Child skill production: Accounting for parental and market-based time and goods investments*

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Abstract

Families invest parental time, home goods/services, and market-based child care in their children. We study these investments, focusing on two issues: the role of parental human capital and the substitutability of inputs in the skill production process. We develop a relative demand estimation strategy that uses intratemporal optimality to estimate the substitutability and relative productivity of different inputs. This approach requires a weak separability assumption on the dynamics of skills, but it does not require data on skills and easily addresses measurement error in inputs. We show how relative demand restrictions can simplify and improve estimation of the dynamics of skill production when incorporating panel data on skill measures. Combining data on relative demand and skill dynamics further allows researchers to test whether beliefs about skill production align with the true technology. Using data from the Child Development Supplement of the PSID, we estimate the skill production technology for American children ages 5–12, finding moderately strong complementarity between inputs. We estimate little effect of parental education on the child production technology: more-educated parents invest more because they have higher incomes and stronger preferences for children's skills. Counterfactual simulations show that the degree of input complementarity we estimate has important implications for policies that subsidize specific inputs or provide free child care.

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1 Introduction

Family investments in children are critical to intergenerational mobility and inequality (Becker and Tomes, 1979, 1986; Cunha et al., 2006; Cunha and Heckman, 2007; Caucutt and Lochner, 2020). Parents spend their own time and money at home investing in their children's human capital. Many also make substantial investments through market-based child care services. This paper studies these investments, focusing on two main issues both theoretically and empirically. First, we study the role of parental human capital in child development through parental wages, productivity differences, and preferences for children's skills. Second, we examine the impacts of tax and subsidy policies on the composition of investment, total effective investment levels, and child skill accumulation. The substitutability of different inputs is critical for both of these issues and plays an important role in our analysis.

Parental investments in children are strongly increasing in family income (see, e.g., Kaushal, Magnuson, and Waldfogel, 2011; Caucutt, Lochner, and Park, 2017). Leibowitz (1974) and Guryan, Hurst, and Kearney (2008) show that high-educated mothers even devote more time to investment-related activities with their children (compared to less-educated mothers), despite their higher wage rates and opportunity costs of time. As discussed by Michael (1973) and Leibowitz (1974), this may reflect income effects associated with greater labor market earnings or home production. More-educated parents may also have more-able children, or they may be more effective at child skill production – in a Hick's neutral sense or in terms of their time devoted to child development.[1] Finally, more-educated parents may have a stronger preference for children's skills or may perceive greater returns to investments or skills (see, e.g., Cunha, Elo, and Culhane, 2022).

A wide range of government policies directly or indirectly influence investment decisions. Welfare policies with work requirements or that claw back gains from working, as well as the structure of income taxation, distort parental time investment margins. Subsidies for sports and arts programs favor family-based investments, while child care incentives support market-based investments. The implications of these policies for child development depend on how families respond by adjusting their investment inputs within periods and over time. These adjustments depend critically on how family-based investments (time and goods/services) interact, how family-based investments interact with market-based child care, and how parental human capital affects the productivity of various investments.

There is an important distinction between the types of inputs parents purchase to foster child devel-

[1]Several recent studies find that investments are more productive for young children with more-educated mothers (e.g., Del Bono et al., 2016; Attanasio, Meghir, and Nix, 2020; Brilli, 2022; Bolt et al., 2023, 2024; Del Boca et al., 2023), while others conclude that parental education only affects child development through higher levels of investment (Cunha and Heckman, 2008; Attanasio et al., 2020; Falk et al., 2021).

opment at home, which often involve direct engagement between parents and children or explicit parental choices about activities (e.g., books, computers, sports or cultural activities), and external child care services where children are not directly engaged with their parents and daily activities are largely directed by others. These expenditures are treated differently by policy and may have different substitution patterns with parental time investments. We distinguish between these two broad types of investment, along with parental time investment, by considering a technology that allows for different patterns of substitution between parental time and purchased goods/services at home and between these home investments and child care purchased from the market. We also allow parental skills to impact the relative productivity of these inputs, as well as the overall productivity of investments in children.

The literature estimating human capital production functions for children has largely focused on dynamic interactions of investments over time (Cunha and Heckman, 2007; Cunha, Heckman, and Schennach, 2010; Del Bono et al., 2016; Pavan, 2016; Attanasio et al., 2017; Caucutt and Lochner, 2020; Agostinelli and Wiswall, 2023), typically reducing investment each period to a single composite input, or imposing assumptions about the substitutability between time and goods investments (Del Boca, Flinn, and Wiswall, 2014, 2016; Brilli, 2022; Lee and Seshadri, 2019; Mullins, 2022; Attanasio et al., 2020) or between home and child care environments (Griffen, 2019; Chaparro, Sojourner, and Wiswall, 2020). Exceptions include recent studies that estimate complementarity between parental time and goods (Abbott, 2022), child care (Moschini, 2023), or a composite of goods and child care (Molnar, 2023).[2]

We use a dynamic model of family investments in children to study differences in investment behavior by parental education and to explore the impacts of policies designed to encourage investments. Our framework allows for several different types of child investment inputs each period and multiple periods of investment. In modeling the technology of skill formation, we allow for flexible substitutability across inputs and incorporate potential effects of parental human capital on the relative productivity of those inputs. Using this framework and data from the Panel Study of Income Dynamics (PSID) and its Child Development Supplement (CDS), we make several distinct contributions to the literature.

First, we establish conditions under which the family decision problem can be decomposed into ***intratemporal*** *and* ***intertemporal*** *components, which facilitates both our theoretical and empirical anal-*

[2]Gayle, Golan, and Soytas (2015) estimate the importance of parental time inputs and required costs of child-rearing in a model with fertility, marriage, and divorce. Fiorini and Keane (2014) estimate the impacts of several categories of children's time use on their cognitive and non-cognitive outcomes, concluding that educational activities with parents are most productive. Todd and Wolpin (2007) estimate the effects of both home inputs and school quality using additively separable value-added models of cognitive achievement. They conclude that differences in school quality (measured by pupil-teacher ratios and teacher salaries) account for a very small proportion of race/ethnicity gaps in achievement. Using a richer dynamic model of skill formation applied to kindergarten children, Agostinelli, Saharkhiz, and Wiswall (2019) find that both home investments and classroom quality (measured by value-added) affect achievement growth; however, differences in home investments explain much more of the cross-sectional variation in skill growth. None of these studies explores the substitutability of home vs. school inputs.

yses.[3] In the intratemporal problem, families choose child input allocations (parental time, household investment goods/services, and market-based child care services) to minimize expenditures given a per-period level of total human capital investment. In the intertemporal problem, families maximize lifetime utility by choosing consumption/savings, leisure, and per-period total human capital investment, given its composite price (determined from the intratemporal problem) and potentially binding borrowing constraints.

Second, we analytically characterize the relationship between parental human capital and family investment inputs, as well as investment responses to input price changes. These results highlight the critical roles of parental human capital in the skill production process and of input substitutability. For example, we show that a wage subsidy (as in the Earned Income Tax Credit) unambiguously increases total investment expenditures and can, with sufficient input complementarity, lead to increases in all investment inputs, including parental time. We also show that the effects of small input price changes on total investment and, therefore, skill growth, do not depend on the substitutability of investments. This result has important implications for direct estimation approaches that rely on independent variation in investment inputs to identify the skill production technology, because it suggests that substantial relative price variation is needed to identify substitution elasticities.

Third, we develop a revealed preference estimation approach that exploits relative demands for inputs to identify the substitutability and relative productivity of different inputs within periods. This approach harkens back to an early literature that estimates input substitutability in firm production.[4] Although it requires a weak separability assumption on the dynamics of skill investment and strictly positive levels of investment inputs and parental work hours, it has three important advantages relative to direct estimation of the full skill production technology (i.e., within-period relative productivity and substitutability of different inputs and the dynamics of investments and skills). First, it does not require a full specification for the dynamics of skill formation, using only within-period relationships to identify the substitutability

[3] This separation is analogous to that of the two-stage budgeting approach commonly used in the labor supply literature (Heckman, 1974; MaCurdy, 1983; Altonji, 1986; Blundell and Walker, 1986). See Gorman (1959) for a general treatment of separability.

[4] Arrow et al. (1961) develops several early estimation approaches for the standard CES production function based on first order conditions under profit maximization. Maddala and Kadane (1966) explore implications of measurement and endogeneity of inputs and their prices for these approaches, while Griliches (1969) studies the impacts of measurement error in the cost of capital for estimates of capital-skill complementarity. Nerlove (1965, 1967) provides an early treatment of different estimation approaches, including efforts based on cost minimization, while Berndt (1976) re-examines several approaches and the importance of data quality. Due to its focus on firm production, this early literature was often concerned about returns to scale, non-competitive factor markets (influencing input prices), and measurement of capital and its price. Given these concerns and that firm revenues and costs were generally well-measured, early studies relied less on relative demand relationships. This literature was not as concerned with measurement error in inputs (other than capital), a key concern in our context (Cunha and Heckman, 2008; Cunha, Heckman, and Schennach, 2010).

of inputs and relative productivity of parental skills (and other factors) for different inputs.[5] Second, it easily addresses measurement error in investment inputs without the need for multiple measures of each input, in contrast to several recent direct estimation approaches (Cunha, Heckman, and Schennach, 2010; Attanasio et al., 2017; Agostinelli and Wiswall, 2023). Third, a relative demand approach only relies on cross-sectional data on investment inputs and their prices, while direct estimation requires panel data on skill measures and investment inputs.

Fourth, we show how relative demand relationships can be used to simplify and improve estimation of the full skill production technology when panel data on skills are available. We begin with a general direct estimation approach that exploits variation in input prices and other exogenous factors (as instruments for investment inputs) but does not require multiple (noisy) measures of each input, which are unavailable in our data. Unfortunately, direct estimation performs quite poorly in our PSID-CDS data given the small sample size, limited variation in relative input prices, and moderate correlations between input levels.[6] Incorporating relative demand restrictions from the intratemporal decision problem not only improves the precision of our estimator, but it also simplifies the problem by enabling us to write the dynamics of skill production as a function of a single (noisily measured) input and relative input prices, rather than as a function of several (noisily measured) inputs. We also show how to test whether parental perceptions of the within-period features of skill production (i.e., relative input productivity and substitutability) are consistent with the true skill production function.

Fifth, using data from the PSID-CDS, we estimate that parental education has modest effects on the productivity of investment inputs for 5–12 year old American children. Observed investment gaps by parental education are driven primarily by differences in overall demand for child investment – stemming from differences in both family resources and preferences for children's skill levels – with differences in parental child-rearing abilities explaining no more than 15% of the gaps. Our results further indicate that differences in the technology of skill production (including differences in total factor productivity or learning ability) by parental education are responsible for little of the growth in skill gaps over ages 5–12. These conclusions are broadly consistent with several previous studies finding that parental education improves the productivity of investments (or total factor productivity) for very young, but not older, children (e.g., Del Bono et al., 2016; Attanasio, Meghir, and Nix, 2020; Brilli, 2022; Bolt et al., 2023, 2024; Del Boca et al., 2023) or that parental education only affects child development through higher

[5]In theory, a fully nonparametric direct approach would not require any assumptions on the technology, but this is generally infeasible due to data constraints. In practice, researchers have assumed that skill dynamics are determined by a Cobb-Douglas (Del Boca, Flinn, and Wiswall, 2014), CES (Cunha, Heckman, and Schennach, 2010) or translog (Agostinelli and Wiswall, 2023) function.

[6]A Monte Carlo analysis shows that these estimation challenges are not specific to our estimator and arise even when inputs are not measured with error.

levels of investment (e.g., Cunha and Heckman, 2008; Agostinelli et al., 2020; Attanasio et al., 2020; Falk et al., 2021).

Sixth, estimates from the PSID-CDS provide robust evidence of moderately strong complementarity between all investment inputs, which produces strong co-movements of inputs in response to policy changes. Estimated elasticities between parental time and purchased goods/services and between these home inputs and child care services range from around 0.2 to 0.5.[7] Counterfactual policy simulations suggest that reductions in the price of home goods/services inputs or market-based child care cause families to expand all types of investment, with much stronger own-price elasticities than cross-price elasticities. A decline in parental wages leads to reductions in all types of investment, including small reductions in parental time investments, due to diminished family resources.[8] The co-movement of inputs in response to price changes contrasts with the commonly assumed Cobb-Douglas specification, which implies greater input subtitutability, even stronger own-price responses, and no cross-price responses. Due to the stronger own-price response, incorrectly assuming a Cobb-Douglas technology would lead one to over-estimate the costs of providing subsidies for child care or goods/services investments. Finally, our analysis suggests that the complementarity of inputs has important implications for the costs of providing free child care.

While the PSID-CDS are the only data we are aware of that contains all of the input measures we consider for a representative sample of American children, it has two important limitations. First, the sample size is fairly small, a problem exacerbated by restrictions associated with our relative demand approach, which limits estimation to families with working parents who purchase strictly positive amounts of investment inputs. Unfortunately, this means that some empirical tests, including whether family perceptions about within-period features of skill production are accurate, do not have much power. Second, achievement and input measures are only available every five years, with no household goods measure available until surveyed children are ages 5–12. This limits the applicability of our results to school-age children and inspires our novel imputation approach to account for missing investments (during non-survey years) using intertemporal optimality and input prices (available for all years).

This paper proceeds as follows. The next section documents investment expenditure patterns by maternal education and household structure using data from the PSID-CDS. Section 3 describes our model of child development and characterizes the effects of parental human capital and input prices on the three types of investment inputs we study. In Sections 4 and 5, we describe our approach for estimating

[7] A few recent studies also find complementarity between different investment inputs (Abbott, 2022; Molnar, 2023; Moschini, 2023). In addition, complementarity between child care and home inputs is consistent with the positive effects of Head Start (Gelber and Isen, 2013) and home visits in Colombia (Attanasio et al., 2020) on family investments.

[8] The resulting declines in achievement growth are broadly consistent with previous evidence on the impacts of the EITC on child achievement (Dahl and Lochner, 2012; Agostinelli and Sorrenti, 2020).

the technology of skill formation followed by the data used in estimation. Section 6 reports our estimation results, while Section 7 presents a counterfactual analysis based on those estimates. Section 8 concludes.

2 Investment Patterns by Parental Education and Marital Status

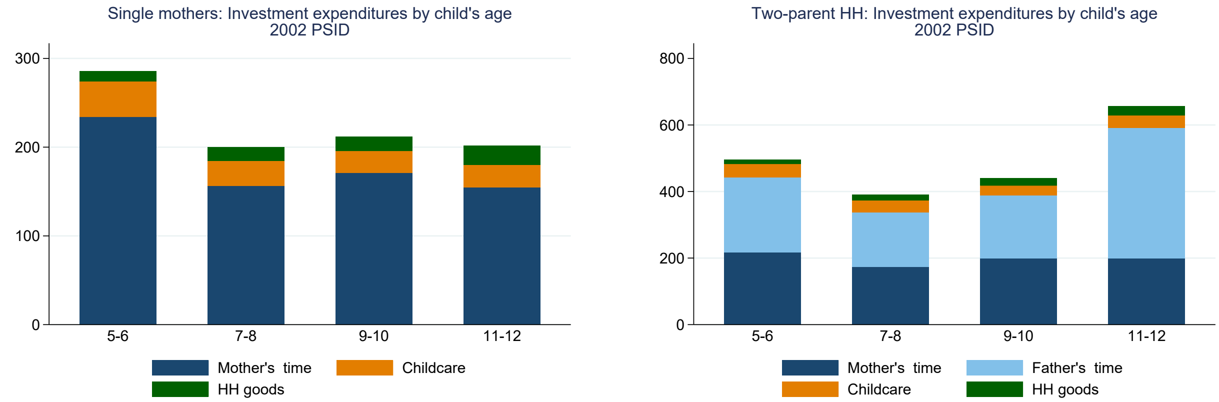
This section provides an overview of weekly child investments – parental time, goods/services, and child care – by parental education for single mothers and two-parent households using the PSID-CDS data.[9] We focus primarily on investment expenditures for three reasons. First, it provides a single metric (dollars/week) for comparing amounts of conceptually different inputs like time vs. goods. Second, it clarifies the costs associated with different forms of child investments. Third, differences in expenditure shares by parental education are informative about the substitutability and relative productivity of different inputs, as we show in the next section.

As with our estimation sample below, this analysis focuses on families with only 1 or 2 children ages 12 or younger, parents ages 18–65, and mothers who were ages 16–45 when their youngest child was born. For reasons made clear in Section 5, this analysis is limited to the 2002 CDS (with expenditures reported in year 2002 dollars) when the children we study were ages 5–12. We will often refer to home (or household) "goods" investments when discussing expenditures on school supplies, books, and toys; although, we also include spending on services like tutoring, lessons, community groups, and sports activities. We use a more limited measure of parental time investment than previous studies using the PSID-CDS (e.g., Del Boca, Flinn, and Wiswall, 2014; Mullins, 2022), dropping such activities as time spent watching television or performing household chores. Our measure includes a broad range of learning, play-based, and social activities. (See Section 5 and Appendix B for more details.) In calculating expenditures for parental time, we multiply active time with children by the parent's reported wage rate, excluding non-working parents.[10] Lastly, for consistency with our empirical analysis later in the paper, we limit the sample to those with positive child care expenditures; although, the patterns we emphasize are quite similar when including families with zero child care spending (see Appendix Figures G-2 and G-3).

Total amounts spent on child investments are sizeable, with single mothers spending about \$225 per week, on average, and two-parent households spending roughly \$425 per week. Spending on each input is 10–20% higher in two-parent families than single-mother households, with the former also benefiting

[9] Caucutt et al. (2020) show similar patterns when combining data from the American Time Use Survey on parental time investment with data from the Consumption Expenditure Survey on household goods/services and child care spending.

[10] We also trim the top/bottom 1% of reported wages to eliminate outliers. About 13% of two-parent households and 7% of single mothers are excluded due to missing wages. Including these parents has negligible effects on average time investing in children and household goods expenditures (by education or overall), while it lowers average spending on child care by less than 15% (similarly across education groups).



(a) Single mothers

(b) Two-parent households

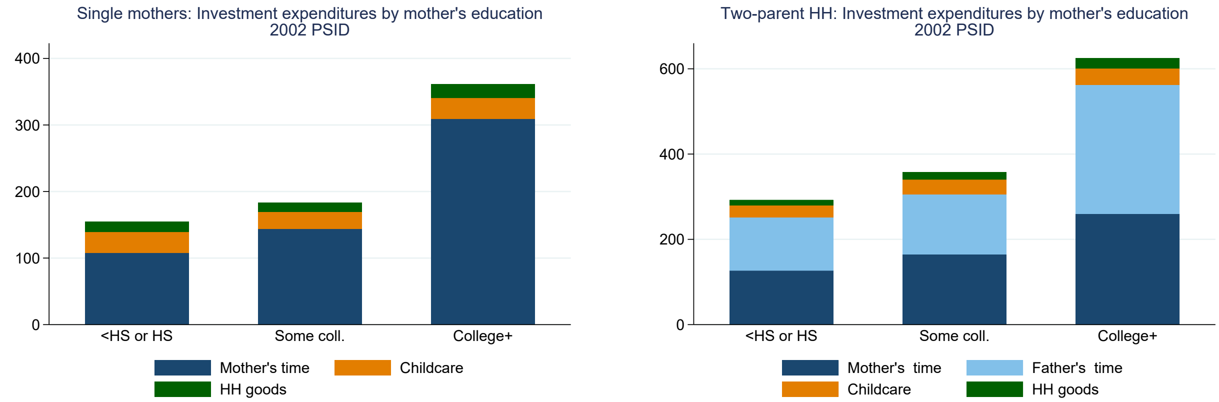
Figure 1: Weekly child investment expenditures by child's age (PSID, 2002)

from father's time expenditures. Figure 1 shows that single mother's weekly expenditures on child investment drop sharply as children begin school full time, remaining fairly stable thereafter. The early drop in expenditures is less pronounced for two-parent households, while father's time expenditures sharply increase as children reach ages 11–12. Single mothers gradually increase the share of expenditures on goods inputs at the expense of time investments, while two-parent households maintain more similar expenditure shares as children age.

Figure 2 shows that investment expenditures are strongly increasing with maternal education. Most notably, time expenditures among college graduate mothers and their husbands are more than double those of their high school counterparts, with much of the difference explained by their higher wage rates. Higher time expenditures for fathers (relative to mothers) are entirely driven by differences in average wage rates. As Figure 3 shows, investment time is increasing in parental education, similar for single and married mothers, and higher for mothers than fathers. Figure 4 reports average expenditure shares on each type of investment input by maternal education, highlighting two features. First, expenditures are dominated by time investments, with single mothers (both parents in two-parent households) contributing about 71% (82%) of their investment expenditures in the form of time. Second, expenditure shares are quite similar across education groups, especially for two-parent households, despite sizeable differences in the total amounts of investment spending.

3 Model

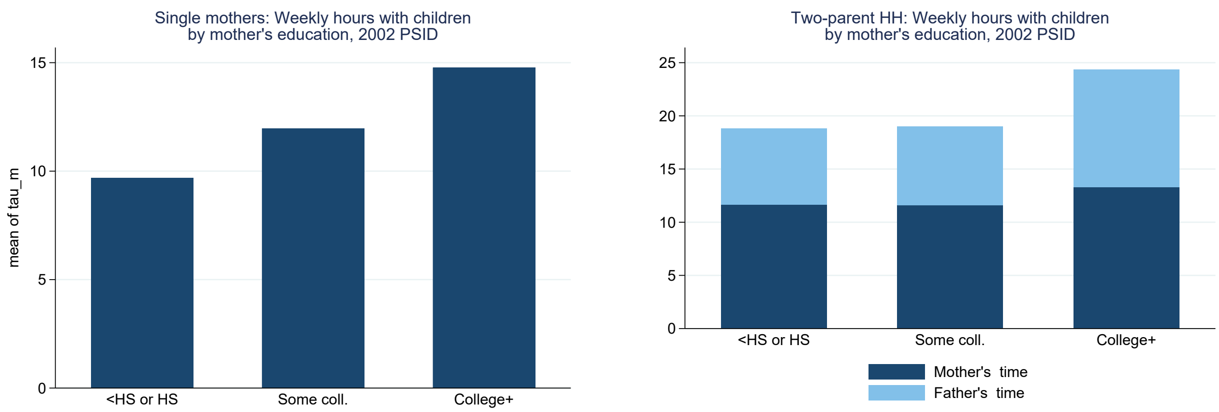
This section develops an economic framework for understanding the multi-input investment and expenditure patterns documented in Section 2 and the impacts of policies that alter the prices of investment



(a) Single mothers

(b) Two-parent households

Figure 2: Weekly child investment expenditures by mother's education (PSID, 2002)



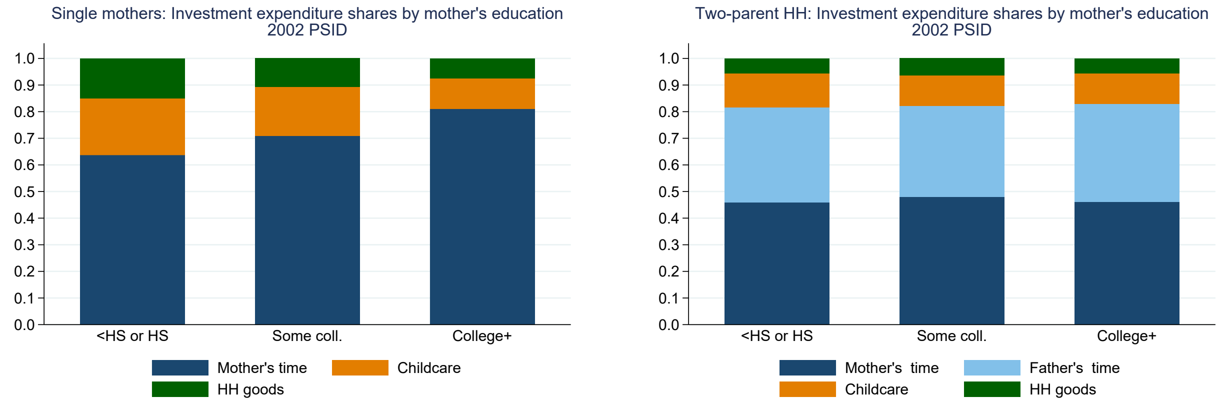
(a) Single mothers

(b) Two-parent households

Figure 3: Weekly investment hours by mother's education (PSID, 2002)

inputs. We begin by establishing conditions that allow the household decision problem to be separated into two distinct (and simpler) problems: (i) an *intratemporal* investment input allocation problem and (ii) an *intertemporal* problem that allocates resources across periods. We then characterize the response of investment inputs to changes in input prices and parental human capital. This theoretical analysis highlights the critical roles of input substitutability and the extent to which parental human capital affects the productivity of different inputs. Insights from this framework guide our estimation strategy.

Model description. Consider two-parent households composed of a mother, father, and child. These households may be heterogenous over the learning ability of the child, θ_t, initial human capital of the



(a) Single mothers

(b) Two-parent households

Figure 4: Expenditure shares by mother's education (PSID, 2002)

child, Ψ_1, and human capital of the mother and father, H_m and H_f, respectively.[11] (Single mother households are identical but without any "father" time, wages, etc.) In every period $t = 1, ..., T$, the household chooses consumption, c_t, assets, A_{t+1}, mother's and father's leisure, $l_{m,t}$ and $l_{f,t}$, respectively, and investments in children.

Child investments take place in the home or in the market. Home investments include time of the mother, $\tau_{m,t}$, time of the father, $\tau_{f,t}$, and goods/services, g_t. Market-based child care is represented by x_t.[12] Child skills evolve according to

$$\Psi_{t+1} = \mathcal{H}_t \left(f_t \left(\tau_{m,t}, \tau_{f,t}, g_t, x_t; H_m, H_f \right); \Psi_t, \theta_t \right), \tag{1}$$

where $f_t(\cdot)$ is strictly increasing and strictly concave in all endogenous investment inputs $(\tau_{m,t}, \tau_{f,t}, g_t, x_t)$ and $\mathcal{H}_t(\cdot)$ is strictly increasing in all of its arguments. The function $f_t(\cdot)$ represents the total skill investment a child receives in period t as a function of all investment inputs that period. We refer to this as the *within-period production function*, because it encompasses all features of skill production related to different inputs within a period, notably the relative productivity and substitutability of inputs. Importantly, we allow these within-period features of skill production to depend on parental human capital, while we treat θ_t as an input-neutral productivity parameter. As is standard in the literature, our recursive specification imposes weak intertemporal separability between inputs across periods, ruling

[11] Because we abstract from schooling inputs, variation in θ_t may also reflect differences in school quality; however, we do not model the choice of neighborhood or private schooling.

[12] Without data on the quality of child care, we do not explicitly model a quantity-quality tradeoff for child care services, implicitly assuming that families purchase the optimal mix given available options. If care is priced according to its productivity in a competitive market, then families would generally be indifferent to the mix. In this case, the price of child care, q_t, reflects the price for a fixed quality of care and unit of time.

out the possibility that an increase in any specific input in one period affects the relative marginal productivities of inputs in other periods. With this intertemporal separability, the function $\mathcal{H}_t(\cdot)$ only determines the dynamics of skill accumulation.

We make one more important assumption on the within-period technology.

Assumption 1. $f_t(\cdot)$ *is homothetic in endogenous inputs* $(\tau_{m,t}, \tau_{f,t}, g_t, x_t)$.

Homotheticity is satisfied by most production functions used in the child development literature, including the nested-CES specification we employ; however, it does restrict the relative marginal productivity of inputs from changing with the total level of investment. Because any returns to scale (and other monotonic transformations) in within-period investment can be incorporated by $\mathcal{H}_t(\cdot)$, we normalize $f_t(\cdot)$ to be homogeneous of degree 1.

Normalizing the time endowment to 1 for each parent, parental hours working are $h_{m,t} = 1 - l_{m,t} - \tau_{m,t}$ and $h_{f,t} = 1 - l_{f,t} - \tau_{f,t}$. A parent's period t wage is given by $W_{j,t} = w_{j,t} H_j$, $j = m, f$, where we distinguish between the part of wages related to skills used in child production (H_j) and an unrelated component ($w_{j,t}$). For expositional purposes, we assume that the component related to child development is fixed over time (e.g. upon a parent finishing school or the child's birth), while the time-varying part, which we often refer to as the price of skill, incorporates wage differences across parents due to factors like labor market experience, discrimination in the labor market, or local wage variation. Let y_t reflect income other than labor earnings (e.g. transfers) in period t. The price of home investment goods is given by p_t, and the price of market child care is given by q_t. Let $\Pi_t \equiv (W_{m,t}, W_{f,t}, p_t, q_t)$ be the vector of all investment input prices faced by the household at time t. Assets at the start of period t are denoted by A_t, and households can borrow/save at interest rate r, subject to borrowing constraints $A_{t+1} \geq A_{min,t}$.

Households have per period preferences over consumption (with price normalized to one) and leisure given by $u(c_t) + v_m(l_{m,t}) + v_f(l_{f,t})$ and discount across periods at the rate $\beta > 0$. In period $T+1$, households have a continuation value, $\tilde{V}(H_m, H_f, A_{T+1}, \Psi_{T+1})$, that depends on parental human capital, parental assets, and child skill.[13] We assume that $\tilde{V}$ is increasing in parental human capital, and strictly increasing and strictly concave in parental assets and child skill. We consider the household's problem for periods $t = 1, ..., T$:

$$V_t(\theta_t, H_m, H_f, A_t, y_t, \Pi_t, \Psi_t)$$
$$= \max_{l_{m,t}, l_{f,t}, \tau_{m,t}, \tau_{f,t}, g_t, x_t, A_{t+1}} u(c_t) + v_m(l_{m,t}) + v_f(l_{f,t}) + \beta V_{t+1}(\theta_{t+1}, H_m, H_f, A_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{t+1})$$

[13] The continuation value, $\tilde{V}$, also depends on all future non-labor income, which we suppress here for ease of notation.

subject to non-negative inputs $(\tau_{m,t}, \tau_{f,t}, g_t, x_t)$, $l_{j,t} \geq 0$ and $l_{j,t} + \tau_{j,t} \leq 1$ for $j = m, f$, child human capital production equation (1),

$$
\begin{aligned}
c_t + W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t} + p_t g_t + q_t x_t + A_{t+1} &= (1+r)A_t + y_t + W_{m,t}(1-l_{m,t}) + W_{f,t}(1-l_{f,t}), \\
A_{t+1} &\geq A_{min,t}, \qquad (2) \\
V_{T+1}(\theta_{T+1}, H_m, H_f, A_{T+1}, y_{T+1}, \Pi_{T+1}, \Psi_{T+1}) &= \tilde{V}(H_m, H_f, A_{T+1}, \Psi_{T+1}). \qquad (3)
\end{aligned}
$$

We assume $u'(\cdot) > 0$, $u''(\cdot) < 0$, $v_j'(\cdot) > 0$, and $v_j''(\cdot) < 0$ for $j = m, f$. We also assume standard Inada conditions for preferences over consumption and leisure. See Appendix A for a full characterization of the problem, including all first order conditions, in the more general case where there may be uncertainty about future parental wages, income, and child abilities.

Separating the Problem. Under certain conditions, the household's problem can be separated into two distinct parts: (i) an *intratemporal* problem choosing child input allocations to minimize expenditures given child's per period total skill investment, $I_t = f_t(\tau_{m,t}, \tau_{f,t}, g_t, x_t; H_m, H_f)$, and (ii) an *intertemporal* problem of maximizing lifetime utility by choosing savings A_{t+1}, leisure $l_{m,t}$ and $l_{f,t}$, and per period child's total skill investment I_t. This separation is possible when three conditions are satisfied (Gorman, 1959). First, if both parents work in the market (i.e., $h_{j,t} > 0$), then the price of time investment is the wage rate, $W_{j,t}$, which is independent of investment choices. In this case, the marginal cost of each input is its fixed price multiplied by the marginal utility of consumption.[14] Second, weak intertemporal separability of inputs in child skill production, coupled with no parental preferences over inputs themselves, implies that the marginal benefit of each input is proportional to its marginal productivity through $f_t(\cdot)$.[15] Third, because $f_t(\cdot)$ is homogeneous of degree 1, the marginal product of each input is homogenous of degree 0 and can be written as a function of input ratios. Under these 3 conditions, the first order conditions with respect to inputs can be used to solve for optimal input ratios as functions only of period t relative input prices, parental human capital, and the within-period production technology $f_t(\cdot)$. This separation result clarifies when assumptions about dynamics (e.g. credit markets, structure of $\mathcal{H}_t(\cdot)$), children's learning ability θ_t, and the nature of preferences (over consumption, leisure, and child skill levels) have no bearing

[14] As discussed in Appendix A.1, for parents who do not work in the market, the price of time investment depends on the marginal rate of substitution between consumption and leisure. Here, the problem cannot be separated into intratemporal and intertemporal problems, because relative input choices would depend on the level of consumption, a fundamentally intertemporal choice. Non-separable preferences for leisure over time or parental learning-by-doing in the labor market would also mean that the price of time is not fully reflected by current wages.

[15] More generally, separating the problem into intratemporal and intertemporal problems requires that parents do not have preferences favoring one input relative to another. Separation of the problem is still possible if families have preferences over total investment, I_t, or child skill levels, Ψ_t, each period.

on the within-period allocation of child investment inputs conditional on total investment that period. (See Appendix A.1 for further details.)

Throughout the rest of this section, we assume that parents are working, so the price of their time is reflected by their wages and the full problem can be separated into intratemporal and intertemporal decisions. We begin with the intratemporal problem, characterizing optimal input choices as a function of per-period total investment, I_t. We further show that this per-period total investment is associated with a composite per-period price, $\bar{p}_t$, that depends on all input prices, parental human capital, and the structure of the within-period investment function $f_t(\cdot)$. Next, we consider the intertemporal problem of choosing total investment, I_t, along with consumption/savings and leisure each period, given the sequence of composite prices, $\bar{p}_t$, faced by the household.

3.1 Intratemporal problem

The intratemporal problem for families with working parents can be written as:

$$\min_{\tau_{m,t},\tau_{f,t},g_t,x_t} W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t} + p_t g_t + q_t x_t \qquad \text{subject to} \qquad I_t = f_t(\tau_{m,t}, \tau_{f,t}, g_t, x_t; H_m, H_f),$$

non-negative inputs $(\tau_{m,t}, \tau_{f,t}, g_t, x_t)$, and $\tau_{j,t} < 1$ for $j = m, f$. Using the fact that $f_{\tau_{j,t}}$, f_{g_t}, and f_{x_t} are homogenous of degree 0, we define input ratios $\Phi_{j,t} \equiv \tau_{j,t}/g_t$ for $j = m, f$, and $\Phi_{x,t} \equiv x_t/g_t$, and write the ratio of first order conditions as three equations in these three unknown ratios:

$$\tilde{W}_{j,t} \equiv \frac{W_{j,t}}{p_t} = \frac{f_{\tau_{j,t}}(\Phi_{m,t}, \Phi_{f,t}, 1, \Phi_{x,t}; H_m, H_f)}{f_{g_t}(\Phi_{m,t}, \Phi_{f,t}, 1, \Phi_{x,t}; H_m, H_f)}, \qquad j = m, f, \tag{4}$$

$$\tilde{q}_t \equiv \frac{q_t}{p_t} = \frac{f_{x_t}(\Phi_{m,t}, \Phi_{f,t}, 1, \Phi_{x,t}; H_m, H_f)}{f_{g_t}(\Phi_{m,t}, \Phi_{f,t}, 1, \Phi_{x,t}; H_m, H_f)}. \tag{5}$$

Equations (4) and (5) implicitly define the input ratios $(\Phi_{m,t}, \Phi_{f,t}, \Phi_{x,t})$ as functions of relative prices $(\tilde{W}_{m,t}, \tilde{W}_{f,t}, \tilde{q}_t)$, parental human capital (H_m, H_f), and the within-period production technology $f_t(\cdot)$. Future child ability levels, parental wages, and other income do not enter this problem, therefore uncertainty about these variables would not affect the intratemporal relative input allocation decision.

Because $f_t(\cdot)$ is homogeneous of degree 1, total investment expenditure $E_t = W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t} + p_t g_t + q_t x_t$ can also be written as $E_t = \bar{p}_t I_t$, where

$$\bar{p}_t = \frac{W_{m,t}\Phi_{m,t} + W_{f,t}\Phi_{f,t} + p_t + q_t\Phi_{x,t}}{f_t(\Phi_{m,t}, \Phi_{f,t}, 1, \Phi_{x,t}; H_m, H_f)} \tag{6}$$

is the composite price of total investment (Gorman, 1959). This composite price depends only on input prices, parental human capital, and the within-period production technology $f_t(\cdot)$.

Much of our analysis considers a nested CES within-period investment function:

$$
\begin{aligned}
f_t &= \left[(\bar{a}_{m,t}[\varphi_m(H_m)\tau_{m,t}]^{\rho_t} + \bar{a}_{f,t}[\varphi_f(H_f)\tau_{f,t}]^{\rho_t} + \bar{a}_{g,t}[\varphi_g(H_m,H_f)g_t]^{\rho_t})^{\gamma_t/\rho_t} + a_{x,t}x_t^{\gamma_t} \right]^{1/\gamma_t} \\
&= \left[\left(a_{m,t}(H_m)\tau_{m,t}^{\rho_t} + a_{f,t}(H_f)\tau_{f,t}^{\rho_t} + a_{g,t}(H_m,H_f)g_t^{\rho_t} \right)^{\gamma_t/\rho_t} + a_{x,t}x_t^{\gamma_t} \right]^{1/\gamma_t}, \qquad (7)
\end{aligned}
$$

where $a_{j,t}(H_j) \equiv \bar{a}_{j,t}[\varphi_j(H_j)]^{\rho_t} > 0$ for $j = m, f$, $a_{g,t}(H_m, H_f) \equiv \bar{a}_{g,t}[\varphi_g(H_m, H_f)]^{\rho_t} > 0$, $\rho_t < 1$, and $\gamma_t < 1$.[16] We highlight three aspects of this specification. First, it allows parental human capital to affect the productivity of household time and goods investments through their respective share parameters. Our specification assumes that parental skills are factor-augmenting, so that parental human capital raises or lowers effective time and goods input levels.[17] (We generally leave conditioning on parental human capital implicit, except when discussing its role.) Second, this specification accommodates flexible substitution patterns between different inputs. The elasticity of substitution between parental time and household goods inputs is constant and given by $\epsilon_{\tau,g,t} \equiv 1/(1-\rho_t)$. By contrast, the elasticity of substitution between market child care services and household goods or parental time investments varies with input levels; however, the elasticity between market child care, x_t, and "composite home investment", $I_{H,t} \equiv \left(a_{m,t}\tau_{m,t}^{\rho_t} + a_{f,t}\tau_{f,t}^{\rho_t} + a_{g,t}g_t^{\rho_t} \right)^{1/\rho_t}$, is constant and given by $\epsilon_{x,H,t} \equiv 1/(1-\gamma_t)$. We generally refer to two inputs as substitutable if their elasticity of substitution is greater than one (e.g. $\epsilon_{\tau,g,t} > 1$ and $\rho_t > 0$) and complementary if their elasticity is less than one. The commonly employed Cobb-Douglas case assumes an elasticity of one across all inputs. Third, this specification for $f_t(\cdot)$ is homogenous of degree 1, as is the technology defining the composite home input, $I_{H,t}$.

This nested CES specification for $f_t(\cdot)$ implies the following optimal input ratios:

$$
\Phi_{j,t} \equiv \frac{\tau_{j,t}}{g_t} = \left(\frac{a_{g,t}}{a_{j,t}} \tilde{W}_{j,t} \right)^{\frac{1}{\rho_t - 1}}, \qquad j = m, f, \qquad (8)
$$

$$
\Phi_{x,t} \equiv \frac{x_t}{g_t} = \left(\frac{a_{g,t}}{a_{x,t}} \right)^{\frac{1}{\gamma_t - 1}} \left(a_{m,t}\Phi_{m,t}^{\rho_t} + a_{f,t}\Phi_{f,t}^{\rho_t} + a_{g,t} \right)^{\frac{\gamma_t - \rho_t}{\rho_t(\gamma_t - 1)}} \tilde{q}_t^{\frac{1}{\gamma_t - 1}}. \qquad (9)
$$

Because of the nested structure of the within-period production function, the price of child care services, q_t, does not affect home input ratios, $\Phi_{m,t}$ and $\Phi_{f,t}$. This simplifies several analytical results, as well as estimation of substitution elasticities.

3.1.1 Expenditure Shares

The intratemporal problem alone allows us to characterize the dependence of expenditure shares on input prices and parental human capital. For simplicity, we consider the case of single mothers and drop

[16] $\bar{a}_m$, $\bar{a}_f$, and $\bar{a}_g$ are strictly positive constants, while $\varphi_m(H_m)$, $\varphi_f(H_f)$, and $\varphi_g(H_m, H_f)$ are strictly positive functions.

[17] Parental human capital may also affect input-neutral child productivity/ability, θ_t, thereby influencing the productivity of all inputs, including child care. We focus here on relative productivity differences.

time t subscripts given the static nature of the intratemporal problem.

Input expenditure ratios are decreasing in their relative prices if and only if the elasticity of substitution between those inputs exceeds one. Households substitute away from an input when its price rises, but relative expenditures on that input only fall if the substitution effects are strong enough to offset the direct effect of the higher price. In the special case of Cobb-Douglas technology for $f_t(\cdot)$, or $\rho = \gamma = 0$ in our nested CES technology, all expenditure shares are invariant to input price changes. More generally, expenditure shares depend on relative prices and substitutability across inputs.

With the nested CES technology of Equation (7), relative home input ratios do not depend on the price of child care services. This implies a simple characterization for the effects of changes in child care prices on expenditure shares: if and only if child care services are complements with the composite home input (i.e., $\gamma < 0$ or $\epsilon_{x,H} < 1$), then an increase in child care prices leads to an increase in the share of expenditures devoted to child care and a reduction in the shares devoted to each home input. The effects of changes in the skill price, w_m, or price of home investment goods, p, depend on the substitutability of all inputs. For example, stronger complementarity between home inputs than between the composite home input, I_H, and child care services (i.e., $\rho < \gamma < 0$) implies that an increase in the skill price, w_m, leads to an increase in the share of expenditures devoted to time inputs and reductions in expenditure shares devoted to each of the other inputs.[18]

Given the patterns exhibited in Figure 4 (especially two-parent households), we highlight two special cases consistent with expenditure shares that are independent of parental human capital: (i) Cobb-Douglas $f(\cdot)$, i.e., $\rho = \gamma = 0$, or (ii) $f(\tau_m, g, x; H_m) = f(\tau_m H_m, g, x)$.[19] Case (i) assumes an elasticity of substitution of one between all endogenous investment inputs, but it places no restrictions on the role of parental human capital in skill production. Of course, this also implies that expenditure shares are independent of input prices. By contrast, case (ii) makes no restrictions on the substitutability of inputs, but it requires that parental skills improve the productivity of parental time with children (relative to the productivity of other inputs) at the same rate they improve parental earnings in the labor market.

More generally, differences in parental human capital affect input expenditure shares through both the price of parental time (i.e., wages) and the effectiveness of different inputs in child skill production. In terms of the latter, parental human capital is factor-augmenting when $\varphi'_m(H_m) > 0$, raising the effective value (or quality) of any given time investment, τ_m. Depending on the substitutability of inputs, an increase in H_m may, therefore, raise ($\rho > 0$) or lower ($\rho < 0$) the marginal productivity of time relative

[18] All expenditure shares respond in the opposite direction under substitutability satisfying $\rho > \gamma > 0$. See Propositions 4 and 5 in Appendix A.2.3 for these results, as well as symmetric results for changes in the price of home inputs, p.

[19] See Appendix A.2.1 for details.

to goods investments, all else equal. Ignoring effects of mother's human capital on the productivity of home goods inputs, when maternal human capital raises productivity relatively more in the labor market than with children, an increase in maternal human capital has qualitatively similar effects on expenditure shares as a rise in w_m, while the opposite is true when skills are relatively more productive for child development. This suggests that the growing expenditure share of single mother's time inputs with maternal education documented in Figure 4(a) is consistent with either: (i) low productivity of parental human capital for child development coupled with input complementarity, or (ii) high productivity of parental human capital for child development coupled with input substitutability.[20]

3.2 Intertemporal Problem

Suppose in every period, $t = 1, ..., T$, along with leisure and assets, the household chooses an amount of total child investment I_t given its per period composite price $\bar{p}_t$ (determined by the intratemporal problem). This problem can be written as follows:

$$V_t(\theta_t, H_m, H_f, A_t, y_t, \Pi_t, \Psi_t) = \max_{l_{m,t}, l_{f,t}, I_t, A_{t+1}} u(c_t) + v(l_{m,t}) + v(l_{f,t}) + \beta\left[V_{t+1}(\theta_{t+1}, H_m, H_f, A_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{t+1})\right]$$

subject to $0 \leq l_{m,t}, l_{f,t} < 1$, $I_t \geq 0$, the borrowing constraint (Equation (2)), final-period value function (Equation (3)), skill production $\Psi_{t+1} = \mathcal{H}_t\left(I_t; \theta_t, \Psi_t\right)$, and budget constraint:

$$c_t + \bar{p}_t(\Pi_t, H_m, H_f)I_t + A_{t+1} = (1+r)A_t + y_t + W_{m,t}(1 - l_{m,t}) + W_{f,t}(1 - l_{f,t}).$$

The standard Euler equation for consumption may be distorted by borrowing constraints, $u'(c_t) \geq \beta(1+r)u'(c_{t+1})$, with strict inequality if and only if Equation (2) binds. As discussed in the literature (e.g., Becker and Tomes, 1986; Cunha and Heckman, 2007; Caucutt and Lochner, 2020), borrowing constraints can distort intertemporal consumption and child investment decisions.

Throughout the rest of this paper, we borrow two assumptions from Del Boca, Flinn, and Wiswall (2014) that facilitate an analytical characterization for total investment.

Assumption 2. $\tilde{V}(\Psi_{T+1}) = \tilde{U}(H_m, H_f, A_{T+1}) + \alpha \ln(\Psi_{T+1})$.[21]

Assumption 3. $\Psi_{t+1} = \theta_t I_t^{\delta_1} \Psi_t^{\delta_2}$.

Under Assumptions 2 and 3, the first order condition for total investment, I_t, can be written as:

$$I_t = \frac{\alpha \beta^{T-t+1} \delta_2^{T-t} \delta_1}{\bar{p}_t u'(c_t)}, \tag{10}$$

[20] See Appendix A.2.3 for a formal characterization of the relationship between parental skills and investment expenditure shares, including when the productivity of home goods inputs also depends on H_m.

[21] To streamline our theoretical discussion, we assume throughout the rest of this section that α is independent of parental human capital. All results hold when α is weakly increasing in parental human capital. Allowing this dependence simply adds a force that further increases investment expenditures with parental human capital.

which has several useful properties.[22] First, it implies that total investment depends only on past decisions (including past investments), current skills, and borrowing constraints through the marginal utility of consumption, $u'(c_t)$. Second, total investment (and its dynamics) depends only on input prices p_t and q_t through the unit price $\bar{p}_t$ (as defined in Equation (6)). Combining Equation (10) with the Euler equation implies that

$$I_{t+1} \geq \left(\frac{\bar{p}_t}{\bar{p}_{t+1}}\right)\left(\frac{1+r}{\delta_2}\right) I_t, \tag{11}$$

where the inequality is strict if and only if the period-t borrowing constraint binds. Investments increase with age if the composite investment price is declining and self-productivity of skills is not too high (i.e., $\delta_2 < 1+r$). Third, if we define the constant $K_t \equiv \alpha\beta^{T-t+1}\delta_2^{T-t}\delta_1$, Equation (10) can be re-arranged to obtain a convenient expression for total investment expenditures:

$$E_t = \bar{p}_t I_t = K_t/u'(c_t). \tag{12}$$

Expenditures are increasing in the discounted value of children's human capital, $\alpha\beta^{T-t+1}$, the productivity of investments, δ_1, self-productivity of child skills, δ_2, and current consumption levels, c_t. The direct link between expenditures and the marginal utility of consumption is useful below in characterizing the effect of parental human capital on investment behavior.[23]

The first order condition for leisure implies $l_{j,t} = L_{j,t}(u'(c_t)W_{j,t})$, where $L'_{j,t}(\cdot) < 0$. Substituting this expression and optimal E_t from Equation (12) into the constrained household's budget constraint in period t ($A_{t+1} = A_{min,t}$) yields

$$c_t = (1+r)A_t + W_{m,t}(1 - L_{m,t}(u'(c_t)W_{m,t})) + W_{f,t}(1 - L_{f,t}(u'(c_t)W_{f,t})) + y_t - \frac{K_t}{u'(c_t)} - A_{min,t},$$

which does not depend on input prices p_t or q_t. Applying the implicit function theorem, Lemma 1 in Appendix A.3.2 shows that current consumption is increasing in parental human capital, returns on parental skills, and non-labor income, while it is independent of the prices of goods inputs and child care.

Because total investment expenditures are inversely proportional to the marginal utility of consumption (see Equation (12)), E_t and c_t respond similarly to changes in input prices and parental skills.

[22] See Appendix A.3.1 for the derivation of Equation (10).

[23] A fourth implication of Assumptions 2 and 3 is that θ_t is additively separable from all other choice and state variables in discounted lifetime utility. Consequently, child learning ability, θ_t, does not affect investment behavior (nor any other decision). Furthermore, uncertainty about future child ability has no effect on family decisions. As discussed in Appendix A, uncertainty about future parental wages or income only affects intertemporal choices through $u'(c_t)$. In the case of binding borrowing constraints, consumption is fully determined by available resources that period, so uncertainty about future wages and income would not affect decisions nor any of the results we discuss. As studied in Abbott (2022), consumption choices for unconstrained families would be affected by uncertainty about future wages and income due to precautionary savings motives. We abstract from this uncertainty here but discuss it further in Appendix A.

Proposition 1. *Total investment expenditures, E_t, are strictly increasing in parental human capital (H_m, H_f), skill prices ($w_{m,t}, w_{f,t}$), and non-labor income (y_t), with $\frac{\partial E_t}{\partial H_j} = \frac{\partial E_t}{\partial w_{j,t}} \frac{w_{j,t}}{H_j} > 0$ for $j \in \{m, f\}$. Total investment expenditures are independent of household goods and child care input prices (p_t, q_t).*

Total investment expenditures are increasing in current income levels. Consequently, increases in parental human capital, skill returns, and other income all lead to increases in total investment expenditures, whereas changes in the prices of home investment goods and child care services have no effect on total expenditures.[24]

Increases in total investment expenditures, E_t, do not necessarily imply increases in total investments, I_t (or child skill accumulation), because increases in input prices and wages raise the composite price of investment, $\bar{p}_t$, as well. These offsetting forces make it difficult to characterize the effects of parental skill returns or human capital on total investment levels without additional assumptions; however, it is straightforward to show that an increase in the price of household goods inputs or child care leads to reductions in total investment, while an increase in non-labor income raises total investment. See Appendix A.3.2 for details.

3.3 Effects of Input Prices and Parental Human Capital on Investment Inputs

The effects of input prices and parental human capital on expenditures devoted to specific inputs depend on both forces discussed thus far: (i) reallocations of expenditure shares (determined from the intratemporal problem) and (ii) changes in the total level of investment expenditures (determined from the intertemporal problem). Effects on specific input levels must also account for any own-price changes.

Because total expenditures are invariant to the prices of home goods and child care services (see Proposition 1), changes in these prices only affect individual input levels through reallocation across inputs. Consistent with downward sloping demand curves, the quantities purchased of home goods and child care services are unambiguously decreasing in their respective prices, even though expenditures on these inputs increase with own-input prices when there is sufficient complementarity (i.e., $\rho_t < \gamma_t < 0$).

Increases in skill prices and parental human capital not only affect expenditure shares, but they also raise total investment expenditures (see Proposition 1). To obtain sharper predictions in what follows, we simplify to single-mother families and assume log preferences for consumption and leisure (i.e., $u(c) = \ln(c)$ and $v_m(l_m) = \psi_m \ln(l_m)$, with $\psi_m > 0$). The results we discuss apply equally to

[24] Among borrowing-constrained households, investment expenditures are independent of all future prices and non-labor income. As shown in Appendix A.3.3, unconstrained households would respond similarly to changes in current or future prices and non-labor income, because the discounted present value of lifetime income determines consumption and expenditure levels. This appendix also discusses the impacts of short-term vs. permanent wage increases for unconstrained families. With log utility over consumption and leisure, a one-time increase in wages has a weaker impact on current expenditures for unconstrained (relative to constrained) families.

the case where borrowing constraints bind or when they never bind. See Appendix A.3.4 for additional details.

Proposition 2. *Suppose current family debts are not too large.*[25] *(A) If* $\min\{\gamma_t, \rho_t\} \geq 0$*, then mother's time investment is strictly decreasing in* $w_{m,t}$*. (B) If* $\rho_t \geq \max\{0, \gamma_t\}$*, then home goods inputs are strictly increasing in* $w_{m,t}$*. (C) If* $\gamma_t \geq 0$*, then market child care is strictly increasing in* $w_{m,t}$*.*

Proposition 2 considers the case of a change in the current skill price; although, a permanent change in the skill price has the same qualitative effects. A rise in the current skill price (e.g., a reduction in labor income taxes) increases both family income and the price of time, where the former implies greater total investment expenditures (Proposition 1). Despite this increase, the higher price of time leads to a reduction in maternal time investment when all inputs are substitutes. If inputs are sufficiently substitutable, then the reduction in time investment is compensated for with an increase in both the home goods input and child care services. Complementarity can lead to the opposite response: The increase in family income associated with higher wages can spur families to increase total investment, and if investment inputs are sufficiently complementary, families will increase all inputs, including parental time. Simulations based on the elasticities (and other parameters) estimated below suggest that this is the case for constrained families.

Finally, we study the effects of maternal human capital on investment input decisions. The relationship between mother's human capital and both home goods and time investments depends quite generally on how maternal skills affect the productivity of inputs and on the substitutability of inputs. When human capital has no effect on child skill production, inputs respond in the same manner to changes in maternal human capital as they do skill prices (see Proposition 2). A more interesting special case arises when parental human capital is equally productive in the labor market as it is in child-rearing but has no effect on the productivity of goods investments. In this scenario, an increase in maternal human capital causes families to substitute away from time investments towards goods investments, leaving expenditure shares unchanged. A positive effect of mother's skill on the productivity of goods inputs counteracts this substitution. See Propositions 7 and 8 in Appendix A.3.4 for details.

While the substitutability of inputs is critical for understanding the impacts of price changes on specific input choices, the next proposition shows that the effects of small input price changes on total investment I_t (and, therefore, child skill accumulation) do not depend on input substitutability.

[25]For constrained families, the required condition on debt is satisfied if borrowing constraints are not growing in discounted present value. For unconstrained families, current debts cannot exceed the present discounted value of all future income.

Proposition 3. *The price elasticities of total investment, I_t, depend on the within-period production function only through input expenditure shares.*[26]

The effects of small changes in input prices on total investment and child skills can be inferred solely from observed input expenditure shares without knowledge of input substitution elasticities. This property implies that, in the presence of limited price variation, direct identification of the production technology is impractical: a result that is borne out in our estimation results below.

More generally, knowledge of the full within-period production function (i.e., input substitutability and relative productivity) is needed to determine the impacts of larger price changes on skill development and to understand observed differences in input choices across families. We now turn to estimation strategies aimed at identifying the skill production process.

4 Estimation Approach

Our empirical analysis begins with a revealed preference approach that uses relative demand for inputs to estimate the within-period production function $f_t(\cdot)$ described in Section 3.1. We then incorporate data on child achievement over time to estimate both $f_t(\cdot)$ and $\mathcal{H}_t(\cdot)$ simultaneously. Because we do not observe multiple (noisy) measures of each investment input, the "direct" estimation approaches of Cunha, Heckman, and Schennach (2010), Attanasio et al. (2017), and Agostinelli and Wiswall (2023) cannot be implemented. Instead, we develop a direct estimation approach that relies on variation in input prices (and other exogenous factors) to instrument for investment inputs. We then show how restrictions from relative demand can be incorporated to simplify and improve estimation (if required conditions are met).[27] Finally, we show how relative demand restrictions can be combined with observations on skill dynamics to test whether beliefs about $f_t(\cdot)$ differ from the actual technology (see, e.g., Boneva and Rauh, 2018; Cunha, Elo, and Culhane, 2022).

4.1 Estimating the Within-Period Production Function, $f_t(\cdot)$

If families are knowledgeable about the within-period skill production function, then relative input demands can be exploited to estimate $f_t(\cdot)$ using data on input choices, input prices, and other household/child characteristics that affect the relative productivity or substitutability of inputs. Our relative demand approach is based on Equations (8) and (9), which requires interior solutions for input choices

[26] This proposition does not rely on log preferences for consumption and leisure or binding borrowing constraints. See Appendix A.4.

[27] In this case, we identify the within-period technology, $f_t(\cdot)$, from intratemporal relationships (between relative inputs and prices) and dynamic features of skill accumulation, $\mathcal{H}_t(\cdot)$, from dynamic relationships (between skills and investments).

and separation of the household problem into intratemporal and intertemporal parts.[28] As discussed in Section 3, we can determine relative input demand from the intratemporal problem alone if parents are working, skill production exhibits weak intertemporal separability of inputs, and $f_t(\cdot)$ is homothetic. Below, we discuss several approaches to address selection into work and concerns arising from families that report zero child care expenditure.

If the conditions for relative demand estimation are met, the approach offers several advantages to direct estimation of the full production function, $\mathcal{H}_t(\cdot)$ and $f_t(\cdot)$. Most notably, it easily addresses measurement error (in multiple inputs), requires no assumptions on $\mathcal{H}_t(\cdot)$ beyond weak intertemporal separability, and does not require panel data on skills. An additional challenge for the direct approach is that it requires independent exogenous variation in inputs to separately identify their relative productivities. Yet, if that exogenous variation is to come from variation in input prices, Proposition 3 suggests that substantial relative price variation is needed to identify elasticities of substitution, because the effects of small price changes on I_t (and, therefore, skills) do not depend on input substitutability. Section 6.2 shows that this is an important practical problem in our context.

Molnar (2023) and Moschini (2023) also use a revealed preference approach to estimate intratemporal features of a more limited production technology with a single unknown elasticity of substitution between inputs. Our richer skill production technology with three distinct types of inputs and flexible substitution patterns across those inputs introduces additional challenges, especially when inputs are measured with error. We address this measurement error, as well as measurement error in wages, and use a few strategies to tackle unobserved heterogeneity in parenting skills and selection into work.[29]

Empirical Specification. Let $Z_{i,t}$ be a set of observed household characteristics for child i at date t, and let $\eta_{m,i}$ and $\eta_{f,i}$ reflect unobserved heterogeneity in the productivity of parental time with children. We estimate the following nested CES within-period production function:

$$f(\tau_{m,i}, \tau_{f,i}, g_i, x_i | Z_{i,t}) = \left[\left(a_m(Z_{i,t}, \eta_{m,i}) \tau_{m,i,t}^{\rho} + a_f(Z_{i,t}, \eta_{f,i}) \tau_{f,i,t}^{\rho} + a_g(Z_{i,t}) g_{i,t}^{\rho} \right)^{\frac{\gamma}{\rho}} + a_x(Z_{i,t}) x_{i,t}^{\gamma} \right]^{\frac{1}{\gamma}},$$

[28] Of course, this approach implicitly assumes that households optimally choose investments based on the decision problem of Section 3 (notably, families cannot have preferences for one type of investment input over another). However, unlike a full structural estimation approach, it does not require specifications for $\mathcal{H}_t(\cdot)$ or preferences over consumption, leisure, and child skills, nor does it require assumptions about credit markets, access to insurance, or the nature of uncertainty about future prices, wages, income, or child ability.

[29] Moschini (2023) takes a time fixed effects approach to address unobserved heterogeneity (and selection into work among parents). Molnar (2023) uses the introduction of a universal child care subsidy in Quebec as an instrument for relative price changes. In related work, Abbott (2022) specifies a similar relative demand function (for his two inputs, time and goods), but he deals with unobserved heterogeneity through estimation of a full dynamic lifecycle model.

assuming factor share parameters $a_j(Z,\eta_j) = \exp(Z\phi_j + \eta_j)$ for $j = m, f$, $a_g(Z) = \exp(Z\phi_g)$, and $a_x(Z) = \exp(Z\phi_x)$.[30] We assume $a_f = 0$ (and exclude father characteristics from $Z_{i,t}$) for single mother households, because we do not generally observe much, if anything, about fathers in these cases.

Following our analysis in Section 3, the expression $a_j(Z,\eta_j)$ maps any factor augmentation by parental human capital ($\varphi_j(H)$) onto observables (Z) and unobservables (η_j).[31] It is worth clarifying the expected sign of the coefficients ϕ_j when $\rho < 0$ ($0 < \epsilon_{\tau,g} < 1$), as we estimate below. If some characteristic Z_k (e.g., education) increases human capital and thereby raises the productivity of parental time (i.e. $\varphi_j'(H) > 0$), then it must be that $\phi_{j,k} < 0$. As discussed in Section 3, when human capital is factor augmenting, an increase in parental human capital raises the total effective time input which may cause parents to spend relatively less time investing in their children.

Our empirical analysis recognizes that investment inputs, as well as parental wage rates, may be measured with error. We use an o superscript to reflect observed measures of these variables, so that $\ln(z^o_{i,t}) = \ln(z_{i,t}) + \xi_{z,i,t}$ for $z \in \{\tau_m, \tau_f, g, x, W_m, W_f\}$. We assume that all idiosyncratic measurement errors are mean zero and independent of all "true" variables (inputs, prices, as well as $Z_{i,t}$ characteristics), unobserved heterogeneity ($\eta_{m,i}, \eta_{f,i}$), and other measurement errors.

Next, let observed wages relative to the price of investment goods be $\tilde{W}^o_{j,i,t} \equiv W^o_{j,i,t}/p_{i,t}$, just as $\tilde{W}_{j,i,t} \equiv W_{j,i,t}/p_{i,t}$ and $\tilde{q}_{i,t} \equiv q_{i,t}/p_{i,t}$. It is also convenient to define the ratio of observed expenditures on parental time and child care relative to observed expenditures on household goods inputs:

$$R_{j,i,t} \equiv \frac{W^o_{j,i,t}\tau^o_{j,i,t}}{p_{i,t}g^o_{i,t}}, \quad \text{for } j \in \{m, f\}, \qquad \text{and} \qquad R_{x,i,t} \equiv \frac{q_{i,t}x^o_{i,t}}{p_{i,t}g^o_{i,t}}.$$

4.1.1 Relative demand for parental time vs. household goods

Based on Equation (8), relative demand for parental time vs. household goods (for working parents) is given by

$$\ln\left(\frac{\tau_{j,i,t}}{g_{i,t}}\right) = \epsilon_{\tau,g}\ln\left(\frac{a_j(Z_{i,t},\eta_{j,i})}{a_g(Z_{i,t})}\right) - \epsilon_{\tau,g}\ln\tilde{W}_{j,i,t}, \quad j = \{m, f\}.$$

Substituting in our assumptions for $a_j(\cdot)$ and $a_g(\cdot)$, incorporating measurement error, and adding $\ln\tilde{W}^o_{j,i,t}$ to both sides, implies the following estimating equation for relative time vs. goods expenditures:

$$\ln(R_{j,i,t}) = Z'_{i,t}\phi_{j,g} + (1-\epsilon_{\tau,g})\ln\tilde{W}^o_{j,i,t} + \tilde{\eta}_{j,i} + \xi_{R_j,i,t}, \quad j = \{m, f\}, \tag{13}$$

where $\phi_{j,g} \equiv \epsilon_{\tau,g}(\phi_j - \phi_g)$, $\tilde{\eta}_{j,i} \equiv \epsilon_{\tau,g}\eta_{j,i}$, $\xi_{R_j,i,t} \equiv \xi_{\tau_j,i,t} - \xi_{g,i,t} + \epsilon_{\tau,g}\xi_{W_j,i,t}$. This shows how relative time vs. goods expenditures depend on their relative prices, as well as characteristics that affect their relative

[30] Parameters ρ and γ may also vary with $Z_{i,t}$, but we abstract from this here for simplicity.

[31] To link our assumptions on $a_m(Z,\eta_m)$ and $a_f(Z,\eta_f)$ to our theoretical analysis of Section 3, suppose human capital for parent $j \in \{m, f\}$ is given by $H_j = \exp(Z\Gamma_j + \tilde{\eta}_j)$, so $\ln(W_{j,t}) = \ln(w_{j,t}) + Z\Gamma_j + \tilde{\eta}_j$, where $w_{j,t}$ is the period t price of skill in labor market. Then, assuming $\varphi_j(H_j) = H_j^{\bar{\varphi}_j}$ implies that $a_j(Z,\eta_j) = exp(Z\phi_j + \eta_j)$ where $\phi_j = \Gamma_j\bar{\varphi}_j\rho$ and $\eta_j = \tilde{\eta}_j\bar{\varphi}_j\rho$.

productivity. Because $\epsilon_{\tau,g} > 0$, household characteristics that raise the productivity of time relative to goods inputs (i.e., $Z_{i,t}$ for which $\phi_j > \phi_g$) will lead to greater relative time investment expenditures, where the effect also depends on the elasticity of substitution between time and goods.[32]

Three econometric challenges arise in estimation of Equation (13). First, unobserved differences in parenting skills $\eta_{j,i}$ may be correlated with wages $W_{j,i,t}$, because skills valued in the labor market may also be productive in child-rearing. Second, measurement error in wages is correlated with observed wages, producing a negative OLS bias for $(1 - \epsilon_{\tau,g})$.[33] Both of these concerns can be addressed using standard instrumental variables techniques.[34] Because wages are only observed for those who work during the year, a third challenge arises from potential selection into work based on $\eta_{j,i}$ if unobserved parental child-rearing skills are correlated with labor market participation. We address this concern in a few ways. First, we estimate Equation (13) conditioning on parents with a high predicted probability of work. As this predicted probability approaches one, our estimates should be consistent. Second, we use available panel data on parental wages to estimate log wage fixed effects for each individual parent. This provides an estimate of unobserved parental skills, which can be included in our set of observed factors affecting relative demand. Finally, we employ a standard Heckman two-step estimator for married mother's time vs. household goods, using variation in father's education and state-specific childcare costs as exclusion restrictions.

4.1.2 Relative demand for child care vs. household goods

Based on Equation (9), relative demand for child care vs. household goods implies the following ratio of expenditures for single mothers:[35]

$$\ln(R_{x,i,t}) = Z'_{i,t}\phi_{x,g} + \left[\frac{\epsilon_{x,H} - \epsilon_{\tau,g}}{1 - \epsilon_{\tau,g}}\right] \ln\left(1 + R_{m,i,t}e^{-\xi_{W_m\tau_m/g,i,t}}\right) + (1 - \epsilon_{x,H}) \ln \tilde{q}_{i,t} + \xi_{x/g,i,t}, \tag{14}$$

where $\xi_{W_m\tau_m/g,i,t} \equiv \xi_{\tau_m,i,t} + \xi_{W_m,i,t} - \xi_{g,i,t}$, and $\xi_{x/g,i,t} \equiv \xi_{x,i,t} - \xi_{g,i,t}$, and

$$\phi_{x,g} \equiv \epsilon_{\tau,g}\left(\phi_x - \frac{\epsilon_{\tau,g}}{\epsilon_{x,H}}\frac{1 - \epsilon_{x,H}}{1 - \epsilon_{\tau,g}}\phi_g\right).$$

[32] We emphasize that ϕ_j reflects the relative productivity of parental time, $\tau_{j,t}$, and not time investment expenditures, $\tau_{j,t}W_{j,t}$. Thus, $\phi_j = 0$ for parental education would imply that time spent investing in children is equally productive for parents of different education levels; although, that time tends to be more costly for more-educated parents.

[33] Measurement error in log wages does not necessarily produce the standard attenuation bias towards zero, because the dependent variable, relative expenditures, is a function of potentially mismeasured wages. Measurement error would lead to standard attenuation bias if we regressed log relative inputs $\ln(\tau^o_{j,i,t}/g^o_{i,t})$ rather than log relative expenditures $R_{j,i,t}$ on observed log relative wages.

[34] With at least 2 periods of data, a time fixed effects strategy can be used to address permanent unobserved heterogeneity (Moschini, 2023); however, this exacerbates concerns about measurement error in wages. For this reason and because we are limited to a single period of data for many families when estimating Equation (13), we pursue alternative approaches.

[35] Appendix E.1 presents an analogous set of results for two-parent households. Section 6.1 discusses families that do not use paid child care.

When both $\epsilon_{\tau,g}$ and $\epsilon_{x,H}$ are between 0 and 1 (as our estimates below suggest), family characteristics that raise the productivity of household goods inputs or lower the productivity of child care will lead to reductions in spending on child care relative to household goods.

We focus on two contrasting sets of assumptions that simplify estimation of equation (14): one assumes that all variation in $R_{m,i,t}$ is real, abstracting from measurement error altogether, while the other assumes that all variation conditional on $(Z_{i,t}, \tilde{W}_{m,i,t})$ reflects measurement error. Appendix E.2 develops an approach based on a second order Taylor approximation to $\ln\left(1 + R_{m,i,t}e^{-\xi_{W_m\tau_m/g,i,t}}\right)$ that allows for both unobserved heterogeneity in $R_{m,i,t}$ and measurement error in inputs and parental wages. Estimates based on this approach (reported in Appendix E.2) are quite similar to those obtained using the simpler approaches discussed here.

In the absence of measurement error in wages, time, and goods inputs (i.e., $\xi_{W_m\tau_m/g,i} = 0$), Equation (14) can be estimated via OLS using the observed expenditure ratio $R_{m,i,t}$. In this case, all observed variation in relative expenditures on time vs. goods conditional on observed characteristics and relative wages, $\tilde{W}_{m,i,t}$, is assumed to be driven by variation in relative productivity (e.g., $\eta_{m,i}$).[36]

Allowing for measurement error in all inputs (but not in wages), we obtain a similar specification provided there is no unobserved heterogeneity in maternal child skill production (i.e., $\eta_{m,i} = 0$):

$$\ln(R_{x,i,t}) = Z'_{i,t}\phi_{x,g} + \left[\frac{\epsilon_{x,H} - \epsilon_{\tau,g}}{1 - \epsilon_{\tau,g}}\right] \ln\left(1 + e^{\ln(\tilde{\Phi}_{m,i,t})}\right) + (1 - \epsilon_{x,H}) \ln \tilde{q}_{i,t} + \xi_{x/g,i,t}, \tag{15}$$

where $\tilde{\Phi}_{m,i,t} \equiv \frac{W_{m,i,t}\tau_{m,i,t}}{p_{i,t}g_{i,t}}$. Absent measurement error in wages and unobserved heterogeneity in maternal child productivity, the predicted values from OLS estimation of Equation (13), $\widehat{\ln(R_{m,i,t})}$, provide consistent estimates of $\ln(\tilde{\Phi}_{m,i,t}) = Z'_{i,t}\phi_{m,g} + (1 - \epsilon_{\tau,g}) \ln \tilde{W}_{m,i,t}$, which can be substituted into Equation (15) in a two-step estimation approach.

4.2 Estimating the Full Skill Production Function, $\mathcal{H}_t(\cdot)$ and $f_t(\cdot)$

To estimate the full child production function (i.e., both $\mathcal{H}_t(\cdot)$ and $f_t(\cdot)$), skill measures in at least two periods are needed. We begin by discussing a general skill technology and standard direct estimation approaches, which rely only on skill dynamics and not the optimality of input choices. We use this structure to clarify key assumptions of our novel estimation approach, which also exploits intratemporal optimality of relative demand. We show how incorporating relative demand moments can both simplify the estimation of skill dynamics, $\mathcal{H}_t(\cdot)$, and improve precision of technology estimates. This is particularly true if families are knowledgeable about the true production function. We can also allow the "perceived"

[36] Variation in $R_{m,i,t}$ could also reflect (unmodeled) variation in preferences or other idiosyncratic factors affecting $\tau_{m,i,t}/g_{i,t}$. This variation would not affect estimates of ϕ_j or $\epsilon_{\tau,g}$ if it is uncorrelated with $Z_{i,t}$ and $\tilde{W}_{m,i,t}$.

technology determining relative input demand to differ from the "actual" technology determining skill production, enabling us to test whether beliefs are accurate.

We confront two major practical challenges common to this literature. First, measures of both investment inputs and children's skills are noisy. Second, skills are only observed several years apart, while investment inputs are not observed every year in between. We address noisy skill measures as much of the literature (Cunha, Heckman, and Schennach, 2010; Agostinelli and Wiswall, 2023), exploiting multiple measurements of latent skills. Noisy inputs present a greater challenge, because $f_t(\cdot)$ is a nonlinear function of several inputs, and we do not observe multiple contemporaneous measures of each input in our data.[37] To address the problem of missing inputs in years between skill measurements, we exploit data on input prices in each year and intertemporal optimality of investment behavior under different assumptions about borrowing/saving.

4.2.1 General Setup and Direct Estimation

We begin with a general skill technology, focusing on single mothers to ease exposition. Let ω reflect parameters of the within-period production function, where we allow variation in this function over time through $Z_{i,t}$; otherwise, we make no assumptions about $\mathcal{H}(\cdot)$ and $f(\cdot)$ for now. Skills satisfy

$$\Psi_{i,t+1} = \mathcal{H}\left(f(\tau_{m,i,t}, g_{i,t}, x_{i,t}|Z_{i,t}, \omega); \Psi_{i,t}, \theta_{i,t}\right),$$

where time-variation in skill dynamics, $\mathcal{H}(\cdot)$, is reflected in $\theta_{i,t}$. Estimation of ω and $\mathcal{H}(\cdot)$ is complicated by measurement error in inputs and past skills, as well as unobserved variation in $\theta_{i,t}$. We assume throughout the rest of this section that $\ln(\theta_{i,t}) = Z_{i,t}\phi_\theta + \xi_{\theta,i,t}$ with $\xi_{\theta,i,t} \perp\!\!\!\perp Z_{i,t}$. Previous studies generally rely on multiple measures of each input and skill, either abstracting from unobserved heterogeneity in $\theta_{i,t}$ or using instruments (e.g., family income or input prices) to address endogeneity of investments (e.g., Cunha, Heckman, and Schennach, 2010; Attanasio et al., 2017; Agostinelli and Wiswall, 2023).

Without multiple measures of each input, we consider a direct estimation approach which assumes that relative input choices are determined by observed variables, $\Omega_{i,t} = (\Pi_{i,t}, Z_{i,t}, A_{i,t}, y_{i,t})$. In this case,

$$\Psi_{i,t+1} = \mathcal{H}\left(f\left(\tau_{m,i,t}, \tau_{m,i,t}\check{\Phi}_{g/\tau_m}(\Omega_{i,t}), \tau_{m,i,t}\check{\Phi}_{x/\tau_m}(\Omega_{i,t})|Z_{i,t}, \omega\right); \Psi_{i,t}, \theta_{i,t}\right), \tag{16}$$

where $\check{\Phi}_{g/\tau_m}(\cdot)$ and $\check{\Phi}_{x/\tau_m}(\cdot)$ are relative investment functions for g/τ_m and x/τ_m, respectively. In principle, these relative investment functions can be estimated (nonparametrically) in an earlier step using data on inputs and Ω_t (e.g., Newey and Powell, 2003), simplifying estimation of skill dynamics by reducing

[37] Unfortunately, this rules out the approaches of Cunha, Heckman, and Schennach (2010), Attanasio et al. (2017), and Agostinelli and Wiswall (2023).

the problem to estimation of a function of a single noisy input, $\tau_{m,i,t}$, rather than a nonlinear function of 3 (4 in the case of two-parent households) noisily measured inputs.[38] If the within-period production function is homogeneous (as is the case for a nested-CES), then $\tau_{m,i,t}$ can be "pulled out" of $f(\cdot)$, further simplifying estimation.

Despite the conceptual appeal of this approach, it provides very imprecise and unstable results in our setting. We, therefore, relegate further details about its implementation and results to Appendix E.5.

4.2.2 Using Relative Demand

Economic theory imposes restrictions on the form of relative investment functions, $\check{\Phi}_{g/\tau_m}(\cdot)$ and $\check{\Phi}_{x/\tau_m}(\cdot)$, which may be helpful in improving precision of the estimates. We consider restrictions from relative demand as determined from the intratemporal problem described in Section 3.1, allowing for the possibility that the "perceived" within-period technology determining relative input demand may differ from the "actual" technology determining the production of skills. This specification is quite general and facilitates testing whether families know the "actual" within-period production function. Finally, we discuss the case when families are knowledgeable about the "actual" $f(\cdot)$, our baseline specification.

Suppose families choose investment inputs based on the nested-CES $f(\cdot)$ in equation (7) but allow for the possibility that they have incorrect beliefs about parameters of the technology, ω. Letting $\breve{\omega}$ reflect the corresponding perceived set of technology parameters, relative investment functions in equation (16) are now determined by equations (8) and (9) such that skill accumulation depends on both "perceived" and "actual" technology parameters $(\breve{\omega}, \omega)$:

$$\Psi_{i,t+1} = \mathcal{H}\left(\underbrace{\tau_{m,i,t}\, f\left(1, \frac{1}{\Phi_{m,t}(\tilde{W}_{m,i,t}, \breve{\omega})}, \frac{\Phi_{x,t}(\tilde{q}_t, \tilde{W}_{m,i,t}, \breve{\omega})}{\Phi_{m,t}(\tilde{W}_{m,i,t}, \breve{\omega})} \big| Z_{i,t}, \omega\right)}_{\equiv \Phi_{I/\tau_m}(\tilde{q}_t, \tilde{W}_{m,i,t}, Z_{i,t}, \breve{\omega}, \omega)}, \Psi_{i,t}, \theta_{i,t}\right). \tag{17}$$

We make three important observations: First, as noted above, homogeneity of the "actual" $f(\cdot)$ simplifies the above expression by enabling $\tau_{m_i,t}$ to be "pulled out" of the function. Second, homogeneity of "perceived" $f(\cdot)$ simplifies investment functions such that they depend only on relative input prices, characteristics determining the relative productivity of investments $Z_{i,t}$, and perceived technology parameters $\breve{\omega}$. Third, given our nested-CES assumption for $f(\cdot)$, the functions $\Phi_{m,t}(\cdot)$ and $\Phi_{x,t}(\cdot)$ are known. Thus, $\breve{\omega}$ could be estimated in an earlier step using the relative demand estimation approach in Section 4.1.

[38] We emphasize that this approach imposes no structure on the investment functions except that *relative* input ratios (but not necessarily investment levels) depend on observed variables.

Equation (17) makes clear that sufficient variation in *relative* input prices (conditional on $Z_{i,t}$) is necessary to estimate "actual" elasticities of substitution as determined by (γ, ρ). If the perceived within-period production function is homogenous, then variation in family income or other factors that impact the desired level of investments should not influence relative input levels and, therefore, does not help in identifying input substitutability; although, it would help in identifying the productivity of total investment, I_t, as embodied in $\mathcal{H}(\cdot)$.[39]

If families are knowledgeable about the actual within-period production technology (i.e., $\breve{\omega} = \omega$), then we can estimate ω using relative demand estimation and only need to estimate skill dynamics in $\mathcal{H}(\cdot)$ using equation (17). Alternatively, both relative demand and skill dynamics can be used jointly to estimate ω and $\mathcal{H}(\cdot)$.

Assuming $\mathcal{H}(\cdot)$ is Cobb-Douglas (Assumption 3) and taking logs of equation (17) yields the following:

$$\ln(\Psi_{i,t+1}) = Z_{i,t}\phi_\theta + \delta_1 \ln\left(\tau_{m,i,t}\ \Phi_{I/\tau_m}(\tilde{q}_t, \tilde{W}_{m,i,t}, Z_{i,t}, \breve{\omega}, \omega)\right) + \delta_2 \ln(\Psi_{i,t}) + \xi_{\theta,i,t}, \tag{18}$$

which is a simple linear equation in unknown parameters $(\phi_\theta, \delta_1, \delta_2)$ when $\breve{\omega} = \omega$ is already known (or estimated from relative demand). This equation is identical to Cobb-Douglas dynamic specifications in the literature used to estimate the dynamics of skill accumulation as a function of a single latent "investment" and past skills, except that it uses a specific input, $\tau_{m,i,t}$, scaled by the ratio of total to maternal time investment, $\Phi_{I/\tau_m}(\cdot)$, derived from relative demand. Measurement error in (log) inputs and past skills can be addressed with an instrumental variables strategy, using secondary measures of inputs and period-t skills as instruments. If $\xi_{\theta,i,t}$ is correlated with $\tau_{m,i,t}$ or $\Psi_{i,t}$ (conditional on $Z_{i,t}$), then additional instruments may be needed to address endogeneity.

4.2.3 Missing Periods between Skill Measurements

Because inputs are only observed five years apart in the PSID-CDS data, we must impute them for intervening periods. To do this, we use the solution for optimal investment based on the two cases described in Section 3.2: non-binding borrowing constraints and no borrowing/saving. In the latter case, we assume preferences are given by $u(c) = \ln(c)$ and $v_j(l_j) = \psi_j \ln(l_j)$, which implies a simple characterization for investment dynamics. We show in Appendix E.4 that skill dynamics over a five-year

[39] While we assume relative inputs are determined by the intratemporal problem of Section 3.1, potentially with incorrect beliefs about $f(\cdot)$, estimated values for $\breve{\omega} \neq \omega$ could also reflect other forms of model misspecification. E.g., mothers may enjoy investment time with their children, distorting estimates of the perceived productivity of τ_m relative to other inputs.

period are given by:

$$\ln(\Psi_{i,t+5}) = \delta_1 \sum_{t=s}^{4} \delta_2^{4-s} \left[\ln\left(\frac{\bar{p}_{i,t}\tau_{m,i,t}\Phi_{I/\tau_m}(\tilde{q}_t, \tilde{W}_{m,i,t}, Z_{i,t}, \breve{\omega}, \omega)}{\bar{p}_{i,t+s}} \right) + \kappa \ln\left(\frac{W_{m,i,t+s} + W_{f,i,t+s} + y_{i,t+s}}{W_{m,i,t} + W_{f,i,t} + y_{i,t}} \right) \right]$$
$$+ Z_{i,t}\bar{\phi}_\theta + \delta_2^5 \ln(\Psi_{i,t}) + \bar{\xi}_{\theta,i,t+5}, \tag{19}$$

where $\kappa = 0$ reflects the unconstrained case and $\kappa = 1$ reflects the constrained case.[40] In the unconstrained case ($\kappa = 0$), the first log term on the right hand side reflects period $t + s$ log total investment, $\ln(I_{i,t+s})$, which is imputed from its period t value using relative composite input prices and then mapped to $\ln(\tau_{m,i,t})$ by the scaling factor $\Phi_{I/\tau_m}(\cdot)$. In the absence of borrowing and saving, investments also vary over time in proportion to full income, as reflected in the second log term.

We address measurement error in latent child skills by using two measures of cognitive skill available in the PSID-CDS, assuming that both scores are linear functions of $\ln(\Psi_{i,t})$ with mean zero measurement errors that are uncorrelated over time and across tests. As is standard, we normalize one factor loading and estimate the other. We use one score to proxy for log skills in Equation (19) and the other as an instrument for that score in forming our moments. We also include covariance restrictions on the two test scores in our set of moments. Because maternal time investment is noisily measured, we use $\ln(\tau^o_{m,i,t+5})$ to instrument for $\ln(\tau^o_{m,i,t})$. Other instruments include all $Z_{i,t}$ characteristics that affect $\theta_{i,t}$. We account for permanent unobserved heterogeneity in $\theta_{i,t}$ and relative productivity parameters using the group fixed effects approach of Bonhomme and Manresa (2015), taking advantage of the PSID's long panel of wages for mothers to classify families into types. See Appendix E for further details.

Our baseline estimates assume families are knowledgeable about the within-period skill production function (i.e., $\breve{\omega} = \omega$), combining relative demand moments with the skill production moments to conduct optimally weighted GMM. In this case, parameters of $f_t(\cdot)$ are jointly determined by the response of both inputs and skills to variation in relative input prices. We can also estimate ω and $\breve{\omega}$ without restricting that they are the same. Relaxing the cross-equation restrictions not only allows us to estimate both "perceived" and "actual" technology parameters, it also allows us to test whether they are equal. Of course, this requires considerable relative price variation in order to identify the production technology from the skill production equation alone. Appendix E.4 provides details on this generalized approach and describes 3 separate tests of the null hypothesis that $\omega = \breve{\omega}$ implemented below.

[40] Because the only time-varying $Z_{i,t+s}$ variable affecting $\theta_{i,t+s}$ in our empirical analysis is child's age, we write the cumulative effects of all $\ln(\theta_{i,t+s})$ as a linear function of $Z_{i,t}$. We also define $\bar{\xi}_{\theta,i,t+5} \equiv \sum_{s=0}^{4} \delta_2^{4-s}\xi_{\theta,i,t+s}$.

5 Data Sources and Construction

We construct a panel dataset on family work behavior, investment in children, and child outcomes from the PSID-CDS. The PSID is a dynastic, longitudinal survey taken annually from 1968 to 1997, and biennially since 1997. The main interview of this survey collects household-level data on economic and demographic variables. The CDS consists of three waves, collected in 1997, 2002, and 2007. The youngest two children in a PSID household between the ages of 0 and 12 at the time of the 1997 survey were considered eligible for interview in the supplement.

Our estimation approach requires merging these data with price variables for home-based goods and child care inputs. We construct these variables from several data sources, as described in Section 5.4.

5.1 Parental Investment and Child Outcomes from the PSID-CDS

In each wave of the CDS, a Primary Caregiver (PCG) is identified in the household with the child and PCG both completing modules of the survey.

Cognitive Outcomes. In all 3 survey waves, assessments of cognitive development were collected for children. We use the *Letter-Word (LW)* and *Applied Problems (AP)* scores from the Woodcock-Johnson battery of tests, which are completed by children ages 3 and older. We normalize the location and scales of these scores using their sample mean and standard deviation for children at age 12.

Time Investment. Measures of time investment come from time diaries completed by CDS children, with assistance from the PCG when necessary. This portion of the survey requires participants to record a detailed, minute-by-minute timeline of their activities for one random weekday and one random weekend day. For each recorded activity, an indicator is provided for whether the mother and/or father actively participated in the activity. Our measure of time investment for each parent is the weighted sum of time each parent actively participates in developmental and social activities with the child, with the weekday receiving weight 5/7 and the weekend day receiving weight 2/7. Notably, our measure excludes time spent with parents engaged in such activities as household chores and watching television. We construct these measures from the 1997, 2002, and 2007 time diaries as detailed in Appendix B.

Child Care Expenditure. In all 3 PCG interviews, respondents for children older than age 5 answer questions about current child care arrangements, costs, and time spent in each arrangement. For children younger than age 5, a retrospective history of arrangements is collected, from which we take all arrangements that are reported as ongoing. We construct a measure of weekly expenditures from these answers.

We also create a second measure from total household expenditures on child care from the main interview, divided by the number of children ages 12 or younger. Both measures are strongly correlated and have similar sample averages. Therefore, we use the average of these measures when both are reported, while we use the available measure when only one is reported. Among families with children ages 5–12 who report spending on child care, 60% report that their main arrangement is a before/after-school program, center-based care, or a non-relative family care provider. Only about 7% report non-relative care within the family's home (e.g., babysitters).

Goods Expenditures. In the 2002 and 2007 PCG interviews, respondents report annual expenditures for the child on school supplies and toys. They also report child participation in private lessons, sports, tutoring, or community groups, along with the costs of these activities. Our measure of market goods expenditures sums over the weekly values of spending on these goods and services.

5.2 Household Variables from the PSID Main Interview

From the main PSID interview, information is collected on household structure, state of residence, as well as the hours of work, earnings, race, and education of household members. We use mothers' childbirth history to construct the number and age of children in the household. While the PSID is available only biennially after 1997, earnings and hours for individuals in "missing" years are available from supplemental interviews after 1997. We combine these data to construct a panel of each mother's marital status, race, education, state of residence, work behavior, and wages through 2017.[41] When the mother is married, we use her spouse's age, education, and wages as measures for the father.

Using the large panel of wage data, we estimate parents' log wage fixed effects from (gender-specific) panel regressions of log wages on individual fixed effects, potential experience and experience-squared, year, and state dummies, where we drop all years with a child age 12 or younger in the household.[42] This effectively nets out differences in average wage rates across states and provides a measure of a parent's value in the labor market at the time he or she leaves school.

[41]We treat cohabiting couples as "married". Wages are imputed as annual earnings divided by annual hours, with the bottom and top 1% of observations set to missing.

[42]Potential experience is given by age - education - 6. Available wage observations from all PSID survey years up to 2017 are used. Whenever we control for these fixed effects in our analysis below, we only include parents with at least 5 wage observations available to estimate the fixed effect. After this restriction, the median number of wage observations used in estimating the fixed effects for both mothers and fathers in our sample is 10.

5.3 Sample Selection

We limit the sample to mothers ages 18–65 who were ages 16–45 in the child's birth year.[43] We consider only child-year observations for children ages 12 or younger, and for households with no more than 2 children ages 12 or younger. Our relative demand estimation of $f(\cdot)$ uses only the 2002 and 2007 surveys, because 1997 does not contain measures of household goods investments. Table 1 reports the characteristics of families for this sample. Because children in the CDS were born in 1997 or earlier, all children used in relative demand estimation are ages 5–12. As discussed below, when estimating the full production function, we also incorporate observations from the 1997 CDS for children ages 3–8.

Table 1: Summary statistics for full sample: 2002 and 2007

	Sample Size	Mean	SD	Min	Max
$\ln(\tilde{W}_{m,t})$	1110	2.44	0.66	-3.07	3.99
$\ln(\tilde{W}_{f,t})$	835	2.93	0.60	1.25	4.90
$\ln(\tilde{q}_t)$	1512	1.10	0.32	0.27	1.89
Child's age	1512	9.53	2.10	5.00	12.00
Mother HS grad	1510	0.33	0.47	0.00	1.00
Mother some coll.	1510	0.32	0.47	0.00	1.00
Mother coll+	1510	0.27	0.44	0.00	1.00
Mother's age	1512	37.56	6.43	21.00	55.00
Father HS grad	951	0.36	0.48	0.00	1.00
Father some coll.	951	0.22	0.42	0.00	1.00
Father coll+	951	0.33	0.47	0.00	1.00
Father's age	937	40.50	7.04	20.00	65.00
Mother white	1499	0.58	0.49	0.00	1.00
Num children age 0-5	1512	0.19	0.42	0.00	2.00
Num of children	1512	2.02	0.73	1.00	6.00
Year = 2007	1512	0.22	0.41	0.00	1.00

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children age 12 and under.

5.4 Price Variables

Relative demand estimation requires prices of child care, q, and prices of home-based goods, p. For child care prices, we draw from annual reports on the cost of child care in the U.S. compiled by Child Care Aware of America (2009–2019) to construct a state-level panel from 2006 to 2018 of hourly prices for 4 year-old care at licensed child care centers. We impute these prices back to 1997 using the average wages of child care workers in each state and year, along with state fixed effects and linear time trends, from the Current Population Survey. Details describing this imputation procedure are provided in Appendix C.

[43] We exclude children whose birth records indicate that they are adopted.

To construct the price of home-based goods, p, we combine data on *Regional Price Parities by State* provided by the Bureau of Economic Analysis, and the *Consumer Price Index* from the Bureau of Labor Statistics. Further details on the construction of this variable can also be found in Appendix C.

Our data collection results in a pair of prices (p, q) for each state and year, which are merged with our PSID-CDS panel using state of residence and calendar year for each PSID household. Variation in goods input prices, p, is modest; however, there is considerable variation in the price of child care across states. Our analysis mainly focuses on the price of child care (and wages) relative to the price of goods inputs. Appendix Figure G-4 shows the distributions of these (log) relative prices in our main sample.[44]

6 Child Production Function Estimates

This section presents estimates of child production functions for children from single- and two-parent homes based on the methods described in Section 4. Our analysis focuses on children ages 12 and under from families with only 1–2 children in that age range.

6.1 Relative Demand Estimates

Parental Time vs. Goods. Estimating relative demand for parental time vs. goods requires wage measures for parents; however, some parents do not work. To alleviate concerns about selection into work, we limit this estimation sample to parents with a relatively high predicted probability of working. To obtain predicted probabilities of work, we use data from 1997, 2002, and 2007 to estimate probit models for working during the year separately for single mothers, married mothers, and married fathers. These models control for the parent's age and education, mother's race, number of children and young children in the household, age of youngest child, and survey year. (See Appendix Table G-2 for estimates.) The median (first quartile) predicted probability of work is 0.81 (0.74) for mothers and 0.96 (0.90) for fathers. We restrict our analysis of relative demand for parental time vs. household goods to women (men) with a predicted probability of work no less than 0.75 (0.9). Unfortunately, this excludes most parents who did not finish high school, so we are unable to estimate the impacts of very low parental education on relative input productivity.[45]

Because the CDS did not collect comprehensive information on household goods expenditures on

[44] Most of the variation in $\tilde{q}$ is driven by differences across states; although, the average price in our sample rose by about 8 percentage points, on average, between 2002 and 2007.

[45] Appendix Table G-1 reports summary statistics (analogous to Table 1) for the subsample with a high predicted probability of work and for the subsample of those respondents with positive wages, time and goods inputs, and with non-missing covariates used in estimation. Table G-3 reports estimates from standard log wage regressions for workers with a high predicted probability of work. The estimates indicate that parents with a college degree earn 50–80% more than those with only a high school degree.

children in 1997, our relative demand estimation uses data from the 2002 and 2007 CDS surveys when children were ages 5–12. Like most of the literature estimating child production functions, we do not observe direct measures of school spending or quality, so our analysis implicitly assumes such differences enter child skill production in an input-neutral way.[46]

Columns (1)–(4) of Table 2 report OLS estimates for several specifications of Equation (13) for all mothers, accounting for potential determinants of the productivity of mother's time with children and/or household goods inputs. As discussed in Section 4, the coefficient on mother's relative log wages provides an estimate of $1 - \epsilon_{\tau,g}$, while coefficients on all other variables provide estimates of $\phi_{m,g} = \epsilon_{\tau,g}(\phi_m - \phi_g)$.

Specifications that condition on maternal age and education (columns (1)–(3)) indicate an elasticity of substitution between maternal time and household goods inputs, $\epsilon_{\tau,g}$, of 0.35–0.4, suggesting significantly stronger complementarity than the Cobb-Douglas case.[47] Column (4), which controls for mother's log wage fixed effects (rather than age and education) suggests a smaller elasticity closer to 0.25. Turning to estimates of $\phi_{m,g}$ and focusing on column (2), we find that mother's education has insignificant effects on the productivity of mother's time relative to household goods inputs. Older children and those with white mothers have a significantly lower factor share for mother's time relative to goods inputs compared to their younger counterparts and those with non-white mothers. While the factor share for mother's time relative to goods appears to be increasing in the number of children (especially young children) in the household, these effects are not statistically significant.[48] Column (4) shows that mothers with high log wage fixed effects (i.e., higher labor market productivity), have a low relative factor share for time compared to goods inputs. Given $\epsilon_{\tau,g} < 1$, this is consistent with factor augmentation where more-skilled mothers require less time investment to achieve the same effective time input.

Columns (5) and (6) of Table 2 report two-stage least squares (2SLS) estimates for time vs. goods relative demand, instrumenting for $\ln(\tilde{W}_{m,t})$. Column (5) uses a measure of predicted log wages from the 2000 Census as an instrument, capturing average differences in log wages by gender, race, education, experience, state, occupation, and industry.[49] Because column (5) includes controls for mother's race, education, and age, these 2SLS estimates mainly exploit variation in average wages across different

[46]Todd and Wolpin (2007) and Agostinelli, Saharkhiz, and Wiswall (2019) find that family inputs are much more important than school quality in explaining variation in skill growth across school-age children. To the extent that school quality is correlated with parental education and affects the productivity of some inputs more than others, these effects will be subsumed in our estimated effects of parental education.

[47]Throughout, we use a statistical significance level of 0.05 when indicating whether an estimate is statistically significant.

[48]In column (3), mother's cognitive ability has insignificant effects on relative demand and its inclusion has little impact on other estimated coefficients. Because over 15% of our sample does not have a reported score, we exclude it more generally.

[49]Predicted log wages are obtained from gender-specific regressions of log wages on an indicator for race (white/non-white), potential experience and experience-squared, educational attainment (<12 years, 12 years, 13–15 years, 16 years, 17+ years), 16 industry dummies, 97 occupation dummies (minor 2000 SOC codes), state dummies, interactions of race and education dummies with experience, and interactions of race and occupation dummies with state dummies.

Table 2: OLS and 2SLS estimates for mother time/goods relative demand

	(1) OLS	(2) OLS	(3) OLS	(4) OLS	(5) 2SLS (pred wage)	(6) 2SLS (state, year)
$\ln(\tilde{W}_{m,t})$	0.645*	0.646*	0.609*	0.758*	0.553*	0.749*
	(0.071)	(0.071)	(0.078)	(0.092)	(0.196)	(0.216)
Married	-0.075	-0.074	-0.121	0.022	-0.071	-0.069
	(0.095)	(0.095)	(0.104)	(0.108)	(0.096)	(0.095)
Child's age	-0.141*	-0.141*	-0.147*	-0.147*	-0.140*	-0.139*
	(0.022)	(0.022)	(0.025)	(0.024)	(0.022)	(0.022)
Mother HS grad	0.099					
	(0.350)					
Mother some coll.	0.106	0.011	-0.043		0.026	-0.018
	(0.351)	(0.102)	(0.117)		(0.113)	(0.117)
Mother coll+	-0.061	-0.157	-0.245		-0.119	-0.218
	(0.357)	(0.112)	(0.131)		(0.155)	(0.164)
Mother's age	-0.008	-0.008	-0.002		-0.007	-0.009
	(0.008)	(0.008)	(0.008)		(0.008)	(0.008)
Mother white	-0.244*	-0.243*	-0.095	-0.328*	-0.233*	-0.249*
	(0.090)	(0.089)	(0.107)	(0.102)	(0.091)	(0.090)
Num. of children ages 0-5	0.156	0.158	0.081	0.163	0.168	0.155
	(0.126)	(0.125)	(0.144)	(0.169)	(0.126)	(0.125)
Num. of children	0.089	0.089	0.090	0.027	0.082	0.097
	(0.062)	(0.062)	(0.068)	(0.066)	(0.063)	(0.063)
Mother's cognitive score			-0.005			
			(0.003)			
Mother's log wage FE				-0.346*		
				(0.114)		
Constant	2.126*	2.213*	2.602*	1.745*	2.398*	1.999*
	(0.469)	(0.355)	(0.449)	(0.366)	(0.520)	(0.553)
R-squared	0.190	0.190	0.167	0.193		
Sample size	727	727	603	562	720	727

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5–12 and only 1–2 children ages 12 and under. Only mothers with predicted probability of work of at least 0.75 included. See Table G-2 for model of predicted probability of work. Column (5) instruments for $\ln(\tilde{W}_{m,i})$ using predicted log wages from 2000 Census as instruments (see text for details), while Column (6) uses state and time indicators for instruments. * significant at 0.05 level.

states, occupations, and industries (including differences in the occupation wage structure across states). Appendix Table G-4 reports first-stage estimates for this instrument and for three related predicted log wage measures that begin with this measure and eliminate average differences across states, occupations, or both. Our main predicted log wage instrument has a first-stage F-statistic of 111.6 and produces a very similar elasticity of substitution (0.45) to its OLS counterpart in column (2). Appendix Tables G-4 and G-5 show that excluding average wage differences by state or by occupation from the instrument yields similar estimates. For comparison purposes, column (6) of Table 2 uses state and time dummies as instruments, assuming that the relative productivity of mother's time vs. household goods inputs is uncorrelated with state-level average wage rates conditional on household composition, maternal age and education. Unfortunately, the first-stage is relatively weak (F-statistic of 2.0), with the estimates implying an elasticity of substitution of 0.25. Both sets of 2SLS estimates produce very similar estimated effects of maternal and child characteristics compared to their OLS counterpart. Hausman tests do not reject equivalence of the 2SLS and OLS estimates.

Appendix Table G-6 shows that OLS and 2SLS estimates are very similar when using the full sample of mothers or those with a very high predicted probability of work (≥ 0.85). These results, plus the estimates from Table 2 column (4), which controls for log wage fixed effects, suggest that selection based on unobserved skill differences does not meaningfully affect our results.

In Table 3, we report estimates for the specification in column (2) of Table 2 separately for single and married mothers, as well as married fathers. The elasticity estimates are remarkably similar across parent types, ranging from 0.29 to 0.37. The effects of child's age on the factor share for parental time vs. household goods is significantly negative for all parent types. Parental education has modest and mostly insignificant effects; although, married mothers with a college degree appear to have a significantly lower factor share for time relative to goods compared to mothers with only a high school degree. Unfortunately, the estimated effects of education are imprecise, making it difficult to draw strong conclusions from the observed patterns. 2SLS estimates (using predicted log wages as an instrument) produce very similar results (see Appendix Table G-7); although, the elasticities of substitution are less precisely estimated and more variable across parent types, ranging from around 0.3 to 0.7. OLS estimates that control for parental log wage fixed effects rather than parental age and education produce elasticity estimates ranging from 0.25 to 0.3 (see Appendix Table G-8).[50]

Appendix Tables G-10, G-11, and G-12 explore heterogeneity in the relative demand for mother's

[50] We obtain similar results for married mothers when estimating a Heckman (1979) two-step selection model using father's age and education and/or child care prices as exclusion restrictions (determining work but not relative input productivity). See Appendix Table G-9.

Table 3: OLS estimates for parental time vs. goods relative demand, by parent type

	(1) All Mothers	(2) Single Mothers	(3) Married Mothers	(4) Married Fathers
$\ln(\tilde{W}_{j,t})$	0.646*	0.711*	0.628*	0.678*
	(0.071)	(0.155)	(0.079)	(0.090)
Married	-0.074			
	(0.095)			
Child's age	-0.141*	-0.162*	-0.132*	-0.107*
	(0.022)	(0.043)	(0.026)	(0.027)
Parent some coll.	0.011	0.198	-0.124	-0.130
	(0.102)	(0.173)	(0.128)	(0.131)
Parent coll+	-0.157	0.009	-0.269*	0.071
	(0.112)	(0.222)	(0.132)	(0.127)
Parent's age	-0.008	-0.014	-0.005	-0.010
	(0.008)	(0.014)	(0.009)	(0.008)
Mother white	-0.243*	-0.413*	-0.170	-0.053
	(0.089)	(0.167)	(0.107)	(0.123)
Num. of children age 0-5	0.158	-0.139	0.291*	0.148
	(0.125)	(0.239)	(0.147)	(0.134)
Num. of children	0.089	0.081	0.107	0.168*
	(0.062)	(0.109)	(0.076)	(0.080)
Constant	2.213*	2.471*	1.982*	1.282*
	(0.355)	(0.691)	(0.429)	(0.434)
R-squared	0.190	0.197	0.194	0.154
Sample size	727	236	491	582

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. Only mothers (fathers) with predicted probability of work of at least 0.75 (0.9) are included. See Table G-2 for model of predicted probability of work. Specifications for mothers (fathers) include mother's (father's) relative wage, education indicators, and age. * significant at 0.05 level.

time vs. goods by child's age, father's wage levels, and prior child achievement levels. These estimates suggest very similar relative demand for children ages 5–8 and 9–12 with nearly identical elasticities of substitution. The same is true for children with high- vs. low-wage fathers, providing support for the assumption that $f_t(\cdot)$ is homogeneous of degree 1, given higher investment levels among high-wage families. Finally, when looking across children with high vs. low prior achievement levels, we estimate weaker complementarity ($\epsilon_{\tau,g}$ of 0.67 vs. 0.32) for lower achieving students; however, a formal F-test fails to reject equality of the relative demand functions.

Nearly all of our estimates of the substitutability between parental time and household goods inputs – regardless of specification, estimation strategy, and parent type – imply an elasticity of 0.2–0.5, significantly stronger complementarity than implied by a Cobb-Douglas technology. The estimated effects of family characteristics on the relative factor shares for time vs. goods are also fairly robust.

Child Care vs. Goods. We next turn to relative demand for child care services vs. household goods inputs, Equation (14). (See its counterpart for two-parent households discussed in Appendix E.1.) An unfortunate practical problem arises here, because child care expenditures are frequently unreported or zero, even among families with parents working significant hours. (By contrast, parental time and household goods inputs are typically reported and positive.) To better understand who reports spending on child care, we estimate the effects of household characteristics and the price of child care on the probability of reporting positive expenditures. As shown in Appendix Table G-13, child care prices have negligible effects on whether someone reports positive expenditures, despite the fact that these prices significantly affect spending on child care relative to home goods inputs (among those who report positive spending) as we show below. We also find that more-educated parents are more likely to report positive child care spending, as are families without older children in the household. These findings are consistent with many families receiving some form of free child care from family (e.g., grandparents, older siblings) or friends.[51] Not only are we unable to determine the value of this free child care, but it is unlikely that families receiving such support satisfy the first order condition for child care services that is central to our relative demand approach. For these families, it seems likely that other constraints (e.g., grandparent time availability) determine the observed amount of child care. Our main analysis omits these families when estimating relative demand for child care vs. household goods; however, Appendix Table G-14 reports similar estimates from a standard Heckman (1979) selection model (using indicators for whether the household head lives in his/her birth state, whether an older relative lives in the household, and whether there are older children in the household as exclusion restrictions).[52]

We focus on relative demand for child care vs. household goods inputs using the sample of all household types (allowing for an intercept shift by marital status).[53] These results are reported in Table 4, which begins with two specifications that only condition on the relative price of child care and family characteristics, ignoring the potential influence of relative parental time vs. goods expenditures, $R_{m,i,t}$ and $R_{f,i,t}$. While these specifications are not generally valid unless $\epsilon_{\tau,g} = \epsilon_{x,H}$, they provide a useful benchmark. Indeed, estimated elasticities of substitution between child care services and the home composite input, $\epsilon_{x,H}$, are within the range of estimates for $\epsilon_{\tau,g}$ at around 0.4 (recall that the coefficient on $\ln(\tilde{q}_i)$ estimates $1 - \epsilon_{x,H}$). One concern with our state-level child care price measure is that variation

[51] Nearly one-quarter of families in our sample reported that relatives were their main source of child care.

[52] Appendix Table G-1 reports summary statistics for the sample of families with positive child care spending. We note that families with zero child care spending are still used in estimating the relative demand for parental time vs. household goods, because that tradeoff is unaffected by the availability of free (but presumably limited) external child care. Reassuringly, we cannot reject that relative demand for time vs. goods is the same for families reporting positive vs. zero child care spending.

[53] See Appendix Table G-15 for specifications estimated separately by marital status. Estimated elasticities of substitution, $\epsilon_{x,H}$, mainly range from 0.2 to 0.6 with slightly (but consistently) lower elasticities for single mothers.

across states may reflect differences in child care quality. To explore this possibility, column (2) includes a measure of required staff/child ratios by state and year from Garcia-Vazquez (2023). While this measure is strongly correlated with $\tilde{q}_{i,t}$ in our sample (correlation of 0.62), column (2) shows that it has negligible effects on relative demand, and its inclusion does not affect other parameter estimates.

Table 4: OLS estimates for child care/goods relative demand, all families

	(1)	(2)	(3)	(4)
$\ln(\tilde{q}_t)$	0.650*	0.598*	0.562*	0.518*
	(0.204)	(0.258)	(0.215)	(0.212)
Married	0.835	0.835	0.080	0.354
	(0.573)	(0.574)	(0.645)	(0.621)
Child's age	-0.124*	-0.121*	-0.077	-0.082
	(0.036)	(0.037)	(0.040)	(0.051)
Mother some coll.	-0.055	-0.062	0.099	0.017
	(0.161)	(0.163)	(0.175)	(0.169)
Mother coll+	-0.315	-0.315	-0.059	-0.216
	(0.169)	(0.169)	(0.181)	(0.179)
Mother's age	0.002	0.002	-0.002	0.002
	(0.013)	(0.013)	(0.015)	(0.014)
Marr. × Father some coll.	0.102	0.104	0.089	0.196
	(0.208)	(0.208)	(0.228)	(0.214)
Marr. × Father coll+	-0.435*	-0.439*	-0.716*	-0.616*
	(0.219)	(0.220)	(0.241)	(0.244)
Marr. × Father's age	-0.017	-0.017	-0.004	-0.011
	(0.015)	(0.015)	(0.017)	(0.015)
Mother white	-0.328*	-0.329*	-0.330*	-0.253
	(0.140)	(0.140)	(0.147)	(0.147)
Num. children ages 0-5	-0.030	-0.029	0.190	0.014
	(0.143)	(0.143)	(0.157)	(0.156)
Num. of children	0.095	0.094	0.129	0.104
	(0.112)	(0.112)	(0.115)	(0.117)
Staff/child ratio		1.760		
		(5.355)		
$\ln(1 + R_{m,t} + Marr. \times R_{f,t})$			0.498*	
			(0.076)	
$\ln(1 + e^{\tilde{\Phi}_{m,t}} + Marr. \times e^{\tilde{\Phi}_{f,t}})$				0.394
				(0.296)
Constant	0.933	0.875	-0.753	-0.304
	(0.591)	(0.617)	(0.690)	(1.146)
R-squared	0.125	0.126	0.296	0.130
Residual sum of squares	453.780	453.633	260.005	384.136
Sample size	347	347	249	310

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5–12 and only 1–2 children ages 12 and under. * significant at 0.05 level.

The remaining two columns of Table 4 account for the effects of relative parental time vs. goods expenditures on the relative demand for child care vs. goods when $\epsilon_{\tau,g} \neq \epsilon_{x,H}$. These specifications (based on Equations 14 and 15) yield very similar estimates for $\epsilon_{x,H}$, ranging from 0.4 to 0.5. Most family characteristics have insignificant effects on the demand for child care relative to goods (i.e., $\phi_{x,g}$). Notably, the estimates suggest weak and insignificant effects of mother's education on the relative factor share for childcare relative to goods inputs but stronger negative effects of father's education.[54]

Summary. Summarizing estimates based on relative demand, we find remarkable consistency across specifications in estimated elasticities of substitution: the elasticity between parental time and home goods inputs and the elasticity between the composite home input and child care both tend to range from 0.2 to 0.5, implying moderately strong complementarity. Most specifications suggest that the factor shares for household goods inputs relative to both child care services and maternal time inputs rise with children's age. We find no consistent patterns for the effects of parental post-secondary schooling on the relative productivity of different inputs.

6.2 Direct Estimation of Production using Skill Dynamics

Following the direct estimation approach described in Section 4.2.1, we use GMM to simultaneously estimate general relative investment functions, $\check{\Phi}_{g/\tau_m}(\cdot)$ and $\check{\Phi}_{x/\tau_m}(\cdot)$, along with equation (16) given our nested-CES specification for $f(\cdot)$ and Cobb-Douglas specification for $\mathcal{H}(\cdot)$.[55] We emphasize that this approach imposes no structure on the investment functions, except that *relative* input ratios (but not necessarily investment levels) depend on observed variables. See Appendix E.5 for details.

Unfortunately, this approach yields extremely imprecise (and numerically unstable) estimates for most parameters. Table G-24 presents bootstrapped 80% confidence intervals for the parameters of a simplified technology, demonstrating the impractically large sampling variance of this estimator.

Appendix F explores the failure of this method using a Monte Carlo simulation calibrated to match empirical levels of variation in prices and inputs, as well as the covariation in inputs and prices captured by our preferred demand estimates. This analysis highlights the difficulty of identifying CES parameters (especially the elasticity of substitution), even in the absence of measurement error and for larger sample sizes than available in the PSID-CDS. Appendix F and results in Tables F-1 and F-2 demonstrate that this poor performance is driven by the optimality of input demand, which induces correlations in inputs,

[54] Appendix Table G-16 explores differences in the relative demand for child care vs. goods separately for children ages 5–8 and 9–12. These results suggest weaker complementarity for younger children; however, sample sizes are small and equality of the specifications across children's ages cannot be rejected.

[55] We address the fact that child skills are noisily measured using a second measurement available in each period. Furthermore, skills are not measured every year, so we must "impute" input levels in years between skill measurements. We discuss this further below in the context of our preferred estimation approach; however, details for this "direct" estimation approach are in Appendix E.5.

and by insufficient variation in relative prices.[56]

6.3 GMM Estimation based on Relative Demand and Skill Dynamics

To estimate the full model, including both $f_t(\cdot)$ and $\mathcal{H}_t(\cdot)$, we now combine relative demand moments with skill production moments using optimally weighted GMM. In addition to the relative demand moments for mother's and father's time vs. goods inputs and child care vs. goods inputs in 2002 and 2007 (underlying OLS and 2SLS estimation in Section 6.1), we also include similar relative demand moments for child care vs. mother's time in 1997, because both of these measures are available in that survey year. To calculate the skill production moments (from 1997 to 2002 and 2002 to 2007), we only include observations for children ages 3–8 (in the earlier years) whose mothers are observed working in each intervening year.[57]

To maintain stability in estimation of the full production function using GMM (as $\rho \to 0$ or $\gamma \to 0$), we normalize $f_t(\cdot)$ so that the productivity constants sum to one within the home input nesting and between that nesting and child care. This means that coefficient estimates on household characteristics, denoted $(\tilde{\phi}_m, \tilde{\phi}_f, \tilde{\phi}_x)$, for the input productivity constants are not directly comparable to their counterparts (ϕ_m, ϕ_f, ϕ_x) in the previous subsection. The new normalization is reported in the notes of Table 5, which presents our main GMM results. Production moments are constructed from Equation (19), which nests our two extreme assumptions of no borrowing/saving ($\kappa = 1$) and unconstrained choices ($\kappa = 0$). Both cases are reported for our preferred specification, which uses a group fixed effects approach (Bonhomme and Manresa, 2015), classifying households based on mothers' wages as described in Appendix E.3. We assume three types, labeling them in ascending order based on their group wage fixed effect (μ_k).[58]

Estimates and standard errors are very similar across the no borrowing/saving and unconstrained cases. Estimated elasticities of substitution are within the ranges of the earlier estimates based on relative demand reported in Section 6.1; however, they are more precise, indicating stronger substitutability between child care and home inputs than between parental time and goods inputs with $\epsilon_{\tau,g} = 0.2$ and $\epsilon_{x,H} = 0.5$.[59] The estimated δ_1 in the no borrowing/saving case implies that a log-point increase in

[56] See Proposition 3 and the surrounding discussion on the importance of sufficient price variation for identification of input substitutability parameters.

[57] Achievement scores are not available for children younger than 3, and those older than 8 will be older than 12 five years later. To use the outcome equation from 1997 to 2002, we require all prices (including wages) between 1997 and 2002; likewise for 2002–2007. When moments include parental time, we continue to restrict samples to those whose parents have a high predicted probability of work; although, the main conclusions are unchanged when this restriction is dropped.

[58] These results are robust to alternative specifications with and without education and unobserved type indicators. See Appendix Tables G-18 and G-19.

[59] The improved precision derives from cross-equation restrictions from estimating all relative demand equations jointly rather than the inclusion of skill production moments. Appendix Table G-17 presents estimates for $f_t(\cdot)$ when all relative demand moment conditions across years and inputs are combined and estimated using GMM. The first specification is

Table 5: Joint GMM Estimation with Demand and Production Moments

	$\epsilon_{\tau,g}$		$\epsilon_{x,H}$		δ_1		δ_2	
	$(\kappa=0)$	$(\kappa=1)$	$(\kappa=0)$	$(\kappa=1)$	$(\kappa=0)$	$(\kappa=1)$	$(\kappa=0)$	$(\kappa=1)$
	0.20^{+}	0.20^{+}	0.50	0.50	0.13	0.17	0.83	0.83
	(0.05)	(0.05)	(0.09)	(0.09)	(0.03)	(0.04)	(0.02)	(0.02)
	$\tilde{\phi}_m$: Mother's Time		$\tilde{\phi}_f$: Father's Time		$\tilde{\phi}_x$: Childcare		$\overline{\phi}_\theta$: TFP	
	$(\kappa=0)$	$(\kappa=1)$	$(\kappa=0)$	$(\kappa=1)$	$(\kappa=0)$	$(\kappa=1)$	$(\kappa=0)$	$(\kappa=1)$
Constant	8.33	8.22	4.25	4.13	−1.21	−1.19	1.30	1.10
	(1.94)	(1.90)	(1.28)	(1.27)	(0.41)	(0.41)	(0.42)	(0.47)
Single	0.08	0.10	-	-	0.61	0.61	-0.15	-0.20
	(0.37)	(0.37)	-	-	(0.21)	(0.21)	(0.06)	(0.07)
Type 2	−0.96	−0.96	-	-	0.08	0.08	0.04	-0.05
	(0.55)	(0.55)	-	-	(0.30)	(0.30)	(0.08)	(0.09)
Type 3	−2.41	−2.41	-	-	0.10	0.10	-0.30	-0.43
	(0.93)	(0.93)	-	-	(0.31)	(0.31)	(0.11)	(0.14)
Mother some coll.	−0.52	−0.50	-	-	0.00	0.01	0.10	0.05
	(0.46)	(0.45)	-	-	(0.20)	(0.20)	(0.07)	(0.07)
Mother coll+	−1.74	−1.71	-	-	−0.22	−0.23	-0.03	-0.08
	(0.74)	(0.74)	-	-	(0.19)	(0.19)	(0.09)	(0.11)
Child's age	−0.61	−0.60	−0.49	−0.48	−0.07	−0.07	-0.13	-0.13
	(0.18)	(0.17)	(0.18)	(0.17)	(0.03)	(0.03)	(0.03)	(0.04)
Num. of children 0-5	0.50	0.46	0.66	0.64	0.07	0.06	0.08	0.06
	(0.30)	(0.30)	(0.42)	(0.42)	(0.12)	(0.12)	(0.04)	(0.05)
Father some coll.	-	-	−1.08	−1.04	-0.01^{+}	-0.00^{+}	0.01	-0.01
	-	-	(0.72)	(0.72)	(0.25)	(0.25)	(0.07)	(0.07)
Father coll+	-	-	−0.98	−0.95	−0.64	−0.63	0.22	0.17
	-	-	(0.68)	(0.68)	(0.24)	(0.23)	(0.07)	(0.08)
Year = 2002	-	-	-	-	-	-	-0.34	-0.30
	-	-	-	-	-	-	(0.05)	(0.06)

Notes: This table reports GMM estimates of the full production function assuming either no binding borrowing constraints ($\kappa = 0$) or no borrowing/saving ($\kappa = 1$). The superscript $^{+}$ indicates, using a Lagrange Multiplier test, rejection at 5% significance of the null hypothesis that an individual parameter enters identically in the demand and production moments. See Appendix E.4 for more details. The function f is specified as: $f = \left[\left(\tilde{a}_{m,i,t}\tau^{\rho}_{m,i,t} + \tilde{a}_{f,i,t}\tau^{\rho}_{f,i,t} + (1-\tilde{a}_{m,i,t}-\tilde{a}_{f,i,t})g^{\rho}_{i,t}\right)^{\gamma/\rho}(1-\tilde{a}_{x,i,t}) + \tilde{a}_{x,i,t}x^{\gamma}_{i,t}\right]^{1/\gamma}$ with $\tilde{a}_{j,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_j)}{1+\exp(Z_{i,t}\tilde{\phi}_m)+\exp(Z_{i,t}\tilde{\phi}_f)}$, $j \in \{m,f\}$ and $\tilde{a}_{x,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_x)}{1+\exp(Z_{i,t}\tilde{\phi}_x)}$. The function $\mathcal{H}$ is specified as $\mathcal{H}(f,\Psi_{i,t},\theta_{i,t}) = \theta_{i,t}f^{\delta_1}\Psi^{\delta_2}_{i,t}$ with $\theta_{i,t} = \exp(Z_{i,t}\phi_\theta + \xi_{\theta,i,t})$. Estimation is based on a sample of $N = 1,365$ children that contribute to at least one moment.

investment raises cognitive skills by 0.17 standard deviations (as measured by the LW score), while the unconstrained case implies a slightly smaller impact. Both estimates of δ_2 are 0.83, indicating strong persistence in skills (i.e., self-productivity).

Next, we relax the assumption that the technology determining relative demand is the same as that determining skill production (e.g., families may hold incorrect beliefs). For each specification, we conduct Lagrange Multiplier tests (Newey and McFadden, 1994) for whether each individual parameter in $f_t(\cdot)$ determining input ratios equals its counterpart in the production function (i.e., whether each $\breve{\omega} = \omega$). The superscript $^+$ indicates rejection of equality for individual parameters at 5% significance. In both cases, we reject individual equality restrictions on only two parameters, the elasticity of substitution between home inputs ($\epsilon_{\tau,g}$) and the contribution of fathers' education to the factor share on childcare. Tables G-20 and G-21 report results when restrictions on parameters that fail the LM test are relaxed. These estimates indicate that when relaxed, the null hypothesis ($\breve{\omega} = \omega$) cannot be rejected based on the difference between parameters (Wald test), nor the change in the GMM criterion (distance metric test). To address concerns that relative demand may be affected by parental preferences for spending time with children, we also consider relaxing the intercept on mothers' factor share in the relative demand equation, finding no evidence to reject the restriction (Appendix Tables G-22 and G-23).

Finally, we highlight several points regarding the estimated share parameters ($\tilde{\phi}_m, \tilde{\phi}_f, \tilde{\phi}_x$). First, our estimates suggest similar relative productivity of parental time and goods investments for married vs. single parents, with a higher factor share of market child care for single mothers. Second, the factor share of mother's time is decreasing in both her education and unobserved type. Given elasticities of substitution less than one, these estimates are consistent with maternal skills improving the effective value of time and reducing its factor share through factor augmentation. We explore the practical implications of this for investment gaps by parental education in the next section. Third, older children have a lower relative productivity of parental time and higher relative productivity of home goods inputs; they also have a lower factor share for child care relative to home inputs.

7 Counterfactual Analysis: Explaining Investment Behavior

In this section, we use our model and the GMM estimates for the case of no borrowing/saving (reported in Table 5) to study key factors driving family investment decisions. In particular, we investigate the

identical to those of Table 5, but excludes moments related to skills and their measurements. Parameters related to $f_t(\cdot)$ are very similar. Specification (2) in Table G-17 uses predicted log wages from the 2000 Census as an instrument for $\ln(\tilde{W}_{m,t})$ and $\ln(\tilde{W}_{f,t})$, analogous to the 2SLS estimates in column (5) of Table 2. Finally, Table G-17 reports that we cannot reject zero correlation between the relative demand residuals across years, suggesting no further role for persistent heterogeneity in factor shares or preferences given our classification procedure.

sources of investment gaps across families and the role of technology in determining investment responses to price changes and to the provision of free child care.[60]

These simulation exercises require calibration of preference parameters (α, ψ_m, ψ_f), which we allow to vary by parental education to fit average maternal time investment levels and parental hours worked depending on whether mothers had attended college or not. We note that preference parameters only affect the levels of investments, not relative input shares nor the composite price of investment. See Appendix D for details on the calibration and simulations, as well as results based on estimates for unconstrained households.

7.1 Family Differences in Investment

There is considerable variation in investment expenditures across families. Studies of the dynamics of skill production commonly assume a single price of "investment", which implicitly attributes all variability in investment expenditures to differences in investment quantities. Yet, we find that the composite price of investment, $\bar{p}_{i,t}$, varies considerably across families. Using our estimated technology and input prices to construct $\bar{p}_{i,t}$ for each child in the 2002 PSID, we find that one-quarter (for single mothers) to one-third (for two-parent families) of the variance in log investment expenditures is explained by variation in log composite prices, which is, in turn, driven almost entirely by differences in parental wages.

Section 2 shows that more-educated parents spend substantially more on investments in their children than less-educated parents. We use our estimates to determine the extent to which these gaps are driven by systematic differences in the productivity of investments, parental wages (i.e., resources), or preferences for children's skills. The first column of Table 6 shows average gaps in total investment expenditures, prices, and quantities for families with college-educated mothers, relative to those with non-college mothers, as implied by the model. Single college-educated mothers spend about 50% more on child investments compared to their non-college-educated counterparts. Among two-parent households, college-educated mothers spend double their less-educated counterparts. Although higher prices explain some of the difference, there are also considerable differences in total investment levels, I_t: college-educated single mothers invest 33% more than their non-college-educated counterparts, while this gap is nearly 40% for two-parent households. The second column shows that accounting for parental wage differences reduces the total investment gap by about one-quarter for single mothers and one-fifth for married mothers. The additional family income associated with higher wages dominates the effects of higher investment prices when it comes to investing in children. The last column shows that the gap for single mothers drops by

[60]These counterfactual exercises exclude families with zero child care expenditures from 2002 PSID, because they are not used for estimation when we rely on log expenditure ratios.

Table 6: Investment Gaps (% Difference) between College and Non-College Mothers (No Borrowing/Saving)

	Baseline	Equalizing:			
		Wages	All Prices	Technology	Preferences
A. Single Mothers					
Total Investment					
Expenditure (E)	47.67	17.91	17.91	47.67	38.49
Price ($\bar{p}$)	13.54	-6.69	-2.35	19.09	13.54
Quantity (I)	33.09	24.60	21.47	28.28	24.88
Mother's Time Investment (τ_m)	23.75	19.39	17.53	23.56	16.11
B. Two-Parent Households					
Total Investment					
Expenditure (E)	95.12	31.26	31.26	95.12	39.73
Price ($\bar{p}$)	42.48	1.59	-0.03	44.98	42.48
Quantity (I)	37.81	30.95	32.82	34.65	-1.27
Mother's Time Investment (τ_m)	26.97	25.13	26.03	31.24	-9.11

about 25%, while the gap for two-parent families is entirely eliminated when only preference parameters are equalized across education groups. Preference parameter differences may reflect different inherent valuations of skill or different views about the value of children's skills later in life (e.g., for college-going or in the labor market).[61] Notably, the impacts of equalizing preferences or wages dwarf the changes induced by equalizing the skill production technology (reported in the fourth column), which accounts for only 15% of the investment gap for single mothers and less than 10% of the gap for two-parent households. Altogether, Table 6 shows that the sizeable investment gaps by maternal education are largely driven by differences in family resources and preferences for (or beliefs about) children's skill development, rather than differences in the productivity of investments. These results further imply that differences in the productivity of investments (including the modest differences in $\theta_{i,t}$ reported in Table 5) explain only a small share of skill growth gaps by maternal education.

7.2 Input Price Changes

Because many policies designed to encourage investments in children (e.g. child care subsidies), as well as many tax and welfare policies, primarily influence family investment decisions through changes in input prices, we next consider the impacts of reducing these prices when children are ages 5–12. In particular, we simulate the effects of separately reducing each input price by 30%.

The first four columns of Table 7 report effects of lowering prices using our estimated nested CES

[61] Higher investments among college-educated families are also consistent with higher perceived returns to investment, δ_1, among these families (Boneva and Rauh, 2018; Cunha, Elo, and Culhane, 2022). While we cannot reject that beliefs about $f_t(\cdot)$ are accurate, we are unable to separately identify beliefs about the dynamics of skills ($\theta_t, \delta_1, \delta_2$) from their true productivity values. We only estimate the latter.

Table 7: Effects of 30% Reduction in Input Prices (No Borrowing/Saving)

	Nested CES				Cobb-Douglas			
	Wages	Wages (Constant income)	Goods	Child Care	Wages	Wages (Constant income)	Goods	Child Care
A. Single Mothers								
Change in Investment at Age 5 (%):								
Total Expenditure (E)	-30.00	0.00	0.00	0.00	-30.00	0.00	0.00	0.00
Mother's Time (τ_m)	-5.63	34.81	1.22	3.79	0.00	42.86	0.00	0.00
Goods (g)	-11.96	25.77	8.67	3.64	-30.00	0.00	42.86	0.00
Child Care (x)	-20.31	13.84	0.66	23.81	-30.00	0.00	0.00	42.86
Total (I)	-9.59	29.16	1.36	7.62	-9.17	29.75	1.58	8.39
Effects on Age 13 Achievement:								
100×Log Achievement (Age 13)	-8.56	19.08	2.01	5.27	-7.94	19.70	2.28	5.65
Value (% Cons. over Ages 5–12)	-5.40	13.29	1.32	3.48	-5.02	13.76	1.50	3.74
B. Two-Parent Households								
Change in Investment at Age 5 (%):								
Total Expenditure (E)	-30.00	0.00	0.00	0.00	-30.00	0.00	0.00	0.00
Mother's Time (τ_m)	-3.21	38.27	0.74	2.00	0.00	42.86	0.00	0.00
Father's Time (τ_f)	-3.14	38.37	0.73	1.95	0.00	42.86	0.00	0.00
Goods (g)	-9.74	28.94	8.16	1.89	-30.00	0.00	42.86	0.00
Child care (x)	-18.64	16.23	0.43	21.85	-30.00	0.00	0.00	42.86
Total (I)	-5.55	34.93	0.87	4.02	-5.13	35.53	1.00	4.36
Effects on Age 13 Achievement:								
100×Log Achievement (Age 13)	-4.83	22.81	1.24	2.76	-2.10	25.54	3.73	5.30
Value (% Cons. over Ages 5–12)	-2.02	10.31	0.53	1.18	-1.15	11.28	1.33	2.00

production function, where the second column reduces wages while holding full income constant (to isolate the pure price effect for comparison with other inputs). Focusing on these columns, we see that a change in the price of any one input causes all inputs to adjust in the same direction due to input complementarity. Cross-price elasticities are substantially weaker than own-price elasticities, but not negligible. For example, a 30% reduction in the price of child care, leads to a 22–24% increase in child care inputs and a 2–4% increase in parental time and home goods inputs. To better understand the role of input complementarity, the last four columns of Table 7 report analogous results using a Cobb-Douglas production function instead of our nested CES.[62] With the Cobb-Douglas specification, changes in home goods or child care prices induce no cross-price effects, and own-input quantities strongly adjust to maintain constant expenditure shares and total expenditure levels. The much stronger own-price responses in the Cobb-Douglas case would lead one to over-predict the costs associated with subsidies for specific inputs. For example, incorrectly assuming a Cobb-Douglas technology would over-state the costs of a 30% subsidy for goods inputs (child care) targeted to single mothers by 31% (15%).

Column (1) shows that the effects of a wage reduction are notably different when full income is not

[62]For comparability, the share parameters of the Cobb-Douglas function are calibrated to match the same expenditure shares as our estimated specification. See Appendix D for details.

held constant. Proposition 1 states that investment expenditures must decline with a reduction in family resources; however, we see an even stronger result: investment levels, even time investments, also fall.

Table 7 also reports the effects of input price changes on child achievement measured at age 13. Due to well-known issues regarding interpretability of test score scales (Cunha, Heckman, and Schennach, 2010), we focus on the consumption equivalent value of the changes in achievement, measured as the percent increase in consumption over ages 5–12 that would make a family indifferent to the change in achievement. For single mothers, increases in investment associated with a 30% reduction in the price of child care (home goods) would raise child achievement at age 13 by an amount valued at 3.5% (1.3%) of ages 5–12 family consumption. The consumption equivalent values of achievement gains are roughly one-third as large for two-parent households. Despite the small share of investment spending devoted to home goods and child care, subsidies for these investments still have important effects on achievement.

While there are sizeable differences in specific input responses to price changes when comparing our estimated nested CES specification with the Cobb-Douglas specification, impacts on total investment and log achievement are fairly similar. As Proposition 3 shows, the effects of marginal changes in input prices on total investment depend on expenditure shares but not elasticities of substitution. Appendix Table D-3 demonstrates that the substitutability of inputs becomes increasingly important as larger price changes are considered. Due to weaker complementarity, the Cobb-Douglas specification over-predicts total investment responses to the 30% input price reductions in Table 7 by up to 17%.

Appendix D.2 considers the case of unconstrained families. As theory predicts (see Appendix A.3.3), a permanent change in wages has effects on investment inputs that are very similar to those for constrained households. By contrast, a temporary wage reduction leads to an increase in parental time investment, because the substitution effect dominates the more modest effect on lifetime income. The increased time investment is paired with an increase in goods investments at the expense of lower child care, and total investment increases over the (temporary) period of reduced wages. Thus, the effects of time-limited policies that impact wage rates depend critically on the extent to which families can borrow and save.

7.3 Free Child Care for Non-College Single Mothers

To further explore the importance of input complementarity for policy, we ask how much it would cost to provide free child care to non-college mothers to raise their total investment levels (over child ages 5–12) to those of their college-educated counterparts. (See Appendix D.3 for details.) Due to complementarity and other investment input responses, we find that a reasonably priced free child care program (costing $115/week) would close the total investment gap. This would produce a modest 0.26 standard deviation improvement in the age-13 skills of children with non-college mothers, preventing additional growth in the skill gap already present at age 5.[63] The public cost of such a program would be modest, because families respond to the savings from access to free child care by increasing their own time and home goods inputs; absent this response, we find that it would be prohibitively costly to eliminate investment gaps with free child care alone.

Assuming a Cobb-Douglas within-period technology, the cost of eliminating the total investment gap would be quite similar to that based on our nested CES specification; however, other inputs respond much less due to the weaker complementarity. As a result, the Cobb-Douglas specification suggests that it would cost about 1/3 less (rather than substantially more) to eliminate the investment gap if private investments were held fixed. This suggests that the substitutability of inputs is critical to understanding endogenous private input responses to free care and whether they lessen or exacerbate the ultimate costs of such a policy.

The complementary response of other inputs reduces public expenditures but comes at a cost to families. For single mothers, this cost is manageable, given their savings on child care. For two-parent households, the story is quite different. Because they spend less on child care to begin with, the beneficial income effects of free care are smaller. As a result, non-college two-parent families are unwilling to increase other investments enough to go along with reasonable amounts of free child care, making it prohibitively costly to eliminate education gaps in total investment by providing free child care alone. By contrast, a Cobb-Douglas specification suggests that it would "only" cost about $330/week to eliminate the gap.

8 Conclusions

Parents spend considerable sums investing in their children in terms of their time, home goods/services inputs, and market-based child care services. We document that more-educated parents spend substantially more on all of these inputs, with parental time the most costly form of investment for all family types. To understand these patterns and to study the impacts of policies that act on input prices, we use a dynamic model of investments in children with multiple inputs each period, flexibility in substitution patterns across those inputs, and several channels through which parental skills may affect the productivity of those inputs.

We show conditions under which the decision problem separates into an intratemporal problem, de-

[63] Few studies evaluate the impacts of free or heavily subsidized child care for school-age children. Seidlitz and Zierow (2020), Schmitz (2022), and Drange and Sandsør (2024) are notable recent exceptions, all finding zero to small (positive) impacts of free center-based after-school child care in Germany and Norway on the cognitive outcomes of 6–10 year olds. While these effects are broadly consistent with our simulation results, it is important to note that our simulations are based on families with positive child care expenditures. The impacts of introducing free public child care on families that rely on family/friends for child care could be larger or smaller depending on the reasons for their choice of care.

termining the relative demand for inputs, and an intertemporal problem, determining the total amount of investment each period. This separation proves useful in estimation of the skill production technology. If relative input choices are driven primarily by relative prices and technology, then relative demand restrictions provide powerful identifying information, fully characterizing the substitutability of investment inputs and their relative productivity under reasonable assumptions on preferences and weak intertemporal separability of skill production.[64] Panel data on skills are not needed to identify these features of the within-period technology, and measurement error in inputs is easily addressed. Combining data on investment inputs and their prices with skill measures over time, we show that one can also exploit relative demand restrictions to simplify estimation of the dynamics of skill production. More generally, with external estimates of relative demand relationships and data on skill accumulation, relative input prices, and only one of many inputs, researchers can follow our approach to estimate features determining the dynamics of skill formation. We also show how restrictions implied by intertemporal optimization (given assumptions about credit markets) can be used to impute missing inputs during periods between observed skill measures. Finally, we show how to relax the assumption that the (perceived) technology driving relative demand is the same as that determining skill production, allowing for the possibility that families hold incorrect beliefs about the development process. This generalization also allows for the possibility that parents place inherent value on specific investment inputs.

Using PSID-CDS data, we find robust evidence that parental time and purchased goods inputs are complements inside the home, while home investments are also (slightly less) complementary with market child care services. Elasticities of substitution range from 0.2 to 0.5, implying much stronger complementarity than implied by the commonly used Cobb-Douglas specification. Our estimates suggest modest effects of parental education on the technology of skill production for 5–12 year old American children. The substantial investment gaps between children of college- vs. high school-educated parents are driven mainly by differences in overall demand for investments, stemming from differences in preferences for children's skills (or perceived returns to investment) and family resources. From a policy perspective, this is, perhaps, the best case scenario, because it suggests that less-educated parents do not suffer from an inability to translate investments into skills.

Our counterfactual policy analysis illustrates the likely impacts of a wide array of policies that affect incentives to invest in various forms, from welfare and tax policies to free child care. First, and most importantly, we find that the estimated patterns of complementarity for investment inputs imply that all inputs move together with any price change; although, cross-price elasticities are generally modest.

[64]By contrast, if differences in relative input choices are driven mainly by heterogeneity in tastes, then relative demand will incorporate these preferences, and a direct estimation approach is needed to identify the actual production technology.

Second, we find that the income effects of changes in wage rates dominate the price effects for credit constrained families. Due to enhanced family resources, policies that subsidize parents' wages would lead to improvements in child achievement that are consistent with estimated effects of EITC expansions on child achievement (Dahl and Lochner, 2012; Agostinelli and Sorrenti, 2020). Third, despite the fact that investment expenditures are dominated by parental time investments, subsidies for home goods/services and child care can produce important effects on child achievement. Fourth, we show that modest spending to finance free child care for non-college parents could eliminate total investment gaps by parental education among single mothers, while eliminating gaps among two-parent households would be prohibitively costly. This sharp difference arises because single mothers reinvest substantial savings from initial outlays on child care to reinforce the policy, while the savings for two-parent households is too modest to adequately finance other matching investments. Finally, we show that incorrectly assuming a Cobb-Douglas specification (which assumes weaker complementarity than we estimate) for the within-period technology of skill production would lead to an over-estimate of own-input responses to price subsidies and, therefore, an over-estimate of their public costs. A Cobb-Douglas technology also predicts stronger total investment and achievement responses to large price changes; however, the substitutability of inputs is unimportant for these outcomes when examining small input price changes.

In this paper, we aim to shed new light on the complex set of decisions parents make regarding how and when to invest in their children. A key challenge is data. The PSID-CDS is the only single data set we are aware of that contains all the input and achievement measures our analysis requires. Yet, the useable sample sizes are small, and both achievement and inputs are infrequently measured. Future research in this area should endeavor to make better use of multiple data sets that may specialize in subsets of needed measures but which contain much larger samples or richer measures of specific inputs or outcomes. Ideally, younger (pre-school) children would be examined, and schooling inputs incorporated for older children, further integrating the literatures on child development and school quality as in Todd and Wolpin (2007) and Agostinelli, Saharkhiz, and Wiswall (2019). Our effort to separate estimation of different parameters into different stages may prove helpful in this regard. Another path forward would be to combine results from several natural or actual experiments, connecting marginal policy effects to primitive parameters of the child production function and/or preferences. Chaparro, Sojourner, and Wiswall (2020) and Mullins (2020) take productive steps in this direction. Richer data may also enable researchers to build on our approach that combines relative demand restrictions with a more flexible production technology using data on both inputs and outcomes (e.g., Cunha, Heckman, and Schennach, 2010; Agostinelli and Wiswall, 2016, 2023) to learn more about parental beliefs vs. the actual productivity of different inputs or the extent to which parents value some inputs (e.g., time with children) over others.[65]

References

Abbott, Brant. 2022. "Incomplete Markets and Parental Investments in Children." Review of Economic Dynamics 44:104–124.

Agostinelli, Francesco, Matthias Doepke, Giuseppe Sorrenti, and Frabrizio Zilibotti. 2020. "It Takes a Village: The Economics of Parenting with Neighborhood and Peer Effects." NBER Working Paper No. 27050.

Agostinelli, Francesco, Morteza Saharkhiz, and Matthew Wiswall. 2019. "Home and School in the Development of Children." NBER Working Paper No. 26037.

Agostinelli, Francesco and Giuseppe Sorrenti. 2020. "Money vs. Time: Familiy Income, Maternal Labor Supply, and Child Development." Working Paper.

Agostinelli, Francesco and Matthew Wiswall. 2016. "Identification of Dynamic Latent Factor Models: The Implications of Re-Normalization in a Model of Child Development." NBER Working Paper No. 22441.

———. 2023. "Estimating the Technology of Children's Skill Formation." Working Paper.

Altonji, Joseph G. 1986. "Intertemporal substitution in labor supply: Evidence from micro data." Journal of Political Economy 94 (3, Part 2):S176–S215.

Arrow, Kenneth J, Hollis B Chenery, Bagicha S Minhas, and Robert M Solow. 1961. "Capital-labor substitution and economic efficiency." Review of Economics and Statistics :225–250.

Attanasio, Orazio, Sarah Cattan, Emla Fitzsimons, Costas Meghir, and Marta Rubio-Codina. 2020. "Estimating the production function for human capital: Results from a randomized controlled trial in Colombia." American Economic Review 110 (1):48–85.

Attanasio, Orazio, Costas Meghir, and Emily Nix. 2020. "Human capital development and parental investment in India." Review of Economic Studies 87 (6):2511–2541.

Attanasio, Orazio, Costas Meghir, Emily Nix, and Francesca Salvati. 2017. "Human capital growth and poverty: Evidence from Ethiopia and Peru." Review of Economic Dynamics 25:234–259.

Becker, Gary S and Nigel Tomes. 1979. "An equilibrium theory of the distribution of income and intergenerational mobility." Journal of Political Economy 87 (6):1153–1189.

———. 1986. "Human Capital and the Rise and Fall of Families." Journal of Labor Economics 4 (3, Part 2):S1–S39.

Berndt, Ernst R. 1976. "Reconciling alternative estimates of the elasticity of substitution." Review of Economics and Statistics :59–68.

[65] See Cunha, Elo, and Culhane (2022) for innovative research on misperceptions using elicited beliefs from parents.

Blundell, Richard and Ian Walker. 1986. "A life-cycle consistent empirical model of family labour supply using cross-section data." Review of Economic Studies 53 (4):539–558.

Bolt, Uta, Eric French, Jamie Hentall-MacCuish, and Cormac O'Dea. 2023. "Intergenerational Altruism and Transfers of Time and Money: A Life Cycle Perspective." Working paper, IFS.

———. 2024. "The Intergenerational Elasticity of Earnings: Exploring the Mechanisms." Working paper, IFS.

Boneva, Teodora and Christopher Rauh. 2018. "Parental beliefs about returns to educational investments—the later the better?" Journal of the European Economic Association 16 (6):1669–1711.

Bonhomme, Stéphane and Elena Manresa. 2015. "Grouped Patterns of Heterogeneity in Panel Data." Econometrica 83.

Brilli, Ylenia. 2022. "Mother's time allocation, child care and child cognitive development." Journal of Human Capital 16 (2):232–272.

Caucutt, Elizabeth, Lance Lochner, Youngmin Park, and Joseph Mullins. 2020. "Child skill production: Accounting for parental and market-based time and goods investments." Working Paper.

Caucutt, Elizabeth M. and Lance Lochner. 2020. "Early and late human capital investments, borrowing constraints, and the family." Journal of Political Economy 128 (3):1065–1147.

Caucutt, Elizabeth M, Lance Lochner, and Youngmin Park. 2017. "Correlation, consumption, confusion, or constraints: Why do poor children perform so poorly?" The Scandinavian Journal of Economics 119 (1):102–147.

Chaparro, Juan, Aaron Sojourner, and Matthew Wiswall. 2020. "Early Childhood Care and Cognitive Development." NBER Working Paper No. 26813.

Child Care Aware of America. 2009–2019. "The US and the High Price of Child Care." Arlington, VA.

Cunha, Flávio, Irma Elo, and Jennifer Culhane. 2022. "Maternal subjective expectations about the technology of skill formation predict investments in children one year later." Journal of Econometrics 231 (1):3–32.

Cunha, Flavio and James Heckman. 2007. "The Technology of Skill Formation." American Economic Review 97 (2):31–47.

Cunha, Flavio and James J. Heckman. 2008. "Formulating, identifying and estimating the technology of cognitive and noncognitive skill formation." Journal of Human Resources 43 (4):738–782.

Cunha, Flavio, James J Heckman, Lance Lochner, and Dimitriy V Masterov. 2006. "Interpreting the evidence on life cycle skill formation." Handbook of the Economics of Education 1:697–812.

Cunha, Flavio, James J. Heckman, and Susanne M. Schennach. 2010. "Estimating the Technology of Cognitive and Noncognitive Skill Formation." Econometrica 78 (3):883–931.

Dahl, Gordon and Lance Lochner. 2012. “The Impact of Family Income on Child Achievement: Evidence from the Earned Income Tax Credit.” American Economic Review 102 (5):1927–1956.

Del Boca, Daniel, Christopher Flinn, Ewout Verriest, and Matthew Wiswall. 2023. “Parenting with Patience: Parental Incentives and Child Development.” Working paper, IZA.

Del Boca, Daniela, Christopher Flinn, and Matthew Wiswall. 2014. “Household choices and child development.” Review of Economic Studies 81 (1):137–185.

———. 2016. “Transfers to Households with Children and Child Development.” Economic Journal 126 (596):F136–F183.

Del Bono, Emilia, Marco Francesconi, Yvonne Kelly, and Amanda Sacker. 2016. “Early Maternal Time Investment and Early Child Outcomes.” Economic Journal 126 (596):F96–F135.

Drange, Nina and Astrid Marie Jorde Sandsør. 2024. “The effects of a free universal after-school program on child academic outcomes.” Economics of Education Review 98:102504.

Falk, Armin, Fabian Kosse, Pia Pinger, Hannah Schildberg-Hörisch, and Thomas Deckers. 2021. “Socioeconomic status and inequalities in children’s IQ and economic preferences.” Journal of Political Economy 129 (9):2504–2545.

Fiorini, Mario and Michael P Keane. 2014. “How the allocation of children’s time affects cognitive and noncognitive development.” Journal of Labor Economics 32 (4):787–836.

Garcia-Vazquez, Martin. 2023. “The Equilibrium Effects of State-Mandated Minimum Staff-to-Child Ratios.” Working Paper.

Gayle, George-Levi, Limor Golan, and Mehmet A Soytas. 2015. “What Accounts for the Racial Gap in Time Allocation and Intergenerational Transmission of Human Capital?” Working Paper.

Gelber, Alexander and Adam Isen. 2013. “Children’s schooling and parents’ behavior: Evidence from the Head Start Impact Study.” Journal of Public Economics 101:25–38.

Gorman, William M. 1959. “Separable utility and aggregation.” Econometrica 27 (3):469–481.

Griffen, Andrew S. 2019. “Evaluating the Effects of Childcare Policies on Children’s Cognitive Development and Maternal Labor Supply.” Journal of Human Resources 54 (3):604–655.

Griliches, Zvi. 1969. “Capital-skill complementarity.” Review of Economics and Statistics :465–468.

Guryan, Jonathan, Erik Hurst, and Melissa Kearney. 2008. “Parental Education and Parental Time with Children.” Journal of Economic Perspectives 22 (3):23–46.

Heckman, James J. 1974. “Effects of child-care programs on women’s work effort.” Journal of Political Economy 82 (2, Part 2):S136–S163.

———. 1979. “Sample selection bias as a specification error.” Econometrica :153–161.

Kaushal, Neeraj, Katherine Magnuson, and Jane Waldfogel. 2011. "How is Family Income Related to Investments in Children's Learning?" In Whither Opportunity? Rising Inequality, Schools, and Children's Life Chances, edited by Greg Duncan and Richard Murnane, chap. 9. New York: Russell Sage Foundation, 187–206.

Lee, Sang Yoon and Ananth Seshadri. 2019. "On the intergenerational transmission of economic status." Journal of Political Economy 127 (2):855–921.

Leibowitz, Arleen. 1974. "Education and home production." American Economic Review 64 (2):243–250.

MaCurdy, Thomas E. 1983. "A simple scheme for estimating an intertemporal model of labor supply and consumption in the presence of taxes and uncertainty." International Economic Review :265–289.

Maddala, GS and Joseph B Kadane. 1966. "Some notes on the estimation of the constant elasticity of substitution production function." Review of Economics and Statistics :340–344.

Michael, Robert T. 1973. "Education in nonmarket production." Journal of Political Economy 81 (2, Part 1):306–327.

Molnar, Timea Laura. 2023. "Costs of Daycare, Complementarities, and Heterogeneous Productivity of Parenting Time in Child Skill Formation." Working Paper.

Moschini, Emily. 2023. "Child Care Subsidies and Child Skill Accumulation in One- and Two-Parent Families." American Economic Journal: Macroeconomics 15 (1):475–516.

Mullins, Joseph. 2020. "A Structural Meta-analysis of Welfare Reform Experiments and their Impacts on Children." Working Paper.

———. 2022. "Designing Cash Transfers in the Presence of Children's Human Capital Formation." Working Paper.

Nerlove, Marc. 1965. Estimation and Identification of Cobb-Douglas Production Functions. Amsterdam: North-Holland.

———. 1967. "Recent Empirical Studies of the CES and Related Production Functions." In The Theory and Empirical Analysis of Production. NBER, 55–136.

Newey, Whitney K and Daniel McFadden. 1994. "Large sample estimation and hypothesis testing." Handbook of econometrics 4:2111–2245.

Newey, Whitney K and James L Powell. 2003. "Instrumental variable estimation of nonparametric models." Econometrica 71 (5):1565–1578.

Pavan, Ronni. 2016. "On the production of skills and the birth-order effect." Journal of Human Resources 51 (3):699–726.

Schmitz, Laura. 2022. "Heterogeneous effects of after-school care on child development." DIW Berlin Discussion Paper.

Seidlitz, Arnim and Larissa Zierow. 2020. "The Impact of All-Day Schools on Student Achievement-Evidence from Extending School Days in German Primary Schools." CESifo Working Paper No. 8618.

Todd, Petra and Kenneth I. Wolpin. 2007. "The Production of Cognitive Achievement in Children: Home, School and Racial Test Score Gaps." Journal of Human Capital 1 (1):91–136.

Online Appendix for "Child skill production: Accounting for parental and market-based time and goods investments"

Elizabeth Caucutt, Lance Lochner, Joseph Mullins, and Youngmin Park

A Analytical Issues

A.1 Separating the household's problem into an intratemporal and intertemporal problem

This appendix focuses on the case in which both parents work (i.e., $h_{m,t} > 0$ and $h_{f,t} > 0$). It also considers the family decision problem under uncertainty about children's future abilities or future parental wages and income. Importantly, this uncertainty has no effect on the intratermporal problem of subsection A.2. Under our main assumptions, uncertainty about children's ability also has no effect on the intertemporal problem of subsection A.3. In the absence of borrowing constraints, uncertainty about future parental wages and income would affect consumption and, therefore, total investment behavior due to precautionary savings motives; however, such uncertainty would not affect decisions during periods in which families are borrowing constrained. We briefly discuss these implications for our characterization of intertemporal decision making (under full certainty) in subsection A.3.

A.1.1 Full problem

The household's problem for periods $t = 1, ..., T$, is given by:

$$\begin{aligned} &V_t(\theta_t, H_m, H_f, A_t, y_t, \Pi_t, \Psi_t) \\ &\quad = \max_{l_{m,t}, l_{f,t}, \tau_{m,t}, \tau_{f,t}, g_t, x_t, A_{t+1}} u(c_t) + v_m(l_{m,t}) + v_f(l_{f,t}) + \beta \mathbb{E}_t V_{t+1}(\theta_{t+1}, H_m, H_f, A_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{t+1}) \end{aligned}$$

subject to non-negative inputs $(\tau_{m,t}, \tau_{f,t}, g_t, x_t)$, $l_{j,t} \geq 0$ and $l_{j,t} + \tau_{j,t} \leq 1$ for $j = m, f$, child human capital production equation (1),

$$\begin{aligned} c_t + W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t} + p_t g_t + q_t x_t + A_{t+1} &= (1+r)A_t + y_t + W_{m,t}(1 - l_{m,t}) + W_{f,t}(1 - l_{f,t}), \\ A_{t+1} &\geq A_{min,t}, \\ V_{T+1}(\theta_{T+1}, H_m, H_f, A_{T+1}, y_{T+1}, \Pi_{T+1}, \Psi_{T+1}) &= \tilde{V}(H_m, H_f, A_{T+1}, \Psi_{T+1}). \end{aligned}$$

We assume $u'(\cdot) > 0$, $u''(\cdot) < 0$, $v_j'(\cdot) > 0$, and $v_j''(\cdot) \leq 0$, $j = m, f$. We also assume standard Inada conditions for preferences over consumption and leisure and that $\tilde{V}$ is strictly increasing and strictly concave in child skill and parental assets.[66] Expectations at time t, denoted by $\mathbb{E}_t$, implicitly integrate over future realizations of children's ability, parental wages, and family income conditional on the current state.

Suppose both parents work in the market, $l_{j,t} + \tau_{j,t} < 1$, $j = m, f$. Let λ_t be the Lagrange multiplier on the period t budget constraint and ξ_t be the Lagrange multiplier on the period t borrowing constraint.

[66] The continuation value, $\tilde{V}$, also depends on all future non-labor income, which we suppress here for ease of notation.

The first order conditions for c_t, $\tau_{j,t}$, g_t, x_t, $l_{j,t}$, A_{t+1}, $j = m, f$ are:

$$\begin{aligned}
\lambda_t &= u'(c_t), && (20)\\
\lambda_t W_{j,t} &= \beta \frac{\partial \mathbb{E}_t V_{t+1}}{\partial \Psi_{t+1}} \frac{\partial \mathcal{H}_t}{\partial f_t} \frac{\partial f_t}{\partial \tau_{j,t}}, \quad j = m, f, && (21)\\
\lambda_t p_t &= \beta \frac{\partial \mathbb{E}_t V_{t+1}}{\partial \Psi_{t+1}} \frac{\partial \mathcal{H}_t}{\partial f_t} \frac{\partial f_t}{\partial g_t}, && (22)\\
\lambda_t q_t &= \beta \frac{\partial \mathbb{E}_t V_{t+1}}{\partial \Psi_{t+1}} \frac{\partial \mathcal{H}_t}{\partial f_t} \frac{\partial f_t}{\partial x_t}, && (23)\\
v_j'(l_{j,t}) &= \lambda_t W_{j,t}, \quad j = m, f, && (24)\\
\lambda_t + \xi_t &= \mathbb{E}_t \left[\lambda_{t+1} \beta (1+r)\right]. && (25)
\end{aligned}$$

Combining the first order conditions for consumption and leisure yields the standard result that the marginal rate of substitution equals the wage rate:

$$v'(l_{j,t}) = u'(c_t) W_{j,t}, \;\; j = m, f. \tag{26}$$

We also have:

$$\begin{aligned}
\lambda_t(c_t + p_t g_t + q_t x_t + A_{t+1} - (1+r)A_t - y_t - W_{m,t}(1 - l_{m,t} - \tau_{m,t}) - W_{f,t}(1 - l_{f,t} - \tau_{f,t})) &= 0, && (27)\\
\xi_t(A_{t+1} - A_{min,t}) &= 0. && (28)
\end{aligned}$$

Note that if a parent does not work, the cost of child time investment is measured by the value of lost leisure, and $v_j'(l_{j,t}) = \beta \frac{\partial \mathbb{E}_t V_{t+1}}{\partial \Psi_{t+1}} \frac{\partial \mathcal{H}_t}{\partial f_t} \frac{\partial f_t}{\partial \tau_{j,t}}$, $j = m, f$.

A.1.2 Intratemporal problem

For $h_{m,t} > 0$ and $h_{f,t} > 0$, the intratemporal problem minimizes expenditures, given I_t:

$$\min_{\tau_{m,t}, \tau_{f,t}, g_t, x_t} W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t} + p_t g_t + q_t x_t$$

subject to non-negative inputs $(\tau_{m,t}, \tau_{f,t}, g_t, x_t)$, $\tau_{m,t} < 1$, $\tau_{f,t} < 1$, and $I_t = f_t(\tau_{m,t}, \tau_{f,t}, g_t, x_t; H_m, H_f)$. Let $\bar{p}_t$ be the Lagrange multiplier on this constraint. The first order conditions for $\tau_{j,t}$, g_t, and x_t, $j = m, f$ are:

$$\begin{aligned}
W_{j,t} &= \bar{p}_t \frac{\partial f_t}{\partial \tau_{j,t}}, \quad j = m, f, && (29)\\
p_t &= \bar{p}_t \frac{\partial f_t}{\partial g_t}, && (30)\\
q_t &= \bar{p}_t \frac{\partial f_t}{\partial x_t}. && (31)
\end{aligned}$$

Substitute these first order conditions into the minimand:

$$E_t = \bar{p}_t \left[g_t \frac{\partial f_t}{\partial g_t} + x_t \frac{\partial f_t}{\partial x_t} + \tau_{m,t} \frac{\partial f_t}{\partial \tau_{m,t}} + + \tau_{f,t} \frac{\partial f_t}{\partial \tau_{f,t}} \right].$$

Because $f_t(\tau_{m,t}, \tau_{f,t}, g_t, x_t)$ is homogenous of degree 1 (Constant Returns to Scale), we have:

$$I_t = f_t(\tau_{m,t}, \tau_{f,t}, g_t, x_t) = \frac{\partial f_t}{\partial g_t} g_t + \frac{\partial f_t}{\partial \tau_{m,t}} \tau_{m,t} + \frac{\partial f_t}{\partial \tau_{f,t}} \tau_{f,t} + \frac{\partial f_t}{\partial x_t} x_t,$$

and, $E_t = \bar{p}_t I_t$.

A.1.3 Intertemporal problem

Suppose in every period, $t = 1, ..., T$, along with leisure and assets, the household chooses an amount of child investment I_t, given a per period composite price $\bar{p}_t$. This problem can be written as follows:

$$V_t(\theta_t, H_m, H_f, A_t, y_t, \Pi_t, \Psi_t) = \max_{l_{m,t}, l_{f,t}, I_t, A_{t+1}} u(c_t) + v(l_{m,t}) + v(l_{f,t}) + \beta \mathbb{E}_t V_{t+1}(\theta_{t+1}, H_m, H_f, A_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{t+1})$$

subject to $0 \leq l_{m,t}, l_{f,t} \leq 1$, $I_t \geq 0$,

$$\begin{aligned}
c_t + \bar{p}_t(\Pi_t, H_m, H_f) I_t + A_{t+1} &= (1+r)A_t + y_t + W_{m,t}(1 - l_{m,t}) + W_{f,t}(1 - l_{f,t}), \\
\Psi_{t+1} &= \mathcal{H}_t\left(I_t, \theta_t, \Psi_t\right), \\
A_{t+1} &\geq A_{min,t}, \\
V_{T+1}(\theta_{T+1}, H_m, H_f, A_{T+1}, y_{T+1}, \Pi_{T+1}, \Psi_{T+1}) &= \tilde{V}(H_m, H_f, A_{T+1}, \Psi_{T+1}).
\end{aligned}$$

The first order conditions for c_t, $l_{j,t}$, I_t, A_{t+1}, $j = m, f$ are:

$$\lambda_t = u'(c_t), \tag{32}$$

$$v_j'(l_{j,t}) = \lambda_t W_{j,t}, \quad j = m, f, \tag{33}$$

$$\lambda_t \bar{p}_t = \beta \frac{\partial \mathbb{E}_t V_{t+1}}{\partial \Psi_{t+1}} \frac{\partial \mathcal{H}_t}{\partial I_t}, \tag{34}$$

$$\lambda_t + \xi_t = \mathbb{E}_t\left[\lambda_{t+1} \beta (1+r)\right]. \tag{35}$$

We also have:

$$\lambda_t(c_t + \bar{p}_t(\Pi_t, H_m, H_f) I_t + A_{t+1} - (1+r)A_t - y_t - W_{m,t}(1 - l_{m,t}) - W_{f,t}(1 - l_{f,t})) = 0, \tag{36}$$

$$\xi_t(A_{t+1} - A_{min,t}) = 0. \tag{37}$$

Comparing first order conditions, we see the separated problem has first order Conditions (32), (33), (35), and (37) corresponding to the full problem Conditions (20), (24), (25), and (28). If we substitute $\bar{p}_t I_t = p_t g_t + q_t x_t + W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t}$, into Condition (36), we have Condition (52). Lastly, noting that $I_t = f_t$, substituting $\bar{p}_t$ from Conditions (29), (30), and (31), separately into Condition (34), yields the full problem Conditions (21), (22), and (23).

A.2 Characterizing the Intratemporal Problem

Given the static nature of the intratemporal problem, we drop time t subscripts throughout this subsection. Because none of the results in this subsection depend on future values of child abilities, parental wages, or family income, uncertainty about their values also plays no role.

A.2.1 Parental skill neutrality

Notice that if $f(\tau_m, \tau_f, g, x; H_m) = f(\tau_m H_m, \tau_f H_f, g, x)$, then we can re-write equations (4) and (5) as follows:

$$\begin{aligned}
\tilde{w}_m \equiv \frac{w_m}{p} &= \frac{f_1(\Phi_m H_m, \Phi_f H_f, 1, \Phi_x)}{f_3(\Phi_m H_m, \Phi_f H_f, 1, \Phi_x)} \\
\tilde{w}_f \equiv \frac{w_f}{p} &= \frac{f_2(\Phi_m H_m, \Phi_f H_f, 1, \Phi_x)}{f_3(\Phi_m H_m, \Phi_f H_f, 1, \Phi_x)} \\
\tilde{q} \equiv \frac{q}{p} &= \frac{f_4(\Phi_m H_m, \Phi_f H_f, 1, \Phi_x)}{f_3(\Phi_m H_m, \Phi_f H_f, 1, \Phi_x)},
\end{aligned}$$

where $f_j(\cdot)$ reflects the partial derivative with respect to argument j. From these 3 equations, we can solve for "effective" input ratios $\Phi_m H_m$, $\Phi_f H_f$, and Φ_x as functions of relative prices $(\tilde{w}_m, \tilde{w}_f, \tilde{q})$ and the technology $f(\cdot)$. Clearly, then, relative expenditure ratios $\tilde{w}_m H_m \Phi_m$, $\tilde{w}_f H_f \Phi_f$, and $\tilde{q}\Phi_x$ depend only on relative prices – and not parental human capital levels – as well. Because none of the relative expenditure ratios depend on parental human capital levels, expenditure shares must also be constant in parental human capital.

A.2.2 Some results for w_m and H_m with CES

Normalizing $\bar{a}_m = \bar{a}_g = a_x = 1$, we have the following for single mothers:

$$\begin{aligned}
f(\tau_m, g, x; H_m) &= \left[([\varphi_m(H_m)\tau_m]^\rho + [\varphi(H_m)g]^\rho)^{\gamma/\rho} + x^\gamma \right]^{1/\gamma} \\
\Phi_m &= \left[\frac{\varphi_g(H_m)}{\varphi_m(H_m)} \right]^{\rho/(\rho-1)} \tilde{W}_m^{1/(\rho-1)} \\
\Phi_x &= \varphi_g(H_m)^{\frac{\rho}{\gamma-1}} \left[(\varphi_m(H_m)\Phi_m)^\rho + \varphi_g(H_m)^\rho \right]^{\frac{\gamma-\rho}{\rho(\gamma-1)}} \tilde{P}_c^{\frac{1}{\gamma-1}}.
\end{aligned}$$

If we define elasticities $\bar{\varphi}_j = \varphi_j'(H_m)H_m/\varphi_j(H_m)$ for $j = m, g$, then

$$\frac{\partial \Phi_m}{\partial w_m} = -\left(\frac{1}{1-\rho} \right) \Phi_m w_m^{-1} \tag{38}$$

$$\frac{\partial \Phi_m}{\partial H_m} = -\left(\frac{1}{1-\rho} \right) \Phi_m H_m^{-1} [1 + \rho(\bar{\varphi}_g - \bar{\varphi}_m)] \tag{39}$$

$$= \frac{\partial \Phi_m}{\partial w_m} \frac{w_m}{H_m} [1 + \rho(\bar{\varphi}_g - \bar{\varphi}_m)] \tag{40}$$

Thus, the ratio of mother's time to goods inputs, Φ_m, does not depend on H_m if $\bar{\varphi}_g = 0$ and $\bar{\varphi}_m = 1/\rho$.

Next, consider the ratio of child care to goods inputs, letting $\chi(H_m) \equiv [\varphi_m(H_m)\Phi_m(H_m)]^\rho + \varphi_g(H_m)^\rho$:

$$\begin{aligned}
\frac{\partial \Phi_x}{\partial H_m} &= \left(\frac{\rho}{\gamma-1} \right) \Phi_x \bar{\varphi}_g H_m^{-1} + \left(\frac{\gamma-\rho}{\rho(\gamma-1)} \right) \Phi_x \frac{\chi'(H_m)}{\chi} \\
&= \left(\frac{\rho}{\gamma-1} \right) \Phi_x \bar{\varphi}_g H_m^{-1} + \left(\frac{\gamma-\rho}{(1-\gamma)(1-\rho)} \right) \Phi_x H_m^{-1} \left[1 + \rho\bar{\varphi}_g - \bar{\varphi}_m - \frac{\varphi_g^\rho(1-\bar{\varphi}_m-\bar{\varphi}_g)}{(\varphi_m\Phi_m)^\rho + \varphi_g^\rho} \right], \qquad (41)
\end{aligned}$$

where the second equality uses equation (39). When parental skills do not affect the productivity of goods inputs (i.e., $\bar{\varphi}_g = 0$), this simplifies considerably to

$$\frac{\partial \Phi_x}{\partial H_m} = \left(\frac{\gamma-\rho}{(1-\gamma)(1-\rho)} \right) \left[\frac{(\varphi_m\Phi_m)^\rho}{(\varphi_m\Phi_m)^\rho + \varphi_g^\rho} \right] \Phi_x H_m^{-1}(1-\bar{\varphi}_m)$$

In this case, the ratio of child care services to goods inputs, Φ_x, does not depend on H_m if $\gamma = \rho$ or $\bar{\varphi}_m = 1$.

Now, consider the effects of H_m on the ratios of expenditures (or expenditure shares):

$$\begin{aligned}
\frac{\partial(\tilde{W}_m\Phi_m)}{\partial H_m} &= w_m\Phi_m + \tilde{W}_m \frac{\partial \Phi_m}{\partial H_m} \\
&= -w_m\Phi_m \left(\frac{\rho}{1-\rho} \right) (1 + \bar{\varphi}_g - \bar{\varphi}_m) \\
\frac{\partial(\tilde{q}\Phi_x)}{\partial H_m} &= \tilde{q} \frac{\partial \Phi_x}{\partial H_m},
\end{aligned}$$

where $\frac{\partial \Phi_x}{\partial H_m}$ is given by equation (41). If $\bar{\varphi}_g = 0$ and $\bar{\varphi}_m = 1$, then the ratio of expenditures for any pair of inputs does not depend on H_m, in which case expenditure shares are also independent of maternal human capital. Regardless of $\bar{\varphi}_m$ and $\bar{\varphi}_g$, the ratio of expenditures on maternal time relative to goods inputs does not depend on H_m if $\rho = 0$, while the ratio of expenditures on child care relative to home goods inputs does not depend on H_m if $\rho = \gamma = 0$. Thus, if $\rho = \gamma = 0$ (i.e., $f_t(\cdot)$ is Cobb-Douglas in all inputs), all expenditure shares are independent of H_m.

A.2.3 Comparative statics results for expenditure shares

For simplicity, we consider the case of single mothers and drop all time subscripts (as we focus on within-period relationships), so

$$f = \left[(a_m \tau_m^\rho + a_g g^\rho)^{\gamma/\rho} + a_x x^\gamma \right]^{1/\gamma}. \tag{42}$$

Total investment expenditures are $E = pg + qx + W_m \tau_m = g\,(p + q\Phi_x + W_m \Phi_m)$, where the latter follows from Equations (8) and (9). Expenditure shares are given by:

$$S_g \equiv \frac{pg}{E} = \frac{p}{p + q\Phi_x + W_m \Phi_m}, \quad S_{\tau_m} \equiv \frac{W_m \tau_m}{E} = \frac{W_m \Phi_m}{p + q\Phi_x + W_m \Phi_m}, \quad S_x \equiv \frac{qx}{E} = \frac{q\Phi_x}{p + q\Phi_x + W_m \Phi_m},$$

where Φ_m and Φ_x are implicitly defined by Equations (8) and (9). Throughout this subsection of the Appendix, define $D \equiv p + q\Phi_x + w_m H_m \Phi_m$.

The following proposition characterizes the effects of child care prices on expenditure shares.

Proposition 4. *If and only if $\gamma < 0$, then q has strictly positive own-price effects on S_x and strictly negative cross-price effects on S_g and S_{τ_m}.*

Proof of Proposition 4: We can differentiate shares with respect to q:

$$\frac{\partial S_g}{\partial q} = \frac{\gamma p \Phi_x}{(1-\gamma) D^2}, \qquad \frac{\partial S_\tau}{\partial q} = \frac{\gamma w_m H_m \Phi_m \Phi_x}{(1-\gamma) D^2}, \qquad \frac{\partial S_x}{\partial q} = \frac{-\gamma [pg + w_m H_m \tau] \Phi_x}{(1-\gamma) D^2}.$$

The stated results in Proposition 4 are immediate from these derivatives. □

Given the nested nature of $f(\cdot)$, the impacts of price changes on home inputs g and τ_m are slightly more complicated, though symmetric.

Proposition 5. *Expenditure shares on home inputs (g or τ_m) are strictly decreasing in their own price (p or w_m) if $\min\{\rho, \gamma\} > 0$ and strictly increasing in their own price if $\max\{\rho, \gamma\} < 0$. Expenditure shares on home inputs are strictly decreasing in the other home input price if $\rho < \min\{0, \gamma\}$, and strictly increasing in the other home input price if $\rho > \max\{0, \gamma\}$. The expenditure share on market child care services is strictly increasing in the price of both home inputs if and only if $\gamma > 0$.*

Proof of Proposition 5: We can differentiate expenditure shares with respect to p:

$$\begin{aligned}
\frac{\partial S_g}{\partial p} &= \frac{-\left\{\rho(1-\gamma)[q\Phi_x a_m \Phi_m^\rho + w_m H_m \Phi_m (a_m \Phi_m^\rho + a_g)] + \gamma(1-\rho) q \Phi_x a_g\right\}}{(1-\gamma)(1-\rho)[a_m \Phi_m^\rho + a_g] D^2} \\
\frac{\partial S_\tau}{\partial p} &= \frac{w_m H_m \Phi_m \left\{p\rho(1-\gamma)[a_m \Phi_m^\rho + a_g] + q\Phi_x(\rho - \gamma) a_g\right\}}{p(1-\rho)(1-\gamma)[a_m \Phi_m^\rho + a_g] D^2} \\
\frac{\partial S_x}{\partial p} &= \frac{\gamma q \Phi_x a_g \left\{p + w_m H_m \Phi_m\right\}}{p(1-\gamma)[a_m \Phi_m^\rho + a_g] D^2},
\end{aligned}$$

and with respect to w_m:

$$\begin{aligned}
\frac{\partial S_g}{\partial w_m} &= \frac{p\left\{q\Phi_x(\rho-\gamma)a_m\Phi_m^\rho + \rho w_m H_m \Phi_m(1-\gamma)[a_m\Phi_m^\rho + a_g]\right\}}{D^2 w_m(1-\gamma)(1-\rho)[a_m\Phi_m^\rho + a_g]} \\
\frac{\partial S_\tau}{\partial w_m} &= -\frac{H_m\Phi_m\left\{p\rho(1-\gamma)[a_m\Phi_m^\rho + a_g] + q\Phi_x[\gamma(1-\rho)a_m\Phi_m^\rho + \rho(1-\gamma)a_g]\right\}}{(1-\rho)(1-\gamma)[a_m\Phi_m^\rho + a_g]D^2} \\
\frac{\partial S_x}{\partial w_m} &= \frac{\gamma q p \Phi_x a_m \Phi_m^\rho}{a_g w_m(1-\gamma)D^2}.
\end{aligned}$$

The stated results in Proposition 5 are immediate from these derivatives. □

Complementarity between both home inputs ($\rho < 0$) and between the home composite input and market child care ($\gamma < 0$) ensures that substitution out of a home input whose price rises is insufficient to compensate for the higher price, leading to a greater expenditure share on that input. If home inputs are not only complementary ($\rho < 0$) but also more complementary than home inputs with market child care ($\rho < \gamma$), then an increase in the price of one home input will cause the expenditure share of the other to fall. The converse of these statements applies when inputs are substitutes. Finally, substitutability between home and market inputs ($\gamma > 0$) implies that an increase in either home input will raise the share of expenditures on child care, while complementarity ($\gamma < 0$) implies the opposite.

In considering the role of parental skills, we assume the following convenient functional forms:

$$\varphi_m(H_m) = H_m^{\bar{\varphi}_m} \qquad \text{and} \qquad \varphi_g(H_m) = H_m^{\bar{\varphi}_g}, \tag{43}$$

where the exponents $\bar{\varphi}_m \geq 0$ and $\bar{\varphi}_g \geq 0$ determine the returns to scale of parental human capital in the production of child skills. Note that the $\bar{\varphi}_j$ here correspond to the elasticities in Section A.2.2. The overall implications of $\bar{\varphi}_g > 0$ on expenditure shares is most transparent when the effect of maternal skills on the productivity of time investment is neutralized by assuming $\bar{\varphi}_m = 1$. The following proposition formally characterizes this case.

Proposition 6. *Suppose $\bar{\varphi}_m = 1$ and $\bar{\varphi}_g > 0$. (A) S_τ is strictly decreasing in H_m if $\rho > \max\{0,\gamma\}$, while it is strictly increasing in H_m if $\rho < \min\{0,\gamma\}$. (B) S_g is strictly decreasing in H_m if $\max\{\rho,\gamma\} < 0$, while it is strictly increasing in H_m if $\min\{\rho,\gamma\} > 0$. (C) S_x is strictly decreasing in H_m if and only if $\gamma > 0$.*

Proof of Proposition 6: Differentiating D with respect to H_m yields:

$$\frac{\partial D}{\partial H_m} = \frac{q\Phi_x[a_m\Phi_m^\rho((\gamma-\rho)(1-\bar{\varphi}_m) + \rho(\gamma-1)\bar{\varphi}_g) + a_g(\rho-1)\gamma\bar{\varphi}_g] + w_m H_m \Phi_m \rho(\gamma-1)(1-\bar{\varphi}_m+\bar{\varphi}_g)[a_m\Phi_m^\rho + a_g]}{(1-\gamma)(1-\rho)H_m[a_m\Phi_m^\rho + a_g]}.$$

Using this, we have

$$\begin{aligned}
\frac{\partial S_g}{\partial H_m} &= \frac{-p\frac{\partial D}{\partial H_m}}{D^2} \\
\frac{\partial S_\tau}{\partial H_m} &= \frac{w_m\Phi_m p\rho(\gamma-1)(1-\bar{\varphi}_m+\bar{\varphi}_g)[a_m\Phi_m^\rho + a_g]}{(1-\gamma)(1-\rho)[a_m\Phi_m^\rho + a_g]D^2} \\
&\quad + \frac{w_m\Phi_m q\Phi_x\left(\gamma(\rho-1)(1-\bar{\varphi}_m)a_m\Phi_m^\rho + (\bar{\varphi}_g(\gamma-\rho) + \rho(\gamma-1)(1-\bar{\varphi}_m))a_g\right)}{(1-\gamma)(1-\rho)[a_m\Phi_m^\rho + a_g]D^2} \\
\frac{\partial S_x}{\partial H_m} &= \frac{\gamma q\Phi_x[pa_m\Phi_m^\rho(1-\bar{\varphi}_m-\bar{\varphi}_g) - pa_g\bar{\varphi}_g + w_m H_m\Phi_m a_m\Phi_m^\rho(1-\bar{\varphi}_m)]}{(1-\gamma)H_m[a_m\Phi_m^\rho + a_g]D^2}.
\end{aligned}$$

The stated results in Proposition 6 are immediate from these derivatives. □

A.3 Characterizing the Intertemporal Problem

A.3.1 Roles of Assumption 1 and 2

The first order condition for I_t is:

$$\beta \mathbb{E}_t \left[\frac{\partial V_{t+1}}{\partial \Psi_{t+1}} \frac{\partial \Psi_{t+1}}{\partial I_t} \right] = \bar{p}_t u'(c_t). \tag{44}$$

Envelope conditions are

$$\frac{\partial V_{t+1}}{\partial \Psi_{t+1}} = \beta \mathbb{E}_{t+1} \left[\frac{\partial V_{t+2}}{\partial \Psi_{t+2}} \frac{\partial \Psi_{t+2}}{\partial \Psi_{t+1}} \right] \quad t = 0, ..., T-1.$$

and

$$\frac{\partial V_{T+1}}{\partial \Psi_{T+1}} = \frac{\partial \tilde{V}}{\partial \Psi_{T+1}}.$$

Combining the envelope conditions for periods $t+1, \ldots, T+1$ and applying the law of iterated expectations gives

$$\frac{\partial V_{t+1}}{\partial \Psi_{t+1}} = \beta^{T-t} \mathbb{E}_{t+1} \left[\frac{\partial \tilde{V}}{\partial \Psi_{T+1}} \prod_{s=t+1}^{T} \frac{\partial \Psi_{s+1}}{\partial \Psi_s} \right].$$

By substituting this into Equation (44), we get

$$\beta^{T-t+1} \mathbb{E}_t \left[\frac{\partial \tilde{V}}{\partial \Psi_{T+1}} \left(\prod_{s=t+1}^{T} \frac{\partial \Psi_{s+1}}{\partial \Psi_s} \right) \frac{\partial \Psi_{t+1}}{\partial I_t} \right] = \bar{p}_t u'(c_t). \tag{45}$$

Assumptions 2 and 3 considerably simply the expression in the expectation operator. Assumption 2 implies

$$\begin{aligned} \frac{\partial \Psi_{t+1}}{\partial I_t} &= \delta_1 \frac{\Psi_{t+1}}{I_t}, \\ \frac{\partial \Psi_{t+1}}{\partial \Psi_t} &= \delta_2 \frac{\Psi_{t+1}}{\Psi_t}, \end{aligned} \tag{46}$$

where the last condition leads to

$$\prod_{s=t+1}^{T} \frac{\partial \Psi_{s+1}}{\partial \Psi_s} = \delta_2^{T-t} \frac{\Psi_{T+1}}{\Psi_{t+1}}. \tag{47}$$

Substituting Equations (46) and (47) into Equation (45) yields

$$\frac{\beta^{T-t+1} \delta_2^{T-t} \delta_1}{I_t} \mathbb{E}_t \left[\Psi_{T+1} \frac{\partial \tilde{V}}{\partial \Psi_{T+1}} \right] = \bar{p}_t u'(c_t).$$

Under Assumption 2, $\frac{\partial \tilde{V}}{\partial \Psi_{T+1}} = \frac{\alpha}{\Psi_{T+1}}$, which implies Equation (10) when substituted into the above expression.[67]

This result makes clear that child ability levels, θ_t, do not impact investment – or any other – decisions due to log separability of θ_t from other inputs in the child production function and log preferences for child skills. As such, uncertainty about children's abilities has no affect on family decisions, or any results that follow.

[67]If $\tilde{V}$ is not logarithmic over final human capital, then the FOC for total investment each period depends on the final level of child skill, which in turn depends on *all* periods of investments, including the current period. This implies that each I_t FOC would generally be a nonlinear function of total investments from all periods, yielding a complex system of nonlinear equations to solve.

A.3.2 Total Expenditures

Uncertainty about future wages or income (but not child ability) would affect unconstrained intertemporal consumption allocations due to precautionary savings motives. Because incorporating this effect would greatly complicate the analysis for unconstrained families with little added insight and because this uncertainty would not impact the behavior of borrowing-constrained families, we abstract from uncertainty throughout the rest of subsection A.3.[68]

To characterize investment behavior when constraints are non-binding throughout parents' lives, we make a simplifying assumption on the continuation value function $\tilde{U}$. This assumption is not necessary for any results for borrowing constrained households.

Assumption 4. *$\tilde{U}(H_m, H_f, A) = \hat{U}((1+r)A + \chi_m H_m + \chi_f H_f)$ where the constants χ_m and χ_f are non-negative and $\hat{U}(\cdot)$ is strictly increasing and strictly concave.*

This assumption represents the case where parents at date $T+1$ value their remaining lifetime wealth as defined by current assets plus the discounted present value of all future earnings represented by $\chi_j H_j$.[69] For ease of exposition, we have suppressed dependence of $\tilde{U}$ and $\hat{U}$ on non-labor earnings, y_t. For constrained families, future income is irrelevant for current decisions. For unconstrained families, we will assume potential non-labor earnings until retirement at T_R. It is useful to define $\Delta(x) \equiv \hat{U}'(x)$, which is a strictly decreasing function given strict concavity of $\hat{U}(\cdot)$.

Lemma 1. *Consumption, c_t, is strictly increasing in parental human capital (H_m, H_f), current skill prices $(w_{m,t}, w_{f,t})$, and current non-labor income (y_t) with $\frac{\partial c_t}{\partial H_j} = \frac{\partial c_t}{\partial w_{j,t}} \frac{w_{j,t}}{H_j} > 0$ for $j \in \{m, f\}$. Consumption, c_t, is independent of current and all future household goods and child care input prices, $\{p_\tau, q_\tau\}_{\tau=t}^{T}$. If borrowing constraints are non-binding in all periods $t, ..., T$, then consumption, c_t, is strictly increasing in all future skill prices and non-labor income, $\{w_{m,\tau}, w_{f,\tau}, y_\tau\}_{\tau=t}^{T}$.*

Proof of Lemma 1: As noted in the text, the budget constraint for constrained households is

$$c_t = (1+r)A_t + W_{m,t}(1 - L_{m,t}(u'(c_t)W_{m,t})) + W_{f,t}(1 - L_{f,t}(u'(c_t)W_{f,t})) + y_t - \frac{K_t}{u'(c_t)} - A_{min,t},$$

where we have defined $l_{j,t} = L_{j,t}(u'(c_t)W_{j,t}))$ for $j = m, f$. Applying the implicit function theorem yields the following: $\partial c_t/\partial p_t = \partial c_t/\partial q_t = 0$,

$$\begin{aligned} \frac{\partial c_t}{\partial w_{j,t}} &= \frac{\left(1 - l_{j,t} - u'(c_t)W_{j,t}L'_{j,t}\right) H_j}{1 + u''(c_t)\left[W_{m,t}^2 L'_{m,t} + W_{f,t}^2 L'_{f,t}\right] - K_t \frac{u''(c_t)}{u'(c_t)^2}} > 0, \quad j \in \{m, f\}, \\ \frac{\partial c_t}{\partial y_t} &= \frac{1}{1 + u''(c_t)\left[W_{m,t}^2 L'_{m,t} + W_{f,t}^2 L'_{f,t}\right] - K_t \frac{u''(c_t)}{u'(c_t)^2}} > 0, \end{aligned}$$

and $\frac{\partial c_t}{\partial H_j} = \frac{\partial c_t}{\partial w_{j,t}} \frac{w_{j,t}}{H_j} > 0$ for $j \in \{m, f\}$.[70]

[68] Uncertainty about future wages and income has no effect on I_t and, therefore, specific investment inputs for borrowing-constrained families, because uncertainty only affects total investment I_t through consumption c_t, which is fully determined by current assets, prices, wages, and income for constrained families.

[69] For example, $\chi_j = \sum_{k=0}^{T_R-(T+1)} (1+r)^{-k} w_{T+1+k}$, assuming individuals retire at date T_R.

[70] The numerator of $\partial c_t/\partial w_{j,t}$ is positive, because $1 - l_{j,t} > 0$ and $L'_{j,t} < 0$, i.e. leisure falls when its marginal cost, $u'(c_t)W_{j,t}$ rises.

For unconstrained households, the convenient assumption that $\beta(1+r)=1$ implies that $c_t = c$, for all t. This simplifies the expressions that follow without altering any important conclusions. Along with Assumption 4, $\beta(1+r)=1$ implies that $A_{T+1} = \frac{\Delta^{-1}(u'(c_T))-\chi_m H_m - \chi_f H_f}{1+r}$. As with the binding constraint case, we can now substitute these expressions into the lifecycle budget constraint and collect consumption terms to obtain:

$$\Upsilon_{T-t}c + (1+r)^{-(T+1-t)}\Delta^{-1}\Big(u'(c)\Big) + \frac{\bar{K}_t}{u'(c)} - \sum_{j=0}^{T-t}(1+r)^{-j}\Bigg[W_{m,t+j}(1-L_{m,t+j}) + W_{f,t+j}(1-L_{f,t+j})\Bigg]$$

$$= (1+r)A_t + (1+r)^{-(T+1-t)}\Big[\chi_m H_m + \chi_f H_f\Big] + \sum_{j=0}^{T_R-t}(1+r)^{-j}y_{t+j},$$

where the constants $\Upsilon_{T-t} \equiv \sum_{j=0}^{T-t}(1+r)^{-j} > 0$ and $\bar{K}_t \equiv \sum_{j=0}^{T-t}(1+r)^{-j}K_{t+j} > 0$, and we recognize the dependence of leisure on its marginal cost, $L_{j,t}(u'(c)W_{j,t})$. This implicitly defines consumption as a function of current and future wages, non-labor income, parental human capital, period t assets, and other preference/technology parameters. We can then use the implicit function theorem to determine how prices, non-labor income, and parental human capital affect consumption. Letting π generically reflect these parameters,

$$\frac{\partial c}{\partial \pi} = \frac{\sum_{j=0}^{T-t}(1+r)^{-j}\left[\frac{\partial W_{m,t+j}}{\partial \pi}\left(1 - l_{m,t+j} - u'(c)W_{m,t+j}L'_{m,t+j}\right) + \frac{\partial W_{f,t+j}}{\partial \pi}\left(1 - l_{f,t+j} - u'(c)W_{f,t+j}L'_{f,t+j}\right)\right]}{\Upsilon_{T-t} + \sum_{j=0}^{T-t}(1+r)^{-j}u''(c)\left[W^2_{m,t+j}L'_{m,t+j} + W^2_{f,t+j}L'_{f,t+j}\right] - \bar{K}_t\frac{u''(c)}{u'(c)^2} + (1+r)^{-(T+1-t)}\frac{u''(c)}{\Delta'(\Delta^{-1}(u'(c)))}}$$

$$+\frac{(1+r)^{-(T+1-t)}\left[\frac{\partial \chi_m}{\partial \pi}H_m + \chi_m\frac{\partial H_m}{\partial \pi} + \frac{\partial \chi_f}{\partial \pi}H_f + \chi_f\frac{\partial H_f}{\partial \pi}\right] + \sum_{j=0}^{T_R-t}(1+r)^{-j}\frac{\partial y_{t+j}}{\partial \pi}}{\Upsilon_{T-t} + \sum_{j=0}^{T-t}(1+r)^{-j}u''(c)\left[W^2_{m,t+j}L'_{m,t+j} + W^2_{f,t+j}L'_{f,t+j}\right] - \bar{K}_t\frac{u''(c)}{u'(c)^2} + (1+r)^{-(T+1-t)}\frac{u''(c)}{\Delta'(\Delta^{-1}(u'(c)))}}.$$

The denominator is strictly positive, because $L'_{k,t} < 0$, $u''(\cdot) < 0$, $\bar{K}_t > 0$, and $\Delta'(\cdot) < 0$. Furthermore, $l_{k,t} < 1$ and $L'_{k,t} < 0$ implies that the numerator terms $\left(1 - l_{k,t+j} - u'(c)W_{k,t+j}L'_{k,t+j}\right)$, $k = m, f$, are strictly positive. Thus, unconstrained consumption is strictly increasing in current and future non-labor income, current and future skill prices, and parental human capital, while it is independent of (current and future) prices for home investment goods and child care services. □

Because $E_t = K_t/u'(c_t)$, total investment expenditures are increasing in current consumption, which is increasing in current income levels. Thus, total investment expenditures are increasing in human capital, current skill prices, and current non-labor income (Proposition 1). When households are borrowing constrained, only current income and prices affect investment behavior. By contrast, unconstrained households can efficiently allocate resources across periods, so total investment expenditures are also increasing in all future levels of non-labor income and skill prices. As a consequence, a permanent increase in skill prices will have greater impacts on current investment expenditures (among unconstrained households) than a one-time increase in the price. Additionally, a single period change in wages or non-labor income in period t will have smaller effects on investment that period when constraints are non-binding compared to when they bind. This is not surprising, because any change in income is spread across all periods (in terms of investment and consumption) when families are unconstrained.

Proof of Proposition 1: The proof is immediate from Lemma 1 given that $E_t = K_t/u'(c_t)$ implies

$$\frac{\partial E_t}{\partial \pi} = -K_t \frac{u''(c_t)}{u'(c_t)^2} \frac{\partial c_t}{\partial \pi} \qquad \text{for} \qquad \pi \in \{p_t, q_t, y_t, w_{m,t}, w_{f,t}, H_m, H_f\}.$$

□

The following corollary shows that increases in the price of household goods inputs or child care lead to reductions in total investment, while an increase in non-labor income raises total investment.

Corollary 1. *Total investment in period t, I_t, is strictly decreasing in the prices of household goods inputs and child care (p_t, q_t), while it is increasing in non-labor income (y_t).*

Proof of Corollary 1: Equation (12) implies that $I_t = \frac{K_t}{\bar{p}_t u'(c_t)}$. Differentiating this with respect to any variable π that affects the composite investment price or consumption implies the following:

$$\frac{\partial I_t}{\partial \pi} = -\frac{I_t}{\bar{p}_t u'(c_t)} \left[\bar{p}_t u''(c_t) \frac{\partial c_t}{\partial \pi} + u'(c_t) \frac{\partial \bar{p}_t}{\partial \pi} \right].$$

Lemma 1 implies that c_t is independent of p_t and q_t, so the fact that $\bar{p}_t$ is increasing in all input prices implies that I_t is decreasing in p_t and q_t. Lemma 1 implies that c_t is increasing in y_t, while $\bar{p}_t$ does not depend on y_t. Together, these imply that I_t is increasing in y_t. □

A.3.3 Responses of Constrained and Unconstrained Families

In this subsection, we compare the responses of constrained and unconstrained single mothers to changes in wages. To simplify the analysis, we assume log utility, $u(c) = \ln(c)$, $v_m(l_m) = \psi_m \ln(l_m)$, with $\psi_m > 0$, and continue to suppose $\beta(1+r) = 1$. Log preferences over consumption imply $E_t = K_t c_t$ (see Equation 12).

Consumption for a constrained mother in period t is given by:

$$c_t^c = \frac{(1+r)A_t + W_{m,t} + y_t - A_{min,t}}{1 + \psi_m + K_t}. \tag{48}$$

If we suppose that the unconstrained mother continues to have the same period utility between $T+1$ and T_R, and we solve her intertemporal problem over the entire horizon, $t...T_R$, she has the lifetime budget constraint:

$$\sum_{j=0}^{T_R-t} (1+r)^{-j} c + \bar{K}_t c = \sum_{j=0}^{T_R-t} (1+r)^{-j} \left[W_{m,t+j} \left(1 - \frac{\psi_m c}{W_{m,t+j}} \right) + y_{t+j} \right] + (1+r)A_t.$$

Solving for c in period t yields

$$c^u = \frac{(1+r)A_t + \sum_{j=0}^{T_R-t} (1+r)^{-j} \left(W_{m,t+j} + y_{t+j} \right)}{\sum_{j=0}^{T_R-t} (1+r)^{-j} \Big(1 + \psi_m \Big) + K_t \sum_{j=0}^{T-t} (1+r)^{-j} \left(\beta \delta_2 \right)^{-j}}.$$

Recall that $\bar{K}_t = K_t \sum_{j=0}^{T-t} (1+r)^{-j} \left(\beta \delta_2 \right)^{-j}$.

Constrained mothers only consider current income when making consumption choices in t, while unconstrained mothers take into account discounted future income through retirement at T_R. If the unconstrained mother's horizon ends when her child leaves home ($T_R = T$), and if future expenditures are constant ($\beta\delta_2 = 1$), the denominator in consumption, c^u, for the unconstrained mother simplifies to

$$(1 + \psi_m + K_t) \sum_{j=0}^{T-t} (1 + r)^{-j}.$$

This is the discounted lifetime analogue to the denominator of the constrained mother.

We begin by comparing one time changes in wages in period t. The constrained household responds:

$$\frac{\partial c_t^c}{\partial w_{m,t}} = \frac{H_m}{1 + \psi_m + K_t} > 0.$$

The unconstrained household responds:

$$\frac{\partial c_t^u}{\partial w_{m,t}} = \frac{H_m}{\sum_{j=0}^{T_R-t} (1+r)^{-j} \Big(1 + \psi_m\Big) + K_t \sum_{j=0}^{T-t} (1+r)^{-j} \left(\beta\delta_2\right)^{-j}} > 0.$$

We can rewrite this as:

$$\frac{\partial c_t^u}{\partial w_{m,t}} = \frac{H_m}{1 + \psi_m + K_t + \sum_{j=1}^{T_R-t} (1+r)^{-j}(1 + \psi_m) + K_t \sum_{j=1}^{T-t} (1+r)^{-j} \left(\beta\delta_2\right)^{-j}} > 0.$$

So we have,

$$\frac{\partial c_t^c}{\partial w_{m,t}} \geq \frac{\partial c_t^u}{\partial w_{m,t}} > 0.$$

The difference between the constrained and the unconstrained derivatives shrinks as t approaches T. At $t = T$, the derivatives are the same if parent's horizon ends at T ($T = T_R$).

Lemma 2. *A one-time increase in wages, leads to a weakly greater increase in current constrained consumption than current unconstrained consumption.*

Next consider a permanent change in wages. Let $W_{m,t} = \Delta w_{m,t} H_m$, where Δ increases in every period. For constrained mothers, current consumption is not impacted by wages rising in every period:

$$\frac{\partial c_t^c}{\partial \Delta} = \frac{w_{m,t} H_m}{1 + \psi_m + K_t} > 0.$$

The current consumption of unconstrained mothers responds to the wage increasing in every period:

$$\frac{\partial c_t^u}{\partial \Delta} = \frac{\sum_{j=0}^{T_R-t} (1+r)^{-j} w_{m,t+j} H_m}{\sum_{j=0}^{T_R-t} (1+r)^{-j} \Big(1 + \psi_m\Big) + K_t \sum_{j=0}^{T-t} (1+r)^{-j} \left(\beta\delta_2\right)^{-j}} > 0.$$

Here, both the numerator and the denominator of the unconstrained derivative are larger than the constrained derivative.

Lemma 3. *If wages are weakly increasing and $\beta\delta_2 \geq 1$, a permanent increase in wages of Δ, leads to a weakly greater increase in current unconstrained consumption than current constrained consumption. If wages are constant, $\beta\delta_1 = 1$, and $T = T_R$, the responses are the same.*

Proof of Lemma 3 For non-negative wage growth, $w_{t+j} \geq w_t$ for all $j \geq 0$ and

$$
\begin{aligned}
\frac{\partial c_t^u}{\partial \Delta} &= \frac{\sum_{j=0}^{T_R-t}(1+r)^{-j} w_{m,t+j} H_m}{\sum_{j=0}^{T_R-t}(1+r)^{-j}(1+\psi_m) + K_t \sum_{j=0}^{T-t}(1+r)^{-j}\left(\beta\delta_2\right)^{-j}} \\
&\geq \frac{w_{m,t} H_m \sum_{j=0}^{T_R-t}(1+r)^{-j}}{\sum_{j=0}^{T_R-t}(1+r)^{-j}(1+\psi_m) + K_t \sum_{j=0}^{T-t}(1+r)^{-j}\left(\beta\delta_2\right)^{-j}} \\
&= \frac{w_{m,t} H_m \sum_{j=0}^{T_R-t}(1+r)^{-j}}{\sum_{j=0}^{T_R-t}(1+r)^{-j}(1+\psi_m + K_t\left(\beta\delta_2\right)^{-j}) - \underbrace{K_t \sum_{j=T-t+1}^{T_R-t}(1+r)^{-j}\left(\beta\delta_2\right)^{-j}}_{=0 \text{ if } T=T_R}} \\
&\geq \frac{w_{m,t} H_m \sum_{j=0}^{T-t}(1+r)^{-j}}{\sum_{j=0}^{T_R-t}(1+r)^{-j}\left(1+\psi_m + K_t\left(\beta\delta_2\right)^{-j}\right)}.
\end{aligned}
$$

If $\beta\delta_2 \geq 1$,

$$
\frac{w_{m,t} H_m \sum_{j=0}^{T_R-t}(1+r)^{-j}}{\sum_{j=0}^{T_R-t}(1+r)^{-j}\left(1+\psi_m + K_t\left(\beta\delta_2\right)^{-j}\right)} \geq \frac{w_{m,t} H_m \sum_{j=0}^{T_R-t}(1+r)^{-j}}{(1+\psi_m + K_t)\sum_{j=0}^{T_R-t}(1+r)^{-j}} = \frac{w_{m,t} H_m}{1+\psi_m+K_t} = \frac{\partial c_t^c}{\partial \Delta}.
$$

Notice that if wages are constant, $\beta\delta_2 = 1$, and $T = T_R$, then $\frac{\partial c_t^u}{\partial \Delta} = \frac{\partial c_t^c}{\partial \Delta}$.

A.3.4 Input Quantities

In this subsection, we discuss comparative statics results for input levels, continuing to abstract from uncertainty about wage and income (in the case of unconstrained families). In what follows, we assume log utility, $u(c) = \ln(c)$, $v_m(l_m) = \psi_m \ln(l_m)$, with $\psi_m > 0$, and $\hat{U}((1+r)A_{T+1} + \chi_m H_m) = \chi_0 \ln((1+r)A_{T+1} + \chi_m H_m)$, with $\chi_0 > 0$.[71] To simplify notation, we consider single mother families. The solution for goods investment when families are borrowing constrained is

$$
g_t = \left(\frac{(1+r)A_t + y_t - A_{min,t} + W_{m,t}}{p_t + q_t \Phi_{x,t} + W_{m,t}\Phi_{m,t}}\right)\left(\frac{K_t}{1+\psi_m+K_t}\right).
$$

[71] If we assume that the mother has the same log period utility functional forms from $T+1$ to T_R, then $\chi_0 = (1+\psi)\sum_{j=0}^{T_R-T-1}(1+r)^{-j}$

When unconstrained, the solution is

$$g_t = \left(\frac{(1+r)A_t + \sum_{j=0}^{T-t}(1+r)^{-j}\left[W_{m,t+j} + y_{t+j}\right] + (1+r)^{t-T-1}\chi_m H_m}{p_t + q_t\Phi_{x,t} + W_{m,t}\Phi_{m,t}} \right) \left(\frac{K_t}{(1+\psi_m)\Upsilon_{T-t} + (1+r)^{t-T-1}\chi_0 + \bar{K}_t} \right).$$

In both cases $\tau_{m,t} = \Phi_{m,t} g_t$ and $x_t = \Phi_{x,t} g_t$.

To facilitate the comparative statics analysis below, it is useful to write the problem in a general way such that our results apply equally to both the constrained and unconstrained cases. To that end, we can write g_t in the following general form:

$$g_t = \tilde{K}_t \left(\frac{\breve{\omega}_t + \bar{W}_t H_m}{p_t + q_t\Phi_{x,t} + w_{m,t} H_m \Phi_{m,t}} \right), \tag{49}$$

where we continue to define $D_t \equiv p_t + q_t\Phi_{x,t} + W_{m,t}\Phi_{m,t}$ (a function of all input prices and H_m). The constant $\tilde{K}_t > 0$ depends on whether constraints are binding or not:

$$\tilde{K}_t = \begin{cases} \frac{K_t}{1+\psi_m+K_t} & \text{if borrowing constrained} \\ \frac{K_t}{(1+\psi_m)\Upsilon_{T-t}+(1+r)^{t-T-1}\chi_0+\bar{K}_t} & \text{if always unconstrained.} \end{cases}$$

The terms collected into $\breve{\omega}_t$ and $\bar{W}_t$ will depend on the particular proposition and constrained vs. unconstrained case as discussed below.

Proof of Proposition 2: Here, we consider the effects of changes in $w_{m,t}$ on g_t, $\tau_{m,t}$, and x_t. We define the $\breve{\omega}_t$ and $\bar{W}_t$ terms in Equation (49) as follows:

$$\breve{\omega}_t = \begin{cases} (1+r)A_t + y_t - A_{min,t} & \text{if borrowing constrained} \\ (1+r)A_t + \sum_{j=0}^{T-t}(1+r)^{-j}y_{t+j} + (1+r)^{t-T-1}\chi_m H_m + \sum_{j=1}^{T-t}(1+r)^{-j}W_{m,t+j} & \text{if always unconstrained} \end{cases}$$

and $\bar{W}_t = w_{m,t} > 0$ in both the constrained and always unconstrained cases. Here, $\breve{\omega}_t$ reflects all currently available resources not earned from current work and is independent of the prices we consider varying here. We require very mild conditions ensuring that $\breve{\omega}_t \geq 0$. In the borrowing constrained case, this condition is always satisfied if borrowing limits are not growing in discounted present value. In the unconstrained case, the condition requires that the vale of current debt not exceed the present discounted value of all future income (from all sources, including returns on human capital beyond year T).

We now differentiate g_t in Equation (49) with respect to $w_{m,t}$:

$$\frac{\partial g_t}{\partial w_{m,t}} = \tilde{K}_t \left(\frac{D_t H_m - (\breve{\omega}_t + w_{m,t} H_m) D_t'}{D_t^2} \right),$$

where D_t' is the derivative of D_t with respect to $w_{m,t}$. Because $D_t H_m > 0$ and $\breve{\omega}_t + w_{m,t} H_m \geq 0$, the numerator is strictly positive if $D_t' \leq 0$. Notice

$$D_t' = \frac{(\gamma-\rho) q_t \Phi_{x,t} a_m \Phi_{m,t}^{\rho}}{w_{m,t}(1-\gamma)(1-\rho)\left[a_m \Phi_{m,t}^{\rho} + a_g\right]} - \frac{\rho H_m \Phi_{m,t}}{1-\rho},$$

which is weakly negative if $\rho \geq \max\{0, \gamma\}$. Therefore, $\frac{\partial g_t}{\partial w_{m,t}} > 0$ if $\rho \geq \max\{0, \gamma\}$, as stated in Proposition 2.

Next, consider the effects of $w_{m,t}$ on $\tau_{m,t}$:

$$\begin{aligned}
\frac{\partial \tau_{m,t}}{\partial w_{m,t}} &= \frac{\partial \Phi_{m,t}}{\partial w_{m,t}} g_t + \frac{\partial g_t}{\partial w_{m,t}} \Phi_{m,t} \\
&= \frac{\Phi_{m,t} \tilde{K}_t}{(1-\rho) w_{m,t} D_t^2} \left\{ \breve{\omega}_t [w_{m,t}(\rho - 1) D_t' - D_t] + w_{m,t} H_m [\rho(D_t' w_{m,t} - D_t) - D_t' w_{m,t}] \right\}.
\end{aligned}$$

We sign $[w_{m,t}(\rho - 1)D_t' - D_t]$ and $[\rho(D_t' w_{m,t} - D_t) - D_t' w_{m,t}]$ separately. First,

$$w_{m,t}(\rho - 1) D_t' - D_t =$$

$$\frac{p_t(1-\gamma)\left[a_m \Phi_{m,t}^{\rho} + a_g\right] + q_t \Phi_{x,t}[(1-\rho) a_m \Phi_{m,t}^{\rho} + (1-\gamma) a_g] + w_{m,t} H_m \Phi_{m,t} (1-\rho)(1-\gamma)\left[a_m \Phi_{m,t}^{\rho} + a_g\right]}{(\gamma - 1)\left[a_m \Phi_{m,t}^{\rho} + a_g\right]} < 0.$$

Because $\breve{\omega}_t \geq 0$, we have $\breve{\omega}_t[w_{m,t}(\rho - 1)D_t' - D_t] \leq 0$. Next,

$$\rho(D_t' w_{m,t} - D_t) - D_t' w_{m,t} = \frac{\rho p_t (1-\gamma)\left[a_m \Phi_{m,t}^{\rho} + a_g\right] + q_t \Phi_{x,t}[\gamma(1-\rho) a_m \Phi_{m,t}^{\rho} + \rho(1-\gamma) a_g]}{(\gamma - 1)\left[a_m \Phi_{m,t}^{\rho} + a_g\right]},$$

which is weakly negative if $\min\{\gamma, \rho\} \geq 0$. Therefore, $\frac{\partial \tau_t}{\partial w_{m,t}} < 0$ if $\min\{\gamma, \rho\} \geq 0$ as stated in Proposition 2.

Finally, consider the effects of $w_{m,t}$ on x_t:

$$\frac{\partial x_t}{\partial w_{m,t}} = \frac{\Phi_{x,t} \tilde{K}_t \left\{ \breve{\omega}_t \Theta_{1,t} + w_{m,t} H_m \Theta_{2,t} \right\}}{w_{m,t}(1-\gamma)(1-\rho)\left[a_m \Phi_{m,t}^{\rho} + a_g\right] D_t^2},$$

where

$$\begin{aligned}
\Theta_{1,t} &= \gamma(1-\rho) a_m \Phi_{m,t}^{\rho} [p_t + w_{m,t} H_m w_{m,t} \Phi_{m,t}] \\
\Theta_{2,t} &= (1-\rho)\left\{ a_m \Phi_{m,t}^{\rho} [p_t + (1-\gamma) q_t \Phi_{x,t} + w_{m,t} H_m \Phi_{m,t}] + (1-\gamma) a_g [p_t + q_t \Phi_{x,t} + w_{m,t} H_m \Phi_{m,t}] \right\} > 0.
\end{aligned}$$

Clearly, $\frac{\partial x_t}{\partial w_{m,t}} > 0$ when $\gamma \geq 0$ as stated in Proposition 2. Also note that if $\breve{\omega}_t = 0$ (e.g. no non-labor income and no borrowing/saving), then $\frac{\partial x_t}{\partial w_{m,t}} > 0$ holds regardless of γ. □

When families are borrowing constrained, permanent changes in wages have identical effects on behavior as changes in current wages. This is not the case when families are unconstrained; although, the results are the same qualitatively. To see this, define $w_{m,t} = \tilde{w}_{mt} \bar{w}_m$ where $\bar{w}_m$ reflects the permanent component of wages. Now define $\breve{\omega}_t$ so that it no longer includes future labor earnings:

$$\breve{\omega}_t = (1+r) A_t + \sum_{j=0}^{T-t} (1+r)^{-j} y_{t+j} + (1+r)^{t-T-1} \chi_m H_m \geq 0,$$

where the conditions on debt that ensure $\breve{\omega}_t \geq 0$ are now stronger than before. (For married couples, $\breve{\omega}_t$ would also include the discounted present value of all spousal wages.) All maternal earnings are now included in $\bar{W}_{m,t} = \sum_{j=0}^{T-t} (1+r)^{-j} w_{m,t+j} > 0$. Based on these definitions and Equation (49), the same approach as above shows that all qualitative properties in Proposition 2 apply to permanent changes in wages, $\bar{w}_m$.

The next two propositions consider the effects of parental human capital on specific input quantities. We begin with the case in which parental human capital does not affect the productivity of home goods inputs.

Proposition 7. *Suppose $\bar{\varphi}_g = 0$. Home goods inputs, g, are strictly increasing in H_m and maternal time investment, τ_m, is strictly decreasing in H_m if any of the following conditions are met: (i) $\bar{\varphi}_m < 1$ and $\rho \geq \gamma \geq 0$, (ii) $\bar{\varphi}_m = 1$, or (iii) $\bar{\varphi}_m > 1$ and $\rho \leq \gamma \leq 0$.*

Next, consider $\bar{\varphi}_g > 0$, so the productivity of home goods investment is increasing in maternal human capital. Recall from that the increase in marginal productivity encourages more skilled mothers to shift their investment portfolio towards home goods if inputs are sufficiently substitutable; otherwise, the factor-augmenting nature of H_m can cause them to turn more to other inputs. To focus on the productivity effects of maternal human capital on home goods investment, consider the case of $\bar{\varphi}_m = 1$, which implies equal productivity of H_m at home and in the labor market.

Proposition 8. *Suppose $\bar{\varphi}_m = 1$ (φ_m is CRS) and $\bar{\varphi}_g > 0$. If $\rho \geq \gamma \geq 0$, then home goods investment is strictly increasing in H_m and parental time investment is strictly decreasing in H_m.*

Proofs of Propositions 7 and 8: In Propositions 7 and 8, we study the effects of H_m on input choices. Here, we continue to use the same family resource decomposition as above for constrained families: $\breve{\omega}_t = (1+r)A_t + y_t - A_{min,t} \geq 0$ and $\bar{W}_{m,t} = w_{m,t}$. For always unconstrained families, we decompose resources into those related and unrelated to mother's human capital as follows:

$$\begin{aligned}
\breve{\omega}_t &= (1+r)A_t + \sum_{j=0}^{T-t}(1+r)^{-j} y_{t+j} \geq 0 \\
\bar{W}_{m,t} &= (1+r)^{t-T-1}\chi_m + \sum_{j=0}^{T-t}(1+r)^{-j} w_{m,t+j} > 0,
\end{aligned}$$

where $\breve{\omega}_t \geq 0$ now requires our strongest condition on the value of debt (i.e., it cannot exceed the discounted value of all non-labor income). Again, for married couples, $\breve{\omega}_t$ would also include the discounted present value of all spousal wages, substantially weakening the condition on debt. The expression $\bar{W}_{m,t}$ corresponds to returns to human capital relevant for the investment decision at time t. For constrained families, it only includes current labor returns, while for unconstrained families, it contains current and all future returns (including the continuation value that depends on maternal human capital).

We denote the derivative of D_t with respect to maternal human capital by $D'_t = q_t \frac{\partial \Phi_{x,t}}{\partial H_m} + w_{m,t}\Phi_{m,t} + w_{m,t} H_m \frac{\partial \Phi_{m,t}}{\partial H_m}$. Consider the effects of changes in H_m on g_t by differentiating Equation (49):

$$\frac{\partial g_t}{\partial H_m} = \tilde{K}_t \left(\frac{D_t \bar{W}_{m,t} - (\breve{\omega}_t + \bar{W}_{m,t} H_m) D'_t}{D_t^2} \right),$$

which is positive if $D'_t \leq 0$. Notice

$$D'_t = \frac{q_t \Phi_{x,t} \left\{ a_m \Phi_{m,t}^{\rho} \left[(\gamma - \rho)(1 - \bar{\varphi}_m) + \rho(\gamma - 1)\bar{\varphi}_g \right] + \bar{\varphi}_g (\rho - 1)\gamma a_g \right\} + w_{m,t} H_m \Phi_{m,t} \rho (\gamma - 1)(\bar{\varphi}_g + 1 - \bar{\varphi}_m)[a_m \Phi_{m,t}^{\rho} + a_g]}{H_m (1-\rho)(1-\gamma)[a_m \Phi_{m,t}^{\rho} + a_g]}.$$

We see that $D'_t \leq 0$, and therefore $\frac{\partial g_t}{\partial H_m} > 0$ if $(\rho - \gamma)(1 - \bar{\varphi}_m) + \rho(1-\gamma)\bar{\varphi}_g \geq 0$, $\gamma \bar{\varphi}_g \geq 0$, and $\rho(\bar{\varphi}_g + 1 - \bar{\varphi}_m) \geq 0$.

When $\bar{\varphi}_g = 0$, we have $(\rho-\gamma)(1-\bar{\varphi}_m) \geq 0$ and $\rho(1-\bar{\varphi}_m) \geq 0$ (Proposition 7). And, when $\bar{\varphi}_g > 0$ and $\bar{\varphi}_m = 1$, we have $\rho \geq 0$ and $\gamma \geq 0$ (Proposition 8).

Next, consider maternal time investment:

$$\begin{aligned}
\frac{\partial \tau_{m,t}}{\partial H_m} &= \Phi_{m,t}\frac{\partial g_t}{\partial H_m} + \frac{\partial \Phi_{m,t}}{\partial H_m} g_t \\
&= \frac{\Phi_{m,t}\tilde{K}_t}{D_t^2 H_m(1-\rho)}\left[\bar{W}_{m,t}H_m\big(\rho(\bar{\varphi}_m - 1 - \bar{\varphi}_g)D_t + (\rho-1)D_t' H_m\big) + \breve{\omega}_t\big((\rho(\bar{\varphi}_m - \bar{\varphi}_g)-1)D_t + (\rho-1)D_t' H_m\big)\right].
\end{aligned}$$

We have two parts of this expression to sign. First:

$$\bar{W}_{m,t}H_m\left\{\rho(\bar{\varphi}_m - 1 - \bar{\varphi}_g)D_t + (\rho-1)D_t' H_m\right\} = \left[\frac{1}{(1-\gamma)[a_m\Phi_{m,t}^\rho + a_g]}\right]\Big\{p_t\rho(\bar{\varphi}_m - \bar{\varphi}_g - 1)(1-\gamma)[a_m\Phi_{m,t}^\rho + a_g] +$$

$$q_t\Phi_{x,t}\big[a_m\Phi_{m,t}^\rho\gamma(1-\rho)(\bar{\varphi}_m - 1) + a_g[(\gamma-\rho)\bar{\varphi}_g + \rho(1-\gamma)(\bar{\varphi}_m - 1)]\big]\Big\},$$

which is positive when: $\rho(\bar{\varphi}_m - \bar{\varphi}_g - 1) \geq 0$, $\gamma(\bar{\varphi}_m - 1) \geq 0$, and $(\gamma-\rho)\bar{\varphi}_g + \rho(1-\gamma)(\bar{\varphi}_m - 1) \geq 0$. It is negative when: $\rho(\bar{\varphi}_g + 1 - \bar{\varphi}_m) \geq 0$, $\gamma(1-\bar{\varphi}_m) \geq 0$, and $(\rho-\gamma)\bar{\varphi}_g + \rho(1-\gamma)(1-\bar{\varphi}_m) \geq 0$.

Second:

$$\breve{\omega}_t\left\{(\rho(\bar{\varphi}_m - \bar{\varphi}_g) - 1)D_t + (\rho-1)D_t' H_m\right\} = \left[\frac{1}{(1-\gamma)[a_m\Phi_{m,t}^\rho + a_g]}\right]\Big\{w_{m,t}H_m\Phi_{m,t}(\rho-1)(1-\gamma)[a_m\Phi_{m,t}^\rho + a_g] +$$

$$p_t(\rho(\bar{\varphi}_m - \bar{\varphi}_g) - 1)(1-\gamma)[a_m\Phi_{m,t}^\rho + a_g] + q_t\Phi_{x,t}\big[a_m\Phi_{m,t}^\rho(1-\rho)(\gamma\bar{\varphi}_m - 1) + a_g[(\gamma-\rho)\bar{\varphi}_g + (1-\gamma)(\rho\bar{\varphi}_m - 1)]\big]\Big\}.$$

Because the first part of the expression in braces $w_{m,t}H_m\Phi_{m,t}(\rho-1)(1-\gamma)[a_m\Phi_{m,t}^\rho + a_g] < 0$, there is always a negative force (independent of parameters) impacting the effect of mother's human capital on time investment when $\breve{\omega}_t > 0$. We can only give cases where the derivative is (strictly) decreasing in mother's human capital. The entire expression related to $\breve{\omega}_t$ is negative when: $(1-\gamma)(1-\rho\bar{\varphi}_m) + \bar{\varphi}_g(\rho-\gamma) \geq 0$, $1 - \gamma\bar{\varphi}_m \geq 0$, and $1 + \rho(\bar{\varphi}_g - \bar{\varphi}_m) \geq 0$.

Altogether, conditions that imply a strictly negative (when $\breve{\omega}_t > 0$) impact of maternal human capital on time investment are as follows:

1. $\rho + \rho(\bar{\varphi}_g - \bar{\varphi}_m) \geq 0$,
2. $\gamma - \gamma\bar{\varphi}_m \geq 0$,
3. $(1-\gamma)\rho(1-\bar{\varphi}_m) + \bar{\varphi}_g(\rho-\gamma) \geq 0$,
4. $(1-\gamma)(1-\rho\bar{\varphi}_m) + \bar{\varphi}_g(\rho-\gamma) \geq 0$,
5. $1 - \gamma\bar{\varphi}_m \geq 0$,
6. $1 + \rho(\bar{\varphi}_g - \bar{\varphi}_m) \geq 0$.

Note that condition 1 implies condition 6, condition 2 implies condition 5, and condition 3 implies condition 4. We are left with conditions 1–3. When $\bar{\varphi}_g = 0$, we have $\rho(1-\bar{\varphi}_m) \geq 0$ and $\gamma(1-\bar{\varphi}_m) \geq 0$ (Proposition 7). And, when $\bar{\varphi}_g > 0$ and $\bar{\varphi}_m = 1$, we have $\rho \geq 0$ and $\rho \geq \gamma$ (Proposition 8). □

A.3.5 Closed form expressions for total investment

If we follow Del Boca, Flinn, and Wiswall (2014) (and several subsequent papers) by assuming log preferences for consumption and leisure (i.e., $u(c) = \ln(c)$ and $v_j(l_j) = \psi_j \ln(l_j)$, $\psi_j \geq 0$, for $j \in \{m, f\}$), then we obtain a closed form expression for total investment among constrained households:

$$I_t = \frac{K_t \left[(1+r)A_t + W_{m,t} + W_{f,t} + y_t - A_{min,t}\right]}{\bar{p}_t \left[1 + \psi_m + \psi_f + K_t\right]}. \tag{50}$$

From this, we see that the dynamics of constrained investment depend on both the dynamics of input prices through $\bar{p}_t$ and the dynamics of "full" family income, $W_{m,t} + W_{f,t} + y_t$.

If we also assume a log continuation utility (i.e., $\tilde{U}(H_m, H_f, A) = \chi_0 \ln(A + \chi_m H_m + \chi_f H_f)$, with χ_0, χ_m, and χ_f all non-negative), then we obtain a very similar closed form expression for total investment in the unconstrained case:

$$I_t = \frac{K_t \left[(1+r)A_t + \sum_{j=0}^{T-t}(1+r)^{-j}\left(W_{m,t+j} + W_{f,t+j} + y_{t+j}\right) + (1+r)^{-(T+1-t)}\left(\chi_m H_m + \chi_f H_f\right)\right]}{\bar{p}_t \left[(1+\psi_m+\psi_f)\Upsilon_{T-t} + (1+r)^{-(T+1-t)}\chi_0 + \bar{K}_t\right]}. \tag{51}$$

Total investment for unconstrained families depends on the discounted present value of lifetime (rather than current) "full" income as well as the continuation value of parental human capital. Also, note that the denominator reflects discounted lifetime sums of $(1 + \psi_m + \psi_f)$ and K_t rather than only their current values. As a result, a single period change in wages or non-labor income in period t will have much smaller effects on investment that period when constraints are not binding compared to when they bind.

A.4 Effects of a Small Price Change

Proof of Proposition 3: From Equation (10), the price elasticity of total investment is

$$\frac{\partial \ln I_t}{\partial \ln \pi_t} = -\frac{\partial \ln \overline{p}_t}{\partial \ln \pi_t} - \frac{\partial \ln u'(c_t)}{\partial \ln \pi_t}, \quad \forall \pi_t \in \Pi_t.$$

First, we show that the second term $\partial \ln u'(c_t)/\partial \ln \pi_t$ does not depend on the within-period production function. Lemma 1 implies that $\partial \ln u'(c_t)/\partial \ln p_t = \partial \ln u'(c_t)/\partial \ln q_t = 0$. Parental wages $W_{m,t}$ and $W_{f,t}$ affect consumption and leisure decisions through the budget constraint. But $\overline{p}_t$ does not play any role in shaping the relationship between parental wages and consumption/leisure choices, so $\partial \ln u'(c_t)/\partial \ln W_{m,t}$ and $\partial \ln u'(c_t)/\partial \ln W_{f,t}$ do not depend on the within-period production function.

Next, we show that the first term $\partial \ln \overline{p}_t/\partial \ln \pi_t$ depends on the within-period production function only through input expenditure shares. Notice that the composite price can be written as the minimum unit cost of production:

$$\overline{p}_t(\Pi_t) = \min_{\tau_{m,t},\tau_{f,t},g_t,x_t} \Big\{ W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t} + p_t g_t + q_t x_t | f_t(\tau_{m,t}, \tau_{f,t}, g_t, x_t) \geq 1 \Big\}.$$

Let $(\underline{\tau}_{m,t}(\Pi_t), \underline{\tau}_{f,t}(\Pi_t), \underline{g}_t(\Pi_t), \underline{x}_t(\Pi_t))$ be the solution to this problem. Then, by application of the envelope theorem (Shephard's Lemma), we have

$$\frac{\partial \overline{p}_t(\Pi_t)}{\partial p_t} = \underline{g}_t(\Pi_t).$$

Therefore, the elasticity of $\overline{p}_t$ with respect to p_t is

$$\frac{\partial \ln \overline{p}_t(\Pi_t)}{\partial \ln p_t} = \frac{p_t \underline{g}_t(\Pi_t)}{\overline{p}_t(\Pi_t)} = \frac{p_t \underline{g}_t(\Pi_t) I_t}{\overline{p}_t(\Pi_t) I_t} = S_{g,t}(\Pi_t).$$

This holds for all input prices. That is,

$$\frac{\partial \ln \overline{p}_t(\Pi_t)}{\partial \ln \pi_t} = S_{\pi,t}(\Pi_t), \quad \forall \pi_t \in \Pi_t.$$

B Time Investment Categories in PSID-CDS

This appendix lists all the activities we include in our parental time investment measure when children are actively engaged with their mother and/or father. This list is based in the 1997 coding, but the categories are very similar in 2002 and 2007.

B.1 Academic investment activities

1. PASSIVE LEISURE - Reading
   - 939 Reading or looking at books; 941 Reading magazines, reviews, pamphlets; 959 Reading newspapers; 942 Reading, NA what; 943 Being read to, listening to a story; 979 Letters (reading or writing) and reading mail)
2. EDUCATIONAL AND PROFESSIONAL TRAINING
   - 519 Other classes, courses, lectures, academic or professional if the child is not a full-time student or NA whether a student, being tutored; 549 Homework (non-computer related), studying, research, reading, related to classes or profession; 569 Other education; "watched a slide program";
3. HOME COMPUTER RELATED ACTIVITIES
   - 501 Lessons in computers (Learning how to use a computer); 504 Using the computer for homework, studying, research, reading related to classes or profession, except for current job; 510 Media, reading the newspaper, stock quotes, weather reports; 511 Library functions (using computer/internet to acquire specialized information); 512 Computer work, getting computer programs to work, reading the manual, repairing computer, setting up computer;

B.2 Health investment activities

1. SERVICES
   - 339 Medical care for self; visits to doctor, dentist, optometrist, including making appointments
2. CARE TO SELF
   - 411 Medical care at home to self; taking care of own sickness.
3. OTHER PERSONAL AND HELPING
   - 488 Receiving child care, a child is the passive recipient of personal care, medical care from parent or other, baby being held, being comforted by a parent

B.3 Play

1. PLAYING/GAMES
    - 866-889 all subcategories

B.4 Arts and crafts with household children

1. HOBBIES
    - 831-835 all subcategories
2. DOMESTIC CRAFTS
    - 841-844 all subcategories
3. ART AND LITERATURE
    - 851-852 all subcategories
4. MUSIC/THEATER/DANCE
    - 861-864 all subcategories
5. CLASSES/LESSONS FOR LEISURE ACTIVITY
    - 887 Lessons in music, singing, instruments

B.5 Sports

1. CLASSES/LESSONS FOR LEISURE ACTIVITY
    - 881 Lessons in dance; 885 Lessons in sports activities such as swimming, golf, tennis, skating, roller skating; 886 Lessons in gymnastics, yoga, judo, body movement; 888 Other lessons, not listed above
2. COMPETITIVE SPORTS-OTHER EDUCATIONAL ACTIVITIES
    - 883 Organized meets, games, practices for team sports
3. ACTIVE LEISURE ACTIVITIES
    - 884 Meets, practices for individual sports
4. ACTIVE SPORTS
    - 801-810, 865 all subcategories
5. OTHER OUT OF DOORS
    - 811-818, 824, 825, 826 all subcategories
6. WALKING
    - 821-823 all subcategories

B.6 Talk and listen

1. PASSIVE LEISURE

   - 962 Other talking/conversations face-to-face conversations, mixed or non-household people in conversation; 963 Conversations with other household members–adults and/or children; 967 Receiving instructions, orders

B.7 Eating

1. INDOOR

   - 108-119 Meal preparation activities

2. CARE TO SELF

   - 439 Meals at home; including coffee, drinking, food from a restaurant eaten at home; 448 Meals away from home eaten at a friend's/relative's home; 449 Meals away from home - eating at restaurants

B.8 Socializing

1. CHILD CARE FOR OTHER HOUSEHOLD CHILDREN

   - 221 Helping children learn (fix things, bake cookies, etc.); 222 Help with homework or supervising homework; 238 Reading to a child; 239 Conversations with or listening to household children only in the context of child care arrangement; 248 Playing with household babies ages 0-2, "playing with baby", indoors or outdoors; 249 Respondents playing indoors with children; 258 Coaching/leading outdoors/non-organizational activities; 259 Respondents playing outdoors with children

2. CARE TO SELF

   - 484 Affection between household and non-household members; giving and getting hugs, kisses, sitting on laps

3. SOCIALIZING

   - 752-799 all subcategories

4. PROFESSIONAL/UNION ORGANIZATIONS

   - 601-602 all subcategories

5. CHILD/YOUTH/FAMILY ORGANIZATIONS

   - 671-672 all subcategories

6. FRATERNAL ORGANIZATIONS

   - 661-662 all subcategories

7. POLITICAL PARTY AND CIVIC PARTICIPATION
    - 621-622 all subcategories
8. SPECIAL INTEREST/IDENTITY ORGANIZATIONS
    - 611-612 all subcategories
9. OTHER MISCELLANEOUS ORGANIZATIONS
    - 689 Other organizations; any activities of an organization not fitting into the above categories
    - 698- 699 Related travel

B.9 Religious activities

1. RELIGIOUS PRACTICE
    - 651-652 all subcategories
2. RELIGIOUS GROUPS
    - 641-644 all subcategories

B.10 Volunteering

1. VOLUNTEER, HELPING ORGANIZATIONS
    - 631-635 all subcategories

B.11 Other activities

1. SERVICES
    - 377 Other professional services; lawyer, counseling (therapy).
2. TRAVEL
    - 597-599 School-related travel; 899 Related travel to sports/active leisure; waiting for related travel; vacation travel
3. ATTENDING SPECTACLES, EVENTS
    - 709-749 all subcategories

C Additional Data Sources

C.1 Child Care Prices

Child care costs for 4-year old center-based care, q, are obtained from annual reports on the cost of child care in the U.S. compiled by Child Care Aware of America (2009–2019).[72] These costs represent the average annual price charged by full-time center providers in each state covering 2006 to 2018. Several values from annual reports were dropped if they were imputed based on previous survey years or were taken from different sources or subsets of locations.

In order to obtain child care cost measures going back to 1997, we use our data (from 2006–2018) to regress state-year child care costs on state fixed effects, a linear time trend, and average state-year hourly wages for child care workers. The estimated coefficient on the linear time trend is 217.5, while the coefficient on average wages for child care workers is 18.8. The state-fixed effects explain most of the variation, and the R^2 statistic for this regression is 0.89. (Regressing the child care price on average wages for child care workers, without state or year fixed effects, yields a coefficient of 576.9 and R^2 statistic of 0.28.) Average wages for child care workers are estimated from the 1992–2019 monthly *Current Population Surveys (CPS)*.[73] We then use the estimated coefficients, including the state fixed effects, to impute child care costs back to 1997 (or for any missing observations) using *CPS* average wages for child care workers for each state and year.

Finally, to put child care prices in roughly hourly terms, consistent with our parental wage measures, we divide our child care cost measures by 33×52, reflecting an average of 33 hours per week spent in family- or center-based child care among young children of employed mothers (Laughlin 2013).

C.2 Household Input Prices

We obtain state-year measures of household-based goods input prices, p, from a combination of goods and services price series from the *Regional Price Parities by State (RPP)* from the U.S. Bureau of Economic Analysis (BEA) and the *Consumer Price Index (CPI-U)* from the U.S. Bureau of Labor Statistics (BLS). The *RPP*'s measure price level differences relative to the U.S. average by state and are available from 2008 to 2017. These indices are divided into several categories: All items; Goods; Services: Rent; and, Other Services.

To create the goods price series by state, we take the U.S. average of the *CPI* for "Commodities" and multiply it by each state's "Goods" *RPP*. This produces price measures by state for 2008–2017. To project back to 1997, we take the regional *CPI* for "Commodities" and use the year-over-year change of this index for each state within its Census region (Northeast, Midwest, South and West), working back from 2008 values. To create the services price series, we follow the same steps, using the "Services: Other" component from the *RPP*'s and the "Services less rent of shelter" index from the CPI. All these prices are year averages using a base year of 2000.

[72] We are grateful to Kristina Haynie of Child Care Aware of America for providing us with a digital compendium of child care prices from all annual reports. Each year, states report the annual prices that child care providers charge for their services. These reports are provided by Child Care Resource and Referral (CCR&R) agencies in each state.

[73] We restrict our *CPS* sample to workers who are at least 18 years old, report either weekly earnings or an hourly wage, and report an occupation of either child care worker or preschool or kindergarten teacher (2010 occupation classification codes 4600 or 2300). Among workers reporting weekly earnings, an hourly wage is calculated from weekly earnings divided by usual hours worked per week. CPS weights are used to calculate state-year average wages.

Finally, we use as our household goods input price, p, a weighted average of these goods and services price series, with a weight of 0.3 on services, reflecting the greater share of goods in the bundle of child investment inputs. For example, we use the 2003–18 *Consumer Expenditure Survey (CEX)* to create a comprehensive measure of potential household investments in children that includes expenditures on "goods" and "services" as described in Appendix C.3 and Appendix Table C-1. Based on this comprehensive measure of household investment inputs, we find that families with 1–2 children, both ages 0–12, spend an average of 35% of all household investment dollars on services. Taking a more limited household investment measure closer to that used in our *PSID-CDS* analysis suggests that families spend, on average, 20% on service-related child investments.

C.3 Consumer Expenditure Survey

The *Consumer Expenditure Survey (CEX)* is a rotating panel gathered by the U.S. Bureau of Labor Statistics. It collects detailed information on consumption, income and household's characteristics, and is representative of the U.S. population. The unit of measurement for the survey is given by Consumer Units. These units are broadly defined as members of a household that are related, or two or more persons living together that use their incomes to make joint expenditures decisions. Each unit is interviewed for up to four times during a 12-month period and is asked to report their expenditures on a detailed set of categories for the preceding three months. After completing the four interviews, each consumer unit is replaced.

The sample we use runs from 2003 to 2018. We exclude consumer units that do not complete all four interviews and those whose key characteristics are inconsistent over time (i.e., changes in age or race of the reference person, or if the family size changes by more than two members), indicating a likely change in families in the unit. We limit our sample to families with parents ages 18–65, mothers who were ages 16–45 when their youngest child was born, and with only 1–2 children (all age 12 or younger).

We use the Universal Classification Codes (UCCs) for expenditure categories to create our household-level investment measures. Our preferred investment measure is composed of two broad categories: investment in goods and in services. Investment in goods includes expenditures on books (for school or other, magazines, etc.), toys, games, musical instruments, and other learning equipment such as computers and accessories for nonbusiness use. The services measure includes admission fees for recreational activities, fees for recreational lessons and tutoring services. We sum the quarterly expenditures reported by each household (across categories and their four interviews) to obtain annual investment measures, then divide by 52 to create weekly measures.

Table C-1 provides a more detailed look at the expenditure categories, along with their average weekly expenditures.[74] We also report household investment expenditure categories consistent with those collected by the *PSID-CDS*. Altogether, the *PSID-CDS* categories aggregate to a weekly expenditure amount of $585.25, roughly 60% of the spending we include from the *CEX*.

[74] We aggregate a few categories, because some categories split over time.

Table C-1: Household Investment Expenditure Categories and Average Weekly Expenditures in the CEX

UCC	Description	PSID CDS	Average Expenditure (2002 dollars)
	Goods:		**561.75**
590220	-Books through book clubs	X	4.41
590230	-Books not through book clubs	X	43.00
590310	-Magazine or newspaper subscription		17.06
590410	-Magazine or newspaper, single copy		6.38
610110	-Toys, games, arts, crafts, tricycles, and battery powered riders	X	203.71
610120	-Playground equipment	X	10.89
610130	-Musical instruments, supplies, and accessories		26.02
660210	-School books, supplies, equipment for elementary, high school	X	24.36
660310	-Encyclopedia and other sets of reference books	X	0.31
660900, 660901	-School books, supplies, equipment for day care, nursery, preschool.	X	2.63
660902	-School books, supplies, and equipment for other schools	X	1.71
660410	-School books, supplies, equipment for vocational and technical schools	X	0.51
670902	-Other school expenses including rentals	X	47.61
690111	-Computers and computer hardware for nonbusiness use		134.65
690112, 690119, 690120	-Computer software and accessories for non-business use		22.48
690117	-Portable memory		2.88
690118	-Digital book readers	X	10.72
690230	-Business equipment for home use		2.43
	Services:		**421.09**
620211, 620212, 620213, 620214, 620215, 620216	-Admission fees for entertainment activities, including movie, theater, concert, opera or other musical series (single admissions and season tickets)		179.22
620310	-Fees for recreational lessons or other instructions	X	223.87
620904	-Rental and repair of musical instruments, supplies, and accessories		2.56
670903	-Test preparation, tutoring services	X	11.53
690113	-Repair of computer systems for nonbusiness use		3.92
	Total Investment		**982.85**

D Details on Counterfactual Analysis

D.1 No Borrowing/Saving

Our main counterfactual analysis assumes that parents have log preferences for consumption and leisure and are borrowing constrained. These assumptions permit a closed form solution for total investment. See Equation (50). We further assume that parents have no non-labor income and cannot borrow or save ($y_t = A_t = A_{min,t} = 0$). Their subjective discounter factor is $\beta = 1/1.02$ and they value their children's achievement at age 13 ($T = 13$). Finally, individuals are endowed with 100 hours per week (5,200 hours per year), which they can use for market work, leisure, or time investment in children.

These assumptions, along with estimated technology parameters and calibrated preference parameters, allow us to simulate investment and achievement for each child in 2002 PSID.

D.1.1 Calibration of Preference Parameters

The utility weights of the Cobb-Douglas utility function (α, ψ_m, and ψ_f) determine how households allocate their resources between consumption, leisure, and child investment in each period. For example, Equation (50) shows that two-parent households spend a fraction $K_t/(1 + \psi_m + \psi_f + K_t)$ of their full income on total investment in children. Therefore, given prices and technology parameters, the preference parameters can be identified from the levels of parental time spent on market work and child investment. We choose the preference parameters so that the model replicates weekly time use patterns from the 2002 PSID.

Table D-1: Calibration Targets: Weekly Hours of Time Investment and Work

	Mother's Education	
	Non-College	College
A. Single Mothers		
Mother's Time Investment	10.04	12.42
Mother's Hours Worked	42.26	38.22
B. Two-Parent Households		
Mother's Time Investment	9.56	12.13
Mother's Hours Worked	38.43	38.58
Father's Hours Worked	43.85	44.03

Table D-2: Calibrated Preference Parameters (No Borrowing/Saving)

	Mother's Education	
	Non-College	College
A. Single Mothers		
α	4.26	5.03
ψ_m	1.26	1.47
B. Two-Parent Households		
α	2.45	3.45
ψ_m	0.53	0.53
ψ_f	0.65	0.57

Tables D-1 and D-2 show calibration targets and calibrated parameters, separately by marital status and mother's education (non-college vs. college). The calibrated parameters imply that college-educated mothers have a stronger preference for their child's skills (α) compared to non-college-educated mothers. College educated single mothers have a higher value of leisure than their non-college counterparts, while the value of leisure is similar among married mothers. College educated fathers have a lower value of leisure than non-college fathers.

D.1.2 Details on Counterfactual Simulations

Investment Gaps by Education. For the "Baseline" column of Table 6, we use wages, other prices, and family characteristics $Z_{i,t}$ for each child observed in the 2002 PSID. When equalizing wages or other prices, we replace them with their averages by marital status (regardless of maternal education). When equalizing technology, we set each family's $Z_{i,t}$ (except marital status) to its value for a hypothetical family with unobserved type 1, non-college-educated parents, a child aged 8, and no children aged 0–5. Finally, when equalizing preferences, we replace the preference parameters of college-educated single mothers and married parents with those of their non-college counterparts.

Input Price Changes. For the price reduction simulations reported in Table 7, we continue to use wages, other prices, and family characteristics $Z_{i,t}$ observed in 2002 PSID, except that we assume children are 5 years old at the time of the price change and there are no other children ages 0–5 in each family. We calculate the effects on achievement at age 13 by simulating investment responses over ages 5–12, taking into account how technology and the investment policy function evolve over time as children get older.

For the Cobb-Douglas production function, we assume that its share parameters vary with marital status, maternal education (non-college vs. college), and child age. These share parameters are calibrated to match the average expenditure shares by marital status, maternal education, and child age that are simulated under the nested CES production function in the absence of price changes.

D.1.3 Additional Counterfactual Simulation Results

Table D-3 reports the percentage change in total investment (at age 5), I_t, in response to input price changes of different magnitudes. Consistent with Proposition 3, responses to small price changes are very similar for the nested CES and Cobb-Douglas specifications, but the differences grow with larger price changes.

D.2 Unconstrained

In this subsection, we provide additional counterfactual analysis without binding borrowing constraints. We continue to assume $u(c) = \ln c$, $v_j(l_j) = \psi_j \ln l_j$ for $j \in \{m, f\}$, $\beta = 1/1.02$, and $y_t = 0$ for all t. In addition, we make assumptions specific to the unconstrained case. As in Appendix A.3.3, we assume that $\beta(1+r) = 1$, parents continue to have the same period utility after period T, and work until period T_R. They live until period T_D and have zero assets at the time of the child's birth, $A_1 = 0$.

Table D-3: Elasticity of Total Investment Quantity with Respect to Input Prices (No Borrowing/Saving)

Price Change	Nested CES				Cobb-Douglas				% Difference between Cobb-Douglas and Nested CES			
	Wages	Wages (Constant income)	Goods	Child Care	Wages	Wages (Constant income)	Goods	Child Care	Wages	Wages (Constant income)	Goods	Child Care
A. Single Mothers												
10% Change	0.28	-0.80	-0.04	-0.23	0.28	-0.80	-0.05	-0.24	0.33	-0.13	6.16	4.72
30% Change	0.32	-0.97	-0.05	-0.25	0.31	-0.99	-0.05	-0.28	-4.34	2.04	16.53	10.12
50% Change	0.38	-1.24	-0.05	-0.29	0.34	-1.32	-0.06	-0.34	-9.72	5.90	31.73	18.27
B. Two-Parent Households												
10% Change	0.16	-0.94	-0.03	-0.12	0.15	-0.94	-0.03	-0.13	-2.04	0.38	4.26	2.54
30% Change	0.18	-1.16	-0.03	-0.13	0.17	-1.18	-0.03	0.00	-7.56	1.71	14.97	-100.00
50% Change	0.23	-1.55	-0.03	-0.15	0.19	-1.61	-0.04	-0.17	-14.15	4.14	30.19	16.89

D.2.1 Model solution

As of $t = 1$, the lifetime budget constraint for single mothers is

$$\sum_{t=1}^{T_D} (1+r)^{-t+1} c_t + \sum_{t=1}^{T} (1+r)^{-t+1} E_t = \sum_{t=1}^{T_R} (1+r)^{-t+1} W_{m,t}(1 - l_{m,t}). \tag{52}$$

Using the optimality conditions for investment and leisure, Equations (10) and (26), and the fact that consumption is constant due to $\beta(1+r) = 1$, we can back out the period consumption from the lifetime budget constraint (52):

$$c = \left[\frac{1-(1+r)^{-T_D}}{1-(1+r)^{-1}} + \frac{1-\delta_2^{-T}}{1-\delta_2^{-1}} \alpha \beta^T \delta_2^{T-1} \delta_1 + \frac{1-(1+r)^{-T_R}}{1-(1+r)^{-1}} \psi_m \right]^{-1} \left[\sum_{t=1}^{T_R} (1+r)^{-t+1} W_{m,t} \right]. \tag{53}$$

From c given by (53), we can calculate investment and leisure and using the optimality conditions (10) and (26).

Because we assume zero asset at child birth ($A_1 = 0$), we can calculate the asset level at child age 5 (A_5) using the period budget constraint as follows:

$$A_{t+1} = (1+r) A_t + W_{m,t}(1 - l_{m,t}) - c_t - E_t.$$

For price-reduction simulations, we assume that the price reduction (for all future periods $t \geq 5$) occurs unexpectedly at child age 5 ($t = 5$), in which case we solve parents' problem as of period $t = 5$, taking A_5 and new prices as given. From the lifetime budget constraint as of $t = 5$, the new level of period consumption after the price reduction is calculated as follows:

$$\begin{aligned} c = & \left[\frac{1-(1+r)^{-(T_D-4)}}{1-(1+r)^{-1}} + \frac{1-\delta_2^{-(T-4)}}{1-\delta_2^{-1}} \alpha \beta^{T-4} \delta_2^{T-5} \delta_1 + \frac{1-(1+r)^{-(T_R-4)}}{1-(1+r)^{-1}} \psi_m \right]^{-1} \\ & \times \left[(1+r) A_5 + \sum_{t=5}^{T_R} (1+r)^{-t+5} W_{m,t} \right]. \end{aligned}$$

Notice that only wage reduction affects the consumption level.

D.2.2 Calibration of Preference Parameters

We assume parents work until age 65 and live until age 80 (based on the average age for two-parent households). Because we do not observe parents' wages over their entire career, we use estimated life-cycle profile of wages, which we construct in the following way. First, using data from PSID, separately for mothers and fathers, we regress log hourly wages on potential experience and experience squared, state and year dummies, and individual fixed effects. Let $\mathcal{W}_j(e_{j,t})$ be the wage of a parent $j \in \{m, f\}$ predicted by their potential experience $e_{j,t}$ in year t.

Next, we construct future and past wages based on the wage in 2002 and predicted wages:

$$W_{j,t} = W_{j,2002} \frac{\mathcal{W}_j(e_{j,t})}{\mathcal{W}_j(e_{j,2002})}, \quad \forall t \neq 2002.$$

Notice that this approach assumes that the gap between actual and predicted wage in 2002 reflects individual fixed effects.

As before, we calibrate the preference parameters (α, ψ_m, ψ_f) separately by marital status and maternal education group by targeting average time spent on investment and market work (presented in Table D-1). The calibrated parameters for the unconstrained case, shown in Table D-4, exhibit patterns that are qualitatively similar to those of the constrained case, except that college-educated single mothers do not have a stronger preference for leisure than their non-college counterparts.

Table D-4: Calibrated Preference Parameters (Unconstrained)

	Mother's Education	
	Non-College	College
A. Single Mothers		
α	7.15	7.16
ψ_m	1.63	1.61
B. Two-Parent Households		
α	3.69	5.72
ψ_m	0.65	0.68
ψ_f	0.76	0.72

D.2.3 Counterfactual Simulations

Tables D-5 and D-6 report counterfactual simulations analogous to Tables 6 and 7 under the assumption that families are unconstrained.

Table D-5: Gaps in Investment (% Difference) between Non-College and College (Unconstrained)

	Baseline	Equalizing:			
		Wages	All Prices	Technology	Preferences
A. Single Mothers					
Total Investment					
Expenditure (E)	49.47	13.64	13.64	49.47	39.57
Price ($\bar{p}$)	14.12	-7.44	-3.20	18.98	14.12
Quantity (I)	32.26	22.08	17.65	28.53	23.50
Mother's Time Investment (τ_m)	23.75	18.23	15.67	22.78	15.55
B. Two-Parent Households					
Total Investment					
Expenditure (E)	104.15	41.69	41.69	104.15	32.97
Price ($\bar{p}$)	48.11	6.86	5.50	49.14	48.11
Quantity (I)	38.09	32.47	34.37	35.57	-10.06
Mother's Time Investment (τ_m)	26.97	26.59	27.52	32.23	-17.31

Table D-6: Effects of 30% Reduction in Prices (Unconstrained)

	Nested CES					Cobb-Douglas				
	Wages (ages 5+)	Wages (ages 5–12)	Wages (Constant income)	Goods	Child Care	Wages (ages 5+)	Wages (ages 5–12)	Wages (Constant income)	Goods	Child Care
A. Single Mothers										
Change in Investment at Age 5 (%):										
Total Expenditure (E)	-29.78	-18.10	0.00	0.00	0.00	-29.78	-18.10	0.00	0.00	0.00
Mother's Time (τ_m)	-5.46	10.57	34.65	1.26	3.85	0.29	17.42	42.86	0.00	0.00
Goods (g)	-11.79	3.02	25.64	8.71	3.70	-29.78	-18.10	0.00	42.86	0.00
Child care (x)	-20.12	-6.77	13.75	0.68	23.88	-29.76	-18.17	0.00	0.00	42.86
Total (I)	-9.47	5.81	28.93	1.40	7.75	-9.02	6.37	29.58	1.62	8.50
Effects on Age 13 Achievement:										
100×Log Achievement (Age 13)	-6.27	2.76	14.65	1.52	3.98	-5.79	3.24	15.13	1.74	4.28
Value (% Cons. over Ages 5–12)	-5.94	2.73	15.39	1.50	3.97	-5.51	3.21	15.93	1.70	4.26
B. Two-Parent Households										
Change in Investment at Age 5 (%):										
Total Expenditure (E)	-29.82	-18.57	0.00	0.00	0.00	-29.82	-18.57	0.00	0.00	0.00
Mother's Time (τ_m)	-3.08	12.90	38.14	0.76	2.05	0.22	16.79	42.86	0.00	0.00
Father's Time (τ_f)	-2.99	12.89	38.24	0.76	1.99	0.26	16.62	42.86	0.00	0.00
Goods (g)	-9.59	5.04	28.84	8.19	1.93	-29.82	-18.51	0.00	42.86	0.00
Child care (x)	-18.52	-5.03	16.14	0.45	21.91	-29.83	-18.37	0.00	0.00	42.85
Total (I)	-5.47	10.12	34.73	0.90	4.12	-5.00	10.62	35.40	1.04	4.43
Effects on Age 13 Achievement:										
100×Log Achievement (Age 13)	-2.65	6.23	18.32	1.76	2.90	-0.46	8.42	20.51	3.75	4.92
Value (% Cons. over Ages 5–12)	-0.86	5.41	14.83	2.19	2.99	0.54	6.90	16.45	3.49	4.33

D.3 Free Child Care Policies

Consider a policy that gives a certain amount of child care, denoted by $\overline{x}_t$, for free. In this case, households' out-of-pocket child care expenditure is a non-linear function of total child care investment x_t:

$$\max\left\{q_t(x_t-\overline{x}_t),0\right\}$$

As a result, the total investment expenditure, $\mathcal{E}_t(I_t)$, also depends on total investment I_t non-linearly. For single mothers, it is given by

$$\mathcal{E}_t(I_t;\Pi_t,\overline{x}_t)=\min_{\tau_{m,t},g_t,x_t}\left\{W_{m,t}\tau_{m,t}+p_tg_t+\max\left\{q_t(x_t-\overline{x}_t),0\right\}\mid f_t(\tau_{m,t},g_t,x_t)\geq I_t\right\}.$$

Let $I_{H,t}(I_t,x_t)$ be the amount of composite home investment (see Section 3.1) that is required to produce I_t for a given level of child care x_t. Because the expenditure on home investment is still a linear function of composite home investment, let $\overline{p}_{H,t}(\Pi_t)$ be the composite price of home investment. Then the total investment expenditure can be expressed as follows:

$$\mathcal{E}_t(I_t;\Pi_t,\overline{x}_t)=\begin{cases}\overline{p}_{H,t}(\Pi_t)I_{H,t}(I_t,\overline{x}_t), & \text{for } I_t<\overline{x}_t/\underline{x}_t(\Pi_t),\\ \overline{p}_t(\Pi_t)I_t-q_t\overline{x}_t, & \text{for } I_t\geq\overline{x}_t/\underline{x}_t(\Pi_t),\end{cases}$$

where $\underline{x}_t(\Pi_t)$ is the cost-minimizing ratio x_t/I_t in the absence of free child care (i.e., $\overline{x}_t=0$) that is defined in Appendix A.4. For high levels of total investment, households invest in child care beyond the free amount and thus behave as if they receive a lump-sum transfer $q_t\overline{x}_t$. At low levels of total investment, however, child care investment is held fixed at the free amount and households optimally choose other investments conditional on $\overline{x}_t$.

With the non-linear total investment expenditure, the optimality condition for total investment, Equation (12), is modified as follows:

$$\mathcal{E}'_t(I_t;\Pi_t,\overline{x}_t)I_t=\frac{K_t}{u'(c_t)}.$$

Using this condition, we solve for I_t numerically.

We consider a policy that gives free child care only to families with non-college mothers in order to close the gap in total investment between non-college and college mothers that is observed in 2002. Results for single mothers, assuming no borrowing/saving, are reported in Table D-7.

Table D-7: Providing Free Child Care to Single Mothers to Eliminate Investment Gaps (Ages 5–12) by Parental Education

	Nested CES			Cobb-Douglas		
	Non-College Mothers		College Mothers	Non-College Mothers		College Mothers
	Baseline	Free Care		Baseline	Free care	
Free Child Care:						
Public Expenditure ($q\overline{x}$)		116.00			115.47	
Quantity ($\overline{x}$)		33.18			33.03	
Investment Quantities:						
Total (I)	11.08	14.75	14.75	10.90	13.82	13.82
Mother's Time (τ_m)	10.04	11.75	12.42	9.99	10.38	12.54
Goods (g)	12.30	14.11	18.96	12.44	12.92	19.86
Child Care (x)	13.17	33.18	18.32	15.39	33.83	19.77

Notes: The table reports average weekly amounts of free child care, $\overline{x}$, (and its cost) provided to single non-college mothers that would be needed to eliminate average differences in total investment, I, (over child ages 5–12) by parental education. Assumes all families are borrowing constrained. The table also reports endogenous responses in other investments, comparing them with baseline amounts for non-college and college mothers.

E Additional Econometrics Details

E.1 Estimation of $f(\cdot)$ for Two-Parent Households

This section discusses estimation of $f(\cdot)$ for two-parent households, dropping t subscripts to simplify the notation. An analogous set of results to those in the text apply; however, the estimating equations are slightly more complicated due to the roles of both father's and mother's time inputs. Relative demand for child care vs. goods in two-parent families implies

$$\ln R_{x,i} = Z_i'\phi_{x,g} + \left[\frac{\epsilon_{x,H} - \epsilon_{\tau,g}}{1-\epsilon_{\tau,g}}\right] \ln\left(1 + R_{f,i}e^{-\xi_{W_f\tau_f/g,i}} + R_{m,i}e^{-\xi_{W_m\tau_m/g,i}}\right) + (1-\epsilon_{x,H})\ln\tilde{q}_i + \xi_{x/g,i}, \quad (54)$$

where $\xi_{\tau_f W_f/g,i} \equiv \xi_{\tau_f,i} + \xi_{W_f,i} - \xi_{g,i}$ and other variables are defined in the main text.

With no measurement error in wages, time or goods inputs (i.e., $\xi_{\tau_f W_f/g,i} = \xi_{\tau_m W_m/g,i} = 0$), Equation (54) can be estimated via OLS.

Incorporating measurement error in all child investment inputs but assuming (i) wages for both parents are measured without error (i.e., $\xi_{W_m,i} = \xi_{W_f,i} = 0$) and (ii) no unobserved heterogeneity in either parent's child production ability (i.e., $\eta_{m,i} = \eta_{f,i} = 0$) yields the following:

$$\ln(R_{x,i}) = Z_i'\phi_{x,g} + \left[\frac{\epsilon_{x,H} - \epsilon_{\tau,g}}{1-\epsilon_{\tau,g}}\right] \ln\left(1 + e^{\ln(\tilde{\Phi}_{fi})} + e^{\ln(\tilde{\Phi}_{m,i})}\right) + (1-\epsilon_{x,H})\ln\tilde{q}_i + \xi_{x/g,i}. \quad (55)$$

As with single mothers, the stated assumptions enable a two-step approach for estimating Equation (55), using predicted values from OLS estimation of Equation (13) for both fathers and mothers, $\widehat{\ln(R_{j,i})}$, in place of $\ln(\tilde{\Phi}_{j,i})$ for $j \in \{m, f\}$.

E.2 Estimation of $f(\cdot)$ with Measurement Error in Wages

This section discusses estimation of $f(\cdot)$ when wages are measured with error. We begin with single mother households. Dropping t subscripts and taking expectations of Equation (14) conditional on observed data yields:

$$\begin{aligned} &E\left[\ln(R_{x,i})\Big| Z_i, R_{m,i}, \tilde{q}_i, g_i^o\right] \\ &= Z_i'\phi_{x,g} + \left[\tfrac{\epsilon_{x,H}-\epsilon_{\tau,g}}{1-\epsilon_{\tau,g}}\right] E\left[\ln\left(1 + R_{m,i}e^{-\xi_{W_m\tau_m/g,i}}\right)\Big| R_{m,i}\right] + (1-\epsilon_{x,H})\ln\tilde{q}_i - E[\xi_{g,i}|g_i^o]. \quad (56)\end{aligned}$$

If the distribution of measurement error in $(W_{m,i}, \tau_{m,i}, g_i)$ were known, we could simply calculate the expectations on the right hand side of the expression and use GMM to estimate $(\epsilon_{\tau,g}, \epsilon_{x,H}, \phi_{x,g})$. Unfortunately, $E\Big[\ln\left(1 + R_{m,i}e^{-\xi_{W_m\tau_m/g,i}}\right)\Big| R_{m,i}\Big]$ does not have a closed form expression. Using a second order Taylor approximation to integrate over measurement error produces

$$\begin{aligned} E\left[\ln(R_{x,i})\Big| Z_i, R_{m,i}, \tilde{q}_i, g_i^o\right] &\approx Z_i'\phi_{x,g} + \left[\frac{\epsilon_{x,H} - \epsilon_{\tau,g}}{1-\epsilon_{\tau,g}}\right] \ln(1 + R_{m,i}) \\ &+ \sigma^2_{W_m\tau_m/g}\left[\frac{\epsilon_{x,H} - \epsilon_{\tau,g}}{1-\epsilon_{\tau,g}}\right]\left(\frac{R_{m,i}}{2(1+R_{m,i})^2}\right) + (1-\epsilon_{x,H})\ln(\tilde{q}_i) + \sigma^2_{\xi_g}\left(\frac{\ln(g_i^o) - E[\ln(g_i^o)]}{Var(\ln(g_i^o))}\right), \quad (57)\end{aligned}$$

where $\sigma^2_{W_m\tau_m/g} \equiv Var(\xi_{\tau_m W_m/g,i})$ and the final term assumes $\xi_{g,i} \sim N(0, \sigma^2_{\xi_g})$. While this expression is only an approximation, it does not require any knowledge of the distribution for $(\xi_{W_m,i}, \xi_{\tau_m,i})$ and is unaffected by unobserved heterogeneity in parental skills. A GMM approach or OLS can be applied to Equation (57) to obtain consistent estimates; although, $\sigma^2_{W_m\tau_m/g}$ is only identified when $\epsilon_{x,H} \neq \epsilon_{\tau,g}$.

As with single mothers, we can account for measurement error in wages and inputs, as well as unobserved heterogeneity in maternal and paternal child productivity, by taking expectations of Equation (54) conditional on observed data:

$$E\left[\ln R_{x,i}\middle|Z_i, R_{f,i}, R_{m,i}, \tilde{q}_i, g_i^o\right]$$
$$= Z_i'\phi_{x,g} + \left[\tfrac{\epsilon_{x,H}-\epsilon_{\tau,g}}{1-\epsilon_{\tau,g}}\right] E\left[\ln\left(1 + R_{f,i}e^{-\xi_{W_f\tau_f/g,i}} + R_{m,i}e^{-\xi_{W_m\tau_m/g,i}}\right)\middle|R_{f,i}, R_{m,i}\right] + (1-\epsilon_{x,H})\ln\tilde{q}_i - E[\xi_{g,i}|g_i^o].$$

Knowledge of measurement error distributions would allow for direct calculation of the conditional expectation terms on the right hand side. Alternatively, a second order Taylor approximation to integrate over measurement error and $\xi_{g,i} \sim N(0, \sigma_g^2)$ yields:

$$\begin{aligned}
E\left[\ln R_{x,i}\middle|Z_i, R_{f,i}, R_{m,i}, \tilde{q}_i, g_i^o\right] &\approx Z_i'\phi_{x,g} + \left[\frac{\epsilon_{x,H}-\epsilon_{\tau,g}}{1-\epsilon_{\tau,g}}\right]\ln(1+R_{f,i}+R_{m,i}) \\
&+ \sigma^2_{W_f\tau_f}\left[\tfrac{\epsilon_{x,H}-\epsilon_{\tau,g}}{1-\epsilon_{\tau,g}}\right]\left(\tfrac{R_{f,i}(1+R_{m,i})}{2(1+R_{f,i}+R_{m,i})^2}\right) + \sigma^2_{W_m\tau_m}\left[\tfrac{\epsilon_{x,H}-\epsilon_{\tau,g}}{1-\epsilon_{\tau,g}}\right]\left(\tfrac{R_{m,i}(1+R_{f,i})}{2(1+R_{f,i}+R_{m,i})^2}\right) \\
&+ \sigma_g^2\left[\tfrac{\epsilon_{x,H}-\epsilon_{\tau,g}}{1-\epsilon_{\tau,g}}\right]\left(\frac{R_{f,i}+R_{m,i}}{2\big(1+R_{f,i}+R_{m,i}\big)^2}\right) - \sigma_g^2\left(\tfrac{\ln(g_i^o)-E[\ln(g_i^o)]}{Var(\ln(g_i^o))}\right) + (1-\epsilon_{x,H})\ln(\tilde{q}_i),
\end{aligned} \tag{58}$$

where $\sigma^2_{W_j\tau_j} \equiv Var(\xi_{W_j} + \xi_{\tau_j})$ for $j \in \{m, f\}$.[75] Based on this moment condition, GMM can be used to efficiently estimate the technology parameters $(\epsilon_{\tau,g}, \epsilon_{x,H}, \phi_{x,g})$ and measurement error variances $\left(\sigma^2_{W_m\tau_m/g}, \sigma^2_{W_m\tau_m/g}, \sigma_g^2\right)$. OLS can also be used; however, there may be some efficiency loss by not imposing parameter restrictions across terms.

Table E-1 contains the results of estimating Equations (57) and (58) together.

E.3 Clustering Routine for Grouped Heterogeneity

For all mothers (indexed by n) in our main dataset, we estimate the wage equation:

$$\ln(W_{n,t}) = \mu_{k(n)} + X_{n,t}\beta + \epsilon_{n,t}$$

where $k(n) \in \{1, 2, ..., K\}$ indicates the mother's fixed, unobserved type and $X_{n,t}$ includes education dummies, a second order polynomial in potential experience, and calendar year dummies.

Let $\mathcal{K} = \{k(1), k(2), ..., k(N)\}$ be the true type of each mother. We estimate the collection of parameters $(\mathcal{K}, \beta, \mu)$ as:

$$\hat{\mathcal{K}}, \hat{\beta}, \hat{\mu} = \arg\min \sum_n \sum_{t=1}^{T_n} (\ln(W_{n,t}) - X_{n,t}\beta - \mu_{k(n)})^2$$

using the iterative clustering routine described in Bonhomme and Manresa (2015), who also demonstrate that this estimator has regular asymptotics.

[75] As with the case for single mothers, these time expenditure measurement error variances are only identified when $\epsilon_{x,H} \neq \epsilon_{\tau,g}$.

Table E-1: OLS estimates for child care/goods relative demand, all families

$\ln(\tilde{q}_t)$	0.665*
	(0.192)
Married	0.997
	(0.689)
Child's age	-0.059
	(0.036)
Mother some coll.	0.201
	(0.158)
Mother coll+	0.392*
	(0.173)
Mother's age	0.012
	(0.013)
Marr. $\times$ Father some coll.	0.052
	(0.202)
Marr. $\times$ Father coll+	-0.345
	(0.218)
Marr. $\times$ Father's age	-0.015
	(0.015)
Mother white	-0.081
	(0.134)
Num. children ages 0-5	0.302*
	(0.140)
Num. of children	-0.039
	(0.104)
$\ln(1 + R_{m,t} + Marr. \times R_{f,t})$	-0.125
	(0.158)
$\frac{R_{m,t}(1+Marr \times R_{f,t})}{2(1+R_{m,t}+R_{f,t})^2}$	-3.355
	(4.324)
$Marr \times \frac{R_{f,t}(1+R_{m,t})}{2(1+R_{m,t}+R_{f,t})^2}$	0.229
	(4.503)
$Marr \times \frac{R_{m,t}+R_{f,t}}{2(1+R_{m,t}+R_{f,t})^2}$	-2.851
	(6.246)
$\frac{\ln(g_t^o)-E(\ln(g_t^o))}{Var(\ln(g_t^o))}$	-0.688*
	(0.084)
Constant	0.246
	(0.796)
R-squared	0.459
Residual sum of squares	199.738
Sample size	249

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5–12 and only 1–2 children ages 12 and under. * significant at 0.05 level.

E.4 Estimation of $f(\cdot)$ and $\mathcal{H}(\cdot)$ using relative demand and skill measures

To maintain stability in estimation of the full production function using GMM (as $\rho \to 0$ or $\gamma \to 0$), we use the following specification:

$$f = \left[\left(\tilde{a}_{m,i,t} \tau_{m,i,t}^{\rho} + \tilde{a}_{f,i,t} \tau_{f,i,t}^{\rho} + (1 - \tilde{a}_{m,i,t} - \tilde{a}_{f,i,t}) g_{i,t}^{\rho} \right)^{\gamma/\rho} (1 - \tilde{a}_{x,i,t}) + \tilde{a}_{x,i,t} x_{i,t}^{\gamma} \right]^{1/\gamma}$$

where

$$\tilde{a}_{j,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_j)}{1 + \exp(Z_{i,t}\tilde{\phi}_m) + \exp(Z_{i,t}\tilde{\phi}_f)}, \; j \in \{m, f\}, \qquad \text{and} \qquad \tilde{a}_{x,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_x)}{1 + \exp(Z_{i,t}\tilde{\phi}_x)}.$$

E.4.1 Intratemporal Moments

We use GMM to jointly estimate all relative demand equations, interacting residuals from the relative demand equations with the appropriate instruments. We use the intratemporal conditions based on equations (8) and (9) to express the ratio of any two observed inputs z_1 and z_2, given parameters ω, prices $\Pi_{i,t}$ and parental marital status, $M_{i,t} \in \{0, 1\}$. Denote this input ratio $\Phi_{z_1/z_2,i,t} = z_{1,i,t}/z_{2,i,t}$.[76] The residuals are given by:

$$\xi_{z_1/z_2,i,t} = \ln \left(\frac{z_{1,i,t}^{o}}{z_{2,i,t}^{o}} \right) - \ln \left(\Phi_{z_1/z_2,i,t} \right), \qquad z_1, z_2 \in \{\tau_m, \tau_f, x, g\}.$$

The moments we use are based on the following set of residuals (with zeros for unobserved values):

$$\xi_i = [\xi_{x/\tau_m,i,97}, \; \xi_{x/\tau_m,i,02}, \; \xi_{x/g,i,02}, \; \xi_{\tau_m/g,i,02}, \; \xi_{\tau_f/g,i,02}, \xi_{x/\tau_m,i,07}, \; \xi_{x/g,i,07}, \; \xi_{\tau_m/g,i,07}, \; \xi_{\tau_f/g,i,07}],$$

interacting each residual $\xi_{z_1/z_2,i,t}$ with the vector of instruments $Z_{z_1/z_2,i,t}$, which include the observable characteristics determining the relevant factor shares ($a_{z_1,i,t}$ and $a_{z_2,i,t}$) along with relative prices (or instruments for relative prices depending on specification). In addition to the relative demand moments for parental time relative to goods and for child care relative to goods for 2002 and 2007 used in Section 4.1, we also include moments for child care relative to mother's time in 1997 (goods inputs are not measured that year). The final vector of moments is:

$$g_{1,N} = \frac{1}{N} \sum_i \begin{bmatrix} \vdots \\ \xi_{z_1/z_2,i,t} \otimes Z_{z_1/z_2,i,t} \\ \vdots \end{bmatrix}, \; \forall \; \xi_{z_1/z_2,i,t} \in \xi_i. \tag{59}$$

Residual Correlation Test Any persistent unobserved heterogeneity that is not accounted for will appear as a correlation in the residual for input ratios across years. We test the null hypothesis of no correlation using:

$$T_N = \sqrt{N} \frac{\sum_i \xi_{x/\tau_m,i,97} \, \xi_{x/\tau_m,i,02}}{\sqrt{s_{x/\tau_m,97}^2 \, s_{x/\tau_m,02}^2}},$$

which is asymptotically $N(0, 1)$ under the null. Here, $\xi_{x/\tau_m,i,t}$ is the residual in the demand for childcare relative to mother's time for child i at time t and $s_{x/\tau_m,t}^2$ is the corresponding sample variance across individuals.

[76] Notice that $\Phi_{\tau_j/g,i,t} = \Phi_{j,i,t}$ where the latter is defined in equation (8) for $j = m, f$ and $\Phi_{x/g,i,t} = \Phi_{x,i,t}$ where the latter is defined in equation (9).

E.4.2 Intertemporal Moments

In order to relate skill measures 5 years apart with missing inputs in the intervening periods, we begin with the first order condition for investment, equation (10), applied to periods t and $t+s$. Together, these imply:

$$\frac{\overline{p}_{t+s}I_{t+s}}{\overline{p}_t I_t} \propto \frac{u'(c_{t+s})}{u'(c_t)}$$

With non-binding borrowing constraints we have:

$$\frac{u'(c_{t+s})}{u'(c_t)} = (\beta(1+r))^{-s}$$

and with no borrowing/saving and log utility:

$$\frac{u'(c_{t+s})}{u'(c_t)} \propto \frac{W_{m,t+s}+W_{f,t+s}+y_{t+s}}{W_m+W_f+y_t}.$$

This gives the relationship:

$$\ln(I_{t+s}) = \mathcal{C}_t + \ln\left(\frac{\overline{p}_t I_t}{\overline{p}_{t+s}}\right) + \kappa \ln\left(\frac{W_{m,t+s}+W_{f,t+s}+y_{t+s}}{W_{m,t}+W_{f,t}+y_t}\right),$$

where $\mathcal{C}_t$ absorbs all factors of proportionality in the above relationships, $\kappa = 1$ in the no borrowing or saving case, and $\kappa = 0$ in the case without binding constraints. Next, using that $I_t = \Phi_{I/\tau_m}\tau_m$, where the constant Φ_{I/τ_m} depends on relative input prices, family characteristics, and technology parameters as defined in equation (17), and iterating on the Cobb-Douglas production function for 5 periods gives:

$$\begin{aligned}\ln(\Psi_{t+5}) \quad &= \quad \sum_{s=0}^{4}\delta_2^{4-s}\left(\mathcal{C}_{t+s} + Z_{t+s}\phi_\theta + \delta_1\left[\ln\left(\frac{\bar{p}_t\tau_{m,t}\Phi_{I/\tau_m}}{\bar{p}_{t+s}}\right) + \kappa\ln\left(\frac{W_{m,t+s}+W_{f,t+s}+y_{t+s}}{W_{m,t}+W_{f,t}+y_t}\right) + \xi_{\theta,t+s}\right]\right) \\ &\quad +\delta_2^5\ln(\Psi_t).\end{aligned}$$

The only time-varying components of Z_{t+s} affecting θ_{t+s} in our empirical analysis is child's age, allowing us to write the entire first term as a function of Z_t. In the case of no borrowing/saving, the first term depends on additional structural parameters $(\alpha, \beta, r, \psi_m, \psi_f)$ through the factor of proportionality $\mathcal{C}_t$; however, age-specific intercept terms can absorb all of these expressions. We use a linear term in age as a first-order approximation. Making these substitutions results in the outcome equation presented in (19).

With the PSID-CDS, we address measurement error in child human capital $(\Psi_{i,t})$ and mother's time using two measures of cognitive ability from the Letter-Word $(LW_{i,t})$ and Applied Problems $(AP_{i,t})$ modules of the Woodcock-Johnson aptitude test. We write these as:

$$S_{i,t} = \lambda_S \ln(\Psi_{i,t}) + \xi_{S,i,t}, \quad S \in \{LW, AP\},\ t \in \{1997, 2002, 2007\}.$$

These measurement assumptions require a normalization on the factor loading for one measure, as in Cunha, Heckman, and Schennach (2010). We set $\lambda_{LW} = 1$, leaving the factor loading on the Applied Problems score (λ_{AP}) to be estimated. Substituting these noisy measures into the outcome equation above gives:

$$\lambda_S^{-1}S_{t+5} = Z_t\overline{\phi}_\theta + \delta_1\sum_{s=0}^{4}\delta_2^{4-s}\left[\ln\left(\frac{\bar{p}_t\tau^o_{m,t}\Phi_{I/\tau_m}}{\bar{p}_{t+s}}\right) + \kappa\ln\left(\frac{W_{m,t+s}+W_{f,t+s}+y_{t+s}}{W_{m,t}+W_{f,t}+y_t}\right)\right] + \delta_2^5 LW_t + \tilde{\xi}_{\Psi,S,t}$$

for $S \in \{AP, LW\}$, where $Z_t \overline{\phi}_\theta$ reflects our approximation for the first term in Equation (19) and $\tilde{\xi}_{\Psi,S,t}$ collects the measurement error terms $\xi_{m,t}$ and $\xi_{AP,t}$, and the innovation term $\tilde{\xi}_{\theta,t+5}$.

Our second set of moments for production parameters are now given by:

$$g_{2,N} = \frac{1}{N} \sum_n \begin{bmatrix} \vdots \\ \tilde{\xi}_{\Psi,S,i,t} \otimes Z_{\Psi,i,t} \\ \vdots \end{bmatrix} \quad t \in \{1997, 2002\}, S \in \{AP, LW\}, \tag{60}$$

where the instrument set $Z_{\Psi,i,t}$ contains all $Z_{i,t}$ that are permitted to influence $\theta_{i,t}$, along with $\ln(\tau^o_{m,i,t+5})$ to instrument for $\ln(\tau^o_{m,i,t})$.

In order to identify the factor loading λ_{AP}, we use the assumption that measurement error is independent over time to write:

$$\lambda_{AP} = \frac{Cov(AP_{i,t+5}, LW_{i,t})}{Cov(LW_{i,t+5}, LW_{i,t})}, \qquad \lambda^2_{AP} = \frac{Cov(AP_{i,t+5}, AP_{i,t})}{Cov(LW_{i,t+5}, LW_{i,t})}.$$

Because we normalize our measurements to have mean zero, these two identifying conditions can be written as the following pair of moments:

$$E\left[(AP_{i,t+5} - \lambda_{AP} LW_{i,t+5}) LW_{i,t}\right] = 0 \qquad \text{and} \qquad E\left[AP_{i,t+5} AP_{i,0} - \lambda^2_{AP} LW_{i,t+5} LW_{i,t}\right] = 0. \tag{61}$$

The full estimation procedure conducts optimally weighted GMM by stacking the moment conditions on input ratios, the moment conditions on the achievement equation, and the moment conditions derived from our measurement assumptions above. The parameters to be estimated are $\omega = (\rho, \gamma, \tilde{\phi}_m, \tilde{\phi}_f, \tilde{\phi}_x)$, $\delta = (\delta_1, \delta_2)$, $\overline{\phi}_\theta$, and λ_{AP}.

E.4.3 Relaxing and Testing Relative Demand and Production Parameters

Within-period production parameters determining skills are given by $\omega = (\rho, \gamma, \phi_m, \phi_f, \phi_x)$. Let $\breve{\omega}$ indicate the parameter values that are perceived by parents in that they determine relative demand but do not necessarily enter the production function (i.e., ω need not equal $\breve{\omega}$).

Letting each relative input ratio depend on relative prices, marital status, and $\breve{\omega}$, implies a moment condition for relative demand that depends only only on perceived $\breve{\omega}$: $g_{1,N}(\breve{\omega})$.

Recall from equation (17) that Φ_{I/τ_m} depends on relative prices, the perceived technology parameters $\breve{\omega}$ through $(\Phi_{m,i,t}, \Phi_{f,i,t}, \Phi_{x,i,t})$, and the true technology parameters, ω. Additionally, the composite price for total investment I_t depends on both perceived and true technology parameters:

$$\bar{p}(\Pi_{i,t}, Z_{i,t}, \omega, \breve{\omega}) = \frac{W_{m,i,t}\Phi_{m,i,t} + W_{f,i,t}\Phi_{f,i,t} + p_{i,t} + q_{i,t}\Phi_{x,i,t}}{\left[\left(a_{m,i,t}\Phi^\rho_{m,i,t} + a_{f,i,t}\Phi^\rho_{f,i,t} + 1 - a_{m,i,t} - a_{f,i,t}\right)^{\gamma/\rho}(1 - a_{x,i,t}) + a_{x,i,t}\Phi^\gamma_{x,i,t}\right]^{1/\gamma}},$$

where we use $a_{g,i,t} = 1 - a_{m,i,t} - a_{f,i,t}$.

Combining moment conditions now gives:

$$g_N(\delta, \overline{\phi}_\theta, \lambda_{AP}, \omega, \breve{\omega}) = \begin{bmatrix} g_{1,N}(\breve{\omega}) \\ g_{2,N}(\delta, \overline{\phi}_\theta, \lambda_{AP}, \omega, \breve{\omega}) \end{bmatrix}.$$

From here, we can directly apply three tests of the null hypothesis that $\breve{\omega} = \omega$ as described in Section 9.2 of Newey and McFadden (1994). The Lagrange Multiplier test statistic is an appropriately normalized derivative of the GMM criterion with respect to the retricted parameters at the restricted estimates ($\omega = \breve{\omega}$). The Wald statistic takes a weighted average of the squared distance between estimates $\hat{\omega}$ and $\hat{\breve{\omega}}$ when they minimize the GMM criterion in an unconstrained way, and the Distance Metric compares the value of the GMM criterion itself at the constrained vs. unconstrained estimates. Each statistic follows a Chi-squared distribution with degrees of freedom equal to the number of constraints. In our analysis, we estimate the constrained model and test individual parameter restrictions using the Lagrange Multiplier statistic. We re-estimate the model by replacing all parameters that fail this test at 5% significance. At those estimates we can conduct the distance metric and Wald tests.

E.5 Direct estimation of $\mathcal{H}(\cdot)$ using relaxed relative demand and skill measures

Following the estimation approach described in Section 4.2.1, we specify a relaxed set of relative demand equations of the form:

$$\check{\Phi}_{j/\tau_m}(\Omega_{i,t}) = Z_{i,t}\beta_j + \Pi_{i,t}\gamma_{j,1} + \gamma_{j,2}\ln(y_{i,t}) \quad j \in \{x, \tau_f, g\}.$$

In addition, in order to handle imputation of input levels over time, we specify that

$$\ln\left(\frac{\tau_{m,i,s}}{\tau_{m,i,t}}\right) = \check{\Phi}_{\tau_m/\tau_m}(\Omega_{i,t}, \Omega_{i,s}) = \beta_\tau Z_{i,t} + (\Pi_{i,s} - \Pi_{i,t})\gamma_{m,1} + \gamma_{m,2}\ln(y_{i,s}/y_{i,t})$$

We first estimate the parameters of this relative demand system using all non-missing input ratios for childcare, father's time, and goods relative to mother's time, in addition to the ratio of mother's time inputs in 2002 relative to 1997 and 2007 relative to 2002. We interact the residuals for each predicted input ratio with the full set of $\Omega_{i,t}$ as instruments.

To then implement the direct production function estimator, we iterate over the five period intervals to derive an outcome equation:

$$\begin{aligned}\ln(\Psi_{i,t+5}) = \sum_{s=0}^{4}\delta_2^{4-s}\Big(&Z_{i,t+s}\phi_\theta + \delta_1\ln(\tau_{m,t}) \\ &+ \delta_1\ln(f(1, \check{\Phi}_{\tau_f/\tau_m,i,t+s}, \check{\Phi}_{g/\tau_m,i,t+s}, \check{\Phi}_{x/\tau_m,i,t+s}; Z_{i,t+s}, \omega)) + \delta_1\ln(\check{\Phi}_{\tau_m/\tau_m,i,t,t+s})\Big) + \tilde{\xi}_{\theta,t+5}\end{aligned} \quad (62)$$

We then implement moment conditions for the residuals in this outcome equation identically to our main approach in Appendix E.4, substituting in values for the input ratios using estimates from the first stage.

Due to numerical instabilities caused by very weak identification of the production parameters, we implement this estimator under the simplifying assumption that factor shares do not vary with $Z_{i,t}$ other than by marital status. To navigate convergence issues, we implement the estimator using an identity waiting matrix and terminating minimization after 100 LBFGS iterations followed by 10 iterations of Newton's method. Table G-24 reports bootstrapped 80% confidence intervals for the parameters of the production function and shows the very poor performance of this estimator.

F A Monte Carlo experiment to evaluate the performance of direct estimation

To better understand the poor performance of the GMM estimation routine when parameters are directly estimated using either a relaxed demand specification (see Section 4.2.1, Appendix E.5 and Table G-24) or a nested CES demand specification (see Section 4.2.2, Appendix E.4 and Tables G-21-G-23), we implement a Monte Carlo simulation in a simplified environment. Results indicate that direct estimation of factor shares and elasticities of substitution is not practical given the amount of variation in inputs available in our data and the sample size of our dataset. This remains true even in the absence of measurement error in inputs, suggesting that the poor performance of direct estimation is not simply an artifact of the particular method that we propose.

F.1 Data Generating Process (DGP)

We assume that an outcome y is generated by a CES function with two inputs:

$$y = \exp(\xi)(ax_1^\rho + (1-a)x_2^\rho)^{\delta/\rho},$$

and that

$$\ln(x_1) \sim N(0, \sigma_x),\ \ \xi \sim N(0, \sigma_\xi).$$

The input x_2 is chosen according to optimal input ratio

$$\frac{x_2}{x_1} = \left(\frac{a}{1-a}\pi\right)^{1/(\rho-1)} \exp(\zeta_1) = \Phi_{2/1}(\pi; a, \rho)\exp(\zeta_1),$$

where π is the relative price of inputs and $\ln(\pi) \sim N(0, \sigma_\pi)$. Let ζ_1 be a preference shock in x_2 (or some unobserved factor affecting relative demand) with $\zeta_1 \sim N(0, \sigma_{\zeta,1})$.

We will compare an estimator with and without measurement error in the second input x_2. In the case with measurement error:

$$\ln(x_2^o) = \ln(x_2) + \zeta_2,\ \ \zeta_2 \sim N(0, \sigma_{\zeta,2}).$$

F.2 Calibration

We choose parameters for this data generating process as follows:

- We choose $a = 0.5$ and $\rho = -3$, following our evidence of moderately strong complementarity between inputs in the data.
- We choose $\sigma_\pi = 0.68$ to match residual variation in childcare prices relative to mothers' wages conditional on $Z_{i,t}$.
- We choose $\sigma_x = 0.73$ to match the variance in residual variation in log mothers' time conditional on $Z_{i,t}$.
- We choose $\sigma_{\zeta,1} = 0.43$ to match the covariance in residuals (using preferred model estimates) for childcare demand relative to mother's time across years (thus capturing time-invariant heterogeneity in relative demand).

- We choose $\sigma_{\zeta,2} = 0.79$ to absorb any additional variation in relative demand residuals not explained by ζ_1.
- We choose $\sigma_\xi = 0.27$ to match half the variance of residuals in the skill outcome equation using preferred model estimates.[77]

F.3 Estimation

We consider four estimators to evaluate the performance of direct estimation relative to our baseline approach. Each estimator performs Nonlinear Least Squares (NLLS) on the implied residual ξ in the outcome equation under differing assumptions regarding measurement error and different cross-equation restrictions between demand and production parameters. As a matter of comparison, recall that the first order conditions for NLLS represent a just-identified GMM estimator, making these results informative for our approach.

Estimator (1). Estimator (1) uses only the outcome equation and true inputs (i.e., without measurement error), but it does not use relative demand restrictions. The NLLS estimator is defined as:

$$\widehat{(a, \rho, \delta)} = \arg\min \sum_i \left(\ln(y_i) - \frac{\delta}{\rho} \ln\left(a x_{1,i}^\rho + (1-a) x_{2,i}^\rho \right) \right)^2 ,$$

where the data $(y_i, x_{1,i}, x_{2,i})_{i=1}^N$ are generated according to the DGP outlined in sections F.1 and F.2.

Estimator (2). Estimator (2) augments Estimator (1) by adding relative demand outcomes to account for measurement error in x_2. In doing so it additionally estimates "perception" parameters $\check{\rho}$ and $\check{a}$ without imposing that these are equal to "actual" production parameters:

$$\begin{aligned}\widehat{(\check{a}, \check{\rho}, a, \rho, \delta)} = \arg\min \Bigg\{ & \sum_i \left(\ln(x_{2,i}^o / x_{1,i}) - \ln(\check{a}/(1-\check{a})) - 1/(\check{\rho}-1) \ln(\pi_i) \right)^2 \\ & + \sum_i \left(\ln(y_i) - \frac{\delta}{\rho} \ln\left(a + (1-a)\Phi_{2/1}(\pi_i; \check{a}, \check{\rho})^\rho \right) - \delta \ln(x_{1,i}) \right)^2 \Bigg\},\end{aligned}$$

where the data $(y_i, x_{1,i}, x_{2,i}^o)_{i=1}^N$ are generated according to the DGP outlined in sections F.1 and F.2. Notice that $x_{2,i}^o$, which includes measurement error, is used in place of actual $x_{2,i}$, here and for the next estimator.

Estimator (3). Estimator (3) minimizes the same objective function as estimator (2) but imposes the cross-equation restriction that $\check{\rho} = \rho$ and $\check{a} = a$, thereby using relative demand information to inform production estimates as in our preferred approach:

$$\begin{aligned}\widehat{(a, \rho, \delta)} = \arg\min \Bigg\{ & \sum_i \left(\ln(x_{2,i}^o / x_{1,i}) - \ln(a/(1-a)) - 1/(\rho-1) \ln(\pi_i) \right)^2 \\ & + \sum_i \left(\ln(y_i) - \frac{\delta}{\rho} \ln\left(a + (1-a)\Phi_{2/1}(\pi_i; a, \rho)^\rho \right) - \delta \ln(x_{1,i}) \right)^2 \Bigg\},\end{aligned}$$

[77] We reduce the variance of the outcome residual to make differences in estimator properties more transparent.

where the data $(y_i, x_{1,i}, x^o_{2,i})_{i=1}^N$ are generated according to the DGP outlined in sections F.1 and F.2.

Estimator (4). Estimator (4) implements the same NLLS routine as estimator (1) using a different DGP from the baseline used for Estimators (1)-(3). For this estimator, we assume that $\ln(x_1) \sim N(0, 0.73)$, as in the baseline DGP, and $\ln(x_2) \sim N(0, 0.87)$, which ensures that x_2 has the same unconditional variance as in the baseline (using $\sigma^2_{x,2} = (1/(\rho - 1))^2 \sigma^2_\pi + \sigma^2_x + \sigma^2_{\zeta,1}$). This alternative DGP allows us to explore the properties of the direct estimator when there is no correlation between inputs, holding equal the overall amount of variation in the data.

F.4 Results and Interpretation

We implement each estimator for 500 Monte Carlo trials using sample sizes of 500, 1000, and 5000, respectively. Table F-1 reports the bias and standard deviation of each parameter in the Monte Carlo sample. Results for estimators (1) and (2) verify that direct estimators for ρ perform very poorly even when inputs are measured perfectly and even for large sample sizes. While these estimators do not exhibit any bias in factor shares (a), they still suffer from substantial sampling variance.

The fact that estimator (4) exhibits meaningful improvements in performance relative to estimator (1) suggests that performance issues are at least partly driven by the correlation between inputs induced by optimal input choices. When relative demand outcomes are used as additional identifying moments for a and ρ in estimator (3), there is a vast improvement in both bias and sampling variance.

Table F-2 presents an identical set of results with relative price variation (σ_π) set to five times the amount of the initial calibration. The reductions in bias and variance of estimated ρ for method (2), which must handle measurement error in the second input using estimated relative input demand, shows the crucial importance of sufficient relative price variation for this method. With this increase in price variation, the performance of all direct estimation methods ((1), (2), and (4)) becomes much more comparable across sample sizes. Unfortunately, estimates of ρ are still quite biased and imprecise, even for a sample size of 5000, highlighting the value of exploiting relative demand.

Table F-1: Results for Monte Carlo Simulation of Direct vs. Demand-based Production Estimation

	Results for ρ							
	Bias				Std. Dev.			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
$N = 500$	17.08	23.76	0.23	13.96	22.49	23.69	1.19	20.67
$N = 1000$	13.73	27.45	0.09	9.96	20.85	23.52	0.70	18.03
$N = 5000$	2.01	26.23	-0.00	0.75	8.05	22.05	0.29	4.54
	Results for a							
	Bias				Std. Dev.			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
$N = 500$	-0.00	-0.01	-0.00	-0.01	0.35	0.41	0.04	0.31
$N = 1000$	0.00	-0.01	0.00	0.02	0.32	0.38	0.03	0.26
$N = 5000$	0.01	0.00	-0.00	-0.00	0.15	0.29	0.01	0.10
	Results for δ							
	Bias				Std. Dev.			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
$N = 500$	-0.00	-0.00	-0.00	-0.00	0.02	0.02	0.02	0.02
$N = 1000$	-0.00	-0.00	0.00	0.00	0.01	0.01	0.01	0.01
$N = 5000$	-0.00	-0.00	-0.00	-0.00	0.00	0.00	0.00	0.01

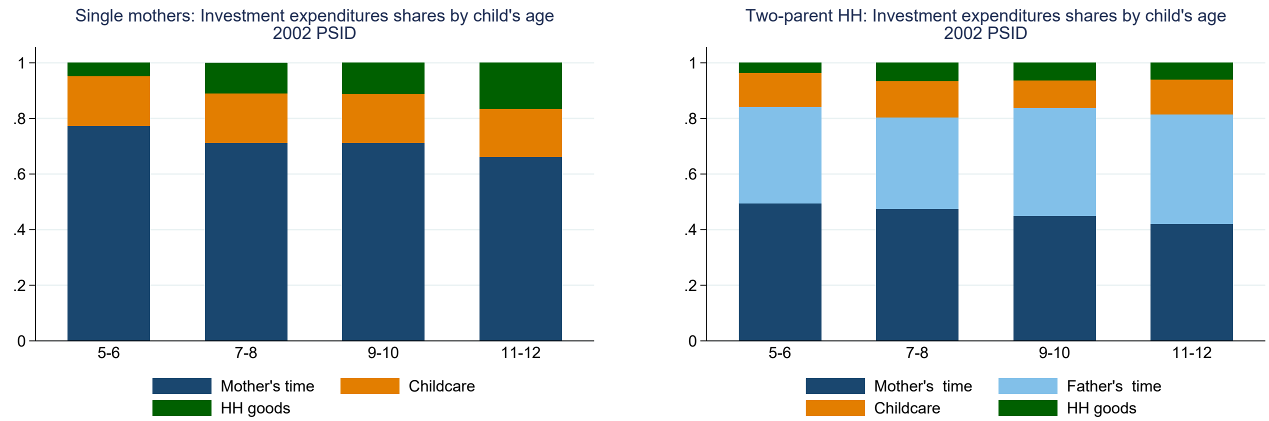
Notes: This table presents results from 500 Monte Carlo samples of each estimator described in Appendix F. Estimator (1) assumes inputs are perfectly measured and performs NLLS on production outcomes. Estimator (2) assumes measurement error in input 2 and uses relative demand observations to impute the second input, but it does not impose cross-equation restrictions between demand and production when performing NLLS. Estimator (3) imposes the cross-equation restrictions. Estimator (4) implements Estimator (1) assuming that x_1 and x_2 have the same unconditional variances but are uncorrelated.

Table F-2: Results for Monte Carlo Simulation with Substantially More Relative Price Variation

	Results for ρ							
	Bias				Std. Dev.			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
$N = 500$	15.40	15.79	0.01	15.84	21.76	21.48	0.19	21.65
$N = 1000$	10.21	11.90	0.01	8.29	18.47	19.01	0.13	17.55
$N = 5000$	1.05	2.67	-0.00	0.96	5.76	8.21	0.06	5.39
	Results for a							
	Bias				Std. Dev.			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
$N = 500$	-0.00	-0.00	0.00	0.02	0.32	0.32	0.04	0.32
$N = 1000$	0.03	0.03	0.00	0.01	0.28	0.29	0.03	0.25
$N = 5000$	0.00	0.02	0.00	0.01	0.12	0.15	0.01	0.11
	Results for δ							
	Bias				Std. Dev.			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
$N = 500$	-0.00	-0.00	0.00	-0.00	0.02	0.02	0.01	0.02
$N = 1000$	-0.00	-0.00	-0.00	-0.00	0.01	0.01	0.01	0.01
$N = 5000$	-0.00	-0.00	-0.00	-0.00	0.01	0.01	0.00	0.01

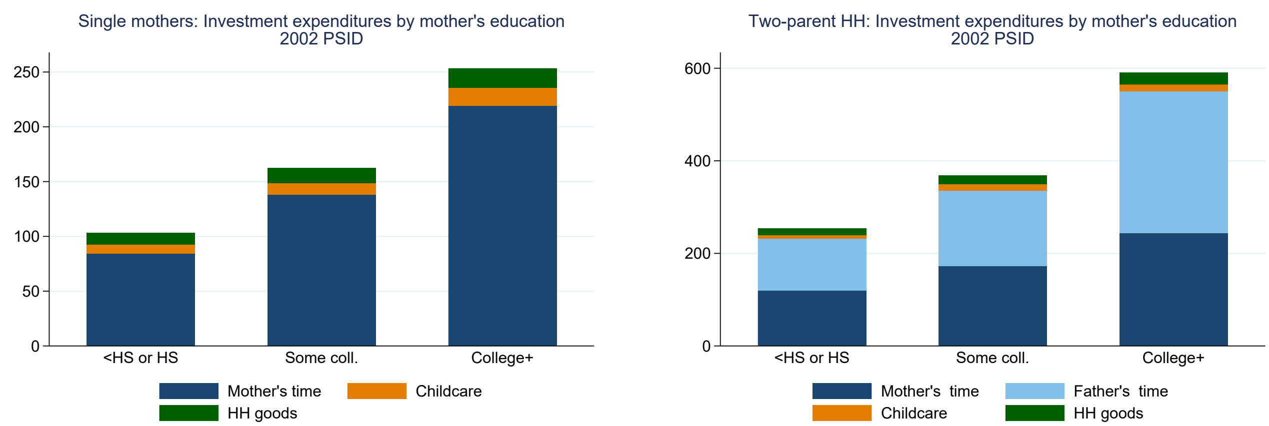
Notes: This table presents identical results to Table F-1 with the only difference being that σ_π is set to five times the calibrated value for the experiments in Table F-1.

G Additional Empirical Results



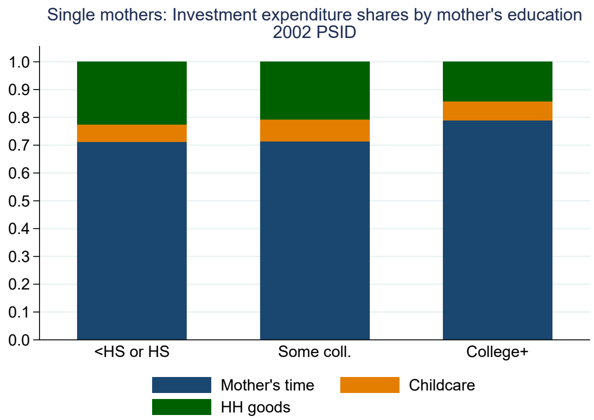
(a) Single mothers (b) Two-parent households

Figure G-1: Expenditure shares by child's age (PSID, 2002)

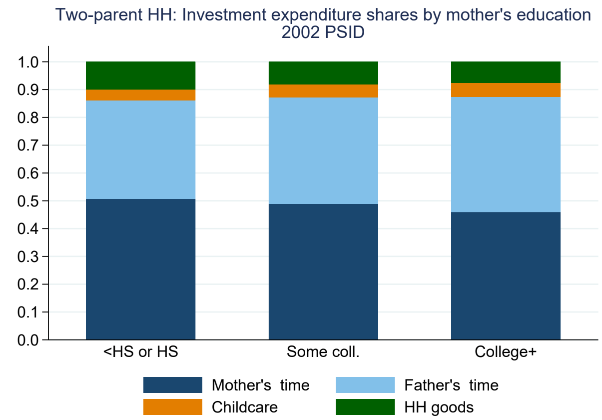


(a) Single mothers (b) Two-parent households

Figure G-2: Weekly child investment expenditures by mother's education, includes families with zero child care spending (PSID, 2002)

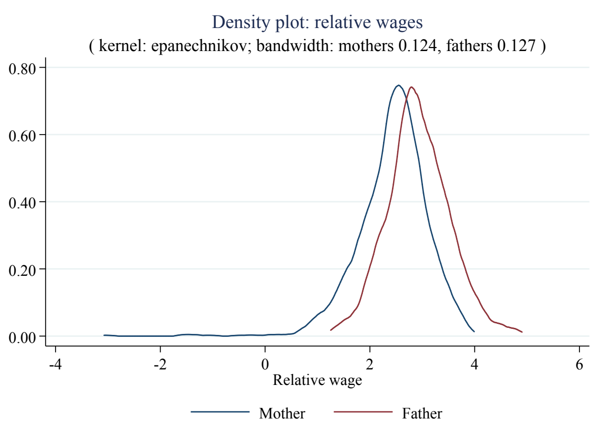


(a) Single mothers

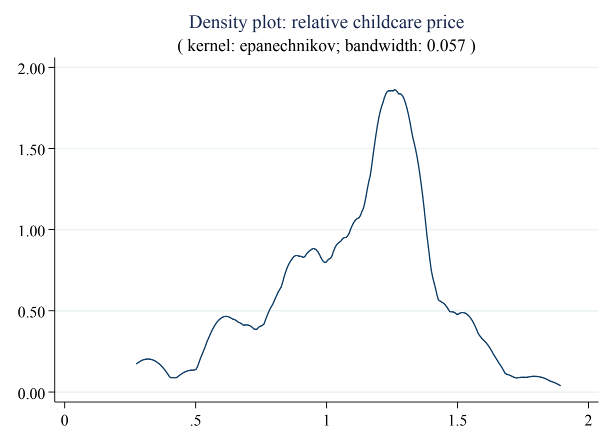


(b) Two-parent households

Figure G-3: Expenditure shares by mother's education, includes families with zero child care spending (PSID, 2002)



(a) $\ln(\tilde{W}_m)$ and $\ln(\tilde{W}_f)$



(b) $\ln(\tilde{q})$

Figure G-4: Distributions of log relative input prices (PSID, 2002 and 2007)

Table G-1: Summary statistics for restricted samples: 2002 and 2007

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	High pred. prob. of work (mothers)			Table 2 (baseline)			Positive child care spending			Table 4 (baseline)		
	Sample Size	Mean	SD	Sample Size	Mean	SD	Sample Size	Mean	SD	Sample Size	Mean	SD
$\ln(\tilde{W}_m)$	928	2.49	0.62	727	2.51	0.62	384	2.56	0.52	322	2.58	0.53
$\ln(\tilde{W}_f)$	662	2.98	0.60	451	2.96	0.56	247	2.86	0.53	213	2.86	0.51
$\ln(\tilde{q})$	1156	1.10	0.32	727	1.10	0.32	423	1.10	0.33	347	1.10	0.33
Child's age	1156	9.60	2.05	727	9.64	2.01	423	8.34	1.95	347	8.36	1.95
Mother HS grad.	1156	0.31	0.46	727	0.29	0.45	422	0.25	0.43	347	0.25	0.44
Mother some coll.	1156	0.35	0.48	727	0.34	0.47	422	0.37	0.48	347	0.35	0.48
Mother coll+	1156	0.33	0.47	727	0.36	0.48	422	0.33	0.47	347	0.35	0.48
Mother's age	1156	37.87	6.36	727	37.92	6.28	423	35.92	6.22	347	36.00	6.36
Father HS grad.	744	0.40	0.49	491	0.42	0.49	265	0.35	0.48	227	0.35	0.48
Father some coll.	744	0.26	0.44	491	0.27	0.44	265	0.23	0.42	227	0.22	0.42
Father coll+	744	0.32	0.47	491	0.30	0.46	265	0.34	0.48	227	0.35	0.48
Father's age	744	40.72	6.99	491	40.51	6.86	264	38.71	6.73	227	38.74	6.84
Mother white	1156	0.57	0.50	727	0.59	0.49	421	0.56	0.50	347	0.56	0.50
Num. children age 0-5	1156	0.14	0.37	727	0.13	0.36	423	0.34	0.50	347	0.34	0.50
Num. of children	1156	1.98	0.70	727	1.97	0.66	423	1.87	0.61	347	1.87	0.58
Year = 2007	1156	0.20	0.40	727	0.19	0.39	423	0.04	0.20	347	0.05	0.22

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5–12 and only 1–2 children ages 12 and under. Columns 1–3 based on sample of mothers with predicted probability of work at least 0.75. Columns 4-6 based on sample used in column (2) of Table 2 (includes those with predicted probability of work at least 0.75, currently working, and no missing covariates). See Table G-2 for model of predicted probability of work. Columns 7–9 based on sample of mothers with positive child care spending. Columns 10–12 based on sample used in Table 4 (includes mothers with positive child care spending and no missing covariates).

Table G-2: Predicted probability (average derivatives) of work probits for parents

	(1) Single Mothers	(2) Married Mothers	(3) Married Fathers
Mother HS grad	0.115*	0.075	-0.024
	(0.038)	(0.045)	(0.026)
Mother some coll.	0.153*	0.111*	-0.008
	(0.039)	(0.047)	(0.027)
Mother coll+	0.271*	0.189*	0.028
	(0.053)	(0.049)	(0.031)
Mother's age	-0.005*	-0.000	0.001
	(0.002)	(0.003)	(0.002)
Mother white	0.047	-0.019	0.056*
	(0.029)	(0.022)	(0.012)
Num children age 0-5	-0.024	-0.034	0.006
	(0.055)	(0.038)	(0.023)
Num of children	-0.004	-0.001	-0.010
	(0.017)	(0.014)	(0.007)
age of youngest child	0.007	0.012	0.002
	(0.009)	(0.007)	(0.004)
year = 2002	0.018	0.074*	0.071*
	(0.028)	(0.021)	(0.012)
year = 2007	0.005	-0.016	0.075*
	(0.047)	(0.040)	(0.017)
Father HS grad		0.123*	0.007
		(0.036)	(0.020)
Father some coll.		0.110*	0.005
		(0.041)	(0.022)
Father coll+		0.023	0.071*
		(0.041)	(0.026)
Father's age		-0.002	-0.002
		(0.002)	(0.001)
Sample size	824	1,753	1,737

Notes: Sample includes families in 1997, 2002, or 2007 PSID-CDS with children ages 5–12 and only 1–2 children ages 12 and under. All specifications include CDS child age dummies. * significant at 0.05 level.

Table G-3: Log wage regressions for parents

	(1) Single Mothers	(2) Married Mothers	(3) Married Fathers
HS graduate			0.307*
			(0.066)
Some college	0.246*	0.257*	0.476*
	(0.053)	(0.049)	(0.069)
College +	0.526*	0.585*	0.781*
	(0.067)	(0.048)	(0.066)
Age	0.068	0.055	0.072*
	(0.038)	(0.033)	(0.020)
Age-squared	-0.001	-0.001	-0.001*
	(0.001)	(0.000)	(0.000)
Mother white	0.133*	-0.044	0.178*
	(0.050)	(0.042)	(0.041)
Constant	0.614	0.963	0.769
	(0.679)	(0.620)	(0.401)
R-squared	0.149	0.179	0.223
Sample size	542	932	1182

Notes: Sample includes families in 1997, 2002, or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. Only mothers (fathers) with predicted probability of work of at least 0.75 (0.9) included. See Table G-2 for model of predicted probability of work. * significant at 0.05 level.

Table G-4: First-stage estimates for mother time/goods relative demand using different predicted wage measures as instruments

	(1) Uses All Variation	(2) Excludes Avg. State Diff.	(3) Excludes Avg. Occ. Diff.	(4) Excludes Avg. State & Occ. Diff.
Mother's predicted log wage	1.084*	1.002*	0.762*	0.373
	(0.103)	(0.110)	(0.178)	(0.209)
Married	-0.045	-0.057	-0.055	-0.058
	(0.046)	(0.047)	(0.049)	(0.049)
Child's age	-0.009	-0.011	-0.012	-0.013
	(0.011)	(0.011)	(0.011)	(0.012)
Mother some coll.	0.191*	0.201*	0.270*	0.271*
	(0.049)	(0.050)	(0.051)	(0.052)
Mother coll+	0.433*	0.448*	0.610*	0.596*
	(0.052)	(0.053)	(0.054)	(0.055)
Mother's age	0.009*	0.012*	0.009*	0.011*
	(0.004)	(0.004)	(0.004)	(0.004)
Mother white	0.026	0.089*	0.073	0.096*
	(0.043)	(0.044)	(0.046)	(0.048)
Num children ages 0-5	0.039	0.032	0.042	0.035
	(0.060)	(0.062)	(0.064)	(0.065)
Num of children	-0.047	-0.055	-0.069*	-0.074*
	(0.030)	(0.030)	(0.031)	(0.032)
Constant	0.295	0.428	0.537	1.323*
	(0.229)	(0.241)	(0.395)	(0.453)
F-statistic Excluded Instrument	111.60	82.83	18.36	3.19
Sample size	720	720	720	720

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5–12 and only 1–2 children ages 12 and under. Only mothers with predicted probability of work of at least 0.75 included. See Table G-2 for model of predicted probability of work. This table reports effects of predicted log wage instruments and other exogenous family characteristics on log relative wages, $\ln(\tilde{W}_{m,t})$, for mothers. Using the 2000 Census, predicted log wages are obtained from gender-specific regressions of log wages on an indicator for race (white/non-white), potential experience and experience-squared, educational attainment (<12 years, 12 years, 13–15 years, 16 years, 17+ years), 16 industry dummies, 97 occupation dummies (minor 2000 SOC codes), state dummies, interactions of race and education dummies with experience, and interactions of race and occupation dummies with state dummies. Column (1) uses predicted log wages. Column (2) eliminates average differences across states from predicted log wages, column (3) eliminates average differences across occupations, and column (4) eliminates average differences across states and occupations. * significant at 0.05 level.

Table G-5: 2SLS estimates for mother time/goods relative demand using different predicted wage measures as instruments

	(1) Uses All Variation	(2) Excludes Avg. State Diff.	(3) Excludes Avg. Occ. Diff.	(4) Excludes Avg. State & Occ. Diff.
$\ln(\tilde{W}_{m,t})$	0.553*	0.359	0.799	-0.450
	(0.196)	(0.226)	(0.457)	(1.243)
Married	-0.071	-0.081	-0.058	-0.123
	(0.096)	(0.097)	(0.098)	(0.127)
Child's age	-0.140*	-0.143*	-0.137*	-0.154*
	(0.022)	(0.023)	(0.023)	(0.031)
Mother some coll.	0.026	0.078	-0.040	0.296
	(0.113)	(0.118)	(0.158)	(0.353)
Mother coll+	-0.119	-0.007	-0.262	0.463
	(0.155)	(0.168)	(0.285)	(0.731)
Mother's age	-0.007	-0.005	-0.010	0.004
	(0.008)	(0.008)	(0.009)	(0.016)
Mother white	-0.233*	-0.218*	-0.251*	-0.158
	(0.091)	(0.092)	(0.096)	(0.138)
Num children ages 0-5	0.168	0.174	0.159	0.201
	(0.126)	(0.127)	(0.127)	(0.150)
Num of children	0.082	0.068	0.101	0.008
	(0.063)	(0.064)	(0.070)	(0.116)
Constant	2.398*	2.800*	1.887	4.479
	(0.520)	(0.572)	(1.002)	(2.606)
Sample size	720	720	720	720

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5–12 and only 1–2 children ages 12 and under. Only mothers with predicted probability of work of at least 0.75 included. See Table G-2 for model of predicted probability of work. Using the 2000 Census, predicted log wages, used as instruments for $\ln(\tilde{W}_{m,t})$, are obtained from gender-specific regressions of log wages on an indicator for race (white/non-white), potential experience and experience-squared, educational attainment (<12 years, 12 years, 13–15 years, 16 years, 17+ years), 16 industry dummies, 97 occupation dummies (minor 2000 SOC codes), state dummies, interactions of race and education dummies with experience, and interactions of race and occupation dummies with state dummies. Column (1) uses predicted log wages as instruments. Column (2) eliminates average differences across states from predicted log wages, column (3) eliminates average differences across occupations, and column (4) eliminates average differences across states and occupations. * significant at 0.05 level.

Table G-6: OLS & 2SLS estimates for mother time/goods relative demand for different selection on predicted probability of work

	(1)	(2)	(3)	(4)	(5)	(6)
	OLS			2SLS (instrument: predicted log wage)		
	Base	All	$Pr(work) \geq 0.85$	Base	All	$Pr(work) \geq 0.85$
$\ln(\tilde{W}_{m,t})$	0.646*	0.662*	0.624*	0.749*	0.413*	0.531*
	(0.071)	(0.065)	(0.101)	(0.216)	(0.185)	(0.235)
Married	-0.074	-0.079	-0.146	-0.069	-0.077	-0.141
	(0.095)	(0.089)	(0.116)	(0.095)	(0.091)	(0.116)
Child's age	-0.141*	-0.131*	-0.146*	-0.139*	-0.133*	-0.146*
	(0.022)	(0.020)	(0.031)	(0.022)	(0.020)	(0.031)
Mother some coll.	0.011	-0.071	0.021	-0.018	-0.003	0.019
	(0.102)	(0.092)	(0.145)	(0.117)	(0.107)	(0.146)
Mother coll+	-0.157	-0.226*	-0.152	-0.218	-0.088	-0.145
	(0.112)	(0.103)	(0.153)	(0.164)	(0.149)	(0.172)
Mother's age	-0.008	-0.006	-0.005	-0.009	-0.004	-0.003
	(0.008)	(0.007)	(0.010)	(0.008)	(0.007)	(0.011)
Mother white	-0.243*	-0.152	-0.283*	-0.249*	-0.128	-0.281*
	(0.089)	(0.084)	(0.111)	(0.090)	(0.086)	(0.111)
Num. children	0.158	0.147	-0.024	0.155	0.164	-0.007
ages 0-5	(0.125)	(0.104)	(0.215)	(0.125)	(0.106)	(0.217)
Num. of children	0.089	0.100	0.106	0.097	0.080	0.091
	(0.062)	(0.056)	(0.080)	(0.063)	(0.058)	(0.081)
Constant	2.213*	1.992*	2.224*	1.999*	2.501*	2.425*
	(0.355)	(0.327)	(0.461)	(0.553)	(0.488)	(0.627)
R-squared	0.190	0.179	0.163	0.187	0.161	0.151
Sample size	727	860	417	727	851	412

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. See Table G-2 for model of predicted probability of work. * significant at 0.05 level.

Table G-7: 2SLS estimates for parental time vs. goods relative demand

	(1) All Mothers	(2) Single Mothers	(3) Married Mothers	(4) Married Fathers
$\ln(\tilde{W}_{j,t})$	0.553*	0.281	0.697*	0.346
	(0.196)	(0.387)	(0.228)	(0.257)
Married	-0.071			
	(0.096)			
Child's age	-0.140*	-0.176*	-0.129*	-0.099*
	(0.022)	(0.045)	(0.026)	(0.027)
Parent some coll.	0.026	0.286	-0.156	-0.020
	(0.113)	(0.189)	(0.142)	(0.154)
Parent coll+	-0.119	0.181	-0.320	0.276
	(0.155)	(0.279)	(0.188)	(0.183)
Parent's age	-0.007	-0.009	-0.007	-0.010
	(0.008)	(0.014)	(0.009)	(0.009)
Mother white	-0.233*	-0.365*	-0.170	0.001
	(0.091)	(0.175)	(0.107)	(0.128)
Num children age 0-5	0.168	-0.121	0.292*	0.154
	(0.126)	(0.240)	(0.147)	(0.135)
Num of children	0.082	0.028	0.110	0.182*
	(0.063)	(0.117)	(0.076)	(0.081)
Constant	2.398*	3.502*	1.858*	2.028*
	(0.520)	(1.086)	(0.582)	(0.683)
R-squared	0.181	0.155	0.195	0.136
Sample size	720	233	487	578

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. Only mothers (fathers) with predicted probability of work of at least 0.75 (0.9) are included. See Table G-2 for model of predicted probability of work. Specification for mothers (fathers) includes mother's (father's) relative wage, education indicators, and age. All columns instrument for $\ln(\tilde{W}_{j,i})$ using predicted log wages from 2000 Census as instruments (see text for details). * significant at 0.05 level.

Table G-8: OLS estimates for parental time vs. goods relative demand including parental log wage fixed effects, by parent type

	(1) All Mothers	(2) Single Mothers	(3) Married Mothers	(4) Married Fathers
$\ln(\tilde{W}_{j,t})$	0.758*	0.767*	0.790*	0.779*
	(0.092)	(0.198)	(0.106)	(0.121)
Married	0.022			
	(0.108)			
Child's age	-0.147*	-0.163*	-0.144*	-0.127*
	(0.024)	(0.053)	(0.027)	(0.030)
Parent's log wage FE	-0.346*	-0.089	-0.503*	-0.171
	(0.114)	(0.198)	(0.141)	(0.122)
Mother white	-0.328*	-0.579*	-0.217	-0.287
	(0.102)	(0.192)	(0.121)	(0.155)
Num children age 0-5	0.163	-0.011	0.222	0.303
	(0.169)	(0.360)	(0.190)	(0.187)
Num of children	0.027	-0.011	0.046	0.169
	(0.066)	(0.117)	(0.082)	(0.092)
Constant	1.745*	2.055*	1.542*	0.959*
	(0.366)	(0.829)	(0.403)	(0.482)
R-squared	0.193	0.215	0.197	0.171
Sample size	562	162	400	413

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. Only mothers (fathers) with predicted probability of work of at least 0.75 (0.9) are included. See Table G-2 for model of predicted probability of work. Specification for mothers (fathers) includes mother's (father's) relative wage, and mother's (father's) log wage fixed effects. * significant at 0.05 level.

Table G-9: Heckman two-step estimates for mother time/goods relative demand (two-parent households)

	(1) father only	(2) father only	(3) P_c only	(4) Both	(5) Both, P(work) $\geq$ .75
A. Relative demand					
$\ln(\tilde{W}_{m,t})$	0.644*	0.640*	0.640*	0.640*	0.633*
	(0.075)	(0.075)	(0.128)	(0.075)	(0.079)
Child's age	-0.134*	-0.133*	-0.171	-0.134*	-0.142*
	(0.028)	(0.027)	(0.112)	(0.027)	(0.030)
Mother HS grad	-0.110				
	(0.248)				
Mother some coll.	-0.335	-0.230	-0.559	-0.237	-0.148
	(0.272)	(0.127)	(0.791)	(0.126)	(0.136)
Mother coll+	-0.472	-0.361*	-0.846	-0.371*	-0.302*
	(0.296)	(0.147)	(1.187)	(0.146)	(0.143)
Mother's age	-0.001	-0.002	0.016	-0.001	-0.004
	(0.009)	(0.009)	(0.048)	(0.009)	(0.010)
Mother white	-0.073	-0.084	0.171	-0.079	-0.154
	(0.111)	(0.108)	(0.651)	(0.107)	(0.112)
Num children age 0-5	0.316*	0.311*	0.469	0.315*	0.283
	(0.126)	(0.125)	(0.467)	(0.125)	(0.149)
Num of children	0.105	0.107	0.176	0.108	0.116
	(0.070)	(0.069)	(0.213)	(0.069)	(0.077)
Year = 2007	0.090	0.086	0.336	0.092	0.084
	(0.133)	(0.133)	(0.606)	(0.133)	(0.139)
Constant	1.952*	1.853*	2.287	1.865*	2.036*
	(0.469)	(0.409)	(1.409)	(0.410)	(0.435)
B. Positive hours worked by mother					
Child's age	0.037	0.038	0.039	0.039	0.053
	(0.032)	(0.032)	(0.031)	(0.032)	(0.038)
Mother HS grad	0.178				
	(0.234)				
Mother some coll.	0.503*	0.347*	0.287*	0.343*	0.424*
	(0.248)	(0.140)	(0.126)	(0.141)	(0.168)
Mother coll+	0.781*	0.625*	0.453*	0.619*	0.669*
	(0.256)	(0.153)	(0.126)	(0.153)	(0.185)
Mother's age	-0.014	-0.013	-0.019	-0.015	-0.018
	(0.015)	(0.015)	(0.010)	(0.015)	(0.018)
Mother white	-0.185	-0.173	-0.268*	-0.192	-0.157
	(0.128)	(0.127)	(0.125)	(0.128)	(0.153)
Num children age 0-5	-0.118	-0.112	-0.154	-0.116	0.170
	(0.139)	(0.139)	(0.135)	(0.139)	(0.212)
Num of children	-0.041	-0.048	-0.061	-0.047	-0.073
	(0.079)	(0.079)	(0.077)	(0.079)	(0.091)
Year = 2007	-0.266	-0.275	-0.219	-0.288*	-0.264
	(0.144)	(0.143)	(0.140)	(0.144)	(0.175)
Father HS grad	0.433*	0.478*		0.494*	0.616
	(0.191)	(0.182)		(0.183)	(0.388)
Father some coll.	0.228	0.269		0.276	0.338
	(0.209)	(0.202)		(0.202)	(0.397)
Father coll+	-0.174	-0.137		-0.135	-0.198
	(0.216)	(0.210)		(0.211)	(0.411)
Father's age	0.007	0.007		0.007	0.013
	(0.013)	(0.013)		(0.013)	(0.015)
$\ln(\tilde{q}_t)$			0.082	0.161	0.116
			(0.158)	(0.162)	(0.190)
Constant	0.349	0.460	1.168*	0.330	0.043
	(0.512)	(0.491)	(0.458)	(0.509)	(0.660)
Inverse Mill's Ratio	-0.196	-0.131	-2.430	-0.186	-0.183
	(0.455)	(0.428)	(5.697)	(0.420)	(0.427)
Num. Pos. Hours	582	582	593	582	491
Sample size	756	756	771	756	610

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. * significant at 0.05 level.

Table G-10: OLS estimates for mother time/goods relative demand by child age

	(1) All	(2) Ages 5-8	(3) Ages 9-12
$\ln(\tilde{W}_{m,t})$	0.646*	0.648*	0.659*
	(0.071)	(0.135)	(0.086)
Married	-0.074	-0.114	-0.036
	(0.095)	(0.158)	(0.120)
Child's age	-0.141*	-0.243*	-0.160*
	(0.022)	(0.079)	(0.048)
Mother some coll.	0.011	0.263	-0.101
	(0.102)	(0.174)	(0.127)
Mother coll+	-0.157	-0.109	-0.180
	(0.112)	(0.192)	(0.139)
Mother's age	-0.008	-0.009	-0.008
	(0.008)	(0.012)	(0.010)
Mother white	-0.243*	-0.125	-0.323*
	(0.089)	(0.144)	(0.115)
Num children ages 0-5	0.158	0.130	0.095
	(0.125)	(0.162)	(0.207)
Num of children	0.089	0.062	0.087
	(0.062)	(0.120)	(0.073)
Constant	2.213*	2.838*	2.491*
	(0.355)	(0.744)	(0.639)
R-squared	0.190	0.170	0.151
Residual sum of squares	826.887	213.167	606.904
F-test equality by child's age (p-value)		0.825	
Sample size	727	224	503

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. Only mothers with predicted probability of work of at least 0.75 included. See Table G-2 for model of predicted probability of work. * significant at 0.05 level.

Table G-11: OLS estimates for mother time/goods relative demand by father's wage

	(1) All	(2) Below Median	(3) Above Median
$\ln(\tilde{W}_{m,t})$	0.716*	0.797*	0.647*
	(0.082)	(0.111)	(0.124)
Child's age	-0.124*	-0.099*	-0.152*
	(0.027)	(0.037)	(0.039)
Mother some coll.	-0.145	-0.293	0.087
	(0.133)	(0.167)	(0.221)
Mother coll+	-0.298*	-0.312	-0.224
	(0.136)	(0.182)	(0.213)
Mother's age	-0.007	-0.008	0.002
	(0.010)	(0.013)	(0.016)
Mother white	-0.119	-0.141	-0.031
	(0.112)	(0.140)	(0.194)
Num children ages 0-5	0.315*	0.373*	0.272
	(0.149)	(0.189)	(0.241)
Num of children	0.084	0.072	0.115
	(0.080)	(0.108)	(0.121)
Constant	1.779*	1.525*	1.576*
	(0.443)	(0.599)	(0.763)
R-squared	0.222	0.258	0.203
Residual sum of squares	457.346	217.931	233.663
F-test equality by father's wage (p-value)		0.786	
Sample size	451	231	220

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. Only mothers with predicted probability of work of at least 0.75 included. See Table G-2 for model of predicted probability of work. * significant at 0.05 level.

Table G-12: OLS estimates for mother time/goods relative demand conditioning on 1997 AP and LW scores

	(1) Baseline	(2) Includes 1997 Achievement	(3) Below Median 1997 Achieve.	(4) Above Median 1997 Achieve.
$\ln(\tilde{W}_{m,t})$	0.557*	0.562*	0.327	0.676*
	(0.117)	(0.118)	(0.195)	(0.142)
Married	-0.118	-0.119	-0.145	-0.054
	(0.147)	(0.148)	(0.210)	(0.211)
Child's age	-0.109*	-0.106	-0.084	-0.121
	(0.054)	(0.054)	(0.083)	(0.070)
Mother some coll.	-0.108	-0.101	-0.303	0.090
	(0.155)	(0.156)	(0.215)	(0.236)
Mother coll+	-0.243	-0.227	-0.113	-0.210
	(0.172)	(0.176)	(0.267)	(0.237)
Mother's age	0.001	0.002	0.018	-0.020
	(0.012)	(0.012)	(0.017)	(0.016)
Mother white	-0.279	-0.268	-0.543*	0.138
	(0.146)	(0.148)	(0.212)	(0.210)
Num children ages 0-5	0.069	0.081	-0.222	0.262
	(0.232)	(0.234)	(0.368)	(0.305)
Num of children	0.120	0.118	0.084	0.130
	(0.089)	(0.089)	(0.127)	(0.128)
1997 Achievement		-0.029		
		(0.068)		
Constant	1.810*	1.741*	1.783	1.983*
	(0.682)	(0.701)	(1.054)	(0.908)
R-squared	0.094	0.095	0.088	0.191
Residual sum of squares	400.478	400.251	207.122	173.339
F-test equality by 1997 Achieve. (p-value)			0.085	
Sample Size	339	339	165	174

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. Only mothers with predicted probability of work of at least 0.75 included. See Table G-2 for model of predicted probability of work. * significant at 0.05 level.

Table G-13: Probit estimates for positive child care expenditures

	Avg. Derivative
$\ln(\tilde{q}_t)$	0.035
	(0.035)
Married	-0.028
	(0.026)
Child's age	-0.040*
	(0.006)
Mother some coll.	0.098*
	(0.025)
Mother coll+	0.120*
	(0.028)
Mother's age	-0.002
	(0.002)
Mother white	-0.029
	(0.025)
Num children age 0-5	0.089*
	(0.027)
Num of children	-0.054*
	(0.022)
Year = 2007	-0.194*
	(0.028)
HH Head lives in same state	-0.015
	(0.024)
Any children ages 13+	-0.045
	(0.033)
Sample size	1,391

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. * significant at 0.05 level.

Table G-14: Heckman two-step estimates for childcare/goods relative demand

	(1)	(2)	(3)
A. Relative demand			
$\ln(\tilde{q}_t)$	0.655*	0.653*	0.456*
	(0.219)	(0.217)	(0.214)
Married	0.825	0.833	0.085
	(0.626)	(0.622)	(0.635)
Child's age	-0.242*	-0.233*	-0.129
	(0.063)	(0.067)	(0.066)
Mother some coll.	0.253	0.232	0.175
	(0.203)	(0.206)	(0.187)
Mother coll+	0.054	0.028	-0.062
	(0.234)	(0.241)	(0.208)
Mother's age	-0.004	-0.004	-0.002
	(0.014)	(0.014)	(0.014)
Marr x Father some coll.	0.071	0.077	0.172
	(0.223)	(0.221)	(0.217)
Marr x Father coll+	-0.455	-0.453	-0.660*
	(0.236)	(0.233)	(0.244)
Marr x Father's age	-0.018	-0.018	-0.006
	(0.016)	(0.016)	(0.016)
Mother white	-0.342*	-0.345*	-0.214
	(0.152)	(0.151)	(0.150)
Num children age 0-5	0.175	0.158	0.126
	(0.177)	(0.179)	(0.184)
Num of children	-0.078	-0.063	0.010
	(0.144)	(0.148)	(0.153)
$\ln(1 + e^{\Phi_{m,t}} + Marr \cdot e^{\Phi_{f,t}})$			0.544
			(0.306)
Constant	1.228	1.206	-0.491
	(0.643)	(0.638)	(1.146)
B. Positive child care expenditure			
$\ln(\tilde{q}_t)$	0.103	0.102	-0.084
	(0.135)	(0.134)	(0.158)
Married	0.144	0.285	-0.191
	(0.379)	(0.375)	(0.469)
Child's age	-0.135*	-0.134*	-0.120*
	(0.023)	(0.023)	(0.034)
Mother some coll.	0.339*	0.351*	0.282*
	(0.101)	(0.101)	(0.118)
Mother coll+	0.501*	0.508*	0.398*
	(0.114)	(0.113)	(0.133)
Mother's age	-0.009	-0.004	0.001
	(0.009)	(0.009)	(0.011)
Marr x Father some coll.	-0.060	-0.067	-0.105

Table G-14 continued from previous page

	(0.135)	(0.135)	(0.154)
Marr x Father coll+	-0.009	-0.009	-0.100
	(0.146)	(0.146)	(0.178)
Marr x Father's age	-0.006	-0.009	-0.001
	(0.009)	(0.009)	(0.011)
Mother white	-0.082	-0.067	0.026
	(0.095)	(0.094)	(0.111)
Num children age 0-5	0.298*	0.301*	0.339*
	(0.106)	(0.105)	(0.129)
Num of children	-0.243*	-0.233*	-0.244*
	(0.094)	(0.094)	(0.111)
Year = 2007	-0.757*	-0.751*	-0.772*
	(0.142)	(0.142)	(0.172)
Household head live in birth state	-0.042	-0.030	-0.149
	(0.092)	(0.092)	(0.107)
Live w/older relative	-0.540*		
	(0.225)		
Any children ages 13+	-0.119	-0.128	-0.313*
	(0.124)	(0.124)	(0.145)
2+ children ages 13+	0.141	0.129	0.116
	(0.217)	(0.217)	(0.250)
$\ln(1 + e^{\Phi_{m,t}} + Marr \cdot e^{\Phi_{f,t}})$			0.322
			(0.186)
Constant	1.286*	1.043*	0.547
	(0.452)	(0.440)	(0.774)
Inverse Mill's ratio	0.948*	0.874	0.467
	(0.421)	(0.460)	(0.426)
Num. pos. child care exp.	338	338	302
Sample size	1318	1318	930

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. * significant at 0.05 level.

Table G-15: OLS estimates for child care/goods relative demand

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Single mothers				Two-parent households			
$\ln(\tilde{q}_t)$	0.853*	0.656*	0.645*	0.737*	0.589*	0.428	0.464	0.609*
	(0.309)	(0.303)	(0.311)	(0.287)	(0.275)	(0.315)	(0.293)	(0.265)
Child's age	-0.132*	-0.048	-0.078	-0.047	-0.119*	-0.112	-0.096	-0.065
	(0.057)	(0.059)	(0.085)	(0.056)	(0.046)	(0.057)	(0.066)	(0.048)
Mother some coll.	0.074	-0.010	0.018	0.260	-0.191	0.076	-0.062	0.146
	(0.260)	(0.261)	(0.289)	(0.259)	(0.215)	(0.252)	(0.227)	(0.214)
Mother coll+	-0.091	-0.196	-0.200	0.291	-0.496*	-0.038	-0.291	0.422
	(0.283)	(0.277)	(0.317)	(0.296)	(0.219)	(0.252)	(0.232)	(0.222)
Mother's age	-0.005	0.006	-0.002	0.016	0.007	-0.024	0.002	-0.014
	(0.017)	(0.017)	(0.017)	(0.016)	(0.024)	(0.030)	(0.026)	(0.025)
Mother white	-0.789*	-0.706*	-0.692*	-0.429	-0.101	-0.107	-0.015	0.086
	(0.240)	(0.229)	(0.260)	(0.229)	(0.177)	(0.198)	(0.189)	(0.168)
Num children age 0-5	-0.289	0.055	-0.164	0.205	0.021	0.200	0.081	0.357
	(0.268)	(0.258)	(0.274)	(0.254)	(0.175)	(0.213)	(0.216)	(0.181)
Num of children	0.056	0.127	0.127	-0.035	0.169	0.103	0.115	-0.073
	(0.171)	(0.157)	(0.168)	(0.156)	(0.151)	(0.171)	(0.167)	(0.145)
$\ln(1 + R_m + Marr. \times R_f)$		0.557*		0.716		0.465*		-0.131
		(0.117)		(0.384)		(0.106)		(0.224)
$\ln(1 + e^{\Phi_m} + Marr. \times e^{\Phi_f})$			0.363				0.311	
			(0.411)				(0.410)	
$\frac{R_m}{2(1+R_m)^2}$				16.853				
				(10.417)				
$\frac{\ln(g^o)-E[\ln(g^o)]}{Var(\ln(g^o))}$				-0.461*				-0.765*
				(0.149)				(0.100)
Father some coll.					0.128	0.115	0.195	0.101
					(0.217)	(0.253)	(0.228)	(0.212)
Father coll+					-0.334	-0.677*	-0.447	-0.208
					(0.212)	(0.242)	(0.247)	(0.211)
Father's age					-0.016	0.017	-0.005	0.006
					(0.020)	(0.024)	(0.021)	(0.021)
$\frac{R_f(1+R_m)}{2(1+R_m+R_f)^2}$								8.015
								(5.819)
$\frac{R_m(1+R_f)}{2(1+R_m+R_f)^2}$								-13.710*
								(6.630)
$\frac{R_m+R_f}{2(1+R_m+R_f)^2}$								0.608
								(8.411)
Constant	1.281	-1.250	-0.075	-2.930	1.380	-0.235	0.140	1.570
	(0.765)	(0.879)	(1.714)	(1.697)	(0.769)	(0.911)	(1.711)	(1.166)
R-squared	0.175	0.385	0.189	0.469	0.131	0.277	0.130	0.510
Sample size	120	94	112	94	227	155	198	155

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. Columns 1-4 report results for single mothers and columns 5-8 report results for two-parent households. * significant at 0.05 level.

Table G-16: OLS estimates for child care/goods relative demand by child age

	(1) Ages 5-8	(2) Ages 9-12	(3) Ages 5-8	(4) Ages 9-12	(5) Ages 5-8	(6) Ages 9-12
$\ln(\tilde{q}_t)$	0.332	0.874*	0.372	0.637	0.156	0.849*
	(0.265)	(0.331)	(0.288)	(0.349)	(0.280)	(0.339)
Married	1.033	0.555	0.933	-1.348	0.173	0.509
	(0.715)	(0.982)	(0.824)	(1.105)	(0.777)	(1.060)
Child's age	-0.235*	0.085	0.023	0.073	-0.135	0.054
	(0.096)	(0.096)	(0.109)	(0.109)	(0.110)	(0.114)
Mother some coll.	0.018	-0.116	0.109	0.098	0.005	0.016
	(0.217)	(0.247)	(0.232)	(0.276)	(0.226)	(0.264)
Mother coll+	-0.524*	-0.031	-0.213	0.168	-0.423	0.060
	(0.217)	(0.279)	(0.236)	(0.302)	(0.224)	(0.306)
Mother's age	0.015	-0.014	0.004	-0.022	0.007	-0.010
	(0.019)	(0.022)	(0.021)	(0.024)	(0.019)	(0.023)
Marr x Father some coll.	0.080	0.165	-0.117	0.265	0.173	0.221
	(0.259)	(0.350)	(0.283)	(0.392)	(0.268)	(0.356)
Marr x Father coll+	-0.214	-0.746*	-0.417	-1.100*	-0.431	-0.809*
	(0.269)	(0.372)	(0.293)	(0.422)	(0.307)	(0.403)
Marr x Father's age	-0.025	-0.008	-0.021	0.029	-0.008	-0.010
	(0.019)	(0.025)	(0.022)	(0.028)	(0.020)	(0.026)
Mother white	-0.226	-0.444	-0.202	-0.422	-0.054	-0.473*
	(0.177)	(0.229)	(0.185)	(0.242)	(0.188)	(0.237)
Num children ages 0-5	0.002	-0.079	0.370	0.074	0.052	0.010
	(0.175)	(0.262)	(0.194)	(0.288)	(0.191)	(0.292)
Num of children	-0.007	0.177	0.033	0.207	-0.077	0.262
	(0.147)	(0.178)	(0.149)	(0.188)	(0.154)	(0.186)
$\ln(1+R_{m,t}+Marr\cdot R_{f,t})$			0.471*	0.474*		
			(0.098)	(0.128)		
$\ln(1+e^{\Phi_{m,t}}+Marr\cdot e^{\Phi_{f,t}})$					0.582	0.020
					(0.371)	(0.492)
Constant	1.834	-1.005	-1.307	-1.659	0.037	-1.045
	(0.936)	(1.235)	(1.169)	(1.341)	(1.574)	(2.007)
R-squared	0.143	0.104	0.317	0.261	0.123	0.118
Residual sum of squares	208.800	225.446	108.628	135.248	170.075	196.242
F-test equality by age (p-value)	0.349		0.411		0.474	
Sample Size	186	161	130	119	163	147

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. * significant at 0.05 level.

Table G-17: Joint GMM Estimation of Relative Demand Moments

	$\epsilon_{\tau,g}$		$\epsilon_{x,H}$		Correl. residuals	
	(1)	(2)	(1)	(2)	(1)	(2)
	0.20	0.37	0.51	0.77	0.88	0.88
	(0.05)	(0.17)	(0.09)	(0.09)		
	$\tilde{\phi}_m$: Mother's Time		$\tilde{\phi}_f$: Father's Time		$\tilde{\phi}_x$: Childcare	
	(1)	(2)	(1)	(2)	(1)	(2)
Constant	8.37	5.55	4.11	3.35	-1.19	-1.45
	(1.97)	(1.67)	(1.28)	(0.76)	(0.40)	(0.28)
Single	0.29	0.11	-	-	0.62	0.63
	(0.38)	(0.21)	-	-	(0.21)	(0.14)
Type 2	-1.14	-0.49	-	-	0.03	0.00
	(0.59)	(0.46)	-	-	(0.29)	(0.20)
Type 3	-2.49	-1.09	-	-	-0.04	-0.09
	(0.96)	(0.86)	-	-	(0.30)	(0.21)
Mother some coll.	-0.44	-0.13	-	-	-0.00	-0.06
	(0.45)	(0.28)	-	-	(0.19)	(0.13)
Mother coll+	-1.81	-0.76	-	-	-0.20	-0.28
	(0.77)	(0.65)	-	-	(0.19)	(0.13)
Child's age	-0.60	-0.34	-0.48	-0.24	-0.07	-0.04
	(0.18)	(0.15)	(0.18)	(0.14)	(0.03)	(0.02)
Num. of children 0-5	0.33	0.16	0.59	0.29	0.09	0.08
	(0.29)	(0.18)	(0.42)	(0.26)	(0.12)	(0.08)
Father some coll.	-	-	-1.08	-0.41	0.06	0.01
	-	-	(0.74)	(0.45)	(0.25)	(0.17)
Father coll+	-	-	-0.84	-0.22	-0.54	-0.41
	-	-	(0.66)	(0.44)	(0.23)	(0.16)

Notes: Specification (1) uses own-relative prices as instruments in each moment condition. Specification (2) uses predicted wages by occupation and state as an instrument for Mothers' and Fathers' wages. The column "Correl. residuals" reports the p-value from a correlation test of the relative demand residuals described in Appendix E.4. The function f is specified as: $f = \left[\left(\tilde{a}_{m,i,t}\tau^{\rho}_{m,i,t} + \tilde{a}_{f,i,t}\tau^{\rho}_{f,i,t} + (1-\tilde{a}_{m,i,t}-\tilde{a}_{f,i,t})g^{\rho}_{i,t}\right)^{\gamma/\rho}(1-\tilde{a}_{x,i,t}) + \tilde{a}_{x,i,t}x^{\gamma}_{i,t}\right]^{1/\gamma}$ with $\tilde{a}_{j,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_j)}{1+\exp(Z_{i,t}\tilde{\phi}_m)+\exp(Z_{i,t}\tilde{\phi}_f)}$, $j \in \{m, f\}$ and $\tilde{a}_{x,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_x)}{1+\exp(Z_{i,t}\tilde{\phi}_x)}$.

Table G-18: Joint GMM Estimation – No Borrowing/Saving ($\kappa = 1$)

	$\epsilon_{\tau,g}$ (1)	(2)	(3)	(4)	$\epsilon_{x,H}$ (1)	(2)	(3)	(4)	δ_1 (1)	(2)	(3)	(4)	δ_2 (1)	(2)	(3)	(4)
	0.30^+	0.26^+	0.20^+	0.18^+	0.53	0.47^+	0.50	0.50	0.15	0.20	0.17	0.16	0.82	0.83	0.83	0.83
	(0.05)	(0.05)	(0.05)	(0.05)	(0.09)	(0.08)	(0.09)	(0.09)	(0.04)	(0.05)	(0.04)	(0.04)	(0.02)	(0.02)	(0.02)	(0.02)
	$\tilde{\phi}_m$: Mother's Time (1)	(2)	(3)	(4)	$\tilde{\phi}_f$: Father's Time (1)	(2)	(3)	(4)	$\tilde{\phi}_x$: Childcare (1)	(2)	(3)	(4)	$\overline{\phi}_\theta$: TFP (1)	(2)	(3)	(4)
Constant	5.43	6.25^+	8.22	12.56	3.44	3.59	4.13	4.29	−1.19	−1.23	−1.19	−1.59	0.76	0.81	1.10	2.00
	(0.77)	(0.99)	(1.90)	(3.68)	(0.81)	(0.90)	(1.27)	(1.40)	(0.31)	(0.42)	(0.41)	(0.61)	(0.50)	(0.55)	(0.47)	(0.45)
Single	0.18	0.21	0.10	0.13	-	-	-	-	0.55	0.61	0.61	0.62	-0.20	-0.19	-0.20	-0.18
	(0.25)	(0.28)	(0.37)	(0.40)	-	-	-	-	(0.20)	(0.22)	(0.21)	(0.21)	(0.06)	(0.07)	(0.07)	(0.07)
Mother some coll.	−0.34	-	−0.50	−0.53	-	-	-	-	0.04	-	0.01	0.04	0.07	-	0.05	0.05
	(0.30)	-	(0.45)	(0.50)	-	-	-	-	(0.18)	-	(0.20)	(0.20)	(0.07)	-	(0.07)	(0.07)
Mother coll+	−1.04	-	−1.71	−1.83	-	-	-	-	−0.20	-	−0.23	−0.20	-0.01	-	-0.08	-0.09
	(0.40)	-	(0.74)	(0.83)	-	-	-	-	(0.18)	-	(0.19)	(0.19)	(0.09)	-	(0.11)	(0.11)
Child's age	−0.40	-0.46^+	−0.60	−0.66	−0.30	−0.35	−0.48	−0.52	−0.06	−0.07	−0.07	−0.07	-0.09	-0.13	-0.13	-0.14
	(0.09)	(0.10)	(0.17)	(0.21)	(0.09)	(0.11)	(0.17)	(0.20)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)	(0.04)
Num. of children 0-5	0.34	0.41	0.46	0.58	0.42	0.48	0.64	0.73	0.09	0.09	0.06	0.08	0.07	0.06	0.06	0.06
	(0.20)	(0.22)	(0.30)	(0.34)	(0.26)	(0.30)	(0.42)	(0.47)	(0.12)	(0.13)	(0.12)	(0.12)	(0.05)	(0.05)	(0.05)	(0.05)
Type 2	-	−0.62	−0.96	-	-	-	-	-	-	0.10	0.08	-	-	-0.06	-0.05	-
	-	(0.38)	(0.55)	-	-	-	-	-	-	(0.32)	(0.30)	-	-	(0.10)	(0.09)	-
Type 3	-	−1.68	−2.41	-	-	-	-	-	-	0.13	0.10	-	-	-0.43	-0.43	-
	-	(0.57)	(0.93)	-	-	-	-	-	-	(0.32)	(0.31)	-	-	(0.14)	(0.14)	-
μ_k	-	-	-	−2.64	-	-	-	-	-	-	-	0.20	-	-	-	-0.52
	-	-	-	(1.09)	-	-	-	-	-	-	-	(0.26)	-	-	-	(0.15)
Father some coll.	-	-	-	-	−0.56	−0.63	−1.04	−1.08	-0.02^+	0.07	-0.00^+	-0.05^+	0.01	-0.00	-0.01	0.00
	-	-	-	-	(0.45)	(0.49)	(0.72)	(0.79)	(0.23)	(0.27)	(0.25)	(0.25)	(0.07)	(0.08)	(0.07)	(0.07)
Father coll+	-	-	-	-	−0.43	−0.40	−0.95	−1.07	-0.62^+	-0.64^+	−0.63	-0.69^+	0.18	0.18	0.17	0.15
	-	-	-	-	(0.40)	(0.43)	(0.68)	(0.75)	(0.21)	(0.23)	(0.23)	(0.24)	(0.07)	(0.09)	(0.08)	(0.08)
Year = 2002	-	-	-	-	-	-	-	-	-	-	-	-	-0.28	-0.26	-0.30	-0.29
	-	-	-	-	-	-	-	-	-	-	-	-	(0.05)	(0.06)	(0.06)	(0.05)

Notes: The superscript $^+$ indicates, using a Lagrange Multiplier test, rejection at 5% significance of the null hypothesis that an individual parameter enters identically in the demand and production moments. See Appendix E.4 for more details. The function f is specified as: $f = \left[\left(\tilde{a}_{m,i,t}\tau^{\rho}_{m,i,t} + \tilde{a}_{f,i,t}\tau^{\rho}_{f,i,t} + (1-\tilde{a}_{m,i,t}-\tilde{a}_{f,i,t})g^{\rho}_{i,t}\right)^{\gamma/\rho}(1-\tilde{a}_{x,i,t}) + \tilde{a}_{x,i,t}x^{\gamma}_{i,t}\right]^{1/\gamma}$ with $\tilde{a}_{j,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_j)}{1+\exp(Z_{i,t}\tilde{\phi}_m)+\exp(Z_{i,t}\tilde{\phi}_f)}$, $j \in \{m, f\}$ and $\tilde{a}_{x,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_x)}{1+\exp(Z_{i,t}\tilde{\phi}_x)}$. The function $\mathcal{H}$ is specified as $\mathcal{H}(f, \Psi_{i,t}, \theta_{i,t}) = \theta_{i,t}f^{\delta_1}\Psi^{\delta_2}_{i,t}$ with $\theta_{i,t} = \exp(Z_{i,t}\phi_\theta + \xi_{\theta,i,t})$.

Table G-19: Joint GMM Estimation – Unconstrained ($\kappa = 0$)

	$\epsilon_{\tau,g}$ (1)	(2)	(3)	(4)	$\epsilon_{x,H}$ (1)	(2)	(3)	(4)	δ_1 (1)	(2)	(3)	(4)	δ_2 (1)	(2)	(3)	(4)
	0.29^+	0.26^+	0.20^+	0.18^+	0.53	0.47^+	0.50	0.50	0.12	0.16	0.13	0.13	0.82	0.84	0.83	0.83
	(0.05)	(0.05)	(0.05)	(0.05)	(0.09)	(0.08)	(0.09)	(0.09)	(0.04)	(0.04)	(0.03)	(0.04)	(0.02)	(0.02)	(0.02)	(0.02)
	$\tilde{\phi}_m$: Mother's Time (1)	(2)	(3)	(4)	$\tilde{\phi}_f$: Father's Time (1)	(2)	(3)	(4)	$\tilde{\phi}_x$: Childcare (1)	(2)	(3)	(4)	$\overline{\phi}_\theta$: TFP (1)	(2)	(3)	(4)
Constant	5.53	6.39	8.33	12.69	3.52	3.72	4.25	4.36	−1.21	-1.24^+	−1.21	−1.61	1.07	1.17	1.30	2.15
	(0.78)	(1.03)	(1.94)	(3.74)	(0.81)	(0.92)	(1.28)	(1.42)	(0.31)	(0.42)	(0.41)	(0.62)	(0.44)	(0.48)	(0.42)	(0.43)
Single	0.18	0.18	0.08	0.10	-	-	-	-	0.55	0.61	0.61	0.61	-0.15	-0.14	-0.15	-0.14
	(0.25)	(0.28)	(0.37)	(0.40)	-	-	-	-	(0.20)	(0.22)	(0.21)	(0.21)	(0.06)	(0.07)	(0.06)	(0.07)
Mother some coll.	−0.36	-	−0.52	−0.54	-	-	-	-	0.04	-	0.00	0.04	0.11	-	0.10	0.09
	(0.30)	-	(0.46)	(0.50)	-	-	-	-	(0.18)	-	(0.20)	(0.20)	(0.06)	-	(0.07)	(0.07)
Mother coll+	−1.07	-	−1.74	−1.86	-	-	-	-	−0.19	-	−0.22	−0.21	0.03	-	-0.03	-0.04
	(0.41)	-	(0.74)	(0.84)	-	-	-	-	(0.18)	-	(0.19)	(0.19)	(0.08)	-	(0.09)	(0.10)
Child's age	−0.42	-0.48^+	−0.61	−0.66	-0.31^+	−0.37	−0.49	−0.53	−0.06	-0.07^+	−0.07	−0.07	-0.10	-0.14	-0.13	-0.15
	(0.09)	(0.11)	(0.18)	(0.21)	(0.09)	(0.11)	(0.18)	(0.21)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Num. of children 0-5	0.35	0.44	0.50	0.61	0.43	0.49	0.66	0.75	0.10	0.10	0.07	0.09	0.08	0.09	0.08	0.07
	(0.20)	(0.23)	(0.30)	(0.35)	(0.27)	(0.30)	(0.42)	(0.47)	(0.12)	(0.13)	(0.12)	(0.12)	(0.04)	(0.05)	(0.04)	(0.05)
Type 2	-	−0.64	−0.96	-	-	-	-	-	-	0.09	0.08	-	-	0.07	0.04	-
	-	(0.39)	(0.55)	-	-	-	-	-	-	(0.32)	(0.30)	-	-	(0.09)	(0.08)	-
Type 3	-	−1.70	−2.41	-	-	-	-	-	-	0.12	0.10	-	-	-0.30	-0.30	-
	-	(0.58)	(0.93)	-	-	-	-	-	-	(0.32)	(0.31)	-	-	(0.12)	(0.11)	-
μ_k	-	-	-	−2.66	-	-	-	-	-	-	-	0.21	-	-	-	-0.41
	-	-	-	(1.10)	-	-	-	-	-	-	-	(0.26)	-	-	-	(0.13)
Father some coll.	-	-	-	-	−0.58	−0.67	−1.08	−1.10	-0.01^+	0.07	-0.01^+	-0.05^+	0.03	0.03	0.01	0.02
	-	-	-	-	(0.45)	(0.50)	(0.72)	(0.80)	(0.23)	(0.27)	(0.25)	(0.25)	(0.07)	(0.07)	(0.07)	(0.07)
Father coll+	-	-	-	-	−0.45	−0.44	−0.98	−1.10	-0.62^+	-0.64^+	−0.64	−0.69	0.24	0.25	0.22	0.19
	-	-	-	-	(0.41)	(0.43)	(0.68)	(0.76)	(0.21)	(0.23)	(0.24)	(0.24)	(0.06)	(0.07)	(0.07)	(0.07)
Year = 2002	-	-	-	-	-	-	-	-	-	-	-	-	-0.32	-0.32	-0.34	-0.33
	-	-	-	-	-	-	-	-	-	-	-	-	(0.04)	(0.05)	(0.05)	(0.05)

Notes: The superscript $^+$ indicates, using a Lagrange Multiplier test, rejection at 5% significance of the null hypothesis that an individual parameter enters identically in the demand and production moments. See Appendix E.4 for more details. The function f is specified as: $f = \left[\left(\tilde{a}_{m,i,t}\tau^{\rho}_{m,i,t} + \tilde{a}_{f,i,t}\tau^{\rho}_{f,i,t} + (1 - \tilde{a}_{m,i,t} - \tilde{a}_{f,i,t})g^{\rho}_{i,t}\right)^{\gamma/\rho}(1 - \tilde{a}_{x,i,t}) + \tilde{a}_{x,i,t}x^{\gamma}_{i,t}\right]^{1/\gamma}$ with $\tilde{a}_{j,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_j)}{1+\exp(Z_{i,t}\tilde{\phi}_m)+\exp(Z_{i,t}\tilde{\phi}_f)}$, $j \in \{m, f\}$ and $\tilde{a}_{x,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_x)}{1+\exp(Z_{i,t}\tilde{\phi}_x)}$. The function $\mathcal{H}$ is specified as $\mathcal{H}(f, \Psi_{i,t}, \theta_{i,t}) = \theta_{i,t} f^{\delta_1}\Psi^{\delta_2}_{i,t}$ with $\theta_{i,t} = \exp(Z_{i,t}\phi_\theta + \xi_{\theta,i,t})$.

Table G-20: Joint GMM Estimation Relaxing Some Parameters Across Relative Demand and Production – Unconstrained ($\kappa = 0$)

	$\epsilon_{\tau,g}$		$\epsilon_{x,H}$		δ_1	δ_2	$2N(Q_N - \tilde{Q}_N)$
	Rel. Dem.	Prod.	Rel. Dem.	Prod.	-	-	-
	0.21	0.55	0.50	-	0.14	0.83	1.98
	(0.05)	(27.77)	(0.09)	-	(0.05)	(0.03)	(0.37)

	$\tilde{\phi}_m$: Mother's Time		$\tilde{\phi}_f$: Father's Time		$\tilde{\phi}_x$: Childcare		$\overline{\phi}_\theta$: TFP
	Rel. Dem.	Prod.	Rel. Dem.	Prod.	Rel. Dem.	Prod.	-
Constant	7.86	-	4.06	-	-1.19	-	1.36
	(1.70)		(1.17)		(0.41)		(0.59)
Single	0.08	-	-	-	0.60	-	-0.13
	(0.34)				(0.21)		(0.14)
Type 2	-0.87	-	-	-	0.07	-	0.01
	(0.50)				(0.30)		(0.14)
Type 3	-2.17	-	-	-	0.10	-	-0.40
	(0.81)				(0.31)		(0.28)
Mother some coll.	-0.47	-	-	-	0.01	-	0.07
	(0.42)				(0.20)		(0.11)
Mother coll+	-1.55	-	-	-	-0.22	-	-0.11
	(0.65)				(0.19)		(0.21)
Child's age	-0.56	-	-0.44	-	-0.07	-	-0.16
	(0.16)		(0.16)		(0.03)		(0.04)
Num. of children 0-5	0.45	-	0.61	-	0.06	-	0.08
	(0.27)		(0.38)		(0.12)		(0.05)
Father some coll.	-	-	-0.96	-	-0.00	-18.19	-0.05
			(0.66)		(0.25)	(1094529195.59)	(0.57)
Father coll+	-	-	-0.84	-	-0.64	-	0.17
			(0.61)		(0.24)		(0.10)
Year = 2002	-	-	-	-	-	-	-0.33
							(0.06)

Notes: This table reports GMM estimates where some parameters are allowed to differ between those determining relative input demand (Rel. Dem.) and those determining actual skill production (Prod.). See Appendix E.4 for more details. The distance metric, $2N(Q_N - \tilde{Q}_N)$, is the difference between the optimally weighted GMM criterion at the restricted estimates and its value at the relaxed estimates. It has a χ^2 distribution with degrees of freedom equal to the number of constraints that are relaxed. Standard errors are indicated in parentheses <u>except</u> for the distance metric, which reports a p-value. The function f is specified as: $f = \left[\left(\tilde{a}_{m,i,t}\tau^{\rho}_{m,i,t} + \tilde{a}_{f,i,t}\tau^{\rho}_{f,i,t} + (1 - \tilde{a}_{m,i,t} - \tilde{a}_{f,i,t})g^{\rho}_{i,t}\right)^{\gamma/\rho}(1 - \tilde{a}_{x,i,t}) + \tilde{a}_{x,i,t}x^{\gamma}_{i,t}\right]^{1/\gamma}$ with $\tilde{a}_{j,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_j)}{1+\exp(Z_{i,t}\tilde{\phi}_m)+\exp(Z_{i,t}\tilde{\phi}_f)}$, $j \in \{m, f\}$ and $\tilde{a}_{x,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_x)}{1+\exp(Z_{i,t}\tilde{\phi}_x)}$. The function $\mathcal{H}$ is specified as $\mathcal{H}(f, \Psi_{i,t}, \theta_{i,t}) = \theta_{i,t} f^{\delta_1}\Psi^{\delta_2}_{i,t}$ with $\theta_{i,t} = \exp(Z_{i,t}\phi_\theta + \xi_{\theta,i,t})$.

Table G-21: Joint GMM Estimation Relaxing Some Parameters Across Relative Demand and Production – No Borrowing/Saving ($\kappa = 1$)

	$\epsilon_{\tau,g}$		$\epsilon_{x,H}$		δ_1	δ_2	$2N(Q_N - \tilde{Q}_N)$
	Rel. Dem.	Prod.	Rel. Dem.	Prod.	-	-	-
	0.21	0.56	0.50	-	0.17	0.82	1.72
	(0.05)	(21.96)	(0.09)	-	(0.05)	(0.03)	(0.42)

	$\tilde{\phi}_m$: Mother's Time		$\tilde{\phi}_f$: Father's Time		$\tilde{\phi}_x$: Childcare		$\overline{\phi}_\theta$: TFP
	Rel. Dem.	Prod.	Rel. Dem.	Prod.	Rel. Dem.	Prod.	-
Constant	7.79	-	3.93	-	-1.18	-	1.21
	(1.69)		(1.17)		(0.41)		(0.62)
Single	0.11	-	-	-	0.60	-	-0.17
	(0.34)				(0.21)		(0.14)
Type 2	-0.87	-	-	-	0.07	-	-0.09
	(0.50)				(0.30)		(0.16)
Type 3	-2.19	-	-	-	0.10	-	-0.55
	(0.82)				(0.31)		(0.30)
Mother some coll.	-0.47	-	-	-	0.01	-	0.02
	(0.42)				(0.20)		(0.12)
Mother coll+	-1.55	-	-	-	-0.22	-	-0.18
	(0.65)				(0.19)		(0.22)
Child's age	-0.56	-	-0.43	-	-0.07	-	-0.16
	(0.15)		(0.16)		(0.03)		(0.04)
Num. of children 0-5	0.43	-	0.60	-	0.05	-	0.07
	(0.27)		(0.38)		(0.12)		(0.05)
Father some coll.	-	-	-0.95	-	0.00	-16.20	-0.08
			(0.67)		(0.25)	(130762285.78)	(0.58)
Father coll+	-	-	-0.83	-	-0.63	-	0.12
			(0.61)		(0.24)		(0.10)
Year = 2002	-	-	-	-	-	-	-0.30
							(0.06)

Notes: This table reports GMM estimates where some parameters are allowed to differ between those determining relative input demand (Rel. Dem.) and those determining actual skill production (Prod.). See Appendix E.4 for more details. The distance metric, $2N(Q_N - \tilde{Q}_N)$, is the difference between the optimally weighted GMM criterion at the restricted estimates and its value at the relaxed estimates. It has a χ^2 distribution with degrees of freedom equal to the number of constraints that are relaxed. Standard errors are indicated in parentheses <u>except</u> for the distance metric, which reports a p-value. The function f is specified as: $f = \left[\left(\tilde{a}_{m,i,t}\tau^{\rho}_{m,i,t} + \tilde{a}_{f,i,t}\tau^{\rho}_{f,i,t} + (1 - \tilde{a}_{m,i,t} - \tilde{a}_{f,i,t})g^{\rho}_{i,t}\right)^{\gamma/\rho}(1 - \tilde{a}_{x,i,t}) + \tilde{a}_{x,i,t}x^{\gamma}_{i,t}\right]^{1/\gamma}$ with $\tilde{a}_{j,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_j)}{1+\exp(Z_{i,t}\tilde{\phi}_m)+\exp(Z_{i,t}\tilde{\phi}_f)}$, $j \in \{m, f\}$ and $\tilde{a}_{x,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_x)}{1+\exp(Z_{i,t}\tilde{\phi}_x)}$. The function $\mathcal{H}$ is specified as $\mathcal{H}(f, \Psi_{i,t}, \theta_{i,t}) = \theta_{i,t} f^{\delta_1}\Psi^{\delta_2}_{i,t}$ with $\theta_{i,t} = \exp(Z_{i,t}\phi_\theta + \xi_{\theta,i,t})$.

Table G-22: Joint GMM Estimation Allowing Time Productivity Share for Mothers to Differ Across Relative Demand and Production – No Borrowing/Saving ($\kappa = 1$)

	$\epsilon_{\tau,g}$		$\epsilon_{x,H}$		δ_1	δ_2	$2N(Q_N - \tilde{Q}_N)$
	Rel. Dem.	Prod.	Rel. Dem.	Prod.	-	-	-
	0.20	-	0.50	-	0.17	0.83	0.00
	(0.05)	-	(0.09)	-	(0.05)	(0.02)	(0.97)

	$\tilde{\phi}_m$: Mother's Time		$\tilde{\phi}_f$: Father's Time		$\tilde{\phi}_x$: Childcare		$\overline{\phi}_\theta$: TFP
	Rel. Dem.	Prod.	Rel. Dem.	Prod.	Rel. Dem.	Prod.	-
Constant	8.19	8.10	4.12	-	-1.19	-	1.11
	(1.89)	(9.53)	(1.27)		(0.41)		(0.73)
Single	0.10	-	-	-	0.61	-	-0.20
	(0.37)				(0.21)		(0.34)
Type 2	-0.96	-	-	-	0.08	-	-0.05
	(0.55)				(0.30)		(0.24)
Type 3	-2.41	-	-	-	0.10	-	-0.43
	(0.92)				(0.31)		(0.45)
Mother some coll.	-0.50	-	-	-	0.01	-	0.05
	(0.45)				(0.20)		(0.16)
Mother coll+	-1.70	-	-	-	-0.23	-	-0.08
	(0.73)				(0.19)		(0.34)
Child's age	-0.59	-	-0.47	-	-0.07	-	-0.13
	(0.17)		(0.17)		(0.03)		(0.09)
Num. of children 0-5	0.46	-	0.64	-	0.06	-	0.06
	(0.29)		(0.42)		(0.12)		(0.06)
Father some coll.	-	-	-1.04	-	-0.00	-	-0.01
			(0.72)		(0.25)		(0.09)
Father coll+	-	-	-0.95	-	-0.63	-	0.17
			(0.67)		(0.24)		(0.12)
Year = 2002	-	-	-	-	-	-	-0.30
							(0.07)

Notes: This table reports GMM estimates where some parameters are allowed to differ between those determining relative input demand (Rel. Dem.) and those determining actual skill production (Prod.). See Appendix E.4 for more details. The distance metric, $2N(Q_N - \tilde{Q}_N)$, is the difference between the optimally weighted GMM criterion at the restricted estimates and its value at the relaxed estimates. It has a χ^2 distribution with degrees of freedom equal to the number of constraints that are relaxed. Standard errors are indicated in parentheses except for the distance metric, which reports a p-value. The function f is specified as: $f = \left[\left(\tilde{a}_{m,i,t}\tau^\rho_{m,i,t} + \tilde{a}_{f,i,t}\tau^\rho_{f,i,t} + (1 - \tilde{a}_{m,i,t} - \tilde{a}_{f,i,t})g^\rho_{i,t}\right)^{\gamma/\rho}(1 - \tilde{a}_{x,i,t}) + \tilde{a}_{x,i,t}x^\gamma_{i,t}\right]^{1/\gamma}$ with $\tilde{a}_{j,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_j)}{1+\exp(Z_{i,t}\tilde{\phi}_m)+\exp(Z_{i,t}\tilde{\phi}_f)}$, $j \in \{m, f\}$ and $\tilde{a}_{x,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_x)}{1+\exp(Z_{i,t}\tilde{\phi}_x)}$. The function $\mathcal{H}$ is specified as $\mathcal{H}(f, \Psi_{i,t}, \theta_{i,t}) = \theta_{i,t} f^{\delta_1} \Psi^{\delta_2}_{i,t}$ with $\theta_{i,t} = \exp(Z_{i,t}\phi_\theta + \xi_{\theta,i,t})$.

Table G-23: Joint GMM Estimation Allowing Time Productivity Share for Mothers to Differ Across Relative Demand and Production – Unconstrained ($\kappa = 0$)

	$\epsilon_{\tau,g}$		$\epsilon_{x,H}$		δ_1	δ_2	$2N(Q_N - \tilde{Q}_N)$
	Rel. Dem.	Prod.	Rel. Dem.	Prod.	-	-	-
	0.20	-	0.50	-	0.14	0.83	0.02
	(0.05)	-	(0.09)	-	(0.04)	(0.02)	(0.88)

	$\tilde{\phi}_m$: Mother's Time		$\tilde{\phi}_f$: Father's Time		$\tilde{\phi}_x$: Childcare		$\overline{\phi}_\theta$: TFP
	Rel. Dem.	Prod.	Rel. Dem.	Prod.	Rel. Dem.	Prod.	-
Constant	8.43	8.80	4.28	-	-1.21	-	1.28
	(1.99)	(11.14)	(1.30)		(0.41)		(0.87)
Single	0.08	-	-	-	0.61	-	-0.14
	(0.37)				(0.21)		(0.27)
Type 2	-0.98	-	-	-	0.08	-	0.04
	(0.56)				(0.30)		(0.24)
Type 3	-2.44	-	-	-	0.10	-	-0.29
	(0.95)				(0.31)		(0.48)
Mother some coll.	-0.53	-	-	-	0.00	-	0.10
	(0.46)				(0.20)		(0.16)
Mother coll+	-1.77	-	-	-	-0.22	-	-0.02
	(0.76)				(0.19)		(0.37)
Child's age	-0.62	-	-0.49	-	-0.07	-	-0.13
	(0.18)		(0.18)		(0.03)		(0.11)
Num. of children 0-5	0.50	-	0.67	-	0.07	-	0.08
	(0.30)		(0.43)		(0.12)		(0.06)
Father some coll.	-	-	-1.10	-	-0.01	-	0.01
			(0.74)		(0.25)		(0.09)
Father coll+	-	-	-1.01	-	-0.64	-	0.21
			(0.69)		(0.24)		(0.11)
Year = 2002	-	-	-	-	-	-	-0.34
							(0.06)

Notes: This table reports GMM estimates where some parameters are allowed to differ between those determining relative input demand (Rel. Dem.) and those determining actual skill production (Prod.). See Appendix E.4 for more details. The distance metric, $2N(Q_N - \tilde{Q}_N)$, is the difference between the optimally weighted GMM criterion at the restricted estimates and its value at the relaxed estimates. It has a χ^2 distribution with degrees of freedom equal to the number of constraints that are relaxed. Standard errors are indicated in parentheses <u>except</u> for the distance metric, which reports a p-value. The function f is specified as: $f = \left[\left(\tilde{a}_{m,i,t}\tau^{\rho}_{m,i,t} + \tilde{a}_{f,i,t}\tau^{\rho}_{f,i,t} + (1 - \tilde{a}_{m,i,t} - \tilde{a}_{f,i,t})g^{\rho}_{i,t}\right)^{\gamma/\rho}(1 - \tilde{a}_{x,i,t}) + \tilde{a}_{x,i,t}x^{\gamma}_{i,t}\right]^{1/\gamma}$ with $\tilde{a}_{j,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_j)}{1+\exp(Z_{i,t}\tilde{\phi}_m)+\exp(Z_{i,t}\tilde{\phi}_f)}$, $j \in \{m, f\}$ and $\tilde{a}_{x,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_x)}{1+\exp(Z_{i,t}\tilde{\phi}_x)}$. The function $\mathcal{H}$ is specified as $\mathcal{H}(f, \Psi_{i,t}, \theta_{i,t}) = \theta_{i,t}f^{\delta_1}\Psi^{\delta_2}_{i,t}$ with $\theta_{i,t} = \exp(Z_{i,t}\phi_\theta + \xi_{\theta,i,t})$.

Table G-24: Direct GMM Estimation of Production Parameters using Relaxed Relative Demand

	ρ (1)	γ (1)	δ_1 (1)	δ_2 (1)
	[-3.00, -0.04]	[-6.79, 2.17]	[-0.13, 0.07]	[0.71, 0.96]
	$\tilde{\phi}_m$: Mother's Time (1)	$\tilde{\phi}_f$: Father's Time (1)	$\tilde{\phi}_x$: Childcare (1)	$\overline{\phi}_\theta$: TFP (1)
Constant	[-3.88, 2.75]	[-3.93, 0.99]	[-2.59, 1.80]	[-4.65, 0.58]
Single	-	-	[-4.40, 2.43]	[-0.01, 1.84]
Mother some coll.	-	-	-	[-3.19, 0.89]
Mother coll+	-	-	-	[-0.16, 2.17]
Father some coll.	-	-	-	[-1.16, 2.97]
Father coll+	-	-	-	[-0.11, 2.35]
Child's age	-	-	-	[-0.01, 0.52]
Num. of children 0-5	-	-	-	[0.45, 4.13]
Year = 2002	-	-	-	[-0.71, -0.25]

Notes: This table reports bootstrapped 80% confidence intervals for GMM estimates of the production function using the "direct" approach outlined in Appendix E.5 and Section 4.2.1. Due to convergence issues, estimates are obtained by terminating minimization after 100 LBFGS iterations followed by 10 Newton iterations. The bootstrapped sample is obtained using 50 samples with replacement of children in the data. The function f is specified as: $f = \left[\left(\tilde{a}_{m,i,t}\tau_{m,i,t}^{\rho} + \tilde{a}_{f,i,t}\tau_{f,i,t}^{\rho} + (1-\tilde{a}_{m,i,t}-\tilde{a}_{f,i,t})g_{i,t}^{\rho}\right)^{\gamma/\rho}(1-\tilde{a}_{x,i,t}) + \tilde{a}_{x,i,t}x_{i,t}^{\gamma}\right]^{1/\gamma}$ with $\tilde{a}_{j,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_j)}{1+\exp(Z_{i,t}\tilde{\phi}_m)+\exp(Z_{i,t}\tilde{\phi}_f)}$, $j \in \{m, f\}$ and $\tilde{a}_{x,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_x)}{1+\exp(Z_{i,t}\tilde{\phi}_x)}$. The function $\mathcal{H}$ is specified as $\mathcal{H}(f, \Psi_{i,t}, \theta_{i,t}) = \theta_{i,t} f^{\delta_1}\Psi_{i,t}^{\delta_2}$ with $\theta_{i,t} = \exp(Z_{i,t}\phi_\theta + \xi_{\theta,i,t})$.