Safe but Fragile: Information Acquisition, Sponsor Support and Shadow Bank Runs *

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Abstract

This paper proposes a theory of shadow bank runs in the presence of sponsor liquidity support. We show that liquidity lines designed to insulate shadow banks from market and funding liquidity risk can be destabilizing, as they provide them with incentives to acquire private information about their assets. This can lead to inefficient market liquidity dry-ups caused by self-fulfilling fears of adverse selection. By lowering asset prices, information acquisition also reduces shadow banks' equity value and may spur inefficient investor runs. We compare different policies that can be used to boost market and funding liquidity. While debt purchases prevent inefficient dry-ups, liquidity injections may backfire by exacerbating adverse selection frictions.

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JEL Classifications: D82, G01, G20

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1 Introduction

The growth of market-based finance over the past decades has given rise to a number of new types of financial intermediaries. These so-called shadow banks operate outside the perimeter of traditional banking regulation and the public financial safety net, even though they are typically created and operated by regulated financial institutions.[1] A key feature in shadow banks' design is the provision of liquidity lines by their sponsoring institutions. Such sponsor support is intended to lower individual shadow banks' susceptibility to investor runs by shielding them from deteriorations in market liquidity conditions. In particular, liquidity lines – by providing shadow banks facing funding withdrawals with a contingent liquidity source – may help them avoid losses that would otherwise accrue from selling assets at discounted prices. The present paper challenges this view by arguing that the provision of such sponsor support can, in fact, be detrimental to aggregate market and funding liquidity conditions. We show this by developing a theory of asset market freezes and investor runs based on shadow banks' ability to acquire private information about their assets.

Contrary to the conventional view outlined above, we argue that market liquidity conditions are not *independent* of shadow banks' choice of liquidity sources since this choice depends on the quality of assets on their balance sheets. Building on Akerlof (1970)'s key insight that asymmetric information can impede trade, we show that access to pre-committed liquidity lines gives shadow banks an incentive to acquire private information about their assets in order to avoid selling good assets at a discount. This can lead to market freezes driven by *endogenous* adverse selection which may, in turn, precipitate panic-driven investor runs. Thus, rather than making the shadow banking sector safer, the presence of liquidity lines may in fact be a *source* of financial fragility by opening the door to market and funding liquidity dry-ups spurred by self-fulfilling fears of adverse selection in asset markets.

[1] See Pozsar et al. (2010) for an overview. A large variety of different types of non-bank financial institutions can be subsumed under the term 'shadow banks,' ranging from money market funds and other open-ended mutual funds that provide funding to off–balance sheet vehicles like SIVs, ABCP conduits or hedge funds.

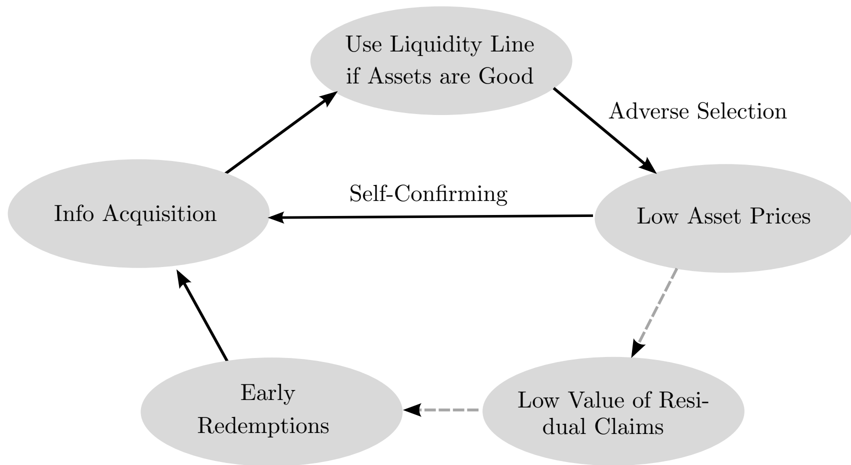
Overview of the Model. Our results are based on a three-date model with three types of risk-neutral agents: shadow banks (e.g. off-balance sheet conduits or mutual funds), wholesale investors, and deep-pocketed asset market traders. Shadow banks (henceforth referred to as "funds" for short) enter the economy with long-term assets, partly financed by redeemable liabilities. These liabilities are held by wholesale investors that can choose to redeem them before funds' assets mature. Funds' assets differ in terms of their payoff at maturity: some pay out a high cash flow (*good type*), while others pay out a low cash flow (*bad type*). Although funds initially do not know their assets' type, they can expend resources to privately learn the type. To obtain the liquidity needed to meet short-term redemptions, funds can then either sell assets in a competitive secondary market or tap a costly liquidity line provided by an (outside) sponsoring institution.[2]

The value of information in this environment stems from funds' ability to hold on to good assets by resorting to their liquidity lines rather than selling them at a discount. Information acquisition by funds also generates an externality as it induces an adverse selection problem in secondary markets that impedes the provision of market liquidity (i.e. lowers asset prices). This leads to a feedback from market prices to information acquisition, as lower prices reduce funds' opportunity costs of using their liquidity lines in case they have good assets.

The key contribution of our paper is to show that this feedback can generate self-fulfilling market liquidity dry-ups. To illustrate the underlying mechanism, suppose a fund faces redemptions and believes that other funds have acquired information (*cf.* the solid lines of Figure 1). If informed funds with good assets opt to finance redemptions using their liquidity lines, the relative share of bad assets in the secondary market increases and asset prices fall. This "lemons discount" raises the value from withholding good assets from the market and, *a fortiori*, the gain from acquiring information. The mere belief that others acquire information thus increases the private surplus from information acquisition, precipitating self-fulfilling market freezes caused by *endogenous* adverse selection.

[2] Off-balance sheet conduits and MMFs had extensive recourse to the balance sheets of their sponsors. This included "liquidity enhancements," or private liquidity lines through which sponsoring institutions could repurchase performing assets if conduits failed to roll over their maturing liabilities.

Figure 1: Model Mechanism



This information-induced dry-up in market liquidity can also raise investors' incentives to redeem their claims early and amplify funds' funding liquidity risk. Funds that sell assets in order to meet early redemptions have to sell increasingly large quantities as prices fall. Early redemptions in this case dilute the claims held by investors at maturity and may lead to self-fulfilling investor runs if asset prices are sufficiently low (*cf.* the dashed line in Figure 1). The increased funding risk raises funds' incentives to acquire information, which further pushes down prices and sparks even more redemptions.

Importantly, financial fragility in our model results from strategic complementarities in funds' information acquisition decisions. In particular, the coordination problem among investors precipitating redemption runs *only* emerges if endogenous adverse selection leads assets to trade at a discount in secondary markets. If markets are liquid, investors' claims at maturity are unaffected by the volume of early redemptions, and their redemption decisions are purely driven by their idiosyncratic liquidity shocks. This distinguishes our model from standard bank run models *à la* Diamond & Dybvig (1983) where fragility stems from strategic complementarities in creditors' withdrawal decisions arising due to *exogenous* discounts banks incur when liquidating assets prematurely.

The strategic complementarities characterizing funds' information acquisition and investors' redemption decisions can lead to multiple Pareto-ranked equilibria. Equilibria *without* information acquisition are characterized by high secondary market prices and low funding risk. These equilibria Pareto-dominate equilibria *with* information acquisition that are characterized by low market prices and high funding liquidity risk.[3] The coordination failure leading funds to acquire information can therefore generate welfare losses due to inefficient liquidity dry-ups. In order to select a unique equilibrium and study the effects of different types of policy interventions, we employ global game techniques by adapting the methodology of Goldstein (2005). This is done by introducing a macroeconomic state that affects the riskiness of assets' cash flows.

We show that depending on the parameters of the model, two different regimes can arise: a *weak dependence* and *strong dependence* regime. In the former, information acquisition by funds leads to a drop in asset prices that may spur panic-driven investor runs. However, no reverse feedback exists and market liquidity risk is unaffected by the volume of early redemptions. In the latter regime, market and funding liquidity mutually reinforce each other. Funding liquidity risk in this case "spills over" and raises funds' incentives to acquire information about their assets, and market liquidity dry-ups are always accompanied by investor runs.

Our model has implications for central bank policies as well as the regulation of shadow banks and their sponsors. Regarding the former, we focus on three specific policy interventions that resemble measures enacted by central banks during the 2007-09 financial crisis to shore up liquidity in securitized asset markets: (i) asset purchases, (ii) outright debt purchases, and (iii) liquidity injections to funds' sponsoring institutions. We show that asset purchases reduce both market and funding liquidity risk, but cannot completely eliminate liquidity dry-ups, implying that the policymaker must sometimes incur losses under such a policy. In contrast, outright debt purchases that completely shield funds from funding liquidity risk can be used to implement the efficient allocation. Liquidity injections to sponsoring institutions, however, may backfire insofar as they can exacerbate the underlying adverse selection problem. With respect to prudential regulation, we argue

[3] As in Hirshleifer (1971), information acquisition has no social value in our model as its only serves to redistribute rents across funds.

that an outright ban on external liquidity support (as recently introduced in the European Union for money market funds) completely eliminates the sources of market and funding illiquidity in our model. That being said, regulations that just raise the marginal cost of liquidity lines (such as additional capital charges for providing liquidity lines stipulated by Basel III) may backfire exactly when central bank liquidity injections work.

Relation to the Literature. Our paper highlights the fragility of financial institutions that rely on market-based liquidity provision and sponsor support to manage their funding risk. In this regard, it relates to a recent literature studying the origins and consequences of sponsor support for non-bank financial institutions. Ordonez (2016) and Segura (2017) focus on the reputational and signalling effects of sponsor support. More closely related to our paper, Parlatore (2016) studies how the interaction between sponsor support and market prices can lead to destabilizing feedback effects. Complementarities in her model arise because fire-sales raise the cost of sponsor support, thereby *lowering* sponsors' incentives to provide support and further pushing down prices. Her model thus highlights how sponsor support (or the lack thereof) can be a source of financial fragility in the presence of cash-in-the-market frictions. Our paper focuses on a different channel and shows how the *active* provision of sponsor support, due to its effect on funds' information acquisition incentives, can lead to self-fulfilling market freezes.

Our paper also builds on the extensive literature studying how adverse selection leads to endogenous liquidity and asset market disruptions, including Eisfeldt (2004), Plantin (2009), Bolton et al. (2011), Kurlat (2013), Malherbe (2014), Heider et al. (2015), and Bigio (2015). Contrary to these papers, which treat asymmetric information as a primitive, adverse selection emerges endogenously in our model due to funds' strategic information acquisition. Related to this point, Gorton & Ordonez (2014), building on Dang et al. (2015), study how information acquisition amplifies aggregate shocks to collateral values. The value of information in their model consists of an information rent that accrues to creditors from liquidating bad collateral at a pooling price.[4] Importantly, the feedback be-

[4]In a related model studying information acquisition by sellers (rather than buyers), Dang et al. (2013)

tween market prices and information acquisition implied by this information rent induces strategic substitutability (rather than strategic complementarity) in information production. Hence, the self-fulfilling liquidity dry-ups that are the focus of our paper cannot arise in Gorton & Ordonez (2014)'s framework. Other recent papers studying strategic complementarities in information acquisition include Fishman & Parker (2015) and Bolton et al. (2016). There, the source of strategic complementarities is different from the one studied here as it operates through rents informed investors extract when buying (rather than selling) assets.

The mutual amplification of market and funding liquidity links our paper to the literature studying the destabilizing effect of margins. For example, in Brunnermeier & Pedersen (2009), Biais et al. (2015) and Kuong (2015), market illiquidity can amplify firm deleveraging due to a fire sale externality. This "margin channel," differs from our "information acquisition channel" in both its empirical and policy implications. First, fire sales resulting from funding constraints that lead prices to decline when firm deleveraging becomes excessive. In contrast, in our model prices decline because some firms opt *not* to sell their assets in secondary markets. Thus, while the "margin channel" suggests that low asset prices should be associated with high trading volumes, our "information acquisition channel" does not.[5] Second, fire sales caused by funding constraints emerge due to a lack of overall liquidity in the economy. Liquidity injections that relax funding constraints therefore dampen price declines caused by fire sales. This contrasts with our framework, where liquidity injections may exacerbate market illiquidity by reinforcing adverse selection.

Finally, our paper draws from the large literature on global games that interprets liquidity dry-ups as the result of a coordination failure (Morris & Shin, 2003, 2004a,b). In particular, global games serve as the workhorse model for studying bank runs (Goldstein

shows that the value of information is the minimum of either the information rent from selling a low payoff security at a high price, or the gain from not selling a high payoff security at a low price. Firms' surplus from information acquisition in our model is similar to the latter. While Dang et al. (2013) focus on optimal security design, we study the feedback between information acquisition and market prices.

[5] This "double whammy" (Tirole, 2011) of declining prices and trading volumes fits well with observed price and trading movements in securitized asset markets during the 2007-09 financial crisis.

& Pauzner, 2005; Rochet & Vives, 2004) as well as the funding risk of non-bank financial institutions such as hedge funds (Liu & Mello, 2011) or mutual funds (Chen et al., 2010; Morris et al., 2017). Compared to this literature, our model studies a novel channel of coordination failure that explicitly ties market and funding liquidity risk to adverse selection caused by strategic information acquisition. Methodologically our analysis is closely related to Goldstein (2005) who first extended global games to a setting with two types of agents and a common fundamental.

2 Information Acquisition and Market Liquidity

2.1 Model Basics

We consider an economy populated by a continuum of risk-neutral financial institutions, indexed by $j \in [0,1]$, that operate for three dates $t \in \{0,1,2\}$. We think of these institutions as non-bank entities like off-balance sheet conduits (e.g. structured investment vehicles, money market funds, or hedge funds). For simplicity, we henceforth refer to these institutions as funds.

Assets. Each fund enters the economy at $t = 0$, holding one unit of a perfectly divisible long-term asset that pays out at $t = 2$. The asset's payoff at maturity consists of two parts: (i) a risky component $\tilde{X}(\theta)$, and (ii) a non-marketable control rent $Q > 0$. The risky component $\tilde{X}(\theta)$ has the following payoff structure:

$$\tilde{X}(\theta) = \begin{cases} X(\theta) & \text{with probability } \pi \\ \theta X(\theta) & \text{with probability } 1-\pi \end{cases}$$

When the realized payoff is $X(\theta)$, the asset is said to be of a *good* type. Otherwise, it is said to be of a *bad* type. The realization of the assets' type is assumed to be i.i.d. across funds. The parameter $\theta \in \Theta \subset [0,1]$ is a macroeconomic state that affects assets' returns in a mean-preserving spread sense: i.e. $\mathbf{E}[\tilde{X}(\theta)] = F$ for all $\theta \in \Theta$ such that

$\frac{\mathrm{d}}{\mathrm{d}\theta}X(\theta) < 0$ and $\frac{\mathrm{d}}{\mathrm{d}\theta}\theta X(\theta) > 0$.[6] This parameter can be interpreted as a measure of aggregate volatility affecting funds' assets, with high (low) values of θ indicating a low (high) degree of macroeconomic uncertainty.

In addition to the risky component, each fund obtains a non-marketable control rent $Q > 0$ per unit of asset under management at $t = 2$. This can be interpreted as additional value created by funds if assets remain on their balance sheet (e.g. due to funds' superior asset management capabilities or trading strategies). Hence, while the *ex ante* book value of the asset is given by F, from the funds' perspective the *ex ante* expected value of assets held until maturity is equal to $F + Q$.

Information Structure. The macroeconomic state θ is drawn at $t = 0$ from a uniform distribution over Θ.[7] The realization of θ becomes common knowledge before the market opens at $t = 1$. At $t = 0$, each fund receives a noisy private signal about the state:

$$\theta_j = \theta + \epsilon_j$$

where ϵ_j is i.i.d. across funds and drawn from a uniform distribution over $[-\epsilon, \epsilon]$.

In addition to observing this noisy signal about θ, funds at $t = 0$ can acquire private information about their assets' type at a fixed cost $\psi > 0$. For simplicity, we assume that by acquiring information funds perfectly observe whether their asset is good or bad.[8] We denote by $\Omega_j \in \{n, g, b\}$ fund j's information set conditional on not acquiring information (n), or acquiring information and verifying its asset's type to be good (g) or bad (b). Correspondingly,

$$\mathbf{E}[\tilde{X}(\theta)|\Omega_j, \theta_j] \in \{F,\, \mathbf{E}(X(\theta)|\theta_j),\, \mathbf{E}(\theta X(\theta)|\theta_j)\}$$

denotes fund j's beliefs at $t = 0$ about its asset's type given its information set.

[6] More explicitly, $X(\theta) = \frac{F}{\pi+(1-\pi)\theta}$ such that $\frac{\mathrm{dX}(\theta)}{\mathrm{d}\theta} = -\frac{(1-\pi)F}{(\pi+(1-\pi)\theta)^2}$ and $\frac{\mathrm{d}\theta\mathrm{X}(\theta)}{\mathrm{d}\theta} = \frac{\pi F}{(\pi+(1-\pi)\theta)^2}$.

[7] The restriction to a uniform distribution is without loss of generality; any other continuous distribution with finite support Θ could be assumed.

[8] Our results would not be altered if we assumed that funds could only observe a noisy signal about the idiosyncratic state of their assets.

Liabilities. Each fund is financed by a distinct unit mass of investors. A fraction $(1-\alpha)$ of each fund's liabilities are irredeemable, e.g. long-term debt or equity shares held by passive investors. The remaining fraction α is held by active investors and is redeemable at $t = 1$.[9] More specifically, we build on Liu & Mello (2011) and model the redemption process as follows: active investors notify their fund about their redemption decision at $t = 0$; their claims are then priced at the current marketable value of the fund, F, and disbursed to investors at $t = 1$. For the moment, we assume that an exogenous share $\lambda \in [0, 1]$ of redeemable liabilities are withdrawn and need to be repaid at $t = 1$. We endogenize active investors' redemption decisions and study the resulting feedback between market and funding liquidity risk in Section 3.

Liquidity Sources. The balance sheet structure described above implies that funds are subject to a standard liquidity mismatch problem: while the long-term asset does not pay out until $t = 2$, funds must finance redemptions of $\alpha\lambda F$ at $t = 1$. We assume that funds can obtain the liquidity needed to meet early redemptions in one of two ways. First, each fund has access to a private liquidity line from which cash can be drawn down at a unit cost of $\kappa > 1$. Alternatively, funds can sell their assets in a competitive secondary market in $t = 1$ at price p. The buyers in the secondary market are large in number, deep-pocketed and risk-neutral and stand ready to purchase assets at their expected value at $t = 1$, i.e. $p = \mathbf{E}[\tilde{X}(\theta)|\theta]$. Since the control rent is non-marketable, funds selling assets to meet early redemptions must necessarily forego the additional payoff Q per unit of asset sold. The sequence of events is summarized in Figure 2.

[9]For example, equity-funded institutions such as mutual funds make use of redemption gates (i.e. temporary suspensions of redemptions) or lock-up periods (i.e. prohibition of redemptions by new investors). Similarly, debt-financed off-balance sheet vehicles issue debt of different maturities. For example, the largest SIVs issued only up to 20% of their liabilities in the form of short-term ABCP and the remaining part in capital notes and medium term notes (Gorton, 2010), while other ABCP-programs issued paper with the option to extend its maturity (Covitz et al., 2013).

Figure 2: Sequence of Events

$t=0$	$t=1$	$t=2$
	true θ revealed	
θ drawn, signals observed	choice of liquidity source	assets pay out
information and redemption choices	market opens, assets trade at p	late redemption payments
	early redemption payments	

2.2 Liquidity Sources and Asset Prices

Liquidity Lines *vs.* Asset Sales. At $t = 1$, funds choose between the two liquidity sources in order to maximize their expected equity value, given their information set Ω_j and the realized macroeconomic state θ. Denote by $V_{\Omega_j}^{LL}$ and $V_{\Omega_j}^{AS}$ the equity value of a fund with information set Ω_j that uses liquidity lines (LL) or asset sales (AS) to obtain liquidity. The expected equity value of a fund that obtains $\ell_j \geq \alpha\lambda F$ units of liquidity at $t = 1$ by selling assets is given by

$$\mathbf{E}[V_{\Omega_j}^{AS}(\ell_j)|\theta] = \mathbf{E}\left[\max\left\{\left(\tilde{X}(\theta) + Q\right)\left(1 - \frac{\ell_j}{p}\right) + (\ell_j - \alpha\lambda F), 0\right\}\middle|\Omega_j, \theta\right] \tag{1}$$

Similarly, the value of a fund choosing to obtain liquidity *via* its liquidity line equals

$$\mathbf{E}[V_{\Omega_j}^{LL}(\ell_j)|\theta] = \mathbf{E}\left[\max\left\{\tilde{X}(\theta) + Q - \kappa\ell_j + (\ell_j - \alpha\lambda F), 0\right\}\middle|\Omega_j, \theta\right] \tag{2}$$

Since $\kappa > 1$, funds meeting early redemptions using their liquidity lines never choose to obtain more liquidity than that needed to meet early redemptions. The choice of ℓ_j for funds selling assets, however, depends on their information set and the size of the control rent, Q. In what follows, we assume that this control rent always exceeds the information rent that informed funds with bad assets could obtain by selling their entire portfolio at a price above their assets' true value. This ensures that informed funds will never choose to sell more assets than what is needed to meet their liquidity needs.[10]

Assumption 1. *Let* $\underline{\theta} \equiv \min\{\Theta\}$*, then the non-transferable control rent* Q *is such that*

$$Q > F - \underline{\theta} X(\underline{\theta})$$

Equations (1) and (2) imply that funds' preference between liquidity lines and asset sales depends on the market price (p) and the cost of liquidity lines (κ). To fix funds' preference ordering over liquidity sources given their information set Ω_j we impose the following restrictions on the cost of liquidity lines and the set of macroeconomic states Θ:

Assumption 2. *The lower and upper bounds of* Θ*,* $\min\{\Theta\} \equiv \underline{\theta}$ *and* $\max\{\Theta\} \equiv \overline{\theta}$*, are such that*

$$\frac{F + Q}{\underline{\theta} X(\underline{\theta})} < \kappa < \frac{X(\overline{\theta}) + Q}{F}$$

where the parameters π *and* Q *are such that* $\Theta \neq \emptyset$*.*

The upper bound on θ corresponds to a standard "lemons condition." It implies that even if assets trade at their *ex ante* expected value ($p = F$), informed funds holding good assets always prefer to meet redemptions by tapping liquidity lines. Effectively, the inequality implies that the cash flow of a good asset is sufficiently large to compensate funds for the cost of the liquidity line, *regardless* of the price at which assets trade in secondary markets. The lower bound on θ, on the other hand, implies that even if assets trade at the lowest possible price ($p = \underline{\theta} X(\underline{\theta})$), uninformed funds prefer to meet redemptions by selling assets in the absence of default.[11]

[10]Technically, the control rent Q guarantees the monotonicity of the surplus function. Without it, funds would have an additional motive to acquire information: i.e. the ability to off-load bad assets at a premium $p - \theta X(\theta)$. Assumption 1 ensures that informed funds never liquidate assets to extract this "lemons rent" because they would forego the (larger) control rent from keeping assets on their balance sheet. This allows us to isolate the strategic complementarities in information acquisition stemming from the "option value" from withholding good assets from the market. Without Assumption 1, the presence of a "lemons rent" could potentially lead funds' information acquisition decisions to become strategic substitutes (since the "lemons rent" decreases when prices fall). How such a "lemons rent" affects information acquisition behaviour has already been studied in the literature, e.g. by Gorton & Ordonez (2014).

[11]The lower bound on θ is a technical condition needed to guarantee the existence of an equilibrium in

In what follows, we also assume that the fraction of irredeemable liabilities is sufficiently large such that funds never become illiquid even if they face full redemptions at $t = 1$, i.e. $\lambda = 1$. In particular, funds never default if the following inequality is satisfied:

$$\alpha F < \min\left\{\frac{\underline{\theta}X(\underline{\theta}) + Q}{\kappa}, \underline{\theta}X(\underline{\theta})\right\} = \frac{F}{\kappa}$$

which implies that the face value of funds' redeemable liabilities is strictly less than the liquidation value of funds with bad assets regardless of whether they obtain liquidity through asset sales or *via* their liquidity lines.[12] This no-default assumption allows us to abstract from gambling incentives driven by funds' limited liability constraint.

Assumption 3. *The fraction of redeemable liabilities is such that* $\alpha < 1/\kappa$.

Lemma 1. *Given Assumptions 1-3, informed funds with good assets strictly prefer the liquidity line, while informed funds with bad assets and uninformed funds strictly prefer asset sales.*

Asset Price. Buyers that purchase assets in the secondary market must break even in expectation. Since the macroeconomic state becomes common knowledge before the market opens at $t = 1$, buyers' expectations are conditioned on the realized value of θ. Their participation constraint is therefore given by

$$p \leq \mathbf{E}[\tilde{X}|\theta] = X(\theta)(\tau + (1-\tau)\theta)$$

where $\tau \in [0, 1]$ denotes the fraction of good assets supplied to the market. Competition in the market ensures that this inequality binds in equilibrium.

The price at which assets trade depends on buyers' beliefs about the share of good assets supplied to the market. Given Lemma 1, only uninformed and informed bad funds

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funds' information acquisition game. The assumption that uninformed funds prefer deleveraging to tapping their liquidity line is also consistent with the fact that off-balance sheet vehicles often relied on "dynamic liquidity management" strategies to manage their funding risk, meaning that they regularly sold assets to obtain liquidity notwithstanding the recourse to their sponsors' balance sheets (Covitz et al., 2013).

[12] The simplification of the right-hand-side of the inequality follows from Assumptions 1 and 2.

supply their assets to the market. Hence, whenever some funds acquire information, the share of good assets traded in the secondary market will be strictly less than the share of good assets in the economy: i.e. $\tau < \pi$. We assume that trading in the secondary market is anonymous, so that market participants cannot infer assets' type based on the quantity fund j supplies to the market.[13] Letting $\sigma \in [0, 1]$ denote the share of funds acquiring information, the fraction of good assets traded in the market is equal to

$$\tau(\sigma) = \frac{\pi(1-\sigma)}{1-\pi\sigma}$$

and the market price can be rewritten as

$$p(\sigma, \theta) = F - (\pi - \tau(\sigma))(1-\theta)X(\theta) \tag{3}$$

By acquiring information, funds induce an asymmetric information friction in secondary markets, as informed funds with good assets withhold these from the market. The resulting adverse selection problem leads assets to trade at a discount compared to their *ex ante* book value. Importantly, this discount is strictly increasing in the fraction of informed funds since the share of good assets traded in the market falls as more funds acquire information, i.e. $\tau'(\sigma) < 0$. Moreover, since the value of bad assets rises when the macroeconomic state improves, the price also increases in the macroeconomic state θ.

Lemma 2. *The secondary market price is strictly decreasing in the fraction of informed funds: $p_\sigma(\sigma, \theta) < 0$, and is strictly increasing in the macroeconomic state: $p_\theta(\sigma, \theta) > 0$.*

2.3 Equilibrium: Information Acquisition

Surplus from Information Acquisition. Given Lemma 1, uninformed funds cover early redemptions by selling assets. The expected equity value of an uninformed fund at $t = 0$ equals $\mathbf{E}[V_n^{AS}(\alpha\lambda F)|\theta_j]$. If a fund acquires information, the asset is verified to be good with probability π and verified to be bad with converse probability. By Lemma 1,

[13] This assumption rules out the possibility of funds using their liquidity lines to signal their type to potential buyers. For an analysis of the signalling effects of liquidity lines see Segura (2017).

funds with good assets always use their liquidity lines, while funds with bad assets opt to sell. The expected surplus from acquiring information at $t = 0$, given signal θ_j, equals

$$\mathbf{E}[S(\sigma, \theta; \lambda)|\theta_j] \equiv \mathbf{E}\left[\pi V_h^{LL}(\alpha\lambda F) + (1-\pi)V_l^{AS}(\alpha\lambda F) - V_n^{AS}(\alpha\lambda F)\middle|\,\theta_j\right]$$

where the subindexes $\{n, h, l\}$ indicate fund j's information set Ω_j at $t = 0$. Using equations (1) and (2), this function can be rewritten as follows

$$\mathbf{E}[S(\sigma, \theta; \lambda)|\theta_j] = \mathbf{E}\left[\pi\left(\frac{X(\theta) + Q}{p(\sigma, \theta)} - \kappa\right)\alpha\lambda F\middle|\,\theta_j\right] \tag{4}$$

The expected surplus from acquiring information can be interpreted as the option value from holding good assets rather than selling them at the pooling price. In particular, informed funds with good assets benefit from using their liquidity lines rather than trading in the market as they only forego κ units of cash flow tomorrow for one unit of liquidity today, compared to $X(\theta) + Q$ units of cash flow tomorrow for $p(\sigma, \theta)$ units of liquidity today. The upper bound on $\overline{\theta}$ (*cf.* Assumption 2) ensures that this difference is positive.

As shown by Lemma 2, the market price declines as more funds become informed due to adverse selection. Lower prices reduce the opportunity cost of using liquidity lines and raise the value from acquiring information. This feedback between the value of information and the market price generates *strategic complementarities* in information acquisition: i.e. for any private signal $\theta_j \in \Theta$, fund $j's$ surplus from acquiring information is strictly increasing in the fraction of funds acquiring information, σ. In addition, because the price increases in the macroeconomic state, the surplus from information strictly decreases in θ.

Lemma 3. *The surplus from information acquisition is increasing in the fraction of informed funds: $S_\sigma(\sigma, \theta; \lambda) > 0$, and decreasing in the macroeconomic state: $S_\theta(\sigma, \theta; \lambda) < 0$.*

In equilibrium, funds choose to acquire information if and only if their expected net surplus from doing so is positive: i.e. $\mathbf{E}[S(\sigma; \theta; \lambda)|\theta_j] - \psi > 0$. In what follows, we impose the following restriction on the relationship between funds' information acquisition costs and the set of macroeconomic states Θ:

Assumption 4. *There exist* $\underline{\theta}_F \in \Theta$ *and* $\overline{\theta}_F \in \Theta$*, such that the costs of acquiring information satisfy*

$$S(1, \overline{\theta}_F; \lambda) < \psi < S(0, \underline{\theta}_F; \lambda)$$

Assumption 4 implies that if the state is above (below) the bound $\overline{\theta}_F$ ($\underline{\theta}_F$), the net surplus from information acquisition is strictly negative (positive) no matter what strategies other funds choose. Moreover, it further implies that there are signals above (below) which it becomes a dominant action to refrain from (engage in) information acquisition.

Equilibrium Definition. Funds choose whether or not to acquire information based on their signal of the macroeconomic state, their beliefs regarding other funds' information acquisition decisions and the expected secondary market price. In equilibrium, the realized share of informed funds and the resulting market price must be consistent with funds' initially held beliefs.

In the absence of fundamental uncertainty – i.e. if the realization of θ was common knowledge at $t = 0$ among funds and investors – the economy would exhibit multiple equilibria for intermediate values of $\theta \in [\underline{\theta}_F, \overline{\theta}_F]$.[14] The noisiness of funds' signals and the implied incomplete information break common knowledge about the macroeconomic state and allow to isolate a unique equilibrium (Morris & Shin, 2003).

A strategy for fund j is defined as a mapping $\sigma_j : \Theta \to [0, 1]$ which specifies for each signal $\theta_j \in \Theta$ fund j's probability to acquire information about its idiosyncratic asset type. A strategy is *monotone*, characterized by a critical threshold $\theta^*_{j,\epsilon}$, whenever the fund acquires information with probability one if and only if $\theta_j < \theta^*_{j,\epsilon}$ and refrains from information acquisition otherwise. A symmetric monotone strategy is a monotone strategy where all funds use the same threshold $\theta^*_{F,\epsilon}$. As shown below, restricting attention to symmetric monotone strategies is without loss of generality.

[14] In one equilibrium, funds expect market prices to be high and refrain from information acquisition implying that market liquidity provision is undistorted by asymmetric information frictions. In a second equilibrium, funds expect market liquidity to dry-up, acquire information and precipitate an adverse selection problem by withholding good assets.

Unique Monotone Equilibrium. By the law of large numbers and using the assumptions of uniformly distributed states and signals, the share of funds acquiring information given a symmetric monotone strategy summarized by $\theta^*_{F,\epsilon}$ is equal to

$$\sigma(\theta^*_{F,\epsilon}, \theta) = \mathbf{Pr}(\theta_j < \theta^*_{F,\epsilon}|\theta) = G\left(\frac{\theta^*_{F,\epsilon} - \theta + \epsilon}{2\epsilon}\right) \tag{5}$$

where $G(x) = \min\{\max\{x, 0\}, 1\}$.

The equilibrium threshold value $\theta^*_{F,\epsilon}$ must be such that a fund observing the signal $\theta_j = \theta^*_{F,\epsilon}$ is just indifferent between acquiring information or not, given that other funds also use the monotone strategy around $\theta^*_{F,\epsilon}$. Given the uniform prior assumption, the posterior belief about θ for a fund receiving signal $\theta^*_{F,\epsilon}$ is uniform over $[\theta^*_{F,\epsilon} - \epsilon, \theta^*_{F,\epsilon} + \epsilon]$. The threshold $\theta^*_{F,\epsilon}$ therefore solves

$$\mathbf{E}[S(\sigma(\theta^*_{F,\epsilon}, \theta), \theta; \lambda)|\theta^*_{F,\epsilon}] = \frac{1}{2\epsilon}\int_{\theta^*_{F,\epsilon}-\epsilon}^{\theta^*_{F,\epsilon}+\epsilon} S(\sigma(\theta^*_{F,\epsilon}, \theta), \theta; \lambda)\mathrm{d}\theta = \psi \tag{6}$$

Changing the variable of integration using the definition of the share of informed funds given by equation (5), this condition can be rewritten as

$$\int_0^1 S(\sigma, \theta(\theta^*_{F,\epsilon}, \sigma); \lambda)\mathrm{d}\sigma = \psi \tag{7}$$

where $\theta(\theta^*_{F,\epsilon}, \sigma) = \theta^*_{F,\epsilon} - \epsilon\left(2G^{-1}(\sigma) - 1\right)$ and $G^{-1}(\sigma) = \inf\{x|G(x) \geq \sigma\}$.

Proposition 1. (Unique Monotone Equilibrium)

1. *There exists a unique monotone equilibrium where funds acquire information if and only if $\theta_j < \theta^*_{F,\epsilon}$, where $\theta^*_{F,\epsilon} \in (\underline{\theta}_F, \overline{\theta}_F)$.*

2. *There are no other equilibria in non-monotone strategies.*

Comparative Statics. The critical state below which funds acquire information, and the corresponding market liquidity risk, is directly affected by the characteristics of funds' assets including their expected cash flow, F, and the non-marketable control rent, Q.

In addition, funds' information acquisition incentives critically depend on their funding liquidity risk, as measured by the fraction of early redemptions, λ.

Corollary 1. *The threshold $\theta^*_{F,\epsilon}$ below which funds acquire information is:*

1. *Either increasing or decreasing in the expected cash flow of the asset: $\partial\theta^*_{F,\epsilon}/\partial F \gtreqless 0$.*

2. *Strictly increasing in the control rent Q: $\partial\theta^*_{F,\epsilon}/\partial Q > 0$.*

3. *Strictly increasing in the fraction of early withdrawals: $\partial\theta^*_{F,\epsilon}/\partial\lambda > 0$.*

Increases in the expected cash flow, F, are associated with two opposing effects: a negative *price effect* and a positive *redemption effect*. The price effect implies that funds have to sell less assets to meet a given amount of redemptions as cash flows increase. This lowers the surplus from information acquisition and tends to lower the equilibrium threshold $\theta^*_{F,\epsilon}$. Larger expected cash flows, however, also imply that investors are entitled to a larger claim if they withdraw early. This redemption effect raises the surplus from information acquisition and tends to push up the threshold $\theta^*_{F,\epsilon}$. Whether the price or the redemption effect dominates depends crucially on the magnitude of the costs of liquidity lines. If κ is sufficiently large, the negative price effect dominates and higher expected cash flows reduce the set of states where the market dries up due to adverse selection.

While changes in expected cash flows have an ambiguous effect on the degree of market liquidity risk, changes in the control rent have an unambiguous effect. In particular, an increase in Q raises the surplus from information acquisition, thereby exacerbating the coordination problem among funds. An increase Q can be more broadly interpreted as a deterioration in the marketability of assets (i.e. the value lost when assets are transferred to a third-party). Viewed in this light, our model suggests that financial institutions specialized in complex and opaque assets that are difficult to bring to market should be more prone to sudden and unexpected deteriorations in market liquidity conditions.

Finally, the set of states where market liquidity dries up due to adverse selection is increasing in the fraction of early redemptions, λ. A larger share of early redemptions raises the surplus from acquiring information and leads funds to acquire information about their assets' type for a larger range of signals. This observation suggests that market and funding illiquidity could at times become mutually reinforcing. As shown in Section 3, this feedback indeed arises as investors' incentives to redeem early increase if they expect price declines to erode the residual equity value of funds using asset sales.

2.4 Discussion of the Modeling Environment

Before turning to the full model with endogenous redemptions, we briefly discuss some of the key elements of the modeling environment and how they map to observed features of the shadow banking sector.

On the Presence of Liquidity Support. The first point that merits discussion is why sponsors would be willing to set up liquidity lines in the first place, rather than forcing funds to always depend on the market to obtain liquidity. Aside from being a realistic feature of shadow banking arrangements,[15] providing such liquidity support, even though socially suboptimal, is always *privately* optimal. The decision to set-up contingent liquidity lines can be thought in terms of a "prisoner's dilemma:" regardless of the liquidity support provided to other funds, an individual sponsor would always choose to "deviate" and set-up a liquidity line for its own fund since it is profitable to tap this line for sufficiently low realizations of the macroeconomic state, θ. Thus, in the absence of a strict commitment device (e.g. a regulatory ban), the emergence of liquidity lines can be rationalized as the result of incentives created by shadow banking arrangements.

Liquidity Lines *vs.* Cash Balances. The assumption that liquidity lines are costly ($\kappa > 1$) can be justified for a number of reasons. For example, providing funds with liquidity may require sponsoring institutions to pass on valuable investment opportunities for which they must be compensated. Funds may nonetheless prefer to use liquidity lines rather than maintaining cash balances since liquidity lines allow funds to avoid paying the liquidity premium implied by holding liquid assets in states of the world where they

[15] Covitz et al. (2013) report that in 2007 around 87% of all ABCP-programs had pre-arranged back-up lines in place.

do not face a liquidity shortfall (Acharya et al., 2013). Moreover, while cash balances constitute a sunk cost if these are stored before liquidity shocks are realized, the costs of contingent liquidity lines are not sunk but rather are only incurred if they are drawn down. These costs therefore directly affect funds' choice of liquidity source when financing early redemptions.

Liquidity Lines *vs.* Debt Issuances. The funds' choice of liquidity source in our model can be more broadly interpreted as a choice between: (i) deleveraging or (ii) borrowing funds from their sponsor at a fixed interest rate. We restrict attention to these two liquidity sources insofar as we consider the model relevant for understanding environments where the issuance of new (debt or equity) securities is not feasible. Like risk-less debt in Gorton & Pennacchi (1990), liquidity lines in our model have the advantage of being an informationally-insensitive source of funds not subject to adverse selection discounts. Consequently, if funds could costlessly issue new securities, they would all optimally choose to issue risk-less debt. Our model assumes that the only way in which funds can obtain such informationally-insensitive financing is by tapping their liquidity lines at a cost, and shows that funds' choice of liquidity source in this case fundamentally depends on their private information.[16] Liquidity lines also differ from new debt issuances as their cost (i.e. the interest charged on funds drawn from the line) are contracted upon before the realization of liquidity shocks and therefore do not react to contemporaneous market information.

On the Absence of Default. The no-default assumption (*cf.* Assumption 3) preserves funds' preference ordering between liquidity lines and asset sales. If this assumption were violated, uninformed funds may prefer to resort to the liquidity line instead of selling assets, gambling that their asset is good. Given funds' limited liability constraint, investors could therefore suffer losses despite funds having received interim support if funds were to default on their liquidity lines. Such a result would be difficult to square with available evidence on sponsor support. For example, Brady et al. (2012) show that there were no

[16] Note that deleveraging is also costly as it requires funds to forgo the non-marketable control rent, Q.

instances during the 2007-09 financial crisis where money market funds "broke the buck" and defaulted *after* having received sponsor support. Similarly, Gorton (2010) documents that structured investment vehicles that received sponsor support did not default.

3 Market Illiquidity and Redemption Risk

3.1 Investors' Redemption Decisions

Active Investors. We follow Diamond & Dybvig (1983) and Liu & Mello (2011) and assume that active investors are subject to idiosyncratic liquidity shocks that affect their valuation for $t = 2$ consumption. In particular, we assume that each active investor faces a liquidity shock with probability μ, implying that a total share $\mu \in (0, 1)$ of active investors becomes *impatient* and always redeem their claims at $t = 1$. The remaining share $(1 - \mu)$ are *patient*: they face no urgent liquidity need, but may nonetheless redeem early if the payoff from doing so exceeds the expected value of their claim at maturity.[17] Investor types are private information, implying that funds cannot condition redemption payments on whether an investor is patient or impatient.

Patient investors, like funds, receive noisy signals about the macroeconomic state θ at $t = 0$. These signals have the same structure as the signals received by funds: $\theta_i = \theta + \epsilon_i$, where i indexes patient investors and ϵ_i is i.i.d. across investors and funds, drawn from a uniform distribution over $[-\epsilon, \epsilon]$. Based on their signals, patient investors form beliefs about funds' expected equity value at maturity, taking the information acquisition behaviour of funds, the resulting market price and the redemption decisions of other patient investors as given.

Surplus from Early Redemption. The total share of active investors redeeming their shares at $t = 1$ is given by $\lambda \in [\mu, 1]$. The value of a claim at maturity equals the *pro-rata* share of a fund's equity value at $t = 2$, denoted by $D_2(\lambda, \theta; \sigma)$. A patient investor who

[17] While our results require the mass of impatient investors to be strictly positive, μ can be arbitrarily small. That is, all our results hold even in the limiting case where $\mu \to 0$.

observes signal θ_i expects the value of claims redeemed at $t = 2$ to equal

$$\mathbf{E}\left[D_2(\lambda,\theta;\sigma)|\,\theta_i\right] = \mathbf{E}\left[\left.\frac{\sigma\pi V_h^{LL}(\alpha\lambda F) + \sigma(1-\pi)V_l^{AS}(\alpha\lambda F) + (1-\sigma)V_n^{AS}(\alpha\lambda F)}{1-\alpha\lambda}\right|\theta_i\right]$$

Using equations (1) and (2), this expression can be rewritten as follows

$$\mathbf{E}\left[D_2(\lambda,\theta;\sigma)|\,\theta_i\right] = \mathbf{E}\left[\left.\frac{1}{1-\alpha\lambda}\left((F+Q)\left(1-\frac{\alpha\lambda F}{p(\sigma,\theta)}\right)+\sigma S(\sigma,\theta;\lambda)\right)\right|\theta_i\right]$$

The expected equity value of an investor's claim at maturity consists of two parts: (i) the residual equity value of funds' portfolios after assets have been sold to cover early redemptions; and (ii) the information rents accruing to informed funds. A patient investor prefers early redemption if and only funds' *ex ante* book value, F, exceeds funds' expected *per capita* equity value at maturity given the signal θ_i. That is,

$$\mathbf{E}[W(\lambda,\theta;\sigma)|\theta_i] \equiv F - \mathbf{E}\left[D_2(\lambda,\theta;\sigma)|\,\theta_i\right] \geq 0 \tag{8}$$

Note that for all values of θ and λ, $W(\lambda,\theta;0) = -Q < 0$: i.e. in the absence of information acquisition by funds, patient investors would never choose to redeem early because they can earn the control rent if assets remain on funds' balance sheets. If $\sigma > 0$, however, asset prices fall below funds' *ex ante* book value, i.e. $p(\sigma,\theta) < F$. A fund that sells assets to cover early redemptions in this case must liquidate more than one unit of asset per claim redeemed at $t = 1$. This erodes the residual value of the fund's portfolio and dilutes the claims of patient investors that hold out until maturity. This effect is counteracted by the fact that a larger share of early redemptions raises the information rents accruing to informed funds with good assets: i.e. $S_\lambda(\sigma,\theta;\lambda) > 0$. However, the former effect always dominates the latter, leading the residual equity value of funds to fall as the fraction of early redemptions rises. In other words, investors' redemption decisions are *strategic complements* whenever $\sigma > 0$. Moreover, since asset prices are increasing in the macroeconomic state, the surplus from early redemption strictly decreases in θ.

Lemma 4. *Investors' surplus from early redemption is increasing in the share of early redemptions and the fraction of informed funds:* $W_\lambda(\lambda, \theta; \sigma) \geq 0$ *and* $W_\sigma(\lambda, \theta; \sigma) > 0$*, and decreasing in the macroeconomic state:* $W_\theta(\lambda, \theta; \sigma) < 0$*.*

For $\sigma > 0$, there exist realizations of the macroeconomic state such that patient investors consider it strictly dominant to redeem early, and states where they consider it strictly dominant to stay invested in the fund until maturity. These regions are bounded (from above and below, respectively), and these bounds are implicitly defined by the following conditions[18]

$$W(\mu, \underline{\theta}_I; \sigma) = 0 \quad \text{and} \quad W(1, \overline{\theta}_I; \sigma) = 0$$

As for funds' information acquisition decision, patient investors must choose whether or not to redeem their claims early based on their signal of the macroeconomic state and their expectations regarding other investors' redemption decisions and the secondary market price. A strategy for investor i is then defined as a mapping $\lambda_i : \Theta \to [0, 1]$ which specifies for each signal $\theta_i \in \Theta$ a probability with which a patient investor i redeems his claim early. As before, we restrict attention to symmetric monotone strategies summarized by a critical signal $\theta^*_{I,\epsilon}$ whereby investors always redeem their claims early with probability one if $\theta_i < \theta^*_{I,\epsilon}$, and never redeem otherwise.

When investors use the symmetric monotone strategy around $\theta^*_{I,\epsilon}$, the law of large numbers and the assumption of uniformly distributed states and signals implies that the share of early redemptions, given a realized state θ, equals

$$\lambda(\theta^*_{I,\epsilon}, \theta) = \mu + (1 - \mu) G\left(\frac{\theta^*_{I,\epsilon} - \theta + \epsilon}{2\epsilon}\right)$$

For fixed values of $\sigma > 0$, the equilibrium threshold $\theta^*_{I,\epsilon}$ is such that an investor who observes $\theta_j = \theta^*_{I,\epsilon}$ is just indifferent between redeeming at $t = 1$ or $t = 2$, given that all

[18] Note that since $W(\lambda, \theta; 0) < 0$ for all $\lambda \in [\mu, 1]$ and $\theta \in \Theta$, it must be that $\lim_{\sigma \to 0} \underline{\theta}_I = \overline{\theta}_I = \underline{\theta}$.

funds use the monotone strategy around $\theta^*_{I,\epsilon}$:

$$\mathbf{E}[W(\lambda(\theta^*_{I,\epsilon},\theta),\theta;\sigma)|\theta^*_{I,\epsilon}] = \frac{1}{2\epsilon}\int_{\theta^*_{I,\epsilon}-\epsilon}^{\theta^*_{I,\epsilon}+\epsilon} \big(F - D_2(\lambda(\theta^*_{I,\epsilon},\theta),\theta;\sigma)\big)\, d\theta = 0$$

3.2 Information Acquisition and Redemption Equilibrium

Joint Equilibrium. The *joint* monotone equilibrium between funds and patient investors is characterized by critical values $\{\theta^{**}_{F,\epsilon},\, \theta^{**}_{I,\epsilon}\}$ such that funds acquire information if and only if $\theta_j < \theta^{**}_{F,\epsilon}$ and patient investors redeem early if and only if $\theta_i < \theta^{**}_{I,\epsilon}$. The equilibrium thresholds $\theta^{**}_{F,\epsilon}$ and $\theta^{**}_{I,\epsilon}$ simultaneously solve the two indifference conditions:

$$\mathbf{E}[S(\sigma,\lambda,\theta)|\,\theta^{**}_{F,\epsilon}] \;=\; \int_0^1 S\left(\sigma, \mu + (1-\mu)G\left(G^{-1}(\sigma) + \frac{\theta^{**}_{I,\epsilon} - \theta^{**}_{F,\epsilon}}{2\epsilon}\right), \theta(\theta^{**}_{F,\epsilon},\sigma)\right) \mathrm{d}\sigma \;=\; \psi \tag{9}$$

and

$$\mathbf{E}[W(\lambda,\sigma,\theta)|\,\theta^{**}_{I,\epsilon}] \;=\; \int_\mu^1 W\left(\lambda, G\left(G^{-1}\left(\frac{\lambda-\mu}{1-\mu}\right) + \frac{\theta^{**}_{F,\epsilon} - \theta^{**}_{I,\epsilon}}{2\epsilon}\right), \theta(\theta^{**}_{I,\epsilon},\lambda)\right) \mathrm{d}\lambda \;=\; 0 \tag{10}$$

where $\theta(\theta^{**}_{I,\epsilon},\lambda) = \theta^{**}_{I,\epsilon} - \epsilon\left(2G^{-1}\left(\frac{\lambda-\mu}{1-\mu}\right) - 1\right)$.

The thresholds defined by conditions (9) and (10) are bounded from above and from below. These bounds are determined by funds' and investors' expected surplus under "extreme beliefs." For funds, they correspond to realizations of the macroeconomic state such that the net expected surplus from information acquisition is equal to zero if funds believe no (all) patient investors redeem their claims early. Formally,

$$\theta^*_{F,\epsilon}(\mu):\; \mathbf{E}\left[S(\sigma(\theta^*_{F,\epsilon},\theta),\mu,\theta)\big|\,\theta^*_{F,\epsilon}\right] = \psi \quad \text{and} \quad \theta^*_{F,\epsilon}(1):\; \mathbf{E}\left[S(\sigma(\theta^*_{F,\epsilon},\theta),1,\theta)\big|\,\theta^*_{F,\epsilon}\right] = \psi$$

Similarly, for investors, they correspond to realizations of θ such that the expected surplus from early redemption is equal to zero if investors believe no (all) funds acquire information

$$\theta^*_{I,\epsilon}(0) = \underline{\theta}, \quad \text{and} \quad \theta^*_{I,\epsilon}(1) : \mathbf{E}[W(\lambda(\theta^*_{I,\epsilon}, \theta), 1, \theta) \big| \, \theta^*_{I,\epsilon}] = 0$$

Notice that patient investors never redeem their claims early if they expect funds to refrain from information acquisition, regardless of the realization of the macroeconomic state. However, this can never arise in equilibrium since it is always dominant for funds to acquire information for sufficiently small realizations of θ as there is always a positive mass μ of impatient investors that redeem their shares early. Broadly speaking, funds' decision to acquire information *induces* a coordination problem among investors and precipitates redemption runs whenever the macroeconomic state is sufficiently low.[19]

Proposition 2. (Joint Monotone Equilibrium)

1. *There exists a unique equilibrium in monotone strategies where equilibrium thresholds are such that* $\theta^{**}_{I,\epsilon} \leq \theta^{**}_{F,\epsilon}$ *with* $\theta^{**}_{I,\epsilon} \in (\underline{\theta}, \theta^*_{I,\epsilon}(1)]$.

2. *There are no other equilibria in non-monotone strategies.*

Efficient Allocation. To study the welfare properties of the equilibrium in Proposition 4, we can compare the thresholds $\{\theta^{**}_{F,\epsilon}, \theta^{**}_{I,\epsilon}\}$ to the first-best thresholds $\{\theta^{sp}_F, \theta^{sp}_I\} \in \Theta^2$ that maximize investors' aggregate utility from consumption, given by

$$\mathcal{U}(\sigma^*, \lambda^*; \theta) = \mathbf{E}_0[\alpha\lambda^* F + (1 - \alpha\lambda^*) D_2(\sigma^*, \lambda^*; \theta)]$$

where $\sigma^* \equiv \sigma(\theta^{sp}_F, \theta)$ and $\lambda^* \equiv \lambda(\theta^{sp}_I, \theta)$. Using funds' value functions (2) and (1), we can rewrite the welfare function as follows

$$\mathcal{U}(\sigma^*, \lambda^*; \theta) = \mathbf{E}_0 \left[F + Q - \alpha\lambda^* F \left(\sigma^* \pi(\kappa - 1) + (1 - \sigma^* \pi) \frac{Q}{p(\sigma^*, \theta)} \right) \right]$$

[19]Technically speaking, even though investors' redemption game taken on its own may not have a lower dominance region (when $\sigma = 0$), the lower dominance region of funds' information acquisition game "induces" a lower dominance region in the investors' game.

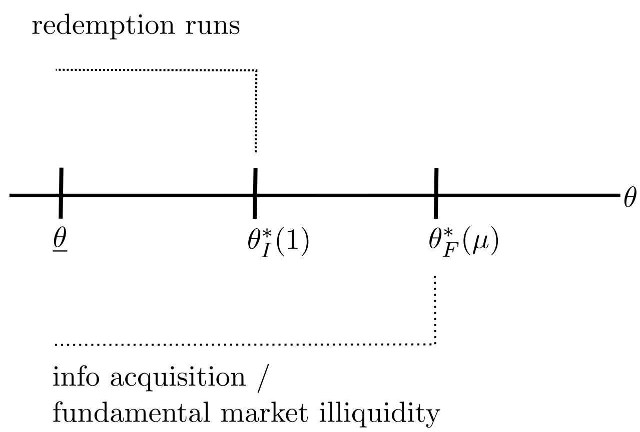
Proposition 3. *The Pareto efficient thresholds are such that $\theta_F^{sp} = \underline{\theta}$ and $\theta_I^{sp} = \underline{\theta}$.*

Discussion of Equilibrium. A comparison of the equilibrium thresholds to the Pareto-efficient thresholds immediately shows that information acquisition is unambiguously inefficient in this economy as it serves only a private *rent-seeking* purpose. While informed good funds avoid the early liquidation of their assets, thereby keeping the control rent Q, Assumption 2 implies that the cost of liquidity lines κ is sufficiently large such that aggregate consumption decreases as more funds become informed.[20] This means that the value of the unrealized gains from trade, due to informed good funds using their liquidity line, always exceeds the value of the foregone control rents from selling assets. The cause of this inefficiency is that funds' incentives are distorted by an externality that operates through changes in the market price, p. Individual funds that acquire information and withhold good assets from the market do not internalize how their behavior affects other funds' option value from holding on to good assets.

For patient investors, in the absence of market liquidity risk, it is never socially (nor privately) optimal to redeem their early since funds' expected equity value at maturity exceeds their *ex ante* book value, F. As funds not acquiring information always opt to meet redemptions by selling assets at their fair value, early redemptions only lead to a destruction of funds' equity value due to the foregone control rent, Q. The coordination failure among investors arises because those that redeem early do not internalize how their decision affects funds' residual equity value, and thereby the payment obtained by investors that redeem at maturity. Importantly, the externality distorting investors' redemption decisions is induced by adverse selection in secondary markets: i.e. funding liquidity risk is a *consequence* of funds' private rent-seeking incentives. Absent information acquisition by funds, inefficient redemption runs would never obtain in equilibrium.

[20] As mentioned above, the cost of liquidity lines may stem from the fact that sponsors must forgo positive net present value investment opportunities in order to disburse liquidity to their funds at $t = 1$.

Figure 3: Weak Dependence Regime



3.3 Feedback between Market and Funding Liquidity

Weak *versus* Strong Dependence. Not only are inefficient redemptions a consequence of information acquisition by funds, but redemption runs can also amplify firms' infomation acquisition incentives and thereby exacerbate market illiquidity. The possibility of such feedback can be illustrated most starkly in the limiting case where agents' private signals become arbitrarily precise: $\epsilon \to 0$. In this case, the behavior of agents becomes degenerate around the realized state and the equilibrium outcome depends on the ordering of the bounds $\theta^*_{I,0}(1)$ and $\theta^*_{F,0}(\mu)$. Following the terminology of Goldstein (2005), we distinguish between a *weak dependence* and a *strong dependence* regime.

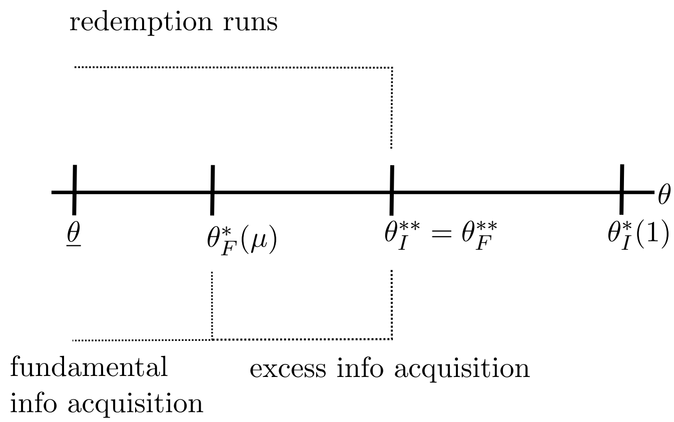
Proposition 4. *For $\epsilon \to 0$, the equilibrium thresholds $\theta^{**}_{F,\epsilon}$ and $\theta^{**}_{I,\epsilon}$ are such that:*

1. *Weak dependence: $\theta^{**}_{I,0} \to \theta^*_{I,0}(1)$ and $\theta^{**}_{F,0} \to \theta^*_{F,0}(\mu)$ if and only if $\theta^*_{I,0}(1) < \theta^*_{F,0}(\mu)$.*

2. *Strong dependence: $\theta^{**}_{I,0} \to \theta^{**}_{F,0} \in [\theta^*_{F,0}(\mu), \theta^*_{I,0}(1)]$ if and only if $\theta^*_{F,0}(\mu) < \theta^*_{I,0}(1)$.*

In the *weak dependence* regime, funds' equilibrium threshold is at its lower bound, $\theta^*_{F,0}(\mu)$. While information acquisition triggers a run by patient investors below $\theta^*_{I,0}(1)$, they abstain from redeeming their claims for all states $\theta \in (\theta^*_{I,0}(1), \theta^*_{F,0}(\mu))$. Hence, in the weak dependence case, funds' information acquisition triggers redemptions, but the coor-

Figure 4: Strong Dependence Regime



dination failure among patient investors does not exert an additional feedback on funds' information acquisition decisions (see Figure 3). In contrast, in the *strong dependence* regime (Figure 4), funds' and investors' thresholds converge: $\theta_{I,0}^{**} \to \theta_{F,0}^{**} \in [\theta_{F,0}^*(\mu), \theta_{I,0}^*(1)]$. Market illiquidity is now always accompanied by redemption runs of patient investors. Higher funding liquidity risk increases funds' incentives to acquire private information and the resulting higher likelihood of market illiquidity further incentivizes investors to redeem early and so on. Thus, the coordination failure among investors "spills over" and amplifies the coordination failure among funds ($\theta_{F,0}^{**} > \theta_{F,0}^*(\mu)$), engendering a destabilizing feedback between redemption risk and market illiquidity.

An interesting implication of Proposition 4 is that redemption runs only arise in cases where market liquidity dries up due to adverse selection. However, secondary market freezes precipitated by funds' decision to acquire private information about their assets need not always result in investor runs (this depends on whether the economy finds itself in the *weak* or *strong dependnece* regime). In other words, while funding illiquidity implies market illiquidity, the converse need not be true. Our model thereby complements the classical bank run literature, e.g. Diamond & Dybvig (1983) and Goldstein & Pauzner (2005), where banks selling assets to meet early withdrawals face an exogenous fire sale discount. More specifically, it proposes a channel through which such fire sale discounts endogenously emerge due to funds' strategic incentives to acquire information about their assets, and shows that while market illiquidity is a necessary condition for runs to arise, it is not always sufficient.

Empirical Implications. The above results have a number of implications regarding the empirical relationship between market and funding liquidity risk in the shadow banking sector. In particular, our results point to a positive correlation between market and funding illiquidity, with the strength of this correlation depending on whether the economy finds itself in the *weak* or *strong dependence* regime. Which regime obtains can be further related to funds' balance sheet characteristics.

Corollary 2. *The economy is more susceptible to the strong dependence regime if: (i) the control rent Q is small; and (ii) the price effect dominates the redemption effect and expected cash flows F are large.*

Recall that strong dependence occurs whenever $\theta^*_{F,0}(\mu) < \theta^*_{I,0}(1)$: i.e. if the coordination problem among funds is relatively muted, funds will react more to the coordination problem among investors. As shown in Corollary 1, a small control rent reduces the coordination problem among funds ($\theta^*_{F,0}(\mu)$ is small). At the same time, a small control rent lowers the foregone cash-flow from withdrawing early, implying that the coordination problem investors will be more pronounced ($\theta^*_{I,0}$ is large). This makes the economy more susceptible to a destabilizing feedback between market and funding illiquidity, and suggests that the correlation between market and funding liquidity risk should be stronger when asset marketability is high.

Similarly, higher expected cash flows make funds less prone to acquire information whenever the negative price effect dominates, and as a result dampen the coordination problem among funds (*cf.* Corollary 1). Since the payment to investors from withdrawing early increases in F, higher expected cash flows also increase their incentives to withdraw and amplify the coordination problem among investors. These comparative static results point to a pro-cyclical correlation between market and funding liquidity. More specifically, they suggest that the shadow banking sector should be especially susceptible to sudden joint collapses in market and funding liquidity conditions at the peak of the cycle.

Finally, the macroeconomic state θ, which measures the variance in cash flows depending on whether assets are *good* or *bad*, can be more broadly interpreted as a proxy for aggregate uncertainty affecting the economy. Our model suggests that market and funding liquidity dry-ups are more likely to occur when the degree of uncertainty is large.[21] This is in line with both macro- and microeconometric studies of the 2007-09 financial crisis. For example, Stock & Watson (2012) find a strong positive correlation between uncertainty and liquidity shocks. Our model provides a specific microeconomic interpretation of this correlation, by showing how high uncertainty induces asset owners on the microeconomic level to produce information that (in the aggregate) can cause a dry-up of markets and runs by investors. This is consistent with microeconometric evidence by Covitz et al. (2013) who show that the incidence of runs on ABCP conduits in 2007-08 was strongly related to macro-financial uncertainty.

4 Policy Implications

Our model allows to analyze different policy measures that can minimize the risk of market and funding liquidity dry-ups. We focus attention on four specific policies: (i) liquidity injections that reduce the cost of private liquidity lines, (ii) asset purchase programs that place a floor on the price at which assets trade, (iii) outright purchases of debt securities, and (iv) prudential regulations that either ban or raise the cost of liquidity lines. The focus on these measures is motivated by policies that central banks and regulators implemented during and following the 2007-09 financial crisis in order to shore up liquidity in financial markets and enhance the stability of the banking *cum* shadow banking sector.

[21] A higher variance in cash flows incentivizes funds to acquire private information about their asset's type since they have more to gain if it turns out to be good. In addition, the higher variance also pushes down market prices when funds acquire information (since good assets are withheld from the market), increasing investors' incentives to redeem their claims early.

Liquidity Injections. We begin by assessing the effect of liquidity injections, e.g. a lowering of interest rates that reduce the cost of funds' liquidity lines. Maintaining the bounds on κ implied by Assumption 2, such a policy has an ambiguous effect on market and funding liquidity risk. Liquidity injections have a (direct) negative effect on market liquidity insofar as they decrease funds' opportunity cost of tapping their liquidity lines. This increases funds' incentives to acquire private information about their assets, thereby amplifying the adverse selection problem in secondary markets. The resulting fall in asset prices increases investors' incentives to redeem their claims early. Concomitantly, however, liquidity injections that lower the cost of liquidity lines increase the residual equity value of informed funds holding good assets and thus decrease investors' incentives to redeem early.[22] This second channel implies an (indirect) positive effect on market liquidity, as fewer early redemptions lower funds' surplus from acquiring information.

Corollary 3. *Liquidity injections that lower the cost of liquidity lines κ can either increase or decrease market and funding liquidity risk:* $\frac{d\theta^{**}_{,F\epsilon}}{d\kappa} \gtrless 0$ *and* $\frac{d\theta^{**}_{I,\epsilon}}{d\kappa} \gtrless 0$.

This ambiguous result relating to reductions in the cost of liquidity lines is broadly consistent with stylized facts regarding the 2007-09 financial crisis. For example, beginning in August 2007, the US Federal Reserve (Fed) adopted "conventional" liquidity measures implemented *via* a lowering of central bank discount rates and short-term repo transactions.[23] These liquidity injections, however, failed to stop the precipitous fall in outstanding ABCP and also failed to prevent the subsequent run on MMFs.

Asset Purchases. Next, we consider the effect of asset purchases resembling, for example, the US Treasury Department's Troubled Asset Relief Program (TARP). In the

[22]The destabilizing effect of liquidity lines has also been pointed out by He & Xiong (2012). In their dynamic debt run model, liquidity lines amplify creditors' incentives to run when asset volatility is high because banks' fundamentals deteriorate while they obtain funds through their liquidity lines. This effect does not arise in our static framework. Instead, cheaper liquidity lines amplify funding withdrawals due to their effect on funds' information acquisition incentives, and thereby the market value of funds' assets.

[23]In the euro area, the ECB injected €95 billion into overnight lending markets on August 9, 2007. Over the following days, the Fed followed suit and injected $62 billion. On September 18, 2007 the Fed supplemented these measures by launching the Term Auction Facility (TAF) which conducted longer-term repurchase transactions totalling $100 billion (Kacperczyk & Schnabl, 2010).

context of our model, this can be thought of as government commitment to purchase assets at a reservation price $q(\theta) > \theta X(\theta)$ for all $\theta \in \Theta$. By placing a floor on asset prices, this policy reduces funds' incentives to acquire information by lowering the option value from withholding good assets from the market. It also reduces investors' incentives to redeem their claims early by raising funds' residual equity value. Even though the floor on asset prices reduces the private surplus from information acquisition, it does not fully eliminate market liquidity risk since funds find it strictly dominant to acquire information for sufficiently small values of θ.[24] Thus, any price guarantee $q(\theta) > \theta X(\theta)$ requires the government to buy bad assets at a price above their fundamental value in some states.

Corollary 4. *Asset price guarantees that place a floor on p decrease market liquidity risk and decrease funding liquidity risk. The expected cost from purchasing assets at price $q(\theta) > \theta X(\theta)$ is equal to:*

$$C^{\mathcal{AP}} = (1-\pi) \int_{\underline{\theta}}^{\max\{\underline{\theta}_F^q(\mu), \theta_{I,0}^q\}} \alpha \left(\mu + (1-\mu)\mathbb{1}_{\theta < \theta_{I,0}^q}\right) F \left(1 - \frac{\theta X(\theta)}{q(\theta)}\right) \mathrm{d}\theta \; > 0$$

Outright Debt Purchases. Finally, we consider the effect of outright purchases of debt securities, such as those conducted by the Federal Reserve under its Commercial Paper Funding Facility (CPFF).[25] In the context of our model, this can be thought of as lowering the fraction of redeemable claims. By committing to purchase claims *at par* at $t = 1$, the government effectively protects funds from funding liquidity risk. In so doing, it lowers funds' incentives to acquire information. Debt purchases also reduce investors' incentives to redeem early by (indirectly) raising asset prices, thereby boosting funds' equity value.

[24] Formally, given some reservation price $q(\theta) > \theta X(\theta)$, the lower dominance region of funds' information acquisition game is given by

$$\underline{\theta}_F^q(\mu): \quad \int_0^1 \pi \left(\frac{X(\underline{\theta}_F^q(\mu)) + Q}{\max\{q(\underline{\theta}_F^q(\mu)), p(\sigma, \underline{\theta}_F^q(\mu))\}} - \kappa\right) \alpha\mu F \mathrm{d}\sigma = \psi$$

[25] This facility provided funding to specially created limited liability company that then bought highly rated unsecured commercial paper or ABCP with short maturities (e.g. three-month) directly from issuers. Under the CPFF the Fed ended up purchasing over \$300 billion worth of commercial paper (Kacperczyk & Schnabl, 2010).

Corollary 5. *Debt purchases that lower the fraction of redeemable claims α decrease market and funding liquidity risk:* $\frac{d\theta^{**}_{F,\epsilon}}{d\alpha} > 0$ *and* $\frac{d\theta^{**}_{I,\epsilon}}{d\alpha} > 0$. *A commitment to buy all redeemable shares implements the efficient allocation.*

If its purchases are unbounded, the government can completely eliminate market liquidity risk by ensuring that no fund acquires information in equilibrium. Debt purchases can therefore be used to implement the efficient allocation described above. Importantly, this policy does not require the government to purchase the totality of funds' outstanding claims, as the absence of market liquidity risk reduces investors' incentives to redeem early. If claims held by the government are treated the same as those held by private investors, such a policy also never requires the government to incur a loss. While the government has to step in and absorb outstanding claims held by impatient investors at $t = 1$, it is always paid back in full at $t = 2$ when assets mature.

Prudential Regulation. In our model, the existence of liquidity lines is the key element incentivizing funds to acquire information. Banning the use of such lines would therefore eliminate the coordination failure leading to market and funding liquidity dry-ups. This stark result rationalizes recent European regulations that prohibits third-party (including sponsor) support for money market funds.[26]

Rather than an outright ban, another possible way to tame recourse to sponsor support is to (marginally) increase its costs. Recent Basel regulations, for example, have increased capital requirements for committed credit lines to funds, making such commitments more expensive. Similarly, Basel III liquidity regulations also raise the costs of liquidity guarantees for off-balance sheet entities as they require banks to hold sufficient unecombered liquid assets against such guarantees. Our model predicts that the effects of such policies would be similarly ambiguous as liquidity injections to sponsoring banks (*cf.* Corollary 3). Regulations that raise the cost of liquidity lines may also give rise to an interesting interaction with central bank policy. For example, suppose *ex ante* regulatory measures

[26] See EU Directive 2017/1131 on the regulation of money market funds. The justification given for this regulation is primarily concerned with the spill-over of problems by MMFs to their sponsors, not with the issue of how their use may lead to a deterioration of market liquidity conditions due to adverse selection.

increase the cost of sponsor support and successfully lower the risk of market and funding liquidity dry-ups. In times of crises, the effects of such policy could be muted if the central bank also chooses to inject liquidity to sponsoring banks. Our model thus suggests that regulations aiming at raising the costs of third party support and central bank policies that lower the costs of liquidity lines may end up inadvertently counteracting each other.

5 Conclusion

This paper proposes a model of (shadow) bank runs based on a feedback between information acquisition and market liquidity. The value of information arises from the option of holding on to good assets by covering redemptions using private liquidity lines rather than selling assets. This generates endogenous adverse selection in secondary markets and reduces market liquidity. Falling prices, in turn, raise investors' incentives to redeem their claims early. This can amplify funding withdrawals and cause market and funding illiquidity to become mutually reinforcing.

Broadly speaking, our paper is motivated by Gorton (2010)'s idea that the run on the shadow banking sector during the 2007-09 financial crisis was caused by a sudden regime switch whereby "informationally insensitive" securities suddenly became "informationally sensitive." A key contribution of our model is to show that such regimes can be sustained by self-fulfilling beliefs about shadow banks' information acquisition behavior. It thereby provides a new framework studying the interaction between information acquisition, market liquidity and funding risk that helps explain the fragility of the shadow banking sector. Although our modelling assumptions make us inclined to think of the funds in our model as shadow banking arrangements, the model can also be applied to more general market-based financial intermediation where fluctuations in the value of intermediaries' assets and liabilities are closely tied to changes in market prices. From this perspective, it highlights the fragility of financial institutions holding complex and opaque securities that rely on a mix of market-based liquidity and third-party support to manage their funding risk.

Appendix

Proofs

Proof of Lemma 1. Before proving the lemma, we show that Assumptions 1 - 3 are not mutually exclusive, i.e. there exist non-empty intervals $\Pi(\kappa)$ and $\mathcal{Q}(\kappa)$ such that any $\pi \in \Pi(\kappa)$ and $Q \in \mathcal{Q}(\kappa)$ satisfy Assumptions 1 and 2. To see this, observe first that we can use Assumption 2 to solve for a largest lower and a smallest upper bound on Θ:

$$\underline{\theta} = \frac{\pi(F+Q)}{\kappa F - (1-\pi)(F+Q)} \quad \text{and} \quad \overline{\theta} = \frac{F - \pi(\kappa F - Q)}{(1-\pi)(\kappa F - Q)}$$

$\Pi(\kappa)$ and $\mathcal{Q}(\kappa)$ must be such that $0 \leq \underline{\theta} < \overline{\theta} \leq 1$. Note first that $\underline{\theta} \geq 0$ and $\overline{\theta} \leq 1$ require $Q \leq (\kappa - 1)F$. Second,

$$\underline{\theta} < \overline{\theta} \;\Leftrightarrow\; (\kappa-1)F((\kappa+1)\pi - 1) < Q((\kappa+1)\pi - 1)$$

Hence, whenever $\pi > (\kappa+1)^{-1}$, the latter implies $Q \geq (\kappa-1)F$ in contradiction to $\underline{\theta} \geq 0$ and $\overline{\theta} \leq 1$. Therefore, $\pi < (\kappa+1)^{-1}$. Substituting the explicit form for $X(\theta) = (\pi + (1-\pi)\theta)^{-1}F$ and the above expression for $\underline{\theta}$ into Assumption 1 and solving for Q yields $Q > \frac{(\kappa-1)F}{\kappa+1}$. Summarizing, any combination of π and Q from $\Pi(\kappa) = (0, (\kappa+1)^{-1})$ and $\mathcal{Q}(\kappa) = (\frac{(\kappa-1)F}{\kappa+1}, (\kappa-1)F)$ satisfies Assumptions 1 and 2.

To prove the Lemma, note that for all Ω_j, we have that $\ell_j^* = \alpha\mu F$ regardless of whether funds use assets sales or the liquidity line to meet redemptions. This follows because for all Ω_j and θ we have that $\frac{d}{d\ell_j}\mathbf{E}[V_{\Omega_j}^{LL}(\ell_j)|\theta] < 0$ since $\kappa > 1$ and $\frac{d}{d\ell_j}\mathbf{E}[V_{\Omega_j}^{AS}(\ell_j)|\theta] < 0$ due to Assumption 1.

For informed funds such that $\Omega_j \in \{g, b\}$ notice that $\mathbf{E}[V_{\Omega_j}^{LL}(\alpha\mu F)|\theta] \gtrless \mathbf{E}[V_{\Omega_j}^{AS}(\alpha\mu F)|\theta]$ implies

$$\max\left\{(\mathbf{E}[\tilde{X}(\theta)|\Omega_j, \theta] + Q) - \alpha\mu F\kappa, 0\right\} \gtrless (\mathbf{E}[\tilde{X}(\theta)|\Omega_j, \theta] + Q)\max\left\{1 - \frac{\alpha\mu F}{p}, 0\right\}, \quad \forall\theta \in \Theta$$

Similarly, for uninformed funds such that $\Omega_j \in \{n\}$ notice that $\mathbf{E}[V_n^{LL}(\alpha\mu F)|\theta] \gtrless \mathbf{E}[V_n^{AS}(\alpha\mu F)|\theta]$ implies

$$\mathbf{E}_0\left[\max\left\{(\tilde{X}(\theta) + Q) - \alpha\mu F\kappa, 0\right\}\middle|\theta\right] \gtrless \mathbf{E}_0\left[(\tilde{X}(\theta) + Q)\max\left\{1 - \frac{\alpha\mu F}{p}, 0\right\}\middle|\theta\right], \quad \forall\theta \in \Theta$$

From Assumption 2, it follows that informed funds holding a good asset prefer the liquidity line while informed funds holding a bad asset and uninformed funds prefer asset sales for all $\alpha\mu F < \min\{(\theta X(\theta) + Q)/\kappa, \theta X(\theta)\}$. Substituting the lower bound for Q implied by Assumption 1, this inequality implies that $\alpha\mu < 1/\kappa$, which must hold due to Assumption 3. $\square$

Proof of Proposition 1. We prove the proposition by showing that funds' information acquisition game satisfies all the properties of Proposition 2.2 in Morris & Shin (2003).

The required properties are:

1. *Action Monotonicity:* $S(\sigma, \theta; \lambda)$ is increasing in σ.

2. *State Monotonicity:* $S(\sigma, \theta; \lambda)$ is decreasing in θ.

3. *Continuity:* $S(\sigma, \theta; \lambda)$ is continuous in both σ and θ.

4. *Finite Expectations of Signals:* The distribution of ϵ_j is integrable.

5. *Uniform Limit Dominance:* There exists $\underline{\theta}_F \in \Theta$, $\overline{\theta}_F \in \Theta$ and such that: (i) $S(\sigma, \theta; \lambda) > \psi$ for all $\sigma \in [0, 1]$ and $\theta \leq \underline{\theta}_F$; and (ii) $S(\sigma, \theta; \lambda) < \psi$ for all $\sigma \in [0, 1]$ and $\theta \geq \underline{\theta}$.

6. *Strict Laplacian State Monotonicity:* There exists a unique θ^*_F solving $\int_0^1 S(\sigma, \theta^*_F; \lambda) d\sigma = \psi$.

Properties 1 and 2 are implied by Lemma 3. Properties 3 and 4 follow from the definition of the surplus function and the uniform distribution of signals, respectively. Property 5 is implied by Assumption 4. Finally, Property 6 follows from the fact that $\int_0^1 S(\sigma, \underline{\theta}_F; \lambda) d\sigma > \psi$, $\int_0^1 S(\sigma, \overline{\theta}_F; \lambda) d\sigma < \psi$ and $\int_0^1 S_\theta(\sigma, \theta; \lambda) d\theta < 0$ for all $\lambda > 0$. It follows that there exists a unique monotone equilibrium and that there are no other equilibria in non-monotone strategies. □

Proof of Corollary 1. Rewrite the equilibrium condition (7) as

$$A(\theta^*_{F,\epsilon}, F, Q, \lambda) \equiv \alpha\lambda\pi F \int_0^1 \left(\frac{X(\theta(\theta^*_{F,\epsilon}, \sigma)) + Q}{p(\sigma, \theta(\theta^*_{F,\epsilon}, \sigma))} - \kappa \right) \mathrm{d}\sigma - \psi = 0$$

From the proof of Proposition 1, $A_{\theta^*_{F,\epsilon}} < 0$. Thus, by the implicit function theorem, for $\tau \in \{F, Q, \lambda\}$,

$$\mathrm{sign}\left\{ \frac{\mathrm{d}\theta^*_{F,\epsilon}}{\mathrm{d}\tau} \right\} = \mathrm{sign}\{A_\tau(\theta^*_{F,\epsilon}, F, Q, \lambda)\}$$

Thus, $A_Q(\theta^*_{F,\epsilon}, F, Q, \tau) = \alpha\lambda\pi F \int_0^1 \left(\frac{1}{p(\sigma, \theta(\theta^*_{F,\epsilon}, \sigma))} \right) \mathrm{d}\sigma > 0$, implying that increases in Q increase the threshold. However,

$$A_F(\theta^*_{F,\epsilon}, F, Q, \tau) = \underbrace{\alpha\lambda\pi \int_0^1 \left(\frac{X(\theta(\theta^*_{F,\epsilon}, \sigma)) + Q}{p(\sigma, \theta(\theta^*_{F,\epsilon}, \sigma)} - \kappa \right) \mathrm{d}\sigma}_{\text{redemption effect } (+)} \underbrace{- \alpha\lambda\pi \int_0^1 \left(\frac{Q}{p(\sigma, \theta(\theta^*_{F,\epsilon}, \sigma))} \right) \mathrm{d}\sigma}_{\text{price effect } (-)} \gtreqless 0$$

The latter can be rewritten as

$$A_F(\theta^*_{F,\epsilon}, F, Q, \tau) = \alpha\lambda\pi \int_0^1 \left(\frac{X(\theta(\theta^*_{F,\epsilon}, \sigma))}{p(\sigma, \theta(\theta^*_{F,\epsilon}, \sigma))} - \kappa \right) \mathrm{d}\sigma \gtreqless 0$$

which is negative, i.e. the negative price effect dominates, if κ becomes sufficiently large.

By the same argument as above, the threshold $\theta^*_{F,\epsilon}$ is strictly increasing in the share of withdrawals since $A_\lambda(\theta^*_{F,\epsilon}, F, Q, \lambda) = \alpha\pi F \int_0^1 \left(\frac{X(\theta(\theta^*_{F,\epsilon}, \sigma)) + Q}{p(\sigma, \theta(\theta^*_{F,\epsilon}, \sigma))} - \kappa \right) \mathrm{d}\sigma > 0$. □

Proof of Lemma 4. We begin by showing that $W_\lambda(\lambda, \theta; \sigma) > 0$ for all $\sigma > 0$. Differentiating the surplus from early redemption with respect to λ yields

$$W_\lambda(\lambda, \theta; \sigma) \propto \mathbf{E}[(F + Q)(F - p(\sigma, \theta)) - \sigma\pi(X + Q - \kappa p(\sigma, \theta))F | \theta_i]$$

Notice that from Assumption 2 and the fact that $p(\sigma, \theta) \geq \theta X(\theta)$, it must be that $\kappa p(\sigma, \theta) \geq (F + Q)$. Hence, we need to show that

$$\mathbf{E}[(F + Q)(F - p(\sigma, \theta)) - \sigma\pi(X - F)F|\theta_i] \geq 0$$

Using the definition of $p(\sigma, \theta)$ and F, this condition can be rewritten as follows

$$\mathbf{E}[(F + Q)(\pi - \tau(\sigma))(1 - \theta)X(\theta) - \sigma\pi(1 - \pi)(1 - \theta)X(\theta)F|\theta_i] \geq 0$$

Substituting in for $\tau(\sigma)$ and rearranging, this inequality implies

$$\frac{F + Q}{1 - \pi\sigma} \geq F$$

which is always satisfied and holds strictly for all $\sigma > 0$. Next, we show that $W_\sigma(\lambda, \theta; \sigma) > 0$. Differentiating the surplus from early redemption with respect to σ yields

$$W_\sigma(\lambda, \theta; \sigma) \propto \mathbf{E}\left[-((F + Q) - \sigma\pi(X(\theta) + Q))\frac{p_\sigma(\sigma, \theta)}{p(\sigma, \theta)} - \pi(X(\theta) + Q - \kappa p(\sigma, \theta))\middle|\theta_i\right]$$

Substituting in for F, this condition can be rewritten as

$$W_\sigma(\lambda, \theta; \sigma) \propto \mathbf{E}\left[-(\pi(1 - \sigma)X(\theta) + (1 - \pi)\theta X(\theta) + (1 - \sigma\pi)Q)\frac{p_\sigma(\sigma, \theta)}{p(\sigma, \theta)} - \pi(X(\theta) + Q - \kappa p(\sigma, \theta))\middle|\theta_i\right]$$

Using the definition of $p(\sigma, \theta)$, this expression can again be rewritten as

$$W_\sigma(\lambda, \theta; \sigma) \propto \mathbf{E}\left[\frac{\pi(1 - \pi)}{1 - \pi\sigma}(1 - \theta)X(\theta) - (1 - \sigma\pi)Q\frac{p_\sigma(\sigma, \theta)}{p(\sigma, \theta)} - \pi(X(\theta) + Q - \kappa p(\sigma, \theta))\middle|\theta_i\right]$$

As before, notice that we must have $\kappa p(\sigma, \theta) \geq (F + Q)$. We therefore need to show that

$$\mathbf{E}\left[\frac{\pi(1 - \pi)}{1 - \pi\sigma}(1 - \theta)X(\theta) - (1 - \sigma\pi)Q\frac{p_\sigma(\sigma, \theta)}{p(\sigma, \theta)} - \pi(1 - \pi)(1 - \theta)X(\theta)\middle|\theta_i\right] > 0$$

Simplifying this condition, we obtain the following inequality

$$\mathbf{E}\left[\pi(1 - \pi)(1 - \theta)X(\theta)\left(\frac{1}{1 - \pi\sigma} - 1\right) - (1 - \sigma\pi)Q\frac{p_\sigma(\sigma, \theta)}{p(\sigma, \theta)}\middle|\theta_i\right] > 0$$

which is always satisfied since $p_\sigma(\sigma, \theta) < 0$. □

Proof of Proposition 2. (i) Unique monotone equilibrium: We show that there exists a unique monotone equilibrium where thresholds are such that $\theta_I^{**} \leq \theta_F^{**}$.

Suppose that funds and investors use monotone strategies around θ_F^{**} and θ_I^{**}. From the proof of Proposition 1, we know that for a fixed value of $\theta_{I,\epsilon}^{**}$ (and hence a fixed value of $\lambda \geq \mu$) there exists a unique threshold $\theta^{**}_{F,\epsilon}(\theta^{**}_{I,\epsilon})$ that solves condition (9). Note that the optimal information acquisition threshold solving equation (9) is weakly increasing in $\theta^{**}_{I,\epsilon}$ with slope given by

$$\frac{\mathrm{d}\theta^{**}_{F,\epsilon}}{\mathrm{d}\theta^{**}_{I,\epsilon}} = \frac{(1-\mu)\int_0^1 S_\lambda(\cdot)\,d\sigma}{(1-\mu)\int_0^1 S_\lambda(\cdot)\,d\sigma - 2\epsilon\int_0^1 S_\theta(\cdot)d\sigma} < 1$$

where the condition follows from application of the implicit function theorem and the fact that $S_\lambda(\cdot) > 0$ and $S_\theta(\cdot) < 0$.

Substituting condition (9) into condition (10) yields

$$H(\theta^{**}_{I,\epsilon}) \equiv \int_\mu^1 W\left(\lambda, G\left(\frac{\lambda-\mu}{1-\mu} + \frac{\theta^{**}_{F,\epsilon}(\theta^{**}_{I,\epsilon}) - \theta^{**}_{I,\epsilon}}{2\epsilon}\right), \theta(\theta^{**}_{I,\epsilon}, \lambda)\right) d\lambda$$

Notice that

$$H(\theta^{*}_{I,\epsilon}(0)) = \int_\mu^1 W\left(\lambda, G\left(\frac{\lambda-\mu}{1-\mu} + \frac{\theta^{**}_{F,\epsilon}(\theta^{*}_{I,\epsilon}(0)) - \theta^{*}_{I,\epsilon}(0)}{2\epsilon}\right), \theta(\theta^{*}_{I,\epsilon}(0), \lambda)\right) d\lambda > 0$$

since $\theta^*_{F,\epsilon}(\mu) > \underline{\theta}$ for all $\mu > 0$, $\underline{\theta}_I(0) = \underline{\theta}$ and $W_\sigma(\cdot) > 0$. Furthermore, we also have that

$$H(\theta^{*}_{I,\epsilon}(1)) = \int_\mu^1 W\left(\lambda, G\left(\frac{\lambda-\mu}{1-\mu} + \frac{\theta^{**}_{F,\epsilon}(\theta^{*}_{I,\epsilon}(1)) - \theta^{*}_{I,\epsilon}(1)}{2\epsilon}\right), \theta(\theta^{*}_{I,\epsilon}(1), \lambda)\right) d\lambda \leq 0$$

where the condition follows from the fact that $G(\cdot) \in [0,1]$ and $W_\sigma(\cdot) > 0$. Hence, by application of the intermediate value theorem, the function $H(\theta^{**}_{I,\epsilon})$ must intersect the x-axis at least once for values of $\theta^{**}_{I,\epsilon} \in (\theta^*_{F,\epsilon}(\mu), \theta^*_{I,\epsilon}(1)]$. Since $\frac{\mathrm{d}\theta^{**}_{F,\epsilon}}{\mathrm{d}\theta^{**}_{I,\epsilon}} < 1$, it follows that

$$H'(\theta^{**}_{I,\epsilon}) = \frac{1}{2\epsilon}\int_\mu^1 W_\sigma(\cdot)\left(\frac{\mathrm{d}\theta^{**}_{F,\epsilon}}{\mathrm{d}\theta^{**}_{I,\epsilon}} - 1\right) d\lambda + \int_\mu^1 W_\theta(\cdot) d\lambda < 0$$

since $W_\theta(\cdot) < 0$, implying that there exists a unique value $\theta^{**}_{I,\epsilon} \in (\theta^*_{F,\epsilon}(\mu), \theta^*_{I,\epsilon}(1)]$ that solves $H(\theta^{**}_{I,\epsilon}) = 0$.

Finally, we show that $\theta^{**}_{I,\epsilon} \leq \theta^{**}_{F,\epsilon}$. By application of the implicit function theorem we have that

$$\frac{d\theta^{**}_{I,\epsilon}}{d\theta^{**}_{F,\epsilon}} = \frac{\int_\mu^1 W_\sigma(\cdot)d\lambda}{\int_\mu^1 W_\sigma(\cdot)d\lambda - 2\epsilon\int_\mu^1 W_\theta(\cdot)} < 1$$

Since $\theta^*_{I,\epsilon}(0) = \underline{\theta}$ and $\theta^*_{F,\epsilon}(\mu) > \underline{\theta}$, the unique fixed point must be such that $\theta^{**}_{I,\epsilon} \leq \theta^{**}_{F,\epsilon}$.

(ii) No other non-monotone equilibria: The argument closely follows the argument in Goldstein (2005). Towards a contradiction, suppose that an alternative non-monotone equilibrium exists where funds acquire information for some signals $\theta_j > \theta^{**}_{F,\epsilon}$ and where patient investors redeem early for some signals $\theta_i > \theta^{**}_{I,\epsilon}$. By the existence of dominance regions there exist bounds θ^N_F and θ^N_I such that funds do not acquire information for $\theta_j > \theta^N_F$ and investors never redeem for $\theta_i > \theta^N_I$. Let σ_N and λ_N denote the fractions of

funds who acquire information and investors who run in this non-monotone equilibrium. They satisfy

$$\sigma_N(\theta) \leq G\left(\frac{\theta_F^N - \theta + \epsilon}{2\epsilon}\right) \quad \text{and} \quad \lambda_N(\theta) \leq \mu + (1-\mu)G\left(\frac{\theta_I^N - \theta + \epsilon}{2\epsilon}\right)$$

A fund whose type is just $\theta_j = \theta_F^N$ must be indifferent between acquiring and not acquiring information:

$$\frac{1}{2\epsilon}\int_{\theta_F^N - \epsilon}^{\theta_F^N + \epsilon} S(\sigma_N(\theta), \lambda_N(\theta), \theta) \mathrm{d}\theta - \psi \;=\; 0$$

Since the surplus from information acquisition is increasing in σ_N and λ_N, it follows that

$$\frac{1}{2\epsilon}\int_{\theta_F^N - \epsilon}^{\theta_F^N + \epsilon} S\left(G\left(\frac{\theta_F^N - \theta + \epsilon}{2\epsilon}\right), G\left(\frac{\theta_I^N - \theta + \epsilon}{2\epsilon}\right), \theta\right) \mathrm{d}\theta - \psi \geq 0$$

Changing variables of integration yields,

$$\int_0^1 S\left(\sigma, G\left(G^{-1}(\sigma) + \frac{\theta_I^N - \theta_F^N}{2\epsilon}\right), \theta(\theta_F^N, \sigma)\right) \mathrm{d}\sigma - \psi \geq 0$$

Comparing this to equation (9) in the text implies

$$\int_0^1 \left[S\left(\sigma, G\left(G^{-1}(\sigma) + \frac{\theta_I^N - \theta_F^N}{2\epsilon}\right), \theta(\theta_F^N, \sigma)\right) - S\left(\sigma, \mu + (1-\mu)G\left(G^{-1}(\sigma) + \frac{\theta_{I,\epsilon}^{**} - \theta_{F,\epsilon}^{**}}{2\epsilon}\right), \theta(\theta_{F,\epsilon}^{**}, \sigma)\right)\right] \mathrm{d}\sigma \geq 0$$

But since $\theta_F^N > \theta_{,F\epsilon}^{**}$ (by assumption) and the surplus function is decreasing in θ the latter can only hold if

$$\theta_I^N - \theta_F^N \;>\; \theta_{I,\epsilon}^{**} - \theta_{F,\epsilon}^{**} \tag{A1}$$

Repeating this line of reasoning for the expected surplus from early redemption implies

$$\theta_F^N - \theta_I^N \;>\; \theta_{F,\epsilon}^{**} - \theta_{I,\epsilon}^{**} \tag{A2}$$

(A2) obviously contradicts (A1), implying that funds will never acquire information at types above $\theta_{F,\epsilon}^{**}$ and investors will never redeem early at types above $\theta_{I,\epsilon}^{**}$. A symmetric argument establishes that agents will not switch at types below $\theta_{F,\epsilon}^{**}$ and $\theta_{I,\epsilon}^{**}$. Thus, a non-monotone equilibrium cannot exist. □

Proof of Proposition 3. The aggregate utility from consumption is given by

$$\mathcal{U}(\sigma^*, \lambda^*; \theta) = \mathbf{E}_0\left[\alpha\lambda^* F + (1 - \alpha\lambda^*) D_2(\sigma^*, \lambda^*; \theta)\right]$$

Using funds' value functions (1) and (2), this can be written as

$$\mathcal{U}(\sigma, \lambda; \theta) = \mathbf{E}_0\left[\alpha\lambda F + (F+Q)\left(1 - \frac{\alpha\lambda F}{p(\sigma,\theta)}\right) + \sigma\left(\pi\left(\frac{X(\theta)+Q}{p(\sigma,\theta)} - \kappa\right)\alpha\lambda F\right)\right]$$

Rearranging the latter yields

$$\mathcal{U}(\sigma,\lambda;\theta)=\mathbf{E}_0\left[F+Q-\alpha\lambda F\sigma\pi(\kappa-1)-\frac{\alpha\lambda F}{p(\sigma,\theta)}\left(F+Q-\sigma\pi(X+Q)-(1-\sigma\pi)p\right)\right]$$

Substituting the definition of $p(\sigma,\theta)$ and rearranging yields the expression in the text

$$\mathcal{U}(\sigma,\lambda;\theta)=\mathbf{E}_0\left[F+Q-\alpha\lambda F\sigma\pi(\kappa-1)-\frac{\alpha\lambda F}{p(\sigma,\theta)}(1-\sigma\pi)Q\right]$$

Notice that $\mathcal{U}_\sigma(\sigma,\lambda;\theta)<0$ for all $\lambda\in[0,1]$ since, by Assumption 2, we must have

$$\kappa>\frac{\theta X(\theta)+Q}{\theta X(\theta)}$$

since $F>\theta X(\theta)$. Given the definition of $\sigma(\theta_F^{sp},\theta)$, it follows that $\theta_F^{sp}=\underline{\theta}$ and $\sigma(\underline{\theta},\theta)=0$ for all $\theta\in\Theta$.

Moreover, we have that $U_\lambda(\sigma,\lambda;\theta)<0$ for any $\sigma\in[0,1]$. Thus, given the definition of $\lambda(\theta_I^{sp},\theta)$, it follows immediately that $\theta_I^{sp}=\underline{\theta}$. □

Proof of Proposition 4. We first prove that $\theta^{**}_{F,\epsilon}\overset{\epsilon\to 0}{\to}\theta^*_{F,0}(\mu)$ and $\theta^{**}_{I,\epsilon}\overset{\epsilon\to 0}{\to}\theta^*_{I,0}(1)$ if and only if $\theta^*_{I,0}(1)<\theta^*_{F,0}(\mu)$. Sufficiency follows by noting that when $\theta^{**}_{I,\epsilon}<\theta^{**}_{F,\epsilon}$, condition (9) implies

$$\lim_{\epsilon\to 0}\mathbf{E}[S(\sigma,\lambda,\theta)|\,\theta^{**}_{F,\epsilon}]=\mathbf{E}[S(\sigma,\mu,\theta)|\,\theta^{**}_{F,\epsilon}]\quad\Leftrightarrow\quad\lim_{\epsilon\to 0}\theta^{**}_{F,\epsilon}=\theta^*_{F,0}(\mu)\tag{A3}$$

Similarly, condition (10) implies

$$\lim_{\epsilon\to 0}\mathbf{E}[W(\lambda,\sigma,\theta)|\,\theta^{**}_{I,\epsilon}]=\mathbf{E}[W(\lambda,1,\theta)|\,\theta^{**}_{I,\epsilon}]\quad\Leftrightarrow\quad\lim_{\epsilon\to 0}\theta^{**}_{I,\epsilon}=\theta^*_{I,0}(1)\tag{A4}$$

Hence, we must have $\theta^*_{I,0}(1)<\theta^*_{F,0}(\mu)$. Necessity then follows from observing that $\theta^{**}_{I,\epsilon}\not<\theta^{**}_{F,\epsilon}$ if $\theta^*_{I,0}(1)\geq\theta^*_{F,0}(\mu)$.

Second, we show that $\theta^{**}_{I,\epsilon}\overset{\epsilon\to 0}{\to}\theta^{**}_{F,0}$ if and only if $\theta^*_{I,0}(1)\geq\theta^*_{F,0}(\mu)$. From above, by contraposition, $\theta^*_{I,0}(1)\geq\theta^*_{F,0}(\mu)$ if and only if $\theta^{**}_{I,0}\geq\theta^{**}_{F,0}$ as $\epsilon\to 0$. But since Proposition 2 implies that $\theta^{**}_{I,\epsilon}\not>\theta^{**}_{F,\epsilon}$, it must be that $\theta^{**}_{I,0}=\theta^{**}_{F,0}$.

Finally, we show that indeed $\theta^{**}_{F,\epsilon}\overset{\epsilon\to 0}{\to}\theta^{**}_{F,0}\in[\theta^*_{F,0}(\mu),\theta^*_{I,0}(1)]$ if $\theta^*_{I,0}(1)\geq\theta^*_{F,0}(\mu)$. From condition (9), we have that

$$\lim_{\epsilon\to 0}\mathbf{E}[S(\sigma,\lambda,\theta)|\,\theta^{**}_{F,\epsilon}]\geq\mathbf{E}[S(\sigma,\mu,\theta)|\,\theta^{**}_{F,\epsilon}]\quad\Leftrightarrow\quad\lim_{\epsilon\to 0}\theta^{**}_{F,\epsilon}\geq\theta^*_{F,0}(\mu)$$

where the inequality follows from $\lim_{\epsilon\to 0}G\left(G^{-1}(\sigma)+\frac{\theta^{**}_{I,\epsilon}-\theta^{**}_{F,\epsilon}}{2\epsilon}\right)\in[0,1]$, $S_\lambda(\cdot)>0$ and $S_\theta(\cdot)<0$. Similarly, using condition (10), we have

$$\lim_{\epsilon\to 0}\mathbf{E}[W(\lambda,\sigma,\theta)|\,\theta^{**}_{I,\epsilon}]\leq\lim_{\epsilon\to 0}\mathbf{E}[W(\lambda,1,\theta)|\,\theta^{**}_{I,\epsilon}]\quad\Leftrightarrow\quad\lim_{\epsilon\to 0}\theta^{**}_{I,\epsilon}\leq\theta^*_{I,0}(1)$$

where the inequality follows from $\lim_{\epsilon\to 0} G\left(G^{-1}\left(\frac{\lambda-\mu}{1-\mu}\right)+\frac{\theta^{**}_{F,\epsilon}-\theta^{**}_{I,\epsilon}}{2\epsilon}\right)\in[0,1]$, $W_\sigma(\cdot)>0$ and $W_\theta(\cdot)<0$. Since $\theta^{**}_{I,\epsilon}\to\theta^{**}_{F,\epsilon}$ as $\epsilon\to 0$, it follows that $\theta^{**}_{F,\epsilon}\to\theta^{**}_{F,0}\in[\theta^*_{F,0}(\mu),\theta^*_{I,0}(1)]$. Clearly, this interval is empty if $\theta^*_{I,0}(1)<\theta^*_{F,0}(\mu)$.

□

Proof of Corollary 2. By Corollary 1, the threshold $\theta^*_{F,0}(\mu)$ is strictly increasing in Q and it is strictly decreasing in F if the negative price effect dominates. Using equation (8), $\theta^*_{I,\epsilon}$ is given by the solution to

$$B(\theta^*_{I,\epsilon},F,Q,1)\equiv F-\int_0^1 D_2(\lambda,\theta(\theta^*_{I,\epsilon},\lambda);1)\mathrm{d}\lambda=0$$

As this is strictly decreasing in $\theta^*_{I,\epsilon}$, we have, for $\tau\in\{F,Q\}$:

$$\mathrm{sign}\left\{\frac{\mathrm{d}\theta^*_{I,\epsilon}(1)}{\mathrm{d}\tau}\right\}=\mathrm{sign}\{B_\tau(\theta^*_{I,\epsilon},F,Q,1)\}$$

Observe that

$$B_Q(\theta^*_{F,\epsilon},F,Q,1)=-\int_0^1\frac{1}{1-\alpha\lambda}\left(1-\frac{\alpha\lambda F}{p(1,\theta(\theta^*_{I,\epsilon},\lambda))}+S_Q(1,\theta(\theta^*_{I,\epsilon},\lambda))\right)\mathrm{d}\lambda<0$$

since $S_Q(\cdot)>0$ (cf. Corollary 1). Thus, as $\theta^*_F(\mu)$ increases and $\theta^*_I(1)$ decreases in Q, for sufficiently small Q, the economy is more susceptible to the strong dependence regime.

Moreover,

$$B_F(\theta^*_{F,\epsilon},F,Q,1)=1-\int_0^1\frac{1}{1-\alpha\lambda}\left(1-\frac{\alpha\lambda F}{p(1,\theta(\theta^*_{I,\epsilon},\lambda))}+S_F(1,\theta(\theta^*_{I,\epsilon},\lambda))\right)\mathrm{d}\lambda>0$$

because $1-\alpha\lambda F/p(\cdot)<1-\alpha\lambda$ and $S_F(\cdot)<0$ (*cf.* Corollary 1). Thus, as $\theta^*_F(\mu)$ decreases in F whenever the negative price effect dominates and $\theta^*_I(1)$ increases in F, the economy is more susceptible to the strong dependence regime when F is sufficiently large. □

Proof of Corollaries 3-5. The equilibrium thresholds solve the following system of equations

$$A(\theta^{**}_{F,\epsilon},\theta^{**}_{I,\epsilon})\equiv\int_0^1 S\left(\sigma,\mu+(1-\mu)G\left(\sigma+\frac{\theta^{**}_{I,\epsilon}-\theta^{**}_{F,\epsilon}}{2\epsilon}\right),\theta(\theta^{**}_{F,\epsilon},\sigma)\right)\mathrm{d}\sigma-\psi=0 \tag{A5}$$

$$B(\theta^{**}_{F,\epsilon},\theta^{**}_{I,\epsilon})\equiv\int_\mu^1 W\left(\lambda,G\left(\frac{\lambda-\mu}{1-\mu}+\frac{\theta^{**}_{F,\epsilon}-\theta^{**}_{I,\epsilon}}{2\epsilon}\right),\theta(\theta^{**}_{I,\epsilon},\lambda)\right)\mathrm{d}\lambda=0 \tag{A6}$$

1. Liquidity Injections. The Jacobian of the system of equations (A5)-(A6) is given by

$$\mathbf{J}=\begin{bmatrix}-\frac{1}{2\epsilon}(1-\mu)\int_0^1 S_\lambda(\cdot)d\sigma+\int_0^1 S_\theta(\cdot)d\sigma & \frac{1}{2\epsilon}(1-\mu)\int_0^1 S_\lambda(\cdot)d\sigma\\ \frac{1}{2\epsilon}\int_\mu^1 W_\sigma(\cdot)d\lambda & -\frac{1}{2\epsilon}\int_\mu^1 W_\sigma(\cdot)d\lambda+\int_\mu^1 W_\theta(\cdot)d\lambda\end{bmatrix}$$

and its determinant is equal to

$$|\mathbf{J}| = \int_0^1 S_\theta(\cdot)d\sigma \int_\mu^1 W_\theta(\cdot)d\lambda - \frac{1}{2\epsilon}\left((1-\mu)\int_0^1 S_\lambda(\cdot)d\sigma \int_\mu^1 W_\theta(\cdot)d\lambda + \int_\mu^1 W_\sigma(\cdot)d\lambda \int_0^1 S_\theta(\cdot)d\sigma\right) > 0$$

where the inequality follows from the fact that $S_\theta(\cdot) < 0$, $W_\theta(\cdot) < 0$, $S_\lambda(\cdot) > 0$ and $W_\sigma(\cdot) > 0$. Application of the implicit function theorem implies that the derivative of the system of equations (A5)-(A6) with respect to κ satisfies

$$\mathbf{J}\begin{bmatrix}\frac{d\theta^{**}_{F,\epsilon}}{d\kappa} \\ \frac{d\theta^{**}_{I,\epsilon}}{d\kappa}\end{bmatrix} = \begin{bmatrix}-\frac{\partial A}{\partial\kappa} \\ -\frac{\partial B}{\partial\kappa}\end{bmatrix}$$

where $\frac{\partial A}{\partial\kappa} = \int_0^1 S_\kappa(\cdot)d\sigma < 0$ by the definition of $S(\sigma,\lambda,\theta)$ and $\frac{\partial B}{\partial\kappa} = \int_\mu^1 W_\kappa(\cdot)d\lambda > 0$ by the definition of $D_2(\lambda,\sigma,\theta)$. By Cramer's rule, we therefore have that

$$\frac{d\theta^{**}_{F,\epsilon}}{d\kappa} = \frac{1}{|\mathbf{J}|}\begin{vmatrix}-\int_0^1 S_\kappa(\cdot)d\sigma & \frac{1}{2\epsilon}(1-\mu)\int_0^1 S_\lambda(\cdot)d\sigma \\ -\int_\mu^1 W_\kappa(\cdot)d\lambda & -\frac{1}{2\epsilon}\int_\mu^1 W_\sigma(\cdot)d\lambda + \int_\mu^1 W_\theta(\cdot)d\lambda\end{vmatrix} \gtrless 0$$

Similarly, we have that

$$\frac{d\theta^{**}_{I,\epsilon}}{d\kappa} = \frac{1}{|\mathbf{J}|}\begin{vmatrix}-\frac{1}{2\epsilon}(1-\mu)\int_0^1 S_\lambda(\cdot)d\sigma + \int_0^1 S_\theta(\cdot)d\sigma & -\int_0^1 S_\kappa(\cdot)d\sigma \\ \frac{1}{2\epsilon}\int_\mu^1 W_\sigma(\cdot)d\lambda & -\int_\mu^1 W_\kappa(\cdot)d\lambda\end{vmatrix} \gtrless 0$$

2. Asset Purchase Programs. Given an asset price guarantee $q(\theta) > \theta X(\theta)$, funds' surplus function (4) implies that funds' equilibrium threshold in this case solves

$$A^{\mathcal{AP}}(\theta^{**}_{F,\epsilon},\theta^{**}_{I,\epsilon}) = \int_0^1 \pi\left(\frac{X(\theta(\theta^{**}_{F,\epsilon},\sigma)) + Q}{\max\{q(\theta(\theta^{**}_{F,\epsilon},\sigma)), p(\sigma,\theta(\theta^{**}_{F,\epsilon},\sigma)\}} - \kappa\right)\alpha\lambda(\theta^{**}_{F,\epsilon},\theta^{**}_{I,\epsilon})d\sigma - \psi = 0$$

Similarly, investors' surplus function (8) in this case solves

$$B^{\mathcal{AP}}(\theta^{**}_{F,\epsilon},\theta^{**}_{I,\epsilon}) = F - \int_\mu^1 D_2(\lambda,\sigma(\theta^{**}_{F,\epsilon},\theta^{**}_{I,\epsilon}),\theta(\theta^{**}_{I,\epsilon},\lambda))d\lambda = 0$$

Taking the limit as $\epsilon \to 0$, we obtain

$$\lim_{\epsilon\to 0}\theta^{**}_{F,\epsilon} = \begin{cases}\underline{\theta}^q_F(\mu) & \text{if } \overline{\theta}^q_I(1) < \underline{\theta}^q_F(\mu) \\ \theta^q_{I,0} \in [\underline{\theta}^q_F(\mu), \overline{\theta}^q_I(1)] & \text{if } \overline{\theta}^q_I(1) \geq \underline{\theta}^q_F(\mu)\end{cases}$$

where

$$\underline{\theta}^q_F(\mu): \quad \int_0^1 \pi\left(\frac{X(\underline{\theta}^q_F(\mu)) + Q}{\max\{q(\underline{\theta}^q_F(\mu)), p(\sigma,\underline{\theta}^q_F(\mu))\}} - \kappa\right)\alpha\mu F d\sigma = \psi$$

and

$$\overline{\theta}^q_I(1): \quad \int_\mu^1 \frac{1}{1-\alpha\lambda}\left((F+Q)\left(1 - \frac{\alpha\lambda F}{\max\{q(\theta^q_I(1)), p(1,\overline{\theta}^q_I)\}}\right) + S^q(1,\lambda,\overline{\theta}^q_I(1))\right)d\lambda = F$$

and $\lim_{\epsilon \to 0} \theta^{**}_{I,\epsilon} = \overline{\theta}^q_I(1)$ or $\lim_{\epsilon \to 0} \theta^{**}_{I,\epsilon} = \theta^q_{I,0} \in [\underline{\theta}^q_F(\mu), \overline{\theta}^q_I(1)]$ depending on whether $\overline{\theta}^q_I(1) \lesseqgtr \underline{\theta}^q_F(\mu)$.

Notice that $\underline{\theta}^q_F(\mu) < \theta^*_{F,0}(\mu)$ since $q(\theta) > \theta X(\theta)$. It follows that asset price guarantees strictly decrease market liquidity risk in both *weak* and *strong dependence* regimes. By putting a lower bound on asset prices, the government also props up funds' residual equity value and thereby strictly decreases funding liquidity risk: i.e. $\overline{\theta}^q_I(1) < \theta^*_{I,0}(1)$. Also, notice that funds still acquire information for values of $\theta < \max\{\underline{\theta}^q_F(\mu), \theta^q_{I,0}\}$, implying that the government will be forced to purchase bad assets at an inflated price in those states. Given some price floor $q > \theta X(\theta)$, the expected cost of asset price guarantees equals

$$\int_{\underline{\theta}}^{\overline{\theta}} \alpha\lambda(\min\{\overline{\theta}^q_I(1), \theta^q_{I,0}\}, \theta) F(1 - \pi\sigma(\max\{\underline{\theta}^q_F(\mu), \theta^q_{I,0}\}, \theta)) \max\left\{1 - \frac{p(\sigma(\max\{\underline{\theta}^q_F(\mu), \theta^q_{I,0}\}, \theta))}{q(\theta)}, 0\right\} d\theta > 0$$

which simplifies to

$$C^{\mathcal{AP}} = \int_{\underline{\theta}}^{\max\{\underline{\theta}^q_F(\mu), \theta^q_{I,0}\}} \alpha \left(\mu + (1-\mu)\mathbb{1}_{\theta < \theta^q_{I,0}}\right) F(1-\pi) \left(1 - \frac{\theta X(\theta)}{q(\theta)}\right) d\theta > 0$$

3. Outright Debt Purchases. We consider outright debt purchases that reduce the fraction of redeemable claims, α. Differentiating the system of equations (A5)-(A6) with respect to α, we obtain

$$\frac{\partial A}{\partial \alpha} = \int_0^1 S_\alpha(\sigma, \cdot) d\sigma > 0 \quad \text{and} \quad \frac{\partial B}{\partial \alpha} = \int_\mu^1 W_\alpha(\lambda; \cdot) d\lambda > 0$$

By the implicit function theorem, we then have

$$\frac{d\theta^{**}_{F,\epsilon}}{d\alpha} > 0 \quad \text{and} \quad \frac{d\theta^{**}_{I,\epsilon}}{d\alpha} > 0$$

so that market liquidity and funding liquidity risk are both increasing in the fraction of redeemable claims. For $\alpha = 0$, the equilibrium thresholds that solve (A5)-(A6) simplify to $\theta^{**}_{F,\epsilon} = \theta^{**}_{I,\epsilon} = \underline{\theta}$. □

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