MINIMIZING ERRORS, MAXIMIZING INCENTIVES: OPTIMAL COURT DECISIONS AND THE QUALITY OF EVIDENCE*

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ABSTRACT

We characterize the best mechanism for a Court to impose liability (and generate incentives) in a setting in which the injurer’s behavior is imperfectly observed and Courts also care about judicial errors. First, we show that the optimal decision rule is an evidentiary standard. Then, we make three main contributions. i) We develop a new methodological approach to deal with this classic problem: rewrite the incentive compatibility constraint in terms of Court errors. This approach can be applied to more general incentives problems, and greatly simplifies the characterization of the optimal standard. ii) We state that the harshness of the optimal evidentiary standards decreases as the quality (informativeness) of the evidence increases. iii) When the informativeness of the evidence is determined by the injurer’s choice of the care technology, the interests of Court and injurer are not aligned. The optimal Court policy is to penalize (even forbid) the use of the less informative care technology.

KEYWORDS: Incentives, evidentiary standards, judicial errors, statistical discrimination and informativeness.

JEL classification numbers: C44, D82, K13, K40

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1 Introduction

In this paper we analyze the design of legal rules and procedures for Courts (juries, agencies and other regulatory bodies) in the regulation of risky activities that may produce negative effects such as environmental pollution, bodily harm to individuals, or financial harm to firms. Intuitively, Courts want to give potential injurers appropriate incentives to exert care. These incentives are typically provided through sanctions and penalties imposed upon deviant behavior. One of the major obstacles for the adequate functioning of such schemes is that, in practice, Courts do not have direct observation of the actual level of care taken by injurers. Instead, Courts have to rely on inferences from the evidence brought before them (witness reports, physical traces of behavior, expert evaluation about the course of events, and so on). We focus on characterizing the best mechanism for the Court to impose liability (and generate incentives) given its limited information on the injurer’s care provided by existing evidence. Although we concentrate on accidents and liability, our analysis has implications for several other areas of law, and more general incentive problems.

We characterize the Court’s optimal decision rule in a principal-agent framework, where the Court acts as principal to the injurer-agent. The injurer chooses the level of care which determines the probability of accident. In case of accident, the Court observes an imperfect signal of the injurer’s actual level of care. Based on this evidence, the Court decides the injurer’s liability.

In such a setting with imperfect information, the Court is concerned not only with the injurer’s incentives to take care, but also by judicial errors. Throughout the paper we take the view, widely shared by legal scholars, that for the Court the costs of convicting the innocent (Type I error) exceed the benefits of acquitting the guilty (Type II error). Under these Court preferences, we show that when there is a monotone relationship between care and evidence, the optimal liability mechanism is an evidentiary standard. That is, in case of accident, if the evidence of care before the Court falls below the standard or threshold, the Court finds the injurer liable; otherwise the
injurer is acquitted. Then, the characterization of the optimal liability mechanism boils down to identifying the optimal evidentiary standard.

In this setting in which Court preferences depend on injurer’s care and on its own decision errors (as in Demougin and Fluet, 2005), we make three main contributions in addition to the characterization of the optimal evidentiary standard in our setting. First, we provide a methodology to deal with general incentive problems in which both performance and decision errors are important. These incentive problems appear in the legal setting and also in others such as optimal promotions, or project selection. Our approach is based on expressing the incentive compatibility constraint in terms of Type I and Type II errors. By doing so, we show that providing incentives is equivalent to keeping a weighted sum of errors below a threshold. In this way we greatly simplify the incentive problem and are able to characterize the optimal evidentiary standard in a straightforward way. Our method can be applied to other settings. For example, in Section 6 we identify a mild condition under which a pay-for-performance principal agent problem can be rewritten in terms of errors and solved using our approach.

Second, the approach of writing incentive problems in terms of decision errors not only reduces complexity but also opens up the possibility of using tools developed for minimizing errors in Bayesian decision problems. This is particularly useful to us as we study how the optimal evidentiary standard depends on the quality of evidence. As evidence is a noisy signal of the injurer’s true care, we analyze how the standard solution of our problem depends on the amount of information (noise) in the evidence. Rewriting the incentive problem in terms of errors identifies a natural candidate to measure information, namely Lehmann (1988)’s criterion of informativeness. This is because Lehmann (1988)’s criterion is strongly connected with the problem of statistical hypothesis testing. This criterion allows us to show generally, without recourse to a parametric family of signals, that the Court’s optimal standard is decreasing in the informativeness of the evidence: more informative evidence leads to less harsh standards. This result is our second main contribution, as it can be applied in several settings with important policy implications. In Sec-
tion 6 we explicitly develop one of the most prominent of these applications, that of statistical discrimination.

Finally, our third main contribution relates to the core of our Court’s problem. A natural extension of our setting is to consider that the quality of the evidence is to some extent endogenous. There are several ways in which an injurer can affect the informativeness of the evidence before the Court. For example, once the accident has occurred, the injurer chooses legal representation. This decision affects the way in which evidence is presented and consequently the quality of the information received by the Court. We discuss these issues in the paper, but our exposition mainly focuses on a different channel for endogenizing the quality of evidence: the choice of care technology by the injurer. For instance, to increase security in a building we could hire security personnel or, alternatively, increase the number of cameras or the quality of the alarm system. These alternative technologies are likely to differ both in their effectiveness but also in how easy it is to show that the right care has been taken (the quality of evidence).

We focus on how technologies differ in terms of how informative they are about care having been taken. If choice exists in this setting, are the interests of the injurer aligned with the Court’s regarding the choice of technology? How does the injurer’s choice of technology affect Court’s optimal decision rule? First, we show that, as expected, the Court always prefers that the injurers choose the more informative technology. More importantly, we find that injurer and Court interests are not aligned: if the Court were to set an equally harsh standard on both technologies, injurers prefer the less informative one. The Court’s optimal reaction to this misalignment is extreme: it applies the optimal standard on the more informative technology and effectively bans the use of the less informative one. In other words, there are no gains from allowing injurers access to a less informative technology. This result is not at all intuitive, since the Court is in effect giving up a technology even if it could modulate its use with a second evidentiary standard. This result has important implications for other policy problems. For example, there are situations in which an injurer is allowed to choose whether to present his case before a general Court or before
a specialized one. The specialized Court has more experience with a certain type of cases which makes it better able to interpret evidence in those cases. This is equivalent to having access to better quality of evidence. Our result says that if there is a clear match between the case and the specialized Court, the legal system should effectively force injurers to present their cases before the specialized Court.

Finally, we also show that if, for reasons of efficiency, it is not feasible to forbid the use of the less informative technology, then it is optimal for the Court to distort the evidentiary standards, making more (less) attractive the more (less) informative technology.

Our work is related to several strands of the literature. We think that the most faithful way to convey the details of those relationships is to discuss them along the way when we present our ideas and the issues that show our links to the earlier literature. A brief sketch is, however, pertinent at this point. Our intention to characterize optimal mechanisms for inducing desirable behavior when Courts have imperfect information about the actions taken\(^1\) is related to a large body of literature\(^2\), starting with the seminal paper of Johnston (1987). Several papers look at the issue from the point of view of deterrence: Fluet (1999) and Lando (2002) analyze the conditions for the evidentiary standard known as "preponderance of the evidence" (the injurer is found guilty if, given all the evidence, the probability of being guilty is higher than the probability of being innocent) to achieve optimal deterrence in an accident setting; Demougin and Fluet (2008) extend the analysis of the deterrence effects of evidentiary standards, and of the active and passive role of the Court, to a setting where parties may manipulate the evidence; Fluet (2010) analyzes in a setting of randomness in the observation of care the deterrence properties of one-sided (such as simple negligence) and two-sided (such as contributory or comparative negligence) liability rules, as well as the issue of uniform vs. case specific standards of proof. We are interested in setting

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\(^1\)Although we mostly present our ideas in the context of accidents and liability, our findings have direct implications for Criminal Law and its enforcement, where imperfect observation by Courts and the possibility of legal error has also been extensively analyzed. See Png (1986), Miceli (1991), Lando (2000), Rizzolli and Saraceno (2010), and Polinsky and Shavell (2008) for a survey.

\(^2\)There is also a body of literature that focuses on optimal Court decisions concerning which party should provide what at trial: Sobel (1985), Hay and Spier (1997), and Gomez (2002).
optimal decision criteria for Courts in order to maximize social preferences that include providing incentives for care and reducing errors in assigning liability. Along this line, the closest papers to ours are Demougin and Fluet (2005, 2006), which thoroughly analyze how the most widely used evidentiary standards both in Common Law and in Civil Law countries, as well as rules excluding evidence from Court’s consideration, affect incentives for care and decision errors. In terms of modelling (specially, the version of the model with continuous effort developed in our application on statistical discrimination), our paper is related to Taylor and Yildirim (2010), which shares with us two important features of our design problem: the role of decision errors in the principal’s objective function, and the impact of signal accuracy on effort choice. Their focus, however, is very different, since they assume a setting with agents with different productivities, and they are interested in whether or not it is optimal for a principal who observes the agent’s type to make use of the information in an evaluation process.

Finally, our analysis of different care technologies relates to Grady (1988, 1994) which identify the dichotomy between durable and non-durable precautions in the functioning of legal liability, and show how Courts, in practice, tend to be harsher in the requirements for care when dealing with non-durable precautions. Depoorter and De Mot (2008) interpret this distinction in terms of memory costs. We offer a different explanation of this differential treatment, based on the diverse informativeness of available care technologies, and the optimality of imposing harsher evidentiary standards on precaution technologies that are less informative to the decision maker.

The paper is structured as follows: Section 2 presents the basic Court’s problem. In Section 3 we establish the optimality of a threshold rule and characterize the optimal standard. In Section 4 we look at the informativeness of evidence and establish the key result: more informative evidence leads to less harsh standards. Then, in Section 5, we extend the basic model to incorporate hetero-

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3 In order to clarify the point, think that some types of precautionary measures extend their risk-reducing effects in a permanent way or at least in a way that extends over time: When an emission producing firm installs a pollutant arrester device, the precautionary measure reduces the likelihood and/or the amount of dangerous pollutants released for the number of years of the working life of the applied technology. Other precautions, however, have only short-lived risk reducing effects: When an employee of the emitting firm checks the functioning of the pollutant arrester, it only controls performance and reduces the risk of external pollution for the shorter period before the next required check-up arrives.
geneity on the type of agent (injurer). We find that more informative evidence allows for greater welfare from less harsh standards. Furthermore, we identify the conflict of interests between Court and injurer and the distortions it generates on the optimal standard, in a setting with endogenous information, where the injurer can select which care technology to use. Finally, we apply our error-based approach to two other problems in Section 6: (i) the general principal-agent problem of setting an employee’s remuneration to encourage effort, and (ii) statistical discrimination in the presence of incentive problems. This is followed by a brief conclusion.

2 The Model

2.1 Accident setting

An agent, the injurer, is engaged in a risky activity that can give rise to accidents. The agent has access to a precaution technology, \( \delta \), and can use this technology with varying degrees of care, \( e \in \{e_L, e_H\} \). The cost of each care level is given by \( c(e) \) where \( c_H(e_H) > c_L(e_L) \). Accidents occur with probability \( p_\delta(e) \) with \( p_\delta(e_L) > p_\delta(e_H) \). The magnitude of the loss is constant and denoted by \( D > 0 \). For the first part of our analysis the precaution technology is fixed so that it will be convenient to drop the \( \delta \) subscripts to reduce notational clutter. Finally, to simplify the presentation and without loss of generality we let \( c(e_H) = c, c(e_L) = 0, p(e_H) = p \in [0, 1] \) and \( p(e_L) = 1 \).

2.2 Evidence of care

In case of accident, the principal, the Court, decides whether or not the injurer is liable and hence has to compensate the victim for the loss from the accident. The Court has no direct observation of the injurer’s care and its decision is based on the evidence brought before it in the form of interviews, reports, documents, etc. This evidence is represented by a signal \( \pi \in [0, 1] \),
an index of the amount of evidence indicating that the agent has taken high care. Formally, 
\(\pi\), is a realization of a random variable \(\Pi\) with distribution function \(f(\pi|e_j)\). This distribution 
depends on the level of care taken by the agent, \(e_j = e_H\) or \(e_L\). For convenience we assume 
that \(f\) is differentiable and non-zero on \([0, 1]\). Let \(F(\pi|e_j)\) denote the corresponding cumulative 
distribution function.

A higher \(\pi\) represents greater evidence that the agent took high care. To ensure that taking 
high care translates into more evidence that the agent took high care, we assume that the signal 
is monotone, that is, \(f(\pi|e)\) satisfies the Monotone Likelihood Ratio Property (MLRP):

\[
\frac{f(\pi|e_H)}{f(\pi|e_L)} \text{ is increasing in } \pi.
\]

This condition ensures that more evidence is “good news” about care (Milgrom (1981)), that 
is, \(\Pr(e_H|\pi)\) is increasing in \(\pi\).

### 2.3 The Court’s decision problem

The Court wants to provide incentives to exert high care (otherwise the problem is trivial). 
We also assume that the Court is concerned with penalizing the innocent. This is a natural 
assumption in this initial version of the model, since convicting an innocent (Type I error) is the 
only error that can arise in equilibrium. Later on, when we generalize our setup, Type II error 
(acquitting the guilty) will also arise in equilibrium.\(^4\)

The Court can commit to a decision rule that is based on the evidence presented after the 
accident. The Court decision rule is represented by the function \(\tau(\pi)\): the probability that the 
Court will rule that the injurer is guilty, and hence has to pay damages equal to \(D\), when faced 
with evidence \(\pi\).\(^5\) For any Court decision rule, \(\tau(\pi)\), the injurer will chose high effort if the cost

\(^4\)In the general setup, we will assume that the social costs of convicting the innocent exceed the benefits of 
convicting one more guilty individual, or in other words, that Type I errors are more serious than Type II errors—a 
criterion shared by most legal scholars, see for example Posner (1999), Miceli, (1991) and Lando (2009).

\(^5\)A Court could implement a more general decision rule by, conditional on the amount of evidence \(\pi\), choosing the 
compensation the injurer has to pay (as a fraction of the maximum liability, that is, full damages) \(\eta D\), \(\eta \in [0, 1]\), as 
well as the probability that compensation is imposed, \(\gamma \in [0, 1]\). Then, \(\tau(\pi)D = \eta(\pi)\gamma(\pi)D\) represents the expected 
compensation to be paid and all results continue to hold.
of high effort plus the corresponding expected liability are lower than the expected liability of low
effort, that is if
\[ p \int \tau(\pi) D F_{\delta}(d\pi|e_H) + c \leq \int \tau(\pi) D F_{\delta}(d\pi|e_L). \quad (IC) \]

From all the rules that satisfy (IC), the Court will choose the one that minimizes Type I error, which is equivalent to minimizing the expected liability of agents taking high effort (the innocent). Therefore the Court’s problem will be:
\[
\min_{\tau(\pi)} p \int \tau(\pi) D F_{\delta}(d\pi|e_H) \quad \text{subject to} \quad (IC).
\]

2.4 Timing

The timing of the model is described as follows: 1) The Court announces its decision rule, \( \tau(\pi) \). 2) The agent chooses his level of care. 3) Nature determines whether an accident happens or not and, the level of evidence \( \pi \), according to the probabilities and information structures described above. 4) Finally, in case of accident, the agent is forced to compensate the victim according to the realized evidence and the Court’s decision rule. In the next section, we analyze the Court’s optimal decision rule.

3 The Court’s Optimal Decision Rule

3.1 Evidentiary standards

Given that we are analyzing a monotone informational setting, it is natural to consider decision rules that are characterized by an evidentiary standard, \( \bar{\pi} \), such that if \( \pi \leq \bar{\pi} \) the Court finds the injurer guilty and makes him pay full damages, while if \( \pi \geq \bar{\pi} \) the Court finds the injurer not guilty and the injurer does not have to pay anything. We refer to these as threshold based rules.\(^6\)

\(^6\)Such rules can be implemented by Courts as negligence rules with appropriate due care and evidentiary standards. Demougin and Fluet (2008) provide a discussion of how such rules could be delegated to a court.
The next proposition shows that such decision rules are optimal.

**Proposition 1** For any decision rule $\tau(\pi)$ that satisfies (IC), there exists a threshold based rule with an evidentiary standard, $\bar{\pi}_\tau$, such that (IC) is satisfied and generates less Type I error:

$$p \int \tau(\pi) \; F(d\pi|e_H) \geq pF(\bar{\pi}_\tau|e_H).$$

That is, for any decision rule that induces high effort there is a threshold rule with a corresponding evidentiary standard that induces high effort and lowers the expected liability cost of agents taking high effort. The sketch of the proof is as follows: start with a rule $\tau(\pi)$ that satisfies (IC). Then, look for the evidentiary standard, $\bar{\pi}_\tau$, that generates the same expected cost for those exerting low effort than under $\tau(\pi)$:

$$\int \tau(\pi) D F(d\pi|e_L) = F(\bar{\pi}_\tau|e_L) D.$$

Then show that a threshold rule based on a standard $\bar{\pi}_\tau$ generates lower expected liability costs for those exerting high effort than under $\tau(\pi)$. This is true because with monotone signals (MLRP), low signals are always more likely to come from low effort, and a threshold rule that concentrates punishment on low signal realizations, is less likely to be punishing those taking high effort. As those exerting low effort are indifferent between the two rules, and those exerting high effort prefer $\bar{\pi}_\tau$, then the threshold rule necessarily satisfies (IC).

Proposition 1 implies that we can concentrate on threshold based rules without loss of generality.\(^7\) This greatly simplifies our analysis, since our mechanism design problem reduces to characterizing the Court’s optimal evidentiary standard, $\bar{\pi}$. The Court’s problem now becomes

$$\min_{\bar{\pi}} pF(\bar{\pi}|e_H) D$$

s.t. $$pF(\bar{\pi}|e_H)D + c \leq F(\bar{\pi}|e_L) D \quad \text{(IC)}$$

\(^7\)The use of threshold rules is the standard approach followed by the law and economics literature for the analysis of negligence under evidentiary uncertainty, see Johnston (1987), Demougin and Fluet (2008).
3.2 Minimizing errors, maximizing incentives

A Court that imperfectly observes the injurer’s actions and uses an evidentiary standard $\tilde{\pi}$ makes Type I errors, which occur with probability $T_I(\tilde{\pi}) = F(\tilde{\pi}|e_H)$; and Type II errors, which occur with probability $T_{II}(\tilde{\pi}) = 1 - F(\tilde{\pi}|e_L)$. This illustrated in the next Figure

[Figure 1 around here]

This figure (and all subsequent ones) is drawn using signals with the following linear information structure which satisfies MLRP:

$$
\begin{align*}
  f_{\delta}(\pi|e_H) &= 1 - \frac{\delta}{2} + \delta \pi, \quad F_{\delta}(\pi|e_H) = \pi - \frac{1}{2} \delta \pi (1 - \pi), \\
  f_{\delta}(\pi|e_L) &= 1 + \frac{\delta}{2} - \delta \pi, \quad F_{\delta}(\pi|e_L) = \pi + \frac{1}{2} \delta \pi (1 - \pi),
\end{align*}
$$

where the parameter $\delta \in [0,2]$ is an index that will be useful later (Figure 1 is drawn using $\delta = 1.75$). As we will see in Section 4, a higher $\delta$ implies that evidence is more informative (a closer relationship between care and observed evidence).

The Court’s problem can be rewritten in terms of Type I and II errors. The rewriting of the objective function is immediate. The incentive compatibility constraint is rewritten as follows:

$$
\begin{align*}
  pF((\tilde{\pi})|e_H)D + c &\leq F((\tilde{\pi})|e_L)D \\
  pF((\tilde{\pi})|e_H)D - F((\tilde{\pi})|e_L)D &\leq -c \\
  pF((\tilde{\pi})|e_H) + 1 - F((\tilde{\pi})|e_L) &\leq 1 - \frac{c}{D} \\
  pT_I(\tilde{\pi}) + T_{II}(\tilde{\pi}) &\leq 1 - \frac{c}{D}.
\end{align*}
$$

Then, the Court’s problem, on Equation (1), is equivalent to the following, more convenient, error minimization problem:

$$
\min_{\tilde{\pi}} T_I(\tilde{\pi})
\quad \text{s.t.} \quad pT_I(\tilde{\pi}) + T_{II}(\tilde{\pi}) \leq 1 - \frac{c}{D} \quad (2)
$$
3.3 The weighted error function.

The relationship between the Court’s evidentiary standard and the agent’s incentive to exert high effort crucially depends on the properties of the weighted error function:\(^8\)

\[
\Phi(\pi) = T_{II} + pT_{I} = 1 - F(\pi |e_L) + pF(\pi |e_H).
\]

**Lemma 1** The weighted error function is positive, continuous and convex, and has a unique minimum on the interval \([0, 1]\) at \(\pi_{\text{min}}\). The function takes values \(\Phi(0) = 1\) and \(\Phi(1) = p\).

The parameter \(p\) determines the amount of error when one applies the strictest standard \((\pi = 1)\). We also refer to this standard as the strict liability regime, as it imposes full liability on the injurer whenever there is an accident. The parameter \(p\) captures the difference in the probability of accident occurrence between the high and low care levels, and hence it may be interpreted as the informativeness of the accident on the level of care. The standard that minimizes the error function, \(\pi_{\text{min}}\), depends on \(p\):\(^9\) Let \(p_{\text{min}} = \frac{f(1|e_L)}{f(1|e_H)}\).

**Corollary 1** The standard that minimizes \(\Phi\), \(\pi_{\text{min}}\), is decreasing with \(p\). For \(p \leq p_{\text{min}}\), \(\pi_{\text{min}} = 1\).

For settings where \(p \leq p_{\text{min}}\) the sole occurrence of the accident is highly informative about the level of care (\(\text{res ipsa loquitur}\)) and increasing the standard leads to greater incentives for high care. On the contrary, if \(p > p_{\text{min}}\) the effect of increasing the standard on effort is not so straightforward. Given a high standard of evidence \((\pi \geq \pi_{\text{min}})\) increasing the standard implies an increase in the total amount of error which reduces the incentives for effort.

As we will see below, we will be mainly interested in the set of standards below \(\pi_{\text{min}}\). Let \(\Phi_D\) be the error function defined on this set, \(D = [0, \pi_{\text{min}}]\), so that \(\Phi_D\) is a decreasing function (and a higher standard increases the incentives to take care).

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\(^8\) The weighted error function corresponds to the deterrence curve in Demougin and Fluet (2005).

\(^9\) This corresponds to the preponderance of evidence standard that is the rule in civil litigation in Common Law jurisdictions. Demougin and Fluet (2005) shows that this standard maximizes deterrence but does not minimize errors. Hence, it does not necessarily maximize social welfare when Court errors are important, as we find in the current paper.
Figure 2 illustrates the shape of the $\Phi$ function for $\delta = 1.75$ (and $p = 0.75$) as well as $\pi_{\min}$, the interval $\mathbb{D}$, and the function $\Phi_{\mathbb{D}}$.

[Figure 2 about here]

3.4 The optimal evidentiary standard

Having analyzed the weighted error function, we can now solve the Court’s problem in Equation (2):

$$\begin{align*}
\min_{\pi} p^{\mathbb{P}_I} \\
\text{s.t. } \Phi(\pi) \leq 1 - \frac{c}{\mathbb{D}},
\end{align*}$$

(3)

and characterize the corresponding optimal standard:

**Proposition 2** There exists a cost level, $c_{\max} = (1 - \Phi(\pi_{\min}))\mathbb{D}$, such that if $c \leq c_{\max}$ then the optimal standard is $\pi^*(c) = \Phi_{\mathbb{D}}^{-1} \left(1 - \frac{c}{\mathbb{D}}\right)$ which is increasing in $c$. If $c > c_{\max}$ high care cannot be induced.

The intuition of this proposition is as follows: For a given cost, there is a set of standards that generates enough incentives to induce high care. As Type I error is monotonically increasing in the evidentiary standard, the Court chooses the minimum of these. If the cost of exerting care increases, it becomes more difficult to induce high care, and the Court has to increase the optimal standard.

Figure 3 illustrates Proposition 2 by characterizing the optimal evidentiary standard when $\delta = 1.75$, $p = 0.75$, and $\mathbb{D} = 0.4$. In Figure 3 we can observe the set of standards that induce high care, $\mathbb{H}(c)$, and the optimal standard, $\pi^*(c)$—the lowest in this set. A higher $c$ (corresponding to the lower green horizontal line at $\frac{c}{\mathbb{D}} = 0.434$) implies a higher optimal evidentiary standard, $\pi^{**}$. [Figure 3 around here]
3.5 Implications: Optimal standards, efficiency and strict liability

We have seen that there is a maximum cost, $c_{\text{max}}$, that determines when it is possible to induce high care. It is interesting to consider whether or not the Court can induce high care when it is efficient to do so, that is, if the costs of exerting care are lower or equal than the expected benefits from reducing the probability of accident. Let $c_{EH}$ be the maximum cost for which it is efficient to induce high care, that is $c_{EH} = (1 - p)D$.

**Lemma 2** If it is efficient to exert high care ($c \leq c_{EH}$), then it is possible to induce high care ($c \leq c_{\text{max}}$), that is $c_{EH} \leq c_{\text{max}}$.

To prove this lemma consider the incentives to take care with the strictest standard, $\bar{\pi} = 1$. This standard implies $\Phi(1) = p$. Replacing $\Phi(1) = p$ in the incentive compatibility constraint we obtain $p \leq 1 - \frac{\delta}{D}$, which is precisely the condition for the cost of high care to be socially optimal.

As the strictest standard corresponds to strict liability, we obtain a known result in the Law and Economics literature: strict liability (the injurer is always liable in case of accident) induces the efficient amount of care since the injurer internalizes all the costs. In other words, as it is always possible to use strict liability, the Court is sure to be able to induce a high level of care when it is efficient to do so.

However, the highest evidentiary standard $\bar{\pi} = 1$ (strict liability) is in general not optimal.

**Corollary 2** The highest evidentiary standard $\bar{\pi} = 1$ is optimal if and only if

- $c = c_{EH}$, that is $c$ is equal to the maximum cost for which it is efficient to induce high effort;
- and
- $p \leq p_{\text{min}}$, the accident is very informative about the level of care.

In the previous discussion, we saw that for $c > c_{EH}$ strict liability does not provide incentives for high care. If $c \leq c_{EH}$, the optimal standard is increasing in the cost of high care (by Proposition
2). Thus, the highest standard may only be optimal for the highest cost of care in the set, that is when \( c = c_{eH} \). Furthermore, from our discussion in the previous section, the monotonicity of incentives for care depends on the value of \( p \). For strict liability to be optimal, incentives for care must be at their maximum at \( \pi = 1 \), and this can only occur if \( \Phi \) is monotone, that is if \( p \leq p_{\text{min}} \). The economic intuition behind this is that there is a lot of Type I error associated with strict liability. Using such a standard will only be optimal if \( p \) is low enough, that is when high care reduces the probability of an accident to such an extent that there is very little Type I error even under strict liability.\(^{10}\)

4 **Informativeness and the Standard of Negligence**

We now turn to what constitutes the central analysis of the paper, namely the link between informativeness of care technologies and the harshness of standards. So far we have assumed the existence of a single precaution technology. Now, we consider that the agent may exert care using different technologies that generate different types of evidence. In particular, we start by assuming that there are two alternative technologies \( \delta \) and \( \delta' \), that both technologies are equally efficient in terms of care (cost and probability of accident are the same, \( c_\delta = c_{\delta'} = c \), and \( p_\delta = p_{\delta'} = p \)) and they only differ in terms of evidence provision (that is, informativeness of the signal, \( \Pi \)).

4.1 *Care technologies and the quality of evidence*

In general, the evidence produced by two different technologies can differ both in its nature as well as in its informativeness about the care of the agent. The nature of the evidence can be different as, for example, one technology may require evidence in the form of witness reports, while the other may require evidence in the form of scientific studies. In order to compare evidence

\(^{10}\)Our model disregards the cost of presenting evidence before the Court. If this process were costly, strict liability may be attractive on grounds of saving administrative costs if it is possible to rule out the presentation of evidence about the parties' care.
standards from different technologies we will use the notion of harshness defined as follows:

**Definition 1** An evidentiary standard \( \tilde{\pi}_\delta \) is harsher than another \( \tilde{\pi}_{\delta'} \) if \( \tilde{\pi}_\delta \) generates more Type I error than \( \tilde{\pi}_{\delta'} \), that is \( T_I(\tilde{\pi}_\delta) \geq T_I(\tilde{\pi}_{\delta'}) \).

Notice that when comparing standards when using the same technology, a harsher standard is one that requires more evidence of care having been taken in order not to be subject to liability.\(^{11}\) To clarify the different roles of the technology and the standard in determining errors we will use the more explicit notation \( T_I(\tilde{\pi}; \delta) \) as needed.

Care technologies may also differ in terms of the informativeness of the evidence they generate. We have assumed that there is a link between evidence and exerted care (MLRP). This link can be stronger (more informative evidence) for some technologies than for others. For the rest of the paper, we will assume that the parameter \( \delta \) ranks technologies according to their informativeness.

There are several notions used in the literature to rank signals according to their informativeness. We use the notion defined in Lehmann (1988):

**Definition 2** Technology \( \delta \) is more (Lehmann) informative than technology \( \delta' \) if

\[
\forall \pi, \quad F_{\delta}^{-1}(F'_{\delta'}(\pi|E_H)|E_H) \geq F_{\delta}^{-1}(F'_{\delta'}(\pi|E_L)|E_L).
\]

This condition is used to define informativeness of signals in many economic problems as it defines informativeness in terms of the value of information in decision making problems: a signal \( X \) is more informative than another \( Y \) if every decision-maker with preferences in a particular class (single-crossing preferences) prefers \( X \) to \( Y \) (Lehmann 1988, Persico 2000, Jewitt 2007). Thus, a signal is more informative if it allows decision-makers to make better decisions (that is to get more value out of their decisions). It is shown by Jewitt (2007) that Lehmann’s notion of informativeness is equivalent to Blackwell sufficiency in a dichotomous setting which is the one used in this analysis. Ganuza and Penalva (2010) provide alternative criteria of informativeness based

\(^{11}\)The harshness of a standard corresponds to the critical level used in hypothesis testing. Applying a harsher standard on evidence is equivalent to applying a more stringent critical level.
on the dispersion of posterior conditional expectations. The weakest of these criteria, integral precision (based on the convex order) is implied by all previously mentioned informativeness criteria. Ganuza and Penalva (2010) show that integral precision is equivalent to Lehmann in dichotomous settings. Then, as dispersion of conditional expectations is easily verified, we use integral precision to prove that the signals in our parametric example are Lehmann ordered (see Appendix).

Lehmann’s notion of informativeness is particularly appropriate in our analysis as it implies that with more informative signals it is possible to construct more powerful hypothesis tests. Next lemma adapts this result to our framework:

**Lemma 3** Let \( \pi^*_\delta \) and \( \pi^*_\delta' \) be defined by \( T_I(\pi^*_\delta; \delta) = T_I(\pi^*_\delta'; \delta') = \alpha \). If technology \( \delta \) is more informative than technology \( \delta' \), then for all \( \alpha \in [0, 1] \), \( \Phi_\delta(\pi^*_\delta) \leq \Phi_\delta(\pi^*_\delta') \).

### 4.2 Optimal standards and the quality of evidence

Lemma 3 allows us to analyze how a change in the informativeness of the care technology affects social welfare and the Court’s choice of the optimal standard that we characterized in the previous section. The next result compares the solutions of the Court’s problem (Equation (3)) for two alternative technologies ranked in terms of their informativeness.

**Theorem 3** Let \( \pi^*_\delta \) and \( \pi^*_\delta' \) be the optimal standards for technologies \( \delta \) and \( \delta' \). If technology \( \delta \) is more informative than \( \delta' \), then \( \pi^*_\delta \) is less harsh than \( \pi^*_\delta' \).

The theorem implies that the optimal harshness of legal standards depends on the informativeness of the care technologies, so that more informative technologies allow for more lenient legal standards.\(^\text{12}\)

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\(^\text{12}\)This result can be applied to ongoing legal discussions on differences in harshness applied to injurers who are using different precaution technologies. Of the many ways in which precaution technologies may differ two are particularly prominent: one, whether precaution technologies are informal or of an organizational nature (for example, whether there is an informal or an organizational system of preventing sexual harassment in the workplace); Two, whether the precaution technology is durable or not (Grady (2009)). A more informative technology is associated...
Furthermore, the Court’s objective and the injurer’s incentives are aligned:

**Corollary 3** If the Court sets the optimal standard for each technology, the more informative one will minimize the costs of the injurer and maximize social welfare.

As the Court is inducing high care, the welfare of both the Court and the injurer is decreasing in Type I error, which is minimized by the more informative technology (Proposition 3).

**Corollary 4** A more informative technology increases the range of costs for which the Court can induce high care: \( c_{\text{max}} \) is increasing in \( \delta \).

With a given technology, the highest cost level for which the Court can induce care is \( c_{\text{max}} \) which solves \( \Phi(\pi_{\text{min}}) = 1 - \frac{\gamma}{\delta} \). As the informativeness of the technology increases, by Proposition 3, the Type II errors induced by the standard \( \pi_{\text{min}} \) can be lowered while keeping the harshness of the standard constant, thereby making high care incentive compatible for cost levels that are slightly higher than \( c_{\text{max}} \).

### 4.3 Implications: Optimal standards and the choice of legal counsel

We have assumed that the injurer only chooses his level of care. In practice the injurer can continue to act in defence of his own interests even after the accident has taken place, and some of these actions can affect the informativeness of the evidence brought before the Court. In this extension, we study how the Court should adjust the legal standard in response to these actions and find that it should apply a harsher evidentiary standard when the defendant takes actions that reduce the informativeness of the evidence. In other words, the guilty defendant may try to obfuscate the evidence, but we will show that in equilibrium this possibility may make the defendant worse off.

This result helps understand the observation of the increasing introduction of organizational precaution technologies in firms, hospitals, etc., even if they are not more effective in reducing risks.

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13 This result helps understand the observation of the increasing introduction of organizational precaution technologies in firms, hospitals, etc., even if they are not more effective in reducing risks.
To illustrate this we analyse the injurer’s choice of legal counsel. Assume that the more skilled lawyer is able to make the guilty appear to be innocent. Formally, let \( g(\pi | e) \) denote the distribution of the signal (evidence) when the injurer took effort level \( e \). We assume that when the injurer is innocent, the evidence brought before the Court is the same as before \( (g(\pi | e_H) = f(\pi | e_H)) \). On the other hand, if the injurer is guilty (he took low care), the lawyer is able to change the distribution of the signal such that with probability \( 1 - q \) it will be the same as before \( (f(\pi | e_L)) \) but with probability \( q \) it has the same distribution as if he had taken due care \( (f(\pi | e_H)) \), where \( q \) indexes the “quality” or the persuasiveness of the lawyer. That is

\[
g(\pi | e_L; q) = qf(\pi | e_H) + (1 - q) f(\pi | e_L).
\]

This implies that the evidence is less informative, the more “skilled” the lawyer.\(^{14}\)

**Lemma 4** For \( q > q' \), \( g(\pi | e; q) \) is less informative than \( g(\pi | e; q') \) according to Lehmann.

Then, how should the Court react to the injurer’s choice of lawyer, if at all?

**Corollary 5** The optimal standard is harsher the more skilled the lawyer is.

This is a direct application of Theorem 3. In order to obtain specific conclusions from this result on the Court and injurer welfare we would need a full-fledged model that specifies the Court’s capacity to adjust the standard to the lawyer’s skill. On the one hand, if the Court is able to design lawyer-specific standards, the optimal reaction would be for the Court to offer a menu of standards that discourages obfuscation. On the other hand, if the Court is unable to adjust the standards in this dimension, the ability of some injurers to obtain “high quality” legal representation would generate negative externalities on injurers with a more limited choice amongst legal counselors.

\(^{14}\)This is because we are assuming that the lawyer’s skill only affects (and reduces) the quality of the evidence when the injurer is guilty. It is natural, that if the lawyer’s skill also affects (and increases) the quality of the evidence in favor of an innocent injurer, the overall effect of the lawyer’s skill over the quality of the evidence is ambiguous.
5 Optimal Standards with a Heterogenous Population

We now proceed to characterize the optimal standard for a population of heterogenous agents, or equivalently, for a situation in which the Court is uncertain about the injurer’s cost of care. The main difficulty of this generalization is that Type II errors will arise in equilibrium.

We start with a single care technology $\delta$ and assume the cost of care for injurer $i$, $c_i$, belongs to $[0, D]$. Costs of care are distributed according to $G(c)$ with pdf $g(c)$ and support $[0, D]$. Then, generally, for any standard set by the Court there will be some agents who will take high care and others who will take low care.

The choice of care in the population is characterized as follows:

**Lemma 5** Given a standard, $\bar{\pi}$, there is a type of agent with cost of care $\bar{c}$, characterized by $\Phi(\bar{\pi}) = 1 - \frac{\bar{c}}{D}$, such that agents with cost $c_i \geq \bar{c}$ prefer to use low care while those with costs $c_i \leq \bar{c}$ prefer to use high care.

This result is illustrated in Figure 4

[Figure 4 around here]

The standard chosen by the Court ($\bar{\pi} = 0.4$) which in Figure 4 is represented by its Type I error, $T_I(\bar{\pi}) = 0.190$, cuts the weighted error curve $\Phi(\pi)$ at $1 - \frac{\bar{c}}{D} = 0.5325$ and thereby identifies the agent type who is indifferent between the two levels of care, $\bar{c}$. The distribution of cost types, $g(c)$, is displayed on the LHS of the Figure, by the y-axis. Agents with $c \geq \bar{c}$ are represented by the red-shaded area under the pdf, while those with $c \leq \bar{c}$ are represented by the green shaded area under the pdf.

Lemma 5 implies that when setting the evidentiary standard the Court has to take into account three factors: (i) the amount of care in the population, which is captured by the level of care of the agent that is indifferent between high and low care, $\bar{c}(\bar{\pi})$, (ii) the amount of Type I error, $T_I(\bar{\pi})$, (iii) the support of the distribution of costs, $[0, D]$. The optimal standard is then determined by the Court in such a way that minimizes the weighted average of Type I and Type II errors.
and (iii) the amount of Type II error, $T_{II}(\bar{\pi})$. We can now describe the Court’s preferences using a social welfare function $W(\bar{c}, T_I, T_{II})$ that depends on these three variables.

Instead of assuming a particular parameterization of the welfare function we will consider a general class of welfare functions. This class of functions, which we refer to as regular welfare functions, is defined as follows:

DEFINITION 3  A Social Welfare Function $W(\bar{c}, T_I, T_{II})$ is regular if: i) $W(\bar{c}, T_I, T_{II})$ is differentiable, increasing in $\bar{c}$ and decreasing in $T_I$ and $T_{II}$; and, ii) For any $\bar{c}$,

$$\frac{\partial W}{\partial T_I} \leq \frac{\partial W}{\partial T_{II}}.$$

The first property establishes that a regular social welfare function values high care and low Type I and Type II errors (differentiability is assumed for convenience but is not necessary for the analysis).\(^{15}\)

The second property establishes that the relative social importance from a change in $T_I$ is everywhere greater than from a change in $T_{II}$. As we discussed in the introduction, this assumption reflects the general fairness concern with the problem of convicting the innocent in legal discourse and practice ("it is better to let the crime of a guilty person go unpunished than to condemn the innocent").\(^{16}\)

As we have changed the Court’s objective function, we have to reconsider the optimality of threshold based rules. Fortunately, we can establish a result similar to Proposition 1.

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\(^{15}\)We do not explicitly introduce the cost of effort since this would complicate the analysis unnecessarily. First, all our results hold if we constrain the Court maximization problem to the set of “efficient standards”, $\pi \in [0, \pi_{eH}]$ where $\bar{c}(\pi_{eH}) = c_{eH}$. Secondly, if $W$ is decreasing in $\bar{c}$ for $c > c_{eH}$, given that $W$ is decreasing in $T_I$ and that $\frac{\partial W}{\partial T_I} \leq \frac{\partial W}{\partial T_{II}}$ then the optimal standard will lie in the “efficient” interval, $\pi^* \in [0, \pi_{eH}]$.

\(^{16}\)In the economically oriented literature on Law enforcement it is customary [Miceli (1991), Lando (2000)] to give asymmetric weight to Type I and Type II errors, due to fairness concerns in society. Moreover, even without imposing an a priori condition on the social preferences over the ratio of one and the other kind of legal errors, there are several reasons why society will be more concerned about mistaken findings of liability: effects on socially valuable activity levels [Kaplow and Shavell (1994); risk aversion of agents [Rizzolli and Stanca (2009)]; and also intrinsic costs of imposing liability [Rizzolli and Saraceno (2010)], which of course are very substantial in the case of non-monetary sanctions such as imprisonment, but are also relevant in the case of monetary damages and penalties.
Proposition 4 Given a regular social welfare function $W(c, T_I, T_{II})$, and a decision rule $\tau(\pi)$, there is a threshold based rule with an evidentiary standard, $\bar{\pi}_\tau$, that generates the same or greater social welfare according to $W(c, T_I, T_{II})$.

The proof follows the same lines as the one for Proposition 1: identify the threshold rule that generates the same $T_{II}$ and show that it generates lower $T_I$. To conclude the proof it suffices to show that the same $T_{II}$ with a lower $T_I$ implies that the incentives for exercising high effort are greater, and hence $\bar{c}$ is higher.\(^{17}\)

With a single agent we have seen that greater informativeness leads to greater social welfare with a less harsh standard (Theorem 3). A similar result can also be obtained in the general setting.

Theorem 5 Consider two technologies $\delta$ and $\delta'$ such that technology $\delta'$ is less informative than $\delta$ and its corresponding optimal standard is $\bar{\pi}'$. i) With technology $\delta$, a set of standards exists that provide greater welfare for all regular social welfare functions; ii) If the standard $\bar{\pi}$ generates greater welfare for all regular social welfare functions under $\delta$ then $\bar{\pi}$ is less harsh than $\bar{\pi}'$, that is $T_I(\bar{\pi}; \delta) \leq T_I(\bar{\pi}'; \delta')$.

Theorem 5 states that when the informativeness of the technology increases, there is a set of standards for which we can guarantee that welfare increases without further assumptions on social welfare. Moreover, the proposition also states that these “better” standards are less harsh (and increase the amount of care in the population).\(^{18}\) In the proof we characterize the set of “better” standards as an interval, $[\pi_{L\delta}, \pi_{H\delta}]$. Furthermore, the extremes of this interval are $\pi_{L\delta} = \Phi_{\pi_i, \delta}^{-1}
\left(1 - \frac{c'}{T_{II}} \right)$ where $c' = (1 - \Phi_{\pi'}(\bar{\pi}'))D$, and $T_I(\pi_{H\delta}) = T_I(\bar{\pi}')$.

\(^{17}\)As can be deduced from the text, the proof uses condition (i) but it does not rely on property (ii) of regular preferences, so that the result holds for all “monotone” social welfare functions.

\(^{18}\)Nevertheless, this does not imply that given a specific social welfare function the optimal policy will be less harsh. It is possible that for a given social welfare function and informativeness, an increase in informativeness may lead to a harsher optimal standard. This is because the Court may prefer to take advantage of the lower errors from the more informative technology to increase care so much that it may end up increasing the harshness of the standard.
In Figure 5 we depict the effect of an increase in the informativeness of the signal (from $\delta' = 1.25$ to $\delta = 1.75$). Note that the x-axis now identifies Type I error (not standards).

Figure 5 illustrates how greater informativeness leads to less error—$\Phi_\delta$ is below $\Phi_{\delta'}$ (Lemma 3). Also, given an arbitrary optimal standard $\bar{\pi}'$ for technology $\delta'$, Figure 5 also identifies the set of standards that with technology $\delta$ generate more welfare for all regular social welfare functions—the section of $\Phi_\delta$ between points B and C (Proposition 5).

### 5.1 Competing technologies and endogenous quality of evidence

So far we have considered what happens when substituting one technology for another. We now consider what happens when there are two (equally efficient) technologies available at the same time so that the choice of technologies (and hence of the informativeness of evidence) is endogenous.

Agents, when facing a choice between two technologies, $\delta$ and $\delta'$, will select the one that provides lower expected costs of precaution. Let $\bar{\pi}$ be the standard applied to agents who choose technology $\delta$ and $\bar{\pi}'$ to those that use $\delta'$. Then, an agent with cost of care $c$ faces the following incentive compatibility constraint:

$$c + p \min \{ F_\delta (\bar{\pi}|e_H) , F_{\delta'} (\bar{\pi}'|e_H) \} D \leq \min \{ F_\delta (\bar{\pi}|e_L) , F_{\delta'} (\bar{\pi}'|e_L) \} D.$$

Rewriting the incentive compatibility constraint in terms of Type I and II errors we obtain:

$$\max \{ T_I (\bar{\pi}; \delta), T_I (\bar{\pi}'; \delta') \} + p \min \{ T_I (\bar{\pi}; \delta), T_I (\bar{\pi}'; \delta') \} \leq 1 - \frac{c}{D}.$$  

Let $\hat{W}$ denote the social welfare function that depends on the standards $\bar{\pi}'$, and $\bar{\pi}$ applied to agents using technologies $\delta'$ and $\delta$ respectively. Then,

$$\hat{W} (\bar{\pi}', \bar{\pi}) = W (\hat{c} (\bar{\pi}', \bar{\pi}), \max \{ T_{II} (\bar{\pi}; \delta), T_{II} (\bar{\pi}'; \delta') \} , \min \{ T_I (\bar{\pi}; \delta), T_I (\bar{\pi}'; \delta') \}).$$
where \( c' (\pi', \pi) \) is the cost of effort for the agent who is indifferent between exerting low or high care, and \( \min \{ T_I (\pi; \delta), T_I (\pi'; \delta') \} \) and \( \max \{ T_{II} (\pi; \delta), T_{II} (\pi'; \delta') \} \) are the equilibrium Type I and Type II errors given the agent’s incentives.

Therefore, the Court’s problem becomes:

\[
\max_{\pi', \pi} \hat{W} (\pi', \pi) \\
\text{s.t. } \max \{ T_{II} (\pi; \delta), T_{II} (\pi'; \delta') \} + p \min \{ T_I (\pi; \delta), T_I (\pi'; \delta') \} \leq 1 - \frac{c}{D}. \tag{4}
\]

The next result illustrates that the presence of competing technologies exacerbates the conflicts of interest between Court and agents and complicates the Court’s decision:

**Lemma 6** If the Court applies an equally harsh standard on both technologies, it is weakly optimal for all agents to use the less informative technology.

If the Court applies an equally harsh standard on both technologies, agents exerting high effort face the same expected cost of care from using either technology and hence are indifferent between the two. But agents exerting low effort strictly prefer the less informative technology, since this technology generates more Type II error.

What we find is that with two competing technologies, social welfare is greater than when only the less informative one is available, but lower than if only the more informative one was available.

**Lemma 7** Consider two technologies \( \delta' \) and \( \delta \) such that technology \( \delta' \) is less informative than \( \delta \), with corresponding optimal standards \( \pi' \) and \( \pi \) respectively. Then,

\[
W (\pi; \delta) \geq \max_{\pi' \in \hat{W}} \hat{W} (\pi', \pi_c) \geq W (\pi'; \delta') .
\]

i) \( W (\pi; \delta) \geq \max_{\pi' \in \hat{W}} \hat{W} (\pi', \pi_c) \). Two cases, a) \( T_I (\pi; \delta) < \min \{ T_I (\pi; \delta), T_I (\pi'; \delta') \} \)

ii) \( \max_{\pi' \in \hat{W}} \hat{W} (\pi', \pi_c) \geq W (\pi'; \delta') \). If \( \pi_c = \pi' \) and \( \pi_c \) is chosen such that \( T_I (\pi_c; \delta) = T_I (\pi'; \delta') \), then \( \max \{ T_{II} (\pi_c; \delta), T_{II} (\pi_c' = \pi'; \delta') \} = T_{II} (\pi_c' = \pi'; \delta') \) and \( \hat{W} (\pi_c, \pi_c) = W (\pi'; \delta') \).
Therefore, the optimal standards \((\bar{\pi}_c', \bar{\pi}_c)\) \(\in\) \(\arg\max\) \(\hat{W}(\pi_c', \bar{\pi}_c)\) generate the same or greater social welfare than \(W(\pi'; \delta')\).

This result states that the presence of two coexisting technologies does not eliminate but can reduce the social welfare improvements that we found in Proposition 5.

[Figure 6 around here]

Figure 6 (combined with Figure 5) helps illustrate Lemma 7. In Figure 5 we saw that with a more informative technology, the Court could increase welfare by selecting any standard corresponding to the section of \(\Phi_\delta\) between points B and C. We find the same B and C points on Figure 6, but if \(\delta\) does not replace \(\delta'\) and agents can still use \(\delta'\) and be subject to the standard \(\pi'\) (point A), standards on technology \(\delta\) that would have led to points between D and B now lead to points on the line between D and A. For points between C and D nothing changes as both \(T_I\) is lower and \(T_{II}\) is greater than with technology \(\delta'\) (point A). Then, injurers of every type prefer the more informative technology, the relevant error function is \(\Phi_\delta\), and social welfare is greater than with technology \(\delta'\). But, for points between D and B, \(T_{II}\) error is lower with technology \(\delta\) than with \(\delta'\) and some injurers with high costs of care prefer to use the less informative care technology. In this case, the injurer type that is indifferent between the two technologies is depicted by the AD line, and on this line social welfare is lower than if technology \(\delta'\) was not available.

Now, consider the Court’s problem (in Equation 4) of choosing optimal standards with competing technologies.

**Lemma 8** In the presence of competing technologies that differ only in terms of their informativeness, an optimal Court policy is to apply the optimal evidentiary standard for those using the more informative technology, and forbid (or impose strict liability on those using) the less informative one.
We can see this in Figure 6 also. Consider the Court’s optimal reaction to the introduction of a more informative technology and let \( \bar{\pi} \) be the optimal standard for \( \delta \) (when technology \( \delta' \) is not available). By Lemma 7, social welfare is maximized if everyone adopts technology \( \delta \) with standard \( \bar{\pi} \). If \( \bar{\pi} \) is a soft standard (a point on \( \Phi_\delta \) to the left of D) then there are no conflicts of interest between the injurer and the Court, as Type I error is lower and Type II error higher than at \( \bar{\pi}' \) (point A). But, if \( \bar{\pi} \) is too harsh (a point on \( \Phi_\delta \) to the left of D), then, not all agents will switch to technology \( \delta \) unless the Court changes the standard for those using \( \delta' \). This is because \( \bar{\pi} \) generates less Type II error than \( \bar{\pi}' \) does with the less informative technology. To make the less informative technology less attractive for those exerting low effort, it is optimal to distort the standard, \( \bar{\pi}' \), making it harsher (and thereby reducing Type II error) until it is below \( T_{II} (\bar{\pi}; \delta) \).

\[ 5.2 \text{ Differences in efficiency between technologies} \]

Up until now we have considered technologies that differed solely in terms of their informativeness, though two care technologies are rarely equal in terms of the costs of care, reduction of ensuing harm, or reduction in the probability of accident. We now suppose that technology \( \delta \) is more informative than \( \delta' \) but consider the possibility that they also differ in other aspects that affect the efficiency with which they reduce risk.

We have seen that courts have incentives to promote the adoption of the more informative technology. If the more informative is also the more efficient technology then the courts’ incentives to promote its adoption and apply harsher standards on the less informative one are even stronger. If, on the other hand, the less informative technology is more efficient, then the Court faces a trade off between the gains from lower errors versus efficiency losses from using the more informative but less effective technology. Depending on the outcome of this trade off the Court would then decide whether to encourage one technology or the other.

An interesting problem arises when differences in efficiency are individual-specific and the Court cannot determine for each agent what is the efficient technology he should be using. To
capture this additional complexity we make the following modelling assumption: the probability of accident is the same for all agents \( (p_\delta = p_{\delta'} = p) \), their cost of effort under both technologies is drawn from the same distribution, \( G(c) \), and agents are randomly (and independently) classified into one of the following three groups:

1. Type A \([\text{inflexible } \delta\text{-types}]\): agents have to assume a (sufficiently high) additional cost for using \( \delta \) such that they will not use \( \delta \) under any circumstances;

2. Type B \([\text{inflexible } \delta\text{-types}]\): agents have to assume a (sufficiently high) additional cost for using \( \delta \) such that they will not use \( \delta \) under any circumstances; and

3. Type F \([\text{flexible types}]\): agents who find \( \delta \) and \( \delta' \) to be equally efficient.

In this case it may not be socially optimal to ban the use of any one of the two technologies.

Let \( \bar{\pi} [\bar{\pi}'] \) be the optimal standards for technology \( \delta [\delta'] \) when it is the only technology in the economy. Also, let \( \bar{\pi}^* \) and \( \bar{\pi}'^* \) denote the optimal standards when both technologies coexist. There are circumstances in which courts would not distort these optimal standards.

**Lemma 9** If the optimal standard for \( \delta \) is more lenient on both the innocent and the guilty, \( T_I (\bar{\pi}; \delta) \leq T_I (\bar{\pi}'; \delta') \) and \( T_{II} (\bar{\pi}; \delta) \geq T_{II} (\bar{\pi}'; \delta') \), then courts optimally apply the standards \( \bar{\pi}^* = \bar{\pi} \) and \( \bar{\pi}'^* = \bar{\pi}' \), Type F agents choose the more informative technology, and social welfare is maximized.

This case coincides with the one we analyzed after Lemma 7 where \( \bar{\pi} \) was "soft", that is, \( \bar{\pi} \) corresponds to a point on the section of \( \Phi_3 \) between points C and D on Figure 6 above. In this situation private and public incentives are aligned as both \( T_I \) is lower and \( T_{II} \) is greater than with technology \( \delta' \), and all flexible agents prefer the more informative technology.

Nevertheless, there are circumstances where flexible types do not find the more informative sufficiently attractive (from the social welfare perspective). Then, the Court needs to weight the costs and benefits of distorting optimal standards. Given the very weak assumptions we have
imposed on the welfare functions it is possible that there may be several local maxima for the social welfare function $W$ as one varies the standard, $\pi$. To simplify the analysis in this section we will focus in the case where there is one global maximum: for all $\delta$, $W(\tilde{c}(\pi; \delta), T_I(\pi; \delta), T_{II}(\pi; \delta))$ is concave in $\pi$.

What we find is that it may be optimal to distort the standards and improve welfare, in order to make the more informative technology more attractive to the flexible types. But, that such distortions take a special form:

**Proposition 6** It is optimal for the Court to (weakly) increase the difference in harshness of the standards applied to the technologies: $T_I(\tilde{\pi}^*; \delta') - T_I(\bar{\pi}; \delta) \geq T_I(\bar{\pi}^*; \delta') - T_I(\bar{\pi}^*; \delta)$.

As we show in the proof of this Proposition, a slightly stronger statement is true, namely, if the standards are not the Court’s preferred ones, it is optimal to either (i) only change (and decrease) the harshness of the standard on $\delta$; (ii) only change (and increase) the harshness of the standard on $\delta'$; or (iii) both decrease the harshness of the standard on $\delta$ and increase the harshness of the standard on $\delta'$. It is never optimal to lower the standard on $\delta$ or increase it on $\delta'$.

Thus, when welfare from flexible types is sufficiently important to warrant distorting the standards, these distortions can be like the ones we saw in previous section: make the standard on the less informative technology harsher, but it could also take the form of softening the standard on the more informative one. In either case, the difference in harshness of standards increases—which implies that generically requiring courts to treat technologies equally is not a good policy in terms of social welfare.

6 Alternative Incentive Problems

So far, the paper has focused on the case where a Court is trying to encourage injurers to take due care to reduce the risk of accidents but the same theoretical structure can be found in other
6.1 A principal-agent problem

Consider a principal who wants to encourage effort from an agent. The agent has a reservation wage equal to $U$ and chooses between two effort levels, $e \in \{e_L, e_H\}$ at a monetary cost of $c$. The agent’s effort affects his productivity, $\pi$, where the distribution of $\pi$ depends on the effort level chosen by the agent, $e$, and is described by the probability distribution $f(\pi|e)$ and the cumulative distribution function $F(\pi|e_H)$, where $f(\pi|e)$ has the same stochastic structure as described in the text, that is, it has the monotone likelihood ratio property. Suppose the principal wants to encourage effort and can do so by offering the agent an incentive contract of the form: a constant wage $w$ and a bonus $B$ with probability $\gamma(\pi)$ where $\gamma : [0, 1] \rightarrow [0, 1]$.

The following condition is sufficient for the principal-agent problem to be equivalent to the Court’s problem:

**Condition:** The principal does not incur the full cost of the bonus.

That is, for the agent the bonus has a higher value than the cost it represents for the principal. This condition captures the idea that the bonus provides some additional value to the agent, which can be pecuniary (for example, the bonus is a signal to the market that increases the agent’s human capital), or non-pecuniary (for example, the psychological value of the recognition of one’s effort). In order for this condition to be satisfied we assume that the principal only pays a fraction $\alpha \in [0, 1)$ of the bonus the agent receives. Then, the principal’s problem is:

$$\min_{w, B} \frac{w}{\gamma(\pi)} + \int_0^1 \alpha B \gamma(\pi) f(\pi|e_H) \, d\pi$$

s.t. $w + \int_0^1 B \gamma(\pi) f(\pi|e_H) \, d\pi - c \geq w + \int_0^1 B \gamma(\pi) f(\pi|e_L) \, d\pi$

and $w + \int_0^1 B \gamma(\pi) f(\pi|e_H) \, d\pi - c \geq U$.

The monotonicity of the signal implies that the optimal compensation policy is a threshold
rule, that is, there is a \( \tilde{\pi} \) such that if \( \pi \geq \tilde{\pi} \) the principal awards the bonus and pays no bonus if \( \pi < \tilde{\pi} \). Among the compensation policies that induce high effort, the principal, wishing to minimize the wage schedule, will reduce overall compensation so as to make the agent indifferent between high effort and the outside option, \( U \), so that the participation constraint will be binding, that is

\[
w + \int_0^1 B_\gamma(\pi)f(\pi|e_H) \, d\pi - c = w + (1 - F(\tilde{\pi}|e_H))B - c = U
\]

\[\iff \Rightarrow w = U + c - (1 - F(\tilde{\pi}|e_H))B.
\]

We can now rewrite the principal’s objective function as

\[
w + \int_0^1 \alpha B_\gamma(\pi)f(\pi|e_H) \, d\pi = U + c - (1 - \alpha)(1 - F(\tilde{\pi}|e_H))B
\]

\[
= U + c - (1 - \alpha)B + (1 - \alpha)F(\tilde{\pi}|e_H)B
\]

\[
= U + c - (1 - \alpha)B + (1 - \alpha)B_T I.
\]

The incentive compatibility constraint can also be rewritten in terms of errors:

\[
(1 - F(\tilde{\pi}|e_H))B + c \geq (1 - F(\tilde{\pi}|e_L))B
\]

\[\iff \Rightarrow 1 + \frac{c}{B} \geq T_I + T_{II}
\]

Let \( a = U + c - (1 - \alpha)B \) and \( b = (1 - \alpha)B \), and the principal’s problem is now:

\[
\min_{\pi} a + bT_I
\]

s.t. \( T_I + T_{II} \leq 1 + \frac{c}{B} \),

which is analogous to the Court’s problem (Equation (3)) namely:

\[
\min_{\pi} pT_I
\]

s.t. \( T_{II} + pT_I \leq 1 - \frac{c}{D} \).
6.2 An incentive model of statistical discrimination

Arrow (1973) and Coate and Loury (1993) describe discrimination as an equilibrium outcome in a situation of asymmetric information regarding employee productivity. They find that if a group of employees expects that their human capital is unlikely to determine whether they are promoted or not (that is, they will be subject to a harsh standard for promotion), they optimally decide not to invest in human capital. A second strand of the literature takes productivity as given and assumes that employers receive signals whose informativeness varies across different groups of employees. This leads to biases in the screening process, and overrepresentation of the more informative group in top positions (Morgan and Brady, 2009). In this application, we combine both approaches, since we deploy a variation of our basic model, where employee productivity will be endogenous, but will be observed by the employer with different accuracy depending on the employee’s type. We find that applying a softer promotion standard to employees with more informative signals generates greater firm value.

In particular, consider the problem of a firm when choosing a promotion policy for its lower-level employees. In contrast to the Court’s problem, in this model the employee (agent) does not choose one of two effort levels, \( e \in \{H, L\} \), but rather chooses a continuous effort level \( e \in [0, 1] \) at a cost \( C(e) \) where \( C(e) \) is a continuously differentiable and convex function of effort.

The employee’s effort choice \( e \) affects future productivity of the firm. In particular, \( e \), determines the distribution of a binary state variable, \( \omega \in \{H, L\} \), where \( e = \text{Pr}\{\omega = H\} \). For example, \( e \) can represent the employee’s private investment in firm-specific human capital, which can be either successful \( \omega = H \) or unsuccessful \( \omega = L \).

The firm does not observe \( e \) or \( \omega \), but instead receives a signal \( \pi \) of the state. The distribution of the signal depends on the state as described in Section 2.2.\(^{19}\) We interpret the signal, \( \pi \), as the employee’s current performance. The firm’s problem is to choose a promotion policy

\(^{19}\)As expected, the distribution \( f(\pi|H) \) in this model corresponds to \( f(\pi|e_H) \) above, and similarly with \( f(\pi|L) \) and \( f(\pi|e_L) \).
based on the employee’s performance, \( \pi \). We concentrate on threshold-based promotion policies. High performance employees (above the threshold, \( \bar{\pi} \)) are promoted (and receive an additional payment, \( B > 0 \)) while low performance employees are not.\(^{20}\) When designing the promotion policy (determining \( \bar{\pi} \)) the objectives of the firm are twofold: (i) To provide incentives to employees so that they exert effort, since effort affects the employee’s performance as well as their future productivity; (ii) To promote highly productive employees while not promoting low productive ones. Not promoting a highly productive employee and promoting a low productive one correspond to Type I and Type II errors (respectively) in our terminology. As it was in the Court’s problem, we assume that firms care more about Type I error. This can be interpreted as it being more difficult to retain productive employees than to undo erroneous promotion decisions.

We will use the standard terminology that describes the firm’s objective as maximizing the present value of profits, and consider a general profit function denoted \( V (e, T_I, T_{II}) \). We define a regular \( V \) function in the same way we defined regular social welfare functions above (Section 5, Definition 3):

**Definition 4** The firm’s present value \( V (e, T_I, T_{II}) \) is regular if: i) \( V (e, T_I, T_{II}) \) is differentiable and increasing in \( e \) and decreasing in \( T_I \) and \( T_{II} \); and, ii) For any \( e \),

\[
\frac{\partial V}{\partial T_I} \geq \frac{\partial V}{\partial T_{II}}.
\]

The firm’s problem is then to maximize \( V \) subject to the restriction that employees’ actions are incentive compatible. The employee’s incentive compatible actions are obtained from a standard risk-neutral incentive problem: employees receive a constant wage, \( w \), plus the possibility of a promotion (and an additional payment, \( B \)). The agent’s decision problem is:\(^{21}\)

\[
\max_{e} w + e \Pr \{ \pi \geq \bar{\pi} | H \} + (1 - e) \Pr \{ \pi \geq \bar{\pi} | L \} - C (e).
\]

\(^{20}\)In a more extensive model, with a standard participation constraint for the employee, it is straightforward to show, using the methods developed above, that the optimal policy can be characterized as a threshold rule.

\(^{21}\)To streamline the presentation we ignore the employee’s participation constraint.
The solution to the optimal level of effort is characterized by the F.O.C.

\[ B \left( \Pr \{ \pi \geq \bar{\pi} | H \} - \Pr \{ \pi \geq \bar{\pi} | L \} \right) = C' (e). \quad (5) \]

Then, the firm’s problem is to choose \( \bar{\pi} \) so as to maximize \( V \) subject to (5), which, when rewritten in terms of errors, becomes:

\[
\max_{\bar{\pi}} V (e, T_I, T_{II}) \\
\text{s.t.} \quad T_I + T_{II} = 1 - \frac{C'(e)}{B}.
\]

(6)

Given that \( C'(e) \) is a monotone increasing function, the sum of Type I and Type II errors determines the equilibrium level of effort. Therefore, this problem is equivalent to the Court’s problem in the heterogenous setting in which the weighted sum of errors determines the marginal type of agent exerting high care: \( T_I + pT_{II} = 1 - \frac{\bar{\pi}}{B} \).

Having established this equivalence, we turn to the problem of statistical discrimination in the workplace. Suppose the firm has two groups of employees, types A and B, and they are facing the incentive problem we have just described. Both groups are equally productive but there is a group-specific effect that makes signals generated by group A more informative than those generated by Group B. This is the standard assumption in the statistical discrimination literature, that in our setting may be justified by the matching in personal characteristics between the employees and their evaluators.\footnote{For example, group A may have the same socio-economic background as that of the firm’s evaluators, which may make it easier for employees to communicate with the evaluators and for the latter to determine the employee’s future contribution to the firm. Similarly, group B may not have access to alternative means of communication or interaction outside the workplace such as extra-curricular sports interests, social clubs, etc, which reduces the precision of the signals available to the firm’s evaluators to make promotion decisions.}

The principal (the firm) can observe the group the employee belongs to, and applies the optimal standard for that group. Using Proposition 5 above, we obtain:

**Proposition 7** Consider that signals from group A are more (Lehmann) informative that those of group B, and let \( \bar{\pi}_B \) be the corresponding optimal standard for group B. Then, i) there exists a set of standards for group A that provide greater value for all regular present value functions than
that generated by employees from group B; ii) If the standard $\bar{\pi}$ for group A generates greater value for all regular present value functions then $\bar{\pi}$ is less harsh than $\bar{\pi}_B$, that is $T_I(\bar{\pi}) \leq T_I(\bar{\pi}_B)$.

The first part of the result is a natural consequence of better information. The second part is more interesting as it states that greater profits are associated with a lower standard for group A employees.

7 Conclusions

In the paper we present a general framework to understand how the quality of evidence about the underlying behavior that the legal system tries to induce from the agents affects the choice of evidentiary standards by the Law, which in turn will determine the incentives that the agents will face in order to adopt the desired levels of behavior. In this setting we analyze the optimal policies in terms of the harshness of the evidentiary standards, and show how some traditional issues, such as the use of strict liability instead of negligence can be presented under a new light.

Moreover, we also explicitly examine the choice of technology by firms engaged in risky activities, given that not all technologies are equally informative of the true level of care in the evidentiary sense. Some technologies are more informative than others. For instance, when precaution is mainly the outcome of policies and investment decisions carried out at the organizational level, and extending to the farthest corners of the entity that poses the risk of harm, typically the level of informativeness is high. On the other side, when precaution decisions are disorganized, taken at the individual level of all agents who may have some influence on the risk, the evidence concerning these precautions would commonly be weaker. From here, our main theoretical analysis considers the optimal choice of evidentiary standards to induce adequate behavior in the presence of a diversity of precautionary technologies. We find that a higher degree of informativeness by a technology should lead the legal system to impose less harsh evidentiary standards to achieve the same level of underlying behavior. The reverse is the case for technologies that are less informa-
tive. Our main results and their extension to heterogeneous populations of agents and competing technologies seems to provide a common explanation to several observed patterns in the evolution of liability for accidents in most developed countries. Our analysis additionally provides a basis for the adjustment of evidentiary standards to actions taken by an injurer after the accident has happened, and more precisely, actions linked to the lawsuit and trial, such as the choice of legal counsel, obstructionist tactics at discovery and other similar types of behavior.
A Appendix

A.1 Proofs

Proof of Proposition 1

Suppose \( t(\pi) \) induces high effort and is not a threshold rule, that is
\[
pD \int_{\pi} t(\pi) f(\pi|e_H) d\pi + c \leq D \int_{\pi} t(\pi) f(\pi|e_L) d\pi
\]
and consider a threshold rule defined by the threshold \( \tilde{\pi}, \tau(\pi) \), that generates the expected cost for low effort
\[
DF(\tilde{\pi}|e_L) \equiv D \int_{\pi} \tau(\pi) f(\pi|e_L) d\pi = D \int_{\pi} t(\pi) f(\pi|e_L) d\pi.
\]

Let \( \psi(\pi) = \tau(\pi) - t(\pi) \). As \( \tau(\pi) = 1 \) for \( \pi \leq \tilde{\pi} \) and \( \tau(\pi) = 0 \) for \( \pi > \tilde{\pi} \) and \( t(\pi) \in [0, 1] \)
then \( \psi(\pi) \geq 0 \) for \( \pi \leq \tilde{\pi} \) and \( \psi(\pi) \leq 0 \) for \( \pi > \tilde{\pi} \). Also, by construction
\[
\int_{\pi} \psi(\pi) f(\pi|e_L) d\pi = 0.
\]

By the MLRP, \( \Pi|e_H \succeq_{tr} \Pi|e_L \), so that applying known results on the preservation of single-crossing properties (for example Athey)
\[
\int_{\pi} \psi(\pi) f(\pi|e_L) d\pi = 0 \Rightarrow \int_{\pi} \psi(\pi) f(\pi|e_H) d\pi \leq 0
\]
\[
\Rightarrow \int_{\pi} \tau(\pi) f(\pi|e_H) d\pi \leq \int_{\pi} t(\pi) f(\pi|e_H) d\pi.
\]
This implies that

a.) If \( t(\pi) \) induces high effort, so does \( \tau(\pi) \):
\[
D \int_{\pi} \tau(\pi) f(\pi|e_L) d\pi = D \int_{\pi} t(\pi) f(\pi|e_L) d\pi 
\geq pD \int_{\pi} t(\pi) f(\pi|e_H) d\pi 
\geq pD \int_{\pi} \tau(\pi) f(\pi|e_H) d\pi.
\]

b.) Type I error is smaller with the threshold rule \( \blacksquare \)

Proof of Lemma 1: The values of \( \Phi \) are obtained by direct evaluation while the existence and uniqueness of the minimum is obtained by looking at the derivative of \( \Phi \):
\[
\Phi'(\pi) = f(\pi|e_L)[p f(\pi|e_H) f(\pi|e_L) - 1].
\]
As the likelihood ratio integrates to one (with respect to $f(\pi|e_L)$) and is monotone, $\Phi$ has at most one sign change (from negative to positive). As the likelihood ratio is increasing it starts off negative so that the minimum of $\Phi$ is either in the interior of $[0, 1]$ or at $\pi = 1$. Uniqueness comes from the differentiability of $f$. ■

Proof of Proposition 2: The level $c_{\text{max}}$ is determined as the solution to $\Phi(\pi_{\text{min}}) = 1 - \frac{c_{\text{max}}}{f}$. For $c > c_{\text{max}}$, for all $\pi \in [0, 1]$, $\Phi(\pi) > 1 - \frac{\tilde{c}}{f}$ so that it is not possible to induce high care. For $c \leq c_{\text{max}}$, let $H(c)$ be the set of $\pi$ that satisfy the incentive compatibility constraint for a given $c$. The set $H(c)$ is a closed interval such that for all $\pi \in H(c)$, $\Phi(c) \leq 1 - \frac{\tilde{c}}{f}$, and the minimum of $H(c) = \Phi^{-1}_B(1 - \frac{\tilde{c}}{f})$. As $\Phi_B$ is decreasing and $1 - \frac{\tilde{c}}{f}$ is decreasing in $c$, $\Phi^{-1}_B$ is increasing in $c$.

Also, as $\Phi'(0) \neq 0$, if $c < c_{\text{max}}$, $H(c)$ is a non-singleton set so that $\min H(c) < 1$. ■

Proof of Lemma 3: We include the proof for completeness. Let $F_{\delta'}(\pi_{\delta'}|e_H) = \alpha = F_{\delta}(\pi_{\delta}|e_H)$. By the definition of Lehmann informativeness

$$F_{\delta}^{-1}(F_{\delta'}(q|e_H)|e_H) \geq F_{\delta}^{-1}(F_{\delta'}(q|e_L)|e_L) \forall q$$

$$\Rightarrow F_{\delta}^{-1}(F_{\delta'}(\pi_{\delta'}|e_H)|e_H) \geq F_{\delta}^{-1}(F_{\delta'}(\pi_{\delta'}|e_L)|e_L).$$

Using $F_{\delta'}(\pi_{\delta'}|e) = \alpha$, the LHS is equal to

$$F_{\delta}^{-1}(\alpha|e_H) = \pi_{\delta}.$$

Applying the mononote transformation $F_{\delta}(\cdot|e_L)$ on both sides of the inequality, we obtain,

$$F_{\delta}(\pi_{\delta}|e_L) \geq F_{\delta'}(\pi_{\delta'}|e_L)$$

$$1 - F_{\delta}(\pi_{\delta}|e_L) \leq 1 - F_{\delta'}(\pi_{\delta'}|e_L)$$

$$T_{II}(\pi_{\delta}) \leq T_{II}(\pi_{\delta'}).$$

Proof of Proposition 3: Let $\pi_{\delta}$ be the standard with technology $\delta$ that is as harsh as $\pi_{\delta'}^*$, with technology $\delta'$. By Lemma (Lehmann) above

$$\Phi_{\delta}(\pi_{\delta}) \leq \Phi_{\delta'}(\pi_{\delta'}^*).$$

As $1 - \frac{\tilde{c}}{f} = \Phi_{\delta}(\pi_{\delta}^*) = \Phi_{\delta'}(\pi_{\delta'}^*) \geq \Phi_{\delta}(\pi_{\delta})$ and $\Phi_B$ is decreasing then $\pi_{\delta}^* \leq \pi_{\delta}$ and $T_{I}(\pi_{\delta}^*) \leq T_{I}(\pi_{\delta}) = T_{I}(\pi_{\delta'}).$ ■

Lemma 10 For any two standards, $\pi$ and $\pi'$, under technologies $\delta$ and $\delta'$ respectively, such that $\tilde{c}(\pi) = \tilde{c}(\pi')$ and $T_{I}(\pi) < T_{I}(\pi')$ then $W(\pi; \delta) \geq W(\pi'; \delta').$
Proof of Lemma 10: For any two $\pi, \pi'$ such that $\bar{c}(\pi) = \bar{c}(\pi')$

$$pT_I(\pi) + T_{II}(\pi) = pT_I(\pi') + T_{II}(\pi').$$

Let $f(x) = W(c, x, 1 - \frac{x}{x_I} - px)$ so that

$$f(T_I(\pi)) = W(c, T_I(\pi), T_{II}(\pi))$$

$$f(T_I(\pi')) = W(c, T_I(\pi'), T_{II}(\pi')).$$

Using

$$f(x) - f(x') = \int_{x'}^{x} f'(y) dy$$

we can write

$$f(T_I(\pi')) - f(T_I(\pi)) = \int_{T_I(\pi)}^{T_I(\pi')} f'(x) dx$$

Using

$$f'(x) = \frac{\partial W}{\partial T_I} - p \frac{\partial W}{\partial T_{II}},$$

the regularity condition

$$\left| \frac{\partial W}{\partial T_I} \right| \geq \left| \frac{\partial W}{\partial T_{II}} \right| > p \left| \frac{\partial W}{\partial T_{II}} \right|$$

implies

$$f'(x) < 0$$

and hence

$$f(T_I(\pi')) - f(T_I(\pi)) < 0$$

$$\iff W(c, T_I(\pi'), T_{II}(\pi')) < W(c, T_I(\pi), T_{II}(\pi)).$$

Proof of Proposition 5: Let $\pi^*$ be the optimal standard with technology $\delta'$ and $c^*$, $T_I^*$, and $T_{II}^*$, the corresponding amount of care, Type I error and Type II error respectively.

We proceed by (i) identifying the standard ($\pi_1$) with the same Type I error that generates more welfare and more activity ($c_1$), (ii) identify the standard ($\pi_2$) that with the same level of activity and less Type I error also provides more welfare, and (iii) show that for all $\pi \in [\pi_2, \pi_1]$ you can get more welfare (with less Type I error and more activity).

(i) An increase in the informativeness of the technology implies that it is possible to set a standard with technology $\delta \pi_I$ that keeps the same harshness ($T_I^*$) and lowers the Type II error ($T_{II} \leq T_{II}^*$). Applying the characterization in the statement, and using Lemma 3, this standard
generates a lower weighted cost of error. Thus, the corresponding cutoff level of care cost is greater 
\((c_1 \geq c^*)\) and these two effects together lead to an increase in welfare:

\[
W(\pi_1) = W(c_1, T_I^*, T_{II}) \geq W(c^*, T_I^*, T_{II}^*) = W(\pi^*).
\]

We assume \(\pi_1 \in D\). If not, then the shape of \(\Phi_\delta\) implies there is a \(\pi_1' \in D\) such that \(\Phi_\delta(\pi_1') = \Phi_\delta(\pi_1) = 1 - \frac{c_2}{D}.\) By Lemma 10 \(W(\pi_1') \geq W(\pi_1') \geq W(\pi^*),\) and we proceed using \(\pi_1'\) instead of \(\pi_1\).

(ii) Similarly, it is possible to set a standard that maintains the same level of activity, \(c^*\). This activity can be achieved by a standard \(\pi_2\) that solves \(\Phi_\delta(D)(\pi_2) = 1 - \frac{c^*}{D}\). The greater informativeness of \(\delta\) means that \(\pi_2\) can be found so that it has lower Type I error—and hence Type II error has to be higher \((T_{II} \leq T_I, T_{II} \geq T_{II}^*)\). Note that this standard \(\pi_2\) is also less harsh that \(\pi_1\) and \(c_2 \leq c_1, T_{II} \leq T_{I},\) and \(T_{II} \geq T_{II}^*\). By Lemma 10 welfare has increased \(W(\pi_2) \geq W(\pi^*)\).

(iii) Let \(\hat{\pi} \in [\pi_2, \pi_1] \subset D,\) and let \(c\) be the corresponding care level \(1 - \frac{c}{D} = \Phi_\delta(\hat{\pi}).\) As \(\Phi_\delta(D)\) is continuous and decreasing, \(c \in [c^*, c_1]\).

Consider evaluating the welfare function at \((c, T_I^*, 1 - \frac{c}{D} - pT_I^*)\) even though \(T_I^*\) and \(T_{II} = 1 - \frac{\hat{\pi}}{D} - pT_I^*\) are not implied by the standard \(\hat{\pi}\). By the monotonicity properties of \(W\)

\[
W\left(c, T_I^*, 1 - \frac{c_1}{D} - pT_I^*\right) \geq W\left(c, T_I^*, 1 - \frac{c}{D} - pT_I^*\right) \geq W\left(c^*, T_I^*, 1 - \frac{c^*}{D} - pT_I^*\right).
\]

If we now compare \(W(\pi, T_I(\hat{\pi}), T_{II}(\hat{\pi}))\) with the welfare obtained using the errors implied by \(\hat{\pi}\), using \(T_I(\hat{\pi}) \leq T_I(\pi_1)\) and Lemma 10

\[
W(\pi, T_I(\hat{\pi}), T_{II}(\hat{\pi})) \geq W(\pi, T_I^*, 1 - \frac{c}{D} - pT_I^*).
\]

PROOF OF PROPOSITION 5: Let \(\pi^*\) be the optimal standard with technology \(\delta'\) and \(c^*, T_I^*,\) and \(T_{II}^*\), the corresponding amount of care, Type I error and Type II error respectively.

We proceed by (i) identifying the standard \((\pi_1)\) with the same Type I error that generates more welfare and more activity \((c_1)\), (ii) identify the standard \((\pi_2)\) that with the same level of activity and less Type I error also provides more welfare, and (iii) show that for all \(\pi \in [\pi_2, \pi_1]\) you can get more welfare (with less Type I error and more activity).

(i) An increase in the informativeness of the technology implies that it is possible to set a standard with technology \(\delta\) \(\pi_{1\delta}\) that keeps the same harshness \((T_I^*)\) and lowers the Type II error
\((T_{1II} \leq T_{II})\). Applying the characterization in the statement, and using Lemma 3, this standard generates a lower weighted cost of error. Thus, the corresponding cutoff level of care cost is greater \((c_1 \geq c^*)\) and these two effects together lead to an increase in welfare:

\[
W(\pi_1) = W(c_1, T_I^*, T_{1II}) \geq W(c^*, T_I^*, T_{II}) = W(\pi^*) .
\]

We assume \(\pi_1 \in \mathbb{D}\). If not, then the shape of \(\Phi_\delta\) implies there is a \(\pi'_1 \in \mathbb{D}\) such that \(\Phi_\delta(\pi'_1) = \Phi_\delta(\pi_1) = 1 - \frac{c_1}{D}\). By Lemma 10 \(W(\pi'_1) \geq W(\pi'_1) \geq W(\pi^*)\), and we proceed using \(\pi'_1\) instead of \(\pi_1\).

(ii) Similarly, it is possible to set a standard that maintains the same level of activity, \(c^*\). This activity can be achieved by a standard \(\pi_2\) that solves \(\Phi_\delta,\mathbb{D}(\pi_2) = 1 - \frac{c^*}{D}\). The greater informativeness of \(\delta\) means that \(\pi_2\) can be found so that it has lower Type I error—and hence Type II error has to be higher \((T_{2II} \leq T_{I}^*, T_{2II} \geq T_{II}^*)\). Note that this standard \(\pi_2\) is also less harsh that \(\pi_1\) and \(c_2 \leq c_1\), \(T_{2II} \leq T_{1II}\), and \(T_{2II} \geq T_{2II}\). By Lemma 10 welfare has increased \(W(\pi_2) \geq W(\pi^*)\).

(iii) Let \(\hat{\pi} \in [\pi_2, \pi_1] \subset \mathbb{D}\), and let \(c\) be the corresponding care level \(1 - \frac{c}{D} = \Phi_\delta(\hat{\pi})\). As \(\Phi_\delta,\mathbb{D}\) is continuous and decreasing, \(c \in [c^*, c_1]\).

Consider evaluating the welfare function at \((c, T_I^*, 1 - \frac{c}{D} - pT_I^*)\) even though \(T_I^*\) and \(T_{II} = 1 - \frac{c}{D} - pT_I^*\) are not implied by the standard \(\hat{\pi}\). By the monotonicity properties of \(W\)

\[
W(c_1, T_I^*, 1 - \frac{c_1}{D} - pT_I^*) \geq W(c, T_I^*, 1 - \frac{c}{D} - pT_I^*) \geq W(c^*, T_I^*, 1 - \frac{c^*}{D} - pT_I^*) .
\]

If we now compare \(W(c, T_I^*, 1 - \frac{c}{D} - pT_I^*)\) with the welfare obtained using the errors implied by \(\hat{\pi}\), using \(T_I(\pi_1) \leq T_I(\pi_2)\) and Lemma 10

\[
W(c, T_I(\hat{\pi}), T_{II}(\hat{\pi})) \geq W(c, T_I^*, 1 - \frac{c}{D} - pT_I^*) .
\]

\[\blacksquare\]

**Proof of Proposition 6**

The Court would ideally like to apply the standards \(\pi^* = \pi\) and \(\pi'^* = \pi'^\) on the inflexible types and have all agents of flexible type select the more informative technology, \(\delta\). If for the Court’s preferred standards, \(\pi^* = \pi\) and \(\pi'^* = \pi'^\), it is incentive compatible for the flexible agents to choose technology \(\delta\) then the result is trivially true.

Suppose it is not incentive compatible for some of the flexible agents to choose technology \(\delta\), and let \(\epsilon'\) be the flexible type that is indifferent between the two technologies.
Now suppose it is optimal for the Court to lower the harshness of the standard on technology $\delta'$, say to $\pi'_a$. Note that this standard implies more Type II error for those using technology $\delta'$ so that technology $\delta'$ becomes more attractive to all flexible agents. As we are in the case where optimality requires that more flexible types adopt the more informative technology then the standard on technology $\delta$, $\pi_a$, must also be lower in order to compensate and stop flexible agents from switching to technology $\delta'$. As both standards are lower, $\bar{\pi}_a < \bar{\pi}$ and $\bar{\pi}'_a < \bar{\pi}'$, by the concavity of the welfare function it is possible to increase both standards, and thereby increasing welfare for all inflexible types, while (weakly) increasing the amount of effort for the marginal flexible agent and increasing welfare for the flexible types. This contradicts the optimality of lowering the standard on $\delta'$. Similar, an increase in the standard on $\delta$ requires a corresponding increase in the standard for $\delta'$ which lowers welfare.

Thus, in order to encourage the flexible types to adopt the more informative technology, the Court could either (i) only change (and decrease) the harshness of the standard on $\delta$; (ii) only change (and increase) the harshness of the standard on $\delta'$; or (iii) both decrease the harshness of the standard on $\delta$ and increase the harshness of the standard on $\delta'$. In all three cases, the difference in harshness of standards increases, which gives the result in the Proposition.

**Proof that signals in parametric example are Lehmann ordered:** The characterization in Ganuza and Penalva (2010) we use is that for signals ordered in terms of supermodular precision with equal priors. Given that in the dichotomous setting supermodular precision implies integral precision, which in turn is equivalent to Blackwell sufficiency, which in turn implies Lehmann (see Ganuza and Penalva (2010) for definitions and details), as our example has supermodular ordered signals, they are also Lehmann ordered.

It is straightforward to see that for all $\delta$ and with equal priors, the marginal distribution of the signals is uniform:

$$p(\pi) = \frac{1}{2} f(\pi|e_L) + \frac{1}{2} f(\pi|e_H)$$

$$= \frac{1}{2} \left(1 + \frac{\delta}{2} - \delta\pi\right) + \frac{1}{2} \left(1 - \frac{\delta}{2} + \delta\pi\right) = 1.$$

Then, for $\delta > \delta'$ to show that the signals are ordered in terms of supermodular precision it suffices (Penalva and Ganuza 2010, Prop 3.ii) to show $f_{\delta} (\pi|e_H) - f_{\delta'} (\pi|e_H)$ is nondecreasing in $\pi$:

$$1 - \frac{\delta}{2} + \delta\pi - \left(1 - \frac{\delta'}{2} + \delta'\pi\right)$$

$$= \frac{1}{2} (\delta - \delta') + \pi (\delta - \delta').$$
References


