Social Security and the search behavior of workers approaching retirement

J. Ignacio García Pérez  Alfonso R. Sánchez Martín
U. Pablo de Olavide, FEDEA & FCEA  U. Pablo de Olavide

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Abstract

This paper explores the links between unemployment, retirement and their associated public insurance programs. It is a contribution to a growing body of literature focused on a better understanding of the labor behavior of advanced-age workers, which has gained importance as the pension crisis looms. The analysis combines a details exploration of the empirical regularities and the development of a new theoretical model. The empirical analysis is based on the newly released Muestra Continua de Vidas Laborales (MCVL), a very large data base of administrative records from the Spanish Social Security. On the other hand, our modeling effort involves a substantial extension of the standard (Mortensen 1986) search model, specifically designed to reproduce the non-stationary environment faced by workers of advanced ages (in the age range 50/65).

We explore the main economic properties of the model via calibrated simulations and compare the findings with the basic qualitative properties of the data. We understand this is a first step in a structural econometric experiment combining our extended model (as the data generating process) and the newly released Spanish Social Security data. We find that the model successfully rationalize a large number of the observable stylized facts. We also find some discrepancies that can be interpreted as evidence of a voluntary use of the unemployment scheme as a route to early retirement.

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1 Introduction

The labor supply behavior of older workers in the OECD countries has been remarkably similar in the last three decades, being dominated by a widespread tendency towards lower rates of employment and participation\textsuperscript{1}. Its coincidence in time with the early manifestations of a more dramatic process (the rapid aging of the population of the developed economies) has, however, allowed it to go relatively unnoticed\textsuperscript{2}. But this perception has changed substantially in recent years, when the connections between the two phenomena have become more apparent. Lower working (and contribution) careers combined with longer periods of enjoyment of the public pensions pose an evident threat to the financial sustainability of PAYG pensions. This gloomy prospect has triggered an intense academic and political debate. The practical results of these debates vary to some degree across countries, but a ubiquitous ingredient in most reform proposals is the need to reverse the historical tendency towards lower employment rates. Examples of these type of policies are the recent delays in the normal retirement age in EEUU or Germany or the explicit targets in the Lisbon-2000 objective\textsuperscript{3}.

The low participation problem has brought under the spotlight the design of some of the best beloved public insurance mechanisms: unemployment benefits and early retirement. The generous unemployment protection in most European countries has traditionally been blamed for its high overall unemployment rate (Ljungqvist and Sargent 1998). It seem reasonable, in any case, to protect our more senior workers of a risk (being fired) that is probably more present at advanced ages than at earlier stages in the life cycle. The optimal design of the system is, however, more difficult, as we have to add to the classical moral hazard problem the documented tendency to use the unemployment scheme as an alternative pathway into retirement (Duvan 2003). Early retirement schemes were also closely related to unemployment at the time of their inception, as most of them where put in place in response to the industrial crises of the seventies. These programs have proved extraordinary popular, both among individuals and corporations. The consequences for the public finances can be, however, devastating, specially in connection with generous minimum pension provisions like the one existing in Spain\textsuperscript{4}.

In this paper we explore the interaction between the labor decisions of older workers and the incentives provided by the pension and unemployment schemes in Spain. Without failing

\textsuperscript{1}There is mounting evidence that in the last few years this tendency has slowed down and in some countries reversed itself.

\textsuperscript{2}Declining birth rates and rising life expectancies have interacted to produce dramatic changes in the size and age structure of Europe’s population in the near future. For example, the OECD predicts that the working-age population (15–64 years) will be 18 % smaller than the current one, and the numbers of those aged over 65 years will increase by 60 %. As a result, the average ratio of retirees to employees will double in Europe from the current 24 % to almost 50 % in 2050. These changes will have profound implications for the European economy and its capability to finance its welfare and health care systems.

\textsuperscript{3}There has been widespread delays in both the early and normal retirement ages in OECD countries. See Casey et al. (2003) for a detailed enumeration. For the EU members, the policy action has revolved around the Lisbon-2000 objectives. They set a explicit target (50%) for the employment rate of workers of more than 50 years of age in 2010.

\textsuperscript{4}Under the minimum pensions scheme large number of workers can avoid the (roughly actuarially fair) penalties for early retirement embedded in the regular pension formula.
to acknowledge the importance of the actions taken by firms, we focus on this work on the study of the behavioral responses on the part of the individuals. The analysis involves two parallel developments. On the one hand, we undertake a extensive empirical analysis of the search and retirement behavior of unemployed workers of advanced age using the newly released Muestra Continua de Vidas Laborales (MCVL hereafter). On the other hand, we formulate a theoretical model of rational decision taking by the individuals. This modeling effort brings together two strands of the literature: search models (traditionally applied to labor decisions early in the working career) and retirement models (applied to the participation decision late in the life-cycle). Individuals in our model are, then, expected utility maximizers that operate in an environment that resembles the institutional details and the sources of uncertainty prevailing at the late stages of the life-cycle.

This model improves on the existing retirement literature (exemplified by eg. Rust and Phelan (1997), French (2005) or Van der Klaauw and Wolpin (2005)) by formally exploring the unemployment path into retirement. Furthermore, most of the more recent modeling efforts have considered the importance of health considerations and the alternative pathway into retirement via disability (eg, Gruber and Kubik (1997) or Rust et al. (2002)). The unemployment route has received less attention, although it seems to be very relevant in European countries like Spain. The paper also contributes to the literature on search models by considering non-participation decisions in a non-stationary environment including the risk of dismissal. The possibility of non-participation in an otherwise standard search model was first analyzed in Van den Berg (1990). More recently, Frijters and Van der Klaauw (2006) estimates a structural, non-stationary search model with non-participation, where the state of inactivity (considered as an absorbing one) is unrelated to the economic conditions. Our analysis improves upon the former by considering the fundamental non-stationary induced by age considerations, and upon the latter by providing a full economic description of the non-participation state (ie, retirement). Furthermore, we include in this literature the risk of dismissal. This has proved important for the search behaviour in non-stationary environments (García Pérez (2006)). It seems to be even more relevant for a proper evaluation of the value of job offers in this context, where the unemployed have a significant alternative in taking the exit way into retirement.

The ultimate target of the development of our formal model is to produce a good environment in which to undertake policy experiments. In our view, the quality of the tool should be judged by its ability to reproduce the empirical data. In this paper we take a first step in this direction by exploring the performance of a calibrated version of the model and by comparing the results with a broad set of stylized facts obtained in the analysis of the MCVL data. Eventually, this process will proceed from this mainly qualitative approach to the truly quantitative one that is need for policy analysis. At that point, the preference parameters and the unobserved distributions should be obtained through structural estimation. Therefore, this paper should be understood as a first step in that direction. Our preliminary findings indicate that a large number of the observable patterns can be rationalized as the optimal reactions of rational individual to the incentive provided by the institutional rules. But there are also indications of the voluntary use

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5The MCVL database is described in detailed in section 2.1
of the unemployment scheme as an early retiring device.

The rest of the paper is organized as follows: firstly we describe the dataset and the basic stylized facts on search and retirement behavior in our sample in section 2. Secondly, we introduce the search model enlarged with an explicit retirement option in section 3. We then propose a baseline example to analyze the basic qualitative properties of the model solutions in section 4. The results of the analysis are presented in section 5, and the paper concludes in section 6.

2 The data set and some descriptive statistics

Our database in this paper is the new Muestra Continua de Vidas Laborales, an administrative dataset based on a random draw from the Spanish Social Security archives. It contains a sample of 4% among all the affiliated workers, working or not, and pensioners in the year 2004. It has information about 1,1 million people which covers their entire labor history and, for pensioners, it also offers the main parameters to calculate their pension.

This database offers information about the personal characteristics of the worker and also about all her employment spells throughout her labor history. We have information about age, gender, occupation, unemployment and employment spells and their respective exact durations. We have available the reasons for each contract ending, the geographical location of the job, the firm’s sector of activity, the type of contract held and whether the contract was signed with a temporary help agency for each spell of employment. Moreover, we have exact information about wages, measured as the “base de cotización” (contribution base) which coincides with the monthly wage for all workers that earn more than the minimum base and less than the maximum one. These two limits are annually decided by Social Security authorities and make the wage to be censored in cases where it is outside these limits.

The duration of the employment spells are built from the dates of beginning and ending the contract and it is measured in months. Moreover, for the periods of non-employment, we can distinguish among the ones when payroll taxes are being paid, that is, when the worker is receiving Unemployment Benefits, and those when the worker contributions to Social Security is not paid, which can be both periods of unemployment without benefits or periods of inactivity. Hence, we use the terminology non-employment to name all these spells of not working within a firm.

The amount of information for each individual in our database is quite large. There exists one different register for each contract held and this makes every change in the contract to need a different register although the employment spell is the same. Hence, we have applied

6We are not considering employment spell durations lower than 15 days in the case that the subsequent period of non-employment is also lower than 15 days in order not to study just very short spells due to reallocation or strong turnover within the firm. Moreover, we are neither studying non-employment spells of less than 30 days. The reason is basically the same: we consider that, given the characteristics of the Spanish labor market for old workers, a transition between two jobs with a period of non-employment of less than one month could be considered as basically a direct job-to-job transition.

7We have first eliminated each incomplete or incorrect register. This may happen because some important information is missing or because it is clearly incorrect (dates of beginning and finishing incompatible, etc.)
some criteria to unify different registers when they refer to the same employment spell or to eliminate simultaneous employment spells, that is when the individual is working with two firms at the same time (we keep only the information about the longer spell). Furthermore, we have also unified each two registers when they correspond to the situation when one contract begins before the previous one has finished.

Finally, we are only considering labor histories of workers within the called “Regimen General”, that is, regular workers being paid by a firm. We are not using information for self-employed neither for workers in Agriculture, Fish and other minor special cases.

Table 1:

<table>
<thead>
<tr>
<th>Number of workers</th>
<th>Female</th>
<th>Males</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unskilled</td>
<td>5,337</td>
<td>26,076</td>
<td>31,413</td>
</tr>
<tr>
<td>Skilled</td>
<td>3,441</td>
<td>11,443</td>
<td>14,884</td>
</tr>
<tr>
<td>Total</td>
<td>8,778</td>
<td>37,519</td>
<td>46,297</td>
</tr>
</tbody>
</table>

Table 2:

<table>
<thead>
<tr>
<th>Year when the Spell Begins</th>
<th>EXIT TO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EMPLOYMENT</td>
</tr>
<tr>
<td>MEN, SKILLED</td>
<td></td>
</tr>
<tr>
<td>1981-1985</td>
<td>5.29</td>
</tr>
<tr>
<td>1986-1990</td>
<td>19.90</td>
</tr>
<tr>
<td>1996-2000</td>
<td>48.43</td>
</tr>
<tr>
<td>MEN, UNSKILLED</td>
<td></td>
</tr>
<tr>
<td>1981-1985</td>
<td>6.52</td>
</tr>
<tr>
<td>1986-1990</td>
<td>20.42</td>
</tr>
<tr>
<td>1991-1995</td>
<td>30.45</td>
</tr>
<tr>
<td>1996-2000</td>
<td>42.61</td>
</tr>
</tbody>
</table>

Tables 1 and 2 shows the distribution of workers fulfilling the previous restrictions and aged more than 52. The sample is much larger for males than for females, given the well known participation behavior of old workers. This, jointly with the quite different behavior of women with respect to men in deciding about their labor supply provide the rationale for our decision to concentrate on males in the rest of the paper. We can see also in Table 1 how the
The presence of low skilled workers is much likely, given their larger firing and unemployment probabilities. In Table 2 we can see that almost two thirds of unemployment spells are taking place in the nineties, specially those which finish as a re-employment transition. The transition from unemployment to retirement is more likely in the second half of the eighties.

In this application, we explore the results for a relatively narrow subsample of workers whose economic incentives are clearly identified. Hence, we do not focus on generality of our results but on getting good empirical counterparts for our theoretical predictions. Given this, we are only considering low skilled males who receive unemployment benefits when unemployed, who have not being in any permanent disability program neither in any long non-participation spell and, finally, who are not being subject to any special agreement with the firm, named in Spain “Convenio Especial”, in order to pay her payroll taxes for being retired with a better pension.\footnote{These agreements between workers and firms, which Labor Authorities has to agreed with, are used by the former to obtain an income larger than the entitled Unemployment Benefits by mean of an extra payment, made by the firm, and also by an extra firm contribution to the Social Security in order to complement her pension rights.}

In Table 3, we have the first picture which shows us how different are workers, depending on whether they decide to exit to a new job or directly to retirement. The former come from much shorter employment spells, with lower wages and also with lower accrued pension rights that the latter. The “pension rights” of an individual is a moving average of his previous labor earnings used by the pension authorities to compute the size of the benefit the individual is entitled to. A formal definition can be found in section 3.1. Moreover, workers who return to employment spend much less time in unemployment and has less access to Unemployment Benefits than the ones who exit to retirement (after 29 months, in average, whereas unemployment duration is 9 months for those who returns to employment). Finally, Table 3 are saying also that workers returning to employment are more likely to be previously working in the construction and service sector whereas those who exit directly to retirement are coming, basically, from Industry.

2.1 Main Empirical regularities

We have explored the transitions of unemployed workers back into work or out of the labor force (into retirement) paying special attention to the role of age, duration in unemployment, pension rights and wages in the immediately preceding job. This has resulted in a relatively large number of empirical patterns, that we summarize (and enumerate for later reference) in the next two (sub)sections. We have also explored the data with the help of reduced-form, competing-risks duration models (comparing the two possible pathways into employment or to retirement). We have made sure that the stylized facts discussed in the main text are not altered once we control for all the observed heterogeneity, although we refer to the these econometric results only occasionally.
<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UNSKILLED MALES</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>EXITING TO EMPLOYMENT</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Last employment’s duration</td>
<td>37.68</td>
<td>76.79</td>
</tr>
<tr>
<td>Unemployment duration</td>
<td>8.93</td>
<td>16.10</td>
</tr>
<tr>
<td>With Unemployment Benefits</td>
<td>71.55%</td>
<td>45%12</td>
</tr>
<tr>
<td>Duration of Unempl. Benefits</td>
<td>8.73</td>
<td>13.35</td>
</tr>
<tr>
<td>Going to the same employer</td>
<td>31.07%</td>
<td>46.28</td>
</tr>
<tr>
<td>Accrued pension rights</td>
<td>9.490.37</td>
<td>4.402.27</td>
</tr>
<tr>
<td>Previous Wage</td>
<td>687.01</td>
<td>471.80</td>
</tr>
<tr>
<td>Working Part time</td>
<td>2.43%</td>
<td>15.39%</td>
</tr>
<tr>
<td><strong>Sector of Activity:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agriculture</td>
<td>2.08%</td>
<td>14.27%</td>
</tr>
<tr>
<td>Industry</td>
<td>19.59%</td>
<td>39.69%</td>
</tr>
<tr>
<td>Construction</td>
<td>48.55%</td>
<td>49.98%</td>
</tr>
<tr>
<td>Services</td>
<td>29.78%</td>
<td>45.73%</td>
</tr>
<tr>
<td><strong>EXITING TO RETIREMENT</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Last employment’s duration</td>
<td>160.47</td>
<td>131.64</td>
</tr>
<tr>
<td>Unemployment duration</td>
<td>28.67</td>
<td>24.14</td>
</tr>
<tr>
<td>With Unemployment Benefits</td>
<td>88.97%</td>
<td>31.33%</td>
</tr>
<tr>
<td>Duration of Unempl. Benefits</td>
<td>25.59</td>
<td>22.61</td>
</tr>
<tr>
<td>Accrued pension rights</td>
<td>13.283.60</td>
<td>5.278.12</td>
</tr>
<tr>
<td>Previous Wage</td>
<td>936.32</td>
<td>605.49</td>
</tr>
<tr>
<td>Working Part time</td>
<td>0.67%</td>
<td>8.15%</td>
</tr>
<tr>
<td><strong>Sector of Activity:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agriculture</td>
<td>6.43%</td>
<td>24.54%</td>
</tr>
<tr>
<td>Industry</td>
<td>43.53%</td>
<td>49.58%</td>
</tr>
<tr>
<td>Construction</td>
<td>23.92%</td>
<td>42.66%</td>
</tr>
<tr>
<td>Services</td>
<td>26.11%</td>
<td>43.92%</td>
</tr>
</tbody>
</table>
2.1.1 Search behavior

The most significant findings regarding the job-acceptance performance of unemployed workers are:  

S1 The re-employment hazard decreases with the length of the unemployment spell (i.e., duration, represented by the integer \( h \))

S2 The re-employment hazard decreases with age.

S3 Accrued pension rights and unemployment duration interact with each other in the following way:

S31 For the long-term unemployed (\( h > 1 \) or more than one year out of work), the re-employment hazard is lower for those with high pension rights.

S32 For the short-term unemployed (\( h = 1 \)), large pension rights are associated with high re-employment hazards at early ages (i.e., in the early fifties). At more advanced ages, however, the relative order of exit intensities changes and it finally resembles that of S31.

S1 and S2 are illustrated in Figure 1. We see that, as is usually found in the search literature, the re-employment hazard is highly decreasing with unemployment duration (much larger for those with a duration of less than one year, \( h=1 \), than for those with an unemployment duration of more than 2 years, \( h=3 \)). We also find a negative pattern in the hazard by age, a dimension that has received much less attention in the previous literature. The conditional probability of returning to work starts at a value close to 15% at the age of 55 and goes down with age (in a roughly linear fashion) until it becomes hardly noticeable just before the normal retirement age. S3 reflects the interaction of pensions institutions and search behavior, and can be clearly appreciated in Figure 2. The left panel shows the re-employment hazard of the short-term unemployed, conditional on the size of the accrued pension rights (we split the sample in three groups according with the percentiles 1/3 and 2/3 of the distribution of pension rights in the sample). The right panel is similar, but for the unemployed with a duration larger than 1 year. The graphs clearly indicate that pension rights are of paramount importance in the job-acceptance process of the unemployed. In particular, being entitled to a large pension reduces the re-employment hazard of an individual with a duration larger than one year to less than one third of the hazard of somebody with less than average pension rights (S31). Strikingly, this pattern is not stable with age for short-term unemployed workers. For those workers we see that at earlier ages the re-employment hazard is higher for those with high pension rights. This hazard, however, goes down with age more rapidly for these workers. Graphically, the hazard for workers with high pension rights crosses that of workers endowed with less pension rights.

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10 By “Hazard” we mean the conditional probability of making a transition (from unemployment to either employment or retirement) at a particular age. All presented hazards are annual.

11 With the possible exception of the very long term unemployed older that 60, who seem to manage to return to the workforce in relatively large numbers. Workers belonging to high-turnover sectors -specially construction-.

12 Apparently, this indicates that paying some of the unemployment benefits in form of future pension rights may induce a higher job-acceptance rate by the unemployed. This possibility is carefully explored later in the
Figure 1: Transition from unemployment to employment: hazard by duration and age

Figure 2: Transition from unemployment to employment: hazard by age, duration (h) and accrued pension rights. (the continuous blue line represents pension rights below the percentile 33%, the dashed-green line those between 33% and 67% and the red-dotted line all the rest)
2.1.2 Retirement behavior

Regarding the transition from unemployment to permanent retirement, we highlight the following empirical regularities:

R1 There is a very marked jump in retirement hazard at the normal retirement age (65).\(^{13}\)

R2 Retirement hazard also jumps at the Early Retirement (60), although the discontinuity is much smaller that at 65.\(^{14}\)

R21 Early retirement is strongly increasing in the duration in unemployment, \(h\).

R22 Early retirement is essentially independent of pension rights.

R3 Retirement hazard is basically constant in the age range 61/64 (intermediate retirement)

R31 Intermediate retirement is not monotone in \(h\), with the hazards ordered in the following way: \(Hz(h=3)<Hz(h=1)<Hz(h=2)\).

R32 Intermediate retirement is monotone increasing on the accrued pension rights, \(\hat{w}\), for longer-term unemployed \((h=\{2,3\})\) and independent of \(\hat{w}\) for the short-term ones \((h=1)\).

R4 The wage obtained in the immediately preceding employment spell is largely irrelevant for the exit to retirement.

These empirical regularities are illustrated in figures 3 and 4. R1 is the single most robust piece of empirical evidence in the entire retirement literature: the pension-system normal retirement age acts as a focal point for clustering withdrawals from the labor force. A quick look at any of the graphs included in the just mentioned figures suffices as confirmation that this is also the case for the unemployed workers in our sample.

We separately explore the incidence of early-retirement at the first pensionable age (60) at the other intermediates ages before 65. It is intuitive that the forces driving workers out of the labor force can differ substantially in either case. A lot of retirement is also probably clustered around sixty due to borrowing constraints: the practical impossibility of anticipating the consumption of future pension income (and the lack of enough accumulated wealth) does paper.

\(^{13}\)The somewhat larger hazard at age 66 is basically due to measurement error in this dataset with respect to the age of the worker.

\(^{14}\)When we refer to a peak in the hazard at 60 we are stressing the drop in the conditional retirement probability observed after that age. Note that, given our identification of retirement with the collection of pension benefits, the absence of retirement before the Early Retirement Age is entirely a matter of construction. In the graphs, however, we see a non negative hazard at earlier ages, indicating the existence of some measurement error -probably in the confusion of old-aged pensions with other type of pensions that can be enjoyed at earlier ages.
Figure 3: Transition from Unemployment to Retirement: hazard by age, duration (h), accrued pension rights (PR) and immediately previous wage.

Figure 4: Transition from Unemployment to Retirement: hazard by age conditional on duration in unemployment, h, and pension rights.
not leave much alternatives to those out of job than to stay in the unemployment scheme.\footnote{The alternative, of course, is to intensify the search effort or to accept the available offers. It means that there is always a behavioral component in this regularity (maybe with the exception of workers with health problems that fail to qualify for a disability pension).} Therefore, we may expect a burst of retirement of long-term unemployed at the exact age of 60, made of all those that would have left before in presence of a \textit{perfect} credit market. The top right panel of Figure 3 strongly confirms this \textit{a priori}: retirement hazard at 60 is monotone increasing in $h$, reaching almost one half of the unemployed with duration higher than 2 years (R21). In these circumstances, it is difficult to discern how other economic incentives work at this age. In particular, it is documented that the early retirement of employees owes a great deal to the effect of minimum pensions (eg. Jiménez Martín and Sánchez Martín (2005)). In our case (unemployed workers), we cannot find evidence of this impact. As stated in R22, the hazard at 60 is virtually independent of the size of the pension rights both in the entire sample (bottom-left panel of Figure 3) or conditional on unemployment duration, $h$ (Figure 4)\footnote{We have checked that this evidence is robust to the discretization of the accrued pension rights.}.

In principle, we should expect to witness the workings of economic incentives more clearly at the intermediate ages. This is roughly confirmed with respect to pension rights and not so much for previous wages. Firstly (R31), the hazard for the unemployed with less than one year of duration is smaller than that of unemployed with more than one but less than two years “on the rolls”. This is consistent with the economics incentives (see the details in section 5.1.2). In contrast, the really long-term unemployed ($h=3$) exhibit the lowest hazard of all, which is at odds with the intuition (and the theory developed in the following section). Secondly (R32), higher pension rights do induce higher retirement hazards (bottom-left panel of Figure 3, although this is less so for $h=1$ (leftmost panel of Figure 4). Finally (R4) there is no clear relation between the size of the immediately preceding wage, $\pi$, and retirement intensities.

All in all, the evidence points to the interesting conclusion that the high incidence of early retirement among the unemployed owes more the results of their search process that to the presence of a generous minimum pensions.

\section{The model}

We model the search and retirement behavior of unemployed workers and the retirement decisions of employees in one particular year, $t$, considering an age range $a \in \{55,65\}$. Time is assumed to be discrete, with one period in the model standing for one year of calendar time. At the beginning of any period individuals of all ages are classified in one of three mutually exclusive labor states: employed, unemployed or retired. Employed workers have a time invariant real wage, $w$, and face a constant probability $\delta$ of being fired at the end of the current period. They have the option to voluntarily leave the labor force and start collecting the pension benefit (if he/she is older than the Early Retirement Age, ERA). Unemployed workers receive one job offer at the beginning of the period with probability $\lambda_{ah}$ (conditional on age, $a$, and the duration, $h$, of the unemployment spell). The offer is fully characterized by the wage, which is a random draw from the invariant distribution $F$. If the offer is rejected, the unemployed can either stay
active for another period or retire immediately (again, after ERA). In case of staying active, the individual receives an unemployment benefit, \( b \), whose amount is contingent in his/her previous wage, \( \pi \), and the duration of the current unemployment spell, \( h \).

In case of retirement, the withdrawal from the labor force is assumed to be permanent. Retirees are granted a pension, \( B \), whose value depends on the age at retirement and on an average of previous wages called “\( \text{Base Reguladora} \)”, \( \bar{w} \). Details on both public programs are provided below. All people are assumed to retire at the age of 70. Retirees take no decisions in this model and face only survival uncertainty, captured by the survival function \( S_a \). Employees and unemployed workers are, of course, exposed to the same mortality risk. Summing up, we need a vector of 4 state variables, \((\pi, \bar{w}, h, a)\), to fully characterize the economic situation of unemployed workers, while a three-state vector, \((\pi, \bar{w}, a)\), is enough to characterize the state of each employee.

### 3.1 Institutions and market arrangements

**Unemployment benefits**

The general unemployment scheme pays a proportion \( b_h \) of the wages enjoyed in the immediately preceding job, although the wage/benefit proportionality is broken by a floor \( b_{\text{min}} \) and a ceiling \( b_{\text{max}} \) on the final benefits. The replacement rates \( b_h \) decrease with the duration, in years, of the current unemployment spell, \( h \). In the present system, those unemployed who have stayed on the rolls for more than 2 years are no longer entitled to receive contributive benefits (they can resort to social assistance programs). However, people older than 52 are entitled to a specific program granting an amount \( b_{\text{min}} \) (75\% of the minimum contribution) till their retirement. In equation (1) we show how all these elements feature in our model:

\[
b(\pi, h) = \begin{cases} 
\text{Max}\{b_{\text{min}}, \text{Min}\{b_1\pi, b_{\text{max}}\}\} & \text{if } h = 1 \\
\text{Max}\{b_{\text{min}}, \text{Min}\{b_2\pi, b_{\text{max}}\}\} & \text{if } h = 2 \\
b_{\text{min}} & \text{if } h = 3 
\end{cases}
\]

(1)

Note that the public employment agency (INEM) takes responsibility of the pay-roll taxes of the unemployed. It contributes the full wages of those unemployed with a duration of less than two years, and the minimum contribution in case of a longer duration. Besides, we do not consider workers covered by special agreements with the pension administration.\[^{17}\]

**The pension system**

The public pension can be claimed at any age after the Early Retirement age, \( N_m \), conditional on a complete withdrawal from labor market activities. The pension benefit of each worker is computed in two steps. First an individual-specific component related to the worker’s previous earnings is calculated. This initial benefit is subsequently compared with the legal minimum and maximum pensions prevailing at each year to determine the final effective payment.

\[^{17}\]Referir a la sección donde se describan los Convenios Especiales
The individual component \((\tilde{B})\) is proportional to the retiree’s pension rights, captured by the average earnings in the \(D\) years immediately before retirement (“Base Reguladora”, \(\hat{w}\)):

\[
\tilde{B}(\hat{w}, a) = \mu(a) \hat{w}_a \quad \text{where} \quad \hat{w}_a = \frac{1}{D} \sum_{i=1}^{D} w_{a-i}
\]  

(2)

the age-dependant replacement rate \(\mu(a)\) reflects a penalty for early retirement:

\[
\mu(a) = \begin{cases} 
\mu_0 & \text{if } a < N_m \\
\mu_0 + \mu_1 (a - \tau_m) < 1 & \text{if } a \in \{N_m, \ldots, N - 1\} \\
1 & \text{if } \tau \geq N
\end{cases}
\]  

(3)

For the unemployed, INEM takes responsibility of the payments to the Social Security on behalf of the worker. As with the unemployment benefit, the scheme is made progressive by the inclusion of floors and ceilings in the payment. This breaks the strict proportionality between the effective benefit \(B(\hat{w}, a)\) and the (average) level of previous wages that would otherwise prevail:

\[
B(\hat{w}, a) = \begin{cases} 
B_{\min} & \text{if } \tilde{B}(w, \hat{w}, a) < B_{\min} \\
\tilde{B}(\hat{w}, a) & \text{if } B_{\min} \leq \tilde{B}(w, \hat{w}, a) \leq B_{\max} \\
B_{\max} & \text{if } B_{\max} < \tilde{B}(w, \hat{w}, a)
\end{cases}
\]  

(4)

The effective benefit is first computed when the individual retires and is kept constant in real terms throughout the rest of his/her life. Note, finally, that we abstract from a number of relatively minor details of the pension and fiscal systems 18.

3.2 Individual Behavior

Every period, unemployed workers take the decision of whether to accept or reject any wage offers they receive and whether to stay active or retire from the labor force. Employees must decide on whether to keep working for the same wage or to retire. In either case we assume individuals are expected utility maximizers, ie. they decide by comparing the expected discounted utility obtained from the associated flows of income and leisure in the different alternatives:

\[
e^*_a = \arg\max E \left[ \sum_{i=a}^{T} \beta^{i-a} u(y_i, e_i) \right]
\]

where \(e^*_a\) stands for the optimal sequence of present and future labor states (from age \(a\) till age \(N\), when we assume retirement is compulsory), \(T\) is the maximum longevity, \(\beta\) is a constant discount factor representing a pure preference for earlier consumption, and individual preferences at every age are represented by an (age-invariant) additively separable CES indirect utility function:

\[
u(y, e) = \left[ \frac{y (1 + \nu(e))}{\eta} \right]^\eta 
\]

\(e = \{E, U, R\}\)

18In particular, contributions, penalties for an insufficient number of contributive years and income taxes
where $\eta$ measures the curvature of the objective function (which, in turn, determines both
the degree of risk aversion and the willingness to substitute income intertemporally) and $\nu$ represents the value attached to the amount of leisure enjoyed at each possible state. A standard
specification, then, would make $\nu(E) < \nu(R)$ (reflecting the foregone leisure and the fixed costs of working) and $\nu(U) < \nu(R)$ (reflecting the utility cost of searching). The relative value of $\nu(E)$
and $\nu(U)$ depends on the relative importance of the cost associated with working and searching
and the possible ‘stigma’ cost of staying unemployed.

As usual, we work with a recursive representation of this discrete-time, optimal control problem. This means that we, firstly, characterize its solution via the value functions associated
with each labor state, and secondly, solve for them by backward induction starting at $\overline{N}$, an age
advanced-enough that we can assume everybody is forced into retirement at that age. At age
$\overline{N}$, then, we just have to keep track of the value function of the retirees:

$$ R_{\overline{N}}(\hat{w}) = \sum_{i=\overline{N}}^{T} S_{\overline{N}}(i) \beta^{i-\overline{N}} u(B(\hat{w}, \overline{N}), R) = \left( \frac{[B(\hat{w}, \overline{N})(1 + l(R))]^\eta}{\eta} \right) \cdot A_{\overline{N}}^{T} \quad (5) $$

where $S_{\overline{N}}(i)$ stands for the survival probability to age $i$ conditional on survival to age $\overline{N}$ and
$A_{i}^{j}$ is the expected discounted value of a constant income flow of one unit starting at age $i$ and
ending at age $j$. To simplify the notation, we denote the (one period ahead) effective discounting
at age $a$, $\beta S_{a}(a+1)$, by $\beta_{a}$ from here onwards.

At any age $a < \overline{N}$, all relevant information is captured by the value functions corresponding
to the three possible labor states: $W, U, R$ representing, respectively, employees, unemployed
and retirees. We review them in turn.

**Employed workers**

Currently employed workers have the option to retire immediately or to stay the same for one
more period. In the latter case, the only risk they face in the future (apart from survival
risk) is the possibility of being fired and become unemployed. This is easily reflected in the
corresponding value function:

$$ W_{a}^{\overline{N}}(w, \hat{w}) = u(w, E) + \beta_{a} [(1 - \delta) W_{a+1}(w, \hat{w}^{\prime}) + \delta U_{a+1}(w, \hat{w}^{\prime}, 1)] \quad (6) $$

where we assume equal next period wages and with next-period pension rights updated as follows:

$$ \hat{w}^{\prime} = \hat{w} + \frac{w - \hat{w}}{D} \quad (7) $$

(as a general rule, primes denote the next period value of any variable). Note that this rule
is only exact under the assumption of constant real wages. In all other cases it is only an
approximation.
For those who find it optimal to retire at age $a$ (with $a \in [N_m, N]$), things do not change in any fundamental way with respect to what we mentioned above for the case $a = N$. In particular, the value function takes a form completely analogous to that in (5). Therefore, the total value of being employed, including the option of retirement, is simply characterized by:

$$W_a(x) = \text{Max}\{ W^e_a(x), R_a(\hat{w}) \}$$

Unemployed workers

For the unemployed we also consider two different value functions. On the one hand, we represent with $U^u_a(\pi, \hat{w}, h)$ the value associated with staying unemployed during the current period (for individuals of age $a$, previous wage $\pi$, pension rights $\hat{w}$ and duration in unemployment $h$). On the other hand, the unemployed at the beginning of the period have the option of leaving the labor force and retire. The total value after adding that possibility to the utility derived from keeping searching for a new job is denoted $U_a(\pi, \hat{w}, h)$, namely, the value attached to be unemployed at the beginning of the period. This is the relevant value for deciding whether to accept or reject any particular job offer. The analytical expression for this latter value function is:

$$U_a(\pi, \hat{w}, h) = \text{Max}\{ U^u_a(\pi, \hat{w}, h), R_a(\hat{w}) \}$$

The value of staying active and continue searching is made up of two elements, a current value $u(b(\pi, h), U)$, and a future value:

$$\beta_a \left\{ \lambda_a h E_w[\text{Max}\{ U_{a+1}(\pi, \hat{w}', h + 1), W_{a+1}(w, \hat{w}') \}] \right\}$$

where $\lambda_a h$ represents the conditional offer arrival rate in the next period and pension rights are updated as in (7). In words, the future value reflects two elements:

- If no offers arrives, he/she has to take the decision of whether to keep searching or retired. Next period value function is then:

$$U_{a+1}(x', h + 1) = \text{Max}\{ U^u_{a+1}(x', h + 1), R_{a+1}(\hat{w}') \}$$

- If an offer of size $w$ arrives, the individual must decide whether to accept or reject it. The optimal behaviour is obtained by comparing $W_{a+1}(w, \hat{w}')$ to $U_{a+1}(x', h')$.

Of course, at $t$, the size of the wage offer is uncertain. Consequently, the individual operates by taking conditional expectations (which is reflected in $E_w[\cdot]$).

Overall, the future value of staying unemployed $U^u_a$ is composed of two elements: an Option Value derived from staying unemployed and so keeping the chances of getting acceptable job offers in the future and the Stop Value of finishing the current unemployment spell by accepting an immediate offer.
The acceptance decision is summarized in the **Reservation Wage**, $w_a(\pi, \hat{w}, h)$: the wage offer that makes the unemployed indifferent between taking the job or staying unemployed, ie:

$$W_a(w_a(\pi, \hat{w}, h), \hat{w}) = U_a(\pi, \hat{w}, h)$$  \hspace{1cm} (9)

With the help of reservation wages, we can rewrite the expected value of the future offers can in a more insightful way:

$$E_w[\max\{U_{a+1}(\pi, \hat{w}', h + 1), W_{a+1}(w, \hat{w}')\}] =$$

$$\int_{w_0(\pi, \hat{w}', h + 1)}^{\infty} W(w, \hat{w}')dF + F(w_{a+1}(\pi, \hat{w}, h + 1))U(\pi, \hat{w}', h + 1)$$

The **Option value** of staying unemployed next period is then $[1 - \lambda'(1 - F(\hat{w}'))]U'$ while the **Stop Value** of accepting next period offers is $\lambda' E_w[W'(w)]$.

### 3.3 Numerical solution method

There are no analytical solutions to the functional equations defining the value functions described in the previous section. Consequently, we employ numerical methods to compute the optimal retirement and search decisions, calculate the value functions and explore the basic properties of the solutions.

Including two continuous states, the value functions are infinite dimensional objects and can only be reproduced in the computer approximately. The use of some numerical approximation method is, then, unavoidable. In particular, we:

- Discretize the continuous state variables when computing the value functions.

  Ie, we build a uniform grid in the State Space $X_N = [\pi_m, \pi_M] \times [\hat{w}_m, \hat{w}_M]$, with $\pi_m$ and $\pi_M$ denoting, respectively, the inferior and superior bounds. $N$ is the number of nodes in the grid. In each iteration $a = \{55, \ldots, 65\}$ we compute:

  $$U_a(x_j, h) j = \{1, \ldots, N\} h = \{1, 2, 3\}$$

- Use linear interpolation whenever a value function is evaluated outside the grid. For example, to compute the reservation wage of an unemployed worker in state $(\pi, \hat{w}, h, a)$ we have to evaluate $W_a(w, \hat{w})$ for any value of $w$ (and not just $w \in \{\pi_m, \ldots, \pi_M\}$). We also have to repeatedly evaluate the future value of staying unemployed with pension rights that do not exactly match the values in the grid.

  We use linear interpolation because, although is more time consuming than other higher order approximation schemes, it guarantees that the shape of the value function is preserved.
4 Calibration

As the model can not be solved analytically, the properties of the solutions must be established through calibrated simulations. In this section we proceed in two steps: (1) we choose a few illustrative cases to uncover the most relevant predictions of the model and (2) check the robustness of the findings through extensive sensitivity simulations. Note that we ultimately hope to estimate some of the parameters of the model.

Our Basic Case is characterized by the following functional forms and parameters:

- **Institutions**

  The parameters describing the pension and unemployment schemes are set to reproduce their direct empirical counterparts as of 2002. Retirement pensions are, then, first available at the age of 60, with an annual early-retirement penalty of 7.5% of the accrued pension rights. \( \hat{w} \) is, in turn, computed as a moving average of the latest 15 years immediately before retirement. The full pension is granted at the normal retirement age of 65. The value of the minimum and maximum pensions are, respectively 6.5 and 23.8 (thousand of current Euros).

  Unemployed workers receive 65% of their previous wages as benefits during the first year (and average of the 70% currently being provided in the first six months, and the 60% thereafter till a maximum of two years). This figure is then reduced to 60% in the second year and just 75% of the minimum contribution (6.2k) in subsequent years. This proportionality is broken by the minimum subsidy just mentioned and a ceiling of two times\(^{19} \) the minimum contribution (12.4 thousand Euros).

- **Preferences**

  Our baseline individual is characterized by pretty standard preference parameter values. We assume him/her to have a high degree of patience, discounting the future by an annual 1 percent (\( \beta = 0.9901 \)). Recall that the presence of life uncertainty increases the effective discount factor of future utility flows in a non-linear fashion (with much more heavy discounting at very advanced ages). Time impatience is important for our purposes because it will lead to earlier retirement.\(^{20} \)

  The relative valuation of leisure when retired/searching/in job is another obvious determinant of retirement and search behavior. We take as our benchmark case that where retirement boosts the utility from income by a 100% (\( \nu(E) = 0 \) vs. \( \nu(R) = 1 \)) (ie, an euro of income is twice as valuable when retired than when working). For the unemployment state, in contrast, we assume an overall negative valuation in comparison with working (\( \nu(E) = 0 \) vs \( \nu(U) = -1/4 \)). This reflects the costs of search and the stigma effect of being unemployed.

\(^{19}\)A compromise between a figure of 2.25 for those with more than two descendants and 1.75 otherwise.

\(^{20}\)The main incentive to stay active till the normal retirement age of 65 is the granting of an 7.5% increase in the value of the pension for each year that retirement is postponed after 60. Very time impatient individuals will not be sensitive to this incentive.
Risk-taking behavior also plays an important role in the search dimension of our problem. Very risk-averse individuals will be wary of rejecting low-quality job offers, as they will be very concerned with the possibility of getting a worse offer (or none at all) in the future. In our baseline case we assume a relatively low degree of risk aversion: $\eta = 0.25$ (implying a CRRA value of 0.75).\(^{21}\)

 Needless to say, there should be a substantial degree of heterogeneity in the distribution of the three properties above among the population. Heterogeneity in the discount factor is briefly considered at the the end of the paper.

**Job-wage offers and dismissal rate**

The properties of the job offers received by the unemployed are largely unobserved and should in principle be estimated jointly with the other unobserved parameters of the model. We insist once more that the calibration provided here is just for illustrative purposes.

Based on the information provided by the Encuesta de Estructura Salarial (EES, 2002) we focus on the subsample of male, blue collar workers age 50 to 65 just employed (without any tenure in the observed job). The size of the wage offers, given this data, are assumed to be lognormally distributed ($F = \text{LogN}(\mu, \sigma)$), with annual mean $\mu = 14.6$ thousand Euros, and standard deviation $\sigma = 6$ thousand Euros.

The separation rate is obtained from the full MCVL sample. We find a 6.4% annual overall rate for blue collar men in the relevant age range. This is, then, the value assigned to our $\delta$ parameter.

Finally, the rate of arrival of job offers is assumed to be linearly dependent on age and unemployment duration:

$$\lambda_{a,1} = \lambda^1_0 - \lambda^1_1(a - 55) \quad \lambda^1_0 = 25\% \quad \lambda^1_1 = 10\%$$

$$\lambda^h_a = \kappa_h \lambda^1_a \quad \kappa_h = \kappa^{h-1} \quad \kappa = 1/2$$

In words, the arrival rate decreases with age, starting at 25% at the age of 50 and losing 1.5 percent per year till the age of 65, when we assume no further offers arrive. The “depreciation” rate implied by the duration of the unemployment spell is much higher. We assume a 50% reduction in the rate of arrival of job offers after staying unemployed for one year (ie, when $h$ increases from 1 to 2). A further 50% decreased is applied if the individual stays unemployed for a second year. It remains constant thereafter.

**Survival process**

We approximately reproduce the INE mortality curve in 2002. We model the mortality hazard rate with a Gompertz distribution ($hm(a) = \alpha_0 e^{\alpha_1 a}$). The estimated parameters

---

\(^{21}\)The rationale for this figure comes from two sources: (1) the available evidence on the structural econometric analysis of retirement- including our own work- and (2) the relatively low rate of acceptance of job offers in the relevant age range.
values are $\alpha_0 = 1.26 \times 10^{-5}$, $\alpha_1 = 0.106$. They imply an annual conditional mortality rate of 1.3% for individuals in their 60s, of 2.8% for individuals in the 70s and 3.9% in the 80s or older.

5 Results

We explore the basic predictions of the model in two ways. We first discuss the details of the optimal retirement and search behavior conditional on the individual state in section 5.1. Alternatively, in section 5.2 we undertake an aggregate simulation and compare the result to the broad empirical trends discussed in section 2.1.

5.1 Theoretical predictions

In this section we summarize the basic properties of the optimal “policy functions” of the model, ie. the optimal retirement and job-acceptance rules of unemployed workers conditional on their age, previous wage, pension rights and duration in unemployment. We start by reviewing the results on the retirement front, and move into job acceptance decisions in the following section. The reader should, in any case, bear in mind that retirement and search behavior interact with each other and cannot be properly understood in isolation.

5.1.1 Unemployed Retirement behavior

Table 4 summarizes the optimal retirement behaviour of our baseline worker. The matrix on the top left corner indicates whether it is optimal to stay active (value 1 in the table) or to retire (red zero) for each combination of previous wage $\pi$ -increasing by row- and pension rights $\hat{w}$ - increasing by column-, at the age of 64 for an individual with less than one year in unemployment ($h=1$). The matrix on the right has a similar layout, but for individuals with more than two years in unemployment ($h=3$). The matrices below this top row correspond to the optimal behavior at earlier ages.

As unemployment does not seem to be a particularly enjoyable state, it seems convenient to start the inquiry by discussing what forces may retain an unemployed worker in the labor market at such advanced ages.\textsuperscript{22} The probability that a jobless worker would stay active are larger (1) the higher the current income; (2) the lower the opportunity cost of the foregone pension (3) the higher the future pension gain by delaying retirement and (4) the higher the chances of getting a job offer in the future (the option value of staying unemployed). This general pattern should be observed at any state.\textsuperscript{23} This means that:

- For each age, $a$, and unemployment duration, $h$, we should expect more red zeroes in Table 4 as we move bottom-up by rows (towards lower forgone pensions) and from right

\textsuperscript{22}For an employed worker, the default “business as usual” option may be to stay working, keeping a high income and sacrificing the potential excess leisure. A priori, things should be very different for unemployed workers, as they have small current income and not very high chances (in Spain) of returning to the labor market.

\textsuperscript{23}These conditions follow from the standard marginal conditions characterizing optimal retirement. See Jiménez Martín and Sánchez Martín (2005) for an example.
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Table 4: Optimal retirement behaviour. 1= stay active; 0= leave the labor force
to left by columns (towards lower current income, as lower \( \pi \) imply lower unemployment benefits). Forces (1) and (2) in the previous enumeration are the main driving forces behind these results. But (3) also plays a small role, as lower previous wages \( \pi \) (and to a lesser extend higher \( \hat{w} \)) imply lower increases in the future pensions brought about by delaying retirement. An exception in the general pattern occurs for workers with very low pension rights, as they may qualify for minimum pensions. This increases the incentives for immediate retirement considerably, as minimum pensions eliminates any further increase in the pension benefit that a worker may get by postponing retirement to a later age. They also increase the opportunity cost of the forgone pensions.

- Ceteris paribus (ie, fixing age, previous wages and pension rights), we should see more retirement for workers with higher duration in unemployment. Graphically, we should see more red zeroes in the right column matrices of Table 4 than in their left column counterparts. This is a consequence of lower current income \( (b \) is decreasing with \( h) \) and lower chances of receiving a job offer in the future \( (\lambda \) is decreasing in \( h) \).

- Finally, with fixed previous wage, pension rights and duration, we should expect more retirement the older the worker (ie, we should see more red zeroes as we move from the matrices on the bottom rows of Table 4 to those on the top rows). Three processes contribute to create a strong dependence of retirement on age. First, the older the worker, the smaller the early retirement penalty and, therefore, the higher the opportunity cost of the foregone pension. Second, the older the worker, the lower the Option Value of staying unemployed (as both the potential number of remaining job offers and the probability attached to each of them go down with age). Last but not least, the higher pensions rights mentioned before imply lower chances of qualifying for the minimum pensions.

5.1.2 Search behavior

A transition from unemployment to a new job is only observed when (1) a wage offer arrives and (2) the offer is acceptable for the unemployed, even when considering the alternative of non participation. In this paper we have little to say about the first process: we take job offers as entirely exogenous and limit ourselves to try to calibrate (estimate in future versions of the paper) them in the best possible way. We concentrate, then, in the behavioral component of the transition: the decision to accept or reject any particular offer. As usual in this literature, reservation wages summarize the behavioral content of this decision. In this section we discuss how the reservation wages changes depending on the individual circumstances by considering four progressively richer environments. Proceeding in this way makes it simpler to identify the origin of the different findings obtained. Thus, we explore four institutional settings in turn:

ST1 Setting 1: Fixed retirement with linear utility and without minimum pensions

ST2 Setting 2: Free retirement for the unemployed, without minimum pensions

ST3 Setting 3: Free retirement for the unemployed, with minimum pensions
ST4 Setting 4: Free retirement for all (General case)

ST1: Optimal search with fixed retirement and linear utility in absence of minimum pensions

Consider an economy where all people in the labor force retire at, say, the normal retirement age $N$. Furthermore, assume that minimum pensions are not available for low income workers and that utility is linear ($\eta = 1$). This simplifications allow for a very clear characterization of some basic properties of reservation wages, as closed-form analytical expressions for $w$ are possible in this simplified environment.

Start with an employed worker of 64 years of age. The value of working at that age, $W_{64}(w, \hat{w})$, is simply $W_{64} = w + \beta_{65} A_{65}^T B_{65}(w, \hat{w})$ with future pension $B_{65} = \hat{w} (\frac{D-1}{D}) + w (\frac{1}{D})$. Therefore, the value function is linear in the previous wage and the accrued pension rights:

$$W_{64}(w, \hat{w}) = w \mathcal{F}_{64} + \hat{w} \mathcal{F}'_{64}$$

Proceeding in a similar fashion we can compute the value of staying unemployed at 64 (assuming $h=1$), $U_{64}(\pi, \hat{w}) = b_1(\pi) + \beta_{65} A_{65}^T B_{65}(\cot(\pi), \hat{w})$, with $\cot(\pi)$ representing the contributions paid by the Employment Office to the Pension System and with future pension $B_{65} = \hat{w} (\frac{D-1}{D}) + \cot(\pi) (\frac{1}{D})$.24 Again, the corresponding value function is linear:

$$U_{64}(\pi, \hat{w}) = \pi \mathcal{G}_{64} + \hat{w} \mathcal{G}'_{64}$$

Obtaining the reservation wage, $w_{64}(\pi, \hat{w})$, is then straightforward:

$$W_{64}(w, \hat{w}) = U_{64}(\pi, \hat{w}) \iff w \mathcal{F}_{64} + \hat{w} \mathcal{F}'_{64} = \pi \mathcal{G}_{64} + \hat{w} \mathcal{G}'_{64} \iff w = (\mathcal{G}_{64}/\mathcal{F}_{64}) \pi$$

Reservation wages at the age of 64 are linear functions of $\pi$. Similarly straightforward calculations at previous ages lead to the following lemma:

**Lemma 1** In a setting with equal retirement ages for employees and unemployed, linear utility and absent minimum pensions (ie, Setting 1), reservation wages conform to the following analytical expressions:

$$w_{a}(\pi, \hat{w}, h) = \begin{cases} (\mathcal{G}^a_0/\mathcal{F}^a_0) \pi & \text{if } h < 3 \text{ and } b(\pi) < \bar{b} \\ \mathcal{G}^a / \mathcal{F}^a & \text{otherwise} \end{cases}$$

implying that, in all cases, reservation wages are (1) independent of accrued pension rights and (2) decreasing in age. For long term unemployed or for those qualifying for the maximum

---

24To simplify notation, we assume $l(R) = 0$. Nothing of substance is missed by incorporating this assumption in this section.
pension, reservation wages are also (3) independent of \( \pi \) and independent of the duration of the unemployment spell. For all other unemployed, reservation wages are (3') increasing in \( \pi \) and decreasing in \( h \).

Proof: TBA

Decreasing reservation wages with age and duration would implied increasing reemployment hazards in both dimensions, a prediction clearly at odds with the empirical evidence. However, a decreasing rate of arrival of job offers (in both age and duration) is enough to square the model predictions with reality. The discrepancies regarding the other two state variables are, however, more difficult to handle. The independence of reservation wages from pension rights is indeed surprising. It reflects the fact that higher pension rights have the same impact on the value functions of employees and unemployed workers (provided they are otherwise identical). It signals that most of the empirical variation in re-employment hazard conditional on \( \hat{w} \) (recall the left-bottom panel of Figure 3) is induced either by variation in retirement or by the minimum pensions. In contrast, the simple model tends to magnify the role of previous wages, whose empirical relevance (right-bottom panel of Figure 3) seems to be pretty small.

ST2: The impact of different retirement ages

When unemployed individuals are allowed to optimize their retirement behavior (but still without minimum pensions) matters change very significantly. In this context pension rights became important while previous wages play a more mute role, which is in much better agreement with the empirical evidence. Unfortunately, we cannot derive general analytical expressions in this setting, although closed-form expressions are still possible once we know the optimal retirement age. This is enough to show the intuition underlying the main result of this section, summarized in the next lemma:

**Lemma 2** Reservation wages are increasing in the accrued pension rights for those unemployed workers whose employed counterparts (employees of the same age and pension rights) retire from the labor force at a more advanced age.

Again, we illustrate this result with the help of an example (with \( \eta = 1 \) just for analytical convenience). Consider again an employed worker of 64 years of age and an unemployed one of the same age. Assume again that the employee retires at 65, but now suppose that the optimal decision for the unemployed in absence of an acceptable job offer is to retire at 64.

In that case, the relevant value function of the unemployed would be: \( U^{64}(\pi, \hat{w}) = B_{64}(\hat{w}) + \beta_{65} A_{65}^T B_{64}(\hat{w}) \) with pension \( B_{64} = \mu_{64} \hat{w} \). The linearity of the value function is preserved:

\[
U(\pi, \hat{w}) = \pi G^{64}_0 + \hat{w} G^{64}_1
\]

Again, we can find an analytical expression for the reservation wage:

\[
W_{64}(\bar{w}, \hat{w}) = U_{64}(\pi, \hat{w}) \leftrightarrow \bar{w} F^{64}_0 + \hat{w} F^{64}_1 = \hat{w} G^{64}_1 \Rightarrow \bar{w} = \frac{1}{F^{64}_0} [G^{64}_1 - F^{64}_1]
\]
The reservation wage is now independent of $\pi$ and increasing in the size of the accumulated pension rights.\textsuperscript{25} The intuition is that higher pension rights increase the value of being in employment/unemployment differently: the impact is stronger on the unemployed (who start enjoying the pension earlier). Larger wage offers are then needed to keep those unemployed workers active in the labor market.

Our simulation predicts that (as assumed in the lemma) employees tend to retire slightly later than their unemployed counterparts. In that situation we find that the reservation wages increase with $\hat{w}$ and, consequently, re-employment hazards decrease with pension rights. In absence of minimum pensions this phenomenon should be observed at advanced ages, close to the normal retirement age (as the incidence of early retirement should be very limited). This can be appreciated in the top panels of Figure 5. Note that the model is now providing a “ranking” of re-employment hazard by pension rights qualitatively similar to that observed in the data.

\textsuperscript{25}This follows if $(G_{64}^{64} - F_{64}^{64} > 0 \Leftrightarrow \mu_{64} + \beta_{65} A_{65} [\mu_{64} - \frac{D}{D}] > 0)$, which is the empirically relevant case.
ST3: The impact of minimum pensions

With the inclusion of minimum pensions the incidence of early retirement increases very substantially for unemployed with (1) low accrued pension rights and (2) low previous wages. Consequently, the mechanism described in the previous paragraph becomes reinforced and its effects become more widespread among low income unemployed workers. Just for illustration, consider again our example of an aged-64 worker, and assume that the minimum pension $B$ is binding in either case (retirement at 64 or 65). As before, suppose delayed retirement in case of employment. The value function will simplify to:

$$W_{64} = w + \beta_{65} A_{65}^T B \\ U_{64}^p = B + \beta_{65} A_{65}^T B$$

It is trivial, then, that the minimum pension is also the reservation wage for these workers. It follows that a generous minimum pension scheme makes the re-employment of low income workers harder: it increases their reservation wages and reduce the corresponding re-employment hazard. The left-bottom panel of Figure 5 illustrate the consequences. At early ages, unemployed workers with low pension rights exhibit lower transition rates than workers in analogous states but with higher pension rights. This “ranking” reverses itself when people approach the normal retirement age. This finding remains after allowing for the optimal retirement of employees, ST4, (right-bottom panel of Figure 5).

Thus, our simple model is able to reproduce the somewhat puzzling stylized fact (S32) of section 2.1: for workers with high pension rights (and short unemployment spells) reemployment hazards are simultaneously large at early ages and low at ages close to the normal retirement age.

Summing up: reservation wages and re-employment probabilities

Figures 6 and 7 provide an overall view of the properties of the reservation wages predicted by the theory and their translation into re-employment probabilities. The most outstanding features are as follows:

- **Pension rights:** At the age of 64 (top-left panel of Figure 6) reservation wages are insensitive to the size of the pension rights for workers with high previous wages. In contrast, they do vary with $\hat{\pi}$ for workers with low previous wage $\pi$. This qualitative difference is due to the different incidence of retirement (higher for the former and lower for the latter), which trigger the effects described in Section 5.1.2. For workers with low $\pi$ the reservation wages are first decreasing in the pension rights and then increasing. The rationalize can be found in the driving factor of the retirement decision: for unemployed with low $\hat{\pi}$, minimum pensions increase the reservation wage (as describe in Section 5.1.2). At earlier ages (left-intermediate and bottom panels of Figure 6) we see that the difference smooths out, with a slightly increasing pattern becoming predominant.

- **Previous wages:** The impact of different retirement ages is also visible in this dimension. For high previous wages the incidence of retirement among the unemployed
Figure 6: Optimal reservation wages $w$ conditional on our state variables at the age of 64 (Top row), 60 (middle row) and 56 (bottom row). The left column display $w$ as a function of pension rights for high (red dash line) and low (blue continuos line) levels of $\pi$. The central column display $w$ as a function of $\pi$ for high (red) and low (blue) levels of pension rights. The right column is similar $w$ as a function of $\pi$ but for the three possible durations $h$.

Figure 7: Top panels: Optimal reservation wages $w$ by age for two levels of pensions rights (left panel); for two levels of the previous wage (central panel) and for the three durations in unemployment (right panel). In all cases the high level is represented with a red-dash line and the low with a blue-continuous line. Bottom panels: similar layout for reemployment hazard.
is smaller, making its behavior more alike to that of the employee. This reinforces the importance of $\pi$ in the reservation wage. This changes drastically when the optimal decision is to retire (which happens for workers with low previous wages). In that case, $\pi$ is essentially insignificant for the job-taking behaviour. A similar pattern is obtained at previous ages.

- **Duration:** The pattern with duration reflects the importance of retirement once more. For low $\pi$, retirement is widespread independently of the length of the unemployment spell. In these circumstances the reservation wages are not affected by $h$ or $\pi$. But for higher $\pi$ we observe a lower incidence of retirement for workers with shorter duration. In that case, the unemployed with higher $h$ are less selective when accepting job offers.

- **Age:** The top panels of Figure 7 summarize the evolution of the reservation wages by age. We do not see much variation, with the exception of ages approaching the normal retirement age. The bottom panels show the effective re-employment hazard by age, serving as illustration of the interaction of the optimal reservation wages with our assumptions regarding the arrival of job offers.

### 5.2 Aggregate model predictions

In this section we estimate the population distribution of our state variables and combine that information with our theoretical policy rules to generate a set of calibrated predictions of the aggregate transitions flows at one particular point in time. The main advantage of this method is its simplicity, as these simulated statistics have straightforward empirical counterparts (discussed in section 2.1).

#### 5.2.1 Aggregate simulations: design

The basic idea is simply to compare the model predictions in term of transition flows with their empirical counterparts, taking the stocks as given.\textsuperscript{26} To implement this simple concept we undertake a sort of Monte-Carlo experiment involving the following steps:

1. Create a large sample of individuals reproducing the empirical distribution of labor states by $\{a, \pi, \hat{w}, h\}$:

   $$\mu(\pi, \hat{w}, a, h) \quad a \in \{50, \ldots, 65\} \quad h \in \{1, \ldots, 3\} \quad \pi \in [\pi_m, \pi_M] \quad \hat{w} \in [\hat{w}_m, \hat{w}_M]$$

2. Simulate the rate of arrival of job offers and the size of the wage proposed, according with the parametric functions included in our model.

\textsuperscript{26}Note that the model also generates predictions in terms of stocks, but we abstract from them at this stage.
3. Let the individual in the simulated sample react to the job offers (and to the alternative to retire from the labor force) in accordance with the model policy functions. Keep records of the job acceptances and retirement decisions.

4. Aggregate the decisions, compute the implicit re-employment and retirement hazards and compare with empirical counterparts.

5.2.2 Aggregate predictions in our calibrated example

Figure 8 plots the re-employment hazards predicted by the model for the empirical sample. It is readily apparent that:

- Reemployment hazard is decreasing with duration, in broad agreement with stylized fact S1;
- Reemployment hazard is decreasing with age, in agreement with stylized fact S2
- Pension rights are predicted to have a small impact. Short-term unemployed workers with high \( \hat{w} \) are predicted to have somewhat smaller re-employment hazard than their analogous with smaller pension rights, especially at ages close to the Normal Retirement Age. That is the only sizable difference. The model’s predictions are, then, at odds with the stylized facts S31 (specially at very early ages) and S32 (for \( h=2 \), pension rights make even an smaller difference than for \( h=1 \)). Note, however that the failure to reproduce S31 is at least partially due to composition effects in the sample, as we showed in section 5.1.2 that the optimal policy functions of the model do reproduce S31).

The failure to reproduce S31/S32 may point to composition effects and self-selection generated from un-observables. If, for example, the value of leisure is unequally distributed in the sample, we may conjecture that the proportion of workers with low \( \nu \) should be higher among those with high pension rights (which are a proxy for permanent income). This would explain the higher re-employment hazard for this group when \( h=1 \). It is probable that some kind of signalling is also present (as high \( \hat{w} \) reveals a higher \( \nu \), it may imply larger wage offers for this group). This would make the screening process more effective for those workers. This latter effect, combined with self-selection would explain why the retirement hazard for longer-term unemployed is lower for those with high pension rights (a more successful screening should produce a larger proportion of people with high \( \nu \) among those with high \( \hat{w} \) that do not accept job offers).

The optimal retirement generated by the model in the empirical sample is showed in Figure 9, which reproduces the hazard out of the labor force by age conditional on the value take by the other state variables of our model (duration, pension rights and previous wages). We highlight the following aspects:

\[27\] Note that it is reasonable to expect a reduction in this hazard at more advanced ages, for the reasons discussed in section 5.1.2
Figure 8: Theoretical prediction, Baseline model. Transition from unemployment to employment: hazard by duration and age; hazard by age, duration \((h)\) and accrued pension rights.

Figure 9: Theoretical prediction, Baseline model. Transition from Unemployment to Retirement: hazard by age, duration, accrued pension rights and immediately previous wage.
• The model reproduces the age-65 spike (top-left panel of Figure 9). The absence of economic incentives to continue working after the normal retirement age seem to be enough to rationalize the very high retirement hazard observed at that age (ie, the stylized fact R1).

• There is a significant amount of retirement at 60 but the model is in troubles to generate a “peak” at 60 (stylized fact R2).

A peak in retirement at the exact age of 60 demands the existence of a mechanism that push workers out at 60 but not at the immediately ensuing ages. Two such mechanisms have been mainly discussed in the literature: minimum pensions and borrowing constraints. The reasons why minimum pensions induce early retirement were discussed in Section 5.1, while the role of credit constrains was explained in Section 2.1.2. In that section we also noticed that there was little evidence supporting a very strong role for minimum pensions in our data. It should not come as a big surprise, then, that a model which only includes minimum pensions has a hard time in mimicking the stylized facts.

The problem is quantitative in that a combination of higher minimum pensions and a higher degree of impatience can certainly deliver an age 60 peak compatible with the data. But, of course, this is not a legitimate way of proceeding, as the value of minimum pensions is not a free parameter of the model. It seems fair to conclude at this point that the failure to reproduce the age-60 peak is a qualitative feature of this specification of the model (in the next section we show how a simple extension of the model can substantially improves its performance in this dimension).

Note, in any case, that the model correctly predicts some aspects of the composition of the early retirees: (1) that long-term unemployed make the bulk of the early retirees (top right panel of Figure 9), in accordance with stylized fact R21; and (2) that workers with both low and high pension rights retire in large numbers at that age (left bottom panel of 9. The main (and fundamental) problem is the absence of workers with average pension rights among the early retirees.

• The incidence of intermediate retirement is decidedly increasing with age.

This is strongly at odds with the evidence (recall stylized fact R3). This also may appear as a quantitative failure at first sight (and so potentially amendable after, eg. the estimation of the model). Unfortunately, after a systematic experimentation with the model we come to the conclusion that only unrealistically low discount factors may reconcile the model predictions and the empirical facts. The failure is, then, qualitative and suggests that some important additional factor must be retaining workers over and above the incentive mechanisms present in the model.

This conjecture is confirmed when exploring the performance of the model in other dimensions. It is successful in predicting higher retirement for those with higher pension rights (with the exception of the very low retirement hazard predicted near 60 for h=2). It is also right when predicting higher retirement for h=2 than for h=1. However, it does not
reproduce the strikingly low hazard observed in the data for $h=3$. It makes sense to try to qualify for a full pension by waiting till 65, but the intermediate income provided by the unemployment benefit is so small in that case that the model simply cannot rationalize these choices. Some other mechanism should be called in to explain this behavior.\footnote{Some of the workers may be wealth enough to afford unemployment, but this is unlikely in a sample of blue collar workers. Some of them may be complementing their incomes with family support or working in the black economy.}

- Wage in the previous employment spell, \( \pi \), does matter for an optimal timing of retirement. In contrast with the evidence (Stylized fact R4), the model predicts larger flows out of the labor force for those with low previous wages (and so, unemployment income).

### 5.2.3 A simulation with unobserved heterogeneity

Some of the problems described in the previous section may be attributed to our assumption of the existence of just one set of preference parameters shared by the entire population. In particular, the observation of high exit rates into retirement at the age of 60 with independence of the size of the accrued pension rights strongly points to some heterogeneity in preferences (or health, previous shocks, ...), resulting in an underlying distribution of the propensity to early retire across the population. This is an important element on the explanation of the peak in early retirement at 60 in the “credit constraint” interpretation of this phenomenon (the peak is formed by workers whose optimal choice would be to leave earlier, but that lack the financial resources to sustain consumption in the intermediate years).

In this section we explore this possibility further, by assuming an unobservable distribution of one of the preference parameters across the population. Such a model should still be considered as fully “standard” and may throw some light on the “failures” detected in the previous section.

As a first experiment we focus on early retirement and propose the coexistence in the population of two groups of workers with different discount factors.\footnote{Again, we only propose a calibrated example to explore the qualitative properties of the solution. This means that we propose a “reasonable” value for the \( \beta \) in each group (we actually choose somewhat extreme values to make the effects of the choice more apparent). In the next section we discuss a simple revealed preference method designed to combine the data and the model to impute the distribution of each group in the sample. We then proceed to discuss the simulated results in detail.}

Revealed preference method

The task consists of imputing the proportion of workers of each type (high/low discount rate, implying a low/high degree of patience), for each combination of our state variables (age, duration in unemployment, pension rights and previous wage). At this stage (before we embark ourselves in the formal estimation of the model) it will be enough for our purposes to make those imputations by combining (in a consistent way) the empirical retirement behavior and the theoretical predictions of the model (conditional on the assumed value of the discount factor).

We proceed as follows:
Figure 10: Theoretical prediction, Baseline model. Transition from unemployment to employment: hazard by duration and age; hazard by age, duration ($h$) and accrued pension rights.

Figure 11: Theoretical prediction, Baseline model. Transition from Unemployment to Retirement: hazard by age, duration, accrued pension rights and immediately previous wage.
1. We discretize the space of our two continuous state variables, pension rights and previous wages; i.e., we build a uniform grid $X_N^G$ in the State Space $X = [\pi_m, \pi_M] \times [\hat{\omega}_m, \hat{\omega}_M]$ (recall Section 3.3).

2. For each combination of age, duration in unemployment and each cell in the grid $X_N^G$ we:

   (a) Compute $\hat{p}_i$ the average empirical retirement hazard among the sample observations lying within the limits of the cell.

   (b) Compute the optimal retirement behavior predicted by the model for each type (Low/High).

   (c) Impute the proportion of type-L individuals in the cell $\theta_{ah,i}$ as follows. If the theoretical prediction for type-L is retirement while for type-H is staying unemployed, make $\theta_{ah,i} = \hat{p}_i$; Conversely, if the theoretical prediction for type-H is retirement while for type-L is staying unemployed, make $\theta_{ah,i} = 1 - \hat{p}_i$; Finally, if the predictions for both types coincide then there is no revelation of preferences in the retirement process and we simply impute a fifty-fifty distribution.

**Model with heterogeneous agents: general qualitative patterns**

In this section we discuss the aggregate predictions of the model including two groups of individuals differing in their discount factor. The values assigned to the annual discount factors of the two types of agents are 10% and -1%. As with our base “calibration” in Section 4, these values are purely for illustrative purposes. We have opted for relatively extreme values for a more clear presentation of the properties of the solution. For the patient individual we have selected the lowest value we are aware of among those reported in academic journals (and coming from the estimation of life cycle models). The reference is (Hurd (1989)). Note that life uncertainty should be added to the pure time discount to get the effective discount applied by the individual to the future utility flows. This means positive discounting after only a few years. On the other hand, we find that results are not very sensitive to the value assumed for the impatient individual. Assuming a 10% discount in this case we estimate the average discount factor in the sample to be 3.9%.

The simulation results are presented in two set of figures. Firstly, Figures 10 and 11 illustrate the basic qualitative features of the solutions in a similar way as their analogous (Figures 8 and 9) in Section 5.2.2. Besides, Figures 12, 13 and 14 allow for a comparison of the levels of the predicted hazards and their empirical counterparts. This provides a first quantitative evaluation of the model and provide some additional insights about its main properties.

We can appreciate some significant differences with the predictions of the model with homogeneous agents:

- The model generates a sizable age-60 spike. As before, workers with long duration in unemployment makes the single largest contribution to the spike, but now (as the left-
bottom panel of Figure 11 makes clear), workers endowed with different levels of pension rights also make sizable contributions to the early-retirement flows. Note, however, that the role played by workers with low $\hat{w}$ (i.e., minimum pensions) is still slightly excessive. This is best appreciated in the top panels of Figure 14. The model also falls short in the prediction of early retirement by short duration unemployed (probably reflecting all the other factors not explicitly included in the model like health problems, family matters, etc). Overall, the reproduction of early retirement behavior seems satisfactory.

- In sharp contrast, the model still fails to deliver a steady rate of exit in the intermediate ages (left panel of Figure 12). A quick inspection of Figures 13 and 14 reveals that the failure is quite general. It is not confined to some particular range of pension rights or duration in unemployment (although it is specially high for the long term unemployed, $h=3$).

- The predictions regarding the level of the previous wage $\pi$ are now much more in line with the data (bottom panel of Figures 11 and 14).

- There are very small differences regarding search behavior. The only reportable qualitative difference seems to be a somewhat higher sensitivity to the size of the pension rights at advanced ages. On the quantitative ground, the model performance is good (right panel of Figure 12).

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Footnote 30: The bottom panels of Figure 13 show the adjustment by age and duration. The model performs well, maybe with the exception of the (strange) high re-employment hazard exhibited by long-term unemployed older that 60. This trait seems to be specific of workers in the construction sector.
Figure 13: Comparison of the theoretical predictions (model with heterogeneous discount factor) and the data: retirement and re-employment hazard by age and duration in unemployment.

Figure 14: Comparison of the theoretical predictions (model with heterogeneous discount factor) and the data: retirement and hazard by age, pension rights and wage in the previous employment spell.
Allowing for some differences derived from the changes in the parameters values used, we think we can safely conclude that the analysis in the previous section provides a clear illustration of the aggregate patterns generated by rational behavior by the unemployed. The match with the data is generally good, but there is one aspect that stand out as problematic: there seem to be a significant group of unemployed workers whose exit rate (into retirement) is significantly smaller than what would be optimal given the incentives provided by the pension and unemployment regulations. This is specially clear at ages close to the normal retirement age. \[31\]

6 Conclusions

To be added

References


Gustman, Alan and Thomas Steinmeier (2002). The social security early retirement age in a structural model of retirement and wealth. Working paper 9183, NBER.


\[31\] At this point we can only conjecture about the identity of the possible various groups of people causing the discrepancy. A small part may be workers with extra resources (wealth, family support, black economy); a small part may reflect measurement error or the communication of deliberately misleading information to the Social Security. One possibility is that part of the sample is made of workers that use unemployment as a “voluntarily” way out the labor force. This may be modeled by assuming \(\nu(U) = \nu(R)\) (the unemployed enjoys the extra-utility of being retired, mostly because it avoids the costs of search) It is also reasonable to assume that the individual receives no job offers. We are undertaking preliminary simulations along these lines at the moment.

