A Model of the Ratings Industry*

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Abstract

This paper examines the reliability of information provided by certification intermediaries, such as rating agencies in the context of both a monopolistic and a duopolistic certification industry. It demonstrates that, in a simple model where the intermediary is concerned about reputation and there is asymmetric information on her ability, the certification intermediary may ignore private information about the quality of the firm and decide instead to conform to the public information. It also shows that an intermediary perceived by the other agents as more talented chooses to act more conservatively by sending unfavourable reports more frequently. However, incentives to send out unfavourable reports and to conform with public information are mitigated by competition in the certification industry. The paper provides a theoretical explanation based on reputational concerns for why a rating agency may exhibit excess sensitivity to the business cycle and for differences in ratings across agencies.

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1 Introduction

Certification intermediaries in financial markets provide information to investors about the value of firms or other economic entities that approach them. Examples of such intermediaries are credit rating agencies and auditing firms. Reputation is the main asset of these intermediaries, since it confers credibility to their announcements and consequently makes firms hire

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their services. The *Economist*\(^1\) summarises the importance of reputation for rating agencies as follows:

"Even more than for accountants and lawyers, rating agencies must trade on their reputations. If, for example, bond investors lose faith in the integrity of rating agencies’ judgements, they will no longer pay attention to their ratings; if rating agencies’ opinions cease to affect the price that borrowers pay for capital, issuers will not pay their fees. So market forces should make rating agencies careful of their good names".

Therefore, one would expect reputational concerns to be a strong motive for them to try hard not to make mistakes and to use all available information, both public (e.g. accounting statements) and non-public (e.g. confidential interviews) when reporting their judgements to investors. But in reality, reputational concerns seem to generate conflicting incentives for certification intermediaries: an accurate report should incorporate private information but reporting according to public beliefs might be the best strategy for intermediaries whose private information is imprecise. This paper takes into account this trade-off and assesses whether certification intermediaries that worry about reputation transmit reliable information, and in what way the structure of the certification industry affects information transmission.

In 2001, credit raters failed to downgrade Enron to below-investment grade until four days before the company filed for bankruptcy. In fact, by the time investors services like Moody’s began cutting Enron’s ratings, bond traders had already been trading Enron at junk levels for several weeks and common stock had dramatically fallen to a seven year low. Quoting Chairman Joe Lieberman:

"In the Enron case (...) credit raters appear to have been no more knowledgeable about the company’s problems than anyone else who was following its fortunes in the newspapers."

WorldCom bonds had also collapsed to junk levels weeks before the company’s rating was downgraded and this happened only about a month before the company disclosed nearly $4 billion in improper accounting.

In the context of the Asian crisis, Reinhart (2002) and Ferri, Lui and Stiglitz (1999) describe how agencies also failed to give warning signals until after the turbulence in the Asian

\(^1\)Use and Abuse of Reputation*, Economist, April 6, 1996.
markets had begun. However, when the crisis was actually spreading, there was widespread downgrading of the Southeast-Asian issuers.

These facts raise several questions regarding the informational value of ratings. Downgrades seem to have reflected information that market participants had already previously incorporated in the pricing process and in some cases, they occurred after the rated entities had themselves disclosed substantially increased risk. Nonetheless, the information rating agencies provide is widely used for purposes that reach far beyond the intention to mitigate asymmetric information among market participants. For example, there are proposals to use ratings for regulatory purposes: the Basel Committee on Banking Supervision intends to see borrowers’ credit ratings included in assessments of the adequacy of bank’s capital. For this reason, it is of foremost importance to understand how rating agencies behave and which mechanisms can be put into practice to increase the credibility of their announcements.

In the model developed below there exists public as well as private information about the quality of the firm and both investors and firms are unsure about the (monopolistic) certification intermediary’s type: she might, or might not, make mistakes when assessing the firm (be untalented or talented). This paper shows that in some situations an untalented certification intermediary chooses to conform to the public information going against what her private information indicates because of fears of being wrong, in which case she would have to bear a heavy reputational cost. As a result, this can happen whenever public information is extreme, i.e. when public information is predominantly very good or very bad. For example, if investors expect a firm to be good and the intermediary’s private information indicates that the firm is bad, there are situations where she chooses to report that the firm is good and vice-versa.

Moreover, whenever the prior belief is not very informative, i.e. for medium values of the prior, conservatism might arise as an untalented intermediary prefers issuing bad reports even though her private signals was positive. And more reputable certification intermediaries, i.e. intermediaries perceived by the market as more talented and that are in fact untalented, tend to issue less favourable reports with greater frequency than more favourable ones for a given prior. This happens because there is an asymmetry of observability in the model: a project issued with an unfavourable report is not undertaken and this limits the learning process about the certification intermediary’s type, which makes sending unfavourable reports a safer option. In addition, the more reputable an intermediary is the less she benefits from issuing a report that turns out to be correct and the higher the loss she incurs into when proven to
Finally, the model concludes that the presence of a potential competitor forces a certification intermediary to issue more favourable reports: it makes her more aggressive and opt for the riskier option more frequently. It can also force more reputable certification intermediaries to abandon their conservative behaviour. The difference is that in the new setting sending an unfavourable report also carries disadvantages: reputation might decrease and this might compromise the chances of being hired next period.

All these conclusions hold even though the model abstracts from conflicts of interest, communication between firms and certification intermediaries, repeated relationships between firms and intermediaries and bribes.

Empirically, several studies addressed the informational value of ratings but the results have been inconclusive. Looking at the US corporate bond market, Katz (1974) finds that bond prices adjust to rating changes and that there is no price movement prior to the announcement of a rating change, suggesting that this change is not anticipated by investor. In contrast, Hettenhouse and Sartoris (1976) and Weinstein (1977) conclude that bond prices react to other information released prior to the rating change. More recently, Heinke and Steiner (2001) examine daily excess Eurobond returns associated with announcements of watchlistings and rating changes by S&P and Moody’s. They find significant price changes up to 100 trading days prior to the rating change. Moreover, bond prices still react to the actual announcements of downgrades but upgrades do not seem to cause any effect in prices. Finally, Amato and Furfine (2003) find that, for a set of observations where a rating has either just been issued or changed, ratings exhibit excess sensitivity to the business cycle.

The model developed here is also related to the literature on reputational concerns and information transmission, in which Benabou and Laroque (1992) and Morris (2001) are major contributors. Both papers build on Crawford and Sobel (1982) and Sobel’s (1985) papers by developing repeated cheap talk models where there is a sender of information, i.e. the equivalent to the certification intermediary in this model, whose type (honest or strategic) is unknown to receivers. Benabou and Laroque (1992) assume the honest sender always reports her signal and, because private information is noisy, they conclude that a strategic sender can manipulate information without risking losing all her credibility as predictions which turn out to be incorrect can always be attributed to an honest mistake. Morris (2001) endogenises the behaviour of the honest sender and shows that she can also have incentives to lie in order to enhance reputation. However, both papers abstract from the role of public information.
Moreover it seems more suitable to assume a sender that is primarily concerned with maximising profits as rating agencies and auditing firms are private companies. Reporting a message that differs from the private signal in this model originates from the fact that the intermediary wants to maximise profits, and therefore her reputation, but is unsure about how much she can trust her private signals.

Reputational concerns and conflicts of interest for investment banks and equity analysts have been covered by Chemmanur and Fulghieri (1994b) and Morgan and Stocken (2003). The former model reputation by investment banks in the equity market, while the latter, develop a static cheap talk model of information transmission for financial analysts. They both assume that compensation is contingent on the message sent, unlike the model developed below where the intermediary fee is paid upfront and before any assessment is performed by the certification intermediary.

There are also papers that address competition and information transmission. Examples are the models by Lizzeri (1999) and Bolton, Freixas and Shapiro (2004). Lizzeri (1999) discusses the role of intermediaries who search out the information of privately informed agents and then decide what to disclose to the uninformed ones. Bolton, Freixas and Shapiro (2004) look at competition between investment banks that help clients, whose type is only known by the bank, to choose the appropriate financial product. Both models abstract from reputational issues whereas the model developed here explicitly models reputation using Bayesian updating.

Finally, this paper is also related to the literature on career concerns, whose seminal papers are Holmstrom (1999) and Holmstrom and Ricart i Costa (1986). Later developments are Scharfstein and Stein (1990) on career concerns and herd behaviour, Prat (2003) on career concerns and transparency and Boot, Milbourn and Thakor (2002) on the delegation of ideas.

The rest of the paper is organised as follows. Section 2.2 describes the basic characteristics of the monopolistic model and section 2.3 looks at a benchmark case. Section 2.4 contains the equilibrium analysis and comparative statics. In Section 2.5 competition is introduced and Section 2.6 concludes. Some proofs are in the Appendix.

\footnote{For example, Moody\'s is a public company and was until 2000 a subsidiary of Dun&Bradstreet, Standard&Poors is a subsidiary of MacGraw-Hill and Fitch, which resulted from the merge of Fitch IBCA Investors Service, Inc. and Duff&Phelps Credit Rating (DCR), is owned by a French conglomerate, FIMALAC SA.}
2 The Model

In this economy, there are three different classes of risk-neutral agents: investors (the market), a certification intermediary (she) and a firm or its manager. The model lasts for two periods and the risk-free interest rate is zero. At each date, there is a firm that needs to undertake an investment project that lasts for one period. At the end of the period the project either succeeds, and its proceeds are distributed to the firm’s holders, or it fails and the firm is liquidated. Market conditions determine the expected liquidation value that to simplify is normalised to zero. The current holders of the firm are liquidity/credit constrained thus the firm cannot undergo the new project unless it succeeds in obtaining financing in the form of an extra loan. In order to obtain this loan the firm needs to be evaluated by a certification intermediary. This may be because creditors are small and dispersed and information is difficult to gather on an individual basis or it might constitute an institutional requirement. The certification intermediary sends a message to investors based on public information and on the information she collects about the quality of the firm. The model studies the distortions to the report that the intermediary issues each period as she considers how it affects her future reputation and profits.

2.1 Agents and Basic Set-up

The firm’s initial market value is $V$, with $V > 1^4$ and the firm needs a loan of $\frac{1}{2}$ to invest in a project essential to the continuation of its activity. The project can be of two types $f$, Good ($f = G$) or Bad ($f = B$). For simplicity, it is assumed that a $G$ project has a payoff of 1, while a $B$ project pays off 0.

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3 Ratings are used in prudential supervision in a large number of countries. For example, of the 12 BIS Basel Committee in Banking Supervision countries, 11 did so in 2000. In the US it goes back to 1931 when regulators either banned some institutional investors from holding securities that fell below a certain grade or specified capital requirements for holding securities that were geared to their ratings. At present, institutional investors, pension and mutual funds and insurance companies, that are among the largest purchasers of fixed-income securities, all use credit ratings to comply with regulatory requirements that require them to maintain certain minimum credit ratings for investments. Financial regulators also use ratings in a similar way for safety regulation of broker-dealers and creditors can demand "ratings triggers" in financial contracts in order to accelerate repayment of an outstanding loan or to secure collateral if the borrower’s rating falls below a certain level.

4 Given that the fees are paid upfront, this assumption is necessary to make sure the firm is has enough resources to pay the intermediary’s fees.
The debt market is characterised by asymmetric information. The firm’s manager, that acts on behalf of the current holders of the firm, knows the project’s type but also that the firm will only continue provided that the project is undertaken and that he will be unemployed otherwise. Therefore, even if a project is of the $B$ type (and generates 0) the manager has incentives to persuade external investors to participate. Investors however, cannot tell good firms from bad ones. The firm’s previous history and general conditions of the economy determine the common prior over the quality of the project. Therefore $\Pr(G)$ equals $\theta$, with $\theta \in (0, 1)$ and for simplicity, it is constant over time. In period 1, before another firm requires certification, the true type of the firm certified in period 0 is revealed.

2.2 Intermediary’s Private Signals

At time $t$, with $t \in \{0, 1\}$, the certification intermediary receives a request for an assessment. She cannot a priori distinguish between Good and Bad firms but by conducting an evaluation of the firm she receives additional noisy information. The intermediary can be of two types: Talented (T) and Untalented (U). A talented intermediary identifies the project’s type with probability 1 (a.s.), while if untalented (U) she only observes a noisy signal about the project.

The certification intermediary knows her own type but investors and firms are uncertain about the intermediary’s ability denoted by $a$, where $a = \{T, U\}$, and must learn about it over time. The intermediary’s private information is given by $s_f$, where $s_G$ is a signal indicating a Good project and $s_B$ is a signal indicating a Bad project (the time subscript is omitted in order to simplify the notation). It is assumed that if the intermediary is talented, which at date 0 occurs with probability $\alpha_0$, with $\alpha_0 \in (0, 1)$, then:

$$\Pr(s_G \mid G, T) = \Pr(s_B \mid B, T) = 1,$$

and

$$\Pr(s_G \mid B, T) = \Pr(s_B \mid G, T) = 0.$$  \hspace{1cm} (1)

If untalented, which at date 0 occurs with probability $1 - \alpha_0$, the signal-generation process is given by:

$$\Pr(s_G \mid G, U) = \Pr(s_B \mid B, U) = 1 - \varepsilon$$

and

$$\Pr(s_G \mid B, U) = \Pr(s_B \mid G, U) = \varepsilon,$$  \hspace{1cm} (2)

where $\varepsilon \in (0, \frac{1}{2})$ and is common knowledge.
After observing the private signal the certification intermediary uses Bayes’ rule to revise her estimate that the project is good. Thus, an intermediary forms her posterior belief using the prior about the project’s type and (1) or (2) depending on her type, according to

\[
\Pr(G \mid s_G, T) = 1, \quad \Pr(G \mid s_B, T) = 0 \tag{3}
\]

and

\[
\Pr(G \mid s_G, U) = \frac{(1 - \varepsilon)\theta}{(1 - \varepsilon)\theta + (1 - \theta)\varepsilon}, \quad \Pr(G \mid s_B, U) = \frac{\varepsilon\theta}{\varepsilon\theta + (1 - \theta)(1 - \varepsilon)}. \tag{4}
\]

Certification intermediaries charge a fee determined endogenously and paid upfront, like common practice with rating agencies. Before evaluating the firm the intermediary is at the same informational level as any potential investor of the firm, i.e. she cannot ex-ante distinguish between the two projects.

Investors value certified firms based on the intermediary’s report, the belief they have that her report is correct, and the other variables that are common knowledge. At the end of the period the state of nature is realised and publicly observed. Investors have then the chance to update their beliefs about the intermediary’s type, by comparing the message sent with the true state in case the project is undertaken. The game is then repeated for one more period with the same intermediary and investors but with a new firm. This concludes the game. Reputation in this context translates the beliefs of investors about the certification intermediary ability, given the message that she sent and the true project’s type or the investor’s initial belief about the project’s true type in case this is not undertaken.

### 2.3 Investors

At each date, the investors’ required repayment to invest in certified debt is derived after a message has been sent by the certification intermediary. The required repayment at time \( t \) depends on the true type, on the intermediary’s message, and on the confidence investors have in her message, as captured by her reputation \( \alpha_t \). This message is given by \( m_f \), where \( f \in \{G, B\} \), \( m_G \) corresponds to a favourable report and \( m_B \) to an unfavourable report (again the time subscript is omitted in order to simplify the notation). The required repayment at time \( t \), for a message \( m_f \) and provided that the firm is of type \( f \), is denoted by \( r_{f f}^t \), with \( r_{f f}^t \in \left[ \frac{1}{2}, 1 \right] \). Using Bayes’ rule to evaluate the various conditional probabilities and given that the financial market is competitive and risk neutral, \( r_{f f}^t \) needs to satisfy the investors participation constraints for each \( m_f \):

\[
\Pr(G \mid m_f, \{\Omega_t\}) r_{f G}^t + \Pr(B \mid m_f, \{\Omega_t\}) r_{f B}^t = \frac{1}{2}
\]
where \( \{\Omega_t\} \) represents the investors’ information set at time \( t \).

Whenever the project is undertaken and fails the liquidation value is zero meaning that \( r_{GB}^t = r_{BB}^t = 0 \). Additionally, in order to simplify the model and focus on the most interesting case it is assumed that investors cannot become involved in a project whose report has been unfavourable\(^5\). As a result, \( r_{GG}^t \) is derived to be equal to

\[
\frac{1}{2} \left( 1 + \frac{\Pr(B) \Pr(m_G | B, \Omega_t)}{\Pr(G) \Pr(m_G | G, \Omega_t)} \right).
\]

Furthermore, \( r_{GG}^t \) needs to be lower than 1 to make sure that firms would like to undertake the project. Hence,

**Lemma 1** A necessary condition for investment to happen is

\[
\Pr(G) \Pr(m_G | G, \Omega_t) > \Pr(B) \Pr(m_G | B, \Omega_t).
\]  \hspace{1cm} (5)

For a given prior belief, the intermediary message needs to be informative. The remainder of this paper considers that investment only takes place if a favourable report is issued, provided that (5) holds.

### 2.4 The Certification Intermediary Fee

The objective of the certification intermediary is to maximise the expected value of her future profits (fee net of any certification costs). The fee is derived as follows. The firm’s manager acts on behalf of the shareholders and knows the project’s type but enjoys private benefit of control. Hence, he is willing to pay any fee to undertake the project and not to reveal the true type in the Bad-project’s case. In particular, the manager of Bad project is willing to pay as much as the manager of a Good project. Because certification is compulsory, at time \( t \) a Good firm is willing to pay a fee \( F_t(\alpha_t) \) up to the amount for which its participation constraint is binding. A higher fee cannot be extracted from Good firms as shareholders would veto it. The surplus for a good message is \( 1 - r_{GG}^t - F_t(\alpha_t) \) and for a bad message \( -F_t(\alpha_t) \). Thus, the Good firm’s participation constraint at \( t \) is the following:

\[
\Pr(m_G | G, \Omega_t) \left( 1 - r_{GG}^t - F_t(\alpha_t) \right) + \Pr(m_G | G, \Omega_t) (-F_t(\alpha_t)) = 0.
\]

\(^5\)This can be an equilibrium condition for certain values of the parameters. But otherwise it can be justified by institutional reasons; for example pensions funds and insurance companies are not allowed to invest in securities rated with a non-investment grade. For other examples see supra note 3.
Looking at the firm participation constraint, two conflicting interests can be identified for the firm (and indirectly for the intermediary): the firm wants the repayment to investors to be as low as possible, and this happens if a more reputable intermediary sends a good message but, on the other hand, a less reputable intermediary is more likely to send a good message necessary for the project to be undertaken.

Given $r^t_{GG}$, the fee at time $t$, $F_t(\alpha_t)$ can be set up to

$$\frac{\Pr(G)\Pr(m_G | G, \{\Omega_t\}) - \Pr(B)\Pr(m_B | B, \{\Omega_t\})}{2\Pr(G)}.$$  \hspace{1cm} (6)

By Lemma 1, this is always positive.

However, for the firm no certification means no project but for the intermediary no certification also means no fee. Both parties have something to lose if the project is not undertaken and this implies that the intermediary might not extract the full surplus of certification from the firm. Hence, the certification intermediary knows she can charge a fraction $\kappa$, with $\kappa \in (0, 1]$, can be thought of as the outcome of bargaining, exogenous to the model, between the intermediary and the firm.

To sum up, the fee charged by the certification intermediary is unique. According to what happens in reality, it is not possible for the certification intermediary to screen among firms by offering a menu of fees $\{F_t(\alpha_t)\}$ and let each firm choose a fee according to its type. This would mean that by choosing a fee the firm would reveal its type and there would be no need for the firm to be assessed and for the intermediary to worry about reputation. The Bad-firm has a monetary surplus associated to the project that equals zero but its manager is ready to pay as much as the Good-firm is paying, in order not to reveal its type, at the expense of the existing holders of the firm\(^6\). Basically, in this model the firm has a passive role. Certification is compulsory, therefore the firm only chooses whether to obtain certification and indirectly whether to undertake the project.

It is also assumed that the firm cannot refuse to make use of the information collected by the intermediary after learning the evaluation that she will report to investors.

2.5 The Certification Intermediary Behaviour

After being hired, the certification intermediary collects information about the firm in the form of the private signal $s_f$. She then balances out the costs and benefits of sending a report that is contrary to the private signal, i.e. she chooses $\Pr(m_G \mid s_B)$ and $\Pr(m_B \mid s_G)$. In order

\(^6\)Remember that the manager will be unemployed unless the project is undertaken.
to ease notation the time subscript is omitted and henceforth Pr($m_G \mid s_B$) is denoted by $\gamma$ and Pr($m_B \mid s_G$) by $\gamma$. There is however an arbitrarily small cost from deviating from the private signal given by $c_t$ that includes, for example, the cost in terms of extra time and effort of commissioning a report where a financial analyst has to disguise private information about the firm\textsuperscript{7}. This cost ensures that the intermediary has incentives to care about reputation even in the last period of the game.

3 Equilibrium Analysis

In a first best world, an intermediary should simply report her private signal. In a world with reputational concerns and where a certification intermediary seeks to maximise expected profits, an equilibrium consists of choices by the intermediary of $\gamma$ and $\gamma$ specifying the probability of sending a message different from the signal received. It also consists of choices by the firm of whether to hire or not the certification intermediary (and indirectly whether to undertake the new project) based on $\alpha_t$, $\gamma$ and $\gamma$ and a system of beliefs formed by investors. Investors choose whether to provide investment funds and the expected repayment based on $\theta$, $\alpha_t$, $\gamma$ and $\gamma$. The model is solved by backwards induction.

3.1 Period 1

3.1.1 Certification Intermediary Optimal Behaviour and Fee

In period 1, since deviating from the private signal is costly and there is no reputational benefit to consider (as the game is over at the end of period 1), the certification intermediary minimises costs by always reporting her signal. Therefore,

**Proposition 1** The certification intermediary never misreports in the last period, regardless of her type.

\textsuperscript{7}It can also include litigation costs, i.e., $c_t$ can be interpreted as the legal cost in case misreporting is discovered times the probability of legal action. Even though legal action is relatively common for auditing firms, lawsuits against rating agencies seem to be quite infrequent. There have however been some cases where rating agencies have been accused of fraud in misreporting or omitting certain facts in their ratings (e.g. the Jefferson County, Colorado, School District case against Moody’s, the Orange County, California case against S&P or the LaSalle Nat’l Bank v. Duff&Phelps Co. case). In addition, agencies have also been investigated by the SEC and the US Department of Justice.

Alternatively, it can be seen as a short-cut to capture in a two period model the impact over the future reputation which would happen in repeated relationships.
Given Proposition 1, probabilities \( \Pr (m_G | G, \{ \Omega_1 \}) \) and \( \Pr (m_G | B, \{ \Omega_1 \}) \) are equal to \( \alpha_1 + (1 - \alpha_1) (1 - \varepsilon) \) and \( (1 - \alpha_1) \varepsilon \) respectively. And using (6) the fee charged in period 1 is

\[
F_1 (\alpha_1) = \kappa \left( \frac{\alpha_1 \varepsilon}{2\theta} + \frac{(\theta - \varepsilon)}{2\theta} \right).
\]

(7)

A higher reputational level of the certification intermediary generates a higher fee as \( \frac{\partial F_1 (\alpha_1)}{\partial \alpha_1} \) is positive. As a result, when hired in period 0, the intermediary acts to maximise \( \alpha_1 \). When investors are unsure about the intermediary’s type they require a higher repayment because they want to be compensated for the probability of an untalented intermediary making a mistake by sending a favourable report for a bad project. In addition to bearing a higher repayment good firms are also uncertain about which message the intermediary is going to send. As a result a lower fee is derived from the firm’s participation constraint.

3.1.2 Posterior beliefs

So far reputation in period 1 has been generally denoted by \( \alpha_1 \) but in fact it varies depending on the message sent and how it relates to the outcome of the project. Therefore, henceforth \( \alpha_{GG} \) denotes the posterior belief that the certification intermediary is talented given that she sent \( m_G \) and the true firm’s type was indeed G, i.e. to denote \( \Pr (T | m_G, G) \); \( \alpha_{GB} \) is used to identify the probability that the certification intermediary is talented given that she sent \( m_G \) but the true type turned out to be B, i.e. \( \Pr (T | m_G, B) \); and finally, \( \alpha_B \) denotes the posterior belief that the certification intermediary is talented given that she sent \( m_B \), i.e. \( \Pr (T | m_B) \). In this case the project is not undertaken and therefore investors and the new firm cannot compare the certification intermediary’s report to the project realisation. The analytical expressions for these probabilities are derived below.

3.2 Period 0

3.2.1 Certification Intermediary Optimal Behaviour

At date 0, a certification intermediary is hired, paid \( F_0 (\alpha_0) \) and collects a private signal about the project. After observing the signal, the intermediary comes up with a posterior belief about the value of the project and must make a decision about what message to send. She makes this decision such that her expected fee in period 1, that depends on her reputational level at the end of period 0, is maximised. The link between periods 0 and 1 is therefore the intermediary’s reputation, which is revised at the end of period 0 in view of whether her forecast was realised or not.
After observing the private signal the certification intermediary uses the Bayes’ rule to revise her estimate about the project’s type according to (3) and (4). A certification intermediary with ability $a$ and a private signal $s_G$ has an expected profit from reporting her signals of:

$$\Pr(G \mid s_G, a) F_1(\alpha_{GG}) + \Pr(B \mid s_G, a) F_1(\alpha_{GB}).$$

But it may be that she decides to send a message different from her private signal even though this implies an extra cost of $c_0$. In this case, the expected profit in period 1 is $F_1(\alpha_B) - c_0$. The intermediary sends the message that generates a higher expected profit in period 1. Looking at the following expression:

$$\pi^e(a) = \Pr(G \mid s_G, a) F_1(\alpha_{GG}) + \Pr(B \mid s_G, a) F_1(\alpha_{GB}) - F_1(\alpha_B) + c_0,$$

in equilibrium, if $\pi^e(a) > (\prec) 0$ the intermediary follows (contradicts) the private signal and if $\pi^e(a) = 0$ there is an equilibrium in mixed strategies.

On the other hand, if the private signal indicates that the project is bad and the message coincides with this private signal, the expected profit in period 1 is simply $F_1(\alpha_B)$, but if the certification intermediary decides to go against her private signal the expected profit is

$$\Pr(G \mid s_B, a) F_1(\alpha_{GG}) + \Pr(B \mid s_B, a) F_1(\alpha_{GB}) - c_0.$$ 

Once more the intermediary looks at

$$\pi^e(a) = \Pr(G \mid s_B, a) F_1(\alpha_{GG}) + \Pr(B \mid s_B, a) F_1(\alpha_{GB}) - c_0 - F_1(\alpha_B).$$

There is no deviation from the private signal for $\pi^e(a) < 0$ and there is an equilibrium in mixed strategies if $\pi^e(a) = 0$. Otherwise, the intermediary contradicts the private signal.

It can also be proven that the talented certification intermediary never misreports. In particular, there cannot be an equilibrium in which the talented certification intermediary always sends a message that goes against her private signal. Looking at pure strategies only, observe that a talented intermediary always has less of an incentive to misreport than the untalented one. So if an intermediary observes $s_G$ and reports $m_B$ the expected profit is $F_1(\alpha_B) - c_0$ regardless of the type. But when a talented intermediary observes $s_B$ and reports $m_G$ her expected profit is $F_1(\alpha_{GB}) - c_0$ which is lower than the expected profit an untalented intermediary, $\Pr(G \mid s_B, a) F_1(\alpha_{GG}) + \Pr(B \mid s_B, a) F_1(\alpha_{GB}) - c_0$ as the fee is increasing in the reputational level and the reputational level increases when the intermediary is correct and decreases otherwise i.e., $\alpha_{GG}$ exceeds $\alpha_{GB}$. Consequently, an untalented
intermediary misreports whenever a talented intermediary does so. Secondly, it can be proven by contradiction that a talented intermediary never misreports. If for signal $s_G$ the talented certification intermediary sends $m_B$, then the untalented certification agent would also choose to send $m_B$. The firm then decides not to hire an intermediary because certification is costly and an unfavourable report implies no investment. And if whenever the signal is $s_B$ the talented certification intermediary sends $m_G$, the untalented certification intermediary would also choose to send $m_G$. If the talented intermediary decides to deviate and be truthful, the untalented might or might not deviate from $m_G$. If she does not, the talented intermediary prefers being truthful because when sending $m_B$ she reveals her type whereas before investors could not distinguish between the two types. In fact, if both types behaved alike investors would be unable to update their prior belief about the intermediary’s type. This would lead to a lower reputational level and consequently to a lower fee. Hence, if the untalented type does not follow the talented type deviation, she always prefers to deviate. If she also sends the true signal $m_B$, then the talented intermediary reconsiders what to do: she can either keep sending $m_B$ or not. But if not the untalented type will again follow because as it was stated in the beginning of this proof a talented intermediary always has less of an incentive to misreport than the untalented one, so if she misreports the other does it as well. And in such case, it is better to be truthful. Hence, in equilibrium the talented intermediary reports her private signal.

This also does not mean that the talented certification intermediary follows a mixed strategy in equilibrium. In this case if a signal $s_G$ is received, the talented certification intermediary is indifferent between sending $m_G$ and $m_B$ and therefore randomises between the two, i.e. $\pi^*(T) = 0$. As a result, the untalented certification intermediary strictly prefers to send $m_B$ as $\pi^*(U) < 0$. This follows from the fact that the noisy private signal makes sending $m_G$ strictly worse for the untalented intermediary than for the talented one (sending $m_B$ gives both types the same profit; i.e. $F_1(\alpha_B) - c_0$ and $F_1(\alpha_{GG})$ exceeds $F_1(\alpha_{GB})$). Hence, every time the talented intermediary chooses to randomise, the untalented intermediary strictly prefers to send $m_B$. This implies that only the talented certification intermediary ever sends $m_G$ and as a result, a favourable report allows firms and investors to identify the intermediary’s type with certainty. Consequently, when this happens she is able to extract the maximum fee in period 1. But this also contradicts the conjectured indifference of the talented certification intermediary between sending $m_G$ and $m_B$. Thus, in equilibrium the talented certification intermediary cannot play a mixed strategy. A similar proof holds when the private signal is
s_B. However in this case, the talented certification intermediary is the only one sending m_B as π^T(U) > 0.

As far as the untalented intermediary is concerned, a mixed strategy independent of θ cannot be an equilibrium. This can be proven by contradiction. If there is a mixed strategy such as π^T(U) = 0 this implies that π^T(U) > 0 because F_1(α_{GG}) is higher than F_1(α_{GB}) and Pr(G | s_G, U) exceeds Pr(G | s_B, U)^8. This means that there is set of priors that makes the untalented intermediary indifferent between reporting favourably or unfavourably when she receives a bad private signal but that makes her report the private signal when this is positive. A similar result holds for π^T(U) = 0. Therefore, it is obvious that the certification intermediary’s optimal behaviour is going to be affected by the prior θ. In fact, it can be proven that the equilibrium is characterised by two values of θ, given by θ_L and θ_H, with 0 < θ_L < θ_H < 1, such that for θ < θ_L the untalented certification intermediary reports m_B when the signal is s_B but plays a mixed strategy if the private signal is s_G; on the other hand, for θ > θ_H the untalented certification intermediary reports m_G when the signal is s_G but plays a mixed strategy when the private signal if s_B. For the remaining set of priors, i.e. θ_L < θ < θ_H, the intermediary reports her private signal. It is then proved that this is indeed the unique equilibrium.

In order to prove this, the way reputation evolves between date 0 and 1 needs to be examined. If θ < θ_L it is above conjectured that a talented certification intermediary always reports her signal, whereas an untalented certification intermediary is expected to report m_B if s_B is observed but plays a mixed strategy if s_G is observed, i.e. reports m_B with probability γ and m_G with probability 1−γ. If m_B is sent, the posterior assessment of her ability is given by

$$\alpha_B = \frac{\alpha_0 (1−\theta)}{\alpha_0 (1−\varepsilon) + (1−\alpha_0) ((1−\varepsilon) \theta + \varepsilon (1−\theta)) \gamma + (\varepsilon \theta + (1−\varepsilon) (1−\theta))}.$$  \hspace{1cm} (10)

And if the certification intermediary reports m_G, her date 1 reputation varies depending on whether the project pays off 1 or 0. These two reputational levels are given by

$$\alpha_{GG} = \frac{\alpha_0}{\alpha_0 + (1−\alpha_0) (1−\gamma)}$$ \hspace{1cm} (11)

and α_GB = 0 respectively. Moreover, α_GB < α_B < α_GG and α_GG > α_0 but α_B only exceeds α_0 when the prior belief θ is relatively low and definitely lower than 1/2. Obviously investors need to be very convinced about the bad quality of the project to be confident about an intermediary’s judgement that is not verifiable.

8And hence π^T(U) − π^T(U) is always positive.
For projects whose $\theta$ exceeds $\theta_H$, the talented certification intermediary reports her signal, whereas the untalented certification intermediary is conjectured to always send $m_G$ when $s_G$ is observed, but sends $m_G$ with probability $\gamma$ and $m_B$ with probability $1 - \gamma$ if $s_B$ is observed. When $m_B$ is sent, the posterior assessment of her ability is

$$\bar{\alpha}_B = \frac{\alpha_0 (1 - \theta)}{\alpha_0 (1 - \theta) + (1 - \alpha_0) (\varepsilon \theta + (1 - \varepsilon) (1 - \theta)) (1 - \gamma)}.$$ (12)

And if the certification intermediary sends $m_G$ her reputation is

$$\bar{\alpha}_{GG} = \frac{\alpha_0}{\alpha_0 + (1 - \alpha_0) ((1 - \varepsilon) + \varepsilon \gamma)}.$$ (13)

or $\bar{\alpha}_{GB} = 0$, depending on whether the project pays off 1 or 0 respectively. In this case $\bar{\alpha}_{GG}$ exceeds $\bar{\alpha}_B$ if $\theta > \frac{\gamma}{\varepsilon (1 - \gamma) + \gamma}$. However, this is always the case because if $\theta$ is lower than that threshold, $\gamma$ equals zero for any $c_0$. But it is also the case there is no $\theta$ compatible with such a $\gamma$. Also, $\bar{\alpha}_{GG} > \alpha_0$ and $\bar{\alpha}_B > \alpha_0$ if $\theta$ is relatively low and always happens for $\theta$ lower than $\frac{1}{2}$. In addition, $\bar{\alpha}_B$ is always lower than $\bar{\alpha}_B$ and $\bar{\alpha}_{GG}$ is always higher than $\bar{\alpha}_{GG}$.

The final step of the proof, i.e. to show that the mixed strategies $(\gamma, 1 - \gamma)$ and $(\gamma, 1 - \gamma)$ do in fact exist, is relegated to the Appendix. The results can be generalised by Proposition 2:

**Proposition 2** The behaviour of the certification intermediary in period 0 is such that:

1. A talented certification intermediary always reports her signal. This means that she reports $m_G$ whenever $s_G$ is observed, and reports $m_B$ whenever $s_B$ is observed.

2. For the untalented certification intermediary, there are $\theta_L$ and $\theta_H$, with $\theta_L > \frac{1}{2}$, such that, for $\theta \in (\theta_L, \theta_H)$, she always reports her signal if $c_0$ is arbitrarily small. Otherwise, and provided that the same condition on $c_0$ is satisfied, the certification intermediary behaves as follows:

- For $\theta$ equal to $\theta_L$ she reports $m_B$ whenever $s_B$ is observed, and reports $m_B$ with probability $\gamma$ and $m_G$ with probability $1 - \gamma$ whenever $s_G$ is observed and for $\theta \in [0, \theta_L)$ she always sends a bad report;
- For $\theta$ equal to $\theta_H$, she reports $m_G$ whenever $s_G$ is observed, and reports $m_G$ with probability $\gamma$ and $m_B$ with probability $1 - \gamma$ whenever $s_B$ is observed and for $\theta \in (\theta_H, 1]$ she always sends a good report.
This proposition establishes that an untalented intermediary may ignore her private signal and decide instead to send a report that fits the expectations created by the public signal. This result is directly related to the issue of conformity. A number of papers such as Bernheim (1994) and Prendergast (1993) discuss this topic. By behaving in this particular way, an agent is basically trying to differentiate himself from the type that he wishes not to be identified with. The goal of the untalented intermediary is to mimic the talented type as by doing so she diminishes the chances of revealing her type. If for example $\theta$ is sufficiently low, there is a relatively high probability that the talented intermediary has received a bad signal and that she will send an unfavourable report. Given that the untalented intermediary cannot trust completely her private signal, there is a critical level of $\theta$, such that she chooses to ignore it with positive probability if it indicates that the firm is good.

On the other hand, it was proven in the Appendix that the threshold $\theta_L$ is higher than $\frac{1}{2}$. For medium values of the prior belief about the project quality, i.e. when the prior is less informative, one would expect the untalented intermediary to report truthfully but in fact, she chooses to report $m_B$ even when this contradicts her private information, i.e. there is an excessive number of bad reports. This happens because there is an asymmetry of observability in the model: a project issued with an unfavourable report is not undertaken and this limits the learning process about the certification intermediary’s type, which makes sending unfavourable reports a safer option.

### 3.2.2 Comparative Statics

A number of interesting results are derived when performing comparative statics in the equilibrium values of $\gamma$ and $\overline{\gamma}$.

**Proposition 3** The equilibrium probabilities $\overline{\gamma}$ and $\gamma$ are monotonic in $\theta$ and greatest for extremely high and low values of $\theta$, i.e. $\frac{\partial \overline{\gamma}}{\partial \theta} > 0$ for $\theta \in (\theta_H, 1)$, and $\frac{\partial \gamma}{\partial \theta} < 0$ for $\theta \in (0, \theta_L)$, with $\overline{\gamma} |_{\theta=\theta_H} = 0$ and $\gamma |_{\theta=\theta_L} = 0$.

This means that the more extreme the prior belief $\theta$ is, the higher the probability of deviation from the private signal. The lower the prior belief, the higher the probability of sending an unfavourable report when facing a good private signal. On the contrary, the higher the prior belief, the higher the probability of sending a favourable report when facing a bad private signal. If the public signal is uninformative, i.e. $\theta = \frac{1}{2}$, an untalented intermediary can either be truthful or conservative, i.e. always reports an unfavourable private signal.
but reports a favourable private signal with a positive probability only, depending on the reputational level.

**Proposition 4** The equilibrium probabilities $\gamma$ and $\underline{\gamma}$ are monotonic and increasing in $\varepsilon$, i.e. $\frac{\partial \gamma}{\partial \varepsilon} > 0$ and $\frac{\partial \underline{\gamma}}{\partial \varepsilon} > 0$.

This is an intuitive result: the less talented an intermediary is, the less she trusts her private signal and the higher the incentive to ignore it and convey with the public information.

**Proposition 5** The equilibrium probability $\gamma$ is concave and decreasing in $\alpha_0$ for sufficiently small levels of $c_0$.

As reputation increases, the intermediary’s difference in future profits from sending the two different messages becomes lower, i.e. $\frac{\partial \gamma}{\partial \alpha_0}$ is negative. For high levels of $\theta$, the higher the initial reputational level $\alpha_0$ the lower the increase in reputation if a favourable report is correct, and the higher the decrease in reputation if it turns out to be wrong. Hence, the more reputable an intermediary is the less incentives she has for gambling by deviating from an unfavourable private signal that has the advantage of generating a non-random level of future profits.

**Proposition 6** The equilibrium probability $\underline{\gamma}$ is always increasing and convex in $\alpha_0$.

The same reasoning as before applies here. Reporting a favourable private signal increases reputation and future profits if the report turns out to be right but if it turns out to be wrong the intermediary’s type is revealed and this generates a high cost in terms of profits next period. Thus because the benefit from deviating is not certain, an intermediary tends to deviate from her private information and send an unfavourable report instead as this turns out to be less risky. As reputation increases, the intermediary’s difference in future profits from sending the two different messages becomes lower, i.e. $\frac{\partial \underline{\gamma}}{\partial \alpha_0}$ is negative. This means that the higher the reputational level the lower the benefits from reporting the private signal and hence, the stronger the incentives to deviate.

This leads to the following corollary:

**Corollary 1** The behaviour of the untalented certification intermediary is such that as her reputational level increases, she tends to issue unfavourable reports more often than favourable ones.
The asymmetry of observability arising from the fact that a good report is always verifiable whereas a bad one is not, combined with the fact that a more reputable intermediary benefits less from gambling, results in a more reputable intermediary being more prone to sending unfavourable reports when the prior is relatively low or relatively high than a less reputable one.

This result is in line with some empirical evidence that suggests that smaller agencies, which are usually regarded as less reputable by investors and firms, tend to rate in a more favourable way. For instance, Cantor and Packer (1997) reveal that, in their sample, DCR and Fitch give systematically higher ratings on jointly rated issues than Moody’s and S&P. They test whether this fact reflects different rating scales or results from selection bias, i.e. only higher-rated firms seek DCR and Fitch ratings. They found limited evidence of selection bias and concluded that observed differences in average ratings seem to reflect differences in rating scales and standards. Also Jewell and Livingston (2000) find that DCR ratings are higher than S&P’s or Moody’s.

### 3.2.3 Fees and Investors

The behaviour described above constitutes the only credible behaviour from the intermediary point of view. And this is going to be useful in period 0 when investors price debt and the firm makes its hiring decision and decides on the fee. Thus, for \( \theta \in [\theta_L, \theta_H] \) the conditional probabilities, repayment to investors and fee remain the same as in period 1 because intermediaries are truthful. If \( \theta \in [0, \theta_L) \), \( \text{pr}(m_G \mid G, \{\Omega_0\}) = \alpha_0 + (1 - \alpha_0)(1 - \varepsilon)(1 - \gamma) \) and \( \text{pr}(m_G \mid B, \{\Omega_0\}) = (1 - \alpha_0)\varepsilon(1 - \gamma) \). And using (6) the fee charged in period 0 is

\[
F_0(\alpha_0) = \kappa \frac{\alpha_0\varepsilon + (\theta - \varepsilon) - (1 - \theta)\varepsilon}{2\theta}.
\]

There are two effects that need to be considered in this case. The firm knows that an intermediary is likely to conform to the public information and that a favourable report is less likely to occur. This has a negative effect on the fee. On the other hand, when the report is indeed favourable, the intermediary is choosing to contradict the public signal so investors believe the intermediary is more likely to be telling the truth but only if the probability that she is making a mistake is not very high, i.e. \( \varepsilon < \theta \). The required repayment to investors is

\(^9\text{Conservatism is also discussed by Zwiebel (1995). The idea is that reputational concerns may lead managers to refrain from undertaking innovations that are first order stochastically dominant because of the downside risk of being fired.} \)
lower and the intermediary can extract a higher fee than if there had not been deviation from
the private signal, i.e. $\gamma = 0$. The lower the difference between $\theta$ and $\varepsilon$, i.e. the higher the
prior belief $\theta$ or the lower the $\varepsilon$, the lower the repayment to investors and the higher the fee.
If on, the other hand, $\theta$ decreases and $\varepsilon$ increases, investors tend to attribute a favourable
message to an honest mistake. They require a higher compensation for this extra risk and
consequently the fee is going to be lower.

Finally, if $\theta \in (\theta_H, 1]$,

$$\Pr(m|G, \{\Omega_0\}) = \alpha_0 + (1 - \alpha_0) (1 - \varepsilon + \varepsilon \gamma)$$

and

$$\Pr(m|B, \{\Omega_0\}) = (1 - \alpha_0) (\varepsilon + (1 - \varepsilon) \gamma).$$

It follows that the fee in period 0 is

$$F_0(\alpha_0) = \kappa \frac{\alpha_0 \varepsilon + (\theta - \varepsilon) + (1 - \alpha_0) \gamma (\theta + \varepsilon - 1)}{2\theta}.$$ 

If $\theta$ exceeds $1 - \varepsilon$ the fee is higher than if the intermediary had followed the private signal,
i.e. $\gamma = 0$. The same logic applies here. As the intermediary is more likely to conform with
the (good) public information, the firm and investors can expect a favourable report more
frequently. But it turns out that $\theta_H$ is always higher than $1 - \varepsilon$\(^{10}\), hence deviations from the
private signal only happen when $\theta$ is very high or the probability of making a mistake $\varepsilon$ is very
low. Therefore, investors are more likely to believe that a favourable report does translate the
intermediary private information. They require a lower repayment and the surplus that the
firm is going to share with the intermediary is higher.

4 Bertrand Competition

So far the focus of this paper has been on the strategic information revelation of a monopolistic
certification intermediary. However, it is common to observe in many markets intermediaries
competing among each other. In the credit rating industry this trend is very likely to increase
in the future given the likely increase in the demand for ratings for regulatory purposes. To
reflect this situation consider again the framework developed in Section 2.2 and introduce a
second certification intermediary\(^{11}\). Thus at each date the existing certification intermediary

\(^{10}\)See Proof of Proposition 2 (part (i) Type $U_B$ randomises for high values of $\theta$) in the Appendix.

\(^{11}\)Most companies are rated by more than one rating agency. Therefore this can be reinterpreted as a firm
seeking for an additional rating that is going to be attributed by one of the remaining rating agencies. For
i faces potential competition of an incoming certification intermediary j. The incoming competitor has entry costs of zero and to simplify the analysis and limit the number of cases to consider they only differ in terms of the initial reputation, i.e. \( \alpha_{i0} \neq \alpha_{j0} \) and \( \alpha_{j0} \) is positive. At each date, intermediaries make simultaneous offers and the firm accepts or refuses the proposals simultaneously.

The repayment to investors is calculated as before but that is not the case for the fee paid by the firm. At each date, the firm’s expected surplus from being certified by intermediary \( \phi \) with \( \phi = i, j \) is

\[
Pr(m_G | G, \{\Omega_t\}, \phi) (1 - r_{\phi t} (m_G, G)) - F_{\phi t} (\alpha_{it}, \alpha_{jt}) .
\]

4.1 Period 1

4.1.1 Certification Intermediary Optimal Behaviour and Fee

In this period no one misreports because this implies an additional cost that is not matched by an additional benefit. Thus,

**Proposition 7** With competition, a certification intermediary continues reporting her private signal in the last period.

Intermediaries make offers to the firm regarding the fee to be charged for certification and this process simply results in the certification intermediary that generates the highest expected surplus setting the price by forcing the other intermediary to set a zero fee. Using the result derived in Proposition 7 and assuming that \( i \) generates the highest expected surplus for the firm, and is consequently hired, the fee charged in period 1 is

\[
Pr(m_G | G, \{\Omega_t\}, i) (1 - r_{it} (m_G, G)) - F_{it} (\alpha_{it}, \alpha_{jt}) = Pr(m_G | G, \{\Omega_t\}, j) (1 - r_{jt} (m_G, G))
\]

or

\[
F_{i1} (\alpha_{i1}, \alpha_{j1}) = \frac{\alpha_{i1} \varepsilon + (\theta - \varepsilon)}{2\theta} - \frac{\alpha_{j1} \varepsilon + (\theta - \varepsilon)}{2\theta}
\]

that simplifies to

\[
F_{i1} (\alpha_{i1}, \alpha_{j1}) = \frac{\varepsilon}{2\theta} (\alpha_{i1} - \alpha_{j1}) .
\]

\text{example, according to Jewell and Livingston (1999 and 2000), show that the DCR and Fitch rating serves as a tie-break in case of split ratings between Moody’s and S&P.}

\text{12You can think of a situation where there is a pool of analysts whose ability is fixed: it can either be 1 or 1 - \varepsilon but the entity that hires them might (or might not) be able to distinguish between the two types of analysts with probability \( \alpha_{i0} (1 - \alpha_{i0}) \).}
After setting a fee such that the fee of the competitor is driven to zero, the firm still has the option not to undertake the project. Therefore, the surplus is divided between the firm and the intermediary according to their bargaining power resulting in the intermediary charging a proportion $\kappa$ of $F_{i1}$. Of course, certification intermediary $i$ is hired because $\alpha_{i1} > \alpha_{j1}$ (initial assumption). In addition, the fee charged now is lower than in the monopolistic case, i.e. $F_{i1} (\alpha_{i1}, \alpha_{j1}) < F_1 (\alpha_{1})$ when $\alpha_{i1} = \alpha_{1}$, as the fee is now the difference between the expected surplus generated by certification by $i$ minus the surplus generated if $j$ had been hired by the firm in proportion $\kappa$.

Negative fees are ruled out in this model, however if this was not the case the results about the fee in period one would not change. If fees could in fact be negative, the intermediary that is not hired in period 0, which is the one with the lowest reputational level, would be willing to pay to have the chance to certify the firm in this period hoping to recover this amount in period one by being hired again. Of course this would only happen if in period one her new reputational level would be higher than her competitor’s, at least in some situations, and consequently, the firm would prefer to hire her instead i.e., if there were chances to recover the negative fee by charging a positive fee in the last period. The competitor, on the other hand, would never set negative fees in the last period as there are no future periods when to recover this "investment". Hence, in period one the intermediary with the highest reputational level would extract as much as possible from the firm. But then the intermediary with the highest reputational level would also be willing to pay to certify in period zero and given that she has more to lose, she would be willing to pay even more, and up to the expected profit in period one and she would end up being hired in period zero. At this point she decides whether to misreport by looking at the expected profit in period one. She would try to extract as much as possible from the firm in the cases where she is the intermediary with the highest reputational level given that her competitor would have no incentives to set negative fees. Hence, in period one intermediaries would always behave according to the differentiated Bertrand competition set-up described above.

Moreover, one could say that because there is competition $\kappa$ could perhaps be lower than in the monopolistic case. But once negative fees are ruled out, one intermediary sets the fee such that the competitor is out of the market even when she sets a zero-fee, i.e. even if all

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13 Remember that the fee increases with the reputational level.
14 And this will happen for sure at least for the case where she sends a good report in period zero that turns out to be correct.
the surplus goes to the firm. So the intermediary with the highest reputational level charges a positive fee that generates at least the same surplus for the firm than the competitor is able to generate with a zero-fee. But even at this point, the firm has the option of not undertaking the project. Therefore, intermediary and firm can bargain on the fee that may decrease even further. But the intermediary that was already out of the market cannot interfere in this bargaining process, since she had already set the fees at the minimum possible level before.

Therefore, $\kappa$ could in fact remain the same or not. In fact, since in competition the intermediary is bargaining over a lower surplus, it could even be the case that she bargains for a higher $\kappa$ to ensure a profit closer to monopoly but, for simplicity, it was assumed that $\kappa$ remains the same.

### 4.1.2 Posterior Beliefs

The posterior beliefs about the certification intermediaries are calculated as before and to distinguish between both intermediaries a subscript $i$ is added to the previous notation. The only difference relative to the previous case is that updating only occurs when an intermediary is hired. Otherwise, her reputational level remains equal to the prior. Also note that $\overline{\alpha}_{iB}$ and $\overline{\alpha}_{iB}$ are higher than $\alpha_{i0}$ for low levels of $\theta$ and definitely if $\theta < \frac{1}{2}$, i.e. sending an unfavourable report results in an increase in reputation only if investors expect the project to be bad, and $\overline{\alpha}_{iB}$ is always higher than $\overline{\alpha}_{iB}$. On the other hand, when they issue favourable reports that turn out to be incorrect the reputational level becomes 0 and increases relative to $\alpha_{i0}$ if they are correct.

### 4.2 Period 0

#### 4.2.1 Certification Intermediary Optimal Behaviour

Assume that $i$ was hired in period 0 and therefore, $\alpha_{i1} = \alpha_{i0}$. The difference in expected profits (in period 1, from sending a favourable and an unfavourable report) when she receives $s_G$ is

$$
\bar{\pi}^i (a) = \Pr (G \mid s_G, a) F_{i1} (\overline{\alpha}_{GG}^i, \alpha_{j0}) + \Pr (B \mid s_G, a) F_{i1} (\overline{\alpha}_{GB}^i, \alpha_{j0}) - F_{i1} (\alpha_B^i, \alpha_{j0}) + c_0.
$$

and the difference in expected profits when she receives $s_B$ is

$$
\bar{\pi}^i (a) = \Pr (G \mid s_B, a) F_{i1} (\overline{\alpha}_{GG}^i, \alpha_{j0}) + \Pr (B \mid s_B, a) F_{i1} (\overline{\alpha}_{GB}^i, \alpha_{j0}) - F_{i1} (\overline{\alpha}_B^i, \alpha_{j0}) - c_0.
$$

(15)
Agent $i$ is going to issue a favourable report (or play a mixed strategy) if $\pi^{ei}(a)$ and $\bar{\pi}^{ei}(a)$ are positive (or equal to zero). Otherwise she issues an unfavourable report. Following the same logic as before the following result can be stated:

**Proposition 8** With competition, the talented certification intermediary never misreports in period 0.

One of the main differences in relation to the previous Section is that now whenever an untalented intermediary is confronted with a realised project type that differs from the report issued, her reputation is driven down to zero and she is not going to be hired in the following period. This happens because if confronted with two intermediaries, one with a reputational level of $\alpha_{j0}$ and the other one with 0, the firm chooses the one with higher posterior about the ability. Therefore:

**Proposition 9** If investors and firms are sure a certification intermediary is untalented, she is never hired independently of the reputational level of her competitor.

On the other hand, if an intermediary issues a correct good report her reputation increases, which means that given that she was hired in period 0, she is necessarily going to be hired in period 1. However, issuing a bad report is no longer a riskless strategy. For high values of $\theta$, issuing a bad report worsens the intermediary’s reputation as the remaining agents believe she is making a mistake. This might or might not affect the intermediary’s possibilities of being hired in the following period depending on how close the initial reputations are.

Summing up, compared with the monopolistic case, the reputational cost of a mistake is now much higher and intermediaries face a considerably lower probability of being hired in the last period when a mistake is discovered. This first effect encourages truth-telling. But there is a second effect: the probability of being hired is determined by the first period announcement and a truthful report that cannot be verified might also affect the intermediary’s chances of being hired, which may in turn lead to less truth-telling. Hence, the crucial point is to study what happens when a bad report implies a decrease in reputation that might compromise future commitments. If not, the following proposition is derived:

**Proposition 10** Competition changes the set of prior beliefs about the quality of the project for which an untalented certification intermediary deviates from the private signal in period 0, when $\alpha_{j0} < \alpha^*_B < \bar{\alpha}^*_B$: she becomes more aggressive and issues favourable reports more frequently than in the monopolistic case.
Proof. In the Appendix. ■

In this case, the intermediary is always hired next period except when a good report is found to be incorrect. However, there is also a monetary effect to consider with competition: the fee is now lower by an amount equivalent to proportion $k$ of the surplus of the competitor relative to the case without competition. But if the intermediary reports an incorrect report she is not going to hired next period and in this case the future payoff is simply zero. Without competition, the intermediary’s type would be revealed but, because the firm has no outside option, she would still be hired but receiving a fee in accordance to her type. So it turns out that the difference between the payoffs in this particular scenario is lower than proportion $k$ of the surplus of the competitor and this makes the decrease in the expected fee relative to the case with no competition lower when a good report rather than a bad report is issued. Hence $\pi^{e1}(U) > \pi^{e}(U)$ and $\pi^{e2}(U) > \pi^{e}(U)$ and reporting favourable messages becomes more frequent than before.

To sum up, the dominant effect in this case is the first one: a lower probability of being hired encourages truth-telling. Intermediaries still conform with public information and ignore private signals when the prior about the quality of the project is extreme, but conservatism is attenuated.

For a low degree of differentiation, i.e. $\alpha_{j0} > \alpha_{iB}$ and/or $\alpha_{j0} > \alpha_{iB}$, competition becomes very aggressive and only when positive reports turn out to be correct the intermediary is hired in the following period. In fact, the intermediary no longer behaves conservatively and does not take into account the effect of the initial prior about the quality of the project when issuing her report. Thus,

**Proposition 11** When $\alpha_{j0} > \alpha_{iB} > \alpha_{iB}$, untalented certification intermediaries always issue favourable reports in period 0.

The second effect is now the dominant one: in order to maximise the probability of being hired in the future the intermediary compromises truth-telling and in the limit only issues favourable reports regardless of her private information.

In the monopolistic case the asymmetry in payoffs observability make more reputable intermediary more prone to sending unfavourable reports for relatively lower or relatively higher priors. A duopolistic structure in the certification industry mitigates this result by introducing more symmetry between sending favourable and unfavourable reports. Sending unfavourable reports is now less beneficial and this affects any untalented intermediary regardless of her
reputation.

5 Conclusion

This paper studies the behaviour of certification intermediaries, in particular it looks at their incentives to report a message that differs from their private signal in a framework where they value reputation. The model identifies a source of incentive conflicts for certification intermediaries. It finds that reputational concerns are not enough to prevent deviations from the private signal, in fact these concerns might end up being the driving force being them. Intermediaries that are sure of their signals always report truthfully but those that cannot trust their private signals may end up ignoring them and sending the report that investors and firms anticipated based on public information, in particular when the public signal is extreme, in an attempt to avoid reputational costs. Despite its simplicity, the model can motivate several patterns of behaviour, in particular, this results provide a theoretical explanation for empirical findings that suggest that ratings agencies exhibit excess sensitivity to the business cycle and in some cases adjust their ratings after market participants have already adjusted their perceptions of creditworthiness.

In the monopolistic setting, the intermediaries with a higher reputation tend to be conservative when issuing their reports but competition forces them to be more aggressive in order to be hired in the following period.

This is relevant for policy-makers. Under proposed revisions to bank capital requirements advanced by the Basel Committee on Banking Supervision, banks using a standardised approach to calculating their minimum required capital will base such requirements, whenever possible, on the credit ratings assigned to the companies to which they lend. To the extent that rating agencies might behave pro-cyclically, bank capital requirements will tend to be higher during downturns, further reducing credit supply during downturns. In addition, the Basel proposal will increase the demand for ratings as will definitely have an impact on the market structure of the industry. The increasing importance of ratings agencies in financial market as a result of regulatory measures demands that these issues should be identified and tackled appropriately.
References


Similar steps to the ones used in Boot, Milbourn and Thakor (2002) are used to solve for the equilibrium.

7.1 The Equilibrium Behaviour of the Untalented Intermediary

Define $\tau \in \{T_G, T_B, U_G, U_B\}$ as the set of possible types, where T and U indicate talented or untalented, and G and B designate the signal received, e.g. $T_G$ is a talented certification intermediary that received a good signal. The set of possible actions is binary: send a favourable report ($m_G$) or send an unfavourable report ($m_B$). Types $U_G$ and/or $U_B$ may randomise across these two actions depending on the value of the prior $\theta$, but $T_G$ and $T_B$ prefer to follow a pure strategy where they always report their private signals. This is proved by identifying the mixed strategy for high and low values of $\theta$ and by proving that the $\theta$ ranges do not overlap.

(i) Type $U_B$ randomises for high values of $\theta$: Let $U_B$ send $m_G$ with probability $\gamma$ and $m_B$ with probability $1 - \gamma$ and assume that the remaining types follow their conjectured equilibrium strategies. The following equation should therefore hold as $U_B$ should be indifferent between sending $m_B$ and $m_G$

$$\Pr (G \mid s_B, U) F_1(\pi_{GG}) + \Pr (B \mid s_B, U) F_1(\pi_{GB}) = F_1(\pi_B) + c_0. \quad (17)$$

The expression becomes clearer by replacing (4), (7) and the different values for $\alpha_1$, that are $\pi_{GG}$, $\pi_{GB}$ and $\pi_B$ and whose expressions are derived above, in (17). It is easily shown that the LHS of (17) is monotonically decreasing in $\gamma$, while the RHS is monotonically increasing in $\gamma$. Moreover, it can be showed that the equality in (17) can only hold for an interior $\gamma \in (0, 1)$ provided that $\theta$ is sufficiently high.
Firstly, observe that at \( \gamma = 0 \) the LHS exceeds the RHS provided that \( \theta \) is sufficiently high. After straightforward manipulation the expression becomes

\[
\kappa \frac{\varepsilon \theta}{\varepsilon \theta + (1 - \varepsilon) (1 - \theta)} \frac{\alpha_0 \varepsilon}{2 \theta \left( \alpha_0 + (1 - \varepsilon) (1 - \alpha_0) \right)} - c_0
\]

\[
= \kappa \frac{2 \theta \left( \alpha_0 (1 - \theta) \varepsilon \right)}{2 \theta \left( \alpha_0 (1 - \theta) + (1 - \alpha_0) \left( \varepsilon \theta + (1 - \varepsilon) (1 - \theta) \right) \right)}.
\]

In order for the LHS to exceed the RHS it is necessary that

\[
(1 - \theta) \left( \varepsilon \theta + (1 - \varepsilon) (1 - \theta) \right) \left( \alpha_0 + (1 - \varepsilon) (1 - \alpha_0) \right)
\]

\[
< \varepsilon \theta \left( \alpha_0 (1 - \theta) + (1 - \alpha_0) \left( \varepsilon \theta + (1 - \varepsilon) (1 - \theta) \right) \right),
\]

and the expression can be simplified as follows

\[
(1 - 2 \theta) (1 - \varepsilon) \left( \alpha_0 + (1 - \varepsilon) (1 - \alpha_0) \right) < \theta^2 \left( (1 - \alpha_0) (2 \varepsilon - 1) - (1 - \varepsilon) \alpha_0 \right). \tag{18}
\]

Secondly, it can be proven that \( \theta \) needs to be higher than \( \frac{1}{2} \) and more precisely higher than \( 1 - \varepsilon \). Noting that as \( \varepsilon < \frac{1}{2} \) the RHS of (18) is always negative, \( 1 - 2 \theta \) needs to be negative to transform the LHS in a negative number and \( \theta \) needs to be high enough for the inequality to occur. But if \( \theta \) equals \( 1 - \varepsilon \) the expression becomes

\[
1 - 2 \varepsilon > (1 - \varepsilon)^2
\]

and this is impossible since \( \varepsilon < \frac{1}{2} \).

Consequently, equality (17) requires that \( \gamma > 0 \) provided that

\[
c_0 < \kappa \frac{\varepsilon \theta \alpha_{GG} \left( \gamma = 0 \right)}{2 \theta \left( \varepsilon \theta + (1 - \varepsilon) (1 - \theta) \right)} - \frac{\kappa \alpha_{BB} \left( \gamma = 0 \right)}{2 \theta} \equiv \tau_{\text{max}}
\]

Now, evaluate (17) at \( \gamma = 1 \). It immediately follows that, independently of \( c_0 \), the LHS of (17) is always smaller than the RHS as the expression simplifies to:

\[
\kappa \left( \frac{\alpha_0 \varepsilon}{2 \theta} \right) \left( \frac{\varepsilon \theta}{\varepsilon \theta + (1 - \varepsilon) (1 - \theta)} \right) - c_0 = \kappa \frac{\varepsilon}{2 \theta}
\]

Thus, there exists \( \gamma \), with \( 0 < \gamma < 1 \). Finally, the posterior beliefs about the certification intermediary need to satisfy a technical condition that ensures \( \pi_B < \pi_{GG} \) and allows for the existence of a mixed strategy no matter how arbitrarily small \( c_0 \) is (otherwise the RHS of (17) would always be higher for \( 0 < \gamma < 1 \)). This condition states that

\[
1 - \varepsilon + \varepsilon \gamma \leq (\varepsilon \theta + (1 - \varepsilon) (1 - \theta)) (1 - \gamma). \tag{19}
\]

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(ii) Types $T_G$, $T_B$ and $U_G$ recommend according to their respective signals for high values of $\theta$: It can be easily shown that $T_B$ strictly prefers to follow her signal (i.e. send an unfavourable report) just by looking at the indifference condition (17) for $U_B$. Since $\text{pr}(G \mid s_B, T) < \text{pr}(G \mid s_B, U)$ and $F_1(\alpha_{GG}) > F_1(\alpha_{GB})$ (because the fee $F_1(\alpha_1)$ is increasing in $\alpha_1$), $T_B$ has strictly less to gain from sending a favourable report than $U_B$. The remaining types, $T_G$ and $U_G$, always send a favourable report as $\text{pr}(G \mid s_G, T) > \text{pr}(G \mid s_G, U) > \text{pr}(G \mid s_B, U) > \text{pr}(G \mid s_B, T)$ by looking at

$$\text{pr}(G \mid s_G, a) F_1(\alpha_{GG}) + \text{pr}(B \mid s_G, a) F_1(\alpha_{GB}) - F_1(\alpha_B) - c_0$$

and realising that it always exceeds

$$\text{pr}(G \mid s_B, a) F_1(\alpha_{GG}) + \text{pr}(B \mid s_B, a) F_1(\alpha_{GB}) - F_1(\alpha_B) - c_0.$$

(iii) Type $U_B$ randomises for low values of $\theta$: For low values of $\theta$, there are two cases: $\theta > \varepsilon$ (Case 1) and $\theta < \varepsilon$ (Case 2).

**Case 1:** This proof mirrors the previous arguments. $U_G$ now sends an unfavourable report with probability $\gamma$ when $\theta$ in the interval $(0, 1)$ is sufficiently low, i.e.

$$\text{pr}(G \mid s_G, U) F_1(\alpha_{GG}) + \text{pr}(B \mid s_G, U) F_1(\alpha_{GB}) = F_1(\alpha_B) - c_0. \quad (20)$$

As before (4), (7) and $\alpha_{GG}$, $\alpha_{GB}$ and $\alpha_B$ (whose expressions are derived above) are used to rewrite expression (20). Following arguments similar to the previous case, it is shown that $0 < \gamma < 1$ provided that $\theta$ is sufficiently low. It can be easily demonstrated that the LHS of (20) is monotonically increasing in $\gamma$, while the RHS is monotonically decreasing in $\gamma$. It needs to be shown that the equality in (20) can only hold for a interior $\gamma \in (0, 1)$ provided that $\theta$ is sufficiently low.

Firstly, observe that at $\gamma = 0$ the RHS exceeds the LHS. After straightforward manipulation the expression becomes

$$\kappa \frac{\alpha_0 \varepsilon}{2 \theta (\alpha_0 + (1 - \alpha_0)(1 - \varepsilon))} \left( (1 - \varepsilon) \theta \right) \frac{(1 - \varepsilon) \theta}{(1 - \varepsilon) \theta + \varepsilon (1 - \theta)} + \frac{\alpha_0 (1 - \theta) \varepsilon}{\kappa \frac{2 \theta (\alpha_0 (1 - \theta) + (1 - \alpha_0)(1 - \varepsilon)(1 - \theta)))} - c_0.$$

In order for the RHS to exceed the LHS it is necessary that

$$(1 - \varepsilon) \theta (\alpha_0 (1 - \theta) + (1 - \alpha_0)(\varepsilon \theta + (1 - \varepsilon)(1 - \theta))) < (1 - \theta) ((1 - \varepsilon) \theta + \varepsilon (1 - \theta)) (\alpha_0 + (1 - \varepsilon)(1 - \alpha_0))$$.
This expression can be simplified as

\[-\theta^2 \alpha_0 < (1 - 2\theta) (\alpha_0 + (1 - \varepsilon) (1 - \alpha_0)),\]

and as \(\alpha_0 + (1 - \varepsilon) (1 - \alpha_0) > \alpha_0\) and \(-\theta^2 < 1 - 2\theta\), the inequality is always satisfied for relatively low values of \(\theta\) and always if \(\theta\) is lower than \(\frac{1}{2}\). Moreover, if \(\theta < \varepsilon\), the equality is always satisfied for any values of the remaining parameters. Consequently, equality (20), requires that \(\gamma > 0\) provided that

\[c_0 < \kappa \frac{(1 - \varepsilon) \theta \alpha_{GG} (\gamma = 0)}{2\theta ((1 - \varepsilon) \theta + \varepsilon (1 - \theta))} - \kappa \frac{\alpha_B (\gamma = 0)}{2\theta} = c_{\text{max}}\]

Now, evaluate (20) at \(\gamma = 1\). It follows that:

\[\kappa \frac{\varepsilon}{2\theta (1 - \varepsilon) \theta + \varepsilon (1 - \theta)} = \kappa \frac{\alpha_0 \varepsilon (1 - \theta)}{2\theta (1 - \theta \alpha_0)} - c_0\]

The LHS exceeds the RHS, regardless of \(c_0\), if:

\[\frac{(1 - \varepsilon) \theta}{(1 - \varepsilon) \theta + \varepsilon (1 - \theta)} > \frac{\alpha_0 (1 - \theta)}{(1 - \theta \alpha_0)}\]

Given that the LHS is increasing in \(\theta\), the RHS is decreasing in \(\theta\) and when \(\theta \to 1\) the LHS exceeds the RHS and otherwise when \(\theta \to 0\), for a given \(\varepsilon\) and \(\alpha_0\) there exists a \(\theta\) lower than 1 such that the for \(\theta > \theta\) the LHS exceeds the RHS and otherwise for \(\theta < \theta\). For example if \(\theta = \frac{1}{2}\), the relationship holds for \(\alpha_0 < \frac{2(1-\varepsilon)}{(2-\varepsilon)}\).

Thus, if \(\theta\) is low enough there exists \(\gamma\) such that \(0 < \gamma < 1\), provided that \(c_0 < c_{\text{max}}\). Finally, the posterior beliefs about the certification intermediary needs to satisfy a technical condition that ensures \(\alpha_B < \alpha_{GG}\) and allows for the existence of a mixed strategy no matter how arbitrarily small \(c_0\). This condition states that

\[(1 - \varepsilon) (1 - \theta) (1 - \gamma) \leq ((1 - \varepsilon) \theta + \varepsilon (1 - \theta)) \gamma + (\varepsilon \theta + (1 - \varepsilon) (1 - \theta))\]

and is always satisfied for any values of the parameters.

**Case 2:** When \(\theta < \varepsilon\) the untalented certification intermediary is not hired in period 1 if her type is revealed at the end of period 0 or her fee simply equals zero (this does not happen for high values of \(\theta\) because it was shown before that "high values of \(\theta\)" means higher than \(\frac{1}{2}\)). \(U_G\) now recommends rejection with probability \(\gamma\), and this is in the interior of \((0, 1)\) if \(\theta\) is sufficiently low. Firstly, observe that at \(\gamma = 0\) the RHS exceeds the LHS provided that \(\theta\) is sufficiently low. After straightforward manipulation the expression becomes

\[\kappa \frac{\theta - \varepsilon}{2\theta} + \kappa \frac{\alpha_0 \varepsilon}{2\theta (\alpha_0 + (1 - \alpha_0) (1 - \varepsilon))} \left(\frac{(1 - \varepsilon) \theta}{(1 - \varepsilon) \theta + \varepsilon (1 - \theta)}\right)
\]

\[= \kappa \frac{\theta - \varepsilon}{2\theta} + \kappa \frac{\alpha_0 \varepsilon (1 - \theta)}{2\theta (\alpha_0 (1 - \theta) + (1 - \alpha_0) ((\varepsilon \theta + (1 - \varepsilon) (1 - \theta))))} - c_0\]
In order for the LHS to exceed the RHS it is necessary that

$$(\theta - \varepsilon)(1 - \theta)(\alpha_0 + (1 - \alpha_0)(1 - \varepsilon))(\alpha_0(1 - \theta) + (1 - \alpha_0)((\varepsilon \theta + (1 - \varepsilon)(1 - \theta)))$$

$$< (1 - 2\theta)(\alpha_0(1 - \varepsilon)(1 - \alpha_0)) + \theta^2\alpha_0,$$

The relationship holds because $\theta < \varepsilon$ and therefore $\theta < \frac{1}{2}$. Now, evaluate (20) at $\gamma = 1$. It follows that:

$$\frac{\kappa \varepsilon}{2\theta(1 - \varepsilon)\theta + \varepsilon(1 - \theta)} - \kappa \frac{(\theta - \varepsilon)\varepsilon(1 - \theta)}{(1 - \varepsilon)\theta + \varepsilon(1 - \theta)} = \kappa \frac{\alpha_0 \varepsilon(1 - \theta)}{2\theta(1 - \theta)} - c_0$$

As the LHS is now higher than before, the conditions derived in the previous case also apply and $\theta$ can be even lower. Thus, if $\theta$ is low there exists $\gamma$, with $0 < \gamma < 1$, provided that $c_0 < c_{\text{max}}$.

(iv) Types $T_G$, $T_B$ and $U_B$ follow their respective signals for low values of $\theta$: Given the equality for $U_G$ in (20), $T_G$ strictly prefers to send a favourable report as $\text{pr}(G \mid s_G, T) > \text{pr}(G \mid s_G, U)$ and $F_1(\omega_{GG}) > F_1(\omega_{GB})$. Similarly, $T_B$ and $U_B$ always send an unfavourable report because $\text{pr}(G \mid s_G, U) > \text{pr}(G \mid s_B, U) > \text{pr}(G \mid s_B, T)$.

### 7.2 Establishing the Distinct $\theta$ Ranges (and Proof of Proposition 3)

Defining $\theta = \theta_H$ as the value of $\theta$ for which (17) holds for $\gamma = 0$ and $\theta = \theta_L$ as the value of $\theta$ for which (20) holds for $\gamma = 0$, it can be demonstrated that $\frac{\gamma}{\theta_H} < 0$ and $\frac{\gamma}{\theta_L} > 0$. Taking the expressions $\pi^e$ and $\pi^c$ it can be shown that $-\frac{\partial \pi^e}{\partial \theta} > 0$. Simple algebra shows that $\frac{\partial \pi^e}{\partial \theta}$ is positive and that $\frac{\partial \pi^c}{\partial \theta}$ is negative. Computing $-\frac{\partial \pi^c}{\partial \theta}$ follows the same logic and is derived to be negative; both $\frac{\partial \pi^e}{\partial \theta}$ and $\frac{\partial \pi^c}{\partial \theta}$ are positive.

From (17) and (20) as $\theta \rightarrow 1$, $\gamma = 1$, and on the other hand, as $\theta \rightarrow 0$, $\gamma = 1$, for $c_0$ sufficiently low. Thus, when $\theta \in (\theta_H, 1)$, there is excessive favourable reports ($\gamma > 0$) and when $\theta \in (0, \theta_L)$, there is excessive unfavourable reports ($\gamma > 0$). It remains to be shown that $\theta_L < \theta_H$. Only then can be stated that there is a region $[\theta_L, \theta_H]$ where there is no deviation from the private signal by the untalented certification intermediary. The equality (20) evaluated at $\theta = \theta_L$ (or $\gamma = 0$) is identical to (17) when this last equality is evaluated at $\theta = \theta_H$ (or $\gamma = 0$), except for the probabilities $\text{pr}(G \mid s_G, U)$ and $\text{pr}(G \mid s_B, U)$. Since for a given $\theta$ $\text{pr}(G \mid s_G, U) > \text{pr}(G \mid s_B, U)$ and these probabilities are increasing in $\theta$, the equalities (20) and (17)) require $\theta_L$ to be lower than $\theta_H$ for $c_0$ sufficiently small.
8 Proof of Proposition 4

The proof is done by implicit differentiation. Starting with \( \frac{\partial \gamma}{\partial e} \), straightforward differentiation, using (17) and the fact that \( \theta_H \) is always higher than \( \frac{1}{2} \), it can be shown that \( \frac{\partial \gamma}{\partial e} \) is always positive. As \( \frac{\partial \gamma}{\partial e} \) is always negative \( \frac{\partial \gamma}{\partial e} = -\frac{\partial \gamma}{\partial e} \) is positive. Turning to \( \frac{\partial \gamma}{\partial e} \), it can be proven by simple algebra that \( \frac{\partial \gamma}{\partial e} \) is negative if \( \theta \leq \frac{1}{2} \). Otherwise, we need to use (21) in the proof; \( \frac{\partial \gamma}{\partial e} \) is positive thus \( \frac{\partial \gamma}{\partial e} = \frac{\partial \gamma}{\partial e} \) is positive.

9 Proof of Propositions 5 and 6

The way the equilibrium values for \( \eta \) and \( \gamma \) vary with \( \alpha_0 \) is determined as follows. Starting with \( \frac{\partial \eta}{\partial \alpha_0} \) is found to be negative if \( \theta_0 \) is sufficiently small and making use of condition (19) in the derivation. The second derivative \( \frac{\partial^2 \eta}{\partial \alpha_0^2} \) is always negative. On the other hand, \( \frac{\partial \eta}{\partial \alpha_0} \) is always decreasing and it can also be proven to be concave in \( \alpha_0 \). Looking at \( \frac{\partial \gamma}{\partial \alpha_0} \) is found to be negative. This can be proven by straightforward derivation, summing and subtracting \( \left( \frac{(1-\varepsilon)\theta}{((1-\varepsilon)\theta+(1-\theta)\varepsilon)} \right) \left( \frac{\alpha_{0H}}{2g} \right) \left( \frac{(1-\theta-(1-\varepsilon)\theta+\varepsilon(1-\theta))\gamma_{B}+(\theta+(1-\varepsilon)(1-\theta))\gamma_{H}}{(\alpha_0(1-\theta)+(1-\alpha_0)((1-\varepsilon)\theta+\varepsilon(1-\theta))\gamma_{B}+(\theta+(1-\varepsilon)(1-\theta))\gamma_{H}} \right) \) and making use of equilibrium condition (20) and of technical condition (21). The second derivative \( \frac{\partial^2 \gamma}{\partial \alpha_0^2} \) is negative and the higher the \( \theta_0 \), the steeper is the slope. As \( \frac{\partial \gamma}{\partial \alpha_0} \) is always positive, \( \frac{\partial \gamma}{\partial \alpha_0} \) is respectively increasing and convex in \( \alpha_0 \).

10 Proof of Propositions 10 and 11

In order to prove how the set of prior beliefs for which there is deviation from the private signal changes there needs to be a comparison between the expected profits functions with and without competition. With competition intermediaries decide whether to announce their private signals by looking at

\[
\pi^e_i (U) = \Pr (G | s_G, U) F_{i1} (\alpha_{iGG}, \alpha_{j0}) + \Pr (B | s_G, U) F_{i1} (\alpha_{iGB}, \alpha_{j0}) - F_{i1} (\alpha_{iB}, \alpha_{j0}) + c_0
\]

and

\[
\pi^{ei} (U) = \Pr (G | s_B, U) F_{i1} (\pi_{iGG}, \alpha_{j0}) + \Pr (B | s_B, U) F_{i1} (\pi_{iGB}, \alpha_{j0}) - F_{i1} (\pi_{iB}, \alpha_{j0}) - c_0.
\]

The fee in period 1 for the case without competition is \( F_{i1} (\alpha_1) = \kappa \left( \frac{\alpha_1 \varepsilon}{2g} + \frac{(\theta - \varepsilon)}{2g} \right) \).

(i) Type \( U_B \) randomises for \( \theta > \theta_H \) in the monopolistic case.
The different fees with competition are $F_{i1}(\overline{x}_{GB}, \alpha_{j0}) = 0,$

$$F_{i1}(\overline{x}_{GG}, \alpha_{j0}) = F_1(\overline{x}_{GG}) - k\left(\frac{\alpha_{j0}\varepsilon}{2\theta} + \frac{(\theta - \varepsilon)}{2\theta}\right)$$

and

$$F_{i1}(\overline{x}_{B}, \alpha_{j0}) = F_1(\overline{x}_{B}) - k\left(\frac{\alpha_{j0}\varepsilon}{2\theta} + \frac{(\theta - \varepsilon)}{2\theta}\right)$$

in case $\overline{x}_{B} > \alpha_{j0}$ and zero otherwise. The remaining probabilities remain the same with and without competition. Consequently, when $\overline{x}_{B} > \alpha_{j0}$

$$\pi^U_i(U) - \pi^e(U) = k\left(\frac{\alpha_{j0}\varepsilon}{2\theta}\right) (\Pr (B | s_B, U)) > 0.$$  

Therefore, when for a given $\theta$ there is an equilibrium in mixed strategies for the monopolistic case, a favourable report is issued with competition. Because $\pi^U_i(U)$ is increasing in $\theta$, an equilibrium in mixed strategies occurs for a lower $\theta$.

If $\overline{x}_{B} < \alpha_{j0}$, $\pi^e_i(U) = \Pr (G | s_iB, U) F_{i1}(\overline{x}_{GG}, \alpha_{j0}) - c_0$. For an arbitrarily small $c_0$, $\pi^e(U)$ is also positive.

(ii) **Type $U_B$ randomises for $\theta < \theta^L$ in the monopolistic case**

In this case, the different fees with competition become $F_{i1}(\overline{x}_{GB}, \alpha_{j0}) = 0,$

$$F_{i1}(\overline{x}_{GG}, \alpha_{j0}) = F_1(\overline{x}_{GG}) - k\left(\frac{\alpha_{j0}\varepsilon}{2\theta} + \frac{(\theta - \varepsilon)}{2\theta}\right)$$

and

$$F_{i1}(\overline{x}_{B}, \alpha_{j0}) = F_1(\overline{x}_{B}) - k\left(\frac{\alpha_{j0}\varepsilon}{2\theta} + \frac{(\theta - \varepsilon)}{2\theta}\right)$$

in case $\overline{x}_{B} > \alpha_{j0}$ and zero otherwise. The remaining probabilities remain the same with and without competition. A similar argument is applied here. When $\overline{x}_{B} > \alpha_{j0}$

$$\pi^e_i(U) = \pi^e(U) = k\left(\frac{\alpha_{j0}\varepsilon}{2\theta}\right) (\Pr (B | s_B, U)) > 0.$$  

Hence, given $\theta$ if $\pi^e(U) = 0$ then $\pi^e_i(U) > 0$. Because $\pi^e_i(U)$ is increasing in $\theta$, a mixed strategy with competition occurs for a lower $\theta$.

If $\overline{x}_{B} < \alpha_{j0}$, $F_{i1}(\overline{x}_{B}, \alpha_{j1}) = 0$ which means that

$$\pi^e_i(U) = \Pr (G | s_G, U) F_{i1}(\overline{x}_{GG}, \alpha_{j0}) + c_0 > 0$$

meaning that a good report is always sent.