Model Secrecy and Stress Tests*

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Abstract

We study whether regulators should reveal the models they use to stress test banks. In our setting, revealing leads to gaming, but not revealing can induce banks to underinvest in socially desirable assets for fear of failing the test. We show that although the regulator can solve this underinvestment problem by making the test easier, some disclosure may still be optimal, which under some conditions takes the simple form of a cutoff rule. We characterize the optimal disclosure policy combined with test difficulty, provide comparative statics, and relate our results to recent policies. We also offer applications beyond stress tests.

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1 Introduction

From the early days of bank stress tests in the wake of the 2008 financial crisis, disclosure has been a key issue of discussion among practitioners, academics, and regulators. Most of the academic discussion has centered around disclosure of the test results to the public (e.g., Goldstein and Sapra (2014), Goldstein and Leitner (2018)). However, academics have paid less attention to another important issue: should regulators disclose the models they use to project bank capital when conducting the test? This issue has recently gained momentum among policy makers and practitioners, leading to a change in the Fed's policy. Under the old policy, the Federal Reserve provided only a broad description of its stress test models. Under the new policy, it provides more information on certain equations and key variables, and illustrates how its models work on hypothetical loan portfolios. Yet, even under the new regime, the Federal Reserve does not fully reveal its models.[1]

An important reason for not revealing the models to the banks is to prevent banks from gaming the test—i.e., taking actions that enable them to pass the test without reducing risk. Indeed, in a speech on September 26, 2016, Former Fed Governor, Daniel Tarullo, said that "Full disclosure would permit firms to game the system—that is, to optimize portfolio characteristics based on the parameters of the model and take risks in areas not well-captured by the stress test just to minimize the estimated stress losses."[2] However, banks have constantly complained about model secrecy, claiming that even their best efforts to prepare for a test could result in unexpected and costly failure.[3] These claims cannot be ignored, particularly given evidence that regulatory uncertainty causes banks to reduce lending (Gissler, Oldfather, and Ruffino (2016)).

We present a stylized framework that allows us to examine the effects of revealing the regulator's stress test models to banks before the test. Our setting has two main forces.

[1] See `https://www.federalreserve.gov/newsevents/pressreleases/bcreg20190205a.htm`.

[2] See `https://www.federalreserve.gov/newsevents/speech/tarullo20160926a.htm`.

[3] See "Fed 'Stress Tests' Still Pose Puzzle to Banks," *Wall Street Journal*, March 12, 2015.

Not revealing reduces gaming, but it can also induce banks to reduce investment in socially desirable assets.

In our model, the bank has better capacity than the regulator to identify and measure risk, but there is a conflict of interest between the bank and the regulator: the bank wants to take more risk than is socially desirable. To be concrete, the bank can invest in a safe asset or a risky asset. The bank knows what the value of the risky asset will be during a crisis (hypothetical stress scenario), but the regulator observes only a noisy signal of that value. This signal represents the asset value predicted by the regulator's model. As we explain in the text, the regulator's signal could also represent one of the parameters in the regulator's model. The bank prefers to invest in the risky asset regardless of its true value during a crisis, but the regulator prefers the risky asset only if this value is sufficiently high. If the bank invests in the safe asset, it always passes the test. If the bank invests in the risky asset, it passes only if the regulator's signal is above some threshold. A bank that fails the test is required to reduce risk.

Our main focus is on whether the regulator should reveal his private signal to the bank before the bank makes its investment decision. We provide results for the benchmark case in which the threshold for passing the test is exogenous, but our main results are for the more practically relevant case in which the passing threshold is chosen by the regulator. This threshold could represent, for example, minimum capital requirements. Our setting also extends to the case in which instead of choosing a portfolio, the bank submits a capital plan to either retain cash (safe action) or pay dividends (risky action).

We first compare between a transparent regime, in which the regulator reveals his signal, and a secret regime, in which the regulator does not reveal his signal. Under the transparent regime, the bank games the test in the sense that when the regulator reveals a passing signal, the bank invests in the risky asset even if it knows that the true value is low. Secrecy mitigates this problem. In particular, fear of failure incentivizes the bank to act more cautiously, investing in the risky asset only if its value exceeds some threshold. However, secrecy can open the door to a new problem: the bank avoids the risky asset not only when it is bad for society but also in some cases when it is good. Our first main result is that if the regulator can freely adjust the passing threshold, then despite this tradeoff, secrecy is always preferred. Intuitively, secrecy prevents gaming, and by setting a sufficiently low passing threshold (an easy-to-pass test), the regulator can also prevent the underinvestment that could result from secrecy.

We then analyze more flexible disclosure rules. Our second main result is that even if the regulator can set the passing threshold optimally, some disclosure may be optimal. The logic behind this result is as follows. The regulator has two tools to induce the bank to reduce risk. First, he can make the test harder by increasing the passing threshold. Second, he can provide partial information. In particular, he can commit to a cutoff disclosure rule, under which he sends a high message if his private signal is above some threshold and a low message if his private signal is below the threshold. The benefit from this disclosure policy is that if the regulator sends the low message, the bank infers that the risky asset is likely to fail the test, and so it reduces investment in this asset.

However, each tool has a social cost. Partial disclosure leads to excessive risk if the regulator sends the high message, while a high passing threshold commits the regulator to sometimes fail the bank even if his model indicates the asset is good. In some cases, full secrecy would require a very high probability of failure to incentivize the bank. But then the regulator can gain by passing the bank more often and mitigating the worsening bank incentives via partial disclosure.

On a technical level, we show that the cutoff disclosure rule described above is optimal even if the regulator can choose multiple cutoffs. However, for some parameter values, the regulator can obtain a better outcome via a nonmonotone disclosure rule, in which messages pool signals from disconnected intervals. Essentially, very low signals are pooled with very high signals, less low with less high, etc. As we explain in Appendix C, this disclosure rule helps reduce the cost of providing incentives to the bank.

We also discuss practical limitations on the regulator's ability to implement the two tools above. One example is when the regulator cannot commit to act according to a prespecified disclosure rule. In this case, the regulator may not be able to implement partial disclosure, and so the relevant comparison might be between a fully transparent regime and a fully secret regime. Another example is when the regulator faces heterogeneous banks but must apply the same passing threshold for everyone. We show that if banks are sufficiently different from one another, then in contrast to our first result, full transparency is preferred to secrecy.

We use our framework to derive comparative statics with respect to the bank's characteristics, such as the bank's cost of failing the test or the bank's appetite for the risky asset (Figure 3 and Figure 4). For example, secrecy combined with an easy test is optimal if the bank's appetite for the risky asset is low, while partial disclosure combined with a hard test is optimal if the bank's appetite towards the risky asset is high. With respect to the information in the regulator's model, the results are ambiguous. A more informative model makes it easier to incentivize the bank, which pushes towards secrecy, but a more informative model also reduces the need to rely on the bank's information, which pushes toward transparency. We also provide some policy implications. For example, greater model transparency does not necessarily require increased capital requirements; and illustrating how the Fed's model works on hypothetical loan portfolios could lead to increased correlation in bank asset holdings (see Section 6).

Finally, we offer applications of our theory beyond stress tests. One application is a firm's board of directors approving a manager's strategic plan. Another application is an investor approving an investment recommendation by a financial advisor. We provide more details in Section 6.

2 Related Literature

Our paper is related to several strands of literature. The first strand studies stress test disclosure. This literature has focused on disclosure of the test results to the public (e.g., Goldstein and Leitner (2018)).[4] In contrast, we focus on disclosure of the regulator's stress test models to the banks before the test. To our knowledge, we are the first paper to offer a formal analysis of this problem. An informal discussion, which includes additional effects that are not studied in our paper, is provided by Goldstein and Leitner (2020). In particular, they distinguish between revealing the models to the public vs. revealing them to the bank.[5] Recent papers have also explored other issues that relate to stress tests, besides disclosure. Colliard (2019) and Leitner and Yilmaz (2019) study the extent to which regulators should rely on banks' internal risk models. Shapiro and Zeng (2019) show that regulators' reputational concerns could lead to inefficiently tough stress tests. Parlatore and Philippon (2018) study the design of stress scenarios.

On the empirical front, there is growing evidence on the effect of stress tests on bank credit supply and the allocations of credit between safe and risky loans (e.g., Acharya, Berger, and Roman (2018) and Cortés et al. (2020)). However, these papers do not discuss welfare implications or the effect of regulatory uncertainty. There is also a large literature documenting the effects of political and regulatory uncertainty on the real economy, including reduced investment.[6] In particular, Gissler, Oldfather, and Ruffino (2016) offer evidence suggesting that uncertainty about the regulation of qualified mortgages caused banks to reduce mortgage lending. This literature is consistent with the idea that model secrecy could induce banks to reduce investment.

[4] A partial list of this growing literature includes Bouvard, Chaigneau, and Motta (2015), Faria-e-Castro, Martinez, and Philippon (2017), Williams (2017), Inostroza and Pavan (2017), Orlov, Zryumov, and Skrzypacz (2018), Corona, Nan, and Zhang (2019), and review papers by Goldstein and Sapra (2014), Leitner (2014), and Goldstein and Leitner (2020). More recent papers include Dogra and Rhee (2018), Quigley and Walther (2020), Inostroza (2019), and Huang (2019).

[5] See also Flannery (2019).

[6] See, for example, Julio and Yook (2012), Fernández-Villaverde et al. (2015), and Baker, Bloom, and Davis (2016).

Our paper also relates to the Bayesian persuasion and information design literature (e.g., Kamenica and Gentzkow (2011) and Bergemann and Morris (2019)). Our results on general disclosure (Appendix C) contribute to a nascent subset of this literature in which the receiver is privately informed, specifically Kolotilin et al. (2017) and Kolotilin (2018). However, the results in these papers cannot be applied in our setting because they assume linear payoffs whereas we allow for nonlinear payoffs. Our paper also relates to Goldstein and Leitner (2018) in the sense that it provides another example in which negative assortative disclosure is optimal.

A number of papers study settings in which an agent is uncertain about how particular actions will be rewarded by the principal. In MacLeod (2003), Levin (2003), and Fuchs (2007), the principal evaluates the agent based on a subjective assessment (formally, a private unverifiable signal). These papers focus on optimal contracting rather than disclosure. Jehiel (2015) provides conditions under which a principal should remain silent about a payoff relevant variable that he privately observes before the agent chooses an action. In his setting, the agent is uninformed, but in our setting the agent (bank) is informed. Ederer, Holden, and Meyer (2018) study a multitask principal-agent problem with an uninformed principal and an informed agent. They provide conditions under which the principal can gain by randomizing between two incentives schemes. Lazear (2006) studies a setting in which a principal can monitor only a limited number of actions that an agent can take. He shows that if agents do not respond much to penalties, the principal can gain by announcing in advance the actions that will be monitored.

Finally, our paper is also related to the literature on delegation and authority in organizations, in which a principal can delegate authority to an informed but biased agent, but cannot design monetary transfers.[7] In our setting, if the regulator (principal) reveals his

[7] A partial list includes Holmstrom (1982), Aghion and Tirole (1997), Dessein (2002), Harris and Raviv (2008), Alonso and Matouschek (2008), Grenadier, A. Malenko, and N. Malenko (2016), and Chakraborty and Yılmaz (2017). See also Leitner and Yilmaz (2019), in which a regulator allocates authority to a bank based on the realization of a signal that the bank produces endogenously.

signal, he effectively restricts the bank's action space to those actions that will surely pass the test. So effectively, the regulator keeps authority. If the regulator does not reveal his signal, he gives the bank more freedom to choose an action, but in contrast to the literature above, the regulator responds to the bank's action using an evaluation process that is based on the regulator's private information. Hence, we can think of our secrecy regime as "delegation with hidden evaluation." In related work, Levit (2020) studies a setting in which an informed principal can take a follow-up action, but in his setting the agent is uninformed, communication with the agent is only via cheap talk, and the principal cannot precommit to taking specific actions.

3 Model

There is a bank and a regulator. The bank can take one of two actions: invest in a safe asset or invest in a risky asset. The payoff from the risky asset depends on the realization of a random variable $\omega \in [\underline{\omega}, \bar{\omega}]$, which represents the value of the risky asset during a crisis. We refer to ω as the state of nature. The bank's payoff is $u(\omega)$ and the regulator's payoff, which represents the value to society, is $v(\omega)$. Both u and v are increasing in ω ($u' > 0$, $v' > 0$) and incorporate the probability of crisis, resulting losses, payoffs during normal times, etc. The payoff from investing in the safe asset does not depend on ω and is normalized to zero for both the bank and regulator. That is, u and v are the relative gains from investing in the risky asset, compared to the safe asset. To save on notation, we use the same letter to denote both a random variable and its realization.

There is a conflict of interest between the bank and the regulator. The bank prefers the risky asset to the safe asset in every state ω, but the regulator prefers the risky asset only if $\omega \geq \omega_r$, where $\omega_r \in (\underline{\omega}, \bar{\omega})$. Formally:

Assumption 1. $u(\omega) \geq 0$ *for all* $\omega \in [\underline{\omega}, \bar{\omega}]$

Assumption 2. $v(\omega) \geq 0$ *if and only if* $\omega \in [\omega_r, \bar{\omega}]$

The conflict of interest captures the idea that the bank does not internalize the social cost associated with risk. For our results, it is not crucial that the bank prefers the risky asset in every state. What matters is that there are states in which the bank prefers the risky asset but the regulator does not.[8]

The next example provides a possible microfoundation for the payoff functions u and v.

Example 1. Suppose the risky asset pays \$2 in normal times and $\omega \in (0, 1)$ during a crisis, and suppose the probability of a crisis is p. The safe asset always pays \$1. Suppose in addition that if the bank's cash holding falls below \$1 (which happens during crisis), there is a social loss $L > 0$. For example, the bank may not be able to make debt payments, which could trigger contagion, or the bank may not be able to continue lending due to debt overhang. (See also Goldstein and Leitner (2018)). Then $u(\omega) = 2(1 - p) + p\omega - 1$ and $v(\omega) = u(\omega) - pL$.[9]

The bank has superior information about the value of the risky asset during a crisis; for simplicity, we assume the bank perfectly observes ω. The regulator does not observe ω, but he observes the realization of a noisy signal $s \in [\underline{s}, \bar{s}]$ of ω. The bank privately observes ω and the regulator privately observes s before the bank makes its investment decision. Everything else is common knowledge. The random variable ω has a cumulative distribution function (CDF) G and density g. Conditional on ω, s has CDF $F(\cdot|\omega)$ and density $f(\cdot|\omega)$. Both $g(\cdot)$ and $f(\cdot|\omega)$ have full support. We also assume:

Assumption 3 (MLRP)**.** *If $\omega' > \omega$, the ratio $f(s|\omega')/f(s|\omega)$ is strictly increasing in s.*

Assumption 3 implies that $1 - F(s|\omega)$ is strictly increasing in ω.[10] That is, the regulator is more likely to observe higher signals when the state ω is higher.

[8] If, there were states ω for which $u(\omega) < 0$ (in contrast to Assumption 1), the bank would never invest in those states, regardless of the regulator's disclosure policy.

[9] Note that u and v are increasing in ω, and if p and L are chosen appropriately, Assumptions 1 and 2 are satisfied.

[10] See Milgrom (1981).

After the bank makes its investment decision, the regulator conducts a stress test. That is, the regulator observes the bank's investment and decides whether to pass or fail the bank. If the bank chooses the safe asset, it always passes the test. If the bank chooses the risky asset, it passes only if the regulator's signal s is above some threshold, which we denote by s_p. This threshold could represent minimum capital requirements. We analyze the case in which s_p is exogenous as well as the case in which s_p is chosen by the regulator. In both cases, the banks knows s_p before it makes an investment decision.

This formulation captures the idea that the bank passes the test if its projected capital—according to the regulator's model—is above some threshold. In general, we can think of the regulator's model as the formula that the regulator uses to assess the bank's capital for a given portfolio and stress scenario. However, in our paper, the bank's portfolio is very simple (safe or risky), and the value of the safe asset is known. Hence, the regulator needs to project only ω, and it is natural to interpret the signal realization s as the output of the regulator's model. Alternatively, s could represent one of the parameters in the regulator's model. For example, s could be a coefficient in a regression that uses historical data to estimate losses on certain type of loan portfolios.

Final payoffs are as follows. If the bank invests in the risky asset and fails the test, the regulator forces the bank to replace the risky asset with the safe asset. In this case, the bank suffers a cost $c > 0$. So, the bank's final payoff is $-c$, and the regulator's final payoff is zero. If the bank invests in the safe asset, both end up with a final payoff of zero. If the bank invests in the risky asset and passes the test, the bank's final payoff is $u(\omega)$ and the regulator's final payoff is $v(\omega)$.

We can interpret the cost c as a cost to the bank's managers from failing the test (e.g., because of a decline in the stock price). The cost c can also be interpreted as an upfront cost that the bank needs to incur before investing in the risky asset and which is already included in $u(\omega)$. If we assume that this cost is a transfer to other economic agents, then c does not affect the regulator's payoff.

Our main results do not depend on the exact specification of final payoffs above. For example, c could depend on ω, u could be flat (and positive), and the regulator's payoff after the bank fails the test need not be zero.[11] We can also capture other consequences of failing the test, which may be relevant in practice. For example, our model maps to a case in which instead of choosing an asset, the bank submits a capital plan which the regulator can either approve or deny (see Appendix A).

The focus of our paper is whether the regulator should reveal his signal s to the bank. We start with the case in which the regulator is restricted to either reveal or not reveal s (Section 4). Then we explore more general disclosure rules (Section 5 and Appendix C). In all cases, the regulator publicly commits to the disclosure policy and to a pass/fail rule, assumptions which we discuss in Section 6. We refer to investment in the risky asset simply as "investing" and investment in the safe asset as "not investing."

The sequence of events is as follows: (i) the regulator publicly commits to a disclosure policy about s and to a passing threshold s_p; (ii) nature chooses ω, the bank observes ω, and the regulator observes s; (iii) the regulator discloses information about s in accordance with his disclosure policy; (iv) the bank chooses the risky asset ("invest") or safe asset ("not invest"); (v) the regulator performs a stress test, and final payoffs are realized.

We solve the game backwards. First, we characterize the bank's investment decision for a given passing threshold and disclosure regime. Then we solve for the optimal disclosure regime for a given passing threshold. Finally, we solve for the optimal passing threshold. If the bank is indifferent between two actions, we assume that it chooses the one that is preferred by the regulator, and if the regulator is also indifferent, we assume that the bank invests. If the regulator is indifferent between multiple passing thresholds, he picks the highest one. Our main results do not depend on these assumptions.

Finally, to simplify the exposition, we focus on the more interesting case in which (i) it is optimal to sometimes fail the bank ($s_p > \underline{s}$) and (ii) if the regulator does not reveal

[11] See Remark 1 and item 6 of Section 6.

his signal, the bank responds by reducing investment. A sufficient condition for this is the following.[12]

Assumption 4. *$E[v(\omega)|\underline{s}] < 0$ and $u(\underline{\omega}) = 0$.*

4 Revealing vs. Not Revealing

In this section we compare between two disclosure regimes: revealing (the regulator reveals his signal s to the bank) and not revealing (the regulator does not reveal his signal to the bank).

4.1 Bank's investment

Let p denote the bank's perceived probability of passing the test upon investment. If the bank invests, its expected payoff is $pu(\omega) - (1-p)c$. If the bank does not invest, its payoff is zero. Hence, the bank invests if and only if

$$pu(\omega) - (1-p)c \geq 0. \tag{1}$$

Consider first revealing. If the regulator reveals a passing signal, then $p = 1$, and the bank's payoff from investing is $u(\omega) \geq 0$. If the regulator reveals a failing signal, then $p = 0$, and the bank's payoff from investing is $-c < 0$. Hence, the bank invests if and only if the regulator observes a passing signal.

Next, consider not revealing. Now the perceived probability p of passing the test depends on ω:

$$p(\omega) \equiv P(s \geq s_p|\omega) = 1 - F(s_p|\omega). \tag{2}$$

From Assumption 3 (MLRP), $p(\omega)$ is increasing in ω. Since $u' > 0$, it then follows that the left-hand-side in Equation (1) is increasing in ω. Hence, the bank follows a cutoff rule,

[12] We provide more details in Lemma E1 and in the proof of Proposition 1.

investing if and only if the state ω is above some threshold. We denote this investment threshold by ω_{NR} ("NR" stands for "not revealing"), and later, we also use $\omega_{NR}(s_p)$ to denote the dependence of ω_{NR} on s_p. For some parameter values, it is optimal for the bank not to invest at all. In this case, we let $\omega_{NR} = \bar{\omega}$, which implies the bank invests with probability 0.

The next lemma summarizes the preceding discussion.

Lemma 1. *1. If the regulator reveals his signal s to the bank, the bank invests if and only $s \geq s_p$.*

2. If the regulator does not reveal his signal s to the bank, there exists $\omega_{NR} \in \Omega$, such that the bank invests if and only if $\omega \geq \omega_{NR}$. The investment threshold ω_{NR} is continuous and increasing in both s_p and c.

The first part in Lemma 1 captures the idea that revealing the regulator's model could lead to gaming. In particular, if the regulator reveals a passing signal $s \geq s_p$, the bank invests in the risky asset even if it knows the asset will perform poorly in a crisis (ω is low)—i.e., the bank games the test. This is consistent with regulators' concerns about gaming, discussed in the introduction. Regulators have also expressed concerns that revealing the regulator's models will cause banks to rely too heavily on them rather than their own models. Consistent with these concerns, Lemma 1 shows that under revealing, the bank's investment depends only on the regulator's signal s, not on its private information ω. (See also the discussion of endogenous information production in Section 6.)

The second part in Lemma 1 captures the idea that not revealing makes the bank more cautious, leading it to avoid investment if $\omega < \omega_{NR}$. The fact that ω_{NR} increases in both s_p and c reflects that the bank becomes more cautious if the test is more difficult to pass or the cost of failing the test is higher.

Remark 1. Our results do not depend on the exact specification of the bank's payoff function u and cost of failing the test c. Any specification such that the left-hand-side in Equation

(1) is increasing in ω will imply that the bank follows a cutoff investment rule and will hence yield the same results.

4.2 Regulator's payoff

We use V_R and V_{NR} to denote the regulator's payoff under revealing and under not revealing, respectively. Later, we also use $V_R(s_p)$ and $V_{NR}(s_p)$ to denote the dependence on s_p.

We derive the regulator's payoffs as follows. Conditional on observing a failing signal $s < s_p$, the regulator's payoff is zero because the bank either does not invest or invests and fails the test. Conditional on observing a passing signal $s \geq s_p$, the regulator's payoff depends on the bank's investment decision from Lemma 1. Under revealing, the bank invests in every state $\omega \geq \underline{\omega}$ and the regulator obtains $\int_{\omega \geq \underline{\omega}} v(\omega) f(\omega|s) d\omega$. Under not revealing, the bank invests only if $\omega \geq \omega_{NR}$ and the regulator obtains $\int_{\omega \geq \omega_{NR}} v(\omega) f(\omega|s) d\omega$. Taking the expectation across all signals $s \in S$ and changing the order of integration, we obtain the following:

Lemma 2. *If the regulator reveals his signal, his payoff is*

$$V_R = \int_{\omega \geq \underline{\omega}} [1 - F(s_p|\omega)] v(\omega) dG(\omega).$$

If the regulator does not reveal his signal, his payoff is

$$V_{NR} = \int_{\omega \geq \omega_{NR}} [1 - F(s_p|\omega)] v(\omega) dG(\omega). \tag{3}$$

The payoffs under the two disclosure regimes are similar, except that the integral for V_R starts at $\underline{\omega}$, whereas the integral for V_{NR} starts at $\omega_{NR} > \underline{\omega}$.[13] This reflects the fact that under not revealing, the bank acts more cautiously, investing in fewer states. The expression inside the integrals reflects the fact that the regulator obtains $v(\omega)$ only if the bank passes

[13] The inequality is strict by Assumption 4.

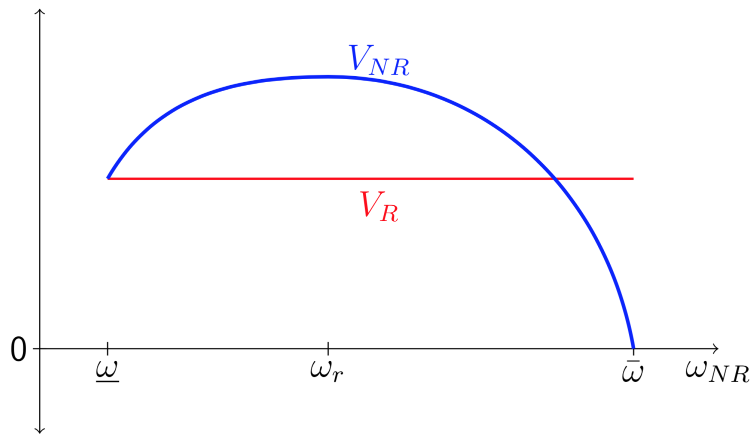


Figure 1: The regulator's payoff under revealing (V_R) and under not revealing (V_{NR}) as a function of the bank's investment threshold ω_{NR}.

the test, which happens with probability $1 - F(s_p|\omega)$.

4.3 Preferred regime

4.3.1 Exogenous passing threshold

To compare between the two disclosure regimes, it is useful to plot the regulator's payoff in both cases, as illustrated in Figure 1. When $\omega_{NR} = \underline{\omega}$, the two payoffs are the same: $V_R = V_{NR}$. The payoff under revealing V_R does not depend on ω_{NR}, but the payoff under not revealing V_{NR} does. As ω_{NR} increases but remains below ω_r, V_{NR} increases because the bank reduces investment in states in which investment is socially undesirable. However, as ω_{NR} increases above ω_r, V_{NR} decreases because the bank reduces investment in states in which investment is socially desirable. In the limit when $\omega_{NR} = \bar{\omega}$, the bank does not invest at all, and so, $V_{NR} = 0$. The figure implies that if $V_R > 0$, not revealing is preferred only if ω_{NR} is sufficiently low; otherwise, revealing is preferred. This is summarized in the proposition below.[14]

[14]If $V_R \leq 0$, then $V_{NR} \geq V_R$ for all ω_{NR}, so not revealing is always preferred.

Proposition 1. *Given a passing threshold $s_p > \underline{s}$ such that $V_R > 0$, there exists $\omega_I \in (\omega_r, \bar{\omega})$ such that:*

(i) If $\omega_{NR} > \omega_I$, the regulator strictly prefers to reveal.

(ii) If $\omega_{NR} \in (\underline{\omega}, \omega_I)$, the regulator strictly prefers not to reveal.

(iii) If $\omega_{NR} = \omega_I$, the regulator is indifferent between revealing and not revealing.

The indifference point ω_I is the unique $\omega' > \underline{\omega}$ that solves

$$\int_{\omega \geq \underline{\omega}} [1 - F(s_p|\omega)]v(\omega)dG(\omega) = \int_{\omega \geq \omega'} [1 - F(s_p|\omega)]v(\omega)dG(\omega).$$

The proposition captures the tradeoff entailed by not revealing. Not revealing reduces or completely eliminates investment in states ω in which investment is not socially desirable. In other words, it reduces "overinvestment." However, it can lead to "underinvestment," in which the bank reduces investment also in states in which investment is socially desirable. If the first effect dominates, not revealing is preferred. If the second effect dominates, revealing is preferred.

4.3.2 Endogenous passing threshold

Now suppose the regulator sets the passing threshold s_p optimally. We let s_p^R denote the passing threshold that the regulator sets if he plans to reveal his signal and s_p^{NR} denote the passing threshold that the regulator sets if he does not plan to reveal. That is, $s_p^R \in \arg\max_{s_p} V_R(s_p)$ and $s_p^{NR} \in \arg\max_{s_p} V_{NR}(s_p)$.

Theorem 1. *If the passing threshold s_p is set optimally, then not revealing is strictly preferred to revealing. That is, $V_{NR}(s_p^{NR}) > V_R(s_p^R)$.*

The basic idea behind Theorem 1 is that by not revealing his signal and adjusting s_p, the regulator can eliminate the overinvestment induced by gaming without inducing underinvestment. In particular, from Proposition 1, we know that with an exogenous passing threshold, revealing is preferred only if not revealing leads to underinvestment. But if the regulator can choose the passing threshold optimally, he can reduce it so that the bank does not act too cautiously. Moreover, a lower s_p also allows the regulator to pass the bank more often, and since the bank does not overinvest, passing more often also benefits the regulator.

We finish this section with two observations:

Lemma 3. *1.* $\omega_{NR}(s_p^{NR}) < \omega_r$

2. There exists $s' < s_p$*, such that* $\int_{\omega \geq \omega_{NR}(s)} v(\omega) f(\omega|s) d\omega > 0$ *for every* $s \in [s', s_p]$

The first part in Lemma 3 says that overinvestment in the risky asset occurs even under the optimal passing threshold. Intuitively, if the bank underinvested, the regulator could gain by reducing s_p, which would allow the regulator to capture the (positive) value of the bank's investment more often and induce the bank to act less cautiously.[15] The second part says that the regulator sometimes fails the bank even though it is suboptimal to do so ex post. Intuitively, the commitment to fail the bank helps reduce overinvestment (see also Section 6).

5 Optimal disclosure

We saw that for a given passing threshold, revealing is preferred to not revealing if the latter leads the bank to act too cautiously. However, if the regulator sets the passing threshold optimally, revealing is strictly dominated. In this section, we show that once we allow for partial disclosure, revealing some information may be optimal even if the regulator sets the passing threshold optimally.

For ease of exposition, we first focus on a simple form of partial disclosure: a *cutoff rule*, which is defined by a threshold s_d, such that the regulator reveals whether the signal realization s is above or below s_d (the subscript "d" stands for "disclosure threshold"). We show that even this simple rule may be preferred to no disclosure.

[15] If $\omega_{NR} = \omega_r$, a lower ω_{NR} worsens the bank's incentives, but this effect is negligible compared to the benefit of approving the bank's investment more often.

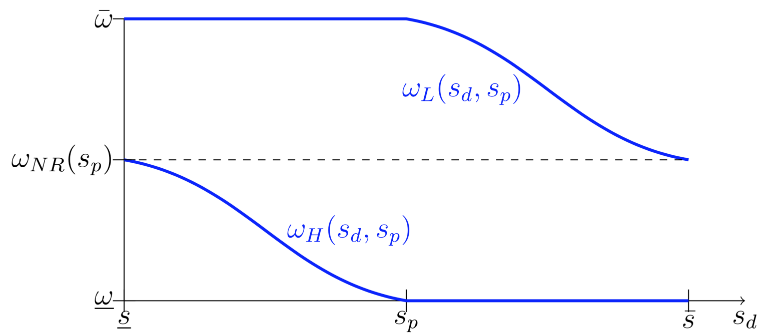


Figure 2: The bank's investment thresholds $\omega_H(s_d, s_p)$ and $\omega_L(s_d, s_p)$ induced by sending a high message and low message, respectively, as functions of s_d while holding s_p fixed.

5.1 Bank investment

Under a cutoff disclosure rule, the regulator sends the bank one of two messages: a "low" message upon observing a signal below s_d ($s < s_d$) and a "high" message upon observing a signal above s_d ($s \geq s_d$). Based on these messages the bank forms posterior beliefs regarding the probability of passing the test (p in Equation 1). Specifically, the posterior probability of passing after a low message is $p_L(\omega) = \Pr(s \geq s_p | s < s_d, \omega)$, and the posterior probability of passing after a high message is $p_H(\omega) = \Pr(s \geq s_p | s \geq s_d, \omega)$. From Assumption 3 (MLRP), $p_L(\omega)$ and $p_H(\omega)$ are increasing in ω.[16]

As in Section 4.1, we can show that the bank follows a cutoff rule, investing if and only if the state ω is above some threshold. We let $\omega_L(s_d, s_p)$ and $\omega_H(s_d, s_p)$ denote the bank's investment threshold following a low message and a high message, respectively. So, if the regulator reveals that $s < s_d$, the bank invests if and only if $\omega \geq \omega_L(s_d, s_p)$. If the regulator reveals that $s \geq s_d$, the bank invests if and only if $\omega \geq \omega_H(s_d, s_p)$.

The next lemma characterizes the two investment thresholds (see also Figure 2). (Recall, $\omega_{NR}(s_p)$ is the bank's investment threshold under no disclosure, as in Section 4.1.)

Lemma 4. *The investment threshold $\omega_L(s_d, s_p)$ equals $\bar{\omega}$ if $s_d \leq s_p$ and strictly decreases to*

[16]In particular, given any message m, the bank's posterior beliefs satisfy MLRP, and as a result, $1 - F(s_p | m, \omega)$ is increasing in ω. The proof of Lemma C1 contains more details.

$\omega_{NR}(s_p)$ as s_d increases from s_p to $\bar{s}$. The investment threshold $\omega_H(s_d, s_p)$ strictly decreases from $\omega_{NR}(s_p)$ to $\underline{\omega}$ as s_d increases from $\underline{s}$ to s_p and equals $\underline{\omega}$ if $s_d \geq s_p$.

The logic behind Lemma 4 and Figure 2 is as follows. If $s_d \leq s_p$, the low message reveals that the regulator observed a failing signal, and so the bank does not invest ($\omega_L = \bar{\omega}$), while the high message pools together the passing signals with some of the failing signals, which leads to some investment. As s_d falls, more failing signals are pooled, and so the bank becomes more cautious, investing in fewer states (higher ω_H). If instead $s_d \geq s_p$, the high message reveals that the regulator observed a passing signal, and so the bank invests in every state ($\omega_H = \underline{\omega}$), while the low message pools together the failing signals with some of the passing signals, which leads to less investment. As s_d increases, more passing signals are pooled, and so the bank becomes less cautious, investing in more states (lower ω_L). If $s = s_p$, the outcome is the same as under full disclosure. If $s \in \{\underline{s}, \bar{s}\}$, the outcome is the same as under no disclosure.

Later, we refer to the case in which $s_d \leq s_p$ as the regulator revealing some of the failing signals, and to the case in which $s_d \geq s_p$ as the regulator revelaing some of the passing signals. In particular, if $s_d \leq s_p$, we obtain the same outcome if instead of sending the low message, the regulator reveals the actual signal realization, and if $s_d \geq s_p$, we obtain the same outcome if instead of sending the high message, the regulator reveals the actual signal realization.

In an alternative interpretation, if the regulator sets $s_d \leq s_p$, he essentially commits to publicly disapprove an asset for investment when his model forecasts a particularly low value for it and simply remain silent otherwise. Similarly, if $s_d \geq s_p$, the regulator publicly approves an asset for investment when his model forecasts a particularly *high* value for it and remains silent otherwise.

5.2 Regulator's payoff

Suppose the regulator chooses a disclosure threshold s_d and a passing threshold s_p. We derive the regulator's payoff as follows.

Conditional on observing a failing signal $s < s_p$, the regulator's payoff is zero, as in Section 4.2. Conditional on observing a passing signal $s \geq s_p$, the regulator's payoff depends on the message sent and the bank's investment rule, as follows:

$$\begin{cases} \int_{\omega \geq \omega_H(s_d,s_p)} v(\omega) f(\omega|s) d\omega & \text{if } s_d < s_p \\ 1_{s \in [s_p, s_d)} \int_{\omega \geq \omega_L(s_d,s_p)} v(\omega) f(\omega|s) d\omega + 1_{s \geq s_d} \int_{\omega_H(s_d,s_p)} v(\omega) f(\omega|s) d\omega & \text{if } s_d \geq s_p \end{cases} \tag{4}$$

Taking the expectation across all signals $s \in S$, rearranging terms, and using the observation that if $s \geq s_p$, $\omega_H(s_d, s_p) = \underline{\omega}$ (Lemma 4), we obtain that:

Lemma 5. *For a given policy (s_d, s_p), the regulator's payoff is:*

$$\begin{cases} \int_{\omega \geq \omega_H(s_d,s_p)} v(\omega)[1 - F(s_p|\omega)] dG(\omega) & \text{if } s_d < s_p \\ \int_{\omega \geq \omega_L(s_d,s_p)} v(\omega)[1 - F(s_p|\omega)] dG(\omega) + \int_{\underline{\omega}}^{\omega_L(s_d,s_p)} v(\omega)[1 - F(s_d|\omega)] dG(\omega) & \text{if } s_d \geq s_p \end{cases}$$

5.3 Exogenous passing threshold

We first solve for the optimal disclosure threshold s_d, for a given passing threshold s_p. If there are multiple solutions, we focus on the highest one (for ease of exposition) and denote it by $s_d(s_p)$. That is, $s_d(s_p)$ is the highest s_d that maximizes the regulator's expected payoff in Lemma 5 for a given s_p.

Proposition 2. *For a given passing threshold s_p:*

1. If no disclosure leads the bank to underinvest ($\omega_{NR}(s_p) > \omega_r$), it is optimal to reveal some of the failing signals. In particular, the regulator sets $s_d(s_p) \in (\underline{s}, s_p)$, so that the bank invests according to the regulator's ideal investment rule: $\omega_H(s_d(s_p), s_p) = \omega_r$.

2. If no disclosure induces the bank to invest according to the regulator's ideal investment *rule ($\omega_{NR}(s_p) = \omega_r$), then no disclosure is optimal: $s_d(s_p) \in \{\underline{s}, \bar{s}\}$.*

3. If no disclosure leads the bank to overinvest ($\omega_{NR}(s_p) < \omega_r$), then either no disclosure is optimal or else it is optimal to reveal some of the passing signals; i.e., $s_d(s_p) \in (s_p, \bar{s})$. In the latter case, $\omega_L(s_d(s_p), s_p) \in (\omega_{NR}(s_p), \omega_r]$. A sufficient condition for partial disclosure to strictly dominate no disclosure is that Equation (5) below holds.

In the first part of Proposition 2, no disclosure leads the bank to be too cautious about investing in the risky asset, and so the purpose of disclosure is to make the bank less cautious. This is done by fully revealing some of the failing signals (setting $s_d < s_p$). Then if the regulator sends the high message, the bank is less worried about failing the test and is induced to invest according to the regulator's ideal investment rule.

In part 2, no disclosure already induces the bank to invest according to the regulator's ideal investment rule. In this case, disclosure can only do harm, because it leads the bank to deviate from the regulator's ideal investment rule.

In part 3, no disclosure leads the bank to be too reckless about investing in the risky asset, and so the purpose of disclosure is to make the bank more cautious. This is done by sending a low message that pools all the failing signals with only *some* of the passing signals (setting $s_d > s_p$). However, this comes at a cost. The high message assures the bank of a passing signal, so the bank acts too recklessly, investing in every state ω. Because of this cost, partial disclosure is not necessarily optimal. Essentially, the cost of partial disclosure is similar to that of full disclosure– it enables gaming. However, with partial disclosure, gaming is a concern only when the high message is sent.

A sufficient condition for partial disclosure to be optimal in part 3 is that revealing the highest signal $\bar{s}$ (and pooling together all the other signals) leads to a better social outcome than no disclosure. In the proof, we show that this condition reduces to

$$-\int_{\underline{\omega}}^{\omega_{NR}} v(\omega) f(\bar{s}|\omega) dG(\omega) < \frac{\partial \omega_L}{\partial s_d}\Big|_{s_d=\bar{s}} v(\omega_{NR})[1 - F(s_p|\omega_{NR})]. \tag{5}$$

The left-hand-side in (5) is the marginal cost of revealing $\bar{s}$. When the bank learns that $s = \bar{s}$ (high message), its investment threshold falls from ω_{NR} to $\underline{\omega}$ –i.e., there is more gaming. The right-hand-side is the marginal benefit of revealing $\bar{s}$. When the regulator reveals that $s < \bar{s}$ (low message), the bank's investment threshold ω_L increases but remains below ω_r; i.e., there is less overinvestment.[17]

An example in which Equation (5) holds (so partial disclosure is optimal) is when the social loss from investing in the risky asset is not too high even in the worst possible state; e.g., if $v(\underline{\omega})$ is not too negative or if L in Example 1 is not too high. In this case, even if the bank games the test, it is not very costly from a social point of view. Another example is when the bank's investment is highly sensitive to partial disclosure (i.e., $\partial\omega_L/\partial s_d$ is very negative). In this case, the bank's investment threshold ω_L rises dramatically in response to even little disclosure, which increases the benefits of partial disclosure.

Under some regularity conditions, we obtain the following comparative statics with respect to the bank's cost of test failure c. If c is intermediate, no disclosure is optimal, but as c moves in either direction (increases or decreases), more disclosure is optimal. Figure 3 illustrates this.[18] When $c = 1$, no disclosure is optimal because it induces the bank to invest according to the regulator's ideal investment rule. As c increases, no disclosure induces the bank to invest too cautiously, and so the regulator reveals some of the failing signals to make the bank less cautious. In this region, a higher c leads to more disclosure, namely a lower s_d. As c decreases, no disclosure induces the bank invest more recklessly, but if the bank is not too reckless, no disclosure continues to be optimal. As c falls further, no disclosure induces the bank to invest too recklessly, and so the regulator reveals some of the passing signals (reduces s_d) to make the bank more cautious.

By adding more structure on the bank's payoff function $u(\omega)$, we can obtain similar comparative statics with respect to other bank's characteristics. For example, suppose

[17] Note that both $\frac{\partial\omega_L}{\partial s_d}$ and $v(\omega_{NR})$ have negative signs.

[18] Here we let $\Omega = [0, 1]$, $S = [0, 1]$, $u(\omega) = (3\omega)^{0.1}$, $v(\omega) = \omega - 0.5$, and $f(s|\omega) = 2[s\omega + (1 - s)(1 - \omega)]$.

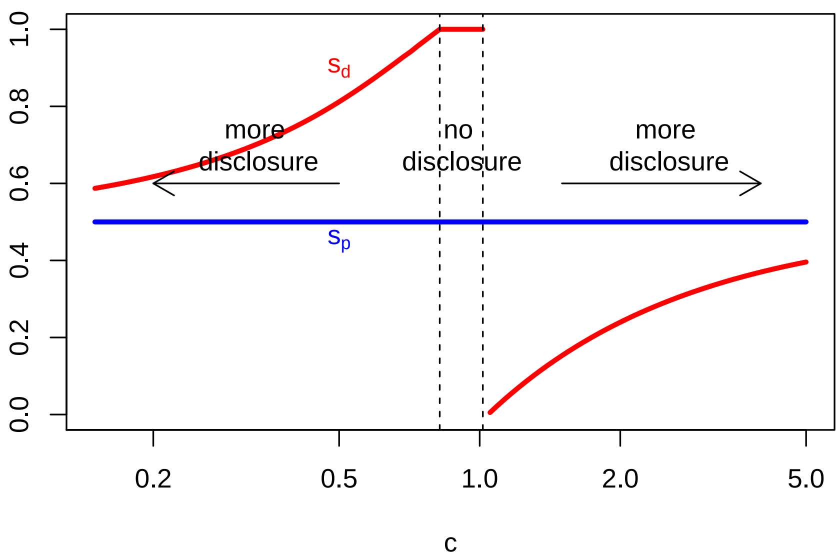


Figure 3: Optimal disclosure threshold s_d as a function of the bank's cost of test failure c, holding s_p fixed.

$u(\omega) = a \cdot \hat{u}(\omega)$, where $a > 0$ represents the bank's appetite towards the risky asset and $\hat{u}(\omega) \geq 0$. Then no disclosure is optimal if a is intermediate, but as a moves in either direction, more disclosure is optimal.[19]

5.4 Endogenous passing threshold

Next, we solve for the optimal passing threshold s_p, namely the (highest) s_p that maximizes the regulator's payoff in Lemma 5 when $s_d = s_d(s_p)$. We denote the optimal s_p by s_p^* and let $s_d^* = s_d(s_p^*)$.

Extending the logic of Lemma 3, we can show that under the optimal passing threshold, the bank overinvests upon receiving the high message. Moreover, the regulator sometimes fails the bank even though it is suboptimal to do so ex post. Formally:

Lemma 6. *1.* $\omega_H(s_d^*, s_p^*) < \omega_r$.

2. There exists $s' < s_p^*$, *such that* $\int_{\omega \geq \omega_L(s_d^*, s_p^*)} v(\omega) f(\omega|s) d\omega > 0$ *for every* $s \in [s', s_p^*]$

The first part in Lemma 6 implies that under the optimal s_p^*, the first two cases in Proposition 2 cannot happen. The first case cannot happen because it would imply that $\omega_H(s_d^*, s_p^*) = \omega_r$. The second case cannot happen because $s_d = \underline{s}$ would imply that $\omega_H(s_d^*, s_p^*) = \omega_{NR}(s_p^*) = \omega_r$. Hence, we obtain the following:

Theorem 2. *If the regulator sets the passing threshold optimally, then either no disclosure is optimal or else it is optimal to reveal some of the passing signals; i.e.,* $s_d^* \in (s_p^*, \bar{s})$. *In the latter case,* $\omega_L(s_d^*, s_p^*) \in (\omega_{NR}(s_p^*), \omega_r]$. *A sufficient condition for partial disclosure to strictly dominate no disclosure is that Equation (5) holds when* $s_p = s_p^{NR}$

The regulator has two tools to mitigate the bank's overinvestment: (i) he can increase the passing threshold s_p; and (ii) he can reveal some of the passing signals (reduce s_d), so

[19]The comparative statics with respect to a are opposite to the those with respect to c because what matters is the ratio $c/u(\omega)$, which determines the bank's investment threshold.

that the bank acts less recklessly upon receiving the low message. Theorem 2 shows that in some cases, it is optimal to combine both tools.

Clearly, the ability to fail the bank is crucial to incentivize the bank. But why would it ever be optimal to combine it with partial disclosure? The answer is that for some parameter values (e.g., if c is low), not revealing anything requires a high probability of failure (high s_p) to incentivize the bank, which wastes valuable investment. In this case, the regulator can gain by passing the bank more often, and the worsening bank incentives can then be mitigated with partial disclosure, so that the marginal benefits and marginal costs of each tool are equated.

Formally, we show in Appendix E that if $s_d^* \in (s_p^*, \bar{s})$, the first-order conditions imply that

$$\begin{aligned} &\frac{\partial s_p}{\partial \omega_L}\Big|_{s_d^*, s_p^*} \int_{\omega \geq \omega_L(s_d^*, s_p^*)} v(\omega) f(s_p^*|\omega) dG(\omega) \\ &= -v(\omega_L^*)[F(s_d^*|\omega_L^*) - F(s_p^*|\omega_L^*)] \\ &\frac{\partial s_d}{\partial \omega_L}\Big|_{s_d^*, s_p^*} \int_{\omega \leq \omega_L((s_d^*, s_p^*)} v(\omega) f(s_d^*|\omega) dG(\omega). \end{aligned} \tag{6}$$

The first line in (6) reflects the marginal cost of providing incentives via the first tool. To increase ω_L, the regulator needs to increase s_p, but then he ends up failing the bank upon observing s_p even though his expected payoff conditional on s_p is positive. The third line reflects the marginal costs of providing incentives via the second tool. To increase ω_L, the regulator needs to decrease s_d, but then the probability of sending the high message (which leads to a social loss) is higher. Finally, the second line reflects the marginal benefit, which is the same for both tools. Increasing ω_L reduces the disutility $v(\omega_L^*)$ from a socially undesirable investment, which happens if the regulator observes a passing signal but sends the low message, i.e., with probability $F(s_d^*|\omega_L^*) - F(s_p^*|\omega_L^*)$.

Under some regularity conditions, the optimal policy is as illustrated in Figure 4. The

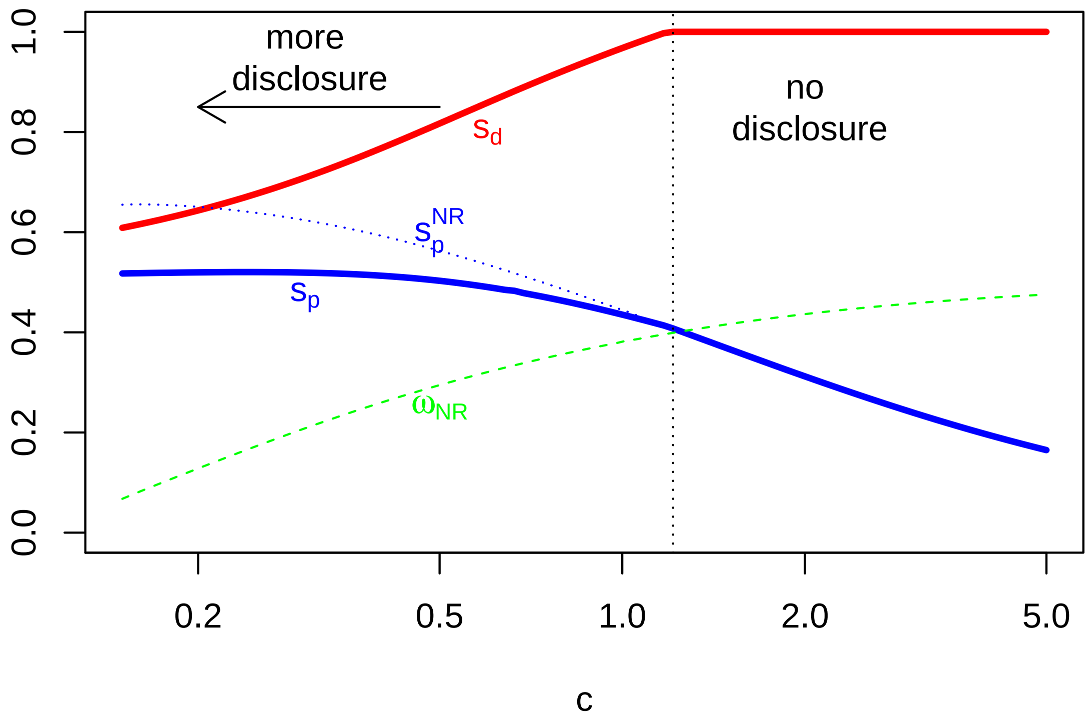


Figure 4: Optimal disclosure threshold s_d and optimal passing threshold s_p, as a function of the bank cost of test failure c, under the same assumptions as Figure 3. In this example, $\omega_r = 0.5$.

figure shows how s_p^* and s_d^* change as a function of c, and it also shows the optimal passing threshold under not revealing s_p^{NR} and the resulting investment threshold $\omega_{NR}(s_p^{NR})$. When c is sufficiently high, the regulator sets a relatively low passing threshold and does not reveal any information. In this case, even a low passing threshold provides incentives to the bank (i.e., ω_{NR} is close to ω_r), so partial disclosure is unnecessary. In contrast, when c is low, no disclosure requires the regulator to set a high passing threshold s_p^{NR} to provide incentives. But since this is costly, the regulator does not increase the passing threshold as much and instead enhances the bank's incentives via partial disclosure. Consistent with Figure 3, as c decreases, the regulator sets a lower s_d.

The comparative statics with respect to a (the bank's appetite towards the risky asset, as defined at the end of Section 5.3) are opposite to that with respect of c. When a is low, the regulator sets a relatively low passing threshold and does not reveal any information. When a is high, the regulator sets a high passing threshold and combines it with partial disclosure.

Remark 2. The comparative statics above are for the case in which the regulator follows a simple cutoff disclosure rule. In Appendix C, we show that a cutoff rule remains optimal even within a larger set of disclosure rules under which the regulator partitions the signal space into nonoverlapping intervals and reveals the interval to which the signal belongs. However, we also show that for some parameter values, the regulator can achieve a better outcome via a nonmonotone disclosure rule in which messages pool signals from disconnected intervals. For example, the regulator sends the high message not only upon observing the highest passing signals but also upon observing some very low failing signals. This type of pooling can help reduce the cost of sending the high message because the bank does not act too recklessly upon receiving that message.

6 Discussion

In this section, we discuss some of the assumptions, interpretations, policy implications, and possible extensions of the model.

1. In our model, the bank chooses a portfolio (safe asset or risky asset). In Appendix A we illustrate that our model also maps to a case in which the bank's portfolio is given, and instead of choosing a portfolio, the bank submits a capital plan, which needs to be approved by the regulator. The capital plan is to either retain cash (safe action) or distribute it as dividends (risky action). In this case, revealing the regulator's signal can induce the bank to retain too little cash, which could result in financial distress. But not revealing can lead the bank to retain too much cash, which could result in wasteful investment (e.g., because of a free cash flow problem as in Jensen (1986)).

2. We assumed that the regulator has full flexibility in adjusting the passing threshold. The result that not revealing is strictly preferred to revealing (Theorem 1) relies on this assumption. However, in practice, the regulator may not have such flexibility, and so revealing might be preferred. One example is when the passing threshold is given exogenously, as in Proposition 1. Another example is when the regulator must apply the same passing threshold to banks with different characteristics. This case could arise because of practical considerations, or because the bank's characteristics are privately observed by the bank. We show that if banks are sufficiently different from one another, then for some parameter values, revealing is strictly preferred to not revealing. We provide a formal statement of this result in Appendix B, but the intuition is simple. The benefit from not revealing the regulator's signal is that by choosing the passing threshold appropriately, the regulator can affect the bank's investment threshold in his favor. But if banks are very different from one another, it is impossible to calibrate the passing threshold to induce desired investment by everyone.

3. Under the optimal policy, the regulator sometimes fails the bank even though it is suboptimal to do so ex-post (Lemma 3 and Lemma 6). As we noted earlier, this commitment helps provide incentives to the bank. But our main results continue to hold even if the regulator cannot commit to act according to a prespecified pass/fail rule. In particular, not revealing continues to dominate revealing, and for some parameter values, partial disclosure continues to be optimal. Intuitively, the regulator's inability to act according to some prespecified rule makes it harder for the regulator to provide incentives to the bank. However, without any further restrictions, the regulator can still provide better incentives and hence achieve a better outcome than full disclosure; and in some cases, he can achieve a better outcome than no disclosure. We provide more details in Appendix D.

4. The analysis of partial disclosure relies on the assumption that the regulator can commit to act according to some prespecified disclosure rule. This commitment is important because without commitment, the regulator would prefer to deviate ex post, reporting the low message instead of the high message. We believe that a commitment to follow prespecified rules, including how much information to reveal, is reasonable in the context of annual stress tests that are conducted by the regulator. In models of repeated interaction, the commitment outcome may also be obtained without commitment.[20] If, however, the regulator cannot commit to act according to a prespecified disclosure rule, the relevant comparison might be simply between a secrecy regime and a fully transparent regime, which do not require this type of commitment.[21]

5. In our model, the regulator has two tools to provide incentives to the bank: the disclosure policy and the passing threshold. In practice, the regulator may be able to use additional tools, such as imposing penalties on banks that fail the test. We can

[20]See Mathevet, Pearce, and Stacchetti (2019) and Best and Quigley (2020).

[21]If not revealing is preferred (for a given s_p), the regulator cannot gain by revealing, because this will worsen the overinvestment problem. If revealing is preferred, we can assign out-of-equilibrium beliefs to rule out a deviation to not revealing.

incorporate this into our setting by assuming that the regulator can affect the bank's private cost of failing the test c. If the regulator has full control over c, he can get arbitrarily close to the first best by setting c close to infinity, passing the banks almost surely, and not revealing anything. However, in the more realistic case in which the regulator does not have full control over the parameter c, the main results in our paper will continue to hold. More generally, if there are multiple tools to incentivize the banks, we believe that as long as these tools cannot be fully adjusted or are costly to adjust, the result that partial disclosure may be optimal will continue to hold.

6. We assumed that if the bank fails the test, the regulator's payoff is zero. This assumption is not crucial for our main results. What is crucial is that there is some social cost of providing incentives by increasing s_p. For example, we could assume that upon failing the bank, the regulator obtains $\alpha v(\omega)$ for some $\alpha \in (0, 1)$. This case could reflect a situation in which the risky asset is transferred to other financial institutions that are less skilled at monitoring the asset but are also less systemically important. If α is not too large, it would still be costly to provide incentives by increasing s_p alone, and so partial disclosure will continue to be optimal.[22]

7. Policy makers have suggested that if the Fed model were to be published, then to counteract gaming, the minimum capital requirement would need to materially increase.[23] Our model suggests that this conclusion is only partially correct. In particular, for some parameter values, the optimal passing threshold under revealing is lower than that under not revealing: $s_p^R < s_p^{NR}$. For example, this could happen if the bank's cost of failing c is low, so the regulator needs to set a very high s_p^{NR} to reduce overinvestment.[24]

[22]For example, under the assumptions of Figure 4 and assuming $c = 0.1$, partial disclosure is optimal whenever $\alpha < 0.76$.

[23]See the departing speech by Fed Governor Daniel Tarullo: https://www.federalreserve.gov/newsevents/speech/tarullo20170404a.htm.

[24]E.g., in Figure 4, $s_p^R = 0.5$, and for a sufficiently low c, $s_p^{NR} > 0.5 = s_p^R$.

8. A widely expressed concern is that disclosing the Fed's models could increase correlations in asset holdings among banks subject to the stress tests (i.e., the largest banks), making the financial system more vulnerable to adverse financial shocks. An extension of our model would suggest that this concern is also valid if the Fed just illustrates how its models work on hypothetical loan portfolios, as under the new policy discussed in the introduction. In particular, the proposed hypothetical portfolios could serve as benchmark portfolios in which too many banks invest, leading to correlated investment. So just as in our basic model, in which the bank could underinvest in a socially valuable risky assets by choosing the safe asset for which the test results are predictable, banks could also underinvest in their idiosyncratic risky portfolios, for which the test results are unpredictable, and overinvest in the benchmark risky portfolio, for which the test results are predictable.

9. A related concern is that revealing the regulator's models will cause banks to exert less effort in developing their own models. A simple extension in which the bank needs to incur a fixed cost to obtain its private signal about ω would imply that the bank will incur this cost only if the regulator does not reveal signal. However, a complete setting that incorporates information production by the bank or by the regulator is beyond the scope of this paper. We believe that in general the conclusions will depend on how we model information production.[25,26]

10. An interesting question is how the optimal disclosure regime changes with respect to

[25] For example, one could think of a setting in which the bank can generate one of two signals: an informative signal that gives the actual realization of ω, or a less informative signal, whose only purpose is to predict the test outcome; e.g., tell whether s is above or below the passing threshold. If the cost of obtaining the second signal is sufficiently low compared to the cost of obtaining the more informative signal, the outcome might be that if the regulator does not reveal his model, the bank generates only the second signal. In this case, revealing the regulator's model could generate a better outcome by saving the inefficient information production by the bank.

[26] See also Leitner and Yilmaz (2019), who show that under some conditions, it is optimal to allow banks to produce two models: a less informative for regulation and a more informative model for their own investment decisions.

the information in the regulator's signal. A more informative signal makes it easier to incentivize the bank, which pushes towards secrecy, but a more informative signal also pushes towards revealing, because the regulator can use his information to force actions without the need to rely on the bank's information. Hence, the relationship between the informativeness of the regulator's signal and the preferred disclosure regime need not be monotone. To illustrate this, we consider a sequence of signals that becomes less informative in the sense of Blackwell (1953). That is, each signal is a garbling of the previous signal. For a fixed passing threshold, we can construct examples in which if the level of garbling is intermediate, not revealing is preferred to revealing, but if the level of garbling is either very high or very low, revealing is preferred.[27]

11. Finally, our setting is an example of a principal-agent problem in which an informed but biased agent takes an action on behalf of a partially informed principal, who can respond to the agent's action after an evaluation process that is based on the principal's private information. In our setting, the agent is the bank and the principal is the regulator, but there are other applications. For example, the agent could be a financial advisor and the principal could be a wealthy individual. Our results suggest that in some cases, the individual could benefit by not sharing his views with the financial advisor about a new investment strategy but replace the advisor if the latter suggests an investment that is deemed too risky by the individual. Similarly, the agent could be the firm's manager and the principal could be the firm's board of directors. In this case, the board could benefit by not expressing their opinions while the manager is working on a strategic plan but use their opinions to disapprove the plan if its value is deemed too low.[28]

[27] See the end of Appendix E.

[28] In both examples, it is natural to assume that the principal cannot commit to an opinion-based decision rule, but as we saw earlier (item 3 in the discussion), our results still hold in this case.

7 Conclusion

We study whether a regulator should reveal his stress tests model to banks before conducting the test. We also explore the interaction between the regulator's disclosure policy and another regulatory tool that can be used to incentivize banks: the threshold for passing the test.

We show that if the regulator has full flexibility in adjusting the passing threshold, not revealing is always preferred to revealing. However, if the regulator cannot freely adjust the passing threshold, then revealing may be preferred. Finally, if the regulator can commit to act according to a disclosure policy that goes beyond just revealing or not revealing, then for some parameter values, some disclosure is optimal even if the regulator can fully adjust the passing threshold. And if we restrict attention to monotone disclosure rules, a simple cutoff rule is optimal. We characterize the optimal cutoff rule and derive comparative statics and policy implications.

Our paper leaves open several questions that could be explored in future work. For example, our framework is static, but because regulators continually update their models, it would be interesting to explore the optimal dynamic disclosure policy. Our framework also assumes the regulator's signal is one dimensional. It would be interesting to explore the case in which the bank can invest in multiple assets, and the regulator's model takes the form of multiple signals that predict the value of each asset.

Appendix

A Capital plans

We provide an example to illustrate that our model maps to a case in which instead of choosing a portfolio, the bank submits a capital plan, which needs to be approved by the regulator.

Suppose the bank has already invested in the risky asset from Example 1. In addition to this asset, the bank has $1 + \delta$ dollars in excess cash, which can either be retained at the bank or paid as dividends. If the bank retains the cash, managers invest it in a negative NPV project that gives only 1. That is, managers waste δ, which could result from a free cash flow problem (Jensen (1986)). Also assume that if the bank's cash holding falls below 1 to $z < 1$, the bank suffers a loss $r(1 - z)$ and the regulator suffers a loss $L(1 - z)$, where $L > r > 0$. These losses, which represent costs of financial distress, can be motivated as in Example 1.

Final payoffs are as follows. If the bank retains the cash, the bank's cash holding never falls below 1. Hence, the final payoff to both the bank and the regulator is $1 + 2(1 - p) + p\omega$. The first term is the payoff from investing the cash, and the other terms represent the expected payoff from the risky asset.

If the bank does not retain the cash, then during a crisis the bank's cash holdings fall below 1 (to ω). So the expected losses to the bank and regulator are $pr(1-\omega)$ and $pL(1-\omega)$, respectively. The final payoffs are the sum of dividend payment, payoff from the risky asset, and expected loss due to financial distress. So the final payoffs to the bank and to the regulator are $1 + \delta + 2(1 - p) + p\omega - pr(1 - \omega)$ and $1 + \delta + 2(1 - p) + p\omega - pL(1 - \omega)$, respectively. Normalizing the payoff from the safe action (of retaining cash) to zero, the functions u and v are given by $u(\omega) = \delta - pr(1 - \omega)$ and $v(\omega) = \delta - pL(1 - \omega)$.[29]

[29]Note that u and v are increasing in ω, and if r and L are chosen appropriately, Assumptions 1 and 2 are

As before, the regulator approves the risky action (in this case, paying dividends) only if the bank passes the test. If the bank retains cash, it always passes the test. Otherwise, it passes the test only if the projected value of its risky asset during a crisis is sufficiently high (i.e., if $s \geq s_p$). As before, if the bank fails the test, the bank's final payoff is $-c < 0$. Recall that c can represent the cost to the bank's managers from failing the test. Here c can also represent an upfront cost that the bank needs to incur to submit a capital plan that involves a dividend distribution, and as before, this cost is a transfer to other agents and does not affect the regulator's payoff.

B Heterogeneous banks

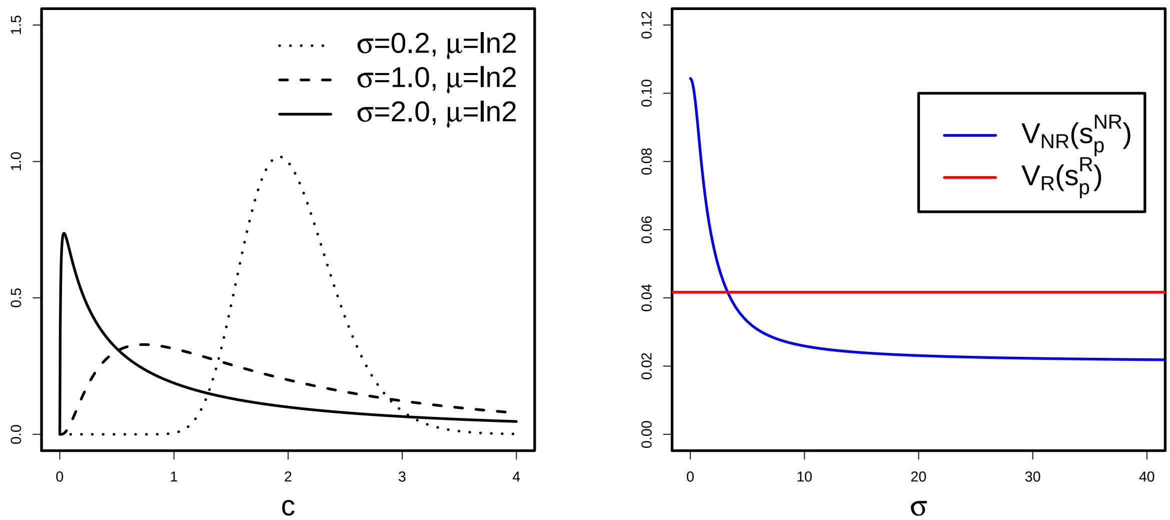
We analyze the case in which the regulator must apply the same passing threshold to banks with different characteristics. Suppose the bank's private cost of failure c is a random variable with a CDF H. The bank observes the realization of c but the regulator does not. Recall that under not revealing, the bank expects to pass the test with probability $p(\omega) = 1 - F(s_p|\omega)$. Rearranging Equation (1), it follows that the bank invests in state ω if and only if $c \leq [F(s_p|\omega)^{-1} - 1]u(\omega)$, i.e., with probability $I(\omega, s_p) \equiv H([F(s_p|\omega)^{-1} - 1]u(\omega))$. Extending the logic of Lemma 2, we obtain that the regulator's payoff under not revealing is:

$$V_{NR} = \int_{\omega \geq \underline{\omega}} I(\omega, s_p)[1 - F(s_p|\omega)]v(\omega)dG(\omega). \tag{B1}$$

The payoff under revealing does not depend on H and is given by $V_R(s_p^R)$, as in Lemma 2. In the special case in which H has all of the mass on a particular c, H is a step function, and (B1) reduces to (3).

To formalize the idea of banks that are sufficiently different from one another, we examine a sequence of distributions H_i that are median-preserving spreads in c, with a limiting

satisfied.



(a) (b)

Figure B1: Panel (a) shows the density of c, when its distribution is lognormal with parameters $\mu = \ln 2$ and various values of σ. Panel (b) shows the regulator's payoff under revealing, $V_R(s_p^R)$, and under not revealing, $V_{NR}(s_p^{NR})$, as a function of σ.

distribution that places half the mass on $c = 0$ and half the mass on $c = \infty$.[30] We show that if $V_R(s_p^R)$ is sufficiently high, as we make precise in the proof, then in the limit, revealing is preferred.

Proposition B1. *If $V_R(s_p^R)$ is sufficiently high, then for any sequence $\{H_i\}_{i=1}^{\infty}$ of distribution functions satisfying*

1. *H_{i+1} is a median-preserving spread of H_i for all $i \in N$; and*

2. *$\lim_{i\to\infty} H_i(c) = \frac{1}{2}$ for all $c > 0$,*

revealing is strictly preferred to not revealing for high enough i.

---

[30] H_b is a *median-preserving spread* of H_a if H_a and H_b have the same median m, $H_b(x) \geq H_a(x)$ for all $x \leq m$, and $H_b(x) \leq H_a(x)$ for all $x \geq m$, with a strict inequality for at least one x.

Figure B1 illustrates the result above for the case in which H is lognormal with parameters $\mu = \ln 2$ and various values of σ, which amounts to fixing the median of H at 2 and increasing uncertainty by increasing σ. For a very low level of uncertainty, not revealing is strictly optimal. For a very high level of uncertainty, revealing is strictly optimal.

C General disclosure

We solve for an optimal disclosure rule for a given passing threshold s_p. To avoid technical issues, we assume that ω and s are drawn from finite sets Ω and S. We denote the elements of Ω by $\omega_1 < \omega_2 < .... < \omega_n$, assume that $\omega_r \in \Omega$, and let i_r denote the $i \in \{1, ..., n\}$ such that $\omega_i = \omega_r$. We use $f(s|\omega)$ and $g(\omega)$ to denote probability mass functions. A *disclosure rule* is defined by a finite set of messages M and a function h that maps each signal $s \in S$ to a distribution over messages. We let $h_m(s)$ denote the probability that the regulator sends message m upon observing s. ($\sum_{m \in M} h_m(s) = 1$ for every $s \in S$.)

C.1 Regulator's problem

As in Section 4.1, we first show that the bank follows a cutoff rule, investing if and only if the state ω is above some threshold. We denote the decision to not invest by the threshold $\omega_{n+1} > \omega_n$ and let $\Omega' \equiv \Omega \cup \{\omega_{n+1}\}$. Formally:

Lemma C1. *For any disclosure rule (M, h), there exists a function $\omega : M \to \Omega'$ such that if the regulator sends message $m \in M$, the bank invests if and only if $\omega \geq \omega(m)$.*

Lemma C1 implies that sending a message is equivalent to sending an investment recommendation $\omega_i \in \Omega'$ such that the bank invests if and only if $\omega \geq \omega_i$. Using a "revelation principle" we can assume, without loss of generality, that the regulator sends only recommendations that the bank obeys.[31] The obedience constraints are that if the bank observes

[31] See Bergemann and Morris (2019).

state ω, and the regulator recommends investment threshold ω_i, then if $\omega < \omega_i$, the bank cannot gain by investing and if $\omega \geq \omega_i$, the bank cannot gain by not investing.

In a slight abuse of notation, we let $h_i(s)$ denote the probability that the regulator recommends $\omega_i \in \Omega'$ upon observing s. We let $v_i(s) \equiv \sum_{\omega \geq \omega_i} v(\omega) f(\omega|s)$.

Lemma C2. *The regulator's problem reduces to choosing a set of functions* $\{h_i : S \longrightarrow [0,1]\}_{i=1,\ldots,n+1}$ *to maximize*

$$\sum_{s \geq s_p} f(s) \sum_{i=1}^{n} v_i(s) h_i(s) \tag{C1}$$

such that

$$u(\omega_{i-1}) \sum_{s \geq s_p} f(s|\omega_{i-1}) h_i(s) - c \sum_{s < s_p} f(s|\omega_{i-1}) h_i(s) \leq 0 \qquad i = 2, .., n+1 \tag{C2}$$

$$\sum_{i=1}^{n+1} h_i(s) = 1 \qquad s \in S. \tag{C3}$$

Equation (C1) is the regulator's expected payoff if the bank's follows the regulator's recommendations. In particular, conditional on observing a failing signal $s < s_p$, the payoff is zero, and conditional on observing a passing signal $s \geq s_p$ and sending recommendation ω_i, the payoff is $v_i(s)$. Equation (C2) says that if the regulator recommends investment threshold ω_i, the bank cannot gain by investing upon observing the lower state ω_{i-1}. To see that, note that by Bayes' rule, the probability of passing the test conditional on state ω and recommendation ω_i is

$$p_i(\omega) \equiv \frac{\sum_{s \geq s_p} f(s|\omega) h_i(s)}{\sum_s f(s|\omega) h_i(s)}. \tag{C4}$$

So the bank cannot gain from investing in state ω_{i-1} if and only if

$$u(\omega_{i-1}) p_i(\omega_{i-1}) - c[1 - p_i(\omega_{i-1}] \leq 0, \tag{C5}$$

which reduces to equation (C2). Equation (C3) simply says that conditional on observing a signal, the regulator sends a recommendation with probability 1. In the proof, we show that the solution to the linear programming problem above also satisfies the other obedience constraints.

C.2 Properties of optimal disclosure rules

If $\omega_{NR}(s_p) \geq \omega_r$, we know from Proposition 2 that a cutoff rule can implement ω_r and hence is optimal. The rest of this section focuses on the case $\omega_{NR}(s_p) < \omega_r$, in which no disclosure leads the bank to act too recklessly, and the purpose of disclosure is to make the bank act more cautiously. We first show that the obedience constraints must be binding and recommended investment thresholds never exceed ω_r. In particular, the regulator never recommends the bank not to invest.

Lemma C3. *If $\omega_{NR}(s_p) < \omega_r$, then under an optimal disclosure rule:*

1. *Equation (C2) is satisfied with equality.*
2. *$h_i(s) = 0$, for every $s \in S$ and every $i > i_r$.*

The next proposition shows that optimal disclosure is "single-peaked." That is, recommended thresholds weakly increase for failing signals $s < s_p$ and weakly decrease for passing signals $s \geq s_p$.

Proposition C1. *If $\omega_{NR}(s_p) < \omega_r$, then under an optimal disclosure rule, the following hold:*

1. *For every $\omega_i > \omega_j$ and $s < s' < s_p$, if $h_i(s) > 0$, then $h_j(s') = 0$.*
2. *For every $\omega_i < \omega_j$ and $s > s' \geq s_p$, if $h_i(s) > 0$, then $h_j(s') = 0$.*

The first part in Proposition C1 says that if the regulator recommends ω_i in some failing signal $s < s_p$, he never makes a lower recommendation in a higher failing signal. The second part says that if the regulator recommends ω_i in some passing signal $s \geq s_p$, he never makes a lower recommendation in a lower passing signal.

The idea behind Proposition C1 is as follows. To induce $\omega_i \leq \omega_r$, the regulator must pool failing signals with passing signals. From equation (C2), the most efficient way to do so is to increase the probability $h_i(s)$ in passing signals $s \geq s_p$ that have a low $f(s|\omega_{i-1})$ and failing signals $s < s_p$ that have a high $f(s|\omega_{i-1})$. In other words, the regulator recommends ω_i in passing signals which a bank that observes ω_{i-1} thinks are relatively less likely, and in failing signals that a bank that observes ω_{i-1} thinks are relatively more likely. By MLRP, higher types ω_i place more weight on higher signals s. This leads to increasing recommendations in failing signals $s < s_p$ and decreasing recommendations in passing signals $s \geq s_p$. For passing signals, an additional force leads to decreasing recommendations. When the regulator observes a higher passing signal $s \geq s_p$, he is less worried about investment in low states ω, because by MLRP, these are less likely. Hence, he can recommend a lower investment threshold.

C.3 Monotone rules

Proposition C1 implies that in general the cutoff disclosure rule from Section 5 need not be optimal. However, a cutoff rule is optimal if we restrict attention to "monotone disclosure rules" under which the regulator partitions the signal space into disjoint intervals and reveals the interval to which the signal belongs.

The idea is as follows. For any set of intervals, the regulator can obtain the same payoff by merging all the intervals that contain only failing signals into one interval, and all the intervals that contain only passing signals into a second interval. Hence, without loss of generality, there are at most three intervals: (i) an interval containing only failing signals, (ii) an interval containing only passing signals, and (iii) an interval containing both passing and failing signals.

If there are two intervals or less, a cutoff rule is optimal. Otherwise, we obtain a contradiction as follows. Suppose there are three intervals, which are defined by the cutoffs s_1 and s_2, where $s_1 < s_p < s_2$, and suppose that the message that the regulator sends upon observing $s \in (s_1, s_2)$ ("the middle message") induces the bank to invest if and only if $\omega \geq \omega'$. If $\omega' < \omega_r$, the regulator can obtain a better outcome by reducing s_1. If $\omega' > \omega_r$, the regulator can obtain a better outcome by increasing s_1. If $\omega' = \omega_r$, it is possible to obtain a better outcome by increasing s_2 and reducing s_1, so that the middle message continues to implement ω_r.

D No Commitment to Pass/Fail Rule

We discuss the case in which the regulator cannot commit to a pass/fail rule. We first show that not revealing continues to be strictly preferred to revealing. As a preliminary, observe that by MLRP, $E[v(\omega)|s]$ is increasing in s, and from the first-order condition for V_R in Lemma 2, s_p^R is the unique $s \in S$ that solves $E[v(\omega)|s] = 0$.

Consider a pure strategy equilibrium in which the regulator passes the bank if and only if $s \in S_p$, where $S_p \subseteq S$. If the regulator reveals his signal, the bank invests if and only if $s \in S_p$. In this case, the bank's action conveys no additional information to the regulator about ω. Hence, the regulator expects to get $E[v(\omega)|s]$ if he passes the bank, and so he passes the bank if and only if $E[v(\omega)|s] \geq 0$. Therefore, the regulator follows a cutoff rule, and the cutoff is the same as under the commitment case. That is, $S_p = \{s : s \geq s_p^R\}$. So the outcome that is obtained under commitment to a pass/fail rule is also obtained without commitment.

Now suppose the regulator does not reveal his signal. Consider an equilibrium in which the bank invests if and only if $\omega \in \Omega_B$, where $\Omega_B \subseteq \Omega$ and $\Omega_B \neq \varnothing$ (empty set).[32] Then the regulator expects to get $E[v(\omega)|s, \omega \in \Omega_B]$ if he passes the bank. By MLRP, $E[v(\omega)|s, \omega \in \Omega_B]$ is strictly increasing in s. Hence, there exists a unique $s_p^{NC} \in S$, such that the regulator passes the bank if and only if $s \geq s_p^{NC}$.[33] ("NC" stands for no commitment

[32] There are also some uninteresting equilibria in which the bank never invests (e.g., because the regulator never passes the bank or because the regulator passes the bank only if he observes a very high signal).

[33] If the regulator always failed the bank, the bank would not invest.

under not revealing.) Following the logic in Section 4.1, the bank will invest if and only if $\omega \geq \omega_{NR}(s_p^{NC})$. That is, $\Omega_B = \{\omega \in \Omega : \omega \geq \omega_{NR}(s_p^{NC})\}$. The regulator's payoff is then $V_{NC} \equiv \int_{s_p^{NC}} E[v(\omega)|s, \omega \in \Omega_B] f(s) ds$. Since $v' > 0$ and $\omega_{NR}(s_p^{NC}) > \underline{\omega}$ (by Assumption 4), $E[v(\omega)|s_p^R, \omega \in \Omega_B] > E[v(\omega)|s_p^R] = 0$. Hence, $s_p^{NC} < s_p^R$, implying that

$$V_{NC} > \int_{s_p^R} E[v(\omega)|s, \omega \in \Omega_B] f(s) ds = \int_{s_p^R} \int_{\omega_{NR}(s_p^{NR})} v(\omega) f(\omega|s) d\omega f(s) ds.$$

Moreover, $\omega_{NR}(s_p^{NC}) < \omega_r$, because if to the contrary $\omega_{NR}(s_p^{NC}) \geq \omega_r$, then $E[v(\omega)|s, \omega \in \Omega_B] > 0$ for every $s \in S$, which implies that $s_p^{NC} = \underline{s}$ and $\omega_{NR}(s_p^{NC}) = \underline{\omega} < \omega_r$, a contradiction. Hence,

$$\int_{s_p^R} \int_{\omega_{NR}(s_p^{NR})} v(\omega) f(\omega|s) d\omega f(s) ds > \int_{s_p^R} \int_{\underline{\omega}} v(\omega) f(\omega|s) d\omega f(s) ds = V_R(s_p^R).$$

Hence, $V_{NC} > V_R(s_p^R)$.

Finally, we can extend the logic above to the case in which the regulator follows a cutoff disclosure rule. In this case, there exist thresholds s_{pH}^{NC}, s_{pL}^{NC}, such that the regulator follows the following strategy: after sending a high message, he passes the bank if and only if $s \geq s_{pH}^{NC}$, and after sending the low message, he passes the bank if and only if $s \geq s_{pL}^{NC}$. We can construct examples in which partial disclosure gives a better outcome than no disclosure, even without commitment to a pass/fail rule.

E Proofs for Main Text

Proof of Lemma 1. All the necessary steps for Part 1 and the cutoff rule in Part 2 are explained in the text. To see why ω_{NR} is continuous and increasing in s_p and c, apply the implicit function theorem to $[1 - F(s_p|\omega_{NR})]u(\omega_{NR}) - F(s_p|\omega_{NR})c = 0$, i.e., equation (1) with $\omega = \omega_{NR}$ and $p = 1 - F(s_p|\omega_{NR})$.

Proof of Lemma 2. Consider first revealing. Conditional on observing a failing signal, the regulator's payoff is zero. Conditional on observing a passing signal, the payoff is $\int_{\omega\geq\underline{\omega}} v(\omega)f(\omega|s)d\omega$. Hence, the expected payoff is

$$\int_{s\geq s_p}\int_{\omega\geq\underline{\omega}} v(\omega)f(\omega|s)d\omega f(s)ds = \int_{\omega\geq\underline{\omega}}\int_{s\geq s_p} v(\omega)f(\omega,s)dsd\omega$$
$$= \int_{\omega\geq\underline{\omega}} v(\omega)\int_{s\geq s_p} f(s|\omega)dsg(\omega)d\omega = \int_{\omega\geq\underline{\omega}} v(\omega)[1-F(s_p|\omega)]dG(\omega)$$

The payoff under not revealing is obtained in a similar fashion, but the integral starts in ω_{NR} rather than $\underline{\omega}$.

Proof of Proposition 1. Observe that V_{NR} is continuous in ω_{NR}, and $\frac{\partial V_{NR}}{\partial\omega_{NR}} = -v(\omega_{NR})[1-F(s_p|\omega_{NR})]$. Hence, $\frac{\partial V_{NR}}{\partial\omega_{NR}}$ has an opposite sign to $v(\omega_{NR})$. So V_{NR} strictly increases from $V_R>0$ over the interval $\omega_{NR}\in(\underline{\omega},\omega_r)$, and strictly decreases to zero over the interval $(\omega_r,\bar{\omega}]$. The first three results follow. The indifference point solves $V_{NR}=V_R$. Finally, since $u(\underline{\omega})=0$ (Assumption 4), we must have $\omega_{NR}>\underline{\omega}$, because if the bank invests when $\omega=\underline{\omega}$, it obtains a negative payoff.

Lemma E1. *$s_p^{NR}>\underline{s}$ and $s_p^R>\underline{s}$.*

Proof. Observe that $\frac{dV_{NR}}{ds_p} = \frac{\partial V_{NR}}{\partial s_p} + \frac{\partial V_{NR}}{\partial\omega_{NR}}\frac{\partial\omega_{NR}}{\partial s_p}$, $\frac{\partial V_{NR}}{\partial\omega_{NR}} = -[1-F(s_p|\omega_{NR}]v(\omega_{NR})$, and

$$\frac{\partial V_{NR}}{\partial s_p} = -\int_{\omega\geq\omega_{NR}} f(s_p|\omega)v(\omega)dG(\omega) = -f(s_p)\int_{\omega\geq\omega_{NR}} v(\omega)f(\omega|s_p)d\omega \qquad \text{(E1)}$$

Since $\omega_{NR}(\underline{s})=\underline{\omega}$, it follows that $\frac{\partial V_{NR}}{\partial s_p}|_{\underline{s}} = -\int_{\omega\geq\underline{\omega}} f(\underline{s}|\omega)v(\omega)dG(\omega) = -f(\underline{s})E[v(\omega)|\underline{s}]>0$ (Assumption 4). Moreover, $v(\underline{\omega})<0$ (Assumption 1), so $\frac{\partial V_{NR}}{\partial\omega_{NR}}|_{\underline{s}}>0$. Finally, since $\frac{\partial\omega_{NR}}{\partial s_p}\geq 0$ (Lemma 1), we obtain $\frac{dV_{NR}}{ds_p}|_{\underline{s}}>0$. Hence, $s_p^{NR}>\underline{s}$. To show that $s_p^R>\underline{s}$, observe that $\frac{dV_R}{ds_p}|_{\underline{s}} = \frac{\partial V_R}{\partial s_p}|_{\underline{s}} = -f(\underline{s})E[v(\omega)|\underline{s}]>0$.

Proof of Theorem 1. Suppose to the contrary that revealing is weakly preferred:

$V_R(s_p^R) \geq V_{NR}(s_p^{NR})$. Since $V_{NR}(s_p^{NR}) \geq V_{NR}(s_p^R)$, it follows that $V_R(s_p^R) \geq V_{NR}(s_p^R)$. Hence, from Proposition 1, $\omega_{NR}(s_p^R) > \omega_r$. Since $\omega_{NR}(\cdot)$ is continuous, increasing, and equals $\underline{\omega}$ at $\underline{s}$, there exists $\hat{s} \in (\underline{s}, s_p^R)$ such that $\omega_{NR}(\hat{s}) = \omega_r$. It then follows from Assumption 2 that

$$
\begin{aligned}
V_R(s_p^R) = \int_{\omega \geq \underline{\omega}} [1 - F(s_p^R|\omega)]v(\omega)dG(\omega) &< \int_{\omega \geq \omega_r} [1 - F(s_p^R|\omega)]v(\omega)dG(\omega) \\
&< \int_{\omega \geq \omega_r} [1 - F(\hat{s}|\omega)]v(\omega)dG(\omega) = V_{NR}(\hat{s}) \leq V_{NR}(s_p^{NR}),
\end{aligned}
$$

which is a contradiction.[34]

Proof of Lemma 3. To prove the lemma, we use the observations in the beginning of Lemma E1. Also, applying the implicit function theorem as in the proof of Lemma 1, we obtain that $\frac{\partial \omega_{NR}}{\partial s_p}|_{s_p^{NR}} \geq 0$, with strict inequality if $\omega_{NR} < \bar{\omega}$.

1. Suppose to the contrary that $\omega_{NR}(s_p^{NR}) \geq \omega_r$. Then $v(\omega_{NR}(s_p^{NR})) \geq 0$ (Assumption 2). Hence, $\frac{\partial \omega_{NR}}{\partial s_p}|_{s_p^{NR}} \geq 0$, $\frac{\partial V_{NR}}{\partial s_p}|_{s_p^{NR}} < 0$, and $\frac{\partial V_{NR}}{\partial \omega_{NR}}|_{s_p^{NR}} \leq 0$. Hence, $\frac{dV_{NR}}{ds_p}|_{s_p^{NR}} < 0$, which implies that $s_p^{NR} = \underline{s}$. But from the proof of Theorem 1, Assumption 4 implies that $s_p^{NR} > \underline{s}$.

2. By Part 1, $\omega_{NR}(s_p^{NR}) < \omega_r$. Hence, $\frac{\partial \omega_{NR}}{\partial s_p}|_{s_p^{NR}} > 0$ and $\frac{\partial V_{NR}}{\partial \omega_{NR}}|_{s_p^{NR}} > 0$. Moreover, $s_p^{NR} < \bar{s}$, because $s_p^{NR} = \bar{s}$ would imply that $\omega_{NR}(s_p^{NR}) = \bar{\omega}$. Hence, $\frac{dV_{NR}}{ds_p}|_{s_p^{NR}} \leq 0$. Hence, $\frac{\partial V_{NR}}{\partial s_p} < 0$, which implies $\int_{\omega \geq \omega_{NR}} v(\omega)f(\omega|s_p^{NR})d\omega > 0$. By continuity, there exists a neighborhood $[s', s_p^{NR}]$ such that $s \in [s', s_p^{NR}]$ implies $\int_{\omega \geq \omega_{NR}(s_p^{NR})} v(\omega)f(\omega|s)d\omega > 0$.

Proof of Lemma 4. As explained in the text, $p_L(\omega)$ and $p_H(\omega)$ are increasing in ω. Consider the low message. If $s_d \leq s_p$, then $p_L(\omega) = 0$ and the bank's payoff (Equation (1)) is $-c < 0$. So the bank does not invest: $\omega_L(s_d, s_p) = \bar{\omega}$. If $s_d > s_p$, then $p_L(\omega) = 1 - \frac{F(s_p|\omega)}{F(s_d|\omega)}$, and the bank's investment threshold ω_L solves $p_L(\omega)[u(\omega) + c] - c = 0$. The left-hand side is increasing in ω and s_d. So by the implicit function theorem, $\frac{\partial}{\partial s_d}\omega_L(s_d, s_p) < 0$. If $s_d = 1$, then $p_L(\omega) = 1 - F(s_p|\omega)$ which is the same as the posteriors under no disclosure. So

[34] The proof also works if $\hat{s}$ is chosen such that $\omega_{NR}(\hat{s}) \in (\omega_r, \omega_I)$, where ω_I is the indifference point in Proposition 1 when $s_p = s_p^R$.

$\omega_L(s_d,s_p)=\omega_{NR}(s_p)$.

Next, consider the high message. If $s_d \leq s_p$, then $p_H(\omega)=\frac{1-F(s_p|\omega)}{1-F(s_d|\omega)}$, and the bank's investment threshold ω_L solves $p_H(\omega)[u(\omega)+c]-c=0$. Since the left-hand-side is increasing in ω and s_d, it follows that $\frac{\partial}{\partial s_d}\omega_H(s_d,s_p)<0$. If $s_d=0$, then $p_H(\omega)=1-F(s_p|\omega)$, which is the same as the posteriors under no disclosure. So $\omega_H=\omega_{NR}(s_p)$. If $s_d>s_p$, then $p_H(\omega)=1$, and the bank's payoff is $u(\omega)>0$, so it's clearly optimal to invest: $\omega_H(s_d,s_p)=\underline{\omega}$.

Proof of Lemma 5. From equation (4) and the observation that conditional on $s<s_p$, the payoff is zero, we obtain that if $s_d<s_p$, the regulator's payoff is

$$\int\limits_{s\geq s_p}\int_{\omega\geq\omega_H(s_d,s_p)} v(\omega)f(\omega|s)f(s)d\omega ds=\int_{\omega\geq\omega_H(s_d,s_p)} v(\omega)\int\limits_{s\geq s_p} f(\omega|s)f(s)dsd\omega$$
$$=\int_{\omega\geq\omega_H(s_d,s_p)} v(\omega)\int\limits_{s\geq s_p} f(s|\omega)dsg(\omega)d\omega=\int_{\omega\geq\omega_H(s_d,s_p)} v(\omega)[1-F(s_p|\omega)]dG(\omega).$$

If $s_d>s_p$, then since $\omega_H(s_d,s_p)=\underline{\omega}$, the regulator's payoff is

$$\int\limits_{s\in(s_p,s_d)}\int_{\omega\geq\omega_L(s_d,s_p)} v(\omega)f(\omega|s)f(s)d\omega ds+\int\limits_{s\geq s_d}\int_{\omega\geq\underline{\omega}} v(\omega)f(\omega|s)f(s)d\omega ds$$
$$=\int\limits_{s\geq s_p}\int_{\omega\geq\omega_L(s_d,s_p)} v(\omega)f(\omega|s)f(s)d\omega ds$$
$$-\int\limits_{s\geq s_d}\int_{\omega\geq\omega_L(s_d,s_p)} v(\omega)f(\omega|s)f(s)d\omega ds+\int\limits_{s\geq s_d}\int_{\omega\geq\underline{\omega}} v(\omega)f(\omega|s)f(s)d\omega ds$$
$$=\int\limits_{s\geq s_p}\int_{\omega\geq\omega_L(s_d,s_p)} v(\omega)f(\omega|s)f(s)d\omega ds+\int\limits_{s\geq s_d}\int_{\underline{\omega}}^{\omega_L(s_d,s_p)} v(\omega)f(\omega|s)f(s)d\omega ds$$
$$=\int_{\omega\geq\omega_L(s_d,s_p)} v(\omega)\int\limits_{s\geq s_p} f(s|\omega)ds\ g(\omega)d\omega+\int_{\underline{\omega}}^{\omega_L(s_d,s_p)} v(\omega)\int\limits_{s\geq s_d} f(s|\omega)ds\ g(\omega)d\omega$$
$$=\int_{\omega\geq\omega_L(s_d,s_p)} v(\omega)[1-F(s_p|\omega)]dG(\omega)+\int_{\underline{\omega}}^{\omega_L(s_d,s_p)} v(\omega)[1-F(s_d|\omega)]dG(\omega).$$

Proof of Proposition 2.

1. In this case, we know from Lemma 4 and its proof that if $s_d \in [\underline{s}, s_p]$, $\omega_H(s_d, s_p)$ is continuous and decreasing in s_d from $\omega_{NR}(s_p) > \omega_r$ to $\underline{\omega} < \omega_r$. Hence, there exists $s_d \in (\underline{s}, s_p)$ such that $\omega_H(s_d, s_p) = \omega_r$. This s_d achieves the maximal attainable payoff to the regulator, because the bank invests according to the regulator's ideal rule whenever it passes the test. Hence, this s_d is optimal. Any other s_d leads to either under- or over-investment for each signal realization s, and hence is suboptimal.

2. If $\omega_{NR}(s_d, s_p) = \omega_r$, so the bank invests according to the regulator's ideal investment rule, then no disclosure achieves the maximal attainable payoff and hence is optimal.

3. Setting $s_d \in (\underline{s}, s_p)$ is suboptimal because it leads to $\omega_H(s_d, s_p) < \omega_{NR}(s_p)$, which reduces the regulator's payoff compared to no disclosure. Hence, either $s_d \in \{\underline{s}, \bar{s}\}$ (no disclosure) or $s_d \in [s_p, \bar{s})$ is optimal, and without loss of generality, $s_d \in [s_p, \bar{s}]$. In this case, the partial derivative of the regulator's payoff in Lemma 5 with respect to s_d is:

$$-\frac{\partial \omega_L}{\partial s_d} v(\omega_L)[F(s_d|\omega_L) - F(s_p|\omega_L)] - \int_{\underline{\omega}}^{\omega_L} v(\omega) f(s_d|\omega) dG(\omega) \tag{E2}$$

A necessary condition for no disclosure to be optimal is that the partial derivative in (E2) evaluated at $s_d = \bar{s}$ is nonnegative. Since $\omega_L(\bar{s}) = \omega_{NR}$ (Lemma 4), the necessary condition reduces to:

$$-\int_{\underline{\omega}}^{\omega_{NR}} v(\omega) f(\bar{s}|\omega) dG(\omega) \geq \frac{\partial \omega_L}{\partial s_d}\Big|_{s_d=\bar{s}} v(\omega_{NR})[1 - F(s_p|\omega_{NR})] \tag{E3}$$

If this necessary condition is violated, partial disclosure is optimal. Hence, equation (5) is a sufficient condition for partial disclosure. Finally, if $s_d \in [s_p, \bar{s})$, then $\omega_L(s_d, s_p) \in (\omega_{NR}(s_p), \omega_r]$, as follows. From Lemma 4, $\omega_L(s_d, s_p) > \omega_{NR}(s_p)$. If to the contrary $\omega_L(s_d, s_p) > \omega_r$ (underinvestment), the regulator can gain by increasing s_d so that $\omega_L(s_d, s_p) = \omega_r$. From the proof of Lemma 4, such s_d exists. Setting this s_d increases the regulator's payoff because upon receiving the low message the bank invests according to the regulator's ideal investment rule, and because a higher s_d increases the probability that the low message rather than the high message is sent.

Proof of Lemma 6.

1. If $s_d \geq s_p$, then by Lemma 4, $\omega_H(s_d, s_p) = \underline{\omega} < \omega_r$. If $s_d < s_p$, then by Lemma 5, the regulator's payoff is $\int_{\omega \geq \omega_H(s_d,s_p)} v(\omega)[1 - F(s_p|\omega)]dG(\omega)$, and we can apply the proof of Lemma 3, replacing $\omega_{NR}(s_P)$ with $\omega_H(s_d, s_p)$.

2. By Theorem 2, there are two cases: $s_d^* \in \{\underline{s}, \bar{s}\}$ or $s_d^* \in (s_p, \bar{s})$. If $s_d^* \in \{\underline{s}, \bar{s}\}$, apply the proof of Lemma 3, part 2. If $s_d^* \in (s_p^*, \bar{s})$, then by Theorem 2, $\omega_L(s_d^*, s_p^*) < \omega_r$, and so $\frac{\partial \omega_L}{\partial s_p} > 0$. The first-order condition $\frac{\partial V}{\partial s_p} = 0$ and equation (E4) then imply that $\int_{\omega \geq \omega_L} v(\omega) f(s_p^*|\omega) dG(\omega) > 0$. Since $\int_{\omega \geq \omega_L} v(\omega) f(\omega|s_p^*) d\omega f(s_p^*) = \int_{\omega \geq \omega_L} v(\omega) f(s_p^*|\omega) dG(\omega)$, the result follows by continuity.

Proof of Theorem 2. All of the necessary steps are explained in the text.

Explanation for Equation (6). Denote the regulator's payoff in Lemma 5 when $s_d \geq s_p$ by V. Then

$$\frac{\partial V}{\partial s_p} = -\frac{\partial \omega_L}{\partial s_p} v(\omega_L)[F(s_d|\omega_L) - F(s_p|\omega_L)] - \int_{\omega \geq \omega_L} v(\omega) f(s_p|\omega) dG(\omega) \tag{E4}$$

$$\frac{\partial V}{\partial s_d} = -\frac{\partial \omega_L}{\partial s_d} v(\omega_L)[F(s_d|\omega_L) - F(s_p|\omega_L)] - \int_{\omega \leq \omega_L} v(\omega) f(s_d|\omega) dG(\omega) \tag{E5}$$

From Theorem 1, we know that full disclosure is suboptimal, i.e., $s_d^* \neq s_p^*$. Hence if $s_d^* \in (s_p^*, \bar{s})$, we must have $\left.\frac{\partial V}{\partial s_d}\right|_{s_d^*, s_p^*} = 0$. Next, by Assumption 4, $s_p^* > \underline{s}$. Moreover, if $s_p = \bar{s}$, the regulator's payoff is zero, which is less than what he obtains under no disclosure (Lemma 3). Hence, $s_p^* \in (\underline{s}, \bar{s})$. Hence, $\left.\frac{\partial V}{\partial s_p}\right|_{s_d^*, s_p^*} = 0$. Since $\frac{\partial \omega_L}{\partial s_d} < 0$ and $\frac{\partial \omega_L}{\partial s_p} > 0$, we can write the first-order condition as in equation (6).

Example (Informativeness of Regulator's Signal). The garbling we consider is a mixture between the original signal $s \in [0, 1]$ that is drawn from a distribution $f(s|\omega)$

satisfying MLRP and a signal that is drawn from a uniform distribution on $[0,1]$, where the mixture weight is $\phi \in (0,1)$. Formally, define the stochastic transformation density $g(s'|s) \equiv \phi \cdot \delta(s'-s) + (1-\phi) \cdot 1$, where $\delta(\cdot)$ is the Dirac delta function. Then $\hat{f}(s|\omega) = \int_0^1 g(s|t) f(t|\omega) dt = \phi \cdot f(s|\omega) + (1-\phi) \cdot 1$ is the density of the garbled signal, and it also satisfies MLRP. If the original signal is garbled k times, the resulting density is $\hat{f}_k(s|\omega) = \phi^k \cdot f(s|\omega) + (1-\phi^k) \cdot 1$, so less weight is placed on $f(s|\omega)$ the more garbling occurs. We let $\alpha \equiv 1 - \phi^k$ be a measure of uninformativeness, define $f_\alpha(s|\omega) = (1-\alpha) f(s|\omega) + \alpha \cdot 1$, and consider the regulator's disclosure policy as α increases from zero to 1.

For example, suppose $\Omega = [0,1]$, $S = [0,1]$, $u(\omega) = \omega^4$, $v(\omega) = \omega - 0.5$, $s_p = 0.5$, $c = 1.1$, and $f(s|\omega)$ is a truncated normal distribution with mean ω and standard deviation 0.1, truncated to the interval $[0,1]$. Numerical computations show in this case that not revealing is strictly preferred if uninformativeness α is in $(.07, .66)$, and revealing is strictly preferred otherwise. So the optimal disclosure regime is nonmonotonic in the uninformativeness of the signal.

F Proof for Appendix B

Proof of Proposition B1. For a given distribution H_i denote $I_i(\omega, s_p) \equiv H_i([F(s_p|\omega)^{-1} - 1]u(\omega))$. So $V_{NR}(s_p^{NR}) = \int_{\underline{\omega}} [1 - F(s_p^{NR}|\omega)] I_i(\omega, s_p^{NR}) v(\omega) dG(\omega)$. Also, as a preliminary, observe that from Proposition 2 and Assumption 2, $V_R(s_p^R) = \int_{\underline{\omega}} [1 - F(s_p^R|\omega)] v(\omega) dG(\omega) \leq \int_{\omega_r} v(\omega) dG(\omega)$. Moreover, $V_R(s_p^R) \geq V_R(\underline{s}) = E[v(\omega)]$, and $V_R(s_p^R) \geq V_R(\bar{s}) = 0$. Hence, $V_R(s_p^R) \geq \max\{E[v(\omega)], 0\}$. Hence, there exists $\nu \in [0,1]$ such that $V_R(s_p^R) = (1-\nu) \int_{\omega_r} v(\omega) dG(\omega) + \nu \max\{E[v(\omega)], 0\}$.[35]

[35]This equation says that $V_R(s_p^R)$ is a weighted average of two extremes: the payoff under a perfectly informative signal (weight $1-\nu$) and the payoff under a perfectly uninformative signal (weight ν). To see that, note that in the first case, the regulator can set the passing threshold so that the bank invests and passes the test if and only if $\omega \geq \omega_r$. So the regulator's payoff is $\int_{\omega \geq \omega_r} v(\omega) dG(\omega)$. In the second case, the regulator either bans investment completely or always approves it. So the regulator's payoff is $\max\{E[v(\omega)], 0\}$.

To prove the proposition, assume that $\nu < 1/2$.[36] So, $V_R(s_p^R) > 0$. Fix a small $\varepsilon > 0$. From the assumptions on $\{H_i\}_{i=1}^{\infty}$, there exists $N > 0$, such that $|I_i(\omega, s) - 1/2| < \varepsilon$ for all $i \geq N$, $\omega \in \Omega$, and $s \in [\underline{s} + \varepsilon, \bar{s} - \varepsilon]$. Suppose $i \geq N$. We will show that if ε is sufficiently small, then $V_{NR}(s_p^{NR}) < V_R(s_p^R)$ for any possible s_p^{NR}. Specifically, if $s_p^{NR} \in [\underline{s} + \varepsilon, \bar{s} - \varepsilon]$, then

$$\begin{aligned} V_{NR}(s_p^{NR}) &< \int_{\underline{\omega}}^{\omega_r} [1 - F(s|\omega)] \left(\frac{1}{2} - \varepsilon\right) v(\omega) dG(\omega) + \int_{\omega_r}^{\bar{\omega}} [1 - F(s|\omega)] \left(\frac{1}{2} + \varepsilon\right) v(\omega) dG(\omega) \\ &= \frac{1}{2} V_R(s_p^{NR}) + \varepsilon \left(\int_{\omega_r}^{\bar{\omega}} [1 - F(s|\omega)] v(\omega) dG(\omega) - \int_{\underline{\omega}}^{\omega_r} [1 - F(s|\omega)] v(\omega) dG(\omega) \right), \end{aligned}$$

where the inequality follows from Assumption 2. Hence, for a small enough ε, $V_{NR}(s_p^{NR}) < V_R(s_p^{NR}) \leq V_R(s_p^R)$. Next, if $s_p^{NR} > \bar{s} - \varepsilon$, then $V_{NR}(s_p^{NR}) \leq \int_{\omega_r}^{\bar{\omega}} [1 - F(s_p^{NR}|\omega))] v(\omega) dG(\omega)$, which is less than $V_R(s_p^R)$, for a small enough ε. Finally, if $s_p^{NR} < \underline{s} + \varepsilon$, then $I_i(\omega, s_p^{NR}) \geq I_i(\omega, \underline{s} + \varepsilon) > \frac{1}{2} - \varepsilon$. Moreover, if ε is small enough, $1 - F(s_p^{NR}|\omega) > \frac{2\nu}{1-2\varepsilon}$ (because $v < 1/2$ implies that $\frac{2\nu}{1-2\varepsilon} < 1$). Hence, $[1 - F(s_p^{NR}|\omega)] I_i(\omega, s_p^{NR}) > \nu$. Hence,

$$\begin{aligned} &= V_{NR}(s_p^{NR}) < \nu \int_{\underline{\omega}}^{\omega_r} v(\omega) dG(\omega) + \int_{\omega_r}^{\bar{\omega}} v(\omega) dG(\omega) \\ &= \nu \int_{\underline{\omega}}^{\bar{\omega}} v(\omega) dG(\omega) + (1 - \nu) \int_{\omega_r}^{\bar{\omega}} v(\omega) dG(\omega) \\ &\leq \nu \max\{E[v(\omega)], 0\} + (1 - \nu) \int_{\omega_r}^{\bar{\omega}} v(\omega) dG(\omega) = V_R(s_p^R). \end{aligned}$$

This concludes the proof.

[36] This assumption says that the weight on the payoff under the more informative signal, as explained in footnote 35, is at least $\frac{1}{2}$.

G Proofs for Appendix C

Proof of Lemma C1. Consider a disclosure rule (M, h). We first show that for any $m \in M$ such that $h_m(s) > 0$ for some $s \geq s_p$, the posterior distribution $f(s|\omega, m)$ satisfies MLRP. That is, if $\omega' > \omega$, the ratio $f(s|\omega', m)/f(s|\omega, m)$ is strictly increasing in s. To see this, observe that $h_m(s) = f(m|s) = f(m|s, \omega) = f(m|s, \omega')$. Hence, from Bayes' rule,

$$\frac{f(s|\omega', m)}{f(s|\omega, m)} = \frac{f(m|s, \omega')f(s|\omega')}{f(m|\omega')}\frac{f(m|\omega)}{f(m|s, \omega)f(s|\omega)} = \frac{f(s|\omega')}{f(m|\omega')}\frac{f(m|\omega)}{f(s|\omega)},$$

which is increasing in s by Assumption 3.

We now prove the lemma. A bank that observes state ω and receives message m forms posterior belief $p_m(\omega) \equiv \Pr(s \geq s_p|\omega, m)$. So the payoff from investing in the risky asset is $u_m(\omega) \equiv u(\omega)p_m(\omega) - c[1 - p_m(\omega)]$. If the bank receives a message m such that $h_m(s) > 0$ for some $s \geq s_p$, then by the result above, $p_m(\omega)$ is strictly increasing in ω. Hence, $u_m(\omega)$ is strictly increasing in ω, and the bank follows a cutoff rule. If instead the bank receives a message m such that $h_m(s) = 0$ for every $s \geq s_p$, then $p_m(\omega) = 0$. Hence, $u_m(\omega) = -c$, implying the bank does not invest regardless of the value of ω.

Proof of Lemma C2. If the bank follows the regulator's recommendations, the regulator's payoff is (C1), as explained in the text. The regulator's problem is to choose a disclosure rule to maximize (C1) such that the bank follows the recommendations. The obedience constraints are as follows. When the bank observes state $\omega \in \Omega$ and obtains recommendation $\omega_i \in \Omega'$, it expects to pass the test with probability $p_i(\omega)$. So the payoff from investing is $u_i(\omega) \equiv u(\omega)p_i(\omega) - c[1 - p_i(\omega)]$. The bank will follow recommendation ω_{n+1}, if and only if $u_{n+1}(\omega) \leq 0$ for every $\omega < \omega_i$, and it will follow recommendation $\omega_i \in \Omega$ if and only if (i) $u_i(\omega) \geq 0$ for every $\omega \in [\omega_i, \omega_n]$, and (ii) $u_i(\omega) \leq 0$ for every $\omega < \omega_i$. By the proof of Lemma C1, $u_i(\omega)$ is either strictly increasing in ω or equals to $-c$. Hence, the obedience constraints reduce to

$$u_i(\omega_i) \geq 0 \qquad \text{if } i \in \{1, ..., n\} \tag{G1}$$

$$u_i(\omega_{i-1}) \leq 0 \qquad \text{if } i \in \{2, ..., n+1\}. \tag{G2}$$

Equation (G2) reduces to (C2), using (C4). Moreover, if the regulator never recommends ω_i (so $h_i(s) = 0$ for every $s \in S$), then (C2) is clearly satisfied Hence, a solution to the regulator's problem satisfies (C2) and (C3).

To complete the proof, we show that if $\{h_i(s)\}_{i,s}$ solves the problem in Lemma C2, then (G1) is satisfied. Suppose to the contrary that there exists $i \in \{1, ..., n\}$ such $u_i(\omega_i) < 0$. If $u_i(\omega_k) < 0$ for every $k \in \{i+1, ..., n\}$, let $j = n+1$. Otherwise, let j be the lowest $k \geq i+1$ such that $u_i(\omega_k) \geq 0$. If $j \leq i_r$, we obtain a contradiction because the regulator can increase his payoff without violating the constraints by recommending ω_j instead of ω_i. If $j > i_r$, there exists a function $q(s)$ that satisfied the following: (i) $u(\omega_{i_r-1}) \sum_{s \geq s_p} f(s|\omega_{i_r-1}) q(s) - c \sum_{s < s_p} f(s|\omega_{i_r-1}) q(s) = 0$; (ii) for every $s < s_p$, $q(s) \leq h_i(s)$, with at least one strict inequality; and (iii) for every $s \geq s_p$, $q(s) = h_i(s)$. The regulator can increase his payoff without violating the constraints if in each state s, instead of recommending ω_i with probability $h_i(s)$, he recommends ω_r with probability $q(s)$ and ω_{n+1} with probability $h_i(s) - q(s)$.

Proof of Lemma C3. Suppose $\{h_i(s)\}_{i,s}$ solves the regulator's problem.

1. We first show there exists $s' \geq s_p$ such that $h_{i_r}(s') < 1$. If not, then

$$\begin{aligned}
&u(\omega_{i_r-1}) \sum_{s \geq s_p} f(s|\omega_{i_r-1}) \cdot 1 - c \sum_{s < s_p} f(s|\omega_{i_r-1}) h_i(s) \\
&\geq u(\omega_{i_r-1}) \sum_{s \geq s_p} f(s|\omega_{i_r-1}) - c \sum_{s < s_p} f(s|\omega_{i_r-1}) > 0,
\end{aligned}$$

where the strict inequality follows since $\omega_{NR} < \omega_r$. But this contradicts (C2). Hence, there also exists $j \neq i_r$ such that $h_j(s') > 0$. Next, we show that $\{h_i(s)\}_{i,s}$ satisfies (C2) with equality. For $i = i_r$, this is true because otherwise, the regulator could improve his payoff without violating the constraints by raising $h_{i_r}(s')$ by some $\Delta > 0$ and reducing $h_j(s')$ by $\Delta > 0$. Now suppose to the contrary that (C2) is slack for some $i \notin i_r$. Then there exists $s'' < s_p$ such that $h_i(s'') > 0$. The regulator can reduce $h_i(s'')$ by some $\Delta > 0$, raise $h_{i_r}(s'')$ by Δ, raise $h_{i_r}(s')$ by $\Delta' \equiv \Delta c f(s''|\omega_{i_r-1})/[u(\omega_{i_r-1})f(s'|\omega_{i_r-1})]$, and reduce $h_j(s')$ by Δ'. If Δ is small enough, then (C2) and (C3) continue to hold, and the regulator increases his payoff by $f(s')\Delta'(v_{i_r}(s') - v_j(s')) > 0$, contradicting optimality.

2. Suppose to the contrary there exists $s \in S$ and $i > i_r$ such that $h_i(s) > 0$. Then there exists $s''' < s_p$ such that $h_i(s''') > 0$. But then the regulator can improve his payoff without violating the constraints by adjusting the disclosure rule in the manner described in part 1. Hence a contradiction.

Proof of Proposition C1. As a preliminary, observe that the Lagrangian of the regulator's problem is $\mathcal{L} = \sum_{s \geq s_p} f(s) \sum_{i=1}^{n} v_i(s)h_i(s) - \sum_{i=2}^{n+1} \lambda_i [u(\omega_{i-1}) \sum_{s \geq s_p} f(s|\omega_{i-1})h_i(s) - c\sum_{s<s_p} f(s|\omega_{i-1})h_i(s)] - \sum_{s \in S} \mu_s \sum_{i=1}^{n+1} h_i(s)$, where $\lambda_i \geq 0$ and μ_s are the lagrange multipliers on (C2) and (C3), respectively. From the definition of $v_i(s)$ and since $f(\omega|s)f(s) = f(s|\omega)g(\omega)$, we obtain that

$$d_i(s) \equiv \frac{\partial \mathcal{L}}{\partial h_i(s)} = \begin{cases} \lambda_i c f(s|\omega_{i-1}) - \mu_s & \text{if } s < s_p \\ \sum_{\omega \geq \omega_i} v(\omega)f(s|\omega)g(\omega) - \lambda_i u(\omega_{i-1})f(s|\omega_{i-1}) - \mu_s & \text{if } s \geq s_p. \end{cases} \tag{G3}$$

The first-order conditions imply the following: (i) if $h_i(s) = 1$, then $d_i(s) \geq 0$; (ii) if $h_i(s) \in (0,1)$, then $d_i(s) = 0$; and (iii) if $h_i(s) = 0$, then $d_i(s) \leq 0$. Also note that by Lemma C3, $h_i(s) > 0$ implies that $\omega_i \leq \omega_r$. We are now ready to prove the proposition.

Suppose $\omega_i > \omega_j$ and $h_i(s) > 0$. If $\lambda_i = 0$, we can show that the regulator never recommends ω_j. Specifically, for every $s'' \geq s_p$, $d_j(s'') - d_i(s'') = \sum_{\omega_j}^{\omega_{i-1}} v(\omega)f(s|\omega)g(\omega) - \lambda_j u(\omega_{i-1})f(s|\omega_{i-1})$, which is negative by Assumptions 2 and 1. Hence, $d_j(s'') < d_i(s'')$, and the first-order conditions imply that $h_j(s'') = 0$ for every $s'' \geq s_p$. Lemma (C3) part 1 then implies that $h_j(s') = 0$ for every $s' < s_p$. The rest of the proof assumes $\lambda_i > 0$.

1. If $s < s' < s_p$, the first order conditions imply that $d_i(s) \geq 0 \geq d_j(s)$. Hence, $\lambda_i c f(s|\omega_{i-1}) \geq \lambda_j c f(s|\omega_{j-1})$. Hence, $\lambda_i c \frac{f(s|\omega_{i-1})}{f(s|\omega_{j-1})} \geq \lambda_j c$. From MLRP, $\frac{f(s'|\omega_{i-1})}{f(s'|\omega_{j-1})} > \frac{f(s|\omega_{i-1})}{f(s|\omega_{j-1})}$. Hence, $\lambda_i c \frac{f(s'|\omega_{i-1})}{f(s'|\omega_{j-1})} \geq \lambda_j c$. Hence, $d_i(s') > d_j(s')$. Hence, $h_j(s') = 0$.

2. Suppose $s > s' \geq s_p$. Following the logic in part 1, it is sufficient to show that $d_i(s) \geq d_j(s)$ implies that $d_i(s') > d_j(s')$. This follows from Assumptions 2 and 1, MLRP, and the observation that

$$\frac{d_i(s) - d_j(s)}{f(s|\omega_{j-1})} = \sum_{\omega_i}^{\omega_{j-1}} v(\omega) \frac{f(s|\omega)}{f(s|\omega_{j-1})} g(\omega) - \lambda_i u(\omega_{i-1}) \frac{f(s|\omega_{i-1})}{f(s|\omega_{j-1})} + \lambda_j u(\omega_{j-1}).$$

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