore, as patience increases, active districts eventually stop being called into the winning coalition with certainty. This produces output losses, as either the gains of including cooperating legislators are not realized, or retaliation takes place.

When there is a relatively large number of active districts, larger than minimum winning coalitions are possible in equilibrium. This happens when the cost of one extra active district is lower than the increase in output it can produce, thus increasing the agenda setter’s payoff. This feature of our model resonates with the large empirical evidence on larger than minimum winning coalitions, and fills a gap in theoretical models of legislative bargaining, where only minimum winning coalitions are possible.

Considering larger than minimum winning coalitions turns around the trade-off be-

is larger than θ_i then the action is successful and we say that the district is active. See Edmond (2013) for a detailed analysis in an application to street protests.

²³An important caveat is that with costly actions, we must assume that destructive districts commit to destroying resources when left out of a winning coalition. Alternatively, extending our legislative bargaining model into a repeated game might explain this as arising from reputational considerations.

tween expropriation of minorities and decision-making costs (Buchanan and Tullock (1962); Harstad (2005)). With myopic agents and a large number of rioters ($r > q - i$), coalitions only include (all) active districts and exclude the passive minority while increasing the size of the pie.

The payoff of becoming a cooperator or retaliator is always positive. Thus, every district that has the option of becoming active will do so if this action is costless, or if the cost is sufficiently low. Our results can be linked to the literature on institutional strength (Scartascini and Tommasi (2012); Levitsky and Murillo (2009)). Districts only cooperate if they get transfers, incentivizing only conditional cooperation. A weak institutional setting, with large potential damage η , many active districts r , and/or myopic agents (small δ) sustains an equilibrium with systematic transfers to retaliation districts. In turn, this leads to greater incentives to become a retaliating member, weakening the institutional framework even further. Our work provides the foundations for a repeated game, in which a share of available resources can be used to invest in strengthening institutions, e.g. by reducing the probability that a district can engage in retaliating activities in the following period. Legislative bargaining can thus introduce persistence to output shocks. We leave the analysis of such an extension for future work.

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6 Appendix

6.1 Proof of Lemma 1

The proof proceeds by contradiction. Suppose $v^+ > v^-$. Then the agenda setter can increase her payoff by reducing m^+ and increasing m^- , keeping $m^+ + m^-$ unaffected. This has no impact on resources to be distributed (the excluded productive district will not increase output by η , but the included destructive district will refrain from destroying resources by η). And the change in the composition of the winning coalition increases the agenda setter's payoff by $\delta(v^+ - v^-) > 0$. Thus, the agenda setter will try to replace productive by destructive districts as much as possible. If no agenda setter includes productive districts unless forced to, then $m^+(i) = \max\{0, q+r^+ - n - \mathbb{1}_{i=+}\}$. If $m^+(i) = 0$, then a destructive district as agenda setter would have the same surplus output as a productive one, the same recognition probability, and would be called into a winning coalition with higher, positive, probability, $\frac{m^-(i)}{r^-} > 0 = \frac{m^+(i)}{r^+}$. Thus, it must be the case that $v^- \geq v^+$. If $m^+(i) = q + r^+ - n - \mathbb{1}_{i=+}$, then a destructive district as agenda setter would have the same surplus output as a productive one, the same recognition probability, and would be called into a winning coalition with higher probability, $\frac{m^-(i)}{r^-} = 1 > \frac{q+r^+ - n - \mathbb{1}_{i=+}}{r^+} = \frac{m^+(i)}{r^+}$. Thus, it must again be the case that $v^- \geq v^+$. Thus, we cannot have $v^+ > v^-$. Similar reasoning rules out $v^+ < v^-$, and we conclude that it must be the case that $v^+ = v^-$.

6.2 Proof of Lemma 2

From first order condition (2) it is immediate that, if $v^0 > v^1$, an agenda setter would never choose to have a passive district in her coalition when an active one is available. If $r \geq q$, no passive is called into the winning coalition, so the value of a passive legislator is just the recognition probability, $\frac{1}{n}$, times the "surplus output" of a passive agenda setter. But an active agenda setter would have a larger surplus output (since she comes from an active district the loss of output, conditional on the same voting majority, is lower), the same recognition probability, and would be called into a winning coalition with higher, positive, probability. Thus, it must be the case that $v^1 \geq v^0$. Consider now the case that $r < q$. A passive district then has positive probability of being called into the winning coalition. But, this probability is 1 for active districts and thus higher than for passive districts (surplus output and recognition probabilities would be the same in this case).

Therefore, it is also the case that $v^1 \geq v^0$.²⁴

6.3 Proof of Lemma 3

The threshold $\delta_q(r)$ is zero when even including all active districts the winning coalition is minimal. This is always the case when $r \leq q - 1$, and is also the case when $r = q$ and the agenda setter is from a active district.

When $r > q$ or $r = q$ and the agenda setter is from a passive district, the threshold $\delta_q(r)$ will be determined by solving the equilibrium under the assumption that $m(i) = q - 1$ and verifying that the agenda setter does not prefer to add an active district into the coalition. For this case, from equations (3), values must satisfy

$$v^0 = \frac{1}{n} [Y - r^- \eta + (q - 1)\eta - (q - 1)\delta v^1], \quad (6)$$

$$v^1 = \frac{1}{n} [Y - r^- \eta + q\eta - (q - 1)\delta v^1] + \frac{n - 1}{n} \left[\frac{n - r}{n - 1} \frac{q - 1}{r} + \frac{r - 1}{n - 1} \frac{q - 1}{r - 1} \right] \delta v^1. \quad (7)$$

From the second equation we can solve for v^1

$$v^1 = \frac{r(Y - r^- \eta + q\eta)}{nr - \delta(n - r)(q - 1)}. \quad (8)$$

Whenever $\delta v^1 > \eta$, the agenda setter will be unwilling to include more than $q - 1$ active districts into her coalition. Thus, δ_q is implicitly determined by $\delta_q v^1 = \eta$,

$$\eta = \delta_q \frac{r(Y - r^- \eta + q\eta)}{nr - \delta_q(n - r)(q - 1)}. \quad (9)$$

Since then RHS of the last equation is increasing in $\delta_q(r)$, the coalition will be minimal when $\delta \geq \delta_q$. Note that it might be the case that $v^1|_{\delta=1} < \eta$, and thus $\delta_q > 1$ If $\delta_q \geq 1$, which might happen for high r^- , coalitions are always larger-than-minimum.

6.4 Proof of Proposition 1

(i) and (ii) Since $\delta \geq \delta_q$, from lemma 2 we are only considering minimum winning coalitions. To determine the threshold $\bar{\delta}$ we solve for a corner equilibrium with $m(i) = \min[q - 1, r - i]$ and verify that the agenda setter does not prefer to reduce the number of active districts included in the coalition. To solve for the value functions, we must

²⁴Equality only holds when $\delta = 0$, or $q = n$, such that $v^0 = v^1 = \frac{Y + r^+ \eta}{n}$.

differentiate whether $m(i) = r - i$ or $m(i) = q - 1$. For the former case, from equations (3),

$$\begin{aligned} v^0 &= \frac{1}{n} [Y + r^+\eta - r\delta v^1 - (q - r - 1)\delta v^0] + \frac{n-1}{n} \left[\frac{n-r-1}{n-1} \frac{q-r-1}{n-r-1} + \frac{r}{n-1} \frac{q-r}{n-r} \right] \delta v^0, \\ v^1 &= \frac{1}{n} [Y + r^+\eta - (r-1)\delta v^1 - (q-r)\delta v^0] + \frac{n-1}{n} \delta v^1. \end{aligned}$$

Where the last equation shows that in this cases all active districts are included in the winning coalition with probability one. Solving we find

$$\begin{aligned} v^0 &= \frac{(Y + r^+\eta)(1 - \delta)(n - r)}{n(n - r)(1 - \delta) + r\delta(n - q)}, \\ v^1 &= \frac{Y + r^+\eta - (q - r)\delta v^0}{n - (n - r)\delta}. \end{aligned} \tag{10}$$

Note that $\frac{dv^0}{d\delta} < 0$. Since expected output is independent of δ , and feasibility implies $rv^1(\delta) + (n - r)v^0(\delta) = Y + r^+\eta$, it must be the case that

$$r \frac{dv^1}{d\delta} + (n - r) \frac{dv^0}{d\delta} = 0.$$

Thus, $\frac{dv^1}{d\delta} > 0$ and $\frac{d\delta(v^1 - v^0)}{d\delta} > 0$. To show that $0 < \bar{\delta} < 1$ we note that $v^0|_{\delta=0} = v^1|_{\delta=0} = \frac{Y+r^+\eta}{n}$, while $v^0|_{\delta=1} = 0$, and $v^1|_{\delta=1} = \frac{Y+r^+\eta}{r}$, implying $v^1|_{\delta=1} - v^0|_{\delta=1} > \eta$. Thus, $\bar{\delta}$ is determined by

$$\begin{aligned} \bar{\delta} (v^1|_{\bar{\delta}} - v^0|_{\bar{\delta}}) &= \eta, \\ \frac{\bar{\delta}(Y + r^+\eta)}{n(1 - \bar{\delta}) + r\bar{\delta}} \left[1 - \frac{1 - \bar{\delta} + \bar{\delta}q/n}{1 + \frac{r\bar{\delta}(n-q)}{(1-\bar{\delta})(n-r)}} \right] &= \eta. \end{aligned} \tag{11}$$

and for $\delta > \bar{\delta}$, the agenda setter would choose $m(i) < r - i$ as the cost of including all active districts in the coalition is higher than the resource cost of excluding some of them.

Turning to the case $m(i) = q - 1$, equations (6) and (7) characterize v^0 and v^1 , from which we get

$$v^1 - v^0 = \frac{\eta}{n} + \frac{q-1}{n} \left[\frac{(n-r)}{r} + 1 \right] \delta v^1$$

From (8) we have that $\frac{dv^1}{d\delta} > 0$ which implies $\frac{d\delta(v^1 - v^0)}{d\delta} > 0$. The threshold $\bar{\delta}$ is characterized

by

$$\eta = \bar{\delta} \left[\frac{\eta}{n} + \bar{\delta} \frac{(q-1)[Y - r^-\eta + q\eta]}{nr - (n-r)(q-1)\bar{\delta}} \right]. \quad (12)$$

Taking into account that $v^1|_{\delta=1} - v^0|_{\delta=1}$ might be lower than η , it might be the case that $\bar{\delta} > 1$. If $\bar{\delta} \geq 1$, then the equilibrium is always at a corner with $m(i) = q - 1$. This will be the case when η is large enough, and for high values of r^- . When $\bar{\delta} < 1$, and $\bar{\delta} < \delta$, the agenda setter prefers to exclude some active districts from the minimum-winning coalition, and the equilibrium is interior.

(iii) and (iv) To determine the threshold $\underline{\delta}$ we solve for a corner equilibrium with $m(i) = r - i$ and verify that the agenda setter does not prefer to reduce the number of active districts included in the coalition.

$$\begin{aligned} v^0 &= \frac{1}{n} [Y + r^+\eta - r\delta v^1], \\ v^1 &= \frac{1}{n} [Y + r^+\eta - (r-1)\delta v^1] + \frac{n-1}{n} \delta v^1. \end{aligned}$$

From the second equation we derive

$$v^1 = \frac{Y + r^+\eta}{n - \delta(n-r)}.$$

It is immediate that $\frac{dv^1}{d\delta} > 0$. An agenda setter will be willing to include all active districts in the coalition as long as the cost of doing so is lower than the damage they could produce on output. Thus, the threshold $\underline{\delta}$ is determined by $\underline{\delta}v^1 = \eta$,

$$\underline{\delta} \frac{Y + r^+\eta}{n - (n-r)\underline{\delta}} = \eta \implies \underline{\delta} = \frac{n\eta}{Y + (n-r^+)\eta}. \quad (13)$$

When $\underline{\delta} < \delta < \delta_q$ the agenda setter will form a coalition with $q - 1 < m(i) < r - i$ active districts.

6.5 Proof of Corollary 1

It is straightforward that active districts have a higher probability of being in the winning coalition when $r > q - 1$, and $\delta \in [0, \delta_q)$, as in this case passive districts are never called into a winning coalition. When $\delta \in [\delta_q, \bar{\delta}]$ such that we have corner equilibria including $m(i) = \max\{q - 1, r - i\}$, if $m(i) = q - 1$, active districts have a positive probability of being in the winning coalition while passive districts are never called into it, while if $m(i) = r - i$ an active district's probability of being in the winning coalition is 1, thus

higher than for a passive district.

We are thus left with the case $\delta \geq \bar{\delta}$. To prove that active districts must have a higher probability of being in the winning coalition we proceed by contradiction and assume that this probability is the same for every district. If this were the case, the probability of being in the winning coalition must be $\frac{q-1}{n}$. This implies

$$m(0) = \frac{rq}{n}, \quad m(1) = \frac{rq}{n} - 1.$$

We now use equations (3) to estimate v^0 and v^1 :

$$\begin{aligned} v^0 \left(1 - \delta \frac{q-1}{n}\right) &= \frac{1}{n} \left[Y - r^- \eta + \frac{rq}{n} \eta - \left(\frac{rq}{n}\right) \delta v^1 - \left(q - \frac{rq}{n} - 1\right) \delta v^0 \right], \\ v^1 \left(1 - \delta \frac{q-1}{n}\right) &= \frac{1}{n} \left[Y - r^- \eta + \frac{rq}{n} \eta - \left(\frac{rq}{n} - 1\right) \delta v^1 - \left(q - \frac{rq}{n}\right) \delta v^0 \right]. \end{aligned}$$

These equations imply

$$(v^1 - v^0) \left(1 - \delta \frac{q-1}{n} - \frac{\delta}{n}\right) = 0. \quad (14)$$

But for an interior solution, as must be the case when $\delta \geq \bar{\delta}$, first order condition (2) implies

$$\delta(v^1 - v^0) = \eta. \quad (15)$$

Equation (14) is generically inconsistent with (15), and would imply that if districts have the same probability of being in the winning coalition they should have the same continuation values, i.e. $v^1 = v^0$. Thus, this tells us that the source of higher ex ante payoffs for active districts is precisely their higher probability of being in the winning coalition.

6.6 Proof of Proposition 2

We start by characterizing equilibria for the two types of interior equilibria: a) for minimum winning coalitions, $\delta > \bar{\delta}$, and b) for larger than minimum winning coalitions, $\delta \in [\underline{\delta}, \delta_q)$.

a) We expect to find multiple interior equilibria since we have a system of three equations, (3), and the indifference condition $\delta(v^1 - v^0) = \eta$, in four unknowns, v^0 , v^1 , $m(0)$, and $m(1)$. Using these three equations leads to a continuum of equilibria characterized by a relation between strategies, say $m(1) = f(m(0))$. This allows us to

write $v^0(m(0))$ and $v^1(m(0))$, which from (3) are given by:

$$\begin{aligned} v^0(m(0)) &= \frac{\frac{1}{n} \left[\tilde{Y}(m(0)) - e(C_m^0) \right]}{1 - \frac{n-1}{n} \rho^0(m(0)) \delta} \\ v^1(m(0)) &= \frac{\frac{1}{n} \left[\tilde{Y}(f(m(0))) - e(C_m^1) \right]}{1 - \frac{n-1}{n} \rho^1(m(0)) \delta} \end{aligned} \quad (16)$$

Because strategies $m(0)$ and $f(m(0))$ satisfy $\delta(v^1 - v^0) = \eta$, for all feasible $m(0)$ we must have that

$$\frac{d \left[\tilde{Y}(m(0)) - e(C_m^0) \right]}{dm(0)} = \frac{d \left[\tilde{Y}(f(m(0))) - e(C_m^1) \right]}{dm(0)} = 0,$$

since agenda setters are indifferent with respect to the composition of their coalitions. We must also have that the total derivatives $\frac{dv^1(m(0))}{dm(0)} = \frac{dv^0(m(0))}{dm(0)}$ (to satisfy $\delta(v^1(m(0)) - v^0(m(0))) = \eta$). Using expressions (16), after some algebra, this implies

$$\frac{dv^0(m(0))}{dm(0)} = \frac{n-1}{n} \delta \frac{v^0}{1 - \frac{n-1}{n} \rho^0 \delta} \frac{d\rho^0}{dm(0)} = \frac{n-1}{n} \delta \frac{v^1}{1 - \frac{n-1}{n} \rho^1 \delta} \frac{d\rho^1}{dm(0)} = \frac{dv^1(m(0))}{dm(0)}.$$

Taking total derivatives for the probabilities of being called into the winning coalition, (4), and (5), and replacing above we get

$$\frac{v^0}{1 - \frac{n-1}{n} \rho^0 \delta} \left(-\frac{r}{n-r} \frac{df(m(0))}{dm(0)} - 1 \right) = \frac{v^1}{1 - \frac{n-1}{n} \rho^1 \delta} \left(\frac{df(m(0))}{dm(0)} + \frac{n-r}{r} \right)$$

If $\frac{df(m(0))}{dm(0)} = -\frac{n-r}{r}$ then $\frac{dv^1(m(0))}{dm(0)} = \frac{dv^0(m(0))}{dm(0)} = 0$. Otherwise we can eliminate from both sides the term $\left(\frac{df(m(0))}{dm(0)} + \frac{n-r}{r} \right)$ and

$$-\frac{n-r}{r} \frac{v^0}{1 - \frac{n-1}{n} \rho^0 \delta} = \frac{v^1}{1 - \frac{n-1}{n} \rho^1 \delta}.$$

But this is absurd since the LHS is negative and the RHS is positive. Thus the only possible solution is that $\frac{df(m(0))}{dm(0)} = -\frac{n-r}{r}$, and v^0 and v^1 are independent of $m(0)$. The intuition for this result comes from the fact that these strategies give legislators the same ex ante probability of being called into the winning coalition, and thus the same ex ante value since the probability of being agenda setter is always $\frac{1}{n}$.

Given that all solutions feature the same ex ante values we can apply a refinement to have a system of four equations in four unknowns. We choose that expected output be

independent of the identity of the agenda setter:

$$-r^- + m(0) = -r^- + m(1) + 1.$$

Using the indifference condition $\delta(v^1 - v^0) = \eta$ to replace v^1 as a function of v^0 in equation (3) for $i = 0$, and the feasibility constraint (which can be used instead of (3) for $i = 1$) we get the following system of two equations in two unknowns

$$v^0 \left(1 - \frac{\delta}{n} \left(\frac{rq - nm(0)}{n - r} \right) \right) = \frac{1}{n} [Y - r^- \eta] \quad (17)$$

$$nv^0 = Y - \left(\frac{r + \delta r^-}{\delta} - m(0) \right) \eta \quad (18)$$

b) The proof mirrors a), with the indifference condition now given by $\delta v^1 = \eta$. From (3) for $i = 1$ we can get the expression for v^1 as a function of $m(0)$ and $m(1)$. Imposing the condition $\delta v^1 = \eta$ for a mixed strategy equilibrium gives a continuum of equilibria characterized by a relation between strategies, $m(1) = f(m(0))$. This allows us to write $v^1(m(0))$, which from (5) is given by:

$$v^1(m(0)) = \frac{\frac{1}{n} [\tilde{Y}(f(m(0))) - e(C_m^1)]}{1 - \frac{n-1}{n} \rho^1(m(0)) \delta}$$

A parallel reasoning as before tells us that both v^1 , and the numerator of the expression above are invariant to changes in $m(0)$ as long as $\delta v^1(m(0)) = \eta$. Thus ρ^1 is independent of $m(0)$, which implies that, as before, $\frac{df(m(0))}{dm(0)} = -\frac{n-r}{r}$. As a corollary we have that v^0 is also independent of $m(0)$ ($\rho^0 = 0$ since passive districts are never called into a winning coalition when this is larger-than-minimum). We apply the same refinement that expected output be independent of the identity of the agenda setter.

Replacing the indifference condition, $v^1 = \frac{\eta}{\delta}$, into (3) for $i = 0$, and into the feasibility constraint, the latter results in

$$\begin{aligned} (n-r)v^0 + r\frac{\eta}{\delta} &= Y - r^- \eta + m(0)\eta \\ (n-r)\frac{1}{n} [Y - r^- \eta] + r\frac{\eta}{\delta} &= Y - r^- \eta + m(0)\eta \\ - [Y - r^- \eta] \frac{r}{n} + r\frac{\eta}{\delta} &= m(0)\eta. \end{aligned} \quad (19)$$

We now continue the proof of our comparative static results with the following lemma, for which $E(m)$ is the expected number of active districts present in interior equilibria.

Note that under our refinement, $E(m) \equiv m_0$.

Lemma 4. For the thresholds characterizing equilibrium types in proposition 1,

$$\begin{aligned}
\text{i) } r > q - 1 : \quad & \frac{d\delta_q}{dq} < 0 \quad , \quad \frac{d\delta_q}{d\eta} > 0 \quad , \quad \frac{d\delta_q}{dr^-} > 0 \quad , \quad \frac{d\delta_q}{dr^+} > 0, \\
& \frac{d\underline{\delta}}{dq} = 0 \quad , \quad \frac{d\underline{\delta}}{d\eta} > 0 \quad , \quad \frac{d\underline{\delta}}{dr^-} > 0 \quad , \quad \frac{d\underline{\delta}}{dr^+} = 0, \\
& \frac{d\bar{\delta}}{dq} < 0 \quad , \quad \frac{d\bar{\delta}}{d\eta} > 0 \quad , \quad \frac{d\bar{\delta}}{dr^-} > 0 \quad , \quad \frac{d\bar{\delta}}{dr^+} > 0, \quad (m(i) = q - 1) \\
\text{ii) } r \leq q - 1 : \quad & \frac{d\bar{\delta}}{dq} > 0 \quad , \quad \frac{d\bar{\delta}}{d\eta} > 0 \quad , \quad \frac{d\bar{\delta}}{dr^-} \leq 0 \quad , \quad \frac{d\bar{\delta}}{dr^+} \leq 0. \quad (m(i) = r - i)
\end{aligned}$$

For interior equilibria,

$$\begin{aligned}
\text{iii) } \delta > \bar{\delta} : \quad & \frac{dE(m)}{d\delta} < 0 \quad , \quad \frac{dE(m)}{dq} > 0 \quad , \quad \frac{dE(m)}{d\eta} > 0 \quad , \quad \frac{dE(m)}{dr^-} > 0 \quad , \quad \frac{dE(m)}{dr^+} > 0, \\
\text{iv) } \delta \in [\underline{\delta}, \delta_q) : \quad & \frac{dE(m)}{d\delta} < 0 \quad , \quad \frac{dE(m)}{dq} = 0 \quad , \quad \frac{dE(m)}{d\eta} > 0 \quad , \quad \frac{dE(m)}{dr^-} > 0 \quad , \quad \frac{dE(m)}{dr^+} > 0.
\end{aligned}$$

Note that i) is straightforward from (9), (12), and (13), and iv) is straightforward from (19). Note that (19) also allows to sign, when possible, the effect on output losses. The proof of iii) is a bit more complicated as there are two equations in the two unknowns, m_0 and v_0 . Nevertheless, after some algebra to replace the derivatives of v_0 with respect to the different parameters we find the above results, which hold since $nm_0 > rq$ for all interior equilibria when $\delta > \bar{\delta}$ (otherwise it would not be the case that $v_1 > v_0$). For ii) the effect of q is straightforward from (11). For η this follows since we established that $\frac{d\delta(v^1 - v^0)}{d\delta} > 0$ in the proof of proposition 1. For r^- and r^+ the effects are ambiguous, as can be see from (11). The intuition is that an increase in the number of active districts has a negative effect on the continuation value of both active and passive districts. For the former due to the dilution of agenda setter rents, while for the latter due to lower probability of being in the winning coalition.

6.7 Proof of Proposition 3

We consider first the case with $r > q - 1$ and $\delta \in [0, \underline{\delta})$, i.e. when all active districts are included in the winning coalition and this is larger-than-minimum. Since there are no output losses, the feasibility constraint implies that for all r

$$rv^1(r) + (n - r)v^0(r) = Y + r^+\eta. \quad (20)$$

Since, from lemma 3, $v^1(r) \geq v^0(r)$, equation (20) implies that $v^1(r) \geq \frac{Y+r^+\eta}{n} \geq v^0(r)$ for all r . Thus, it must be the case that $v^1(r) \geq v^0(r-1)$ for all r (note that if $\delta > 0$, then $v^1(r) > v^0(r)$, and thus $v^1(r) > v^0(r-1)$).

Next we consider the case $r > q-1$, and $\delta \in [\underline{\delta}, \delta_q)$, i.e. when there is a larger-than-minimum winning coalition but not all active districts are included in it. Since the agenda setter in these equilibria satisfies the first order condition (2) for an interior equilibrium, and $m(i) > q-1$, it must be the case that

$$\eta - \delta v^1(r) = 0.$$

Considering that the RHS of equation (20) now reflects an output loss, $Y - r^-\eta + m(r)\eta$, we infer that

$$v^1(r) = \frac{\eta}{\delta} \geq \frac{Y}{n} - \frac{r^- - m(r)}{n} \eta \geq v^0(r).$$

Thus, we find that $v^1(r) \geq v^0(r-1)$ for all r .

We now consider the case $\delta \in [\delta_q, \bar{\delta}]$ such that we have corner equilibria including $m(i) = \max\{q-1, r-i\}$ active districts. If $m(i) = r-i$ then output is efficient and we can apply the logic of the case with $r > q-1$ and $\delta \in [0, \underline{\delta}]$. Thus, we consider that $m(i) = q-1$ and there are output losses. The value functions $v^1(r)$ and $v^0(r)$ for this case must satisfy equations (6) and (7). Thus,²⁵

$$\begin{aligned} v^1(r) &= \frac{r(Y - r^-\eta + q\eta)}{nr - \delta(n-r)(q-1)}, \\ v^0(r-1) &= \frac{1}{n} \left[Y - (r^- - 1)\eta + (q-1)\eta - \frac{(q-1)\delta(r-1)(Y - (r^- - 1)\eta + q\eta)}{n(r-1) - \delta(n-r+1)(q-1)} \right], \\ &= \frac{(Y - r^-\eta + q\eta)[r-1 - \delta(q-1)] - \delta(q-1)(r-1)\eta}{n(r-1) - \delta(n-r+1)(q-1)}. \end{aligned}$$

Thus,

$$\begin{aligned} v^0(r-1) &= v^1(r) \frac{r - (1 + \delta(q-1))}{r} \frac{nr - \delta(n-r)(q-1)}{n(r-1) - \delta(n-r+1)(q-1)} \\ &\quad - \frac{\delta(q-1)(r-1)\eta}{n(r-1) - \delta(n-r+1)(q-1)} \\ &\leq v^1(r) \frac{r - (1 + \delta(q-1))}{r} \frac{nr - \delta(n-r)(q-1)}{n(r-1) - \delta(n-r+1)(q-1)}. \end{aligned}$$

²⁵In what follows we assume that the district evaluating the action would be a destructive type. The analysis is similar for a district with the option to be productive.

Since the term multiplying $v^1(r)$ in the last expression is smaller than one (this follows since $r > q - 1$), it is always the case that $v^1(r) \geq v^0(r - 1)$ for all r .

We are left now with the last case, $\delta \geq \bar{\delta}$, i.e. interior equilibria that imply minimum winning coalitions. Since the agenda setter in these equilibria satisfies the first order condition (2) for an interior equilibrium, and $m(i) < q - 1$, it must be the case that

$$\eta - \delta(v^1(r) - v^0(r)) = 0. \quad (21)$$

The RHS of the feasibility constraint, (20), now is given by $Y - r^-\eta + m(r)\eta$. Using equation (21) to write the LHS of the feasibility constraint in terms of $v^0(r)$ we have

$$nv^0(r) + r\frac{\eta}{\delta} = Y - r^-\eta + m(r)\eta.$$

Using this last equation for r and $r - 1$ and equation (21) we get²⁶

$$n[v^1(r) - v^0(r - 1)] = \eta \left(\frac{n - 1}{\delta} - 1 \right) + \eta(m(r) - m(r - 1)).$$

Since the first term in the RHS is positive, if $m(r) \geq m(r - 1)$, then $v^1(r) > v^0(r - 1)$. We prove this by contradiction. If $m(r) < m(r - 1)$, we can show that there are strategies that result in higher values v^0 and v^1 , implying that a choice of $m(r) < m(r - 1)$ is suboptimal. For this we consider strategies that imply $m'(r) = m(r - 1)$, which is a feasible option. We write feasibility constraints for $m(r)$ and $m'(r)$, using (21) to substitute $v^1(r)$ in terms of $v^0(r)$,

$$\begin{aligned} nv^0(r) + r\frac{\eta}{\delta} &= Y - r^-\eta + m(r)\eta, \\ nv^{0'}(r) + r\frac{\eta}{\delta} &= Y - r^-\eta + m(r - 1)\eta \end{aligned}$$

Subtracting these two equations we get

$$n[v^{0'}(r) - v^0(r)] = [m(r - 1) - m(r)]\eta > 0.$$

Thus proving that $m(r) < m(r - 1)$ is not optimal. This completes the proof that $v^1(r) \geq v^0(r - 1)$ for all r and all δ .

²⁶Again, in what follows we assume that the district evaluating the action would be a destructive type. The analysis is similar for a district with the option to be productive.