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ABSTRACT

We study the aggregate implications of sectoral shocks in a multi-sector New Keynesian model featuring sectoral heterogeneity in price stickiness, sector size, and input-output linkages. We calibrate a 341 sector version of the model to the United States. Both theoretically and empirically, sectoral heterogeneity in price rigidity (i) generates sizable GDP volatility from sectoral shocks, (ii) amplifies both the "granular" and the "network" effects, (iii) alters the identity and relative contributions of the most important sectors for aggregate fluctuations, (iv) can change the sign of fluctuations, (v) invalidates the Hulten Theorem, and (vi) generates a frictional origin of aggregate fluctuations.

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I Introduction

Identifying aggregate shocks that drive business cycles might be difficult (Cochrane (1994)). A recent literature advances the possibility that shocks at the firm or sector level may be the origin of aggregate fluctuations. This view stands in contrast to the "diversification argument" of Lucas (1977), which conjectures idiosyncratic shocks at a highly disaggregated level average out.[1] In contrast, Gabaix (2011) argues the diversification argument does not readily apply when the firm-size distribution is fat-tailed, which is the empirically relevant case for the United States. Intuitively, shocks to disproportionately large firms matter for aggregate fluctuations, known as the "granular" effect. In a similar vein, Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) focus on sectoral shocks and show that input-output relationships across sectors can mute the diversification argument if measures of sector centrality follow a fat-tailed distribution. They label this channel the "network" channel. Thus, either through a granular or network channel, microeconomic shocks to small numbers of firms or sectors may drive aggregate fluctuations.[2]

Prices are the key transmission mechanism of sectoral technology shocks to the economy. But this prior work assumes prices are flexible which might not be an innocuous assumption. A large literature documents prices might be sticky in the short run and prices for different goods change at different frequencies. In fact, nominal price rigidities are a leading explanation for the real effects of nominal shocks. For these reasons, we study whether and how price rigidity affects the aggregate importance of microeconomic shocks.

To fix ideas, consider a multi-sector economy without linkages across sectors and consider a positive productivity shock to one sector. Marginal costs in this sector decrease and prices will fall in the absence of pricing frictions. But consider what happens if prices do not adjust. Demand for goods remains unchanged, so production remains unchanged. Therefore, regardless of the size of the sector, the contribution of its shocks to aggregate fluctuations is zero (except for some general equilibrium effects).[3] A similar logic applies to production networks. Following a positive productivity shock, a price cut in one sector will propagate downstream by decreasing production costs. This, in turn, will trigger price cuts in other sectors. But, if prices do not change in the shocked sector, marginal costs of downstream firms remain unchanged and there

[1] Dupor (1999) takes a perspective similar to Lucas (1977) and implicitly so does anyone who models aggregate shocks driving aggregate fluctuations.

[2] A fast-growing literature has followed. Some recent examples are Acemoglu, Akcigit, and Kerr (2016); Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017); Atalay (2015); Baqaee (2016); Bigio and La'O (2016); Caliendo, Parro, Rossi-Hansberg, and Sarte (2014); Carvalho and Gabaix (2013); Carvalho and Grassi (2015); Di Giovanni, Levchenko, and Méjean (2014); Di Giovanni, Levchenko, and Méjean (2016); Foerster, Sarte, and Watson (2011); Ozdagli and Weber (2016); Grassi (2017); and Baqaee and Farhi (2017).

[3] First, lower demand for inputs in the shocked sector decreases wages. Second, higher profits of firms in the shocked sector increase household income. However, these effects are small up to a first-order approximation.

is no propagation regardless of the centrality of the shocked sector.

In the data, prices are neither fully rigid nor fully flexible, and substantial heterogeneity of price rigidity exists across sectors in the United States (see Bils and Klenow (2004); Klenow and Kryvtsov (2008); Nakamura and Steinsson (2008)). How does the heterogeneity in nominal price rigidity interact with the granular effect of Gabaix (2011) and the network effect of Acemoglu et al. (2012) in affecting the power of microeconomic shocks to generate sizable aggregate fluctuations? Do price rigidities distort the identity of sectors that are the origin of aggregate fluctuations? Can price rigidity create a "frictional" origin of aggregate fluctuations, conceptually different from the granular or the network origins the literature already describes?

We study these questions in a multi-sector New Keynesian model in which firms produce output using labor and intermediate inputs. Our answers are yes and yes: heterogeneity in price rigidities changes the identity of sectors from which aggregate fluctuations originate, and generates GDP volatility from sectoral shocks independent of the sector-size distribution and network centrality. Our model follows Basu (1995) and Carvalho and Lee (2011), but we make no simplifying assumptions on the steady-state distribution of sectoral size, input-output linkages, and the sectoral distribution of price-setting frictions that we model following Calvo (1983).[4] Sectoral productivity shocks are the only source of variation in our model.

As a first step, we analytically study in a simplified version of our model the distortionary role of price rigidity on the granular and network origins of aggregate fluctuations. Up to a log-linear approximation, GDP is a linear combination of sectoral shocks, and the model nests Gabaix (2011) and Acemoglu et al. (2012) as special cases. When we abstract from intermediate inputs and price stickiness, we recover the granularity effect of Gabaix (2011): the ability of microeconomic shocks to generate aggregate fluctuations depends on the fat-tailedness of the sector-size distribution, which we measure by sector GDP.

But price stickiness introduces three new effects. First, it dampens the level of aggregate volatility originating from any shock, both sector-specific and aggregate. This is a conventional effect in New Keynesian models. Second, a novel relative effect arises: the sectoral distribution of price rigidity distorts the relative importance of sectors for aggregate fluctuations. In particular, a sector is important when it is large, as in Gabaix (2011), and/ or when its prices are much more flexible than most prices in the economy. Consider a scenario in which the sector-size distribution is fat-tailed and size is negatively correlated with price rigidity; that is, larger sectors are more likely to have more flexible prices. In this case, shocks to large sectors become even more

[4]We argue below that our results are most likely stronger in models with state-dependent pricing. Any price-adjustment technology should deliver our key results, as long as price rigidity is heterogeneous. We choose Calvo pricing as an expository tool, and for computational reasons.

important for aggregate volatility than in a frictionless economy, because large sectors are now *effectively* larger. They pass through *relatively* more of a shock. In other words, the distribution of the multipliers mapping sectoral shocks into aggregate fluctuations is more fat-tailed than implied by the pure sector-size distribution alone. The opposite holds if sector size is positively correlated with price rigidity. The diversification argument of Lucas (1977) might even gain bite due to sticky prices, although the conditions necessary for the granular channel hold in a frictionless economy. In short, whenever the ranking of flexible sectors in a sticky-price economy is sufficiently different from the ranking of the most important sectors in an economy with flexible prices, price rigidity will distort the effect of sectoral shocks on aggregate fluctuations. Third, because heterogeneous price rigidity re-weights the importance of sectors for aggregate fluctuations, it may even distort the sign the business cycles and not only generate inertia, as is standard with aggregate shocks.

We reach similar results for the network effect of Acemoglu et al. (2012). We know from Acemoglu et al. (2012), with flexible prices, microeconomic shocks are more important for macroeconomic volatility if the distribution of sector centrality is more fat-tailed: large suppliers of intermediate inputs (first-order interconnection) and/ or large suppliers to large suppliers of intermediate inputs (second-order interconnection) are important for aggregate volatility. With price stickiness, the most flexible sectors among large suppliers of intermediate inputs and/ or the most flexible sectors among large suppliers to the most flexible large intermediate input suppliers are now the most important determinants of aggregate volatility. Thus, the multipliers of sectoral shocks to aggregate volatility may be more or less fat-tailed than the distribution of sector centrality. Heterogeneity in price rigidity invalidates the Hulten (1978) result that holds in Gabaix (2011) and Acemoglu et al. (2012): sector (or firm) total sales is no longer a sufficient statistic for the importance of GDP volatility.

In a second step, we show the quantitative importance of sectoral shocks to drive aggregate fluctuations. We calibrate the model to the input-output tables of the Bureau of Economic Analysis (BEA) at the most disaggregated level and the micro data underlying the Producer Price Index (PPI) from the Bureau of Labor Statistics (BLS). After merging these two datasets, we end up with 341 sectors. We conduct a series of experiments within our 341-sector economy. Price rigidity does indeed substantially affect the importance of microeconomic shocks for aggregate fluctuations. We base the following discussion on relative multipliers, that is, multipliers of sectoral productivity shocks on GDP volatility relative to the multiplier of aggregate productivity shocks on GDP volatility, because the effect of aggregate shocks is not invariant to the distribution of price rigidity, sectoral GDP, and input-output linkages.

In our first experiment, we match sectoral GDP shares but assume equal input-output linkages across sectors. The relative multiplier of sectoral productivity shocks on GDP volatility increases from 11% when prices are flexible to 33.7% when price stickiness across sectors follows the empirical distribution. In the second experiment, we match input-output linkages to the U.S. data but assume equal sector sizes. Now, the relative multiplier increases from 8% with flexible prices to 13.2%. In a third experiment, differences in the frequency of price changes are the only source of heterogeneity across sectors. The relative multiplier of sectoral shocks is now 12.4%, more than twice as large compared to the multiplier in an economy with complete symmetry and equal price stickiness across sectors. This result suggests a "frictional" origin of aggregate fluctuations: heterogeneity in price stickiness alone can generate aggregate fluctuations from sectoral shocks.

Overall, when all three heterogeneities are present, the relative multiplier on GDP volatility is 32%, almost six times larger than in an economy with complete symmetry across sectors. The six-fold increase of the relative multiplier underscores the relevance of microeconomic shocks for aggregate fluctuations, and shows heterogeneities in sector size, input-output structure, and price stickiness are intricately linked and reinforce each other.

But price rigidity does not only contribute to the importance of micro shocks driving aggregate volatility. Differences in price rigidity across sectors have also strong effects on the identity and contribution of sectors driving aggregate fluctuations. For instance, the identity of the two most important sectors for aggregate volatility shifts from "Real Estate," and "Wholesale Trading" with flexible prices to "Oil and Gas Extraction" and "Dairy cattle and milk production" with heterogeneous sticky prices when we only consider network effects. When we also allow for sectoral heterogeneity in sector size, the two most important sectors with flexible prices are "Retail trade" and "Real Estate" but "Monetary authorities and depository credit intermediation" and "Wholesale Trading" with sticky prices.

At an abstract level, our analysis does not only show that the size or centrality of nodes in the network matters for the macro effect of micro shocks, but also the frictions that affect the capacity of nodes to propagate shocks. This point goes beyond sticky prices. Nonetheless, we focus on sticky prices for two reasons. First, prices are the central transmission mechanism of sectoral shocks in production networks. Of course, nodes may be differentially important for GDP fluctuations due to other frictions, such as financial frictions. But pricing frictions are first order because they determine the propagation mechanism through the production networks. They simultaneously affect the transmission of shocks on top of all other frictions by directly influencing demand and supply. Second, price stickiness is a measurable friction at a highly disaggregated level.

The frictional origin of fluctuations also goes beyond production networks in a closed economy; it applies to all networks with heterogeneous effects of frictions across nodes, for example, in international trade networks, financial networks, or social networks. Our work is thus related to an extensive literature that we do not attempt to summarize here; instead, we only highlight the papers that are most closely related below.

A. Literature review

Long and Plosser (1983) pioneer the microeconomic origin of aggregate fluctuations, and Horvath (1998) and Horvath (2000) push this literature forward. Dupor (1999) argues microeconomic shocks matter only due to poor disaggregation. Gabaix (2011) invokes the firm size-distribution, and Acemoglu et al. (2012) assert the sectoral network structure of the economy to document convincingly the importance of microeconomic shocks for macroeconomic fluctuations: under empirically plausible assumptions, microeconomic shocks do matter. Barrot and Sauvagnat (2016), Acemoglu et al. (2016), and Carvalho, Nirei, Saito, and Tahbaz-Salehi (2016) provide empirical evidence for the importance of idiosyncratic shocks for aggregate fluctuations, and Carvalho (2014) synthesizes this literature.

The distortionary role of frictions (and price rigidity in particular) is at the core of the business-cycle literature that conceptualizes aggregate shocks as the driver of aggregate fluctuations, including the New Keynesian literature. However, to the best of our knowledge, no parallel study of the distortionary role of frictions exists when aggregate fluctuations have microeconomic origins. That said, a few recent papers include frictions in their analyses. Baqaee (2016) shows entry and exit of firms coupled with CES preferences may amplify the aggregate effect of microeconomic shocks. Carvalho and Grassi (2015) study the effect of large firms in a quantitative business-cycle model with entry and exit. Bigio and La'O (2016) study the aggregate effects of the tightening of financial frictions in a production network. Despite a different focus, we share our finding with Baqaee (2016) and Bigio and La'O (2016) that the Hulten theorem does not apply in economies with frictions.

Our model shares building blocks with previous work studying pricing frictions in production networks. Basu (1995) shows frictions introduce misallocation resulting in nominal demand shocks looking like aggregate productivity shocks. Carvalho and Lee (2011) develop a New Keynesian model in which firms' prices respond slowly to aggregate shocks and quickly to idiosyncratic shocks, rationalizing the findings in Boivin et al. (2009). We build on their work to answer different questions and relax assumptions regarding the production structure to quantitatively study the interactions of different heterogeneities.

Nakamura and Steinsson (2010), Midrigan (2011), and Alvarez, Le Bihan, and Lippi (2016), among many others, endogenize price rigidity to study monetary non-neutrality in multi-sector menu-cost models. Computational burden and calibration issues make such an approach infeasible in our highly disaggregated model which is why we study the effect of disaggregation on monetary non-neutrality in a multi-sector Calvo model in a companion paper (Pasten, Schoenle, and Weber (2016)). Bouakez, Cardia, and Ruge-Murcia (2014) estimate a Calvo model with production networks, using data for 30 sectors, and find heterogeneous responses of sectoral inflation to a monetary policy shock, but do not study the questions we pose in this paper.

Other recent applications of networks in different areas of macroeconomics and finance are Gofman (2011), who studies how intermediation in over-the-counter markets affects the efficiency of resource allocation, Di Maggio and Tahbaz-Salehi (2015), who study the fragility of the interbank market, Ozdagli and Weber (2016), who show empirically that input-output linkages are a key propagation channel of monetary policy shocks to the stock market, and Kelly, Lustig, and Van Nieuwerburgh (2013), who study the joined dynamics of the firm-size distribution and stock return volatilities. Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016) and Herskovic (2015) study the asset-pricing implications of production networks.

II Model

Our multi-sector model has households supplying labor and demanding goods for final consumption, firms with sticky prices producing varieties of goods with labor and intermediate inputs, and a monetary authority setting nominal interest rates according to a Taylor rule. Sectors are heterogeneous in three dimensions: their final goods production, input-output linkages, and the frequency of price adjustment.

A. Households

The representative household solves

$$\max_{\{C_t, L_{kt}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \sum_{k=1}^{K} g_k \frac{L_{kt}^{1+\varphi}}{1+\varphi} \right),$$

subject to

$$\begin{aligned} \sum_{k=1}^{K} W_{kt} L_{kt} + \sum_{k=1}^{K} \Pi_{kt} + I_{t-1} B_{t-1} - B_t &= P_t^c C_t \\ \sum_{k=1}^{K} L_{kt} &\leq 1, \end{aligned}$$

where C_t and P_t^c are aggregate consumption and aggregate prices, respectively. L_{kt} and W_{kt} are labor employed and wages are paid in sector $k = 1, ..., K$. Households own firms and receive net income, Π_{kt}, as dividends. Bonds, B_{t-1}, pay a nominal gross interest rate of I_{t-1}. Total labor supply is normalized to 1.

Households' demand of final goods, C_t, and goods produced in sector k, C_{kt}, are

$$C_t \equiv \left[\sum_{k=1}^{K} \omega_{ck}^{\frac{1}{\eta}} C_{kt}^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{1}$$

$$C_{kt} \equiv \left[n_k^{-1/\theta} \int_{\Im_k} C_{jkt}^{1-\frac{1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}. \tag{2}$$

A continuum of goods indexed by $j \in [0, 1]$ exists with total measure 1. Each good belongs to one of the K sectors in the economy. Mathematically, the set of goods is partitioned into K subsets $\{\Im_k\}_{k=1}^{K}$ with associated measures $\{n_k\}_{k=1}^{K}$ such that $\sum_{k=1}^{K} n_k = 1$.[5] We allow the elasticity of substitution across sectors η to differ from the elasticity of substitution within sectors θ.

The first key ingredient of our model is the vector of weights $\Omega_c \equiv [\omega_{c1}, ..., \omega_{cK}]$ in equation (1). These weights show up in households' sectoral demand:

$$C_{kt} = \omega_{ck} \left(\frac{P_{kt}}{P_t^c} \right)^{-\eta} C_t. \tag{3}$$

All prices are identical in steady state, so $\omega_{ck} \equiv \frac{C_k}{C}$, where variables without a time subscript are steady-state quantities. In our economy, C_t represent the total production of final goods, that is, GDP. The vector Ω_c represents steady-state sectoral GDP shares satisfying $\Omega_c' \iota = 1$ where ι denotes a column-vector of 1s. Away from the steady state, sectoral GDP shares depend on the gap between sectoral prices and the aggregate price index, P_t^c:

$$P_t^c = \left[\sum_{k=1}^{K} \omega_{ck} P_{kt}^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{4}$$

[5] The sectoral subindex is redundant, but it clarifies exposition. Note we can interpret n_k as the sectoral share in gross output.

We can interpret P_t^c as GDP deflator. Households' demand for goods within a sector is given by

$$C_{jkt} = \frac{1}{n_k}\left(\frac{P_{jkt}}{P_{kt}}\right)^{-\theta} C_{kt} \text{ for } k = 1, ..., K. \tag{5}$$

Goods within a sector share sectoral consumption equally in steady state. Away from the steady state, the demand of goods within a sector is distorted by the gap between a firm's price and the sectoral price, defined as

$$P_{kt} = \left[\frac{1}{n_k}\int_{\Im_k} P_{jkt}^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \text{ for } k = 1, ..., K. \tag{6}$$

The household first-order conditions determine labor supply and the Euler equation:

$$\frac{W_{kt}}{P_t^c} = g_k L_{kt}^{\varphi} C_t^{\sigma} \text{ for all } k, j, \tag{7}$$

$$1 = \mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} I_t \frac{P_t^c}{P_{t+1}^c}\right]. \tag{8}$$

We implicitly assume sectoral segmentation of labor markets, so labor supply in equation (7) holds for a sector-specific wage $\{W_{kt}\}_{k=1}^K$. We choose the parameters $\{g_k\}_{k=1}^K$ to ensure a symmetric steady state across all firms.

B. Firms

A continuum of monopolistically competitive firms exists in the economy operating in different sectors. We index firms by their sector, $k = 1, ...K$, and by $j \in [0, 1]$. The production function is

$$Y_{jkt} = e^{a_{kt}} L_{jkt}^{1-\delta} Z_{jkt}^{\delta}, \tag{9}$$

where a_{kt} is an i.i.d. productivity shock to sector k with $\mathbb{E}[a_{kt}] = 0$ and $\mathbb{V}[a_{kt}] = v^2$ for all k, L_{jkt} is labor, and Z_{jkt} is an aggregator of intermediate inputs:

$$Z_{jkt} \equiv \left[\sum_{k'=1}^{K} \omega_{kk'}^{\frac{1}{\eta}} Z_{jk}\left(k'\right)^{1-\frac{1}{\eta}}\right]^{\frac{\eta}{\eta-1}}. \tag{10}$$

$Z_{kjt}(r)$ is the amount of goods firm j in sector k uses in period t as intermediate inputs from sector r.

The second key ingredient of our model is heterogeneity in aggregator weights $\{\omega_{kk'}\}_{k,k'}$. We denote these weights in matrix notation as Ω, satisfying $\Omega\iota = \iota$. The demand of firm jk for goods produced in sector k' is given by

$$Z_{jkt}\left(k'\right) = \omega_{kk'} \left(\frac{P_{k't}}{P_t^k}\right)^{-\eta} Z_{jkt}. \tag{11}$$

We can interpret $\omega_{kk'}$ as the steady-state share of goods from sector k' in the intermediate input use of sector k, which determines the input-output linkages across sectors in steady state. Away from the steady state, the gap between the price of goods in sector k' and the aggregate price relevant for a firm in sector k, P_t^k distorts input-output linkages:

$$P_t^k = \left[\sum_{k'=1}^{K} \omega_{kk'} P_{k't}^{1-\eta}\right]^{\frac{1}{1-\eta}} \text{ for } k = 1, ..., K. \tag{12}$$

P_t^k uses the sector-specific steady-state input-output linkages to aggregate sectoral prices.

The aggregator $Z_{jk}\left(k'\right)$ gives the demand of firm jk for goods in sector k':

$$Z_{jk}\left(k'\right) \equiv \left[n_{k'}^{-1/\theta} \int_{\Im_{k'}} Z_{jkt}\left(j', k'\right)^{1-\frac{1}{\theta}} dj'\right]^{\frac{\theta}{\theta-1}}. \tag{13}$$

Firm jk's demand for an arbitrary good j' from sector k' is

$$Z_{jkt}\left(j', k'\right) = \frac{1}{n_{k'}} \left(\frac{P_{j''k't}}{P_{k't}}\right)^{-\theta} Z_{jk}\left(k'\right). \tag{14}$$

In steady state, all firms within a sector share the intermediate input demand of other sectors equally. Away from the steady state, the gap between a firm's price and the price index of the sector it belongs to (see equation (6)) distorts the firm's share in the production of intermediate input. Our economy has $K + 1$ different aggregate prices, one for the household sector and one for each of the K sectors. By contrast, the household sector and all sectors face unique sectoral prices.

The third key ingredient of our model is sectoral heterogeneity in price rigidity. Specifically, we model price rigidity à la Calvo[6] with parameters $\{\alpha_k\}_{k=1}^K$ such that the pricing problem of firm jk is

$$\max_{P_{jkt}} \mathbb{E}_t \sum_{s=0}^{\infty} Q_{t,t+s} \alpha_k^\tau \left[P_{jkt} Y_{jkt+s} - MC_{kt+s} Y_{jkt+s}\right].$$

[6]The computational burden with several hundred sector-specific state variables imposes prohibitive limitations on most endogenous price adjustment technologies, such as menu costs. However, as will become clear, our main point about the importance of heterogeneous-price stickiness is, to a first order, due to the relative difference across firms, a modeling feature that endogenous price adjustment technologies will preserve.

Marginal costs are $MC_{kt} = \frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta} e^{-a_{kt}} W_{kt}^{1-\delta}\left(P_t^k\right)^{\delta}$ in reduced form after imposing the optimal mix of labor and intermediate inputs:

$$\delta W_{kt} L_{jkt} = (1-\delta)\, P_t^k Z_{jkt}, \tag{15}$$

and $Q_{t,t+s}$ is the stochastic discount factor between period t and $t+s$.

We assume the elasticities of substitution across and within sectors are the same for households and all firms. This assumption shuts down price discrimination among different customers, and firms choose a single price P_{kt}^*:

$$\sum_{\tau=0}^{\infty} Q_{t,t+\tau}\alpha_k^s Y_{jkt+\tau}\left[P_{kt}^* - \frac{\theta}{\theta-1} MC_{kt+\tau}\right] = 0, \tag{16}$$

where $Y_{jkt+\tau}$ is the total production of firm jk in period $t+\tau$.

We define idiosyncratic shocks $\{a_{kt}\}_{k=1}^K$ at the sectoral level, and it follows that the optimal price, P_{kt}^*, is the same for all firms in a given sector. Thus, aggregating among all prices within sector yields

$$P_{kt} = \left[(1-\alpha_k)\, P_{kt}^{*1-\theta} + \alpha_k P_{kt-1}^{1-\theta}\right]^{\frac{1}{1-\theta}} \text{ for } k = 1, ..., K. \tag{17}$$

C. Monetary policy, equilibrium conditions, and definitions

The monetary authority sets nominal interest rates according to a Taylor rule:

$$I_t = \frac{1}{\beta}\left(\frac{P_t^c}{P_{t-1}^c}\right)^{\phi_\pi}\left(\frac{C_t}{C}\right)^{\phi_y}. \tag{18}$$

Monetary policy reacts to inflation, P_t^c/P_{t-1}^c, and deviations from steady state total value-added, C_t/C. We do not model monetary policy shocks.

Bonds are in zero net supply, $B_t = 0$, labor markets clear, and goods markets clear such that

$$Y_{jkt} = C_{jkt} + \sum_{k'=1}^{K}\int_{\Im_{k'}} Z_{j'k't}(j,k)\, dj', \tag{19}$$

implying a wedge between gross output Y_t and GDP C_t.

III Theoretical Results in a Simplified Model

We can derive closed-form results for the importance of sectoral shocks for aggregate fluctuations in a simplified version of our model. Given the focus of the paper, we study log-linear deviations from steady state. We report the steady-state solution and the full log-linear system solving for the equilibrium in the Online Appendix. All variables in lower cases denote log-linear deviations from steady state.

A. Simplifying Assumptions

We make the following simplifying assumptions:

(i) Households have log utility, $\sigma = 1$, and linear disutility of labor, $\varphi = 0$. Thus,

$$w_{kt} = p_t^c + c_t;$$

that is, the labor market is integrated and nominal wages are proportional to nominal GDP.

(ii) Monetary policy targets constant nominal GDP, so

$$p_t^c + c_t = 0.$$

(iii) We replace Calvo price stickiness by a simple rule: all prices are flexible, but with probability λ_k, a firm in sector k has to set its price before observing shocks. Thus,

$$P_{jkt} = \begin{cases} \mathbb{E}_{t-1}\left[P_{jkt}^*\right] & \text{with probability } \lambda_k, \\ P_{jkt}^* & \text{with probability } 1-\lambda_k, \end{cases}$$

where $\mathbb{E}_{t-1}$ is the expectation operator conditional on the $t-1$ information set.

Solution We show in the Online Appendix that under assumptions (i), (ii), and (iii), c_t is given by

$$c_t = \chi' a_t, \tag{20}$$

where $\chi \equiv (\mathbb{I}-\Lambda)\left[\mathbb{I}-\delta\Omega'(\mathbb{I}-\Lambda)\right]^{-1}\Omega_c$. Λ is a diagonal matrix with price-rigidity probabilities $[\lambda_1, ..., \lambda_K]$ as entries, and $a_t \equiv [a_{1t}, ..., a_{Kt}]'$ is a vector of sectoral productivity shocks. Recall Ω_c and Ω represent in steady state the sectoral GDP shares and intermediate-input shares.

A linear combination of sectoral shocks describes the log-deviation of GDP from its steady state up to a first-order approximation. Thus, aggregate GDP volatility is

$$v_c = v\sqrt{\sum_{k=1}^{K}\chi_k^2} = \|\chi\|_2\, v, \tag{21}$$

because all sectoral shocks have the same volatility; that is, $\mathbb{V}[a_{kt}] = v^2$ for all k. $\|\chi\|_2$ denotes the Euclidean norm of χ.

Thus, χ is a vector of multipliers from the volatility of sectoral productivity shocks to GDP volatility. We will refer to these multipliers as *sectoral multipliers* in the following.

Below, we study the effect of heterogeneous price rigidity on the scale of aggregate volatility v_c in an economy with a given number of sectors K. We also investigate the effect on the rate of decay of v_c as the economy becomes increasingly more disaggregated, $K \to \infty$.

We use the following definition:

Definition 1 *A given random variable X follows a **power-law distribution with shape parameter** β when $\Pr(X > x) = (x/x_0)^{-\beta}$ for $x \geq x_0$ and $\beta > 0$.*

B. The Granular Effect and Price Rigidity

We now study the interaction of price rigidity with the granular effect of Gabaix (2011). The granular effect studies the role of the firm-size distribution on the importance of microeconomic shocks as the origin of aggregate volatility. Gabaix (2011) measures firm size by total sales, which includes sales as final goods and as intermediate inputs. By contrast, the setup of our model and data requirements have us study sectors instead of firms. However, this difference is only nominal.

We shut down intermediate inputs, that is, $\delta = 0$, to disentangle the contribution of sales as final goods from sales as intermediate inputs. Hence, sector size only depends on sectoral consumption. With $\delta = 0$, our expressions mirror the ones in Gabaix (2011) in special cases. We study the effect of intermediate inputs, that is, the network effect of Acemoglu et al. (2012) below.

When $\delta = 0$,

$$\chi = \left(\mathbb{I} - \Lambda'\right)\Omega_c,$$

or, simply, $\chi_k = (1 - \lambda_k)\,\omega_{ck}$ for all k.

Recall $\omega_{ck} = C_k / \sum_{k=1}^{K} C_k$. Hence, steady-state sectoral GDP shares fully determine sectoral multipliers only when prices are flexible. In general, sectoral multipliers also depend on the sectoral distribution of price stickiness. Sales are no longer a sufficient statistic for the importance of sectors for aggregate volatility breaking the Hulten (1978) result in the Gabaix (2011) framework.

The following lemma presents our first result for homogeneous price stickiness across sectors.

Lemma 1 *When $\delta = 0$ and $\lambda_k = \lambda$ for all k, then*

$$v_c = \frac{(1-\lambda)\, v}{\overline{C}_k K^{1/2}} \sqrt{\mathbb{V}(C_k) + \overline{C}_k^2},$$

where $\overline{C}_k$ and $\mathbb{V}(\cdot)$ are the sample mean and sample variance of $\{C_k\}_{k=1}^K$.[7]

This lemma follows from equation (21) when $\delta = 0$. As in Gabaix (2011), the volatility of GDP in an economy with K sectors depends on the cross-sectional dispersion of sector size, here measured by $\mathbb{V}(C_k)$. Price rigidity only has a scale effect on volatility depending on whether productivity shocks are sectoral or aggregate. The scale effect follows from equation (20): if $\delta = 0$, and all sectoral shocks are perfectly correlated, then $v_c = (1-\lambda)\, v$. Active monetary policy can correct this scale effect.

The next proposition determines the rate of decay of v_c as the economy becomes increasingly more disaggregated, $K \to \infty$, in the presence of homogeneous price stickiness.

Proposition 1 (Granular effect) *If $\delta = 0$, $\lambda_k = \lambda$ for all k, and $\{C_k\}_{k=1}^K$ follows a power-law distribution with shape parameter $\beta_c \geq 1$, then*

$$v_c \sim \begin{cases} \frac{u_0}{K^{\min\{1-1/\beta_c, 1/2\}}} v & \text{for } \beta_c > 1 \\ \frac{u_0}{\log K} & \text{for } \beta_c = 1 \end{cases}$$

where u_0 is a random variable independent of K and v.

Proof. See Online Appendix. ■

Proposition 1 revisits the central idea of the granular effect: when the size distribution of sectors is fat-tailed, given by $\beta_c < 2$, aggregate volatility v_c converges to zero at a rate slower than the Central Limit Theorem implies, which is $K^{1/2}$. The rate of decay of v_c becomes slower as $\beta_c \to 1$. Intuitively, when the size distribution of sectors is fat-tailed, few sectors remain disproportionately large at any level of disaggregation. Gabaix (2011) documents that a power-law distribution with shape parameter close to 1 characterizes the upper tail of the empirical distribution of firm sizes.[8] Thus, contrary to Dupor (1999), sectoral shocks can generate sizable aggregate fluctuations even if we study sectors at a highly disaggregated level. Homogeneous price rigidity plays no role for this result, except for the scale effect we discuss in Lemma 1.

We now study the case of heterogeneous price rigidity across sectors.

[7] We define $\mathbb{V}(X_k)$ of a sequence $\{X_k\}_{k=1}^K$ as $\mathbb{V}(X_k) \equiv \frac{1}{K}\sum_{k=1}^K \left(X_k - \overline{X}\right)^2$. The definition of the sample mean is standard.

[8] We find similar results with sectoral data.

Lemma 2 *When $\delta = 0$ and price rigidity is heterogeneous across sectors, then*

$$v_c = \frac{v}{\overline{C}_k K^{1/2}} \sqrt{\mathbb{V}\left(\left(1-\lambda_k\right) C_k\right) + \left[\left(1-\overline{\lambda}\right) \overline{C}_k - \mathbb{COV}\left(\lambda_k, C_k\right)\right]^2},$$

where $\overline{\lambda}$ is the sample mean of $\{\lambda_k\}_{k=1}^K$ and $\mathbb{COV}(\cdot)$ is the sample covariance of $\{\lambda_k\}_{k=1}^K$ and $\{C_k\}_{k=1}^K$.[9]

For a fixed number of sectors, Lemma 2 states the volatility of GDP depends on the sectoral dispersion of the convoluted variable $(1-\lambda_k)C_k$ as well as the covariance between sectoral price rigidity and sectoral GDP. This result holds independently of the dependence between price rigidity and sectoral GDP. The dependence between sectoral consumption and price rigidity is, however, important for the rate of decay of v_c as $K \to \infty$.

Proposition 2 *If $\delta = 0$, λ_k, and C_k are independently distributed, the distribution of λ_k satisfies*

$$\Pr\left[1-\lambda_k > y\right] = \frac{y^{-\beta_\lambda} - 1}{y_0^{-\beta_\lambda} - 1} \text{ for } y \in [y_0, 1], \beta_\lambda > 0,$$

and C_k follows a power-law distribution with shape parameter $\beta_c \geq 1$, then

$$v_c \sim \begin{cases} \frac{u_0}{K^{\min\{1-1/\beta_c, 1/2\}}} v & \text{for } \beta_c > 1 \\ \frac{u_0}{\log K} & \text{for } \beta_c = 1, \end{cases}$$

where u_0 is a random variable independent of K and v.

Proof. See Online Appendix. ■

Proposition 2 shows price rigidity does not affect the rate of decay of v_c as $K \to \infty$ when λ_k and C_k are independent. The independence assumption and the lower bound in the support of the distribution of the frequency of price adjustment, λ_k, explain this result. If λ_k were unbounded below, $(1-\lambda_k) C_k$ would follow a Pareto distribution with shape parameter equal to the minimum of the shape parameters of the distributions of C_k and $1-\lambda_k$. But under the assumptions of Proposition 2, the convoluted variable $(1-\lambda_k) C_k$, follows a Pareto distribution with shape parameter of the distribution of C_k.

Price rigidity is still economically important despite the irrelevance for the rate of convergence. Lemma 2 implies price rigidity distorts the identity and the contribution of the most important sectors for the volatility of GDP. The distortion arising from price rigidity is

[9] We define $\mathbb{COV}(X_k, Q_k)$ of sequences $\{X_k\}_{k=1}^K$ and $\{Q_k\}_{k=1}^K$ as $\mathbb{COV}(X_k, Q_k) \equiv \frac{1}{K}\sum_{k=1}^K \left(X_k - \overline{X}\right)\left(Q_k - \overline{Q}\right)$.

central for policy makers who aim to identify the microeconomic origin of aggregate fluctuations, for example, for stabilization purposes.

Proposition 2 assumes a specific functional form for the distribution of λ_k, because we cannot prove more general results. We show in the appendix that the distributional assumption characterizes the empirical marginal distribution of sectoral frequencies well. The distribution is Pareto with a theoretically bounded support (that is not binding in our sample of sectors).

We now move to the central result in this section.

Proposition 3 *Let $\delta = 0$. The distributions of λ_k and C_k are not independent such that the following relationships hold:*

$$\lambda_k = \max\left\{0, 1 - \phi C_k^{\mu}\right\} \text{ for some } \mu \in (-1, 1), \ \phi \in (0, x_0^{-\mu}), \tag{22}$$

and C_k follows a power-law distribution with shape parameter $\beta_c \geq 1$.

If $\mu < 0$,

$$v_c \sim \begin{cases} \frac{u_1}{K^{\min\{1-(1+\mu)/\beta_c, 1/2\}}} v & \text{for } \beta_c > 1 \\ \frac{u_2}{K^{\min\{-\mu, 1/2\}} \log K} v & \text{for } \beta_c = 1. \end{cases} \tag{23}$$

If $\mu > 0$,

$$v_c \sim \begin{cases} \frac{u_2}{K^{\min\left\{1 - \frac{1+\mathbf{1}\{K \leq K^*\}\mu}{\beta_c}, \frac{1}{2}\right\}}} v & \text{for } \beta_c > 1 \\ \frac{u_2}{K^{-\mathbf{1}\{K \leq K^*\}\mu} \log K} v & \text{for } \beta_c = 1, \end{cases} \tag{24}$$

for $K^* \equiv x_0^{-\beta_c} \phi^{-\beta_c/\mu}$.

Proof. See Online Appendix. ■

Proposition 3 studies the implications of the interaction between sector size and price rigidity on the rate of decay of GDP volatility, v_c, as the economy becomes more disaggregated. First, consider the case in which $\mu < 0$, that is, when larger sectors have more rigid prices. When $\beta_c \in (\max\{1, 2(1+\mu)\}, 2)$, then v_c decays at rate $K^{1/2}$. In general, when $\beta_c \in [1, 2)$, a positive relationship between sectoral size and price stickiness slows down the rate of decay of v_c despite the bounded support of the price-stickiness distribution.

Next, consider the case in which larger sectors have more flexible prices ($\mu > 0$). Equation (22) and the bounded support of the frequency of price adjustment generates a kink such that sectors with value added larger than $\phi^{-1/\mu}$ have perfectly flexible prices. This kink generates a kink in the rate of decay of aggregate volatility, v_c. If $\beta_c \in [1, 2)$, v_c decays at a rate faster than when sector size and price stickiness are independently distributed, as long as the number of sectors is weakly smaller than K^*, $K \leq K^*$. If the number of sectors is sufficiently large, price rigidity is irrelevant for the rate of decay of v_c, as in Proposition 2. Intuitively, sector size and price rigidity are independent for any sector with value added larger than $\phi^{-1/\mu}$. For $K > K^*$, the probability is high enough that sectors with value added larger than $\phi^{-1/\mu}$ dominate the upper tail of the distribution of multipliers $\chi_k = (1 - \lambda_k)\, C_k$.

The central question now becomes: What is a sufficiently large number of sectors empirically; that is, how large is the threshold K^*? We can answer this question within the context of Proposition 3. When $K > K^*$, a high density of sectors with fully flexible prices exists. In our calibration with 341 sectors, the finest level of disaggregation our data allow, no sector has fully flexible prices. Thus, when larger sectors tend to have more flexible prices, the price-setting frictions slow down the rate of decay of aggregate volatility v_c for any level of disaggregation with at most 341 sectors.

With no kink in the relationship between sectoral size and price stickiness for large sectors, price stickiness slows down the rate of decay of v_c for any level of disaggregation, just as in the case of $\mu < 0$.

For expositional convenience, we have assumed a deterministic relationship between sectoral size and price stickiness. However, if this relationship is stochastic, we trivially find price rigidity distorts the identity of the most important sectors for GDP volatility—even if price rigidity is irrelevant for the rate of decay of GDP volatility.

The next corollary summarizes the results of this section.

Corollary 1 *In an economy in which sectors have heterogeneous sectoral GDP shares but no input-output linkages, sectoral heterogeneity of price rigidity distorts the magnitude of aggregate volatility generated by idiosyncratic sectoral shocks as well as the identity of sectors from which aggregate fluctuations originate.*

C. The Network Effect and Price Rigidity

We now study how price rigidity affects the network effect of Acemoglu et al. (2012). We assume a positive intermediate input share, $\delta \in (0, 1)$, but shut down the heterogeneity of sectoral GDP shares, that is, $\Omega_c = \frac{1}{K}\iota$. The vector of multipliers mapping sectoral shocks into aggregate volatility now solves

$$\chi = \frac{1}{K} (\mathbb{I} - \Lambda) \left[\mathbb{I} - \delta\Omega' (\mathbb{I} - \Lambda)\right]^{-1} \iota. \tag{25}$$

This expression nests the solution for the "influence vector" in Acemoglu et al. (2012) when prices are fully flexible, that is, $\lambda_k = 0$ for all $k = 1, ..., K$.[10]

[10]The only difference here is $\chi'\iota = 1/(1-\delta)$, because Acemoglu et al. (2012) normalize the scale of shocks such that the sum of the influence vector equals 1.

In general, however, a non-trivial interaction between price rigidity and input-output linkages across sectors exists. To study this interaction, we follow Acemoglu et al. (2012) and use an approximation of the vector of multipliers truncating the effect of input-output linkages at second-order interconnections:

$$\chi \simeq \frac{1}{K}\left(\mathbb{I}-\Lambda\right)\left[\mathbb{I}+\delta\Omega'\left(\mathbb{I}-\Lambda\right)+\delta^{2}\left[\Omega'\left(\mathbb{I}-\Lambda\right)\right]^{2}\right]\iota.$$

Let us first assume homogeneous price rigidity across sectors.

Lemma 3 *If $\delta \in (0,1)$, $\Omega_c = \frac{1}{K}\iota$, and $\lambda_k = \lambda$ for all k, then*

$$v_c \geq \frac{(1-\lambda)\,v}{K^{1/2}}\sqrt{\kappa+\delta'^{2}\mathbb{V}\left(d_k\right)+2\delta'^{3}\mathbb{COV}\left(d_k,q_k\right)+\delta'^{4}\mathbb{V}\left(q_k\right)}, \tag{26}$$

where $\kappa \equiv 1+2\delta'+3\delta'^{2}+2\delta'^{3}+\delta'^{4}$, $\delta' \equiv \delta\left(1-\lambda\right)$, $\mathbb{V}\left(\cdot\right)$ and $\mathbb{COV}\left(\cdot\right)$ are the sample variance and covariance statistics across sectors, and $\{d_k\}_{k=1}^{K}$ and $\{q_k\}_{k=1}^{K}$ are the *outdegrees* and *second-order outdegrees*, respectively, defined for all $k=1,...,K$ as

$$\begin{aligned} d_k &\equiv \sum_{k'=1}^{K}\omega_{k'k}, \\ q_k &\equiv \sum_{k'=1}^{K}d_{k'}\omega_{k'k}. \end{aligned}$$

Lemma 3 follows from equation (21), $d=\Omega'\iota$ and $q=\Omega'^{2}\iota$. We have an inequality, because the exact solution for the multipliers χ is strictly larger than the approximation. Acemoglu et al. (2012) coin the terms "outdegrees" and "second-order outdegrees" to measure the centrality of sectors in the production network. In particular, d_k is large when sector k is a large supplier of intermediate inputs. In turn, q_k is large when sector k is a large supplier of large suppliers of intermediate inputs.

Similarly to Lemma 1, homogeneous price rigidity across sectors only has a scale effect on aggregate volatility for a given level of disaggregation. Thus, as in Acemoglu et al. (2012), aggregate volatility from idiosyncratic shocks is higher if the production network is more asymmetric, that is, if a higher dispersion of outdegrees and second-order outdegrees exists across sectors.

The next proposition shows results for the rate of decay of v_c as $K \to \infty$ under the assumption of homogeneous price rigidity.

Proposition 4 (Network effect) *If $\delta \in (0,1)$, $\lambda_k = \lambda$ for all k, $\Omega_c = \frac{1}{K}\iota$, the distribution* *of outdegrees* $\{d_k\}$*, second-order outdegrees* $\{q_k\}$*, and the product* $\{d_k q_k\}$ *follow power-law distributions with respective shape parameters* $\beta_d, \beta_q, \beta_z > 1$ *such that* $\beta_z \geq \frac{1}{2}\min\{\beta_d, \beta_q\}$*, then*

$$v_c \geq \begin{cases} \frac{u_3}{K^{1/2}} v & \text{for } \min\{\beta_d, \beta_q\} \geq 2, \\ \frac{u_3}{K^{1-1/\min\{\beta_d,\beta_q\}}} v & \text{for } \min\{\beta_d, \beta_q\} \in (1,2), \end{cases}$$

where u_3 *is a random variable independent of* K *and* v*.*

Proof. See Online Appendix. ■

Proposition 4 summarizes the network effect: the rate of decay of aggregate volatility depends on the distribution of measures of network centrality and their interaction. Thus, if some sectors are disproportionately central in the production network, sectoral idiosyncratic shocks have sizable effects on aggregate volatility even if sectors are defined at a highly disaggregated level. The fattest tail among the distributions of outdegrees and second-order outdegrees bounds the rate of decay of aggregate volatility, if the positive relation between outdegrees and second-order outdegrees is not too strong.

Acemoglu et al. (2012) document in the U.S. data that $\beta_d \approx 1.4$ and $\beta_q \approx 1.2$. We find slightly higher numbers in the data we use in our calibration.[11]

As before, homogeneous price rigidity across sectors only has a scale effect on GDP volatility. However, it has one implication worth noticing. Because $\beta_q < \beta_d$ in the U.S. data, the distribution of second-order outdegrees contributes the most to the slow decay of v_c when K is large. Lemma 3 implies this contribution is quantitatively less important as price rigidity increases, because $\delta'/\delta = 1 - \lambda$.

Next, we turn to our results for the case of heterogeneous price rigidity across sectors.

Lemma 4 *If* $\delta \in (0,1)$*,* $\Omega_c = \frac{1}{K}\iota$*, and price rigidity is heterogeneous across sectors, then*

$$v_c \geq \frac{v}{K^{1/2}} \left[\begin{array}{c} \left(\frac{1}{K}\sum_{k=1}^{K}(1-\lambda_k)^2\right)\left[\widetilde{\kappa} + \delta^2 \mathbb{V}\left(\widetilde{d}_k\right) + 2\delta'^3 \mathbb{COV}\left(\widetilde{d}_k, \widetilde{q}_k\right) + \delta'^4 \mathbb{V}\left(\widetilde{q}_k\right)\right] \\ -\left(\frac{1}{K}\sum_{k=1}^{K}(1-\lambda_k)^2\right)\left[2\delta^2\left(1+\widetilde{\delta}+\widetilde{\delta}^2\right)\mathbb{COV}\left(\lambda_k, \widetilde{d}_k\right) + \delta^4 \mathbb{COV}\left(\lambda_k, \widetilde{d}_k\right)^2\right] \\ +\mathbb{COV}\left((1-\lambda_k)^2, \left(1+\delta\widetilde{d}_k + \delta^2\widetilde{q}_k\right)^2\right) \end{array} \right]^{\frac{1}{2}}, \tag{27}$$

where $\widetilde{\kappa} \equiv 1 + 2\widetilde{\delta} + 3\widetilde{\delta} + 2\widetilde{\delta} + \widetilde{\delta}$, $\widetilde{\delta} \equiv \delta\left(1-\overline{\lambda}\right)$, $\overline{\lambda}$ is the sample mean of $\{\lambda_k\}_{k=1}^{K}$, $\mathbb{V}(\cdot)$ and $\mathbb{COV}(\cdot)$ are the sample variance and covariance statistics across sectors, and $\left\{\widetilde{d}_k\right\}_{k=1}^{K}$ and $\{\widetilde{q}_k\}_{k=1}^{K}$ are the **modified outdegrees** and **modified second-order outdegrees**, respectively, defined for

[11] Given these numbers, we abstract from the case in which $\min\{\beta_d, \beta_q\} = 1$ in Proposition 4.

all $k = 1, ..., K$ as

$$\widetilde{d}_k \equiv \sum_{k'=1}^{K} (1 - \lambda_{k'}) \omega_{k'k},$$
$$\widetilde{q}_k \equiv \sum_{k'=1}^{K} (1 - \lambda_{k'}) \widetilde{d}_{k'} \omega_{k'k}.$$

Lemma 4 follows from equation (21), $\widetilde{d} = \Omega' (\mathbb{I} - \Lambda) \iota$ and $\widetilde{q} = [\Omega' (\mathbb{I} - \Lambda)]^2 \iota$, which we label the vectors of modified outdegrees and modified second-order outdegrees, respectively. These statistics measure the centrality of sectors in the production network after adjusting nodes by their degree of price rigidity. In particular, $\widetilde{d}_k$ is high either when sector k is a large supplier of intermediate inputs and/ or when it is a large supplier of the most flexible sectors. Similarly, $\widetilde{q}_k$ is large when sector k is a large supplier of the most flexible sectors, which are in turn large suppliers of the most flexible sectors.

The lower bound for v_c in Lemma 4 collapses to the one in Lemma 3 if price rigidity is homogeneous across sectors. The first line on the right-hand side of equation (27) is similar to the one in equation (27) in Lemma 3 with two differences. First, by Jensen's inequality,

$$\frac{1}{K} \sum_{k=1}^{K} (1 - \lambda_k)^2 \geq \left(1 - \overline{\lambda}\right)^2.$$

The muting effect of price rigidity on aggregate volatility is weaker if price rigidity is heterogeneous across sectors relative to an economy with $\lambda_k = \overline{\lambda}$ for all k.

Second, we now compute key statistics using modified outdegrees, that is, $\widetilde{d}$ and $\widetilde{q}$ instead of d and q. To see the implications, note

$$\widetilde{d}_k = \left(1 - \overline{\lambda}\right) d_k - K \mathbb{COV} \left(\lambda_{k'}, \omega_{k'k}\right),$$
$$\widetilde{q}_k = \left(1 - \overline{\lambda}\right)^2 q_k - K \mathbb{COV} \left(\lambda_{k'}, \widetilde{d}_k \omega_{k'k}\right) - \left(1 - \overline{\lambda}\right) \sum_{k'=1}^{K} \omega_{k'k} \mathbb{COV} \left(\lambda_s, \widetilde{d}_s \omega_{sk'}\right).$$

The dispersion of $\widetilde{d}$ is higher than the dispersion of $\left(1 - \overline{\lambda}\right) d$ when $\mathbb{COV}\left(\lambda_{k'}, \omega_{k'k}\right)$ is more dispersed across sectors and when it is negatively correlated with d. In words, the dispersion of $\widetilde{d}$ is high when the intermediate input demand of the most flexible sectors is highly unequal across supplying sectors, and when large intermediate input-supplying sectors are also large suppliers to flexible sectors. Similarly, the dispersion of $\widetilde{q}$ is higher than the dispersion of $\left(1 - \overline{\lambda}\right)^2 q$ when $\mathbb{COV}\left(\lambda_{k'}, \widetilde{d}_k \omega_{k'k}\right)$ is more dispersed and is negatively correlated with q.

The second and third lines on the right-hand side of the lower bound for v_c in of equation (27) capture new effects. In particular, volatility of GDP is higher when $\mathbb{COV}\left(\lambda_k, \widetilde{d}_k\right) < 0$, that is, if sectors with high modified outdegree, $\widetilde{d}_k$, are the most flexible sectors (second line), and if Jensen's inequality effect is stronger (third line).

Analyzing the rate of decay of v_c as $K \to \infty$ is more complicated compared to a case with no intermediate inputs, $\delta = 0$.

Proposition 5 *If $\delta \in (0,1)$, $\Omega_c = \frac{1}{K}\iota$, price rigidity is heterogeneous across sectors, the distribution of modified outdegrees $\left\{\widetilde{d}_k\right\}$, modified second-order outdegrees $\{\widetilde{q}_k\}$, and the product $\left\{\widetilde{d}_k\widetilde{q}_k\right\}$ follow power-law distributions with respective shape parameter $\widetilde{\beta}_d, \widetilde{\beta}_q, \widetilde{\beta}_z > 1$ such that $\widetilde{\beta}_z \geq \frac{1}{2}\min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\}$, then*

$$v_c \geq \begin{cases} \frac{u_4}{K^{1/2}}v & \text{for } \min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\} \geq 2, \\ \frac{u_4}{K^{1-1/\min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\}}}v & \text{for } \min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\} \in (1,2), \end{cases}$$

where u_4 is a random variable independent of K and v.

Proof. See Online Appendix. ■

Proposition 5 resembles Proposition 3 in the context of production networks. If sectors with the most sticky (flexible) prices are also the most central in the price-rigidity-adjusted production network such that $\min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\} > (<) \min\{\beta_d, \beta_q\}$, GDP volatility decays at a faster (slower) rate than when price rigidity is homogeneous across sectors or independent of network centrality. Also as before, regardless of the effect of price rigidity on the rate of decay of v_c as $K \to \infty$, price rigidity distorts the identity of the most important sectors driving GDP volatility originating from idiosyncratic shocks through the network effect.

The following corollary summarizes the findings of this section.

Corollary 2 *In an economy characterized as a production network, sectoral heterogeneity of price rigidity distorts the scale of aggregate volatility generated by idiosyncratic sectoral shocks as well as the identity of sectors from which aggregate fluctuations originate.*

The details of the analysis are different from the details in Section III*B*., but the main message is identical: the inefficiency that price rigidity introduces can dampen aggregate fluctuations just as in an economy with aggregate shocks. However, it also changes the sectoral origin of aggregate fluctuations. Sectoral multipliers may become larger or smaller.

Importantly, one can also think of these sectoral multipliers as relative weights to compute aggregate fluctuations, weighting the effect of sectoral shocks (see equation (20)). It is trivial to see that re-weighting sectoral shocks of potentially opposite signs can easily change the sign of business cycles, relative to a frictionless economy.

D. Relaxing Simplifying Assumptions

We now discuss the implications of further relaxing the simplifying modeling assumptions we made to derive the results in Sections III*B.*, III*C.*, and IIIIII

Non-linear Disutility of Labor When $\varphi > 0$, labor supply and demand jointly determine wages such that

$$w_{kt} = c_t + p_t^c + \varphi l_{kt}$$

becomes the log-linear counterpart to equation (7). Thus, with monetary policy targeting $c_t + p_t^c = 0$, it no longer holds that sectoral productivity shocks have no effect on sectoral wages.[12]

We now describe these effects one by one. First, the log-linear version of the production function implies

$$l_{kt}^d = y_{kt} - a_{kt} - \delta \left(w_{kt} - p_t^k \right).$$

Hence, conditioning on sectoral gross output, shocks in sector k have direct effects on labor demand in sector k and indirect effects on all other sectors to the extent the sector-specific aggregate price of intermediate inputs, $\{p_t^k\}_{k=1}^K$, changes (which depends on input-output linkages).

Second, aggregating demand for goods by households and firms implies sectoral gross output depends on total gross output y_t and prices according to

$$y_{kt} = y_t - \eta \left(p_{kt} - \left[(1-\psi) p_t^c - \psi \widetilde{p}_t \right] \right).$$

Hence, conditioning on total gross output, shocks in sector k affect sectoral gross output through the effects on the relative price between sectoral prices and the GDP deflator, p_t^c, and sectoral prices and the economy-wide aggregate price for intermediate goods, $\widetilde{p}_t$.

$\widetilde{p}_t$ is given by

$$\widetilde{p}_t = \sum_{k'=1}^{K} \zeta_{k'} p_{k't},$$

which uses steady-state shares of sectors, ζ_k, in the aggregate production of intermediate inputs

[12] The Online Appendix contains details of the derivations.

as weights,

$$\zeta_k \equiv \sum_{k'=1}^{K} n_{k'} \omega_{k'k}.$$

$\{n_k\}_{k=1}^{\infty}$ are the shares of sectors in aggregate gross output (which coincides with the measure of firms in each sector):

$$n_k = (1-\psi)\,\omega_{ck} + \psi \zeta_k \text{ for all } k = 1, ..., K.$$

$\psi \equiv \frac{Z}{Y}$ is the fraction of total gross output used as intermediate input in steady state.

Third, the response of total gross output y_t to the shocks depends on the response of value added, c_t, and production of intermediate inputs, z_t, according to

$$y_t = (1-\psi)\, c_t + \psi z_t,$$

such that z_t solves

$$z_t = (1+\Gamma_c)\, c_t + \Gamma_p \left(p_t^c - \widetilde{p}_t\right) - \Gamma_a \sum_{k'=1}^{K} n_{k'} a_{k't}.$$

$\Gamma_c \equiv \frac{(1-\delta)(\sigma+\varphi)}{(1-\psi)+\varphi(\delta-\psi)}, \Gamma_a \equiv \frac{1+\varphi}{(1-\psi)+\varphi(\delta-\psi)}, \Gamma_p \equiv \frac{1-\delta}{(1-\psi)+\varphi(\delta-\psi)}.$

Thus, another channel through which sectoral productivity shocks affect labor demand is through their effects on the aggregate demand for intermediate inputs.

To sum up, equation (20) still gives the solution for c_t, but the vector of multipliers χ is now

$$\chi \equiv (\mathbb{I}-\Lambda)\left[\gamma_1 \mathbb{I} + \gamma_2 \aleph \iota'\right]\left[\mathbb{I} - \varphi\left[\gamma_3 \iota \Omega_c' + \gamma_4 \iota \vartheta' - \gamma_5 \iota'\right](\mathbb{I}-\Lambda) - \gamma_6 \Omega' (\mathbb{I}-\Lambda)\right]^{-1} \Omega_c, \tag{28}$$

with $\gamma_1 \equiv \frac{1+\varphi}{1+\delta\varphi}$, $\gamma_2 \equiv \frac{\psi(1-\delta)\Gamma_a}{1+\delta\varphi}$, $\gamma_3 \equiv \frac{(1-\delta)[(1-\psi)\eta-1]}{1+\delta\varphi}$, $\gamma_4 \equiv \frac{\psi(1-\delta)(\eta-\Gamma_p)}{1+\delta\varphi}$, $\gamma_5 \equiv \frac{\gamma_2}{\Gamma_a}$, $\gamma_6 \equiv \delta\gamma_1$, $\aleph \equiv (n_1, ..., n_K)'$, and $\vartheta = (\zeta_1, ..., \zeta_K)'$.

Relative to the solution for χ in equation (25), multipliers take a richer functional form, capturing all three channels that elastic labor demand introduces. Although the interaction between the GDP shares and network effects is more involved, the distortionary effect of heterogeneous price rigidity works similarly as above.

Pricing Friction and Monetary Policy Rule. Calvo pricing frictions result in serial correlation in the response of prices even when shocks are i.i.d.:

$$p_{kt} = (1+\beta+\kappa_k)^{-1}\left[\kappa_k mc_{kt} + \beta \mathbb{E}\left[p_{kt+1}\right] + p_{kt-1}\right] \text{ for } k = 1, ..., K,$$

where $\kappa_k \equiv (1-\alpha_k)(1-\beta\alpha_k)/\alpha_k$.

A Taylor rule of the form

$$i_t = \phi_\pi^c \left(p_t^c - p_{t-1}^c\right) + \phi_c c_t$$

offsets some of the serial correlation that price rigidity introduces.

Serial correlation in price responses implies GDP is now given by

$$c_t = \sum_{\tau=0}^{\infty} \sum_{k=1}^{K} \rho_{k\tau} a_{kt-\tau}.$$

Hence, we have to redefine multipliers χ_k for $k = 1, ..., K$,

$$\chi_k \equiv \sqrt{\sum_{\tau=0}^{\infty} \rho_{k\tau}^2}, \tag{29}$$

such that $v_c = \|\chi\|_2 \, v$ still holds.

In contrast to our simplified model, χ does not capture the effect of sectoral shocks on GDP c_t and aggregate volatility v_c. We thus adjust the definition of χ to simplify the comparison between our simplified model and our quantitative analysis below.

IV Data

This section describes the data we use to construct the input-output linkages, and sectoral GDP, and the micro-pricing data we use to construct measures of price stickiness at the sectoral level.

A. Input-Output Linkages and Sectoral Consumption Shares

The Bureau of Economic Analysis (BEA) produces input-output tables detailing the dollar flows between all producers and purchasers in the United States. Producers include all industrial and service sectors, as well as household production. Purchasers include industrial sectors, households, and government entities. The BEA constructs the input-output tables using Census data that are collected every five years. The BEA has published input-output tables every five years beginning in 1982 and ending with the most recent tables in 2012. The input-output tables are based on NAICS industry codes. Prior to 1997, the input-output tables were based on SIC codes.

The input-output tables consist of two basic national-accounting tables: a “make” table and a “use” table. The make table shows the production of commodities by industry. Rows present industries, and columns present the commodities each industry produces. Looking across columns for a given row, we see all the commodities a given industry produces. The sum of the entries comprises industry output. Looking across rows for a given column, we see all industries producing a given commodity. The sum of the entries is the output of a commodity. The use table contains the uses of commodities by intermediate and final users. The rows in the use table contain the commodities, and the columns show the industries and final users that utilize them. The sum of the entries in a row is the output of that commodity. The columns document the products each industry uses as inputs and the three components of value added: compensation of employees, taxes on production and imports less subsidies, and gross operating surplus. The sum of the entries in a column is industry output.

We utilize the input-output tables for 2002 to create an industry network of trade flows. The BEA defines industries at two levels of aggregation: detailed and summary accounts. We use the detailed levels of aggregation to create industry-by-industry trade flows. The BEA also provides the data to calibrate sectoral GDP shares directly.

The BEA provides concordance tables between NAICS codes and input-output industry codes. We follow the BEA's input-output classifications with minor modifications to create our industry classifications. We account for duplicates when NAICS codes are not as detailed as input-output codes. In some cases, an identical set of NAICS codes defines different input-output industry codes. We aggregate industries with overlapping NAICS codes to remove duplicates.

We combine the make and use tables to construct an industry-by-industry matrix that details how much of an industry's inputs other industries produce. We use the make table ($MAKE$) to determine the share of each commodity each industry k produces. We define the market share ($SHARE$) of industry k's production of commodity as

$$SHARE = MAKE \odot (\mathbb{I} - MAKE)^{-1}_{k,k'} .$$

We multiply the share and use tables (USE) to calculate the dollar amount that industry k' sells to industry k. We label this matrix revenue share ($REVSHARE$), which is a supplier industry-by-consumer industry matrix,

$$REVSHARE = SHARE \times USE.$$

We then use the revenue-share matrix to calculate the percentage of industry k' inputs purchased from industry k, and label the resulting matrix $SUPPSHARE$:

$$SUPPSHARE = REVSHARE \odot \left((\mathbb{I} - MAKE)^{-1}_{k,k'} \right)' . \tag{30}$$

The input-share matrix in this equation is an industry-by-industry matrix and therefore consistently maps into our model.[13]

B. Frequencies of Price Adjustments

We use the confidential microdata underlying the producer price data (PPI) from the BLS to calculate the frequency of price adjustment at the industry level.[14] The PPI measures changes in prices from the perspective of producers, and tracks prices of all goods-producing industries, such as mining, manufacturing, and gas and electricity, as well as the service sector. The BLS started sampling prices for the service sector in 2005. The PPI covers about 75% of the service sector output. Our sample ranges from 2005 to 2011.

The BLS applies a three-stage procedure to determine the sample of goods. First, to construct the universe of all establishments in the United States, the BLS compiles a list of all firms filing with the Unemployment Insurance system. In the second and third stages, the BLS probabilistically selects sample establishments and goods based on either the total value of shipments or the number of employees. The BLS collects prices from about 25,000 establishments for approximately 100,000 individual items on a monthly basis. The BLS defines PPI prices as "net revenue accruing to a specified producing establishment from a specified kind of buyer for a specified product shipped under specified transaction terms on a specified day of the month." Prices are collected via a survey that is emailed or faxed to participating establishments. Individual establishments remain in the sample for an average of seven years until a new sample is selected to account for changes in the industry structure.

We calculate the frequency of price adjustment at the goods level, FPA, as the ratio of the number of price changes to the number of sample months. For example, if an observed price path is \$10 for two months and then \$15 for another three months, one price change occurs during five months, and the frequency is 1/5. We aggregate goods-based frequencies to the BEA industry level.

The overall mean monthly frequency of price adjustment is 22.15%, which implies an average duration, $-1/\log(1 - FPA)$, of 3.99 months. Substantial heterogeneity is present in the frequency across sectors, ranging from as low as 4.01% for the semiconductor manufacturing sector (duration of 24.43 months) to 93.75% for dairy production (duration of 0.36 months).

[13] Ozdagli and Weber (2016) follow a similar approach.

[14] The data have been used before in Nakamura and Steinsson (2008); Goldberg and Hellerstein (2011); Bhattarai and Schoenle (2014); Gorodnichenko and Weber (2016); Gilchrist, Schoenle, Sim, and Zakrajšek (2016); Weber (2015); and D'Acunto, Liu, Pflueger, and Weber (2016), among others.

V Calibration

We calibrate the steady-state input-output linkages of our model Ω to the U.S. input-output tables in 2002. The same 2002 BEA data also allow us to directly calibrate sectoral GDP shares, Ω_C. The Calvo parameters match the frequency of price adjustments between 2005 and 2011, using the micro data underlying the PPI from the BLS. After we merge the input-output and the frequency-of-price-adjustment data, we end up with 341 sectors.

The detailed input-output table has 407 unique sectors in 2002. We lose sectors for three reasons. First, some sectors produce almost exclusively final goods, so the data do not contain enough observations of such goods to compute frequencies of price adjustment. Second, the goods some sectors produce do not trade in a formal market, so the BLS has no prices to record. Examples of missing sectors are (with I/O industry codes in parentheses) "video tape and disc rentals" (532230), "bowling centers" (713950), "military armored vehicle, tank, and tank component manufacturing" (336992), and "religious organizations" (813100). Third, the data for some sectors are not available at the six-digit level.

We show results for several calibrations of our model. **MODEL1** has linear disutility of labor, $\varphi = 0$, and monetary policy targeting constant nominal GDP. This model is the closest parametrization of our full-blown New Keynesian model to the simplified model we study in Section III, with the modeling of the pricing friction as the only difference.[15]

MODEL2 is identical to **MODEL1**, but we set the inverse-Frisch elasticity to $\varphi = 2$.

MODEL3 is an intermediate case in which $\varphi = 0$, and monetary policy that follows the Taylor rule we specified in Section II with parameters $\phi_c = 0.33/12 = 0.0275$ and $\phi_\pi = 1.34$.

In **MODEL4**, monetary policy follows this same Taylor rule, but we set the inverse-Frisch elasticity to $\varphi = 2$.

These calibrations are at a monthly frequency, so the discount factor is $\beta = 0.9975$ (implying an annual risk-free interest rate of about 3%). We set the elasticity of substitution across sectors to $\eta = 2$ and within sectors to $\theta = 6$ following Carvalho and Lee (2011). We will also report robustness results for the elasticities, setting $\theta = 11$, which is equivalent to a 10% markup. We also set $\delta = 0.5$ so the intermediate inputs share in steady state is $\delta * (\theta - 1)/\theta = 0.42$, which matches the 2002 BEA data.

[15] We interpret the frequencies of price adjustments as the probability a sector can adjust prices after the shock.

VI Quantitative Results

A. Multipliers

We provide quantitative evidence by first studying the multipliers that map sectoral shocks into aggregate GDP volatility.

Table 1 reports the magnitude of multipliers, $\|\chi\|$, for different experiments. We formally define the multiplier, χ, in equation (29). We report multipliers in levels but also relative to the multiplier that maps aggregate productivity shocks into aggregate GDP volatility (we will sometimes refer to the latter as "aggregate multiplier"). Price rigidity has a mechanical effect on aggregate volatility, dampening volatility originating from idiosyncratic, but also aggregate shocks. The relative multiplier controls for the general dampening effect of price rigidity on aggregate volatility and allows an easier comparison of how the different heterogeneities interact.

A.1 Multipliers: Flexible Prices

We start in Panel A of Table 1 with MODEL1, which corresponds to the simplified model of Section III except for the modeling of the pricing friction; that is, it features Calvo price stickiness, a constant nominal GDP target in the monetary policy rule, and linear disutility of labor. Column (1) assumes flexible prices to isolate the quantitative strength of the pure granular effect due to the empirical distribution of sectoral GDP, the pure network effect due to the empirical input-output structure of the U.S. economy, and their joint effect.

We start with an economy in which all sectors are homogeneous, that is, when they have equal size and uniform input-output linkages. As the model in Section III suggests, the multiplier equals $K^{-1/2}$ for K=341 and it is 5.42% of the aggregate multiplier, which equals 1. The multiplier is 0.2047 when we calibrate sector size Ω_C to U.S. data, but shut down intermediate input use, $\delta = 0$. This calibration isolates the granular effect. GDP volatility increases by a factor of 4 with sectoral heterogeneity in size relative to uniform GDP shares across sectors, showing a strong granular effect from idiosyncratic shocks for aggregate volatility.

Intermediate inputs ($\delta = 0.5$) with homogeneous steady-state input-output linkages, Ω, mute the strength of the granular channel of idiosyncratic shocks. As line (3) shows, the relative multiplier is now 11% rather than almost 20%.

In line (4), we focus on the fully heterogeneous network channel for aggregate fluctuations; that is, we impose equal GDP shares across sectors but calibrate Ω to the actual, heterogeneous U.S. input-output tables. The multiplier is now 0.0801. The network channel increases the multiplier by 50% relative to the multiplier in an economy with a homogeneous steady-state input-output structure (0.0536), but the network channel in isolation is smaller than the granular channel for aggregate fluctuations originating from final goods production.

The last lines study granular and network channels jointly. The multiplier is now 16.88%, indicating the potential of idiosyncratic shocks to be a major driving force behind aggregate fluctuations. The multiplier in this case is 50% larger than in an economy in which we calibrate GDP shares to U.S. data but impose a symmetric input-output structure across sectors. The overall effect is remarkably close to the one predicted by Gabaix's (2011) measure of total sales.

A.2 Multipliers: Homogeneous Sticky Prices

We next allow for rigid prices in column (2) of Table 1 but impose homogeneous price stickiness across sectors. Specifically, we calibrate the sectoral Calvo parameter to the average frequency of price adjustment in the United States for all sectors.

Comparing columns (1) and (2) across rows, we find price rigidity reduces the level of aggregate volatility sectoral shocks generate by an order of magnitude, just as our model in Section III predicts (the scale effect). However, sticky prices also tend to dampen aggregate volatility due to aggregate shocks in general. Hence, we focus our discussion on relative multipliers. Multipliers relative to an aggregate productivity shock are similar to the case with flexible prices in column (1), with two exceptions: (i) introducing homogeneous input-output linkages offsets the granular channel less than under flexible prices (compare rows (2) and (3) across columns (1) and (2)), and (ii) the granular effect in row (4) becomes slightly weaker (going from 8.01% to 6.09%). We expect these results based on our analysis in Section III. Pricing frictions mitigate the network effect of second-order outdegrees more than the network effect of first-order outdegrees. Because the distribution of second-order outdegrees is more fat-tailed than outdegrees, even homogeneous pricing stickiness across sectors reduces the quantitative strength of the network effect.

A.3 Multipliers: Heterogeneous Sticky Prices

Column (3) of Table 1 presents the main results of this section. The calibration captures the empirical sector-size distribution, the actual input-output structure of the U.S. economy at the most granular level, as well as detailed, heterogeneous output price stickiness across sectors, and allows us to analyze the relevance of idiosyncratic shocks for aggregate fluctuations.

The calibration empirically confirms our theoretical predictions of Section III. First, across rows, we see heterogeneous price rigidity increases the *level* of aggregate fluctuations originating from idiosyncratic shocks by at least 95% and up to 180% relative to the case of equal price stickiness in column (2).

Second, heterogeneity in price stickiness alone increases the *relative* multiplier of sectoral shocks on GDP volatility: the relative multiplier goes from 5.42% to 12.36% in a calibration with equal GDP shares across sectors and homogeneous input-output linkages (see row (1)). Heterogeneous price stickiness thus increases the relative multiplier by more than the network effect, which generates a relative multiplier of 8.01% and 6.09% depending on whether prices are flexible or homogeneously sticky across sectors (see row (4) in columns (1) and (2)). Thus, heterogeneous price rigidity creates a “frictional” channel of aggregate volatility independent of the “granular” or “network” channels in the literature.

Third, the interaction between the granular effect of heterogeneous sector sizes and the frictional channel is strong. In a calibration without intermediate inputs, $\delta = 0$, we find a relative multiplier of 37.18% instead of 20.47% when price rigidity is equal for all sectors (see rows 2 in columns (2) and (3)). If $\delta = 0.5$ and input-output linkages are equal for all sectors, the relative multiplier is 33.73%, whereas it is only 11% in an economy with flexible prices and 17% in an economy with equal price stickiness across sectors (see row 3).

Fourth, a strong interaction exists between the network channel of aggregate fluctuations and the frictional channel: the relative multiplier is 13.16% with sticky prices calibrated to the U.S. economy, whereas it is only 8.01% in a flexible-price economy and 6.09% in an economy with equal price stickiness (compare row (4) across columns).

Overall, when our model matches the sector-size distribution, the input-output linkages of the U.S. economy, and the distribution of price rigidity across sectors, the relative multiplier that maps sectoral productivity shocks into aggregate volatility equals almost a third of the multiplier of an aggregate productivity shock, which is an almost 90% increase compared to a relative multiplier of 16.88% in a frictionless economy (last row) and almost six times larger than in an economy with homogeneous sectors.

We find across calibrations that (i) heterogeneity in price stickiness alone can generate large aggregate fluctuations from idiosyncratic shocks, (ii) homogeneous input-output linkages mute the granular effect relative to an economy without intermediate input use, and (iii) introducing heterogeneous price stickiness across sectors in an economy with sectors of different sizes but homogeneous input-output linkages increases the size of the relative multipliers. Section III in the online appendix explains the economics behind these findings within our simplified model.

A.4 Multipliers: Alternative Model Specifications

Panels B to D of Table 1 report similar results for three alternative model specifications. MODEL2 drops the assumption of a linear disutility of labor. MODEL3 assumes a standard Taylor rule instead of a monetary policy targeting constant nominal GDP, whereas MODEL4 additionally drops the assumption of linear disutility of labor. The level of the multipliers differs from MODEL1 in Panel A, but the relative multipliers are almost identical across different calibrations for flexible and homogeneously sticky prices. Under heterogeneous price stickiness, the levels of the multipliers are similar across calibrations, with some differences in relative multipliers when we drop the assumption of linear disutility of labor.

Table 2 reports multipliers in levels and relative to the aggregate multiplier for the same four models, but studies only the impact effect of sectoral shocks on GDP. Multipliers differ only slightly relative to the ones in reported in Table 1, suggesting the Calvo assumption introduces only a small degree of persistence relative to the simple specification of price rigidity we study in the simplified model of Section III.

We follow Carvalho and Lee (2011) in the calibration of deep parameters, but one might be concerned a low elasticity of substitution within sector might partially drive our findings.[16] Table 3 shows our findings across models and calibrations barely change when we increase the within-sectors elasticity of substitution, θ, from a baseline value of 6 to 11, which reduces the markup from 20% to 10%.[17]

In our baseline analysis, we have to drop the construction sectors because the BLS does not provide pricing information at the six-sector NAICS level. Table A.1 in the Online Appendix reports results for a calibration in which we assign the same price stickiness measure to all six-digit construction sectors. Results are similar to the one we discussed above. In untabulated results, we also find similar results for a model with binding ZLB.

B. Distorted Idiosyncratic Origin of Fluctuations

One of the key modeling insights is that heterogeneity in price stickiness can change the identity and relative importance of sectors. Table 4 shows introducing heterogeneity in the frequency of price adjustment across sectors indeed changes the identity and relative contribution of the five most important sectors for the multiplier for different calibrations of MODEL1. Relative contributions sum to 1, and the entries in Table 4 tell us directly the fraction of the multiplier coming from the reported sectors.[18]

[16] We thank Susanto Basu for raising this point.
[17] We also lower δ from 0.5 to 0.46 to match the ratio of C/Y in the data.
[18] Results are similar for the other models.

In column (1), we calibrate Calvo parameters to the sectoral frequency of price changes in the United States, but impose equal GDP shares across sectors, and input-output linkages are homogeneous. The five most important sectors are the five sectors with most the flexible prices, which are mostly commodities and farming products: "Dairy cattle and milk production" (112120), "Alumina refining and primary aluminum production" (33131A), "Primary smelting and refining of copper" (331411), "Oil and gas extraction" (211000), and "Poultry and egg production" (1121A0). The relative contributions to the overall multiplier range from 7.62% to 5.49%. If all sectors were perfectly identical, all sectors would have a contribution of 0.29% (341^{-1}).

Columns (2) and (3) in Table 4 show how price rigidity affects the granular channel of idiosyncratic shocks. Column (2) assumes flexible prices but steady-state sectoral GDP shares that match the data. Column (3) also matches the sectoral frequency of price changes. The identity and relative contribution changes in both calibrations: with flexible prices, the most important sectors and are "Retail trade" (4A0000), "Wholesale trade" (420000), and "Real estate" (531000), with relative contributions of 33.33%, 14.43%, and 10.89%, respectively; once we allow for rigid prices, the most important sectors are "Monetary authorities and depository credit intermediation" (52A000), "Wholesale trade" (420000), and "Oil and gas extraction" (211000) with relative contributions of 25.33%, 18.74%, and 13.50%.

Columns (4) and (5) of Table 4 studies the distortion price rigidity introduces on the network effect of aggregate fluctuations. Column (4) assumes flexible prices and steady-state input-output linkages calibrated to the U.S. input-output tables, whereas column (5) also matches sectoral frequencies of price adjustment. Again, the identity of the five most important sectors changes completely: with flexible prices, the most important sectors are "Wholesale trade" (420000), followed by "Real estate" (531000), "Electric power generation, transmission, and distribution" (221100), "Monetary authorities and depository credit intermediation" (52A000), and "Retail trade" (4A0000), with relative contributions of 24.53%, 7.61%, 3.66%, 3.26%, and 2.90%.

Once we allow for sticky prices, the contributions of the five most important sectors range from 9.85% to 5.76% which in descending order are "Electric power generation, transmission, and distribution" (211000), " Dairy cattle and milk production" (112120), "Petroleum refineries" (324110), "Primary smelting and refining of copper" (331411), and "Cattle ranching and farming" (1121A0). In short, energy sectors become the most important, followed by farming sectors, once we allow for price stickiness to follow the empirical distribution.

Finally, we compare columns (6) and (7) in Table 4 to see how the introduction of heterogeneous price stickiness across sectors changes the importance and identity of sectors for the multiplier when both the granular and network channels are at work.

The relative contributions of the five most important sectors with flexible prices are: "Retail trade" (4A0000), "Real estate" (531000), "Wholesale trade" (420000), "Monetary authorities and depository credit intermediation" (52A000), and "Telecommunications" (517000). With flexible prices, "Retail trade" and "Real estate" jointly account for 45% of the aggregate fluctuations originating from shocks at the micro level.

When we turn on heterogeneous sticky prices, instead, "Real estate" no longer belongs to the top five sectors and the contribution of "Retail trade" drops to 10%. Interestingly, "Monetary authorities and depository credit intermediation" becomes the most important sector, with a contribution of almost 30% when we allow for heterogeneities along all three dimension in the most realistic calibration.

The discussion so far focused on the identity and contribution of the five most important sectors. The distorting nature of heterogeneous price stickiness, however, is a more general phenomenon. Figure 1 is a scatter plot of the sectoral rank in the contribution to aggregate fluctuations originating from sectoral shocks. The y-axis plots the rank in an economy with heterogeneous price stickiness, and the x-axis plots the rank in an economy with identical price stickiness across sectors, while we allow for heterogeneity in input-output linkages and consumption shares in both cases. We see the introduction of differential price stickiness changes the rank of sectors throughout the whole distribution, and drastically so for some sectors.

The evidence on the changing identity and relative importance of sectors for aggregate fluctuations originating from sectoral shocks we observe in Table 4 and Figure 1 underlines the importance of studying granular, network, and frictional channels in combination. A central bank that aims to stabilize sectoral prices of "big" or "central" sectors might make systematic policy mistakes if it does not take into account the "frictional" origin of aggregate fluctuations.

VII Concluding Remarks

This paper studies the "frictional origin" of aggregate fluctuations originating from sectoral shocks when nominal output prices are heterogeneously rigid across sectors. We do so theoretically and quantitatively in a detailed, calibrated multi-sector New Keynesian model with intermediate inputs.

Heterogeneity in price rigidity has first-order effects: it generates a frictional origin of aggregate fluctuations, it amplifies the granular and network channels of idiosyncratic shocks, and it changes the identity and relative importance of sectors for aggregate fluctuations originating from sectoral shocks. Importantly, sector sales are no longer a sufficient statistic for a sector's contribution to aggregate volatility, as in Hulten (1978). Interestingly, in our most realistic calibration, we moreover find "Monetary authorities and depository credit intermediation" to be the most important sector for aggregate fluctuations originating from shocks at the micro level.

To date, the implications of price rigidity, and frictions in general, for the granular and network effects remain largely unexplored. Our analysis suggests price rigidity has direct and important implications for the modeling and understanding of business cycles. The interaction also has important implications for the conduct of monetary policy. A central bank that aims to stabilize sectoral prices of "big" or "central" sectors might make systematic policy mistakes if it does not take into account the "frictional" origin of aggregate fluctuations. Although beyond the scope of this paper, future work may explore optimal monetary policy in our heterogeneous production economy.

To make the points of this paper, we assumed exogenously given price rigidity. We conjecture that endogenizing price adjustment may amplify our results, because sectors hit by larger shocks might adjust prices more frequently. However, the exact result may depend on the respective price-adjustment technology. We leave this extension to future research.

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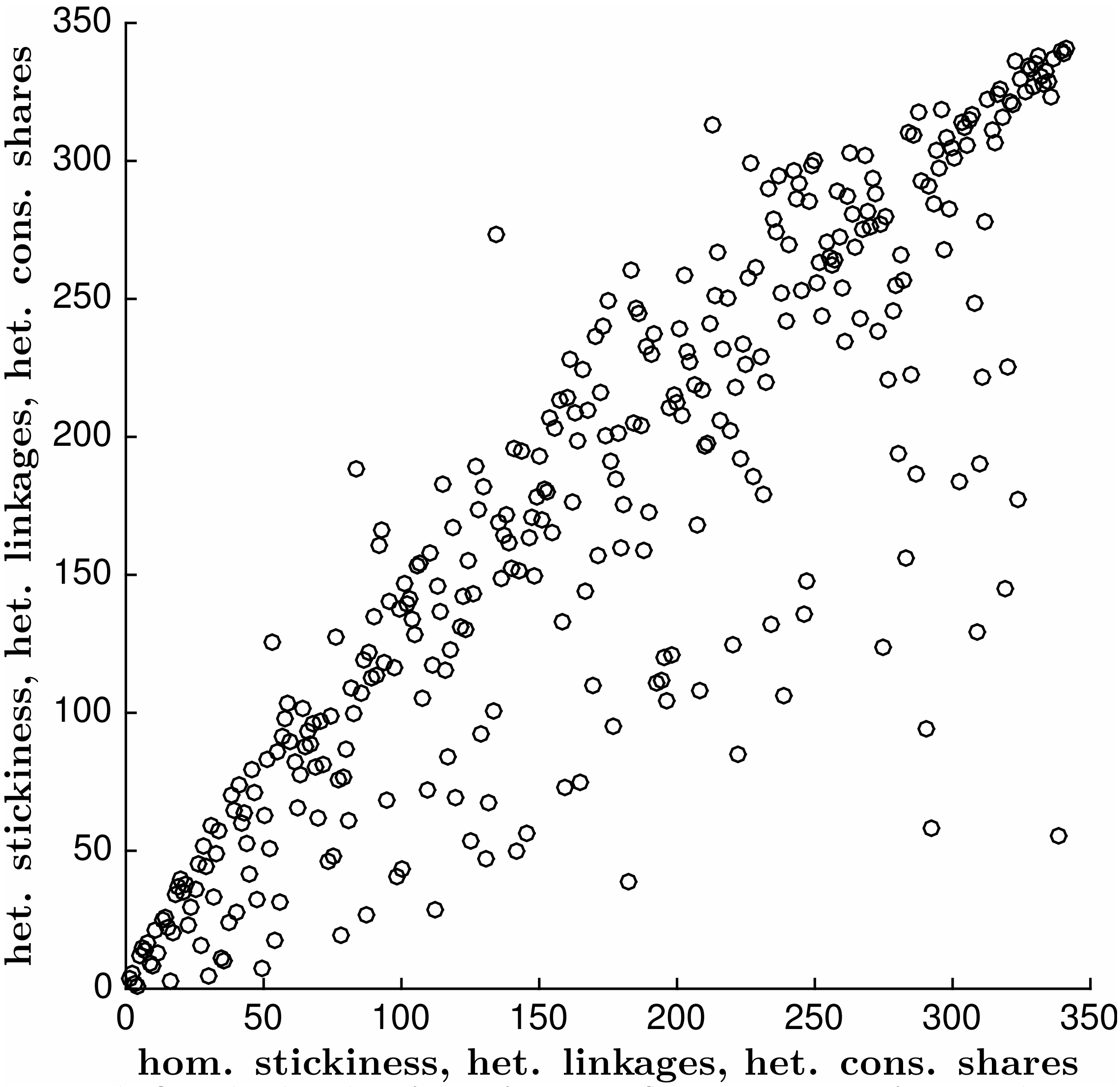
Figure 1: **Ranking of Sectors: heterogeneous versus homogeneous Price Stickiness**



This figure plots the ranking of sectors for aggregate fluctuations originating from sectoral shocks for an economy with heterogeneous price stickiness across sectors (y-axis) and an economy with identical price stickiness for all sectors (x-axis). We assume heterogeneous GDP shares and input-output linkages calibrated to the United States in both cases.

Table 1: **Multipliers of Sectoral Shocks into Aggregate Volatility**

This table reports multipliers of sectoral productivity shocks on GDP volatility, $\|\chi\|$, with relative multipliers, $\|\chi\|$, in parenthesis. The latter are relative to the multiplier of an aggregate productivity shock on GDP volatility. Ω_c represents the vector of GDP shares and Ω the matrix of input-output linkages. We calibrate a 341-sector version of our model to the input-output tables and sector size from the BEA and the frequencies of price adjustment from the BLS.

			Flexible Prices		Homogeneous Calvo		Heterogeneous Calvo	
			(1)		(2)		(3)	
					Panel A: MODEL1			
(1)	hom Ω_c	hom Ω	0.0542	(5.42%)	0.0017	(5.42%)	0.0045	(12.36%)
(2)	het Ω_c	$\delta = 0$	0.2047	(20.47%)	0.0105	(20.47%)	0.0226	(37.18%)
(3)	het Ω_c	hom Ω	0.1126	(11.26%)	0.0054	(17.00%)	0.0114	(33.73%)
(4)	hom Ω_c	het Ω	0.0801	(8.01%)	0.0019	(6.09%)	0.0054	(13.16%)
(5)	het Ω_c	het Ω	0.1688	(16.88%)	0.0060	(18.96%)	0.0117	(31.78%)
					Panel B: MODEL2			
(1)	hom Ω_c	hom Ω	0.0542	(5.42%)	0.0031	(5.42%)	0.0049	(8.16%)
(2)	het Ω_c	$\delta = 0$	0.2047	(20.47%)	0.0223	(20.47%)	0.0320	(30.28%)
(3)	het Ω_c	hom Ω	0.1162	(11.62%)	0.0088	(15.36%)	0.0137	(22.97%)
(4)	hom Ω_c	het Ω	0.0780	(7.80%)	0.0037	(6.38%)	0.0062	(9.22%)
(5)	het Ω_c	het Ω	0.1700	(17.00%)	0.0105	(18.30%)	0.0154	(24.14%)
					Panel C: MODEL3			
(1)	hom Ω_c	hom Ω	0.0542	(5.42%)	0.0011	(5.42%)	0.0059	(15.26%)
(2)	het Ω_c	$\delta = 0$	0.2047	(20.47%)	0.0081	(20.47%)	0.0265	(46.11%)
(3)	het Ω_c	hom Ω	0.1126	(11.26%)	0.0037	(18.51%)	0.0132	(44.82%)
(4)	hom Ω_c	het Ω	0.0801	(8.01%)	0.0012	(6.00%)	0.0068	(15.87%)
(5)	het Ω_c	het Ω	0.1688	(16.88%)	0.0040	(19.60%)	0.0128	(39.81%)
					Panel D: MODEL4			
(1)	hom Ω_c	hom Ω	0.0542	(5.42%)	0.0025	(5.42%)	0.0063	(9.40%)
(2)	het Ω_c	$\delta = 0$	0.2047	(20.47%)	0.0226	(20.47%)	0.0384	(33.09%)
(3)	het Ω_c	hom Ω	0.1162	(11.62%)	0.0074	(16.10%)	0.0158	(26.80%)
(4)	hom Ω_c	het Ω	0.0780	(7.80%)	0.0029	(6.23%)	0.0076	(10.20%)
(5)	het Ω_c	het Ω	0.1700	(17.00%)	0.0086	(18.58%)	0.0169	(26.59%)

Table 2: **Multipliers of Sectoral Shocks into Aggregate Volatility: Impact Response**

This Table reports the impact multipliers of sectoral productivity shocks on GDP volatility, $\|\chi\|$, with relative multipliers, $\|\chi\|$, in parentheses. The latter are relative to the multiplier of an aggregate productivity shock on GDP volatility. Ω_c represents the vector of GDP shares and Ω the matrix of input-output linkages. We calibrate a 341-sector version of our model to the input-output tables and sector size from the BEA and the frequencies of price adjustment from the BLS.

			Flexible Prices		Homogeneous Calvo		Heterogeneous Calvo	
			(1)		(2)		(3)	
					Panel A: MODEL1			
(1)	hom Ω_c	hom Ω	0.0542	(5.42%)	0.0008	(5.42%)	0.0042	(13.92%)
(2)	het Ω_c	$\delta = 0$	0.2047	(20.47%)	0.0058	(20.47%)	0.0192	(42.79%)
(3)	het Ω_c	hom Ω	0.1126	(11.26%)	0.0029	(19.47%)	0.0096	(40.89%)
(4)	hom Ω_c	het Ω	0.0801	(8.01%)	0.0008	(5.45%)	0.0049	(14.67%)
(5)	het Ω_c	het Ω	0.1688	(16.88%)	0.0030	(20.01%)	0.0094	(36.68%)
					Panel B: MODEL2			
(1)	hom Ω_c	hom Ω	0.0542	(5.42%)	0.0018	(5.42%)	0.0045	(8.77%)
(2)	het Ω_c	$\delta = 0$	0.2047	(20.47%)	0.0153	(20.47%)	0.0277	(31.63%)
(3)	het Ω_c	hom Ω	0.1162	(11.62%)	0.0055	(16.77%)	0.0116	(25.22%)
(4)	hom Ω_c	het Ω	0.0780	(7.80%)	0.0019	(5.87%)	0.0055	(9.66%)
(5)	het Ω_c	het Ω	0.1700	(17.00%)	0.0062	(18.85%)	0.0125	(25.45%)
					Panel C: MODEL3			
(1)	hom Ω_c	hom Ω	0.0542	(5.42%)	0.0011	(5.42%)	0.0043	(13.44%)
(2)	het Ω_c	$\delta = 0$	0.2047	(20.47%)	0.0081	(20.47%)	0.0211	(40.37%)
(3)	het Ω_c	hom Ω	0.1126	(11.26%)	0.0037	(18.15%)	0.0101	(38.82%)
(4)	hom Ω_c	het Ω	0.0801	(8.01%)	0.0011	(5.60%)	0.0051	(14.15%)
(5)	het Ω_c	het Ω	0.1688	(16.88%)	0.0039	(19.43%)	0.0100	(35.02%)
					Panel D: MODEL4			
(1)	hom Ω_c	hom Ω	0.0542	(5.42%)	0.0025	(5.42%)	0.0048	(8.49%)
(2)	het Ω_c	$\delta = 0$	0.2047	(20.47%)	0.0226	(20.47%)	0.0323	(30.42%)
(3)	het Ω_c	hom Ω	0.1162	(11.62%)	0.0073	(15.95%)	0.0127	(24.18%)
(4)	hom Ω_c	het Ω	0.0780	(7.80%)	0.0028	(6.09%)	0.0060	(9.40%)
(5)	het Ω_c	het Ω	0.1700	(17.00%)	0.0085	(18.52%)	0.0141	(24.69%)

Table 3: **Multipliers of Sectoral Shocks into Aggregate Volatility: High Elasticity of Substitution**

This table reports multipliers of sectoral productivity shocks on GDP volatility, $\|\chi\|$, with relative multipliers, $\|\chi\|$, in parenthesis. The latter are relative to the multiplier of an aggregate productivity shock on GDP volatility. Ω_c represents the vector of GDP shares and Ω the matrix of input-output linkages. We calibrate a 341-sector version of our model to the input-output tables and sector size from the BEA and the frequencies of price adjustment from the BLS. We increase the elasticity of substitution across sectors, θ, from a baseline value of 6 to 11.

			Flexible Prices		Homogeneous Calvo		Heterogeneous Calvo	
			(1)		(2)		(3)	
					Panel A: MODEL1			
(1)	hom Ω_c	hom Ω	0.0542	(5.42%)	0.0018	(5.42%)	0.0048	(12.40%)
(2)	het Ω_c	$\delta = 0$	0.2047	(20.47%)	0.0105	(20.47%)	0.0226	(37.18%)
(3)	het Ω_c	hom Ω	0.1195	(11.95%)	0.0058	(17.34%)	0.0123	(34.05%)
(4)	hom Ω_c	het Ω	0.0764	(7.64%)	0.0020	(5.96%)	0.0057	(13.11%)
(5)	het Ω_c	het Ω	0.1710	(17.10%)	0.0064	(19.11%)	0.0125	(32.20%)
					Panel B: MODEL2			
(1)	hom Ω_c	hom Ω	0.0542	(5.42%)	0.0035	(5.42%)	0.0055	(7.81%)
(2)	het Ω_c	$\delta = 0$	0.2047	(20.47%)	0.0223	(20.47%)	0.0320	(30.28%)
(3)	het Ω_c	hom Ω	0.1215	(12.15%)	0.0101	(15.39%)	0.0153	(22.12%)
(4)	hom Ω_c	het Ω	0.0754	(7.54%)	0.0042	(6.33%)	0.0068	(8.84%)
(5)	het Ω_c	het Ω	0.1717	(17.17%)	0.0120	(18.31%)	0.0173	(23.58%)
					Panel C: MODEL3			
(1)	hom Ω_c	hom Ω	0.0542	(5.42%)	0.0012	(5.42%)	0.0064	(15.26%)
(2)	het Ω_c	$\delta = 0$	0.2047	(20.47%)	0.0081	(20.47%)	0.0265	(46.11%)
(3)	het Ω_c	hom Ω	0.1195	(11.95%)	0.0041	(18.65%)	0.0142	(44.92%)
(4)	hom Ω_c	het Ω	0.0764	(7.64%)	0.0013	(5.90%)	0.0073	(15.81%)
(5)	het Ω_c	het Ω	0.1710	(17.10%)	0.0043	(19.66%)	0.0138	(40.28%)
					Panel D: MODEL4			
(1)	hom Ω_c	hom Ω	0.0542	(5.42%)	0.0030	(5.42%)	0.0070	(8.86%)
(2)	het Ω_c	$\delta = 0$	0.2047	(20.47%)	0.0226	(20.47%)	0.0384	(33.09%)
(3)	het Ω_c	hom Ω	0.1215	(12.15%)	0.0088	(15.95%)	0.0177	(25.27%)
(4)	hom Ω_c	het Ω	0.0754	(7.54%)	0.0034	(6.21%)	0.0083	(9.67%)
(5)	het Ω_c	het Ω	0.1717	(17.17%)	0.0103	(18.52%)	0.0192	(25.60%)

Table 4: **Contribution of Sectors to Multiplier of Sectoral Shocks on GDP Volatility**

This table reports the contribution of five most important sectors to the multiplier of sectoral shocks on GDP volatility for MODEL1 and the identity of sectors in parentheses. The different columns represent calibrations which match the frequency of price adjustments (λ), the distribution of consumption shares (Ω_c), or the actual input-output matrix (Ω). 1121A0: Cattle ranching and farming; 112120: Dairy cattle and milk production; 211000: Oil and gas extraction; 221100: Electric power generation, transmission, and distribution; 324110: Petroleum refineries; 33131A: Alumina refining and primary aluminum production; 331411: Primary smelting and refining of copper; 336111: Automobile manufacturing; 4A0000: Retail trade; 420000: Wholesale trade; 517000: Telecommunications; 52A000: Monetary authorities and depository credit intermediation; 531000: Real estate; 621A00: Offices of physicians, dentists, and other health practitioners; 622000: Hospitals

λ (1)	Ω_c (2)	λ, Ω_c (3)	Ω (4)	λ, Ω (5)	Ω_c, Ω (6)	λ, Ω_c, Ω (7)
7.62% (112120)	33.33% (4A0000)	25.33% (52A000)	24.53% (420000)	9.85% (211000)	23.22% (4A0000)	28.64% (52A000)
6.33% (33131A)	14.43% (420000)	18.74% (420000)	7.61% (531000)	6.69% (112120)	21.99% (531000)	20.71% (420000)
6.32% (331411)	10.89% (531000)	13.50% (211000)	3.66% (221100)	6.23% (324110)	16.85% (420000)	11.46% (324110)
5.92% (211000)	6.69% (621A00)	11.55% (324110)	3.26% (52A000)	6.15% (331411)	6.25% (52A000)	10.49% (4A0000)
5.49% (1121A0)	5.72% (622000)	10.73% (336111)	2.90% (4A0000)	5.76% (1121A0)	5.58% (517000)	9.24% (336111)

Online Appendix: Price Rigidities and the Granular Origins of Aggregate Fluctuations

Ernesto Pasten, Raphael Schoenle, and Michael Weber

Not for Publication

I Steady-State Solution and Log-linear System

A. Steady-State Solution

Without loss of generality, set $a_k = 0$. We show below conditions for the existence of a symmetric steady state across firms in which

$$W_k = W, Y_{jk} = Y,\ L_{jk} = L,\ Z_{jk} = Z, P_{jk} = P \text{ for all } j, k.$$

Symmetry in prices across all firms implies

$$P^c = P^k = P_k = P$$

such that, from equations (1), (2), and (10) in the main body of the paper and (13),

$$\begin{aligned} C_k &= \omega_{ck} C, \\ C_{jk} &= \frac{1}{n_k} C_k, \\ Z_{jk}\left(k'\right) &= \omega_{kk'} Z, \\ Z_{jk}\left(j', k'\right) &= \frac{1}{n_{k'}} Z_{jk}\left(k'\right). \end{aligned}$$

The vector $\Omega_c \equiv [\omega_{c1}, ..., \omega_{cK}]'$ represents steady-state sectoral shares in value-added C, $\Omega = \{\omega_{kk'}\}_{k,k'=1}^{K}$ is the matrix of input-output linkages across sectors, and $\aleph \equiv [n_1, ..., n_K]'$ is the vector of steady-state sectoral shares in gross output Y.

It also holds that

$$\begin{aligned} C &= \sum_{k=1}^{K} \int_{\Im_k} C_{jk} dj, \\ Z_{jk} &= \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{jk}\left(j', k'\right) dj' = Z. \end{aligned}$$

From Walras' law in equation (19) and symmetry across firms, it holds

$$Y = C + Z. \tag{A.1}$$

Walras' law and results above yield, for all j, k:

$$\begin{aligned} Y_{jk} &= C_{jk} + \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{j'k'}\left(j, k\right) dj' \\ Y &= \frac{\omega_{ck}}{n_k} C + \frac{1}{n_k} \left(\sum_{k'=1}^{K} n_{k'} \omega_{k'k} \right) Z, \end{aligned}$$

so $\aleph$ satisfies

$$\begin{aligned} n_k &= \left(1 - \psi\right) \omega_{ck} + \psi \sum_{k'=1}^{K} n_{k'} \omega_{k'k}, \\ \aleph &= \left(1 - \psi\right) \left[I - \psi \Omega'\right]^{-1} \Omega_c, \end{aligned}$$

for $\psi \equiv \frac{Z}{Y}$. Note by construction $\aleph' \iota = 1$, which must hold given the total measure of firms is 1.

Steady-state labor supply from equation (7) is

$$\frac{W_k}{P} = g_k L_k^{\varphi} C^{\sigma}.$$

In a symmetric steady state, $L_k = n_k L$, so this steady state exists if $g_k = n_k^{-\varphi}$ such that $W_k = W$ for all k. Thus, steady-state labor supply is given by

$$\frac{W}{P} = L^{\varphi} C^{\sigma}. \tag{A.2}$$

Households' budget constraint, firms' profits, production function, efficiency of production (from equation (15)) and optimal prices in steady state respectively are

$$CP = WL + \Pi \tag{A.3}$$

$$\Pi = PY - WL - PZ \tag{A.4}$$

$$Y = L^{1-\delta} Z^{\delta} \tag{A.5}$$

$$\delta WL = (1-\delta) PZ \tag{A.6}$$

$$sP = \frac{\theta}{\theta - 1} \xi W^{1-\delta} P^{\delta} \tag{A.7}$$

for $\xi \equiv \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta} \right)^{-\delta}$.

Equation (A.7) solves

$$\frac{W}{P} = \left(\frac{\theta - 1}{\theta \xi} \right)^{\frac{1}{1-\delta}}. \tag{A.8}$$

This latter result together with equations (A.5), (A.6), and (A.7) solve

$$\frac{\Pi}{P} = \frac{1}{\theta} Y.$$

Plugging this result in equation (A.4) and using equation (A.1) yields

$$\begin{aligned} C &= \left[1 - \delta \left(\frac{\theta - 1}{\theta} \right) \right] Y \\ Z &= \delta \left(\frac{\theta - 1}{\theta} \right) Y, \end{aligned} \tag{A.9}$$

such that $\psi \equiv \delta \left(\frac{\theta - 1}{\theta} \right)$.

This result and equation (A.7) gives

$$L = \left[\delta \left(\frac{\theta - 1}{\theta} \right) \right]^{-\frac{\delta}{1-\delta}} Y,$$

where Y solves from this latter result, equations (A.2), (A.9) and (A.8):

$$Y = \left(\frac{\theta - 1}{\theta \xi} \right)^{\frac{1}{(1-\delta)(\sigma+\varphi)}} \left[\delta \left(\frac{\theta - 1}{\theta} \right) \right]^{\frac{\delta \varphi}{(1-\delta)(\sigma+\varphi)}} \left[1 - \delta \left(\frac{\theta - 1}{\theta} \right) \right]^{-\frac{\sigma}{\sigma+\varphi}}.$$

B. Log-linear System

B.1 Aggregation

Aggregate and sectoral consumption (interpreted as value-added) given by equations (1) and (2) are

$$
\begin{aligned}
c_t &= \sum_{k=1}^{K} \omega_{ck} c_{kt}, \\
c_{kt} &= \frac{1}{n_k} \int_{\Im_k} c_{jkt} dj.
\end{aligned} \tag{A.10}
$$

Aggregate and sectoral production of intermediate inputs are given by

$$
\begin{aligned}
z_t &= \sum_{k=1}^{K} n_k z_{kt}, \\
z_{kt} &= \frac{1}{n_k} \int_{\Im_k} z_{jkt} dj,
\end{aligned} \tag{A.11}
$$

where (10) and (13) imply that $z_{jk} = \sum_{r=1}^{K} \omega_{kr} z_{jk}(r)$ and $z_{jk}(r) = \frac{1}{n_r} \int_{\Im_r} z_{jk}(j', r) \, dj'$.

Sectoral and aggregate prices are given from equations (4), (6), and (12),

$$
\begin{aligned}
p_{kt} &= \int_{\Im_k} p_{jk} dj \text{ for } k = 1, ..., K \\
p_t^c &= \sum_{k=1}^{K} \omega_{ck} p_{kt}, \\
p_t^k &= \sum_{k'=1}^{K} \omega_{kk'} p_{k't}.
\end{aligned}
$$

Aggregation of labor is

$$
\begin{aligned}
l_t &= \sum_{k=1}^{K} l_{kt}, \\
l_{kt} &= \int_{\Im_k} l_{jkt} dj.
\end{aligned} \tag{A.12}
$$

B.2 Demands

Households' demand for sectors and goods in equations (3) and (5) for all $k = 1, ..., K$ become

$$
\begin{aligned}
c_{kt} - c_t &= \eta \left(p_t^c - p_{kt}\right), \\
c_{jkt} - c_{kt} &= \theta \left(p_{kt} - p_{jkt}\right).
\end{aligned} \tag{A.13}
$$

In turn, firm jk's demand for sectors and goods in equation (11) and (14) for all $k, r = 1, ..., K$,

$$
\begin{aligned}
z_{jkt}\left(k'\right) - z_{jkt} &= \eta \left(p_t^k - p_{k't}\right), \\
z_{jkt}\left(j', k'\right) - z_{jkt}\left(k'\right) &= \theta \left(p_{k't} - p_{j'k't}\right).
\end{aligned} \tag{A.14}
$$

Firms' gross output satisfies Walras' law,

$$
y_{jkt} = (1 - \psi)\, c_{jkt} + \psi \sum_{k'=1}^{K} \int_{\Im_{k'}} z_{j'k't}\,(j, k)\, dj'. \tag{A.15}
$$

Total gross output follows from the aggregation of equations (19),

$$
y_t = (1 - \psi)\, c_t + \psi z_t. \tag{A.16}
$$

B.3 IS and labor supply

The household Euler equation in equation (8) becomes

$$
c_t = \mathbb{E}_t\left[c_{t+1}\right] - \sigma^{-1}\left\{i_t - \left(\mathbb{E}_t\left[p_{t+1}^c\right] - p_t\right)\right\}.
$$

The labor supply condition in equation (7) is

$$
w_{kt} - p_t^c = \varphi l_{kt} + \sigma c_t. \tag{A.17}
$$

B.4 Firms

Production function:

$$
y_{jkt} = a_{kt} + (1 - \delta)\, l_{jkt} + \delta z_{jkt} \tag{A.18}
$$

Efficiency condition:

$$w_{kt} - p_t^k = z_{jkt} - l_{jkt} \tag{A.19}$$

Marginal costs:

$$mc_{kt} = (1 - \delta)\, w_{kt} + \delta p_t^k - a_{kt} \tag{A.20}$$

Optimal reset price:

$$p_{kt}^* = (1 - \alpha_k \beta)\, mc_{kt} + \alpha_k \beta \mathbb{E}_t \left[p_{kt+1}^* \right]$$

Sectoral prices:

$$p_{kt} = (1 - \alpha_k)\, p_{kt}^* + \alpha_k p_{kt-1}$$

B.5 Taylor rule:

$$i_t = \phi_\pi \left(p_t^c - p_{t-1}^c \right) + \phi_c c_t$$

II Solution of Key Equations in Section III

A. Solution of Equation (25)

Setting $\sigma = 1$ and $\varphi = 0$ in equation (A.17) yields

$$w_{kt} = c_t + p_t^c = 0,$$

where the equality follows from the assumed monetary policy rule, so equation (A.20) becomes

$$mc_{kt} = \delta p_t^k - a_{kt}.$$

Here, sectoral prices for all $k = 1, ..., K$ are governed by

$$\begin{aligned} p_{kt} &= (1 - \lambda_k)\, mc_{kt} \\ &= \delta\,(1 - \lambda_k)\, p_t^k - (1 - \lambda_k)\, a_{kt}, \end{aligned}$$

which in matrix form solves

$$p_t = -\left[\mathbb{I} - \delta\,(\mathbb{I} - \Lambda)\,\Omega\right]^{-1}(\mathbb{I} - \Lambda)\, a_t.$$

$p_t \equiv [p_{1t}, ..., p_{Kt}]'$ is the vector of sectoral prices, Λ is a diagonal matrix with the vector $[\lambda_1, ..., \lambda_K]'$ in its diagonal, Ω is the matrix of input-output linkages, and $a_t \equiv [a_{1t}, ..., a_{Kt}]'$ is the vector of realizations of sectoral technology shocks.

The monetary policy rule implies $c_t = -p_t^c$, so

$$c_t = \left(\mathbb{I} - \Lambda'\right)\left[\mathbb{I} - \delta\left(\mathbb{I} - \Lambda'\right)\Omega'\right]^{-1}\Omega_c' a_t.$$

Solution of Equation (28)

Setting $\sigma = 1$ and $\varphi > 0$ in (A.17) yields

$$w_{kt} = \varphi l_{kt}^s + c_t + p_t^c = \varphi l_{kt}^d,$$

which follows from the assumed monetary policy rule.

Labor demand is obtained from the production function in equation (A.18), the efficiency condition for production in equation (A.19), and the aggregation of labor in equation (A.12):

$$l_{kt}^{d} = y_{kt} - a_{kt} - \delta \left(w_{kt} - p_t^k \right).$$

y_{kt} follows from equations (A.10), (A.11), (A.13), (A.14), and (A.15):

$$y_{kt} = y_t - \eta \left(p_{kt} - \left[(1-\psi)\, p_t^c + \psi \sum_{k=1}^{K} n_k p_t^k \right] \right),$$

where

$$\widetilde{p}_t \equiv \sum_{k=1}^{K} n_k p_t^k = \sum_{k=1}^{K} \zeta_k p_{kt},$$

with $\zeta_k \equiv \sum_{k'=1}^{K} n_{k'} \omega_{k'k}$.

To solve for y_t, we use equations (A.11), (A.12), (A.16) and $y_t = \sum_{k=1}^{K} \int_{\Im_k} y_{jkt} dj$ to get

$$y_t = c_t + \psi \left[\Gamma_c c_t - \Gamma_p \left(\widetilde{p}_t - p_t^c \right) - \Gamma_a \sum_{k=1}^{K} n_k a_{kt} \right],$$

where $\Gamma_c \equiv \frac{(1-\delta)(1+\varphi)}{(1-\psi)+\varphi(\delta-\psi)}, \Gamma_p \equiv \frac{1-\delta}{(1-\psi)+\varphi(\delta-\psi)},\ \Gamma_a \equiv \frac{1+\varphi}{(1-\psi)+\varphi(\delta-\psi)}$.

Putting together all these equations, sectoral wages solve

$$w_{kt} = \frac{\varphi}{1+\delta\varphi} \left[\begin{array}{c} (1+\psi\Gamma_c)\, c_t - a_{kt} - \psi\Gamma_a \sum_{k'=1}^{K} n_{k'} a_{k't} \\ \left[(1-\psi)\,\eta + \psi\Gamma_p \right] p_t^c + \psi \left(\eta - \Gamma_p \right) \widetilde{p}_t + \delta p_t^k - \eta p_{kt} \end{array} \right].$$

With this expression, the solution to equation (28) follows the same steps as the solution to equation (25).

III The Network Effect, the Granular Effect, and Price Rigidity

We find in Section VI across calibrations that (i) heterogeneity in price stickiness alone can generate large aggregate fluctuations from idiosyncratic shocks, (ii) homogeneous input-output linkages mute the granular effect relative to an economy without intermediate input use, and (iii) introducing heterogeneous price stickiness across sectors in an economy with sectors of different sizes but homogeneous input-output linkages increases the size of the relative multipliers.

Consider the economy of Section III with sectors of potentially different sizes, input-output linkages, and price rigidities. In this general case, the vector of multipliers up to second-order terms has elements

$$\chi_k \geq (1-\lambda_k)\left[\omega_{ck} + \delta\widehat{d}_k + \delta^2\widehat{q}_k\right]. \tag{A.21}$$

As before, ω_{ck} denotes the size of sector k, $\widehat{d}_k$ the **generalized outdegree** of sector k, and $\widehat{q}_k$ the **generalized second-order outdegree** of sector k:

$$\begin{aligned} \widehat{d}_k &\equiv \sum_{k'=1}^{K} \omega_{ck'}\left(1-\lambda_{k'}\right)\omega_{k'k}, \\ \widehat{q}_k &= \sum_{k'=1}^{K} \widehat{d}_{k'}\left(1-\lambda_{k'}\right)\omega_{k'k}. \end{aligned} \tag{A.22}$$

These two terms embody the effects of heterogeneity of GDP shares, I/O linkages, and price rigidity across sectors. The multiplier of idiosyncratic shocks as in the previous derivations equals $\|\chi\|_2 = \sqrt{\sum_{k=1}^{K}\chi_k^2}$, and $\sum_{k=1}^{K}\chi_k$ represents the multiplier of an aggregate shock.

Lemma 5 *If input-output shares and sector sizes are homogeneous across sectors, that is, $\omega_{ck} = \omega_{kk'} = 1/K$ for all k, k', but price stickiness λ_k is heterogeneous, then the multiplier of idiosyncratic shocks is*

$$\|\chi\|_2 = \frac{1}{K\left(1-\delta\left(1-\overline{\lambda}\right)\right)}\sqrt{\sum_{k=1}^{K}\left(1-\lambda_k\right)^2},$$

where $\overline{\lambda} \equiv \frac{1}{K}\sum_{k=1}^{K}\lambda_k$. The multiplier is increasing in the dispersion of price stickiness across sectors. The aggregate multiplier is invariant to allowing for price dispersion. Thus, the multiplier of sectoral shocks relative to aggregate shocks is increasing in the dispersion of price rigidity.

Proof. See Online Appendix. ■

The intuition for the previous proposition follows exactly as in Section III *B.*. The next proposition considers the case in which only input-output linkages are restricted to be homogeneous:

Lemma 6 *If input-output linkages are homogeneous across sectors, that is,* $\omega_{kk'} = 1/K$ *for all* k, k'*, but sector size* ω_{ck} *is unrestricted and prices are frictionless* ($\lambda_k = 0$)*, then given* $\text{var}(\omega_{ck}) \geq var(1/K)$*, the multiplier of sectoral shocks relative to the aggregate shock multiplier,*

$$\frac{\sqrt{\sum_{k=1}^{K}\left(\omega_{ck}+\frac{\delta/K}{1-\delta}\right)^2}}{(1-\delta)^{-1}} \leq \sqrt{\sum_{k=1}^{K}\omega_{ck}^2},$$

is smaller than when input-output linkages are shut down ($\delta = 0$)*, ceteris paribus.*

Proof. See Online Appendix. ■

Allowing for price stickiness ($\lambda_k > 0$) is an important case in the calibrations. We study this case next. Relative to the previous proposition, we find that the introduction of heterogeneous price rigidity leads to an overall increase in the relative multiplier.

Lemma 7 *If input-output linkages are homogeneous across sectors, that is,* $\omega_{kk'} = 1/K$ *for all* k, k'*, but sector size* ω_{ck} *is unrestricted, and price rigidity is heterogeneous* ($\lambda_k > 0$)*, then given* $\overline{\lambda} \equiv \frac{1}{K}\sum_{k=1}^{K}\lambda_k$*,* $\overline{\overline{\lambda}} \equiv \frac{1}{K}\sum_{k=1}^{K}\omega_{ck}\lambda_k$ *the multiplier of idiosyncratic shocks relative to aggregate shocks,*

$$\frac{\|\chi\|_2}{\sum\limits_{k=1}^{K}\chi_k} = \frac{\sqrt{\sum_{k=1}^{K}(1-\lambda_k)^2\left(\omega_{ck}+\frac{\delta\left(1-\overline{\overline{\lambda}}\right)/K}{1-\delta\left(1-\overline{\lambda}\right)}\right)^2}}{\left(1-\overline{\overline{\lambda}}\right)/\left[1-\delta\left(1-\overline{\lambda}\right)\right]},$$

is (1) increasing in the simple and weighted average of heterogeneous price rigidity across sectors, $1-\overline{\lambda}$*, and* $1-\overline{\overline{\lambda}}$*, (2) increasing in the covariance of price rigidity and sector size,* $cov((1-\lambda_k, \omega_{ck})$*, and (3) increasing in the dispersion of sectoral price flexibility,* λ_k*.*

Proof. See Online Appendix. ■

The first effect (1) is an effect due to the average degree of price rigidity in the economy. The last two effects (2) and (3) capture the effect of heterogeneity in the interaction with sector size and the dispersion of price flexibility.

IV Proofs

Most proofs below are modifications of the arguments in Gabaix (2011), Proposition 2, which rely heavily on the Levy's Theorem (as in Theorem 3.7.2 in Durrett (2013) on p. 138).

Theorem 8 (Levy's Theorem) *Suppose $X_1, ..., X_K$ are i.i.d. with a distribution that satisfies*

(i) $\lim_{x\to\infty} \Pr[X_1 > x] / \Pr[|X_1| > x] = \theta \in (0,1)$

(ii) $\Pr[|X_1| > x] = x^{-\zeta} L(x)$ *with* $\zeta < 2$ *and* $L(x)$ *satisfies* $\lim_{x\to\infty} L(tx)/L(x) = 1$.

Let $S_K = \sum_{k=1}^{K} X_k$,

$$a_K = \inf\{x : \Pr[|X_1| > x] \leq 1/K\} \text{ and } b_K = K\mathbb{E}\left[X_1 \mathbf{1}_{|X_1| \leq a_K}\right] \tag{A.23}$$

As $K \to \infty$, $(S_K - b_K)/a_K \xrightarrow{d} u$ *where* u *has a nondegenerated distribution.*

A. Proof of Proposition 1

When $\delta = 0$ and $\lambda_k = \lambda$ for all k,

$$\|\chi\|_2 = \frac{1-\lambda}{K^{1/2}\overline{C}_k}\sqrt{\frac{1}{K}\sum_{k=1}^{K} C_k^2}.$$

Given the power-law distribution of C_k, the first and second moments of C_k exist when $\beta_c > 2$, so

$$K^{1/2}\|\chi\|_2 \longrightarrow \frac{\sqrt{\mathbb{E}\left[C_k^2\right]}}{\mathbb{E}\left[C_k\right]}.$$

In contrast, when $\beta_c \in (1,2)$, only the first moment exists. In such cases, by the Levy's theorem,

$$K^{-2/\beta_c}\sum_{k=1}^{K} C_k^2 \xrightarrow{d} u_0^2,$$

where u_0^2 is a random variable following a Levy's distribution with exponent $\beta_c/2$ since $\Pr\left[C_k^2 > x\right] = x_0^{\beta} x^{-\beta_c/2}$.

Thus,

$$K^{1-1/\beta_c}\|\chi\|_2 \xrightarrow{d} \frac{u_0}{\mathbb{E}\left[C_k\right]}.$$

When $\beta_c = 1$, the first and second moments of C_k do not exist. For the first moment, by

Levy's theorem,

$$\left(\overline{C}_k - \log K\right) \xrightarrow{d} g,$$

where g is a random variable following a Levy distribution.

Since the second moment is equivalent to the result above,

$$(\log K) \|\chi\|_2 \xrightarrow{d} u'.$$

B. Proof of Proposition 2

Let λ_k and C_k be two independent random variables distributed as specified in the Proposition, the counter-cumulative distribution of $z_k = (1-\lambda_k) C_k$ is given by

$$f_Z(z) = \int_z^{z/y_0} f_{C_k}(z/y) f_{1-\lambda_k}(y)\, dy,$$

which follows as Pareto distribution with shape parameter β_c. The proof of the Proposition then follows the proof of Proposition 1 for

$$\|\chi\|_2 = \frac{1}{K^{1/2}\overline{C}_k}\sqrt{\frac{1}{K} z_k^2}. \tag{A.24}$$

C. Proof of Proposition 3

As specified in the proposition, λ_k and C_k are related through $Z_k = (1-\lambda_k) C_k = \phi C_k^{1+\mu}$. When $\mu < 0$, Z_k is distributed Pareto with shape parameter $\beta_c/(1+\mu)$. Proceeding similarly to the proof of Proposition 1, when $\beta_c > \max\{1, 2(1+\mu)\}$, both $\mathbb{E}\left[Z_k^2\right]$ and $\mathbb{E}[C_k]$ exist, so $v_c \sim v/K^{1/2}$. When $\beta_c \in (1, \max\{1, 2(1+\mu)\})$, $\mathbb{E}[C_k]$ exist but $\mathbb{E}\left[Z_k^2\right]$ does not.

Applying Levy's theorem,

$$K^{-2(1+\mu)/\beta_c} \sum_{k=1}^{K} Z_k^2 \xrightarrow{d} u^2.$$

Thus, $v_c \sim \frac{u_1}{K^{1-(1+\mu)/\beta}} v$.

When $\beta_c = 1$, the last result also holds. But now $\mathbb{E}[C_k]$ does not exist. As in Proposition 1, $\left(\frac{1}{K}\sum_{k=1}^{K} C_k - \log K\right) \xrightarrow{d} g$. Thus, if $\mu \in [-1/2, 0]$, $v_c \sim \frac{u_2}{K^{-\mu}\log K} v$, whereas if $\mu \in (-1, -1/2)$, $v_c \sim \frac{u_2}{K^{1/2}\log K} v$.

The proposition for $\mu < 0$ is then obtained by rearranging terms.

When $\mu > 0$, Z_k is distributed piece-wise Pareto such that

$$\Pr\left[Z_k \geq z\right] = \begin{cases} x_0^{\beta_c} z^{-\beta_c} & \text{for } z > \phi^{-2/\mu} \\ z_0^{-\beta_c/(1+\mu)} z^{-\beta_c/(1+\mu)} & \text{for } z \in \left[z_0^2, \phi^{-2/\mu}\right]. \end{cases}$$

We now follow the same steps as in the proof of Proposition 1. When $\beta_c > 2(1+\mu)$, $\mathbb{E}\left[Z_k^2\right]$ and $\mathbb{E}\left[C_k\right]$ exist, so $v_c \sim v/K^{1/2}$. When $\beta_c \in (1, 2(1+\mu))$, $\mathbb{E}\left[C_k\right]$ exists but $\mathbb{E}\left[Z_k^2\right]$ does not. Applying Levy's theorem,

$$\frac{1}{a_K} \sum_{k=1}^{K} Z_k^2 \xrightarrow{d} u^2,$$

where

$$a_K = \begin{cases} x_0^2 K^{2/\beta_c} & \text{for } K > K^* \\ z_0^2 K^{2(1+\mu)/\beta_c} & \text{for } K \leq K^* \end{cases}$$

for $K^* \equiv x_0^{-\beta_c} \phi^{-\beta_c/\mu}$. Thus, $v_c \sim \frac{u_1}{K^{1-\frac{1+1\{K\leq K^*\}\mu}{\beta_c}}} v$ for some random variable u_1.

When $\beta_c = 1$, $\left(\frac{1}{K}\sum_{k=1}^{K} C_k - \log K\right) \xrightarrow{d} g$, so now $v_c \sim \frac{u_2}{K^{-1\{K\leq K^*\}\mu}\log K} v$ for some random variable u_2, completing the proof.

D. Proof of Proposition 4

When $\delta \in (0,1)$, $\lambda_k = \lambda$ for all k, and $\Omega_c = \frac{1}{K}\iota$, we know from equation XX that

$$\begin{aligned} \|\chi\|_2 &\geq \frac{1-\lambda}{K}\sqrt{\sum_{k=1}^{K}\left[1+\delta' d_k + \delta'^2 q_k\right]^2} \\ &\geq (1-\lambda)\sqrt{\frac{1+2\delta'+2\delta'^2}{K} + \frac{\delta'^2}{K^2}\sum_{k=1}^{K}\left[d_k^2 + 2\delta' d_k q_k + \delta'^2 q_k^2\right]}. \end{aligned}$$

Following the same argument as in Proposition 2,

$$\begin{aligned} K^{-2/\beta_d}\sum_{k=1}^{K} d_k^2 &\longrightarrow u_d^2, \\ K^{-2/\beta_q}\sum_{k=1}^{K} q_k^2 &\longrightarrow u_q^2, \\ K^{-1/\beta_z}\sum_{k=1}^{K} d_k q_k &\longrightarrow u_z^2, \end{aligned}$$

where u_d^2, u_q^2 and u_z^2 are random variables. Thus, if $\beta_z \geq 2\min\{\beta_d, \beta_q\}$,

$$v_c \geq \frac{u_3}{K^{1-1/\min\{\beta_d,\beta_q\}}} v$$

where u_3^2 is a random variable.

E. Proof of Proposition 5

Analogous to the proof of Proposition 4.

F. Proof of Proposition 5

This proposition follows from the dispersion of price stickiness across sectors entirely analogously as in the discussion of the granularity effect in Section III *B.*].

G. Proof of Proposition 6

This proposition follows entirely analogously to the previous ones. Note that the inequality holds if $\mathrm{var}(\omega_{ck} \geq 1/K$.

H. Proof of Proposition 7

This follows directly from the given expression, as well as from Jensen's Inequality.

V Distribution of the Frequency of Price Changes

Here, we report the shape parameters of the power law distribution for the frequencies of price adjustments following Gabaix and Ibragimov (2011). To do so, we compute the OLS estimator of the empirical log-counter cumulative distribution on the log sequence of the variables using the data in the upper 20% tail. The shape parameter of the sectoral distribution of frequency of price changes is 2.5773 (st dev 0.4050); that is, the distribution of the frequency of price adjustment is not fat-tailed.

Table A.1: **Multipliers of Sectoral Shocks into Aggregate Volatility: with construction**

This table reports multipliers of sectoral productivity shocks on GDP volatility, $\|\chi\|$, with relative multipliers, $\|\chi\|$, in parenthesis. The latter are relative to the multiplier of an aggregate productivity shock on GDP volatility. Ω_c represents the vector of GDP shares and Ω the matrix of input-output linkages. We calibrate a 341-sector version of our model to the input-output tables and sector size from the BEA and the frequencies of price adjustment from the BLS.

			Flexible Prices		Homogeneous Calvo		Heterogeneous Calvo	
			(1)		(2)		(3)	
					Panel A: MODEL1			
(1)	hom Ω_c	hom Ω	0.0535	(5.35%)	0.0016	(5.35%)	0.0044	(11.94%)
(2)	het Ω_c	$\delta = 0$	0.1922	(19.22%)	0.0093	(19.22%)	0.0190	(32.57%)
(3)	het Ω_c	hom Ω	0.1067	(10.67%)	0.0047	(16.01%)	0.0096	(29.50%)
(4)	hom Ω_c	het Ω	0.0793	(7.93%)	0.0018	(6.02%)	0.0053	(12.89%)
(5)	het Ω_c	het Ω	0.1604	(16.04%)	0.0053	(17.88%)	0.0096	(27.87%)
					Panel B: MODEL2			
(1)	hom Ω_c	hom Ω	0.0535	(5.35%)	0.0029	(5.35%)	0.0049	(7.93%)
(2)	het Ω_c	$\delta = 0$	0.1922	(19.22%)	0.0197	(19.22%)	0.0263	(25.84%)
(3)	het Ω_c	hom Ω	0.1100	(11.00%)	0.0078	(14.49%)	0.0114	(19.72%)
(4)	hom Ω_c	het Ω	0.0773	(7.73%)	0.0034	(6.31%)	0.0061	(8.95%)
(5)	het Ω_c	het Ω	0.1614	(16.14%)	0.0093	(17.27%)	0.0125	(20.78%)
					Panel C: MODEL3			
(1)	hom Ω_c	hom Ω	0.0535	(5.35%)	0.0010	(5.35%)	0.0058	(14.83%)
(2)	het Ω_c	$\delta = 0$	0.1922	(19.22%)	0.0070	(19.22%)	0.0227	(41.68%)
(3)	het Ω_c	hom Ω	0.1067	(10.67%)	0.0032	(17.44%)	0.0112	(40.56%)
(4)	hom Ω_c	het Ω	0.0793	(7.93%)	0.0011	(5.93%)	0.0067	(15.68%)
(5)	het Ω_c	het Ω	0.1604	(16.04%)	0.0034	(18.47%)	0.0107	(36.12%)
					Panel D: MODEL4			
(1)	hom Ω_c	hom Ω	0.0535	(5.35%)	0.0027	(5.35%)	0.0069	(8.61%)
(2)	het Ω_c	$\delta = 0$	0.1922	(19.22%)	0.0195	(19.22%)	0.0319	(28.39%)
(3)	het Ω_c	hom Ω	0.1149	(11.49%)	0.0076	(15.05%)	0.0148	(22.02%)
(4)	hom Ω_c	het Ω	0.0747	(7.47%)	0.0031	(6.13%)	0.0082	(9.41%)
(5)	het Ω_c	het Ω	0.1628	(16.28%)	0.0089	(17.48%)	0.0157	(22.27%)