Central Bank Digital Currency and Financial Stability*

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Abstract

We develop a model of financial intermediation with remunerated Central Bank Digital Currency (CBDC) as consumers' alternative to bank deposits and an endogenous risk of bank runs. Echoing widespread concerns, higher CBDC remuneration raises bank fragility by increasing consumers' withdrawal incentives. On the other hand, it also induces banks to offer more attractive deposit contracts in order to retain funding, thereby reducing fragility. This results in a U-shaped relationship between bank fragility and CBDC remuneration. We evaluate policy proposals aimed at mitigating the financial-stability risks of CBDC, such as holding limits and contingent CBDC remuneration.

Keywords: Central Bank Digital Currency, Bank Fragility, Financial Stability, CBDC Remuneration, Global Games.

JEL Codes: D82, G01, G21.

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1 Introduction

Central banks around the globe (Kosse and Mattei, 2022) are researching the costs and benefits of central bank digital currency (CBDC). These efforts are a response to the declining importance of cash as means of payment and the challenges associated with the proliferation of new forms of private digital money such as stablecoins. While CBDC aims to preserve the role of public money and fend off threats to monetary sovereignty, some policy makers are concerned about its potentially adverse effects on the financial system (Ahnert et al., 2022) and on the business model of traditional banks (Vives, 2019).

One issue that has received particular attention is the effect of CBDC on financial stability (Bank for International Settlements, 2020). Its status as safe asset with potentially positive remuneration—a key difference to physical cash—could render it an attractive store of value and thus increase the risk of bank runs during crisis times. The recent episode involving U.S. regional banks highlights that bank runs continue to be an important real-world phenomenon.

This paper aims to inform this debate by incorporating remunerated CBDC in an otherwise standard global-games bank-run model (Goldstein and Pauzner, 2005; Vives, 2005; Carletti et al., 2023). At the initial date, a profit-maximizing bank with access to profitable but risky long-term investment opportunities raises uninsured deposits. At the interim date, consumers receive a noisy private signal about the investment's profitability ("economic fundamentals") and decide whether to withdraw their balances or roll them over. Funds that are not kept in the monopolist bank can be held either in cash or CBDC. In line with the existing literature, we assume that CBDC holdings are possibly remunerated, which improves consumers' outside option and thus curtails the bank's market power in the deposit market (e.g., Chiu et al. 2023; Andolfatto 2021; Whited et al. 2023).

When making their withdrawal decisions, consumers trade off the value of keeping their funds in the bank and converting them into CBDC. Accordingly, our model allows us to study how the terms of the deposit contract and CBDC remuneration affect the probability of a bank run (our measure of bank fragility).

In this economy, an increase in CBDC remuneration has two effects. First, it makes withdrawals at the interim date more attractive by offering a higher payoff from storing funds with the central bank for consumption at the final date. This *direct effect* makes the bank more fragile, consistent with the line of argument underlying the ongoing policy debate. Second, a higher CBDC remuneration improves consumers' outside option at the funding stage, and therefore induces the bank to offer more attractive deposit contracts. As a consequence, consumers have lower incentives to withdraw their funds at the interim date. This *indirect effect* renders the bank more stable.

In equilibrium, the total effect of CBDC remuneration on bank fragility depends on the relative strengths of these two countervailing forces. The indirect effect dominates if and only if the elasticity of the run probability with respect to the bank deposit rate exceeds one. A sufficient condition for this to obtain is that the profitability of the bank's investment project is high relative to the level of remuneration on CBDC. In this case, bank fragility is minimized (corresponding to maximized utilitarian welfare) for a strictly positive level of CBDC remuneration.

Importantly, CBDC remuneration has redistributive effects. A higher CBDC rate moves rents from banks (whose expected profits shrink) towards depositors (who earn higher deposit rates). As long as the CBDC rate is not too high, this redistribution is socially desirable because it helps to make banks more stable.

Next, we examine the potential effects of two CBDC design features that have received some attention in the policy debate. Various central banks (including the European Central Bank and the Bank of England) have proposed the introduction of individual holding limits with the aim of reducing financial stability concerns associated with CBDC (Bindseil et al., 2021; Bank of England, 2023). In our model, holding limits reduce the effective remuneration that consumers earn on withdrawn funds because only a fraction of them can be held in CBDC, with the remainder being held as cash. Accordingly, if the interest on CBDC can be set freely, then holding limits cannot improve the equilibrium in terms of welfare. However, they become a relevant tool if CBDC remuneration is exogenous from a financial stability perspective, for example because it mainly aims at monetary policy objectives outside of the model. Building on our previous insights, holding limits can then improve social welfare for high levels of CBDC remuneration. However, it is optimal not to impose any holding limits if the rate paid on CBDC is low. This result serves as a cautionary note to policymakers, suggesting that holding limits are not the panacea for the financial stability implications of CBDC.

Alternatively, we study the possibility that CBDC remuneration is contingent on the state of the financial system, i.e. that CBDC holdings earn a lower rate in crisis times. We distinguish between two dimensions of such a policy, namely the threshold (in terms of interim withdrawals) at which it enters into effect as well as the reduction in remuneration relative to tranquil times. We show that a more restrictive design along the first dimension always reduces fragility (and is related to a partial suspension of convertibility), and provide conditions for this effect to obtain along the second dimension. Intuitively, this policy tool dampens the direct effect by reducing the return from withdrawing deposits. On the other hand, it has little impact on the ex-ante return from holding CBDC, so its impact on the indirect effect is weak. We conclude that contingent remuneration can be an effective tool to reduce unwanted financial stability implications of CBDC.

Finally, we extend the model in three directions. First, we limit the bank's market power in the deposit market by assuming that the deposit contract is determined by Nash bargaining with depositors. In line with the intuition from the baseline model, a decline in bank market power weakens the indirect effect by reducing the bank's incentives to adjust the deposit rate in response to changes in CBDC remuneration. We show analytically that a higher CBDC rate always increases bank fragility with perfect competition, because the deposit rate is already quite high and not very responsive to changes in CBDC remuneration. We also provide numerical examples showing that our baseline result still holds provided that the bank has sufficient market power in the deposit market.

Second, we extend our model to allow for bank risk-taking on the asset side, which broadens the analysis to a more complete notion of financial stability.

Specifically, we assume that the bank can exert costly monitoring effort to increase the probability that the investment project is profitable. In this context, we define financial stability as the probability that the bank survives, i.e. there is no run *and* investment succeeds. We show analytically that higher CBDC remuneration affects these two separate components in different ways: It induces the bank to increase its monitoring intensity, but at the same time increases fragility because the exogenous deposit contract removes the indirect effect from the baseline model. Accordingly, the overall effect on financial stability is ambiguous. However, we provide a numerical example showing that an increase in CBDC remuneration can lead to an increase in financial stability, consistent with our main analysis.

Third, we explore the robustness of our results in a traditional bank-run model in which banks act as liquidity providers (Diamond and Dybvig, 1983). This shifts the focus from the long-term deposit rate (in the main model) to the interim repayment, which is a measure of bank liquidity provision. Following closely the global-games version of this model in Goldstein and Pauzner (2005), we show that a direct and indirect effect with opposite signs are still present. On the one hand, a higher CBDC rate increases the incentives to run (direct effect) by increasing the ultimate consumption value of interim withdrawals. On the other hand, banks respond to the introduction of CBDC by changing the terms of the deposit contract, which makes them less susceptible to runs (indirect effect). As in Keister and Monnet (2022), higher CBDC remuneration also reduces bank liquidity provision, i.e. it reduces the short-term deposit rate.

Literature. Our paper is part of a fast-growing literature on CBDC. An overview of recent work is found in Ahnert et al. (2022). A key feature of our model is that the bank is not passive, but instead adjusts its behaviour (here its deposit rates) in response to the introduction to CBDC. This channel is also present in recent papers that study the effects of CBDC on credit supply (Keister and Sanches, 2022; Andolfatto, 2021; Chiu et al., 2023; Whited et al., 2023; Brunnermeier and Niepelt, 2019). In contrast, our focus is on financial stability.

Several other papers connect CBDC to financial stability. Using a Diamond and Dybvig (1983) model, Fernández-Villaverde et al. (2021) and Fernández-Villaverde et al. (2023) study the implications for bank runs. They show that, by fostering a flow of deposits out of the banking system into the central bank, the introduction of CBDC completely removes the risk of bank runs, as also shown in Skeie (2020), while creating a trade-off for the central bank between efficiency and price stability. Keister and Monnet (2022) also consider the implications of CBDC for bank runs, but focus on the efficacy of government interventions. In their framework, CBDC allows the central bank to have more accurate information about the health of the banking sector and thus to intervene promptly to mitigate the risk of a run. In Williamson (2022), the fragility of banks induced by the introduction of CBDC is efficient.

A key difference relative to these papers is our use of global-games methods to uniquely pin down the probability of a bank run. This approach allows us to study how CBDC design affects bank fragility, both directly via withdrawal incentives and indirectly via the bank's response in deposit rates. Global games were introduced by Carlsson and van Damme (1993), and have been widely applied to study run-like behaviour (e.g. Rochet and Vives, 2004; Goldstein and Pauzner, 2005; Vives, 2014; Bouvard et al., 2015; Liu, 2016; Ahnert, 2016; Eisenbach, 2017; Ahnert et al., 2019; Liu, 2023; Carletti et al., 2023; Schilling, 2023). Morris and Shin (2003) and Vives (2005) survey the theory and applications of global games.

2 Model

The model builds on Goldstein and Pauzner (2005) and Carletti et al. (2023). The economy extends over three dates $t = 0, 1, 2$ and is populated by a bank and a unit continuum of consumers indexed by $i \in [0, 1]$. There is a single divisible good for consumption and investment. All agents are risk neutral and do not discount the future. Consumers are endowed with one unit of funds at $t = 0$ only.

At $t = 0$, the bank has access to a profitable but risky investment technology. Investment returns $L \in (0, 1)$ if liquidated at $t = 1$ (the liquidation value) and $R\theta$ upon maturity at $t = 2$, where $\theta \sim U[0, 1]$ represents the economic fundamental and $R > 2$ is a constant that reflects the return from lending. To finance investment, the bank raises funds from consumers in exchange for demandable deposit contracts.[1] The bank chooses the deposit contract that maximizes expected profits. The contract specifies a repayment $r_1 \geq 1$ at $t = 1$ and $r_2 > r_1$ at $t = 2$.[2]

Debt is demandable, so depositors can withdraw their funds before the bank's investment mature. At $t = 1$, each depositor receives a noisy private signal about the fundamental

$$s_i = \theta + \varepsilon_i, \tag{1}$$

with $\varepsilon_i \sim U[-\varepsilon, +\varepsilon]$. In addition to being informative about the profitability of the bank's investment project, it also provides information about the signals (and withdrawal actions) of other depositors. As is standard in much of the global-games literature, we assume vanishing noise, $\varepsilon \to 0$, to simplify the analysis.[3]

The bank satisfies interim withdrawals by liquidating investment. Let $n \in [0, 1]$ be the fraction of consumers who withdraw at $t = 1$. When the liquidation proceeds at $t = 1$ are insufficient to meet withdrawals, $n > \overline{n} \equiv \frac{L}{r_1}$, the bank is bankrupt due to illiquidity. Otherwise, it continues to operate until $t = 2$. If the bank cannot meet the remaining withdrawals, $n > \widehat{n} \equiv \frac{R\theta - r_2}{R\theta \frac{r_1}{L} - r_2}$, it is bankrupt due to insolvency, where $\widehat{n}$ solves the insolvency condition

$$R\theta \left(1 - \frac{\widehat{n} r_1}{L}\right) = (1 - \widehat{n})\, r_2. \tag{2}$$

The left-hand side is the return on the part of the project that was not liquidated at $t = 1$, and the right-hand side represents the remaining withdrawals at $t = 2$.

[1] Bank debt is assumed to be demandable, which arises endogenously with liquidity needs (Diamond and Dybvig, 1983) or as a commitment device to overcome agency conflicts (Calomiris and Kahn, 1991; Diamond and Rajan, 2001). Accordingly, uninsured deposits refer to any short-term or demandable debt instrument, which includes uninsured retail deposits and insured deposits when deposit insurance is not credible (Bonfim and Santos, 2020). Three quarters of U.S. commercial bank funding are deposits, half of which are uninsured (Egan et al., 2017).

[2] We relax the lower bound on the short-term deposit rate r_1 in Section 5.3.

[3] The assumption of vanishing private noise ensures a unique equilibrium and simplifies the analysis of the bank's choice of deposit rate at the initial date. In a similar framework, Vives (2014) studies the properties of the multiple equilibria that arise when this assumption is relaxed.

Bankruptcy is costly and we assume zero recovery for simplicity.[4]

As alternatives to bank deposits, consumers can store their wealth in CBDC or cash. A deep-pocketed central bank offers consumers deposits with a per-period gross return $\omega \geq 1$, while cash is unremunerated.[5] Accordingly, consumers strictly prefer CBDC over cash as long as $\omega > 1$. They are indifferent for $\omega = 1$, so that this case is equivalent to a model without CBDC.

Our main interpretation is that ω represents the remuneration of CBDC. However, it could also capture other benefits relative to cash such as the reduced risk of theft or the additional convenience derived from digital payment means, such as the ability to settle e-commerce transactions.[6]

Relative to an economy with only deposits and cash, the introduction of CBDC has two effects. First, it improves the outside option of consumers deciding at $t = 0$ whether to deposit funds with the bank from 1 to ω^2 (the compound return on CBDC over two periods). Second, it pays interest ω on funds withdrawn from the bank at $t = 1$. Table 1 summarizes the timeline of the economy.

$\boldsymbol{t = 0}$	$\boldsymbol{t = 1}$	$\boldsymbol{t = 2}$
1. CBDC design	**1.** Fundamental shock	**1.** Investment matures
2. Bank sets rates	**2.** Private signals	**2.** Consumption
3. Consumers deposit	**3.** Withdrawal choice	

Table 1: Timeline

[4] Bankruptcy costs are large. For example, James (1991) measures the losses associated with bank failure as the difference between the book value of assets and the recovery value net of direct expenses associated with failure. These losses amount to about 30% of failed banks' assets.

[5] We abstract from both raising funds (e.g. via taxation) and an investment choice of the central bank at $t = 0$. This is without loss of generality in our model because the central bank disburses no funds on the equilibrium path. At $t = 0$, the bank sets deposit rates high enough such that consumers prefer bank deposits over CBDC. A run leads to costly liquidation of assets, so no funds are re-deposited with the central bank at $t = 1$. Nonetheless, the option of remunerated CBDC affects both withdrawal incentives at $t = 1$ and the deposit rate at $t = 0$.

[6] In principle, ω could also reflect an improved outside option arising from the existence of alternative stores of value such as treasury bonds or money market funds. However, these do not have a means-of-payment functionality enjoyed by cash, bank deposits, and CBDC.

3 Equilibrium

To solve for the equilibrium, we work backwards. First, for a given deposit contract and CBDC remuneration, we characterize a bank failure threshold $\theta^*(r_1, r_2, \omega)$. Next, we solve for the bank's choice of deposit contract $(r_1^*(\omega), r_2^*(\omega))$ for a given remuneration. Finally, we study how remuneration affects overall bank fragility.

3.1 Bank fragility

We use global-games methods to solve for the unique equilibrium at the withdrawal stage, building on Goldstein and Pauzner (2005) and Carletti et al. (2023). To characterize individual withdrawal decisions, we start by establishing the dominance bounds that yield ranges of the fundamental θ for which consumers have a dominant strategy. As in Goldstein and Pauzner (2005), we assume that there exists a threshold $\overline{\theta}$ such that the liquidation value is $L = R$ for $\theta > \overline{\theta}$, where we assume $\overline{\theta} \to 1$.[7] When $\theta \geq \overline{\theta}$, it is optimal for a depositor not withdraw irrespective of the withdrawal decision of all other depositors, i.e., waiting until date 2 is a dominant action.[8] Hence, the range $[\overline{\theta}, 1]$ identifies the upper dominance region.[9]

Second, withdrawing is a dominant strategy when $\theta < \underline{\theta}$. This (lower dominance) bound solves

$$R\underline{\theta} - r_2 = 0, \tag{3}$$

so that $\underline{\theta} = \frac{r_2}{R} \in (0, 1)$.[10] The intuition is as follows. When no other depositor withdraws ($n = 0$), the bank is always liquid at $t = 1$ and insolvent at $t = 2$ for $R\theta < r_2$. Therefore, withdrawing yields a payoff of r_1, while not withdrawing returns zero. So running on the bank is a dominant strategy for $\theta < \underline{\theta}$ (bankruptcy).

[7] This assumption can be viewed as a liquidation value of the bank's project that depends on the fundamental θ. Specifically, it is equal to $L < 1$ for $\theta \in [0, \overline{\theta})$ and to R for $\theta \in [\overline{\theta}, 1]$. The cutoff value $\overline{\theta}$ can be arbitrarily close to (but strictly below) 1.

[8] Given the technical assumption $L = R$, in the upper dominance region a depositor receives r_2 at date 2 and ωr_1 at date 1. The condition $r_2 > \omega r_1$ must hold to ensure that runs do not occur with certainty and the banker does not make zero profits with probability 1.

[9] An endogenous upper dominance region arises in Kashyap et al. (2023), where depositors receive signals about the asset's interim liquidation value instead.

[10] The bank always chooses $r_2^* < R$. Otherwise, deposit-taking would be unprofitable.

In the intermediate range $(\underline{\theta}, \overline{\theta})$, a consumer's decision to withdraw depends on what she expects the other consumers to do. Using global-games techniques, we can solve for the bank failure threshold, characterized in the next proposition.

Proposition 1. ***Failure threshold.*** *There exists a unique fundamental threshold $\theta^* \in \left(\underline{\theta}, \overline{\theta}\right)$. Each consumer withdraws their deposits and the bank fails if and only if $\theta < \theta^*$, where*

$$\theta^* \equiv \underline{\theta}\frac{r_2 - \omega L}{r_2 - \omega r_1} > \underline{\theta}. \tag{4}$$

The threshold θ^* decreases in L and R, increases in ω and r_1, and is non-monotonic in r_2: $\frac{\partial \theta^*}{\partial L} < 0$, $\frac{\partial \theta^*}{\partial R} < 0$, $\frac{\partial \theta^*}{\partial \omega} > 0$, $\frac{\partial \theta^*}{\partial r_1} > 0$, $\frac{\partial \theta^*}{\partial r_2} < 0$ if and only if $r_2 < r_2^{max}$.

Proof. See Appendix A, which also defines the threshold r_2^{max}. □

Under vanishing noise, the bank failure threshold θ^* corresponds to the probability of a bank run, which we thus use as our measure of bank fragility. A higher liquidation value L or higher profitability R reduce depositors' incentives to run.

The terms of the deposit contract (r_1, r_2) also affect the failure threshold. As in Diamond and Dybvig (1983) and Goldstein and Pauzner (2005), a higher short-term deposit rate increases fragility. Liquidity provision by the bank $(r_1 > L)$ gives rise to strategic complementarity in consumer withdrawal decisions, so that both panic runs and fundamental runs exist, $\theta^* > \underline{\theta}$.

Moreover, the relationship between the long-term deposit rate r_2 and bank fragility is non-monotonic: when the deposit rate is low, higher rates reduces fragility while the opposite holds for high deposit rates. Two opposing factors are at play. On the one hand, a higher long-term deposit rate implies that depositors receive a higher payoff when they wait and the bank is solvent. On the other hand, a higher long-term rate makes it more likely for the bank to be insolvent.

All else equal, the probability of a bank run increases with CBDC remuneration, since it increases the payoff from storing wealth outside the bank between $t = 1$ and $t = 2$, and thus makes withdrawing more attractive. However, this direct effect, $\frac{\partial \theta^*}{\partial \omega}$, fails to capture the overall impact because r_1 and r_2 are held fixed.

As we show below, changes in CBDC remuneration induce the bank to adjust the terms of the deposit contract, which in turn affects θ^*. To see this formally, we can use total differentiation:

$$\frac{d\theta^*}{d\omega} = \frac{\partial \theta^*}{\partial \omega} + \frac{\partial \theta^*}{\partial r_1}\frac{dr_1^*}{d\omega} + \frac{\partial \theta^*}{\partial r_2}\frac{dr_2^*}{d\omega}. \tag{5}$$

We next study these indirect effects of CBDC remuneration on bank fragility via the equilibrium deposit rates r_1^* and r_2^*.

3.2 Deposit rates

Since a bank run leads to zero profits, the bank internalizes the effects of the deposit contract on fragility, $\theta^* = \theta^*(r_1, r_2)$. With vanishing noise, $\epsilon \to 0$, consumer behaviour is fully symmetric. For $\theta > \theta^*$, there are no interim withdrawals and the investment matures at $t = 2$ with a return $R\theta$. The banker pays the promised return r_2 to consumers and pockets the difference, $R\theta - r_2$. For $\theta < \theta^*$, all consumers withdraw at $t = 1$ and the bank makes zero profits. Using $\overline{\theta} \to 1$, the banker's problem at $t = 0$ is therefore[11]

$$\max_{r_1 \geq 1, r_2} \Pi \equiv \int_{\theta^*}^{1} (R\theta - r_2)\, d\theta \tag{6}$$

$$\text{s.t. } V \equiv \int_{\theta^*}^{1} r_2\, d\theta - \omega^2 \geq 0. \tag{7}$$

Equation (7) is the consumers' participation constraint. The first term is the expected payoff from keeping funds in the bank until $t = 2$, which is the long-term deposit rate in case there is no bank run. The second term reflects the outside option, which is to store wealth in remunerated CBDC for a per-period return ω.

The following proposition characterizes the bank deposit rates in equilibrium.

Proposition 2. ***Deposit rates.*** *Let $\omega < \widetilde{\omega}$ and $R > \underset{\sim}{R}$. The equilibrium deposit rates are $r_1^* = 1$ and $r_2^* < r_2^{max}$. The long-term deposit rate solves $V(r_2^*) \equiv 0$ (the*

[11] Expected bank profits can be written as $\Pi = (1 - \theta^*) \left[\frac{R}{2}(1 + \theta^*) - r_2\right]$, which is naturally interpreted as the probability of no run times the expected bank profits conditional on no run.

participation constraint binds), increases in CBDC remuneration, and decreases in the liquidation value and investment profitability: $\frac{dr_2^*}{d\omega} > 0$, $\frac{dr_2^*}{dL} < 0$, *and* $\frac{dr_2^*}{dR} < 0$.

Proof. See Appendix B, which also defines the bounds $\widetilde{\omega}$ and $\underset{\sim}{R}$. □

A higher short-term deposit rate r_1 reduces expected bank profits because the bank is more fragile (see Proposition 1). This also tightens consumers' participation constraint, since they are repaid less often. Accordingly, the bank chooses the lowest possible value for r_1, which is independent of CBDC remuneration.

In general, the long-term deposit rate r_2^* is pinned down by either the bank's first-order condition or the consumer participation constraint. The bounds on R and ω are sufficient conditions for the participation constraint to bind. Intuitively, they ensure that the bank has a large enough margin to adjust the deposit contract.[12] We henceforth assume that these conditions are met.

An increase in CBDC remuneration improves consumers' outside option, both at the initial and interim dates. Accordingly, to remain attractive and guarantee consumer participation, the bank needs to offer a more attractive long-term deposit rate r_2^*. A higher liquidation value or investment profitability has the opposite effect. Because they reduce bank fragility, consumer participation can be satisfied with a lower long-term deposit rate.

Since it can exert market power, the bank offers a deposit rate r_2 lower than the one that minimizes fragility, which would be $r_2 = r_2^{max}$. While such a lower rate renders the bank more fragile, it also makes it more profitable in the states of the world in which the bank survives, thus maximizing expected bank profits.

Combining Propositions 1 and 2, a change in CBDC remuneration ω has two opposing effects on bank fragility θ^* in equilibrium. On the one hand, a higher remuneration leads to a higher incentive to withdraw at $t = 1$ and thus a larger threshold θ^*. On the other hand, the bank responds to higher remuneration by

[12]These assumptions ensure that the bank can always raise its deposit rate in response to higher CBDC remuneration, so financial intermediation continues to be feasible in our model. See Fernández-Villaverde et al. (2021) for a model in which CBDC crowds out all bank deposits.

increasing deposit rates r_2^* at $t = 0$, which reduces bank fragility ceteris paribus. The overall effect of a change in ω on θ^* depends on which of these two effects dominates. The next result offers some insight into their relative strength.

Lemma 1. ***Elasticity of the failure threshold.*** *Let* $\eta \equiv -\frac{r_2}{\theta^*}\frac{\partial \theta^*}{\partial r_2}$ *be the elasticity of the failure threshold with respect to the deposit rate. Higher CBDC remuneration reduces bank fragility,* $\frac{d\theta^*}{d\omega} < 0$*, if and only if* $\eta > 1$.

Proof. See Appendix C. □

Lemma 1 states that the indirect effect of higher CBDC remuneration dominates the direct effect whenever the failure threshold θ^* is very elastic to changes in the bank deposit rate r_2. That is, higher CBDC remuneration needs to induce a sufficiently strong increase in deposit rates for overall fragility to fall. The elasticity η depends on equilibrium deposit rates and, thus, ultimately on parameters.

We now state our main positive result on CBDC remuneration and bank fragility. It is also shown in Figure 1.

Proposition 3. ***CBDC remuneration and bank fragility.*** *Bank fragility is U-shaped in CBDC remuneration with a unique minimum* $\omega_{min} > 1$.

Proof. See Appendix C. □

This proposition shows that the introduction of a positively remunerated CBDC can lead to a lower probability of runs (relative to an economy in which only cash is available to depositors as alternative to bank deposits). Furthermore, as illustrated in Figure 1, bank fragility is a convex function of CBDC remuneration: It first decreases with ω and then increases as a consequence of the two offsetting forces represented by the direct and indirect effects described above.

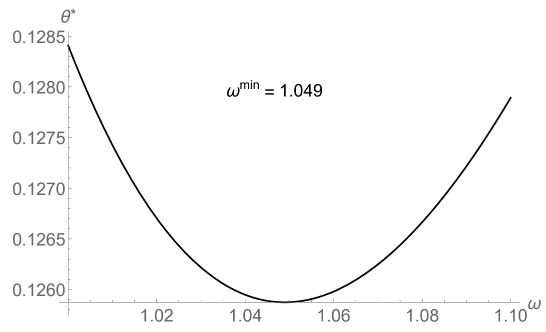

Figure 1: Bank failure threshold θ^* and CBDC remuneration ω. Parameters: $L = 0.9$, $R = 15$, so $\widetilde{\omega} \approx 1.32$.

4 CBDC design

Financial stability concerns feature prominently in the policy debate on CBDC (Bank for International Settlements, 2020). Accordingly, several central banks have advanced concrete proposals on specific CBDC design features that aim to mitigate potentially adverse effects. While our results in the previous section have shown that such concerns may already be mitigated by an appropriate level of CBDC remuneration, this section aims to link our model explicitly to this debate.

In order to do so, we consider a central bank who operates as a constrained planner and takes the informational friction and the privately optimal behaviour of consumers and the bank as given. Throughout we assume that the central bank can commit to a CBDC design and maximizes social welfare. Specifically, the central bank aims to maximize utilitarian welfare W, which is given by the sum of expected bank profits and expected consumer payoffs:

$$W \equiv \int_{\theta^*}^{1} (R\theta - r_2) d\theta + \int_{\theta^*}^{1} r_2 d\theta = \frac{R}{2} \left[1 - (\theta^*)^2\right]. \quad (8)$$

Accordingly, maximizing welfare is equivalent to minimizing fragility in our economy. Following Proposition 3, a central bank that can freely set CBDC remuneration will choose $\omega^* = \omega_{min}$.

While the case where the central bank can freely set CBDC remuneration represents a useful benchmark, it is rather unrealistic because most central banks do not consider CBDC to be a policy tool (European Central Bank, 2020; Bank of England, 2023). Even if they did, monetary policy considerations would likely be the overriding determinant of the level of CBDC remuneration (see, e.g., Lilley and Rogoff, 2020; Jiang and Zhu, 2021). Therefore, in what follows we assume that the CBDC rate is fixed and study two design features that have emerged in the current policy debate as remedies to tackle the financial stability concerns associated with the introduction of CBDC: holding limits and contingent remuneration.

4.1 Holding limits

The introduction of individual holding limits has been proposed by various policy makers (e.g. Bindseil et al., 2021; Bank of England, 2023) as potential tool to limit the attractiveness of CBDC as store of value and thus for reducing run incentives.[13] In order to provide a formal analysis of this tool in the context of our model, we modify the baseline framework and assume that consumers can only hold a proportion γ of their wealth in a CBDC.[14]

The fact that holding limits imply that not all withdrawn funds earn the same return potentially complicates depositors' withdrawal decision at $t = 1$. They can either withdraw only some of their funds at $t = 1$ and hold them in CBDC, or withdraw all of their funds and hold a portfolio of CBDC and cash. However, the following Lemma shows that partial withdrawals are never optimal. Intuitively, depositors withdraw in equilibrium only when the bank fails, so it is optimal for each consumer to fully withdraw in this case.

Lemma 2. ***No partial withdrawals.*** *Under holding limits, depositors never*

[13] The European Central Bank stated that it is exploring individual-specific holding limits in the context of its digital euro project. An amount of 3,000 EUR has been forwarded (see "Digital euro will protect consumer privacy, ECB executive pledges", Financial Times, 20 June 2021). Similarly, the Bank of England has recently proposed a holding limit of 10,000-20,000 GBP in a public consultation. Other proposals include tiered remuneration, as suggested in Bindseil (2020). If the second tier is remunerated at zero or below, this is equivalent to holding limits in our model because consumers would prefer to hold amounts exceeding the first tier in cash.

[14] In practice, policy makers are considering nominal limits (Bindseil et al., 2021). In our model, all consumers are identical at both $t = 0$ and $t = 1$ in equilibrium, so a proportional limit is equivalent to nominal limit. However, nominal and proportional holding limits may have different implications when consumers are heterogeneous.

withdraw only a fraction of their funds.

Proof. See Appendix D. □

Since depositors always withdraw their entire balance when running on the bank, the effective per-period remuneration on wealth held outside of the bank changes from ω (without holding limits) to

$$\omega^{HL} \equiv \gamma\omega + 1 - \gamma, \tag{9}$$

because the remaining $1-\gamma$ must be held in cash. Proposition 4 provides the results of both a positive and normative analysis of holdings limits on CBDC.

Proposition 4. ***Holding limits.*** *Holding limits, $\gamma < 1$, increase (reduce) bank fragility for low (high) levels of CBDC remuneration ω. Hence, the central banks optimally sets holding limits as*

$$\gamma^* = \begin{cases} \frac{\omega_{min}-1}{\omega-1} & \text{if } \omega > \omega_{min} \\ 1 & \text{if } \omega \leq \omega_{min} \end{cases}$$

Proof. See Appendix D. □

From a positive perspective, the introduction of holding limits reduces the pass-through of CBDC remuneration to consumers' outside option at both $t=0$ and $t=1$. In line with our previous analysis, this leads to two opposing effects on bank fragility. At the interim date, holding limits reduce the return that consumers earn on withdrawn funds. Since only part of their wealth held outside the bank can be stored in remunerated CBDC, the remainder must be held as cash and earns a return of 1. This corresponds to the direct effect and makes the bank less fragile. However, holding limits soften the competition that the bank faces at the funding stage, and thus imply a lower equilibrium deposit rate r_2^*. This increases consumers' withdrawal incentives at $t=1$ and thus makes the bank more fragile (the "indirect" effect).

The overall effect on bank fragility depends on which of these two effects dominates, which is determined by the (exogenous) level of CBDC remuneration. As shown in Figure 2, holding limits only have a beneficial effect on bank fragility when the level of CBDC remuneration is sufficiently high. Since holding limits reduce the responsiveness of the deposit rate to changes in CBDC remuneration, they also limit the beneficial effects of higher CBDC remuneration that are attained when ω is sufficiently low.

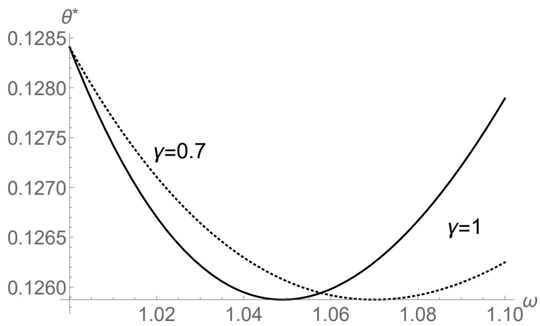

Figure 2: Bank failure threshold θ^*, CBDC remuneration ω, and holding limits γ. The solid line captures an economy without holding limits ($\gamma = 1$), while the dotted line captures an economy in which consumers can hold 70% of their funds in CBDC. Parameters: $L = 0.9$, $R = 15$.

Building on this insight, the socially optimal holding limit depends on the level of CBDC remuneration. Whenever ω exceeds the social optimum ω_{min}, the central bank uses holding limits to implement this optimum by setting $\gamma = \frac{\omega_{min}-1}{\omega-1}$. Otherwise, it is optimal to not impose any holding limit. Overall, Proposition 4 contains a note of caution for policymakers: the calibration of CBDC remuneration and holding limits should avoid inefficient outcomes via higher bank fragility. Whenever CBDC remuneration is high, holding limits can be used as a tool to achieve the constrained-efficient outcome. However, this is not true for low levels of CBDC remuneration.

Finally, holding limits also have a distributional impact. Since they decrease the effective remuneration of consumers' outside option, they reduce the competitive pressure of CBDC remuneration on bank deposit rates, and thus lead to higher bank profits.

4.2 Contingent remuneration

Next, we consider the possibility that CBDC remuneration can be contingent on the state of the financial system. The underlying idea is that a reduction of CBDC remuneration in crisis times can serve as a tool to reduce depositors' withdrawal incentives whenever they become acute (see Bindseil, 2020; Bindseil et al., 2021). To study the effectiveness of this policy, we modify the baseline model to specify CBDC remuneration as follows. In the first period, CBDC balances earn $\omega_1 = \omega$, as before. By contrast, the second period remuneration depends on consumers' withdrawals,

$$\omega_2(n) = \begin{cases} \omega & \text{if } n \leq \widetilde{n} \\ \underline{\omega} & \text{if } n > \widetilde{n}, \end{cases} \tag{10}$$

where $\underline{\omega} \in [1, \omega]$ is the reduced CBDC remuneration in crisis times, which is characterized by withdrawals of at least $\widetilde{n}$ depositors. The lower bound on $\underline{\omega}$ arises because we continue to assume that cash is available as alternative storage at each date.[15] Moreover, we focus on $\widetilde{n} < \overline{n}$ because a reduced remuneration cannot have a beneficial effect when the bank is illiquid at the interim date.

A stricter intervention is captured by lower values of the policy parameters $\widetilde{n}$ or $\underline{\omega}$.[16] While their impact on depositors' withdrawal incentives for a given deposit rate r_2 is similar, they affect depositors' participation constraint (and thus the bank's choice r_2^*) in different ways.

To see this difference formally, it is useful to first consider the marginal depositor's payoff upon withdrawal at $t = 1$, which can be written as

$$\pi_{1,CR} = \omega \int_0^{\widetilde{n}} r_1 dn + \underline{\omega} \int_{\widetilde{n}}^{\overline{n}} r_1 dn \leq \omega \int_0^{\overline{n}} r_1 dn = \pi_1, \tag{11}$$

[15] This is consistent with most central banks already having committed to continue supplying cash after a potential CBDC introduction. We abstract from the inconvenience associated with the handling and storage of physical cash, which may in practice enable central banks to set a slightly negative CBDC rate (the "effective lower bound") without triggering a complete substitution by consumers into cash.

[16] As the policy is implemented based on observed withdrawals at the final date (i.e. after consumers' interim-date choices), contingent remuneration does not require superior information by the central bank at the interim date. Moreover, this specification rules out any potential complications associated with an endogenous public signal. See also Angeletos et al. (2006).

where the subscript CR refers to contingent remuneration. As before, $\overline{n}$ is the bank's illiquidity threshold, so π_1 is the equivalent in the baseline model without contingent remuneration (see also Appendix A). For a given deposit rate r_2, a decrease in both $\widetilde{n}$ and $\underline{\omega}$ yields a lower expected payoff from withdrawing, and thus a lower run threshold relative to the baseline model, $\theta^*_{CR}(r_1, r_2) < \theta^*(r_1, r_2)$.

Recall that the deposit rate is pinned down by depositors participation constraint. With contingent remuneration, it reads[17]

$$V_{CR} = \int_0^{\theta^*_{CR}} r_2 d\theta - \omega \left(\int_0^{\theta^*_{CR}} \underline{\omega} d\theta + \int_{\theta^*_{CR}}^1 \omega d\theta \right) \geq 0. \tag{12}$$

Unlike $\widetilde{n}$, the reduced remuneration rate $\underline{\omega}$ does not only affect depositors' participation constraint indirectly via a change in the run threshold, but also directly. Hence, its impact on the deposit rate r_2 is more complex, as detailed below.

Proposition 5. ***Contingent remuneration.*** *A more restrictive design of contingent remuneration affects bank fragility as follows.*
(1) $\frac{d\theta^*_{CR}}{d\widetilde{n}} > 0$*;*
(2) $\frac{d\theta^*_{CR}}{d\underline{\omega}} > 0$ *if*

$$(1 - \theta^*_{CR}) \frac{r_2^2(1 - L)}{RL(r_2 - \frac{\pi_{1,CR}}{L})} (\overline{n} - \widetilde{n}) + \omega \theta^*_{CR} \frac{\partial \theta^*_{CR}}{\partial r_2} > 0.$$

Moreover, $\frac{d\theta^*_{CR}}{d\underline{\omega}} < 0$ *for* $\widetilde{n} \to \overline{n}$.

Proof. See Appendix E. □

Similar to the effect of CBDC remuneration in the main model, there is both a direct and an indirect effect of the policy parameters $\widetilde{n}$ and $\underline{\omega}$ on bank fragility. An earlier reduction of CBDC remuneration upon withdrawals (a lower $\widetilde{n}$) directly benefits financial stability. Moreover, the cutoff $\widetilde{n}$ does not directly enter the participation constraint of investors (as it only enters via the failure

[17] We use the fact that with vanishing noise, either all or no depositors runs, so that the reduced (full) CBDC rate is earned in the second period in case of failure (survival) for any $\widetilde{n} \in [0, 1]$.

threshold), resulting in a weak indirect effect. Thus, the direct effect always dominates and fragility is unambiguously reduced. This result is reminiscent to a partial suspension of convertibility in run models, but also considers the effects on ex-ante deposit rates. However, this ex-ante effect is always weak and dominated by the effect on withdrawal incentives, yielding the first result of the proposition.

The effect of a reduced remuneration (a lower $\underline{\omega}$) is generally more complex. As intended, the lower return on withdrawn funds reduces withdrawal incentives. However, $\underline{\omega}$ enters the participation constraint directly (see equation (12)), which strengthens the indirect effect that operates via equilibrium deposit rates.

Unfortunately, the total effect of CBDC remuneration is difficult to sign in general. For the special case where $\widetilde{n} \to \overline{n}$, however, the beneficial direct effect of contingent remuneration on withdrawal incentives vanishes because the bank is always illiquid and fails. We are thus left with an indirect effect of contingent remuneration, which lowers equilibrium deposit rates and therefore raises bank fragility. Figure 3 offers a numerical example that shows that a lower remuneration during financial turmoil can lower bank fragility.

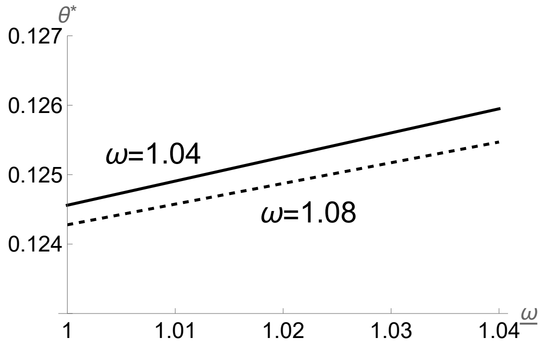

Figure 3: Bank failure threshold θ^*, CBDC remuneration ω, and reduced remuneration $\underline{\omega}$. The solid line and dashed lines are drawn for different values of CBDC remuneration, respectively $\omega = 1.04$ and $\omega = 1.08$. Parameters: $L = 0.9$, $R = 15$.

Interestingly, introducing contingent remuneration does not affect the impact of CBDC remuneration ω on financial fragility. This is shown in Figure 4. In line with the above results, a decrease in $\underline{\omega}$ simply leads to a downward shift in the curve illustrating the impact of CBDC remuneration on fragility, thus stressing the beneficial effect of this design feature.

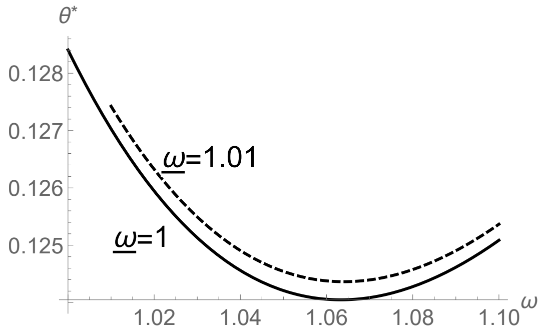

Figure 4: Bank failure threshold θ^*, CBDC remuneration ω, and reduced remuneration $\underline{\omega}$. The solid line captures an economy in which consumers receive a zero remuneration on CBDC in the event of a run, while the dotted line captures an economy in which they still receive a positive remuneration but below that they receive when there is no run. Parameters: $L = 0.9$, $R = 15$.

5 Extensions

In this section, we consider three extensions of the baseline model. We first limit the bank's market power in the deposit market, then examine its risk-taking incentives on the asset side, and finally consider its liquidity provision to depositors.

5.1 Limited market power in the deposit market

So far, we have considered a bank that acts as a monopolist in the deposit market. In this subsection, we relax this assumption.[18] This approach is partly motivated by theoretical work on the effects of CBDC on bank credit supply, which reaches different conclusions depending on the level of competition in deposit markets (Keister and Sanches, 2022; Andolfatto, 2021; Chiu et al., 2023).

More specifically, we assume that the deposit contract is determined by Nash bargaining between the bank and depositors. This approach is attractive because it allows us to model the degree of deposit market competition by varying the bank's bargaining power $\beta \in (0, 1)$. Formally, the deposit contract is the solution

[18] A large literature documents imperfect competition in retail deposit markets, including Neumark and Sharpe (1992), Hannan and Berger (1997), and Drechsler et al. (2017).

to

$$\max_{r_1,r_2} \left(\int_{\theta^*}^{1} R\theta - r_2 d\theta \right)^{\beta} \left(\int_{\theta^*}^{1} r_2 d\theta - \omega^2 \right)^{1-\beta}. \tag{13}$$

where the first (second) bracket represents the bank's (depositors') surplus in excess of their outside option. As in the main text, $r_1^* = 1$ follows immediately.

For $\beta \to 1$, this collapses to the baseline model where the bank maximizes expected profits subject to depositors' participation constraint. By contrast, $\beta \to 0$ corresponds to a model with perfect competition, where the deposit rate r_2 maximizes the expected value of the deposit claim subject to non-negative bank profits. The following proposition summarizes the resulting implications of this polar case for the effect of CBDC remuneration on bank fragility.

Proposition 6. ***Perfect competition in the deposit market.*** *For $\beta \to 0$, an increase in CBDC remuneration increases bank fragility, $\frac{d\theta^*}{d\omega} > 0$.*

Proof. See Appendix F. □

Depositors' gross surplus from the deposit contract is $(1 - \theta^*)r_2$, stating that the deposit rate r_2 is earned whenever the bank survives (this happens with probability $1-\theta^*$). Under perfect competition, the value of the deposit contract is maximized. The resulting equilibrium deposit rate is higher than under monopoly. In fact, it exceeds the level r_2^{max} beyond which an increase in the deposit rate raises the risk of bank failure (see also Proposition 1).[19]

As in the baseline model, higher CBDC remuneration affects bank fragility both directly (via the failure threshold) and indirectly (via the deposit contract). As before, the direct effect is positive. However, the indirect effect is no longer unambiguously negative, but its sign varies with parameters. In any case, deposit rates are so high that the indirect effect is of second order, so that the overall effect of higher CBDC remuneration is to unambiguously increase bank fragility.

[19] Bank profits evaluated at the competitive deposit rate are positive. That is, the value of the deposit claim is maximized for a deposit rate below the rate at which bank profits are zero.

We now turn to the intermediate case of limited market power, $0 < \beta < 1$. The following result states how the equilibrium deposit rates are pinned down.

Proposition 7. ***Deposit rates with limited market power of banks.*** *The deposit rate r_2^* solves*

$$\frac{\beta}{\int_{\theta^*}^{1} \left(R\theta - r_2^*\right) d\theta} \left(1 - \theta^* + (R\theta^* - r_2^*)\frac{\partial \theta^*}{\partial r_2}\right) = \frac{1-\beta}{\int_{\theta^*}^{1} r_2 d\theta - \omega^2} \left(1 - \theta^* - r_2^* \frac{\partial \theta^*}{\partial r_2}\right), \tag{14}$$

where $\theta^* = \theta^*(r_2^*)$. For high enough market power of banks, $\beta > \underline{\beta}$, the deposit rate increases in CBDC remuneration, i.e., $\frac{dr_2^*}{d\omega} > 0$.

Proof. See Appendix F. □

Intuitively, a decline in the bank's bargaining power dampens the impact of CBDC remuneration on deposit rates, since the bank is no longer a monopolist. Unfortunately, a full analytical characterization of the general case is analytically difficult. To provide some additional insights, Figure 5 provides numerical examples which show that the U-shaped relationship between bank fragility and CBDC remuneration is preserved when the bank's bargaining power in the deposit market is sufficiently high, whereas a monotonically increasing relationship is obtained for more competitive deposit markets.

5.2 Bank risk-taking on the asset side

So far we have considered a fragile liability side of banks (uninsured deposits) as a source of financial instability. However, financial instability can also be the result of banks' risk-taking decisions on their asset side (e.g., risk choices and asset substitution). In this extension, we allow for such risk-taking on the asset side.[20] Accordingly, building on Carletti et al. (2023) to account for the interactions between asset and liability side of bank balance sheet, we extend the baseline

[20] Monnet et al. (2023) also study the effect of CBDC on bank risk-taking. They find that banks respond to the introduction of CBDC by becoming safer, even excessively safe.

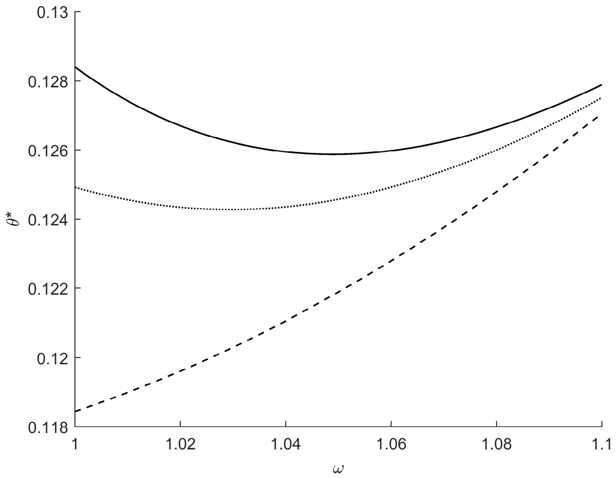

Figure 5: Bank failure threshold θ^*, CBDC remuneration ω, and bargaining power in the deposit market β. The solid line is for monopoly, the dotted line for high market power of the bank, and the dashed line are for low market power of the bank in the deposit market. Parameters: $L = 0.9$, $R = 15$, $\beta \in \{1, 0.998, 0.9\}$.

model assuming that at $t = 0$, the bank chooses its monitoring effort. Consistently with the literature (e.g., Holmstrom and Tirole (1997), Hellmann et al. (2000), Morrison and White (2005), Dell'Ariccia and Marquez (2006), Allen et al. (2011), DellAriccia et al. (2014)), the effort q fully determines the probability of success of bank investment, whose return changes to

$$P = \begin{cases} R\theta & \text{w.p. } q \\ 0 & \text{w.p. } 1-q \end{cases}.$$

Higher monitoring leads to a higher success probability, but it entails a non-pecuniary cost $\frac{c}{2}q^2$. To keep the analysis tractable, we consider an exogenous deposit contract (r_1, r_2).[21] This assumption shuts down the channel along which higher CBDC remuneration improves financial stability in the main text and allows us to focus on how CBDC remuneration ω affects bank risk choices q^* instead.[22]

To solve the model, we proceed as in the main text.[23] We begin by deriving the endogenous run threshold θ^*_q, so that all depositors run on the bank at $t = 1$

[21] The deposit contract is such that the participation constraint of investors holds.

[22] Given that we assume an exogenous deposit contract (partly for tractability), we cannot really distinguish between monitoring and screening.

[23] We continue to assume that $\overline{\theta} \to 1$ and $\epsilon \to 0$. Moreover, we require that $qr_2 > \omega r_1$ to rule out a certain bank run. See also the discussion about dominance bounds in Section 3.1.

if and only if $\theta < \theta_q^*$. Following the same steps as in Section 3.1, we obtain that

$$\theta_q^* = \frac{r_2}{R} \frac{q r_2 - \omega L}{q r_2 - \omega r_1}. \tag{15}$$

Better monitoring increases the probability that the bank is able to repay depositors at $t = 2$, and therefore reduces incentives to run ($\frac{\partial \theta_q^*}{\partial q} < 0$). This means that lower risk on the asset side of the bank leads to lower risk on its liability side.

Taking the run threshold θ_q^* into account, we then solve for the bank's optimal choice of monitoring effort q at $t = 0$. The bank solves

$$\max_q \Pi_q \equiv q \int_{\theta_q^*}^{1} (R\theta - r_2)\, d\theta - \frac{cq^2}{2}. \tag{16}$$

Provided that c is sufficiently high, there exists a unique and interior solution q^*, which is given by the solution to the following first-order condition

$$FOC_q \equiv \int_{\theta_q^*}^{1} (R\theta - r_2)\, d\theta - q \frac{\partial \theta_q^*}{\partial q} \left(R\theta_q^* - r_2\right) - cq = 0. \tag{17}$$

The bank's risk choice at $t = 0$ reflects a trade-off. The last term in equation (17) reflects the marginal cost of monitoring effort. The other two terms represent the marginal benefit of higher monitoring. First, more monitoring increases the probability that the project is successful, so that the bank reaps the residual claim $R\theta - r_2$ more often (provided there is no bank run, $\theta > \theta_q^*$). Second, the bank benefits from the interaction between the bank's asset and liability sides. An increase in monitoring reduces depositors' incentives to run, so that costly bankruptcy can be avoided.

Since we allow for risk-taking on the asset side of the balance sheet, it is important to note that there are now two potential sources of bank failure: bank runs and an unsuccessful investment project. We can therefore measure financial stability by the overall probability that the bank survives, which we define as

$$\Phi^* \equiv q^* \left(1 - \theta_q^*\right).$$

The following proposition shows that an increase in CBDC remuneration affects its two separate components in opposite ways.

Proposition 8. ***Risk taking on the asset side.*** *Higher CBDC remuneration improves monitoring,* $\frac{dq^*}{d\omega} > 0$*, but also increases fragility,* $\frac{d\theta_q^*}{d\omega} > 0$*.*

Proof. See Appendix G. □

Changes in CBDC remuneration affect the marginal benefit of bank monitoring, which follows directly from the first-order condition (17). The direction of this effect depends both on the direct effect of CBDC remuneration on the run threshold ($\frac{\partial \theta^*}{\partial \omega}$) as well as the threshold's sensitivity to changes in monitoring ($\frac{\partial^2 \theta^*}{\partial q \partial \omega}$). Proposition 8 states that the overall effect is always positive, that is an increase in CBDC remuneration always leads to higher bank monitoring ($\frac{dq^*}{d\omega} > 0$) and thus renders the bank's asset side more stable.

The effect of CBDC remuneration of the probability of a bank run can be written as

$$\frac{d\theta_q^*}{d\omega} = \frac{\partial \theta_q^*}{\partial \omega} \left[1 - \frac{\omega}{q} \frac{dq^*}{d\omega} \right]. \tag{18}$$

Just like in the main text, an increase in CBDC remuneration affects the run threshold θ_q^* both directly and indirectly. However, in this case, the indirect operates through bank monitoring, $\frac{dq^*}{d\omega}$, and not through the deposit contract (which is assumed to be exogenous). While these two effects go in opposite directions, Proposition 8 states that the direct effect always dominates, so that a higher CBDC remuneration always increases the risk of bank runs.

Since CBDC affects both aspects of financial stability in opposite ways, its overall effect on financial stability is ambiguous. While it is difficult to derive sufficient conditions analytically, Figure 6 provides a numerical example for which the beneficial effect of higher bank monitoring dominates. Accordingly, higher CBDC remuneration can increase financial stability, consistent with the main text.

To isolate the impact on asset-side risk, we have assumed an exogenous deposit contract for tractability. If the deposit contract were endogenous, two

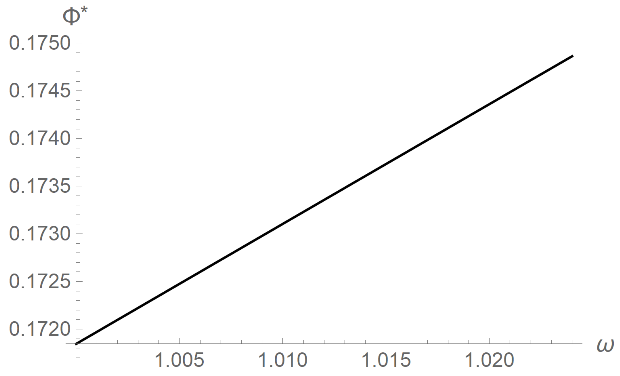

Figure 6: Financial stability Φ^* and CBDC remuneration ω. Parameters: $L = 0.9$, $R = 20$, $c = 0.1$, $r_1 = 1$, and $r_2 = 6$.

additional effects would arise. First, as shown in Section 3, higher remuneration of CBDC would increase the deposit rate, which directly reduces the bank's margin and, in turn, its incentive to exert monitoring effort. Second, the reduced fragility due to the higher deposit rate would translate into higher expected profits, and thus induce a higher risk management effort. These additional effects, which come from the interaction of the asset and liability sides of the bank balance sheet, go in opposite directions, thus making the characterization of the overall effect of CBDC on financial stability even more involved.

5.3 Liquidity provision

So far, we have studied risk-neutral depositors with no marginal value for interim liquidity. In this section, we relax this assumption by considering a traditional bank-run model in which banks provide liquidity to risk-averse depositors (Diamond and Dybvig, 1983). As a consequence, the interim deposit rate r_1 is now the center of attention. To facilitate a positive analysis, we incorporate remunerated CBDC into the seminal global-games model of bank runs with liquidity provision developed by Goldstein and Pauzner (2005).

As before, there is a unit mass continuum of consumers endowed with 1 unit of funds who can choose to deposit their wealth into a bank at date 0 or into CBDC at both date 0 and date 1. The latter earns $\omega \geq 1$ per period. Unlike in the main text, and closely following Goldstein and Pauzner (2005), we assume that consumers are risk-adverse with utility function $u(c)$ such that $u(0) = 0$, $u'(c) > 0$, $u''(c) < 0$, and coefficient of relative risk aversion $RRA > 1$ for any $c \geq 1$. While consumers are identical ex-ante, a fraction $\lambda \in (0,1)$ is hit by a preference shock at date 1 and must consume immediately. We refer to them as early or impatient, while the remaining $1-\lambda$ depositors are called late or patient because they are indifferent between consuming at dates 1 and 2.

At date 0, the bank raises funds in exchange for a deposit contract $\{r_1, r_2\}$, where r_1 is the promised repayment upon withdrawal at the interim date and $r_2 > r_1$ is the promised repayment upon withdrawal at the final date. The bank uses the resources raised initially to invest in the risky asset. The project can be liquidated at par at date 1 (i.e. $L = 1$), and yields a return R with probability $p(\theta) = \theta$ at date 2 (and zero otherwise). Unlike in the main text, banks operate in a perfectly competitive environment and maximize depositors' expected utility at $t = 0$. As a result, late depositors waiting until the final date are just residual claimants of the bank and therefore receive a pro-rata share of the bank's available resources. If the bank has insufficient resources to repay r_1 in the case of a run at the interim date, the early withdrawing depositors also receive a pro-rata share of the available resources. This small technical deviation from the sequential service constraint assumed in Goldstein and Pauzner (2005) improves tractability.

We again solve the model by working backwards, starting with depositors' withdrawal choices. We have the following result.

Proposition 9. ***Failure threshold.*** *There exists a unique threshold equilibrium. When $\varepsilon \to 0$, all depositors withdraw and the bank fails if and only if $\theta < \theta^*$, where*

$$\theta^* = \frac{\int_{\lambda}^{\overline{n}} u(\omega r_1)\, dn + \int_{\overline{n}}^{1} u\left(\frac{\omega}{n}\right) dn}{\int_{\lambda}^{\overline{n}} u\left(\frac{1-nr_1}{1-n} R\right) dn}. \tag{19}$$

The failure threshold θ^* increases in both r_1 and ω, i.e. $\frac{\partial \theta^*}{\partial r_1} > 0$ and $\frac{\partial \theta^*}{\partial \omega} > 0$.

Proof. See Appendix H. □

A higher CBDC remuneration increases a patient depositor's expected payoff from withdrawing at the interim date, which raises bank fragility. This result corroborates the direct effect from our main analysis. As usual, a higher short term deposit rate also raises bank fragility.

Having characterized the run probability θ^*, we turn to the choice of deposit contract. A competitive bank chooses r_1 to maximize depositors' expected utility[24]

$$\max_{r_1} \int_0^{\theta^*} [\lambda u(1) + (1-\lambda) u(\omega)]\, d\theta + \int_{\theta^*}^1 \left[\lambda u(r_1) + (1-\lambda)\theta u\left(\frac{1-\lambda r_1}{1-\lambda} R\right)\right] d\theta. \tag{20}$$

The first term in Equation (20) represents depositors' expected utility upon a run. All depositors withdraw from the bank at date 1 and receive the pro-rata share of 1. Impatient consumers need to consume immediately and therefore obtain $u(1)$, while impatient depositors keep their funds in their CBDC account for one period and receive $u(\omega)$ at date 2. The second term in (20) captures depositors' expected utility in the absence of a run. Impatient depositors withdraw at date 1 and receive $u(r_1)$, while patient depositors wait until the final date and receive the pro-rata share $\frac{1-\lambda r_1}{1-\lambda} R$ provided the project is successful (with probability θ). We have the following result.

Proposition 10. ***Deposit rates.*** *The equilibrium deposit rate $r_1^* > 1$ is the solution to the following first-order condition:*

$$\begin{aligned} FOC \equiv & -\frac{\partial \theta^*}{\partial r_1}\left[\lambda u(r_1) + (1-\lambda)\theta^* u\left(\frac{1-\lambda r_1}{1-\lambda} R\right) - \lambda u(1) - (1-\lambda) u(\omega)\right] + \\ & + \lambda \int_{\theta^*}^1 \left[u'(r_1) - \theta R u'\left(\frac{1-\lambda r_1}{1-\lambda} R\right)\right] d\theta = 0. \end{aligned} \tag{21}$$

The deposit rate decreases in ω, $\frac{dr_1^*}{d\omega} < 0$, thus lowering bank fragility indirectly.

Proof. See Appendix I. □

As in Goldstein and Pauzner (2005), the equilibrium deposit rate r_1 trades off the marginal cost associated with a higher run probability (the first term in (21))

[24] We are here implicitly focusing on values of ω that are not too large. Very large values of ω such that the utility from a run is higher than that in the absence of a run implies that runs occur all the time (so $\theta^* = 1$) and also make the choice of the deposit contract irrelevant.

with the marginal benefit due to better risk-sharing (the second term in (21)). Moreover, $r_1^* > L = 1$ implies that the bank provides liquidity in equilibrium, i.e., the deposit rate exceeds the liquidation value of its investments. This gives rise to strategic complementarities in depositors' withdrawal decisions.

The level of CBDC remuneration ω affects the trade-off between more runs and greater risk-sharing. While a higher CBDC rate reduces the costs associated with a run (because it increases patient depositors' consumption), it also decreases the benefit from risk-sharing, due to a higher run threshold, ceteris paribus. Since the latter effect dominates, an increase in CBDC remuneration leads to an overall lower reduction in the deposit rate.

We have thus shown that an indirect effect of CBDC remuneration on bank fragility also obtains in this framework. As in the main text, it operates through the deposit contract, and moves in the opposite direction than the direct effect.

6 Conclusion

The aim of this paper is to examine the impact of CBDC on financial stability. We develop a global-games model of financial intermediation and bank runs in which a remunerated CBDC provides consumers with an alternative to bank deposits (and cash). Consistent with concerns among policymakers, a higher CBDC remuneration raises bank fragility by increasing consumers' withdrawal incentives. However, it also induces the bank to offer more attractive deposit contracts in an effort to retain funding, which reduces fragility. Accordingly, the overall relationship between bank fragility and CBDC remuneration is U-shaped.

Within this framework, we evaluate several policy proposals aimed at allaying financial stability concerns connected to an introduction of CBDC. We find that a positive remuneration of CBDC can be socially desirable because it lowers bank fragility. When CBDC remuneration is exogenous from a financial stability perspective (e.g. when it is determined by monetary objectives or bound by previous commitments), holding limits can be socially beneficial for a high CBDC rate.

However, they are ineffective for low levels of CBDC remuneration. Contingent remuneration, which lowers rates in times of financial distress, can be effective in reducing withdrawal incentives without having a large effect on deposit rates.

We extend the model to allow for imperfect competition in the deposit market, bank risk-taking on its asset side, and liquidity provision. These analyses support the robustness of our baseline results. Further extensions may generate additional insights. For example, one could consider the role of bank equity and liquid reserves on the interaction between CBDC remuneration and financial stability. One could also examine how CBDC affects the bank's funding mix, as cheaper but more unstable wholesale deposits may become attractive, or the supply of deposits may change. Exploring general equilibrium effects via the response of asset prices and exchange rates after the introduction of CBDC may be interesting. Finally, the interaction of CBDC design features (remuneration, holding limits, and contingent remuneration) with traditional tools for mitigating run risk, such as lender of last resort policies, is an interesting avenue for further research.

A Proof of Proposition 1

The proof builds on Carletti et al. (2023). The only difference is that our model exhibits global strategic complementarity (depositor's incentive to withdraw at $t = 1$ monotonically increases in the number of depositors withdrawing). The arguments in their proofs establish that, in the limit of $\epsilon \to 0$, there is a unique threshold value of the fundamental, denoted as θ^*, below which all consumers choose to withdraw from the bank. We first prove the existence of a unique equilibrium and then study its comparative statics.

Existence and uniqueness. For $\theta \in (\underline{\theta}, \overline{\theta})$, a depositor's decision to withdraw depends on the withdrawal choices of others. Suppose that all depositors use a threshold strategy s^*. Then, the fraction of depositors withdrawing at $t = 1$, $n(\theta, s^*)$, equals the probability of receiving a signal below s^*:

$$n(\theta, s^*) = \begin{cases} 1 & \text{if } \theta \leq s^* - \epsilon, \\ \frac{s^* - \theta + \epsilon}{2\epsilon} & \text{if } s^* - \epsilon < \theta \leq s^* + \epsilon, \\ 0 & \text{if } \theta > s^* + \epsilon. \end{cases} \tag{22}$$

Thus, a depositor's withdrawal decision is characterized by the pair of thresholds $\{s^*, \theta^*\}$, which solves the following system of equations:

$$R\theta^* \left(1 - \frac{n(\theta^*, s^*) r_1}{L}\right) - (1 - n(\theta^*, s^*)) r_2 = 0, \tag{23}$$

$$r_2 Pr(\theta > \theta^* | s^*) = \omega r_1 Pr(\theta > \theta_n | s^*), \tag{24}$$

where $\theta_n = s^* + \epsilon - 2\epsilon \frac{L}{r_1}$ is the solution to $n(\theta, s^*)\, r_1 = L$.

Condition (23) identifies the level of fundamentals θ at which the bank is just able to repay the promised repayment to non-withdrawing depositors. Hence, it pins down the cutoff θ^*. Condition (24), instead, states that at the signal threshold s^* a depositor is indifferent between withdrawing at $t = 1$ and waiting until $t = 2$, since the expected payoff at $t = 2$, as captured by the LHS in (24), is equal to the expected $t = 1$ payoff, which is captured by the RHS in (24). Hence, given θ^* from (23), it pins down the threshold signal s^*. Note that the payoff at $t = 1$ is received whenever the bank is liquid, while the payoff at $t = 2$ is received whenever the bank is solvent. Differentiating the LHS of (23) with respect to θ, we obtain

$$R\left(1-\frac{n(\theta,s^*)r_1}{L}\right)-\frac{\partial n(\theta,s^*)}{\partial\theta}\left[R\theta\frac{r_1}{L}-r_2\right]>0, \tag{25}$$

for any $\theta>\underline{\theta}$ since $r_1>L$ and $\frac{\partial n(\theta,s^*)}{\partial\theta}\leq 0$. Taking the derivative of (23) with respect to $n(.)$, we obtain $-R\theta\frac{r_1}{L}+r_2<0$ for any $\theta>\underline{\theta}$ because $r_1>L$. Overall, this implies that the LHS in (23) monotonically increases with θ and the signal s_i and so does the LHS in (24). Furthermore, rearranging (23) as $R\theta^*-r_2-n(\theta^*,s^*)\left[R\theta^*\frac{r_1}{L}-r_2\right]=0$, it follows that (23) is negative at $\theta=\underline{\theta}$ and positive at $\theta=\overline{\theta}$. Using (24), this means that at $\theta=\underline{\theta}$, a depositor expects to receive 0 when waiting and thus strictly prefers to withdraw. At $\theta=\overline{\theta}$ such that the LHS in (23) is strictly positive, a depositor expects to receive $r_2>\omega\, r_1$ when waiting. Since ωr_1 exceeds the RHS in (24), it follows that, at $\theta=\overline{\theta}$, a depositor strictly prefer not to withdraw.

Overall, the analysis above implies that $\underline{\theta}<\theta^*<\overline{\theta}$ and analogously that the threshold signal s^* falls within the range $\left(\underline{\theta}+\epsilon,\overline{\theta}-\epsilon\right)$. Given that $\underline{\theta}>0$ and $\overline{\theta}\to 1$, it follows that the equilibrium pair $\{\theta^*,s^*\}$ falls in the range $(0,1)$.

To obtain a closed-form expression, we perform a change of variable using (22) from which we obtain $\theta(n)=s^*+\epsilon(1-2n)$. At the limit, when $\epsilon\to 0$, $\theta(n)=s^*$, which identifies the run threshold and it is equal to the solution to

$$\int_0^{\widehat{n}(\theta^*)} r_2 dn=\int_0^{\overline{n}}\omega r_1\, dn\equiv\pi_1\Rightarrow\widehat{n}\left(\theta^*\right)r_2=\omega L, \tag{26}$$

where π_1 is the expected payoff from withdrawing at $t=1$. Solving for θ^* yields the closed-form expression stated. And $\theta^*>\underline{\theta}$ directly follows from $L<1\leq r_1$.

Comparative statics. To complete the proof, we study how bank fragility θ^* changes with deposit rates r_1 and r_2, as well as CBDC remuneration ω, liquidation value L, and the investment profitability R. We have the following:

$$\frac{\partial\theta^*}{\partial r_1}=\frac{\omega\theta^*}{(r_2-\omega r_1)}>0, \tag{27}$$

$$\frac{\partial\theta^*}{\partial r_2}=\frac{1}{R}\frac{r_2-\omega L}{r_2-\omega r_1}-\frac{\underline{\theta}\omega(r_1-L)}{(r_2-\omega r_1)^2}=\frac{r_2^2-2\omega r_1 r_2+\omega^2 L r_1}{R(r_2-\omega r_1)^2}, \tag{28}$$

$$\frac{\partial\theta^*}{\partial\omega}=\underline{\theta}\frac{r_2(r_1-L)}{(r_2-\omega r_1)^2}>0,\quad \frac{\partial\theta^*}{\partial L}=-\frac{\omega\underline{\theta}}{r_2-\omega r_1}<0,\quad \frac{\partial\theta^*}{\partial R}=-\frac{\theta^*}{R}<0, \tag{29}$$

where we used $r_1>L$ and $r_2>\omega r_1$. To establish the sign of $\frac{\partial\theta^*}{\partial r_2}$, we need to determine the sign of the numerator since the denominator is positive. The numerator is negative whenever $r_2^A < r_2 < r_2^B$, where r_2^A and r_2^B denote the roots of the associated quadratic equation $r_2^2 - 2\omega r_1 r_2 + \omega^2 L r_1 = 0$ since $\Delta = 4\omega^2 r_1^2 - 4\omega^2 L > 0$. The two roots are equal to:

$$r_2^{A/B} = \omega r_1 \left(1 \pm \sqrt{1 - \frac{L}{r_1}}\right). \tag{30}$$

The smaller root r_2^A is inadmissible as it implies $r_2 < \omega r_1$, a contradiction. Thus, only the bigger root $r_2^B > \omega r_1$ is admissible. Since this value is the maximum of the relevant deposit rates considered by the bank, as we will show shortly, we label it $r_2^{max} \equiv r_2^B$. To summarize, $\frac{\partial \theta^*}{\partial r_2} < 0$ if $r_2 < r_2^{max}$ and $\frac{\partial \theta^*}{\partial r_2} > 0$ if $r_2 > r_2^{max}$.

B Proof of Proposition 2

A higher short-term deposit rate r_1 increases fragility (Proposition 1), so it reduces expected bank profits because the bank is solvent less often, $\frac{\partial \Pi}{\partial r_1} = (R\theta^* - r_2)\frac{\partial \theta^*}{\partial r_1} < 0$. A higher short-term deposit rate also tightens the participation constraint of consumers because they are repaid less often, $\frac{\partial V}{\partial r_1} = -r_2 \frac{\partial \theta^*}{\partial r_1} < 0$. Thus, $r_1^* = 1$.

The proof of the remaining claims is in several steps. We first derive sufficient conditions for the participation constraint of consumers to bind in equilibrium. Then, we derive comparative statics of the equilibrium deposit rate. As they would be useful later, we state some partial derivatives (evaluated at r_1^*):

$$\frac{\partial \theta^*}{\partial r_2} = \frac{(r_2 - \omega)^2 - \omega^2(1-L)}{R(r_2 - \omega)^2} = \frac{1}{R} - \frac{\omega^2(1-L)}{R(r_2-\omega)^2}, \tag{31}$$

$$\frac{\partial^2 \theta^*}{\partial \omega \partial r_2} = -\frac{2(1-L)\omega r_2}{R(r_2-\omega)^3} < 0, \tag{32}$$

$$\frac{\partial^2 \theta^*}{\partial r_2^2} = \frac{2(1-L)\omega^2}{R(r_2-\omega)^3} > 0. \tag{33}$$

B.1 Binding participation constraint of consumers

Step 1: We derive bounds on the deposit rate chosen by the bank. A profit-maximizing bank never chooses a rate that entails $\theta^* = 1$. If a run is certain, the bank is certain to make zero (expected) profits. Hence, the bank chooses $r_2 > r_2^{min}$ where r_2^{min} solves

$\theta^*(r_2^{min}) \equiv 1$, yielding a lower bound on the deposit rate:

$$r_2^{min} \quad = \quad \frac{R+\omega L}{2} - \sqrt{\left(\frac{R+\omega L}{2}\right)^2 - R\omega}. \tag{34}$$

We have shown in Proposition 1 that bank fragility decreases in the long-term deposit rate as long as $r_2 < r_2^{max}$. We now impose constraints on parameters to ensure that the participation constraint of consumers is slack at $r_2 = r_2^{max}$, that is $V(r_2^{max}) > 0$. Note that $\theta^*(r_2^{max}) = \frac{\omega}{R}\left(1+\sqrt{1-L}\right)^2$ and $V(r_2^{max}) = \omega\left(1+\sqrt{1-L}\right) - \frac{\omega^2}{R}\left(1+\sqrt{1-L}\right)^3 - \omega^2$, resulting in a lower bound on investment profitability:

$$R > \underline{R}_1 \equiv \frac{\omega\left(1+\sqrt{1-L}\right)^3}{1+\sqrt{1-L}-\omega}. \tag{35}$$

An upper bound on CBDC remuneration ensures that the denominator of $\underline{R}_1$ is always positive:

$$\omega < \widetilde{\omega} \equiv 1+\sqrt{1-L}. \tag{36}$$

Note that $r_2^{min} < r_2^{max}$, which justifies our labels, and ensures that the bank does not always fail, $\theta^*(r_2^{max}) < 1$, which is the economically interesting case.

Step 2: We can write marginal bank profits as

$$\frac{d\Pi}{dr_2} \equiv -\frac{\partial \theta^*}{\partial r_2}\left(R\theta^* - r_2\right) - \int_{\theta^*}^{1} d\theta. \tag{37}$$

Since $(R\theta^* - r_2) = r_2\omega\frac{(r_1-L)}{r_2-\omega r_1} > 0$ and $1-\theta^* > 0$ (given the bounds on r_2) as well as the parameter constraints ensuring that higher long-term deposit rates reduce bank fragility, there is a non-trivial trade-off for the bank: higher long-term deposit rates make the bank more stable but also reduce its profit margin.

Evaluating marginal profits at $r_2 = r_2^{\max}$ (where, by definition, $\frac{\partial\theta^*}{\partial r_2} = 0$), gives $\frac{d\Pi}{dr_2} < 0$. Moreover, $\frac{\partial \Pi}{\partial r_2} < 0$ for all $r_2 > r_2^{max}$. Thus, the bank chooses a deposit rate $r_2 < r_2^{max}$ if feasible (i.e. if the participation constraint of consumers holds). Given the parameter constraints on investment profitability and CBDC remuneration (see step 1), the participation constraint is indeed slack, so the bank chooses a rate $r_2^* < r_2^{max}$ (establishing an upper bound on the deposit rate).

Similarly, evaluating at $r_2 = r_2^{\min}$ (where, by definition, $\theta^* = 1$) gives $\frac{d\Pi}{dr_2} > 0$.

Furthermore, at $r_2 = r_2^{\min}$, we also have $V < 0$ (i.e. the participation constraint is violated), so the bank always chooses a higher deposit rate, $r_2^* > r_2^{\min}$ (establishing a lower bound on the deposit rate).

Step 3: Next, we show that expected bank profits Π are globally concave. As a result, the unconstrained choice of deposit rate that ignores the participation constraint of consumers, denoted by r_2^Π and solving $\frac{d\Pi}{dr_2} \equiv 0$, is unique. To establish global concavity, we show that the second-derivative is always negative:

$$\frac{d^2\Pi}{dr_2^2} \equiv -\frac{\partial^2\theta^*}{\partial r_2^2}\left(R\theta^* - r_2\right) - \left(\frac{\partial\theta^*}{\partial r_2}\right)^2 R + 2\frac{\partial\theta^*}{\partial r_2} < 0,$$

because $\frac{\partial\theta^*}{\partial r_2} < 0$ and $\frac{\partial^2\theta^*}{\partial r_2^2} > 0$.

Consider $\underset{\sim}{r}_2 = \omega^2$, which solves the participation constraint (PC) of investors with no bank failure. Since the bank sometimes fails, $\theta^* > 0$, $\underset{\sim}{r}_2$ is clearly a lower bound on the value that solves the binding PC, $r_2^{PC} > \underset{\sim}{r}_2$. This bound is helpful in establishing sufficient conditions for the relevant equilibrium condition to be the binding PC. By global concavity of Π, a sufficient condition for $r_2^\Pi < \underset{\sim}{r}_2$ is $\frac{d\Pi(\underset{\sim}{r}_2)}{dr_2} < 0$. Intermediate results are $\theta^*(\underset{\sim}{r}_2) = \frac{\omega^2(\omega-L)}{R(\omega-1)}$, $R\theta^*(\underset{\sim}{r}_2) - \underset{\sim}{r}_2 = \frac{\omega^2(1-L)}{\omega-1}$, and $\frac{\partial\theta^*(\underset{\sim}{r}_2)}{\partial r_2} = \frac{\omega^2-2\omega+L}{R(\omega-1)^2}$. Thus, we can express $\frac{d\Pi(\underset{\sim}{r}_2)}{dr_2} < 0$ as a lower bound:

$$R > \underline{R}_2 \equiv \frac{\omega^2}{\omega-1}\left[-\frac{1-L}{(\omega-1)^2}\left(\omega^2 - 2\omega + L\right) + \omega - L\right] = \omega^2\left(1 + \frac{(1-L)^2}{(\omega-1)^3}\right). \tag{38}$$

As a result, we have shown that $r_2^{PC} > r_2^\Pi$. Finally, we verify that $\underset{\sim}{r}_2 \geq r_2^{min}$. Rewriting $\theta^*(\underset{\sim}{r}_2) < 1$ yields another lower bound on profitability:

$$R > \underline{R}_3 \equiv \frac{\omega^2(\omega-L)}{\omega-1}. \tag{39}$$

Since $\omega < \widetilde{\omega}$, which implies $\omega^2 - 2\omega + L < 0$, we can rank these bounds $\underline{R}_2 > \underline{R}_3$. Thus, we can drop the bound $\underline{R}_3$. Taking stock, we define $\underset{\sim}{R}$ as the largest of all lower bounds on the investment returns (see below for the definition).

B.2 Existence of a unique deposit rate, comparative statics

Having established that the deposit rate r_2^* corresponds to the solution to the binding participation constraint, we next prove its existence and uniqueness. Recall that the net value of the deposit claim is $V = \int_{\theta^*}^{1} r_2 \, d\theta - \omega^2$. So, $V(r_2^*) \equiv 0$. Note that $V(r_2^{min}) = -\omega^2 < 0$ and $V(r_2^{max}) > 0$ given the parameter constraints on R and ω. Differentiating V with respect to r_2:

$$\frac{dV}{dr_2} = -\frac{\partial \theta^*}{\partial r_2} r_2 + (1 - \theta^*) > 0, \tag{40}$$

so a higher (long-term) deposit rate increases the value of the deposit claim for two reasons: consumers receive a high payment in the absence of a bank run and the bank is less fragile (Proposition 1). Given the monotonicity of V in r_2 and its change of signs from the bound r_2^{min} to r_2^{max}, a solution for r_2^* exists and is unique.

Next, we study the comparative statics of r_2^*. First, consider CBDC remuneration ω, using the implicit function theorem (IFT), $\frac{dr_2}{d\omega} = -\frac{\frac{\partial V}{\partial \omega}}{\frac{\partial V}{\partial r_2}}$. The denominator is positive, as shown in (40). Hence, the sign of $\frac{dr_2}{d\omega}$ is the opposite of the sign of the numerator:

$$\frac{\partial V}{\partial \omega} = -2\omega - \frac{\partial \theta^*}{\partial \omega} r_2 < 0. \tag{41}$$

It follows that r_2 monotonically increases in CBDC remuneration ω:

$$\frac{dr_2^*}{d\omega} = \frac{2\omega + \frac{\partial \theta^*}{\partial \omega} r_2}{1 - \theta^* - \frac{\partial \theta^*}{\partial r_2} r_2} > 0. \tag{42}$$

Finally, we derive the comparative statics of the equilibrium deposit rate with respect to investment characteristics. Using the implicit function theorem again, the results $\frac{dr_2^*}{dL} < 0$ and $\frac{dr_2^*}{dR} < 0$ follow from $\frac{\partial V}{\partial L} = -r_2 \frac{\partial \theta^*}{\partial L} > 0$ and $\frac{\partial V}{\partial R} = -r_2 \frac{\partial \theta^*}{\partial R} > 0$.

C Proof of Lemma 1 and Proposition 3

We first prove the lemma and then the proposition. Using the expression for $\frac{dr_2}{d\omega}$ in Equation (42), we expand the expression for $\frac{d\theta^*}{d\omega}$:

$$\frac{d\theta^*}{d\omega} = \frac{\partial \theta^*}{\partial \omega} + \frac{\partial \theta^*}{\partial r_2} \frac{dr_2^*}{d\omega} = \frac{\partial \theta^*}{\partial \omega} + \frac{\partial \theta^*}{\partial r_2} \frac{2\omega + \frac{\partial \theta^*}{\partial \omega} r_2^*}{1 - \theta^* - r_2^* \frac{\partial \theta^*}{\partial r_2}}. \tag{43}$$

Since the denominator of the second term is positive, we get $\frac{d\theta^*}{d\omega} < 0$ whenever $\frac{\partial\theta^*}{\partial\omega}\left(1-\theta^*-r_2^*\frac{\partial\theta^*}{\partial r_2}\right)+\frac{\partial\theta^*}{\partial r_2}\left(2\omega+\frac{\partial\theta^*}{\partial\omega}r_2^*\right)<0$. This inequality simplifies to

$$\frac{\partial\theta^*}{\partial\omega}(1-\theta^*)+2\omega\frac{\partial\theta^*}{\partial r_2}<0. \tag{44}$$

Using the equilibrium deposit rate to replace $1-\theta^*=\frac{\omega^2}{r_2^*}$ and the fact that $\frac{\partial\theta^*}{\partial r_2}=\frac{1}{r_2}\left[\theta^*-\omega\frac{\partial\theta^*}{\partial\omega}\right]$, we can re-express this condition as:

$$\theta^*+r_2^*\frac{\partial\theta^*}{\partial r_2}<0, \tag{45}$$

which has the intuitive interpretation of an elasticity. In particular, the elasticity of the failure threshold with respect to deposit rate, $\eta=-\frac{r_2^*}{\theta^*}\frac{\partial\theta^*}{\partial r_2}$, has to exceed 1 for the indirect effect to dominate and thus $\frac{d\theta^*}{d\omega}<0$, where r_2^* solves $V(r_2^*)=0$.

Using $1-\theta^*=\frac{\omega^2}{r_2^*}$ to rewrite Condition (44) yields $\omega\frac{\partial\theta^*}{\partial\omega}+2r_2\frac{\partial\theta^*}{\partial r_2}<0$. Inserting the expressions for the partial derivatives dividing by the positive common term $\frac{r_2^*}{R(r_2^*-\omega)^2}$, we obtain $\eta>1$ if and only if $\omega r_2^*(1-L)+2\left[(r_2^*)^2-2\omega r_2^*+\omega^2L\right]<0$. Rewriting yields the following condition with a quadratic term:

$$h(r_2^*,\omega)\equiv(r_2^*)^2-\frac{3+L}{2}\omega r_2^*+\omega^2L<0. \tag{46}$$

We turn to the proof of the proposition. First, we determine whether $\frac{d\theta^*}{d\omega}<0$ when evaluated at $\omega=1$ is possible. Using condition (46), this boils down to $(r_2^*)^2-\frac{3+L}{2}r_2^*+L<0$. Thus, we can find the roots $r_2^C\equiv\frac{\omega}{4}\left(3+L-\sqrt{L^2-10L+9}\right)$ and $r_2^D\equiv\frac{\omega}{4}\left(3+L+\sqrt{L^2-10L+9}\right)$ such that $h<0$ if and only if $r_2^C<r_2^*<r_2^D$. Since $r_2^C<\omega$ is inadmissible, r_2^D is the relevant root, which is independent of R.

Second, we impose parameter constraints to ensure $r_2^D\in\left(r_2^{min},r_2^{max}\right)$. Using the expression for r_2^{max} as given in (30) and evaluating it at $r_1=1$ and $\omega=1$, $r_2^D<r_2^{max}$ can be expressed as $\frac{1-L}{4}+\sqrt{1-L}>\frac{1}{4}\sqrt{L^2-10L+9}$. Squaring and rewriting yields $8(1-L)(1+\sqrt{1-L})>0$, which always holds for $L<1$. Moreover, for $r_2^D>r_2^{min}$ to hold at $\omega=1$, it suffices to show that $\theta^*(\omega=1,r_2=r_2^D)<1$. This yields another lower bound on profitability:

$$R>\underline{R}_4\equiv\frac{r_2^D(r_2^D-L)}{r_2^D-1}. \tag{47}$$

Third, r_2^* decreases in R, while r_2^D is independent of it. Thus, there exists a critical value, $\underline{R}_5$, such that $r_2^* < r_2^D$ for all $R > \underline{R}_5$. Importantly, $\underline{R}_5 < \infty$. One can easily show that $r_2^D > 1 > L$ because $\sqrt{L^2 - 10L + 9} > 1 - L$ can be rearranged by squaring to $8(1 - L) > 0$. By contrast, $r_2^* \to 1$ for $R \to \infty$ since $\theta^* \to 0$ and thus $r_2^* \to 1$ for a given $L < 1$ and $\omega = 1$.

The reader may notice that the bound $\underline{R}_2$ characterized in the proof of Proposition 2 converges to ∞ as $\omega \to 1$, thus becoming the binding bound on profitability. However, it is important to stress that this simple sufficient condition is quite restrictive. In fact, the numerical example in the main text shows that our results also hold for much lower levels of the investment profitability R.

Fourth, we show that $\frac{d\theta^*}{d\omega} > 0$ for large ω. Recall that $\frac{dr_2^*}{d\omega} > 0$ and $r_2^* < r_2^{max}$. Then, we can denote ω^{max} such that $r_2^* \to r_2^{max}$ when $\omega \to \omega^{max}$. In this limit, Condition (45) is violated because $\frac{\partial\theta^*}{\partial r_2} \to 0$ when $r_2 \to r_2^{max}$. Thus, $\frac{d\theta^*}{d\omega} > 0$.

Note that $\omega^{max} < \widetilde{\omega}$. To see this, recall that (i) $\underline{R}_1 = +\infty$ at $\omega = \tilde{\omega}$ and (ii) $\underline{R}_1 < \infty$ for any $\omega < \widetilde{\omega}$. That is, for any $\omega < \widetilde{\omega}$, there exists a finite $\underline{R}_1$ such that the participation constraint binds exactly at $r_2 = r_2^{max}$. Hence, $\frac{\partial \underline{R}_1}{\partial\omega} > 0$ implies that there exists an $\omega < \widetilde{\omega}$ and $R > \underline{R}_1$ for which $r_2^* = r_2^{max}$, denoted as ω^{max}.

Taken these steps together, we have $\frac{d\theta^*}{d\omega} > 0\big|_{\omega=1} < 0$ and $\frac{d\theta^*}{d\omega}\big|_{\omega^{max}} > 0$. Hence, there is at least a value of ω, denoted as ω^{min}, at which θ^* is minimized.

Fifth, we show that ω^{min} is unique. The value ω^{min} solves $h(r_2^*, \omega_{min}) = 0$, where $h(r_2, \omega)$ is given in (46). Since r_2^* is a function of ω, $h(r_2(\omega), \omega)$ is a polynomial where ω is the main variable. The degree of the polynomial determines the number of possible values ω^{min}. Since $\frac{d\theta^*}{d\omega}\big|_{\omega=1} < 0$ and $\frac{d\theta^*}{d\omega}\big|_{\widetilde{\omega}} > 0$, the number of solutions ω^{min} must be odd. To determine the degree of the polynomial $h(r_2(\omega), \omega)$, it is useful to characterize a closed-form solution for r_2^*. Since r_2^* solves $V(r_2^*, \omega) = 0$ given in (7). Substituting the expression for θ^* from (4), we obtain:

$$r_2^3 - r_2^2(R + \omega L) + r_2 R\omega(\omega + 1) - R\omega^3 = 0. \tag{48}$$

Equation (48) has three roots, which solve the corresponding depressed cubic equation

$$y^3 + Py + Q = 0, \tag{49}$$

where $y = r_2 - \frac{R+\omega L}{3}$, $P = \frac{3R\omega(1+\omega)-(R+\omega L)^2}{3}$ and $Q = \frac{-2(R+\omega L)^3+9(R+\omega L)R\omega(\omega+1)-27R\omega^3}{27}$. We focus on parameters such that $4P^3 + 27Q^2 > 0$, so there is only one real root: $y = \sqrt[3]{-\frac{Q}{2} + \sqrt{\frac{Q^2}{4} + \frac{P^3}{27}}} + \sqrt[3]{-\frac{Q}{2} - \sqrt{\frac{Q^2}{4} + \frac{P^3}{27}}}$. The expression pinning down y and, in turn, r_2^* is a function of ω. One can show that ω only appears at a power of 1. This implies that $h(r_2(\omega), \omega)$ has at most two roots, of which only one can be in the range $1 < \omega < \tilde{\omega}$. Since the derivative is initially negative and eventually positive, there must be an odd number of crossings with zero within $[1, \tilde{\omega}]$. Hence, ω_{min} is unique.

D Proof of Lemma 2 and Proposition 4

Proof of Lemma. To simplify the exposition, the proof is done under the assumption of $r_1^* = 1$. This is without loss of generality because $r_1^* = 1$ emerges in equilibrium, as in the main analysis, given that the run threshold increases in r_1.

Under partial withdrawals, the depositor wishes to redeposit the maximum amount γ with the central bank at $t = 1$, so it withdraws deposits worth γ from the bank at $t = 1$. The remaining $1 - \gamma$ are kept in the bank until $t = 2$. Thus, the bank requires resources worth γ at $t = 1$ to meet withdrawals. The proof considers separately the cases of $\gamma > L$ and $\gamma \leq L$ because this ranking of aggregate resources determines the presence of strategic complementarity in withdrawal decisions. For each case, we compare a depositor's expected payoffs from partial and full withdrawals. Recall that full withdrawals imply that γ is held in CBDC and $1 - \gamma$ in cash.

Case 1: When $\gamma > L$, the bank does not have enough resources to meet total withdrawals when all depositors partially withdraw. This gives rise to strategic complementarity in depositors' withdrawal decisions. Hence, we follow the same steps as in the main text to compute the run threshold, denoted as θ_{PW}^* for partial withdrawals (PW). The relevant equations are the insolvency condition, $R\theta\left(1 - \frac{n\gamma}{L}\right) - (1 - n\gamma)\, r_2 = 0$, which pins down $\widehat{n}_{PW} = \widehat{n}/\gamma$, and the indifference condition:

$$\gamma \int_0^{\widehat{n}_{PW}} r_2 dn = \gamma\omega \int_0^{\overline{n}_{PW}} dn, \tag{50}$$

where $\overline{n}_{PW} = \overline{n}/\gamma$ scales similarly as $\widehat{n}_{PW}$. As a result, we obtain the same indifference condition as before, $\widehat{n} r_2 = \omega \overline{n}$.

Consider next the case of full withdrawals (FW), assuming that a depositor withdraws the entire amount when choosing to run on the bank. Thus, the thresholds for illiquidity and insolvency are unchanged relative to the baseline model, $\overline{n}_{FW} = \overline{n}$ and $\widehat{n}_{FW} = \widehat{n}$, but the indifference condition changes to:

$$\int_0^{\widehat{n}_{FW}} r_2 dn = [\gamma\omega + (1-\gamma)] \int_0^{\overline{n}_{FW}} dn. \tag{51}$$

We can now compare partial and full withdrawals. The LHS of (50) is the same as the LHS of (51). For $\omega > 1$ and $\gamma < 1$, the RHS of (51) is smaller than the RHS of (50). As a result, bank fragility is lower under full withdrawals, $\theta^*_{FW} < \theta^*_{PW}$. The second benefit of full withdrawal is that depositors get more of their funds out in a run: $\gamma\omega$ with partial withdrawals and $\gamma\omega + (1-\gamma)$ with full withdrawals. As a result, the expected payoff for depositors is higher under full withdrawals. Hence, partial withdrawals are never privately optimal. Given the linearity of depositors' expected payoffs in γ and ω, it follows that any partial withdrawal $\gamma + x$ with $x > 0$ is dominated by an even larger partial equal to $\gamma + y$, with $x < y \leq 1 - \gamma$. Therefore, full withdrawals are optimal.

Case 2: Consider now $\gamma \leq L$, a situation in which partial withdrawals does not give rise to any coordination failure since the bank has enough asset liquidity. Thus, only fundamental-driven runs occur when $\theta < \underline{\theta}$, which is the same under the partial and full withdrawal assumption and equal to $\underline{\theta} = \frac{r_2}{R}$, as before.

For any $\theta < \underline{\theta}$, any resource kept at the bank is lost due to costly bankruptcy. Hence, it immediately follows that it is never optimal to only withdraw a fraction γ of deposits. This concludes the proof of the Lemma.

Proof of Proposition. Introducing holding limits affects consumers' decisions at $t = 0$ and $t = 1$. At $t = 0$, holding limits changes the consumers' participation constraint to:

$$\int_{\theta^*}^1 r_2 \, d\theta \geq (\omega^{HL})^2 = [1 + \gamma(\omega - 1)]^2 \,. \tag{52}$$

The left-hand side is the value of the deposit claim to the consumer (unchanged relative to the main text). The right-hand side is the expected return of holding CBDC, which differs from the main text because only a fraction γ of funds can be held in CBDC. At $t = 0$ an consumer invests γ in CBDC and $1 - \gamma$ in storage/cash. At $t = 1$, the initial investment returns ω on the γ units, whose a fraction γ is held in the CBDC account while the remainder is held in storage/cash. Thus, the analysis in the main text is a special case for no holding limits, $\omega^{HL}(\gamma = 1) = \omega$.

At $t = 1$, holding limits only affects a depositor's expected payoff from withdrawing, $r_1\omega^{HL}$, so they have the intended effect of directly reducing withdrawal incentives by lowering the remuneration of the withdrawn funds stored until $t = 2$. Thus, the effective CBDC remuneration with holding limits is $\omega^{HL} \equiv 1 + \gamma(\omega - 1)$. Once this transformation is made, the economy is identical to the one without holding limits with the only difference that ω is replaced by ω^{HL}.

The bank run threshold is $\theta^*_\gamma = \frac{r_2}{R}\frac{r_2 - L\omega^{HL}}{r_2 - r_1\omega^{HL}}$, where θ^*_γ increases in γ because $\frac{\partial\theta^*_\gamma}{\partial\gamma} = \frac{r_2}{R}\frac{r_2(\omega-1)(r_1-L)}{(r_2-r_1(1+\gamma(\omega-1)))^2} > 0$ whenever $\omega > 1$. This result captures the "common wisdom" about holding limits: introducing them (i.e., setting $\gamma < 1$) reduces bank fragility, effectively mitigating the direct effect of CBDC remuneration on fragility.

However, introducing holding limits also affects the sensitivity of the run threshold to changes in r_2, thus leading to a potential ambiguous effect on fragility when banks respond to the introduction of CBDC. The derivative of the threshold θ^*_γ with respect to r_2 is now a function of γ and equal to $\frac{\partial\theta^*_\gamma}{\partial r_2} = \frac{\theta^*_\gamma}{r_2} - \frac{r_2}{R}\frac{(r_1-L)\omega^{HL}}{(r_2-r_1\omega^{HL})^2}$. Hence, the total effect of holding limits on fragility is thus not obvious and again depends on both a direct effect (via lower withdrawal incentives) and an indirect effect (via equilibrium deposit rates).

E Proof of Proposition 5

We derive the total effect of changes in policy parameters on fragility. There is the usual direct effect and an indirect effect via deposit rates. As in the main model, we start with the direct effect on withdrawal incentives at $t = 1$. In the main model, $\pi_1 = \omega r_1 \overline{n} = \omega L$, while with contingent remuneration (CR) we have $\pi_{1,CR}$, as given in the main text. As a result, the failure threshold in the withdrawal subgame is lower with contingent remuneration, $\theta^*_{CR} < \theta^*$, which is the intended objective of the policy. To see this, we write the failure threshold as a function of the expected payoff from withdrawing:

$$\theta^* = \frac{r_2}{R}\frac{r_2 - \pi_1}{r_2 - \pi_1\frac{r_1}{L}}, \tag{53}$$

so $\frac{d\theta^*}{d\pi_1} = \frac{r_2^2(r_1-L)}{RL(r_2-\pi_1 r_1/L)^2} > 0$ and $\frac{\partial \pi_{1,CR}}{\partial \underline{\omega}} = (\overline{n}-\widetilde{n})r_1 > 0$ and $\frac{\partial \pi_{1,CR}}{\partial \widetilde{n}} = (\omega-\underline{\omega})r_1 > 0$. Thus, contingent remuneration achieves its direct objective of reducing withdrawal incentives, the more so, the more restrictive the policy is (lower $\underline{\omega}$ and lower $\widetilde{n}$).

Since $\frac{d\pi_{1,CR}}{dr_1} = \widetilde{n}(\omega - \underline{\omega}) > 0$ and $\frac{\partial \widetilde{n}}{\partial r_1} < 0$, higher short-term deposit rates again increase fragility, $\frac{d\theta^*_{CR}}{dr_1} > 0$. Moreover, when evaluated at $r_1 = 1$, we have

$$\frac{\partial \theta^*_{CR}}{\partial r_2} = \frac{(r_2 - \frac{\pi_{1,CR}}{L})^2 - \left(\frac{\pi_{1,CR}}{L}\right)^2(1-L)}{R(r_2 - \frac{\pi_{1,CR}}{L})^2} = \frac{1}{R} - \frac{\pi^2_{1,CR}(1-L)}{RL^2(r_2 - \frac{\pi_{1,CR}}{L})^2} \tag{54}$$

$$\pi_{1,CR} = \underline{\omega}L + \widetilde{n}(\omega - \underline{\omega}), \tag{55}$$

so $\pi_{1,CR}$ is a sufficient statistic for the effect of intervention parameters on fragility at $t = 1$ and comprises the effects of both $\widetilde{n}$ and $\underline{\omega}$.

As in the main model, there is also an effect of CR on the ex-ante deposit rate and, thus, an indirect effect on fragility. For vanishing private noise, the remuneration of CBDC is ω for $\theta > \theta^*$ and $\underline{\omega}$ for $\theta < \theta^*$, resulting in the changed participation constraint of consumers given in the main text. Expected bank profits change to

$$\Pi_{CR} = \int_{\theta^*_{CR}}^{1} (R\theta - r_2)d\theta. \tag{56}$$

The incentives of the bank and consumers are again aligned in setting the lowest feasible short-term deposit rate, $r^*_{1,CR} = 1$, because $\frac{d\Pi_{CR}}{dr_1} < 0$ and $\frac{dV_{CR}}{dr_1} = \frac{d\theta^*_{CR}}{dr_1}[-r_2+\omega(\omega-\underline{\omega})] < 0$ because $r_2^* > \omega^2$ in any equilibrium. Using the same steps as in the main text, we find that $r^*_{2,CR}$ solves a binding participation constraint.

There is an asymmetry in policy parameters on the ex-ante choices. Lowered CBDC remuneration $\underline{\omega}$ affects the value of the deposit claim directly, while the intervention threshold $\widetilde{n}$ only affects the ex-ante choice via its effect on fragility. Formally, changes in $\widetilde{n}$ translate into changes in $\pi_{1,CR}$, so we study how the latter affects fragility. To derive the effect of the intervention on the deposit rate, we use the IFT and the following partial derivatives:

$$\frac{\partial V_{CR}}{\partial r_2} = 1 - \theta^*_{CR} - \left[r_2 - \omega(\omega - \underline{\omega})\right]\frac{\partial \theta^*_{CR}}{\partial r_2} > 0, \tag{57}$$

$$\frac{\partial V_{CR}}{\partial \pi_{1,CR}} = -\left[r_2 - \omega(\omega - \underline{\omega})\right]\frac{\partial \theta^*_{CR}}{\partial \pi_{1,CR}} < 0, \tag{58}$$

$$\frac{\partial V_{CR}}{\partial \underline{\omega}} = -\left[r_2 - \omega(\omega - \underline{\omega})\right]\frac{\partial \theta^*_{CR}}{\partial \pi_{1,CR}}\frac{\partial \pi_{1,CR}}{\partial \underline{\omega}} - \omega\theta^*_{CR} < 0. \tag{59}$$

Thus, the effects on the deposit rate are:

$$\frac{dr^*_{2,CR}}{d\pi_{1,CR}} = \frac{\left[r_2 - \omega(\omega - \underline{\omega})\right] \frac{\partial \theta^*_{CR}}{\partial \pi_{1,CR}}}{1 - \theta^*_{CR} - \left[r_2 - \omega(\omega - \underline{\omega})\right] \frac{\partial \theta^*_{CR}}{\partial r_2}} > 0, \tag{60}$$

$$\frac{dr^*_{2,CR}}{d\underline{\omega}} = \frac{\left[r_2 - \omega(\omega - \underline{\omega})\right] \frac{\partial \theta^*_{CR}}{\partial \pi_{1,CR}} \frac{\partial \pi_{1,CR}}{\partial \underline{\omega}} + \omega \theta^*_{CR}}{1 - \theta^*_{CR} - \left[r_2 - \omega(\omega - \underline{\omega})\right] \frac{\partial \theta^*_{CR}}{\partial r_2}} > 0. \tag{61}$$

Totally differentiating the failure threshold with respect to each policy parameter (taking into account the indirect effect via deposit rates) yields the following:

$$\frac{d\theta^*_{CR}}{d\pi_{1,CR}} = \frac{(1 - \theta^*_{CR}) \frac{\partial \theta^*_{CR}}{\partial \pi_{1,CR}}}{1 - \theta^*_{CR} - \left[r_2 - \omega(\omega - \underline{\omega})\right] \frac{\partial \theta^*_{CR}}{\partial r_2}} > 0, \tag{62}$$

$$\frac{d\theta^*_{CR}}{d\underline{\omega}} = \frac{(1 - \theta^*_{CR}) \frac{\partial \theta^*_{CR}}{\partial \pi_{1,CR}} \frac{\partial \pi_{1,CR}}{\partial \underline{\omega}} + \omega \theta^*_{CR} \frac{\partial \theta^*_{CR}}{\partial r_2}}{1 - \theta^*_{CR} - \left[r_2 - \omega(\omega - \underline{\omega})\right] \frac{\partial \theta^*_{CR}}{\partial r_2}}, \tag{63}$$

whose denominator is positive. To obtain the inequality in the proposition, we simply substitute the expressions for $\frac{\partial \theta^*_{CR}}{\partial \pi_{1,CR}}$ and $\frac{\partial \pi_{1,CR}}{\partial \underline{\omega}}$. For $\widetilde{n} \to \overline{n}$, the beneficial direct effect vanishes because $\frac{\partial \pi_{1,CR}}{\partial \underline{\omega}} \to 0$. As the detrimental indirect effect via lower deposit rates remains bounded away from zero, the overall effect on fragility is detrimental.

F Proof of Propositions 6 and 7

Perfect competition. Consider perfect competition, $\beta \to 0$. It implies that the bank maximizes the expected return of its deposit claim (which is equivalent to maximizing V) subject to non-negative profits, $\Pi \geq 0$. Our approach will be to consider the unconstrained problem and then check whether bank profits are indeed non-negative. The first-order condition pins down the equilibrium deposit rate r_2^*:

$$H(r_2^*) \equiv \left.\frac{dV}{dr_2}\right|_{r_2 = r_2^*} = 0. \tag{64}$$

Since $H(r_2^{max}) > 0$, we deduce that $r_2^{max} < r_2^*$ and, as a result, fragility increases in the deposit rate around the equilibrium, $\left.\frac{\partial \theta^*}{\partial r_2}\right|_{r_2 = r_2^*} > 0$. Since

$$\frac{\partial H}{\partial r_2} \equiv -2 \frac{\partial \theta^*}{\partial r_2} - r_2 \frac{\partial^2 \theta^*}{\partial r_2^2} < 0, \tag{65}$$

a unique global maximum exists. Using the IFT and

$$\frac{\partial H}{\partial \omega} \equiv -\frac{\partial \theta^*}{\partial \omega} - r_2 \frac{\partial^2 \theta^*}{\partial r_2 \partial \omega}, \tag{66}$$

we obtain

$$\frac{dr_2^*}{d\omega} = \frac{\frac{\partial \theta^*}{\partial \omega} + r_2 \frac{\partial^2 \theta^*}{\partial r_2 \partial \omega}}{-2\frac{\partial \theta^*}{\partial r_2} - r_2 \frac{\partial^2 \theta^*}{\partial r_2^2}}, \tag{67}$$

so $\frac{dr_2^*}{d\omega} > 0$ whenever $r_2^* < 3\omega$, which arises from inserting the partial derivatives into (66) and re-arranging. To obtain a sufficient condition for $r_2^* < 3\omega$, note that $H(3\omega) < 0$ suffices. Using $\theta^*(r_2 = 3\omega) = 3\omega\frac{3-L}{2R}$ and $\frac{\partial \theta^*}{\partial \omega}\big|_{r_2=3\omega} = \frac{3+L}{4R}$, we obtain a sufficient condition for $\frac{dr_2^*}{d\omega} > 0$ under perfect competition, which is $R < 3\omega\frac{9-L}{4}$.

Next, we turn from the effect of CBDC remuneration on deposit rates to its effect on bank fragility. Using the total derivative of fragility, $\frac{d\theta^*}{d\omega} = \frac{\partial \theta^*}{\partial \omega} + \frac{\partial \theta^*}{\partial r_2}\frac{dr_2^*}{d\omega}$, we obtain $\frac{d\theta^*}{d\omega} > 0$ (after some rearrangement) whenever $-\frac{\partial \theta^*}{\partial \omega}\frac{\partial \theta^*}{\partial r_2} - r_2\frac{\partial \theta^*}{\partial \omega}\frac{\partial^2 \theta^*}{\partial r_2^2} + r_2\frac{\partial \theta^*}{\partial r_2}\frac{\partial^2 \theta^*}{\partial r_2 \partial \omega} < 0$, which always holds given the signs of the various partial derivatives already established. Hence, higher CBDC remuneration increases fragility.

Finally, we need to establish that $\Pi(r_2^*) \geq 0$. Note that expected profits can be written as $\Pi = (1-\theta^*)\left[\frac{R}{2}(1+\theta^*(r_2)) - r_2\right]$, where the first factor is strictly positive because of $\theta^* < 1$ from the upper-dominance region. The second factor is also positive because $r_2^* \in (\omega, R)$ and the following result:

$$\frac{R}{2}(1+\theta^*) - r_2 = \frac{R}{2} + \frac{r_2(r_2 - \omega L)}{2(r_2 - \omega)} - r_2 = \frac{R - r_2}{2} + \frac{r_2}{2}\frac{\omega(1-L)}{r_2 - \omega} > 0, \tag{68}$$

so expected profits at the equilibrium deposit rate are strictly positive.

Imperfect competition. For $\beta \in (0,1)$, the equilibrium deposit rate r_2^* arises from taking logs and differentiating, yielding the expression in Equation (14). That is, the first-order condition that pins down r_2^* is written as $\Lambda(r_2^*) = 0$, where

$$\begin{aligned} \Lambda(r_2) \equiv\ & -\beta\left(1 - \theta^* + (R\theta^* - r_2)\frac{\partial \theta^*}{\partial r_2}\right)\left[\int_{\theta^*}^1 r_2 d\theta - \omega^2\right] + \cdots \\ & \cdots + (1-\beta)\left(1 - \theta^* - r_2\frac{\partial \theta^*}{\partial r_2}\right)\int_{\theta^*}^1 (R\theta - r_2)\, d\theta. \end{aligned} \tag{69}$$

Thus, the partial derivative $\frac{\partial \Lambda}{\partial \omega}$ contains both positive and negative terms as well as an ambiguous one. All non-positive terms are multiplied by $(1-\beta)$, so a high enough value of β suffices for $\frac{\partial \Lambda}{\partial \omega} > 0$ and, thus, $\frac{dr_2^*}{d\omega} > 0$ from the IFT. To see this, note that

$$\begin{aligned}\frac{\partial \Lambda}{\partial \omega} &= -\beta\left(-\theta_\omega + (R\theta^* - r_2)\theta_{r\omega} + R\theta_\omega\theta_r\right)\left[\int_{\theta^*}^1 r_2 d\theta - \omega^2\right] + \cdots \\ &\cdots - \beta\left(1 - \theta^* + (R\theta^* - r_2)\theta_r\right)\left[-r\theta_\omega - 2\omega\right] + \cdots \\ &\cdots + (1-\beta)\int_{\theta^*}^1 (R\theta - r_2)\, d\theta \left[-\theta_\omega - r\theta_{r\omega}\right] - (1-\beta)\theta_\omega(R\theta^* - r_2)\left(1 - \theta^* - r_2\theta_r\right)\end{aligned} \tag{70}$$

where subscripts on θ denote first- and second-order partial derivatives of θ^* and recall that $\theta_\omega > 0$ and $\theta_{r\omega} < 0$.

G Proof of Proposition 8

This proof has three parts. First, we derive the run threshold. This part uses the same argument as the proof of Proposition 1. The threshold θ_q^* corresponds to the solution to $\int_0^{\widehat{n}(\theta)} qr_2 dn = \int_0^{\overline{n}} \omega r_1 dn$, because the bank repays depositors r_2 at $t = 2$ only when the project succeeds, where both $\widehat{n}(\theta)$ and $\overline{n}$ are independent of q and identical to the corresponding cutoffs in the main text. Some algebra yields the threshold θ_q^* stated in the proposition. Differentiating this threshold with respect to q and ω, we obtain:

$$\frac{\partial \theta_q^*}{\partial q} = \frac{r_2}{R}\frac{r_2 q r_2 - r_2\omega r_1 - q r_2 r_2 + \omega L r_2}{(q r_2 - \omega r_1)^2} = -\frac{r_2^2}{R}\frac{\omega(r_1 - L)}{(q r_2 - \omega r_1)^2} < 0, \tag{71}$$

$$\frac{\partial \theta_q^*}{\partial \omega} = \frac{r_2}{R}\frac{-L q r_2 + L\omega r_1 + q r_2 r_1 - \omega L r_1}{(q r_2 - \omega r_1)^2} = \frac{r_2}{R}\frac{q r_2(r_1 - L)}{(q r_2 - \omega r_1)^2} > 0. \tag{72}$$

Second, we solve for the bank's choice at $t = 0$. Differentiating the expected profits (16) with respect to q, we obtain Equation (17). A high enough c ensures that the solution q^* is interior and unique (because $SOC_q < 0$ for high c).

Third, and finally, we study how an increase in CBDC remuneration affects financial stability. Note that the monitoring effort q^* directly depends on CBDC remuneration. Formally, the overall effect of a change in CBDC remuneration on bank monitoring effort can be expressed as follows (because of the IFT):

$$\frac{\partial q^*}{\partial \omega} = -\frac{\frac{\partial FOC_q}{\partial \omega}}{SOC_q}. \tag{73}$$

Since $SOC_q < 0$, the sign of $\frac{dq^*}{d\omega}$ is equal to the sign of $\frac{\partial FOC_q}{\partial \omega}$, which is equal to

$$
\begin{aligned}
\frac{\partial FOC_q}{\partial \omega} &= -\frac{\partial \theta_s^*}{\partial \omega}\left(R\theta_q^* - r_2\right) - q\frac{\partial \theta_q^*}{\partial q}\frac{\partial \theta_s^*}{\partial \omega}R - q\frac{\partial^2 \theta_q^*}{\partial q \partial \omega}\left(R\theta_q^* - r_2\right) \\
&= -\left[\frac{\partial \theta_q^*}{\partial \omega} + q\frac{\partial^2 \theta_q^*}{\partial q \partial \omega}\right]\left(R\theta_q^* - r_2\right) - q\frac{\partial \theta_q^*}{\partial q}\frac{\partial \theta_q^*}{\partial \omega}R \\
&= -\left[\frac{\partial \theta_q^*}{\partial \omega} + q\frac{\partial^2 \theta_q^*}{\partial q \partial \omega}\right]\left(R\theta_q^* - r_2\right) + \omega\left(\frac{\partial \theta_s^*}{\partial \omega}\right)^2 R,
\end{aligned}
$$

where

$$
\begin{aligned}
\frac{\partial^2 \theta_q^*}{\partial q \partial \omega} &= -\frac{r_2^2}{R}(1-L)\frac{(qr_2-\omega)^2 + 2\omega(qr_2-\omega)}{(qr_2-\omega)^4} = -\frac{r_2^2}{R}(1-L)\frac{qr_2+\omega}{(qr_2-\omega)^3} < 0 \\
&= -\frac{\partial \theta_q^*}{\partial \omega} - 2\omega\frac{qr_2^2}{R}\frac{(1-L)}{(qr_2-\omega)^3}.
\end{aligned}
$$

Substituting the expressions for θ_q^*, $\frac{\partial \theta_s^*}{\partial \omega}$, $\frac{\partial^2 \theta_q^*}{\partial q \partial \omega}$, and $\frac{\partial \theta_q^*}{\partial q}$, we obtain

$$
\frac{\partial FOC_q}{\partial \omega} = 2\omega\frac{qr_2^2}{R}\frac{(1-L)}{(qr_2-\omega)^3}\left(R\theta_s^* - r_2\right) + \omega\left(\frac{\partial \theta_q^*}{\partial \omega}\right)^2 R = \frac{1}{R}q\omega r_2^3(L-1)^2\frac{qr_2+2\omega}{(qr_2-\omega)^4} > 0,
$$

which implies, in turn, that $\frac{dq^*}{d\omega} > 0$.

Finally, we move on to the total effect of CBDC remuneration on bank fragility:

$$
\frac{d\theta^*}{d\omega} = \frac{\partial \theta^*}{\partial \omega}\left[1 - \frac{\omega}{q}\frac{dq^*}{d\omega}\right] > 0, \tag{74}
$$

where the sign arises because $\frac{\partial \theta^*}{\partial \omega} > 0$ and one can show that

$$
\left[1 - \frac{\omega}{q}\frac{dq^*}{d\omega}\right] = 1 + \frac{\omega}{q}\frac{\frac{\partial^2\left(q\int_{\frac{r_2}{R}\frac{qr_2-\omega L}{qr_2-\omega}}^{1}(R\theta - r_2)d\theta - \frac{cq^2}{2}\right)}{\partial q \partial \omega}}{\frac{\partial^2\left(q\int_{\frac{r_2}{R}\frac{qr_2-\omega L}{qr_2-\omega}}^{1}(R\theta - r_2)d\theta - \frac{cq^2}{2}\right)}{\partial q^2}} > 0.
$$

H Proof of Proposition 9

The proof builds on Goldstein and Pauzner (2005) because introducing ω does not affect the properties of the relevant functions.

All impatient depositors withdraw, while patient depositors decide between withdrawing and re-depositing with the central bank to earn ω and keeping their funds in the bank. The lower dominance bound is the value of θ that makes a patient depositor indifferent when only impatient depositors withdraw ($n = \lambda$):

$$u(\omega r_1) \equiv \underline{\theta} u\left(c_{2\lambda}\right), \tag{75}$$

where the consumption level is $c_{2\lambda} \equiv R\frac{1-\lambda r_1}{1-\lambda}$. Similarly, $c_{2n} \equiv R\frac{1-nr_1}{1-n}$. The depositor's utility differential is $v(\theta, n) = \theta u(c_{2n}) - u(\omega r_1)$ if the bank is liquid at $t = 1$, $\lambda \leq n \leq \overline{n}$. Otherwise, $\overline{n} < n \leq 1$, $v(\theta, n) = 0 - u\left(\frac{\omega}{n}\right)$ because all withdrawing depositors receive an equal share of the liquidation proceeds and patient depositors will redeposit these funds, earn a return ω, and then consume at $t = 2$. One-sided strategic complementarity applies, so we apply the results in Goldstein and Pauzner (2005). The equilibrium failure threshold arises from the indifference condition of the marginal depositor for $\epsilon \to 0$:

$$\theta^* \int_{\lambda}^{\overline{n}} u(c_{2n})dn = \int_{\lambda}^{\overline{n}} u(\omega r_1)dn + \int_{\overline{n}}^{1} u\left(\frac{\omega}{n}\right) dn, \tag{76}$$

so rewriting yields Condition (19). We turn to the comparative statics:

$$\begin{aligned}
\frac{\partial \theta^*}{\partial r_1} &= \frac{\int_{\lambda}^{\overline{n}} u'\left(r_1\omega\right)\omega dn + \theta^* \int_{\lambda}^{\overline{n}} u'\left(c_{2n}\right) R\frac{n}{1-n}dn}{\int_{\lambda}^{\overline{n}} u\left(c_{2n}\right) dn} > 0 \\
\frac{\partial \theta^*}{\partial \omega} &= \frac{\int_{\lambda}^{\overline{n}} u'\left(r_1\omega\right) r_1 dn + \int_{\overline{n}}^{1} u'\left(\frac{\omega}{n}\right)\frac{1}{n}dn}{\int_{\lambda}^{\overline{n}} u\left(c_{2n}\right) dn} > 0 \\
\frac{\partial^2 \theta^*}{\partial r_1 \partial \omega} &= \frac{\int_{\lambda}^{\overline{n}} \left[u''\left(r_1\omega\right)\omega r_1 + u'\left(r_1\omega\right)\right] dn}{\int_{\lambda}^{\overline{n}} u\left(c_{2n}\right) dn} + \frac{\partial \theta^*}{\partial \omega}\frac{\int_{\lambda}^{\overline{n}} u'\left(c_{2n}\right) R\frac{n}{1-n}dn}{\int_{\lambda}^{\overline{n}} u\left(c_{2n}\right) dn}
\end{aligned}$$

I Proof of Proposition 10

We first prove that $r_1^* > 1$ and then we show that $\frac{dr_1}{d\omega} < 0$. Consider the FOC in the proposition. Evaluating (21) at $r_1 = 1$, we obtain

$$\frac{\partial \theta^*}{\partial r_1}(1-\lambda)[\underline{\theta} u(R) - u(\omega)] + \lambda \int_{\underline{\theta}}^{1} \left[u'(1) - \theta R u'(R)\right] d\theta \quad = \quad \lambda \int_{\underline{\theta}}^{1} \left[u'(1) - \theta R u'(R)\right] d\theta > 0,$$

as $\theta^* = \underline{\theta}$ when $r_1 = 1$, $\underline{\theta} u(R) - u(\omega) = 0$ from the definition of $\underline{\theta}$ at $r_1 = 1$, and $u'(1) - \theta R u'(R) > 0$ given that $RRA > 1$. Hence, this cannot be optimal.

Next, we prove that r_1 decreases in ω. As a first step, we show that an increase in ω always reduces depositors' expected utility for a given deposit rate r_1, $\frac{\partial EU}{\partial \omega} < 0$. To see this, it is useful to rewrite depositors' expected utility as

$$\begin{aligned} EU(r_1, \omega, \theta^*(r_1, \omega)) &= \lambda u(r_1) + (1-\lambda) \frac{u\left(\frac{1-\lambda r_1}{1-\lambda} R\right)}{2} - \lambda \theta^* \big[u(r_1) - u(1)\big] \\ &\quad -(1-\lambda)\theta^* \left(\frac{\theta^*}{2} u\left(\frac{1-\lambda r_1}{1-\lambda} R\right) - u(\omega)\right), \end{aligned} \tag{77}$$

where the first and second term capture the expected utility of early and late consumers when there is no run, the third term represents the expected cost of a run for early consumers, and the fourth term captures the expected cost of a run for late consumers.

An increase in ω increases θ^*, and therefore unambiguously increases the expected cost of a run for early depositors. It also affects the cost of a run for late depositors in two opposite ways. First, as runs become more likely, the expected cost increases. Second, late depositors' expected utility in the event of a run increases, which reduces the expected cost. When the first effect dominates the second, an increase in ω unambiguously reduces depositors' expected utility. We show below that this is always the case in our setting. To see this, we start by focusing on the case in which an increase in ω has a beneficial effect on the cost of runs. We refer to this as *the best case scenario.*

In the best case scenario, an increase in ω can make the second term vanish, i.e. late depositors expect to incur no cost from a run. Denote this level of CBDC remuneration by ω_E. However, early depositors still incur a run cost that is increasing in ω. Furthermore, in this extreme case, ω is so high that running is always optimal. To see this, notice that $u(\omega r_1) \geq u(\omega)$ for any $r_1 \geq 1$, so that when $\frac{\theta^*}{2} u\left(\frac{1-\lambda r_1}{1-\lambda} R\right) = u(\omega) < u(\omega r_1)$. For any smaller increase in CBDC remuneration (so $\omega < \omega_E$), the expected cost of a run for both early and late depositors increases. Hence, the expected utility of depositors unambiguously decreases with ω for all possible (but fixed) values of r_1.

Equipped with this first result, we now prove that r_1^* decreases in ω. Consider two values of ω, with $\omega_A < \omega_B$ and denote as r_{1A}^* and r_{1B}^*, the solution to the bank maximization problem when $\omega = \omega_A$ and $\omega = \omega_B$, respectively. We want to show that $r_{1A}^* > r_{1B}^*$. Since r_{1A}^* is the equilibrium deposit rate, any increase in r_1 decreases the expected utility. Furthermore, panic-driven runs are inefficient, so any increase in $\theta^*(.)$ lowers expected utility, $\frac{\partial EU}{\partial \theta^*} < 0$. Finally, recall that an increase in ω negatively affects expected utility, as just shown. Denote by $EU(r_{1A}^*, \omega_A, \theta^*(r_{1A}^*, \omega_A))$ the expected utility at the optimum for $\omega = \omega_A$.

Take now $\omega = \omega_B$. Keeping the deposit rate fixed at $r_1 = r_{1A}^*$, the change from ω_A to ω_B lowers expected utility, as shown above, due to the increase in the probability of a run. Hence, acknowledging that, irrespective of the associated cost, panic runs are always inefficient, the bank could unambiguously increase utility by reducing r_1. Hence, r_1 must decrease as ω increases, so $r_{1A}^* > r_{1B}^*$.

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