Low Emission Zones: The Rationale for Bang-Bang Solutions*

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Abstract

Low Emission Zones (LEZs)—restricted areas for vehicles that fail to meet certain environmental standards—are becoming an increasingly popular policy tool to encourage the adoption of low-emission vehicles. Despite this, little is known about their optimal design, particularly regarding which vehicles to restrict and when, and their geographic extension. Unlike price-based instruments (e.g., taxes), LEZs generate convex welfare functions, leading to "bang-bang" solutions: (i) all restricted vehicles should be treated equally, although some (the transition technologies) may become restricted later than others, and (ii) the LEZ should be either as large as possible or not implemented at all if it falls below a minimum scale. Combining reduced-form and structural analysis, we assess Madrid's LEZ through the lens of this bang-bang rationale.

Keywords: Low Emission Zones, Air Pollution, Electric Vehicles, Transition Technologies, Pigouvian Taxes

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1 Introduction

Road transport is a leading source of greenhouse gases and local air pollutants. The associated health, economic, and societal consequences are substantial and well-documented. An increasingly popular policy tool to handle these external costs is the creation of low emission zones (LEZs): urban areas that restrict access to vehicles that fail to meet certain environmental standards. Initially introduced in several German cities in 2008, there are now over 500 LEZs across Europe, and many cities around the world, including Shanghai, Beijing, Haifa, Seoul, and Jakarta, are adopting similar strategies. Even some U.S. cities have begun experimenting with LEZs or are planning to introduce them.

An often overlooked aspect of LEZs is the significant variation in their design, particularly concerning the geographic scope of the covered area and the types of restricted vehicles. For example, the LEZ in Berlin, implemented in 2008, spans 88 km^2, restricts some, but not all, gasoline and diesel cars, and imposes different geographic restrictions among its restricted cars. In contrast, the LEZ in Madrid, known as Madrid Central and implemented a decade later, covers a much smaller area, of just 5 km^2, but restricts all gasoline and diesel cars, making no geographic distinction among them. Are both designs equally optimal from a social point of view? Should some cars be restricted more than others? And, is there an optimal LEZ size—not too large, not too small?

Surprisingly, little is known about these policy-design questions. This paper is a first attempt to address them, using Madrid Central (MC) as both motivation and evidence. Since a LEZ's ultimate goal is to encourage the adoption of cleaner vehicles, we initiate our analysis in Section 2 by looking at how MC has impacted the adoption of hybrids and electric vehicles (EVs), the only vehicles allowed to enter MC. Looking at sales of new units and using different cities in Spain to build a synthetic control (Jones and Marinescu, 2022; Abadie, 2021), our difference-in-difference estimates indicate that MC was effective in accelerating the adoption of hybrids but not of EVs.

This finding raises additional policy questions that are relevant beyond MC. For example, was it a good idea to exempt hybrids entirely? Would it have been more effective to impose some restrictions on hybrids as well, albeit less stringent than those on gasoline and diesel cars—perhaps by allowing hybrids to operate within a smaller LEZ, following the discriminatory design in Berlin? This approach might have increased the appeal of EVs relative to hybrids, but it would also have made hybrids less attractive compared to gasoline and diesel cars. Ultimately, what would have been the socially optimal choice?

We shed light on these questions in Section 3, where we use a simple dynamic model with three horizontally differentiated car types whose emission rates differ. Hybrids emit less than gasoline cars but more than EVs, which are assumed to be emission-free. According to the model, the answer to the above questions follow a "bang-bang" rationale: all restricted vehicles should be treated equally, and the LEZ should be either as large as possible or not implemented at all if it falls below a minimum scale. Within this bang-bang rationale, hybrids may still qualify as a transition technology—kept unrestricted as electric vehicles for some time, and then as restricted as gasoline cars afterward.

Where does this bang-bang logic come from? Unlike price-based instruments, such as taxes, which operate under an efficient rationing rule, LEZs operate under an inefficient (e.g., proportional) rationing rule, leading to convex welfare functions.[1] To see this, imagine a planner who introduces a small pollution tax on dirty vehicles. The welfare impact of this tax is confined to the marginal drivers who switch to cleaner cars in response to the tax. Their welfare loss from moving away from their preferred (i.e., no-intervention) car choice is smaller than the social welfare gain from reduced emissions. Importantly, social welfare is unaffected by the tax impact on inframarginal drivers—those who do not switch. The planner finds it optimal to increase the tax up to its Pigouvian value, where the (concave) welfare function reaches its maximum level.

Imagine now a small LEZ that only restricts access to gasoline cars. Unlike the tax, the welfare impact of the LEZ extends to all inframarginal individuals who continue driving gasoline cars. These individuals will have to either change their transport mode (e.g., to public transport) if they want to enter the LEZ, reroute their trips around it, or cancel some trips altogether. Given their large number, the welfare losses from these inframarginal individuals can be significant compared to the net welfare gains from the few marginal individuals who switch from gasoline cars to either hybrids or EVs. This makes this small LEZ potentially a bad idea—unless it can be expanded above a minimum scale, enough to significantly reduce the number of inframarginal gasoline-car drivers.

This minimum scale corresponds to the LEZ size that achieves the same welfare as the no-intervention outcome. Thus, when the pollution harm is not too large, no LEZ is better than any LEZ smaller than such a minimum scale. In contrast, when the pollution harm is relatively large, the minimum scale can be very small, even zero, meaning that almost any LEZ would be beneficial. However, the convexity of the welfare function would call for the largest possible (i.e., politically feasible) LEZ.

This bang-bang logic—aiming to reduce the number of inframarginal consumers of restricted cars as much as possible—also explains why, once restricted, all such cars should be treated equally; that is, subject to the largest possible LEZ. It likewise explains why treating hybrids as a transition technology can be optimal within a LEZ design. In anticipation of their lower future value, some individuals switch away from hybrids today, even if they are not yet restricted.[2] As a result, there will be fewer inframarginal consumers driving hybrids tomorrow, when the restriction becomes active. With fewer inframarginal consumers, it becomes less costly to impose a restriction on these not-so-clean vehicles.[3]

In our baseline model, we assume that all inframarginal individuals are equally affected by the LEZ, which explains why rationing is inefficient.[4] This could be ameliorated if the LEZ's impacts across individuals were correlated with their preferences for cars,

[1] See Tirole (1988) for more on rationing rules.

[2] Some individuals will switch to electric vehicles while others to gasoline cars. However, the shift to gasoline cars is less relevant, as those are already restricted.

[3] Our definition of a transition technology differs sharply from alternative notions, which are often based on the idea that electric vehicles remain too expensive or unfamiliar to serve as a viable outside option for many gasoline drivers. Under this alternative view, hybrids should remain unrestricted until these concerns dissipate. While our definition is not in conflict with that view, it is fundamentally different.

[4] Another source of inefficiency arises from the asymmetric treatment of polluting cars. While LEZs provide more favorable treatment to hybrids compared to taxes, they impose harsher restrictions on gasoline cars.

such that those most willing to switch cars (i.e., those with the weakest preferences for gasoline cars) are also those most affected by the LEZ. Whether such a correlation exists—and in the "right" direction—is an empirical question, as is whether it would be enough to eliminate the convexity of the welfare function that underpins the bang-bang rationale.[5]

We turn to these empirical questions, along with questions on the optimality of MC, including the hybrids' exemption, in Section 4. We start our structural analysis by incorporating more consumer heterogeneity than our simple theory model allows, for instance, by letting consumer preferences and LEZ impacts be a function of income and distance from the centroid of MC. We find that...[TBC]

Our paper contributes to two strands of the literature. First, we contribute to the expanding literature evaluating the performance of LEZ programs around the world (e.g., Wolff 2014, Gehrsitz 2017, Zhai and Wolff 2021, Sarmiento, Wägner and Zaklan 2023, Galdon-Sanchez et al. 2023). These studies, which typically focus on the short-term impacts of LEZs, find that they reduce pollution within the restricted area but do not necessarily decrease traffic. However, none of them examine the long-term effects on green car adoption, which are crucial for evaluating the policy-design questions at the core of our analysis.

Much of the literature focuses on comparing the performance of different policy instruments, typically categorized as either market-based (e.g., taxes, emissions trading) or command-and-control (e.g., emission and technology standards). While Low Emission Zones (LEZs) clearly fall within the command-and-control (CAC) category, they differ from other instruments in this group due to the bang-bang rationale underlying their optimal design. Under most standard CAC instruments, the welfare function retains its concavity property, meaning the bang-bang rationale does not apply (see, e.g., Ellerman et al. 2000, Montero 2005). However, the rationale does hold in other driving-restriction programs, such as the vintage restrictions analyzed in Barahona, Gallego and Montero (2020). This is not surprising since they are built under a similar inefficient-rationing logic.

The rest of the paper is organized as follows. Section 2 contains the reduced-form analysis of MC. Section 3 contains the theory model. The structural analysis is in Section 4. We conclude in Section 5.

2 Madrid Central

The Madrid City Council implemented traffic restrictions on non-residents' polluting cars in the Centro district in May 2018 and announced that it would fully enforce the restriction from November 2018 onward, which later became known as Madrid Central

[5]We find the convexity property to be quite persistent as we explore other extensions of our baseline model, such as relaxing the full market-coverage assumption by allowing drivers to complete some of their trips using public transport. In these extensions, we also study how the introduction of price-based instruments, whether taxes or subsidies, in combination with LEZs can recover the concavity of the welfare function.

(MC).[6]. The goal of this section is to study the impact of MC on the faster adoption of cleaner vehicles, a LEZ's ultimate goal.

2.1 Data Description

We utilize two datasets for our reduced-form analysis: vehicle registration and demographic data. The vehicle registration data utilized in our study are provided by the Directorate-General for Traffic and contain information on all vehicles registered in Spain between 2015 and 2019. This dataset offers detailed information on car registrations, including vehicle characteristics such as model, emissions standard, vehicle tax, first registration date, and registration location by postal code. We identify two car categories that are allowed freely to enter the LEZ: (1) battery-fuelled or fuel-cell electric vehicles (henceforth labeled as EV); (2) all electric vehicles such as plug-in-hybrid electric vehicles and others that are not included in EV categories (henceforth labeled as Hybrids). We focus only on newly registered cars (excluding used cars that change ownership) because almost all electric vehicles are new. These two car categories are our primary focus.

For the control variables, we merged the registration data with detailed demographic and income distribution data from the Spanish Statistical Office at the zip-code level for 2017 (INE, 2020). This allows us to examine the impact of LEZs on the adoption of greener vehicles while also considering the population's characteristics in different areas, such as their income level, age distribution, and household size.

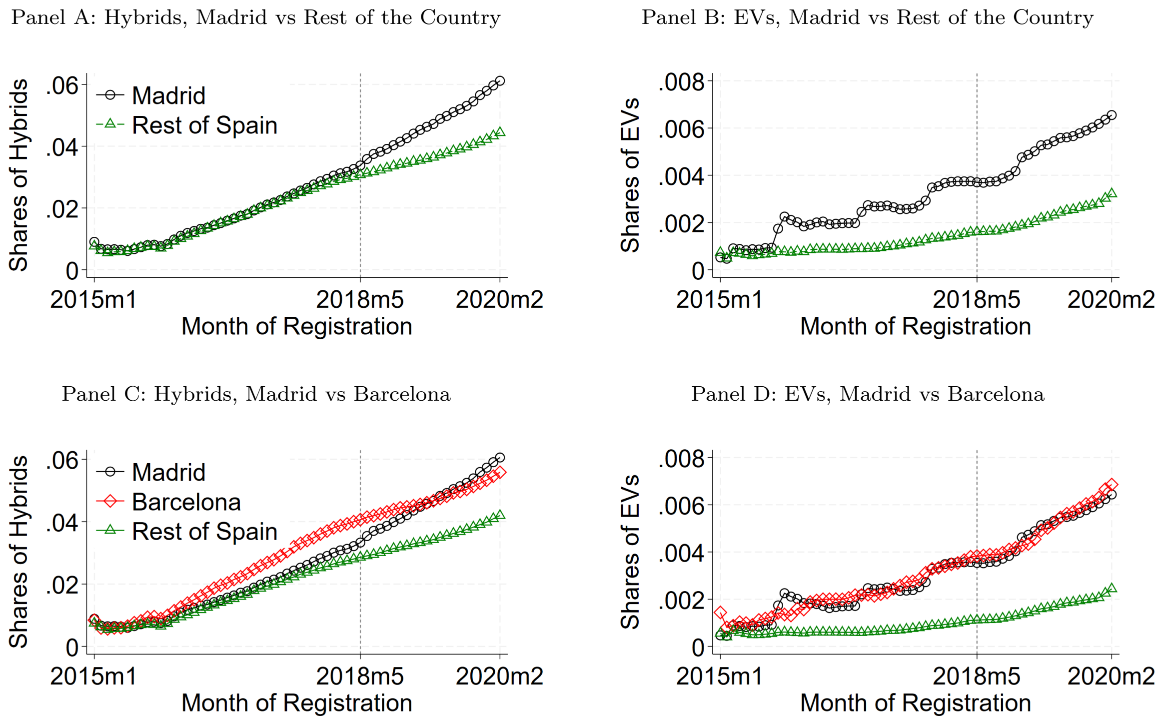
The data on vehicle registrations is then aggregated on a monthly basis and by province due to the limited number of daily registrations at the zip code level, resulting in 3,120 observations, covering 60 months for 52 provinces.[7] Figure 1 (panels A and B) presents the evolution of the cumulative share of newly registered green cars (Hybrids and EVs) over total car registrations in Madrid since January 2015 in Madrid versus the rest of the country. The share of Hybrids goes up much faster in Madrid versus the rest of the country after the introduction of the LEZ in May 2018. However, the rest of the country fails to reproduce the trajectory of EV adoption in Madrid even before the introduction of the LEZ.

Figure 1 (panels C and D) also compares the evolution of green car adoption in Madrid versus Barcelona, given the similarities between the two cities. Before May 2018, the figure highlights two main observations: (1) Barcelona did not closely mirror Madrid in terms of Hybrids adoption, but (2) it exhibits a similar trend as Madrid for EV adoption. For Hybrids, the requisite parallel trends necessary for difference-in-differences models are not evident in the pre-LEZ data. In contrast, visual inspection suggests that they may be present for EVs. To address these concerns, we employ the synthetic control approach (Abadie, 2021) to assess the impact of the LEZ on Hybrid and EV adoption. In the case of EVs, we anticipate that Barcelona will carry the most weight in constructing the synthetic control for Madrid.

[6] https://www.telemadrid.es/madrid-central/Madrid-Central-cronologia-fechas-clave-0-2070092975--20181123024819.html

[7] The six zip codes corresponding to MC are excluded from the analysis since the residents in this area are not affected by the LEZ rules, and they are too small to be the control regions.

Figure 1: Trends of hybrid and EV adoptions



Notes: These figures show the increasing trends of green car registration before and after Madrid Central. It shows the evolution of the share of newly registered Hybrids and EVs over total new car registrations in Madrid since January 2015 versus the rest of the country (panels A and B) and versus Barcelona (panels C and D). Particularly, the trend became relatively steeper in the Madrid region right after Madrid Central was announced in May 2018.

2.2 Madrid vs. Synthetic Madrid

We let $J+1$ be the total number of provinces in Spain, where each province is indexed by j, and we set $j=1$ to signify Madrid province, the treated unit. To create a synthetic Madrid, we compute a weighted average of the control provinces $j=2,...,J+1$, and denote it as a vector of weights $W=(w_2,...,w_{J+1})'$ where $0 \leq w_j \leq 1$ and $w_2+...+w_{J+1}=1$. A specific set of weights characterizes every possible synthetic control, and we select W so that the difference between Madrid and the control units concerning the number of critical predictors of the outcome variable and the outcome variable itself is minimized in the pre-treatment period.

For the outcome variables, we use the monthly share of hybrids and EVs relative to the total new cars registered at the municipality level. We normalize these monthly shares relative to the corresponding month of the year 2015, as follows:

$$ShareHybrid_{pmy} = \frac{Hybrid_{pmy}}{N_{pmy}} - \frac{Hybrid_{pmy=2015}}{N_{pmy=2015}} \tag{1}$$

where $ShareHybrid_{my}$ is the share of Hybrids (or EVs) in province p, at month m and year y. $Hybrid$ is the total number of Hybrids (or EVs), and N is the total number of new cars registered.

For the predictors of either Hybrids or EVs adoption, we use 2017 demographic characteristics averaged to the province level, such as the total number of households, purchasing power in Euro, highest purchasing power in the province, the fraction of the population aged 15 to 59, fraction of population below the age of 15, fraction of male population, and household size. The predictors are also assigned weights to give more relevance to important predictors of the outcome variable. The weights for each predictor for hybrids and EVs adoptions are presented in Table 1.

Table 1: Share of Car Registration Predictors Means Before May 2018

		Hybrids		EVs	
	Madrid	Weight	Synthetic	Weight	Synthetic
Car per household	1.42	0.02	0.63	0.04	0.73
Mean Purchasing Power (EUR)	16392	0.04	15186	0.22	15913
Number of Households	9913	0.00	5785	0.05	6601
Share of Population age 15-59	0.62	0.76	0.62	0.02	0.60
Share of Population below age 15	0.17	0.09	0.16	0.06	0.16
Average Number of Household Members	3.28	0.01	3.15	0.60	3.22
Shares of Male	0.49	0.08	0.50	0.02	0.50

Notes: Car per household is the total number of cars divided by the number of households.

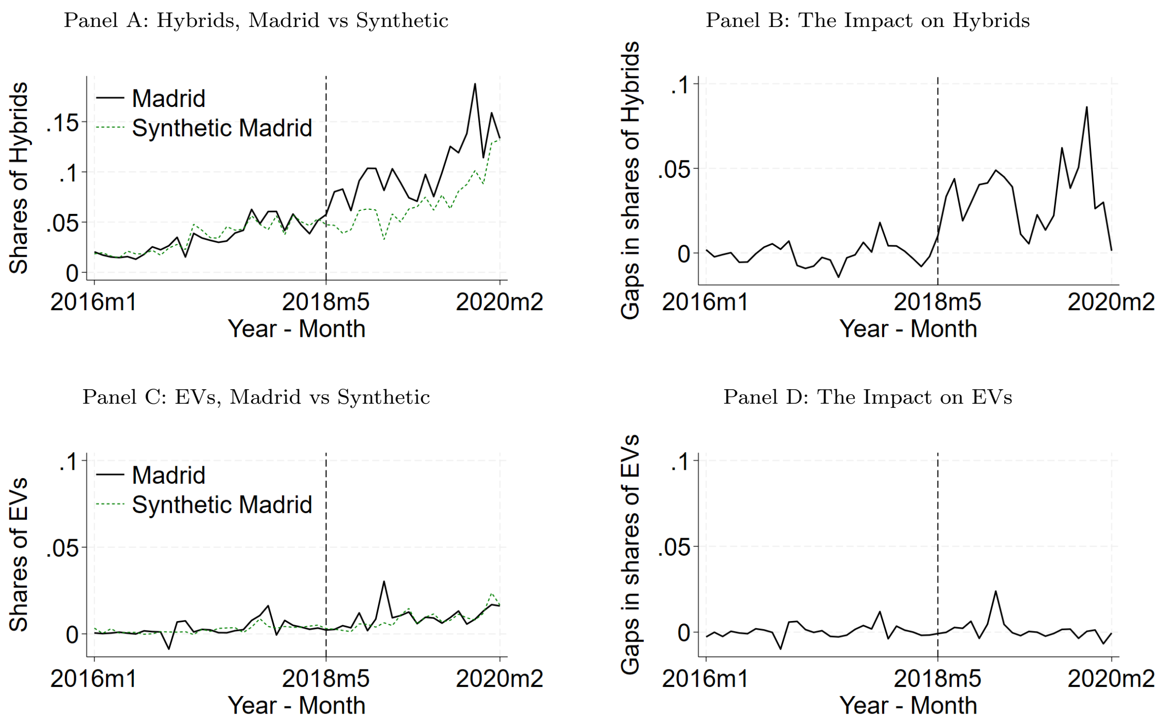
2.3 Impact of MC

The effects of MC on new car registrations are assessed based on the monthly averages after May 2018 in comparison between Madrid and its synthetic control. Barcelona holds the highest weight in the synthetic Madrid, which is expected given the similarity between Madrid and Barcelona in key characteristics such as purchasing power. The other province that receives a non-zero weight is Las Palmas, located on an island particularly suited for EVs, given its size. This is compounded by the higher transportation costs of importing gasoline.

Figure 2 (panel A) shows that the weighted average of provinces in the control group closely mirrors the trajectory of hybrids adoption in Madrid before the introduction of the LEZ. The gap between Madrid and the synthetic Madrid (panel B) suggests that the LEZ had a positive effect on Hybrid adoption, leading to a notable increase of 0.031 in Hybrid registration shares (compared to 0.03 in May 2018 in Figure **??**, representing a 100% increase in Hybrid adoption).

Figure 2 (panel C and D) also shows the trends in EVs registered in Madrid versus its synthetic (in this case, Barcelona receives the full weight). The gap between Madrid and Barcelona suggests that the LEZ had a positive impact on EVs adoption in Madrid. In particular, it led to an increase of 0.004 in EV registration shares (compared to 0.006 in May 2018, representing a 67% increase). However, prior to May 2018, we see a slightly positive difference in EV adoption in Madrid versus Barcelona, indicating positive trends even before May 2018. This is consistent with the increasing trends before May 2018 shown in Figure 1, casting doubt on whether the LEZ impacted truly impacted EV adoption. We further confirm this evidence through our placebo checks in Section .

Figure 2: Green adoption in Madrid vs. Synthetic Madrid



Notes: Panel A shows the trends of Hybrids adoption in Madrid versus synthetic Madrid, with the root-mean-square error (RMSE) of the difference in each pre-period year between treatment and synthetic control is 0.006. Panel B plots the effect of MC on hybrids adoption. Panels C and D report similar evidence for EVs adoption, with RMSE of 0.003.

2.4 Placebo Effects of MC

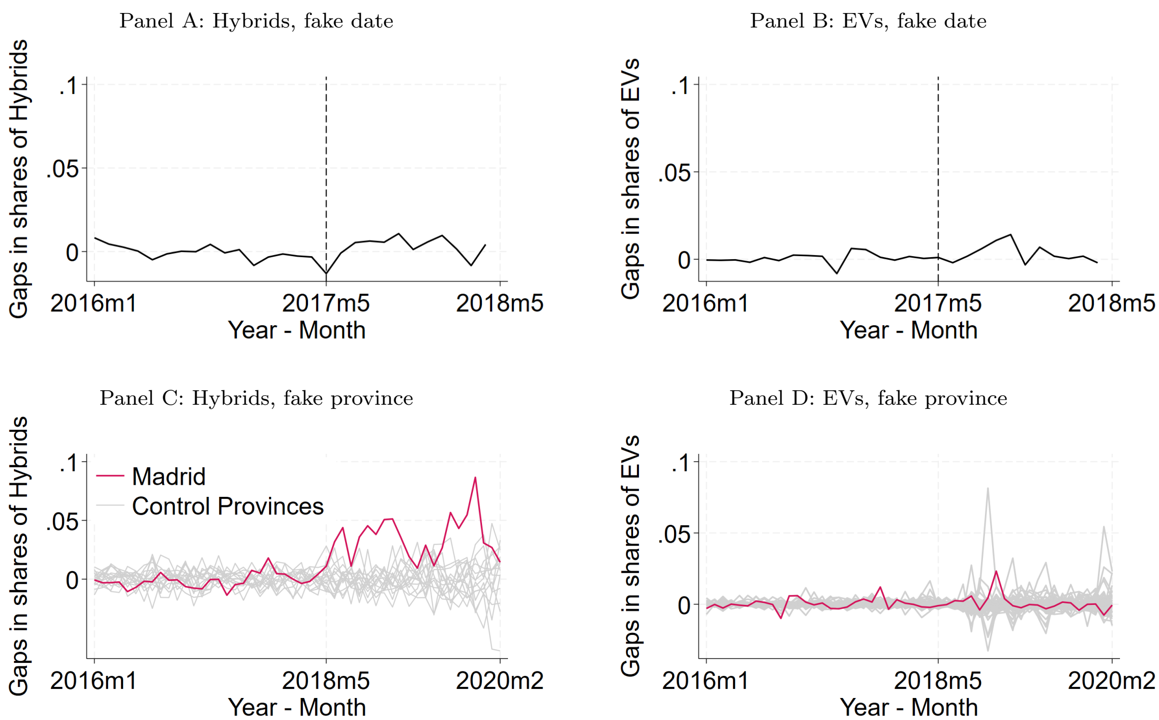
We conducted two placebo tests to ensure the validity of our findings: one involves a fake timing of the announcement of the LEZ instead of May 2018, and the other involves a fake province as the treated province instead of Madrid. We aimed to ascertain whether this placebo treatment would not lead to a post-placebo-treatment divergence in the trajectory of Hybrids or EVs registered between Madrid and its synthetic control.

Our placebo tests confirm our causal interpretation of the impacts of the LEZ on hybrids adoption but raise doubts concerning the actual causal effect of the LEZ on EV adoption. In particular, the placebo tests suggest that the increasing trends in EV adoption in Madrid (Figure 2, Panels C and D) are not primarily due to the LEZ. Intuitively, for those who buy a new vehicle in order to access the LEZ, acquiring a Hybrid, which is cheaper than an EV, may suffice. We elaborate on these two placebo checks in detail below.

For the placebo in the timing of the treatment, we use data prior to the announcement of the LEZ and randomly pick the fake timing of the treatment. In this case, we have used

May 2017 as the fake LEZ time. We conduct similar analyses as earlier. Panel A in Figure 3 shows that the fake LEZ does not explain the divergence in Hybrid adoption between Madrid and its synthetic control. In contrast, Panel B shows an average treatment effect for EV adoption over the post-treatment period of 0.0045. This figure is almost the same as in the baseline analysis, suggesting that EV registration in Madrid depicts increasing trends even before the Madrid Central announcement. Therefore, the effects depicted in Panel D of Figure 2 are not caused by the LEZ.

Figure 3: Placebo Tests



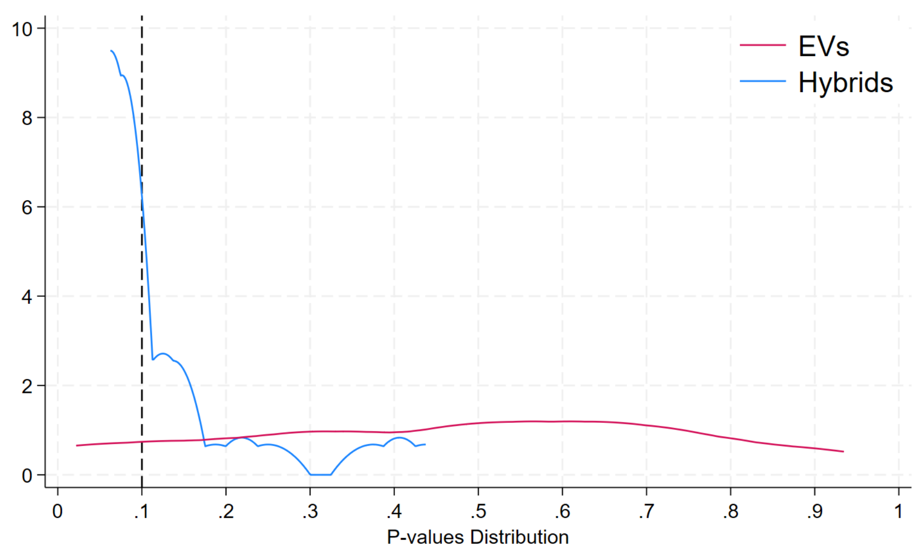
Notes: These figures illustrate the impact of MC on green car adoption (similar to 2) and the placebo effects using fake treatment units. The x-axis for each plot are the same as in the respective plot with treatment effects. In Panels A and B, we use May 2017 as the fake LEZ time. In Panels C and D, we iteratively reassign the treatment to control units where no intervention took place. Provinces with a pre-treatment MSPE (mean squared prediction error) two times higher than Madrid's are excluded.

For the placebo in the treatment unit, we iteratively reassigned the treatment to control units where no intervention occurred. The purpose is to test whether the treatment effect observed in Madrid was attributable to the intervention and not to other province-specific factors that may have affected the outcome.[8] Panel C and D in Figure 3 depict the difference between the treatment and placebo effects. For Hybrids, the registration gap shown in Panel C during the post-LEZ period was the largest compared to other placebo effects using fake provinces as the treatment, further supporting the validity of the findings regarding the causal effect of the LEZ on Hybrid adoption. In contrast, for EVs, Panel D shows a similar magnitude of both treatment and placebo effects, suggest-

[8]All the provinces with a pre-treatment MSPE at least five times larger than Madrid's pre-treatment MSPE are excluded in order to exclude provinces that do not look like Madrid province pre-treatment.

ing that the LEZ does account for the changes in the increase of EV adoption. In Figure 4, we present the distribution of the p-values of the treatment effect for each month-year. In particular, we compute the p-values for each period by comparing the main treatment effect (using Madrid as the treatment and the correct timing of MC) to the empirical distribution of placebo estimates, similar to Jones and Marinescu (2022). The p-values for the impact of MC on hybrid car adoption are less than 0.1, indicating statistical significance at the 90% level for most months. In contrast, the p-values for the impact on EV adoption are not statistically significant for most months.

Figure 4: P-values of the treatment effect



3 A Simple Model of Car Choices

Our reduced-form results raise several policy questions that are relevant beyond MC. For instance, was it a good idea to exempt hybrids as well? Would it have been better to restrict hybrids too, but to a lesser extent than gasoline and diesel cars, perhaps by letting the latter face a larger LEZ, as done in Berlin, for example? This could have certainly helped make the EV option more attractive relative to hybrids, but would it have been the socially optimal thing to do?

3.1 Preliminaries

To shed light on these questions, we develop a simple dynamic model with infinitely-lived drivers and three types of vehicles in the market—electric (E), hybrid (H) and gasoline (G)—offered in three distinct and equidistant points on a circle of perimeter equal to 1. The cost of producing these cars is c^j for $j = E, H, G$. Given our focus on adoption (extensive margin), we will assume that all cars are used the same; hence, a vehicle's total emissions can be proxied by its emissions rate, as detailed below.[9] We adopt this Salop framework to communicate our insights in the clearest possible way, at times with corner solutions (e.g., when no single unit of a particular type is sold in equilibrium). We adopt a more flexible (horizontal-differentiation) framework in the structural estimation following this section, where we allow for an arbitrary number of vehicles, of different sizes.

Drivers are uniformly distributed with density 1 on the circle. A driver located at $x \in [0, 1]$, who at the beginning of a given period holds a car of type $j = E, H, G$ and age $a = 1, 2, ..., T_j$, where T_j is the (endogenous) age of the oldest car in the market, derives a surplus of

$$u_a^j(x) = v_a^j - \gamma|x - x^j|,$$

where v_a^j is the value the individual obtains from driving the car during one period, which is decreasing with age (i.e., $v_a^j > v_{a+1}^j$), x_j marks car j's "location" (with $x^E = 0$, $x^H = 1/3$ and $x^G = 2/3$), and $\gamma > 0$ is the (per-period) horizontal-differentiation parameter (i.e., the transportation cost). We assume that the market is always fully covered, i.e., all drivers hold on to one vehicle in each period, and no more than one, in all equilibria.

Vehicles also differ in how much they pollute each period. A gasoline vehicle of age a emits $e_a^G = e_a$ units of pollution per period, with $e_{a+1} \geq e_a$, while a hybrid vehicle of age a emits $e_a^H = \alpha e_a$ per period, with with $\alpha \in (0, 1)$. Electric vehicles are emission-free, i.e., $e_a^E = 0$. The social cost of a unit of emission is normalized to 1, so emissions capture the social cost of pollution.

3.2 The No-Intervention Outcome

Consider first the case in which drivers face no environmental policies. Let U_a^j be the value (i.e., lifetime utility flow) of an individual located at x with a car of type j and age a. As in Adda and Cooper (2000), the set of Bellman's equations that must hold in equilibrium is given by

$$\begin{aligned}
U_1^j(x) &= u_1^j(x) - p_1^j + \delta U_2^j(x) \\
&\vdots \\
U_a^j(x) &= u_a^j(x) + \delta U_{a+1}^j(x) \\
&\vdots \\
U_{T_j}^j(x) &= u_{T_j}^j(x) + \delta U_1^j(x) + \delta z^j
\end{aligned}$$

[9]We relax this assumption in the appendix, showing that results remain qualitatively unchanged when we let restricted cars to be driven less, in proportion to the LEZ's size.

where $\delta < 1$ is the discount factor, p_1^j is the price of a new (i.e., age 1) car and z^j its scrap value. Note that the car is scrapped at the beginning of $T_j + 1$ (or end of T_j).[10]

In this model, where individuals only differ in their horizontal preferences, there is no reason for individuals to trade their second-hand cars. Individuals hold on to their cars until they are scrapped, which is when they purchase a new car. This implies that the price of second-hand cars can be obtained from the indifference condition

$$U_1^j(x) = U_a^j(x) - p_a^j$$

for $a > 1$, which says that after scrapping their cars individuals should be indifferent whether to buy a new car or a second-hand car.[11] Solving the Bellman's equations, we arrive at the following lemma.

Lemma 1 *The lifetime value of holding on to a car of type j for an individual located at x, $U_1^j(x)$, is given by*

$$U_1^j(x) = \frac{1}{1-\delta^{T_j}} \left[\sum\nolimits_{a=1}^{T_j} \delta^{a-1} u_a^j + \delta^{T_j} z^j - p_1^j \right]$$

where T_j is the age of the oldest type-j car in the market.

The terms in brackets correspond to the net present value of owning a type-j car, a cycle that repeats every T_j periods. To ensure that $T_j + 1$ is the optimal scrapping age requires that age-T_j cars will not be scrapped, and that age-$T_j + 1$ cars are never kept. These two optimality conditions respond to the values of the primitives (i.e., v_a^j, p_1^j, z^j, and δ). For example, it can be shown (see the Appendix) that if it is optimal to scrap cars at age 3, then the following two conditions must hold

$$v_2^j > \frac{1}{1+\delta} \left(v_1^j + \delta v_2^j + z^j - p_1^j \right) > v_3^j. \tag{2}$$

The inequality on the left says that individuals want to run their cars for more than one period, whereas the inequality on the right indicates that running the car for an additional (i.e., third) period is not worthwhile. In this case, it is better to scrap the car and get a brand new one (or, which is the same, acquire a "one-period-old" one and run it only for one period).

[10] Note that for simplicity we are assuming that all cars survive until they are scrapped at the end of their lives, but we could easily let them exit the market at some exogenous rate due to crashes, fatal malfunctioning, etc.; a rate that may vary with the vehicle's type and age. For example, if we denote by ξ_a^j the probability that a car of type j and age a survives to the next period, the corresponding Bellman's equation would change to $U_a^j(x) = u_a^j(x) + \xi_a^j \delta U_{a+1}^j(x) + (1-\xi_a^j)\delta(U_1^j(x) + z^j)$.

[11] This condition can be conveniently rewritten as

$$p_a = v_a + \delta U_{a+1} - U_1$$

which is expression (3) in Adda and Cooper (2000).

Let us denote by $\tilde{x}^{jk}$ the location of drivers who are indifferent between owning vehicle j and $k \neq j$, i.e., $U_1^j(\tilde{x}^{jk}) = U_1^k(\tilde{x}^{jk})$. Drivers located between $\tilde{x}^{jk}$ and $\tilde{x}^{jl}$, with $l \neq j, k$, will own a type-j vehicle, so from Lemma 1 we obtain that the total demand for new type-j vehicles is equal to

$$q_1^j = \int_{\tilde{x}^{jk}}^{\tilde{x}^{jl}} dx = \frac{1}{3} + \frac{1}{2\Gamma}\left(2(V_j + Z_j - P_j) - (V_k + Z_k - P_k) - (V_l + Z_l - P_l)\right), \quad (3)$$

for $j, k, l = E, H, G$, with $j \neq k, l$, and where $\Gamma = \gamma/(1-\delta)$, $V_j = \sum_{a=1}^{T_j} \delta^{a-1} v_a^j/(1-\delta^{T_j})$, $Z_j = \delta^{T_j} z^j/(1-\delta^{T_j})$, and $P_j = p_1^j/(1-\delta^{T_j})$. The demand for j is decreasing in its own (lifetime) price P_j and increasing in its (lifetime) service value V_j and (lifetime) scrap value Z_j. Moreover, because vehicle types are substitutes, demand for j is increasing in P_k and P_l, and decreasing in V_k, V_l, Z_k and Z_l.

Since (3) also corresponds to the per-period demand for second-hand cars, total welfare, in present value terms, is given by

$$W = \sum_{j=E,H,G} \left((V_j + Z_j - C_j - E_j) q_1^j - \int_{\tilde{x}^{jk}}^{\tilde{x}^{jl}} \Gamma |x - x_j| dx \right), \quad (4)$$

for $j, k, l = E, H, G$, with $j \neq k, l$, and where $C_j = c^j/(1-\delta^{T_j})$ is j's (lifetime) production cost, and $E_j = \sum_{a=1}^{T_j} \delta^{a-1} e_a^j/(1-\delta^{T_j})$ is its (lifetime) emissions harm.[12]

3.3 Introduction to Policy Interventions

We are interested in the impact of policy interventions on the adoption of low-emission vehicles. Since our focus is on correcting the environmental externality, we assume that cars are sold at cost or with a constant, uniform markup.[13]

We examine two interventions: Pigouvian taxes and LEZs. We consider Pigouvian taxes (i.e., taxes set at their first-best levels) not because we view them as a feasible policy option, but rather as a benchmark for evaluating the performance of LEZs. Other price-based instruments could also be considered, such as subsidies for the purchase of new low-emission vehicles and for the earlier retirement of polluting ones.[14]

Both types of interventions—Pigouvian taxes and LEZs—affect the market-equilibrium outcome, although in different ways. This can be seen with the help of (3). Taxes work

[12] Note that cars can have different lifetimes.

[13] Giving oligopoly rents to car dealers this way will have no welfare consequences, and hence, no impact on policy design. Adding market power in a different way would not be neutral in terms of policy design.

[14] Note, however, that these alternative instruments do not replicate the work of taxes. Subsidies for the purchase of low-emission vehicles may fail to implement the first-best by extending their lifespans beyond the first-best level. Scrappage subsidies may help shorten the lifespan of existing gasoline cars in the short run, but in the long-run, permanent scrappage subsidies could have the opposite effect—prolonging their use (or delaying their phaseout) by making the purchase of new gasoline cars more attractive.

by moving (lifetime) prices P_j away from C_j. For example, if each period a Pigouvian tax $\tau_a^j = e_a^j$ is added to the registration of a vehicle of type j, then $P_j = C_j + E_j$.[15]

A LEZ policy—which resembles a proportional rationing scheme—works differently by reducing the (lifetime) service value of the restricted car from v_a^j to $(1 - s_a^j)v_a^j$, where $s_a^j < 1$ is the extension of the LEZ affecting vehicle j of age a.[16] One could argue that two different LEZs, of size s^G affecting gasoline cars and of size s^H affecting hybrids, could replicate the work of two different (lifetime) taxes, τ^G and τ^H. This is true in terms of market shares of new cars, as can be inferred from (3), but not in terms of welfare.

The reason is that a LEZ policy works through the intensive margin by destroying welfare from inframarginal consumers, those who continue holding restricted cars. In contrast, taxes do not trigger such welfare destruction; they work at the margin without affecting inframarginal consumers (besides the tax payment).[17] Thus, the difference between taxes and LEZs boils down to efficient vs inefficient rationing. This difference will prove crucial in the design of a LEZ, as we show next.[18]

As with any intervention in a durable-good market, it takes time for the composition of the car fleet to adjust to its new (steady-state) equilibrium. Given this gradual adjustment, one can entertain the idea that the optimal LEZ design could also evolve alongside the fleet until reaching the new steady-state equilibrium.

To better understand how to arrive at the optimal LEZ design, we proceed sequentially: first, we study the case of "short-lived" vehicles, where the fleet adjusts instantly to a policy shock; then, we consider the more realistic case of "long-lived" vehicles. Without loss of generality we simplify the analysis by assuming that cars are symmetric except for their pollution levels: all cars have the same production cost c, provide the same service value v_a, and have the same scrap value z. We also assume that pollution is age-independent, i.e., $e_a = e$ for all a.[19]

3.4 Equilibrium with Short-Lived Vehicles

Suppose that cars last only one period, after which they are scrapped. This is equivalent to assuming long-lived vehicles with myopic consumers, who place no weight in future payoffs. In the absence of a policy shock, the market is equally shared by the three types of vehicles. We examine first how these shares change under a fist-best intervention and then under a LEZ intervention.

[15] Note that T_j may have also changed (i.e., decreased) as a result of the tax.

[16] Under our working assumption that policies have no impact on the intensive margin, this destruction of welfare does not come from fewer trips but from less desirable trips; for example, longer trips to bypass the LEZ, shopping trips outside the LEZ instead of into the LEZ, etc.

[17] We assume that taxes are returned to individuals in a lump-sum fashion.

[18] Differences extend to their distinct impacts on the intensive margin, that is, on the amount of driving. Gasoline taxes, for example, ration the least valuable trips, while LEZs ration trips regardless of their value, which is more akin to proportional rationing. We will save for these intensive margin considerations later.

[19] This is a reasonable assumption when the main focus is on global (i.e., carbon) emissions (see, e.g., Jacobsen et al. 2023). We will nevertheless discuss the implication of relaxing this assumption by assigning more weight to local pollutants.

3.4.1 First-best solution

The welfare maximizing (i.e., first-best) solution can be implemented through Pigouvian taxes reflecting the social cost of emissions relative to electric vehicles, $p_1^G = c + e$, $p_1^H = c + \alpha e$ and $p_1^E = c$. These prices move market shares away from the equal split under the no-intervention solution. Depending on consumer preferences (captured by the transportation cost γ) and emission costs (captured by e and α), Pigouvian taxes can give rise to corner solutions where, as stated in our first proposition, some vehicle types are not sold in equilibrium.[20]

Proposition 1 *(FB solution with short-lived vehicles) Consider the following emission thresholds:*

$$e_G^{FB} \equiv \frac{2\gamma}{3(2-\alpha)} \text{ and } e_H^{FB} \equiv \frac{\gamma}{2\alpha}.$$

Under the FB intervention:

(i) All three type of vehicles are sold in equilibrium if $e \leq e_G^{FB}$.

(ii) Only electric vehicles are sold in equilibrium if $e \geq \max\left\{e_G^{FB}, e_H^{FB}\right\}$.

(iii) Otherwise, only hybrids and electric vehicles are sold in equilibrium.

Figure 5 provides a graphical representation of the FB solution as a function of α and e (for now, only pay attention to the dashed-line thresholds e_G^{FB} and e_H^{FB}). Under the FB intervention, gasoline cars are sold in equilibrium only when their emissions are sufficiently low, $e \leq e_G^{FB}$ (regions IV and V in the figure). Otherwise, Pigouvian taxes are so high that consumers prefer not to buy them. The condition for positive sales of gasoline cars is less demanding the stronger the consumers' (horizontal) preferences are, here captured by γ. It is also less demanding the higher α as the tax on hybrids is higher. On the other extreme, if the emissions of gasoline cars and hybrids are sufficiently high, only EVs are sold in equilibrium (regions I and II in the figure). If this is the case, hybrids (and hence gasoline cars) are not part of the FB solution. Yet, as we will show next, hybrids can be part of a second-best (i.e., LEZ) solution when the FB is not feasible.[21]

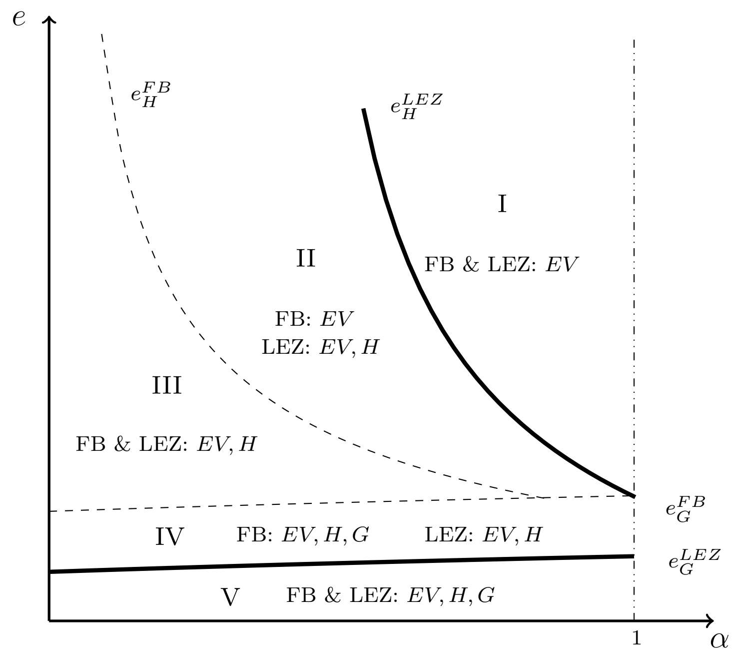
3.4.2 Low emission zones

LEZs restrict car access to a specific area within the city depending on a vehicle's emission rate. Trivially, a LEZ never replicates the FB solution, given its inefficient rationing. Although highly unrealistic, there is only one case where LEZs replicate the FB: when the FB solution includes only electric vehicles in the market and authorities can deploy

[20] Note that these corner solutions are not meant to capture what may actually happen in reality but rather to better illustrate the differences between first-best and LEZ interventions.

[21] In the Appendix, we extend the analysis to consider the interaction between LEZs and subsidies or taxes, and possibly under costly public funds.

Figure 5: First-Best and LEZ Solutions



a sufficiently large LEZ, essentially, banning gasoline cars and hybrids from circulation altogether.

There are two dimensions to a LEZ design: its **stringency** (i.e., the type of cars that are restricted; either no cars, only gasoline cars, or both gasoline cars and hybrids), and its **size**, $s \in [0, \bar{s}]$, where $\bar{s}$ is the largest LEZ that is (politically) feasible to implement. We let $\bar{s} \leq \gamma/3v_1$, which is the size of a LEZ that would completely displace gasoline cars from the market.[22]

For now, we restrict attention to a LEZ design that makes no distinction between restricted cars, so if both hybrids and gasoline cars happen to be restricted, then $s^G = s^H = s$. Later, we will consider designs that may vary by vehicle type, provided that more than one type is restricted. Since EVs are pollution-free, it is never optimal to limit their access to the LEZ. Hence, without loss of generality, we set $v_1^E = v_1$.

Our second proposition characterizes, for a given LEZ of size s, which vehicles (if any) should be restricted.

Proposition 2 *(LEZ solution with short-lived vehicles) Consider a LEZ of size s and the following emission thresholds:*

$$e_G^{LEZ}(s) \equiv \frac{2\gamma - 3v_1 s}{3(2-\alpha)} < \frac{2\gamma}{3(2\alpha - 1)} \equiv e_H^{LEZ}.$$

Then, it is optimal to:

[22] The size of a LEZ that would completely displace gasoline and hybrid cars is $2\gamma/3v_1$. It seems realistic, however, to restrict attention to LEZs not larger than $\gamma/3v_1$, or more generally, not larger than $\Gamma/3V$. Looking at larger LEZs would only introduce additional cases without adding new insights.

(i) restrict no vehicle if $e \leq e_G^{LEZ}$

(ii) restrict only gasoline cars if $e_G^{LEZ} < e \leq e_H^{LEZ}$*, and*

(iii) restrict both gasoline and hybrid vehicles if $e > e_H^{LEZ}$*.*

The LEZ thresholds stated in the above proposition correspond to the solid lines depicted in Figure 5. Restricting gasoline vehicles entails a trade-off. On the one hand, gasoline car users suffer a loss as they cannot access the restricted area. On the other, there is a reduction in emissions as consumers substitute gasoline cars for hybrids and EVs. Therefore, it is optimal to restrict gasoline cars when their emissions are high relative to the cost of reducing choice (proxied by γ). The critical emission threshold e_G^{LEZ} also depends on the size of the LEZ: a bigger LEZ reduces the market share of gasoline cars, thus reducing the mass of drivers who are restricted from entering the LEZ. Hence, for small s, the mass of (inframarginal) gasoline-car users is large, and it is not optimal to restrict them (neither hybrid users) unless their emissions are substantial. In this case, the welfare loss inflicted on inframarginal gasoline users is more than compensated by the emission-reduction gain from marginal gasoline users who switch to cleaner cars.

For a given size s, the LEZ policy is a blunt instrument that does not induce the FB level of green car adoption. In fact, if s is small, gasoline car sales are above their FB level. However, there is a critical size of the LEZ above which there is over-shooting, with gasoline car sales falling below their FB level.

Similarly, restricting hybrid vehicles entails a trade-off between the welfare loss of constraining hybrid users and the impact on emissions. However, unlike gasoline cars, restricting hybrids does not necessarily lead to lower emissions: some sales are shifted to cleaner electric vehicles, but others to more polluting gasoline cars as hybrids lose their advantage relative to gasoline cars. In particular, if hybrids are sufficiently clean relative to gasoline cars, i.e., if $\alpha < 1/2$, the second effect implies that net emissions would increase. Hence, in this case, it is never optimal to restrict hybrids from accessing the LEZ. Otherwise, if $\alpha > 1/2$, net emissions would decrease, making it optimal to restrict hybrids, but only when the emissions reduction is high relative to the cost of limiting car choice (proxied by γ). The emissions threshold does not depend on the size of the LEZ directly given that, conditionally on restricting gasoline cars (for which s has to be sufficiently large), welfare is proportional to s whether hybrids are restricted or not.

Importantly, allowing hybrids into the LEZ gives hybrids an advantage relative to gasoline cars. This serves to strengthen the power of the LEZ to induce customers to switch away from gasoline cars. In this sense, hybrids can be viewed as second-best technologies: they play a role in a second-best LEZ policy but not in the FB solution. Depicted as Region II in Figure 5, there are parameter values for which hybrids would not be sold under Pigouvian taxes (Proposition 1) and yet, they are allowed to enter the LEZ (Proposition 2). Our following corollary characterizes when this is the case for a given LEZ of size s.

Corollary 1 *(Second-best technology) For a given LEZ of size* s*, hybrids are given access to the LEZ even though they do not belong to the FB solution if and only if* $\max\left\{e_G^{FB}, e_H^{FB}\right\} < e \leq e_H^{LEZ}$*.*

It follows that hybrids should be considered second-best technologies only when gasoline cars are very polluting, and hybrids are not so polluting relative to gasoline cars. There is a similar implication with regard to gasoline cars, but going in the opposite direction. As illustrated in Region IV of the figure, more gasoline cars are sold under Pigouvian taxation than under the LEZ policy.

Given the optimal access policy to the LEZ, we now characterize its optimal size.

Proposition 3 *(Optimal LEZ with short-lived vehicles) Let $\bar{s} \leq \gamma/3v_1$ be the largest (politically feasible) LEZ and consider the thresholds $\tilde{s}(e)$ and $\underline{e}$ given by:*

$$e_G^{LEZ}(\tilde{s}) = e \text{ and } \underline{e} \equiv e_G^{LEZ}(\gamma/3v_1) = \frac{\gamma}{3(2-\alpha)}$$

Then, it is optimal to have:

(i) no LEZ if either $e \leq \underline{e}$ or $\underline{e} < e \leq e_G^{LEZ}$ and $\bar{s} < \tilde{s}(e)$

(ii) a LEZ of the maximum possible size $\bar{s}$ and only restrict gasoline vehicles if $e_G^{FB} < e \leq e_H^{LEZ}$ or $\underline{e} < e \leq e_G^{FB}$ and $\bar{s} \geq \tilde{s}(e)$, and

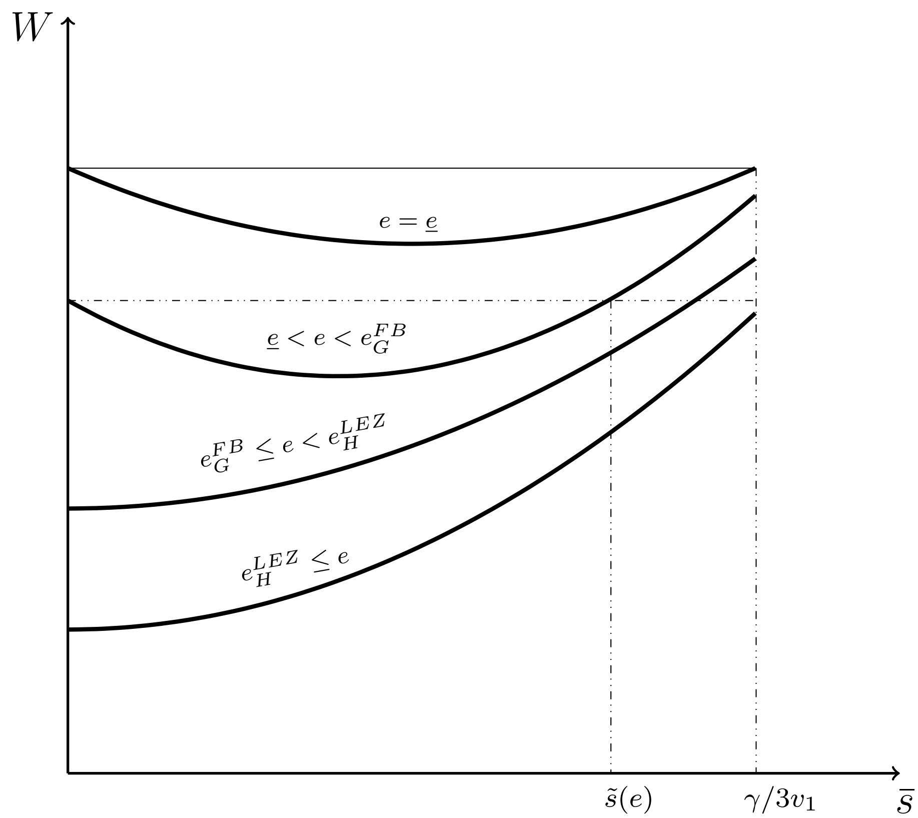
(iii) a LEZ of the maximum possible size $\bar{s}$ and restrict gasoline vehicles and hybrids if $e > e_H^{LEZ}$

As illustrated in Figure 6 (and formally shown in the proof of the proposition), welfare is a convex function of the size of the LEZ. The reason is that the larger the LEZ, the lower the sales of restricted vehicles. Hence, limiting access to these vehicles becomes increasingly more appealing, given that the utility loss would affect fewer (inframarginal) users. Thus, the size of the optimal LEZ is a bang-bang solution: either to have no LEZ ($s = 0$), if it cannot be made large enough, or to have the largest possible LEZ ($s = \bar{s}$).[23] In sum, intermediate-sized LEZ are never optimal.[24]

Figure 6: Optimal LEZ Size

[23]Note that $\tilde{s}(e) = 0$ for $e = e_G^{FB}$, so if $e_G^{FB} \leq e < e_H^{LEZ}$, any LEZ restricting gasoline cars leads to welfare gains.

[24]This bang-bang property is unrelated to the Salop structure. It is due to inefficient rationing.



LEZs contribute to increasing welfare if they are powerful enough to affect purchase decisions away from polluting vehicles, even at the cost of reducing value for inframarginal consumers. If emissions are large enough, any gain on marginal sales compensates for the inframarginal loss. This explains why, when e is large enough, it is always optimal to create a LEZ and make it as large as possible. However, if emissions are not high enough (like the two upper curves in Figure 6), the LEZ has to be large enough so that the impact on the marginal sales compensates for the loss of inframarginal users, whose mass is lower the larger the size of the LEZ. This explains why it is optimal to make the LEZ as large as possible, provided the maximum size is large enough (part (ii) of the proposition), or not to create a LEZ altogether (part (i) of the proposition).

Our theory so far provides three testable hypotheses. The first is whether letting hybrids enter MC is a sensible idea. The answer to this question requires estimates of e and α (the latter, in turn, depends on substitution patterns between gasoline and both hybrid and electric vehicles). The second hypothesis, conditional on finding support for the first, is whether MC is large enough to justify its restriction upon gasoline cars. The third is whether a bigger MC would be welfare-enhancing, considering the various trade-offs discussed above.

One could extend our theory to advance a fourth hypothesis, which is whether the social planner could gain from a LEZ design that considers different sizes for hybrids and gasoline cars, say s^G and s^H. Convexity of the welfare function, however, implies that this is never optimal, as our following proposition establishes.

Proposition 4 *If it is possible to set* $s^H \in [0, s^G]$*, the optimal LEZ involves either*

$s^H = s^G$ *or* $s^H = 0$.

Like Propositions 2 and 3, this proposition also follows a bang-bang logic: hybrids should be treated either like electric vehicles ($s^H = 0$) or like gasoline cars ($s^H = s^G$). This invites a fifth hypothesis: in a dynamic environment, where the fleet adjusts gradually to a policy shock, could it be optimal to treat hybrids like electric vehicles for a certain number of periods, after which they should be treated like gasoline cars? A positive answer would make hybrids a transition technology according to our theory.[25] We explore this possibility next.

3.5 Equilibrium with Long-Lived Vehicles

The first-best (FB) design remains unchanged with the introduction of long-lived vehicles, so we focus here only on LEZ designs. For that purpose, suppose that all cars last for two periods, regardless of their type or whether a LEZ policy is in place, meaning they are scrapped at the end of the second period or the beginning of the third.[26] We leave for the extensions the case where some cars' lifespans may change in response to the policy.

In this two-period cycle, it will take at most three periods for the fleet composition to reach its new steady-state after the introduction of a LEZ in period $t = 1$ (it will take only two periods if the LEZ design is time-invariant). Therefore, the planner's welfare function can be written as

$$W = \sum\nolimits_{t=1}^{\infty} \delta^{t-1} W_t = W_1 + \delta W_2 + \frac{\delta^2}{1-\delta} W_3 \tag{5}$$

where W_t is the per-period welfare in $t = 1, 2, 3, ...$To simplify notation, from now on we will assume that all individuals, including the social planner, discount the future at $\delta = 1/2$.

Consider a LEZ policy introduced at $t = 1$. During this first period, second-hand cars affected by the policy will outnumber their new counterparts. This implies that, in equilibrium, the price of these second-hand cars must fall enough to make their drivers indifferent between them and new cars. If the policy is adjusted at $t = 2$——for example, by adding more restricted cars or extending its size——new and second-hand cars of each type will continue to be in different proportions. Only at $t = 3$ new and second-hand cars of each type will be present in equal numbers, marking the point at which the fleet reaches its new steady state.

Thus, if at $t = 1$ the social planner commits to a time-invariant LEZ of size s, the short-lived analysis from Proposition 2 extends to this dynamic environment with only minor notational changes.[27]

Proposition 5 *(Time-invariant LEZ with long-lived vehicles) Consider a time-invariant LEZ of size* $s < \bar{s} \equiv \Gamma/3V = \gamma/(2v_1 + v_2)$ *introduced at* $t = 1$ *and the following emission*

[25] As we will see below, our view of transition technology differs sharply from that based on changes in the relative cost of electric vehicles overtime.

[26] Formally, we are assuming that expression (2) holds at all times, with $v_a^j = (1 - s_a^j)v_a$.

[27] We also leave for the extensions the case where the planner cannot commit.

thresholds:

$$\hat{e}_G^{LEZ}(s) \equiv \frac{4\gamma(v_1+v_2)-(2v_1+v_2)^2 s}{3(2v_1+v_2)(2-\alpha)} < \frac{4\gamma(v_1+v_2)}{3(2v_1+v_2)(2\alpha-1)} \equiv \hat{e}_H^{LEZ}.$$

Then, it is optimal to:

(i) restrict no vehicle if $e \leq \hat{e}_G^{LEZ}$,

(ii) restrict only gasoline cars if $\hat{e}_G^{LEZ} < e \leq \hat{e}_H^{LEZ}$, *and*

(iii) restrict both gasoline and hybrid vehicles if $e > \hat{e}_H^{LEZ}$.

In a dynamic environment, however, the planner has the option to commit to a LEZ that evolves over time. In principle, the LEZ can evolve in both stringency (i.e., the type of cars that are restricted) and size. It is not difficult to anticipate from Propositions 2 and 3 that the short-lived analysis regarding gasoline cars remains unchanged: if it is optimal to restrict gasoline cars, they should be restricted to the maximum extent—that is, from the very first day, $t = 1$, and across the largest possible area.

Interestingly, this parallel does not extend to hybrids. It may be optimal to treat them gradually (i.e., as a transition technology)—yet still within a bang-bang logic—by keeping them unrestricted like electric vehicles for a certain number of periods, after which they become as restricted as gasoline cars. As the next proposition establishes, in our dynamic environment where cars last for two periods, this translates into placing no restriction on hybrids during the first period, $t = 1$, and applying the gasoline-level restriction from the second period onward, $t = 2, 3, ...$

Proposition 6 *(Hybrids as a transition technology) Consider a LEZ of size* $s < \bar{s}$, *introduced at t=1, and the following emission thresholds:*

$$\hat{e}_T^{LEZ} \equiv \frac{16\gamma(v_1+v_2)+v_1(v_1+8v_2)s}{6(5v_1+4v_2)(2\alpha-1)} \text{ and } \hat{e}_{TT}^{LEZ} \equiv \max\left\{\hat{e}_G^{LEZ}, \hat{e}_T^{LEZ}\right\} < \hat{e}_H^{LEZ}$$

where $\hat{e}_G^{LEZ}$ *and* $\hat{e}_H^{LEZ}$ *are as defined in Proposition 5.*

Then, it is optimal to:

(i) restrict no vehicle if $e \leq \hat{e}_G^{LEZ}$

(ii) restrict only gasoline cars from $t = 1$ *onward if* $\hat{e}_G^{LEZ} < e \leq \hat{e}_{TT}^{LEZ}$

(iii) restrict gasoline cars from $t = 1$ *onward, and treat hybrids as a transition technology (i.e., as electric cars during* $t = 1$, *and as gasoline cars from* $t = 2$ *onward) if* $\hat{e}_{TT}^{LEZ} < e \leq \hat{e}_H^{LEZ}$, *and*

(iv) restrict both gasoline and hybrid vehicles from $t = 1$ *onward if* $e > \hat{e}_H^{LEZ}$.

To convey some intuition about the emergence of a transition technology, imagine a market with only two types of cars: clean (or electric) and dirty (in our setting, this means letting $\alpha = 1$ and placing hybrids and gasoline cars at $x = 1/2$). Consider two LEZ designs. Under a time-invariant design introduced at $t = 1$, there will be an emission threshold—say, e_D^{LEZ}—determining whether dirty cars should be restricted from $t = 1$ onward or not at all. Under an alternative LEZ design, still announced at $t = 1$, there will be a lower emission threshold—say, $e_{DD}^{LEZ} < e_D^{LEZ}$—determining whether dirty cars should be restricted from $t = 2$ onward or not at all.[28] According to our theory, it would be optimal to treat polluting cars as a transition technology whenever $e \in \left(\hat{e}_{DD}^{LEZ}, \hat{e}_D^{LEZ}\right)$. Within this range, dirty cars are not so polluting as to warrant immediate restriction, but neither are they clean enough to remain unrestricted indefinitely.

Why does this transition technology emerge? Announcing at $t = 1$ that polluting cars will be restricted in the future has two effects. On the one hand, it has an immediate impact on car choice: in anticipation of their lower future value, some individuals switch right away from dirty cars, even if they are not yet restricted. As a result, there will be fewer inframarginal consumers driving dirty cars tomorrow, when the restriction becomes active. With fewer inframarginal consumers, it becomes less costly to impose a restriction on these dirty cars.

This same logic extends to hybrids, with the only caveat that when individuals move away from hybrids at $t = 1$, some switch to electric vehicles while others switch to gasoline cars. However, the shift to gasoline cars is less relevant, as those are already restricted. In other words, the mere announcement that hybrids will be restricted in the future makes the LEZ instrument—at least with respect to hybrids and for a limited time (in our case, one period)—work more like a price instrument: it affects consumer choices at the margin without destroying value for inframarginal consumers, i.e., those who continue driving hybrids in $t = 1$.

Note that $\hat{e}_T^{LEZ}$ in Proposition 6 is increasing in s, thereby reducing the range within which hybrids qualify as a transition technology. The reason is that a larger LEZ leaves fewer inframarginal consumers affected by the LEZ to protect, reducing the need for a transition technology to ease their pain. In a way, the size of the LEZ and the option to treat hybrids as a transition technology act as substitutes.[29]

It should be clear by now that our definition of a transition technology differs sharply from alternative notions, which are often based on the idea that electric vehicles remain too expensive or unfamiliar to serve as a viable outside option for many gasoline drivers. Under this alternative view, hybrids should remain unrestricted until these concerns dissipate. While our definition is not in conflict with that view, it is fundamentally different. For one, it is developed in a context in which primitives (i.e., costs, preferences, and pollution rates) do not change over time. For another, it applies specifically in the context of a LEZ, not of a FB intervention. Neither have alternative notions a bite in a FB context—if anything, taxes on polluting cars should be even higher to account

[28] These are the only two relevant LEZ designs to consider. It is easy to see that announcing the restriction on dirty cars to begin at $t = 3$ instead of $t = 2$ is strictly dominated by the other two designs.

[29] On the other hand, if s is sufficiently small—and α sufficiently large—such that $\hat{e}_T^{LEZ} \leq \hat{e}_G^{LEZ}$, then hybrids should be restricted whenever gasoline cars are, whether at $t = 2$ under a transition-technology mode, when $e \in \left(\hat{e}_G^{LEZ}, \hat{e}_H^{LEZ}\right)$, or at $t = 1$, when $e \geq \hat{e}_H^{LEZ}$.

for potential learning-by-doing and network externalities associated with the adoption of electric vehicles. For the same reason, alternative notions should have no bite in a LEZ context either.

Finally, it is important to emphasize that the bang-bang rationale behind our notion of transition technology is not an artifact of the assumption that cars last only two periods. In settings where cars last for more than two periods, there will still be a point in time when hybrids shift abruptly from being as unrestricted as electric vehicles to being as restricted as gasoline cars. As we discuss next, this rationale also holds when relaxing other assumptions, such as allowing a car's scrapping age to respond to a policy intervention.

3.6 Extensions

We can think of three (to be treated in the appendix or online Appendix)

1. Cars' lifespans may change in response to a policy shock. Does this change the bang-bang rationale.
2. Commitment....Again this doesn't change bang-bang rationale
3. LEZ size that can vary over time.....

4 Structural Analysis

- Motivated by our (reduced-form) empirical and theory results, we now proceed to estimate a flexible demand model
- Every quarter, consumers choose whether to buy a new car or not
- The utility that consumer i gets from buying car j in quarter h and city c is given by

$$u_{ijtc} = -\alpha_i p_{jt} - \delta_i X_j + \delta_j + \delta_t + \delta_c + \xi_{jtc} + \epsilon_{ijtc}$$

  where $\alpha_i = \pi_\alpha Inc_{m(i)}$, $\delta_i = \pi_\delta Inc_{m(i)} + \psi' LEZ_{m(i),t}$, and $\epsilon_{ijt} \sim EVI$

  - $X_j \in \{g, h, e\}$: vehicle type (gas, hybrid, electric).
  - $Inc_{m(i)}$: average income of zip-code $m(i)$ where i lives.
  - $LEZ_{m(i),t}$: indicator variable for zip-codes close to Madrid Central in the post-policy period.
  - $\delta_j, \delta_t, \delta_c$: car model, quarter, and city fixed effects.

4.1 Structural Model: Identification

- We estimate the model following BLP (1995) and Petrin (2002)
- Instruments for prices:

1. Real exchange rate between Spain and manufacturing country (Grieco et al 2023)
2. Price of lithium and steel interacted with the car's weight

- **Micro-moments:**

  - $\mathbb{E}[i$ chooses car with $\{p_{jt} > \bar{p}\} \,|Inc]$,
  - $\mathbb{E}[i$ chooses car with $\{X_j = x\} \,|Inc]$ for $x \in \{g, h, e\}$,
  - $\mathbb{E}[i$ chooses car with $\{X_j = x\} \,|LEZ]$ for $x \in \{g, h, e\}$.

4.2 Counterfactuals

What would be the effects on car sales and welfare if...

1. ... **hybrids were also restricted** from entering the LEZ?

   - We can modify ψ so that the cost induced to gasoline cars also applies to hybrid cars

2. ...the **LEZ area were larger** (smaller distance between municipalities and the LEZ)?

   - We can modify ψ so increase the penalty that LEZ imposes on owners of restricted cars

3. ...there were **higher subsidies for EV** adoption? How does the LEZ policy and subsidies compare in effectiveness/welfare impacts?

   - We can reduce prices p_jt for EVs and eliminate the penalty for restricted cars

5 Conclusions

In this paper, we have analyzed the extent to which the introduction of Madrid Central's Low Emission Zone (LEZ) has influenced the adoption of hybrid and electric vehicles (EVs) within the city. By leveraging a synthetic control method, we compared Madrid's car registration trends with a carefully constructed synthetic Madrid.

Our findings indicate that the implementation of the LEZ has had a significant impact on the adoption of hybrid vehicles. Specifically, there was a notable increase in the registration of hybrids post-implementation, reflecting a shift in consumer preferences towards less polluting vehicles. This suggests that the LEZ was successful in steering car buyers away from gasoline vehicles, aligning with the policy's objective of reducing urban emissions. However, the impact on EV adoption is less clear-cut. While there was an observed increase in EV registrations, our placebo tests raised questions about the causal relationship between the LEZ and this increase.

Overall, our study highlights the complexity of policy interventions in urban transportation. The positive effect on hybrid adoption demonstrates that well-designed environmental zones can influence consumer behavior and support sustainability goals. Nonetheless, the ambiguous results for EVs suggest that additional measures, perhaps in the form of targeted incentives or infrastructure improvements, may be necessary to fully realize the potential of LEZs in fostering the adoption of electric vehicles.

Future research could explore these dynamics more deeply, considering factors such as consumer preferences, economic incentives, and the availability of charging infrastructure to provide a more comprehensive understanding of how to best design and implement urban mobility policies.

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Appendix

A.2 Proof of Proposition 1

Part (i). Suppose that e is not so high that all types of cars are sold in equilibrium. From (3) we obtain that their market shares are given by

$$q_1^E = \frac{1}{3} + \frac{(1+\alpha)e}{2\gamma},\ q_1^H = \frac{1}{3} + \frac{(1-2\alpha)e}{2\gamma} \text{ and } q_1^G = \frac{1}{3} - \frac{(2-\alpha)e}{2\gamma}$$

Hence, a necessary condition for gasoline cars to enjoy a positive share is

$$e < e_G^{FB} \equiv \frac{2\gamma}{3(2-\alpha)}.$$

Part (ii). Suppose $e > e_G^{FB}$, so there are no gasoline cars sold under the FB intervention. From (3), we obtain that the market shares of the remaining types are given by

$$q_1^E = \frac{1}{2} + \frac{\alpha e}{\gamma} \text{ and } q_1^H = \frac{1}{2} - \frac{\alpha e}{\gamma}.$$

Hence, a necessary condition for hybrids to enjoy a positive share is

$$e < e_H^{FB} \equiv \frac{\gamma}{2\alpha}.$$

Note, however, that the difference

$$e_G^{FB} - e_G^{FB} = \frac{(6-7\alpha)\gamma}{6\alpha\,(2-\alpha)}$$

is positive as long as $\alpha < 6/7$. So for $\alpha \geq 6/7$, the condition for hybrids to enjoy a positive share reduces to $e < e_G^{FB}$.
Part (iii). It follows from Parts (i) and (ii).

A.3 Proof of Proposition 2

Part (i). Let $v^G = (1-s^G)v_1$ and $v^H = (1-s^H)v_1$. Assuming for now that v^G and v^H are such that all models are sold in equilibrium, take the derivative of the welfare expression (4) with respect to v^G and v^H to obtain (recall that $v_e = v$)

$$\frac{\partial W}{\partial v^G} = \frac{1}{6\gamma}(6v^G - 3v^H - 3v_1 + 2\gamma - 3(2-\alpha)e) \quad (6)$$

$$\frac{\partial W}{\partial v^H} = \frac{1}{6\gamma}\left(6v^H - 3v^G - 3v_1 + 2\gamma - 3(2\alpha-1)e\right) \quad (7)$$

respectively. These expressions show that welfare is convex in v^G and v^H, and that their difference

$$\frac{\partial W}{\partial v^G} - \frac{\partial W}{\partial v^H} = -\frac{3}{2\gamma}\left((1-\alpha)\,e + \left(v^H - v^G\right)\right) \tag{8}$$

is negative when evaluated at $v^H = v^G = v_1$. This indicates that any LEZ policy must start by restricting gasoline vehicles first.

To assess when this is the case, we compare welfare levels with and without restricting gasoline vehicles only by a LEZ of size s. Writing the welfare function as $W\left(s^H, s^G\right)$ for convenience, with $s^H = 0$ and $s^G = s$, we obtain

$$W(0,0) - W(0,s) = \frac{v_1 s}{6\gamma}\left(2\gamma - 3v_1 s - 3(2-\alpha)\,e\right)$$

which indicates that having no LEZ is optimal, as opposed to a LEZ of size s, when

$$e \leq e_G^{LEZ}(s) \equiv \frac{2\gamma}{3(2-\alpha)} - \frac{v_1 s}{2-\alpha} = e_G^{FB} - \frac{v_1 s}{2-\alpha}.$$

Part (ii). Suppose that $e > e_G^{LEZ}(s)$. To assess the benefit of adding hybrids to the LEZ restriction, we compare welfare levels with and without restricting them, conditional on gasoline cars already restricted. Simple algebra yields

$$W(0,s) - W(s,s) = \frac{v_1 s}{6\gamma}\left(2\gamma - 3(2\alpha-1)\,e\right)$$

which indicates that adding hybrids to the LEZ is optimal when

$$e \geq e_H^{LEZ} \equiv \frac{2\gamma}{3(2\alpha-1)}.$$

Note that $e_H^{LEZ} > e_G^{LEZ}(s)$ for all s, as expected.
Part (iii). It follows from Parts (i) and (ii).

A.4 Proof of Corollary 1 (Second-best technologies)

Simply compare Propositions 1 and 2 and the overlap of parameter values for which hybrids are allowed to enter the LEZ but do not belong to the FB solution.

A.5 Proof of Proposition 3

We evaluate welfare $W(s^H, s^G)$ at various solutions for given s^H and s^G.
Part (i). Suppose $e < e_H^{LEZ}$. We know from Proposition 2 that it is optimal to set $s^H = 0$. Let $s^G = s$ and consider two cases. First, consider $e < \underline{e} = e_G^{LEZ}(\gamma/3v_1) = \frac{\gamma}{3(2-\alpha)} < e_G^{LEZ}(s)$. In this case $W(0,0) > W(0,s)$ for any $s \leq \gamma/3v_1$, so it is optimal not to create a LEZ, i.e., set $s = 0$. Second, consider $\underline{e} < e < e_H^{LEZ}$. Note that $\partial W(0,s)/\partial s^2 = v_1^2/\gamma > 0$,

so $W(0,s)$ is convex in s (see also proof of Proposition 2). This implies that we need to compare welfare evaluated at both extremes: $W(0,0) - W(0,\bar{s}) > 0$ if

$$e < e_G^{LEZ}(\bar{s}) \equiv \frac{2\gamma}{3(2-\alpha)} - \frac{v_1}{2-\alpha}\bar{s},$$

which requires

$$\bar{s} < \frac{2\gamma - 3(2-\alpha)e}{3v_1} \equiv \tilde{s}.$$

Part (ii). For the same logic as above, if $\underline{e} < e < e_H^{LEZ}$ and $\bar{s} > \tilde{s}$ then $W(0,0) - W(0,\bar{s}) < 0$ so the optimal size of the LEZ is $\bar{s}$.

Part (iii). Last, if $e > e_H^{LEZ}$, conditionally on having a LEZ of size s, the optimal policy is to limit hybrids and gasoline vehicles. The welfare function $W(s,s)$ is convex in s. From the proof of Proposition 2 above, we know that $W(0,\bar{s}) < W(\bar{s},\bar{s})$ if $e > e_H^{LEZ}$, which completes the proof.

A.6 Proof of Proposition 4

By contradiction. Let's suppose there exist some $\hat{s}^H \in (0, s^G)$ such that $W(\hat{s}^H, s^G) > \max\{W(0,s^G), W(s^G,s^G)\}$. Consider first the case in which $W(\hat{s}^H, s^G) > W(0,s^G) > W(s^G,s^G)$. Compute the derivative

$$\frac{\partial W(s^H,s^G)}{\partial s^H} = \frac{v(-2\gamma - 3e + 6\alpha e + 6v_1 s^H - 3v_1 s^G)}{6\gamma} \tag{9}$$

and cross-derivative

$$\frac{\partial W(s^H,s^G)}{\partial s^H \partial s^G} = -\frac{v_1^2}{2\gamma} < 0.$$

Since the cross-derivative is negative, there is less of a reason to restrict hybrids. If so, it must be true that (9) is positive at $s^H = \hat{s}^H$ Otherwise, it would not be possible to explain the increase in welfare from $W(0,s^G)$ to $W(\hat{s}_h, s^G)$ and the fact that (9) is increasing in s^H (for that increase one would need that (9) takes negative values as s^H is increased from $s^H = 0$ to $s^H = \hat{s}^H$, and those negative values occur as one increases s^H). But if (9) is negative at $s^H = \hat{s}^H$, one wants to continue all the way to $s^H = s^G$, a contradiction with $W(0,s^G) > W(s^G,s^G)$.

The proof of the second case $W(s^H,s^G) > W(s^G,s^G) > W(0,s^G)$ follows the same logic.

A.6 Proof of Proposition 5

It follows the proof of Proposition 6 while imposing $s_1^H = s_2^H = s^H$.

A.7 Proof of Proposition 6

The location of the indifferent consumer between the different types of cars will vary overtime and whether cars are new or second-hand. Denote by $\tilde{x}^{jk}_{a,t}$ the location of the consumer that at time $t \geq 1$ is indifferent between a car type $j = E, H, G$ of age $a = 1, 2$ and a car type $k \neq j$ of the same age a. Because cars survive until they are scrapped, we have that $\tilde{x}^{jk}_{2,t+1} = \tilde{x}^{jk}_{1,t}$ for all $t \geq 1$. We also know that at $t = 3$ the fleet reaches its steady state, so $\tilde{x}^{jk}_{1,t-1} = \tilde{x}^{jk}_{a,t} = \tilde{x}^{jk}_{a,t+1}$ for all a and $t \geq 3$.

To find $\tilde{x}^{jk}_{a,t}$ we use the set of Bellman's equations adjusted to account for a LEZ policy affecting gasoline cars and possibly hybrids. For example, to find $\tilde{x}^{EH}_{1,1}$, we start by solving the system of equations pertaining to drivers of electric vehicles (recall that $x^E = 0$)

$$U^E_1(\tilde{x}^{EH}_{1,1}) = v_1 - \gamma(\tilde{x}^{EH}_{1,1} - x^E) - c + \delta U^E_2(\tilde{x}^{EH}_{1,1}) \tag{10}$$

and

$$U^E_2(\tilde{x}^{EH}_{1,1}) = v_2 - \gamma\tilde{x}^{EH}_{1,1} + \delta z + \delta U^E_1(\tilde{x}^{EH}_{1,1}) \tag{11}$$

from where we obtain $U^E_1(\tilde{x}^{EH}_{1,1})$ and $U^E_2(\tilde{x}^{EH}_{1,1})$. Then, and allowing hybrids to face different restrictions today ($t = 1$) and tomorrow ($t \geq 2$), we solve the system (recall that $x^H = 1/3$)

$$U^H_1(\tilde{x}^{EH}_{1,1}) = (1 - s^H_1)v_1 - \gamma|\tilde{x}^{EH}_{1,1} - x^H| - c + \delta U^H_2(\tilde{x}^{EH}_{1,1}) \tag{12}$$

and

$$U^H_2(\tilde{x}^{EH}_{1,1}) = (1 - s^H_2)v_2 - \gamma|\tilde{x}^{EH}_{1,1} - x^H| + \delta z + \delta U^H_1(\tilde{x}^{EH}_{1,1}) - \delta(s^H_2 - s^H_1)v_1 \tag{13}$$

where s^H_t is the size of the LEZ affecting hybrids in period t, with $s^H_2 \geq s^H_1$ (recall that $s^H_t = s^H_2$ for all $t \geq 3$). Note that the last term in (13) captures the fact that $U^H_1(\tilde{x}^{EH}_{1,1})$ in period $t \geq 2$ is lower than in period $t = 1$ because the LEZ restriction is possibly larger in $t \geq 2$ than in $t = 1$. Solving (12) and (13) we obtain $U^H_1(\tilde{x}^{EH}_{1,1})$ and $U^H_2(\tilde{x}^{EH}_{1,1})$. Finally, $\tilde{x}^{EH}_{1,1}$ is obtained from solving $U^E_1(\tilde{x}^{EH}_{1,1}) = U^H_1(\tilde{x}^{EH}_{1,1})$.

To obtain $\tilde{x}^{EH}_{1,2}$, the location of the indifferent consumer between hybrids and EVs in period $t = 2$, we proceed much as before but for a minor adjustment. Since EVs are never restricted, $U^E_1(\tilde{x}^{EH}_{1,2})$ is obtained directly from the system (10)–(11), simply changing $\tilde{x}^{EH}_{1,1}$ for $\tilde{x}^{EH}_{1,2}$. The system to find $U^H_1(\tilde{x}^{EH}_{1,2})$ is slightly different than before

$$U^H_1(\tilde{x}^{EH}_{1,2}) = (1 - s^H_2)v_1 - \gamma|\tilde{x}^{EH}_{1,2} - x^H| - c + \delta U^H_2(\tilde{x}^{EH}_{1,2})$$

and

$$U^H_2(\tilde{x}^{EH}_{1,2}) = (1 - s^H_2)v_2 - \gamma|\tilde{x}^{EH}_{1,2} - x^H| + \delta z + \delta U^H_1(\tilde{x}^{EH}_{1,2})$$

Solving the above system to obtain $U^H_1(\tilde{x}^{EH}_{1,2})$ and making $U^E_1(\tilde{x}^{EH}_{1,2}) = U^H_1(\tilde{x}^{EH}_{1,2})$ lead to $\tilde{x}^{EH}_{1,2}$.

We skip the solution for the remaining indifference locations, namely, $\tilde{x}^{EG}_{1,1}$, $\tilde{x}^{EG}_{1,2}$, $\tilde{x}^{HG}_{1,1}$ and $\tilde{x}^{HG}_{1,2}$. These indifference locations will be a function of the primitives and the

LEZ design, which is given by the triplet (s^G, s_1^H, s_2^H). Note that since gasoline cars are restricted from $t = 1$ onward, $\tilde{x}_{1,1}^{EG} = \tilde{x}_{1,2}^{EG}$, reducing the number of indifference locations to the following five (after making $\delta = 1/2$)

$$\begin{aligned}
\tilde{x}_{1,1}^{EH} &= \frac{1}{12\gamma}\left(2\gamma + 3v_1 s_1^H + (v_1 + 2v_2)s_2^H\right), \\
\tilde{x}_{1,2}^{EH} &= \frac{1}{6\gamma}\left(\gamma + (2v_1 + v_2)s_2^H\right), \\
\tilde{x}_{1,1}^{HG} &= \frac{1}{12\gamma}\left(6\gamma + (4v_1 + 2v_2)s^G - 3v_1 s_1^H - (v_1 + 2v_2)s_2^H\right), \\
\tilde{x}_{1,2}^{HG} &= \frac{1}{6\gamma}\left(3\gamma + (2v_1 + v_2)s^G - (2v_1 + v_2)s_2^H\right), \text{ and} \\
\tilde{x}_{1,t}^{EG} &= \frac{1}{6\gamma}\left(5\gamma - (2v_1 + v_2)s^G\right).
\end{aligned}$$

Plugging these indifference locations into the welfare function (5), we obtain an expression that can be conveniently written as function the LEZ design as follows,

$$W(s^G, s_1^H, s_2^H) = W_1(s^G, s_1^H, s_2^H) + \delta W_2(s^G, s_1^H, s_2^H) + \frac{\delta^2}{1-\delta} W_3(s^G, s_2^G)$$

The rest of the proof—finding the emission thresholds $\hat{e}_G^{LEZ}$, $\hat{e}_H^{LEZ}$, and $\hat{e}_T^{LEZ}$—follows the same steps found in the proof of Proposition 2, so it can be omitted.